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Abstract

It has been largely acknowledged that monetary policy can affect borrowers and lenders differently. This paper investigates whether the distributional effects of monetary policy are an inherent feature of monetary economies with private credit instruments. In our framework, both money and credit instruments can potentially be used as media of exchange to overcome trading frictions in decentralized markets. Entrepreneurs have access to productive projects but face credit constraints due to limited pledgeability of their returns. Monetary policy affects the liquidity premium on private credit and thereby influences the cost of borrowing and the level of investment, but any attempt to ease borrowing constraints results in suboptimal decentralized-market trading activity. We show that this policy trade-off is *not* an inherent feature of monetary economies with private credit instruments. If we consider a richer set of aggregate liquidity management instruments, such as the payment of interest on inside money and capital requirements, it is possible to implement an efficient allocation.

Keywords: monetary theory and policy, liquidity premium, Friedman rule, investment, bank lending channel

JEL Classification: E32, E42, E52, G28

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1. INTRODUCTION

The distributional effects of changes in the money supply have long been recognized by many economists as one of the main problems for monetary policy formulation and implementation. Throughout monetary history, the conflicting views of different segments of the population regarding the desirable monetary arrangements have been at the center of contemporary policy discussions. For instance, the free silver movement in the United States provides a vivid example of the distributional effects of monetary arrangements and their political implications. Following the demonetization of silver in 1873, the ensuing monometallic standard based on gold resulted in a secular downward trend in the price level as the U.S. economy expanded vigorously following the resumption of gold payments in 1879.[1] It has been argued that the ensuing deflation had opposite effects on borrowers and lenders. For instance, an important group of borrowers who suffered from the adherence to the gold standard and strongly favored the remonetization of silver were the Midwest farmers. Opposing the free silver movement were mainly bank depositors (i.e., savers) in urban centers.[2]

More recently, many economists have argued that the monetary policies followed by major central banks since the financial crisis aim at reducing the cost of borrowing for households and firms to boost consumption and investment. Some critics have pointed out that, although these policies can benefit borrowers, they reduce the welfare of asset holders by depressing the real return on liquid assets.

These important episodes clearly highlight the potential welfare consequences of the distributional effects of monetary arrangements on borrowers and lenders. Given the possibility of these effects, it is crucial to develop a theoretical framework to evaluate the welfare properties of alternative monetary arrangements in the presence of private credit instruments. The development of a tractable framework that is explicit about the underlying trading frictions that prevent certain contracts from emerging in equilibrium allows us to answer

[1] See Friedman and Schwartz (1963).

[2] See Rockoff (1990).

a number of interesting questions: Is the presence of distributional effects an inevitable feature of monetary economies with private credit instruments? Is it possible to design an institutional arrangement that is consistent with an efficient allocation? Does it require government intervention? If so, what are the required policy instruments?

To answer these questions, we build on the work of Lagos and Wright (2005) to construct a search-theoretic model in which both fiat money and private credit instruments can potentially be used as media of exchange to overcome trading frictions in decentralized markets. In our framework, entrepreneurs have access to productive projects but face credit constraints due to limited pledgeability of their returns, as in Kiyotaki and Moore (2005) and Holmstrom and Tirole (2011). As we will see, these features of the environment can potentially create a situation in which borrowers and lenders will benefit from antagonistic monetary policies.

As a useful benchmark, we start our analysis by considering an arrangement with the property that money and credit are divorced. By this we mean that privately issued credit claims cannot circulate as a medium of exchange so that fiat money is the only asset that can serve this purpose. Not surprisingly, we show that monetary policy does not affect the allocation in the credit market and that private credit creation does not influence the total supply of liquid claims in the economy. As a result, the effects of monetary policy are unambiguous so that the government can induce the efficient level of decentralized-market trading activity by implementing the Friedman rule. Although optimal monetary policy can eliminate the opportunity cost of holding liquid assets, the ensuing equilibrium allocation is inefficient because the collateral constraint implies that a positive measure of entrepreneurs with socially productive projects will end up credit constrained, with aggregate investment below the socially optimum.

Subsequently, we consider an economy without any exogenous restriction on inside money creation so that private credit claims can circulate as a medium of exchange (or can be used as collateral for the issuance of liquid claims). We start by characterizing a *pure* inside-money equilibrium (i.e., an equilibrium with the property that fiat money is not valued) and show that the ensuing allocation is inefficient. Depending on the parameters, the equilibrium real interest rate can be lower than or equal to the rate of time preference. If it is lower, then the agents inefficiently economize on liquid assets for transaction purposes. If it is equal to the rate of time preference, then entrepreneurs with socially productive projects end up credit constrained.

Then, we characterize equilibria with the property that both fiat money and private credit claims circulate as a means of payment. In these equilibria, the real interest rate is typically higher than in the pure inside-money equilibrium, as in Tirole (1985) and Farhi and Tirole (2012). A key property of this inside- and outside-money economy is that the cost of borrowing for entrepreneurs equals the real return on liquid assets when both assets serve as media of exchange, implying that monetary policy can affect the level of investment in the economy. Because the government can control the real return on liquid assets in a stationary equilibrium, it is possible to induce the socially efficient level of investment by setting the real return on liquid assets below the rate of time preference, which relaxes the collateral constraint.

Thus, the only way to reduce the cost of borrowing for entrepreneurs to achieve the efficient level of investment is by deviating from the Friedman rule, which necessarily results in suboptimal decentralized-market trading activity. In other words, a monetary policy that implements a real return on liquid assets below the rate of time preference results in an equilibrium allocation with the property that borrowers (i.e., entrepreneurs) are better off and asset holders (i.e., buyers) are worse off. The existence of this policy trade-off explains why different groups in society can favor antagonistic policies. Basically, the borrowers want the government to choose a monetary policy that implies a low real interest rate, whereas the asset holders want the government to induce a high real interest rate. In the context of our initial example, the analysis in this paper rationalizes the choice of a bimetallic standard by borrowers and of a monometallic standard by asset holders.

So far, we have shown that a policy trade-off can arise in an economy with the property that both fiat money and private credit instruments are used as media of exchange. Our next step in the paper is to show that the previously described trade-off does *not* arise when we consider a broader set of aggregate liquidity management instruments. In particular, we show that it is possible to set the cost of borrowing for entrepreneurs below the real return on liquid assets in a stationary equilibrium if the government is willing to pay interest on inside money. If the government sets up a facility to make interest payments on privately issued loans, then it is possible to eliminate the opportunity cost of holding liquid assets and at the same time lower the cost of borrowing for entrepreneurs to relax the collateral constraint. As a result, the government can implement an efficient allocation by selecting a nominal interest rate on outside money below the nominal interest rate on inside money.

Finally, we show that, depending on the parameters, fiat money cannot be valued in equilibrium when the real interest rate equals the rate of time preference in a pure inside-money equilibrium. In this case, the previously described policy instruments cannot be effective unless we supplement them with capital requirements on inside-money creation. The introduction of capital requirements has the effect of neutralizing some of the collateral that is endogenously created to serve as media of exchange. Then, we show that it is possible to select a sufficiently low capital requirement that is consistent with an equilibrium with both fiat money and private credit claims circulating as media of exchange, so that the government can implement an efficient allocation by using the previously described policy instruments.

Our results show that the distributional effects of monetary policy are not an inevitable feature of monetary economies with private credit instruments. Although distributional effects across borrowers and lenders can be a problem for monetary policy implementation, it is possible to consider a broader set of aggregate liquidity management instruments to construct a monetary arrangement that is not necessarily characterized by the aforementioned distributional effects, allowing the government to implement an efficient allocation.

Our paper contributes to the literature on private liquidity creation. Recent contributions to this literature include Geromichalos, Licari, and Suarez-Lledo (2007), Farhi and Tirole (2012), and Rocheteau and Wright (2013).[3] What makes our analysis different from these papers is that we consider an economy in which entrepreneurs with productive projects face

[3] Other important papers in the literature include Champ, Smith, and Williamson (1996), Berentsen, Camera, and Waller (2007), and Williamson (2012).

a collateral constraint and privately created assets can serve as a medium of exchange. As a result, liquidity creation and credit-market activity are intertwined so that distributional effects can potentially arise in equilibrium.

