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Stress Tests and Information Disclosure*

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Abstract

We study an optimal disclosure policy of a regulator who has information about banks' ability to overcome future liquidity shocks. We focus on the following trade-off: Disclosing some information may be necessary to prevent a market breakdown, but disclosing too much information destroys risk-sharing opportunities (Hirshleifer effect). We find that during normal times, no disclosure is optimal, but during bad times, partial disclosure is optimal. We characterize the optimal form of this partial disclosure. We also relate our results to the debate on the disclosure of stress test results.

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1 Introduction

In the new era of financial regulation following the crisis of 2008, central banks around the world will conduct periodic stress tests for financial institutions to assess their ability to withstand future shocks. A key question that occupies policymakers and bankers is whether the results of the stress tests should be disclosed and, if so, at what level of detail. The debate over this question is summarized in an article in the *Wall Street Journal* from March 2012. In this article, Fed Governor Daniel Tarullo expresses support for wide disclosure, saying that "the disclosure of stress-test results allows investors and other counterparties to better understand the profiles of each institution." On the other hand, the Clearing House Association expresses the concern that making the additional information public "could have unanticipated and potentially unwarranted and negative consequences to covered companies and U.S. financial markets."[1]

A classic concern about disclosure in the economics literature is based on the Hirshleifer effect (Hirshleifer, 1971). According to the Hirshleifer effect, greater disclosure might decrease welfare because it reduces risk-sharing opportunities for economic agents. This is indeed a relevant concern in the context of banks and stress tests. A large literature (e.g., Allen and Gale, 2000) studies risk-sharing arrangements among banks. If banks are exposed to random liquidity shocks, they will create arrangements among themselves or with outside markets to insure against such shocks. If more information about the state of each individual bank and its ability to withstand future shocks is publicly disclosed, then such risk-sharing opportunities will be limited, generating a welfare loss.

While this concern may provide credible content to the "unwarranted and negative consequences" referred to in the above quote from the Clearing House Association, it is hard to deny that greater disclosure that "allows investors and other counterparties

[1] See "Lenders Stress over Test Results," *Wall Street Journal*, March 5, 2012.

to better understand the profiles of each institution" appears to be crucial at times. In particular, as was clear during the recent financial crisis, when aggregate conditions seem bleak, the lack of disclosure might lead to a breakdown in financial activity. In the context of risk sharing and insurance, if the aggregate state of the financial sector is perceived to be weak, banks would not be able to insure themselves against undesirable outcomes (see, e.g., Leitner, 2005). In this case, some disclosure on certain banks might be necessary to enable some risk sharing and its welfare-improving effects.

In this paper, we study a model to analyze these forces and provide guidance for optimal disclosure policy in light of these forces. In the model, financial institutions suffer a loss if their future capital falls below a certain level. Part of the future capital of the financial institution can be forecasted based on current analysis and will become clear to policymakers conducting stress tests. However, there are also future shocks that cannot be forecasted with such an analysis. Financial institutions can engage in risk-sharing arrangements to guarantee that their capital does not fall below the critical level.

These risk-sharing arrangements work well if the overall state of the financial industry is perceived to be strong. In this case, no disclosure by the regulator is needed. Consistent with the Hirshleifer effect, disclosure can be even harmful because it prevents optimal risk-sharing arrangements from taking place. However, if, on average, banks are perceived to have capital below the critical level, then risk-sharing arrangements that insure them against falling below that level cannot arise without some disclosure. In this case, partial disclosure emerges as the optimal solution.

To study optimal disclosure rules in bad times, we distinguish between two different cases. First, we consider an environment where the information discovered by the regulator in the stress test is not already known to the bank. This is a reasonable assumption if the information involves assessment of bank exposure to aggregate conditions or to the state of other banks, and those are known to the regulator, who

analyzes many banks, and not to the individual banks themselves. In this case, we show that it is optimal to create two scores – a high score and a low score – and to give the high score to a group of banks whose average forecastable capital is equal to the critical level, and a low scores to other banks. This is similar to the Bayesian persuasion solution proposed by Kamenica and Gentzkow (2011).

By providing disclosure that separates some bad banks from the others, the regulator enables risk sharing among the remaining banks. Importantly, for this to work, the regulator must not provide too much information. It is sufficient to use only two scores and classify banks as "good" or "bad." Providing more detailed information about the "bad" banks does not hurt, but the regulator must not provide more information about "good" banks. In particular, within the group of "good" banks, there are some "bad" banks as well; pooling these banks together enables risk sharing.

Interestingly, the disclosure rule is not necessarily monotone; i.e., it is not always the case that banks below a certain threshold are classified as "bad" and others are classified as "good." There is a gain and a cost from including a bank in the "good" group. The gain is enabling the bank to participate in the risk sharing, preventing a welfare-decreasing drop in capital. The cost is that placing the bank in the "good" group takes resources, thereby preventing other banks from being in that group. The allocation of banks into the "good" group depends on the gain-to-cost ratio, and this does not always generate a monotone rule; it depends on the distribution of shocks that banks are exposed to. We provide conditions under which the disclosure rule is monotone.

The second environment we consider is one where the information discovered by the regulator in the stress test is known to the bank itself but not to the outside market. In this case, pooling banks into two groups will not generally work. Banks whose forecastable level of capital is significantly above the critical level will refuse to participate in a risk-sharing arrangement with a group whose average forecastable

capital is just at the critical level. Hence, in this case, the optimal disclosure rule has multiple scores. As before, one score is reserved for banks that are revealed to be below the critical capital level, and these banks are shunned from risk-sharing arrangements. Other scores pool together banks below the critical level with a bank above the critical level to enable risk sharing. Different scores are required to accommodate the different reservation utilities of different banks above the critical level of capital.

Interestingly, in this environment, non-monotonicity becomes a general feature of optimal disclosure rules. When considering banks below the critical level of capital, it turns out that the stronger ones will be pooled with a bank whose level of capital is only slightly above the critical level (hence receiving a moderate score), while the weaker ones will be pooled with a bank whose level of capital is significantly above the critical level (hence receiving a high score). As we show in this paper, the increase in cost from pooling with a moderately strong bank to pooling with a very strong bank is not significant for the weakest banks but is significant for the moderately weak banks, and this leads to the non-monotonicity result.

In summary, our paper generates the following results about optimal disclosure rules. First, no disclosure is optimal during good times, but partial disclosure is optimal during bad times. Second, partial disclosure takes the form of different scores pooling together banks of different levels of strength. The number of scores increases as we move from a case in which banks do not already have the information revealed in the stress test to the case in which they do possess this information. Third, non-monotonicity appears to be a pervasive feature of optimal disclosure rules, such that a given score pools together strong banks with weak banks.

1.1 Related literature

The literature on disclosure of regulatory information is reviewed in a recent paper by Goldstein and Sapra (2012), which highlights the disadvantages of disclosure.

Morris and Shin (2002) show that disclosure might be bad if economic agents share strategic complementarities and wish to act like each other even though it is not socially optimal. Providing a public signal then makes them place a too large weight on it because it provides information not only about fundamentals but also about what others know about the fundamentals. However, Angeletos and Pavan (2007) show that this conclusion may not hold when agents share strategic substitutes or when coordination is socially desirable. Leitner (2012) shows that disclosing too much information may reduce the regulator's ability to extract information about complex contracts that banks enter with one another. In his setting, it is optimal to reveal partial information. The regulator should set a position limit for each bank and reveal only whether the bank has reached its limit; however, the regulator should not reveal the exact position that the bank has entered. The idea that disclosing information may reduce the regulator's ability to collect information from banks also appears in Prescott (2008). Bond and Goldstein (2012) show that disclosure of information by the government to the market might harm the government's ability to learn from the market. Hence, the government may want to disclose information only on variables on which it cannot learn from the market. Increased disclosure might also be harmful due to the adverse effect it might have on the ex-ante incentives of bank managers, as in the traditional corporate-finance literature emphasizing the tension between ex-post and ex-ante optimal actions (e.g., Burkart, Gromb, and Panunzi, 1997). Our paper analyzes a different tradeoff involving risk-sharing opportunities, which are at the heart of financial activity.

In a related paper, Lizzeri (1999) studies the optimal disclosure policy of an intermediary who is hired by a firm to certify the quality of its products.[2] Lizzeri (1999) shows that a monopolist intermediary may choose to restrict the flow of information and reveal only the minimum information that is required for an efficient exchange.

[2] See also Kartasheva and Yilmaz (2012), who extend Lizzeri's framework by adding different outside options for firms as well as information asymmetries among potential buyers.

Disclosing less information allows the intermediary to extract more rents from firms that are being rated. Instead, in our setting, providing less information allows for better risk sharing.

There is also an extensive literature that studies information disclosure by firms, particularly whether the regulator should mandate firms to disclose information.[3] Our paper contributes to this literature by illustrating a case in which the regulator would like to restrict information flow from firms. A strong firm ignores the fact that revealing information destroys risk-sharing opportunities for weak firms, but the regulator takes this negative externality into account.

In a different context, Marin and Rahi (2000) provide a theory of market incompleteness, which is based on the tradeoff between adverse selection and the Hirshleifer effect. Adverse selection favors an increase in the number of securities because it reduces information asymmetries among agents. The Hirshleifer effect favors a reduction in the number of securities. Our paper does not talk about security design but instead discusses how the regulator should pool banks into groups to enable risk sharing. Because the utility function in our setting exhibits some convexity (a bank suffers a loss if its capital falls below a certain level), two groups may be necessary even when banks do not have private information. When banks have private information, more groups are necessary to accommodate the different reservation utilities of banks above the critical level.

