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A Tractable Model of the Demand for Reserves Under Nonlinear Remuneration Schemes*

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Abstract

We propose a tractable model of the demand for reserves under nonlinear remuneration schemes that can encompass quota systems and voluntary reserve target frameworks, among other possibilities. We show how such remuneration schemes have several favorable properties regarding interest-rate control by the central bank. In particular, wider tolerance bands can reduce rate volatility due to variations in the supply of reserves, both large and small, although they may curtail trading in the interbank market.

Keywords: Bank reserves, monetary policy implementation.
J.E.L. codes: E41, E42.

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1 Introduction

The remuneration of reserves at a central bank can play a key role in the implementation of the desired level of interest rates. Recently, several central banks have put in effect nonlinear remuneration schemes, in which reserves are awarded different rates for certain portions of the holdings. Examples of such remuneration schemes are quota systems, such as those currently in use in Norway and Switzerland, and the voluntary reserve target framework used by the Bank of England from 2006 to 2008. The Federal Reserve System also has the capacity to implement a nonlinear remuneration profile because it can set different rates for required and excess reserves.

In this paper, we propose a tractable model of the demand for reserves under a fairly general specification for their remuneration. The baseline remuneration scheme in the model consists of three "tracks" or "bands:" Reserve holdings within a *tolerance band* of a *reserve target* are remunerated at the *target rate*. Reserves in excess of the top of the tolerance band are remunerated at a lower rate; if reserves are below the bottom of the tolerance band, the deficiency is penalized at a higher rate. A scheme with only two bands (such as quotas systems) is captured as a special case of the more general specification.

In the tradition of Poole (1968), we assume banks do not have full control of end-of-period reserve holdings due to the presence of late payment shocks. Along the demand curve, banks trade off the gains from arbitraging between the market and target rates against the inherent risk introduced by the late payment shocks and the concave remuneration scheme. The choice of a Laplace distribution for net payment shocks leads to explicit solutions for the demand and inverse demand for reserves as well as an intuitive formulation for the key trade-off outlined previously.[1]

We show that supplying an amount of reserves equal to the aggregate of reserve targets ensures that the effective funds rate equals the target rate. The result holds quite generally, including in environments with heterogeneous banks, and does not require precise knowledge of the structural parameters. Moreover, the remuneration scheme can be scaled up easily without compromising implementation by scaling up the reserve targets correspondingly.

Of course, central banks do not have full control of the supply of reserves, because there are several autonomous factors that may unexpectedly shift the total of reserves available to banks. It is also possible that policymakers deem that the aggregate of reserve targets is not the desirable level of supplied reserves in some circumstances. We thus next characterize interest-rate control in response to shocks to the supply of reserves.

The most striking feature is that the tolerance bands flatten the demand for reserves around the target, containing interest-rate volatility. We derive the effect of small changes in the supply of reserves in terms of the spread between the excess and penalty rates, the

[1] The Laplace distribution for net payments arises naturally if we assume that *gross* transfers are distributed exponentially.

width of the tolerance band, and the dispersion in payment shocks. For medium-size shocks to the supply of reserves, we show how deviations in the interest rate are tightly bound—and how wider tolerance bands tighten these bounds. Finally, in response to large deviations in the supply of reserves, the remuneration scheme acts similarly to a corridor system, with the remuneration rate for excess reserves acting as a floor and the penalty rate for deficiencies acting as a ceiling.

Wider tolerance bands, however, may reduce participation and volume in the funds market. Trading in the model, as it is in Poole (1968), is driven by the concavity of the remuneration scheme. Wider tolerance bands extend the strictly linear portion of the payoffs, which reduces interest-rate volatility but also the gains from trade. The smaller gains of trade, in turn, render the market volume more sensitive to trading costs. The spread between the target and penalty rate acts similarly: A narrow spread improves interest-rate control but discourages active reserve management.

This paper is firmly in the tradition of Poole (1968), modeling the demand for reserves as driven by payment considerations in the context of a centralized market. This approach has proven quite fruitful, with a long body of work based on the U.S. federal funds market. Some notable examples are Ho and Saunders (1985), Hamilton (1996), and Furfine (2000), among many others. These papers do not consider nonlinear remuneration schemes centered around the interest-rate target; instead, the demand for reserves is pinned down by reserve requirements as in a classic corridor system.[2]

There is an incipient literature studying remuneration schemes akin to the one analyzed here. An early contribution is the analysis by Whitesell (2006) of voluntary reserve targets. More recent contributions studying models of voluntary reserve targets are Jackson and Noss (2015) and Baughman and Carapella (2016).

Starting with Furfine (1999) and Ashcraft and Duffie (2007), researchers have set to capture the trading arrangements in interbank markets, which typically operate over the counter (i.e., without a centralized clearing platform). Indeed, over-the-counter trading can explain several key features of these markets, from intraday patterns to dispersion in rates. See, among many others, Afonso and Lagos (2015), Bech and Klee (2011), Berentsen and Monnet (2008), Armenter and Lester (forthcoming), Bech and Monnet (2016), and Ennis and Weinberg (2013). Modeling the microfoundations of trade, however, comes at a cost, because these models are not very tractable and cannot be easily extended. Researchers thus may prefer to posit a centralized market, as it is done here and in Martin et al. (2013), knowing that as long as trading is not sparse, it is an effective approximation.

The paper is structured as follows. Section 2 describes the remuneration scheme studied here. Section 3 derives the demand for reserves and discusses the key trade-off

[2]See Ennis and Keister (2008) for a concise treatment of implementation frameworks for monetary policy and their nomenclature.

that pins down the desired level of reserve holdings. The results regarding interest-rate control are documented in Section 4. Section 5 explores the implications of heterogeneity, and Section 6 introduces a meaningful participation decision in trading. Finally, Section 7 concludes with suggestions for further research. All proofs and derivations are in the Appendix.

2 Remuneration of reserves

Let $M' > 0$ be the end-of-period reserve holdings and $M^* > 0$ the reserve target. Let $R(M', M^*)$ denote the remuneration of the end-of-period reserve holdings given the reserve target.

The holdings are remunerated as follows:

- If reserve holdings are within a tolerance band $\delta \geq 0$ of the target,

$$M' \in [(1-\delta)M^*, (1+\delta)M^*],$$

  then the reserve balances earn rate i^*,

$$R(M', M^*) = i^* M'.$$

  Throughout the paper, we will refer to the remuneration rate of compliant reserve balances, i^*, as the target rate, under the assumption that the remuneration scheme would be indeed centered around the rate the central bank desires to implement.

