WORKING PAPER NO. 13-9/R
WORKER FLOWS AND JOB FLOWS: A QUANTITATIVE INVESTIGATION

Shigeru Fujita
Federal Reserve Bank of Philadelphia

Makoto Nakajima
Federal Reserve Bank of Philadelphia

May 2014
Worker Flows and Job Flows: A Quantitative Investigation∗

Shigeru Fujita† and Makoto Nakajima‡

This Version: May 2014

Abstract

This paper explores the sources of differences in the cyclical ity of worker flows and job flows, using a multiple-worker firm matching model with on-the-job search, where heterogeneous firms operate decreasing-returns-to-scale production technology. We show that the calibrated model successfully replicates (i) countercyclical worker flows between employment and unemployment, (ii) procyclical job-to-job transitions, and (iii) opposite movements of job creation and destruction rates over the business cycle. Job creation and destruction rates measured by net employment changes behave differently from total hiring and separation rates, because separations occur at firms with positive net employment changes and, similarly, firms that are shrinking on net may hire workers. The introduction of on-the-job search into the model is essential to simultaneously match the cyclical ity of worker flows and job flows.

JEL Classification: E24, E32, J63, J64

Keywords: job flows, worker flows, multiple-worker firm, and search and matching.

∗For helpful comments, we thank seminar and conference participants at Bank of Japan, CEMFI, Cleveland Fed, Deutsche Bundesbank, European Central Bank, 2009 Far East and South Asia Econometric Society Meeting, Kansas City Fed/ NYU-Stern Joint Workshop for Economists Working in Parallel, New York/Philadelphia Workshop on Quantitative Macroeconomics, 2009 North American Econometric Society Meeting, Simon Fraser University, 2009 Society for Computational Economics Meeting, University of British Columbia, University of Pennsylvania Wharton School, and ZEW Conference on Recent Developments in Macroeconomics. The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System. This paper is available free of charge at www.philadelphiafed.org/research-and-data/publications/working-papers/.

†Research Department, Federal Reserve Bank of Philadelphia. Ten Independence Mall, Philadelphia, PA 19106. E-mail: shigeru.fujita@phil.frb.org.

‡Research Department, Federal Reserve Bank of Philadelphia. Ten Independence Mall, Philadelphia, PA 19106. E-mail: makoto.nakajima@phil.frb.org.
1 Introduction

Worker flows and job flows behave differently over the business cycle. It is well known that worker flows between employment and unemployment are countercyclical. Separations into unemployment increase during recessions because the transition rate into unemployment increases; hires from unemployment also rise because the increases in the separation flow raise the unemployment pool rapidly, thus increasing hires as well.\(^1\) The behavior of job flows is different. Job destruction is countercyclical, whereas job creation is procyclical.

In this paper, we investigate the sources of the differences in the cyclicality of worker flows and job flows, using a search/matching model, where each firm operates a decreasing returns-to-scale production technology and is subject to aggregate as well as idiosyncratic productivity shocks. Each firm hires multiple workers, in contrast to the canonical matching model of Mortensen and Pissarides (1994), where a worker-firm match is taken to be the unit of analysis. One apparent reason that job flows behave differently from worker flows (between employment and unemployment) is that firms can hire workers not only from the unemployment pool but also from other firms. Similarly, workers can separate to other employers as well as into unemployment.\(^2\) Mortensen (1994) addresses this issue in a single-worker matching model by adding on-the-job search (OJS) into the Mortensen and Pissarides (1994) model. We introduce OJS and associated job-to-job transitions into the multiple-worker firm environment.

There are two reasons why we need the multiple-worker firm environment for our research interest. The first is that job flows are defined by establishment-level net employment changes within a certain period (usually a quarter). To be consistent with this measurement, we need a model with a meaningful notion of the establishment, which can hire many workers. Moreover, net employment changes over a quarterly period can be quite different from gross separations and hires that occur throughout the period. We address the time aggregation issue by solving the model at higher frequency and analyzing the cyclicality of job flow series constructed in the same way as in the actual data. The second reason is more substantive. That is, worker separations occur at establishments that are “creating” jobs (positive net employment changes) and, similarly hires occur at establishments that are “destroying” jobs (negative net employment changes). In our model, when a firm aims to achieve the optimal employment level by laying off existing workers or hiring new workers, it explicitly takes into account the worker attritions due to job-to-job transitions. Empirically speaking, net employment changes are indeed different from underlying hires and separations.\(^3\) In a single-worker environment, however, one cannot distinguish between worker flows and job flows even after incorporating job-to-job transitions.

Our model is an extension of the work by Cooper et al. (2007) and Elsby and Michaels (2013), who also consider the multiple-worker firm environment with random labor matching.

\(^1\)The job finding rate from unemployment drops significantly during recessions, lowering hires, but this effect is dominated by the increase in the inflow.

\(^2\)Flows between employment and out-of-the-labor force are another reason, but we do not consider these flows in the present paper.

\(^3\)See, for example, Burgress et al. (2000) and Davis et al. (2012).
Our extension is to incorporate OJS in a similar environment. In our model, firms are subject to idiosyncratic and aggregate productivity shocks. Hiring workers is subject to search frictions, and actively shedding workers (layoffs) requires the firm to pay a firing cost. The introduction of OJS considerably complicates the model. In the presence of OJS, we need to keep track of the wage distribution of all workers and the wage offer distribution of the vacancy-posting firms. When aggregate uncertainty is present, these distributions are time varying. The firm needs to know information about the wage distribution, because it tells the firm the acceptance probability of its wage offer (if the firm is hiring). The firm also needs to know (regardless of whether it is hiring or not) the wage offer distribution, because it tells the firm how many workers will leave through job-to-job transitions for a given level of wage that the firm is currently paying to its employees. We solve for the dynamic stochastic equilibrium of this challenging environment by applying the standard tool developed for heterogeneous agent models with uninsurable income risk (see, e.g., Krusell and Smith (1998)).

The calibrated model successfully matches key cyclical features of worker flows and worker transition rates. First, it matches the procyclicality of the job finding rate of unemployed workers (unemployment-to-employment transition rate, UE transition rate) and the countercyclicality of the separation rate into unemployment (employment-to-unemployment transition rate, EU transition rate). Second, the model matches countercyclical worker flows between employment and unemployment. Third, the model also generates procyclical job-to-job transitions.4

Next, we also show that our model replicates the procyclical job creation rate and countercyclical job destruction rate, with these two variables being strongly negatively correlated with each other.5 We use our model as a laboratory to explore the underlying linkages between worker flows and job flows. In particular, we investigate how similarly or differently job creation and destruction rates behave relative to total hiring and separation rates.6 Note that the job flow data are collected on a quarterly basis, while hiring and separation rates are typically measured at monthly frequency. The difference in the data collection frequency itself can be a source of the differences in their cyclical behavior. Within our model economy, we separate out the effect of the data collection frequency and the underlying economic forces.

We find that the difference in the cyclicality between the total separation rate and the quarterly job destruction rate is particularly large: The total separation rate is procyclical and its correlation with aggregate output is higher than 0.4. The reason for this procyclicality is that, even though the EU transition rate is strongly countercyclical, the job-to-job transition rate is strongly procyclical, with the average volume of the latter being larger.

---

4See, for example, Elsby et al. (2009), Fujita and Ramey (2009), and Shimer (2012) for empirical facts on the transition rates between employment and unemployment and Fallick and Fleischman (2004) and Nagypál (2008) for facts on job-to-job transitions.

5Throughout the paper, we use the term “job flows” to represent rates of job creation and destruction. The job creation (destruction) rate is defined as a sum of net employment changes at the establishments that are expanding (shrinking) over a quarterly period, normalized by the total employment stock.

6The total separation rate is simply the sum of the EU separation rate and the job-to-job transition rate. The hiring rate is similarly defined as the total number of hires divided by aggregate employment.
The job destruction rate is countercyclical and its correlation with output is $-0.38$. The economic reason for this difference is that job-to-job transitions (as a part of separations) occur not just at firms that are “destroying” jobs but also at the firms that are “creating” jobs. However, by construction, the job destruction rate considers only the firms that reduced employment on net. Those shrinking establishments are more likely to be the ones that need to appeal to layoffs (EU separations). Accordingly, the contribution of the EU separation rate to the cyclical movement of the job destruction rate is larger, which pushes its correlation with output into a negative direction. When the effect of time aggregation present in the quarterly measure of the destruction rate is taken into account, the negative correlation reaches the level consistent with the observed data. The discrepancy between the job creation rate and the hiring rate is less stark but still significant. Both job creation and hiring rates are procyclical, but the procyclicality of the hiring rate is stronger. The job creation rate tends to increase in booms because there are more firms with positive net employment changes. However, during booms, the number of hires required to achieve the same level of employment growth also increases, because firms lose more workers through quits (job-to-job transitions) to higher-paying firms. We also find that time aggregation reduces the procyclicality of the job creation rate, making the difference in the cyclicality of the job creation rate and the hiring rate significant.

To illustrate the importance of job-to-job transitions in our model, we also solve the model without job-to-job transitions and compare its quantitative properties with those in our model. This exercise highlights the important property of our model that makes vacancies more volatile and persistent relative to the model without job-to-job transitions. As in standard labor search/matching models, higher aggregate productivity by itself increases vacancies in our model. In the absence of job-to-job transitions, more vacancy postings simply mean a higher job finding rate of those who are in the unemployment pool. In our model, on the other hand, more vacancies also imply an increased pace of job-to-job transitions, through which workers move up to higher-paying firms. When a firm that intends to expand employment loses its workers through job-to-job transitions, it needs to post more vacancies to make up for the number of workers who left the firm through job-to-job transitions. When this firm hires workers from lower-paying firms, those lower-paying firms (as far as these firms are also hiring) also need to refill those positions by hiring more workers. This “vacancy chain” introduced earlier by Akerlof et al. (1988) makes vacancies in our model more volatile and persistent than in the model without job-to-job transitions.

Let us now discuss where our paper stands in relation to the literature. As mentioned above, this paper is closely related to studies by Cooper et al. (2007) and Elsby and Michaels (2013), who also analyze and quantitatively evaluate a multiple-worker firm matching model. The key difference from these papers is the presence of OJS in our model. There are several other papers that study the directed search environment with decreasing returns to scale (e.g., Kaas and Kircher (2013) and Schaal (2012)). Kaas and Kircher (2013) analyze the environment without OJS, and Schaal (2012) adds OJS to the model. However, Schaal (2012) focuses more on the recent Great Recession episode in the presence of the uncertainty shock. On the other hand, our paper follows more closely the traditional random matching
environment with continuous wage renegotiation and looks more generally at the model’s cyclical features with a particular attention to worker flows and job flows.\footnote{A recent work by Moscarini and Postel-Vinay (2013) analyzes and solves the random-matching, wage-posting model under the presence of the aggregate shock. But worker transitions into unemployment are exogenous, and their research interest is different from ours.}

In terms of the economic interest, our paper is related to the work by Mortensen (1994), who attempts to replicate worker flows and job flows simultaneously, as in this paper. He does so, however, in a single-worker firm matching model with OJS and thus faces several limitations we have discussed above. Veracierto (2009) provides a synthesis of the different strands of the literature (in particular, the Mortensen-Pissarides random-matching framework and the Lucas and Prescott (1974) island framework) and discusses the cyclical properties of worker transition rates and job flows. However, his model does not allow for OJS, and thus no job-to-job transitions exist in his model. Our analysis of the differences between total separation/hiring rates and job flows is an important part of our paper that is distinct from his paper.\footnote{Note, however, that his paper looks at broader statistics that we do not consider in our paper. For example, his model is a full-fledged RBC model with physical capital and risk aversion. He can therefore assess the broader macroeconomic implications of this model. See also Veracierto (2007), who studies normative aspects of a similar environment without aggregate uncertainty.}

This paper proceeds as follows. In the following section, we summarize the business-cycle features of worker flows and job flows by looking at standard business-cycle statistics. Section 3 presents our model. In Section 4, we briefly discuss the solution algorithms to solve for the steady-state equilibrium and the dynamic stochastic equilibrium. Details of the algorithms are presented in the Appendix. Section 5 discusses the calibration strategy. Section 6 discusses this paper’s main quantitative results in detail. In Section 7, we conduct some experiments that shed more light on the importance of job-to-job transitions in our model. In particular, we compare the quantitative properties of the model with and without job-to-job transitions. Section 8 presents the results under alternative calibrations to show that our main results are robust with respect to several plausible alternative calibrations. The final section concludes the paper by discussing some micro-level counterfactual properties of our model and potentially useful extensions to overcome those problems.

