WORKING PAPERS
RESEARCH DEPARTMENT

WORKING PAPER NO. 13-46
FISCAL STIMULUS AND DISTORTIONARY TAXATION

Thorsten Drautzburg
Federal Reserve Bank of Philadelphia

Harald Uhlig
University of Chicago

First Draft: January 2, 2010
This Draft: November 13, 2013

RESEARCH DEPARTMENT, FEDERAL RESERVE BANK OF PHILADELPHIA

Ten Independence Mall, Philadelphia, PA 19106-1574 • www.philadelphiafed.org/research-and-data/

Fiscal Stimulus and Distortionary Taxation*

Thorsten Drautzburg
Federal Reserve Bank
of Philadelphia

Harald Uhlig†
University of Chicago

First draft: January 2nd, 2010
This revision: November 13, 2013

Abstract

We quantify the fiscal multipliers in response to the American Recovery and Reinvestment Act (ARRA) of 2009. We extend the benchmark Smets-Wouters (2007) New Keynesian model, allowing for credit-constrained households, the zero lower bound, government capital, and distortionary taxation. The posterior yields modestly positive short-run multipliers around 0.53 and modestly negative long-run multipliers around -0.36. We explain the central empirical findings with the help of a simple three equation New Keynesian model with sticky wages and credit-constrained households.

Keywords: Fiscal Stimulus, New Keynesian model, liquidity trap, zero lower bound, fiscal multiplier

JEL codes: E62, E63, E65, H20, H62

---

*This research has been supported by the NSF grant SES-0922550.

†Address: Thorsten Drautzburg, Federal Reserve Bank of Philadelphia, Email: tdrautzburg@gmail.com. Harald Uhlig, Department of Economics, University of Chicago, 1126 East 59th Street, Chicago, IL 60637, U.S.A, email: huhlig@uchicago.edu. Additional affiliations for H. Uhlig: CentER, NBER and CEPR. We are grateful for the useful feedback from a number of seminar and conference audiences. The views expressed herein are the authors' views only. They do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia, the Federal Reserve System, or its Board of Governors. This paper is available free of charge at `www.philadelphiafed.org/research-and-data/publications/working-papers/`.

1 Introduction

Five years after the financial crisis, fiscal policy is at the heart of policy debates. Many European governments struggle with the debt they accumulated because of their fiscal response to the financial crisis. In the US, the debt ceiling, and cuts to transfer programs or direct tax increases to finance the government deficit are key policy issues that point to the costs of increased government spending. At the same time, it is often argued that the effect of government spending on GDP and unemployment is larger in a financial crisis, making discretionary spending more attractive. In this paper, we bring both aspects of "fiscal stimulus" together. We quantify the size, uncertainty, and sensitivity of fiscal multipliers in response to a fiscal stimulus, as in the American Recovery and Reinvestment Act (ARRA) of 2009 in the United States, using an extension of a benchmark New Keynesian model.

Purists might disagree with the focus on the size of fiscal multipliers. Policy should care about welfare, rather than derivative measures such as GDP or unemployment. Policy should solve a Mirrlees-Ramsey problem and use the best combinations of available tools to maximize welfare, subject to constraints imposed by markets and the asymmetry of information. We do not disagree. Indeed, there is considerable literature on these topics. We address welfare issues in section 6, but they are not the main focus of this paper.

Many public debates focus on the effects of fiscal spending on GDP and unemployment. Economists have the tools to answer these questions, and therefore, perhaps they should. Several recent papers have addressed these issues. This paper seeks to contribute to this emerging literature. We quantify the relative importance of the optimistic analysis of fiscal policy in New Keynesian models of the zero lower bound (ZLB) (e.g., Eggertsson (2010) and Christiano et al. (2011)) relative to the pessimistic assessment of fiscal policy in a neoclassical growth model with distortionary taxes as in Uhlig (2010b). We therefore use a model that adds many New Keynesian frictions to the backbone of the neoclassical growth model with distortionary taxes. Building on Smets and Wouters (2007), frictions in our model include sticky prices and wages, but we also incorporate the ZLB and credit-constrained consumers.

In a nutshell, we find: While the benchmark long-run multiplier is modestly negative, rather than substantially negative as in the pure neoclassical model, and while the precise answer is sensitive to some key assumptions and uncertain parameters, much of the pessimistic assessment survives indeed. For a benchmark parameterization, we find modestly positive short-run multipliers with a posterior mean of 0.53 and modestly negative long-run multipliers centered around -0.36. The multiplier is particularly sensitive to the fraction of transfers given to rule-of-thumb consumers, the anticipated length of the zero lower bound, and is nonlinear in the degree of price and wage stickiness.

Using a simple three-equation analogue to our full New Keynesian model, we show that three effects are crucial for understanding our results. First, higher future distortionary taxes have negative neoclassical wealth effects today. Because taxes respond only slowly to the deficit, disincentives to work and inflationary pressure tend to arise after the ZLB. Depressed private consumption after the ZLB lowers private demand at the ZLB as agents smooth consumption. While this first effect may suggest that government expenditure should be financed immediately, this ignores the second effect of taxes in our model. The second effect is a traditional Keynesian demand effect of taxes, which comes from rule-of-thumb agents consuming their current net income each period. This demand effect can push the multiplier below one even if taxes are only adjusted at the ZLB. Both negative effects of distortionary taxes contrast with the influential analysis of fiscal policies at the ZLB by Eggertsson (2010). He shows that tax increases can increase consumption demand through a substitution effect: Labor tax increases may lower the real interest rate by raising inflation at the ZLB. This positive intertemporal substitution effect is the third key effect in the model. While the overall effect of deficit-financed stimulus on private demand is therefore theoretically ambiguous, we find that the overall effect is negative in our estimated quantitative model.

We view the following elements as important for a quantitative analysis of fiscal stimulus at the ZLB. First, the ZLB has to be incorporated: In Eggertsson (2010) and Christiano et al. (2011) monetary policy and its restrictions due to the ZLB on interest rates matter substantially for the effectiveness of fiscal stimulus. Second, credit-constrained agents are important for analyzing government expenditure and transfers. Galí et al. (2007) have made this point for government consumption, while Coenen et al. (2012) argue that transfers to credit-constrained households matter substantially, similar to our paper. In contrast to our paper, they abstract from distortionary taxes and the empirically observed build-up in the stimulus spending. Third, the analysis of government expenditure programs has long recognized that fiscal stimulus takes time in practice. This is also true for the ARRA, despite calls for immediate actions as in Spilimbergo et al. (2008). It therefore serves as a useful benchmark and example for the speed at which fiscal policy tools can be deployed, as emphasized by Cogan et al. (2010). Beyond these elements with a Keynesian flavor, we acknowledge that government expenditures are financed eventually with distortionary taxes, creating costly disincentive effects, a point emphasized by Uhlig (2010b). Finally, model coefficients are uncertain and results are sensitive to specific assumptions. We therefore use Bayesian estimation techniques as well as sensitivity analysis to quantify the uncertainty in our answers. As Leeper et al. (2011) have pointed out, the New Keynesian model we use here together with its prior is already an important determinant of our answers. This is desirable: The model assumptions should be crucial. The Bayesian estimation serves to quantify the results more sharply and to inform us about the overall posterior uncertainty.

The analysis here has much in common with, and is inspired by, Cogan et al. (2010), but there are a number of important differences. First, our analysis emphasizes the determinants of the fiscal multiplier, both qualitatively and quantitatively. Second, we consider a more detailed fiscal sector to better reflect the ARRA stimulus, even though we also start from the Smets and Wouters (2007) New Keynesian model. The fiscal sector in our model differs along several dimensions to better reflect the empirical stimulus. In our model, revenues are raised through distortionary taxation, and we introduce credit-constrained consumers in our benchmark model, breaking Ricardian equivalence and making transfers a central part of our analysis. Indeed, transfers comprise 59% of the ARRA, but their analysis has (necessarily) been absent in Cogan et al. (2010). Another sizable part of the ARRA takes the form of government investment; we therefore extend the model to feature government capital. Finally, we study the interaction with monetary policy, when constrained by a ZLB. We focus on a deterministic duration of the ZLB but show in an extension that our results are robust to endogenizing its duration. Besides these modeling extensions, we discuss the intuition behind the main determinants of the fiscal multiplier. In our analysis, we highlight the importance of rigid wages rather than rigid prices in the presence of credit-constrained households: With rigid wages and constrained households, labor taxes have an additional aggregate demand effect, absent from a model with only sticky prices.

Our focus is on the positive analysis of the ARRA, not on optimal fiscal policy. Indeed, optimal policy need not involve government expenditure at all, as pointed out by Correia et al. (2010). They have shown that when consumption tax rates are a policy instrument, adjusting tax rates can substitute for adjusting interest rates, thereby circumventing the ZLB.

We compute the welfare effects of the policy intervention separately for credit-constrained and unconstrained agents. The effects on unconstrained agents are significantly negative but small because they are close to their unconstrained optimum. As credit-constrained agents have a higher rate of time preference, we consider a range of rates of time preference, up to 30% higher than those of unconstrained agents on an annual basis. If agents are not too impatient, the welfare gains through higher short-run consumption are more than offset by the disutility of hours worked and the lower consumption in the transition back to the balanced growth path. However, starting at rates of time preference about 15% higher than that of unconstrained agents, the welfare effects become significantly positive for constrained agents.

The New Keynesian models underlying our analysis have also been criticized considerably for their lack of a financial sector, a feature likely to be important for understanding the events of 2008 (see Uhlig, 2010a; Krugman, 2009; Buiter, 2009). We agree with this critique and therefore feature a financial friction per the "short cut" of allowing for time-varying wedges between the central bank interest rate, government bond rates, and the return to private capital, following Hall (2011). Our estimates show that these wedges are indeed the key to understanding the recession. Understanding their nature more deeply should therefore be high on the research agenda, but it is not the focus of this paper and is beyond its scope. In our paper, incorporating these financial frictions enables us to plausibly endogenize the ZLB duration.

Several recent contributions have analyzed fiscal policy in the workhorse three-equation New Keynesian model in more detail. Farhi and Werning (2012) show that multipliers are strictly increasing in price flexibility and backloading of the stimulus spending during the ZLB. Our simple model shows that when taking the financing of fiscal stimulus into account, more flexibility can, in contrast, lower the financed spending multiplier. Braun et al. (2012) and Mertens and Ravn (2013) analyze a non-linear version of the workhorse model. Braun et al. (2012) question whether standard log-linearization is appropriate for analyzing fiscal policy at the ZLB since the economy may be far away from its steady state. Their non-linear analysis implies substantially smaller multipliers. Mertens and Ravn (2013) argue that the cause of the ZLB matters and propose a sunspot shock as the underlying cause. A sunspot ZLB equilibrium implies in their analysis that labor tax cuts are actually more effective than government spending at stimulating the economy.[1] While these mechanisms are outside our model, we view their pessimistic analysis of the effectiveness of fiscal stimulus as complementary to our findings. Fernandez-Villaverde et al. (2011) point out that at the ZLB, future labor market reforms are particularly important, as the positive wealth effect stimulates demand during the ZLB. This argument is the flip side of our finding that the negative wealth effect due to distortionary taxes also matters for low short-run multipliers.

Aside from the contributions previously cited, the analysis here is related to a number of additional structural quantitative contributions, notably by

[1] Aruoba and Schorfheide (2012) estimate a small-scale sunspot model of the ZLB, but find that sunspot regime switches played only a minor role for the US.

Erceg and Linde (2010), as well as Leeper and various co-authors (Davig and Leeper, 2009; Leeper et al., 2010, 2009). In a model that also features distortionary taxes, rule-of-thumb consumers, and financial frictions, Erceg and Linde (2010) point out that the marginal multiplier differs from the average multiplier: If the stimulus is successful, the economy leaves the binding ZLB earlier, and the effect of additional spending is reduced. We address this issue by endogenizing the duration of the ZLB in robustness tests. A key difference is their focus on the short run when the effects of adjusting distortionary taxes instead of transfers matter less. Leeper et al. (2010) allow future government consumption and transfers to adjust in order to rebalance the government budget, and they find that adjusting spending and transfers in addition to taxes raises the multiplier. Leeper et al. (2009) point out the importance of productive government investment and government capital, Davig and Leeper (2009) allow for fiscal policy to switch between passive and active regimes in a New Keynesian model. Interestingly, they find the largest difference in multipliers due to switches in the monetary policy regime, which we address by varying the ZLB duration.

Besides the structural literature, a growing body of literature has empirically analyzed multipliers and employment effects of fiscal policy. A contribution closely related to our paper is by Conley and Dupor (2012), who investigate the ARRA employment effects at the state level. They find that only government employment has increased significantly, while private employment effects are not significantly different from zero. Even though they consider a ZLB episode, their findings are in line with the aggregate time series evidence in Barro and Redlick (2011) and Ramey (2011). In their sample, Barro and Redlick (2011) find short-run multipliers significantly below one that do not vary significantly with the business cycle. Ramey (2011) concludes that government spending does not tend to increase private activity – suggesting that the multiplier is not larger than one. While her finding on the overall private-sector activity is in line with our main finding, she does not find significant effect of marginal tax rates on private activity, unlike our model. This may be because tax rates respond only slowly to deficits, as in our model, and may therefore be hard to detect in an estimation exercise. Also, Barro and Redlick (2011) do find significant negative effects of higher marginal tax rates on GDP. Other estimates of multipliers also point to the importance of financing effects, even at the state level, where fiscal policy can be analyzed as if interest rates were held constant. While Shoag (2010) finds multipliers significantly in excess of unity at the state level using pension windfalls, Clemens and Miran (2012) find multipliers smaller than one when using variation in state spending financed by states themselves. Clemens and Miran (2012) explain this difference with the importance of Ricardian elements, in line with the theoretical analysis in Farhi and Werning (2012).

The paper is structured as follows. Section 2 gives an overview of the model. Section 3 discusses the estimation and calibration procedure. It also provides a decomposition of the shocks driving the 2007–2009 recession. Section 4 presents the main results on the multiplier. Section 5 provides a sensitivity analysis and explains empirical results using an extended version of the three-equation New Keynesian model. Section 6 discusses welfare effects. The analysis is complemented by a detailed Technical Appendix which provides all model details as well as code for replicating our results or calculating other fiscal experiments. Section 7 concludes.

2 The model

The model is an extension of Smets and Wouters (2007), and we refer the reader to that paper and to the Technical Appendix for the complete details. Here we provide a brief overview and describe the extensions.

The Smets and Wouters (2007) model is a New Keynesian model, set in discrete time. There is a continuum of households. Workers supply homogeneous labor. Unions differentiate the labor supplied by households and set wages for each type of labor in monopolistic competition. Wages are Calvo-sticky. There is a continuum of intermediate good firms. They supply intermediate goods in monopolistic competition. They set prices. Prices are Calvo-sticky. Final goods use intermediate goods. Final goods are produced in perfect competition. Households have preferences for final goods, allowing for habit formation and leisure. Capital is produced by investing in final goods, subject to adjustment costs: Given installed capital and previous-period investment, the marginal product of investment for producing new capital is decreasing. There is variable capital utilization.

We extend the model with several features: We constrain the interest rate set by the central bank to be nonnegative. The government raises revenues with distortionary taxation. We introduce credit-constrained consumers. There is productive government capital. We introduce a wedge between various returns as a stand-in for financial frictions. We adopt the notation convention that variables indexed as t are known in period t.

2.1 The zero lower bound

The monetary authority follows a Taylor-type rule, but interest rates may be held constant for a deterministic period of time. We alternatively also consider interest bounded below by a constant slightly above zero, resulting in an endogenous ZLB duration. It is easier to describe these scenarios in their log-linearized form: For the original version, the reader is referred to the Technical Appendix.

In our benchmark scenario, the central bank keeps the interest rate at its historical level of 2008:4 for k quarters. Households fully anticipate this policy. Let $\hat{R}_t^{TR}$ denote the log-deviation of the shadow Taylor Rule return, given by:

$$\begin{aligned}\hat{R}_t^{TR} &= \psi_1(1-\rho_R)\hat{\pi}_t + \psi_2(1-\rho_r)(\hat{y}_t - \hat{y}_t^f) \\ &\quad +\psi_3\Delta(\hat{y}_t - \hat{y}_t^f) + \rho_R\hat{R}_{t-1}^{TR} + ms_t\end{aligned}$$

where $\hat{\pi}_t$ is the log-deviation for inflation, $\hat{y}_t$ is the log-deviation for output, $\hat{y}_t^f$ is the log-deviation in the flexible-price version of the economy and ms_t is a shock to the interest rate set by the central bank.

The effective interest rate in our benchmark scenario is then given by:

$$\hat{R}_t^{FFR} = (1 - ZLB_t)\hat{R}_t^{TR} + ZLB_t\hat{R}_0^{FFR},$$

where ZLB_t is an indicator function modeling that takes the value of one while the ZLB binds and is otherwise zero. At the ZLB, the central bank return equals its historical starting value, $\hat{R}_0^{FFR}$.

When endogenizing the ZLB duration, the central bank sets the log-deviation of the central bank return to

$$\hat{R}_t^{FFR} = \max\{(1 - \bar{R}^{FFR}) + \bar{\epsilon}, \hat{R}_t^{TR}\},$$

where $\bar{R}^{FFR}$ is the steady state nominal return and $\bar{\epsilon} > 0$ is a constant set slightly above zero, for technical reasons (and set to $\bar{\epsilon} = \frac{0.25}{400}$ in the numerical calculations, implying a lower bound of 25 basis points for the central bank interest rate). To implement this specification, we need to estimate the counterfactual level of the interest rate R_t^{TR} when the ZLB is binding. The estimation is discussed in section 3.2.

2.2 Households, distortionary taxation, and financial frictions

A fraction $1-\phi$ of the household is unconstrained and solves an infinite-horizon maximization problem. The preferences of such a household, j, are

$$U = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^s \left(\frac{1}{1-\sigma}\left(c_t(j) - h\, c_{t-1}^{aggr}\right)^{1-\sigma}\right) \exp\left(\frac{\sigma-1}{1+\nu}\, n_t(j)^{1+\nu}\right)\right], \quad (2.1)$$

where $c_t(j)$ is consumption of household j, $n_t(j)$ is its labor supply and c_t^{aggr} is aggregate consumption. $h \in [0,1)$ captures external habit formation, σ denotes the inverse of the intertemporal elasticity of substitution, and ν equals the inverse of the labor supply elasticity. Households discount the future by $\beta \in (0,1)$.

Following Trabandt and Uhlig (2011), we assume that the government provides transfers and collects linear taxes on labor income, capital income net of depreciation as well as consumption, adapted to the model here. The budget constraint of household j is therefore given by

$$
\begin{aligned}
&(1+\tau^c)c_t(j) + x_t(j) + \frac{B_t^n(j)}{R_t^{gov}P_t} \\
&\leq \; s_t^{\text{unconstr}} + \frac{B_{t-1}^n(j)}{P_t} + (1-\tau_t^n)\frac{W_t}{P_t}\left(n_t(j) + \lambda_{w,t} n_t^{\text{(aggr)}}\right) + \\
&\quad + \left((1-\tau^k)\left(\frac{R_t^k u_t(j)}{P_t} - a(u_t(j))\right) + \delta\tau^k\right)\left((1-\omega_{t-1}^k)k_{t-1}^p(j) + \omega_{t-1}^k k_{t-1}^{p,aggr}\right) + \frac{\Pi_t^p}{P_t},
\end{aligned}
$$

and the capital accumulation constraint is given by

$$
k_t^p(j) \;=\; \frac{(1-\delta)}{\mu}k_{t-1}^p(j) + q_{t+s}^x\left(1 - \xi\left(\frac{x_t(j)}{x_{t-1}(j)}\right)\right)x_t(j),
$$

where $c_t(j)$ is consumption, $x_t(j)$ is investment, $B_{t-1}^n(j)$ are nominal government bond holdings, $n_t(j)$ is labor, $k_{t-1}^p(j)$ is private capital, and $u_t(j)$ is capacity utilization, all of household j and chosen by household j. R_t^{gov} is the nominal return for the one-period government bond from t to $t+1$ set at date t, $n_t^{\text{(aggr)}}$ is aggregate labor, P_t is the aggregate price level, W_t is aggregate wages, $\lambda_{w,t}$ is the aggregate markup from union-determined wages, R_t^k is the undistorted return on capital, and ω_t^k is a friction or wedge on private capital markets. In the budget constraint, note that ω_t^k enters as a variable known at date $t-1$, so that the distortions to future capital returns affect investment in the current period. Also note that individual losses due to this wedge are redistributed in the aggregate, so that the wedge distorts investment decisions but does not destroy aggregate resources directly. Π_t^p is nominal firm profits, q_{t+s}^x is an investment-specific technology parameter; $\xi(\cdot)$ captures adjustment costs, satisfying $\xi(\mu) = \xi'(\mu) = 0, \xi'' > 0$; τ^c, τ^n, τ^k are taxes; s_t^{unconstr} are real transfers to unconstrained households, all taken as given by household j; and $a(\cdot)$ represents the strictly increasing and strictly convex cost function of varying capacity utilization. In particular, note that taxing capital net of depreciation implies deducting a depreciation rate that depends on capacity utilization. Furthermore, the household receives labor income both directly from working as well as indirectly from the surplus that unions charge on labor; both sources of labor income are taxed.

We assume that the interest rate R_t^{gov} on government bonds, which unconstrained households can freely trade, equals the federal funds rate R_t^{FFR} up to an exogenous friction or wedge ω_t^{gov}:

$$R_t^{gov} = (1+\omega_t^{gov})R_t^{FFR}.$$

Our government bond shock takes the place of the discount rate shock to β in Smets and Wouters (2007) and models such as Christiano et al. (2011). It is a reduced form to capture preference for liquidity among investors. It also captures lower borrowing costs for the government when ω_t^{gov} is low. In contrast to the liquidity preference shock ω_t^{gov}, ω_t^k is a financial friction which captures shocks in the intermediation between households and firms.[2] By including two different financial friction shocks, our model is flexible enough to allow the data to determine the relative importance of the two shocks.

We assume that a fraction $\phi \in (0, 0.5)$ of the households is credit-constrained. In their version of the budget constraint, $B_{t-1}^n(j) = 0$, $x_t(j) = 0$ and $k_{t-1}^p(j) = 0$, i.e., these households do not save or borrow. They receive profit income from intermediate producers (which equals zero in the steady state). Put differently, the budget constraint of a credit-constrained household j is

$$\begin{aligned} &(1+\tau^c)c_t(j) \\ &\leq \quad s_t^{\text{constr}} + (1-\tau_t^n)\frac{W_t}{P_t}\left(n_t(j) + \lambda_{w,t}n_t^{(\text{aggr})}\right) + \frac{\Pi_t^p}{P_t} \end{aligned}$$

where s_t^{constr} is the transfers to credit-constrained agents. As a justification, one may suppose that credit-constrained households discount the future substantially more steeply and are thus uninterested in accumulating government bonds or private capital, unless their returns are extraordinarily high. Conversely, these households find it easy to default on loans and are therefore not able to borrow. We hold the identity of credit-constrained households and

[2] Ilut and Schneider (2011) propose increased ambiguity as an alternative interpretation which also increases wedges between safe and risky assets.

thereby their fraction of the total population constant. Note that we allow the transfers s_t^{constr} to constrained households to differ from the transfers s_t^{unconstr} to the unconstrained households.

Wages are set by unions on behalf of the households, recognizing that each differentiated wage is Calvo-sticky. Since the majority of workers in the unions are unconstrained households, wages are set according to their preferences. Firms hire workers randomly from both types of households, so that labor supplied by both types of households is the same in equilibrium.

2.3 Government capital and policy feedback rules

As the ARRA contains government investment, we wish to feature government capital as productive input. In order to maintain the assumption of perfect competition at the firm level, we also wish to keep the final goods production function to have constant returns to scale. We therefore assume that government capital K_{t-1}^g enters private production as an externality for individual intermediate-goods firm, similar to Barro and Sala-i Martin (1992). To obtain an aggregate constant-returns-to-scale production function before fixed costs, we assume that the externality of K_{t-1}^g at the firm level is relative to aggregate output, before fixed costs.

Specifically, the technology of intermediate firm i is given by

$$Y_t(i) = \tilde{\epsilon}_t^a \left(\frac{K_{t-1}^g}{\int_0^1 Y_t(\iota) d\iota + \Phi \mu^t} \right)^{\frac{\zeta}{1-\zeta}} (K_t^{\text{eff.}}(i))^\alpha (\mu^t n_t(i))^{1-\alpha} - \mu^t \Phi,$$

where Φ are fixed costs, $K_t^{\text{eff.}}$ is effective capital used by firm i, created from aggregate private capital,

$$K_t^{\text{eff.}} = u_t k_{t-1}^p (1 - \phi)$$

(assuming symmetric choices for the unconstrained households), where ϵ_t^a is an exogenous, stochastic component of TFP, and where the services of government capital K_{t-1}^g are subject to congestion: What matters is the ratio of government capital to average gross output (i.e., inclusive of the fixed costs). Without price dispersion, the resulting aggregate production function is:

$$Y_t = \epsilon_t^a {K_{t-1}^g}^{\zeta} K_t^{s\alpha(1-\zeta)} (\mu^t n_t)^{(1-\alpha)(1-\zeta)} - \mu^t \Phi, \quad \epsilon_t^a \equiv (\tilde{\epsilon}_t^a)^{1-\zeta},$$

where TFP in terms of the private factors of production is

$$TFP = \epsilon_t^a {K_{t-1}^g}^{\zeta} \mu^{(1-\alpha)(1-\zeta)t}.$$

We assume that the accumulation of government capital is symmetric to the accumulation of private capital (i.e., is subject to a similar technology),

$$k_t^g = \frac{(1-\delta)}{\mu} k_{t-1}^g + q_t^g \left(1 - S_g\left(\frac{x_t^g + \epsilon_t^{x,g}}{x_{t-1}^g + \epsilon_{t-1}^{x,g}}\right)\right)(x_t^g + \epsilon_t^{x,g})$$

where $S_g(\mu) = S_g'(\mu) = 0, S_g''(\cdot) > 0$ represent adjustment costs, $q_t^{x,g}$ is a shock to the government-investment-specific technology parameter, and $\epsilon_t^{x,g}$ is additional, exogenous government investment. We assume that the capacity utilization of government capital, and therefore its depreciation, is constant. We assume that the government chooses investment to maximize the present discounted value of output net of investment costs, except for a discretionary fiscal stimulus, denoted by $\epsilon_t^{x,g}$ and set to zero at the steady state. In other words, the first-order condition of the government determines optimal government investment, while actual government investment may be higher by some amount chosen along the stimulus path. To enforce the expansion of government investment, we stipulate that the government cannot undo the stimulus investment for the first 12 periods but has to provide at least a replacement for the depreciated ARRA investment – otherwise, the optimality condition would imply complete crowding out.

We assume the following feedback rule for labor tax rates, following Uhlig (2010b) (for the full detail, see the Technical Appendix): Break the period-by-period government budget constraint into two parts. On the "right side," there is a "deficit" d_t, prior to new debt and labor taxes

$$\begin{aligned} d_t &= \text{gov.spend.+subs.}_t + \text{old debt repaym.}_t \\ &\quad -\text{cons.tax rev.,cap.tax rev.}_t - \bar{\tau}^l \,\text{lab.income}_t, \end{aligned}$$

which is financed on the "left side" with labor tax revenues and new debt,

$$\tau_t^l \,\text{lab.income}_t + \text{new debt}_t = d_t.$$

Along the balanced growth path, there is a path for the debt level as well as the deficit $\bar{d}_t$. The labor tax rate is then assumed to solve

$$(\tau_t^l - \bar{\tau}^l)\,\text{lab.income}_t = \psi_\tau(d_t - \bar{d}_t) + \epsilon_{\tau,l}$$

where $\epsilon_{\tau,l}$ is a labor tax shock. While Leeper et al. (2010) have shown that different feedback rules matter for policy analysis, we focus on this simple rule to analyze in detail the effect of distortionary tax financing.

2.4 Shocks

We assume that there are 10 stochastic processes driving the economy. Unless stated otherwise, the processes follow independent AR(1)'s in logs: (1) Technology $\tilde{\epsilon}_t^a$, (2) Gov.bond wedge ω_t^{gov}: financial friction wedge between financial funds rate (FFR) and gov't bonds, (3) Priv. bond wedge ω_t^k: financial friction wedge between gov't bond returns and a component of the returns to private capital, (4) Gov. spending plus net export. Co-varies with technology, (5) Investment specific technology q_t^x (rel. price), (6) Gov. investment specific technology q_t^g (rel. price), (7) Monetary policy ms_t, (8) Labor tax rates $\epsilon_{\tau,l}$, (9) markup for prices: ARMA(1,1), and (10) markup: wages: ARMA(1,1).

For the stimulus plan, we proceed differently: Shortly, we add an exogenous multi-year and perfectly foreseen component to government consumption, transfers, and government investment. Formally, these series are given by, for example, $g_t = g_t^{ARRA} + g_t^{stoch}$, with g_t^{ARRA} predetermined at $t = 0$ and as in

Figure 1. g_t^{stoch} is our usual AR(1) process. To incorporate transfer and to decompose government spending into consumption and investment we classify the various spending categories according to the ARRA. We use the estimates by the CBO (CBO, 2009) for the effects of the ARRA by budget title as the source. We take the annual time path for these expenditures directly from the CBO, whereas the distribution within each year is proportional to the Cogan et al. (2010, CCTW) path within each year. Appendix 8.1 contains the details on the components. Figure 1 presents a graphical overview. Essentially, we decompose the CCTW government spending path into a separate consumption and investment path, and add transfers. Importantly, many of the transfers are "front-loaded," so that they occur earlier than government spending, while the "stimulus" government investment occurs later.[3]

This paper vs CCTW: Aggregate | Our "stimulus" in detail

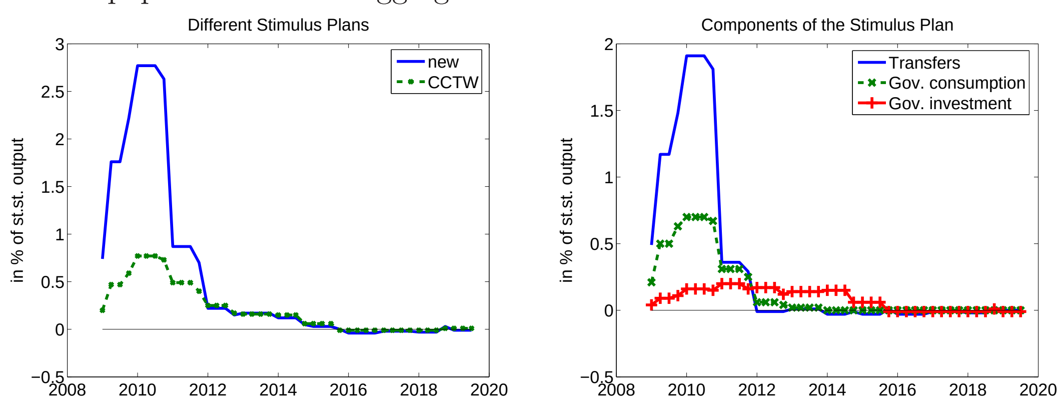


Figure 1: *Our three stimulus components compared with Cogan et al. (2010).*

Furthermore, we consider as our benchmark that the central bank will leave the federal funds rate unchanged at near zero for eight quarters, and that this is fully anticipated as of the 2009:1. A duration of eight quarters already lies above the implied median ZLB duration of six quarters with an endogenous ZLB according to equation (2.1), as shown in Figure 16 in the Appendix. Such a duration is also consistent with Gust et al. (2013). They

[3]We classify transfers to state and local governments as consumption. To the extent that these transfers are spent only partially by local governments with the remainder being an intergovernmental transfer, the effective stimulus program would weigh transfers and investment more, lowering the overall multiplier from the right panel in Figure 7.

find in their non-linear model of the ZLB that less than 3% of ZLB spells last 12 quarters or more and 78% of spells last 4 quarters or less. However, if more negative shocks hit the economy, a longer duration would be generated. We therefore consider other deterministic durations as well. For the numerical calculations, the relaxation algorithm proposed by Juillard (1996) and implemented in Dynare is particularly convenient for the type of forward-simulation (rather than estimation) performed here. By solving a time-varying system of equations backward from terminal conditions, it allows us to incorporate anticipated shocks even when they multiply coefficients, for example, to "switch off" the interest rate rule temporarily.

3 Estimation and Analysis

3.1 Data and Estimation

We solve the model using a log-linear approximation and Dynare. The first-order conditions and their log-linearized versions are in a Technical Appendix, available upon request. We estimate the model, using the following 10 time series: (1) Output: Chained 2005 real GDP, growth rates, (2) Consumption: Private consumption expenditure, growth rates, (3) Investment: private fixed investment, growth rates, (4) Government investment, growth rates, (5) Hours worked: Civilian employment index × average nonfarm business weekly hours worked index, demeaned log, (6) Inflation: GDP deflator, quarterly growth rates, (7) Wages: Nonfarm Business, hourly compensation index, growth rates, (8) FFR: Converted to quarterly rates, (9) Corporate-Treasury bond yield spread: Moody's Baa index – 10 year Treasury bond at quarterly rates, demeaned, (10) Dallas Fed gross federal debt series at par value, demeaned log.

Sources and details for the data are described in Appendix 8.1. We use an updated version of the Smets-Wouters dataset, for the range 1947:2-2009:4, using quarterly data and four periods for the start-up. In difference to the original dataset, we classify consumer durables as investment expenditure. The estimation of the model uses data from 1948:2 up to 2008:4, with the additional four quarters for comparison of the model prediction to the actual evolution and the first four quarters used to presample. Our earlier starting date is motivated by Barro and Redlick (2011), who find that the multiplier is significantly below one only after including the World Wars in their sample. While we cannot go this far back, we choose the longer sample, as it includes the Korean war as well as the Vietnam war buildup. Figure 2 shows the additional evidence from the larger fluctuation in fiscal expenditures available in this larger sample.

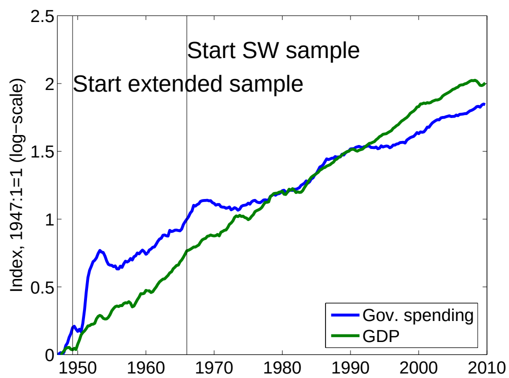


Figure 2: *Comparing our extended sample to the original Smets-Wouters data set. Note the additional variation in government spending in the larger sample.*

We fixed ("calibrated") several parameters a priori. For tax rates and the debt-GDP ratio, we relied on Trabandt and Uhlig (2011). Time averages of government spending components were obtained from the NIPA, Table 3.1 (quarterly), lines 35 (investment), 16 (consumption), and transfers (17). Government consumption includes net exports (line 2 minus line 14 in Table 4.1). To obtain ratios relative to GDP, GDP data from line 1, Table 1.1.5 was used. Following Smets and Wouters (2007), the Kimball curvature parameter is taken from Eichenbaum and Fisher (2007), who set it to roughly match it to their data on the empirical frequency of price adjustment. Following Cooley and Prescott (1995), the depreciation rate is derived from the law of motion for capital and their observation of $\frac{\bar{x}}{\bar{k}} = 0.0076$ at quarterly frequency. The complete list of calibrated parameters, and their comparison to the corresponding parameters in Smets and Wouters (2007), if available, is in Table 1. We calibrate the share of credit-constrained agents in the population to 0.25, a share typical for DSGE models used in the IMF comparison project (Coenen et al., 2012). All agents receive the same per-capita transfers. We estimate our model

Table 1: *Calibrated parameters*

	SW (1966:1–2004:4)	This paper (1948:2–2008:4)
Depreciation δ	0.025	0.0145
Wage markup λ_w	0.5	0.5
Kimball curvature goods mkt. $\hat{\eta}_p$	10	10
Kimball curvature labor mkt. $\hat{\eta}_w$	10	10
Capital tax τ^k	n/a	0.36
Consumption tax τ^c	n/a	0.05
Labor tax τ^n	n/a	0.28
Share credit constrained ϕ	n/a	0.25
Gov. spending, net exports-GDP $\frac{g}{\bar{y}}$	0.18	0.153
Gov. investment-GDP $\frac{\bar{x}^g}{\bar{y}}$	n/a	0.04
Debt-GDP $\frac{\bar{b}}{\bar{y}}$	n/a	$4\times$ 0.63

using Dynare and a fairly standard Bayesian prior. Details on the estimation can be found in Appendix 8.2. The estimates largely agree with those found by Smets and Wouters (2007), leaning somewhat more to more endogenous persistence: The habit parameter is slightly higher, as are estimates of price and wage stickiness, for example. Like these authors' estimates, our estimates also yield a rather small capital share: Our posterior mean is 0.24, while they found 0.19. This is at odds with calibrated values in the literature, see, e.g., Cooley and Prescott (1995), and may play a substantial role in calculating the long-horizon impact of distortionary taxation. We investigate this issue in our sensitivity analysis. The calibrated government investment-to-GDP ratio as well as the estimated growth trend $\mu \approx 1.005$ imply a government share in production of $\zeta \approx 2.30$ percent. The implied Frisch elasticity of labor supply is small at 0.30.[4] The posterior mean for σ implies that there is a mild complementarity between hours and consumption of unconstrained agents, meaning that unconstrained agents want to work less when their consumption falls, as in Christiano et al. (2011).

[4]The Frisch elasticity is given by $(\nu+(1-\sigma^{-1})\bar{n}^{\nu+1})^{-1} = (\nu+(1-\sigma^{-1})\frac{1-\alpha}{(1-h)\bar{C}^{RA}/\bar{Y}})^{-1}$.

3.2 Decomposing the 2007–2009 recession

The model allows the decomposition of movements in our 10 macroeconomic time series into 10 ten shocks that caused them. Beyond helping to understand what caused the recent recession, this analysis is crucial for simulating stimulus effects when the ZLB has an endogenous duration: The interest rate prescribed by the Taylor rule depends on the levels of inflation, the output gap, the change in interest rates, and monetary policy shocks. By estimating the historical state of the economy, we can estimate and simulate the level of the desired interest rate and thereby compute the implied duration of the ZLB.

