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COMPETITIVE THEORIES FOR ECONOMIES WITH GENERAL TRANSACTIONS TECHNOLOGY

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ABSTRACT

In this paper, we describe and compare two approaches to analyzing transactions costs in a general equilibrium setting. In the first approach, which we label the *transactions costs* approach, the commodity space is the same as that used in models without transactions costs. In the second approach, which we label the *valuation equilibrium* approach, the commodity space is chosen so that the exchange problem can be formulated as an instance of the abstract exchange model described in Debreu (1954). We argue that the valuation equilibrium approach provides a tractable framework for quantitative studies of the effects of transactions costs on economy-wide resource allocation.

Motivation

There are many activities that we think facilitate exchange between decision-making units (individuals and firms). The variety of tasks performed by workers in a bank, both in providing payment services and in matching borrowers and lenders, is a good example of such activities. If a change in regulation or technology changes the cost of performing these tasks, we might wish to know how much the community gains or loses as a consequence. Unfortunately, getting an answer to these sorts of questions is more complicated than usual because standard economic theory ignores costs of exchange.

In this paper, we examine an environment in which it takes resources to reallocate goods across individuals. It turns out that going from the description of the environment to representation of the environment as a competitive economy is not straightforward. In particular, there are (at least) two different ways of formulating the associated exchange problem, and each has different consequences for the nature of the associated competitive equilibria. This paper is wholly concerned with a statement of these two approaches and a comparison of the type of results available for each.

In section I, we describe an environment with a transactions technology. In section II, we formulate the exchange problem for this environment, assuming that individuals trade in competitive markets in each of the goods. This formulation, which we label the *transactions costs approach*, is closely related to that of Kurz (1974) and Heller and Starr (1976) and is one way of dealing with transactions costs in a general equilibrium setting. In section III, we reformulate the exchange problem as one in which individuals trade lotteries defined over the space of goods rather than the goods themselves. So reformulated, the exchange problem turns out to be a special case of Debreu's (1954) valuation equilibrium. For this reason, we call this formulation the *valuation equilibrium approach.*[1] In section IV, we present an example to show that the predictions of the two formulations do not, in general, coincide. Section V gives some numerical examples illustrating the valuation equilibrium approach. Section VI concludes.

I. The Economic Environment

There are a finite number of commodities indexed by n =1,2,...,N and a finite number of consumer-types indexed by i =1,2,...,I. The measure of consumer-type i is λ^i and $\Sigma_i\lambda^i = \lambda$. The utility function of consumer-type i is $u^i(c):R^N_+ \rightarrow R$, where $c=(c_1,c_2,...,c_N)$ is a non-negative vector of commodities consumed. The functions u^i are assumed to be strictly increasing and strictly concave. Each consumer-type has a non-negative endowment vector of commodities $\omega^i = (\omega^i_1,\omega^i_2,...,\omega^i_N)$.

There is a technology that permits transactions in goods between consumers. Suppose that a consumer of type i wishes to consume a vector of goods c. Let the N-vector of goods he delivers to some set of other consumers be given by $d = \max\{\omega^i - (c_1,c_2,...,c_N), 0\}$ and the N-vector that he receives from some set of other consumers be given by $r = \max\{(c_1,c_2,...,c_N) - \omega^i, 0\}$. The transactions technology is described by a function $t(d,r)$: $R^N_+ \times R^N_+ \rightarrow R^N_+$, which specifies the minimum input requirements for a consumer of type i to make the delivery d and absorb the receipt r. If we allow one of the goods to be the consumer’s time, the time-costs of carrying out different types of transactions are part of this environment. For the type of

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[1] In our earlier paper (Chatterjee and Corbae (1995)) we noted that the abstract exchange model described in Debreu (1954) can be used to analyze environments with (possibly) non-convex transactions technologies. This paper provides a more detailed comparison of the two different approaches to modelling transactions costs in a general equilibrium setting.

applications we have in mind, it is important that t encompass *fixed* input requirements for transactions. That is, the set $T \equiv \{(d,r,f): t(d,r) \le f\} \subset R^N_+ \times R^N_+ \times R^N_+$ is permitted to be non-convex.

As an example of this transactions technology, consider the case where N=2 and interpret goods as dated consumption of a single good. Then, part of what T describes is the (possibly fixed) input requirements for making or taking out a loan.