2 Cyclicality of Worker Flows and Job Flows

This section reviews the cyclical properties of worker flows and job flows. While one can find the cyclical properties of worker flows and job flows in the literature, the two sets of data are usually discussed in isolation. We discuss them together and highlight the differences in their cyclicality. Let us first review the definitions of the series.

2.1 Measurement

Job flows. The job flow series are measured from the Business Employment Dynamics (BED) data, which are based on the administrative records of the Quarterly Census of Em-
employment and Wages (QCEW). The coverage of the QCEW is very broad, representing 98% of employment on nonfarm payrolls. The administrative records are linked across quarters to provide a longitudinal history for each establishment. The linkage process makes it possible to calculate net employment gains at opening and expanding establishments and net employment losses at closing and contracting establishments. The measures of job flows were originally developed by Davis et al. (1996): Job creation (destruction) is defined as the sum of net employment gains (losses) over all establishments that expand (contract) or start up (shut down) between the two sampling dates. Since we are interested in business-cycle fluctuations of the series, we use the series that trace net employment changes over a quarterly period. Normalizing creation and destruction by aggregate employment yields rates of job creation and destruction, respectively.\(^9\) As mentioned above, we use the term “job flows” to represent “rates” unless otherwise explicitly mentioned. The sample period of the job flow series starts at 1992Q3 and ends at 2011Q4.

**Worker flows and transition rates.** We use the Current Population Survey (CPS) to measure worker flows and worker transition rates. The CPS asks whether the worker is employed and, if non-employed, whether he or she is engaged in active job search activities (i.e., unemployed) during the preceding month. While the CPS is designed to provide a snapshot of the U.S. labor market for each month, one can use its longitudinal component to obtain measures of worker flows. We use the flow series constructed by the BLS.\(^{11}\) Worker flows between employment and unemployment come from comparison of the labor market status at each monthly survey. To be specific, transition rates between employment and unemployment are, respectively, measured by

\[
\frac{EU_t}{E_{t-1}} \quad \text{and} \quad \frac{UE_t}{U_{t-1}},
\]

where \(EU_t\) (\(UE_t\)) refers to the number of workers who switch their labor market status from “employed” (“unemployed”) to “unemployed” (“employed”) between month \(t - 1\) and \(t\). \(EU_t\) and \(UE_t\) represent separations into unemployment and hires from unemployment, respectively. The definitions in Equation (1) give the EU transition rate and UE transition rate, respectively. The sample period for the BLS data is January 1990 to December 2011.\(^{12}\)

We also consider job-to-job transitions. Measuring job-to-job transitions in the CPS became feasible after the CPS redesign in 1994. Specifically, the dependent coding, which asks the individual if he or she is currently employed by the same employer as in the previous month, made it possible to measure job-to-job transitions. Fallick and Fleischman (2004)

---

\(^9\)The BED series are available at www.bls.gov/bdm/.

\(^{10}\)More precisely, average employment between the beginning and the end of the quarter is used for normalization.

\(^{11}\)The data are available at www.bls.gov/cps/cps_flows.htm. Fujita and Ramey (2006) also construct worker flow series that are comparable to the BLS series. The cyclicity of the two data sets is very similar. See Fujita and Ramey (2006) for the data construction details and measurement issues in the CPS.

\(^{12}\)In the BLS data, the flow that occurs from \(t - 1\) and \(t\) is dated at \(t\). Due to that convention, the BLS flow data start at February 1990.
are the first to exploit this data structure for measuring job-to-job transitions in the CPS.\footnote{Moscarini and Thompson (2007) explore several measurement issues of CPS-based job-to-job transitions and correct some measurement issues that existed in Fallick and Fleischman (2004). While their adjustments alter the overall level of job-to-job transitions somewhat, the time-series behavior is not significantly affected. We thus use the readily available series by Fallick and Fleischman (2004).}

Denoting the worker flow corresponding to those who are employed at different employers between $t - 1$ and $t$ by $EE_t$, we can write the job-to-job transition rate as

$$\frac{E E_t}{E_{t-1}}.$$  \hfill (2)

The data are updated regularly and the sample period for our analysis is January 1994 to December 2011. All monthly worker flows and transition rates are converted into quarterly series by time averaging.\footnote{Throughout the paper, we consider only worker flows and transition rates measured by the CPS. An alternative data source for worker flows is the JOLTS (Job Openings and Labor Turnover Survey). However, the JOLTS worker flows are not linked to workers’ labor force status and thus cannot be coherently linked to unemployment dynamics. Moreover, the JOLTS data only start at December 2000 and thus cover a much shorter time period than the CPS data. In principle, JOLTS worker flows are equivalent to the sum of all transitions into or out of employment that we can measure in the CPS. Although our analysis omits the flows into and out of the out-of-the-labor force and focuses on the transitions between employment and unemployment, the cyclicality of the JOLTS-based hiring (separation) rate, which includes hires from (separations into) the out-of-the-labor force, is largely consistent with the CPS-based hiring (separation) rate obtained by adding only UE (EU) transitions and job-to-job transitions and normalizing it by the employment stock.}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{chart.png}
\caption{Job Creation and Destruction Rates}
\end{figure}

Notes: The data are taken from the BLS Business Employment Dynamics and cover the private business sector.
2.2 Business-Cycle Statistics

Unimportance of entry and exit. First, consider Figure 1, where we plot the time series of job flows. The figure shows not only the total rates of job creation and destruction but also their breakdowns into expansion, entry, contraction, and exit. The intention is to show the unimportance of the extensive margins for the business-cycle fluctuations of job flows. According to the data, roughly 75% of total job flows come from expansion or contraction of the existing establishments at a quarterly frequency. More important, cyclical fluctuations of job flows are mostly accounted for by expansion or contraction. For instance, the correlation between the total job creation (destruction) rate and the expansion (contraction) rate is higher than 0.95. It is important to recognize that these two facts do not imply the unimportance of entry and exit at a lower frequency. However, Figure 1 establishes our point that extensive margins are not important at the quarterly frequency. We thus abstract away from the extensive margin in our model on the basis of the quarterly evidence presented in Figure 1.

Cyclicality. Table 1 characterizes the cyclicality of worker flows and job flows using standard business-cycle statistics. The original series are logged and then detrended using the HP filter with smoothing parameter of 1,600. As mentioned above, original worker flows and transition rates are monthly series. We render them quarterly by simple averaging. The real GDP series is used as a cyclical indicator to gauge each variable’s volatility and cyclicality. We can summarize the characteristics of the labor market flows as follows:

- The EU (separation) transition rate is countercyclical, while UE (job finding) and EE (job-to-job) transition rates are procyclical.
- The UE transition rate is somewhat more volatile than the EU transition rate.\(^{16}\)
- The EU flow is somewhat more volatile than the other two flows (UE and job-to-job flows).
- The job destruction rate is countercyclical and the job creation rate is procyclical. But the correlations are weaker in general.
- The job destruction rate is somewhat more volatile than the job creation rate.
- Worker flows are more volatile than job flows.

\(^{15}\)The frequency of the measurement is important because, at a quarterly frequency, entrants become incumbents after a quarter, but the same entrants measured at annual frequency become incumbents only after one year. Thus, the share of job flows accounted for by entrants and exits becomes larger when measured at a lower frequency. They also become more important cyclicality-wise.

\(^{16}\)Shimer (2012) and Hall (2005) argue that the separation rate into unemployment is roughly constant over the business cycle. Fujita and Ramey (2006, 2009), Fujita (2011), Elsby et al. (2009), and Yashiv (2007) argue otherwise.
Table 1: Business Cycle Statistics for Worker Flows and Job Flows

<table>
<thead>
<tr>
<th></th>
<th>Standard Deviation</th>
<th>Relative Standard Deviation</th>
<th>Correlation with Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Worker flows</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E to U</td>
<td>0.065</td>
<td>5.496</td>
<td>−0.800</td>
</tr>
<tr>
<td>E to E</td>
<td>0.060</td>
<td>4.950</td>
<td>0.744</td>
</tr>
<tr>
<td>U to E</td>
<td>0.046</td>
<td>3.824</td>
<td>−0.687</td>
</tr>
<tr>
<td>Transition rates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EU transition rate</td>
<td>0.071</td>
<td>5.966</td>
<td>−0.840</td>
</tr>
<tr>
<td>EE transition rate</td>
<td>0.056</td>
<td>4.620</td>
<td>0.698</td>
</tr>
<tr>
<td>UE transition rate</td>
<td>0.080</td>
<td>6.731</td>
<td>0.860</td>
</tr>
<tr>
<td>Job flows</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Creation rate</td>
<td>0.036</td>
<td>3.060</td>
<td>0.447</td>
</tr>
<tr>
<td>Destruction rate</td>
<td>0.045</td>
<td>3.838</td>
<td>−0.450</td>
</tr>
<tr>
<td>Stocks</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>0.116</td>
<td>9.712</td>
<td>−0.889</td>
</tr>
<tr>
<td>Vacancies</td>
<td>0.121</td>
<td>10.153</td>
<td>0.862</td>
</tr>
</tbody>
</table>


Table 1 also shows volatilities of the unemployment rate and vacancies. As is well known in the literature, these two variables are quite volatile when compared with the volatility of labor productivity. The same is true with respect to output volatility. Lastly, a well-known fact about the cyclicality of unemployment and vacancies, i.e., the Beveridge curve, can also be observed from each variable’s correlation with output.

3 Model

Our model is an extension of those developed by Cooper et al. (2007), Elsby and Michaels (2013), and Mortensen (2010). Time is discrete. There are two types of agents: firms and workers. Both are infinitely lived and risk neutral. The total measure of firms is normalized to one. The total measure of workers is denoted by $L$. 
3.1 Timing

The timing of events is summarized in Figure 2. At the beginning of the period, a firm’s idiosyncratic states are characterized by \((x, n)\), where \(x\) represents idiosyncratic productivity and \(n\) represents the number of its workers. In addition, there is aggregate uncertainty in the economy in the form of a shock to aggregate productivity \(z\). As will be clear later, each firm’s decision is also influenced by the economy-wide joint distribution of \(x\) and \(n\) and is written as \(m(x, n)\). We summarize the aggregate states by \(s = \{z, m\}\). The stochastic processes for \(z\) and \(x\) are, respectively, denoted by \(G_z(z'|z)\) and \(G_x(x'|x)\). Since we formulate the model recursively, we drop time subscript from all variables and follow the convention that a primed variable denotes the variable at the beginning of the next period. Note, however, as indicated in Figure 2, the firm enters into the current period with the employment level \(n\) and produces with \(n'\) after labor turnover is completed in the current period. It then starts the next period with \(n'\).

After the realization of productivities, firms make the separation/hiring decision. The hiring decision is subject to a search friction, which is discussed below. Hires include those from other firms (job-to-job transitions) as well as those from unemployment. Similarly, separations include worker flows to other hiring firms and to unemployment. As described in Figure 2, worker turnover occurs within a period. For example, vacancies posted at the beginning of the period after the realizations of productivities can be filled in the same period before production. After all worker flows are completed, wage negotiations between the employer and employees take place and then the firm produces.

3.2 Technology and Wage Bargaining

The following decreasing-returns-to-scale production technology is available to all firms:

\[ y = zxxn'^\alpha. \]  

Again, \(n'\) represents the number of workers who engage in production at the firm, whose beginning-of-the-period employment level was \(n\). We consider the following bargaining envi-
ronment. First, we assume that the workers engage in the bargaining as a group (or a union). We further assume that the outside option for the union and the firm is to halt production for one period. The latter assumption is borrowed from Hall and Milgrom (2008). Although Hall and Milgrom apply their bargaining protocol to the single-worker firm environment with linear technology, where each individual worker and the firm bargain over the surplus, their main insight comes from the wage negotiations between the union and the firm and thus is naturally extended to our multiple-worker environment.\(^{17}\)

Under the Nash bargaining between the union and the firm, the surplus sharing rule can be written as

\[
\eta \left[ \alpha z n' \alpha - wn \right] = (1 - \eta) \left[ wn - bn \right],
\]

where \(\eta\) is the bargaining power of the union. The expression inside the square brackets on the left-hand side is the surplus for the firm, which equals the current-period total profits. Similarly, the expression on the right-hand side is the surplus for the union, which simply aggregates the individual-level current-period surplus. Solving the above equation for wage gives

\[
w(x, n', s) = \eta z n'^{\alpha-1} + (1 - \eta) b. \tag{4}
\]

Note that having this simple analytical expression for wages greatly simplifies our quantitative analysis, particularly in our environment with on-the-job search. Equation (4) simply states that wages are determined as the weighted average of the worker’s outside option \((b)\) and average product \(y/n'\).\(^{18}\)

### 3.3 Search and Matching

Due to the search friction, only a fraction of job openings are filled every period. There is a flow vacancy posting cost, as in the standard model. However, we assume that the marginal cost of posting a vacancy is increasing in the number of vacancies posted at the firm level to facilitate the calibration of our model (see the discussion in Section 5.1). We also introduce the firing cost that applies when the firm decides to shed workers beyond job-to-job transitions. The specific formulation of these features is described below.