The rest of the paper is organized as follows. Section 2 describes the environment. In Section 3, we characterize efficient allocations. Section 4 describes the set of equilibrium allocations for an economy with an exogenous separation between money and credit. Section 5 removes this exogenous restriction and analyzes an economy with inside money. Section 6 considers a broader set of policy instruments to achieve efficiency. In Section 7, we discuss the welfare consequences of capital requirements. Section 8 concludes.

2. ENVIRONMENT

Time is discrete and continues forever. Each period is divided into two subperiods in which economic activity will differ. There is a frictionless centralized market in the first subperiod, while trade is decentralized in the second subperiod. A perishable commodity is produced and consumed in each subperiod. We refer to the commodity produced in the first subperiod as the CM good and to the commodity produced in the second subperiod as the DM good. The CM good can also be used as input for an intertemporal production technology to be described next.

The economy is populated by a large number of three types of agents, referred to as buyers, sellers, and entrepreneurs. Buyers and sellers are infinitely lived agents, whereas entrepreneurs live for two periods only. In each period, a new generation of entrepreneurs is born. We normalize to one the size of each group.

Entrepreneurs are consumers of the CM good and only participate in the centralized market, remaining idle in the decentralized market. An entrepreneur wants to consume only in the second period of his life and is endowed at birth with an indivisible and nontradable project that requires exactly one unit of the CM good as input and that pays off $\gamma \in \mathbb{R}_+$ units of the CM good in the following period. Let $G(\gamma)$ denote the distribution of payoffs across the population of entrepreneurs born in period t. The support of the distribution is

$[0, \bar{\gamma}]$.

Buyers and sellers participate in both markets in each period. All buyers and all sellers are able to produce the CM good in the first subperiod using a linear production technology that requires labor as input. In the second subperiod, buyers want to consume but cannot produce, whereas sellers are able to produce but do not want to consume. For this reason, we refer to a consumer of the DM good as a *buyer* and to a producer of the DM good as a *seller*. In the decentralized market, a buyer is randomly matched with a seller with probability $\alpha \in [0, 1]$ and vice versa, so trade is bilateral.

There is a continuum of buyers with measure one, each with preferences represented by

$$U^b \left(x_t^b, q_t \right) = x_t^b + u \left(q_t \right).$$

Here $x_t^b \in \mathbb{R}$ denotes net consumption of the CM good, and $q_t \in \mathbb{R}_+$ denotes consumption of the DM good. Assume that $u : \mathbb{R}_+ \to \mathbb{R}_+$ is increasing, strictly concave, and continuously differentiable, with $u(0) = 0$ and $u'(0) = \infty$.

There is a continuum of sellers with measure one, each with preferences represented by

$$U^s \left(x_t^s, q_t \right) = x_t^s - w \left(q_t \right).$$

Here $x_t^s \in \mathbb{R}$ denotes net consumption of the CM good, and $q_t \in \mathbb{R}_+$ denotes production of the DM good. Assume that $w : \mathbb{R}_+ \to \mathbb{R}_+$ is increasing, convex, and continuously differentiable, with $w(0) = 0$. An entrepreneur born in period t derives utility from consumption in period $t + 1$. All agents have the same subjective discount factor $\beta \in (0, 1)$. Finally, suppose that $\bar{\gamma} > \beta^{-1}$.

We assume that buyers and sellers are anonymous (i.e., their identities are unknown and their trading histories are privately observable), which precludes credit in the decentralized market. Because there is no scope for trading future promises in this market, a medium of exchange is essential to achieve allocations that we could not achieve without it.

Throughout the analysis, we also assume that only a fraction $\theta \in (0, 1)$ of an entrepreneur's project can be pledged as collateral to secure a loan in the credit market. As a result, entrepreneurs will face a collateral constraint in the credit market when borrowing funds to finance their project.

3. EFFICIENT ALLOCATIONS

It is helpful to start our analysis by characterizing efficient allocations. To be clear, we characterize *unconstrained* efficient allocations by assuming that the planner can overcome the previously described frictions when choosing an allocation. Let $\gamma_t^* \in [0, \bar{\gamma}]$ denote the marginal entrepreneur above which the social planner chooses to undertake all investment projects in period t, and let $x_t \in \mathbb{R}_+$ denote the entrepreneur's average consumption in old age. The planner's problem consists of choosing an allocation $\left\{x_t^b, x_t^s, x_t, \gamma_t^*, q_t\right\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t \left\{x_t^b + x_t^s + \alpha \left[u\left(q_t\right) - w\left(q_t\right)\right]\right\}$$

subject to the resource constraint

$$x_t^b + x_t^s + x_t + 1 - G\left(\gamma_t^*\right) = \int_{\gamma_{t-1}^*}^{\bar{\gamma}} \gamma g\left(\gamma\right) d\gamma,$$

and the required level of utility for the entrepreneur

$$x_t \geq U_t^e,$$

with the required values $\{U_t^e\}_{t=0}^{\infty}$ taken as given.

The first-order conditions are given by

$$u'\left(q_t\right) = w'\left(q_t\right)$$

and

$$g\left(\gamma_t^*\right)\left(1 - \beta\gamma_t^*\right) = 0.$$

Then, the solution to the planner's problem implies that a socially efficient allocation must satisfy

$$q_t = q^*$$

and

$$\gamma_t^* = \beta^{-1}$$

at all dates $t \geq 0$, with q^* denoting the surplus-maximizing quantity satisfying $u'(q^*) = w'(q^*)$. Along the socially optimal path, total investment is given by

$$1 - G\left(\beta^{-1}\right)$$

in every period.

4. SEPARATING MONEY FROM CREDIT

In this section, we characterize equilibrium allocations by assuming an institutional arrangement with the property that money and credit are divorced. By this we mean an arrangement in which privately issued claims cannot be used as media of exchange. We believe this is a useful benchmark to start our analysis because it clearly illustrates the roles of money and credit in the decentralized economy. Following the tradition in the literature, we refer to this arrangement as a narrow-banking economy.

4.1. Credit Market

Let $1 + r_t$ denote the real interest rate on a privately issued one-period loan contract. Given this interest rate, an entrepreneur of type $\gamma \in [0, \bar{\gamma}]$ born in period t has a profitable project only if

$$\gamma - (1 + r_t) \geq 0. \tag{1}$$

Because only a fraction $\theta \in (0, 1)$ of the project's payoff is pledgeable, the entrepreneur is subject to the pledgeability restriction

$$1 + r_t \leq \theta\gamma. \tag{2}$$

In other words, the promised repayment cannot exceed the value of the entrepreneur's pledgeable assets.

Let $\hat{\gamma}_t \in \mathbb{R}_+$ denote the entrepreneur whose project's payoff satisfies the pledgeability restriction with equality in period t, given the market interest rate $1 + r_t$. Then, the marginal type $\hat{\gamma}_t$ satisfies

$$\hat{\gamma}_t = \frac{1 + r_t}{\theta}. \tag{3}$$

As we can see, the total demand for loans is then given by

$$1 - G(\hat{\gamma}_t) = 1 - G\left(\frac{1 + r_t}{\theta}\right).$$

4.2. Asset Demand

Start with the buyer's portfolio problem. Let $m_t \in \mathbb{R}_+$ denote the buyer's real balances and let $d_t \in \mathbb{R}_+$ denote the real value of his liquid portfolio. In the narrow-banking economy, we have

$$d_t = (1 + i^m) \rho_t m_t, \tag{4}$$

where $\rho_t \in \mathbb{R}_+$ is the reciprocal of the inflation rate between dates t and $t+1$ and $i^m \in \mathbb{R}_+$ is the nominal interest rate on money balances. Because the government chooses the interest rate on money balances to influence the equilibrium allocation in the economy, we refer to the interest rate i^m as the policy rate.