II. The Transactions Cost (TC) Formulation

In this section, we formulate the exchange problem for our environment in the standard way and adopt the commodity space used in general equilibrium models *without* transactions technologies. In particular, we assume that there are competitive markets in each of the N goods. Unlike the standard general equilibrium model, the presence of the transactions technology implies that it is costly to participate in these markets. Since costs of using markets is how most economists define transactions costs, we call this the *transactions costs approach.*

Following Heller and Starr, let b^i_n (s^i_n) denote the non-negative quantity purchased (sold) of good n by an agent of type i, with $b^i=(b^i_1,b^i_2,...,b^i_N)$ and $s^i=(s^i_1,s^i_2,...,s^i_N)$. In the terminology of the previous section, s^i is a vector of deliveries and b^i is a vector of receipts. Let f^i_n denote the non-negative quantity of good n used by an agent of type i as an input in the transactions technology, with $f^i = (f^i_1,f^i_2,...,f^i_N)$. Finally, let p_n denote the price of consumption good n, with $p = (p_1,p_2,...,p_N)$. Then, each agent type maximizes his utility subject to the following constraints:

(2.1) $c^i = \omega^i + b^i - s^i - f^i \ge 0.$

(2.2) $f^i - t(s^i,b^i) \geq 0$.

(2.3) $p(b^i - s^i) \leq 0$.

The demand correspondence of agent type i, $\gamma(p,i)$, is the set of (b^i, s^i, f^i), which maximizes $u(c^i)$ subject to the constraints imposed by (2.1) - (2.3). Even though $u^i(c)$ are strictly concave, the possible non-concavity of t implies that $\gamma(p,i)$ could contain more than one element and it need not be convex.

Since we have a continuum of agents, we use the approach in Hildenbrand (1974) to state the conditions for a competitive equilibrium. Let $E = [0,\lambda]$ and let μ be the Lebesgue measure. Let $h : E \rightarrow \{1,2,...,I\}$ be some Borel measurable function that assigns each agent e to some type. Since an agent e is of type h(e), the demand correspondence of this agent is $\gamma(p,h(e))$.

Definition. A *competitive equilibrium with transaction costs* is a price system $p^*>0$, and a triplet of μ-integrable functions $b^*: E \rightarrow R^N_+$, $s^*: E \rightarrow R^N_+$, and $f^*: E \rightarrow R^N_+$ satisfying $(b^*,s^*,f^*) \varepsilon \gamma(p^*,h(e))\ \forall e \varepsilon E$, such that $\int_E b^*(e)\mu(de) = \int_E s^*(e)\mu(de)$.

An observable implication of this equilibrium is the allocation of consumption across agents. It is natural to express this allocation as a set of probability distributions, one for each agent type, over the space of consumption bundles. To accomplish this, let $E^i = \{e \varepsilon E: h(e)=i\}$, let $\omega(e): E \rightarrow R^N_+$ be the (μ-integrable) function such that $\omega(e) = \omega^i$ for $e \varepsilon E^i$, and define $c^{i*}(e): E^i \rightarrow R^N_+$, as $\omega^i + b^*(e) - s^*(e) - f^*(e)$. Thus, $c^{i*}(e)$ gives the equilibrium consumption vector of agent e of type i and is a μ-integrable function. Then, $\nu(B) = \mu\{e \in E^i : c^{i*}(e) \in B\}$ for every subset B of R^N_+ gives the equilibrium distribution of agents of type i over consumption bundles.

When the transactions technology set T is not convex, there are no general results on existence or optimality of competitive equilibrium, although existence theorems for special non-convex technologies can be devised (as in Heller and Star). From the point of view of application, this is a serious drawback of the TC approach.

III. The Valuation Equilibrium (VE) Formulation

In this section, we present an alternative formulation of the resource allocation problem. To motivate this approach, note that in the TC approach consumer choices are constrained not only by feasibility (2.1) and the prices (2.3) but also by the transactions technology (2.2). This feature makes the TC approach different from the standard general equilibrium approach in which consumer choices are constrained only by feasibility and prices. In other words, in the standard approach, the “goods” being produced by the transactions technology t would be priced through the market system. A key feature of the valuation equilibrium approach is that the pricing of the output of the transactions technology is indeed done through the market.