Finally, the idea that risk-sharing arrangements may break down when aggregate conditions are bleak relates to Leitner (2005). He shows that in this case, it is optimal for banks to remain unlinked rather than form a financial network. In one interpretation of our model, we show how the disclosure policy affects the financial networks that banks form.

[3] A partial list of this literature includes Grossman (1981), Diamond (1985), Fishman and Hagerty (1990, 2003), and Admati and Pfleiderer (2000).

2 A model

2.1 The bank

There are three dates $t = 0, 1, 2$. A bank has an asset that yields a random cash flow at date 1 and no cash flows afterward. This cash flow is the sum of two random variables $\tilde{\theta}$ and $\tilde{\varepsilon}$, where $\tilde{\theta}$ is referred to as the bank's type and $\tilde{\varepsilon}$ is the bank's idiosyncratic risk, which is independent of its type. At date 0, the bank can sell the asset in a perfectly competitive market for an amount x, which will be derived endogenously. The amount of cash available for the bank at date 1, which we denote by z, is therefore $z = \tilde{\theta} + \tilde{\varepsilon}$ if the bank keeps the asset, and $z = x$ if the bank sells the asset. Everyone is risk neutral, and the risk-free rate is normalized to be zero percent; therefore, x equals the expected value of the asset $\tilde{\theta} + \tilde{\varepsilon}$, conditional on the information available to the market.

The bank's date-2 payoff is:

$$R(z) = \begin{cases} z & \text{if } z < 1 \\ z + r & \text{if } z \geq 1. \end{cases} \tag{1}$$

This payoff function is a reduced form to capture the general idea that banks suffer a loss when their cash holdings fall below some threshold. The payoff function can also represent a project that yields a positive net present value $r > 0$ but requires a minimum level of investment. For various reasons (e.g., projects cash flows are nonverifiable), the bank cannot finance the project if it does not have sufficient cash in hand. For convenience, we stick to the project interpretation, but the reader can think of other interpretations.

The bank acts to maximize its expected payoff at $t = 2$: $E[R(z)]$. As will be clear later, this provides incentives for banks to sell their assets in the financial market for an amount of at least one dollar. This is an insurance to guarantee that the bank can later make the investment. More generally, selling the asset can be thought of as

engaging in a risk-sharing arrangement.[4]

The random variables $\tilde{\theta}$ and $\tilde{\varepsilon}$ are drawn at date 0, and we denote their realizations by θ and ε, respectively. The bank's type $\tilde{\theta}$ is drawn from a finite set $\Theta \subset \mathbb{R}$ according to a probability distribution function $p(\theta) = \Pr(\tilde{\theta} = \theta)$. The idiosyncratic risk $\tilde{\varepsilon}$ is drawn from a cumulative distribution function F that satisfies $E(\tilde{\varepsilon}) = 0$; for simplicity, we assume that F is continuous. The probability structure (i.e., the functions p and F) is common knowledge.

The planner observes θ. The market observes neither θ nor ε. As for the bank, we focus on two cases:

(1) The bank observes neither θ nor ε.

(2) The bank observes θ but not ε.

The first case captures the idea that the government may have some information advantage relative to banks. This is a plausible assumption when asset values depend on future government actions or when asset values depend on interactions among banks, and the government's ability to collect information from multiple banks allows it to come up with better estimates. The second case captures the idea that the government and banks share the same information, which is unobservable to other market participants. For example, the bank may know its ability to withstand future liquidity shocks, and the government can find out this information by conducting stress tests.

Denote the lowest type by $\theta_{\min}$ and the highest type by $\theta_{\max}$. We assume that $\theta_{\max} > 1$, so if information on θ were publicly available, at least some types could sell their assets for more than one dollar and invest in their projects. We also assume that:

[4] We rule out partial insurance in which a bank with type $\theta < 1$ sells its asset for a price 1, which is paid with probability θ (i.e., the bank transfers the asset with probability 1 but receives payment with probability that is less than 1). This can be motivated by assuming that banks enter risk-sharing arrangements by forming links as in Leitner (2005). In his model, the bank's investment can succeed only if all the banks to which it is linked invest as well; hence, helping just a fraction of the banks in the network does not help.

Assumption 1: $F(1-\theta_{\min}) < 1$ and $F(1-\theta_{\max}) > 0$.

This implies that for any type realization there is a positive probability that the asset cash flow will be more than 1; but there is also a positive probability that the asset cash flow will be less than 1.

2.2 Disclosure rules

The planner's problem is to choose a disclosure rule, as defined below, to maximize total surplus, taking as given the effect of disclosure on the bank's ability to sell its asset for at least one dollar. Since the market breaks even on average, maximizing total surplus is the same as maximizing the bank's expected utility.

Formally, a *disclosure rule* is a set of "scores" S and a function that maps each type to a distribution over scores. Since Θ is assumed to be finite, we also assume that S is finite. Denote by $g(s|\theta)$ the probability that the planner assigns a score $s \in S$ when he observes type θ; that is, $g(s|\theta) = \Pr(\tilde{s} = s|\tilde{\theta} = \theta)$. (For every $\theta \in \Theta$, $\sum_{s\in S} g(s|\theta) = 1$.) For example, full disclosure is obtained when for every type θ, the planner assigns some score $s_\theta \in S$ with probability 1, such that $s_\theta \neq s_{\theta'}$ if $\theta \neq \theta'$. No disclosure is obtained when the planner assigns the same distribution over scores to all types; e.g., each type obtains the same score.

For use below, denote $\mu(s) = E[\tilde{\theta} + \tilde{\varepsilon}|\tilde{s} = s)]$, which is the expected value of the bank's asset conditional on the bank obtaining score s. Since $\tilde{\varepsilon}$ is independent of $\tilde{\theta}$, and since $E(\tilde{\varepsilon}) = 0$, we obtain that

$$\mu(s) = E[\tilde{\theta}|\tilde{s} = s] = \sum_{\theta\in\Theta} \theta \Pr(\tilde{\theta} = \theta|\tilde{s} = s) = \frac{\sum_{\theta\in\Theta} \theta p(\theta) g(s|\theta)}{\sum_{\theta\in\Theta} p(\theta) g(s|\theta)}, \tag{2}$$

where the last equality follows from Bayes' rule.

2.3 Sequence of events

We assume that the planner can commit to assigning scores according to the disclosure rule chosen. Hence, the sequence of events is as follows:

$t = 0$: (a) The planner announces its disclosure rule.

(b) The bank's type θ is realized and observed by the planner.

(c) The planner assigns the bank a score s, according to the disclosure rule, and publicly announces the score.

(d) The market offers to purchase the asset at a price $x(s)$.

(e) The bank either keeps the asset or sells it for a price $x(s)$.

$t = 1$: The bank invests if its available cash z is above 1.

$t = 2$: The bank obtains $R(z)$.

The planner's disclosure rule and assigned scores specify a game between the bank and the market. We focus on perfect Bayesian equilibria of this game. Specifically, the bank chooses whether to sell or keep the asset to maximize its expected profits, conditional on its information, and the market chooses a price $x(s)$ that equals the expected value of the asset conditional on public information, taking as given the bank's equilibrium strategy. We assume that if the bank is indifferent between selling and not selling, it sells. The planner chooses a disclosure rule that maximizes the bank's expected utility, taking as given the equilibrium strategies of the market and of the bank.

Finally, note that there is a big difference between the bank and the planner even in the second case in which the bank and the planner share the same information about θ. The bank maximizes its ex-post utility after θ is realized. The planner maximizes the bank's ex-ante utility before θ is realized. If there are many banks, one can think of $p(\theta)$ as the fraction of banks with a realization of θ. In this case, maximizing the bank's ex-ante utility is the same as maximizing the sum of banks' ex-post utilities. Hence, the bank and the planner have different objective functions ex post: the bank cares only about its own utility, while the planner cares about the sum of utilities of all banks.

3 Bank does not observe its type

We start with the case in which the bank observes only the score s. We solve the game backward. One observation that simplifies the analysis is that the bank's decision of whether to sell the asset depends on s but not on θ or ε. Hence, the fact that the bank sells the asset does not convey any additional information to the market. Consequently, the market sets a price $x(s) = \mu(s)$, which is the expected value of the bank's asset conditional on the bank obtaining score s. Given that, the bank's decision is as follows:

Lemma 1 *In equilibrium, the bank sells the asset if and only if* $\mu(s) \geq 1$.

The proof of Lemma 1 and all other proofs are in the appendix. The idea behind Lemma 1 is simple. If $\mu(s) > 1$, selling guarantees that the bank will have sufficient funds to invest in its positive NPV project; hence, the bank is happy to replace the asset's random cash flow with its expected value. If instead, $\mu(s) < 1$, the bank prefers to keep the asset because if the bank sells the asset, the bank will surely not have sufficient funds to invest, but if the bank keeps the asset, there is a positive probability that the asset's cash flow will turn out to be high and the bank will have sufficient funds. Essentially, due to the payoff structure in (1), the bank acts as a risk-loving agent when the expected payoff is below 1 and as a risk-averse agent when the expected payoff is above 1. This follows from the fact that the bank receives a "bonus" on its assets when the value of the assets is above 1 (or alternatively, the bank receives a "penalty" when the value falls below 1).