The meeting technology takes the following Cobb-Douglas form:

\[
M = \mu S^{\psi} V^{1-\psi},
\]

\(^{17}\)To motivate their wage-setting mechanism, they specifically discuss the wage negotiations between General Motors and the United Auto Workers.

\(^{18}\)In the earlier version of our paper, we applied Stole and Zwiebel (1996a,b) bargaining to current-period output. However, the resulting rule that splits the marginal surplus between the firm and a worker in the firm does not guarantee \(w \geq b\) by itself. Stole and Zwiebel (1996b) in fact show that, in their static environment, the firm chooses employment such that \(w = b\). Although this result does not literally hold in our dynamic environment, we still find that the wage distribution is close to degenerate at the point \(w = b\), making it difficult to obtain the stable numerical solution. Our current formulation, on the other hand, guarantees that \(w\) is greater than \(b\). Consequently, the resulting wage distribution is smooth, which greatly helps us avoid running into technical issues when solving the model numerically.
where $S$ is the efficiency-weighted number of job seekers, $V$ is the aggregate number of job openings, and $\mu$ is a scaling parameter. We normalize search efficiency of each unemployed worker at 1 and assume that each on-the-job seeker searches for a job at a reduced efficiency of $\gamma \in [0, 1]$. This specification allows us to abstract away from the search decision of the employed workers, while giving us the flexibility of matching the volume of job-to-job transitions in our quantitative exercise.

Recall that $L$ is a fixed measure of the labor force. Thus $S$ can be written as

$$S = \gamma L(1 - U) + LU,$$

where $U$ is the unemployment rate. Given the meeting technology, the contact probability for each vacancy posted is written as

$$q(\theta) = \frac{\mu S^\psi V^{1-\psi}}{V} = \mu \theta^{-\psi},$$

where $\theta = \frac{V}{S}$ is labor market tightness in this economy. Similarly for workers, the contact probability per unit of search is written as

$$f(\theta) = \frac{\mu S^\psi V^{1-\psi}}{S} = \mu \theta^{1-\psi}.$$

While unemployed workers meet a potential employer with this probability each period, the contact probability of employed workers, denoted by $f_e(\theta)$, is reduced by a factor of $\gamma$, as in

$$f_e(\theta) = \gamma f(\theta).$$

We make two important assumptions that simplify our analysis significantly. First, we assume that the workers are myopic in the sense that the worker’s job acceptance decision (when contacted by a different firm) is based on the comparison of the wage paid by the current employer and the wage offered by another firm. Similarly, given that workers are myopic, quits into unemployment do not occur in equilibrium since the wage is always higher than the flow value of unemployment (see also the discussion in footnote 18). We make the assumption solely because it is practically impossible to solve the model with forward-looking workers. We discuss below the challenges we face in solving such a model. The second assumption is that the firm does not respond to the outside offer the workers receive through OJS, for example, as assumed in Mortensen (2010). In other words, we require the equal treatment of the equally productive workers.\(^\text{19}\)

Let the CDF of wages of all employed workers be $H(w)$ with $H(w) = 0$ and $H(\bar{w}) = 1$. Next, let $K(w)$ be the CDF of wages offered by hiring (vacancy posting) firms with $K(w) = 0$ and $K(\bar{w}) = 1$. Given these objects, we can express the vacancy filling rate (also known as the vacancy yield) of a firm offering the wage $w$ as

$$h(w, s) = q(\theta) \frac{U + \gamma(1 - U)H(w)}{U + (1 - U)\gamma}. \quad (5)$$

---

\(^{19}\)See the working paper version of Mortensen (2010) for a more detailed discussion on this “non-response” policy. See Postel-Vinay and Robin (2002) and Cahuc et al. (2006) for models that allow for a counteroffer from the current employer when a worker receives an outside offer.
Note that the vacancy filling rate depends not only on aggregate productivity $z$ but also on the distribution of the firm type $m(x, n)$. When a firm is posting vacancies, each job opening receives an application at rate $q(\theta)$. If the worker is unemployed, the acceptance rate is 1 given that we calibrate the value of $b$ so that it is lower than the wage in any possible states; and if the worker is employed, the offer is accepted with probability $H(w)$, the probability that the worker is currently employed at a firm paying a wage less than $w$.

The quit rate from a firm paying the wage $w$ is written as

$$k(w, s) = f_e(\theta)(1 - K(w)). \quad (6)$$

The quit rate for the firm can be expressed as a product of the contact probability $f_e(\theta)$ and the probability that the wage offered by the poaching firm is higher than the wage paid by the current employer $w$.

As mentioned above, we adopt the assumption that workers are myopic, thus making the job acceptance/rejection decision dependent only on wages. In principle, the decision of forward-looking workers should be based on the present discounted values. However, it is simply too time consuming to solve the model with forward-looking workers (especially because we need to solve such a model numerous times for the purpose of calibration). Imagine computing the value of a worker who is employed at a particular firm. Given that the worker can receive a job offer from any hiring firms, the calculation involves integrating the values of the worker with respect to the “value offer” distribution, which is endogenous and time-varying in the presence of the aggregate shock. The value of the worker then influences the decision of the firm that currently employs this worker. This firm makes its employment decision, taking into account the distribution of the values of all workers in the economy and the value-offer distribution of the hiring firms. These distributions are akin to $H(w)$ and $K(w)$ and enter into the firm’s decision problem for the same reasons that $H(w)$ and $K(w)$ enter into the firm’s problem in our model. After solving for the value-based decisions of a large number of workers and firms, taking the endogenous distributions as given, we then need to ensure the convergence of these distributions. We tried to solve the individual worker and firm problems in the environment without the aggregate uncertainty, assuming a seemingly plausible guess for the two distributions (parameterized by the beta distributions) and found that the convergence of the value functions itself takes a tremendous amount of time (more than a week). Given this experience, we decided that it is simply too time consuming to calibrate and quantitatively evaluate such a model.

Computational time is especially long because of the presence of aggregate uncertainty in the model and the fact that the model frequency is monthly (which implies a high discount factor). Because the main purpose of the paper is to study business-cycle properties of labor market flows and also because it is important to be able to construct the data out of our model in exactly the same manner as in the actual data (e.g., worker flows are constructed using monthly data while job flows are constructed using quarterly data), we cannot drop either the aggregate shock or the monthly frequency.

Compared to this process, solving our model with myopic workers is manageable. In particular, it is not necessary to solve for the worker value function that involves the integration of the future values with respect to the offer-value distribution. Accordingly, the step
to ensure the consistency between the distribution perceived by the worker and the actual distribution can also be avoided. Solving our model with good initial conditions still takes two to three days on a state-of-the-art computational environment. And solving the model with forward-looking workers would be exponentially more time consuming.

Lastly, note that having an analytical expression for the wage function (without a forward-looking variable such as market tightness) does not help us avoid the steps described for the economy with forward-looking workers. Our wage expression results from our bargaining environment and this itself does not imply workers care only about wages in the job acceptance/rejection decision. However, our wage-setting scheme is one of the plausible schemes that can be adopted under the assumption of myopic workers.

3.4 Optimal Employment Decision

The firm makes hiring and separation decisions by maximizing the present discounted value of flow profits:

$$\Pi(x, n, s) = \max_{n' \geq 0} \left\{ z xn'^{\alpha} - w(x, n', s)n' - \mathbb{I}_{n' > (1 - k(w(x, n', s), s))n} C^H(x, n, n', s) \right.\left. - \mathbb{I}_{n' < (1 - k(w(x, n', s), s))n} C^L(x, n, n', s) + \beta \int \int \Pi(x', n', s')dG_z(x'|x)dG_z(z'|z) \right\},$$

under the forecasting functions $m' = \Phi_m(s)$, $\theta = \Phi_\theta(s)$, $H(w) = \Phi_H(s)$, and $K(w) = \Phi_K(s)$. The firm uses $\Phi_m(s)$ to evaluate the value in the next period. The remaining three forecasting functions are needed to construct $h(w, s)$ and $k(w, s)$. $C^H(x, n, n', s)$ and $C^L(x, n, n', s)$ are the total costs of hiring and laying off workers, respectively. The total hiring cost takes the following form:

$$C^H(x, n, n', s) = \kappa_0 \left( v + \frac{\kappa_1}{2} v^2 \right),$$

where $v$ represents the number of vacancies posted by the firm and is written as

$$v = \frac{n' - (1 - k(w(x, n', s), s))n}{h(w(x, n', s), s)},$$

where $h(w, s)$ is a vacancy yield introduced in Equation (5). The total cost of layin off workers is written as

$$C^L(x, n, n', s) = \tau \left[ (1 - k(w(x, n', s), s))n - n' \right],$$

where $\tau$ is the per-worker cost of laying off workers. The above expression implies that the firm does not need to pay the firing cost when it loses workers through quits (job-to-job transitions).

The present discounted value of profits $\Pi(\cdot)$ is a function of the four state variables including the type distribution $m(x, n)$. The first two terms on the right-hand side correspond
to the flow profits to the firm. The wage function takes the form derived earlier, Equation (4). We assume that the total hiring cost $C_H$ is quadratic in the number of vacancies with $\kappa_0 > 0$ and $\kappa_1 > 0$, as in Equation (8).\footnote{The presence of the linear portion in the hiring cost implies that the marginal cost of posting a vacancy jumps to a positive value when the firm decides to hire and it increases with the number of vacancies. In the absence of the quadratic term, it is the same as the standard linear cost model where the marginal cost is constant at the positive value $\kappa_0$.}

Note that the firm loses $k(w, s)n$ workers through job-to-job transitions. Paying a higher wage has the effect of saving the hiring cost since it reduces quits. Observe also that the firm posts $1/h(w, s)$ vacancies per hire, given that each vacancy is filled with probability $h(w, s)$. Thus paying a higher wage has another effect that the offered wage is more likely to be accepted by on-the-job seekers at other firms. The total cost of layoffs $C_L$ is linear in the reduction of the number of employees net of job-to-job transitions. The quit rate $k(w, s)$ also affects the layoff decision because separations through quits allow firms to avoid paying the firing cost. Moreover, in reducing employment to $n'$, the firm knows that it has an offsetting upward effect on the wage, thus reducing the quit rate $k(w, s)$. The last term of the expression (7) gives the expected value of the future stream of profits discounted by $\beta$.

As in Elsby and Michaels (2013), the optimal employment decision of the firm is characterized by an $(s, S)$ rule, with the inaction region $(n^*, n)$ characterized by the following first-order conditions:

\begin{align}
&\alpha z x_n n^* - w - w_n n^* - C^H_3(x, n, n^*, s) + \beta \int \int \Pi_n(x', n^*, s')dG_x(x'|x)dG_z(z'|z) = 0, \\
&\alpha z x n^* - w - w_n n^* - C^L_3(x, n, n^*, s) + \beta \int \int \Pi_n(x', n^*, s')dG_x(x'|x)dG_z(z'|z) = 0,
\end{align}

where $C^H_3(x, n, n^*, s)$ and $C^L_3(x, n, n^*, s)$ are derivatives of each function with respect to its third argument ($n'$) and are expressed as

\begin{align}
C^H_3(x, n, n') &= \kappa_0 \left[ 1 + \kappa_1 \frac{n' - (1 - k(w, s))n}{h(w, s)} \right] \\
&\times \left[ \frac{(1 + nk'(w, s)w_n(x, n', s))h(w, s) - (n' - (1 - k(w, s))n)h(w, s)w_n(x, n', s)}{h^2(w, s)} \right], \\
C^L_3(x, n, n') &= -\tau \left[ k_w(w, s)w_n(x, n', s)n + 1 \right].
\end{align}

Note that, under the assumptions of no job-to-job transitions ($\gamma = 0$), no firing cost ($\tau = 0$), and the linear vacancy posting cost ($\kappa_1 = 0$), these two equations reduce to those presented in Elsby and Michaels (2013), save for some minor differences. The envelope conditions are
written as
\[
\Pi_n(x, n, s) = \begin{cases} 
\kappa_0 \frac{1-k(w, s)}{h(w, s)} \left[ 1 + \kappa_1 \frac{n'-(1-k(w, s))n}{h(w, s)} \right] & \text{if } \bar{n} < n^*, \\
\frac{1-k(w, s)}{1-n_k(w, s)w_n(x,n',s)} \left[ \alpha x n'^{\alpha-1} - w(x, n', s) - w_n(x, n', s)n' + \beta \int \int \Pi_n(x', \bar{n}, s')dG_zdG_x \right] & \text{if } \bar{n} \in [n^*, \bar{n}^*], \\
-\tau [1-k(w, s)] & \text{if } \bar{n} > \bar{n}^*,
\end{cases}
\]

where \( \bar{n} = (1-k(w, s)) \) and \( w = w(x, n', s) \).