A transactions technology allows consumers to undertake different patterns of trade. Thus, the “goods” being produced are the different patterns of trade made possible by the technology. In the valuation equilibrium approach, each pattern of trade is regarded as a “good” itself. For the consumer, the quantity associated with a “good” is the *probability* with which the consumer would want that particular pattern of trade; for a firm operating the transactions technology, the quantity associated with the “good” is the measure of consumers who desire that pattern of trade.

Although the valuation approach does not require it, for expositional simplicity we work with discrete probability distributions. Let $\tilde{C} \subset R_+^N$ be a finite set of grid points (discrete approximation) of $[0,c_{1,max}]\times[0,c_{2,max}]\times...[0,c_{N,max}]$. The upper bounds, $c_{n,max}$, $n \in \{1,2,...,N\}$, are assumed to be large but finite. We assume that $\tilde{C}$ contains the endowment point of each consumer-type. Let Ω be the set of endowment points. Let $C=\tilde{C}\times\Omega$ be the *underlying commodity space*. A point $c \in C$ is $(c_1,c_2,...,c_{2N})$; by $c_j(c)$, we mean the jth element of c.

Consumers choose *probabilities* for each element in C. The role of Ω in the underlying commodity space is to allow consumers to signal to the Walrasian auctioneer the type of transactions they must make if they end up consuming the bundle $c_1,c_2,...,c_N$. For instance, if a consumer puts positive probability on some c, it will signal that the consumption of the associated sub-vector $(c_1(c),c_2(c),...,c_N(c))$ requires the deliveries $d = \max \{(c_{N+1},c_{N+2},...,c_{2N}) - (c_1,c_2,...,c_N), 0\}$ and the receipts $r = \max \{(c_1,c_2,...,c_N) - (c_{N+1},c_{N+2},...,c_{2N}), 0\}$. The cost to the consumer of putting positive probability on c will depend on the nature of the implied delivery and receipt vectors.

The *commodity space* in the VE approach is taken to be the normed linear space L formed by the set of all functions $z: C \rightarrow R$ (the norm is assumed to be the sup norm). Note that L is finite-dimensional, with dimension equal to the cardinality of C. The environment of section I can then be given the following representation.

<u>Consumers</u>

The endowment point of consumer of type i is taken to be the degenerate probability distribution $e^i \in L$ such that:

(3.1) $e^i(\omega_1^i,\omega_2^i,...,\omega_N^i,\omega_1^i,\omega_2^i,...,\omega_N^i)=1$ and $e^i(c)=0\ \forall$ other c.

The consumption possibility set of consumer of type i is a set $X^i \subset L$ with the property that for all $x \varepsilon X^i$:

(3.2) $x(c) \geq 0\ \forall c\ \varepsilon\ C$ and $\Sigma_{c\varepsilon C}\ x(c) = 1$

(3.3) $x(c)>0 \Rightarrow (c_{N+1}(c), c_{N+2}(c),...,c_{2N}(c)) = \omega^i$.

Thus, the consumption possibility set of each consumer is a set of restricted probability distributions over C. The restriction (3.3) requires that a consumer of a given type always correctly signal his implied deliveries and receipts.

The utility function of consumer-type i defined over $x\ \varepsilon\ X^i$ is:

(3.4) $U^i(x) = \Sigma_{c\varepsilon C}\ u(c_1(c), c_2(c),...,c_N(c))x(c)$.

We note X^i and $U^i(x)$ satisfy the following key properties:

(X.1) X^i is compact and convex.

(X.2) if $x, \hat{x}\ \varepsilon\ X^i$, $U^i(x) > U^i(\hat{x})$, and $\theta\ \varepsilon\ (0,1)$, then $U^i(\theta x + (1-\theta)\hat{x}) > U^i(\hat{x})$.

(X.3) $U^i(x)$: $X^i \rightarrow R$ is continuous.

<u>Firm</u>

The firm in this formulation is a clearinghouse. It "takes in" the points in C that are assigned negative weights ($y(c) < 0$) and "distributes" the points in C that are assigned positive weight ($y(c) > 0$). The absolute value of these weights should be thought of as the "number" (or measure) of commitments to take in or distribute a point. The physical constraint on the clearinghouse is that it cannot distribute more of any good than it takes in.