Financial frictions are important determinants of nominal interest rates. They drive a wedge between returns on government bonds, private capital, and the FFR. In our model, we can decompose the gap between the federal funds rate R_t^{FFR} and the return on private capital R_t^{kp}, up to first order, into ω_t^b and ω_t^k, after rescaling appropriately:[5]

$$1 = \frac{1}{(1+\omega_t^{gov})} \frac{\pi_{t+1}}{R_t^{FFR}} (1-\omega_t^k) R_t^{kp}.$$

Here, ω_t^b represents the wedge between the government bond yield and the federal funds rate, whereas the latter is the wedge between private and government bonds. Since the spread between private and government bonds is observable, we estimate only ω_t^b. Our estimation results are consistent with the intuition that financial frictions were indeed central to the 2007–2009 recession. As Figure 3 and Table 2 document, shocks to these wedges indeed played a large role in understanding the recent recession, accounting alone for over 100% of the decline in output, in stark contrast to their small contribution to

[5]To see this, note that the first-order conditions of the households imply:

$$\begin{aligned} 1 &= \beta E_t \left[\frac{u_{c,t+1}}{u_{c,t}} \frac{R_t^{gov}}{\pi_{t+1}} \right] = \beta E_t \left[\frac{u_{c,t+1}}{u_{c,t}} (1+\omega_t^{gov}) \frac{R_t^{FFR}}{\pi_{t+1}} \right] \\ &= \beta E_t \left[\frac{u_{c,t+1}}{u_{c,t}} \Big((1-\omega_t^k)((1-\tau^k)(r_{t+1}^k u_{t+1} - a(u_{t+1})) + \delta\tau^k) + (1-\delta)\frac{Q_{t+1}}{Q_t} \Big) \right] \end{aligned}$$

where Q_t is the price of capital. Simplify this expression by assuming a constant Q_t, constant utilization, and ignoring uncertainty. The first line can then be substituted into the second. Defining $R_t^{kp} \equiv (r_{t+1}^k - \tau^k(r_{t+1}^k - \delta)) + (1-\delta)$ yields the above equation.

the full-sample variance of output as well as other included time series. While both financial friction shocks are important in our estimation, the government bond shock contributes more than twice as much as the private bond shock. In our interpretation, this means that liquidity considerations were more important than shocks to intermediation. In the model, the government bond shock depresses output, consumption, and private as well as government investment, whereas the shock to the spread between private bonds and government bonds leads to a decline in consumption only with some delay, and actually increases government investments. Both shocks furthermore result in a modest decline in the federal funds rate. Figure 13 in the Technical Appendix shows these impulse responses for a one-standard deviation shock to the two wedges.

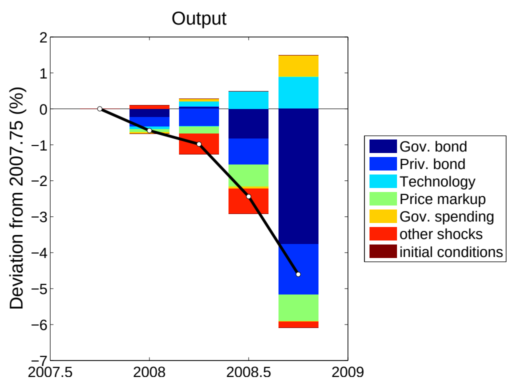


Figure 3: *Historical Shock Decomposition: Output. Results are at the posterior median. 2007:4 is the NBER recession date.*

Besides being important for extracting the historical starting conditions of the economy in 2009, allowing for a bond premium shock also changes the posterior over structural parameters. Del Negro et al. (2013) also estimate a much higher degree of nominal rigidities in their model than in Smets and Wouters (2007), once they introduce a bond premium shock in their model. As discussed in detail in Del Negro et al. (2013), the higher degree of stickiness is key to explaining the lack of deflation in the recent recession. We discuss in detail in Section 5.5 how stickiness affects our analysis.

Table 2: *Historical decomposition of recent recession and overall variance decomposition for output. All numbers are at the Bayesian posterior mean.*

	2008:4 vs. 2007:4 Historical decomposition		Total Sample Variance decomposition
Shock	total	percent	percent
Gov. bond	-3.76	81.69	5.11
Priv. bond	-1.41	30.63	1.38
Technology	0.89	-19.44	19.23
Price markup	-0.74	16.14	6.68
Gov. spending	0.60	-12.95	3.49
Priv. inv.	-0.30	6.57	14.04
Labor tax	-0.26	5.60	19.63
Monetary pol.	0.22	-4.69	17.37
Wage markup	0.14	-3.11	8.38
Gov. inv.	0.03	-0.65	4.59
Initial values	-0.01	0.22	n/a
Sum	-4.60	100.00	100.00

3.3 Implications for unemployment

Since unemployment, in addition to GDP growth, is at the center of many public debates, we back out a predicted change in the unemployment rate from the model. To that end, we regress the quarterly unemployment rate on the hours-worked measure used to estimate our model and use the implied OLS estimate to infer the effect on the unemployment rate. The fit is reasonable with an R^2 of 0.77. We neglect the additional parameter uncertainty introduced because of the uncertain estimates of the regression coefficients.[6]

3.4 Computing multipliers

Our main focus is on the fiscal multiplier (i.e., the ratio of output changes to the total stimulus-planned change in spending and transfers). Note that due to the eventual balancing of the government budget, there will also be an induced movement in tax rates as a "secondary" effect. As is customary, we shall not include these secondary movements in the denominator (i.e., in

[6]Table 12 and Figure 22 in the Technical Appendix provide estimation details.

quantifying the stimulus-planned changes). As this is a dynamic model, the horizon plays a role. Following Uhlig (2010b), we use the net present value fiscal multiplier φ_t, dividing the net present value of output changes up to some horizon t by the change in government spending and transfers until the same time. I.e., we shall use

$$\varphi_t = \sum_{s=1}^{t} \left(\mu^s \prod_{j=1}^{s} R_{j,ARRA}^{-1} \right) \hat{y}_s / \sum_{s=1}^{t} \left(\mu^s \prod_{j=1}^{s} R_{j,ARRA}^{-1} \right) \hat{g}_s \tag{3.1}$$

where φ_t: horizon-t multiplier, $R_{j,ARRA}$ is the government bond return, from $j-1$ to j, $\hat{y}_s$ is the output change at date s due to ARRA in percent of the balanced-growth GDP path and $\hat{g}_s$: ARRA spending at date s in percent of the balanced-growth GDP path.

When analyzing our results, we report the posterior median, as well as confidence bands covering 90 percent or 67 percent of the posterior probability.

4 Main Results

4.1 Description of benchmark results

Figure 4 contains our benchmark results for output, the unemployment rate, the federal funds rate, inflation, government debt, and consumption.[7] These graphs are perhaps reminiscent of the information shown in the official White House piece by Bernstein and Romer (2009). However, we include an important piece of information, which is missing there. The short-run debt dynamics shown here induce long-run debt-and-tax dynamics, shown in Figure 5. The increase in labor tax rates long after the fiscal stimulus phase has finished induces the decline of output for many years to come.

The resulting fiscal multiplier therefore declines with the horizon. The fiscal multipliers for the shorter horizon, shown in the left panel of Figure 6, can therefore be quite misleading in terms of assessing the long-term costs of

[7] Results for the consumption of both types of agents, real wages, tax rates, and investment are shown in Figure 14.

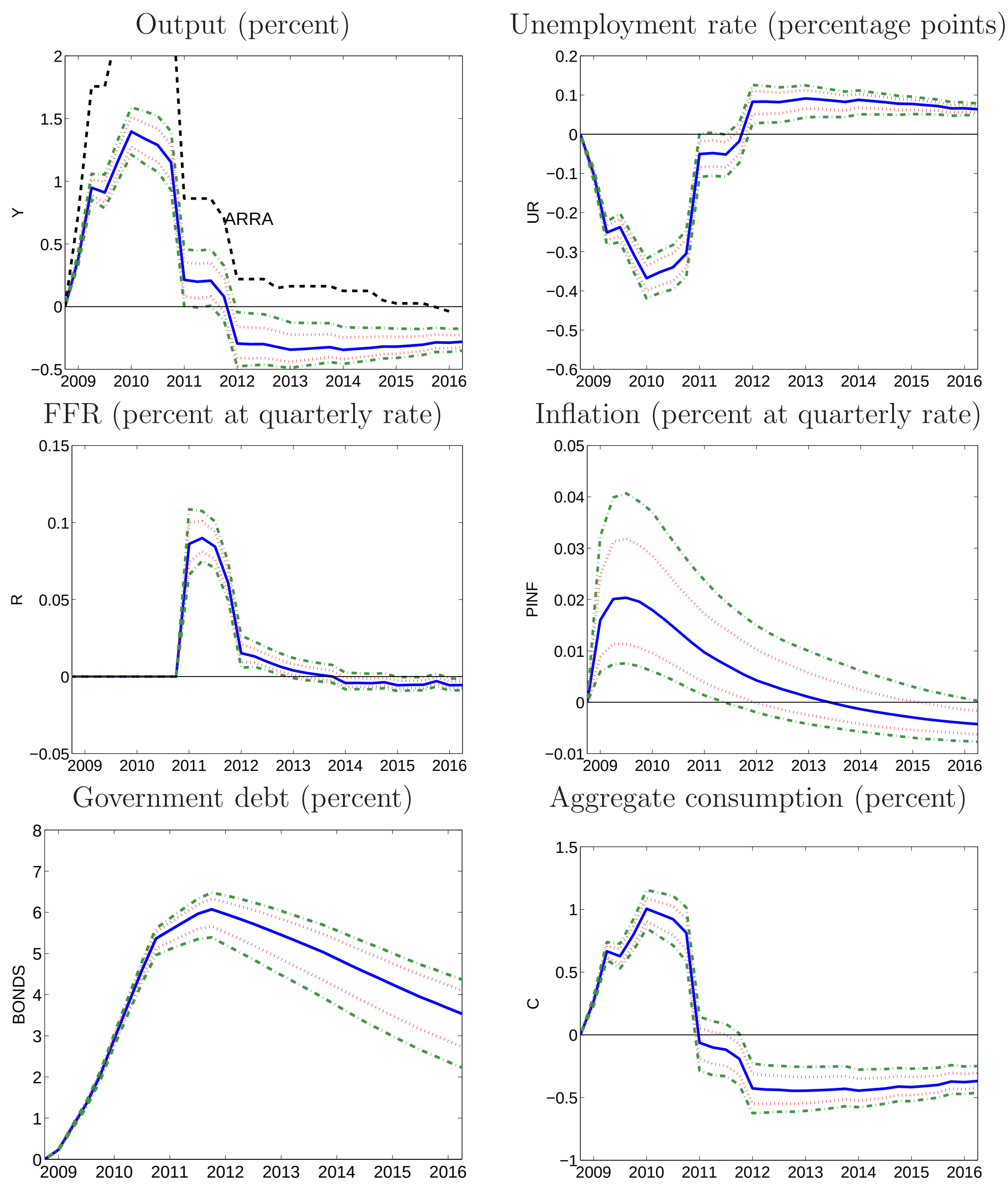


Figure 4: *Benchmark impact of ARRA*

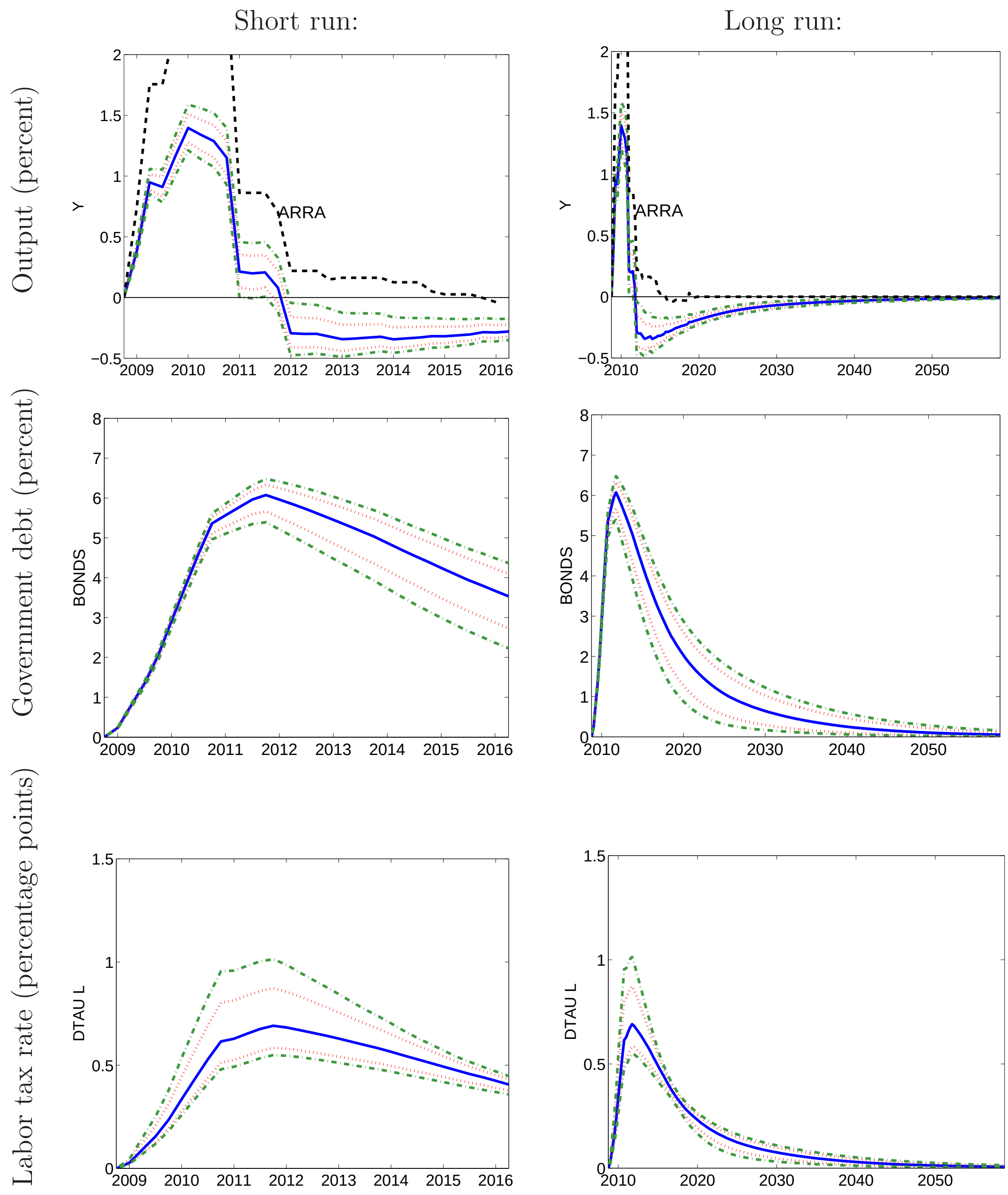


Figure 5: *Short- and long-run impact of ARRA*

fiscal stimulus. Indeed, the long-run multipliers are considerably smaller or negative, as compared with the short-run multipliers, shown in the right panel of Figure 6. These results are qualitatively in line with Uhlig (2010b), though the results are quantitatively rather different: The long-run fiscal multipliers are negative there and here, but considerably more negative there. One may be tempted to read the difference as "relief," compared with the pessimistic scenario in Uhlig (2010b). Note, however, that the model here is heavily tilted toward a model in which fiscal stimulus is often thought to work well. We therefore believe that the negative long-run effects of fiscal stimulus should give pause to arguments in its favor. Even at the short horizon, the benchmark multiplier is just around 0.5.

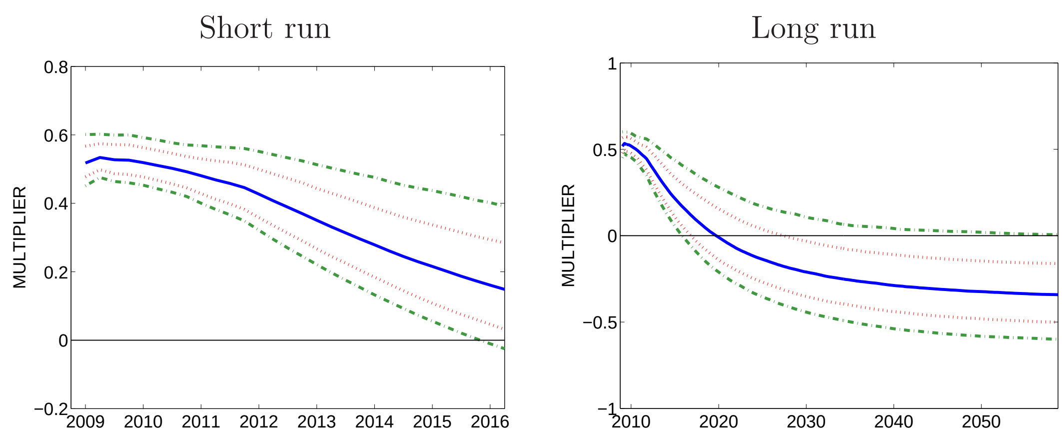


Figure 6: *Short-run and long-run fiscal multipliers in the benchmark parameterization*

We departed from the original Smets-Wouters model in order to model the fiscal stimulus in more detail by being able to distinguish between money spent on government transfers, consumption, and investment. Our results imply that each component indeed affects the economy differently: Transfers to credit-constrained agents are similar to government consumption, whereas transfers to unconstrained households mainly increases the need to raise distortionary taxes. Discretionary government investment increases private sector productivity but may also crowd out optimal government investment, effectively lowering the size of the long-term debt burden faced by households.

The right panel in Figure 7 shows that in our benchmark model, the government investment component contributes to a positive multiplier, whereas the government consumption and transfer components lower the overall multiplier below zero. Importantly, note that the simulation for the consumption-only stimulus shows that our model is capable of generating short-run multipliers in excess of unity, but that the composition of the ARRA pushes the overall short-run multiplier below unity.

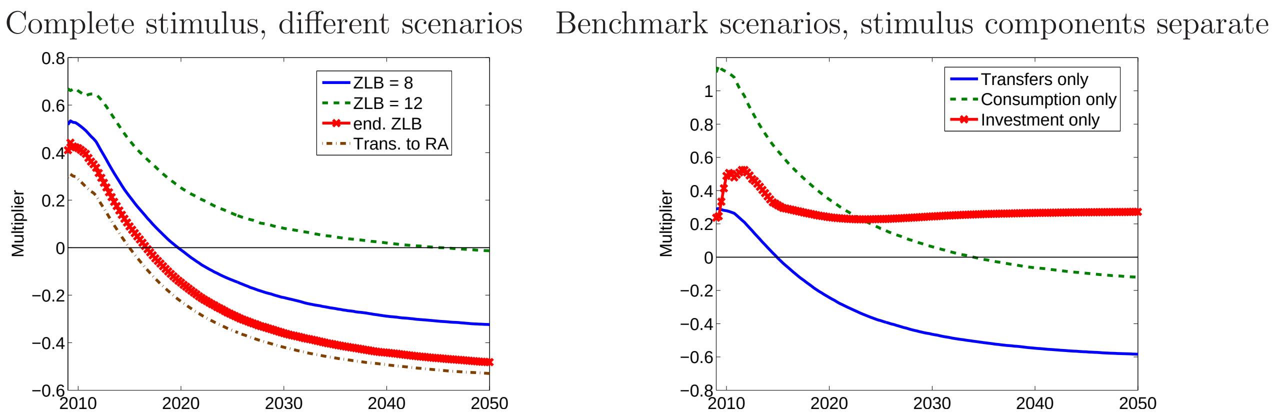


Figure 7: *Comparison of long-run multipliers: posterior medians*

4.2 Robustness of the fiscal multiplier

The qualitative findings of a short-run multiplier below one and a long-run multiplier around zero, or even negative, are robust to a wide range of different modeling assumptions. Table 3 and Table 4 provide an overview. Compared to our benchmark analysis, we change (1) the duration of the binding ZLB, (2) the type of distortionary taxes, (3) the fraction of RoT agents, (4) targeted transfers, (5) the degree of nominal rigidities, (6) the capital share, and (7) the speed of budget balancing. Except with perfectly targeted transfers, the median short-run multiplier is below one, implying that government spending partially crowds out private activity rather than increasing it. The long-run multiplier is sensitive to the type of taxation, but unless the ZLB is expected to last for more than three years, the median long-run multiplier is negative with labor taxes.

Table 3: Long-run fiscal multipliers as $t \to \infty$: sensitivity

Scenario	5 percent	16.5 percent	median	83.5 percent	95 percent
Benchmark	-0.63	-0.53	-0.36	-0.18	-0.01
Lump-sum tax	0.35	0.45	0.61	0.79	0.94
Consumption tax	0.22	0.32	0.48	0.65	0.78
0 quarters ZLB	-0.95	-0.87	-0.74	-0.62	-0.51
12 quarters ZLB	-0.43	-0.30	-0.05	0.22	0.46
20 quarters ZLB	-0.28	-0.11	0.33	1.17	2.21
Endogenous ZLB, Taylor rule	-0.77	-0.67	-0.52	-0.33	-0.11
15 p.c. RoT population share	-0.77	-0.67	-0.52	-0.35	-0.18
35 p.c. RoT population share	-0.47	-0.36	-0.17	0.02	0.22
0 p.c. RoT transfer share	-0.76	-0.68	-0.56	-0.45	-0.35
100 p.c. RoT transfer share	-0.39	-0.16	0.24	0.69	1.12
Stickiness scale=0.10	-1.07	-0.99	-0.89	-0.79	-0.72
Stickiness scale=1.15	-0.67	-0.56	-0.38	-0.19	0.03
Capital share $\alpha = 0.35$	-1.00	-0.86	-0.65	-0.42	-0.20
Tax adjustment speed $\psi_\tau = 0.025$	-0.60	-0.49	-0.32	-0.15	0.02
Tax adjustment speed $\psi_\tau = 0.050$	-0.66	-0.55	-0.39	-0.22	-0.06

Table 4: One-year fiscal multipliers: sensitivity

Scenario	5 percent	16.5 percent	median	83.5 percent	95 percent
Benchmark	0.46	0.49	0.53	0.57	0.60
Lump-sum tax	0.55	0.58	0.62	0.67	0.71
Consumption tax	0.57	0.59	0.63	0.68	0.71
0 quarters ZLB	0.16	0.19	0.23	0.27	0.30
12 quarters ZLB	0.58	0.61	0.67	0.74	0.80
20 quarters ZLB	0.64	0.69	0.80	1.01	1.29
Endogenous ZLB, Taylor rule	0.32	0.36	0.43	0.50	0.58
15 p.c. RoT population share	0.40	0.42	0.46	0.50	0.53
35 p.c. RoT population share	0.53	0.56	0.61	0.65	0.70
0 p.c. RoT transfer share	0.26	0.27	0.30	0.32	0.34
100 p.c. RoT transfer share	1.05	1.11	1.21	1.32	1.40
Stickiness scale=0.10	0.08	0.10	0.13	0.17	0.19
Stickiness scale=1.15	0.45	0.47	0.50	0.53	0.56
Capital share $\alpha = 0.35$	0.45	0.48	0.53	0.59	0.63
Tax adjustment speed $\psi_\tau = 0.025$	0.49	0.51	0.55	0.59	0.62
Tax adjustment speed $\psi_\tau = 0.050$	0.44	0.47	0.50	0.54	0.57

5 Understanding multipliers

To understand the main qualitative features of our model, a modified version of the three-equation New Keynesian textbook model is useful. The key is the addition of rule-of-thumb agents with rigid wages instead of rigid prices. This model helps to explain the estimated effects of the ZLB duration, distortionary taxes, the role of RoT agents, the stimulus composition, and stickiness.

We show that when wages are sticky, workers are off their labor supply curve, and changes in labor taxes affect the disposable income of RoT agents and thereby affect aggregate demand directly. This novel aggregate demand effect can partially crowd out private consumption even at a persistent ZLB and push the multiplier below one. Deficit-financing stimulus avoids this problem, but at the cost of depressed future output, as higher taxes lower work incentives and tighter monetary policy in response to inflation hit the economy.

The simple model abstracts from additional mechanisms in the full model, such as capital and the timing of the stimulus. Capital accumulation as in Uhlig (2010b) is an important omission from the simple model. It amplifies the distortionary effects of increased labor taxes: The predicted negative effects on output after exit from the ZLB and the phasing out of the stimulus lower the return on capital and cause investment to fall. The timing of the stimulus relative to the duration of the ZLB also matters, as shown by Woodford (2011): When government spending persists beyond the duration of the ZLB, it lowers the multiplier. Since a small portion of the stimulus persists for up to six years, this timing effect is also present in our empirical model. Timing is, however, not crucial for our main quantitative results: Figure 19 in the Appendix compares labor-tax- with transfer-financing when the entire stimulus is spent uniformly over the first four quarters. When transfers are adjusted, the multiplier is larger than one, whereas the median multiplier with distortionary taxes falls in the long run to almost minus one.

5.1 Simple New Keynesian model

We modify the textbook New Keynesian model (e.g., Galí, 2008, ch. 3,6) to resemble our baseline model by including rule-of-thumb agents as in Galí et al. (2007), with sticky wages as in Smets and Wouters (2007). We abstract from steady state government consumption and debt. The result is a modified version of the standard three-equation model consisting of an Euler equation, a Phillips curve, and an interest rate rule.

In difference to the full model, there is no capital and no indexation of wages or prices. We consider log-utility ($\sigma = 1$). The fraction ϕ of RoT households does not receive profit income, so that their individual share of income and consumption is equal to $\frac{\epsilon_p - 1}{\epsilon_p}$, the labor share. $\frac{\epsilon_p}{\epsilon_p - 1}$ is the steady state markup in the goods market. Away from the ZLB, the monetary authority follows a Taylor rule with a constant intercept and reacts only to current inflation with a coefficient γ_π. In deviations from the steady state:

$$\hat{R}_t = (1 - \mathbf{1}_{ZLB,t})\gamma_\pi \hat{\pi}_t. \tag{5.1}$$

Lemma 1 in Appendix D.9.8 shows that the Taylor Principle $\gamma_\pi > 1$ is sufficient to guarantee stability despite the presence of RoT agents if $\phi \frac{\epsilon_p - 1}{\epsilon_p}(1 + \nu) < 1$.

We consider spending processes as in Eggertsson (2011) and Christiano et al. (2011): The government targets the stimulus perfectly to the length of the ZLB: $\hat{g}_t = \mathbf{1}_{ZLB,t}\bar{g}$. In the steady state, all fiscal variables are zero.

The description of this simple economy is then completed by specifying the government tax policy. Starting from the zero steady state, a stylized representation of our tax policy rule is:

$$\text{labor share} \times d\tau_t^n + d\tau_t^c - \hat{s}_t^e = \psi_\tau(\hat{g}_t + \hat{s}_t^x), \quad \hat{d}_t = (1 - \psi_\tau)(\hat{g}_t + \hat{s}_t^x) \qquad \text{at ZLB,} \tag{5.2a}$$

$$\text{labor share} \times d\tau_t^n + d\tau_t^c - \hat{s}_t^e = (1 - \psi_\tau)\hat{d}_{t-1}, \quad \hat{d}_t = 0 \qquad \text{after ZLB,} \tag{5.2b}$$

where transfers $\hat{s}_t$ are the sum of endogenous transfers $\hat{s}_t^e$, and exogenous stimulus transfers $\hat{s}_t^x$. ψ_τ governs the tax adjustment speed as before.

Appendix D.9.3 derives the Phillips curve and the Euler equation in the limit of perfectly flexible wages, $\lim \zeta_w \searrow 0$, and sticky prices $\zeta_p > 0$ as:

$$\hat{y}_t = \frac{\hat{g}_t + \phi_c \frac{\epsilon_p}{\epsilon_p - 1}\hat{s}_t}{1-\phi_c(1+\nu)} + \mathbb{E}_t\left[\hat{y}_{t+1} - \frac{\hat{g}_{t+1} + \phi_c \frac{\epsilon_p}{\epsilon_p-1}\hat{s}_{t+1}}{1-\phi_c(1+\nu)}\right] - \frac{1}{1-\phi_c(1+\nu)}(\hat{R}_t - \mathbb{E}_t[\hat{\pi}_{t+1} + d\tau^c_{t+1} - d\tau^c_t]) \tag{5.3a}$$

$$\hat{\pi}_t = \beta\mathbb{E}_t[\hat{\pi}_{t+1}] + \lambda_p\left(\Big(\nu + (1-\phi_c(1+\nu))\Big)\hat{y}_t - \hat{g}_t - \phi_c\frac{\epsilon_p}{\epsilon_p - 1}\hat{s}_t + d\tau_t^n + d\tau_t^c\right) \tag{5.3b}$$

where $\lambda_p = \frac{(1-\zeta_p)(1-\beta\zeta_p)}{\zeta_p}$ is a measure of price flexibility and $\phi_c = \phi \times \text{labor share} = \phi\frac{\epsilon_p - 1}{\epsilon_p}$ is the RoT consumption share.

When wages are sticky $\zeta_w > 0$ but prices are fully flexible, $\lim \zeta_p \searrow 0$, the Phillips curve and the Euler equation are (cf. Appendix D.9.4):

$$\hat{y}_t = \frac{\hat{g}_t + \phi_c\frac{\epsilon_p}{\epsilon_p-1}\hat{s}_t - \phi_c(d\tau_t^n + d\tau_t^c)}{1-\phi_c} + \mathbb{E}_t\left[\hat{y}_{t+1} + \frac{-\hat{g}_{t+1} - \phi_c\frac{\epsilon_p}{\epsilon_p-1}\hat{s}_{t+1} + \phi_c(d\tau^n_{t+1} + d\tau^c_{t+1})}{1-\phi_c}\right] - (\hat{R}_t - \mathbb{E}_t[\hat{\pi}_{t+1} + d\tau^c_{t+1} - d\tau^c_t]) \tag{5.4a}$$

$$\hat{\pi}_t = \beta\mathbb{E}_t[\hat{\pi}_{t+1}] + \lambda_w\left((1+\nu)\hat{y}_t + \frac{-\hat{g}_t - \phi_c\frac{\epsilon_p}{\epsilon_p-1}\hat{s}_t + \phi_c(d\tau_t^n + d\tau_t^c)}{1-\phi_c} + d\tau_t + d\tau_t^c)\right) \tag{5.4b}$$

where $\lambda_w = \frac{(1-\zeta_w)(1-\beta\zeta_w)}{\zeta_w}$ is a measure of wage flexibility.

Without RoT agents, $\phi_c = 0$, equations (5.3) and (5.4) are isomorphic and the source of nominal rigidity is irrelevant. Once there is a positive mass of RoT agents, the source of nominal rigidity makes a qualitatively difference. Transfers also drop out of the private-sector equilibrium conditions without RoT agents because of Ricardian Equivalence.

Taxes only have a direct effect on output under sticky wages through the Euler equation. The direct contemporaneous effect is due to reduced income and therefore consumption demand of RoT agents when taxes increase and gross wages are sticky. Known future taxes also lower future demand by RoT agents, but for given output, this requires unconstrained agents to consume more. Trying to smooth consumption, unconstrained agents demand higher consumption today. This wealth effect results, all else being equal, in higher current output. In contrast, with flexible wages, net real wages are invariant to labor taxes and no demand effect arises. Indirect effects of taxes result from the pass-through of taxes to price inflation through the Phillips curve under both sticky prices and sticky wages.

Present in both Euler equations is a traditional Keynesian multiplier: For given output and wages, when the government consumes $\hat{g}$ extra units of output, hours rise by $\hat{n} = \hat{g}$ given the labor-only production function and hence the real income by RoT agents rises. That raises aggregate demand by an additional $\phi_c \hat{g}$ units, increasing labor demand further. This adds up to a total effect of $\sum_{j=0}^{\infty} (\phi_c)^j = \frac{1}{1-\phi_c}$ per unit of government purchases $\hat{g}$ under sticky wages in (5.4a). If wages are flexible, unions demand additional increases in the real wage which cause the multiplier to rise to $\frac{1}{1-\phi_c(1+\nu)}$ in (5.3a).

When analyzing our empirical results through the lens of the simple model, we often consider limits of a slow speed of tax adjustment ψ_τ and high degree of nominal rigidities ζ_w, ζ_p, in line with the full model posterior mean. Numerical examples use the following quarterly calibration: We set the rate of time preference to $\beta^{-1} - 1 = 0.01$ and choose $\zeta_w = \zeta_p = \frac{4}{5}$ for price and wage stickiness. We set the elasticity of substitution between intermediate inputs equal to $\epsilon_p = 3$ to generate a labor share of $\frac{\epsilon_p - 1}{\epsilon_p} = \frac{2}{3}$. The fraction of RoT agents is set to 0.25. We choose a reaction coefficient of $\gamma_\pi = 2$ for the monetary authority and impose a Frisch elasticity of $\nu^{-1} = 1$.

5.2 Duration of the ZLB

Our empirical findings imply a short-run multiplier smaller than unity but strictly increasing in the duration of the ZLB. Despite the "paradox of toil" in Eggertsson (2011) we show that this finding can be explained by our simple model with RoT agents when wages, but not prices, are sticky.

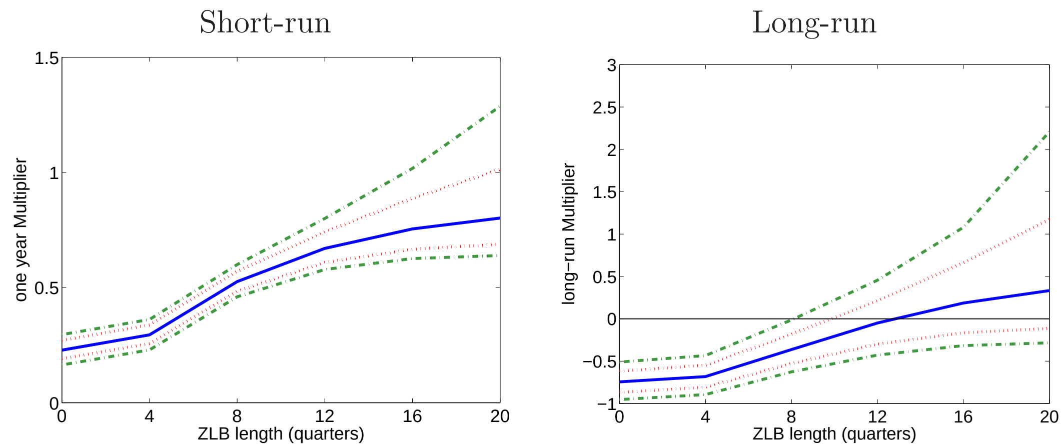


Figure 8: *Sensitivity of multipliers to the deterministic ZLB duration*

Figure 8 shows the posterior distribution of short-run and long-run multipliers when varying the exogenous duration of the ZLB from zero to five years. The median short-run multiplier increases monotonically from 0.23 without a binding ZLB to 0.80 with a duration of five years, while the median long-run multiplier increases from -0.74 to +0.33. However, the posterior uncertainty also increases significantly: With a five-year ZLB length, the 90% credible set for the long-run multiplier ranges from -0.28 to +2.21.

Our simple sticky wage model reproduces the fact that short-run multipliers are below unity even with a persistent ZLB. To that end, we adopt the Markov specification for the ZLB in Eggertsson (2011) and assume immediate taxation to match his analysis of increasing labor taxes during the recession. Proposition 1 shows that the short-run multiplier can be below one with sticky wages even in the presence of a persistent ZLB, but not when only prices are sticky. The simple model attributes the low multiplier to the negative aggregate demand effect from taxing RoT agents. This effect dominates both the expected inflation effect, which increases demand via the neoclassical substitution effect, as well as the Keynesian multiplier effect due to RoT agents.

Proposition 1. *Consider the model in Section* 5.1*. Assume immediate taxation* ($\psi_\tau = 1$)*, that the Taylor Principle holds and that the ZLB is a nonrecurrent Markov state which persists with probability* μ*:* $\Pr\{\boldsymbol{1}_{ZLB,t} = 1|\boldsymbol{1}_{ZLB,t-1} =$

$1\} = \mu, \Pr\{\mathbf{1}_{ZLB,t} = 1 | \mathbf{1}_{ZLB,t-1} = 0\} = 0$. *In the case of sticky prices and flexible wages, also assume that* $0 < \phi < \frac{\frac{\epsilon_p}{\epsilon_p - 1}}{1+\nu}$. *Consider the case of financing through distortionary labor taxes:* $\hat{s}_t = d\tau_t^c = 0$.

(a) For sufficiently small persistence of the ZLB, μ, the impact multiplier $\frac{y_{ZLB}}{g}$ *is strictly smaller than one under flexible prices and sticky wages and strictly larger than one under flexible wages and sticky prices.*

(b) The multiplier increases monotonically in the expected duration of the ZLB with either sticky wages or sticky prices in the region of determinacy.

Proof: See Appendix D.9.8, *page* LX.

5.3 Distortionary taxes

The premise of this paper is that distortionary taxes matter for the effects of government spending. Figure 9 shows that this is indeed the case: The multiplier is significantly lower with distortionary labor taxes than with lump-sum taxes. This difference is most pronounced in the long run (right panel), when the multiplier is centered around +0.6 with lump-sum taxation and around -0.45 with labor taxes. Short-run multipliers with consumption taxes are about as high as they are with transfer financing, but long-run multipliers are lower than with transfer financing.

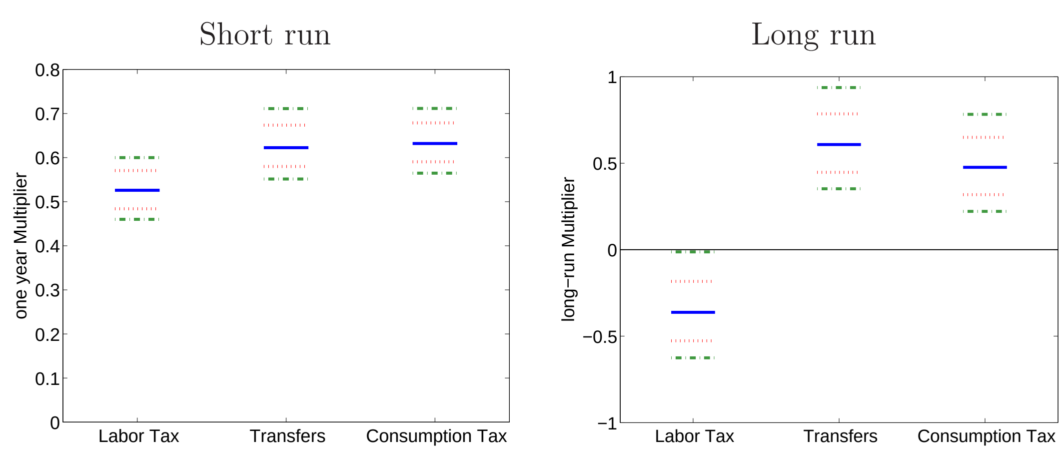


Figure 9: *Fiscal multipliers. Comparing distortionary labor taxes (benchmark) to consumption and lump-sum taxation*

To understand the results on taxation, we turn again to the simple model with sticky wages. Fixing the duration of the ZLB at one period, we focus first on the type of taxation while allowing for a varying speed of tax adjustment. Part (a) of Proposition 2 implies that the multiplier is higher with lump-sum taxation than it is with labor taxes. Lump-sum taxes have the same Keynesian demand effects through RoT agents, but do not cause the monetary authority to raise interest rates due to inflationary pressure. Labor taxes therefore depress future output and consumption by lowering demand in the short run via a negative wealth effect that is absent with lump-sum taxes. Because increasing consumption taxes generate consumer price inflation, short-run multipliers can be higher with consumption taxes than they are with transfers when taxes are adjusted slowly. However, with sufficiently sticky wages, long-run multipliers are the highest with transfer financing. The reason is that the future consumption tax hike lowers consumption demand, and the monetary authority does not react much with sufficiently sticky wages.

Proposition 2. *Consider the model in Section* 5.1 *with sticky wages and flexible prices. Assume the Taylor Principle is satisfied:* $\gamma_\pi > 1$.