Note that the first-order conditions, Equations (10) through (13), imply that firms need to know the derivatives of the vacancy filling rate \( h(w, s) \) and the quit rate \( k(w, s) \). These derivatives capture the effects that (i) offering a higher wage raises the acceptance rate of job-to-job movers and (ii) paying higher wages to the existing workers lowers the quit rate. Note also that, in order to know these objects, the firm needs to know the wage and wage offer distributions \( H(w) \) and \( K(w) \), both of which are endogenously moving along with the aggregate shock.

### 3.5 Equilibrium

The equilibrium of the model economy with the aggregate shock consists of the value function \( \Pi(x, n, s) \), the employment decision rule \( g(x, n, s) \), the wage function \( w(x, n', s) \), and the forecasting functions \( \Phi_m(s), \Phi_\theta(s), \Phi_H(s), \text{ and } \Phi_K(s) \), such that

1. \( g(x, n, s) \) maximizes the value of the firm, and \( \Pi(x, n, s) \) is the associated optimal value function.

2. Workers make the following job acceptance decision: An unemployed worker accepts a wage offer \( w \) if \( w > b \); and a worker currently employed at wage \( w \) accepts the wage offer \( w' \) if \( w' > w \).

3. The outcome of the bargaining between the firm and its employees, \( w(x, n', s) \), is characterized by (4).

4. The forecasting functions \( \Phi_m(s), \Phi_\theta(s), \Phi_H(s), \text{ and } \Phi_K(s) \) are consistent with the optimal employment decision of individual firms.

### 4 Computation

The details of the computational algorithms are presented in the Appendix. Here we summarize the algorithms used to solve for the steady-state equilibrium and the dynamic stochastic equilibrium. As mentioned before, one complication in solving the model is that the wage and wage offer distributions are endogenous objects that go into the decision problem. A conceptually straightforward way of dealing with the situation is to check the convergence
on the mass of workers at all grid points of wage. We propose a more efficient parametric method without sacrificing accuracy: We use the beta distribution to approximate these functions.\textsuperscript{21} Remember that the derivatives of the two distributions are in the first-order conditions. Approximating these distributions by the beta distributions allows us to compute these derivatives analytically.

To ease the notation, let us define the expected marginal profit function after the employment decision is completed in the current period as

\[
D(x, n', z, m') = \int \int \Pi_n(x', n', z', m')dG_x(x'|x)dG_z(z'|z).
\]  

(15)

To approximate this function, we replace the type distribution \(m\) by the aggregate unemployment rate \(U\). The idea is the same as the solution technique used to solve heterogeneous agent models with the uninsurable income risk, where the information in the wealth distribution is captured well with its mean. The \(D\) function is approximated by a piecewise linear function of the continuous state variables \(n', z\), and \(U\) for each discretized value of idiosyncratic productivity \(x\).

### 4.1 Steady-State Equilibrium

In the steady-state equilibrium, the aggregate state variable \(s\) is time-invariant and thus can be dropped. The first stage to solve for the steady-state equilibrium is to iterate on \(\pi^*(x, n)\), \(n^*(x, n)\), and \(D(x, n')\) for given guesses of \(H(w)\) and \(K(w)\) (which are parameterized by the beta distributions) and market tightness \(\theta\), using the first-order conditions (10) and (11).\textsuperscript{22} The reason that this step requires the iteration also on \(\pi^*\) and \(n^*\) is because the firm makes the labor adjustment decision by taking into account its effect on the current-period wage, which in turn influences the quit rate and the acceptance rate of job-to-job movers, thereby having a feedback effect on the labor adjustment decision.

Once we obtain the convergence on the optimal employment adjustment function, \(n' = g(x, n)\), and the \(D(x, n')\) function, the second stage of the algorithm simulates the economy to obtain the invariant distribution of \(m(x, n)\). Using this invariant type distribution and the wage function, we can actually compute \(H(w)\) and \(K(w)\), from which we update the parameters of the beta distributions. The labor market variables such as vacancies posted and the number of job seekers are also obtainable at this point, given that we have the information on the employment adjustment policy and the type distribution \(m(x, n)\). The entire process repeats until the convergence on the employment policy function, the expected marginal profit function \(D(x, n')\), and the parameters of the beta distributions is achieved.

\textsuperscript{21}The beta distribution is characterized by two parameters over an interval \([0, 1]\). We transform wage values into this interval. The bounds of the interval in the wage space need to be determined endogenously, and thus this procedure requires the convergence on four parameters for each distribution function.

\textsuperscript{22}Strictly speaking, \(\pi^*(x, n)\) actually does not depend on \(n\) because the marginal firing cost is constant. But we are using more general notation here.
4.2 Dynamic Stochastic Equilibrium

In the presence of aggregate uncertainty, current-period market tightness $\theta$ depends not only on realized aggregate productivity $z$, but also on the type distribution $m(x, n)$. In making the employment adjustment decision, each firm therefore needs to know the relationship between $\theta$ and $m(x, n)$ as well as $z$. As mentioned above, it is assumed that the firms use only the mean of the distribution (aggregate employment and thus equivalently unemployment, $U$) to summarize the information in $m(x, n)$, following the methodology often used in models of uninsurable income risk (e.g., Krusell and Smith (1998)). Further, calculating and updating the $D(x, n', z, m')$ function requires the firms to form the forecast for the next-period type distribution. Given our assumption about the approximate equilibrium, this entails forecasting next-period aggregate unemployment, $U'$, using current-period unemployment and realized aggregate productivity. Note also that calculation of $D(x, n', z, U')$ requires the prediction of $H(w)$ and $K(w)$ as well. This process is carried out by postulating the forecasting rule for each parameter of the two beta distributions.

The algorithm starts with guessing a set of coefficient values of the forecasting rules. Given these rules, we can solve the individual firms’ problem, following the procedure used to solve for the steady-state equilibrium (except that those functions now depend on the aggregate state variables). Once we achieve the convergence on the $D(x, n', z, m')$ function and the employment policy function $g(x, n, z, U)$, we simulate a large panel dataset from which we can obtain a long time series of $\{z, U, \theta, H(w), K(w)\}$. By using these objects, we can update the forecasting rules by running OLS regressions. The algorithm stops when the convergence on the coefficients on those forecasting rules is achieved.

5 Calibration

One period in the model is assumed to be one month. The exogenous productivity processes follow the standard AR(1) processes:

$$\ln z' = \rho_z \ln z + \varepsilon'_z,$$
$$\ln x' = \rho_x \ln x + \varepsilon'_x,$$

where $\varepsilon_x \sim N(0, \sigma^2_x)$ and $\varepsilon_z \sim N(0, \sigma^2_z)$. These processes are then approximated by a finite-state, first-order Markov chain.\(^{23}\)

While we use the finite-state approximation in calculating the conditional expectation with respect to aggregate uncertainty (when we solve the firm’s problem), we maintain the original AR(1) process in the simulation stage so that the process has a continuous state space. This enables us to generate the smooth impulse response functions presented below.

\(^{23}\)
Table 2: Model Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ψ</td>
<td>Elasticity of matching function with respect to job seekers</td>
</tr>
<tr>
<td>α</td>
<td>Curvature of the production function</td>
</tr>
<tr>
<td>β</td>
<td>Time discount factor</td>
</tr>
<tr>
<td>η</td>
<td>Worker bargaining power</td>
</tr>
<tr>
<td>μ</td>
<td>Scale parameter of the matching function</td>
</tr>
<tr>
<td>κ₀</td>
<td>Parameter of the vacancy posting cost</td>
</tr>
<tr>
<td>κ₁</td>
<td>Parameter of the vacancy posting cost</td>
</tr>
<tr>
<td>τ</td>
<td>Firing cost</td>
</tr>
<tr>
<td>γ</td>
<td>Search intensity of on-the-job seekers</td>
</tr>
<tr>
<td>b</td>
<td>Flow outside benefit</td>
</tr>
<tr>
<td>ρₓ</td>
<td>Persistence of the idiosyncratic productivity process</td>
</tr>
<tr>
<td>σₓ</td>
<td>Standard deviation of the idiosyncratic shock</td>
</tr>
<tr>
<td>ρₗ</td>
<td>Persistence of the aggregate productivity process</td>
</tr>
<tr>
<td>σₗ</td>
<td>Standard deviation of the aggregate shock</td>
</tr>
<tr>
<td>L</td>
<td>Labor force (population) size</td>
</tr>
</tbody>
</table>

We partition the model parameters into two groups: The one determined exogenously to the model without solving the model and the other determined by matching the empirical moments. Table 2 provides the summary of the model parameters.

5.1 Parameters Set Exogenously

First, the time discount factor β is set to 0.996, which implies the quarterly discount factor of 0.99, a standard value used in the business-cycle literature. The curvature of the production function α is set to 0.67. This appears to be a value commonly used in the literature, for example, by Cooper et al. (2007). We will later consider a somewhat higher value (0.72) to check the sensitivity of the results with respect to this parameter. Worker bargaining power η and the elasticity of the matching function with respect to unemployment 1 − ψ are set to 0.5. We consider an alternative value for η later. The persistence parameter of aggregate productivity is set to 0.983, which implies a quarterly autocorrelation of 0.95, following the convention of the business-cycle literature. The parameter values so far are relatively uncontroversial.

The flow outside option value b is set to 0.4. It is well known that this parameter plays an important role in amplifying the shock in the standard model with linear production technology (Shimer (2005), Costain and Reiter (2008), and Hagedorn and Manovskii (2008)). Elsby and Michaels (2013) show that, in the environment with decreasing-returns-to-scale production technology, the model can generate larger volatility for a given level of the ratio between b and average labor productivity. Note that b = 0.4 itself does not represent the ratio to productivity given that productivity is endogenous in our decreasing-returns-to-scale...
environment. In our benchmark calibration, this flow outside option value amounts to 66% of average labor productivity and 79% of the average wage. Note also that we do not use \( b \) to target these values (they are computed only ex post). Nevertheless, these ratios appear reasonable, considering the various values used in the literature.\(^{24}\)

Lastly, \( \kappa_1 \), one of the two parameters that characterize the vacancy posting cost, is set to 0.1. The other parameter for the vacancy posting cost \( \kappa_0 \) is endogenously determined as described in the next section. But we want to mention here that, with our specification of the vacancy posting cost, the marginal cost of posting a vacancy is \( \kappa_0 + \kappa_0 \kappa_1 v \) and the chosen values for \( \kappa_0 \) and \( \kappa_1 \) imply a fairly small slope. Thus our calibration implies that our model is close to the case with a linear vacancy posting cost. We nevertheless introduce the curvature to the vacancy posting cost for a technical reason that, when the cost is linear (i.e., \( \kappa_1 = 0 \)), the wage distribution is so compressed that it becomes difficult to obtain the stable numerical solution to our model featuring OJS, which directly links the wage distribution with job-to-job transitions. We also considered a calibration with a higher value of \( \kappa_1 = 0.15 \), and the results are little affected.

### 5.2 Parameters Set Endogenously

First, we use the scale parameter of the matching function \( \mu \) and one of the two parameters of the vacancy posting cost \( \kappa_0 \) to achieve the average levels of the monthly UE transition rate \( f(\theta) \) and job filling rate \( q(\theta) \) at around 0.25 and 0.9, respectively. The former number is based on the time-series data on the UE transition rate computed from the CPS labor flow data for the period January 1990 and December 2011.\(^{25}\) The latter is based on the evidence by Davis et al. (2013), who show that the daily job filling rate fluctuates at around 7%, which translates into the monthly filling rate of 0.9.\(^{26}\) We set \( \mu \) at 0.424 and \( \kappa_0 \) at 0.030 to hit these targets as closely as possible.