Thus, the aggregate production possibility set is a set $Y \subset L$, with the property that for all $y\ \varepsilon\ Y$:

(3.5) $y(c) < 0 \Rightarrow c = (\omega^i, \omega^i)$ for some $i = \{1,2,...,I\}$

(3.6) $\Sigma_{c\varepsilon C}\{c_j(c) + \Sigma_{c\varepsilon C} t_j(d(c),r(c))\}y(c) \leq 0\ \forall\ j = 1,2,...,N$

where $d(c) = \max\{(c_{N+1}(c),c_{N+2}(c),...,c_{2N}(c))- (c_1(c),c_2(c),...,c_N(c)), 0\}$, and

$r(c) = \max \{(c_1(c),c_2(c),...,c_N(c)) - (c_{N+1}(c),c_{N+2}(c),...,c_{2N}(c)),0\}.$

Y satisfies the following property:

(Y.1) Y is a convex cone.

To complete the formulation, we follow Debreu and define the *price system* for this economy as an element of the set of continuous linear functionals v: L → R. Since L is finite-dimensional, each v has a unique inner-product representation: $v(x) = \Sigma_{c \varepsilon C}\, p(c)x(c)$; here p(c) is the value of the point c, and the price of the lottery is its expected value.

Equilibrium

To apply Debreu's definition of a valuation equilibrium, we restrict consumers of each type to choose the same consumption bundle (see Hornstein and Prescott (1993) for a discussion of this restriction). With this restriction, we have:

Definition: A *valuation equilibrium* is (x^{i*}), y^* and a price system v^* such that (i) for every i, x^{i*} maximizes $U^i(x)$ s.t. $x \varepsilon X^i$ and $v^*(x) \leq v^*(e^i)$, (ii) y^* maximizes $v^*(y)$ subject to $y \varepsilon Y$, and (iii) $\Sigma_i \lambda^i (x^{i*} - e^i) = y^*$.

In this valuation equilibrium, consumers and the firm get together and trade lotteries. During this process, no transactions in goods take place. Once a valuation equilibrium is established, the lotteries are run and each consumer learns of the point c he must consume. If a consumer of type i learns that he must consume the point $\hat{c}$, he must deliver to the clearinghouse (firm) the vector of goods $d = \max \{\omega^i-(c_1(\hat{c}),c_2(\hat{c}),...,c_N(\hat{c})),0\}$ and receive from it the vector of goods $r = \max \{(c_1(\hat{c}),c_2(\hat{c}),...,c_N(\hat{c}))-\omega^i,0\}$. Condition (iii) in the definition of a valuation equilibrium ensures that there is enough of each good being delivered to the clearinghouse to meet all distributions as well as transactions costs.

In this description, transactions costs are experienced by the clearinghouse. However, the underlying commodity space allows the price of lotteries to reflect these costs so that consumers can make informed decisions. To see the clearest example of this, suppose that consumer-types i and i' wish to obtain (as part of their chosen lotteries) the *same* vector of goods $(c_1,c_2,...,c_N)$ with the *same* probability. Since the endowment of type i is different from that of type i', this would involve a different pattern of exchange for the two types. Therefore, i and i' will assign the same probability to *different* points in C, points that differ only in their (N+1) to 2N elements. This feature allows the function p(c) to capture any differences in the cost of input requirements of the different transactions made by consumer-types i and i'.

Unlike the TC approach, a significant advantage of the VE approach is that results from standard general equilibrium theory apply. The proof of existence can be supplied by adapting well-known arguments (see, for instance, Hornstein and Prescott (1993)). Also, under the properties X.1 - X.3 and Y.1, the first and second welfare theorems hold.

The first welfare theorem assures us that a valuation equilibrium is Pareto-optimal (i.e., all gains from trade are exhausted). The second welfare theorem assures us that every Pareto-optimal allocation is a valuation equilibrium for some distribution of endowments. Thus, the set of valuation equilibria consistent with economy-wide resource constraints corresponds to the solution of constrained linear programming problems.

IV. On the Equilibrium Predictions of the TC and VE Formulations

An observable implication of a TC equilibrium is a set of probability distributions of agents, one for each agent-type, over the space of consumption bundles. A valuation equilibrium implies such a set of distributions directly because each member of a given type chooses (by assumption) the same lottery over the space C. By the law of large numbers, the realized distribution of members over C will be identical to the common probability distribution chosen by each member. This similarity raises the question: can equilibrium consumption allocations from the TC formulation be supported as part of a valuation equilibrium? In this section we provide an example for which the answer is no. Therefore, the predictions of the two formulations do not, in general, coincide.