The expected utility for a bank of type θ, given that the planner follows a disclosure rule (S, g), is then

$$u(\theta) \equiv \sum_{s:\mu(s)<1} E[R(\theta + \tilde{\varepsilon})]g(s|\theta) + \sum_{s:\mu(s)\geq 1} R(\mu(s))g(s|\theta). \tag{3}$$

The first term represents the cases in which the bank keeps the asset, and the second term represents the cases in which the bank sells the asset.

The planner's problem is to choose a disclosure rule (S, g) to maximize the bank's ex-ante expected utility $\sum_{\theta \in \Theta} p(\theta) u(\theta)$.

Denote the probability that a bank of type θ sells the asset by $h(\theta)$; that is, $h(\theta) = \sum_{s:\mu(s) \geq 1} g(s|\theta)$. As noted earlier, this is the probability that a bank of type θ can engage in a risk-sharing arrangement.

Lemma 2 *The planner's problem reduces to finding a function $h : \Theta \to [0, 1]$ to maximize*

$$\sum_{\theta \in \Theta} p(\theta) \Pr(\tilde{\varepsilon} < 1 - \theta) h(\theta), \tag{4}$$

subject to the constraint

$$\sum_{\theta \in \Theta} p(\theta)(\theta - 1) h(\theta) \geq 0. \tag{5}$$

The objective function (4) represents the benefits from risk sharing. The planner maximizes the probability that banks with a low realization of cash flow will be able to sell their assets and guarantee that they have the necessary amount to invest and receive the net present value r.

Constraint (5) captures the idea that risk sharing is possible only if there are sufficient resources. Formally, for every score s that induces the bank to sell its asset, we must have $\mu(s) \geq 1$ (Lemma 1). It then follows from equation (2) that for every such score, we must have $\sum_{\theta \in \Theta} p(\theta)(\theta - 1) g(s|\theta) \geq 0$. Summing over all scores with $\mu(s) \geq 1$, we obtain constraint (5).

One can think of constraint (5) as the planner's resource constraint. The planner would like to implement an outcome in which every bank engages in risk sharing. However, the planner faces a constraint that the average cash flow of banks that participate in risk sharing must be at least 1. Essentially, the planner implements a transfer of resources from types with $\theta > 1$ to types with $\theta < 1$, so a high type sells

its asset for less than what the asset is truly worth, and a low type sells its asset for more than what the asset is worth.

Effectively, the only effect of the disclosure rule is to determine whether a bank is going to sell the asset or not. Since we know that banks sell when $\mu(s) \geq 1$ and do not sell otherwise, we can focus on a disclosure rule that assigns at most two scores: a "low" score s_0 such that $\mu(s_0) < 1$ and a "high" score s_1 such that $\mu(s_1) \geq 1$. Types that obtain a high score sell the asset, and types that obtain a low score keep the asset. In this case, $h(\theta)$ is the probability that type θ obtains the high score.

Proposition 1 below characterizes the optimal disclosure rule. The derivation of the result is as follows (the proof contains more details):

When $\theta \geq 1$, increasing $h(\theta)$ increases the objective function and relaxes the constraint; hence, the optimal disclosure rule is such that $h(\theta) = 1$ for every $\theta \geq 1$. In contrast, when $\theta < 1$, increasing $h(\theta)$ increases the objective function but *tightens* the constraint. If $E(\tilde{\theta}) \geq 1$, assigning $h(\theta) = 1$ for every $\theta \in \Theta$ satisfies the constraint and hence is optimal. Otherwise, the resource constraint is binding, and the optimal disclosure rule depends on the "gain-to-cost ratio"

$$G(\theta) \equiv \frac{\Pr(\tilde{\varepsilon} < 1 - \theta)}{1 - \theta}. \tag{6}$$

The numerator reflects the gain from increasing $h(\theta)$, and the denominator reflects the cost. The gain is that type θ can invest in its project even if it has a low realization of cash flow. The cost is that type θ requires resources in the amount $1 - \theta$.

Since the problem is linear, it is optimal to assign $h(\theta) = 1$ to types with high gain-to-cost ratios and $h(\theta) = 0$ to types with low ratios. In other words, types with high gain-to-cost ratios obtain the high score, s_1, and types with low gain-to-cost ratios obtain the low score, s_0. Since there is a finite number of types, there could also be a type that obtains the high score with a probability $h(\theta) \in (0, 1)$. To simplify the exposition, we focus on the case in which $G(\theta_1) \neq G(\theta_2)$ if $\theta_1 \neq \theta_2$, so there is at most one such type. The probability that this type obtains the high score is such

that the resource constraint is satisfied with equality.

For use below, we order the types in $\{\theta \in \Theta : \theta < 1\}$ according to their gain-to-cost ratios $G(\theta)$, such that b_1 is the type with the highest ratio, b_2 is the type with the second highest ratio, and so on. Also, let l^* be the largest integer i, such that $E(\theta|\theta \geq 1 \cup \theta \in \{b_1, ..., b_i\}) \geq 1$. Then the type that could have $h(\theta) \in (0, 1)$ is type b_{l^*+1}.

Proposition 1 *Assume that the bank does not observe its type.*

(i) If $E(\tilde{\theta}) \geq 1$, the optimal disclosure rule is such that $h(\theta) = 1$ for every $\theta \in \theta$.

(ii) If $E(\tilde{\theta}) < 1$, the optimal disclosure rule is such that

$$h(\theta) = \begin{cases} 1 \text{ if } \theta \geq 1 \text{ or } \theta \in \{b_1, ..., b_{l^*}\} \\ 0 \text{ if } \theta < 1 \text{ and } \theta \notin \{b_1, ..., b_{l^*}, b_{l^*+1}\}. \end{cases} \tag{7}$$

(For type b_{l^*+1}, $h(b_{l^*+1})$ is found from the resource constraint: $h(b_{l^*+1})p(b_{l^*+1})(1 - b_{l^*+1}) = \sum_{\theta \geq 1 \text{ or } \theta \in \{b_1, ..., b_{l^*}\}} p(\theta)(\theta - 1)$.)

The first part in Proposition 1 says that if there are sufficient resources, every bank must obtain a score that induces selling; that is, every bank obtains a score, such that $\mu(s) \geq 1$. This can be implemented by giving all banks the same score; i.e., no disclosure. This can also be implemented by assigning more than one score such that the average cash flows of a bank receiving each score is at least 1. In particular, in the special case $\theta_{\min} \geq 1$, the optimal disclosure rule can be implemented by assigning a different score to each type; i.e., full disclosure.

The second part says that if there are insufficient resources, the planner must assign at least two scores, a high score, s_1, and a low score, s_0. The high score pools all the types that are at or above 1 with some type that are below 1, such that the average cash flows of banks receiving the high score equals 1. In this case, full disclosure is suboptimal because under full disclosure, only types above 1 sell their assets, whereas under the optimal disclosure rule, some types that are below 1 also sell their assets.

Corollary 1 *Assume that the bank does not observe its type:*

1. *Full disclosure is optimal if and only if* $\theta_{\min} \geq 1$.
2. *No disclosure is optimal if and only if* $E(\tilde{\theta}) \geq 1$.

In general, the banks that obtain the low score in the second part of Proposition 1 are not necessarily the lowest types. In other words, the banks that are shunned from risk-sharing arrangements are not necessarily the lowest types. However, if the gains-to-cost function $G(\theta)$ is increasing when $\theta < 1$, then types that obtain low scores are the low types. In this case, the optimal disclosure rule involves a cutoff, such that types above the cutoff obtain a high score and types below the cutoff obtain a low score. A sufficient condition for this to happen is that the probability distribution of the idiosyncratic risk satisfies the following condition:

Condition 1 $F(\varepsilon)/\varepsilon$ *is decreasing when* $\varepsilon > 0$.

Corollary 2 *If* $E(\tilde{\theta}) < 1$, *and if Condition (1) is satisfied, the optimal disclosure rule involves a cutoff such that types below the cutoff obtain a low score (and hence do not engage in risk sharing) and types above the cutoff obtain a high score (and hence engage in risk sharing).*

Any probability distribution function that is concave on the positive region satisfies Condition (1). Examples are a normal distribution with mean zero and a uniform distribution. Also note that condition (1) is equivalent to saying that $\frac{F(\varepsilon)}{\varepsilon} > F'(\varepsilon)$ for every $\varepsilon > 0$.

Finally, we assumed above that all types of banks have the same r, that is, the same investment opportunities. The results extend easily to the case in which r depends on the bank's type according to some function $r(\theta)$. In this case, the gain-to-cost ratio becomes $r(\theta)G(\theta)$. Everything else being equal, the gain of giving a high score is higher if the bank's continuation value is higher. Hence, if $r'(\theta) > 0$, the optimal rule may involve a cutoff even if Condition (1) does not hold.

4 Bank observes its type

So far, we assumed that the bank does not observe its type. We showed that it is possible to implement the optimal disclosure rule with two scores, such that the planner pools everyone who sells under the same score. In this section, we show that this conclusion may no longer be true when the bank observes its type. The difference is that now each type has a "reservation price," i.e., a minimum price at which it is willing to sell. When different types have different reservation prices, the planner may need to assign more than two scores to distinguish among them. We also discuss how the planner should assign these multiple scores to low types who are pooled with high types.

