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CREDIT ENFORCEMENT CYCLES**

Lukasz A. Drozd
Research Department
Federal Reserve Bank of Philadelphia

Ricardo Serrano-Padial
Drexel University

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Credit Enforcement Cycles

Lukasz A. Drozd and Ricardo Serrano-Padial*

ABSTRACT

Empirical evidence suggests that widespread financial distress, by disrupting enforcement of credit contracts, can be self-propagatory and adversely affect the supply of credit. We propose a unifying theory that models the interplay between enforcement, borrower default decisions, and the provision of credit. The central tenets of our framework are the presence of capacity constrained enforcement and borrower heterogeneity. We show that, despite heterogeneity, borrowers tend to coordinate their default choices, leading to fragility and to credit rationing. Our model provides a rationale for the comovement of enforcement, default rates and credit seen in the data.

Keywords: contract enforcement, enforcement capacity, default spillovers, credit crunch, credit cycles, global games, heterogeneity

JEL codes: D82, D84, D86, G21, O16, O17, O43.

*Drozd: Federal Reserve Bank of Philadelphia, Ten Independence Mall, Philadelphia, PA 19106 (email: lukaszadrozd@gmail.com). Serrano-Padial (corresponding author): School of Economics, Drexel University, 3220 Market St, Philadelphia, PA 19147 (email: rspadial@gmail.com). An earlier version was circulated under the title “Financial Contracting with Enforcement Externalities.” We thank Jason Donaldson, Pedro Gete, Patrick Kehoe, Marzena Rostek, Joel Sobel, Marek Weretka, and audiences at CEMFI, CIDE, Drexel, Georgetown, Indiana, NBER micro-to-macro group, UNC-Chapel Hill, Oxford, University of Pennsylvania, Pompeu Fabra, Philadelphia Fed, Queen Mary, Royal Holloway, St. Louis Fed, Surrey, Virginia, Wharton, and UW-Madison for their comments. Drozd acknowledges the financial support of the Vanguard Research Fellowship from Rodney L. White Center for Financial Research. The views expressed in this paper are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or the Federal Reserve System. This paper is available free of charge at www.philadelphiafed.org/research-and-data/publications/working-papers.

1 Introduction

Effective contract enforcement has long been considered the cornerstone of well-functioning credit markets (La Porta et al., 1998). Despite its importance, the strength of enforcement is hardly a stable feature of the economy and can be severely compromised in times of widespread financial distress. For example, Figure 1 shows the evolution of unresolved default cases in court during the 2007-10 financial crisis in three major countries affected by the crisis: the U.S., Italy and Spain.¹ In all three countries, the initial surge in new default cases resulted in a groundswell of pending cases and had a lasting impact on the efficacy of enforcement.² As is well known, credit also tightened during this time period.³

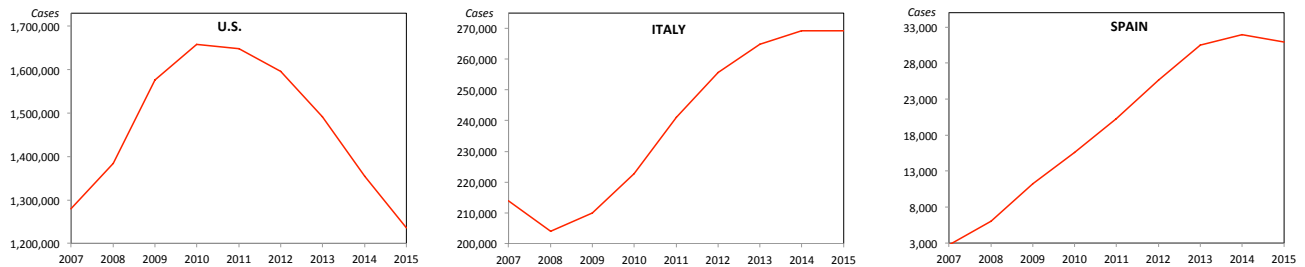


Figure 1: Default Cases Pending in Court

Motivated by the observed variation in the efficacy of enforcement and credit market outcomes, we propose a novel theory that links enforcement, default rates and credit supply. The central element of our theory is that individual incentives to default are tied to the aggregate

¹Figure 1 uses annual court data on pending cases. Data refer to individual and corporate bankruptcies in the U.S. (source: U.S. courts, <http://www.uscourts.gov/report-name/bankruptcy-filings>); commercial property executions in Italy (source: Italian Ministry of Justice, <https://reportistica.dgstat.giustizia.it/>); and commercial bankruptcies in Spain (source: Spain's Ministry of Justice <http://www6.poderjudicial.es>). We start the series at 2007 because the three countries underwent bankruptcy law reforms during the period 2003-05.