- If reserve holdings exceed the top of the tolerance band,

$$M' > (1+\delta)M^*,$$

  then holdings in excess of $M^*(1+\delta)$ get remunerated at rate, i^x (x for excess),

$$R(M', M^*) = i^*(1+\delta)M^* + i^x \left(M' - M^*(1+\delta)\right).$$

- If reserve holdings are below the bottom of the tolerance band,

$$M' < (1-\delta)M^*,$$

  then the deficiency to $M^*(1-\delta)$ is charged at rate, i^p (p for penalty),

$$R(M', M^*) = i^*(1-\delta)M^* - i^p \left(M^*(1-\delta) - M'\right).$$

The remuneration of reserves is then a continuous, piece-wise linear function, capturing the three remuneration "tracks" or "bands" and the associated rates. The remuneration scheme is fully described by the parameters $\{M^*, \delta, i^*, i^x, i^p\}$. The specification allows the special case of no tolerance bands, $\delta = 0$, and thus only two tracks.

2.0.1 Equivalent implementations

We described the remuneration scheme as provided by the central bank, assuming that it automatically computes and transfers the stated retribution of reserves based on a target and rates. The previous specification, however, captures several implementation schemes that are thus equivalent for the purposes of the analysis here. We briefly discuss two of these alternative implementations.

Quotas. The remuneration scheme above can be described without a reserve target or tolerance bands. We can instead specify the threshold levels for reserves, say, $M_1 \geq M_2$, that trigger each of the remuneration tracks. This definition fits better remuneration systems based on quotas.

We can easily map a reserve target into a quota system by setting $M_1 = (1 - \delta)M^*$ and $M_2 = (1 + \delta)M^*$. Mapping from a quota system to the previous description is straightforward as well, with the reserve target given by the midpoint between thresholds,

$$M^* = \frac{M_1 + M_2}{2}$$

and the tolerance band by half the difference between thresholds,

$$\delta = \frac{M_2 - M_1}{2}.$$

Facilities. An alternative implementation is to combine standing facilities for banks to deposit and borrow at the rates i^x, i^p, respectively, with a large lump-sum penalty associated with reserve holdings outside the tolerance band. The rationale is that no bank would incur such a penalty and instead would adjust its reserve balances to always be in the tolerance band, resulting in an identical remuneration function R as described previously. Of course, this assumes that: (1) banks have unlimited access to both facilities, and (2) there is no form of stigma associated with borrowing from the central bank.

2.1 Assumptions on rates

The first assumption is indispensable if the remuneration of reserves is to encourage banks to bring their holdings in line with the targets:

$$i^x < i^* < i^p. \tag{1}$$

The penalty rate is not a penalty unless it exceeds i^*, and similarly the excess rate i^x must provide an incentive to keep reserve holdings in check and hence must be below i^*. An immediate implication of the above is that the remuneration function is strictly concave in M' and strictly concave in M^*. Figure 1 displays the remuneration function under Assumption (1).

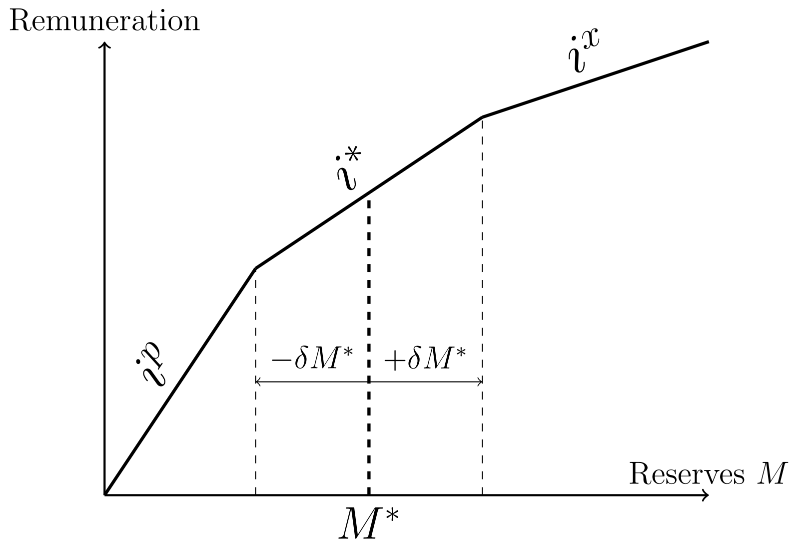

Figure 1: Reserve Remuneration

To understand how the remuneration scheme will map into interest-rate control, it is useful to look at the marginal rate as a function of reserves. Figure 2 does exactly this. For reserve holdings in deficiency of the bottom of the tolerance band, additional funds reduce the penalty borrowing and hence are valued at i^p. Similarly, reserve holdings in excess of the top of the tolerance band only earn i^x. These two rates will work as the upper and lower bound on the interest rate as in a classic corridor system.

We impose an additional assumption on the rate schedule:

$$i^* = \frac{i^x + i^p}{2}. \tag{2}$$

The assumption delivers a symmetry between the excess and penalty rates. It has the virtue of simplicity, both in terms of communication of the framework and also for the analytic results to follow. Since it links all three rates, in what follows, we will express all formulas in terms of i^*, i^p, knowing that one can obtain i^x from (2) simply as

$$i^x = i^* - (i^p - i^*).$$

2.2 Rewriting the remuneration function

The symmetry assumption (2) allows us to collapse the remuneration function into a simple form:

$$R(M', M^*) = i^* M' - (i^p - i^*) \max\left\{|M' - M^*| - \delta M^*, 0\right\}. \tag{3}$$

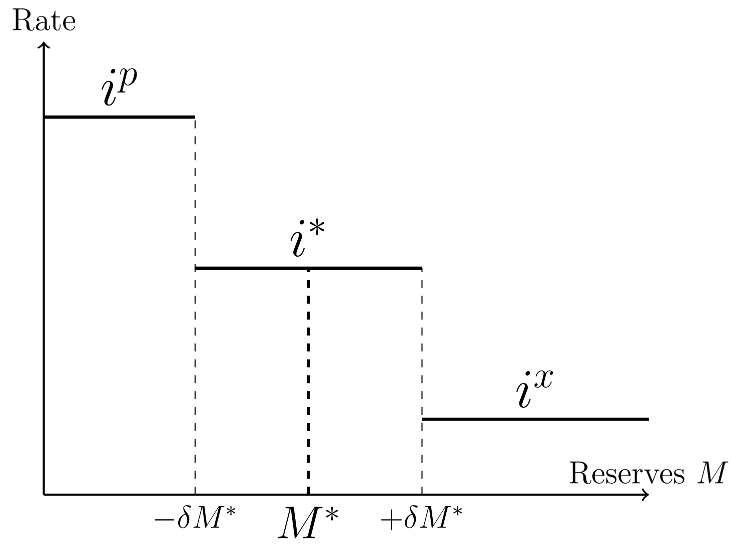

Figure 2: Marginal Rates

The second term acts as a penalty. It is zero unless reserve holdings are further from the target than allowed by the tolerance band. Under the symmetry assumption (2), it does not matter in which direction the deviation from target is because, in either case, a "penalty spread" equal to $i^p - i^*$ is applied.

2.3 Some properties of the remuneration function

There are a couple of immediate properties of the remuneration function that are worth highlighting before any additional assumptions are made.