The search intensity parameter of employed workers \( \gamma \) is selected to match the average job-to-job transition rate in the Fallick and Fleischman (2004) data that cover the period between January 1994 and December 2011. The average job-to-job transition rate over this period is around 2.5% in the data, and we roughly match this number by setting \( \gamma = 0.1145 \).

Next, we calibrate the two parameters of the firm-level productivity process, \( \rho_x \) and \( \sigma_x \), by referring to the following two statistics. First, the persistence parameter is selected to match the average “one-quarter persistence measure” of the job destruction rate. This statistic is proposed by Davis et al. (1996) and gives the percentage of newly destroyed jobs at time \( t \) that do not reappear at the next sampling date. They report the historical average of this measure for the manufacturing sector for the period 1972Q2 through 1988Q4 at 0.72.\(^{27}\) We

\(^{24}\)In the calibration by Elsby and Michaels (2013), the value of \( b \) relative to average productivity is 61%, and thus our calibration is close to theirs along this dimension.

\(^{25}\)Note that the model includes job-to-job transitions to which a different transition rate applies because employed workers do not necessarily accept all offers. Because unemployed workers accept all offers in our model, \( f(\theta) \) in the model corresponds to the UE transition rate in the data.

\(^{26}\)Fujita and Ramey (2012) also use the same target value, based on the evidence by Barron et al. (1997).

\(^{27}\)Unfortunately, empirical evidence on this measure is available only for the manufacturing sector. The
compute the same statistic using the simulated data, and our choice $\rho_x = 0.9075$ allows us to roughly match this statistic. The standard deviation $\sigma_x$ is assigned to match the dispersion of the employment growth distribution. Davis et al. (2006) calculate employment-weighted cross-sectional dispersion (standard deviation) of annual employment growth rates using the Longitudinal Business Database (LBD) for the period 1978 through 2001. Their figure shows that the standard deviation fluctuates roughly around 0.60. Our model generates a value close to this target with $\sigma_x = 0.1091$. Note that the original measure is based on net employment changes over an annual interval. We therefore construct the corresponding statistic in our model, taking net employment changes over a 12-month period.

Next, the labor-force size $L$ is set to the level that is consistent with the average EU separation rate of 1.5%. The idea is as follows. Once we solve for the employment decisions of individual firms, we know the number of employed workers and flows into and out of unemployment. By selecting $L$ (thus the number of the unemployed), we can set the unemployment rate at a certain level, which in turn is equivalent to setting the separation rate at a target level (i.e., 1.5%), given that the job finding rate is already targeted at 25%.²⁸

The size of the aggregate shock $\sigma_z$ is set to 0.0026 and selected by matching the standard persistence measure of job creation is defined similarly as the percentage of newly created jobs at time $t$ that remain filled at the next sampling date one quarter later. Davis et al. (1996) report that job creation persistence in manufacturing is 0.68 over the same period.

²⁸We describe our calibration procedure as if we target only two transition rates between employment and unemployment. But it is equivalent to targeting the unemployment rate instead of the EU separation rate.
deviation of the aggregate output series. Over the sample period 1990 to 2011, the standard deviation of the logged and HP-filtered real GDP series is 0.0119. \( \sigma_z = 0.0026 \) allows us to roughly match this level of volatility.

The last parameter to be determined is the level of the firing cost \( \tau \). We use this parameter to match the volatility of the UE transition rate \( f(\theta) \). One may think that this parameter is most effective in controlling the volatility of the EU transition rate, given that, in the model, the firing cost applies to those who are separating into unemployment. However, under the presence of job-to-job transitions, the link between the firing cost and the volatility of the EU transition rate is more complicated. We find that it is easier to match the volatility of \( f(\theta) \) than that of the EU separation rate. We will discuss the underlying economic reasons for this result in Section 7.1. Note that this parameter \( \tau \) represents the per-worker resource cost associated with layoffs. The chosen value \( \tau = 0.13 \) in our benchmark calibration implies that this resource cost amounts to roughly 25% of the average monthly wage. It is small, which consistent with the empirical evidence that firing costs are small in the United States.

6 Main Results

This section presents the main results of the paper. We first show that the model matches the first moments of the observed data reasonably well. We also make sure that the model is capable of capturing key cross-sectional relationships between job flows and worker flows, namely, the “hockey stick” hiring and separation functions recently studied by Davis et al. (2012). We then examine if the model can replicate the cyclicality of worker flows, transition rates, and job flows.

6.1 First-Moment Properties

The first-moment properties of the model under the benchmark calibration are summarized in Table 4. Recall that we calibrate the model to achieve the average levels of the three worker transition rates and job flow persistence. While we are unable to match the first moments exactly, the model-based averages are reasonably close to the corresponding target levels. The unemployment rate in the model is somewhat higher than the average level used in the literature (e.g., 6%) but this occurs because, in our calibration, the EU transition rate is on the high side and the UE transition rate on the low side (relative to the values used in the literature), both of which raise the average unemployment rate. This itself does not have any material impact on our results. Job creation and destruction rates in the model, which are not directly targeted in our calibration, fluctuate around 10%. Overall, the calibrated model replicates the observed first moments reasonably well.

In the last row, we also report the ratio of the flow outside option value to average labor productivity. We report this number only because of the literature’s interest in the volatility puzzle of Shimer (2005). Elsby and Michaels (2013) show that their model with

\[ \text{We use the post-1990 output series, simply because other series we use are available from (or shortly after) 1990. See the notes to Table 1.} \]
decreasing-returns-to-scale technology generates a larger magnification with a smaller value of the outside option value relative to average productivity. Our number (0.66) is close to their corresponding number (0.61). Note, however, that our model differs significantly from theirs, mainly due to the presence of OJS. Their model is a natural extension of Mortensen and Pissarides (1994), and thus the comparison to the literature on the volatility puzzle is more direct.

### 6.2 Worker Flows and Job Flows in the Cross Section

A recent paper by Davis et al. (2012) characterizes the cross-sectional relationship between worker flows and job flows, using the “hockey-stick” functions. Here we show that our model replicates the main features of this well-established empirical relationship, using the simulated observations from the steady-state version of our model.\(^{30}\) In Panel (a) of Figure 3, the horizontal axis measures net employment growth over a quarterly period. Thus the firms located below (above) zero are destroying (creating) jobs over the quarterly period.

The vertical axis measures the total hiring and separation rates over the same quarterly period. Note that, in our model, the total hiring (or separation) rate corresponds to the sum of hires from (or separations into) unemployment and other firms (job-to-job transitions), normalized by the employment stock. The quarterly measures add up all hires (separations) that occur during the quarterly period. The circles in the scatter plot indicate the relationship between the hiring rate and net employment growth at individual firms, while the triangles show the relationship between the separation rate and net employment growth.

---

\(^{30}\)Our calculation is based on a random sample of 10,000 establishments (out of all 1 million establishments) over a quarterly period in the steady-state equilibrium.
Figure 3: Worker Flow Rates as a Function of Net Employment Growth Rates

Notes: Based on the 10,000 simulated firm-level observations over quarterly (Panel (a)) and monthly (Panel (b)) periods in the steady-state equilibrium. Blue solid and red dashed lines represent predictions from the cubic polynomial regressions.

Blue solid and red dashed lines represent approximations of these relationships, based on the cubic polynomial regressions. Panel (b) plots the same relationships at the frequency of the model (i.e., monthly) and thus removes the effect of time aggregation that is present at a quarterly frequency. The empirical result by Davis et al. (2012) measures net employment growth at a quarterly frequency and thus corresponds to the result in Panel (a).

This figure provides an important insight into why worker flows and job flows can be different from each other. One can clearly see on the left-hand side of Panel (a) that there are some firms that are shrinking on net (destroying jobs) at a quarterly frequency, yet hiring some workers, represented by the circles. Observe, however, that the hiring rates at the firms that are shrinking at a rapid pace are normally zero. Further, when the data are sampled at a monthly frequency, the circles that indicated positive hiring rates at shrinking firms in Panel (a) disappear, implying that separation and job destruction rates are nearly equivalent at a monthly frequency (note that they are only “nearly” equivalent for the reason discussed below). In other words, positive hiring rates that are observed at the shrinking firms are due to time aggregation.\(^{31}\) In the empirical relationship reported by Davis et al. (2012), there are many shrinking establishments that are hiring on a gross basis, and the extent of this

\(^{31}\)There are many logical possibilities that positive hires are observed at firms that are shrinking on net at a quarterly frequency. For example, a firm hires some workers in the first month, but sheds workers in the following two months due to negative shocks and thus shrinks on net.
happening is clearly more significant than can be seen in our simulated quarterly data. For example, Figure 6 in their paper shows that, even at the firms that are cutting employment by 60%, the gross hiring rate is higher than 10%. This type of pattern simply cannot be explained by time aggregation only, as indicated by our result. As discussed by Davis et al. (2012), nontrivial positive hiring at net shrinking firms in the actual data is likely due to the fact that there are some essential positions that the firm needs to refill (after workers leave the firm), even though firm size is rapidly shrinking on net. Such a feature is simply not present in our model.

Note that it is incorrect to say, however, that separations and job destruction are the same to the left of zero in the model (even at a monthly frequency). The equivalence holds only at the firms that are making relatively large employment adjustments. One can see in Panel (b) that there is a small region to the immediate left of zero where nonzero hires are observed. One can also see that, over the same small region, there are more separations than net employment changes. The same pattern can be more clearly seen when the effect of time aggregation is considered in Panel (a). While this region looks small, there are a large number of firms located in this region. For example, 68% of firms are located in the region between 0 and -5% in the steady-state equilibrium. This phenomenon corresponds to the scenario where the firm wants to reduce its employment (thus destroying jobs), but quits (job-to-job transitions) are more than enough to achieve the target employment level. Thus, these firms end up needing to hire at least some workers. In other words, when a firm loses its workers through job-to-job transitions (which generate gross separations), the firm finds it optimal to partially replace these workers. In this case, both hiring and separations coexist within a period, and employment growth is negative. This case is unlikely to occur when a firm receives a large negative shock, in which case the firm is willing to cut its workforce beyond quits.

Next, consider the firms that are located to the right of zero net employment growth (thus creating jobs). One can see in Panel (a) that these firms experience a nontrivial number of separations even when they are expanding their employment at a rapid pace. The same is true even at the monthly frequency displayed in Panel (b), and thus time aggregation is not the reason for the observed nontrivial separations. In the model, even at the firms that are growing rapidly, there are always workers leaving for higher-paying firms. The pattern of separations existing over the entire range of positive employment growth is consistent with the empirical fact presented by Davis et al. (2012), although the model misses the fact that some separations at these expanding firms are classified as layoffs in the real data. In the model, these separations are all job-to-job transitions and thus are interpreted more naturally as quits.

In summary, the conceptual differences between job flows and worker flows arise because (i) the firms that are destroying jobs on net may hire workers and (ii) separations occur at the firms that are creating jobs. The first difference exists only at the firms with small negative employment growth, whereas the second difference arises over the entire range of employment growth rates (except at the firms that are paying the highest wage in the economy).
Table 5: Second-Moment Properties of the Model: Benchmark Calibration

<table>
<thead>
<tr>
<th></th>
<th>Standard Deviation</th>
<th>Relative Standard Deviation</th>
<th>Correlation with Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Worker flows</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E to U</td>
<td>0.097</td>
<td>8.235</td>
<td>-0.778</td>
</tr>
<tr>
<td>E to E</td>
<td>0.088</td>
<td>7.406</td>
<td>0.991</td>
</tr>
<tr>
<td>U to E</td>
<td>0.075</td>
<td>6.344</td>
<td>-0.599</td>
</tr>
<tr>
<td>Transition rates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EU transition rate</td>
<td>0.104</td>
<td>8.758</td>
<td>-0.826</td>
</tr>
<tr>
<td>EE transition rate</td>
<td>0.078</td>
<td>6.607</td>
<td>0.986</td>
</tr>
<tr>
<td>UE transition rate</td>
<td>0.080</td>
<td>6.810</td>
<td>0.983</td>
</tr>
<tr>
<td>Job flows</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Creation rate</td>
<td>0.029</td>
<td>2.457</td>
<td>0.125</td>
</tr>
<tr>
<td>Destruction rate</td>
<td>0.029</td>
<td>2.492</td>
<td>-0.350</td>
</tr>
<tr>
<td>Stocks</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>0.137</td>
<td>11.588</td>
<td>-0.905</td>
</tr>
<tr>
<td>Vacancies</td>
<td>0.122</td>
<td>10.310</td>
<td>0.943</td>
</tr>
</tbody>
</table>

Notes: Based on the simulation of a panel of 1 million establishments over 1,200 (monthly) periods. Worker flows, worker transition rates, unemployment rate, and vacancies are converted into quarterly data by time averaging. Job flows are based on net employment changes over a quarter. All observations are logged and HP filtered with smoothing parameter of 1,600.