²For example, foreclosure delays increased substantially in the U.S. during the crisis (Cordell et al., 2015). Similarly, in Italy, timelines to write off bad debt from banks' balance sheets went up from under 4 years to over 6 years between 2007 and 2011 (Bank of Italy, 2013). Carpinelli et al. (2016) document that judiciary backlogs were the primary factor behind the slower resolution of non-performing loans during this time period in Italy.

³The majority of banks tightened lending standards in the U.S., Italy and Spain during the 2007-10 crisis. Source: Federal Reserve Bank of St. Louis (U.S.) and ECB Euro Area Bank Lending Survey (Italy, Spain).

default rate because enforcement is an exhaustible resource. The goal of our theory is to uncover the mechanism governing such a link and explore its consequences for the propagation of financial shocks.

The basic premise of our theory is supported by the empirical literature. First, a growing body of micro-level studies shows that the strength of debt enforcement affects borrowers' incentives to default. For example, [Schiantarelli, Stacchini and Strahan \(2016\)](#) report that firms selectively default on loans from banks that are in jurisdictions with weak enforcement.⁴ In a similar vein, [Iverson \(2017\)](#) finds that a higher caseload in U.S. bankruptcy courts is associated with higher recidivism and creditor losses.⁵ In the context of the mortgage market, [Chan et al. \(2016\)](#) and [Zhu and Pace \(2015\)](#), among others, document that foreclosure delays spur mortgage defaults.⁶ Second, a large empirical literature links enforcement to the provision of credit. Examples include the micro-level studies by [Jappelli, Pagano and Bianco \(2005\)](#), [Safavian and Sharma \(2007\)](#), [Ponticelli \(2016\)](#) and [Rodano, Serrano-Velarde and Tarantino \(2016\)](#), along with the cross-country evidence in [La Porta et al. \(1998\)](#), [Djankov, McLiesh and Shleifer \(2007\)](#), [Djankov et al. \(2008\)](#) and [Bae and Goyal \(2009\)](#).

Modeling the interconnected nature of these phenomena raises two important issues that our theory overcomes. First, the fact that higher default rates incentivize default by weakening enforcement suggests the presence of strategic complementarities in borrowers' decisions to default. This raises the issue of equilibrium indeterminacy, which in turn questions whether

⁴[Ippolito et al. \(2016\)](#) also find evidence of coordinated defaults triggered by liquidity shocks that hit European banks in 2007.

⁵[Favara, Schroth and Valta \(2012\)](#) show that cross-country variations in enforcement affect firms' strategic default choices.

⁶Specifically, [Chan et al. \(2016\)](#) estimate that a foreclosure delay of nine months is associated with a 40% higher default rate on underwater mortgages while controlling for a wide array of confounding factors. [Zhu and Pace \(2015\)](#) find that a delay of three months increases the probability of default by 30%. The work by [Mayer et al. \(2014\)](#) confirms that strategic considerations are an important factor determining households' default decisions, and that by [Gupta \(2016\)](#) and [Goodstein et al. \(2017\)](#) find evidence of mortgage default spillovers not explained by changes in house prices or in credit supply. Using a quantitative model, [Herkenhoff and Ohanian \(2015\)](#) estimate that foreclosure delays added 25% to the delinquency rate during the crisis.

one can offer precise guidance on how outcomes depend on economic fundamentals. The second issue stems from the fact that, in any realistic setting, borrowers are widely heterogeneous in terms of their intrinsic propensity to default, since borrowers' financial state naturally differs in the presence of random shocks and varying borrower characteristics, such as equity, income, etc.⁷ Taking into account agent heterogeneity is crucial, as heterogeneity is known to alleviate the coordination problem among borrowers that strategic complementarities bring about.⁸