(a) The impact multiplier is strictly lower when financed with labor taxes rather than lump-sum taxes if $\psi_\tau < 1$ *and equal otherwise. The long-run multiplier is lower with both labor taxes and consumption taxes than it is with lump-sum taxes if* $\psi_\tau < 1$ *and wages are sufficiently sticky* ($\zeta_w \nearrow 1$).
If taxes are adjusted sufficiently slowly, $\psi_\tau < \bar{\psi}^n < 1$, *the impact multiplier is higher with consumption taxes than it is with labor taxes. The impact multiplier is higher with consumption taxes than it is with transfer financing if* $\psi_\tau < \bar{\psi}^s$, *where* $\bar{\psi}^s < \bar{\psi}^n$.

The following results assume financing through labor taxes: $\hat{s}_t = d\tau_t^c = 0$.

(b) If wages are sufficiently sticky ($\zeta_w \nearrow 1$) *and* $\phi > 0$, *increasing the tax adjustment speed* ψ_τ *lowers the impact multiplier. Without RoT agents,* $\phi = 0$, *increasing the tax adjustment speed* ψ_τ *increases the impact multiplier.*

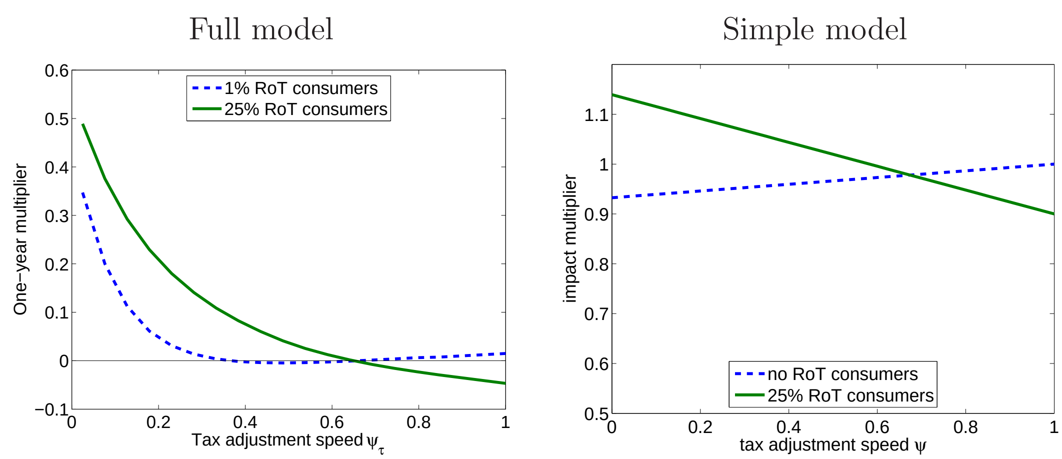
(c) Lowering the labor share $\frac{\epsilon_p - 1}{\epsilon_p}$ *lowers the multiplier if the impact multiplier is positive. A sufficient condition is that wages are sufficiently sticky*

($\zeta_w \nearrow 1$). *If the impact multiplier is nondecreasing in the labor share, the long-run multiplier is strictly lower for lower labor shares for all* $\psi_\tau < 1$.

Proof: See Appendix D.9.8, *page* LXIII.

Two important parameters governing the effect of distortionary taxes are the tax adjustment speed ψ_τ and the labor share as the tax base. Parts (b) and (c) of Proposition 2 characterize the effect of both. Without RoT agents, it is always better to not rely on deficit financing and to repay government debt immediately: This avoids the negative wealth effects coming from the response of the monetary authority to future inflation. However, with very sticky wages and RoT agents, the demand effect dominates and adjusting taxes more slowly increases the short-run multiplier. The negative wealth effects of distortionary taxes are more pronounced the smaller the tax base (i.e., the smaller the labor share). Since output in both periods falls, the long-run multiplier is more sensitive to the labor share than the short-run multiplier.[8]

In the full model, increasing the budget-balance speed ψ_τ leads to lower



Note: The full model results set the habit parameter to $h = 0.5$ and otherwise uses the posterior mean for the simulation.

Figure 10: *Multipliers as a function of tax adjustment speed and rule-of-thumb consumers*

multipliers, as shown in Tables 3 and 4, within the range of stable parameters.

[8] A smaller labor share also lowers the Keynesian multiplier effect $\frac{1}{1-\phi_c} = \frac{1}{1-\phi\times\text{labor share}}$.

This range is, however, very narrow. To allow for a comparison over a wider range, we set the degree of habit formation to $h = 0.5$, as compared with the posterior median of $h = 0.85$, so that adjusting taxes fully and immediately is consistent with a unique locally bounded equilibrium. Figure 10 shows in the left panel the effect of varying the speed of tax adjustment in the full model with adjusted habit (solid blue line).[9] In line with the results in the simple model (right panel), adjusting taxes faster lowers the multiplier with RoT agents and the estimated high degree of wage stickiness. Without RoT agents, the simple model implies that the short-run multiplier is almost flat, but slightly increasing in the tax adjustment speed. This implication holds only for high-enough tax adjustment speeds in the empirical model.[10]

Figure 11 reveals that long-run multipliers are quite sensitive to the capital share. The long-run multiplier declines monotonically in the capital share, as predicted in Proposition 2. In the simple model this is explained by two channels. First, textbook economic analysis shows that taxes are less distortionary when raised from a wider base. Second, a higher capital share lowers the Keynesian multiplier effect, which is based on labor income. The estimated capital share is around 0.24 rather than 0.35, often used in the calibration literature (see Cooley and Prescott (1995)).

5.4 Role of rule-of-thumb agents

Rule-of-thumb households are important in two respects. First, a sizable portion of the population violates Ricardian equivalence. Their presence amplifies stimulus effects on output via a traditional Keynesian multiplier. Second, the distribution of transfers between these households and the unconstrained households has aggregate effects. It turns out that the second effect is more important than the first.

Table 5 analyzes the relative importance of the population share and the

[9] See Figure 20 in the Appendix for analogous results for the long-run multiplier and Figure 23 for an illustration of the model generating the non-monotonicity.

[10] Appendix D.9.7 explains the difference in a three-period version of the simple model because the inflationary pressure upon exit from the ZLB is no longer proportional to ψ_τ but to $\psi_\tau(1-\psi_\tau)$ so that the cost of taxation is minimized for intermediate values of ψ_τ.

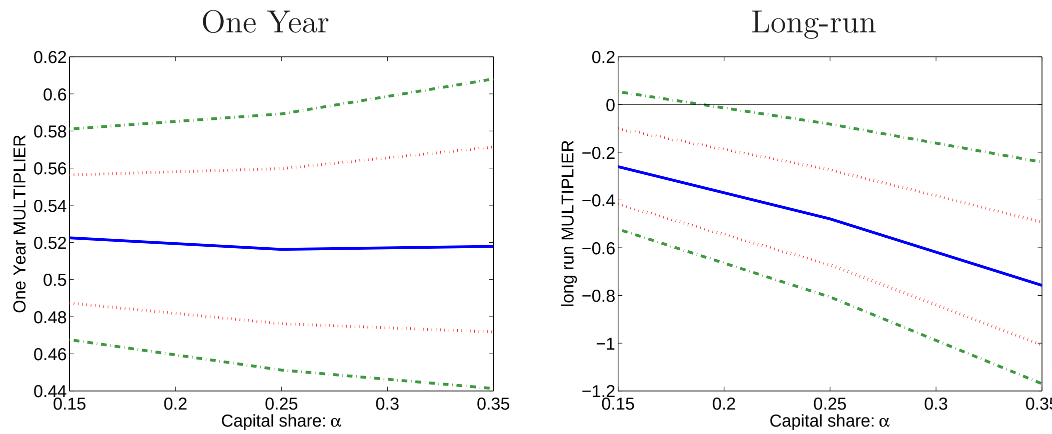


Figure 11: *Short-run and long-run fiscal multipliers: sensitivity to the capital share*

distribution of transfer payments. Panel (a) of Table 5 shows the change in the fiscal multipliers, when we change the share of the population that is credit-constrained. In this experiment, the transfers are equally distributed across the population (i.e., the share of the transfers to the credit-constrained population equals the share of that population). This confounds two effects, however. The first effect is the mere rise in the share of credit-constrained households, keeping their share of transfer receipts the same. This is shown in panel (b) of Table 5. The second effect comes from the share of transfers received by the RoT households. Panel (c) of Table 5 therefore varies the share of transfers received by these households but keeps their share of the population constant at the benchmark value of 25 percent. While the second experiment has a rather modest impact on the short-run multiplier, the third experiment has a larger short-run impact. Both effects contribute to raising long-run multipliers considerably. For example, and for the last experiment, the median long-run fiscal multiplier changes from -0.51 to 0.29, as the fraction of targeted transfers is varied from zero to 100 percent.

Proposition 3 states that when the budget is balanced slowly, the simple sticky-wage model reproduces our empirical findings: First, more RoT agents in the population increase the multiplier. Traditional Keynesian multiplier logic explains this effect. Second, the transfer multiplier lies below the gov-

Table 5: *Sensitivity of multipliers to the credit-constrained fraction of the population and their transfer share. Panel (a): All households receive the same amount of transfers (i.e., fraction of constrained households and total transfers rise together). Panel (b), only the fraction of constrained household rises. Panel (c): only the share of transfers going to constrained households rises.*

(a) Constant transfers/household			
RoT transfer share = RoT fraction =	0.10	0.25	0.40
One-year multiplier	0.33	0.54	0.82
Long-run multiplier	-0.62	-0.31	0.12
(b) Varying RoT fraction, fixed absolute RoT transfers			
RoT transfer share = 0.25, RoT fraction =	0.10	0.25	0.40
One year multiplier	0.45	0.54	0.66
Long-run multiplier	-0.53	-0.31	-0.03
(c) Fixed RoT fraction, varying absolute RoT transfers			
RoT fraction = 0.25, RoT transfer share =	0	0.25	1.00
One year multiplier	0.31	0.54	1.23
Long-run multiplier	-0.51	-0.31	0.29

ernment spending multiplier because the cost of financing is the same, while only the fraction targeted to RoT agents translates into spending.

Proposition 3. *Consider the model in Section* 5.1*. Assume the Taylor Principle is satisfied:* $\gamma_\pi > 1$*. Consider the case of financing through distortionary labor taxes:* $\hat{s}^e_t = d\tau^c_t = 0$*.*

(a) If taxes are adjusted sufficiently slowly ($\psi_\tau < \frac{\epsilon_p - 1}{\epsilon_p}$)*, the impact multiplier increases strictly in the share of RoT agents* ϕ*.*

(b) The multiplier on transfers is strictly increasing in the fraction of transfers RoT agents receive and weakly smaller than the government spending multiplier.

Proof: See Appendix D.9.8*, page* LXIV*.*

One may wish to conclude from this that "fiscal stimulus" in the form of transfers to constrained agents may be quite effective in increasing output. That may be so. However, the modeling of the credit-constrained agents is done here with the simple short cut of assuming that these agents do not keep savings and cannot borrow. For a more sophisticated exercise, the bounds to borrowing and savings should be endogenized, and may actually depend on the size and persistence of the government transfers.[11] Furthermore, microdata can potentially be informative about the degree to which households are credit-constrained or refrain from saving. A more detailed investigation is called for, if future "fiscal stimulus" programs are to focus on this particular group.

5.5 Nominal rigidities

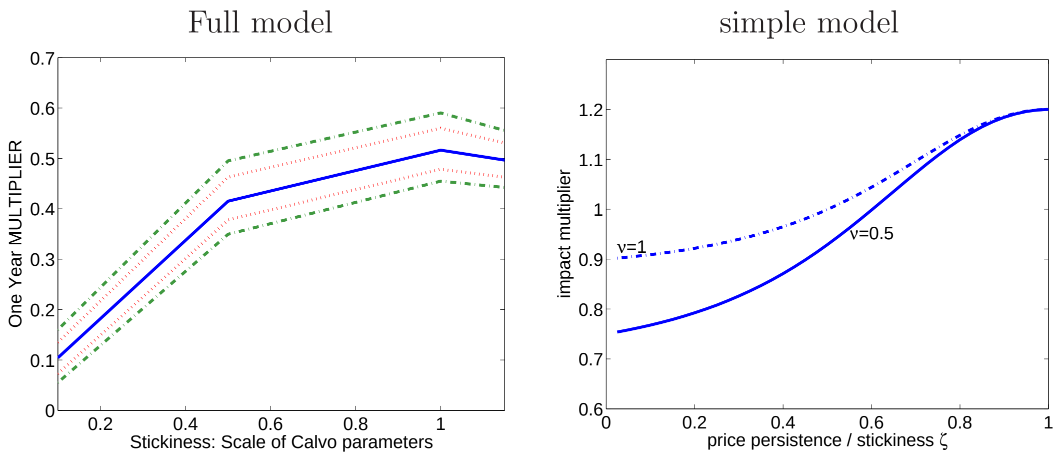
Our empirical findings suggest that the multiplier is increasing in the degree of nominal rigidities over the relevant range. This finding is apparently at odds with the literature: As Farhi and Werning (2012) show, the multiplier falls monotonically in the degree of of nominal rigidities in the workhorse New Keynesian model with Ricardian equivalence. The simple sticky wage model in this paper shows that in the presence of distortionary taxes, the cost of adjusting taxes after exiting the ZLB overturns this result.

The left panel in Figure 12 documents that the short-run multiplier is increasing in the degree of stickiness, over most of the range of stickiness.[12] The estimated nominal rigidities are very strong, but lower than the estimates in Del Negro et al. (2013): The mean estimates are $\zeta_p = 0.81$ and $\zeta_w = 0.83$ for the Calvo parameter for prices and wages. In Figure 12, we consider values of 10% to 115% of these mean estimates, scaling both parameters proportionately. The median short-run multiplier increases from a value of 0.1, when stickiness is only 10% of the mean estimates, to slightly above 0.5 at the posterior mean, before falling slightly when increasing stickiness further.

The simple sticky wage model reproduces the main finding, as the right panel of Figure 12 illustrates for different labor supply elasticities. Proposi-

[11] Oh and Reis (2012) investigate the role of targeted transfers as fiscal stimulus in an incomplete markets model with borrowing constraints. However, in their model, the increased transfers last only one period and are conditioned on exogenous attributes only. Even so, their model implies an upper limit for targeted transfers thresholds to avoid transfers that "would turn the rich into poor and vice versa" (p. S61).

[12] Cf. Figure 17 in the Appendix for the corresponding long-run multipliers and the implied inflation response.



Note: The full model uses the posterior mean for the simulation except for the habit parameter set to $h = 0.5$.

Figure 12: *Short-run multipliers as a function of stickiness*

tion 4 shows that this is a generic outcome of our sticky wage model with slow taxation. In the simple model, the negative effect of more flexible wages on the impact multiplier of tax-financed government expenditure is due to more monetary tightening upon exit from the ZLB. The resulting negative wealth effect depresses private consumption also in the short run, lowering the multiplier.

Proposition 4. *Consider the model in Section* 5.1*. Assume the Taylor Principle is satisfied:* $\gamma_\pi > 1$*. Consider the case of financing through distortionary labor taxes:* $\hat{s}_t = d\tau_t^c = 0$*.*

If $\psi_\tau < 1$ *and* $\phi < \frac{\epsilon_p}{\epsilon_p - 1}\frac{1}{1+\nu}$*, increasing wage flexibility* ($\zeta_w \searrow 0$) *lowers the impact multiplier.*

Proof: See Appendix, page LXIV*.*

6 Welfare effects

Multipliers are silent on welfare implications of the ARRA package. If the output increase is driven by a disproportionate increase in hours worked, consumers are likely to be worse off, even if the multiplier is large and positive.

Given perfect foresight of the stimulus plan, we calculate the compensating variation in lifetime consumption along the balanced growth path that makes consumers indifferent between ARRA and the modified historic growth path. Let $\Gamma_i \times 100$ be the percentage of consumption without the stimulus that consumers of type $i, i \in \{RA, RoT\}$ would be willing to give up each period to implement ARRA. We provide an explicit formula in the Technical Appendix for backing out Γ_i from the net present value of future utility changes. The discount rates for each consumer type are crucial here.

Two caveats complicate the welfare calculation. First, the calculation is numerically challenging because, at our estimates, the effective discount factor $\beta_{RA}\mu^{1-\sigma}$ is close to unity so that convergence is slow. Numerical error is important to address because we are relating the cost of an intervention over about 10 years to lifetime consumption so that errors of a small magnitude might be important for the results. Second, our parameter estimates are only directly applicable to unconstrained households, whereas we need to consider both types of households in the welfare analysis. If constrained households are sufficiently impatient and receive a high weight in the social welfare function, the results presented previously could be overturned if constrained agents value the initial consumption increase enough. The calibration of the discount rate for the constrained households is a challenge, however. Lawrance (1991) finds that rates of time preference vary by 7 percent on an annual basis across rich and poor households. Using data on individual choices between lump-sum and annuity payments, Warner and Pleeter (2001) find differences in annual rates of time preference of up to 30 percent, depending on various characteristics. We therefore consider two discount factors for the RoT agents by adding 7% or 30% to the annual discount factor of unconstrained agents, i.e.,

$$1/\beta_{RoT} \in \{1/\beta_{RA} + 0.07/4, 1/\beta_{RA} + 0.3/4\}$$

noting that our model is for quarterly data. We also vary over a wider range.

The welfare effects are small but significantly negative for unconstrained households, according to our calculations in Table 6. The median effect on constrained agents is -0.02 percent, independent of the length of the ZLB, with the 90 percent posterior confidence intervals ranging from -0.03 percent to -0.01 percent. The small magnitude is unsurprising given that small deviations from the optimum have small effects on the welfare of unconstrained agents. Unconstrained agents have less leisure and, for most parameter values considered, also have lower consumption, explaining the negative sign.

The effect on constrained agents is ambiguous, as lines two and three in Table 6 show. If the discount factor of the RoT agents is just 7% higher than that of the unconstrained agents, the welfare effect is negative, but it is significantly positive, if their discount factor is at least 15%–20% higher than that of unconstrained agents, as shown in Figure 21 in the Appendix.

Table 6: *Welfare effects ($\Gamma\times100$) of stimulus: Lifetime-consumption equivalent of compensating variation. Posterior median (90% confidence interval).*

Scenario	8 quarters ZLB	0 quarters ZLB	12 quarters ZLB
Unconstrained agents	-0.02(-0.03,-0.01)	-0.02(-0.03,-0.01)	-0.02(-0.03,-0.01)
RoT, 7% higher annual DF	-0.07(-0.13,-0.00)	-0.14(-0.22,-0.08)	-0.08(-0.16,0.03)
RoT, 30% higher annual DF	0.63(0.39,0.95)	0.45(0.22,0.65)	0.58(0.24,0.97)

7 Conclusions

We have quantified the size, uncertainty and sensitivity of fiscal multipliers in response to the ARRA. To that end, we have extended the benchmark Smets and Wouters (2007) New Keynesian model, allowing for credit-constrained households, a central bank constrained by the ZLB, government capital, and a government raising taxes with distortionary taxation. We have distinguished between short-run and long-run multipliers. For a benchmark parameterization, we find modestly positive short-run multipliers with a posterior mean of 0.53 and modestly negative long-run multipliers centered around -0.36. These multipliers below one can be explained by negative neoclassical wealth and Keynesian demand effects of distortionary labor taxes in a simple New Keynesian model with credit-constrained agents and sticky wages.

The multiplier is particularly sensitive to the type of taxes used to finance the ARRA, the fraction of transfers given to credit-constrained households, the anticipated length of the ZLB, and to the capital share. The multiplier is nonlinear in the degree of price and wage stickiness. Reasonable specifications are consistent with substantially negative multipliers within a short time frame. Furthermore, the policy intervention may lower the welfare of agents in the economy. Unconstrained agents would have a higher lifetime utility without the ARRA, and even impatient constrained agents may be better off without the intervention: The disutility of hours worked during the expansion and lower consumption in the transition to the long-run offset short-run gains from higher consumption.

References

Aruoba, S. Borogan and Frank Schorfheide (2012). "Macroeconomic Dynamics Near the ZLB: A Tale of Two Equilibria". University of Pennsylvania, mimeo.

Barro, Robert J. and Charles J. Redlick (2011). "Macroeconomic Effects From Government Purchases and Taxes". *The Quarterly Journal of Economics*, 126(1):51–102.

Barro, Robert J. and Xavier Sala-i Martin (1992). "Public Finance in Models of Economic Growth". *The Review of Economic Studies*, 59(4):645–661.

Bernstein, Jared and Christina Romer (2009). "The Job Impact of the American Recovery and Reinvestment Act". *press release, January 9, Office of the Vice President Elect*.

Braun, R. Anton, Lena Mareen Körber, and Yuichiro Waki (2012). "Some unpleasant properties of log-linearized solutions when the nominal rate is zero". Working Paper 2012-05, Federal Reserve Bank of Atlanta.

Buiter, Willem (2009). "The unfortunate uselessness of most 'state of the art' academic monetary economics". *www.voxeu.org, March 6, 2009*.

Bullard, James and Kaushik Mitra (2002). "Learning about monetary policy rules". *Journal of Monetary Economics*, 49(6):1105–1129.

CBO, Congressional Budget Office (2009). "Cost estimate for conference agreement for H.R.1." *http://www.cbo.gov/ftpdocs/99xx/doc9989/hr1conference.pdf*.

Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo (2011). "When is the government spending multiplier large?" *Journal of Political Economy*, 78–121.

Clemens, Jeffrey and Stephen Miran (2012). "Fiscal Policy Multipliers on Subnational Government Spending". *American Economic Journal: Economic Policy*, 4(2):46–68.

Coenen, G., C. J. Erceg, C. Freedman, D. Furceri, M. Kumhof, R. Lalonde, J. Linde, D. Muir, S. Mursula, A. Mourougane, J. Roberts, W. Roeger, S. Snudden, M. Trabandt, and J. in't Veld (2012). "Effects of Fiscal Stimulus in Structural Models". Tech. Rep. 1.

Cogan, John F., Tobias Cwik, John B. Taylor, and Volker Wieland (2010). "New Keynesian versus old Keynesian government spending multipliers". *Journal of Economic Dynamics and Control*, 34(3):281 – 295.

Conley, Timothy J. and Bill Dupor (2012). "The American Recovery and Reinvestment Act: Solely a Government Jobs Program?" Ohio State University, unpublished.

Cooley, Thomas F. and Edward C. Prescott (1995). "Economic Growth and Business Cycles". In Cooley, Thomas F. (ed.) "Frontiers of Business Cycle Research", chap. 1, 1–38. Princeton University Press.

Correia, Isabel, Emmanuel Fahri, Juan Pablo Nicolini, and Pedro Teles (2010). "Unconventional Fiscal Policy at the Zero Bound". *draft, Banco de Portugal*.

Davig, Troy and Eric M. Leeper (2009). "Monetary-Fiscal Policy Interactions and Fiscal Stimulus". NBER Working Papers 15133, National Bureau of Economic Research.

Del Negro, Marco, Marc P. Giannoni, and Frank Schorfheide (2013). "Inflation in the Great Recession and New Keynesian models". Staff Reports 618, Federal Reserve Bank of New York.

Eggertsson, Gauti B. (2010). "The Paradox of Toil". *draft, Federal Reserve Bank of New York*.

——— (2011). *What Fiscal Policy is Effective at Zero Interest Rates?*, 59–112. University of Chicago Press.

Eichenbaum, Martin and Jonas D.M. Fisher (2007). "Estimating the frequency of price re-optimization in Calvo-style models". *Journal of Monetary Economics*, 54(7):2032–2047.

Erceg, Christopher J. and Jesper Linde (2010). "Is There a Fiscal Free Lunch in a Liquidity Trap?" CEPR Discussion Paper 7624.

Farhi, Emmanuel and Ivan Werning (2012). "Fiscal Multipliers: Liquidity Traps and Currency Unions". NBER Working Papers 18381, National Bureau of Economic Research.

Fernandez-Villaverde, Jesus, Pablo A. Guerron-Quintana, and Juan Rubio-Ramirez (2011). "Supply-Side Policies and the Zero Lower Bound". NBER Working Papers 17543, National Bureau of Economic Research.

Galí, Jordi (2008). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press, Princeton, NJ.

Galí, Jordi, J. David Lopez-Salido, and Javier Valles (2007). "Understanding the Effects of Government Spending on Consumption". *Journal of the European Economic Association*, 5(1):227–270.

Gust, Christopher, David López-Salido, and Matthew E. Smith (2013). "The Empirical Implications of the Interest-Rate Lower Bound". Federal Reserve Board.

Hall, Robert E. (2011). "The Long Slump". *American Economic Review*, 101(2):431–69.

Ilut, Cosmin and Martin Schneider (2011). "Ambiguous Business Cycles". *draft, Duke University*.

Juillard, Michel (1996). "Dynare: A program for the resolution and simulation of dynamic models with forward variables through the use of a relaxation algorithm". *CEPREMAP, Couverture Orange 9602*.

Kimball, Miles S (1995). "The Quantitative Analytics of the Basic Neomonetarist Model". *Journal of Money, Credit and Banking*, 27(4):1241–77.

Krugman, Paul (2009). "How Did Economists Get It So Wrong?" *New York Times, September 2, 2009*.

Lawrance, Emily C (1991). "Poverty and the Rate of Time Preference: Evidence from Panel Data". *Journal of Political Economy*, 99(1):54–77.

Leeper, Eric M., Michael Plante, and Nora Traum (2010). "Dynamics of fiscal financing in the United States". *Journal of Econometrics*, 156(2):304–321.

Leeper, Eric M., Nora Traum, and Todd B. Walker (2011). "Clearing up the Fiscal Multiplier Morass: A Bayesian Perspective". *Draft, University of Indiana.*

Leeper, Eric M., Todd B. Walker, and Shu-Chun Susan Yang (2009). "Government Investment and Fiscal Stimulus in the Short and Long Runs". NBER Working Papers 15153, National Bureau of Economic Research.

Mertens, Karel and Morten O. Ravn (2013). "Fiscal Policy in an Expectations Driven Liquidity Trap". draft, Cornell University.

Oh, Hyunseung and Ricardo Reis (2012). "Targeted transfers and the fiscal response to the great recession". *Journal of Monetary Economics*, 59, Supplement:S50 – S64.

Ramey, Valerie (2011). "Government Spending and Private Activity". *draft.*

Shoag, Daniel (2010). "The Impact of Government Spending Shocks: Evidence on the Multiplier from State Pension Plan Returns". *draft, Harvard University.*

Smets, Frank and Raf Wouters (2003). "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area". *Journal of the European Economic Association*, 1(5):1123–1175.

——— (2007). "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach". *The American Economic Review*, 97(3):586–606.

Spilimbergo, Antonio, Steve Symansky, Olivier Blanchard, and Carlo Cottarelli (2008). "Fiscal Policy for the Crisis". *IMF Staff Position Note SPN/08/01.*

Trabandt, Mathias and Harald Uhlig (2011). "The Laffer Curve revisited." *Journal of Monetary Economics*, 58(4):305–327.

Uhlig, Harald (2010a). "Economics and Reality". Working Paper 16416, National Bureau of Economic Research.

——— (2010b). "Some Fiscal Calculus". *American Economic Review, Papers and Proceedings*, 100:30–34.

Warner, John T. and Saul Pleeter (2001). "The Personal Discount Rate: Evidence from Military Downsizing Programs". *American Economic Review*, 91(1):33–53.

Woodford, Michael (2011). "Simple Analytics of the Government Expenditure Multiplier". *American Economic Journal: Macroeconomics*, 3(1):1–35.

8 Appendix

8.1 Data

The different series come from the NIPA tables, the FRED 2 database and the Bureau of Labor Statistics (BLS) database. Federal debt data is taken from database of the Federal Reserve Bank of Dallas. Nominal series for wages, consumption, government expenditures, and private investment are deflated with the general GDP deflator.

Generally we follow Smets and Wouters (2007) when creating our dataset with the following exceptions: we use civilian noninstitutionalized population throughout, although the series is not seasonally adjusted before 1976. The base year for real GDP is 2005 instead of 1996. We include durables consumption in investment instead of consumption. Using the same definition, all series but real wages exhibit a correlation of almost 100 percent across the two datasets. For the change in real wages, the correlation is 0.9. Including durables consumption in investment causes the correlation for the investment series to drop to 0.70 and for consumption to drop to 0.78.

Since no data for the Corporate-Treasury bond yield spread is available before 1953:1 we set it to zero for the missing periods. We use the secondary market rate for three-months T-bills before 1954:3, as the FFR is not available.

The categorization of the various stimulus components is shown in detail in tables 9, 10, and 11 in the Technical Appendix. Our source is the CBO (2009), specifically "Table 2: Estimated cost of the conference agreement for H.R. 1, the American Recovery and Reinvestment Act of 2009, as posted on the website of the House Committee on Rules." The annual time path for these expenditures is taken from CBO (2009) and the annual sum for each component is split across quarters in proportion to the aggregate series in Cogan et al. (2010).

8.2 Estimation

Tables 7 and 8 contain the results from estimating our model, using Dynare and a Bayesian prior.

Table 7: *Estimation, part 1. The calibrated government investment-to-GDP ratio as well as the estimated growth trend μ implies a government share in production of $\zeta = 2.30$ percent.*

	Prior	Prior mean (s.d.)	SW model 66:1-08:4	Our model 49:2-08:4
Adj. cost $S''(\mu)$	norm	4.000 (1.500)	5.93 (1.1)	4.51 (0.78)
Risk aversion σ	norm	1.500 (0.375)	1.42 (0.11)	1.17 (0.08)
Habit h	beta	0.700 (0.100)	0.7 (0.04)	0.85 (0.02)
Calvo wage ζ_w	beta	0.500 (0.100)	0.77 (0.05)	0.83 (0.03)
Inv. labor sup. ela. ν	norm	2.000 (0.750)	1.96 (0.54)	2.16 (0.51)
Calvo prices ζ_p	beta	0.500 (0.100)	0.69 (0.05)	0.81 (0.03)
Wage indexation ι_w	beta	0.500 (0.150)	0.62 (0.1)	0.41 (0.08)
Price indexation ι_p	beta	0.500 (0.150)	0.26 (0.08)	0.28 (0.07)
Capacity util.	beta	0.500 (0.150)	0.59 (0.1)	0.43 (0.07)
$1+\frac{\text{Fix. cost}}{Y} = 1+\lambda_p$	norm	1.250 (0.125)	1.64 (0.08)	1.94 (0.05)
Taylor rule infl. ψ_1	norm	1.500 (0.250)	2 (0.17)	1.63 (0.18)
same, smoothing ρ_R	beta	0.750 (0.100)	0.82 (0.02)	0.92 (0.02)
same, LR gap ψ_2	norm	0.125 (0.050)	0.09 (0.02)	0.13 (0.03)
same, SR gap ψ_3	norm	0.125 (0.050)	0.24 (0.03)	0.2 (0.02)
Mean inflation (data)	gamm	0.625 (0.100)	0.76 (0.09)	0.58 (0.08)
100×time pref.	gamm	0.250 (0.100)	0.16 (0.05)	0.12 (0.04)
Mean hours (data)	norm	0.000 (2.000)	1.07 (0.95)	0.04 (0.69)
Trend $(\mu - 1) * 100$	norm	0.400 (0.100)	0.43 (0.02)	0.48 (0.01)
Capital share α	norm	0.300 (0.050)	0.19 (0.02)	0.24 (0.01)
Gov. adj. cost $S_g''(\mu)$	norm	0.000 (0.500)	n/a	7.11 (1.09)
Budget bal speed $\frac{\psi_\tau - 0.025}{0.175}$	beta	0.25 (0.1637)	n/a	0.05 (0.04)
Mean gov. debt	norm	0.000 (0.500)	n/a	-0.16 (0.51)
Mean bond spread	gamm	0.500 (0.100)	n/a	0.47 (0.04)

Table 8: *Estimation, part 2*

	Prior	Prior mean (s.d.)	SW model 66:1-08:4	Our model 49:2-08:4
s.d. tech.	invg	0.100 (2.000)	0.46 (0.03)	0.47 (0.02)
AR(1) tech.	beta	0.500 (0.200)	0.95 (0.01)	0.95 (0.01)
s.d. bond	invg	0.100 (2.000)	0.24 (0.03)	0.95 (0.04)
AR(1) bond ρ_q	beta	0.500 (0.200)	0.27 (0.1)	0.67 (0.03)
s.d. gov't	invg	0.100 (2.000)	0.54 (0.03)	0.36 (0.02)
AR(1) gov't	beta	0.500 (0.200)	0.98 (0.01)	0.98 (0.01)
Cov(gov't, tech.)	norm	0.500 (0.250)	0.53 (0.09)	0.3 (0.04)
s.d. inv. price	invg	0.100 (2.000)	0.43 (0.04)	1.25 (0.1)
AR(1) inv. price	beta	0.500 (0.200)	0.73 (0.06)	0.56 (0.06)
s.d. mon. pol.	invg	0.100 (2.000)	0.24 (0.02)	0.22 (0.01)
AR(1) mon. pol.	beta	0.500 (0.200)	0.16 (0.07)	0.22 (0.05)
s.d. goods m-up	invg	0.100 (2.000)	0.14 (0.01)	0.32 (0.02)
AR(1) goods m-up	beta	0.500 (0.200)	0.89 (0.04)	0.91 (0.05)
MA(1) goods m-up	beta	0.500 (0.200)	0.73 (0.08)	0.96 (0.02)
s.d. wage m-up	invg	0.100 (2.000)	0.26 (0.02)	0.23 (0.02)
AR(1) wage m-up	beta	0.500 (0.200)	0.97 (0.01)	0.97 (0.01)
MA(1) wage m-up	beta	0.500 (0.200)	0.91 (0.03)	0.92 (0.02)
s.d. Tax shock	invg	0.100 (2.000)	n/a	1.44 (0.08)
AR(1) tax shock	beta	0.500 (0.200)	n/a	0.98 (0.01)
s.d. gov. inv. price	invg	0.100 (2.000)	n/a	0.79 (0.08)
AR(1) gov. inv. price	beta	0.500 (0.200)	n/a	0.97 (0.01)
s.d. bond spread	invg	0.100 (2.000)	n/a	0.08 (0)
AR(1) bond spread	beta	0.500 (0.200)	n/a	0.91 (0.02)

Technical Appendix

A Additional results

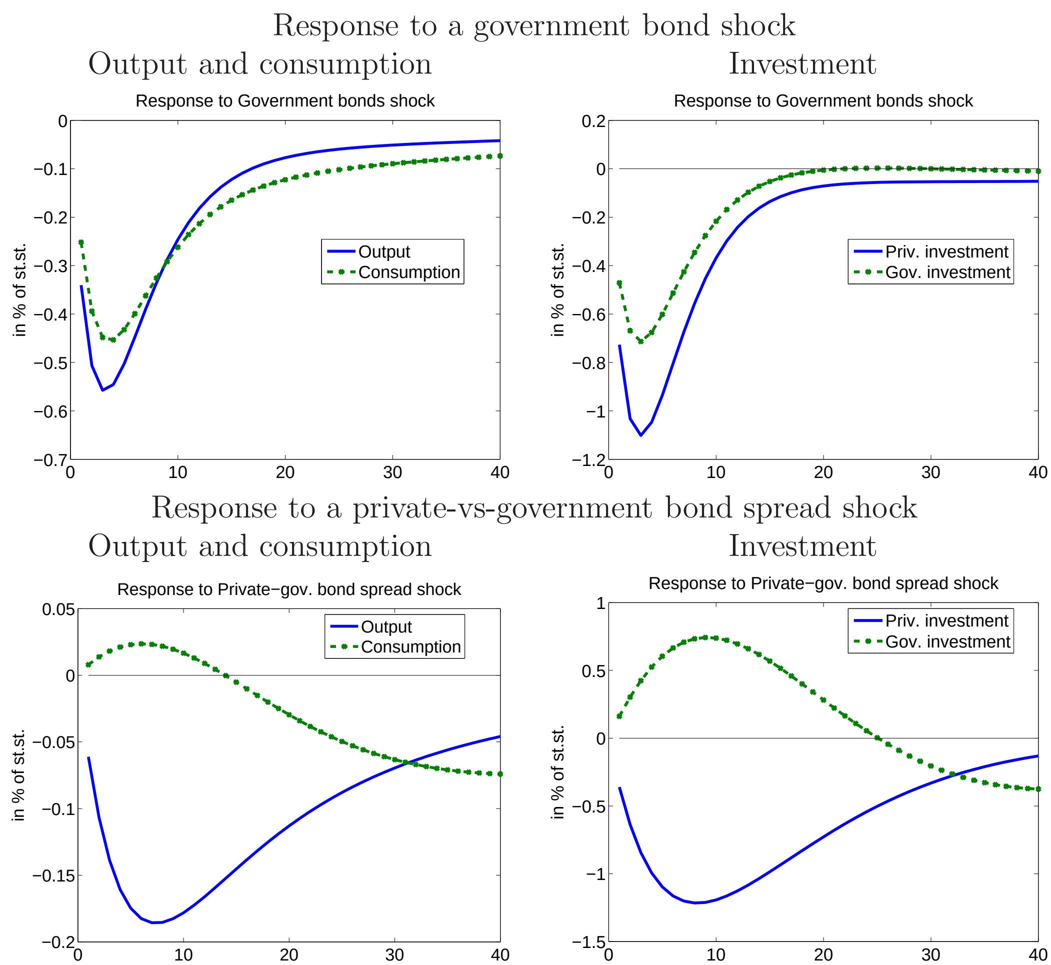


Figure 13: Response to the bond shocks

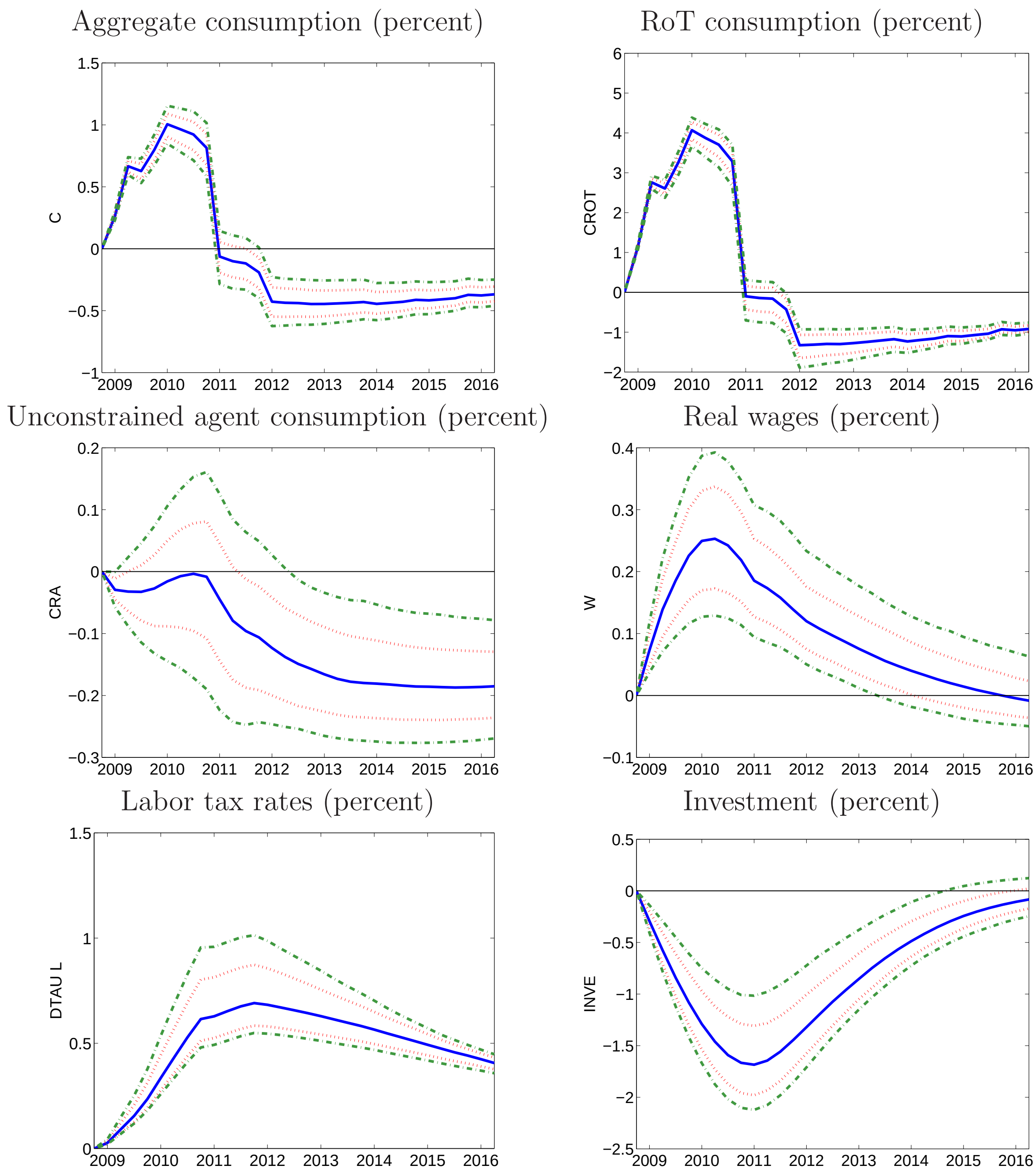


Figure 14: Benchmark impact of ARRA: consumption, investment, tax rates, and real wages