6.3 Cyclicality of Worker Flows and Transition Rates

Table 5 presents the same second-moment statistics discussed earlier in Table 1. As we mentioned in the calibration section, we choose \( \tau \) to match the volatility of the UE transition rate. In the observed data, its standard deviation is 6.7 times as large as output volatility. We roughly match this value in our simulated data. Other cyclical measures in this table are not directly targeted. First, observe that the EU transition rate in our model is somewhat too volatile. In the data, the EU separation rate is less volatile than the UE transition rate, whereas in our model it is more volatile.\(^{32}\)

Note that the large volatility of the EU transition rate makes not only EU worker flow more volatile but also UE flow more volatile. Large fluctuations in the flow into unemployment also implies large fluctuations in the flow out of unemployment with a lag (due to search frictions). The larger volatility of separations into unemployment (together with the fluctuations in \( f(\theta) \) comparable to the data) makes the volatility of the unemployment rate somewhat larger than that of the observed data.

The model replicates very well the correlation pattern with respect to output. We documented earlier the countercyclicality of the EU separation rate and the procyclicality of the UE and EE transition rates. The model naturally reproduces this pattern. Next, the

---

\(^{32}\)The same pattern is found in a single-worker matching model with endogenous separation. See, for example, Fujita and Ramey (2012) and Fujita and Moscarini (2013).
model also replicates the cyclical pattern of gross worker flows: countercyclicality of flows between employment and unemployment and procyclicality of the job-to-job flow. As discussed in Section 2, hires from unemployment are countercyclical because the increase in the separation flow increases the size of the unemployment pool in recessions. This effect is counteracted by the decline in the UE transition (job finding) rate, and thus the countercyclical of the UE hiring flow is weaker than that of the EU separation flow (although the UE hiring flow is still strongly countercyclical in both the model and the data).

The model also replicates the Beveridge curve, a strong negative correlation between unemployment and vacancies, which is indicated by the countercyclical of unemployment and the procyclicality of vacancies shown in the last two rows of the table. We discuss the correlations of job flows with output, referring also to Table 6.

Figure 4 presents the impulse response functions to a one-standard-deviation negative aggregate productivity shock.\(^{33}\) Panel (a) presents the responses of worker transition rates. The EU transition rate increases sharply on impact. While a substantial part of the initial increase is reversed in the following period, it stays at a level higher than the steady-state level for an extended period. The UE and the job-to-job transition rates decline on impact and stay persistently low. Panel (b) presents the responses of worker flows. Not surprisingly, the behavior of the EU flow is very similar to that of the EU transition rate. The hiring flow from unemployment declines initially, reflecting the drop in the UE transition rate, but quickly reverses its course and goes up to a level higher than the steady-state level. These latter movements are responsible for the UE flow’s negative correlation with output.\(^{34}\) In contrast, the job-to-job flow stays below the steady-state level, exhibiting a mild hump-shaped response followed by an initial sharp drop.

Panel (c) shows that the model is capable of generating the Beveridge curve. The presence of job-to-job transitions in the model plays an important role in generating the persistent declines in vacancies. Vacancies are much less persistent in the model without job-to-job transitions, as discussed later. In Panel (d), we present responses of aggregate output and labor productivity along with the exogenous driving process. Reflecting the persistent hump-shaped increases in the unemployment rate, aggregate output also exhibits the hump-shaped response, hitting its lowest level in the seventh month after the shock. In our model, labor productivity, measured by aggregate output divided by aggregate employment, is an endogenous variable. Labor productivity does not decline as much as the driving process since marginal product of labor increases as firms reduce employment. Also, labor productivity

\(^{33}\)Strictly speaking, impulse responses in our model are not symmetric because the model is nonlinear. We checked to see if the model exhibits significant asymmetry, by comparing the responses to the positive and negative shocks, and found that only minor asymmetry exists in our model. A recent paper by Petrosky-Nadeau and Zhang (2013) argues that impulse responses in the standard labor matching model with linear technology exhibit large asymmetry. However, the asymmetry in their model is due to the zero lower bound of the number of vacancies, which tends to bind when a large negative productivity shock hits the economy, especially if the calibration by Hagedorn and Manovskii (2008) is adopted. In our model, however, even when the economy is hit by a large recessionary shock, there are still a large number of firms posting vacancies. In other words, the zero lower bound for the number of vacancies never binds in our model economy.

\(^{34}\)Fujita (2011) presents impulse response functions of worker transition rates and flows based on the identified VARs, and they are similar to those presented in Figure 4.
Figure 4: Impulse Response Functions in the Benchmark Model
Notes: Plotted are responses to a one-standard-deviation negative aggregate shock, expressed as log deviations from the steady-state levels.

reverses more quickly, as indicated by the difference between the pink solid line and the green dashed line in Panel (d).

6.4 Cyclicality of Job Flows
Table 5 also shows that job creation and destruction rates fluctuate much less than worker flows do. One can also see that the job creation rate is weakly procyclical and the destruction rate is countercyclical. To better understand these results, Table 6 reports the same second-moment statistics for quarterly job flows (as shown in the previous table), monthly job flows, and rates of total separations and hires. Note that, in the model, we can construct “monthly
Table 6: Job Flows vs. Worker Flows

<table>
<thead>
<tr>
<th></th>
<th>Standard Deviation</th>
<th>Relative Standard Deviation</th>
<th>Correlation With Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Creation rate (Q)</td>
<td>0.029</td>
<td>2.457</td>
<td>0.125</td>
</tr>
<tr>
<td>Creation rate (M)</td>
<td>0.031</td>
<td>2.604</td>
<td>0.354</td>
</tr>
<tr>
<td>Hiring rate (M)</td>
<td>0.035</td>
<td>2.972</td>
<td>0.571</td>
</tr>
<tr>
<td>Destruction rate (Q)</td>
<td>0.029</td>
<td>2.492</td>
<td>−0.350</td>
</tr>
<tr>
<td>Destruction rate (M)</td>
<td>0.022</td>
<td>1.870</td>
<td>−0.038</td>
</tr>
<tr>
<td>Separation rate (M)</td>
<td>0.020</td>
<td>1.664</td>
<td>0.423</td>
</tr>
</tbody>
</table>

Notes: The letter in parentheses indicates the data collection frequency (quarterly or monthly). Monthly job flows are constructed by applying the same idea as for the quarterly job flows to monthly employment changes: job creation (destruction) = sum of employment changes at expanding (shrinking) establishments over a monthly period (normalized by employment). Hiring rate: all hires (sum of UE and EE worker flows) as a fraction of employment. Separation rate: all separations (sum of EU and EE worker flows) as a fraction of employment.

job flows” using the monthly interval in lieu of the quarterly interval. Even at the monthly frequency, job creation and destruction rates are not the same as total separation and hiring rates, as discussed earlier (Section 6.2).

Table 6 indicates that volatilities of total separation/hiring rates are much smaller than the three components of worker flows. In Table 5, one can see the EU separation rate is more than eight times as volatile as output, and the job-to-job transition rate is almost seven times as volatile. However, Table 6 shows that the total separation rate, which is the sum of the two series, is only 1.7 times as volatile. A similar pattern can be seen for the hiring rate as well. The volatilities of the job creation and destruction rates (especially at a monthly frequency) are roughly comparable to those of the total hiring rate and separation rate, respectively.

This result is easily understood by noting that the countercyclicality of EU and UE flows is countered by the procyclicality of the job-to-job flow. In Figure 5, we plot impulse responses of job flows and separation/hiring rates. In Panel (b) of Figure 4, we plot responses of three worker flows. One can clearly see that EU and job-to-job flows are offsetting each other’s movements and thus creating a less volatile total separation flow. On the hiring side, although UE and job-to-job flows are moving in the same direction in the first few months after the shock, they move in the opposite direction thereafter, again offsetting each other’s movements.\(^\text{36}\)

\(^\text{35}\)As noted before, the total hiring rate is computed as the sum of the hiring rate from unemployment (the UE flow normalized by employment) and from other jobs (which is equivalent to the EE transition rate). Although the volatility of the hiring rate from unemployment is not reported in the table, it is close to the volatility of the UE flow itself (reported on the third row), given that the volatility of the denominator (employment) is small enough.

\(^\text{36}\)In Panel (b) of Figure 5, both separations and hires are normalized by employment, in order to be consistent with job creation and destruction rates, while worker flows in Panel (b) of Figure 4 are not normalized by employment. This difference is not very important for our discussion, because changes in
While volatilities are of a similar magnitude for all three rows in Table 6, the correlation with output differs considerably. First, consider the difference between the job destruction rate and the total separation rate. The total separation rate is procyclical even though the job destruction rate is countercyclical. The procyclicality of the separation rate implies that the procyclicality of job-to-job transitions dominates the countercyclicality of the EU separation rate on net. Note that the volatility of the EU transition rate is somewhat larger than that of the job-to-job transition rate (see Table 5), when the two transition rates are examined separately. However, the average level of the job-to-job transition rate is larger and thus it carries a larger weight when the total separation rate is considered. Panel (b) of Figure 5 shows that the total separation rate initially increases due to the increase in the EU transition rate in response to the negative shock. However, from around the fourth month, the total separation rate stays below the steady-state level for an extended period. This latter pattern makes the correlation with output positive.

The procyclicality disappears when the job destruction rate is considered. The large part of the change comes from the difference between the monthly job destruction rate and the total separation rate (the correlation coefficient drops from 0.423 to −0.038). The selection of firms in measuring the job destruction rate plays an important role here. Note that, as discussed earlier in regard to Panel (b) of Figure 3, a nontrivial number of separations exist at expanding firms and these separations consist only of job-to-job transitions. The flip side of this is that when only the shrinking firms are selected, the share of layoffs (EU transitions) employment have only a minor impact on the behavior of total hiring and separation rates.
to total separations is larger. This pushes its cyclical to the countercyclical direction.\(^{37}\)

By comparing the responses of the job destruction rate and the total separation rate, one can see that (i) the initial increase in the job destruction rate is larger, (ii) the job destruction rate stays above the steady-state level for a longer period, and (iii) the negative deviation from the steady-state level is smaller for the job destruction rate after the response turns negative. These three factors contribute to making the job destruction rate a countercyclical variable. Table 6 also shows that time aggregation plays an important role in the countercyclicality of the job destruction rate. This effect comes from the fact that, when the three-month interval is considered, the selection effect discussed above becomes stronger. That is, the selection of the firms with negative net employment growth over the longer-run period is more likely to include those that need to appeal to layoffs.

On the hiring/job creation side, note first that we do not match the magnitude of the positive correlation with output: The model generates the correlation 0.125, while the correlation of the empirical series is higher than 0.4. However, the issue here has more to do with a timing of the comovement. As discussed above, the output response exhibits a clear hump shape, while changes in the job creation rate are concentrated in the short run. We

---

\(^{37}\)Remember, however, that at firms with small negative employment growth rates, not all separations take the form of layoffs (EU separations). At those firms, separations can consist only of job-to-job transitions. This is discussed earlier with respect to Panel (b) of Figure 3.
therefore view that the model’s underlying economic forces are largely consistent with the empirical evidence.

The (monthly) job creation rate and the hiring rate behave similarly in terms of correlations with output. However, the procyclicality of the hiring rate is stronger. In order to understand this difference, we plot in Figure 6 how the hiring and separation functions differ in the boom and recession periods. The main differences in the hiring and separation functions lie on the right-hand side of the graph. That is, the hiring and separation functions both shift up (down) in the boom (recession) period. Note that the difference in the separation function reflects the strong procyclicality of job-to-job transitions (recall that all separations at expanding firms are job-to-job transitions at least at monthly frequency). This means that, for a given level of net employment growth, the firm needs to hire more workers in the boom period, which is reflected in the upward shift in the hiring function. Note that the aggregate job creation rate can be viewed as an integral of the area under the 45-degree line over the region of positive net employment growth rates, with its distribution being the weighting function. The fluctuations of the job creation rate then can be understood as resulting from the procyclical shifts in the growth distribution. The aggregate hiring rate is, however, influenced not only by the procyclical shifts in the distribution, but also by the procyclical movements in the hiring function itself illustrated in Figure 6. Again, the upward shift of the hiring function in a boom results from job-to-job transitions that necessitate more hires for a given level of the growth rate. This additional force creates stronger procyclicality of the aggregate hiring rate.