Our theory incorporates these ingredients, strategic complementarities and borrower heterogeneity, by adding two key features to an otherwise standard model of debt-financed entrepreneurial activity: (i) the presence of an economy-wide enforcement capacity, and (ii) stochastic investment returns that are private information of borrowers. The first feature implies that high default rates can exhaust the enforcement capacity, severing enforcement. Weak enforcement, in turn, strengthens individual incentives to default, thus introducing complementarities in default choices. The second feature implies that borrowers are heterogeneous in terms of their propensity to default on their debt. We overcome equilibrium indeterminacy by focusing on a global games formulation of the coordination problem faced by borrowers. Specifically, while borrowers in our model are aware of the feedback loop between default and enforcement, they retain some idiosyncratic uncertainty about the link between enforcement levels and the aggregate default rate. To provide a complete characterization of equilibrium, we tackle the challenges associated with the presence of borrower heterogeneity by generalizing existing results from [Sakovics and Steiner \(2012\)](#) and [Frankel, Morris and Pauzner \(2003\)](#).

⁷For instance, the micro-level studies on mortgage default find substantial variation in the default response to foreclosure delays by loan-to-value, which exhibits substantial heterogeneity in the borrower population. In the corporate context, [Schiantarelli et al. \(2016\)](#) and similar studies find the (heterogeneous) financial state of firms to be a key determinant of their incentive to strategically default in response to weak enforcement.

⁸Tackling complementarity under heterogeneity is challenging, and the current literature offers limited guidance in this respect. The few studies addressing coordination of defaults in credit markets, such as [Bond and Rai \(2009\)](#), feature homogeneous agents. The macroeconomic literature on enforcement frictions under heterogeneity abstracts from the feedback loop between enforcement and aggregate default rates. For instance, the workhorse model of enforcement frictions ([Bernanke et al., 1999](#)) assumes a constant cost of enforcement, while [Herkenhoff and Ohanian \(2015\)](#) study the effects of foreclosure delays on default choices, assuming that delays are exogenous.

Our analysis yields two main insights. First, despite borrower heterogeneity, we show that both default rates and enforcement levels are fragile to changes in economic fundamentals because, in equilibrium, some agents coordinate their default choices, leading to a discontinuous response of the aggregate default rate. In the context of the model without idiosyncratic uncertainty, which has multiple equilibria, this fragility corresponds to a switch from the equilibrium with the lowest default rate and strong enforcement to the equilibrium with the highest default rate and weak enforcement. Second, since credit is the only tool lenders have to avoid a spike in default rates caused by a worsening of fundamentals, credit tightens in response to shocks, leading to the propagation of shocks through a credit crunch.

The intuition behind the discontinuous response of the default rate is an endogenous clustering of equilibrium strategies among heterogeneous agents, which partially undoes the “smoothing” effect of agent heterogeneity on default rates. Clustering emerges due to the way beliefs are formed in a population of heterogeneous agents. Specifically, agents whose payoff from defaulting is lower (i.e., those with high project returns), and hence who have a lower intrinsic propensity to default, rationally anticipate that, whenever they find default optimal, so must agents with a higher propensity to default. As a result, when the agents characterized by a higher payoff from defaulting have a sufficiently large presence in the population, the agents who are less prone to default expect the default rate to go up substantially and enforcement to be weaker. The expectation of weaker enforcement, in turn, induces them to cluster on the same equilibrium strategy as the high-propensity agents, leading to a snowball effect that counters the effect of heterogeneity on default incentives.

We illustrate the quantitative implications of clustering by endogenizing enforcement capacity, which is optimally chosen by a planner at a cost, and parameterizing the model to match pre-crisis default rates and credit levels in the U.S. By doing so, we show that, without considerable heterogeneity, the model cannot match observed default rates and exhibits unrealistically

large default clusters. We find that a stylized shock that exogenously increases the default rate on pre-existing loans, thus reducing the available enforcement capacity for new loans, leads to a potentially severe credit crunch when shocks are large. Specifically, a shock size equivalent to the average variation in the default rate of commercial loans leads to a decline in credit supply by 5%, while a surge in defaults similar in magnitude to the one experienced by the U.S. in 2008-09 lowers credit supply by as much as 30%. Furthermore, the propagation does not depend on the likelihood of these shocks, either because the planner does not build enough capacity to prevent a credit drop when an unlikely shock hits, or because the planner increases capacity in anticipation of a likely shock, thereby leading to a credit expansion when the shock does not hit.