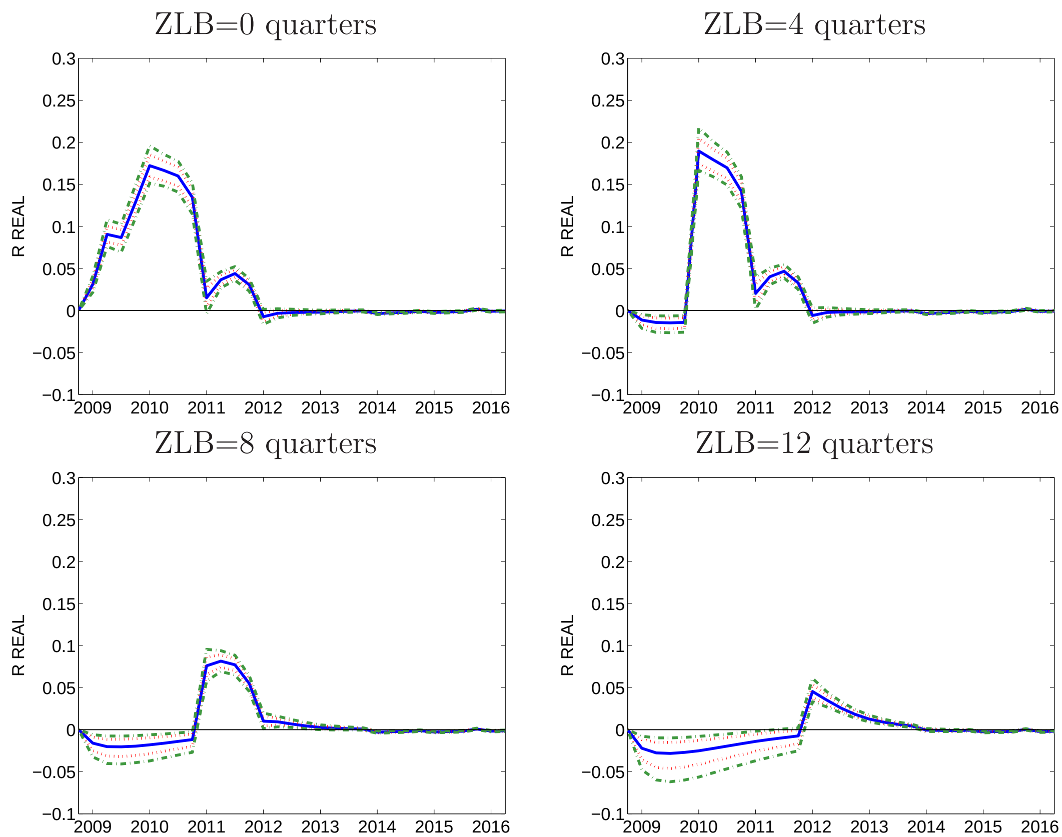


Figure 15: Impact of ARRA on real interest rates for varying ZLB length

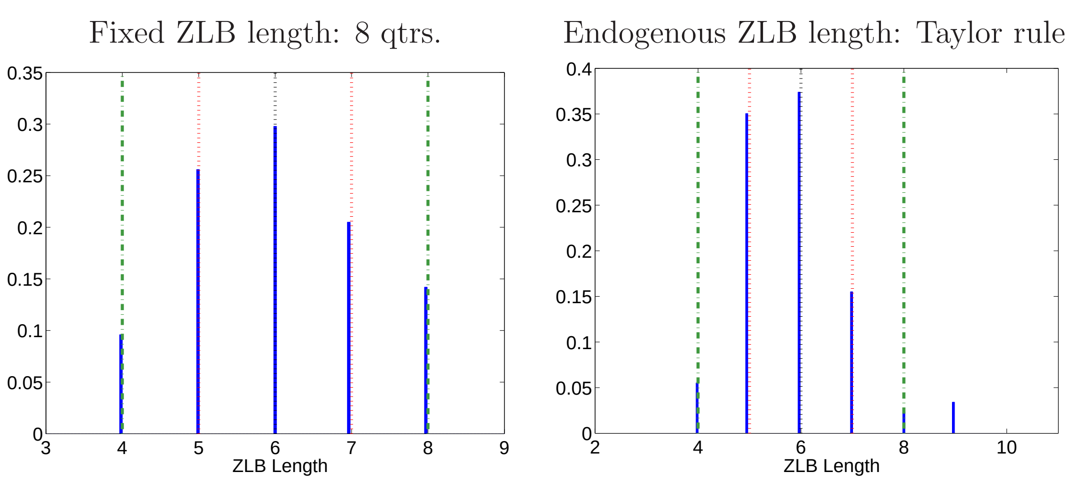


Figure 16: ZLB duration implied by Taylor rule

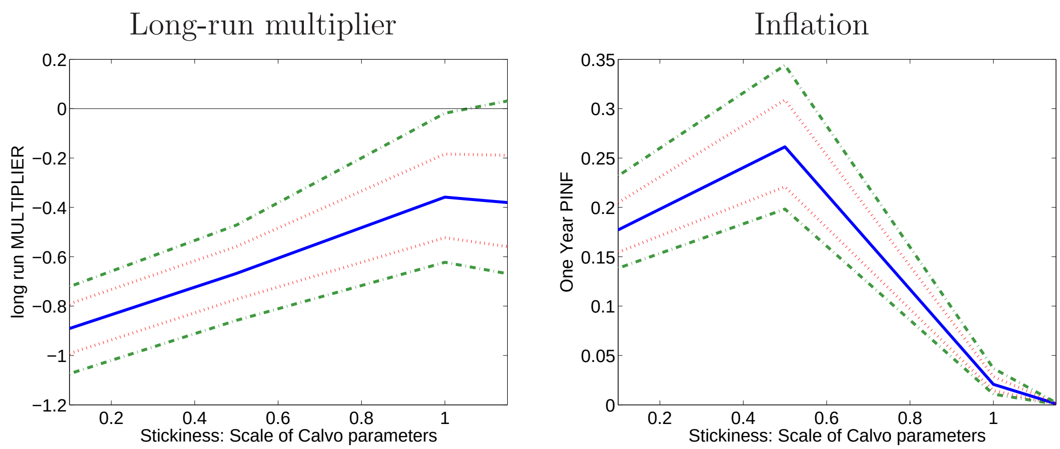


Figure 17: Long-run multiplier and inflation response: sensitivity to price and wage stickiness

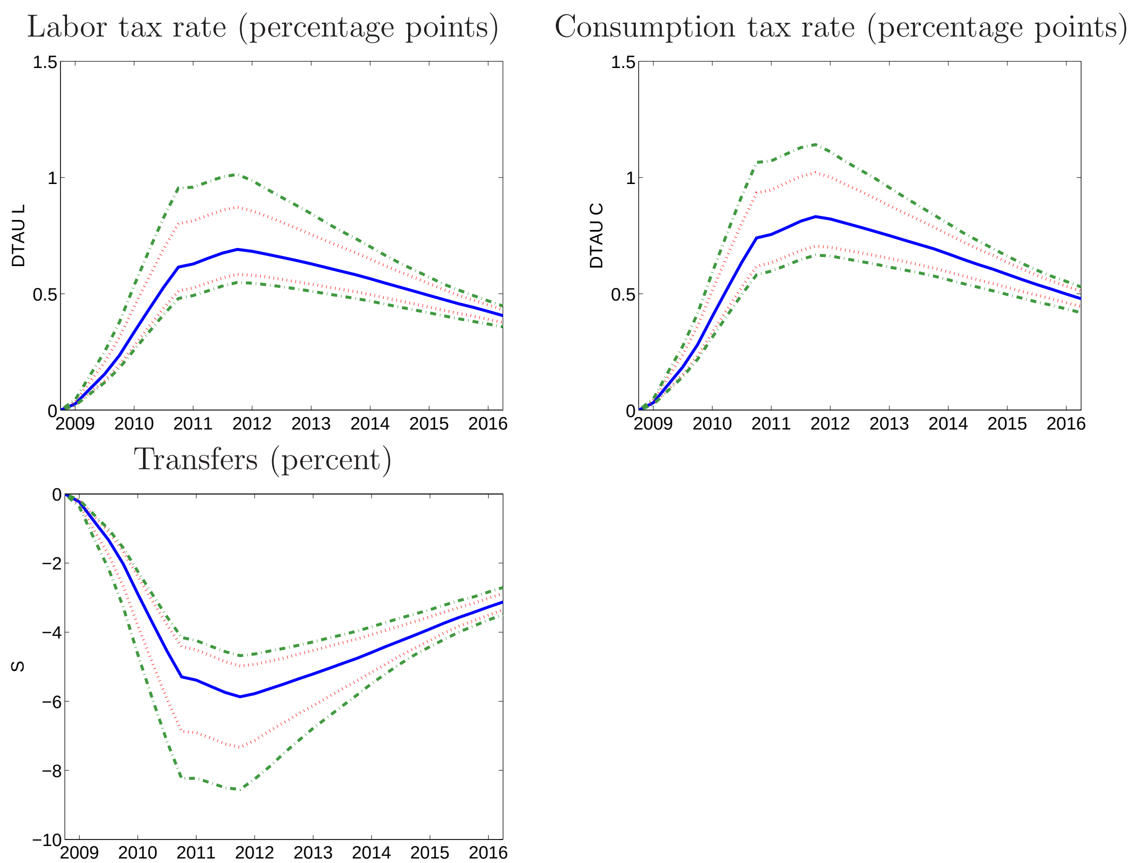


Figure 18: Changes in tax rates and lump-sum transfers due to stimulus

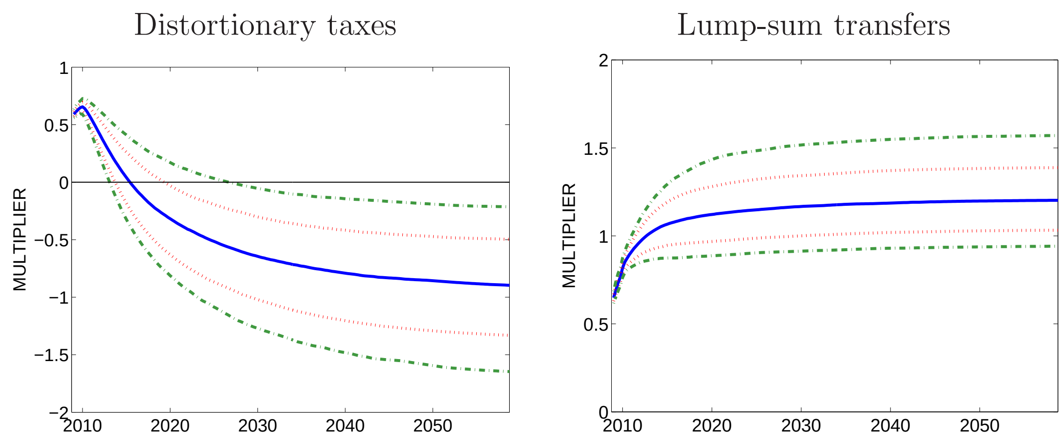


Figure 19: Fiscal multipliers: Stimulus spent uniformly over first four quarters. Comparing labor taxes (benchmark) and lump-sum taxation

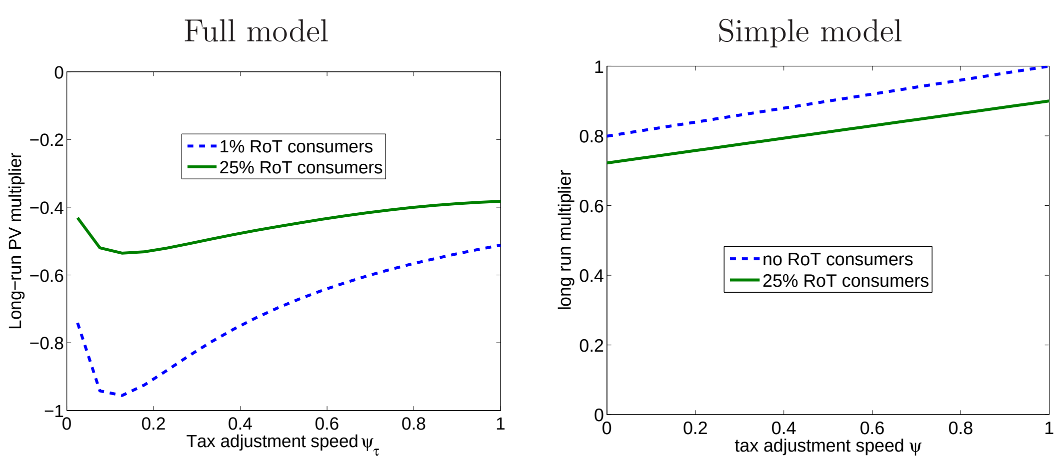


Figure 20: Multipliers as a function of tax adjustment speed and rule-of-thumb consumers

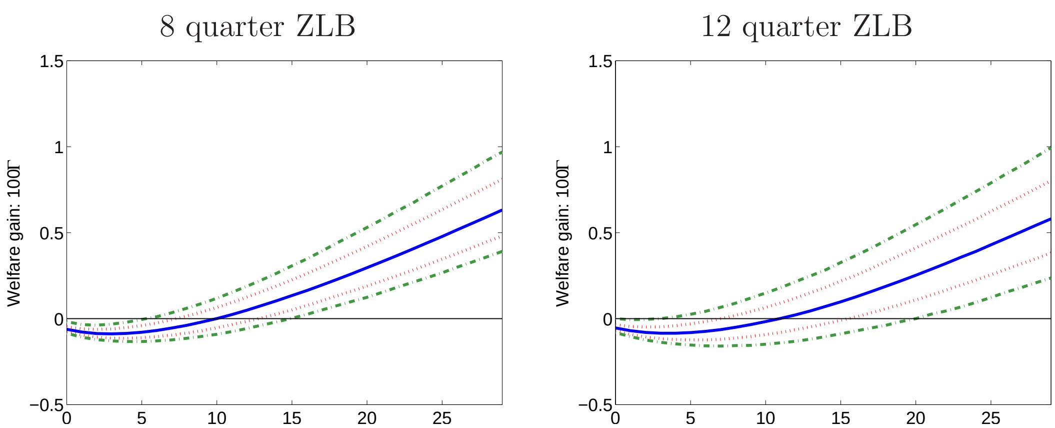


Figure 21: Long-run welfare gains from stimulus: 8 and 12 quarter ZLB, varying annual rate of time preference as compared with unconstrained agents

B Categorizing stimulus spending

Table 9: Categorizing the stimulus – government consumption

Item	Amount (bn USD)	Share
Dept. of Defense	4.53	0.59
Employment and Training	4.31	0.56
Legislative Branch	0.03	0
National Coordinator for Health Information Technology	1.98	0.26
National Institutes of Health	9.74	1.26
Other Agriculture, Food, FDA	3.94	0.51
Other Commerce, Justice, Science	5.36	0.69
Other Dept. of Education	2.12	0.28
Other Dept. of Health and Human Services	9.81	1.27
Other Financial Services and Gen. Govt	1.31	0.17
Other Interior and Environment	4.76	0.62
Special Education	12.2	1.58
State and Local Law Enforcement	2.77	0.36
State Fiscal Relief	90.04	11.68
State Fiscal Stabilization Fund	53.6	6.95
State, Foreign Operations, and Related Programs	0.6	0.08
Other	2.55	0.33
Consumption	209.64	27.2

Table 10: Categorizing the stimulus –government investment

Item	Amount (bn USD)	Share
Broadband Technology Opportunities Program	4.7	0.61
Clean Water and Drinking Water State Revolving Fund	5.79	0.75
Corps of Engineers	4.6	0.6
Distance Learning, Telemedicine, and Broadband Program	1.93	0.25
Energy Efficiency and Renewable Energy	16.7	2.17
Federal Buildings Fund	5.4	0.7
Health Information Technology	17.56	2.28
Highway Construction	27.5	3.57
Innovative Technology Loan Guarantee	6	0.78
NSF	2.99	0.39
Other Energy	22.38	2.9
Other Transportation	20.56	2.67
Investment	136.09	17.66

Table 11: Categorizing the stimulus – transfers

Item	Amount (bn USD)	Share
Assistance for the Unemployed	0.88	0.11
Economic Recovery Programs, TANF, Child Support	18.04	2.34
Health Insurance Assistance (spending)	25.07	3.25
Health Insurance Assistance (revenue)	-0.39	-0.05
Low Income Housing Program	0.14	0.02
Military Construction and Veteran Affairs	4.25	0.55
Other housing assistance	9	1.17
Other Tax Provisions	4.81	0.62
Public housing capital fund	4	0.52
Refundable Tax Credits	68.96	8.95
Student financial assistance	16.56	2.15
Supplemental Nutrition Assistance Program	19.99	2.59
Tax Provisions	214.56	27.84
Unemployment Compensation	39.23	5.09
Transfers and tax cuts	425.09	55.15

C Backing out the unemployment rate

To back out the model implications for the unemployment rate, we regress the time series for hours worked used for the model estimation on the average quarterly unemployment rate. Table 12 shows the regression results. Figure 22 displays the actual and fitted unemployment rate. Multiplying hours worked on the OLS regression coefficient gives the implied change in the unemployment rate.

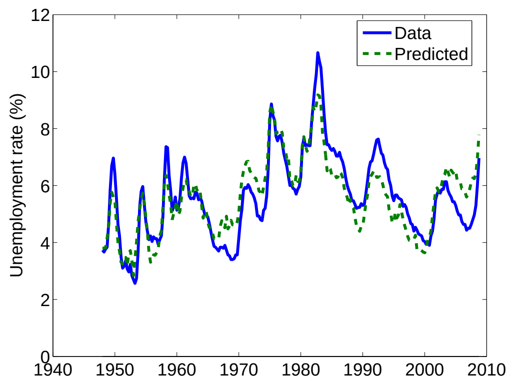


Figure 22: Regression of quarterly unemployment rate on the model-implied employment measure: actual vs. predicted unemployment rate

Table 12: OLS regression estimates of unemployment rate on the model-implied employment measure

	Constant	Employment (lab_t)	R^2
Unemployment Rate (UR_t)	5.60	-0.46	0.77
	(5.51, 5.69)	(-0.49, -0.43)	

Sample period: 1948:1 – 2008:4. Unemployment rate is the arithmetic mean over the quarter. Labor input in the model is measured as $\text{lab}_t \equiv \log \frac{\text{Avg. hours}_t \times \text{Employment}_t}{\text{Population}_t} - \text{mean}$; 95 percent OLS confidence intervals in parentheses.

D Model appendix

Apart from the model extensions due to the introduction of government capital, rule-of-thumb consumers, and distortionary taxation, the following model appendix follows mostly the appendix of Smets and Wouters (2007), with minor changes to unify the notation.

D.1 Production

Final goods are produced in a competitive final goods sector that uses differentiated intermediate inputs, supplied by monopolistic intermediate producers.

D.1.1 Final goods producers

The representative final goods producer maximizes profits by choosing intermediate inputs $Y_t(i), i \in [0,1]$, subject to a production technology that generalizes a CES production function: Objective:

$$\max_{Y_t, Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \quad \text{s.t.} \quad \int_0^1 G\left(\frac{Y_t(i)}{Y_t}; \tilde{\epsilon}_t^{\lambda,p}\right) di = 1. \tag{D.1}$$

$G(\cdot)$ is the Kimball (1995) aggregator, which generalizes CES demand by allowing the elasticity of demand to increase with relative prices: $G' > 0, \quad G'' < 0, \quad G(1; \tilde{\epsilon}_t^{\lambda,p}) = 1$. $\tilde{\epsilon}_t^{\lambda,p}$ is a shock to the production technology which changes the elasticity of substitution.

Denote the Lagrange multiplier on the constraint by Ξ_t^f. If a positive solution to equation (D.1) exists it satisfies the following conditions:

$$\begin{aligned} [Y_t] \quad P_t &= \Xi_t^f \frac{1}{Y_t} \int_0^1 G'\left(\frac{Y_t(i)}{Y_t}; \tilde{\epsilon}_t^{\lambda,p}\right) \frac{Y_t(i)}{Y_t} di, \\ [Y_t(i)] \quad P_t(i) &= \Xi_t^f \frac{1}{Y_t} G'\left(\frac{Y_t(i)}{Y_t}; \tilde{\epsilon}_t^{\lambda,p}\right). \end{aligned}$$

From these two equations, we obtain an expression for the aggregate price index and intermediate inputs. The price index is given by:

$$P_t = \int_0^1 \frac{Y_t(i)}{Y_t} P_t(i) di. \tag{D.2}$$

Solving for intermediate input demands:

$$Y_t(i) = Y_t G'^{-1}\left(\frac{P_t(i)Y_t}{\Xi_t^f}\right) = Y_t G'^{-1}\left(\frac{P_t(i)}{P_t}\int_0^1 G'\left(\frac{Y_t(j)}{Y_t};\tilde{\epsilon}_t^{\lambda,p}\right)\frac{Y_t(j)}{Y_t}dj\right). \tag{D.3}$$

For future reference, note that the relative demand curves $\mathrm{y}_t(i) \equiv \frac{Y_t(i)}{Y_t}$ are downward sloping in the relative price $\frac{P_t(i)}{P_t}$ with an decreasing elasticity as the relative quantity increases. For simplicity, the dependence of the $G(\cdot)$ aggregator on the shock $\tilde{\epsilon}_t^{\lambda,p}$ is suppressed:

$$\eta_p(\mathrm{y}_t(i)) \equiv -\frac{P_t(i)}{Y_t(i)}\frac{d\mathrm{y}_t(i)}{dP_t(i)}\Big|_{dY_t=d\Xi_t^f=0} = -\frac{G'(\mathrm{y}_t(i))}{\mathrm{y}_t(i)G''(\mathrm{y}_t(i))} \tag{D.4}$$

$$\begin{aligned}
\hat{\eta}_p(\mathrm{y}_t(i)) &\equiv \frac{P_t(i)}{\eta_p(\mathrm{y}_t(i))}\frac{d\eta_p(\mathrm{y}_t(i))}{dP_t(i)} = 1+\eta_p+\eta_p\frac{G'''(\mathrm{y}_t(i))}{G''(\mathrm{y}_t(i))}\mathrm{y}_t(i)\\
&= 1+\eta_p(\mathrm{y}_t(i))\left(2+\frac{G'''(\mathrm{y}_t(i))}{G''(\mathrm{y}_t(i))}\mathrm{y}_t(i)-1\right)\\
&= 1+\eta_p(\mathrm{y}_t(i))\left(\frac{2+\frac{G'''(\mathrm{y}_t(i))}{G''(\mathrm{y}_t(i))}\mathrm{y}_t(i)}{1-\eta_p(\mathrm{y}_t(i))^{-1}}(1-\eta_p(\mathrm{y}_t(i))^{-1})-1\right)\\
&\equiv 1+\frac{1+\lambda^p(\mathrm{y}_t(i))}{\lambda^p(\mathrm{y}_t(i))}\left(\frac{1}{[1+\lambda^p(\mathrm{y}_t(i))]A_p(\mathrm{y}_t(i))}-1\right),
\end{aligned} \tag{D.5}$$

where the last line defines the markup λ_t^p and the parameter A_p as

$$\lambda_t^p(\mathrm{y}_t(i)) \equiv \frac{1}{\eta_p(\mathrm{y}_t(i))-1}, \qquad A_p(\mathrm{y}_t(i)) \equiv \frac{\lambda^p(\mathrm{y}_t(i))}{2+\frac{G'''(\mathrm{y}_t(i))}{G''(\mathrm{y}_t(i))}\mathrm{y}_t(i)}.$$

The model will be parameterized in terms of $\hat{\epsilon}(1)$, the change in the own-price elasticity of demand along the balanced growth path. To that end, it is convenient to solve for A_p in terms of the markup and the $\hat{\epsilon}$:

$$A_p(\mathrm{y}) = \frac{1}{\lambda^p(\mathrm{y})\hat{\eta}_p(\mathrm{y})+1}. \tag{D.6}$$

Finally, note that in the Dixit-Stiglitz case $G(\mathrm{y}) = \mathrm{y}^{\frac{1}{1+\lambda^p}}$ so that the elasticity of demand is constant at $\eta_p(\mathrm{y}) = \frac{1}{\lambda^p}+1\forall\mathrm{y}$ and consequently $\hat{\eta}_p = 0$.

D.1.2 Intermediate goods producers

There is a unit mass of intermediate producers, indexed by $i \in [0,1]$. Each producer is the monopolistic supplier of good i. They rent capital services

K_t^{eff} and hire labor n_t to maximize profits intertemporally, taking as given rental rates R_t^k and wages W_t. Given a Calvo-style pricing friction, their profit-maximization problem is dynamic.

Production is subject to a fixed cost and the gross product is produced using a Cobb-Douglas technology at the firm level. Government capital K_t^g increases total factor productivity in each firm, but is subject to a congestion effect as overall production increases, similar to the congestion effects in the AK model in Barro and Sala-i Martin (1992). Firms fail to internalize the effect of their decisions on public sector productivity. Net output is therefore given by:

$$Y_t(i) = \tilde{\epsilon}_t^a \left(\frac{K_{t-1}^g}{\int_0^1 Y_t(j)dj + \Phi\mu^t} \right)^{\frac{\zeta}{1-\zeta}} K_t^{eff}(i)^{\alpha} [\mu^t n_t(i)]^{1-\alpha} - \mu^t\Phi, \tag{D.7}$$

where $\Phi\mu^t$ represent fixed costs which grow at the rate of labor augmenting technical progress and $K_t(i)^{eff}$ denotes the capital services rented by firm i. $\tilde{\epsilon}_t^a$ denotes a stationary TFP process.

To see the implications of the congestion costs, consider the symmetric case that $Y_t(i) = Y_t, K_t^{eff}(i) = K_t^{eff} \forall i$, which is the case along the symmetric balanced growth path and in the flexible economy. We then obtain the following aggregate production function:

$$Y_t = \epsilon_t^a {K_{t-1}^g}^{\zeta} {K_t^{eff}}^{\alpha(1-\zeta)} [\mu^t n_t]^{(1-\alpha)(1-\zeta)} - \mu^t\Phi, \quad \epsilon_t^a \equiv (\tilde{\epsilon}_t^a)^{1-\zeta}. \tag{D.8}$$

Choose units such that $\bar{\epsilon}^a \equiv 1$.

To solve a firm's profit maximization problem, note that it is equivalent to minimizing costs (conditional on operating) and then choosing the quantity optimally. Consider the cost-minimization problem first:

$$\min_{K_t(i), n_t(i)} W_t n_t(i) + R_t^k K_t(i) \text{ s.t. (D.7)}.$$

Denote the Lagrange multiplier on the production function by MC_t: Producing a marginal unit more raises costs (the objective) by MC_t. The static FOC are necessary and sufficient, given $Y_t(i)$:

$$[n_t(i)] \quad MC_t(i)(1-\alpha)\frac{Y_t(i) + \mu^t\Phi}{n_t(i)} = W_t,$$
$$[K_t(i)] \quad MC_t(i)\alpha\frac{Y_t(i) + \mu^t\Phi}{K_t(i)} = R_t^k.$$

The FOC can be used to solve for the optimal capital-labor ratio in production and marginal costs:

$$\frac{k_t(i)}{n_t(i)} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}, \tag{D.9}$$

$$MC_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}\frac{W_t^{1-\alpha}(R_t^k)^\alpha \mu^{-(1-\alpha)t}}{\left(\frac{K_{t-1}^g}{Y_t+\mu^t\Phi}\right)^{\frac{\zeta}{1-\zeta}}\epsilon_t^a}, \tag{D.10}$$

$$mc_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}\frac{w_t^{1-\alpha}(r_t^k)^\alpha}{\left(\frac{\mu k_{t-1}^g}{y_t+\Phi}\right)^{\frac{\zeta}{1-\zeta}}\epsilon_t^a},$$

$$mc_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}\frac{w_t^{1-\alpha}(r_t^k)^\alpha}{\left(\frac{\mu k_{t-1}^g}{y_t+\Phi}\right)^{\frac{\zeta}{1-\zeta}}\epsilon_t^a}, \tag{D.11}$$

where lower-case letters denote detrended, real variables, as applicable:

$$k_t \equiv K_t\mu^{-t}, y_t \equiv Y_t\mu^{-t}, w_t \equiv \frac{W_t}{\mu^t P_t}, r_t^k \equiv \frac{R_t^k}{P_t}, mc_t \equiv \frac{MC_t}{P_t}.$$

For future reference, it is useful to detrend the FOC:

$$w_t = mc_t(i)(1-\alpha)\frac{y_t(i)+\Phi}{n_t(i)}, \tag{D.12a}$$

$$r_t^k = mc_t(i)\alpha\frac{y_t(i)+\Phi}{k_t(i)}. \tag{D.12b}$$

Given the solution to the static cost-minimization problem, the firm maximizes the present discounted value of its profits by choosing quantities optimally, taking as given its demand function (D.3), the marginal costs of production (D.10), and the Calvo-style price-setting friction. The Calvo-friction implies that a firm can re-set its price in each period with probability $1-\zeta_p$ and otherwise indexes its price to an average of current and past inflation $\prod_{l=1}^{s}\pi_{t+l-1}^{\iota_p}\bar{\pi}^{1-\iota_p}$. In each period t that the firm can change its prices it chooses:

$$P_t^*(i) = \arg\max_{\tilde{P}_t(i)} \mathbb{E}_t \sum_{s=0}^{\infty}\zeta_p^s\frac{\bar{\beta}^s\xi_{t+s}P_t}{\xi_t P_{t+s}}\Big[\tilde{P}_t(i)\Big(\prod_{l=1}^{s}\pi_{t+l-1}^{\iota_p}\bar{\pi}^{1-\iota_p}\Big) - MC_{t+s}(i)\Big]Y_{t+s}(i),$$

subject to (D.3) and (D.10). $\frac{\bar{\beta}^s\xi_{t+s}}{\xi_t}$ denotes the (noncredit-constrained) representative household's stochastic discount factor and $\pi_t \equiv \frac{P_t}{P_{t-1}}$ denotes period t inflation.

To solve the problem, it is useful to define $\chi_{t,t+s}$ such that in the absence of further price adjustments prices evolve as $P_{t+s}(i) = \chi_{t,t+s} P_t^*(i)$:

$$\chi_{t,t+s} = \begin{cases} 1 & s = 0, \\ \prod_{l=1}^{s} \pi_{t+l-1}^{\iota_p} \bar{\pi}^{1-\iota_p} & s = 1, \dots, \infty. \end{cases}$$

Using the definition $\mathrm{y}_{t+s}(i) = \frac{Y_{t+s}(i)}{Y_{t+s}}$ yields therefore:

$$\frac{d(Y_{t+s}(i)[P_{t+s}(i) - MC_{t+s}(i)])}{d\tilde{P}_t(i)} = \mathrm{y}_{t+s}(i) Y_{t+s} \left(\chi_{t,t+s}[1 - \eta_p(\mathrm{y}_{t+s}(i))] + \eta_p \frac{MC_{t+s}(i)}{P_t(i)} \right).$$

The first order condition is then given by:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\bar{\beta}^s \xi_{t+s} P_t}{\xi_t P_{t+s}} \mathrm{y}_{t+s}(i) Y_{t+s} \left([1 - \eta_p(\mathrm{y}_{t+s}(i))] \chi_{t,t+s} + \eta_p \frac{MC_{t+s}(i)}{P_t(i)} \right) = 0 \tag{D.13}$$

For future reference, it is useful to rewrite the FOC as follows:

$$\frac{P_t^*(i)}{P_t} = \frac{\mathbb{E}_t \sum_{s=0}^{\infty} (\mu \bar{\beta} \zeta_p)^s \frac{\xi_{t+s}}{\lambda_p(\mathrm{y}_{t,t+s}(i)) \xi_t} y_{t,t+s}(i) \frac{\eta_p(\mathrm{y}_{t,t+s}(i))}{\eta_p(\mathrm{y}_{t,t+s}(i)) - 1} mc_{t+s}(i)}{\mathbb{E}_t \sum_{s=0}^{\infty} (\mu \bar{\beta} \zeta_p)^s \frac{\xi_{t+s}}{\lambda_p(\mathrm{y}_{t,t+s}(i)) \xi_t} \frac{\chi_{t,t+s}}{\prod_{l=1}^{s} \pi_{t+s}} y_{t,t+s}(i)} \tag{D.14}$$

where $\mathrm{y}_{t,t+s}(i) = G'^{-1} \left(\frac{P_t^* \chi_{t,t+s} Y_{t+s}}{\Xi_{t+s}^f} \right), Y_{t,t+s}(i) = \mathrm{y}_{t,t+s}(i) Y_{t+s}$.

Noting that measure $1 - \zeta_p$ of firms changes prices in each period and that each firm faces a symmetric problem, the expression for the aggregate price index (D.2) can be expressed recursively as a weighted average of adjusted and indexed prices:

$$P_t = (1 - \zeta_p) P_t^* G'^{-1} \left(\frac{P_t^* Y_t}{\Xi_t^f} \right) + \zeta_p \pi_{t-1}^{\iota_p} \bar{\pi}^{1-\iota_p} P_{t-1} G'^{-1} \left(\frac{\pi_{t-1}^{\iota_p} \bar{\pi}^{1-\iota_p} P_{t-1} Y_t}{\Xi_t^f} \right). \tag{D.15}$$

This expression uses that price distribution of nonadjusting firms at t is the same as that of all firms at time $t - 1$, adjusted for the shrinking mass due to price adjustments. The optimal price equals the average price along the deterministic balanced growth path, which is normalized to unity:

$$\bar{P}^* = \bar{P} = 1.$$

Similarly, along the deterministic growth path, the price is a constant markup over marginal cost:

$$\frac{\bar{P}^*}{\bar{P}} = \frac{\eta_p}{\eta_p - 1}\overline{mc} = (1 + \bar{\lambda}_p)\overline{mc} = 1 \tag{D.16}$$

Finally, the assumption of monopolistic competition in the presence of free entry requires zero profits along the balanced growth path. Real and detrended profits of intermediate producer i are given by:

$$\Pi_t^p(i) = \frac{P_t(i)}{P_t} y_t(i) - w_t n_t(i) - r_t^k k_t(i) = \frac{P_t(i)}{P_t} y_t(i) - mc_t(i)[y_t(i) + \mu^t \Phi]$$

Integrating over all $i \in [0, 1]$ and using the definition of the price index (D.2) yields:

$$\Pi_t^p = y_t - w_t \int_0^1 n_t(i)di - r_t^k \int_0^1 k_t(i)di \tag{D.17a}$$

$$= y_t - mc_t \left(\int_0^1 y_t(i)di + \Phi \right) = y_t - mc_t \left(y_t \int_0^1 \frac{P_t(i)}{P_t} di + \Phi \right) \tag{D.17b}$$

Using the expression for the steady state markup, equation (D.16), the zero-profit condition (D.17b) implies that along the symmetric balanced growth path:

$$0 = \bar{\Pi}^p = \bar{y} - \frac{\bar{y} \int_0^1 \frac{P(i)}{P} di + \Phi}{1 + \bar{\lambda}_p} = \bar{y} - \frac{\bar{y} + \Phi}{1 + \bar{\lambda}_p} \quad \Rightarrow \frac{\Phi}{\bar{y}} = \bar{\lambda}_p. \tag{D.18}$$

D.1.3 Labor packers

Intermediate producers use a bundle of differentiated labor inputs, $\ell \in [0, 1]$, purchased from labor packers. Labor packers aggregagte, or pack, differentiated labor, which they purchase from unions. They are perfectly competitive and face an analogous problem to final goods producers:

$$\max_{n_t, n_t(\ell)} W_t n_t - \int_0^1 W_t(\ell) n_t(\ell) d\ell \quad \text{s.t.} \quad \int_0^1 H\left(\frac{n_t(\ell)}{n_t}; \tilde{\epsilon}_t^{\lambda,w} \right) d\ell = 1, \tag{D.19}$$

where $H(\cdot)$ has the same properties as $G(\cdot)$: $H' > 0, H'' < 0, H(1) = 1$.