We first derive banks' reservation prices. Define

$$\rho(\theta) = \begin{cases} \max\{1, \theta - r\Pr(\tilde{\varepsilon} < 1-\theta)\} \text{ if } \theta \geq 1 \\ \min\{1, \theta + r\Pr(\tilde{\varepsilon} \geq 1-\theta)\} \text{ if } \theta < 1. \end{cases} \tag{8}$$

Then,

Lemma 3 *A bank of type θ will sell its asset if and only if the price is at least $\rho(\theta)$.*

We refer to $\rho(\theta)$ as type θ's reservation price. As illustrated in Figure 1, the reservation price is increasing in θ. For high types, $\theta > 1$, the reservation price is lower than the true value θ because these types are willing to pay a premium $r\Pr(\tilde{\varepsilon} < 1-\theta)$ to guarantee that they will have the minimum amount necessary for investment. But the price must be at least one for this type of insurance to work. Low types, $\theta < 1$, should also have at least one dollar if they want to insure themselves, but the very low types may be willing to sell their assets for even less than one dollar. Such a sale goes against insurance, so the very low types will be willing to do so only if the price is strictly higher than the true value.

If $\rho(\theta_{\max}) = 1$, so the highest reservation price is one, the optimal disclosure rule from Section 3 remains optimal. The case $\rho(\theta_{\max}) = 1$ happens when $\theta_{\max} - r\Pr(\tilde{\varepsilon} <$

$1-\theta_{\max}) \leq 1$; i.e., when r is sufficiently high, so the cost of not obtaining insurance is very high, or when $\theta_{\max}$ is sufficiently low, so the cost of selling at a price of 1 rather than the true value $\theta_{\max}$ is not too high.

Proposition 2 *If $\theta_{\max} - r\Pr(\tilde{\varepsilon} < 1-\theta_{\max}) \leq 1$, i.e., r is sufficiently high or $\theta_{\max}$ is sufficiently low, Proposition 1 continues to hold even if banks observe their types.*

The rest of this section focuses on the more interesting case $\rho(\theta_{\max}) > 1$. We first establish that:

Lemma 4 *Under an optimal disclosure rule:*

1. Every type $\theta \geq 1$ sells its asset with probability 1.

2. Whenever type $\theta \geq 1$ receives score s, the price is $x(s) = \mu(s)$.

3. If the highest type that obtains score s is less than 1, then every type keeps its asset upon obtaining score s.

The idea behind the first part in Lemma 4 is that if a type $\theta \geq 1$ did not sell its asset, the planner could strictly increase the utility of that type, without affecting the utilities of other types, by fully revealing θ's type. Then the market would offer to buy the asset of type θ at a price θ, and type θ would accept the offer.

The second part in Lemma 4 follows from the first part and the observation that the reservation price is increasing in θ. These imply that every type sells its asset upon obtaining score s, and hence selling does not convey any additional information to the market.

The third part in Lemma 4 reflects the fact that if there is no type above 1 that obtains score s, the price $x(s)$ must be less than 1. But then banks will sell only if the price is strictly above their true value. However, this cannot be an equilibrium outcome, since the market would lose money. Note that this result holds under any disclosure rule, not only an optimal one.

For use below, denote the types in Θ by $\theta_{\max} = \theta_1 > \theta_2 > ... > \theta_m = \theta_{\min}$ and suppose that $\theta_k \geq 1 > \theta_{k+1}$, so there are exactly k types at or above 1. Denote $\rho_i = \rho(\theta_i)$.

Denote by S_i the set of scores that type θ_i obtains with a positive probability but higher types do not obtain; that is, $S_i = \{s \in S : g(s|\theta_i) > 0 \text{ and } g(s|\theta') = 0 \text{ for every } \theta' > \theta\}$. From Lemma 3 and Lemma 4, we know that for each $i \in \{1, ..., k\}$ and $s \in S_i$, we must have

$$x(s) = \mu(s) \geq \rho_i. \tag{9}$$

That is, if the highest type that obtains score s is type $\theta_i \geq 1$, the expected cash flow conditional on obtaining score s must be at least as high as type θ_i's reservation price. From equation (2), equation (9) reduces to

$$\sum_{\theta \in \Theta} p(\theta)(\theta - \rho_i) g(s|\theta) \geq 0. \tag{10}$$

Equation (10) is a generalization of the resource constraint (5).

As in Corollary 1, full disclosure is optimal only if there are no types below 1. No disclosure is optimal only if there are sufficient resources, but the condition for no disclosure changes to $E(\tilde{\theta}) \geq \rho_1$, so that equation (9) holds for the highest type.

The rest of this section focuses on the case in which resources are scarce, so the optimal disclosure rule is such that there is at least one type that keeps its asset with a positive probability. A sufficient condition for this to happen is that $E(\tilde{\theta}) < 1$. In this case, all resource constraints are binding. In particular, if the highest type that obtains score s is $\theta_i \geq 1$, the price must equal ρ_i. This means that all lower types that obtain score s also sell for a price ρ_i. An implication of this is that if types $\theta_i > \theta_j \geq 1$ have different reservation prices (which is the case when $\rho_i > 1$), the planner must assign them different scores. Formally,

Proposition 3 *Suppose $E(\tilde{\theta}) < 1$. Under an optimal disclosure rule, types that are above 1 and that have different reservation prices must obtain different scores.*

Intuitively, if types $\theta_i > \theta_j \geq 1$ have different reservation prices but the same score, the sale price depends on the reservation price of the highest type. This means that the lowest type sells the asset for more than its reservation price and, therefore, ends up with more resources than it requires. But this is a waste of resources without any gain. The planner can do better by assigning the lower type its own score, so that this type ends up with less resources. This frees up resources that can be used to subsidize types with $\theta < 1$. This, in turn, increases the probability that these low types invest in their projects.

It follows that when $E(\tilde{\theta}) < 1$, and $\rho_1 > \rho_2 > ... > \rho_k$, the planner must assign at least $k+1$ scores, $s_0, s_1, ..., s_k$, such that for each $i \in \{1, ., , , .k\}$, score s_i pools together type θ_i with a type (or types) that are below 1, and score s_0 pools together only types that are below 1. A bank sells its asset if and only if $s \neq s_0$. When a bank obtains score $s_i \neq s_0$, the bank sells the asset at a price ρ_i. Since $\rho_1 > \rho_2 > ... > \rho_k$, it is natural to think of score s_1 as the highest, score s_2 as the second highest, etc. We can assume, without loss of generality, that scores $s_0, s_1, ..., s_k$ are the only scores.[5]

Next we discuss how the planner should assign scores to types that are below 1; that is, how the planner should pool types that are below 1 with types that are above 1. Suppose first that there is only one type above 1, type θ_1. The analysis is similar to the the one in Section 3, but now the gains-to-cost ratio depends on ρ_1:

$$G_1(\theta) \equiv \frac{\Pr(\tilde{\varepsilon} < 1 - \theta)}{\rho_1 - \theta}. \tag{11}$$

In particular, the gain of pooling type $\theta < 1$ with type $\theta_1 > 1$ is the same as in Section 3, but the cost is higher, since type θ ends up with $\rho_1 > 1$ rather that 1. This reflects the fact that when a low type is pooled with a high type, the market price reflects the reservation price of the highest type.

Suppose now that there are two types that are above 1, $\theta_1 > \theta_2 > 1$. The gain from pooling type $\theta < 1$ with either type θ_1 or type θ_2 is the same. However, the

[5] Lemma A-2 in the appendix provides more details.

cost is different: it is less costly to pool type θ with type θ_2 because then type θ ends up with less resources. The "net" benefit of pooling type θ with type θ_2 rather than with type θ_1 is

$$\frac{G_2(\theta)}{G_1(\theta)} = \frac{\Pr(\tilde{\varepsilon} < 1-\theta)}{\rho_2 - \theta} \frac{\rho_1 - \theta}{\Pr(\tilde{\varepsilon} < 1-\theta)} = \frac{\rho_1 - \theta}{\rho_2 - \theta} > 1. \tag{12}$$

Since the net benefit is higher when θ is higher, the planner would prefer to pool type θ_2 with higher types (among those with $\theta < 1$) and type θ_1 with lower types. Hence, if, for example, $\theta' < \theta'' < 1$, we may obtain an outcome in which type θ' is pooled with type θ_1 and sells its asset for price ρ_1, and type θ'' is pooled with type θ_2 and sells it asset for a price ρ_2. In this case, the lower types sells for a higher price; that is, the lower type obtains a higher score.

The intuition above extends to the case in which there are more than two types above 1. Formally,

Proposition 4 *Suppose $E(\tilde{\theta}) < 1$ and $\theta' < \theta'' < 1$. Under an optimal disclosure rule, if there is a positive probability that type θ' obtains score $s' \neq s_0$ and type θ'' obtains score $s'' \neq s_0$, then the prices must satisfy $x(s'') \leq x(s')$. In other words, among the types $\theta < 1$ that sell their assets, lower types obtain higher scores.*

Propositions 3 and 4 imply that when banks observe their types, the sale price is increasing in type when $\theta > 1$ but decreasing in type when $\theta < 1$. Hence, non-monotonicity is a general feature of optimal disclosure rules. (In contrast, in Section 3, all types that sell their assets sell for the same price, and only the probability of selling the asset could be non-monotone.) The next example illustrates this.