7 Importance of Job-to-Job Transitions

Our model differs from those of Cooper et al. (2007) and Elsby and Michaels (2013) mainly because ours explicitly incorporates job-to-job transitions. In this section, we further investigate the implications of incorporating job-to-job transitions into the model. More specifically, we first examine the quantitative properties of the model without job-to-job transitions by setting \( \gamma = 0 \). That model is similar to the model of Elsby and Michaels (2013).\(^{38}\) This exercise demonstrates that incorporating job-to-job transitions is crucial in matching the cyclicality of worker flows and job flows simultaneously. Further, we will point out an important mechanism that contributes to generating larger volatility and persistence in our model. Thereafter, we discuss how the quantitative properties of the model change when the firing cost is lowered. This exercise is useful for demonstrating the interaction between job-to-job transitions and the size of the firing cost.

7.1 Comparison to the Model without Job-to-Job Transitions

Table 7 presents the cyclical properties of the model when the search intensity of employed workers is set to zero (third and fourth columns). In calibrating the model, we match

\(^{38}\)Our model differs from theirs even without job-to-job transitions mainly because of the different wage-setting rules. The specifications of the shock processes and hiring/firing costs are also different.
Table 7: Second-Moment Properties: No OJS and Lower Firing Cost

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>No OJS</th>
<th>Lower τ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Relative</td>
<td>Corr. w/</td>
<td>Relative</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>Output</td>
<td>SD</td>
</tr>
<tr>
<td>Worker flows</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E to U</td>
<td>8.235</td>
<td>−0.778</td>
<td>7.604</td>
</tr>
<tr>
<td>E to E</td>
<td>7.406</td>
<td>0.991</td>
<td>n.a.</td>
</tr>
<tr>
<td>U to E</td>
<td>6.344</td>
<td>−0.599</td>
<td>4.917</td>
</tr>
<tr>
<td>Transition rates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EU transition rate</td>
<td>8.758</td>
<td>−0.826</td>
<td>7.960</td>
</tr>
<tr>
<td>EE transition rate</td>
<td>6.607</td>
<td>0.986</td>
<td>n.a.</td>
</tr>
<tr>
<td>UE transition rate</td>
<td>6.810</td>
<td>0.983</td>
<td>4.669</td>
</tr>
<tr>
<td>Job flows</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Creation rate</td>
<td>2.457</td>
<td>0.125</td>
<td>5.339</td>
</tr>
<tr>
<td>Destruction rate</td>
<td>2.492</td>
<td>−0.350</td>
<td>7.972</td>
</tr>
<tr>
<td>Stocks</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemp. rate</td>
<td>11.588</td>
<td>−0.905</td>
<td>9.181</td>
</tr>
<tr>
<td>Vacancies</td>
<td>10.310</td>
<td>0.943</td>
<td>2.871</td>
</tr>
</tbody>
</table>

Notes: See notes to Table 5. The model with no OJS is solved and calibrated by setting \( \gamma = 0 \). Recalibrated parameters are set to the following values: \( \kappa_0 = 0.9637 \), \( L = 5.3395 \), and \( \sigma_z = 0.0051 \). The first two parameters are used to match the average levels of UE and EU transition rates. The last parameter is used to match the output volatility. Parameters not listed are unchanged from the benchmark calibration. The model with a lower firing cost is recalibrated to satisfy the same steady-state moment conditions and output volatility condition as in the benchmark calibration. Recalibrated parameters are set to the following values: \( \tau = 0.0065 \), \( \kappa_0 = 0.078 \), \( \gamma = 0.1145 \), \( \sigma_x = 0.1091 \), \( \sigma_z = 0.0031 \), and \( L = 5.638 \).

the average levels of UE and EU transition rates by using \( \kappa_0 \) and \( L \), as in the benchmark calibration. We also adjust the size of the standard deviation of the aggregate shock so that the model generates the output volatility of the same magnitude as before. The remaining parameters stay the same. In particular, we do not adjust the level of \( \tau \), and thus the model does not exactly match the volatility of the UE transition rate. However, it is within a reasonable range.

First, note that, even without OSJ, the model matches the correlation pattern of EU and UE worker flows and transition rates, which is not surprising. However, without OJS, the model fails to replicate the cyclical patterns in job flows. In the absence of job-to-job transitions, the job destruction rate is the equivalent to the EU transition rate at monthly frequency. Thus, the difference in the correlation coefficients with output arises solely due to time aggregation in the measurement of the job destruction rate (−0.827 vs. −0.830). In the model without OJS, the effect of time aggregation is small. The job creation rate is now strongly negatively correlated with output. This result is quite intuitive because job creation
Figure 7: Impulse Response Functions in the Model without Job-to-Job Transitions

Notes: Plotted are responses to a one-standard-deviation negative aggregate shock, expressed as log deviations from the steady-state levels.

now needs to happen only through hires from unemployment, which is countercyclical.\(^{39}\)

Next, the volatility of job flows is too high in this model relative to the empirical counterpart. Recall that, in our model with OJS, the procyclical movements of job-to-job transitions are offset by the countercyclicality of worker flows between employment and unemployment, thereby making the volatility of total separation/hiring rates smaller. In our model with OJS, this offsetting effect was clearly the reason why job flows are much less volatile than

\(^{39}\)Note that, in the model without OJS, gross separations and hires are equivalent to net employment changes at monthly frequency, given that no workers separate from expanding firms and no shrinking firms hire workers.
each component of worker flows, and without job-to-job transitions, this effect disappears.

Another important result here is a significantly lower volatility of vacancies in the model without job-to-job transitions. The UE transition rate consequently is also significantly less volatile.\(^{40}\) To understand this result, Figure 7 plots the same impulse response functions as plotted in Figure 4 in our model. Panel (c) of Figure 7 shows that, although vacancies decline initially in response to the negative aggregate shock, this initial effect diminishes relatively quickly. The UE transition rate (plotted in Panel (a)), which again is proportional to market tightness, declines in a hump-shaped manner because of the persistent increases in unemployment. However, compared to its response plotted in Panel (a) of Figure 4, the deviation from the steady-state level is smaller. Comparing Figures 4 and 7, one can see that the presence of job-to-job transitions plays an important role in generating the persistent and more volatile behavior in vacancies in our model.\(^{41}\) In Panel (c) of Figure 4, vacancies stay persistently low after the initial negative response; this persistent decline coincides with a persistent decline in job-to-job transitions (see Panels (a) and (b)). The mechanism behind this result was discussed earlier in relation to how the cross-sectional hiring and separation functions move over the business cycle (Figure 6). That is, in a recession, job-to-job transitions slow down, which reduces the expanding firms’ need to post vacancies. As shown in Figure 6, the separation function shifts down in a recession, reducing the hiring rate at a given level of the net expansion rate. The opposite effect operates in a boom period. That is, more vacancies need to be posted at a given expansion rate. This phenomenon in our model is akin to the “vacancy chain” introduced by Akerlof et al. (1988). When a firm seeking to expand its employment loses workers to higher-paying firms, it hires workers from lower-paying firms (or from unemployment). This poaching from lower-paying firms creates another chain of vacancy posting at the firms located at the lower end of the wage distribution. Discussions so far demonstrate the qualitative and quantitative importance of job-to-job transitions in the multiple-worker firm setting, not only for the cyclicality of job flows and worker flows, as discussed in the previous section, but also for an understanding of vacancy dynamics.

### 7.2 Firing Cost and Job-to-Job Transitions

Let us now discuss the quantitative properties of the model when the level of the firing cost is reduced. As mentioned in the calibration section, \(\tau\) is chosen to match the UE transition rate. This second exercise explains the mechanism behind the procedure. Specifically, we set \(\tau\) to 0.065, which is half the level in the benchmark calibration. Note that we recalibrate the model in this case so that the model matches all the moment conditions (such as mean

\(^{40}\)Note that, in the model without job-to-job transitions, the UE transition rate is proportional to labor market tightness given that the job acceptance probability of the unemployed workers is 1. The unemployment rate remains relatively volatile, mainly because the separation rate remains volatile.

\(^{41}\)As presented in Table 4, the flow outside option value (relative to the average productivity) in our benchmark model is 0.656. In the model without OJS, the corresponding value turns out to be 0.583. While our benchmark model uses a higher outside option value, the difference in this value is not large enough to explain the difference in the volatility.
levels of all transition rates) except for the condition for the volatility of the UE transition rate. The results are summarized in the last two columns of Table 7.

We can first make the following two observations. First, the volatility of vacancies is significantly reduced under this alternative calibration. This reduced volatility of vacancies translates into lower volatilities of UE and job-to-job transition rates. Second, the volatility of the EU separation rate is much less affected. The first result can be understood as follows. When the firing cost is set lower, the hiring cost needs to be raised in order for the model to generate the same steady-state worker transition rates. More specifically, the level of $\kappa_0$ required to match the mean transition rates turns out to be more than twice that of the value in the benchmark calibration (0.078 vs. 0.03). This is intuitive, because lowering $\tau$ by itself has the effect of raising average worker transition rates. The otherwise higher worker transition rates need to be countered by raising the level of the hiring cost. The higher hiring cost then has the effect of lowering the volatility of vacancies.

The question is why the volatility of the EU transition rate is not affected very much even when $\tau$ is lowered. The result comes from the two opposing effects that the lower firing cost has on the volatility of the layoff (EU transition) rate. The first is a direct effect: the lower firing cost raises the volatility of the EU transition rate. In the standard model without job-to-job transitions such as Elsby and Michaels (2013), only this effect would be present, and thus $\tau$ can be used to control the volatility of the EU transition rate. However, in our model with job-to-job transitions, the reduced volatility of vacancies discussed in the previous paragraph has the effect of suppressing the volatility of the layoff rate.

In our model, there is an important link between volatilities of vacancies and the layoff rate. Remember that the firm does not need to pay the firing cost when workers leave the firm through job-to-job transitions. The procyclical job-to-job transitions, which are driven by fluctuations in vacancies, imply that the chance of the firm needing to appeal to layoffs (even if they are costly) increases (declines) in a recession (boom) for a given target level of employment growth. Consider a firm that is seeking to reduce employment through layoffs on top of job-to-job transitions. These are the firms with zero hiring rates on the negative side of Figure 6. For these firms, the (negative) employment growth rate is simply the sum of the quit (job-to-job transition) rate and the layoff (EU transition) rate. Thus, for a given rate of net employment reduction, an increase in the quit rate during a boom means that the firm does not need to fire as many workers as in other periods. In terms of volatilities of these variables, smaller fluctuations in the quit rate (driven by the fluctuations in vacancies) mean smaller fluctuations in the layoff rate. In other words, the composition of separations (quits vs. layoffs) fluctuates more (less) with a higher (lower) volatility of vacancies to achieve the same rate of employment reduction.

The explanation so far applies to how each firm mixes quits and layoffs to achieve a given negative employment growth rate. As mentioned above, the direct effect of a lower firing cost would be to raise the volatility of the EU transition rate. One can think of this direct effect as resulting from a larger fluctuation in the employment growth distribution over the business cycle. The insensitivity of the volatility of the EU transition rate with respect to the level of the firing cost results from the two opposing effects. The side effect of this feature
Table 8: Second-Moment Properties: Sensitivity Analysis

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>( \alpha = 0.72 )</th>
<th>( \psi = 0.6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Relative</td>
<td>Corr. w/ SD Output</td>
<td>Relative Corr. w/ SD Output</td>
</tr>
<tr>
<td>Worker flows</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E to U</td>
<td>8.235</td>
<td>-0.778</td>
<td>8.827 -0.750</td>
</tr>
<tr>
<td>E to E</td>
<td>7.406</td>
<td>0.991</td>
<td>8.826 0.984</td>
</tr>
<tr>
<td>U to E</td>
<td>6.344</td>
<td>-0.599</td>
<td>6.879 -0.559</td>
</tr>
<tr>
<td>Transition rates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EU transition rate</td>
<td>8.758</td>
<td>-0.826</td>
<td>9.387 -0.801</td>
</tr>
<tr>
<td>EE transition rate</td>
<td>6.607</td>
<td>0.986</td>
<td>7.974 0.976</td>
</tr>
<tr>
<td>UE transition rate</td>
<td>6.810</td>
<td>0.983</td>
<td>8.273 0.973</td>
</tr>
<tr>
<td>Job flows</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Creation rate</td>
<td>2.457</td>
<td>0.125</td>
<td>2.867 0.257</td>
</tr>
<tr>
<td>Destruction rate</td>
<td>2.492</td>
<td>-0.350</td>
<td>2.356 -0.152</td>
</tr>
<tr>
<td>Stocks</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemp. rate</td>
<td>11.588</td>
<td>-0.905</td>
<td>13.168 -0.903</td>
</tr>
<tr>
<td>Vacancies</td>
<td>10.310</td>
<td>0.943</td>
<td>12.901 0.935</td>
</tr>
</tbody>
</table>

Notes: See notes to Table 5. The model is recalibrated for both cases to match the steady-state statistics and output volatility. \( \tau \) is kept at the same value and thus the volatility of the UE transition rate is not targeted in the two alternative calibrations. Recalibrated parameters are set to the following values (those not listed are set to the same values as in the benchmark calibration). \( \alpha = 0.72: \kappa_0 = 0.011, \gamma = 0.115, \sigma_x = 0.093, \sigma_z = 0.0020, \) and \( L = 10.147 \). \( \psi = 0.6: \mu = 0.424 \) and \( \sigma_z = 0.0020. \)

is that, in our model with job-to-job transitions, it is difficult to control the volatility of the EU transition rate.