The FOC yield differentiated labor demand, analogous to intermediate goods demand (D.3):

$$n_t(\ell) = n_t H'^{-1}\left(\frac{W_t(\ell) n_t}{\Xi_t^n}\right) = n_t H'^{-1}\left(\frac{W_t(\ell)}{W_t}\int_0^1 H'\left(\frac{n_t(l)}{n_t};\tilde{\epsilon}_t^{\lambda,w}\right)\frac{n_t(l)}{n_t}dl\right). \tag{D.20}$$

Given the aggregate nominal wage $W_t = \int_0^1 \frac{n_t(\ell)}{n_t} w_t(\ell)d\ell$, labor packers are willing to supply any amount of packed labor n_t. Labor demand elasticity behaves analogously to the intermediate goods elasticity:

$$\eta_w(\mathrm{n}_t(\ell)) \equiv -\frac{W_t(\ell)}{n_t(\ell)}\frac{d\mathrm{n}_t(\ell)}{dW_t(\ell)}\Big|_{dn_t = d\Xi_t^l = 0} = -\frac{H'(\mathrm{n}_t(\ell))}{\mathrm{n}_t(\ell)H''(\mathrm{n}_t(\ell))} \tag{D.21}$$

$$\hat{\eta}_w(\mathrm{n}_t(\ell)) \equiv \frac{W_t(\ell)}{\eta_w(\mathrm{n}_t(\ell))}\frac{d\eta_w(\mathrm{n}_t(\ell))}{dW_t(\ell)} = 1 + \frac{1+\lambda^w(\mathrm{n}_t(\ell))}{\lambda^w(\mathrm{n}_t(\ell))}\left(\frac{1}{[1+\lambda^w(\mathrm{n}_t(\ell))]A_w(\mathrm{n}_t(\ell))} - 1\right), \tag{D.22}$$

where $\mathrm{n}_t(\ell) \equiv \frac{n_t(\ell)}{n_t}$ and the markup is defined as $\lambda_t^n(\mathrm{n}_t(\ell)) \equiv \frac{1}{\eta_w(\mathrm{n}_t(\ell))-1}$. $A_w(\mathrm{n}_t(\ell)) \equiv \frac{\lambda^w(\mathrm{n}_t(\ell))}{2+\frac{H'''(\mathrm{n}_t(\ell))}{H''(\mathrm{n}_t(\ell))}\mathrm{n}_t(\ell)}$ can be equivalently expressed as:

$$A_w(\mathrm{n}) = \frac{1}{\lambda^w(\mathrm{n})\hat{\eta}_w(\mathrm{n}) + 1}. \tag{D.23}$$

D.2 Households

There is a measure one of households in the economy, indexed by $j \in [0,1]$, endowed with a unit of labor each. Households are distributed uniformly over the real line (i.e., the measure of households is the Lebesgue measure Λ). We distinguish two types of households: intertemporally optimizing households $j \in [0, 1-\phi]$ and rule-of-thumb households $j \in (1-\phi, 1]$, so that they have measures $\Lambda([0,1-\phi]) = 1-\phi$ and $\Lambda([0,\phi]) = \phi$, respectively.

Households have preferences over streams of consumption and hours worked, $\{C_{t+s}(j), n_{t+s}(j)\}_{s=0}^{\infty}$, which are represented by the life-time utility function U_t:

$$U_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{1-\sigma}\big(C_{t+s}(j) - hC_{t+s-1}\big)^{1-\sigma}\right] \exp\left[\frac{\sigma-1}{1+\nu} n_{t+s}(j)^{1+\nu}\right]. \tag{D.24}$$

Here $h \in [0,1)$ captures external habit formation, σ denotes the inverse of the intertemporal elasticity of substitution, and ν equals the inverse of the labor supply elasticity. Households discount the future by $\beta \in (0,1)$, where β varies by household type.

The fraction $1-\phi$ of the labor force that is not credit-constrained maximizes its life-time utility subject to a lifetime budget constraint and a capital accumulation technology. The remainder of the labor force, i.e., a fraction ϕ, is credit constrained (or rule-of-thumb): they cannot save or borrow.

D.2.1 Intertemporally optimizing households

The intertemporally optimizing households choose consumption $\{C_{t+s}(j)\}$, investment in physical capital $\{X_{t+s}(j)\}$, physical capital $\{K^p_{t+s}(j)\}$, capacity utilization $\{u_{t+s}(j)\}$, nominal government bond holdings $B^n_{t+s}(j)$, and labor supply $\{n_{t+s}(j)\}$ to maximize (D.24) subject to a sequence of budget constraints (D.25), the law of motion for physical capital (D.26), and a no-Ponzi constraint. Households take prices $\{P_{t+s}\}$, nominal returns on government bonds $\{q^b_{t+s}R_{t+s}\}$, the nominal rental rate of capital $\{R^k_{t+s}\}$, and nominal wages $\{W_{t+s}\}$ as given.

The budget constraint for period $t+s$ is given by:

$$
\begin{aligned}
&(1+\tau^c_{t+s})C_{t+s}(j) + X_{t+s}(j) + \frac{B^n_{t+s}(j)}{R^{gov}_{t+s}P_{t+s}} \leq \\
&\qquad S_{t+s} + \frac{B^n_{t+s-1}(j)}{P_{t+s}} + (1-\tau^n_{t+s})\frac{[W^h_{t+s}n_{t+s}(j) + \lambda_{w,t+s}n_{t+s}W^h_{t+s}]}{P_{t+s}} + \\
&+\left[(1-\tau^k_{t+s})\left(\frac{R^k_{t+s}u_{t+s}(j)}{P_{t+s}} - a(u_{t+s}(j))\right) + \delta\tau^k_{t+s}\right][(1-\omega^k_{t+s-1})K^p_{t+s-1}(j) + \omega^k_{t+s-1}K^{p,agg}_{t+s-1}] + \frac{\Pi^p_{t+s}\mu^{t+s}}{P_{t+s}},
\end{aligned}
\tag{D.25}
$$

where $(\tau^c_{t+s}, \tau^k_{t+s}, \tau^n_{t+s})$ represent taxes on consumption expenditure, capital income, and labor income, respectively. The wage received by households differs from the one charged to labor packers because of union profits — union profits $\lambda_{w,t+s}n_{t+s}W^h_{t+s}$ are taken as given by households. Households also receive nominal lump-sum transfers $\{S_{t+s}\}$. $a(\cdot)$ represents the strictly increasing and strictly convex cost function of varying capacity utilization, whose first derivative in the case of unit capacity utilization is normalized as $a'(1) = \bar{r}^k$.[13] At unit capacity utilization, there is no additional cost: $a(1) = 0$. $\Pi^p_{t+s}\mu^{t+s}$ are nominal profits, which households also take as given.

There is a financial market friction present in the budget constraint; $\omega^k_{t+s} \neq 0$ represents a wedge between the returns on private and government bonds, and is a pure financial market friction — if $\omega^k_{t+s} > 0$ then households obtain less than one dollar for each dollar of after-tax capital income they receive, representing agency costs. Agency costs are reimbursed directly to unconstrained households, so that the friction has no effect on aggregate resources. This financial market friction is similar to a shock in Smets and Wouters (2003), who introduce it ad hoc in the investment Euler equation and motivate it as a short-cut to model informational frictions that disappear at the steady state.

[13] $\bar{r}^k$ represents the real steady state return on capital services.

Physical capital evolves according to the following law of motion:

$$K^p_{t+s}(j) = (1-\delta)K^p_{t+s-1}(j) + q^x_{t+s}\left[1 - S\left(\frac{X_{t+s}(j)}{X_{t+s-1}(j)}\right)\right] X_{t+s}(j), \quad \text{(D.26)}$$

where new investment is subject to adjustment costs described by $S(\dot{)}$. These costs satisfy $S(\mu) = S'(\mu) = 0, S'' > 0$. The relative price of investment changes over time, as captured by the exogenous $\{q^x_{t+s}\}$ process. Physical capital depreciates at rate δ.

For future reference, note that the effective capital stock is given by the product of capacity utilization and physical capital stock:

$$K^{eff}_{t+s}(j) = K^p_{t+s-1}(j)u_{t+s}(j). \quad \text{(D.27)}$$

To obtain the aggregate capital stock, multiply the above quantity by $(1-\phi)$.

The solution to the household's problem is characterized completely by the law of motion for physical capital (D.26) and the following necessary and sufficient first order conditions. To derive these conditions, denote the Lagrange multipliers on the budget constraint (D.25) and the law of motion (D.26) by $\beta^t(\Xi_t, \Xi^k_t)$ – replacing the household index j by a superscript RA.

$$[C_t] \quad \Xi_t(1+\tau^c_t) = \exp\left(\frac{\sigma-1}{1+\nu}(n^{RA}_t)^{1+\nu}\right)[C^{RA}_t - hC^{RA}_{t-1}]^{-\sigma}$$

$$[n_t] \quad \Xi_t(1-\tau^n_t)\frac{W^h_t}{P_t} = \exp\left(\frac{\sigma-1}{1+\nu}(n^{RA}_t)^{1+\nu}\right)(n^{RA}_t)^{\nu}[C^{RA}_t - hC^{RA}_{t-1}]^{1-\sigma}$$

$$[B_t] \quad \Xi_t = \beta q^b_t R_t \mathbb{E}_t\left(\frac{\Xi_{t+1}}{P_{t+1}/P_t}\right)$$

$$[K^p_t] \quad \Xi^k_t = \beta\mathbb{E}_t\left(\Xi_{t+1}\left[\tilde{q}^k_t\left((1-\tau^k_{t+s})\left[\frac{R^k_{t+1}}{P_{t+1}}u_{t+1} - a(u_{t+1}) + \delta\tau^k_{t+1}\right] + (1-\delta)\frac{\Xi^k_{t+1}}{\Xi_{t+1}}\right]\right)\right.$$

$$[X_t] \quad \Xi_t = \Xi^k_t q^x_t\left(1 - S\left(\frac{X^{RA}_t}{X^{RA}_{t-1}}\right) - S'\left(\frac{X^{RA}_t}{X^{RA}_{t-1}}\right)\left(\frac{X^{RA}_t}{X^{RA}_{t-1}}\right)\right)$$
$$+ \beta\mathbb{E}_t\left(\frac{\Xi^k_{t+1}}{\Xi_t}q^x_{t+1}S'\left(\frac{X^{RA}_{t+1}}{X^{RA}_t}\right)\left(\frac{X^{RA}_{t+1}}{X^{RA}_t}\right)^2\right)$$

$$[u_t] \quad \frac{R^k_{t+1}}{P_t} = a'(u^{RA}_{t+1}).$$

By setting $a'(1) \equiv \bar{r}^k$ we normalize steady state capacity utilization to unity: $\bar{u} \equiv 1$.

For what follows, it is useful to detrend these first order conditions and the law of motion for capital. To that end, use lower-case letters to denote detrended and real variables, as exemplified in the following definitions:

$$k_t^{RA} \equiv \frac{K_t^{RA}}{\mu^t}, w_t \equiv \frac{W_t}{P_t\mu^t}, w_t^h \equiv \frac{W_t^h}{P_t\mu^t}, r_t^k \equiv \frac{R_t^k}{P_t}, \xi_t \equiv \Xi_t\mu^{\sigma t}, Q_t \equiv \frac{\Xi_t^k}{\Xi_t}, \bar{\beta} = \beta\mu^{-\sigma}.$$

μ denotes the gross trend growth rate of the economy. For future reference, note that government expenditure is normalized differently: $g_t = \frac{G_t}{\bar{Y}\mu^t}$. Substituting in for the normalized variables yields:

$$\xi_t(1+\tau_t^c) = \exp\left(\frac{\sigma-1}{1+\nu}(n_t^{RA})^{1+\nu}\right)[c_t^{RA} - (h/\mu)c_{t-1}^{RA}]^{-\sigma} \tag{D.29a}$$

$$\xi_t(1-\tau_t^n)w_t^h = \exp\left(\frac{\sigma-1}{1+\nu}(n_t^{RA})^{1+\nu}\right)(n_t^{RA})^{\nu}[c_t^{RA} - (h/\mu)c_{t-1}^{RA}]^{1-\sigma} \tag{D.29b}$$

$$\xi_t = \bar{\beta}R_t^{gov}\mathbb{E}_t\left(\frac{\xi_{t+1}}{P_{t+1}/P_t}\right) \tag{D.29c}$$

$$Q_t = \bar{\beta}\mathbb{E}_t\left(\frac{\xi_{t+1}}{\xi_t}\Big[\tilde{q}_t^k\big((1-\tau_{t+1}^k)[r_{t+1}^k u_{t+1} - a(u_{t+1})] + \delta\tau_{t+1}^k\big) + (1-\delta)Q_{t+1}\Big]\right) \tag{D.29d}$$

$$\begin{aligned} 1 &= Q_t q_t^x\left(1 - S\Big(\frac{x_t^{RA}\mu}{x_{t-1}^{RA}}\Big) - S'\Big(\frac{x_t^{RA}\mu}{x_{t-1}^{RA}}\Big)\Big(\frac{x_t^{RA}\mu}{x_{t-1}^{RA}}\Big)\right) \\ &+ \bar{\beta}\mathbb{E}_t\left(\frac{\xi_{t+1}}{\xi_t}Q_{t+1}q_{t+1}^x S'\Big(\frac{x_{t+1}^{RA}\mu}{x_t^{RA}}\Big)\Big(\frac{x_{t+1}^{RA}\mu}{x_t^{RA}}\Big)^2\right) \end{aligned} \tag{D.29e}$$

$$r_{t+1}^k = a'(u_{t+1}^{RA}). \tag{D.29f}$$

The detrended law of motion for physical capital is given by

$$k_t^{p,RA} = \frac{(1-\delta)}{\mu}k_{t-1}^{p,RA} + q_t^x\left[1 - S\Big(\frac{x_t^{RA}}{x_{t-1}^{RA}}\mu\Big)\right]x_t^{RA}. \tag{D.30}$$

Combining the FOC for consumption and hours worked gives the static optimality condition for households:

$$\frac{1-\tau_t^n}{1+\tau_t^c}w_t^h = (n_t^{RA})^{\nu}[c_t^{RA} - (h/\mu)c_{t-1}^{RA}]. \tag{D.31}$$

Combining (D.29a) for two consecutive periods and using (D.29c) gives the consumption Euler equation:

$$\mathbb{E}_t\left(\frac{\xi_{t+1}(1+\tau^c_{t+1})}{\xi_t(1+\tau^c_{t+1})}\right)=\mathbb{E}_t\left(\exp\left(\frac{\sigma-1}{1+\nu}\left(\frac{n^{RA}_{t+1}}{n^{RA}_t}\right)^{1+\nu}\right)\left[\frac{c^{RA}_{t+1}-(h/\mu)c^{RA}_t}{c^{RA}_t-(h/\mu)c^{RA}_{t-1}}\right]^{-\sigma}\right). \tag{D.32}$$

Equation (D.29d) is the investment Euler equation. The FOC for capital (D.29e) can be used to compute the shadow price of physical capital Q_t.

Using the investment Euler equation shows that along the deterministic balanced growth path the value of capital equals unity (since $S'(\mu)=S(\mu)=0$ and $\bar{q}^x=1$). From the consumption Euler equation and $\bar{q}^b=1$. we obtain the interest rate paid on government bonds under balanced growth. Finally, the pricing equation for capital and the investment Euler equation pin down the rental rate on capital. Summarizing:

$$\bar{Q}=1, \tag{D.33a}$$

$$\bar{R}=\bar{\beta}^{-1}\bar{\pi}, \tag{D.33b}$$

$$1=\bar{\beta}[(1-\bar{\tau}^k)\bar{r}^k+\delta\bar{\tau}^k+(1-\delta)],$$

$$\Leftrightarrow \quad \bar{r}^k=\frac{\bar{\beta}^{-1}-1+\delta(1-\bar{\tau}^k)}{1-\bar{\tau}^k}. \tag{D.33c}$$

The bond premium shock q^b_t differs from a discount factor shock, although it results in an observationally equivalent consumption Euler equation – if time preference were time-varying, the period utility function would become:

$$\left[\frac{1}{1-\sigma}\big(C_{t+s}(j)-hC_{t+s-1}\big)^{1-\sigma}\right]\exp\left[\frac{\sigma-1}{1+\nu}n_{t+s}(j)^{1+\nu}\right]\prod_{l=1}^{s}\check{q}^b_{t+l-1},$$

so that the ratio $\frac{\check{\xi}_{t+1}}{\xi_t}$ would be proportional to $\check{q}^b_t$, so that the consumption Euler equation is unchanged. The effects differ, however, insofar that the present formulation on basis of the government discount factor also affects the investment Euler equation and the government budget constraint.

For measurement purposes, it is useful to rewrite the linearized FOC for capital, after substituting out for the discount factor. It shows that the private bond shock represents the premium paid for private bonds over government bonds holding the rental rate on capital fixed:

$$\frac{\bar{r}^k(1-\bar{\tau}^k)\mathbb{E}_t(\hat{r}^k_{t+1})+(1-\delta)\mathbb{E}_t(\hat{Q}_{t+1})}{\bar{r}^k(1-\bar{\tau}^k)+\delta\bar{\tau}^k+1-\delta}-\hat{Q}_t=\left(\hat{R}_t-\mathbb{E}_t[\pi_t]\right)+\hat{q}^b_t+\hat{q}^k_t.$$

Note: The shock $\tilde{q}^k_t$ in the budget constraint has been rescaled here; $\hat{q}^k_t$ is the deviation of the rescaled shock from its steady state value.

D.2.2 Credit-constrained or rule-of-thumb households

A fraction $\phi \in (0, 0.5)$ of the households is assumed to be credit-constrained. As a justification, one may suppose that credit-constrained households discount the future substantially more steeply, and they are thus uninterested in accumulating government bonds or private capital, unless their returns are extraordinarily high. Conversely, these households find it easy to default on loans, and they are therefore not able to borrow. We hold the identity of credit-constrained households, and thereby their fraction of the total population, constant.

Rule-of-thumb households face a static budget constraint in each period and are assumed to supply the same amount of labor as intertemporally optimizing households. Given

$$n^{RoT}_{t+s}(j) = n^{RA}_{t+s} = n_{t+s},$$

consumption follows from the budget constraint in each period:

$$(1+\tau^c_{t+s})C^{RoT}_{t+s}(j) \leq S^{RoT}_{t+s} + (1-\tau^n_{t+s})\frac{W^h_{t+s}n^{RoT}_{t+s}(j) + \lambda_{w,t+s}W^h_{t+s}n_{t+s}}{P_{t+s}} + \Pi^p_{t+s}\mu^{t+s}. \tag{D.34}$$

Rule-of-thumb households receive transfers, labor income including union profits, and profits made by intermediate goods-producing firms.

Removing the trend from the budget constraint (D.34), omitting the j index, and solving for (detrended) consumption:

$$c^{RoT}_{t+s} = \frac{1}{(1+\tau^c_{t+s})}\left(s^{RoT}_{t+s} + (1-\tau^n_{t+s})[w^h_{t+s}n^{RoT}_{t+s} + \lambda_{w,t+s}w^h_{t+s}n_{t+s}] + \Pi^p_{t+s}\right). \tag{D.35}$$

From the budget constraint (D.34), the following steady state relationship holds:

$$\bar{c}^{RoT} = \frac{\bar{s}^{RoT} + (1-\bar{\tau}^n_t)\bar{w}\bar{n}}{1+\bar{\tau}^c}. \tag{D.36}$$

We assume that:

$$\bar{s}^{RoT} = \bar{s}. \tag{D.37}$$

D.2.3 Households: labor supply, wage setting

Households supply homogeneous labor to unions, which differentiate labor into varieties indexed by $\ell \in [0, 1]$ and sell it to labor packers. In doing so, unions take aggregate quantities (i.e., households' cost of supplying labor and aggregate labor demand and wages) as given. Unions maximize the expected present discounted value of net-of-tax wage income earned in excess of the cost of supplying labor. In the presence of rule-of-thumb households, unions act as if they were maximizing surplus for the intertemporally optimizing households only. If the mass of rule-of-thumb households is less than the mass of intertemporally optimizing households, i.e., $\phi < 0.5$, which is satisfied in the parameterizations used, a median-voter decision rule justifies this assumption.

The labor unions problem is analogous to that of price-setting firms, with the marginal rate of substitution between consumption and leisure in the representative household taking the role of marginal costs in firms' problems. From the FOC $[C_t]$ and $[n_t]$, the marginal rate of substitution is given by $\frac{U_{n,t+s}}{\Xi_{t+s}} = (n_t^{RA})^{\nu}[C_t^{RA} - hC_{t-1}^{RA}](1+\tau_t^c)$. Whenever a union has the chance to reset the wage it charges, it chooses $W_t^*(\ell)$:

$$W_t^*(\ell) = \arg\max_{\tilde{W}_t(\ell)} \mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_w)^s \frac{\bar{\beta}^s \xi_{t+s}}{\xi_t} \left[(1-\tau_{t+s}^n) \frac{W_{t+s}(\ell)}{P_{t+s}} + \frac{U_{n,t+s}}{\Xi_{t+s}} \right] n_{t+s}(\ell), \tag{D.38}$$

subject to the labor demand equation (D.20). $1-\zeta_w$ denotes the probability that a union can reset its wage. If it cannot adjust, wages are adjusted according to a moving average of past and steady state inflation and labor productivity growth:

$$W_{t+s}(\ell) = W^*{}_t(\ell) \prod_{v=1}^{s} \mu(\pi_{t+v-1})^{\iota_w} \bar{\pi}^{1-\iota_w} \equiv W_t^*(\ell)\chi_{t,t+s}^w.$$

Using that $n_t = n_t^{RA}$, the first order condition is given by

$$\begin{aligned}0 = \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\bar{\beta}^s \xi_{t+s}}{\xi_t \lambda^w(\mathrm{n}_{t,t+s}(\ell))} \frac{n_{t+s}(\ell)}{W_t^*(\ell)} \Bigg(& (1-\tau_{t+s}^n) \frac{W_t^*(\ell)\chi_{t,t+s}^w(\ell)}{P_{t+s}} \\ & - [1+\lambda^w(\mathrm{n}_{t+s}(\ell))](1+\tau_{t+s}^c) n_{t+s}^{\nu}[C_{t+s}^{RA} - hC_{t+s-1}^{RA}] \Bigg)\end{aligned} \tag{D.39}$$

and can be equivalently expressed as

$$\frac{W_t^*(\ell)}{P_t} = \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\bar{\beta}^s \xi_{t+s}}{\xi_t \lambda^w(\mathrm{n}_{t,t+s}(\ell))} n_{t+s}(\ell)[1+\lambda^w(\mathrm{n}_{t+s}(\ell))](1+\tau_{t+s}^c) n_{t+s}^{\nu}[C_{t+s}^{RA} - hC_{t+s-1}^{RA}]}{\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\bar{\beta}^s \xi_{t+s}}{\xi_t \lambda^w(\mathrm{n}_{t,t+s}(\ell))} n_{t+s}(\ell)(1-\tau_{t+s}^n) \frac{\chi_{t,t+s}^w(\ell)}{P_{t+s}/P_t}}. \tag{D.40}$$

Aggregate wages evolve as:

$$W_t = (1-\zeta_w)W_t^* H'^{-1}\left(\frac{W_t^* n_t}{\Xi_t^n}\right) + \zeta_w \pi_{t-1}^{\iota_w}\bar{\pi}^{1-\iota_w} W_{t-1} H'^{-1}\left(\frac{\pi_{t-1}^{\iota_w}\bar{\pi}^{1-\iota_w} W_{t-1} n_t}{\Xi_t^n}\right). \tag{D.41}$$

Along the deterministic balanced growth path, the detrended desired real wage is given by a constant markup over the marginal rate of substitution. Given constant inflation, the symmetric deterministic growth path also implies, from equation (D.41), that the desired real wage equals the actual real wage:

$$\bar{w} = \bar{w}^* = (1+\bar{\lambda}_w)\bar{w}^h = (1+\bar{\lambda}_w)\frac{1+\bar{\tau}^c}{1-\bar{\tau}^n}\bar{n}^\nu \bar{c}^{RA}[1-h/\mu], \tag{D.42}$$

where the second equality uses (D.31).

D.3 Government

The government sets nominal interest R_t according to an interest rate rule, purchases goods and services for government consumption G_t, pays transfers S_t to households, and provides public capital for the production of intermediate goods, K_t^g. It finances its expenditures by levying taxes on capital and labor income, a tax on consumption expenditure, and a tax on one period nominal bond issues. We consider a setup in which monetary policy is active in the neighborhood of the balanced growth path.

D.3.1 Fiscal policy

In modeling the government sector, we take as given the tax structure along the balanced growth path as in Trabandt and Uhlig (2011), who used NIPA data to compute the capital and labor income and consumption expenditure tax rates for the US. Off the balanced growth path, we follow Uhlig (2010b) in assuming that labor tax rates adjust gradually to balance the budget in the long run, whereas in the short run much of any additional government expenditure is tax financed.

The government flow budget constraint is given by:

$$G_t + X_t^g + S_t + \frac{B_{t-1}}{P_t} \leq \frac{B_t}{R_t^{gov} P_t} + \tau_t^c C_t + \tau_t^n n_t \frac{W_t}{P_t} + \tau_t^k \left[u_t \frac{R_t^k}{P_t} - a(u_t) - \delta\right] K_{t-1}^p. \tag{D.43}$$

Detrended, the government budget constraint is given by:

$$\bar{y}g_t + x_t^g + s_t + \frac{b_{t-1}}{\mu\pi_t} \leq \frac{b_t}{R_t^{gov}} + \tau_t^c c_t + \tau_t^n n_t w_t + \tau_t^k k_t^s r_t^k - \tau_t^k[a(u_t) + \delta]\frac{k_{t-1}^p}{\mu}. \tag{D.44}$$

Government consumption $g_t = \frac{G_t}{\bar{y}\mu^t}$ is given exogenously and is stochastic, driven by genuine spending shocks as well as by technology shocks.

By introducing a wedge between the FFR and government bonds, we capture both short-term liquidity premia as well as changes in the term structure of government debt. Since the latter is absent with only one-period bonds, in the estimation, the bond premium may also reflect differences in the borrowing cost due to a more complex maturity structure.[14]

Labor tax rates have both a stochastic and a deterministic component. They adjust deterministically to ensure long-run budget balance at a speed governed by the parameter $\psi_\tau \in [\underline{\psi}_\tau, 1]$, where $\underline{\psi}_\tau$ is some positive number large enough to guarantee stability. To simplify notation denote the remaining detrended deficit prior to new debt and changes in labor tax rates as d_t:

$$d_t \equiv \bar{y}g_t + x_t^g + \bar{s} + s_t^{exo} + \frac{b_{t-1}}{\mu\pi_t} - \bar{\tau}^c c_t - \bar{\tau}^n w_t n_t - \bar{\tau}^k k_t^s r_t^k + \bar{\tau}^k \delta \frac{k_{t-1}^p}{\mu}.$$

In the baseline case, labor tax rates are adjusted according to the following rule:

$$(\tau_t^n - \bar{\tau}^n)w_t n_t + \epsilon_t^\tau = \psi_\tau(d_t - \bar{d}), \tag{D.45}$$

where ϵ_t^τ is an exogenous shock to the tax rate.

In general:

$$\psi_\tau(d_t - \bar{d}) - \epsilon_t^\tau = \begin{cases} (\tau_t^n - \bar{\tau}^n)w_t n_t & \text{Baseline}, \tau_t^c = \tau_t^k = s_t^{endo} = 0, \\ (\tau_t^c - \bar{\tau}^c)c_t & \text{Alternative 1}, \tau_t^n = \tau_t^k = s_t^{endo} = 0, \\ (\tau_t^k - \bar{\tau}^k)k_t^s(r_t^k - \delta) & \text{Alternative 2}, \tau_t^n = \tau_t^c = s_t^{endo} = 0, \\ -(s_t^{endo} - \bar{s}) & \text{Alternative 3}, \tau_t^n = \tau_t^c = \tau_t^k = 0. \end{cases} \tag{D.46}$$

Debt issues are then given by the budget constraint or equivalently as the residual from (D.45): $\frac{b_t}{R_t^{gov}} = (1 - \psi_\tau)(d_t - \bar{d}) + \epsilon_t^\tau$.

Government investment is chosen optimally for a given tax structure. Given the congestion effect of production on public infrastructure, a tax on production would be optimal (Barro and Sala-i Martin, 1992). Similarly, we neglect

[14]Historical data from the Board of Governors of the Federal Reserve System implies a maturity between 10 and 22 quarters with an average between 16 and 20 quarters (The Federal Reserve Board Bulletin, 1999, Figure 4).

the potential cost of financing of productive government expenditure via distortionary taxes. To motivate this assumption, note that along the balanced growth path, government capital can be completely debt-financed or privatized and financed through government bond issues, whereas other government expenditures, such as transfers, that are not backed by real assets have to backed by the government's power to levy taxes.

Formally, the government chooses investment and capital stock to maximize the present discounted value of output net of investment expenditure along the balanced growth path:

$$\max_{\{K^g_{t+s},X^g_{t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{\Xi_{t+s}}{\Xi_t}[Y_{t+s} - X^g_{t+s}],$$

given K^g_{t-1} and subject to the aggregate production function (D.8) and to the capital accumulation equation:

$$K^g_{t+s} = (1-\delta)K^g_{t+s-1} + q^{x,g}_{t+s}\left[1 - S_g\left(\frac{[X^g_{t+s} + \tilde{u}^{x,g}_{t+s}]}{[X^g_{t+s-1} + \tilde{u}^{x,g}_{t+s-1}]}\right)\right](X^g_{t+s} + \tilde{u}^{x,g}_{t+s}). \tag{D.47}$$

The government is subject to similar adjustment costs as the private sector is $S_g(\mu) = S'_g(\mu) = 0, S''_g > 0$, and investment is subject to shocks to its relative efficiency $q^{x,g}_{t+s}$. We assume that government capital depreciates at the same rate as private physical capital. $\tilde{u}^{x,g}$ represents exogenous shocks to government investment spending, such as stimulus spending.

Denote the Lagrange multiplier on (D.47) at time $t+s$ as $\beta^s\frac{\Xi^g_{t+s}}{\Xi_t}$. Then the first order conditions are:

$$\begin{aligned}
[X^g_t] \quad 1 = {} & \frac{\Xi^g_t}{\Xi_t}q^x_t\left(1 - S_g\left(\frac{[\tilde{u}^{x,g}_t + X^g_t]}{[\tilde{\epsilon}^{x,g}_{t-1} + X^g_{t-1}]}\right) - S'_g\left(\frac{[\tilde{\epsilon}^{x,g}_t + X^g_t]}{[\tilde{\epsilon}^{x}_{t-1} + X^g_{t-1}]}\right)\left(\frac{[\tilde{u}^{x,g}_t + X^g_t]}{[\tilde{\epsilon}^{x,g}_{t-1} + X^g_{t-1}]}\right)\right) \\
& + \beta\mathbb{E}_t\left(\frac{\Xi^g_{t+1}}{\Xi_t}q^x_{t+1}S'_g\left(\frac{[\tilde{\epsilon}^{x,g}_{t+1} + X^g_{t+1}]}{[\tilde{u}^{x,g}_t + X^g_t]}\right)\left(\frac{[\tilde{\epsilon}^{x,g}_{t+1} + X^g_{t+1}]}{[\tilde{u}^{x,g}_t + X^g_t]}\right)^2\right) \\
[K^g_t] \quad \frac{\Xi^g_t}{\Xi_t} = {} & \beta\mathbb{E}_t\left(\frac{\Xi_{t+1}}{\Xi_t}\zeta\frac{Y_t + \mu^t\Phi}{K^g_{t-1}} + (1-\delta)\frac{\Xi^g_{t+1}}{\Xi_t}\right).
\end{aligned}$$

Defining the shadow price of government capital as $Q^g_t \equiv \frac{\Xi^g_t}{\Xi_t}$ and detrending, the first order conditions can be equivalently written as:

$$1 = Q^g_t q^x_t\left(1 - S_g\left(\frac{[\epsilon^{x,g}_t + ^g_t]\mu}{[\epsilon^{x,g}_{t-1} + x^g_{t-1}]}\right) - S'_g\left(\frac{[\epsilon^{x,g}_t + x^g_t]\mu}{[\tilde{\epsilon}^{x}_{t-1} + x^g_{t-1}]}\right)\left(\frac{[\epsilon^{x,g}_t + x^g_t]\mu}{[\epsilon^{x,g}_{t-1} + x^g_{t-1}]}\right)\right)$$

$$+\bar{\beta}\mathbb{E}_t\left(Q^g_{t+1}\frac{\xi_{t+1}}{\xi_t}q^x_{t+1}S'_g\left(\frac{[\epsilon^{x,g}_{t+1}+x^g_{t+1}]\mu}{[\epsilon^{x,g}_t+x^g_t]}\right)\left(\frac{[\tilde{\epsilon}^{x,g}_{t+1}+x^g_{t+1}]\mu}{[\epsilon^{x,g}_t+x^g_t]}\right)^2\right) \tag{D.48a}$$

$$Q^g_t=\bar{\beta}\mathbb{E}_t\left(\frac{\xi_{t+1}}{\xi_t}\zeta\frac{y_t+\Phi}{k^g_{t-1}/\mu}+\frac{\xi_{t+1}}{\xi_t}(1-\delta)Q^g_{t+1}\right), \tag{D.48b}$$

where $\epsilon^{x,g}_t\equiv\frac{1}{\mu}\tilde{\epsilon}^{x,g}_t$ denotes the detrended investment spending shock.

Along the balanced growth path, $S_g(\mu)=S'_g(\mu)=0,\bar{q}^{x,g}=1,\bar{\epsilon}^{x,g}=0$ ensure that the shadow price of capital equals unity. Introduce r^g_t as shorthand for the implied rental rate on government capital:

$$r^g_t=\zeta\frac{y_t+\Phi}{k^g_t/\mu}. \tag{D.49}$$

In the steady state, from (D.48b):

$$\bar{r}^g=\bar{\beta}^{-1}-(1-\delta). \tag{D.50}$$

Equation (D.48b) determines the optimal ratio of government capital to gross output. Importantly, the law of motion for government capital (D.47) and (D.48b) evaluated at the balanced growth path allow us to back out the share of government capital in the aggregate production function, for any given government investment to net output ratio $\frac{\bar{x}^g}{\bar{y}}$. From the law of motion along the balanced growth path:

$$\bar{x}^g=\left(1-\frac{1-\delta}{\mu}\right)\bar{k}^g \quad\Leftrightarrow\quad \frac{\bar{x}^g}{\bar{y}}=[\mu-(1-\delta)\frac{\bar{k}^g}{\mu\bar{y}},$$

From the equation for r^g_t, we have that $\frac{\bar{k}^g}{\mu\bar{y}}=\zeta\frac{\bar{y}+\Phi}{\bar{y}}\frac{1}{\bar{r}^g}$. Combined with the previous equation this allows us to solve for the government capital share ζ:

$$\zeta=\frac{\bar{y}}{\bar{y}+\Phi}\frac{\bar{r}^g}{1-(1-\delta)}\frac{\bar{x}}{\bar{y}}. \tag{D.51}$$

D.3.2 Monetary policy

The specification of the interest rate rule follows Smets and Wouters (2007). The Federal Reserve sets interest rates according to the following rule:

$$\frac{R^{FFR}_t}{\bar{R}}=\left(\frac{R^{FFR}_{t-1}}{\bar{R}}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\psi_1}\left(\frac{Y_t}{Y^f_t}\right)^{\psi_2}\right]^{1-\rho_R}\left(\frac{Y_t/Y_{t-1}}{Y^f_t/Y^f_{t-1}}\right)^{\psi_3}\epsilon^r_t, \tag{D.52}$$

where ρ_R determines the degree of interest rate smoothing and Y_t^f denotes the level of output that would prevail in the economy in the absence of nominal frictions and with constant markups (i.e., the flexible output level). $\psi_1 > 1$ determines the reaction to deviations of inflation from its long-run average, and $\psi_2, \psi_3 > 0$ determines the reaction to the deviation of actual output from the flexible economy output and to the change in the gap between actual and flexible output.

Due to financial market frictions, the return on government bonds differs from the FFR:

$$R_t^{gov} = R_t^{FFR}(1 + \omega_t^b).$$

The flexible economy is the limit point of the economy characterized above with $\zeta_p = \zeta_w = 0$ and no markup shocks: $\epsilon_t^{\lambda,p} = \epsilon_t^{\lambda,w} = 0$. From the pricing and wage-setting rules, this limiting solution implies:

$$\frac{P_t^f(i)}{P_t^f} = [1 + \lambda_p(\mathrm{y}_t^f(i))]mc_t^f(i), \tag{D.53}$$

$$\frac{W_t^f(\ell)}{P_t^f} = [1 + \lambda_w(\mathrm{n}_t^f(\ell))]\frac{1 + \tau_t^c}{1 - \tau_t^{n,f}} n_t^{f\,\nu}[C_t^f - hC_{t-1}^f], \tag{D.54}$$

where the superscript f denotes variables in the flexible economy. Given that final goods are the numeraire and given that firms are symmetric and can freely set their prices:

$$1 = P_t^f = P_t^f(i) = [1 + \lambda_p(1)]mc_t^f(i) \quad \forall t, \tag{D.55}$$

implying that marginal costs are constant for all firms.

Similarly, since all unions face a symmetric problem and can freely reset wages we have that, using that the numeraire equals unity and dividing by trend growth:

$$\frac{W_t^f(\ell)}{\mu} = \frac{W_t^f}{\mu} = w_t^f = [1 + \lambda_w(1)]\frac{1 + \tau_t^c}{1 - \tau_t^{n,f}} n_t^{f\,\nu}[c_t^f - (h/\mu)c_{t-1}^f]. \tag{D.56}$$

Money does not enter explicitly in the economy: the Federal Reserve supplies the amount of money demanded at interest rate R_t.

D.4 Exogenous processes

The exogenous processes are assumed to be log-normally distributed and, with the exception of government spending shocks, independent. Government spending shocks are correlated with technology shocks. Shocks to the two markup processes follow an ARMA(1,1) process, whereas the other shocks are AR(1) processes.

$$\log \epsilon_t^a = \rho_a \log \epsilon_{t-1}^a + u_t^a, \qquad u_t^a \overset{iid}{\sim} \mathcal{N}(0, \sigma_a^2) \tag{D.57a}$$

$$\log \epsilon_t^r = \rho_r \log \epsilon_{t-1}^r + u_t^r, \qquad u_t^r \overset{iid}{\sim} \mathcal{N}(0, \sigma_r^2) \tag{D.57b}$$

$$\log g_t = \log g_t^a + \tilde{u}_t^g, \tag{D.57c}$$

$$\log g_t^a = (1-\rho_g) \log \bar{g} + \rho_g \log g_{t-1}^a + \sigma_{ga} u_t^a + u_t^g, \qquad u_t^a \overset{iid}{\sim} \mathcal{N}(0, \sigma_a^2) \tag{D.57d}$$

$$\log s_t^{exo} = \tilde{u}_t^s, \tag{D.57e}$$

$$\log \epsilon_t^\tau = \rho_\tau \log \epsilon_{t-1}^\tau + u_t^\tau, \qquad u_t^\tau \overset{iid}{\sim} \mathcal{N}(0, \sigma_\tau^2) \tag{D.57f}$$

$$\log \tilde{\epsilon}_t^{\lambda,p} = \rho_{\lambda,p} \log \tilde{\epsilon}_{t-1}^{\lambda,p} + u_t^{\lambda,p} - \theta_{\lambda,p} u_{t-1}^{\lambda,p}, \qquad u_t^{\lambda,p} \overset{iid}{\sim} \mathcal{N}(0, \sigma_{\lambda,p}^2) \tag{D.57g}$$

$$\log \tilde{\epsilon}_t^{\lambda,w} = \rho_{\lambda,w} \log \tilde{\epsilon}_{t-1}^{\lambda,w} + u_t^{\lambda,w} - \theta_{\lambda,w} u_{t-1}^{\lambda,w}, \qquad u_t^{\lambda,w} \overset{iid}{\sim} \mathcal{N}(0, \sigma_{\lambda,w}^2) \tag{D.57h}$$

$$\log(1+\omega_t^b) \equiv \log q_t^b = \rho_b \log q_{t-1}^b + u_t^b, \qquad u_t^b \overset{iid}{\sim} \mathcal{N}(0, \sigma_b^2) \tag{D.57i}$$

$$\log(1-\omega_t^k) \equiv \log q_t^k = \rho_k \log q_{t-1}^k + u_t^k, \qquad u_t^k \overset{iid}{\sim} \mathcal{N}(0, \sigma_k^2) \tag{D.57j}$$

$$\log q_t^x = \rho_x \log q_{t-1}^x + u_t^x, \qquad u_t^x \overset{iid}{\sim} \mathcal{N}(0, \sigma_x^2) \tag{D.57k}$$

$$\log q_t^{x,g} = \rho_{x,g} \log q_{t-1}^{x,g} + u_t^{x,g}, \qquad u_t^{x,g} \overset{iid}{\sim} \mathcal{N}(0, \sigma_{x,g}^2) \tag{D.57l}$$

Three shocks are deterministic and used for policy counterfactuals only:

$$\tilde{u}_t^s, \tilde{u}_t^g, \tilde{u}_t^{x,g}.$$

D.5 Equilibrium conditions

D.5.1 Aggregation

From the final goods producers' problem (D.1) and using the zero-profit condition in the competitive market, net output in nominal and real terms is given by:

$$P_tY_t = \int_0^1 P_t(i)Y_t(i)di \quad \Leftrightarrow \quad Y_t = \int_0^1 \frac{P_t(i)}{P_t}Y_t(i)di.$$

Outside the flexible economy, relative prices differ from unity, so that output is not simply the average production of intermediates. To a first order, however, price dispersion is irrelevant because $y_t(i) \approx y_t - \eta_p(1)y_t\left(\frac{P_t(i)}{P_t} - 1\right)$, so that the dispersion term averages out in the aggregate $\int_0^1 y_t(i)di \approx y_t$.