Example 1 Suppose that there are eight types: $\theta_1 > \theta_2 > 1 > \theta_3 > ... > \theta_8$. Suppose that $\rho_1 > \rho_2 \geq 1$ and $E(\tilde{\theta}) < 1$. Then we need at least three scores: s_0, s_1, and s_2. Suppose the gains-to-cost functions that are associated with score s_1 and score s_2 are both increasing in θ; that is, the functions $G_1(\theta) = \frac{\Pr(\tilde{\varepsilon}<1-\theta)}{\rho_1-\theta}$ and

$G_2(\theta) = \frac{\Pr(\tilde{\varepsilon}<1-\theta)}{\rho_2-\theta}$ are both increasing in θ (see Figure 2). Suppose

$$p_2(\theta_2 - \rho_2) = p_3(\rho_2 - \theta_3) + \frac{1}{3}p_4(\rho_2 - \theta_4) \tag{13}$$

$$p_1(\theta_1 - \rho_1) = \frac{2}{3}p_4(\rho_1 - \theta_4) + \frac{1}{5}p_5(\rho_1 - \theta_5) \tag{14}$$

As will become clear, equation (13) is the resource constraint that is associated with score s_2, and equation (14) is the resource constraint that is associated with score s_1.

The optimal disclosure rule is as follows. (Each element in the table is the probability of assigning score s to type θ.)

	θ_8	θ_7	θ_6	θ_5	θ_4	θ_3	θ_2	θ_1
s_1				$\frac{1}{5}$	$\frac{2}{3}$			1
s_2					$\frac{1}{3}$	1	1	
s_0	1	1	1	$\frac{4}{5}$				

To see why, note that since $G_1(\theta)$ and $G_2(\theta)$ are both increasing in θ, score s_0 is given to low types. (Note that since $\rho_1 > \rho_2$, $G_1(\theta)$ is below $G_2(\theta)$ for every $\theta < 1$.) Regarding scores s_1 and s_2, we know from Proposition 3 that with probability 1, type θ_1 obtains score s_1, and type θ_2 obtains score s_2. As for the other types, which are below 1, we know from Proposition 4 that score s_2 is given to higher types compared with score s_1. It then follows from equation (13) that score s_2 is first given to type θ_3. Since there are remaining resource even if type θ_3 obtains score θ_3 with probability 1, score s_2 is also given to type θ_4, but only with probability $\frac{1}{3}$. This exhausts all resources that type θ_2 contributes. Similarly, score s_1 is given to the next highest types until all resources are exhausted. Hence, type θ_4 obtains score s_1 with probability $\frac{2}{3}$ (so that it sells its asset with probability 1), and type θ_5 obtains score s_1 with probability $\frac{1}{5}$, so that the resource constraint (14) is satisfied with equality. All remaining types obtain score s_0. ■

Note that while the sale price in Example 1 is non-monotone in type, the probability of selling the asset is monotone. In particular, as in Corollary 2, there exists a cutoff such that types above the cutoff sell their asset, and types below the cutoff do

not sell. This follows since we assumed in the example that the gains-to-cost function that is associated with each score $s \neq s_0$ is increasing in θ. A sufficient condition for this to happen is that condition 1 holds and ρ_1 is sufficiently low.[6] However, if ρ_1 is sufficiently high, condition 1 implies that the gains-to-cost function $G_1(\theta)$ is decreasing in θ.[7] In this case, there does not exist a cutoff such that types above the cutoff sell and types below the cutoff do not sell. Hence, we obtain two forms of non-monotonicity: First, the probability of selling the price does not increase in type. Second, the sale price does not increase in type. The next example illustrates this.

Example 2 Consider Example 1 but assume that ρ_1 is sufficiently high, so that $G_1(\theta)$ is decreasing in θ. In addition, instead of equation (14), assume that

$$p_1(\theta_1 - \rho_1) = p_8(\rho_1 - \theta_8) + \frac{1}{10} p_7(\rho_1 - \theta_7), \tag{15}$$

which will be the resource constraint that is associated with score s_1. In this case, the optimal disclosure rule is

	θ_8	θ_7	θ_6	θ_5	θ_4	θ_3	θ_2	θ_1
s_1	1	$\frac{1}{10}$						1
s_2					$\frac{1}{3}$	1	1	
s_0		$\frac{9}{10}$	1	1	$\frac{2}{3}$			

In particular, as before, score θ_2 is assigned to type θ_3 and type θ_4, such that the resource constraint (13) is binding. However, since the gains-to-cost function that is associated with score s_1 is decreasing in type, score s_1 is given to the lowest type. Hence, type θ_8 obtains score s_1 with probability 1, and type θ_7 obtains score s_1 with probability $\frac{1}{10}$. Then the resource constraint (15) is satisfied with equality. The remaining score s_0, is given to all remaining types (those in the middle). Hence, the probability of selling the asset $(1 - s_0)$ is non-monotone. ■

[6] To see that, note that $G_i(\theta)$ increases when $\theta < 1$ if and only if $F(\varepsilon)/(\varepsilon + \rho_i - 1)$ is decreasing when $\varepsilon > 0$, or equivalently, if for every $\varepsilon > 0$, $\frac{F(\varepsilon)}{F'(\varepsilon)} > \varepsilon + \rho_i - 1$. By continuity, if ρ_i is sufficiently small ($\rho_i \downarrow 1$), condition 1 implies $\frac{F(\varepsilon)}{F'(\varepsilon)} > \varepsilon + \rho_i - 1$.

[7] In particular, $\frac{F(\varepsilon)}{F'(\varepsilon)} < \varepsilon + \rho_i - 1$ for every $\varepsilon > 0$, so $G_1(\theta)$ is decreasing when $\theta < 1$.

5 Conclusion

Our paper provides a model of an optimal disclosure policy of a regulator, who has information about banks (e.g., the regulator has conducted stress tests). The regulator's disclosure policy affects whether banks can take corrective actions, particularly whether banks can engage in risk-sharing arrangements to protect themselves against the possibility that their future capital falls below some critical level. We show that during normal times, no disclosure is necessary, but during bad times, partial disclosure is needed. Partial disclosure takes the form of different scores pooling together banks of different levels of strength. Two scores are sufficient if banks do not have the information that the regulator has. In this case, the optimal disclosure rule may take a simple form, such that banks whose forecasted capital is below some threshold obtain the low score and banks whose forecasted capital is above the threshold obtain the high score; we provide conditions for this to happen. More than two scores may be needed if a bank shares the same information that the regulator has about the bank. In this case, the optimal disclosure rule is non-monotone: among the strong banks, the stronger banks obtain higher scores, but among the weak banks that are pooled with strong banks, the weaker banks obtain higher scores.

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Appendix

Proof of Lemma 1. From the text, the equilibrium price is $x(s) = \mu(s)$. If the bank sells the asset at price $\mu(s)$, its final payoff is $R(\mu(s))$. If the bank keeps the asset, its (expected) final payoff, conditional on its information, is $E[R(\tilde{\theta} + \tilde{\varepsilon}|\tilde{s} = s)] = \mu(s) + r\Pr(\tilde{\theta} + \tilde{\varepsilon} \geq 1|\tilde{s} = s)$. Hence, if $\mu(s) \geq 1$, it is optimal to sell, since $R(\mu(s)) = \mu(s) + r > E[R(\tilde{\theta} + \tilde{\varepsilon}|\tilde{s} = s)]$. If $\mu(s) < 1$, it is optimal to keep the asset, since $R(\mu(s)) = \mu(s) < E[R(\tilde{\theta} + \tilde{\varepsilon}|\tilde{s} = s)]$. The strict inequality follows from Assumption 1. Q.E.D.

Proof of Lemma 2. The planner's problem is to find a disclosure rule (S, g) to maximize $\sum_{\theta\in\Theta} p(\theta)u(\theta)$. Since equation (3) reduces to

$$u(\theta) = \sum_{s:\mu(s)<1} [\theta + r\Pr(\tilde{\varepsilon} \geq 1 - \theta)]g(s|\theta) + \sum_{s:\mu(s)\geq 1} [\mu(s) + r]g(s|\theta),$$

it follows that:

$$\begin{aligned}\sum_{\theta\in\Theta} p(\theta)u(\theta) &= \sum_{\theta\in\Theta} p(\theta) \sum_{s:\mu(s)<1} \theta g(s|\theta) + \sum_{\theta\in\Theta} p(\theta) \sum_{s:\mu(s)<1} r\Pr(\tilde{\varepsilon} \geq 1 - \theta)g(s|\theta) \\ &\quad + \sum_{\theta\in\Theta} p(\theta) \sum_{s:\mu(s)\geq 1} \mu(s)g(s|\theta) + \sum_{\theta\in\Theta} p(\theta) \sum_{s:\mu(s)\geq 1} rg(s|\theta).\end{aligned}$$