The first property is that the scale of the remuneration of reserves can be increased simply by increasing both holdings and target proportionally. Formally, the remuneration function satisfies

$$R(\alpha M', \alpha M^*) = \alpha R(M', M^*)$$

for any $\alpha \geq 0$. While hardly surprising, this property makes it clear that there are no scale effects to the reserve scheme as designed previously. It is thus equally well suited for small or large reserve holdings.

The second property is a bit more technical, stating that the remuneration function is globally concave. For any $\alpha \in [0, 1]$ the remuneration function satisfies

$$\alpha R(M_1', M_1^*) + (1 - \alpha) R(M_2', M_2^*) \leq R(M_\alpha', M_\alpha^*)$$

where $M_\alpha' = \alpha M_1' + (1 - \alpha) M_2'$ and $M_\alpha^* = \alpha M_1^* + (1 - \alpha) M_2^*$. Once we introduce some uncertainty in the banks' problem, the concavity in R is instrumental to align demand

and supply of reserves at the desired rate.

The Appendix contains proofs for both properties.

3 Demand for reserves with uncertainty

We now introduce a layer of uncertainty in the banks' holdings of reserves. Consider a bank starting start the period with M_0 reserve holdings and a pre-specified target, M^*. Then, the bank has a chance to trade funds F to adjust its reserve holdings mid-day, which we denote as $M = M_0 + F$, at the funds rate i^f.

The bank is then subject to a *net payments* shock, $P \in \Re$, late in the period, which impacts its end-of-period reserve holdings,

$$M' = M + P.$$

The idea is that the payment shock cannot be fully anticipated by the bank, which is captured by having P being a random variable, distributed according to cumulative density function G with mean zero. Note that by assumption, the bank cannot undo the payment shock by trading additional funds.

Hence at the time the bank decides on funds trades, it must look at its *expected* remuneration,

$$R^e(M, M^*) \equiv E\left\{R\left(M + P, M^*\right)\right\} = \int R\left(M + P, M^*\right) dG(P).$$

The late payment shocks imply that the bank is cautious about taking large positions in the funds market. For example, if the funds rate is slightly above the remuneration rate for complying reserves, $i^f > i^*$, the bank will look to achieve reserve holdings below M^* to take advantage of a higher rate, but it will not drive reserve holdings to the bottom of the tolerance band as it would do if there was no uncertainty. Now the bank is worried that a late payment would push its reserve holdings out of the tolerance band, with the associated penalty rate, and wipe out the profits associated with the spread $i^f - i^*$.

We are now set to state the bank's problem of maximizing expected returns taking as given the funds rate i^f and the reserve target M^*,

$$\max_{F \geq -M_0} R^e(M_0 + F, M^*) - i^f F.$$

Clearly the above problem can be stated directly in terms of reserve holdings after trading in the funds market has finalized, $M = M_0 + F$,

$$\max_{M \geq 0} R^e(M, M^*) - i^f(M - M_0).$$

Given the concavity properties previously established, the above problem is concave and first-order conditions will be both necessary and sufficient to derive the demand for reserves.

3.1 Net payment shocks

We assume that net payment shocks, P, follow a Laplace distribution centered at 0 and scale parameter $\beta > 0$. The c.d.f. for $P \leq 0$ is

$$G(P) = \frac{1}{2}\exp\left(\frac{P}{\beta}\right)$$

and for $P > 0$

$$G(P) = 1 - \frac{1}{2}\exp\left(\frac{-P}{\beta}\right).$$

The Laplace distribution has sound structural foundations. It arises naturally if payments and obligations are independent and memory-less; that is, having received \$10 million in payments does not change the distribution of obligations or *further* payments. In this case, payments and obligations follow an exponential distribution. If in addition both payments and obligations have the same mean β, then the difference between them (i.e., the net payments) are distributed according to a Laplace distribution with mean zero and scale parameter β.

The key property of the Laplace distribution is that expected absolute value of payments in excess of a threshold take a simple form, namely, a constant β over the threshold, $E\{|P||x \leq |P|\} = x + \beta$. This allows us to simplify the expected remuneration substantially in many instances.

3.2 Demand for reserves

We now characterize the trade-off underlying the bank's reserve holdings given a funds rate, tracing the bank's demand curve which, in turn, can be inverted to obtain the equilibrium funds rate as a function of the supply of reserves.

We will first focus on the case that the funds rate is not too different from the remuneration rate i^* or, alternatively, when the supply of reserves is not too far from the reserve target M^*. This implies that the desired reserve holdings by the bank stay in the tolerance band. If the funds rate departs substantially from i^*, the bank may want to set reserve holdings outside the tolerance band—a case we will address later. To ease notation, let $\mathbf{T}$ denote the tolerance band, i.e., the interval $[(1-\delta)M^*, (1+\delta)M^*]$.

The Appendix shows that for all $M \in \mathbf{T}$, the expected remuneration function is

$$R^e(M, M^*) = i^*M - \beta\,(i^p - i^*)\Pr(M' \notin \mathbf{T}). \tag{4}$$

The term $\Pr(M' \notin \mathbf{T})$ is the probability that end-of-period reserve holdings step out of the tolerance band, in which case, the penalty spread $i^p - i^*$ applies. The coefficient β captures the expected amount by which reserves would be *outside* of the tolerance band and is naturally linked to the volatility in gross payments. The probability that end-of-period reserve holdings step out of the tolerance band is given by

$$\Pr(M' \notin \mathbf{T}) = \exp\left(\frac{-\delta M^*}{\beta}\right)\cosh\left(\frac{M^* - M}{\beta}\right) \tag{5}$$

where cosh is the hyperbolic cosine function, which is amenable to analytic results.

We now solve the optimal reserve holdings by the bank,

$$\max_{M \geq 0} R^e(M, M^*) - i^f(M - M_0).$$

Under the assumption that the desired reserve holdings are inside the tolerance band, the necessary and sufficient condition for the previous problem is

$$\sinh\left(\frac{M - M^*}{\beta}\right) = \exp\left(\frac{\delta M^*}{\beta}\right)\frac{i^* - i^f}{i^p - i^*}, \tag{6}$$

where sinh is the hyperbolic sine function. The above condition may appear a tad obscure, but it can be easily recast to illustrate the key trade-off faced by the bank and to derive the demand function and its inverse.

3.2.1 Trade-off

Let us start by taking a look at the trade-off the bank faces when managing its reserve balances. The Appendix shows that (6) can be easily expressed in terms of probabilities as

$$i^* - i^f = (i^p - i^*)\left(\Pr\{M' \geq (1+\delta)M^*\} - \Pr\{M' \leq (1-\delta)M^*\}\right). \tag{7}$$

The bank is balancing the gains from arbitraging any spread between the remuneration rate of complying reserves and the funds rate, $i^* - i^f$, against the balance of risks of missing the tolerance band above or below, weighted by the penalty spread.

If the funds rate is spot on the target, $i^* = i^f$, (7) implies that the bank chooses a symmetric risk profile,

$$\Pr(M' \geq (1+\delta)M^*) = \Pr(M' \leq (1-\delta)M^*).$$

Given the symmetry in the distribution of net payments, the bank adjusts its demand on reserve holdings to be right on target, $M = M^*$. We will encounter the one-to-one relationship between the funds rate being on target, $i^* = i^f$, and reserve holdings being equal to target, $M = M^*$, again and again in the analysis.