Consider the buyer's portfolio problem. Let $J_t(d_{t-1}, l_{t-1})$ denote the value function for a buyer who enters the centralized market in period t holding a portfolio of liquid assets with real value $d_{t-1} \in \mathbb{R}_+$ and a portfolio of loans with face value $l_{t-1} \in \mathbb{R}_+$. Let $V_t(d_t, l_t)$ denote the value function in the decentralized market. The Bellman equation for the buyer can be described as

$$J_t(d_{t-1}, l_{t-1}) = \max_{(x_t, m_t, l_t) \in \mathbb{R} \times \mathbb{R}_+^2} [x_t + V_t(d_t, l_t)]$$

subject to (4) and

$$x_t + m_t + l_t = d_{t-1} + (1 + r_{t-1}) l_{t-1} + \tau_t.$$

Here $\tau_t \in \mathbb{R}$ denotes the real value of lump-sum government transfers. The value $V_t(d_t, l_t)$ satisfies

$$V_t(d_t, l_t) = \alpha [u(q_t(d_t)) + \beta J_{t+1}(d_t - z_t(d_t), l_t)] + (1 - \alpha) \beta J_{t+1}(d_t, l_t),$$

where $q_t(d_t)$ denotes production of the DM good in exchange for a monetary transfer with real value $z_t(d_t)$. As we will see, the terms of trade depend only on the real value of the buyer's liquid portfolio, which is one of the key properties of the Lagos-Wright framework.

Throughout the analysis, we assume that the terms of trade are determined by Nash bargaining. For simplicity, we restrict attention to the special case in which the buyer has all the bargaining power. In what follows, nothing hinges on this particular choice of the bargaining protocol. The bargaining problem can be described as

$$\max_{(q,z)\in\mathbb{R}^2_+} \left[u\left(q\right)-\beta z\right]$$

subject to the seller's participation constraint

$$-w\left(q\right)+\beta z\geq 0$$

and the liquidity constraint

$$z\leq d_t,$$

where $z\in\mathbb{R}_+$ denotes the real value of the asset transfer to the seller. The solution to this problem is given by

$$q\left(d_t\right)=\begin{cases} w^{-1}\left(\beta d_t\right) \text{ if } d_t<\frac{w(q^*)}{\beta} \\ q^* \text{ if } d_t\geq\frac{w(q^*)}{\beta}\end{cases}$$

and

$$z\left(d_t\right)=\begin{cases} d_t \text{ if } d_t<\frac{w(q^*)}{\beta} \\ \beta^{-1}w\left(q^*\right) \text{ if } d_t\geq\frac{w(q^*)}{\beta}.\end{cases}$$

Given these solutions, the portfolio problem in the centralized market can be written as

$$\max_{(m_t,l_t)\in\mathbb{R}^2_+} \left\{-m_t-l_t+\alpha\left[u\left(q_t\left(d_t\right)\right)-\beta z_t\left(d_t\right)\right]+\beta\left[d_t+\left(1+r_t\right)l_t\right]\right\}. \tag{5}$$

subject to (4). The slope of the objective function with respect to m_t is

$$-1+\beta\left(1+i^m\right)\rho_t\left[\alpha\frac{u'\left(w^{-1}\left(\beta\left(1+i^m\right)\rho_t m_t\right)\right)}{w'\left(w^{-1}\left(\beta\left(1+i^m\right)\rho_t m_t\right)\right)}+1-\alpha\right]$$

if $0<m_t<\frac{w(q^*)}{\beta(1+i^m)\rho_t}$. The slope is

$$-1+\beta\left(1+i^m\right)\rho_t$$

if $m_t>\frac{w(q^*)}{\beta(1+i^m)\rho_t}$. The slope of the objective function with respect to l_t is

$$-1+\beta\left(1+r_t\right)$$

for any interior value $l_t > 0$. As we can see, the solution to the portfolio problem depends on the real return on money, given by $(1+i^m)\,\rho_t$, and the real interest rate, given by $1+r_t$.

In what follows, it is convenient to define the function $L : \mathbb{R}_+ \to \mathbb{R}_+$ by

$$L(d) = \begin{cases} \alpha \frac{u'\left(w^{-1}(\beta d)\right)}{w'\left(w^{-1}(\beta d)\right)} + 1 - \alpha \text{ if } \beta d \le w(q^*) \\ 1 \text{ if } \beta d > w(q^*). \end{cases}$$

Then, the first-order condition with respect to m_t can be written as

$$1 = \beta(1+i^m)\,\rho_t L\left((1+i^m)\,\rho_t m_t\right).$$

If $(1+i^m)\,\rho_t < \beta^{-1}$, then the previous condition implicitly defines the demand for real balances as a function of the real return on money.

In the narrow-banking economy, agents cannot use privately issued loans as a medium of exchange or pledge them as collateral for issuing claims that can serve as a means of payment. As a result, the supply of loans is zero if $1+r_t < \beta^{-1}$. If $1+r_t = \beta^{-1}$, then the supply of loans is a correspondence $[0,\infty)$, given that lenders will be indifferent.

Consider now the seller's portfolio problem. Let $N_t(d_{t-1}, l_{t-1})$ denote the value function for a seller who enters the centralized market in period t holding a portfolio of liquid assets with real value $d_{t-1} \in \mathbb{R}_+$ and a portfolio of loans with face value $l_{t-1} \in \mathbb{R}_+$. Let $H_t(d_t, l_t)$ denote the value function in the decentralized market. The Bellman equation for the seller can be written as

$$N_t(d_{t-1}, l_{t-1}) = \max_{(x_t, m_t, l_t) \in \mathbb{R}\times\mathbb{R}^2_+} \left[x_t + H_t(d_t, l_t)\right]$$

subject to (4) and

$$x_t + m_t + l_t = d_{t-1} + (1+r_{t-1})\,l_{t-1}.$$

The value $H_t(d_t, l_t)$ satisfies

$$H_t(d_t, l_t) = \alpha\left[-w\left(q_t\left(d_t^+\right)\right) + \beta N_{t+1}\left(d_t - z_t\left(d_t^+\right), l_t\right)\right] + (1-\alpha)\,\beta N_{t+1}(d_t, l_t),$$

where $d_t^+ \in \mathbb{R}_+$ denotes the real value of the liquid assets of the buyer with whom the seller is matched in the decentralized market. Because of quasi-linear preferences, the seller's asset holdings do not influence the terms of trade in bilateral meetings.

A seller is unwilling to produce in exchange for money balances in the centralized market when $(1+i^m)\,\rho_t < \beta^{-1}$. If $(1+i^m)\,\rho_t = \beta^{-1}$, then the seller is indifferent. In this case, we can assume, without loss of generality, that the seller does not hold money balances across periods. A seller is unwilling to produce to make a loan in the credit market if $1+r_t < \beta^{-1}$. The seller is indifferent when $1+r_t = \beta^{-1}$. In this case, we can similarly assume, without loss of generality, that the seller decides not to be a lender in the credit market.

4.3. Equilibrium

To provide an equilibrium definition, we need to specify government behavior. Let $\bar{m}_t \in \mathbb{R}_+$ denote the real value of the money supply in period t. The government budget constraint can be written as

$$(1+i^m)\,\rho_{t-1}\bar{m}_{t-1} + \tau_t = \bar{m}_t.$$

Suppose the government keeps the money supply constant over time. Then, we obtain the law of motion

$$\bar{m}_t = \rho_{t-1}\bar{m}_{t-1}. \tag{6}$$

As previously mentioned, the government implements monetary policy by setting the interest rate on money balances.

In what follows, it is helpful to explicitly define the demand for real balances. If $(1+i^m)\,\rho_t < \beta^{-1}$, then we have

$$m\,(\rho_t, i^m) = \frac{1}{(1+i^m)\,\rho_t} L^{-1}\left(\frac{1}{\beta\,(1+i^m)\,\rho_t}\right).$$

If $(1+i^m)\,\rho_t = \beta^{-1}$, then the demand for real balances is a correspondence $[w\,(q^*)\,, \infty)$.

The market-clearing condition in the money market implies

$$\bar{m}_t = m\,(\rho_t, i^m)\,.$$

Then, the law of motion (6) implies the equilibrium relation

$$m\left(\rho_{t+1}, i^m\right) = \rho_t m\,(\rho_t, i^m)\,, \tag{7}$$

which determines the evolution of the reciprocal of the inflation rate ρ_t along the equilibrium trajectory. Note that the initial choice $\rho_0 \in \mathbb{R}_+$ is arbitrary.