8 Results under Alternative Calibrations

In this section, we examine the sensitivity of the model’s quantitative properties with respect to the following two parameters: the curvature parameter of the production function \( \alpha \) and the elasticity parameter (with respect to job seekers) of the matching function \( \psi \). Specifically, we raise \( \alpha \) from 0.67 to 0.72. Changing this parameter appears to be a sensible thing to examine not only because the original value is only loosely chosen but also because it is a priori conceivable that the change in the marginal product schedule may make significant differences in our results, for example, through the changes in the wage schedule and the firm-size distribution. Next, we change \( \psi \) from 0.5 to 0.6, given that the literature often uses different values within a certain range for this parameter. For these two changes, we

---

42We also considered the effect of changing the worker bargaining power \( \eta \) from 0.5 to 0.6. The basic conclusion from this exercise is the same as for the two cases, and thus we focus on the effects of the two parameters.

43See Brügemann (2008) for a discussion on this issue.
recalibrate the model by matching all steady-state statistics and the output volatility. We keep the value of the firing cost parameter the same as in the benchmark model. In other words, we do not match the volatility of the UE transition rate in these calibrations. (See notes to Table 8 for the parameter values.)

Table 8 presents the results. The model under a higher $\alpha$ performs reasonably well: All correlation patterns are very close to those under the benchmark calibration. Note, however, that the model is somewhat “too volatile.” In particular, the relative volatility of the UE transition rate now goes up to 8.3, while in the benchmark calibration it was 6.8, which is calibrated to match the observed one. Matching the volatility of the UE transition rate requires adjusting $\tau$. In this case, the model’s overall behavior will become closer to that under the benchmark calibration.

The properties of the model with $\psi = 0.6$ remain relatively close to those under the benchmark calibration. The main differences lie in the higher volatility of the EU transition rate and the lower volatility of the UE transition rate (compared to the benchmark case). These changes make both the job creation rate and the job destruction rate countercyclical, given that the effect of the EU transition rate on these variables becomes stronger. However, the correlation patterns of the remaining variables stay intact and consistent with the empirical evidence. Overall, the sensitivity analysis here demonstrates that the model’s overall features are not significantly affected by the alternative parameter values.

9 Conclusion

In this paper, we have studied the quantitative properties of a multiple-worker matching model with OJS. We show that the model is capable of replicating the overall cyclical patterns of worker flows and job flows simultaneously. Procyclical job-to-job transitions, coupled with countercyclical worker flows between unemployment and employment, are important for this success. However, it is also shown that the cyclical features of total separation and hiring rates differ significantly from those of job flows. The difference arises because job-to-job transitions (as a part of separations) occur at the firms that are “creating” jobs on net and, similarly, hires occur at firms that are “destroying” jobs on net. Further, we also find that our model with job-to-job transitions exhibits the “vacancy chain” whereby poaching of workers by higher-paying firms prompts the further increase in vacancy posting at lower-paying firms that are trying to expand. This feature is absent in the model without OJS.

Our results, however, are by no means perfect. One issue not discussed here is that the model misses important features of the cross-sectional employment growth distribution studied by Davis et al. (2012) (our calibration only matches the dispersion of the annual growth rate distribution). Davis et al. (2012) show that, at a quarterly frequency, roughly 15% of establishments report no net employment change. Our model is simply unable to capture this pattern because the firm always loses some workers through job-to-job transitions, even when the firm’s idiosyncratic state does not change. More generally speaking, firm-level employment adjustments in our model are too responsive to the shocks: there are more firms that are making relatively large adjustments than implied by the observed data.
Introduction of nonconvex capital adjustment costs could work as a device to slow down employment adjustments, thus making the model more closely match the data. This extension would be a challenging yet fruitful avenue for future research. We also abstract away from the employed workers’ search decision by assuming they are looking for a job with a fixed reduced search intensity. However, it is clearly the case in reality that the search intensity of workers at struggling (and thus shrinking) firms is higher. Faberman and Nagypál (2008) and Davis et al. (2012) call this the “abandon ship” effect. Endogenizing the search decision would allow us to further analyze the cross-sectional heterogeneity and time-series behavior of quits and layoffs.

Appendices

A Computation

Details of the numerical procedure to solve for the steady-state equilibrium and the dynamic stochastic equilibrium are as follows.

A.1 Steady-State Equilibrium

1. Guess equilibrium market tightness $\theta$, and $\Omega_H$ and $\Omega_K$, the parameters of beta distributions that characterize $H(w)$ and $K(w)$, respectively. Each of $\Omega_H$ and $\Omega_K$ includes four parameters: two of them determine the upper and lower bound of the distribution and the other two are the shape parameters for the beta distribution.\(^{44}\)

2. A guess of $\theta$ immediately gives $q(\theta)$ and $f(\theta)$. A guess of $\Omega_H$ is used to construct the approximated wage distribution $H(w)$. A guess of $\Omega_K$ gives the approximated wage offer distribution $K(w)$. Using $H(w)$ and $K(w)$, we can compute $h(w)$ and $k(w)$.

3. Guess $D(x, n')$, the expected marginal profit function of a firm with type $(x, n')$. Guess also the optimal employment adjustment rule $n' = g(x, n)$. This function is used to obtain the wage function.

4. Use the first-order conditions (10) and (11) to obtain the $(s, S)$ band, $\bar{n}^*(x, n)$ and $\underline{n}^*(x, n)$, of the firm’s employment adjustment rule. These two functions are used to update the firm’s optimal employment adjustment rule $n' = g(x, n)$.

5. Using the updated optimal employment adjustment rule $n' = g(x, n)$, (14), and (15), update the firm’s marginal profit function $D(x, n')$.

6. Check convergence of $D(x, n')$ and $n' = g(x, n)$, based on the distance between the old and updated functions. If convergence is obtained, go to the next step. Otherwise, update $D(x, n')$ and $n' = g(x, n)$ and go back to step 4.

\(^{44}\)Note that the beta distribution is defined on $[0, 1]$ and we map wages into this interval.
7. Using the optimal employment adjustment rule \( n' = g(x, n) \) and the stochastic process for \( x \), simulate the economy until the invariant type distribution \( m(x, n) \) is obtained.

8. Using \( m(x, n) \), update \( H(w) \) and \( K(w) \) as follows:
   \[
   H(w) = \frac{\int 1[w > w(x, g(x, n))] g(x, n) dm}{\int g(x, n) dm} \tag{16}
   \]
   \[
   K(w) = \frac{\int 1[w > w(x, g(x, n))] \max[g(x, n) - (1 - k(w(x, g(x, n)))) n, 0] dm}{\int \max[g(x, n) - (1 - k(w(x, g(x, n)))) n, 0] dm}, \tag{17}
   \]
   where \( 1 \) is an indicator function. These distributions can be used to obtain updated parameters \( \Omega_H \) and \( \Omega_K \).

9. Compute the total labor force (population) consistent with the stationary distribution \( m \) and the unemployment rate \( U \) as
   \[
   L = \frac{\int g(x, n) dm}{1 - U}.
   \]
   Note that \( U \) is fixed at a targeted level. In other words, we don’t need to find an equilibrium \( U \) because we impose it to be exactly our target value. The efficiency-weighted number of searchers \( S \), the total number of vacancies (normalized by the labor force) \( V \), and the labor market tightness \( \theta \) are calculated by
   \[
   S = \gamma(1 - U) + U, \tag{18}
   \]
   \[
   V = \frac{1}{L} \int \frac{\max[g(x, n) - (1 - k(w(x, g(x, n)))) n, 0]}{h(w(x, g(x, n)))} dm, \tag{19}
   \]
   \[
   \theta = \frac{V}{S}. \tag{20}
   \]

10. Check convergence of \( \{\theta, \Omega_H, \Omega_K\} \). If the distance between the guess and the updated numbers is smaller than the predetermined value, then stop. Otherwise, update \( \{\theta, \Omega_H, \Omega_K\} \) and go back to step 2.

### A.2 Dynamic Stochastic Equilibrium

1. Set up the equilibrium forecasting functions. Note that the type distribution \( m \) is replaced by \( U \). Also note that \( H(w) \) and \( K(w) \) are parameterized by the beta distributions. Let \( \Omega_H = \{\omega^H_i\}_{i=1,\ldots,4} \) and \( \Omega_K = \{\omega^K_i\}_{i=1,\ldots,4} \) be set of parameters associated with \( H(w) \) and \( K(w) \), respectively. \( \{U', \theta, \Omega_H, \Omega_K\} \) is the list of variables to be forecast through the following forecasting rules:
   \[
   \log U' = \phi_0^H + \phi_1^H \log z + \phi_2^H \log U \tag{21}
   \]
   \[
   \log \theta = \phi_0^\theta + \phi_1^\theta \log z + \phi_2^\theta \log U \tag{22}
   \]
   \[
   \log \omega^H_i = \phi_0^{2+i} + \phi_1^{2+i} \log z + \phi_2^{2+i} \log U \quad i = 1, \ldots, 4 \tag{23}
   \]
   \[
   \log \omega^K_i = \phi_0^{6+i} + \phi_1^{6+i} \log z + \phi_2^{6+i} \log U \quad i = 1, \ldots, 4 \tag{24}
   \]
Let \( \Phi \) be the vector of the parameters \( \{\phi_0^i, \phi_1^i, \phi_2^i\}_{i=1,...,10} \).

2. With a guess for \( \Phi \), and current \( z \) and \( U \), we can predict \( q(\theta), f(\theta), H(w), K(w), \) and \( U' \).

3. Guess the expected marginal profit of the firm with a type \((x,n')\) under the aggregate state \((z,U')\), \( D(x,n',z,U') \).

4. Using the first-order conditions (10) and (11), compute \((s,S)\) band \( \mathbf{u}^*(x,n,z,U) \) and \( \mathbf{u}^*(x,n,z,U) \). These two functions characterize the firm’s optimal employment adjustment rule \( n' = g(x,n,z,U) \).

5. Using the optimal employment adjustment rule \( n' = g(x,n,z,U) \) and the envelope conditions (14), update the firm’s marginal profit function \( D(x,n',z,U') \) from (15).

6. Check convergence of \( D(.) \). If the distance between the initial and updated \( D(.) \) and \( g(.) \) are smaller than a predetermined tolerance level, go to the next step. Otherwise, update \( D(.) \) and \( g(.) \) and go back to step 3.

7. Using the optimal employment adjustment rule \( n' = g(x,n,z,U) \) and the stochastic processes for \( z \) and \( x \), simulate the economy for \( T = T_0 + T_1 \) periods. The economy consists of a panel of 1 million establishments over 1,320 periods with \( T_0 = 120 \) and \( T_1 = 1,200 \). The simulation starts with the steady-state distribution of \( m(x,n) \). The unemployment rate in the initial period can be obtained from the distribution. In each period, compute \( H(w) \) and \( K(w) \) following the formulas (16) and (17). The unemployment rate in each period is calculated as

\[
U = \frac{L - \int g(x,n,z,U)dm(x,n)}{L}. \tag{25}
\]

The number of vacancies \( V \), the number of job seekers \( S \), and the labor market tightness \( \theta \) in each period are calculated by the formulas (18), (19), and (20).

8. Using the sequence \( \{z_t, U_t, \theta_t, H_t(w), K_t(w)\}_{t=T_0+1,...,T} \), run OLS regressions (21) through (24) and obtain the new set of coefficients \( \Phi \).

9. Check convergence of \( \Phi \). If the distance between the old and new \( \Phi \) is smaller than a predetermined tolerance level, then stop. Otherwise, update \( \Phi \) and go back to step 2.

References