In the presence of heterogeneous labor, the measurement of labor supply faces similar issues because:

$$n_t = \int_0^1 \frac{W_t(\ell)}{W_t}n_t(\ell)d\ell,$$

which, by analogy to the above argument for output, generally differs from average hours. However, to a first order:

$$\int_0^1 n_t(\ell)d\ell \approx n_t \tag{D.58}$$

Noncredit constrained households are indexed by $j \in [0, 1-\phi]$, and there is measure $1-\phi$ of these households in the economy. Each noncredit-constrained household supplies $K_t(j) = K_t^{RA}$ units of capital services, so that total holdings of capital capital and government bonds per intertemporally optimizing household are given by $\frac{1}{1-\phi}$ times the aggregate quantity. Similarly, household investment is a multiple of aggregate investment. To see this, note that aggregate quantities of bond holdings B_t, investment X_t, physical capital K_t^p, and capital services K_t are computed as:

$$K_t = \int_0^{1-\phi} K_t(j)\Lambda(dj) = K_t(1-\phi)^{-1}\Lambda([0, 1-\phi]) = K_t.$$

Aggregate consumption is given by:

$$C_t = \int_0^1 C_t(j)\Lambda(dj) = \int_0^{1-\phi} C_t^{RA}\Lambda(dj) + \int_{1-\phi}^1 C_t^{RoT}\Lambda(dj) = (1-\phi)C^RA_t + \phi C_t^{RoT}. \tag{D.59}$$

Given the consumption of rule-of-thumb agents (D.36), that of intertemporally optimizing agents is given by:

$$\bar{c}^{RA} = \frac{\bar{c} - \phi\bar{c}^{RoT}}{1-\phi}. \tag{D.60}$$

Similarly, aggregate transfers are given by

$$S_t = (1-\phi)S_t^{RA} + \phi S_t^{RoT}, \tag{D.61}$$

where equation (D.37) implies that:

$$\bar{s} = \bar{s}^{RA} + \bar{s}^{RoT}.$$

Aggregate labor supply coincides with individual labor supply of either type of household.

D.5.2 Market clearing

Labor market clearing requires that labor demanded by intermediaries equals labor supplied by labor packers:

$$\int_0^1 n_t(i)di = n_t = n_t \int_0^1 \frac{W_t(\ell)}{W_t} n_t(\ell)d\ell,$$

where $n_t(\ell)$ is measured in units of the differentiated labor supplies and n_t is measured in units which differs from those supplied by households.

Adding the government and the budget constraints of the two types of households, integrated over $[0, 1-\phi]$ and $(1-\phi, 1]$, respectively, and substituting $\int_0^1 n_t(j)W_t^h(1+\lambda_{t,w})dj = W_t n_t$, which results from combining the labor packers' zero-profit condition with the union problem into the household budget constraint, yields the following equation:

$$\begin{aligned} C_{t+s} + X_{t+s}(j) + G_t + X_{t+s}^g = {} & n_t \frac{W_{t+s}}{P_{t+s}} \\ & + \left[\frac{R_{t+s}^k u_{t+s}}{P_{t+s}} - a(u_{t+s})\right] K_{t+s-1}^p + \frac{\Pi_{t+s}^p \mu^{t+s}}{P_{t+s}}, \end{aligned}$$

Detrending and substituting in for real profits from (D.17a) and using that $w_t \int_0^1 n_t(i)di = w_t n_t$ yields:

$$c_{t+s} + x_{t+s} + \bar{y}g_{t+s} + x_{t+s}^g = y_{t+s} - a(u_{t+s})\mu k_{t+s-1}^p, \tag{D.62}$$

which is the goods market clearing condition: Production is used for government and private consumption, government and private investment, as well as variations in capacity utilization.

D.6 Linearized equilibrium conditions

D.6.1 Firms

Log-linearizing the production function around the symmetric balanced growth path:

$$\hat{y}_t = \frac{\bar{y}+\Phi}{\bar{y}}\left(\hat{\epsilon}^a_t + \zeta \hat{k}^g_{t-1} + \alpha(1-\zeta)\hat{k}_t + (1-\alpha)(1-\zeta)\hat{n}_t\right). \tag{D.63}$$

The capital-labor ratio is approximated by (D.9):

$$\hat{k}_t = \hat{n}_t + \hat{w}_t - \hat{r}^k_t, \tag{D.64}$$

where symmetry around the balanced growth path was used.

Marginal costs in (D.65) are approximated by:

$$\widehat{mc}_t = (1-\alpha)\hat{w}_t + \alpha \hat{r}^k_t - \frac{1}{1-\zeta}\left(\zeta \hat{k}^g_t - \zeta \frac{\bar{y}}{\bar{y}+\Phi}\hat{y}_t + \hat{\epsilon}^a_t\right)\left(\frac{k^g_t}{y_t+\Phi}\right)^{\frac{\zeta}{1-\zeta}} \tilde{\epsilon}^a_t, \tag{D.65}$$

and in the flexible economy from (D.55):

$$\widehat{mc}^f_t = 0 \tag{D.66}$$

To log-linearize the pricing FOC (D.14), note that, to a first order, the common terms in numerator and denominator, i.e., $\frac{\xi_{t+s} y_{t,t+s}(i)}{\lambda_p(\mathrm{y}_{t+s}(\mathrm{i}))\xi_t}$, cancel out, using equation (D.16). As a preliminary step, notice that in the absence of markup shocks:

$$\begin{aligned}
\overline{mc} d\left(\frac{\eta_p(\mathrm{y}_{t+s}(i))}{1-\eta_p(\mathrm{y}_{t+s}(i))}\right)\Big|_{\mathrm{y}_{t+s}(i)=1} &= \overline{mc}\frac{\bar{\eta}_p}{1-\bar{\eta}_p}\frac{-1}{1-\bar{\eta}_p}\frac{d\eta_p(\mathrm{y}_{t+s}(i))|_{\mathrm{y}_{t+s}(i)=1}}{\bar{\eta}_p} \\
&= -\bar{\lambda}_p \hat{\eta}_p(1) d\left(\frac{P^*_t(i)}{P_{t+s}}\right)\Big|_{\frac{P_{t+s}(i)}{P_{t+s}}=1}, \\
d\left(\frac{P_{t+s}(i)}{P_{t+s}}\right)\Big|_{\frac{P^*_t(i)}{P_{t+s}}=1} &= d\left(\frac{\chi_{t,t+s}}{\prod_{l=1}^{s}\pi_{t+l}}\right) + d\left(\frac{P^*_t(i)}{P_t}\right).
\end{aligned}$$

Notice that from (D.22):

$$1 + \bar{\lambda}_p \hat{\eta}_p = \frac{1}{\bar{A}_p}.$$

To simplify notation and to address markup shocks, use $\bar{\epsilon}^{\lambda,p} = 1$ and define:

$$p^*_t(i) \equiv \frac{P^*_t(i)}{P_t},$$
$$\hat{\epsilon}^{\lambda,p}_{t+s} \equiv \frac{\partial}{\partial \epsilon^{\lambda,p}_{t+s}} \left(\frac{\eta_p(\mathrm{y}_{t+s}(i))}{1-\eta_p(\mathrm{y}_{t+s}(i))} \right)\Big|_{\mathrm{y}_{t+s}(i)=1} \hat{\hat{\epsilon}}^{\lambda,p}_{t+s} = \frac{\eta_p(1)}{[1-\eta_p(1)]^2} \left(\frac{G'_\epsilon(1)}{G'(1)} - \frac{G''_\epsilon(1)}{G''(1)} \right).$$

Now, taking a first-order approximation of (D.14) and using symmetry yields:

$$\begin{aligned}
0 &= \mathbb{E}_t \sum_{s=0}^{\infty} (\mu\bar{\beta}\zeta_p)^s \Big[\hat{p}^*_t(i) + \sum_{l=1}^{s} [\iota_p \hat{\pi}_{t+l-1} - \hat{\pi}_{t+l}] \Big] (1 + \bar{\lambda}_p \hat{\eta}(1)) \\
&\quad - [\widehat{mc}_{t+s} + \hat{\epsilon}^{\lambda,p}_{t+s}] \\
\Leftrightarrow \frac{1}{1-\bar{\beta}\zeta_p\mu} \frac{1}{\bar{A}_p} \hat{p}^*_t &= \mathbb{E}_t \sum_{s=0}^{\infty} (\mu\bar{\beta}\zeta_p)^s [\widehat{mc}_{t+s} + \hat{\epsilon}^{\lambda,p}_{t+s}] - \sum_{l=1}^{s} [\iota_p \hat{\pi}_{t+l-1} - \hat{\pi}_{t+l}] \frac{1}{\bar{A}_p} \\
&= \widehat{mc}_t + \hat{\epsilon}^{\lambda,p}_t - \frac{\bar{\beta}\mu\zeta_p}{1-\bar{\beta}\mu\zeta_p} \frac{1}{\bar{A}_p} [\iota_p \hat{\pi}_t - \mathbb{E}_t \hat{\pi}_{t+1}] \\
&\quad + \mu\bar{\beta}\zeta_p \mathbb{E}_t \mathbb{E}_{t+s} \sum_{s=0}^{\infty} (\mu\bar{\beta}\zeta_p)^s [\widehat{mc}_{t+1+s} + \hat{\epsilon}^{\lambda,p}_{t+1+s}] - \sum_{l=1}^{s} [\iota_p \hat{\pi}_{t+l} - \hat{\pi}_{t+1+l}] \frac{1}{\bar{A}_p} \\
&= \widehat{mc}_t + \hat{\epsilon}^{\lambda,p}_t - \frac{\bar{\beta}\mu\zeta_p}{1-\bar{\beta}\mu\zeta_p} \frac{1}{\bar{A}_p} [\iota_p \hat{\pi}_t - \mathbb{E}_t \hat{\pi}_{t+1}] + \mu\bar{\beta}\zeta_p \mathbb{E}_t \hat{p}^*_{t+1}.
\end{aligned}$$

Now, linearizing the evolution of the price index (D.15):

$$\hat{p}^*_t = \frac{\zeta_p}{1-\zeta_p} [\hat{\pi}_t - \iota_p \hat{\pi}_{t-1}] \quad \Leftrightarrow \quad \hat{\pi}_t = \frac{1-\zeta_p}{\zeta_p} \hat{p}^*_t + \iota_p \hat{\pi}_{t-1}.$$

Forwarding the equation once and substituting in and solving for $\hat{\pi}_t$ yields:

$$\hat{\pi}_t = \frac{\iota_p}{1+\iota_p\bar{\beta}\mu} \hat{\pi}_{t-1} + \frac{1-\zeta_p\bar{\beta}\mu}{1+\iota_p\bar{\beta}\mu} \frac{1-\zeta_p}{\zeta_p} \bar{A}_p (\widehat{mc}_t + \hat{\epsilon}^{\lambda,p}_t) + \frac{\bar{\beta}\mu}{1+\iota_p\bar{\beta}\mu} \mathbb{E}_t \hat{\pi}_{t+1}. \quad \text{(D.67)}$$

D.6.2 Households

The law of motion for capital (D.26) and the fact that individual capital holdings are proportional to aggregate capital holdings imply:

$$\hat{k}^p_t = \left(1 - \frac{\bar{x}}{\bar{k}^p}\right) \hat{k}^p_{t-1} + \frac{\bar{x}}{\bar{k}^p} (\hat{x}_t + \hat{q}^x_{t+s}). \quad \text{(D.68)}$$

From (D.27), capital services evolve as:

$$\hat{k}_t = \hat{u}_t + \hat{k}^p_{t-1}. \tag{D.69}$$

From the static optimality condition (D.31)

$$\hat{w}^h_t = \nu \hat{n}_t + \frac{\hat{c}^{RA}_t - (h/\mu)\hat{c}^{RA}_{t-1}}{1-h/\mu} + \frac{d\tau^n_t}{1-\bar{\tau}^n} + \frac{d\tau^c_t}{1+\tau^c}. \tag{D.70}$$

In the flexible economy, given the absence of markup shocks equation (D.56) implies:

$$\hat{w}^f_t = \nu \hat{n}^f_t + \frac{\hat{c}^{RA,f}_t - (h/\mu)\hat{c}^{RA,f}_{t-1}}{1-h/\mu} + \frac{d\tau^{n,f}_t}{1-\bar{\tau}^n} + \frac{d\tau^{c,f}_t}{1+\bar{\tau}^c}. \tag{D.71}$$

In the presence of rigidities, the dynamic wage-setting equation (D.40) can be linearized as in the derivation of (D.67), recognizing that the analogue to marginal costs is given by (D.70):[15]

$$\begin{aligned}
\hat{w}_t = {} & \frac{\hat{w}_{t-1}}{1+\bar{\beta}\mu} + \frac{\bar{\beta}\mu \mathbb{E}_t[\hat{w}_{t+1}]}{1+\bar{\beta}\mu} \\
& + \frac{(1-\zeta_w\bar{\beta}\mu)(1-\zeta_w)}{(1+\bar{\beta}\mu)\zeta_w}\bar{A}_w\left[\frac{1}{1-h/\mu}[\hat{c}_t - (h/\mu)\hat{c}_{t-1}] + \nu\hat{n}_t - \hat{w}_t + \frac{d\tau^n_t}{1-\tau_n} + \frac{d\tau^c_t}{1+\tau_c}]\right] \\
& - \frac{1+\bar{\beta}\mu\iota_w}{1+\bar{\beta}\mu}\hat{\pi}_t + \frac{\iota_w}{1+\bar{\beta}\mu}\hat{\pi}_{t-1} + \frac{\bar{\beta}\mu}{1+\bar{\beta}\mu}\mathbb{E}_t[\hat{\pi}_{t+1}] + \frac{\hat{\epsilon}^{\lambda,w}_t}{1+\bar{\beta}\mu}.
\end{aligned} \tag{D.72}$$

[15] Here, the analogy with marginal costs holds only to a first order. Noting that common terms drop out the first order condition (D.39) and using (D.42) as well as $A_w \equiv [1+\bar{\lambda}_w\hat{\eta}_w(1)]^{-1}$ linearizes as follows:

$$\begin{aligned}
0 = {} & \mathbb{E}_t\left(\sum_{s=0}^{\infty}(\zeta_w\mu\bar{\beta})^s\frac{\bar{n}}{\bar{\lambda}_w}\bar{w}^*\left(\left[\hat{w}^*_t + \sum_{l=1}^{s}(\iota_w\hat{\pi}_{t+l-1} - \hat{\pi}_{t+l})\right](1+\bar{\lambda}\hat{\eta}_w(1)) - \bar{\lambda}_w\hat{\eta}_w(1)\hat{w}_{t+s} + \hat{w}^h_{t+s} + \hat{\hat{\epsilon}}^{\lambda,w}_{t+s}\right)\right) \\
\propto {} & \frac{1}{1-\zeta_w\mu\bar{\beta}}A_w^{-1}[\hat{w}^*_t + \iota_w\hat{\pi}_t - \mathbb{E}_t(\hat{\pi}_{t+1})] \\
& + \mathbb{E}_t\left(\sum_{s=0}^{\infty}(\zeta_w\mu\bar{\beta})^s\left(\left[A_w^{-1}\sum_{l=1}^{s-1}(\iota_w\hat{\pi}_{t+l} - \hat{\pi}_{t+l+1})\right](1+\bar{\lambda}\hat{\eta}_w(1)) - [A_w^{-1}-1]\hat{w}_{t+s} - \hat{w}^h_{t+s} - \hat{\hat{\epsilon}}^{\lambda,w}_{t+s}\right)\right) \\
\propto {} & \frac{1}{1-\zeta_w\mu\bar{\beta}}A_w^{-1}[\hat{w}^*_t + \iota_w\hat{\pi}_t - \mathbb{E}_t(\hat{\pi}_{t+1}) - \zeta_w\mu\bar{\beta}\mathbb{E}_t(w^*_{t+1})] - \hat{w}^h_t - \hat{\hat{\epsilon}}^{\lambda,w}_t - (1-A_w^{-1})\hat{w}_t
\end{aligned}$$

Log-linearizing the law of motion for aggregate wages (D.41) around the symmetric balanced growth path yields:

$$\hat{w}^*_t = \frac{1}{1-\zeta_w}[\hat{w}_t - \zeta_w\hat{w}_{t-1} - \zeta\iota_w\hat{\pi}_{t-1} + \zeta_w\hat{\pi}_t.$$

Substituting this equation into the above for $\hat{w}^*_t, \hat{w}^*_{t+1}$ and re-arranging yields (D.72).

From the consumption Euler equation (D.32):

$$
\begin{aligned}
&\mathbb{E}_t[\hat{\xi}_{t+1} - \hat{\xi}_t] + \mathbb{E}_t[d\tau^c_{t+1} - d\tau^c_t] = \\
&= \mathbb{E}_t\left((\sigma-1)\bar{n}^{1+\nu}[\hat{n}_{t+1} - \hat{n}_t] - \frac{\sigma}{1-h/\mu}\left[\hat{c}^{RA}_{t+1} - \left(1+\frac{h}{\mu}\right)c^{RA}_t + \frac{h}{\mu}\hat{c}^{RA}_{t+1}\right]\right) \\
&= \frac{1}{1-h/\mu}\mathbb{E}_t\big((\sigma-1)\frac{\bar{n}^{1+\nu}[\bar{c}^{RA} - h/\mu\bar{c}^{RA}]}{\bar{c}^{RA}}[\hat{n}_{t+1} - \hat{n}_t] \\
&\qquad\qquad - \sigma\left[\hat{c}^{RA}_{t+1} - \left(1+\frac{h}{\mu}\right)c^{RA}_t + \frac{h}{\mu}\hat{c}^{RA}_{t+1}\right]\big) \\
&= \frac{1}{1-h/\mu}\mathbb{E}_t\big((\sigma-1)\frac{1}{1+\bar{\lambda}_w}\frac{1-\bar{\tau}^n}{1+\tau^c}\frac{\bar{w}\bar{n}}{\bar{c}^{RA}}[\hat{n}_{t+1} - \hat{n}_t] \\
&\qquad\qquad - \sigma\left[\hat{c}^{RA}_{t+1} - \left(1+\frac{h}{\mu}\right)c^{RA}_t + \frac{h}{\mu}\hat{c}^{RA}_{t+1}\right]\big),
\end{aligned}
$$

where the last equality uses (D.42). Solving for current consumption growth:

$$
\begin{aligned}
\hat{c}^{RA}_t = &\frac{1}{1+h/\mu}\mathbb{E}_t[\hat{c}^{RA}_{t+1}] + \frac{h/\mu}{1+h/\mu}\hat{c}^{RA}_{t-1} + \frac{1-h/\mu}{\sigma[1+h/\mu]}\mathbb{E}_t[\hat{\xi}_{t+1} - \hat{\xi}_t + (d\tau^c_{t+1} - d\tau^c_t)] \\
&- \frac{[\sigma-1][\bar{w}\bar{n}/\bar{c}]}{\sigma[1+h/\mu]}\frac{1}{1+\lambda_w}\frac{1-\tau^n}{1+\tau^c}(\mathbb{E}_t[\hat{n}_{t+1}] - \hat{n}_t). \qquad \text{(D.73)}
\end{aligned}
$$

The remaining households' FOC linearize as:

$$
\mathbb{E}_t[\hat{\xi}_{t+1} - \hat{\xi}_t] = -\hat{q}^b_t - \hat{R}_t + \mathbb{E}_t[\hat{\pi}_{t+1}], \qquad \text{(D.74a)}
$$

$$
\begin{aligned}
\hat{Q}_t &= -\hat{q}^b_t - (\hat{R}_t - \mathbb{E}_t[\pi_{t+1}]) + \frac{1}{\bar{r}^k(1-\tau^k) + \delta\tau^k + 1 - \delta}\times \\
&\times\Big[(\bar{r}^k(1-\tau^k) + \delta\tau^k)\hat{q}^k_t - (\bar{r}^k - \delta)d\tau^k_{t+1} + \qquad \text{(D.74b)} \\
&\qquad + \bar{r}^k(1-\tau^k)\mathbb{E}_t(\hat{r}^k_{t+1}) + (1-\delta)\mathbb{E}_t(\hat{Q}_{t+1})\Big], \qquad \text{(D.74c)}
\end{aligned}
$$

$$
\hat{x}_t = \frac{1}{1+\bar{\beta}\mu}\left[\hat{x}_{t-1} + \bar{\beta}\mu\mathbb{E}_t(\hat{x}_{t+1}) + \frac{1}{\mu^2 S''(\mu)}[\hat{Q}_t + \hat{q}^x_t]\right], \qquad \text{(D.74d)}
$$

$$
\hat{u}_t = \frac{a'(1)}{a''(1)}\hat{r}^k_t \equiv \frac{1-\psi_u}{\psi_u}\hat{r}^k_t. \qquad \text{(D.74e)}
$$

For the credit-constrained households, (D.35) implies the following linear consumption process: consumption evolves as

$$\hat{c}_t^{RoT} = \frac{1}{1+\tau^c}\left(\frac{\bar{s}^{RoT}}{\bar{c}^{RoT}}\hat{s}_t + \frac{\bar{w}\bar{n}}{\bar{c}^{RoT}}[(1-\tau^n)(\hat{w}_t+\hat{n}_t) - d\tau_t^n] - d\tau_t^c + \frac{\bar{y}}{\bar{c}^{RoT}}\frac{d\Pi_t^p}{\bar{y}}\right), \tag{D.75}$$

where the change in profits is given by:

$$\frac{d\Pi_t^p}{\bar{y}} = \frac{1}{1+\lambda_p}\hat{y}_t - \widehat{mc}_t.$$

D.6.3 Government

The financing need evolves as:

$$\begin{aligned}\frac{dd_t}{\bar{y}} = \frac{1}{\mu}\Bigg[&\mu[\hat{g}_t^a + \hat{g}^s] + \mu\frac{\bar{s}}{\bar{y}}\hat{s}_t^{exog} + \frac{\bar{b}}{\bar{y}}\frac{\hat{b}_{t-1} - \hat{\pi}_t}{\bar{\pi}} - \mu\tau^n\frac{\bar{w}\bar{n}}{\bar{c}}\frac{\bar{c}}{\bar{y}}\Big(\hat{w}_t + \hat{n}_t\Big)\\ &- \mu\tau_c\frac{\bar{c}}{\bar{y}}\hat{c}_t - \tau^k[\bar{r}^k r_t^k + (r_t^k - \delta)\hat{k}_{t-1}^p]\mu\frac{\bar{k}}{\bar{y}}\Bigg].\end{aligned} \tag{D.76}$$

In the benchmark case of distortionary labor taxes, labor tax rates evolve according to (D.45), which is linearized as:

$$\begin{aligned}\bar{\tau}^n\frac{\bar{w}\bar{n}}{\bar{c}}\frac{\bar{c}}{\bar{y}}\left[\frac{d\tau_t^n}{\tau_n}\right] + \hat{\epsilon}_t^\tau &= \psi_\tau\frac{dd_t}{\bar{y}}\\ &= \frac{\psi_\tau}{\mu}\Bigg[\mu[\hat{g}_t^a + \hat{g}^s] + \mu\frac{\bar{s}}{\bar{y}}\hat{s}_t^{exog} + \frac{\bar{b}}{\bar{y}}\frac{\hat{b}_{t-1} - \hat{\pi}_t}{\bar{\pi}} - \mu\tau^n\frac{\bar{w}\bar{n}}{\bar{c}}\frac{\bar{c}}{\bar{y}}\Big(\hat{w}_t + \hat{n}_t\Big)\\ &\qquad - \mu\tau_c\frac{\bar{c}}{\bar{y}}\hat{c}_t - \tau^k[\bar{r}^k r_t^k + (r_t^k - \delta)\hat{k}_{t-1}^p]\mu\frac{\bar{k}}{\bar{y}}\Bigg].\end{aligned} \tag{D.77}$$

In general, tax rates, or endogenous transfers, satisfy from (D.46):

$$\bar{\tau}^n\frac{\bar{w}\bar{n}}{\bar{c}}\frac{\bar{c}}{\bar{y}}\left[\frac{d\tau_t^n}{\tau_n}\right] + \tau^c\frac{\bar{c}}{\bar{y}}\frac{d\tau_t^c}{\tau^c} + \tau^k\frac{[\bar{r}^k - \delta]\bar{k}}{\bar{y}}\frac{d\tau_t^k}{\tau^k} - \frac{\bar{s}}{\bar{y}}\hat{s}_t^{endog} + \hat{\epsilon}_t^\tau = \psi_\tau\frac{dd_t}{\bar{y}}. \tag{D.78}$$

Debt holdings are determined from the budget constraint (D.44):

$$\begin{aligned}\frac{1}{\bar{R}}\frac{\bar{b}}{\bar{y}}[\hat{b}_t - \hat{R}_t - \hat{q}_t^b] = (1-\psi_\tau)\frac{dd_t}{\bar{y}} - \bar{\tau}^n\frac{\bar{w}\bar{n}}{\bar{c}}\frac{\bar{c}}{\bar{y}}\left[\frac{d\tau_t^n}{\tau_n}\right] - \tau^c\frac{\bar{c}}{\bar{y}}\frac{d\tau_t^c}{\tau^c}\\ - \tau^k\frac{[\bar{r}^k - \delta]\bar{k}}{\bar{y}}\frac{d\tau_t^k}{\tau^k} + \frac{\bar{s}}{\bar{y}}\hat{s}_t^{endog} - \hat{\epsilon}_t^\tau\end{aligned} \tag{D.79}$$

The linearized counterpart to the law of motion for government capital (D.47) is given by:

$$\hat{k}^g = \left(1 - \frac{\bar{x}^g}{\bar{k}^g}\right)\hat{k}^g_{t-1} + \frac{\bar{x}^g}{\bar{k}^g}\hat{q}^{x,g}_t + \frac{\bar{x}^g}{\bar{k}^g}[\hat{x}^g_t + \hat{\epsilon}^{xg}_t], \tag{D.80}$$

where $u^{x,g}_t \equiv \frac{\tilde{u}^{x,g}_t}{\bar{x}^g}$.

The marginal product of government capital (D.49) is approximated by:

$$\hat{r}^g_t = \frac{\bar{y}}{\bar{y} + \Phi}\hat{y}_t - \hat{k}^g_{t-1}. \tag{D.81}$$

The shadow price of government capital (D.48b) has the following linear approximation:

$$\hat{Q}^g_t = -(\hat{R}_t + \hat{q}^b_t - \mathbb{E}_t[\pi_{t+1}]) + \frac{1}{\bar{r}^g + 1 - \delta}[\bar{r}^g\mathbb{E}_t(\hat{r}^g_{t+1}) + (1-\delta)\mathbb{E}_t(\hat{Q}^g_{t+1})]. \tag{D.82}$$

The Euler equation for government investment (D.48a) is approximated as:

$$\hat{x}^g_t = \frac{1}{1+\bar{\beta}\mu}\left[\hat{x}_{t-1} + u^{xg}_{t-1} + \bar{\beta}\mu\mathbb{E}_t([\hat{x}^g_{t+1} + u^{xg}_{t+1}]) + \frac{1}{\mu^2 S''_g(\mu)}[\hat{Q}^g_t + \hat{q}^{x,g}_t]\right] - u^{xg}_t. \tag{D.83}$$

The monetary policy rule (D.52) is approximated by:

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)[\psi_1\hat{\pi}_t + \psi_2(\hat{y}_t - \hat{y}^f_t)] + \psi_3\Delta(\hat{y}_t - \hat{y}^f_t) + \hat{\epsilon}^r_t \tag{D.84}$$

D.6.4 Exogenous processes

The shock processes (D.57) are linearized as

$$\hat{\epsilon}^a_t = \rho_a\hat{\epsilon}^a_{t-1} + u^a_t, \tag{D.85a}$$
$$\hat{\epsilon}^r_t = \rho_r\hat{\epsilon}^r_{t-1} + u^r_t, \tag{D.85b}$$
$$\hat{g}_t = \hat{g}^a_t + \tilde{u}^g_t, \tag{D.85c}$$
$$\hat{g}^a_t = \rho_g\hat{g}^a_{t-1} + \sigma_{ga}u^a_t + u^g_t, \tag{D.85d}$$
$$\hat{s}_t = \tilde{u}^s_t, \tag{D.85e}$$
$$\hat{\epsilon}^\tau_t = \rho_\tau\hat{\epsilon}^\tau_{t-1} + u^\tau_t, \tag{D.85f}$$
$$\hat{\hat{\epsilon}}^{\lambda,p}_t = \rho_{\lambda,p}\hat{\hat{\epsilon}}^{\lambda,p}_{t-1} + u^{\lambda,p}_t - \theta_{\lambda,p}u^{\lambda,p}_{t-1}, \tag{D.85g}$$
$$\hat{\hat{\epsilon}}^{\lambda,w}_t = \rho_{\lambda,w}\hat{\hat{\epsilon}}^{\lambda,w}_{t-1} + u^{\lambda,w}_t - \theta_{\lambda,w}u^{\lambda,w}_{t-1}, \tag{D.85h}$$
$$\hat{q}^b_t = \rho_b\hat{q}^b_{t-1} + u^b_t, \tag{D.85i}$$

$$\hat{q}_t^k = \rho_k \hat{q}_{t-1}^k + u_t^k, \tag{D.85j}$$
$$\hat{q}_t^x = \rho_x \hat{q}_{t-1}^x + u_t^x, \tag{D.85k}$$
$$\hat{q}_t^{x,g} = \rho_{x,g} \hat{q}_{t-1}^{x,g} + u_t^{x,g}. \tag{D.85l}$$

D.6.5 Aggregation

Aggregate consumption (D.59) and transfers (D.61) are linearized as:

$$\hat{c}_t = (1-\phi)\frac{\bar{c}^{RA}}{\bar{c}}\hat{c}_t^{RA} + \phi\frac{\bar{c}^{RoT}}{\bar{c}}\hat{c}_t^{RoT}, \tag{D.86}$$
$$\hat{s}_t = (1-\phi)\frac{\bar{s}^{RA}}{\bar{s}}\hat{s}_t^{RA} + \phi\frac{\bar{s}^{RoT}}{\bar{s}}\hat{s}_t^{RoT}. \tag{D.87}$$

D.6.6 Market clearing

Goods market clearing:

$$\hat{y}_t = \frac{\bar{c}}{\bar{y}}\hat{c}_t + \frac{\bar{x}}{\bar{y}}\hat{x}_t + \frac{\bar{x}^g}{\bar{y}}\hat{x}_t^g + \hat{g}_t + \frac{\bar{r}^k\bar{k}}{\bar{y}}\hat{u}_t. \tag{D.88}$$

D.6.7 Solution

In addition to the exogenous processes in (D.85), the economy with frictions is reduced to 21 variables, whereas the flexible economy is characterized by 19 variables only, given perfectly flexible prices and wages. Table 13 on page XXXVIII lists the remaining variables and the corresponding equations. For the flexible economy, all variables other than those with an "n/a" entry have an ^{f} superscript. The markup shock processes affect only the economy with frictions. Table 14 on page XXXIX lists the steady state relationships that enter the linearized equations.

D.7 Measurement equations

For the estimation of the model, the following measurement equations are appended to the model:

$$\Delta Y_t = 100(\hat{y}_t - \hat{y}_{t-1}) + 100(\mu - 1), \tag{D.89a}$$
$$\Delta C_t = 100(\hat{c}_t - \hat{c}_{t-1}) + 100(\mu - 1), \tag{D.89b}$$
$$\Delta X_t = 100(\hat{x}_t - \hat{x}_{t-1}) + 100(\mu - 1), \tag{D.89c}$$
$$\Delta X_t^g = 100(\hat{x}_t^g - \hat{x}_{t-1}^g) + 100(\mu - 1), \tag{D.89d}$$

Variable	Economy with frictions	Economy without frictions
$\hat{c}$	(D.86)	(D.86)
$\hat{c}^{RA}$	(D.73)	(D.73)
$\hat{c}^{RoT}$	(D.75)	(D.75)
$\hat{x}$	(D.74a) in (D.74d)	(D.74d), (D.74a)
$\hat{k}^p$	(D.68)	(D.68)
$\hat{k}$	(D.69)	(D.69)
$\hat{u}$	(D.74e)	(D.74e)
$\hat{Q}$	(D.74a) in (D.74c)	(D.74c), (D.74a)
$\hat{r}^k$	(D.64)	(D.64)
$\hat{x}^g$	(D.74a) in (D.83)	(D.83), (D.74a)
$\hat{k}^g$	(D.80)	(D.80)
$\hat{Q}^g$	(D.74a) in (D.82)	(D.82), (D.74a)
$\hat{r}^g$	(D.81)	(D.81)
$\begin{cases} d\tau^n, d\tau^c \\ d\tau^k, \hat{s}^{endo} \end{cases}$	one variable according to (D.78) & (D.76) other variables = 0	(D.78) & (D.76) other variables = 0
$\hat{b}$	(D.79)	(D.79)
$\hat{R}$	(D.84)	indirectly via (D.66)
$\hat{\pi}$	(D.67)	=0
$\widehat{mc}$	(D.65)	=0
$\hat{w}$	(D.72)	(D.71)
$\hat{y}$	(D.88)	(D.88)
$\hat{n}$	(D.63)	(D.63)

Table 13: Unknowns and equations

Constant	Equation	Expression
$\frac{\bar{c}}{\bar{y}}$	(D.62)	$1-\frac{\bar{x}}{\bar{y}}-\frac{\bar{x}^g}{\bar{y}}-g$
$\frac{\bar{c}^{RA}}{\bar{y}}$	(D.60)	$\frac{\bar{c}-\phi\bar{c}^{RoT}}{\bar{y}(1-\phi)}$
$\frac{\bar{c}^{RoT}}{\bar{y}}$	(D.36)	$\frac{\bar{s}^{RoT}+(1-\tau^n)\bar{w}\bar{n}}{\bar{y}(1+\tau^c)}$
$\frac{\bar{x}}{\bar{k}^p}$	(D.30)	$1-\frac{1-\delta}{\mu}$
$\frac{\bar{x}}{\bar{k}}$	(D.30)	$\mu-(1-\delta)$
$\frac{\bar{k}}{\bar{y}}$	(D.8)	$\left(\frac{\bar{y}+\Phi}{\bar{y}}\right)^{\frac{1}{1-\zeta}}\left(\frac{\bar{k}^g}{\bar{y}}\right)^{\frac{-\zeta}{1-\zeta}}\left(\frac{\bar{k}}{\bar{n}}\right)^{1-\alpha}$
$\bar{u}$	normalization	$a'^{-1}(\bar{r}^k)$
$\bar{\beta}$	definition	$\beta\mu^{-1}$
$\bar{r}^k$	(D.33c)	$\frac{\bar{\beta}^{-1}-\delta\tau^k-(1-\delta)}{1-\tau^k}$
$\frac{\bar{k}^g}{\bar{y}}$	(D.47)	$\left(1-\frac{1-\delta}{\mu}\right)^{-1}\frac{\bar{x}^g}{\bar{y}}$
ζ	(D.51)	$\frac{\bar{y}}{\bar{y}+\Phi}\frac{\bar{r}^g}{1-(1-\delta)/\mu}\frac{\bar{x}}{\bar{y}}$
$\bar{r}^g$	(D.50)	$\bar{\beta}^{-1}-(1-\delta)$
$\bar{R}$	(D.33b)	$\bar{\beta}^{-1}\bar{\pi}$
$\overline{mc}$	(D.16)	$(1+\bar{\lambda}_p)^{-1}$
$\bar{\lambda}_p$	(D.18)	$\frac{\Phi}{\bar{y}}$
$\bar{w}$	(D.11)	$\frac{\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)}{(1+\lambda_w)^{\frac{1}{(1-\zeta)(1-\alpha)}}}\frac{\left(\frac{\bar{k}^g}{\bar{y}}\right)^{\frac{\zeta}{(1-\zeta)(1-\alpha)}}}{\bar{r^k}^{\frac{\alpha}{1-\alpha}}}$
$\frac{\bar{w}\bar{n}}{\bar{y}}$	$[n_t(i)]$, $[K_t(i)]$,(D.16),(D.18)	$1-\bar{r}^k\frac{\bar{k}}{\bar{y}}$
$\frac{\bar{k}}{\bar{n}}$	(D.9)	$\frac{\alpha}{1-\alpha}\frac{\bar{w}}{\bar{r}^k}$

Table 14: Steady state relationships

$$\Delta \frac{W_t}{P_t} = 100(\hat{w}_t - \hat{w}_{t-1}) + 100(\mu - 1), \tag{D.89e}$$

$$\hat{\pi}_t^{obs} = 100\hat{\pi}_t + 100(\bar{\pi} - 1), \tag{D.89f}$$

$$\hat{R}_t^{obs} = 100\hat{R}_t + 100(\bar{R} - 1), \tag{D.89g}$$

$$\hat{q}_t^{k,obs} = 100\hat{q}_t^k + \bar{\hat{q}}^{k,obs}, \tag{D.89h}$$

$$\hat{n}_t^{obs} = 100\hat{n}_t + \bar{\hat{n}}^{obs}, \tag{D.89i}$$

$$\hat{b}_t^{obs} = 100\hat{b}_t + \bar{\hat{b}}^{obs}. \tag{D.89j}$$

The constants give the inflation rate $\bar{\pi}$ along the balanced growth path and the trend growth rates. $100(\mu - 1)$ represents the deterministic net trend growth imposed on the data. Note that apart from the trend growth rate and the constant nominal interest rate, the discount factor can be backed out of the constants:

$$\beta = \frac{\bar{\pi}}{\bar{R}} \mu^{\sigma}.$$

The constant terms in the measurement equation are necessary even if the data are demeaned for the particular observation sample because the allocation in the flexible economy cannot be attained in the economy with frictions. Given a nonzero output gap, other variables also will deviate from zero. To see why, notice that for the allocations to be the same in both the economy with frictions and in its frictionless counterpart required that the Calvo constraints on price- and wage-setting were slack – otherwise the equilibrium allocation would differ from that in the flexible economy. Slack Calvo constraints, in turn, required that aggregate prices and wages be constant, which implied a constant real wage. Finally, a constant real wage would be inconsistent with the allocation in the flexible economy.