How does the bank adjust the demand for reserves when there is an arbitrage opportunity $i^* \neq i^f$? Exactly as we would expect. If, for example, the funds rate is below the remuneration rate, $i^* > i^f$, the bank will look to profit from the spread $i^* - i^f$ by increasing its desired reserve holdings, increasing the risk of exceeding the top of the tolerance band and decreasing the risk of falling short of the bottom of the tolerance band,

$$\Pr\left(M' \geq (1+\delta)M^*\right) > \Pr\left(M' \leq (1-\delta)M^*\right).$$

The adjustment in the demand of reserves is only modest, since the bank will only tolerate some risk asymmetry; that is, as dictated by (7), when the risk balance equals the ratio of the spread between $i^* - i^f$ and the penalty spread.

3.2.2 Demand function

We turn to derive the demand function for reserves, that is, expressing the desired reserve holdings as a function of the funds rate as well as the rest of parameters. Some straightforward manipulation of (6) delivers

$$M = M^* - \beta \sinh^{-1}\left(\exp\left(\frac{\delta M^*}{\beta}\right)\frac{i^f - i^*}{i^p - i^*}\right). \tag{8}$$

This expression is perhaps not immediate but proves very useful for comparative statics. The intercept is naturally the target, M^*, which is achieved when $i^f = i^*$ since $\sinh^{-1}(0) = 0$. The demand function is decreasing with i^f everywhere, and its slope is approximately equal to β if the funds rate is close to i^*, highlighting the first-order effect coming from the dispersion in net payments. It can also be shown that the demand for reserves is strictly increasing in the reserve target, as one would expect.[3]

The demand function, however, responds in more complex ways to the parameters of the reserve scheme. In particular, the parameters amplify or attenuate the response in the demand of reserves to changes in the spread $i^f - i^*$ but have no effect when the funds rate is equal to the target, $i^f = i^*$, as the demand remains pinned to the reserve target, $M = M^*$.

3.2.3 Inverse demand function

Finally, the *inverse demand function*, specifying the funds rate as a function of the reserve holdings, can be easily obtained from (6) and is equal to

$$i^f = i^* - (i^p - i^*)\Pr\left(|P| \geq \delta M^*\right)\sinh\left(\frac{M - M^*}{\beta}\right). \tag{9}$$

The specification highlights the importance of net payment shocks. The probability that a net payment shock exceeds by itself the tolerance band, $\Pr\left(|P| \geq \delta M^*\right)$, is central to

[3]See Appendix for derivations.

the slope of the inverse demand function. Note that $\Pr(|P| \geq \delta M^*)$ depends only on the variance of net payments β, the reserve target M^*, and the tolerance band width δ, but not the choice of reserve holdings themselves. A noteworthy property of the inverse demand function is that the funds rate is more sensitive to changes in M for smaller probabilities $\Pr(|P| \geq \delta M^*)$ and less sensitive for larger penalty spreads—in line with the results from the demand function. Perhaps more important, (9) shows that the inverse demand function can be shifted one to one with the rate schedule as long as tolerance band, penalty spreads, and reserve targets stay constant.

Assuming that the bank described previously acts as "representative," the inverse demand equation (9) can be used to study movements in the supply of reserves, by equating $M^s = M$. We can then compute the marginal effect of a movement in the supply,

$$\frac{di^f}{dM^s} = -\beta^{-1}(i^p - i^*) \Pr(|P| \geq \delta M^*) \cosh\left(\frac{M^s - M^*}{\beta}\right) < 0. \tag{10}$$

This expression shows that the interest-rate sensitivity to supply changes is linked to the dispersion of volatility relative to the tolerance band, $\Pr(|P| \geq \delta M^*)$ as well as the penalty rate. Indeed, these are the only terms that matter if $M^s \approx M^*$, since $\cosh 0 = 1$.

It is possible to recast (10) in terms of probabilities as

$$\beta \frac{di^f}{dM^s} = (i^p - i^*) \Pr(M' \notin \mathbf{T}). \tag{11}$$

This expression is not useful analytically but delivers a semblance of a rule of thumb regarding the interest-rate sensitivity.

3.3 Demand for reserves outside the tolerance band

The previous formulas apply when the demand for reserves lies within the tolerance band, $M \in \mathbf{T}$. Once the desired reserves step out of the tolerance band, the bank evaluates the risks differently, because even if the net payments are zero, it will not earn the target rate on the margin.

Fortunately, this poses no difficulty for the model. The inverse demand function is continuous for all reserve holdings, with reserve holdings being strictly within the tolerance band for an open interval of rates that includes i^*.

For reserve holdings below the bottom of the tolerance band, $M < (1-\delta)M^*$, the inverse demand function is given by

$$i^f = i^p - (i^p - i^*) \cosh\left(\frac{\delta M^*}{\beta}\right) \exp\left(\frac{M - M^*}{\beta}\right). \tag{12}$$

For reserve holdings above the top of the tolerance band, $M > (1+\delta)M^*$, the inverse demand function is then

$$i^f = i^x + (i^p - i^*) \cosh\left(\frac{\delta M^*}{\beta}\right) \exp\left(-\frac{M - M^*}{\beta}\right). \tag{13}$$

Evaluating (12) and (13) at the respective boundaries of the tolerance band, we confirm the continuity of the inverse demand function.[4]

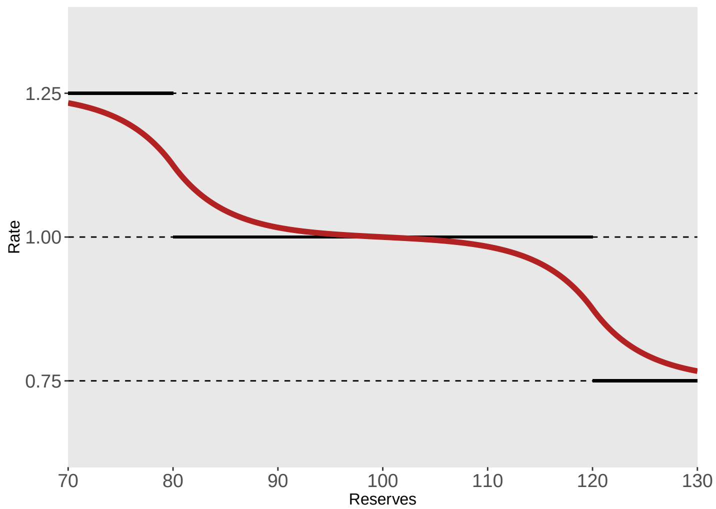

Figure 3: Demand for Reserves

The complete inverse demand function is illustrated in Figure 3 for an arbitrary choice of parameters.[5] The most striking feature of the demand is how it flattens in the middle of the tolerance band, ensuring that the funds rate stays close to the target i^* even if the supply of reserves deviates moderately from the target. It is also easy to see that rates will be bounded by i^x and i^p for all values of M. These will indeed be the key results discussed in the next Section on interest-rate control.