The sum of the first and third terms in the right-hand-side of the equation above reduces to $E(\tilde{\theta})$, as follows:

$$\begin{aligned}&\sum_{\theta\in\Theta} p(\theta) \sum_{s:\mu(s)<1} \theta g(s|\theta) + \sum_{\theta\in\Theta} p(\theta) \sum_{s:\mu(s)\geq 1} \mu(s)g(s|\theta) \\ =\ &\sum_{\theta\in\Theta} \theta p(\theta) \sum_{s:\mu(s)<1} g(s|\theta) + \sum_{s:\mu(s)\geq 1} \mu(s) \sum_{\theta\in\Theta} p(\theta)g(s|\theta) \\ =\ &\sum_{\theta\in\Theta} \theta p(\theta) \sum_{s:\mu(s)<1} g(s|\theta) + \sum_{s:\mu(s)\geq 1} \sum_{\theta\in\Theta} \theta p(\theta)g(s|\theta) \\ =\ &\sum_{\theta\in\Theta} \theta p(\theta) \sum_{s:\mu(s)<1} g(s|\theta) + \sum_{\theta\in\Theta} \theta p(\theta) \sum_{s:\mu(s)\geq 1} g(s|\theta) = E(\tilde{\theta}),\end{aligned}$$

where the third line follows from equation (2). Hence,

$$\begin{aligned}\sum_{\theta\in\Theta} p(\theta)u(\theta) &= E(\tilde{\theta}) + \sum_{\theta\in\Theta} p(\theta) r \Pr(\tilde{\varepsilon} \geq 1-\theta) \sum_{s:\mu(s)<1} g(s|\theta) + r \sum_{\theta\in\Theta} p(\theta) \sum_{s:\mu(s)\geq 1} g(s|\theta) \\ &= E(\tilde{\theta}) + \sum_{\theta\in\Theta} p(\theta) r \Pr(\tilde{\varepsilon} \geq 1-\theta)[1-h(\theta)] + r \sum_{\theta\in\Theta} p(\theta) h(\theta) \\ &= E(\tilde{\theta}) + \sum_{\theta\in\Theta} p(\theta) r \Pr(\tilde{\varepsilon} \geq 1-\theta) + r \sum_{\theta\in\Theta} p(\theta)[1-\Pr(\tilde{\varepsilon} \geq 1-\theta)] h(\theta)\end{aligned}$$

Hence,

$$\sum_{\theta\in\Theta} p(\theta)u(\theta) = E(\tilde{\theta}) + r \sum_{\theta\in\Theta} p(\theta) \Pr(\tilde{\varepsilon} \geq 1-\theta) + r \sum_{\theta\in\Theta} p(\theta) \Pr(\tilde{\varepsilon} < 1-\theta) h(\theta) \quad \text{(A-1)}$$

The first two terms in the right-hand side of (A-1) are exogenous and are not affected by the planner's disclosure rule. Only the third term is endogenous and affected by the planner's disclosure rule. Hence, maximizing $\sum_{\theta\in\Theta} p(\theta)u(\theta)$ is equivalent to maximizing (4).

From Lemma A-1 below, we can focus, without loss of generality, on disclosure rules with only two scores, s_0, and s_1, such that $\mu(s_0) < 1$ and $\mu(s_1) \geq 1$. From Lemma 1, we know that $h(\theta) = g(s_1|\theta)$. Hence, the relevant constraint is $\mu(s_1) \geq 1$. From equation (2), the constraint $\mu(s_1) \geq 1$ reduces to $\sum_{\theta\in\Theta} p(\theta)(\theta-1)g(s_1|\theta) \geq 0$, which is equivalent to constraint (5). Q.E.D.

Lemma A-1 *Assume that the bank does not observe its type. For every disclosure rule (S, g), we can construct a disclosure rule that induces the same probability that a bank of type θ sells its asset (i.e., $h(\theta)$) but that uses only two scores, s_0, s_1, such that $\mu(s_0) < 1$ and $\mu(s_1) \geq 1$.*

Proof of Lemma A-1. For a given disclosure rule (S, g), define a disclosure rule $(\tilde{S}, \tilde{g})$, such that $\tilde{S} = \{s_0, s_1\}$ and such that for every $\theta \in \Theta$, $\tilde{g}(s_0|\theta) = \sum_{s:\mu(s)<1} g(s|\theta)$ and $\tilde{g}(s|\theta) = \sum_{s:\mu(s)\geq 1} g(s|\theta)$. From Lemma 1, we need to show that $\mu_{\tilde{g}}(s_1) \geq 1$ and $\mu_{\tilde{g}}(s_0) < 0$, where the subscript $\tilde{g}$ indicates that the expected values are given

disclosure rule $(\tilde{S}, \tilde{g})$. To see why $\mu_{\tilde{g}}(s_1) \geq 1$, observe that:

$$\begin{aligned}
\mu_{\tilde{g}}(s_1) &= \frac{\sum_{\theta\in\Theta} \theta p(\theta)\tilde{g}(s_1|\theta)}{\sum_{\theta\in\Theta} p(\theta)\tilde{g}(s_1|\theta)} = \frac{\sum_{\theta\in\Theta} \theta p(\theta) \sum_{s:\mu(s)\geq 1} g(s|\theta)}{\sum_{\theta\in\Theta} p(\theta) \sum_{s:\mu(s)\geq 1} g(s|\theta)} \\
&= \frac{\sum_{s:\mu(s)\geq 1} \sum_{\theta\in\Theta} \theta p(\theta) g(s|\theta)}{\sum_{s:\mu(s)\geq 1} \sum_{\theta\in\Theta} p(\theta) g(s|\theta)} = \frac{\sum_{s:\mu(s)\geq 1} \mu(s) \sum_{\theta\in\Theta} p(\theta) g(s|\theta)}{\sum_{s:\mu(s)\geq 1} \sum_{\theta\in\Theta} p(\theta) g(s|\theta)} \\
&\geq \frac{\sum_{s:\mu(s)\geq 1} \sum_{\theta\in\Theta} p(\theta) g(s|\theta)}{\sum_{s:\mu(s)\geq 1} \sum_{\theta\in\Theta} p(\theta) g(s|\theta)} = 1,
\end{aligned}$$

where the first and fourth equalities follow from equation (2) and the second equality follows from the definition of $\tilde{g}$. Similarly, we can show that $\mu_{\tilde{g}}(s_0) < 1$. Q.E.D.

Proof of Proposition 1.

Part (A): Assigning $h(\theta) = 1$ for every $\theta \in \theta$ achieves the maximal attainable value for the objective function and satisfies the planner's resource constraints. Any other disclosure rule reduces the value of the objective function, by Assumption 1.

Part (B): First, by Assumption 1, it is clearly (uniquely) optimal to set $h(\theta) = 1$ for every $\theta \geq 1$. In addition, if $h(b_j) > 0$ for some j, it is optimal to set $h(b_i) = 1$ for every $i < j$. To see why, suppose, by contradiction, that under an optimal disclosure rule there exists $i < j$, such that $h(b_j) > 0$ but $h(b_i) < 1$. Consider a small $\Delta > 0$, let $\Delta' = \frac{P(b_i)}{P(b_j)} \frac{1-b_i}{1-b_j} \Delta$, and consider an alternate disclosure rule in which we increase $h(b_i)$ by Δ and reduce $h(b_j)$ by Δ'. We obtain a contradiction to the optimality of the original by showing that the alternate rule increases the value of the objective function without violating the resource constraint. In particular, since type b_i has a higher gains-to-cost ratio than type b_j, it follows that $\Delta P(b_i) \Pr(\tilde{\varepsilon} < 1 - b_i) > \Delta \frac{P(b_i)}{P(b_j)} \frac{1-b_i}{1-b_j} P(b_j) \Pr(\tilde{\varepsilon} < 1 - b_j)$, and so the alternate rule increases the value of the objective function. In addition, since $\Delta P(b_i)(b_i - 1) = \Delta \frac{P(b_i)}{P(b_j)} \frac{1-b_i}{1-b_j} P(b_j)(b_j - 1)$, the resource constraint remains unchanged.

Since $\theta_{\max} > 1$, the resource constraint is slack if $h(\theta) = 0$ for every $\theta < 1$. Hence, under the optimal disclosure rule, there exists i, such that $h(b_j) > 0$. Denote the

lowest such j by j^*. Then $h(b_i) = 0$ when $i > j^*$, and it follows from above that $h(b_i) = 1$ when $i < j^*$. Finally, note that if $j^* \neq l^*$, it is possible to increase the objective function without violating the constraint. Q.E.D.

Proof of Corollary 1.

Part 1: Under full disclosure, type θ is offered a price θ, and hence, type θ sells its asset if and only if $\theta \geq 1$ (Lemma 1). Hence, under full disclosure, $h(\theta) = 1$ if and only if $\theta \geq 1$. If $\theta_{\min} \geq 1$, then $E(\tilde{\theta}) \geq 1$ and full disclosure is optimal by the first part of Proposition 1. If $\theta_{\min} < 1$, then either $E(\tilde{\theta}) \geq 1$, and full disclosure is suboptimal by the first part of Proposition 1, or else $E(\tilde{\theta}) < 1$ and full disclosure is suboptimal by the second part of Proposition 1. In particular, under full disclosure, $h(\theta) = 0$, for every $\theta < 1$, while under the optimal disclosure rule, there must exist $\theta' > 0$, such that $h(\theta') > 0$. The last statement follows since $\theta_{\max} > 1$.

Part 2: Under no disclosure, every bank is offered a price $E(\tilde{\theta})$. Hence, it follows from Lemma 1 that under no disclosure, the bank will sell its asset if and only if $E(\tilde{\theta}) \geq 1$. Hence, if $E(\tilde{\theta}) \geq 1$, we know from the first part of Proposition 1 that no disclosure is optimal. If $E(\tilde{\theta}) < 1$, we know from the second part of Proposition 1 that no disclosure is suboptimal because under the optimal disclosure rule, at least some banks sell (since $\theta_{\max} > 1$.) Q.E.D.

Proof of Corollary 2. From Proposition 1, it is sufficient to show that if condition 1 holds, $G(\theta) = \frac{F(1-\theta)}{1-\theta}$ is increasing in θ whenever $\theta < 1$. Denote $\varepsilon = 1 - \theta$. Then we need to show that $\frac{F(\varepsilon)}{\varepsilon}$ is decreasing in ε whenever $\varepsilon > 0$. This follows from condition 1. Q.E.D.