Because the distribution $G(\gamma)$ is invariant across generations, the marginal type $\hat{\gamma}_t$ and the real interest rate r_t will remain constant along the equilibrium trajectory. In addition, note that the market-clearing condition in the credit market can be satisfied if and only if

$$1 + r_t = \frac{1}{\beta} \tag{8}$$

holds at all dates $t \geq 0$. Now we can provide a formal definition of equilibrium.

Definition 1 *A narrow-banking equilibrium can be defined as a sequence $\{\rho_t, r_t\}_{t=0}^{\infty}$ satisfying (7) and (8) at all dates $t \geq 0$.*

As we can see, the equilibrium conditions in the credit market are not influenced by the equilibrium conditions in the money market and vice versa. This dichotomy occurs because we have considered an institutional arrangement with the property that privately issued loans cannot be "securitized" to be used as a means of payment in the decentralized market.

Given an initial condition $\rho_0 \in \mathbb{R}_+$, the dynamic system (7) pins down the equilibrium sequence $\{\rho_t\}_{t=0}^{\infty}$. In particular, the stationary sequence $\rho_t = 1$ at all dates $t \geq 0$ solves (7). In a stationary equilibrium, the level of real balances is

$$m(1, i^m) = \frac{1}{(1 + i^m)} L^{-1}\left(\frac{1}{\beta(1 + i^m)}\right),$$

and the quantity traded in the decentralized market is implicitly defined by

$$\alpha \frac{u'(q(i^m))}{w'(q(i^m))} + 1 - \alpha = \frac{1}{\beta(1 + i^m)}. \tag{9}$$

Note that $q'(i^m) > 0$ for any $0 < 1 + i^m < \beta^{-1}$ so that the government can induce a higher level of decentralized-market output by raising the policy rate. In equilibrium, the buyers act as savers who demand a liquid instrument to serve as a temporary store of value. By increasing the policy rate, the government lowers the opportunity cost of holding liquid assets for transaction purposes. As a result, trading activity in the decentralized market picks up. Despite the possibility of stimulating DM output, monetary policy has no effect on investment decisions in the centralized market.

4.4. Welfare

Our next step is to verify whether an efficient allocation can be implemented in the narrow-banking economy. If we take the limit $i^m \to \beta^{-1} - 1$, then we have

$$q\left(i^m\right) \to q^*$$

so that the surplus-maximizing quantity is traded in the decentralized market. The policy rate $i^m \to \beta^{-1} - 1$ is consistent with the Friedman rule, which eliminates the opportunity cost of holding liquid assets for transaction purposes. As in standard search-theoretic exchange models with divisible money, the Friedman rule induces a socially efficient level of decentralized-market trading activity.[4]

As we have seen, monetary policy has no effect on the credit market. Because entrepreneurs cannot pledge the full value of their projects, the allocation in the credit market is suboptimal. Because condition (8) must be satisfied in every period $t \geq 0$, we have

$$\hat{\gamma}_t = \frac{1}{\theta\beta} > \frac{1}{\beta}$$

at all dates $t \geq 0$. Given the equilibrium interest rate, entrepreneurs in the range $\gamma \in \left[\frac{1}{\beta}, \frac{1}{\theta\beta}\right]$ have a profitable project but are unable to borrow in the credit market because of the limited pledgeability of their return. As a result, total investment is below the socially optimum,

$$1 - G\left(\frac{1}{\theta\beta}\right) < 1 - G\left(\frac{1}{\beta}\right),$$

so the narrow-banking economy does not deliver an efficient allocation for any choice of the policy rate.

5. INSIDE MONEY

In this section, we study the properties of equilibrium allocations without portfolio restrictions. As we will see, inside money will emerge in equilibrium because the agents are free to use privately created assets as media of exchange. For this reason, we refer to this

[4] The optimality of the Friedman rule has also been established in the large-household model of Shi (1997).

arrangement as an inside-money economy. Although the agents are not subject to portfolio restrictions, we will show that an inside-money economy will not deliver an efficient allocation for any level of the policy rate. As we will see, the government will face a trade-off between easing borrowing constraints for entrepreneurs and promoting efficient exchange.

5.1. Asset Demand

In the absence of portfolio restrictions, all types of assets can serve as media of exchange. In this case, the real value of the agent's portfolio of liquid assets is given by

$$d_t = (1 + r_t)\, l_t + (1 + i^m)\, \rho_t m_t. \tag{10}$$

Then, the buyer's portfolio problem can be described as

$$\max_{(m_t, l_t) \in \mathbb{R}^2_+} \{-m_t - l_t + \alpha [u(q(d_t)) - \beta z(d_t)] + \beta d_t\}$$

subject to (10). An *interior* solution is characterized by the first-order conditions

$$1 + r_t = (1 + i^m)\, \rho_t \tag{11}$$

and

$$\frac{1}{\beta(1 + r_t)} = L(d_t), \tag{12}$$

with the real value of liquid assets given by (10). Condition (11) is a no-arbitrage condition that says that if inside and outside assets are equally useful in facilitating exchange in the decentralized market, then their yields must be the same.

5.2. Equilibrium

The market-clearing condition in the credit market implies

$$l_t = 1 - G\left(\frac{1 + r_t}{\theta}\right).$$

Given the no-arbitrage condition (11), the real value of liquid assets can be written as

$$d_t = (1 + r_t)\left[1 - G\left(\frac{1 + r_t}{\theta}\right) + m_t\right]. \tag{13}$$

The law of motion for real balances (6) and the no-arbitrage condition (11) imply

$$m_{t+1} = \left(\frac{1 + r_t}{1 + i^m} \right) m_t. \tag{14}$$

In addition, the real interest rate on loans cannot exceed the rate of time preference so that the boundary condition

$$1 + r_t \leq \beta^{-1} \tag{15}$$

must hold at all dates $t \geq 0$. Given these conditions, we can now provide a formal definition of equilibrium.

Definition 2 *An inside-money equilibrium is a sequence $\{m_t, d_t, r_t\}_{t=0}^{\infty}$ satisfying (12)-(15) in every period $t \geq 0$.*

The previously described equilibrium relations imply that credit-market conditions influence the value of money and vice versa. Because the loans issued in the credit market to finance capital investments can be used as collateral to create liquid claims, the equilibrium interest rate on loans will be influenced by the demand for liquid assets and vice versa. As a result, monetary policy can potentially influence the allocation in the credit market and the level of investment in the economy.

5.3. Equilibrium Without Valued Fiat Money

It is helpful to start our analysis by characterizing equilibria with the property that fiat money is not valued. Some authors refer to this type of allocation as the bubble-free equilibrium; see Tirole (1985) and Farhi and Tirole (2012). If we set $m_t = 0$ at all dates $t \geq 0$, then the equilibrium sequence $\{d_t, r_t\}_{t=0}^{\infty}$ is such that $(d_t, r_t) = \left(d^N, r^N\right)$ at all dates $t \geq 0$, with the pair $\left(d^N, r^N\right)$ satisfying

$$\frac{1}{\beta \left(1 + r^N\right)} = L\left(d^N\right)$$

and

$$d^N = \left(1 + r^N\right) \left[1 - G\left(\frac{1 + r^N}{\theta}\right)\right].$$

Then, an inside-money equilibrium without valued fiat money can be defined as a real interest rate $1+r^N$ satisfying

$$\Omega\left(1+r^N\right)=0, \tag{16}$$

where the function $\Omega:\left[0,\beta^{-1}\right]\to\mathbb{R}$ is given by

$$\Omega\left(y\right)\equiv L\left(y\left[1-G\left(\frac{y}{\theta}\right)\right]\right)-\frac{1}{\beta y}.$$

There are two possibilities.