Finally, our analysis has important policy implications. When enforcement is constrained, the most effective way to reduce its adverse effect on the economy is to prevent default by agents who are most prone to default, even though the cost of doing so might be high. This intervention mitigates the snowball effect underlying the aforementioned clustering of defaults. An example of such a policy is the 2009 Home Affordable Modification Program (HAMP), which targeted U.S. homeowners who could prove that they were underwater and had not yet defaulted. The success of this program has been documented by [Agarwal et al. \(2017\)](#).

In terms of related literature, the issue of coordination of defaults due to limited enforcement was first addressed by [Bond and Rai \(2009\)](#), who studied a micro-finance model featuring a population of homogeneous borrowers. Their analysis showed the possibility of a “borrower run” on a single lender driven by the link between the default rate and the continuation value that the lender may be able to deliver after the run takes place and depletes its funds. As in our model, they tackled the equilibrium multiplicity by using global games. Whereas they focus on the survival of the lending institution as a function of its financial health and the amount of credit, our goal is to provide a mechanism underlying the comovement of default rates, credit

and enforcement levels, in an economy with heterogeneous agents.⁹ Independently, our paper complements the literature on fire sales and propagation of shocks (Shleifer and Vishny, 2011). Enforcement capacity in this context loosely corresponds to the ability of financial intermediaries to absorb widespread liquidation of collateral. The connection between enforcement levels and default incentives then finds its parallel in the link between the price of assets and incentives to sell collateral. Our analysis could thus be broadly considered as providing a modeling framework to study collateral externalities. An example of such a study is the recent model by Dávila and Korinek (2017), who explored how collateral externalities affect credit.¹⁰

In addition, our paper contributes to the study of global games featuring agent heterogeneity by extending the characterization result established by Sakovics and Steiner (2012). They focus on a particular environment in which, by construction, all agents take the same action in equilibrium. In contrast, we characterize the endogenous emergence of decision clusters as a function of agent heterogeneity — in the class of games studied by Sakovics and Steiner (2012) payoff heterogeneity is restricted to guarantee that all agents cluster on the same strategy.

2 Theoretical framework

The economy is populated by a measure one of ex-ante identical and risk-neutral entrepreneurs, who are seeking funds from lenders to invest in a risky project. The return on their project depends on the resources invested and a random i.i.d. return w , distributed according to continuous cdf $F(w)$, which is their private information. This assumption implies that entrepreneurs are ex-post heterogeneous in their project returns.

Funding comes exclusively in the form of state non-contingent, defaultable loans. Loans are

⁹In the context of sovereign debt crises, Arellano and Kocherlakota (2014) use a 2-firm model to illustrate how coordinated defaults in the private sector can trigger sovereign default when asset liquidation is exogenously limited to one firm.

¹⁰Coordination problems arising from limited enforcement have also been studied in the crime (Bond and Hagerty, 2010) and tax evasion literatures (Bassetto and Phelan, 2008).

described by a tuple (b, \bar{b}) , where b is the loan amount, and \bar{b} is repayment. Each entrepreneur has seed capital equal to y and makes a total investment of $y + b$ in the project. Final output is $(y + b)w$. The lending market is competitive and lenders effectively maximize the utility of entrepreneurs subject to a break-even constraint.

To secure funding, entrepreneurs use projects as collateral for the loan. The liquidation value of the project is $\mu(y + b)w$, where $\mu < 1$. Liquidation can be either *enforced* or *non-enforced*. If liquidation is enforced, the entire liquidation value goes to lenders. If liquidation is not enforced, entrepreneurs retain a portion $0 < \gamma < 1$ of the liquidation value at the lender's expense.