Proof of Lemma 3. Suppose a bank is offered a price x, and the bank knows that it is type θ. If the bank sells the asset, it obtains $R(x)$. If the bank keeps the asset, it obtains $E[R(\theta + \tilde{\varepsilon})]$. Hence, the bank sells if and only if

$$R(x) \geq E[R(\theta + \tilde{\varepsilon})]. \tag{A-2}$$

Observe that $E[R(\theta+\tilde{\varepsilon})] = \theta + r\Pr(\tilde{\varepsilon} \geq 1-\theta)$, and $R(x) = \begin{cases} x+r \text{ if } x \geq 1 \\ x \text{ if } x < 1 \end{cases}$. Hence, if $\theta \geq 1$, then $E[R(\theta+\tilde{\varepsilon})] \geq 1$, and so equation (A-2) can hold only if $x \geq 1$. In this case, equation (A-2) reduces to $x + r \geq \theta + r\Pr(\tilde{\varepsilon} \geq 1-\theta)$, which reduces to $x \geq \theta - r\Pr(\tilde{\varepsilon} < 1-\theta)$. If instead $\theta < 1$, then whenever $x \geq 1$, we clearly have $E[R(\theta+\tilde{\varepsilon})] < x + r$, so equation (A-2) holds; and if $x < 1$, equation (A-2) reduces to $x \geq \theta + r\Pr(\tilde{\varepsilon} \geq 1-\theta)$. Q.E.D.

Proof of Proposition 2. First observe that since $\theta_{\max} > 1$, the condition $\theta_{\max} - r\Pr(\tilde{\varepsilon} < 1-\theta_{\max}) \leq 1$ is equivalent to $\rho(\theta_{\max}) = 1$. Since $\rho(\theta)$ is increasing in θ, every type will agree to sell a price 1.

Consider any disclosure rule (g, S). If $\mu(s) \geq 1$, the market price will be $x(s) = \mu(s)$, and every type will sell. If $\mu(s) < 1$, the price must be below 1, since otherwise everyone will sell, and the market will lose money. But then only types below 1 may sell, and the proof of Part 3 in Lemma 4 implies that in equilibrium, no type sells upon receiving score s. Hence, Lemma 1 continues to hold, and the bank's ex-ante expected utility given disclosure rule (g, S) is the same as in Section 3. Hence, Proposition 1 continues to hold. Q.E.D.

Proof of Lemma 4

Part 1. The proof is by contradiction. Consider an optimal disclosure rule (S, g) and suppose there exists a type $\theta' \geq 1$ and a score $s' \in S$, such that $g(s'|\theta') > 0$ and such that type θ' does not sell its asset upon obtaining score s'.

Consider an alternate disclosure rule $(\tilde{S}, \tilde{g})$, in which we add a score $\tilde{s} \notin S$ that type θ' obtains instead of score s'. Specifically, $\tilde{S} = S \cup \{\tilde{s}\}$ and $\tilde{g}(s|\theta) = \begin{cases} g(s|\theta) \text{ if } \theta \neq \theta' \text{ and } s \neq s' \\ 0 \text{ if } \theta = \theta' \text{ and } s = s' \\ g(s|\theta) \text{ if } \theta = \theta' \text{ and } s = \tilde{s}. \end{cases}$. Under the alternate rule, the only type that obtains score $\tilde{s}$ is θ'. Hence, $x(s') = \theta'$. Since $\rho(\theta') \leq \theta'$, type θ' sells its asset upon obtaining score $\tilde{s}$. Hence, the alternate rule increases the probability that type θ invests in its project, while keeping the probabilities that each of the other types invests un-

changed. Hence, the alternate rule increases the bank's ex ante expected utility. But this contradicts the optimality of the original disclosure rule (S, g).

Part 2. Consider an optimal disclosure rule (S, g) and suppose there exist a type $\theta \geq 1$ and a score $s \in S$, such that $g(s|\theta) \geq 0$. From part 1, we know that type θ sells the asset upon obtaining score s. Hence, $\rho(\theta) \leq x(s)$. From part 1, we also know that every type $\theta' > \theta$ such that $g(s|\theta') > 0$ sells. Finally, every type $\theta' < \theta$ such that $g(s|\theta') > 0$ sells, since $\rho(\theta') < \rho(\theta) \leq x(s)$. Hence, every type that obtains score s sells the asset upon obtaining the score. Consequently, selling does not convey any additional information to the market, and the market sets a price $x(s) = \mu(s)$, which is based only on the information that is contained in the score.

Part 3. The proof is by contradiction. (Note that it applies to the equilibrium that is induced by any disclosure rule, not necessarily the optimal.) Suppose that the highest type that obtains score s is less than 1 (that is, $g(s|\theta) = 0$ for every $\theta \geq 1$), and suppose that the equilibrium that is induced by disclosure rule g is such that some types sell upon obtaining score s. Denote the highest type that sells by θ'. ($\theta' < 1$.) The sale price must satisfy $x(s) \leq \theta'$, so that the market expected profits are non-negative. Since $\theta' < \rho(\theta') \leq 1$, we obtain that $x(s) < \rho(\theta')$. But this contradicts the fact that type θ' sells. Q.E.D.

Lemma A-2 *Assume that the bank observes its type. For every disclosure rule (S, g) that is optimal, we can construct a disclosure rule that induces the same probability that a bank of type θ sells its asset (and hence, is also optimal) but that uses at most $k+1$ scores, $s_0, s_1, s_2, ..., s_k$ such that when $s_i \neq s_0$, the highest type that obtains score s_i is type θ_i.*

Proof of Lemma A-2 Suppose (S, g) is an optimal disclosure rule. For every $i \in \{1, ..., k\}$, define $S_i = \{s : \mu(s) \in [\rho_i, \rho_{i-1})\}$, where $\rho_0 = \infty$. Let $(\tilde{S}, \tilde{g})$ be a

disclosure rule with $k+1$ scores $\tilde{S} = \{s_0, s_1, s_2, ..., s_k\}$, such that for every $\theta \in \Theta$,

$$\tilde{g}(s_i|\theta) = \begin{cases} \sum_{s \in S_i} g(s|\theta) & \text{if } i \in \{1, 2, .., k\} \\ \sum_{s \notin \cup_{i=1}^{k} S_i} g(s|\theta) & \text{if } i = 0 \end{cases}$$

Under disclosure rule (S, g), type $\theta_i \geq 1$ sells the asset upon obtaining score s if and only if $\mu(s) \geq \rho_i$. This happens with probability $\sum_{j=1}^{i} \sum_{s \in S_j} g(s|\theta)$. Type $\theta < 1$ sells if and only if $\mu(s) \geq \rho_k$, which happens with probability $\sum_{j=1}^{k} \sum_{s \in S_j} g(s|\theta)$. Following similar steps as in the proof of Lemma A-1, we obtain that (i) $\mu_{\tilde{g}}(s_0) < \rho_k$, and (ii) for every $i \in \{1, 2, ..., k\}$, $\mu_{\tilde{g}}(s_i) \in [\rho_i, \rho_{i-1})$. Hence, the probability that type θ sells the asset under disclosure rule $(\tilde{S}, \tilde{g})$ is the same as under disclosure rule (S, g). Q.E.D.

Lemma A-3 *Suppose banks know their types. For $i \in \{1, ..., k\}$, denote $h_i(\theta) = \sum_{s \in S_i} g(s|\theta)$. The planner's problem reduces to finding a set of functions $\{h_i : \Theta \longrightarrow [0, 1]\}_{i=1,...,k}$ to maximize*

$$\sum_{\theta \in \Theta} p(\theta) \Pr(\tilde{\varepsilon} < 1 - \theta) \sum_{i=1}^{k} h_i(\theta), \tag{A-3}$$

such that the following constraints hold:

(i) For every type $\theta \in \Theta$,

$$\sum_{i=1}^{k} h_i(\theta) \leq 1. \tag{A-4}$$

(ii) For every $i \in \{1, ..., k\}$,

$$\sum_{\theta \in \Theta} p(\theta)(\theta - \rho_i) h_i(\theta) \geq 0. \tag{A-5}$$

(iii) For every $i \in \{1, ..., k\}$, $h_i(\theta) = 0$ if $\theta > \theta_i$.

Proof of Lemma A-3. Maximizing the bank's ex-ante expected utility $\sum_{\theta \in \Theta} p(\theta) u(\theta|g)$ is equivalent to maximizing (A-3). (The proof is an extension of the proof of Lemma

2. More details to be added.) The first constraint says that the probability that a bank obtains a score $s \in \cup_{i=1}^{k} S_i$ is at most 1. The second constraint follows by summing the resource constraints for each $s \in S_i$. The third constraint follows from the definition of S_i. Q.E.D.

Lemma A-4 *If $E(\tilde{\theta}) < 1$, there must be a type $\theta' < 1$ that keeps its asset (i.e., obtains score s_0) with a positive probability.*

Proof of Lemma A-4. The proof is by contradiction. Consider the planner's problem in Lemma A-3. Suppose that no type obtains score s_0 with a positive probability; that is, $\sum_{i=1}^{k} h_i(\theta) = 1$ for every type $\theta \in \Theta$. Then since $\rho_i \geq 1$ for every $k \geq 1$, it follows that $\sum_{i=1}^{k} \sum_{\theta \in \Theta} p(\theta)(\theta - \rho_i) h_i(\theta) \leq \sum_{i=1}^{k} \sum_{\theta \in \Theta} p(\theta)(\theta - 1) h_i(\theta) = \sum_{\theta \in \Theta} p(\theta)(\theta - 1) \sum_{i=1}^{k} h_i(\theta) = E(\tilde{\theta}) - 1 < 0$. However, summing up all k resource constraints, we obtain $\sum_{i=1}^{k} \sum_{\theta \in \Theta} p(\theta)(\theta - \rho_i) h_i(\theta) \geq 0$. Hence, a contradiction. Q.E.D.