An Example

Assume that I=2, $\lambda^i=1\ \forall$ i, and N=3. The utility function of all agents is $\ln(c_1) + 0.6\ln(c_2) + 0.4\ln(c_3)$. The endowment of consumer-type 1 is (2.5,0.5,0.5) and of consumer-type 2 is (1.0, 3.22, 3.10). The exchange technology is of a fixed-input type, where t(0,0) = 0, t(d,r) = 0.3 if max {d,r} >> 0, and t(d,r) = 0.1 otherwise.

The equilibrium consumption allocation using the TC approach turns out to be the following pair of probability distributions for the two consumer types:

$\{\pi^1(1.37, 1.37, 1.17) = 1\}$

and

$\{\pi^2(1.68, 3.22, 1.43) = 0.40,$

$\pi^2(1.77, 1.77, 3.10,) = 0.60\}$

In other words, in the TC equilibrium for this economy, consumers of type 1 transact in all three goods while a fraction (0.4) of consumers of type 2 transact in goods 1 and 3 and the remaining type 2 consumers transact in goods 1 and 2.

For this pair to show up as part of an equilibrium using the VE approach, it must be the case that the lottery chosen by type 1 is the degenerate distribution:

$\{x^1(1.37, 1.37, 1.17, 2.5, 0.5, 0.5)=1$

$x^1(c) = 0$ for all other $c\varepsilon C\}$

and the lottery chosen by type 2 places a 40 percent weight on the node in the underlying commodity space corresponding to the allocation chosen in the TC equilibrium

$\{x^2(1.68, 3.22, 1.43, 1.0, 3.22, 3.10) = 0.40,$

$x^2(1.77, 1.77, 3.10, 1.0, 3.22, 3.10) = 0.60$, and

$x^2(c) = 0$ for all other $c\varepsilon C\}$.

Furthermore, the corresponding inputs and outputs of the clearinghouse that supports this allocation would be given by the following:

$\{y(2.50, 0.50, 0.50, 2.50, 0.50, 0.50) = -1,$

$y(1.37, 1.37, 1.17, 2.50, 0.50, 0.50) = 1,$

$y(1.00, 3.22, 3.10, 1.00, 3.22, 3.10) = -1,$

$y(1.68, 3.22, 1.43, 1.00, 3.22, 3.10) = 0.40$

$y(1.77, 1.77, 3.10, 1.00, 3.22, 3.10)= 0.60,$

$y(c) = 0$ for all other $c\varepsilon C\}$.

In this allocation, consumers of type 2 are indifferent between a consumption allocation involving transactions in goods 1 and 2 and one involving transactions in goods 1 and 3. However, this means that the lottery for consumers of type 2 cannot be optimal because by agreeing to receive the average consumption of good 1 for certain they can, ex-ante, make themselves better off. More specifically, let $\tilde{c}_1$=0.40(1.68)+0.60(1.77) denote the average consumption of agent-type 2 in the TC equilibrium. Consider the following lottery for agent-type 2:

$\{\tilde{x}^2(\tilde{c}_1, 3.22, 1.43, 1.0, 3.22, 3.10)=0.40,$

$\tilde{x}^2(\tilde{c}_1, 1.77, 3.10, 1.0, 3.22, 3.10)=0.60$, and

$\tilde{x}^2(c) = 0$ for all other $c \varepsilon C\}$.

Let y' be given by:

$\{y'(2.5,0.5,0.5,2.5,0.5,0.5)=-1,$

$y'(1.37, 1.37, 1.17, 2.50, 0.50, 0.50) = 1,$

$y'(1.00, 3.22, 3.10, 1.00, 3.22, 3.10) = -1,$

$y'(\tilde{c}_1, 3.22, 1.43, 1.00, 3.22, 3.10) = 0.40$

$y'(\tilde{c}_1, 1.77, 3.10, 1.00, 3.22, 3.10) = 0.60,$

$y'(c) = 0$ for all other $c \varepsilon C\}$.

It is easy to check that $(x^1, \tilde{x}^2)$, y' is attainable and that $U^2(\tilde{x}^2) > U^2(x^2)$. This contradicts the Pareto-optimality of a valuation equilibrium. Hence, no prices supporting (x^i),y can exist.