Case 1. $1-G\left(\frac{1}{\theta\beta}\right)<w\left(q^*\right)$

In this case, we have $\Omega\left(\beta^{-1}\right)>0$. Then, an inside-money equilibrium without valued fiat money exists if and only if there exists $0<1+r<\beta^{-1}$ such that $\Omega\left(1+r\right)<0$. It is possible that a solution to (16) does not exist because there is no guarantee that the function $\Omega:\left[0,\beta^{-1}\right]\to\mathbb{R}$ crosses the horizontal axis at least once (i.e., there may not exist $0<1+r<\beta^{-1}$ such that $\Omega\left(1+r\right)<0$). Note that

$$\Omega'\left(y\right)=L'\left(y\left[1-G\left(\frac{y}{\theta}\right)\right]\right)\left[1-G\left(\frac{y}{\theta}\right)-\frac{y}{\theta}g\left(\frac{y}{\theta}\right)\right]+\frac{1}{\beta y^2}$$

for any $0<y<\beta^{-1}$. As we can see, the sign of the derivative of Ω is ambiguous so that there is no guarantee that an inside-money equilibrium without valued fiat money exists unless we make additional assumptions on the distribution function $G\left(\gamma\right)$.

If an equilibrium exists, then it satisfies $d^N<\frac{w(q^*)}{\beta}$ and $r^N<\beta^{-1}-1$. The condition $1-G\left(\frac{1}{\theta\beta}\right)<w\left(q^*\right)$ implies that, given the real interest rate β^{-1}, the per capita value of the loan portfolio is not sufficiently large to support the surplus-maximizing quantity q^* in the decentralized market. In this equilibrium, the liquidity constraint is binding and the real interest rate is below β^{-1}.

Depending on the parameters, the equilibrium real interest rate can imply efficient investment. This occurs when $\frac{\theta}{\beta}$ solves (16) so that $1+r^N=\frac{\theta}{\beta}$ holds in equilibrium. Note that $\frac{\theta}{\beta}$ is the interest rate that completely neutralizes the collateral restriction, implying that any type $\gamma\geq\frac{1}{\beta}$ undertakes his project in equilibrium, as in the social planner's solution. Another possibility is to have $1+r^N<\frac{\theta}{\beta}$ so that total investment is above the socially optimum. Finally, we can also have $1 + r^N > \frac{\theta}{\beta}$, which implies that total investment is below the socially optimum.

Case 2. $1 - G\left(\frac{1}{\theta\beta}\right) \geq w\left(q^*\right)$

Because $\Omega\left(\beta^{-1}\right) = 0$, it follows that $1+r^N = \beta^{-1}$ is an inside-money equilibrium without valued fiat money. Note that the ensuing allocation is the same as that of a narrow-banking economy when the government implements the Friedman rule ($i^m \to \beta^{-1}-1$). The condition $1 - G\left(\frac{1}{\theta\beta}\right) \geq w\left(q^*\right)$ implies that, given the real interest rate β^{-1}, the per capita value of the loan portfolio is sufficiently large to support the surplus-maximizing quantity q^* in the decentralized market. In this equilibrium, the liquidity constraint is slack, and the real interest rate equals the rate of time preference.

5.4. Equilibria with Valued Fiat Money

In this subsection, we characterize equilibria with the property that fiat money is positively valued. Tirole (1985) and Farhi and Tirole (2012) refer to these allocations as bubbly equilibria. It is clear that an equilibrium with valued fiat money exists if and only if $1 - G\left(\frac{1}{\theta\beta}\right) < w\left(q^*\right)$. In what follows, restrict attention to stationary equilibrium allocations. In this case, an inside-money equilibrium with valued fiat money can be defined as a triple (m, d, r) satisfying

$$1 + r = 1 + i^m, \tag{17}$$

$$d = \left(1 + i^m\right)\left[1 - G\left(\frac{1 + i^m}{\theta}\right) + m\right], \tag{18}$$

and

$$\frac{1}{\beta\left(1 + i^m\right)} = L\left(d\right), \tag{19}$$

with the policy rate $1 + i^m$ in the range $\left(0, \beta^{-1}\right)$.

Given that $d < \frac{w(q^*)}{\beta}$, it follows that

$$L\left(d\right) = \alpha \frac{u'\left(w^{-1}\left(\beta d\right)\right)}{w'\left(w^{-1}\left(\beta d\right)\right)} + 1 - \alpha$$

and

$$\beta d = w(q).$$

Then, condition (19) becomes (9), which implicitly defines $q(i^m)$. As we have seen, $q'(i^m) > 0$ so that decentralized-market trading activity is strictly increasing in the policy rate.

An inside-money equilibrium with valued fiat money exists if and only if

$$\Gamma(1+i^m) \equiv \frac{w(q(i^m))}{\beta} - (1+i^m)\left[1 - G\left(\frac{1+i^m}{\theta}\right)\right] > 0.$$

Note that $\Gamma(\beta^{-1}) = \frac{w(q^*)}{\beta} - \frac{1}{\beta}\left[1 - G\left(\frac{1}{\theta\beta}\right)\right] > 0$ and $\Gamma(1+r^N) = 0$. Then, there exist critical values $\bar{\imath}$ and $\bar{\imath}'$, with $\bar{\imath} \geq \bar{\imath}' > r^N$, such that an inside-money equilibrium exists provided $1+i^m \in [1+r^N, 1+\bar{\imath}'] \cup [1+\bar{\imath}, \beta^{-1}]$. As we will see, we will typically have $[1+r^N, 1+\bar{\imath}'] \cup [1+\bar{\imath}, \beta^{-1}] = [1+r^N, \beta^{-1}]$.

The no-arbitrage condition (17) implies that the policy rate determines the real interest rate in the credit market, given that inside money is created in equilibrium. Consequently, a lower policy rate reduces the cost of borrowing for entrepreneurs, which results in a larger investment amount.

5.5. Welfare

Suppose initially that $1 - G\left(\frac{1}{\theta\beta}\right) < w(q^*)$. Then, monetary policy can be effective if fiat money is positively valued in equilibrium. To investigate the welfare consequences of monetary policy in the inside-money economy, it is helpful to start with the Friedman rule. Taking the limit $i^m \to \beta^{-1} - 1$ implies $q(i^m) \to q^*$ and $1 + r = \beta^{-1}$. The ensuing allocation is associated with efficient decentralized-market trading activity, but investment is below the socially optimal level. As in the narrow-banking economy, the Friedman rule results in inefficiently low investment.

Given that the policy rate influences the cost of borrowing for entrepreneurs in the inside-money economy, it is possible to choose the policy rate that implements the efficient level of investment. As we have seen, the no-arbitrage condition

$$1 + i^m = 1 + r$$

must hold in a stationary equilibrium with valued fiat money. To obtain efficient investment, we need to set

$$1 + i^m = \frac{\theta}{\beta} < \frac{1}{\beta}.$$

By selecting a policy rate lower than the rate of time preference, it is possible to induce the efficient level of investment. The government is able to lower the borrowing cost to $\theta\beta^{-1}$, which perfectly neutralizes the adverse effects of the collateral restriction on credit creation. Despite the possibility of inducing the efficient level of investment, the quantity traded in the decentralized market is suboptimal, given that the opportunity cost of holding liquid claims for transaction purposes becomes strictly positive.

The previous analysis shows that the only way to reduce the cost of borrowing for entrepreneurs to achieve the efficient level of investment is by setting the policy rate below the Friedman rule, which necessarily results in suboptimal decentralized-market trading activity. In other words, a policy rate below the Friedman rule induces an allocation with the property that borrowers (entrepreneurs) are better off and asset holders (buyers) are worse off. The existence of this policy trade-off explains why different groups in society can favor antagonistic policies. Basically, the borrowers want the government to select a low policy rate, whereas the asset holders (i.e., savers) want the government to select a high policy rate.

Suppose now that $1 - G\left(\frac{1}{\theta\beta}\right) \geq w(q^*)$. In this case, we have shown that the unique inside-money equilibrium allocation is the same as that of the narrow-banking economy at the Friedman rule, which eliminates the opportunity cost of holding liquid assets but fails to induce the socially optimal level of investment.