Lemma A-5 *If $E(\tilde{\theta}) < 1$, then under an optimal disclosure rule, all resource constraints are binding.*

Proof of Lemma A-5. The proof is by contradiction. Suppose (S, g) is an optimal disclosure rule and suppose there exists a score s, such that the highest type that obtains score s is θ_i and such that the resource constraint that is associated with score s is not binding; that is, $\sum_{\theta \in \Theta} p(\theta)(\theta - \rho_i) g(s|\theta) > 0$. Since $E(\tilde{\theta}) < 1$, we know from Lemma A-4 that there exists type $\theta' < 1$ that obtains score s_0 with a positive probability. Consider an alternate disclosure rule in which the planner reduces the probability that type θ' obtains score s_0 by a small Δ and increases the probability that type θ' obtains score s by Δ. The alternate rule increases the value of the objective function without violating any of the constraints. But this contradicts the optimality of the original disclosure rule. Q.E.D.

Proof of Proposition 3. Consider the planner's problem in Lemma A-3. We can assume, without loss of generality, that $\rho_1 > \rho_2 > ... > \rho_k$. We want to show that if $E(\tilde{\theta}) < 1$, then $h_i(\theta_i) = 1$ for every $i \in \{1, ..., k\}$. The proof is by contradiction. Suppose there exists $i \in \{1, ..., k\}$, such that $h_i(\theta_i) < 1$. From Lemma 4, we know that θ_i sells its asset with probability 1. Hence, there must be $j < i$, such that $h_j(\theta_i) > 0$. We obtain a contradiction by showing that there is an alternate solution that increases the value of the objective function in Lemma A-3 without violating the constraints.

Case 1: $\rho_j \geq \theta_i$. Consider alternating the original solution as follows: Reduce $h_j(\theta_i)$ by a small amount Δ and increase $h_i(\theta_i)$ by the same amount. Since $\rho_i \leq \theta_i$, increasing $h_i(\theta_i)$ weakly relaxes the resource constraint i, and since $\rho_j \geq \theta_i$, reducing $h_j(\theta_i)$ weakly relaxes the resource constraint j. In addition, at least one of these two constraints is strictly relaxed: if $\theta_i = 1$, then $\rho_j > \theta_i$, and constraint j is strictly relaxed; otherwise $\rho_i < \theta_i$, and constraint i is strictly relaxed. Finally, the value of the objective function and all other constraints remain unchanged. But this contradicts Lemma A-5.

Case 2: $\rho_j < \theta_i$. In this case, θ_i adds resources to the resource constraint j, and reducing $h_j(\theta_i)$ tightens the constraint. Since the resource constraint j is binding (Lemma A-5), there must be a type $\theta'' < \rho_j$, such that $h_j(\theta'') > 0$; this type takes resources from constraint j. Fix a small $\Delta > 0$ and let $\Delta' = \frac{p(\theta_i)(\theta_i - \rho_j)}{p(\theta'')(\rho_j - \theta'')}\Delta$; observe that $\Delta' > 0$. Consider an alternate solution in which for type θ_i, we reduce $h_j(\theta_i)$ by Δ but increase $h_i(\theta_i)$ by Δ, and for type θ'', we reduce $h_j(\theta'')$ by Δ' but increase $h_i(\theta'')$ by Δ'. Under the alternate rule, the probability that each type sells its asset remains unchanged, so the objective function remains unchanged. The resource constraint j remains unchanged since $-p(\theta_i)(\theta_i - \rho_j)\Delta - p(\theta'')(\theta'' - \rho_j)\Delta' = 0$. In contrast, since

$\rho_j > \rho_i$ (as $j < i$), the resource constraint i is loosened:

$$\begin{aligned}
p(\theta_i)(\theta_i - \rho_i)\Delta + p(\theta'')(\theta'' - \rho_i)\Delta' &= p(\theta_i)(\theta_i - \rho_i)\Delta + p(\theta'')(\theta'' - \rho_i)\frac{p(\theta_i)(\theta_i - \rho_j)}{p(\theta'')(\rho_j - \theta'')}\Delta \\
&> p(\theta_i)(\theta_i - \rho_i)\Delta + p(\theta'')(\theta'' - \rho_j)\frac{p(\theta_i)(\theta_i - \rho_j)}{p(\theta'')(\rho_j - \theta'')}\Delta \\
&= p(\theta_i)(\theta_i - \rho_i)\Delta - p(\theta_i)(\theta_i - \rho_j)\Delta \\
&= p(\theta_i)(\rho_j - \rho_i)\Delta > 0.
\end{aligned}$$

All other constraints remain unchanged. But this contradicts Lemma A-5. Q.E.D.

Proof of Proposition 4. Consider the planner's problem in Lemma A-3. The proof is by contradiction. Suppose $(h_i)_{i=1,\ldots,k}$ is an optimal solution, such that $h_i(\theta) > 0$ for some type $\theta < 1$, and suppose, by contradiction, that there exists $\theta' < \theta$ and $j > i$, such that $h_j(\theta') > 0$. Assume, without loss of generality, that $\rho_j < \rho_i$.

Fix a small $\Delta > 0$, and let $\Delta' = \frac{p(\theta)(\theta-\rho_i)}{p(\theta')(\theta'-\rho_i)}\Delta$; observe that $\Delta' > 0$. Consider alternating the original solution as follows: For type θ, reduce $h_i(\theta)$ by Δ and increase $h_j(\theta)$ by Δ. For type θ', reduce $h_j(\theta')$ by Δ' and increase $h_i(\theta')$ by Δ'. Under the alternate rule, the probability that each type sells its asset remains unchanged, so the objective function remains unchanged. The resource constraint i remains unchanged since $-\Delta p(\theta)(\theta - \rho_i) + \Delta' p(\theta')(\theta' - \rho_i) = 0$. The resource constraint j is loosened since

$$\begin{aligned}
\Delta p(\theta)(\theta - \rho_j) - \Delta' p(\theta')(\theta' - \rho_j) &= \Delta p(\theta)(\theta - \rho_j) - \Delta\frac{p(\theta)(\theta - \rho_i)}{(\theta' - \rho_i)}(\theta' - \rho_j) \\
&= \Delta p(\theta)[(\theta - \rho_j) - \frac{(\theta - \rho_i)}{(\theta' - \rho_i)}(\theta' - \rho_j)] \\
&= \Delta p(\theta)\frac{(\theta - \rho_j)(\theta' - \rho_i) - (\theta - \rho_i)(\theta' - \rho_j)}{(\theta' - \rho_i)} \\
\Delta p(\theta)\frac{\left(\rho_i - \rho_j\right)(\theta' - \theta)}{(\theta' - \rho_i)} &> 0,
\end{aligned}$$

where the inequality follows since $\rho_i > \rho_j \geq 1 > \theta > \theta'$. All other constraints remain unchanged. So the alternate solution gives the same value for the objective but relaxes one of the resource constraints. But this contradicts Lemma A-5. Q.E.D.

Figure 1: The figure illustrates the reservation price $\rho(\theta)$ as a function of θ.

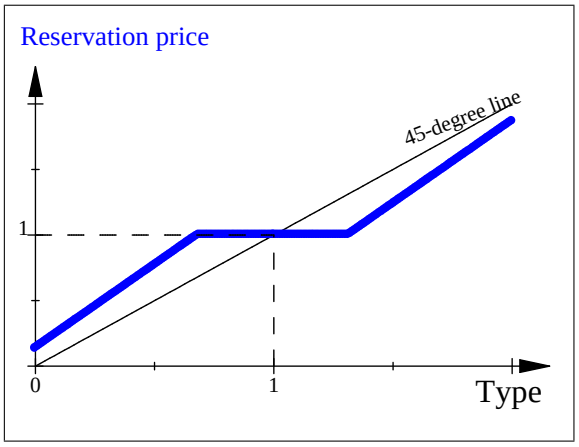
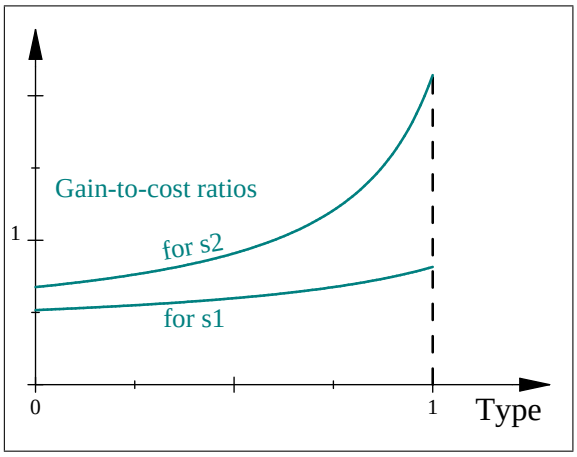
Figure 2: The figure illustrates the gain-to-cost functions that are associated with the highest score s_1 and the second highest score s_2.