D.8 Welfare implications

To evaluate welfare implications, we approximate the compensating variation in terms of quarterly consumption of each type of agent separately as well as the population-weighted average.

Independent of whether a household is constrained or not, equation (D.24) gives the preferences of the household. Using the log-linearized model solution around the deterministic balanced growth path, the lifetime utility of any time path of consumption and hours worked can be computed as:

$$U_t(\{\hat{c}_{t+s}, \hat{n}_{t+s}\}) = \sum_{s=0}^{\infty} \beta^s \left[\frac{(\mu^{1-\sigma})^{t+s}}{1-\sigma} \left(\bar{c} \exp[\hat{c}_{t+s}] - \frac{h}{\mu} \bar{c} \exp[\hat{c}_{t+s-1}] \right)^{1-\sigma} \right]$$

$$
\begin{aligned}
&\qquad\qquad \times \exp\left[\frac{\sigma-1}{1+\nu}(\bar{n}\exp[\hat{n}_{t+s}])^{1+\nu}\right] \\
&= (\mu^{1-\sigma})^t \sum_{s=0}^{\infty}[\beta\mu^{1-\sigma}]^s \left[\frac{\bar{c}^{1-\sigma}}{1-\sigma}\big(\exp[\hat{c}_{t+s}] - \frac{h}{\mu}\exp[\hat{c}_{t+s-1}]\big)^{1-\sigma}\right] \\
&\qquad\qquad \times \exp\left[-\frac{\bar{n}^{1+\nu}}{1+\nu}\exp[(1+\nu)\hat{n}_{t+s}]\right]^{1-\sigma} \\
&= (\mu^{1-\sigma})^t \frac{\bar{c}^{1-\sigma}}{1-\sigma} \\
&\times \sum_{s=0}^{\infty}[\beta\mu^{1-\sigma}]^s \left[\left(e^{\hat{c}_{t+s}} - \frac{h}{\mu}e^{\hat{c}_{t+s-1}}\right)\exp\Big[-\frac{\bar{n}^{1+\nu}}{1+\nu}\exp[(1+\nu)\hat{n}_{t+s}]\Big]\right]^{1-\sigma}.
\end{aligned}
\tag{D.90}
$$

Now we can compute the compensating variation between to paths of consumption and leisure, with and without the fiscal stimulus, as:

$$
\Gamma = \left[\frac{\sum_{s=0}^{\infty}[\beta\mu^{1-\sigma}]^s\left(e^{\hat{c}^{ARRA}_{t+s}} - \frac{h}{\mu}e^{\hat{c}^{ARRA}_{t+s-1}}\right)\exp\left[-\frac{\bar{n}^{1+\nu}}{1+\nu}\exp[(1+\nu)\hat{n}^{ARRA}_{t+s}]\right]\Big)^{1-\sigma}}{\sum_{s=0}^{\infty}[\beta\mu^{1-\sigma}]^s\left(e^{\hat{c}^{wo}_{t+s}} - \frac{h}{\mu}e^{\hat{c}^{wo}_{t+s-1}}\right)\exp\left[-\frac{\bar{n}^{1+\nu}}{1+\nu}\exp[(1+\nu)\hat{n}^{wo}_{t+s}]\right]\Big)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}} - 1.
\tag{D.91}
$$

An individual with discount factor β would be willing to give up a fraction Γ of consumption in each period to live in an otherwise identical world with the fiscal stimulus in place.

For large s, the deviations from the balanced growth path are numerically indistinguishable from zero. However, since $\beta\mu^{1-\sigma}$ is in practice close to unity, even for $s = 1,000$, the infinite sum has not converged. We therefore approximate:

$$
\begin{aligned}
&\sum_{s=0}^{\infty}[\beta\mu^{1-\sigma}]^s \left[\left(e^{\hat{c}_{t+s}} - \frac{h}{\mu}e^{\hat{c}_{t+s-1}}\right)\exp\Big[-\frac{\bar{n}^{1+\nu}}{1+\nu}\big(\exp[(1+\nu)\hat{n}_{t+s}]\big)\Big]\right]^{1-\sigma} \\
&\qquad \approx \sum_{s=0}^{T}[\beta\mu^{1-\sigma}]^s \left[\left(e^{\hat{c}_{t+s}} - \frac{h}{\mu}e^{\hat{c}_{t+s-1}}\right)\exp\Big[-\frac{\bar{n}^{1+\nu}}{1+\nu}\big(\exp[(1+\nu)\hat{n}_{t+s}]\big)\Big]\right]^{1-\sigma} \\
&\qquad\quad + \frac{[\beta\mu^{1-\sigma}]^{T+1}}{1-\beta\mu^{1-\sigma}}(1-h/\mu)^{1-\sigma},
\end{aligned}
$$

for some large T. In practice, we use $T = 1,000$ but checked the results for $T = 5,000$.

To obtain $\bar{n}^{1+\nu}$, multiply equation (D.42) by $\bar{n}$ and divide by $\bar{y}$. This shows that $\bar{n}^{1+\nu} = \frac{\bar{w}\bar{n}}{\bar{y}}\frac{1}{(1+\lambda^w)}\frac{1}{\bar{c}^{RA}/\bar{y}}\frac{1}{1-\frac{h}{\mu}}\frac{1-\bar{\tau}^n}{1+\tau^c}$, which is in terms of the constants in Table 14.

D.9 Simple New Keynesian Model

D.9.1 Setup

There is a unit mass of agents, a fraction ϕ of which are constrained to be rule-of-thumb (RoT) and consume their period labor income only. Intermediate goods are produced in monopolistic competition, while final goods are produced competitively as an aggregate of all intermediate goods $i \in [0,1]$. The profits of intermediate goods producers go only to unconstrained agents. A competitive aggregate of differentiated labor is the only input into intermediate goods production. Differentiated labor of type $j \in [0,1]$ is provided by trade unions, which differentiate households' homogeneous labor. Under the maintained assumption of $\phi < \frac{1}{2}$, wages are set by the union to maximize the intertemporal utility of unconstrained households. All agents provide the same amount of labor and receive an equal share of labor income inclusive of the markup.

Agents' flow utility is given by:

$$U(C(j), N(j)) = \log C(j) - \frac{N(j)^{1+\nu}}{1+\nu}. \tag{D.92}$$

In what follows, we drop the j index, unless needed, as we assume that agents insure completely against idiosyncratic labor income risk, and allocations are therefore independent of j.

A fraction $\phi \in [0,1)$ of agents are rule-of-thumb, and they consume their period labor income net of taxes τ_t^n and transfers S_t, subject to a consumption tax of τ_t^c:

$$C_t^{RoT} = \frac{(1-\tau_t^n)N_t W_t + s_t P_t}{(1+\tau_t^c)P_t}. \tag{D.93}$$

Note that it is important that rule-of-thumb agents do not earn the same income as unconstrained agents. If they did earn real income equal to total production, in equilibrium, consumption of unconstrained and constrained agents would coincide.

The remaining $1-\phi$ agents maximize expected lifetime utility, discounted at rate β:

$$\max_{\{C_t, N_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t), \tag{D.94}$$

given initial nominal bond holdings B_{-1}. The maximization is subject to the budget constraint:

$$\frac{P_t C_t}{1+\tau_t^c} + B_t = R_{t-1}B_{t-1} + (1-\tau_t^n)N_t W_t + \Pi_t + s_t P_t, \tag{D.95}$$

where P_t is the price level and i_t is the nominal interest rate. Π_t denotes lump-sum transfers (e.g., from profit income). C_t and P_t are aggregates over individual varieties $j \in [0,1]$.

A competitive final goods producer aggregates varieties according to a standard Dixit-Stiglitz aggregator with elasticity of substitution ϵ_p:

$$Y_t = \left(\int_0^1 y_t(i)^{1-\frac{1}{\epsilon_p}} di\right)^{\frac{\epsilon_p}{\epsilon_p - 1}}.$$

Similarly, a competitive labor aggregator provides units of labor supply according to a Dixit-Stiglitz aggregator with elasticity of substitution ϵ_w:

$$N_t = \left(\int_0^1 y_t(j)^{1-\frac{1}{\epsilon_p}} dj\right)^{\frac{\epsilon_p}{\epsilon_p - 1}}.$$

Producers of variety i have a constant returns to scale, labor-only production function: $Y_t(i) = N_t(i)$. They adjust prices subject to a Calvo friction. Opportunities for price adjustment arrive at rate $1-\zeta$.

Market clearing implies that:

$$Y_t = \phi C_t^{RoT} + (1-\phi)C_t^u + G_t. \tag{D.96}$$

The intertemporal equilibrium condition for unconstrained households implies that:

$$1 = \beta \mathbb{E}_t \left[\frac{e^{r_t}}{\pi_{t+1}\frac{1+\tau_{t+1}^c}{1+\tau_t^c}} \frac{C_t^u}{C_{t+1}^u} \right]. \tag{D.97}$$

The intratemporal equilibrium condition for unconstrained households implies:

$$MRS_t = N_t^{\nu} C_t^u. \tag{D.98}$$

Real marginal cost is given by:

$$MC_t = \frac{W_t}{P_t}. \tag{D.99}$$

A Taylor rule subject to the ZLB governs the interest rate:

$$R_t = (1 - 1_{ZLB,t})e^{\gamma_\pi \pi_t}. \tag{D.100}$$

In general, there are both Calvo price-setting and wage-setting frictions, as specified by (D.101) and (D.102):

$$\text{Sticky prices: } \sum_{k=0}^{\infty} \zeta_p^k \mathbb{E}_t[SDF_{t,t+k} Y_{t+k}(i)(P_t^*(i) - \frac{\epsilon_p}{\epsilon_p - 1} P_{t+k} MC_t)] = 0, \tag{D.101a}$$

$$P_t^{1-\epsilon_p} = \zeta_p P_{t-1}^{1-\epsilon_p} + (1-\zeta_p) P_t^{*1-\epsilon_p}, \tag{D.101b}$$

$$\text{Sticky wages: } \sum_{k=0}^{\infty} \zeta_w^k \mathbb{E}_t \Big[SDF_{t,t+k} N_{t+k}(j) \Big(\frac{1-\tau_{t+k}^n}{1+\tau_{t+k}^c} \frac{W_t^*(j)}{P_{t+k}} - \frac{\epsilon_w}{\epsilon_w - 1} MRS_{t+k} \Big) \Big] = 0, \tag{D.102a}$$

$$W_t^{1-\epsilon_w} = \zeta_w W_{t-1}^{1-\epsilon_w} + (1-\zeta_w) W_t^{*1-\epsilon_w}, \tag{D.102b}$$

where the stochastic discount factor is given by $SDF_{t,t+k} = \beta^k \frac{C_t}{C_{t+k}}$.

For the remainder of the analytical section we consider, however, the limit of either flexible prices $\zeta_p \to 0$ or flexible wages $\zeta_w \to 0$. In the limit of flexible prices, (D.101) implies that $P_t^*(i) = \frac{\epsilon_p}{\epsilon_p - 1} P_t MC_t \forall i$ and $P_t = P_t^*$: real wages are constant and equal the inverse of the markup: $MC_t = \frac{W_t}{P_t} = \frac{\epsilon_p - 1}{\epsilon_p}$. Similarly, for flexible wages (D.102) implies that the real net wage is a constant markup over the marginal rate of substitution: $\frac{W_t}{P_t} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{1+\tau_t^c}{1-\tau_t^n} MRS_t$.

The government financing requirement $D_t = P_t d_t$ evolves as follows:

$$P_t d_t = P_t(g_t + s_t^x) + B_{t-1} + \bar{\tau} W_t N_t, \tag{D.103}$$

where s_t^x are exogenous transfers as part of the stimulus. Together with the endogenous transfers s_t^e, which may be adjusted to finance deficits, they sum up to total transfers: $s_t = s_t^e + s_t^x$. We assume zero initial debt, $B_0 = 0$.

Taking government expenditure as exogenous, we consider policy rules of the following form for $\psi_\tau \in [0, 1]$:

$$\begin{aligned} &B_t = (1-\psi_\tau) P_t d_t && \Big((\tau_t - \bar{\tau}) W_t N_t + P_t c_t \tau_t^c - P_t s_t^e \Big) = \psi_\tau D_t && \text{if } G_t \neq 0 \\ &B_t = 0 && \Big((\tau_t - \bar{\tau}) W_t N_t + P_t c_t \tau_t^c - P_t s_t^e \Big) = D_t && \text{if } G_t = 0. \end{aligned} \tag{D.104}$$

D.9.2 Log-linearized equations

Log-linearize the model around a zero tax, zero government spending and transfers, and zero inflation steady state. In steady state, rule-of-thumb agents' consumption share equals their population share times the labor share in real income (i.e., $\phi_c \equiv \phi\frac{\epsilon-1}{\epsilon}$; steady state profits amount to $\frac{1}{\epsilon}$ of real income).

Consumption of rule-of-thumb consumers from (D.93):

$$c_t^{RoT} = n_t + w_t - p_t - d\tau_t^n + \frac{\epsilon_p}{\epsilon_p - 1}s_t - d\tau_t^c, \tag{D.105}$$

where $d\tau_t^n, d\tau_t^c$ is in percentage of the steady state wage rate, while s_t is in percentage of steady state output.

The intratemporal marginal rate of substitution follows from (D.98):

$$mrs_t = c_t^u + \nu n_t. \tag{D.106}$$

The intertemporal Euler equation (D.97) implies:

$$0 = \mathbb{E}_t[c_t^u - c_{t+1}^u + r_t - \pi_{t+1} - (d\tau_{t+1}^c + d\tau_t^c)]. \tag{D.107}$$

Marginal costs (D.99) evolve as:

$$mc_t = w_t - p_t. \tag{D.108}$$

The resource constraint (D.96) implies:

$$y_t = \phi_c c_t^{RoT} + (1 - \phi_c)c_t^u + g_t, \qquad g_t \equiv \frac{dg_t}{\bar{y}}. \tag{D.109}$$

Since price and wage dispersion do not matter to a first order, the production function implies:

$$y_t = n_t. \tag{D.110}$$

A piecewise linear approximation to the Taylor rule (D.100):

$$r_t = (1 - 1_{ZLB,t})\gamma_\pi \pi_t. \tag{D.111}$$

Under sticky prices and flexible wages (note that mc is in deviation from its steady state value $-\log\frac{\epsilon_p}{\epsilon_p-1}$):

$$p_t^* = (1 - \beta\zeta_w)\sum_{k=0}^{\infty}(\beta\zeta_p)\mathbb{E}_t[mc_{t+k} + p_{t+k}] \tag{D.112a}$$

$$\pi_t \equiv p_t - p_{t-1} = (1 - \zeta_p)(p_t^* - p_{t-1}) \tag{D.112b}$$

$$mc_t = w_t - p_t = mrs_t + d\tau_t^n + d\tau_t^c \tag{D.112c}$$

Equations (D.112) imply:[16]

$$\pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + \frac{(1-\zeta_p)(1-\beta\zeta_p)}{\zeta_p}mc_t \equiv \beta\mathbb{E}_t[\pi_{t+1}] + \lambda_p(mrs_t + d\tau_t^n + d\tau_t^c). \tag{D.113}$$

Under sticky wages and flexible prices:

$$w_t^* = (1-\beta\zeta_w)\sum_{k=0}^{\infty}(\beta\zeta_w)\mathbb{E}_t[mrs_{t+k} + p_{t+k} + d\tau_{t+k}^n + d\tau_{t+k}^c] \tag{D.114a}$$

$$w_t = \zeta_w w_{t-1} + (1-\zeta_w)w_t^* \tag{D.114b}$$

$$w_t = p_t. \tag{D.114c}$$

[16] To see this, note that the equations in (D.112) imply:

$$\begin{aligned}
p_t^* - p_{t-1} &= (1-\beta\zeta_w)\sum_{k=0}^{\infty}(\beta\zeta_p)\mathbb{E}_t[mc_{t+k} + p_{t+k} - p_{t-1}] \\
&= \beta\zeta_p\mathbb{E}_t[p_{t+1}^* - p_t] + (1-\beta\zeta_p)mc_t + \pi_t \\
\Leftrightarrow \pi_t = (1-\zeta_p)(p_t^* - p_{t-1}) &= \beta\zeta_p(1-\zeta_p)\mathbb{E}_t[p_{t+1}^* - p_t] + (1-\zeta_p)(1-\beta\zeta_p)(mrs_t + d\tau_t^n + d\tau_t^c) \\
&\quad + (1-\zeta_p)\pi_t \\
\Leftrightarrow \zeta_p\pi_t &= \beta\zeta_p\mathbb{E}_t[\pi_{t+1}] + (1-\zeta_p)(1-\beta\zeta_p)(mrs_t + d\tau_t^n + d\tau_t^c) \\
\Leftrightarrow \pi_t &= \beta\mathbb{E}_t[\pi_{t+1}] + \frac{(1-\zeta_p)(1-\beta\zeta_p)}{\zeta_p}mc_t \equiv \beta\mathbb{E}_t[\pi_{t+1}] + \lambda_p(mrs_t + d\tau_t^n + d\tau_t^c).
\end{aligned}$$

Similar to (D.113), the equations in (D.114) can be shown to imply:[17]

$$\begin{aligned}\pi_t &= \mathbb{E}_t[\pi_{t+1}] + \frac{(1-\zeta_w)(1-\beta\zeta_w)}{\zeta_w}(mrs_t + d\tau_t^n + d\tau_t^c) \\ &\equiv \mathbb{E}_t[\pi_{t+1}] + \lambda_w(mrs_t + d\tau_t^n + d\tau_t^c). \end{aligned} \tag{D.115}$$

Using that $\bar{c} = \bar{y}$ in steady state, the government tax rule (D.104) becomes:

$$\begin{aligned} b_t &= (1-\psi_\tau)d_t \qquad & \left(d\tau_t^n \frac{\bar{w}\bar{n}}{\bar{p}\bar{y}} + d\tau_t^c - s_t^e\right) &= \psi_\tau d_t \qquad & \text{if } G_t \neq 0 \\ b_t &= 0 \qquad & \left(d\tau_t^n \frac{\bar{w}\bar{n}}{\bar{p}\bar{y}} + d\tau_t^c - s_t^e\right) &= d_t \qquad & \text{if } G_t = 0, \end{aligned} \tag{D.116}$$

where the financing requirement evolves as, given the assumption of zero steady state tax rates:

$$d_t = g_t + s_t^x + b_{t-1}, \qquad b_0 = 0. \tag{D.103'}$$

D.9.3 Simplified equilibrium conditions: sticky prices and flexible wages

First, solve this model under the assumption of sticky prices and flexible wages. Use the intratemporal equilibrium condition (D.112c) for $w_t - p_t - d\tau_t^n - d\tau_t^c = c_t^u + \nu n_t$ to substitute in the equation for RoT consumption (D.105) to get

$$c_t^{RoT} = c_t^u + (1+\nu)n_t + \frac{\epsilon_p}{\epsilon_p - 1}s_t. \tag{D.117}$$

[17] Use that $mrs_{t+k} = c_{t+k} + \nu n_{t+k}$ to rewrite recursively:

$$\begin{aligned} w_t^* &= \beta\zeta_w \mathbb{E}_t[w_{t+1}^*] + (1-\beta\zeta_w)(w_t - (w_t - p_t - mrs_t) + d\tau_t) \\ \Leftrightarrow w_t^* - w_{t-1} &= \beta\zeta_w(\mathbb{E}_t[w_{t+1}^* - w_t] + w_t - w_{t-1}) + (1-\beta\zeta_w)(w_t - w_{t-1} - (w_t - p_t - mrs_t) + d\tau_t^n + d\tau_t^c) \\ \Leftrightarrow w_t - w_{t-1} &= \beta\zeta_w(\mathbb{E}_t[w_{t+1} - w_t] + (w_t - w_{t-1})(1-\zeta_w)) \\ &\quad + (1-\zeta_w)(1-\beta\zeta_w)(w_t - w_{t-1} - (w_t - p_t - mrs_t) + d\tau_t^n + d\tau_t^c) \\ &= \beta\zeta_w\mathbb{E}_t[w_{t+1} - w_t] + (1-\zeta_w)(w_t - w_{t-1}) \\ &\quad - (1-\zeta_w)(1-\beta\zeta_w)((w_t - p_t - mrs_t) - d\tau_t^n + d\tau_t^c) \\ \Leftrightarrow w_t - w_{t-1} &= \beta\mathbb{E}_t[w_{t+1} - w_t] + \frac{(1-\zeta_w)(1-\beta\zeta_w)}{\zeta_w}(mrs_t + d\tau_t) \\ &\equiv \mathbb{E}_t[w_{t+1} - w_t] + \lambda_w(mrs_t + d\tau_t^n + d\tau_t^c). \end{aligned}$$

Because $w_t = p_t$, it follows trivially that $w_t - w_{t-1} = \pi_t$ for all t.

Use equation (D.117) in the resource constraint (D.109) and the production function $y_t = n_t$. Then re-arrange to get

$$\begin{aligned} c_t^u &= \frac{y_t - \phi_c c_t^u - \phi_c(1+\nu)y_t - g_t - \phi_c \frac{\epsilon_t}{\epsilon_t - 1} s_t}{(1-\phi_c)} \\ \Leftrightarrow c_t^u &= (1 - \phi_c(1+\nu))y_t - g_t - \phi_c \frac{\epsilon_t}{\epsilon_t - 1} s_t. \end{aligned} \tag{D.118}$$

Use this equation in the Euler equation to get the aggregate sticky price supply schedule:

$$\begin{aligned} y_t = & \frac{g_t + \phi_c \frac{\epsilon_t}{\epsilon_t - 1} s_t}{1 - \phi_c(1+\nu)} + \mathbb{E}_t[y_{t+1} - \frac{g_{t+1} + \phi_c \frac{\epsilon_t}{\epsilon_t - 1} s_{t+1}}{1 - \phi_c(1+\nu)}] \\ & - \frac{1}{1 - \phi_c(1+\nu)}(r_t - \mathbb{E}_t[\pi_{t+1} + d\tau_{t+1}^c - d\tau_t^c]). \end{aligned} \tag{D.119}$$

Note that (D.119) is independent of the labor tax rate because workers are on their labor supply curve.

Use the intratemporal condition and market clearing in the expression for marginal cost:

$$mc_t = d\tau_t^n + d\tau_t^c + \Big(\nu + (1 - \phi_c(1+\nu))\Big)y_t - g_t - \phi_c \frac{\epsilon_p}{\epsilon_p - 1} s_t. \tag{D.120}$$

Use the marginal cost equation (D.120) in the pricing equation to obtain the Phillips curve:

$$\pi_t = \beta \mathbb{E}_t[\pi_t] + \lambda \left(d\tau_t^n + d\tau_t^c + \Big(\nu + (1 - \phi_c(1+\nu))\Big)y_t - g_t - \phi_c \frac{\epsilon_p}{\epsilon_p - 1} s_t \right). \tag{D.121}$$

D.9.4 Simplified equilibrium conditions: sticky wages and flexible prices

To solve the model under the assumption of flexible prices but sticky wages, note that the first step in the derivation for sticky prices and flexible wages does not apply. The marginal rate of substitution is not equalized to the real wage. Instead, the real wage is constant by (D.114c). Using $w_t = p_t$, substitute the RoT consumption function (D.105) into the resource constraint (D.109) after using again that $y_t = n_t$. Solve for the consumption of unconstrained agents:

$$(1 - \phi_c)c_t^u = y_t - g_t - \phi_c(y_t - d\tau_t^n - d\tau_t^c - \frac{\epsilon_p}{\epsilon_p - 1} s_t)$$

$$= (1-\phi_c)y_t - g_t + \phi_c(d\tau_t^n + d\tau_t^c - \frac{\epsilon_p}{\epsilon_p - 1}s_t)$$

$$\Leftrightarrow c_t^u = y_t + \frac{-g_t + \phi_c(d\tau_t^n + d\tau_t^c - \frac{\epsilon_p}{\epsilon_p-1}s_t)}{1-\phi_c}. \tag{D.122}$$

Substituting the MRS (D.106) and (D.122) in the sticky wage pricing equation (D.115) yields the sticky wage New Keynesian Phillips curve:

$$\pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + \lambda_w\left((1+\nu)y_t + \frac{-g_t - \phi_c\frac{\epsilon_p}{\epsilon_p-1}s_t + \phi_c(d\tau_t^n + d\tau_t^c)}{1-\phi_c} + d\tau_t + d\tau_t^c\right). \tag{D.123}$$

Using (D.122) in the Euler equation (D.107) to get the sticky wage version of the New Keynesian IS curve:

$$y_t = \frac{g_t + \phi_c\frac{\epsilon_p}{\epsilon_p-1}s_t - \phi_c(d\tau_t^n + d\tau_t^c)}{1-\phi_c} - (r_t - \mathbb{E}_t[\pi_{t+1} + d\tau_{t+1}^c - d\tau_t^c])$$
$$+ \mathbb{E}_t[y_{t+1} + \frac{-g_{t+1} - \phi_c\frac{\epsilon_p}{\epsilon_p-1}s_{t+1} + \phi_c(d\tau_{t+1}^n + d\tau_{t+1}^c)}{1-\phi_c}]. \tag{D.124}$$

D.9.5 Persistent ZLB, immediate taxation

We now consider the case of a persistent ZLB and immediate taxation ($\psi_\tau = 1$) through labor taxes only, i.e., $s_t = d\tau_t^c = 0$.

The effect of the ZLB is modeled as a persistent nonrecurrent Markov process, similar to Eggertsson (2011): The economy starts at the ZLB and remains in it with *iid* probability μ: $\Pr\{\mathbf{1}_{ZLB,t+1} = 1|\mathbf{1}_{ZLB,t} = 1\} = \mu$. Government expenditure follows the same Markov process with $G_t = g\bar{Y}\mathbf{1}_{ZLB,t}$. As a consequence of the tax rule (D.116), it then follows that taxes are also Markov: $(\tau_t - \bar{\tau})\frac{\bar{w}\bar{N}}{\bar{p}} = g\bar{Y}\mathbf{1}_{ZLB,t}$ and $d\tau_t^n = \frac{\epsilon_p}{\epsilon_p-1}g$.

Lemma 1 in Section D.9.8 implies that under the Taylor Principle, and the assumption that $\phi_c(1+\nu) < 1$ if prices are sticky, the economy jumps back to its steady state, after the ZLB becomes slack. Lemma 2 in Section D.9.8 then implies that for μ small enough, there is a unique Markov equilibrium. Assuming that the determinacy condition on μ in Lemma 2 is satisfied, the equilibrium conditions can be solved forward to solve for this Markov equilibrium.

Consider the case of sticky prices and flexible wages first. Take expectations in the sticky price Phillips curve (D.121) under the described Markov-structure to solve for inflation at the ZLB:

$$\pi_{ZLB}^{fw}(1-\beta\mu) = \lambda_p\left(\frac{1}{\epsilon-1}g + (1+\nu)(1-\phi_c)y\right), \tag{D.125}$$

using that $d\tau_t^n - g = \frac{1}{\epsilon_p - 1} g$.

Using the flexible wage ZLB inflation (D.125) in the sticky price and flexible wage Euler equation (D.119) and taking expectations under the Markov structure yields an expression for output with flexible wages during the ZLB:

$$y_{ZLB}^{fw} = \frac{\frac{1}{1-\phi_c(1+\nu)} + \frac{\lambda_p}{1-\phi_c(1+\nu)} \frac{\mu}{(1-\beta\mu)(1-\mu)} \left(\frac{1}{\epsilon - 1}\right)}{1 - \frac{\lambda_p}{1-\phi_c(1+\nu)} \frac{\mu}{(1-\beta\mu)(1-\mu)} (1+\nu)(1-\phi_c)}. \tag{D.126}$$

Now consider sticky wages. Taking expectations in the sticky wage Phillips curve (D.123) under the Markov structure yields the flexible price inflation rate at the ZLB:

$$\pi_{ZLB}^{fp}(1-\beta\mu) = \lambda_w \left(y_{ZLB} - \frac{g}{1-\phi_c} + \sigma \frac{\phi_c}{1-\phi_c} \tau_{ZLB} + \nu y_{ZLB} + \tau_{ZLB}) \right). \tag{D.127}$$

Using the expression in the flexible price and sticky wage Euler equation (D.124) yields flexible price output at the ZLB:

$$\begin{aligned} y_{ZLB}^{fp} &= \mu y_{ZLB}^{fp} + \frac{g(1-\mu) - \phi_c(1-\mu)\tau_{ZLB}}{1-\phi_c} + \mu \pi_{ZLB}^{fp} \\ &= \frac{(1-\mu)\frac{1-\frac{\epsilon}{\epsilon-1}\phi_c}{1-\phi_c} - \frac{\mu}{1-\beta\mu}\lambda_w \frac{1}{1-\phi_c} + \frac{\mu}{1-\beta\mu}\lambda_w \left(\frac{\phi_c}{1-\phi_c} + 1\right) \frac{\epsilon}{\epsilon-1}}{1 - \mu - \frac{\mu}{1-\beta\mu}\lambda_w(1+\nu)} g \\ &= \frac{\frac{1-\frac{\epsilon}{\epsilon-1}\phi_c}{1-\phi_c} + \frac{\mu}{(1-\beta\mu)(1-\mu)}\lambda_w \frac{1}{1-\phi_c}\frac{1}{\epsilon-1}}{1 - \frac{\mu}{(1-\beta\mu)(1-\mu)}\lambda_w(1+\nu)} g. \end{aligned} \tag{D.128}$$

D.9.6 One-period ZLB, slow taxation

In this section, we focus on the case of sticky wages and flexible prices. We consider the case of slow taxation when the ZLB binds for a single period. We also assume that government expenditures last only for one period. Throughout, we make use of the fact that when the ZLB is slack, Lemma 1 implies that, under the Taylor Principle, there is a unique locally bounded equilibrium from period two onward. Therefore, the expectations in the period-one Euler equation and the Phillips curve are pinned down, and we can solve for the unique equilibrium output and consumption in period one.

Since in period two the ZLB is slack, interest rates react to the inflation caused by higher taxes. Inflation follows from the NK Phillips curve (D.123) after using that $\pi_3 = 0$:

$$\pi_2 = \lambda \left(y_2 + \nu y_2 + \frac{\phi_c}{1-\phi_c}(d\tau_2^n + d\tau_2^c - \frac{\epsilon_p}{\epsilon_p - 1} ds_2) + d\tau_2^n + d\tau_2^c \right). \quad \text{(D.129)}$$

This implies that the monetary authority raises interest rates in the second period in response, as prescribed by the Taylor rule (D.111).

Since a fraction ψ_τ of the period one deficit is repaid using taxes in period one and the remainder is repaid in period two:

$$\left(\frac{\bar{w}\bar{n}}{\bar{p}\bar{y}} d\tau_1^n + d\tau_1^c - s_1 \right) = \psi_\tau d_1 (= \psi_\tau g) \qquad b_1 = (1-\psi_\tau) d_1 \quad \text{(D.130a)}$$

$$\left(\frac{\bar{w}\bar{n}}{\bar{p}\bar{y}} d\tau_2^n + d\tau_2^c - s_2 \right) = d_2 (= (1-\psi_\tau) d_1 = (1-\psi_\tau) g) \qquad b_2 = 0. \quad \text{(D.130b)}$$

Solve the model backward. The tax rate follows from (D.130). The inflation rate is still given by (D.129). The only modification in the derivation of period two output comes from the different tax rate and its aggregate demand and inflation effect.:

$$\begin{aligned}
y_2 &= -\frac{\phi_c}{1-\phi_c}(d\tau_2^n + d\tau_2^c - \frac{\epsilon_p}{\epsilon_p - 1} s_2) - \gamma_\pi \pi_2 - d\tau_2^c \\
&= -\frac{\phi_c}{1-\phi_c}(d\tau_2^n + d\tau_2^c - \frac{\epsilon_p}{\epsilon_p - 1} s_2) - \gamma_\pi \lambda \left(y_2 + \frac{\phi_c}{1-\phi_c}(d\tau_2^n + d\tau_2^c - \frac{\epsilon_p}{\epsilon_p - 1} s_2) + \nu y_2 + d\tau_2^n + d\tau_2^c \right) \\
&\quad - d\tau_2^c \\
&= -\underbrace{\frac{\frac{\phi_c}{1-\phi_c} + \gamma_\pi \lambda \left(\frac{\phi_c}{1-\phi_c} + 1 \right)}{1 + \gamma_\pi \lambda (1+\nu)}}_{\equiv m_2^n} \underbrace{\frac{\epsilon}{\epsilon - 1}(1-\psi_\tau) g}_{= d\tau_2^n} \quad \text{labor tax only} && \text{(D.131a)} \\
&= -\underbrace{\frac{\frac{\phi_c}{1-\phi_c} + \gamma_\pi \lambda \left(\frac{\phi_c}{1-\phi_c} + 1 \right) + 1}{1 + \gamma_\pi \lambda (1+\nu)}}_{\equiv m_2^c} \underbrace{(1-\psi_\tau) g}_{= d\tau_2^c} \quad \text{consumption tax only} && \text{(D.131b)} \\
&= -\underbrace{\frac{\frac{\phi_c}{1-\phi_c} + \gamma_\pi \lambda \frac{\phi_c}{1-\phi_c}}{1 + \gamma_\pi \lambda (1+\nu)} \frac{\epsilon_p}{\epsilon_p - 1}}_{\equiv m_2^s} \underbrace{(1-\psi_\tau) g}_{= -s_2} \quad \text{transfers only} && \text{(D.131c)}
\end{aligned}$$

Now use the period one Euler equation (D.124):

$$y_1 = \frac{g + \phi_c \frac{\epsilon_p}{\epsilon_p - 1} s_1 - \phi_c (d\tau_1^n + d\tau_1^c)}{1-\phi_c} + y_2 - \phi_c \frac{\frac{\epsilon_p}{\epsilon_p - 1} s_2 - (d\tau_2^n + d\tau_2^c)}{1-\phi_c} + \pi_2 + d\tau_2^c - d\tau_1^c$$

$$
\begin{aligned}
&= \frac{g + \phi_c \frac{\epsilon_p}{\epsilon_p - 1} s_1 - \phi_c (d\tau_1^n + d\tau_1^c)}{1 - \phi_c} + \left(\phi_c \frac{\frac{\epsilon_p}{\epsilon_p - 1} s_2 - \phi_c (d\tau_2^n + d\tau_2^c)}{1 - \phi_c} - \gamma_\pi \pi_2 - d\tau_2^c \right) \\
&- \phi_c \frac{\frac{\epsilon_p}{\epsilon_p - 1} s_2 - (d\tau_2^n + d\tau_2^c)}{1 - \phi_c} + \pi_2 + d\tau_2^c - d\tau_1^c \\
&= \frac{g + \phi_c \frac{\epsilon_p}{\epsilon_p - 1} s_1 - \phi_c (d\tau_1^n + d\tau_1^c)}{1 - \phi_c}. - (\gamma_\pi - 1)\pi_2 - d\tau_1^c
\end{aligned} \tag{D.132}
$$

Under the assumption of labor taxes only:

$$
\begin{aligned}
y_1^n &= \frac{g - \phi_c d\tau_1^n}{1 - \phi_c} - (\gamma_\pi - 1)\lambda \Big(\Big(\frac{\phi_c}{1 - \phi_c} d\tau_2^n + d\tau_2^n \Big) - (1 + \nu) m_2^n d\tau_2^n \Big). \\
&= \frac{1 - \psi_\tau \phi_c \frac{\epsilon}{\epsilon - 1}}{1 - \phi_c} g - (\gamma_\pi - 1)\lambda \Big(\frac{1}{1 - \phi_c} - (1 + \nu) m_2^\tau \Big) \frac{\epsilon_p}{\epsilon_p - 1} (1 - \psi_\tau) g \\
&= \frac{1 - \psi_\tau \phi_c \frac{\epsilon}{\epsilon - 1}}{1 - \phi_c} g - (\gamma_\pi - 1)\lambda \frac{\frac{1 - \phi_c (1 + \nu)}{1 - \phi_c}}{1 + \gamma \lambda_w (1 + \nu)} \frac{\epsilon_p}{\epsilon_p - 1} (1 - \psi_\tau) g.
\end{aligned} \tag{D.133}
$$

Now, assume consumption taxes only:

$$
\begin{aligned}
y_1^c &= \frac{g - \phi_c d\tau_1^c}{1 - \phi_c} - (\gamma_\pi - 1)\lambda \Big(\Big(\frac{\phi_c}{1 - \phi_c} d\tau_2^c + d\tau_2^c \Big) - (1 + \nu) m_2^c d\tau_2^c \Big) - d\tau_1^c \\
&= \frac{1 - \phi_c \psi}{1 - \phi_c} g - (\gamma_\pi - 1)\lambda \Big(\frac{1}{1 - \phi_c} - (1 + \nu) m_2^c \Big) (1 - \psi) g - \psi g \\
&= \frac{1 - \phi_c \psi}{1 - \phi_c} g + (\gamma_\pi - 1)\lambda \frac{\frac{\nu}{1 - \phi_c}}{1 + \gamma_\pi \lambda_w (1 + \nu)} (1 - \psi) g - \psi g \\
&= \frac{1 - \psi}{1 - \phi_c} g + (\gamma_\pi - 1)\lambda \frac{\frac{\nu}{1 - \phi_c}}{1 + \gamma_\pi \lambda_w (1 + \nu)} (1 - \psi) g.
\end{aligned} \tag{D.134}
$$

Period one consumption tends to be higher with consumption tax when compared with labor taxes through the inflation channel, but it is lower through an aggregate demand channel when taxes are adjusted rapidly (i.e., when ψ is high).

$$
\frac{y_1^c - y_1^n}{g} = -\psi \frac{1 - \phi_c \frac{\epsilon_p}{\epsilon_p - 1}}{1 - \phi_c} + \frac{(\gamma_\pi - 1)\lambda_w}{1 + \gamma_p i \lambda_w (1 + \nu)} \frac{\nu + \frac{\epsilon_p}{\epsilon_p - 1}(1 - \phi_c (1 + \nu))}{1 - \phi_c} (1 - \psi).
$$

Hence:

$$
y_1^c > y_1^n
$$

$$\Leftrightarrow \psi < \frac{(\gamma_\pi - 1)\lambda_w(\nu + \frac{\epsilon_p}{\epsilon_p - 1}(1 - \phi_c(1+\nu))}{(1 - \phi_c\frac{\epsilon_p}{\epsilon_p-1})(1 + \gamma_\pi\lambda_w(1+\nu)) + (\gamma_\pi - 1)\lambda_w(\nu + \frac{\epsilon_p}{\epsilon_p-1}(1 - \phi_c(1+\nu))} \in (0,1). \tag{D.135}$$

Define $\bar{\psi}^n$ as the threshold in (D.135). For period two:

$$\frac{y_2^c - y_2^n}{g} \propto -\left(1 - \phi_c\frac{\epsilon_p}{\epsilon_p - 1} - \gamma_\pi\lambda_w\frac{1}{\epsilon_p - 1}\right)(1 - \psi). \tag{D.136}$$

Now, assume transfers only:

$$\begin{aligned} y_1^s &= \frac{g + \phi_c\frac{\epsilon_p}{\epsilon_p-1}s_1}{1 - \phi_c} - (\gamma_\pi - 1)\lambda\Big(\Big(\frac{\phi_c}{1-\phi_c}\frac{\epsilon_p}{\epsilon_p - 1}s_2\Big) - (1+\nu)m_2^s s_2\Big) \\ &= \frac{1 - \phi_c\psi\frac{\epsilon_p}{\epsilon_p-1}}{1-\phi_c} - (\gamma_\pi - 1)\lambda\Big(\frac{\phi_c}{1-\phi_c}\frac{\epsilon_p}{\epsilon_p - 1} - (1+\nu)m_2^s\Big)(1-\psi)g \\ &= \frac{1 - \phi_c\psi\frac{\epsilon_p}{\epsilon_p-1}}{1-\phi_c} + (\gamma_\pi - 1)\lambda\frac{\epsilon_p}{\epsilon_p - 1}(1-\psi)\frac{\phi_c}{1-\phi_c}\frac{\nu}{1 + \gamma_\pi\lambda_w(1+\nu)}(1-\psi)g \end{aligned} \tag{D.137}$$

Thus:

$$\frac{y_1^s - y_1^n}{g} = (\gamma_\pi - 1)\lambda\frac{1}{1 + \gamma\lambda_w(1+\nu)}\frac{\epsilon_p}{\epsilon_p - 1}(1 - \psi_\tau) > 0. \tag{D.138}$$

The difference between transfer- and consumption-tax financed multipliers:

$$\frac{y_1^s - y_1^c}{g} = \frac{1 - \phi_c\frac{\epsilon_p}{\epsilon_p-1}}{1-\phi_c}\psi - \frac{(\gamma_\pi - 1)\lambda_w\nu}{1 + \gamma_\pi\lambda_w(1+\nu)}\frac{1 - \phi_c\frac{\epsilon_p}{\epsilon_p-1}}{1-\phi_c}(1-\psi).$$

Thus:

$$y_1^s - y_1^c > 0 \quad \Leftrightarrow \quad \psi > \frac{(\gamma_\pi - 1)\lambda_w\nu}{(1 + \gamma_\pi\lambda_w(1+\nu) + (\gamma_\pi - 1)\lambda_w\nu} \in (0,1). \tag{D.139}$$

Define $\bar{\psi}^s$ as the threshold in (D.139). For period two:

$$\frac{y_2^s - y_2^c}{g} = \frac{1 - \phi_c\frac{\epsilon_p}{\epsilon_p-1}}{1-\phi_c}\frac{1 + \gamma_\pi\lambda_w}{1 + \gamma_\pi\lambda_w(1+\nu)}(1 - \psi_\tau). \tag{D.140}$$

Note that $\bar{\psi}^n > \bar{\psi}^s$. To see this, write:

$$\bar{\psi}^n = \frac{1}{1 + (1 - \phi_c \frac{\epsilon_p}{\epsilon_p - 1}) \frac{1+\gamma\lambda_w(1+\nu)}{(\gamma_\pi - 1)\lambda_w(\nu + \frac{\epsilon_p}{\epsilon_p-1}(1-\phi_c(1+\nu))}}$$

$$\bar{\psi}^s = \frac{1}{1 + \frac{1+\gamma\lambda_w(1+\nu)}{(\gamma_\pi - 1)\lambda_w \nu}}.$$

Thus, $\bar{\psi}^n > \bar{\psi}^s$ if $\frac{(\nu + \frac{\epsilon_p}{\epsilon_p-1}(1-\phi_c(1+\nu))}{1-\phi_c\frac{\epsilon_p}{\epsilon_p-1}} > \nu$ or

$$\bar{\psi}^n > \bar{\psi}^s \quad \Leftrightarrow \quad \nu \frac{1 - \phi_c \frac{\epsilon_p}{\epsilon_p-1}}{1 - \phi_c \frac{\epsilon_p}{\epsilon_p-1}} + \frac{\epsilon_p}{\epsilon_p - 1} \frac{1 - \phi_c}{1 - \phi_c \frac{\epsilon_p}{\epsilon_p-1}} > \nu, \tag{D.141}$$

which is satisfied given $\phi = \phi_c \frac{\epsilon_p}{\epsilon_p-1} < 1$.