5.6. Example

To better illustrate the properties of the inside-money economy, it is helpful to consider an example with the functional forms $u(q) = 2\sqrt{q}$ and $w(q) = q$. In this case, (11) and (12) imply

$$m_t = \beta(1 + r_t) - 1 + G\left(\frac{1 + r_t}{\theta}\right) \equiv H(1 + r_t).$$

Note that $H(0) = -1$, $H\left(\beta^{-1}\right) = G\left(\frac{1}{\theta\beta}\right)$, and $H'(1+r) > 0$. Then, we can use (14) to obtain the dynamic system:

$$H\left(1+r_{t+1}\right) = \left(\frac{1}{1+i^m}\right)\left(1+r_t\right) H\left(1+r_t\right). \tag{20}$$

Start with the inside-money equilibrium without valued fiat money. In this case, we have

$$H\left(1+r^N\right) = 0 \Leftrightarrow \beta\left(1+r^N\right) - 1 + G\left(\frac{1+r^N}{\theta}\right) = 0.$$

Note that

$$1+r^N < \frac{\theta}{\beta} \Leftrightarrow G\left(\frac{1}{\beta}\right) > 1-\theta.$$

If $G\left(\beta^{-1}\right) > 1-\theta$, then the equilibrium without valued fiat money implies overinvestment. Because the portfolio of private loans is the sole asset class serving as collateral for the creation of liquid claims, the equilibrium real interest rate ends up being excessively low from a social perspective. As a result, entrepreneurs with inefficient projects are able to invest in spite of the collateral constraint, and buyers inefficiently economize on liquid assets due to the opportunity cost of holding them in a portfolio for transaction purposes.

Note that (20) implicitly defines a strictly increasing mapping $1+r_{t+1} = f\left(1+r_t\right)$. As we have seen, $1+r^N$ is a fixed point. There exists another fixed point, given by $1+r_t = 1+i^m$, provided $i^m > r^N$. This stationary equilibrium corresponds to the allocation with positively valued fiat money.

To characterize nonstationary equilibria, it is helpful to consider the uniform distribution on $[0, \bar{\gamma}]$. Then, we have

$$H\left(1+r_t\right) = \left(\beta + \frac{1}{\theta\bar{\gamma}}\right)\left(1+r_t\right) - 1$$

so that the pure inside-money interest rate is given by

$$1+r^N = \frac{\theta\bar{\gamma}}{1+\beta\theta\bar{\gamma}}.$$

The dynamic system (20) becomes

$$\left(\beta + \frac{1}{\theta\bar{\gamma}}\right)\left(1+r_{t+1}\right) - 1 = \left(\frac{1}{1+i^m}\right)\left(\beta + \frac{1}{\theta\bar{\gamma}}\right)\left(1+r_t\right)^2 - \left(\frac{1}{1+i^m}\right)\left(1+r_t\right)$$

so that

$$1 + r_{t+1} = \left(\frac{1}{1+i^m}\right)(1+r_t)^2 - \left(\frac{\theta\bar{\gamma}}{1+\beta\theta\bar{\gamma}}\right)\left[\left(\frac{1}{1+i^m}\right)(1+r_t) - 1\right] \equiv f(1+r_t).$$

Note that

$$f'(1+r_t) = \left(\frac{1}{1+i^m}\right)\left[2(1+r_t) - \frac{\theta\bar{\gamma}}{1+\beta\theta\bar{\gamma}}\right]$$

and

$$f(0) = \frac{\theta\bar{\gamma}}{1+\beta\theta\bar{\gamma}} = 1 + r^N.$$

As we have seen, there exist two fixed points: $1 + r^N$ and $1 + i^m$. Given these stationary solutions, it is possible to construct nonstationary equilibria. In particular, starting from any point $1 + r_0$ in the open interval $(1 + r^N, 1 + i^m)$, the dynamic system (20) generates an equilibrium interest rate sequence that converges monotonically to $1 + r^N$ so that the economy converges to the pure inside-money regime. Figure 1 depicts the dynamic system with $\beta = .96$, $\bar{\gamma} = 1.5$, $\theta = .9$, and $i^m = 0.01$.

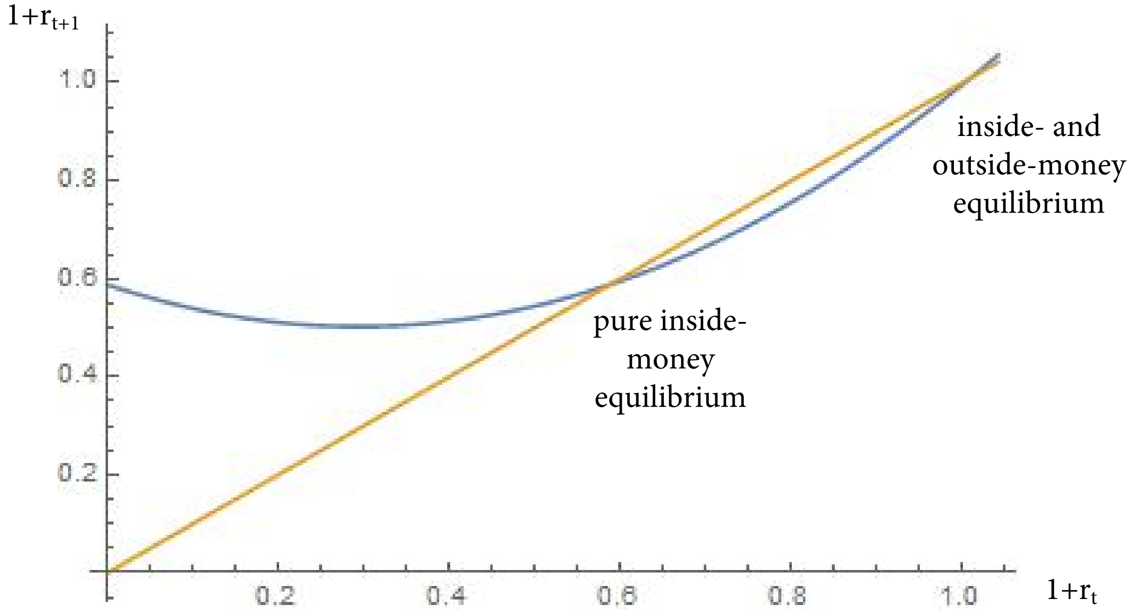

Figure 1

5.7. Money-Growth Rule

So far, we have assumed that the government implements monetary policy by setting the nominal interest rate on money balances, keeping the money supply constant by levying a lump-sum tax to withdraw the new money injected through interest payments. An alternative way to implement policy is by following a money-growth rule of the form:

$$\bar{M}_t = (1+\omega)\,\bar{M}_{t-1},$$

where $\bar{M}_t \in \mathbb{R}_+$ denotes the per-capita money supply in period t and $\omega \geq \beta - 1$ denotes the money growth rate. In this case, we can derive the law of motion

$$m_t = (1+\omega)\,\rho_{t-1} m_{t-1},$$

which describes the evolution of real balances along the equilibrium trajectory. Then, an inside-money equilibrium can be defined as a sequence $\{\rho_t, m_t, d_t, r_t\}_{t=0}^{\infty}$ satisfying

$$\rho_t = 1 + r_t,$$

$$\frac{1}{\beta\,(1+r_t)} = L\,(d_t)\,,$$

$$d_t = (1+r_t)\left[1 - G\left(\frac{1+r_t}{\theta}\right)\right] + (1+r_t)\,m_t,$$

$$m_{t+1} = (1+\omega)\,(1+r_t)\,m_t,$$

together with the boundary condition $1 + r_t \leq \beta^{-1}$.

In particular, a stationary equilibrium with valued fiat money can be defined as a triple (m, d, r) satisfying

$$1 + r = \frac{1}{1+\omega},$$

$$d = \frac{1}{(1+\omega)}\left[1 - G\left(\frac{1}{\theta\,(1+\omega)}\right)\right] + \frac{m}{1+\omega},$$

and

$$\frac{1+\omega}{\beta} = L\,(d)\,.$$

The no-arbitrage condition implies that a higher money growth rate is associated with a lower steady-state real interest rate. Because the inflation rate equals the money growth rate in a stationary equilibrium, we find that a higher steady-state inflation rate reduces the cost of borrowing for entrepreneurs but increases the opportunity cost of holding liquid assets, lowering the quantity traded in the decentralized market. The alternative monetary policy rule described in this section implies that the group of borrowers can collectively benefit from a higher inflation rate, but asset holders end up getting a smaller surplus in the decentralized market due to the higher opportunity cost of holding liquid assets associated with a deviation from the Friedman rule.