This example makes clear that for the environment described in section I, the TC approach involves an arbitrary restriction on trade. Because all members of a type get identical utility but (possibly) different consumption bundles, there will generally be gains from lotteries. There is nothing in the description of the environment that rules out such lotteries.

V. Examples of Valuation Equilibria

In this section we compute valuation equilibria for an economy with preference and endowment patterns identical to the one studied in the previous section but equipped with different transactions technologies. For convenience, we summarize again the features of the environment that are the same across these economies.

The economic environment has N = 3, I = 2, and $\lambda^i = 1\ \forall i$. Both consumer-types have utility function $\ln(c_1) + 0.6\ln(c_2) + 0.4\ln(c_3)$. The endowment of consumer-type 1 is (2.5, 0.5, 0.5) and the endowment of consumer-type 2 is (1.0, 3.22, 3.1).

Example 1: Arrow-Debreu

In this example we assume that there are no input requirements for transacting. Thus, t(d,r) = 0 for all d,r. Below, we report the lotteries chosen by each consumer type, the production point chosen by the firm, and the prices of the lotteries. As one would expect, the lotteries chosen by the consumers in this environment are degenerate and the equilibrium allocations are identical to the standard Arrow-Debreu allocations.

Allocations

$\{x^{1*}(1.49, 1.58, 1.53, \omega^1_1, \omega^1_2, \omega^1_3) = 1$

$x^{1*}(c) = 0$ for all other $c \varepsilon C\}$

$\{x^{2*}(2.01, 2.14, 2.07, \omega^2_1, \omega^2_2, \omega^2_3) = 1$

$x^{2*}(c) = 0$ for all other $c \varepsilon C\}$.

$\{y^{*}(\omega^1_1, \omega^1_2, \omega^1_3, \omega^1_1, \omega^1_2, \omega^1_3) = -1$

$y^{*}(1.49, 1.58, 1.53\ \omega^1_1, \omega^1_2, \omega^1_3) = 1$

$y^{*}(\omega^2_1, \omega^2_2, \omega^2_3, \omega^2_1, \omega^2_2, \omega^2_3) = -1$

$y^{*}(2.01, 2.14, 2.07, \omega^2_1, \omega^2_2, \omega^2_3) = 1$

$y^{*}(c) = 0$ for all other $c \varepsilon C\}$.

$p^* = (1, 0.56, 0.39)$, $p^*(c) = \Sigma_n p^*_n(c_n + t_n(d(c), r(c)))$, $v^*(x) = \Sigma_c p^*(c)x(c)$

Example 2: Medium Transactions Costs

In this example the transactions technology is assumed to be the following: t(0,0) = 0; t(d,r) = (0.15,0,0)' if max{d,r} >> 0; t(d,r) =(0.05,0,0)' otherwise. In other words, it is assumed that only good 1 is needed as input in the transactions technology and that it requires more of good 1 to transact in all three goods than in one or two goods.

Allocations

$\{x^{1*}(1.40, 1.47, 1.61, \omega_1^1, \omega_2^1, \omega_3^1) = 1$

$x^{1*}(c) = 0$ for all other $c \varepsilon C\}$

$\{x^{2*}(1.83, 2.11, \omega_3^2, \omega_1^2, \omega_2^2, \omega_3^2) = 0.17$

$x^{2*}(1.83, 2.11, 1.92, \omega_1^2, \omega_2^2, \omega_3^2) = 0.83$

$x^{2*}(c) = 0$ for all other $c \varepsilon C\}$.

$p^* = (1.0,0.52,0.38)$, $p^*(c) = \Sigma_n p_n^*(c_n+t_n(d(c),r(c)))$, $v^*(x) = \Sigma_{c\varepsilon C} p^*(c)x(c)$

Since the equilibrium production point can be inferred from the lotteries chosen by each consumer-type, we have suppressed it. The important point to notice is that while consumers of type 1 continue to choose a degenerate lottery, consumers of type 2 find it optimal to choose a non-degenerate one. In particular, 17 percent of consumers of type 2 end up with a consumption pattern that involves transactions in only goods 1 and 2. The other 83 percent transact in all three goods.

It is also of some interest to note the value of goods underlying the valuation equilibrium relative to the Arrow-Debreu equilibrium. Since good 1 is now valuable for transaction purposes as well, the price of goods 2 and 3, in terms of good 1, falls.