4 Interest-rate control

We now ask at what level the supply of reserves should be set to ensure that the funds rate equals the target rate and, if the supply of reserves exhibits small or large deviations from such level, by how much the funds rate drifts from the target rate in response.

The previous section already stated the answer to the first question. The funds rate is equal to the target rate, $i^f = i^*$, if and only if the supply of reserves is equal to the reserve target, $M^s = M^*$. The result is immediate given equations (8) and (9): The demand for

[4]For the special case of no tolerance bands, $\delta = 0$, the inverse demand function is given by (12) and (13) everywhere, noting that $\cosh 0 = 1$.

[5]The reserve target is set $M^* = 100$, tolerance bands are 20%, and the target rate is set at 1%, with the penalty and excess rates being 25 basis points above and below, respectively. The payment parameter β is 5.

reserves (8) is indeed equal to M^* when $i^f = i^*$, and the inverse demand confirms that if $M^s = M^*$, then the market rate is equal to the target rate. The result is extraordinarily robust: It does not depend on how wide the tolerance band is or how large the penalty spread, or even the expected value of gross payments. As discussed in the next section, it also holds in environments with heterogeneous banks.

Of course, central banks do not have full control of the supply of reserves, because there are several autonomous factors that may unexpectedly shift the total of reserves available to banks. It is also possible that policymakers deem that the aggregate of reserve targets is not the desirable level of supplied reserves in some circumstances. The next step is to check how the funds rate responds to deviations in the supply of reserves, both large and small.

Let us start with small deviations in the supply of reserves, $M \approx M^*$. From the inverse demand (9), we obtain

$$\frac{di^f}{dM} = -\beta^{-1}(i^p - i^*)\,\Pr\left(|P| \geq \delta M^*\right). \tag{14}$$

This expression shows that the slope of the inverse demand function at $M^s = M^*$ is linked to the dispersion in late payment shocks, the width of the tolerance band, and the penalty spread $i^p - i^*$. A flatter inverse demand function naturally implies that the interest rate is less sensitive to movements in the supply. To get a better grasp of the magnitude, we can express the absolute change in the interest rate in terms of a percentage change in the supply of reserves from its target M^*, again in absolute terms:

$$\frac{d|i^f|}{dM}\,|M - M^*| = \frac{|M - M^*|}{\beta}(i^p - i^*)\,\Pr\left(|P| \geq \delta M^*\right).$$

The wider the tolerance bands, the smaller the interest-rate sensitivity, as $\Pr\left(|P| \geq \delta M^*\right)$ unambiguously decreases. Similarly, a narrower penalty spread flattens the inverse demand. Note how the magnitude of the deviation in the supply of reserves is measured against the expected size of gross payments, β.

The connection between interest-rate control and the tolerance bands is further highlighted for our next result, concerning medium-size movements in the supply of reserves. In the Appendix, we show that if the supply of reserves is within the tolerance band,

$$(1 - \delta)M^* \leq M^s \leq (1 + \delta)M^*,$$

then the deviations of the funds rate from the target are bounded from above and below by

$$\left|i^f - i^*\right| \leq \frac{1}{2}(i^p - i^*)\,\Pr\left(|P| \geq 2\delta M^*\right). \tag{15}$$

This is quite a remarkable result, which formalizes the flat region for the inverse demand function around M^*, apparent in Figure 3. The bounds given by (15) are indeed quite tight: If, say, the penalty spread is 25 basis points and the probability that a payment shock spans the full width of the tolerance band—that is, 2δ—is about 5 percent, the bounds are smaller than 1 basis point.

Furthermore, a wider tolerance band not only makes the bound on interest-rate deviations tighter, as the term $\Pr(|P| \geq 2\delta M^*)$ decreases, but it also extends the range of deviations in the supply of reserves for which the bound (15) applies. The intuition is quite simple. The tolerance bands make banks comfortable taking risks and arbitraging any deviation $i^f \neq i^*$ by borrowing or lending quite far from the target and thus putting pressure on the rate to return to the target. This gives the market additional capacity for deviations in the supply of reserves from the reserve target.

The final result proves that the remuneration scheme delivers robust interest-rate control even in the event of large deviations. It can be easily confirmed from (12) and (13) that rates are always bounded below by i^x and above by i^p. Thus, these two rates work akin to a corridor or channel system: The remuneration scheme incents banks to seek to lend aggressively if market rates approach i^p or to borrow if rates approach i^x. It should be noted, however, that we have not considered the possibility that a bank may find itself with an overnight overdraft—for which penalty rates are typically much higher. In the event of drastically scarce reserves, banks could be looking to borrow at rates above i^p to avoid the overdraft penalty rates.

5 Heterogeneous banks

So far, we have derived the demand functions and results on interest-rate control assuming all banks are identical. We now relax this assumption and explore the implications of heterogeneous banks. We index each bank by subscript $j \in J$ and its corresponding target and payment dispersion by $\{M_j^*, \beta_j\}$. There is no need to entertain heterogeneity in initial balances because they are irrelevant for the demand of reserves.

The first result is that implementing the target rate, $i^f = i^*$, remains as simple as before. As long as the supply of reserves is set equal to the aggregate of reserve targets,

$$M^s = \sum_J M_j^*,$$

the funds rate will be equal to the target rate i^*. The reason is that the demand for reserves (8) of bank j is always equal to M_j^* if $i^f = i^*$. Heterogeneity in payment dispersion, β_j, affects the slope of the bank's demand curve around i^* but not its level. There is no other equilibrium: If the funds rate traded, say, below the target rate, then *all* banks would demand reserves in excess of their targets, and the funds market would not clear.

We can also derive an exact aggregation result if the reserve targets of each bank are proportional to the scale of its payment shocks,

$$M_j^* \propto \beta_j.$$

This is reasonable if banks differ mainly in size, and targets are set proportionally to, say, total deposits or assets. Under this assumption, the demand for reserves for bank j can be written as

$$M_j = M_j^* - \beta_j \sinh^{-1}\left(\alpha \frac{i^f - i^*}{i^p - i^*}\right) \tag{16}$$

where

$$\alpha = \exp\left(\frac{\delta M_j^*}{\beta_j}\right)$$

is constant across banks by assumption. We can then simply add up the demand of all banks to obtain

$$\sum_J M_j = \sum_J M_j^* - \left(\sum_J \beta_j\right) \sinh^{-1}\left(\alpha \frac{i^f - i^*}{i^p - i^*}\right). \tag{17}$$

We can now construct a "representative" bank simply by taking the average of targets and payment dispersion parameters, that is,

$$\beta_A = \frac{1}{J} \sum_J \beta_j$$

and

$$M_A^* = \frac{1}{J} \sum_J M_j^*.$$

Since (17) is linear, simple manipulation obtains

$$M_A = M_A^* - \beta_A \sinh^{-1}\left(\alpha \frac{i^f - i^*}{i^p - i^*}\right),$$

which is indeed the demand of reserves of a bank with parameters $\{M_A^*, \beta_A\}$ as given by (8).