As previously mentioned, adherence to the gold standard in the last quarter of the 19th century resulted in a secular deflation in Western Europe and in the United States. Several authors in the literature have argued that borrowers (e.g., farmers) suffered severe economic losses because their ex-post debt burdens were larger than initially expected. Our framework predicts that the group of borrowers will end up with a smaller share of the surplus in a deflationary environment, but the mechanism at work here is different. In our analysis, a secular deflation tightens the collateral constraint by raising the ex-ante cost of borrowing for entrepreneurs, leading to equilibrium credit rationing. In particular, a rate of deflation associated with a real interest rate greater than the critical value $\theta\beta^{-1}$ implies an allocation with the property that a positive measure of borrowers will have socially productive projects but will be unable to undertake these investments because of the collateral constraint. In addition, the entrepreneurs who can satisfy the collateral constraint will end up getting a smaller surplus because of the higher real interest rate.

6. INTEREST ON INSIDE MONEY

As we have seen, the inside-money economy fails to implement an efficient allocation when the unique policy instrument is the interest rate on money balances. The government faces a trade-off between easing borrowing constraints for entrepreneurs and promoting efficient exchange in the decentralized market. If the government chooses a policy rate lower than that consistent with the Friedman rule, it can reduce the cost of borrowing for entrepreneurs and raise the level of investment in the economy. However, the buyer's liquidity constraint will bind in the decentralized market, resulting in suboptimal trading activity.

Our goal in this section is to consider a broader set of policy instruments that can influence aggregate liquidity creation. In particular, we start by supplementing the previously described monetary policy with the option to pay interest on privately created assets. As we will see, the interest payments on inside money will create a wedge between the cost of borrowing for entrepreneurs and the rate of return for asset holders, breaking the previously described policy trade-off.

Suppose that the government sets up a facility to make nominal interest payments on privately issued loans. Let $i^l \in \mathbb{R}_+$ denote the *nominal* interest rate paid on loans presented at the facility for the collection of interest payments. In reality, one would expect that interest payments on privately issued loans would be made to financial intermediaries who would effi ciently hold the loan portfolio. In our framework, there is no essential role for financial intermediation, so the agents are indifferent between holding assets directly and depositing them with an intermediary. As a result, any agent holding privately created assets can go directly to the facility to receive interest payments. It is possible to modify our framework to introduce a welfare-improving role for fi nancial intermediation. In this case, the interest payments would be made to intermediaries who would hold the portfolio of assets on behalf of the agents. This would be a more realistic way to make interest payments on inside money but the required changes are beyond the scope of this paper.

In the previously described regime, the buyer's portfolio problem can be described as

$$\max_{(m_t,l_t)\in\mathbb{R}^2_+} \left\{-m_t - l_t + \alpha\left[u\left(q\left(d_t\right)\right) - \beta z\left(d_t\right)\right] + \beta d_t\right\}$$

subject to

$$d_t = \left(1 + r_t + i^l \rho_t\right) l_t + \left(1 + i^m\right) \rho_t m_t.$$

An interior solution is characterized by the first-order conditions

$$1 + r_t + i^l \rho_t = \left(1 + i^m\right) \rho_t, \tag{21}$$

$$\frac{1}{\beta\left(1+r_t+i^l\rho_t\right)}=L\left(d_t\right), \tag{22}$$

and

$$d_t=\left(1+r_t+i^l\rho_t\right)\left[1-G\left(\frac{1+r_t}{\theta}\right)+m_t\right]. \tag{23}$$

In addition, we must have

$$m_{t+1}=\rho_t m_t \tag{24}$$

at all dates $t\geq 0$. Given these changes in the equilibrium conditions, it is now possible to provide a formal definition of equilibrium when the government pays interest on inside money.

Definition 3 *An inside-money equilibrium with valued fiat money can be defined as a sequence* $\{m_t,d_t,r_t,\rho_t\}_{t=0}^{\infty}$ *satisfying (21)-(24) and the boundary conditions* $\left(1+i^m\right)\rho_t\leq\beta^{-1}$ *and* $1+r_t+i^l\rho_t\leq\beta^{-1}$ *at all dates* $t\geq 0$.

In what follows, we want to show that it is possible to choose a policy combination $\left(i^l,i^m\right)$ that implements an efficient allocation when $1-G\left(\frac{1}{\beta}\right)<w\left(q^*\right)$. In particular, we claim that the policy choice $\left(i^m,i^l\right)=\left(\frac{1-\beta}{\beta},\frac{1-\theta}{\beta}\right)$ is consistent with an efficient allocation. To verify this claim, note that a stationary equilibrium allocation associated with any feasible policy choice $\left(i^m,i^l\right)$ must satisfy

$$1+r+i^l=1+i^m$$

and

$$\frac{1}{\beta\left(1+i^m\right)}=L\left(\left(1+i^m\right)\left[1-G\left(\frac{1+r}{\theta}\right)+m\right]\right),$$

with $m>0$ and

$$1+i^m\leq\beta^{-1}.$$

By setting $\left(i^m,i^l\right)=\left(\frac{1-\beta}{\beta},\frac{1-\theta}{\beta}\right)$, we get

$$1+r=\frac{\theta}{\beta}$$

and

$$m=w\left(q^*\right)-1+G\left(\frac{1}{\beta}\right)>0.$$

The surplus-maximizing quantity is traded in the decentralized market, and the real interest rate is precisely that which perfectly neutralizes the collateral constraint so that all entrepreneurs with type $\gamma \geq \frac{1}{\beta}$ undertake their project in equilibrium. Thus, it is possible to implement an equilibrium with the property that the real return on liquid assets is $\frac{1}{\beta}$ and the cost of borrowing for entrepreneurs is $\frac{\theta}{\beta} < \frac{1}{\beta}$, achieving an efficient allocation. It is helpful to summarize the previously derived result in the following proposition.

Proposition 4 *The policy combination* $\left(i^m, i^l\right) = \left(\frac{1-\beta}{\beta}, \frac{1-\theta}{\beta}\right)$ *is consistent with a socially efficient inside-money equilibrium provided* $1 - G\left(\frac{1}{\beta}\right) < w\left(q^*\right)$*.*

As we have seen, it is possible to implement an efficient allocation when the government has the ability to pay interest on inside assets. The basic idea behind the previously described policy choice is to drive a wedge between the cost of borrowing for entrepreneurs and the rate of return for asset holders. By paying interest on inside assets, the government can relax the collateral constraint and at the same time eliminate the opportunity cost of holding liquid assets for transaction purposes.

7. CAPITAL REQUIREMENTS

The previous section described an optimal liquidity policy under the assumption $1 - G\left(\frac{1}{\beta}\right) < w\left(q^*\right)$. In this section, we show that an efficient allocation can also be implemented when $1 - G\left(\frac{1}{\beta}\right) \geq w\left(q^*\right)$, provided the government supplements the previously described optimal policy with capital requirements on inside money creation.

To facilitate the interpretation of our results, suppose that a financial intermediary can be formed in the centralized market in every period to issue claims backed by a portfolio of assets consisting of fiat money and privately issued loans. As previously mentioned, the introduction of financial intermediaries in our framework is innocuous, given that the agents will be indifferent between holding assets directly and depositing them with intermediaries. Because of free entry into financial intermediation, an intermediary will offer a deposit contract that maximizes the utility of depositors. Suppose that any intermediary is required to hold equity equal to $1 - \delta$ times the real value of the loan portfolio. As a result, only a fraction $\delta \in [0,1]$ of the loan portfolio can be used as collateral for securing claims that can circulate as a medium of exchange. Let $d_t \in \mathbb{R}_+$ denote the real value of liquid claims and let $e_t \in \mathbb{R}_+$ denote equity. The real value of the intermediary's assets per depositor is given by

$$\left(1 + r_t + i^l \rho_t\right) l_t + \left(1 + i^m\right) \rho_t m_t.$$

Then, the intermediary's budget constraint is

$$e_t + d_t = \left(1 + r_t + i^l \rho_t\right) l_t + \left(1 + i^m\right) \rho_t m_t. \tag{25}$$