Example 3: High Transactions Costs

In this example, the transactions technology is similar to the one in the previous example except that more of good 1 is needed to transact. Thus, $t(0,0) = 0$; $t(d,r) = (0.3,0,0)'$ if $\max\{d,r\} >> 0$; $t(d,r) = (0.1,0,0)'$ otherwise.

Allocations

$\{x^{1*}(1.46, 1.55, \omega_3^1, \omega_1^1, \omega_2^1, \omega_3^1) = 0.37$

$x^{1*}(1.46, 1.55, 1.32, \omega_1^1, \omega_2^1, \omega_3^1) = 0.63$

$x^{1*}(c) = 0$ for all other $c \varepsilon C\}$

$\{x^{2*}(1.70, \omega_2^2, 1.56, \omega_1^2, \omega_2^2, \omega_3^2) = 0.23$

$x^{2*}(1.70, 1.86, \omega_3^2, \omega_1^2, \omega_2^2, \omega_3^2) = 0.67$

$x^{2*}(1.70, 1.86, 1.56, \omega_1^2, \omega_2^2, \omega_3^2) = 0.10$

$x^{2*}(c) = 0$ for all other $c \varepsilon C\}$.

$p^* = (1.0,0.56,0.44)$, $p^*(c) = \Sigma_n p_n^*(c_n+t_n(d(c),r(c)))$, $v^*(x) = \Sigma_{c\varepsilon C} p^*(c)x(c)$

In this case, consumers of both types choose non-degenerate lotteries. This particular example is of some interest because the environment is identical to the one for which we reported the equilibrium using the TC approach. In that equilibrium, all consumers of type 1 transacted in all goods. However, in the valuation equilibrium, only 63 percent do; the rest transact in goods 1 and 2 only. Similarly, while none of the consumers of type 2 transact in all three goods in the TC equilibrium, 10 percent of these consumers do so in the valuation equilibrium.

Interestingly enough, the high cost equilibrium has higher relative prices for goods 2 and 3 (in terms of goods 1) than the medium cost equilibrium, despite the fact that more of good 1 is needed to transact in the high cost equilibrium. Clearly, the reason must lie in how the market demand and supply of goods 2 and 3 are altered by the increase in the input requirements for transactions.

Table 1 gives some sense of these factors by describing the participation pattern in each of the goods across these three economies. The top panel shows that all consumers transact in all three goods in the zero-cost Arrow-Debreu equilibrium. The middle panel shows that in the medium cost economy, only 83 percent of consumers of type 2 transact in good 3. This means, of course, that the market supply of good 3 is lower in this economy than in the Arrow-Debreu economy. Similarly, the bottom panel shows that in the high cost economy, there are fewer people transacting in goods 2 and 3 relative to the medium cost economy and the Arrow-Debreu economy.

The value of resources spent on transactions is also noted in the Table. It is obviously zero in the Arrow-Debreu economy but rises to 8 percent of the value of all goods in the medium cost economy and to 10 percent in the high cost economy.

VI. Conclusions

There are at least three advantages to employing the valuation equilibrium approach to problems with general transactions technologies rather than the more common transactions costs approach. First, it enables one to make a connection with standard general equilibrium theory even when transactions technology is non-convex and so it’s straightforward to check the conditions that guarantee existence and optimality of competitive equilibrium. Second, the approach is amenable to integration with environments in which there are other constraints on trade. For instance, Townsend (1987) uses lotteries and linear programs to study resource allocation with private information. Finally, there are substantial computational benefits in that both allocations and prices can be obtained through solutions to *linear* programming problems.

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Table 1 : Equilibrium Participation Patterns

Arrow-Debreu

	Good 1	Good 2	Good 3
Consumer-type 1	1	1	1
Consumer-type 2	1	1	1

Percent Value-Added in the Transactions Sector 0 %

Transaction Technology with Medium Costs

	Good 1	Good 2	Good 3
Consumer-type 1	1	1	1
Consumer-type 2	1	1	0.83

Percent Value-Added in the Transactions Sector 8 %

Transaction Technology with High Costs

	Good 1	Good 2	Good 3
Consumer-type 1	1	1	0.63
Consumer-type 2	1	0.77	0.33

Percent Value-Added in the Transactions Sector 10 %