We can thus derive the inverse demand function for this case as simply

$$i^f = i^* - (i^p - i^*) \Pr(|P| \geq \delta M_A^*) \sinh\left(\frac{M_A - M_A^*}{\beta_A}\right),$$

and the previous results and comparative statics carry over.

Unfortunately, there is no comparable aggregation result for the general case. In fact, it turns out that whether the heterogeneity makes the aggregate demand for reserves more or less elastic depends on how the dispersion in payments and the reserve target are jointly distributed. Letting

$$\alpha_j = \exp\left(\frac{\delta M_j^*}{\beta_j}\right),$$

the first-order Taylor expansion of the demand for reserves of bank j, around $i^f = i^*$, is given by

$$M_j \approx M_j^* - \beta_j \alpha_j \frac{i^f - i^*}{i^p - i^*}. \tag{18}$$

We can view β_j, α_j as random variables as distributed across the bank population. Aggregating the approximation to the bank's demand for reserves, we obtain

$$M_A - M_A^* = -\left(\beta_A \alpha_A + Cov\left(\beta_j, \alpha_j\right)\right) \frac{i^f - i^*}{i^p - i^*} \tag{19}$$

where variables with subscript A are the simple averages, and the covariance term invokes the law of large numbers.

The approximation (19) to the aggregate demand function for reserves provides some interesting insights. We showed that when the reserve targets of each bank are proportional to the scale of its payment shocks, then we could obtain an exact aggregation result. Equation (19) shows that heterogeneity has no first-order effects if deviations on the ratio of targets-to-payments scale are unrelated to the dispersion of payments. Formally, if $Cov\left(\beta_j, \alpha_j\right) = 0$, then the aggregate demand for reserves approximates to

$$M_A - M_A^* = -\beta_A \alpha_A \frac{i^f - i^*}{i^p - i^*},$$

which is indeed equal to the first-order Taylor expansion of the demand of the representative bank constructed previously on the assumption that α_j was constant. In other words, for heterogeneity to reshape the slope of the demand curve, there must be a systematic relationship between the dispersion of payments β_j and α_j, as given by the covariance term.

For example, if banks with a large dispersion in payment shocks (i.e., high β_j) manage their reserves with a relatively high turnover (low α_j), then the slope of the demand for reserves is flatter, which in turn implies that interest rates are more sensitive to deviations in the aggregate supply. This is intuitive. Banks with large payment volatility but (relatively) low targets will not be very proactive in arbitraging any spread between the funds and the target rate, $i^f - i^*$. Banks with low payment volatility would be a bit more proactive but limited anyway by their own tolerance bands. As a result, the overall capacity of the system to absorb deviations in the supply of reserves diminishes.

6 Market participation

We have shown that a remuneration scheme with generous tolerance bands performs admirably in terms of interest-rate control. Is there a downside to wider and wider tolerance bands? One may conjecture that, as the remuneration scheme becomes flat, banks become lackadaisical regarding reserve management and do not participate in the funds market, instead letting their reserves be what they may after net payments.

To address this possibility, we consider the problem of a bank with target M^* and beginning-of-the-period M_0 reserve holdings. The bank can choose to access the market to adjust its midday reserve holdings to the level of its choosing at the prevalent market rate. To do so, however, the bank must incur a cost $\phi > 0$, which may capture anything from commissions to overhead costs. The bank thus may choose to not access the funds market and leave its midday reserve holdings unadjusted at M_0.

To solve this problem, we need first to compute the expected value of accessing the funds market. Assume the prevalent rate is i^f and define

$$V(M_0, M^*) = \max_M R^e(M, M^*) - i^f(M - M_0). \tag{20}$$

This is the expected remuneration after the bank optimally adjusts its midday reserves given the rate i^f. The Appendix shows that, if the funds rate is on target, $i^f = i^*$, the value of accessing the market takes a simple form,

$$V(M_0, M^*) = i^f M_0 - \beta(i^p - i^*) \exp\left(-\frac{\delta M^*}{\beta}\right). \tag{21}$$

This can also be written in terms of probability over net payments as we have done before:

$$V(M_0, M^*) = i^f M_0 - \beta(i^p - i^*) \Pr\left(|P| \geq \delta M^*\right). \tag{22}$$

There are two noteworthy properties. First, the expected remuneration scales up with M_0 one to one, yet it is strictly below $i^f M_0$. The reason is the concavity of the remuneration scheme that, coupled with the stochastic net payment shocks, decreases the expected remuneration. Second, wider tolerance bands *increase* the expected remuneration, simply by decreasing the probability that net payments drive end-of-period reserves outside the tolerance band, where they earn a lower rate.

We now solve for the expected remuneration associated with not accessing the funds market. Assuming that $M_0 \in \mathbf{T}$, the expected remuneration is

$$R^e(M_0, M^*) = i^f M_0 - \beta(i^p - i^*) \exp\left(-\frac{\delta M^*}{\beta}\right) \cosh\left(\frac{M^* - M_0}{\beta}\right).$$

A bank with beginning-of-the-period reserves M_0 and reserve target M^* will access the funds market if

$$V(M_0, M^*) - \phi \geq R^e(M_0, M^*). \tag{23}$$

This holds whenever

$$\beta(i^p - i^*) \exp\left(-\frac{\delta M^*}{\beta}\right) \left(\cosh\left(\frac{M^* - M_0}{\beta}\right) - 1\right) \geq \phi. \tag{24}$$

As shown in the Appendix, the market access problem has a simple solution: The bank will access the funds market whenever its beginning-of-the-period reserves, M_0, are more than Δ from the target, M^*, that is, whenever $M_0 \leq M^* - \Delta$ or $M_0 \geq M^* + \Delta$. Otherwise, the bank leaves its reserve holdings at M_0. This is certainly intuitive: The closer M_0 is to the target M^*, the less to be gained from adjusting reserve holdings to M^*.

We now look at how the region of inactivity defined by Δ, $M_0 \in [M^* - \Delta, M^* + \Delta]$, varies with the width of the tolerance band. Confirming the previous conjecture, a wider tolerance band *decreases* participation in the funds market. The Appendix shows that Δ is indeed increasing in δ. By easing the danger of end-of-period reserves to step out of the tolerance band, banks feel less pressure to adjust their position and incur the cost ϕ.

There is thus a possible trade-off when setting the tolerance band. A wide tolerance band would reduce interest-rate volatility in response to deviations in the supply of reserves but would limit participation in the market and thus trading volumes. The same trade-off applies to the penalty spread: A narrow spread does help interest-rate control but discourages participation.

7 Conclusions and further research

We have proposed a model of the demand for reserves under a fairly general specification for the remuneration of reserves. We have highlighted several desirable properties regarding interest-rate control, mainly connected to the presence of tolerance bands. We have noted, however, the concern that wider tolerance bands may reduce trading in the funds market.