Last, compare long-run consumption tax multipliers to long-run transfer multipliers by computing a weighted average of (D.139) and (D.140):

$$\begin{aligned}
\frac{y_1^s + \beta y_2^s - (y_1^c + \beta y_2^c)}{g} &= \frac{1 - \phi_c \frac{\epsilon_p}{\epsilon_p-1}}{1 - \phi_c} \\
&\quad \times \left(\psi - (1-\psi) \frac{(\gamma_\pi - 1)\lambda_w \nu}{1 + \gamma_\pi \lambda_w (1+\nu)} + \beta(1-\psi) \frac{1 + \gamma_\pi \lambda_w}{1 + \gamma_\pi \lambda_w(1+\nu)} \right) \\
&= \frac{1 - \phi_c \frac{\epsilon_p}{\epsilon_p-1}}{1 - \phi_c} \left(\psi + (1-\psi) \frac{\beta + \lambda_w(\beta\gamma_\pi - (\gamma_\pi - 1)\nu)}{1 + \gamma_\pi \lambda_w (1+\nu)} \right).
\end{aligned} \tag{D.142}$$

D.9.7 Three-period labor tax sticky wage model

We also consider an alternative "three-period" version of the labor tax rule:

$$\frac{W_1 N_1}{P_1 Y_1}(\tau_1^n - \bar{\tau}^n) = \psi_\tau d_1 \qquad b_1 = (1 - \psi_\tau) \qquad d_1 = g, \tag{D.143a}$$

$$\frac{W_2 N_2}{P_2 Y_2}(\tau_2^n - \bar{\tau}^n) = \psi_\tau d_2 \qquad b_2 = (1 - \psi_\tau) \qquad d_2 = b_1, \tag{D.143b}$$

$$\frac{W_3 N_3}{P_3 Y_3}(\tau_3^n - \bar{\tau}^n) = d_3 (= (1 - \psi_\tau)^2 g) \qquad b_3 = 0 \qquad d_3 = b_2. \tag{D.143c}$$

The corresponding linear 3-period tax rule (D.143) becomes:

$$\frac{\bar{w}\bar{n}}{\bar{p}\bar{y}} d\tau_1 = \psi_\tau d_1 (= \psi_\tau g) \qquad b_1 = (1 - \psi_\tau) \qquad d_1 = g, \tag{D.144a}$$

$$\frac{\bar{w}\bar{n}}{\bar{p}\bar{y}}d\tau_2 = \psi_\tau d_2 (= \psi_\tau(1-\psi_\tau)g) \qquad b_2 = (1-\psi_\tau) \qquad d_2 = b_1, \tag{D.144b}$$
$$\frac{\bar{w}\bar{n}}{\bar{p}\bar{y}}d\tau_3 = d_3 (= (1-\psi_\tau)^2 g) \qquad b_3 = 0 \qquad d_3 = b_2. \tag{D.144c}$$

Using that the economy is in its steady state in period 4 ($y_4 = \pi_4 = 0$), the model can again be solved backward, iterating on the New Keynesian IS equation (D.124) and the NK Phillips curve (D.123). Solving for the output levels yields:

$$\frac{y_3}{g} = -\frac{\gamma\lambda\left(1+\sigma\frac{\phi_c}{1-\phi_c}\right)+\sigma\frac{\phi_c}{1-\phi_c}}{\sigma+\gamma\lambda(\sigma+\nu)}\frac{\epsilon}{\epsilon-1}(1-\psi_\tau)^2 \tag{D.145a}$$

$$\begin{aligned}\frac{y_2}{g} &= -\frac{\lambda\sigma(\gamma(1+\beta)-1)\left(1-\frac{\phi_c\nu}{1-\phi_c}\right)}{(\sigma+\gamma\lambda(\sigma+\nu))^2}\times\frac{\epsilon}{\epsilon-1}(1-\psi_\tau)\\ &-\frac{\lambda\gamma\left(\frac{1}{1-\phi_c}-\beta\gamma\left(1-\frac{\phi_c\nu}{1-\phi_c}+\frac{\phi_c}{1-\phi_c}(2\gamma(\nu+\sigma)-(1+\nu))\right)\right)+\frac{\phi_c\sigma^2}{1-\phi_c}+(\gamma\lambda)^2\left(1+\frac{\phi_c\sigma}{1-\phi_c}\right)(\sigma+\nu)}{(\sigma+\gamma\lambda(\sigma+\nu))^2}\\ &\times\frac{\epsilon}{\epsilon-1}(1-\psi_\tau)\psi_\tau\end{aligned} \tag{D.145b}$$

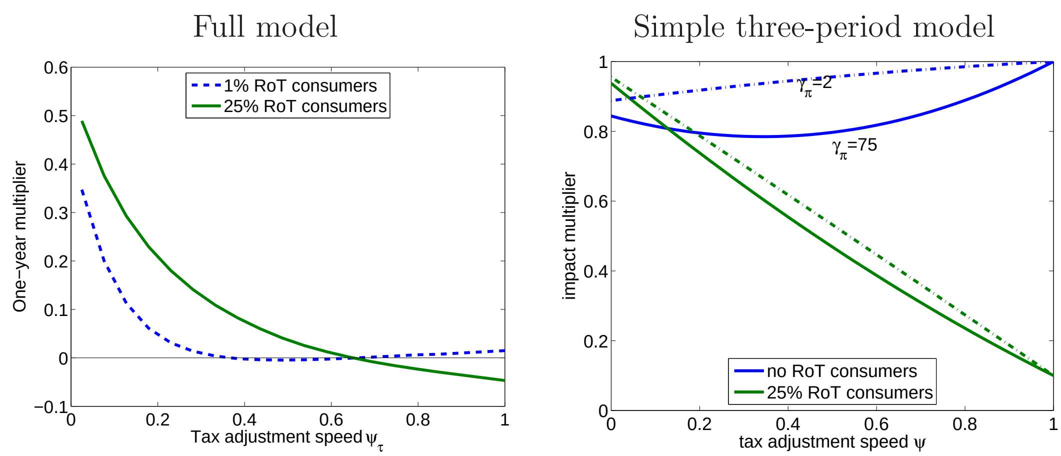
$$\begin{aligned}\frac{y_1}{g} &= 1-(\gamma-1)\lambda\Big(1-\frac{\phi_c\nu}{1-\phi_c}\Big)\frac{((1+\beta+\lambda)\sigma+\lambda\nu)}{(\sigma+\gamma\lambda(\nu+\sigma))^2}\frac{\epsilon}{\epsilon-1}(1-\psi_\tau)^2\\ &-(\gamma-1)\lambda\Big(1-\frac{\phi_c\nu}{1-\phi_c}\Big)\frac{1}{\sigma+\gamma\lambda(\nu+\sigma)}\frac{\epsilon}{\epsilon-1}(1-\psi_\tau)\psi_\tau-\frac{\phi_c}{1-\phi_c}\frac{\epsilon}{\epsilon-1}\psi_\tau\end{aligned} \tag{D.145c}$$

From equation (D.145c) it can be seen that the nonmonotonicity between the extremes of $\psi_\tau \in 0, 1$ stems from the term involving $\psi_\tau(1-\psi_\tau)$, the term proportional to the tax increase in period 2, whose influence is maximized at the intermediate value of $\psi_\tau = \frac{1}{2}$.

$$-(\gamma-1)\lambda\Big(1-\frac{\phi_c\nu}{1-\phi_c}\Big)\frac{1}{\sigma+\gamma\lambda(\nu+\sigma)},$$

If $\phi_c(1+\nu) < 1$ and given the Taylor Principle $\gamma > 1$ this term is strictly decreasing in both γ and λ (the slope of the Phillips curve, which tends to increase in wage flexibility). Thus, an aggressive central bank or very flexible wages induce a nonmonotonicity in the impact multiplier in the absence of RoT agents. Economically, the accumulation of debt, and the corresponding tax increase, causes the most inflationary pressure in period two for intermediate values of ψ_τ. This leads to an aggressive response by the central bank, which actually causes consumption to fall in period two because of a negative substitution effect as real interest rates increase. This causes private consumption in the first period also to fall because agents desire to smooth consumption and increase their savings demand.

Figure 23 provides a comparison of the simple three period model with the full empirical model. It shows that if the reaction of the monetary authority in the intermediate period two is strong enough, the simple three-period model can reproduce the qualitative feature of a nonmonotone reaction to the speed of tax adjustment.



Note: The full model results set the habit parameter to $h = 0.5$ and otherwise uses the posterior mean for the simulation. The simple model uses $\beta = 1.01^{-1}, \nu = 1, \zeta = \frac{4}{5}, \epsilon_p = 3$ and $\gamma_\pi = 2$ for the two-period model and $\gamma_\pi = 25$ for the three period model.

Figure 23: Multipliers as a function of tax adjustment speed and rule-of-thumb consumers

D.9.8 Proofs

Lemma 1. *If the Taylor Principle is satisfied ($\gamma_\pi > 1$), outside of the ZLB the locally bounded equilibrium is unique under both sticky prices and flexible wages and under sticky wages and flexible prices.*

Proof: Consider the case of sticky prices and flexible wages. Since the ZLB is nonrecurrent, outside of the ZLB the equilibrium is characterized by an operational Taylor rule (D.111) *and by equations* (D.124) *and* (D.123). *Writing in matrix form and substituting the Taylor rule, the system can be written as*

$$\underbrace{\begin{bmatrix} 1 & \frac{\gamma_\pi}{1-\phi_c(1+\nu)} \\ -\lambda_w(1-\phi_c(1+\nu)+\nu) & 1 \end{bmatrix}}_{\equiv A_p} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 1 & \frac{1}{1-\phi_c(1+\nu)} \\ 0 & \beta \end{bmatrix}}_{\equiv B_p} \mathbb{E}_t \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{1-\phi_c(1+\nu)} \\ \lambda & -1 \end{bmatrix} \begin{bmatrix} \tau_t \\ g_t \end{bmatrix}. \tag{D.146}$$

Uniqueness of the locally bounded equilibrium requires both eigenvalues $\Lambda_{1,2}$ of $C_p \equiv A_p^{-1} B_p$ to lie inside the unit circle. The characteristic equation for Λ can be written as $\Lambda^2 - \mathrm{tr}(C_p)\Lambda + \det(C_p) = 0$. Following standard textbook analysis of eigenvalues on the unit circle yields the sufficient and necessary conditions in Bullard and Mitra (2002, Appendix A) for local uniqueness:

$$|\det(C_p)| < 1 \qquad |\mathrm{tr}(C_p)| < \det(C_p) + 1. \tag{D.147}$$

Here, the determinant satisfies

$$\det(C_p) = \frac{\beta(1-\phi_c(1+\nu))}{(1-\phi_c(1+\nu)) + \gamma_\pi \lambda_p(\nu + (1-\phi_c(1+\nu)))} \in (0,1).$$

The trace is strictly positive under the assumption of $\phi_c(1+\nu) < 1$ and satisfies:

$$\begin{aligned}
\mathrm{tr}(C_p) &= \frac{(1+\beta)(1-\phi_c(1+\nu)) + \lambda_p(\nu + (1-\phi_c(1+\nu))}{(1-\phi_c(1+\nu)) + \gamma_\pi \lambda_p(\nu + (1-\phi_c(1+\nu))} \\
&< \frac{(1-\phi_c(1+\nu)) + \gamma_\pi \lambda_p(\nu + (1-\phi_c(1+\nu))) + \beta(1-\phi_c(1+\nu))}{(1-\phi_c(1+\nu)) + \gamma_\pi \lambda_p(\nu + (1-\phi_c(1+\nu)))} \\
&= 1 + \frac{\beta(1-\phi_c(1+\nu))}{(1-\phi_c(1+\nu)) + \gamma_\pi \lambda_p(\nu + (1-\phi_c(1+\nu)))} \\
&= 1 + \det(C_p),
\end{aligned}$$

where the inequality follows from the Taylor Principle.

Now consider the case of flexible prices and sticky wages. In this case, the matrices are given by:

$$A_w \equiv \begin{bmatrix} 1 & \gamma_\pi \\ -\lambda_w(1+\nu) & 1 \end{bmatrix}, \qquad B_W \equiv \begin{bmatrix} 1 & 1 \\ 0 & \beta \end{bmatrix}.$$

Proceeding analogously by defining $C_w \equiv A_w^{-1} B_w$ yields the following conditions for uniqueness:

$$\begin{aligned}
\det(C_w) &= \frac{\beta}{1 + \gamma_\pi \lambda_w(1+\nu)} \in (0,1) \\
\mathrm{tr}(C_w) &= \frac{\beta}{1 + \gamma_\pi \lambda_w(1+\nu)} + \frac{1 + \lambda_w(1+\nu)}{1 + \gamma_\pi \lambda_w(1+\nu)}
\end{aligned}$$

$$< \frac{\beta}{1+\gamma\lambda_w(1+\nu)} + \gamma_\pi \frac{1+\lambda_w(1+\nu)}{1+\gamma_\pi\lambda_w(1+\nu)} = \det(C_w)+1,$$

where the inequality holds again because of the Taylor principle.

Lemma 2. *Suppose the Taylor Principle is satisfied, the ZLB is nonrecurrent, and taxation is immediate ($\psi_\tau = 1$). Assume $\phi_c(1+\nu) < 1$ if wages are flexible and prices are sticky. If and only if the numerator of the Euler equations at the ZLB,* (D.126) *and* (D.128)*, are positive, there is a unique locally bounded Markov equilibrium in $\boldsymbol{1}_{ZLB,t}$ at the ZLB.*

Proof: Since the ZLB is nonrecurrent, by Lemma 1, if $\boldsymbol{1}_{ZLB,t} = 0$, there is a unique bounded equilibrium. Moreover, since all exogenous variables are zero outside of the steady state with immediate taxation, $y_t = \pi_t = 0$ is the unique bounded solution conditional on exit from the ZLB. Conditional on $\boldsymbol{1}_{ZLB,t} = 1$, the dynamics can therefore by described by:

$$\begin{aligned} A_{j|\gamma_\pi=0}\begin{bmatrix} y_{ZLB,t} \\ \pi_{ZLB,t}\end{bmatrix} &= B_j\mathbb{E}_t\begin{bmatrix} y_{t+1} \\ \pi_{t+1}\end{bmatrix} + D_j\begin{bmatrix}\tau_t \\ g_t\end{bmatrix}, \quad j=p,w. \\ &= B_j\mu\begin{bmatrix} y_{ZLB,t+1} \\ \pi_{ZLB,t+1}\end{bmatrix} + D_j\begin{bmatrix}\tau_t \\ g_t\end{bmatrix}, \quad j=p,w, \end{aligned}$$

where the second equality uses the Markov structure of the equilibrium. $j = p, w$ indexes the matrices defined in the proof of Lemma 1*.*

Define $C_{j,ZLB} = A^{-1}_{j|\gamma_\pi=0}B_j\mu$. Proceeding as in the proof of Lemma 1*, uniqueness of the locally bounded Markov equilibrium is equivalent to $|\det(C_{j,ZLB})| < 1$ and $|\operatorname{tr}(C_{j,ZLB})| < 1+\det(C_{j,ZLB})$.*

Consider the case of flexible wages and sticky prices. Then $\det(C_{p,ZLB}) = \beta\mu^2 \in (0,1)$. $|\operatorname{tr}(C_{j,ZLB})| < 1+\det(C_{j,ZLB})$ is equivalent to

$$\begin{aligned} &\mu(1+\beta) + \frac{\lambda_p\mu(\nu+1-\phi_c(1+\nu))}{1-\phi_c(1+\nu))} < 1+\beta\mu^2 \\ \Leftrightarrow\ &\frac{\mu(1+\beta)+\lambda_p\mu(\nu+1-\phi_c(1+\nu))}{1-\phi_c(1+\nu))} < 1+\beta\mu^2-\mu(1+\beta) = (1-\beta\mu)(1-\mu) \end{aligned}$$

Simplifying the left-hand side and dividing through by the right-hand side yields

$$\frac{\mu}{(1-\beta\mu)(1-\mu)}\frac{\lambda_p}{1-\phi_c(1+\nu)}(\nu+1)(1-\phi_c) < 1,$$

which is equivalent to the denominator in (D.126) *being positive.*

Now consider the case of flexible prices and sticky wages. Again, $\det(C_{w,ZLB}) = \beta\mu^2 \in (0,1)$. The trace condition $\operatorname{tr}(C_{j,ZLB})| < 1+\det(C_{j,ZLB})$ is equivalent *to:*

$$\mu(1+\beta)+\mu\lambda_w(1+\nu)<1+\mu^2\beta$$
$$\Leftrightarrow \mu\lambda_w(1+\nu)<1+\mu^2\beta-\mu(1+\beta)=(1-\beta\mu)(1-\mu)$$

Dividing through by $(1-\beta\mu)(1-\mu)$ *yields:*

$$\frac{\mu}{(1-\beta\mu)(1-\mu)}\lambda_w(1+\nu)<1,$$

which is equivalent to the denominator of (D.128) *being positive.*

Proposition 1. *Consider the model in Section* 5.1. *Assume immediate taxation* ($\psi_\tau=1$)*, that the Taylor Principle holds and that the ZLB is a nonrecurrent Markov state which persists with probability* μ: $\Pr\{\boldsymbol{1}_{ZLB,t}=1|\boldsymbol{1}_{ZLB,t-1}=1\}=\mu, \Pr\{\boldsymbol{1}_{ZLB,t}=1|\boldsymbol{1}_{ZLB,t-1}=0\}=0$. *In the case of sticky prices and flexible wages, also assume that* $0<\phi<\frac{\frac{\epsilon_p}{\epsilon_p-1}}{1+\nu}$. *Consider the case of financing through distortionary labor taxes:* $\hat{s}_t=d\tau_t^c=0$.

(a) For sufficiently small persistence of the ZLB, μ*, the impact multiplier* $\frac{y_{ZLB}}{g}$ *is strictly smaller than one under flexible prices and sticky wages and strictly larger than one under flexible wages and sticky prices.*

(b) The multiplier increases monotonically in the expected duration of the ZLB with either sticky wages or sticky prices in the region of determinacy.

Proof: The Taylor Principle guarantees local uniqueness of the equilibrium around the steady state. For μ *small enough, the local uniqueness extends to the system at the ZLB and the derivation of* (D.148) *is valid. This allows to impose that the model returns to the steady state with probability* $1-\mu$ *in which case the unique Markov equilibrium at the ZLB is given by the following solution to the pairs of Euler equations and Phillips curves* (D.128) *and* (D.126)*:*

$$y_{ZLB}^{fw}=\frac{\frac{1}{1-\phi_c(1+\nu)}+\frac{\lambda_p}{1-\phi_c(1+\nu)}\frac{\mu}{(1-\beta\mu)(1-\mu)}\left(\frac{1}{\epsilon-1}\right)}{1-\frac{\lambda_p}{1-\phi_c(1+\nu)}\frac{\mu}{(1-\beta\mu)(1-\mu)}(1+\nu)(1-\phi_c)}\quad \zeta_p>0,\zeta_w=0, \tag{D.126}$$

$$\begin{aligned}
y_{ZLB}^{fp}&=\frac{(1-\mu)\frac{1-\frac{\epsilon}{\epsilon-1}\phi_c}{1-\phi_c}-\frac{\mu}{1-\beta\mu}\lambda_w\frac{1}{1-\phi_c}+\frac{\mu}{1-\beta\mu}\lambda_w\left(\frac{\phi_c}{1-\phi_c}+1\right)\frac{\epsilon}{\epsilon-1}}{1-\mu-\frac{\mu}{1-\beta\mu}\lambda_w(1+\nu)}g\\
&=\frac{\frac{1-\frac{\epsilon}{\epsilon-1}\phi_c}{1-\phi_c}+\frac{\mu}{(1-\mu)(1-\beta\mu)}\lambda_w\frac{1}{1-\phi_c}\frac{1}{\epsilon-1}}{1-\frac{\mu}{(1-\mu)(1-\beta\mu)}\lambda_w(1+\nu)}g\quad \zeta_p=0,\zeta_w>0. \qquad (\text{D.128})
\end{aligned}$$

(a) Flexible wage case: The limit $\mu \searrow 0$ in (D.126) *yields*

$$\lim_{\mu \searrow 0} y_{ZLB}^{fw} = \frac{1}{1-\phi_c(1+\nu)} g > g,$$

given that $\phi_c \equiv \phi \frac{\epsilon_p - 1}{\epsilon_p} < (1+\nu)^{-1}$.

(a) Flexible price case: From equation (D.128) *it follows that*

$$\lim_{\mu \searrow 0} y_{ZLB}^{fp} = \frac{1 - \frac{\epsilon}{\epsilon-1}\phi_c}{1-\phi_c} g < g,$$

given $\phi_c = \phi \frac{\epsilon-1}{\epsilon} \in [0,1)$. The result follows by continuity for μ small enough.

(b) Flexible price case: Note that $A(\mu) \equiv \frac{\mu}{(1-\mu)(1-\beta\mu)}$ is strictly increasing in $\mu \in [0,1)$. Hence $\frac{d}{d\mu}\frac{y_1}{g} = A'(\mu)\frac{d}{dA(\mu)}\frac{y_1}{g}$. The latter expression is given by:

$$\frac{d}{dA(\mu)}\frac{y_1}{g}$$
$$= \frac{\lambda_w \frac{1}{\epsilon_p - 1}\big(1 - A(\mu)\lambda_w(1+\nu)\big) + A(\mu)\lambda_w(1+\nu)\left(1 - \frac{\epsilon_p}{\epsilon_p-1}\phi_c + A(\mu)\lambda_w \frac{1}{\epsilon_p-1}\right)}{\big(1 - A(\mu)\lambda_w(1+\nu)\big)^2 (1-\phi_c)} > 0$$

in the region of determinacy satisfying $1 > \frac{\mu}{(1-\mu)(1-\beta\mu)}\lambda_w(1+\nu)$ from Lemma 2.

(b) Flexible wage case: Define $A(\mu)$ as above.

$$\frac{d}{dA(\mu)}\frac{y_1}{g}$$
$$= \lambda_p \frac{\frac{1}{\epsilon_p-1}\left(1 - \frac{1}{1-\phi_c(1+\nu)}\lambda_p A(\mu)(1+\nu)(1-\phi_c)\right) + \frac{1}{1-\phi_c(1+\nu)}\lambda_p(1+\nu)(1-\phi_c)(1+\lambda_p A(\mu)\frac{1}{\epsilon_p-1})}{\left(1 - \frac{1}{1-\phi_c(1+\nu)}\lambda_p A(\mu)(1+\nu)(1-\phi_c)\right)^2 (1-\phi_c(1+\nu))} > 0,$$

in the region of determinacy satisfying $1 > \frac{1}{1-\phi_c(1+\nu)}\lambda_p A(\mu)(1+\nu)(1-\phi_c)$ from Lemma 2.

Figure 8 shows illustrates the Proposition for a numerical example with the parameter values described in Section 5. The case with sticky wages (solid blue line) qualitatively matches the quantitative model. We find that the multiplier is smaller than one for short durations of the ZLB, but it it increases monotonically in the expected duration of the ZLB. It exceeds unity here with an expected duration of little more than 1.5 quarters already, but given that an expected duration of four quarters takes the model into the indeterminacy region, the scale is not readily comparable. In contrast, the simple sticky price model (dashed green line) implies that the multiplier is uniformly above unity.

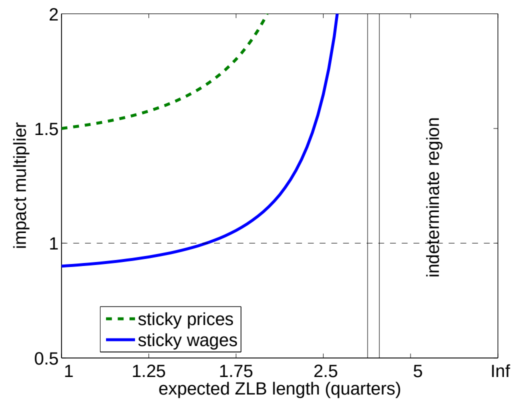


Figure 24: Short-run fiscal multipliers in simple model

Proposition 2. *Consider the model in Section* 5.1 *with sticky wages and flexible prices. Assume the Taylor Principle is satisfied:* $\gamma_\pi > 1$*.*

(a) The impact multiplier is strictly lower when financed with labor taxes rather than lump-sum taxes if $\psi_\tau < 1$ *and equal otherwise. The long-run multiplier is lower with both labor taxes and consumption taxes than it is with lump-sum taxes if* $\psi_\tau < 1$ *and wages are sufficiently sticky* $(\zeta_w \nearrow 1)$*.*
If taxes are adjusted sufficiently slowly, $\psi_\tau < \bar{\psi}^n < 1$*, the impact multiplier is higher with consumption taxes than it is with labor taxes. The impact multiplier is higher with consumption taxes than it is with transfer financing if* $\psi_\tau < \bar{\psi}^s$*, where* $\bar{\psi}^s < \bar{\psi}^n$*.*

The following results assume financing through labor taxes: $\hat{s}_t = d\tau_t^c = 0$*.*

(b) If wages are sufficiently sticky $(\zeta_w \nearrow 1)$ *and* $\phi > 0$*, increasing the tax adjustment speed* ψ_τ *lowers the impact multiplier. Without RoT agents,* $\phi = 0$*, increasing the tax adjustment speed* ψ_τ *increases the impact multiplier.*

(c) Lowering the labor share $\frac{\epsilon_p - 1}{\epsilon_p}$ *lowers the multiplier if the impact multiplier is positive. A sufficient condition is that wages are sufficiently sticky* $(\zeta_w \nearrow 1)$*. If the impact multiplier is nondecreasing in the labor share, the long-run multiplier is strictly lower for lower labor shares for all* $\psi_\tau < 1$*.*

Proof: Under the Taylor Principle, there is a locally unique bounded equilibrium outside of the ZLB due to Lemma 1*. The above backward induction is therefore valid.*

(a) Under the Taylor Principle, the inequality (D.138) *implies that the impact multiplier is higher with transfer financing than with labor taxes.*
The results for the consumption tax as compared with the labor tax and transfer financing follow from (D.135)*, and* (D.139)*, whereas the comparison of thresholds follows from* (D.141)*.*

Since the long-run multiplier here is defined as $\frac{y_1+\beta y_2}{g}$*, the result for labor taxes compared with transfers follows immediately from comparing* (D.131c) *and* (D.131a)*, which imply that period two output is higher with transfer financing. Since by* (D.138) *period one output is also higher, the result for the long-run multiplier is immediate.*
Comparing the long-run transfer financed multiplier to the consumption tax multiplier implies from (D.142) *that their difference is proportional to:*

$$\psi + (1-\psi)\frac{\beta + \lambda_w(\beta\gamma_p i - (\gamma_\pi - 1)\nu)}{1+\gamma_\pi\lambda_w(1+\nu)},$$

which is strictly positive for all $\psi \geq 0$ *if* $\beta+\lambda_w(\beta\gamma_p i-(\gamma_\pi-1)\nu) > 0$*. Sufficient for this is that* $\zeta_w \nearrow 1$ *so that* $\lambda_w \searrow 0$*. By continuity, the result holds for a neighborhood of sticky wages.*

(b) For $\lambda_w \searrow 0$ *and* $\phi_c > 0$*,* (D.133) *yields* $\frac{y_1}{g} = \frac{1-\phi_c\frac{\epsilon_p}{\epsilon_p-1}\psi_\tau}{1-\phi_c}$ *and hence*

$$\lim_{\lambda_w \searrow 0} \frac{d}{d\psi_\tau}\frac{y_1}{g} = -\frac{\phi_c\frac{\epsilon_p}{\epsilon_p-1}}{1-\phi_c} < 0.$$

From (D.133) $\lim_{\phi_c \searrow 0}\frac{y_1}{g} = 1-(\gamma_\pi-1)\lambda\frac{\nu}{1+\gamma_\pi\lambda_w\left(1+\nu\right)}\frac{\epsilon_p}{\epsilon_p-1}(1-\psi_\tau)$ *is strictly increasing in* ψ_τ *given* $\gamma_\pi > 1$*.*

(c) To see the effect of changing the labor share on the impact multiplier, it is useful to make the dependence of the consumption share of RoT agents $\phi_c = \phi\frac{\epsilon_p-1}{\epsilon_p}$ *on the labor share explicit in* (D.133)*. Rewrite:*

$$\frac{y_1}{g} = \frac{1}{1-\phi\frac{\epsilon_p-1}{\epsilon_p}}\left(1-\psi_\tau\phi-(\gamma_\pi-1)\lambda\frac{\left(\frac{\epsilon_p}{\epsilon_p-1}-\phi(1+\nu)\right)}{1+\gamma\lambda_w(1+\nu)}(1-\psi_\tau)\right).$$

Define $\frac{y_1}{g} \equiv \frac{1}{1-\phi\frac{\epsilon_p-1}{\epsilon_p}} \times P$*. Clearly, in the limit of perfectly sticky wages,* $P = 1-\phi\psi_\tau > 0$*. Hence by continuity, for sufficiently sticky wages the multiplier is strictly positive.*

Now differentiate the impact multiplier with respect to the labor share:

$$\frac{d}{d\frac{\epsilon_p-1}{\epsilon_p}}\frac{y_1}{g} = \phi\left(\frac{1}{1-\phi\frac{\epsilon_p-1}{\epsilon_p}}\right)^2 \times P + \frac{1}{1-\phi\frac{\epsilon_p-1}{\epsilon_p}} \times (\gamma_\pi-1)\lambda\frac{\left(\frac{\epsilon_p}{\epsilon_p-1}\right)^2}{1+\gamma\lambda_w(1+\nu)}(1-\psi_\tau) > 0,$$

using that the Taylor Principle holds. Thus, the impact multiplier is increasing in the labor share.
If the impact multiplier is nondecreasing in the labor share, it is sufficient to

show that the period 2 multiplier is increasing in the labor share for the long-run multiplier to increase in the labor share. To see the effect of the labor share on the period 2 multiplier $\frac{y_2}{g}$, *rewrite* (D.131a) *as:*

$$\frac{y_2}{g} = -\frac{1}{\frac{\epsilon_p-1}{\epsilon_p}\left(1-\phi\frac{\epsilon_p-1}{\epsilon_p}\right)}\underbrace{\frac{1+\gamma_\pi\lambda_w}{1+\gamma_\pi\lambda_w(1+\nu)}(1-\psi)}_{\geq 0}.$$

Since $\frac{d}{d\frac{\epsilon_p-1}{\epsilon_p}} - \frac{1}{\frac{\epsilon_p-1}{\epsilon_p}\left(1-\phi\frac{\epsilon_p-1}{\epsilon_p}\right)} = +\frac{1-2\phi_c}{\left(\frac{\epsilon_p-1}{\epsilon_p}\right)^2\left(-\phi\frac{\epsilon_p-1}{\epsilon_p}\right)^2} > 0$, $\frac{y_2}{g}$ *is increasing in the labor share given* $\phi_c < \phi < \frac{1}{2}$.

Proposition 3. *Consider the model in Section* 5.1. *Assume the Taylor Principle is satisfied:* $\gamma_\pi > 1$. *Consider the case of financing through distortionary labor taxes:* $\hat{s}^e_t = d\tau^c_t = 0$.

(a) If taxes are adjusted sufficiently slowly ($\psi_\tau < \frac{\epsilon_p-1}{\epsilon_p}$)*, the impact multiplier increases strictly in the share of RoT agents* ϕ.

(b) The multiplier on transfers is strictly increasing in the fraction of transfers RoT agents receive and weakly smaller than the government spending multiplier.

Proof: Under the Taylor Principle, the steady state is the locally unique bounded equilibrium outside of the ZLB. Thus, the above backward induction is valid.

(a) Since $\phi_c = \frac{\epsilon_p}{\epsilon_p-1}\phi$, *we can equivalently consider an increase in* ϕ *or* ϕ_c. *An increase in* ϕ_c *affects the multiplier as follows:*

$$\begin{aligned}\frac{d}{d\phi_c}\frac{y_1^\tau}{g} &= \frac{d}{d\phi_c}\frac{1-\psi_\tau\frac{\epsilon_p}{\epsilon_p-1}\phi_c}{1-\phi_c} - \frac{(\gamma_\pi-1)\lambda_w}{1+\gamma_\pi\lambda_w(1+\nu)}\frac{\epsilon_p}{\epsilon_p-1}(1-\psi_\tau)\frac{d}{d\phi_c}\frac{1-\phi_c(1+\nu)}{1-\phi_c}\\ &= \frac{1-\psi_\tau\frac{\epsilon_p}{\epsilon_p-1}}{(1-\phi_c)^2} + \frac{(\gamma_\pi-1)\lambda_w}{1+\gamma_\pi\lambda_w(1+\nu)}\frac{\epsilon_p}{\epsilon_p-1}(1-\psi_\tau)\frac{\nu}{(1-\phi_c)^2}.\end{aligned}$$

Because the second term is strictly positive under the Taylor Principle, a sufficient condition for the entire expression to be positive is for ψ_τ *to be sufficiently small:* $1 > \psi_\tau\frac{\epsilon_p}{\epsilon_p-1}$.

(b) From the tax rule (D.130)*, the path of labor taxes is the same for equal spending on stimulus transfer* s^x_t *and government spending* g_t. *Thus from the Phillips curve* (D.123)*, period two inflation is the same with transfers or government spending. Given that period two output is unchanged, but only a fraction* $\phi = \phi_c\frac{\epsilon_p}{\epsilon_p-1}$ *of transfers is spent during period one, it follows from the Euler equation* (D.124)*, that period one output and hence the impact multiplier*

is strictly lower with transfers. Since period two output is unchanged, the long-run multiplier is also strictly lower.

Proposition 4. *Consider the model in Section* 5.1*. Assume the Taylor Principle is satisfied:* $\gamma_\pi > 1$*. Consider the case of financing through distortionary labor taxes:* $\hat{s}_t = d\tau_t^c = 0$*.*

If $\psi_\tau < 1$ *and* $\phi < \frac{\epsilon_p}{\epsilon_p - 1}\frac{1}{1+\nu}$*, increasing wage flexibility (*$\zeta_w \searrow 0$*) lowers the impact multiplier.*

Proof: Under the Taylor Principle, the steady state is the locally unique bounded equilibrium outside of the ZLB. Thus, the above backward induction is valid.

From (D.133) *and under the Taylor Principle,* $\frac{d}{d\lambda_w}\frac{y_1}{g} \propto -\frac{1-\phi_c(1+\nu)}{1-\phi_c}(1-\psi_\tau)$ *since* $\frac{d}{\lambda_w}(\gamma_\pi - 1)\frac{\epsilon_p}{\epsilon_p - 1}(1-\psi_\tau)\frac{\lambda_w}{1+\lambda_w\gamma_\pi(1+\nu)} > 0$*. Using* $\phi_c = \phi\frac{\epsilon_p - 1}{\epsilon_p}$*, it follows that* $\frac{d}{d\lambda_w}\frac{y_1}{g} < 0$ *if* $\phi < \frac{\epsilon_p}{\epsilon_p - 1}\frac{1}{1+\nu}$*.*