The capital requirement imposes the restriction

$$e_t \geq (1 - \delta) \left(1 + r_t + i^l \rho_t\right) l_t$$

so that (25) implies the following restriction on the real value of liquid claims:

$$d_t \leq \delta \left(1 + r_t + i^l \rho_t\right) l_t + \left(1 + i^m\right) \rho_t m_t.$$

It is now straightforward to describe the intermediary's portfolio problem in the centralized market. In particular, the intermediary solves the following problem:

$$\max_{(m_t, l_t) \in \mathbb{R}_+^2} \left\{ -m_t - l_t + \alpha \left[u\left(q\left(d_t\right)\right) - \beta z\left(d_t\right) \right] + \beta \left[\left(1 + r_t + i^l \rho_t\right) l_t + \left(1 + i^m\right) \rho_t m_t \right] \right\}$$

subject to

$$d_t = \delta \left(1 + r_t + i^l \rho_t\right) l_t + \left(1 + i^m\right) \rho_t m_t.$$

If $\left(1 + i^m\right) \rho_t < \beta^{-1}$ and $1 + r_t + i^l \rho_t < \beta^{-1}$, then we obtain an interior solution characterized by the first-order conditions

$$\frac{1}{\beta\left(1 + i^m\right) \rho_t} - 1 = \frac{1}{\delta} \left[\frac{1}{\beta\left(1 + r_t + i^l \rho_t\right)} - 1 \right], \tag{26}$$

$$\frac{1}{\beta\left(1 + r_t + i^l \rho_t\right)} = L_\delta\left(d_t\right), \tag{27}$$

and

$$d_t = \delta \left(1 + r_t + i^l \rho_t\right) \left[1 - G\left(\frac{1 + r_t}{\theta} \right) \right] + \left(1 + i^m\right) \rho_t m_t, \tag{28}$$

where the function $L_\delta : \mathbb{R}_+ \to \mathbb{R}_+$ is given by

$$L_\delta(d) = \begin{cases} \alpha\delta \frac{u'\left(w^{-1}(\beta d)\right)}{w'\left(w^{-1}(\beta d)\right)} + 1 - \alpha\delta \text{ if } \beta d \leq w(q^*) \\ 1 \text{ if } \beta d > w(q^*). \end{cases}$$

Given these changes in the equilibrium conditions, it is now possible to provide a formal definition of equilibrium.

Definition 5 *An inside-money equilibrium with valued fiat money can be defined as a sequence* $\{m_t, d_t, r_t, \rho_t\}_{t=0}^{\infty}$ *satisfying (24), (26)-(28), and the boundary conditions* $(1+i^m)\rho_t \leq \beta^{-1}$ *and* $1 + r_t + i^l \rho_t \leq \beta^{-1}$ *at all dates* $t \geq 0$.

Suppose $1 - G\left(\frac{1}{\beta}\right) \geq w(q^*)$. We claim that there exists a capital requirement $\delta^* < 1$ such that the policy combination $\left(i^l, i^m, \delta\right) = \left(\frac{1-\theta}{\beta}, \frac{1-\beta}{\beta}, \delta^*\right)$ is consistent with an efficient allocation. To verify this claim, start with the combination $\left(i^l, i^m\right) = \left(\frac{1-\theta}{\beta}, \frac{1-\beta}{\beta}\right)$ and select an arbitrary capital requirement $\delta \in [0,1]$. Then, there is an associated stationary allocation with the real value of liquid claims given by

$$d = \delta \frac{1}{\beta}\left[1 - G\left(\frac{1}{\beta}\right) + m\right].$$

If $\delta = 1$, then

$$\frac{1}{\beta}\left[1 - G\left(\frac{1}{\beta}\right)\right] \geq \frac{w(q^*)}{\beta},$$

which necessarily implies $m = 0$.

Consider a fixed value $m > 0$. Because $1 - G\left(\frac{1}{\beta}\right) \geq w(q^*)$, there exists a critical value $\delta^*(m) < 1$ such that

$$\delta^*(m)\left[1 - G\left(\frac{1}{\beta}\right) + m\right] = w(q^*).$$

It is clear that $\delta^*(m)$ is strictly decreasing in m. Then, we have just shown that it is possible to construct a stationary allocation with $q = q^*$, $m > 0$, and $1 + r = \frac{\theta}{\beta}$ provided $\left(i^l, i^m\right) = \left(\frac{1-\theta}{\beta}, \frac{1-\beta}{\beta}\right)$ and

$$\delta^*(m) = \frac{w(q^*)}{1 - G\left(\beta^{-1}\right) + m}.$$

Hence, an efficient allocation can be consistent with the equilibrium conditions when the government pays interest on inside money and imposes capital requirements on private portfolios. We summarize this result in the following proposition.

Proposition 6 *Given any $m > 0$, the policy combination*

$$\left(i^l, i^m, \delta\right) = \left(\frac{1-\theta}{\beta}, \frac{1-\beta}{\beta}, \frac{w\left(q^*\right)}{1 - G\left(\beta^{-1}\right) + m}\right)$$

is consistent with a socially efficient inside-money equilibrium when $1 - G\left(\frac{1}{\beta}\right) \geq w\left(q^*\right)$.

The analysis developed in this section shows that capital requirements are essential for efficiency when the supply of productive projects is abundant because a sufficiently large fraction of the privately created collateral has to be "neutralized" via capital requirements to ensure the coexistence of inside and outside money in equilibrium, so that government liquidity policy can be effective.

8. CONCLUSION

We have constructed a model with the property that both money and private credit instruments can potentially be used as media of exchange to overcome trading frictions in decentralized markets. In our framework, entrepreneurs have access to productive projects but face credit constraints due to limited pledgeability of their returns. When private credit instruments serve as media of exchange, monetary policy can lower the cost of borrowing for entrepreneurs, which relaxes the collateral constraint and boosts investment in the economy. But the government faces a trade-off between easing borrowing constraints for entrepreneurs and promoting efficient exchange in decentralized markets.

This trade-off implies that the welfare of borrowers and that of lenders move in opposite directions. Although these distributional effects can occur in an inside-money economy, our results have shown that they are not an inevitable feature of monetary economies with private credit instruments. As we have seen, it is possible to consider a broader set of aggregate liquidity management instruments to construct a monetary arrangement that is not necessarily characterized by the aforementioned distributional effects, allowing the government to implement an efficient allocation.

REFERENCES

[1] A. Berentsen, G. Camera, C. Waller. "Money, credit and banking," *Journal of Economic Theory* 135 (2007), pp. 171-195.

[2] B. Champ, B. Smith, S. Williamson. "Currency elasticity and banking panics: Theory and evidence," *Canadian Journal of Economics* 29 (1996), pp. 828-864.

[3] E. Farhi, J. Tirole. "Bubbly liquidity," *Review of Economic Studies* 79 (2012), pp. 678-706.

[4] M. Friedman, A. Schwartz. *A Monetary History of the United States, 1867-1960*, Princeton University Press, 1963.

[5] A. Geromichalos, J. Licari, J. Suarez-Lledo. "Asset prices and monetary policy," *Review of Economic Dynamics* 10 (2007), pp. 761-79.

[6] B. Holmstrom, J. Tirole. *Inside and Outside Liquidity*. Cambridge, MA: MIT Press, 2011.

[7] N. Kiyotaki, J. Moore. "Liquidity and asset prices" *International Economic Review* 46 (2005), pp. 317-349.

[8] R. Lagos, R. Wright."A unified framework for monetary theory and policy analysis," *Journal of Political Economy* 113 (2005), pp. 463-84.

[9] G. Rocheteau, R. Wright. "Liquidity and asset-market dynamics," *Journal of Monetary Economics* 60 (2013), pp. 275-294.

[10] H. Rockoff. "The 'Wizard of Oz' as a monetary allegory," *Journal of Political Economy* 98 (1990), pp. 739-760.

[11] S. Shi. "A divisible search model of fiat money,"*Econometrica*, 65 (1997), pp. 75-102.

[12] J. Tirole. "Asset bubbles and overlapping generations," *Econometrica* 53 (1985), pp. 1071-1100.

[13] S. Williamson. "Liquidity, monetary policy, and the financial crisis: A New Monetarist approach," *American Economic Review* 102 (2012), pp. 2570-2605.