We envision two avenues for future research. First, we would like to incorporate balance-sheet considerations into the demand for reserves. For example, banks may hold reserves to satisfy regulatory requirements or be reluctant to borrow or lend unsecured funds.[6] If the supply of reserves is at a particularly high level, risk considerations may also come into play. Incorporating these additional considerations is a must for any model of voluntary reserve targets, since these considerations—and not the remuneration scheme—will pin down the level of the targets.

Second, reserve-averaging periods are known to be an important factor in stabilizing day-to-day volatility in interest rates. An immediate question is how reserve averaging interacts with a remuneration scheme as described here. Does it enhance or diminish the effect of tolerance bands? It is perhaps fair to conjecture that a modest reserve-averaging period can incent trading in the funds market without compromising interest-rate control.

[6]See Potter (2013) and Armenter and Lester (forthcoming).

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APPENDIX

A Properties of the remuneration function

Proposition A.1. *The remuneration function satisfies* $R(\alpha M', \alpha M^*) = \alpha R(M', M^*)$ *for any* $\alpha \geq 0$.

Proof. The result follows immediately from noting that the conditions $M' > (1+\delta)M^*$ and $M' < (1-\delta)M^*$ imply their counterparts for $\alpha M', \alpha M^*$. □

Proposition A.2. *For any* $\alpha \in [0,1]$ *the remuneration function satisfies*

$$\alpha R(M_1', M_1^*) + (1-\alpha)R(M_2', M_2^*) \leq R(M_\alpha', M_\alpha^*)$$

where $M_\alpha' = \alpha M_1' + (1-\alpha)M_2'$ *and* $M_\alpha^* = \alpha M_1^* + (1-\alpha)M_2^*$.

Proof. Note that $\max\{x, 0\}$ is a convex function, and hence

$$\begin{aligned}\alpha \max\left\{|M_1' - M_1^*| - \delta M_1^*, 0\right\} + (1-\alpha)\max\left\{|M_2' - M_2^*| - \delta M_2^*, 0\right\} \\ \leq \max\left\{\alpha|M_1' - M_1^*| + (1-\alpha)|M_2' - M_2^*| - \delta M_\alpha^*, 0\right\}.\end{aligned}$$

Since $\max\{x, 0\}$ is also non-decreasing and $|x|$ is convex,

$$\begin{aligned}\max\left\{\alpha|M_1' - M_1^*| + (1-\alpha)|M_2' - M_2^*| - \delta M_\alpha^*, 0\right\} \\ \leq \max\left\{|M_\alpha' - M_\alpha^*| - \delta M_\alpha^*, 0\right\}.\end{aligned}$$

Recalling that the penalty enters with a negative sign in the remuneration function, we obtain the desired result. □

B Expected remuneration

We include here the algebra to derive the expectation for the nonlinear term in (3):

$$\Psi^e = \int \max\left\{|M + P - M^*| - \delta M^*, 0\right\} dG(P). \tag{25}$$

The first step is to split the support for P depending on the sign of $M + P - M^*$:

$$\begin{aligned}
\Psi^e &= \int_{-\infty}^{M^*-M} \max\left\{(1-\delta)M^* - M - P, 0\right\} dG(P) \\
&+ \int_{M^*-M}^{\infty} \max\left\{M + P - (1+\delta)M^*, 0\right\} dG(P).
\end{aligned}$$

Noting that $(1-\delta)M^* - M - P \geq 0$ implies $P \leq M^* - M$ and $M + P - (1+\delta)M^* \geq 0$ implies $P \geq M^* - M$, we obtain

$$\begin{aligned}
\Psi^e &= -\int_{-\infty}^{(1-\delta)M^*-M} \left\{M + P - (1-\delta)M^*\right\} dG(P) \\
&+ \int_{(1+\delta)M^*-M}^{\infty} \left\{M + P - (1+\delta)M^*\right\} dG(P).
\end{aligned}$$

Isolating the terms with P,

$$\begin{aligned}
\Psi^e &= \left((1-\delta)M^* - M\right) G\left((1-\delta)M^* - M\right) \\
&+ \left(M - (1+\delta)M^*\right)\left(1 - G\left((1+\delta)M^* - M\right)\right) \\
&- \int_{-\infty}^{(1-\delta)M^*-M} P dG(P) \\
&+ \int_{(1+\delta)M^*-M}^{\infty} P dG(P).
\end{aligned}$$

To solve further, we need to distinguish three cases: $M \in \mathbf{T}$, $M < (1-\delta)M^*$, and $M > (1+\delta)M^*$.

B.1 Desired reserve holdings are compliant

Assume $M \in \mathbf{T}$. Using the key property of the Laplace distribution, $M \geq (1-\delta)M^*$ implies that

$$\int_{-\infty}^{(1-\delta)M^*-M} P \frac{dG(P)}{G((1-\delta)M^* - M)} = (1-\delta)M^* - M - \beta$$

and $M \leq (1+\delta)M^*$ implies

$$\int_{(1+\delta)M^*-M}^{\infty} P \frac{dG(P)}{1-G((1+\delta)M^*-M)} = (1+\delta)M^* - M + \beta.$$

The above implies that

$$\Psi^e = \beta \left(G((1-\delta)M^* - M) + 1 - G(M^*(1+\delta) - M) \right)$$

and noting that $G((1-\delta)M^* - M) + 1 - G(M^*(1+\delta) - M)$ is equal to the probability that $M + P$ steps out of the tolerance band, we obtain (4).

Finally, we relate the probability to its analytic solution below:

$$\begin{aligned}
& G((1-\delta)M^* - M) + 1 - G(M^*(1+\delta) - M) \\
&= \frac{1}{2} \exp\left(\frac{-\delta M^*}{\beta}\right) \left(\exp\left(\frac{M^* - M}{\beta}\right) + \exp\left(-\frac{M^* - M}{\beta}\right) \right) \\
&= \exp\left(\frac{-\delta M^*}{\beta}\right) \cosh\left(\frac{M^* - M}{\beta}\right).
\end{aligned}$$

B.2 Desired reserves are below the bottom of the tolerance band

Now consider the case $M < (1-\delta)M^*$. Since $M \leq (1+\delta)M^*$, then

$$\int_{(1+\delta)M^*-M}^{\infty} P \frac{dG(P)}{1-G((1+\delta)M^*-M)} = (1+\delta)M^* - M + \beta.$$

Noting that P has mean zero, the law of conditional expectations implies that

$$E\left\{P | P \leq (1-\delta)M^* - M\right\} = -\frac{\Pr\left(P > (1-\delta)M^* - M\right)}{\Pr\left(P \leq (1-\delta)M^* - M\right)} E\left\{P | P > (1-\delta)M^* - M\right\}.$$

Since $(1-\delta)M^* - M > 0$, the Laplace distribution is back at work and

$$\int_{-\infty}^{(1-\delta)M^*-M} P \frac{dG(P)}{G((1-\delta)M^*-M)} = -\frac{1-G((1-\delta)M^*-M)}{G((1-\delta)M^*-M)} \left((1-\delta)M^* - M + \beta\right).$$

Now we obtain that

$$\Psi^e = (1-\delta)M^* - M + \beta \left(1 - G((1-\delta)M^* - M) + 1 - G((1+\delta)M^* - M)\right).$$

Analytically,

$$\Psi^e = (1-\delta)M^* - M + \beta \exp\left(\frac{M - M^*}{\beta}\right) \cosh\left(\frac{\delta M^*}{\beta}\right).$$

B.3 Desired reserves are above the top of the tolerance band

Now consider the case $M > (1+\delta)M^*$. Since $M > (1-\delta)M^*$,

$$\int_{-\infty}^{(1-\delta)M^*-M} P\frac{dG(P)}{G((1-\delta)M^*-M)} = (1-\delta)M^* - M - \beta.$$

The same steps as in the previous case leads to

$$\int_{(1+\delta)M^*-M}^{\infty} P\frac{dG(P)}{1-G((1+\delta)M^*-M)} = -\frac{G((1+\delta)M^*-M)}{1-G((1+\delta)M^*-M)}\left((1+\delta)M^*-M-\beta\right).$$

Now we obtain that

$$\Psi^e = M - (1+\delta)M^* + \beta\left(G((1-\delta)M^*-M) + G((1+\delta)M^*-M)\right)$$

or

$$\Psi^e = M - (1+\delta)M^* + \beta\exp\left(-\frac{M-M^*}{\beta}\right)\cosh\left(\frac{\delta M^*}{\beta}\right).$$

C Demand for reserves

C.1 General problem

Substitute $M = (1-\delta)M^*$ in (6) to obtain

$$\exp\left(-\frac{\delta M^*}{\beta}\right)\sinh\left(-\frac{\delta M^*}{\beta}\right) = \frac{i^* - i^f}{i^p - i^*}.$$

Re-arranging

$$i^f = i^* - \frac{1}{2}(i^p - i^*)\left(\exp\left(-2\frac{\delta M^*}{\beta}\right) - 1\right)$$

or

$$i^f = i^* + \frac{1}{2}(i^p - i^*)\Pr\left(|P| \geq 2\delta M^*\right).$$

Substituting instead $M = (1+\delta)M^*$,

$$i^f = i^* - \frac{1}{2}(i^p - i^*)\left(1 - \exp\left(-2\frac{\delta M^*}{\beta}\right)\right)$$

or

$$i^f = i^* - \frac{1}{2}(i^p - i^*)\Pr\left(|P| \geq 2\delta M^*\right).$$

These two expressions define the range of interest rates such that (6) applies.

The first-order condition associated with $M < (1-\delta)M^*$ is

$$\frac{i^* - i^f}{i^p - i^*} + \left(1 - \cosh\left(\frac{\delta M^*}{\beta}\right)\exp\left(\frac{M - M^*}{\beta}\right)\right) = 0. \tag{26}$$

The inverse demand function is then

$$i^f = i^p - (i^p - i^*)\cosh\left(\frac{\delta M^*}{\beta}\right)\exp\left(\frac{M - M^*}{\beta}\right). \tag{27}$$

Evaluated at $M = (1-\delta)M^*$,

$$\frac{i^* - i^f}{i^p - i^*} + \left(1 - \cosh\left(\frac{\delta M^*}{\beta}\right)\exp\left(\frac{-\delta M^*}{\beta}\right)\right) = 0$$

or

$$\frac{i^* - i^f}{i^p - i^*} + \frac{1}{2}\left(1 - \exp\left(-\frac{2\delta M^*}{\beta}\right)\right) = 0.$$

This can be rewritten as

$$i^f = i^* + \frac{1}{2}(i^p - i^*)\Pr\left(|P| \geq 2\delta M^*\right).$$

This confirms that the inverse demand function is continuous.

Turning to the case $M > (1+\delta)M^*$, the inverse demand function is then

$$i^f = i^x + (i^p - i^*)\cosh\left(\frac{\delta M^*}{\beta}\right)\exp\left(-\frac{M - M^*}{\beta}\right). \tag{28}$$

We can again evaluate at $M = (1+\delta)M^*$ to confirm that we obtain

$$i^f = i^* + \frac{1}{2}(i^p - i^*)\Pr\left(|P| \geq 2\delta M^*\right).$$

C.2 Value function

Define

$$V(M_0, M^*) = \max_M R^e(M, M^*) - i^f(M - M_0)$$

where the dependence of V on interest rates and the tolerance band is left implicit.

For the case $M \in \mathbf{T}$, note (6) can be written as

$$\exp\left(-\frac{\delta M^*}{\beta}\right)\left(\exp\left(\frac{M - M^*}{\beta}\right) - \cosh\left(\frac{M - M^*}{\beta}\right)\right) = \frac{i^* - i^f}{i^p - i^*}.$$

Substituting for $(i^p - i^*)\exp\left(-\frac{\delta M^*}{\beta}\right)\cosh\left(\frac{M-M^*}{\beta}\right)$ in (4), we obtain

$$V(M_0, M^*) = (i^* - i^f)M + i^f M_0 + \beta(i^p - i^*)\left(\frac{i^* - i^f}{i^p - i^*} - \exp\left(-\frac{(1+\delta)M^* - M}{\beta}\right)\right). \quad (29)$$

If $i^f = i^*$, this simplifies to

$$V(M_0, M^*) = i^f M_0 - \beta(i^p - i^*)\exp\left(-\frac{\delta M^*}{\beta}\right).$$

D Market access decision

Noting that $\cosh(x) = \cosh(-x)$, we can write (24) for a strict equality in terms of the absolute value of $|M^* - M_0| = \Delta$,

$$\beta(i^p - i^*)\exp\left(-\frac{\delta M^*}{\beta}\right)\left(\cosh\left(\frac{\Delta}{\beta}\right) - 1\right) = \phi. \quad (30)$$

The function $h(\Delta) = \cosh\left(\frac{\Delta}{\beta}\right) - 1 : \Re_+ \to \Re_+$ is strictly increasing and a full range in $\Re_+$. Hence, there exists always a solution $\Delta \geq 0$. Indeed, $h(0) = 0$ so $\Delta > 0$ for all parameters considered.

Using the implicit function theorem, it is possible to sign the comparative statics for $\Delta > 0$ as a function of δ, cost ϕ, and the spread $i^p - i^*$:

$$\begin{aligned}
\frac{\partial \Delta}{\partial \delta} &= \left(\beta(i^p - i^*)\sinh\left(\frac{\Delta}{\beta}\right)\right)^{-1} \phi M^* \exp\left(\frac{\delta M^*}{\beta}\right) > 0, \\
\frac{\partial \Delta}{\partial (i^p - i^*)} &= -\left(\beta(i^p - i^*)\sinh\left(\frac{\Delta}{\beta}\right)\right)^{-1} \frac{\phi}{i^p - i^*} < 0 \\
\frac{\partial \Delta}{\partial \phi} &= \left(\beta(i^p - i^*)\sinh\left(\frac{\Delta}{\beta}\right)\right)^{-1} > 0.
\end{aligned}$$