The distinction between enforced and non-enforced liquidation conveys the idea that entrepreneurs can extract rents when liquidation is either delayed or disrupted. For example, entrepreneurs may divert resources after they default on the loan and before the project is taken over by lenders. Alternatively, time may eat up the value of the project that lenders receive, meanwhile serving the entrepreneurs. Finally, bargaining power in an out-of-court *renegotiation* settlement may crucially depend on the threat-point of legal enforcement.¹¹

The second key assumption in our model is that the probability that liquidation is enforced is endogenous and depends on the aggregate default rate. Specifically, we assume that the total mass of projects that can be enforced is constrained by the economy-wide *enforcement capacity*, $X \in [0, 1]$. Hence, if the default rate exceeds the capacity, enforcement probability falls below one. While enforcement capacity is fixed at the lending stage, it is optimally chosen by a benevolent planner before the lending market opens. When choosing enforcement capacity, the planner faces a trade-off between investing in enforcement and using available resources for some other purpose. The planner's choice captures the idea that the enforcement infrastructure, e.g., the bankruptcy court system, is typically established by the government.

To analyze how fluctuations in enforcement affect the provision of credit, we introduce shocks

¹¹Favara, Schroth and Valta (2012) provide evidence that weak enforcement leads to a higher incidence of renegotiation between defaulting firms and creditors.

that randomly deplete the enforcement capacity accumulated by the planner before the credit market opens. Accordingly, enforcement capacity available to sustain new loans is $X = X_o - s$, where X_o is the ex-ante choice of the planner and s is a random aggregate shock. The presence of a shock to capacity conveys in a simple way the basic idea that poor performance of pre-existing loans tangles part of the enforcement infrastructure and diverts enforcement resources away from the new loans market (we obtain qualitatively similar results if we instead introduce a shock to the distribution of project returns).

The timing of events is as follows: First, the planner chooses initial capacity X_o and the shock s is realized. Second, the credit market opens and lenders compete to extend loans to entrepreneurs. Finally, entrepreneurs privately observe their project returns and simultaneously decide whether to repay their loans or default, at which point enforcement takes place.

In what follows, we begin by discussing the repayment decision of entrepreneurs and the enforcement of loans in default. We refer to this stage as the *enforcement game*. We then work backward and derive its implications for the supply of credit by lenders and the planner's choice of enforcement capacity. To ease exposition all proofs are relegated to the Appendix.

2.1 Enforcement game

At the enforcement stage, loan terms and the enforcement capacity are given, and entrepreneurs simultaneously decide whether to pay back the loan or default after privately observing their project returns. To simplify notation, we first make the following observation. The repayment amount \bar{b} corresponds to the minimum return level necessary to repay the loan given by $\bar{w} = \bar{b}/(y+b)$. Hence, in what follows, we conveniently represent loans by the tuple (b, \bar{w}) , as an agent with $w < \bar{w}$ does not have enough funds to pay back the loan, making her default non-strategic. Only the agents with $w \geq \bar{w}$ have the choice of whether or not to default.

Let $a \in \{0, 1\}$ denote an entrepreneur's decision to repay the loan, where $a = 1$ means

repayment and $a = 0$ indicates default. Let $m = 1$ denote enforcement and $m = 0$ the lack of it. The entrepreneur's payoff at the enforcement stage is

$$u(a, w, m) := \begin{cases} (y + b)(w - \bar{w}) & a = 1 \\ (1 - m)\gamma\mu(y + b)w & a = 0. \end{cases} \quad (1)$$

Hence, her repayment decision solves

$$\max_{a \in \{0,1\}} \{E(P)u(w, a, 1) + (1 - E(P))u(w, a, 0)\}, \quad (2)$$

where $E(P)$ denotes the expected probability that liquidation is enforced after defaulting. The solution to this problem implies that the entrepreneurs' repayment decision is governed by a cutoff rule: An entrepreneur repays her loan if $E(P)$ exceeds some value $\theta(w)$. In what follows, we refer to this cutoff value as the agent's *propensity to default*.

Lemma 1. *The default decision of an agent with contract (b, \bar{w}) is¹²*

$$a = \begin{cases} 1 & \text{if } E(P) \geq \theta(w) \\ 0 & \text{otherwise,} \end{cases} \quad (3)$$

where

$$\theta(w) := 1 - \frac{1}{\mu\gamma} \left(1 - \frac{\bar{w}}{w}\right). \quad (4)$$

As Lemma 1 shows, the propensity to default $\theta(w)$ is strictly decreasing in project returns. Apart from capturing a natural feature of default incentives, this property will play an important role in the characterization of equilibrium. In addition, the agents whose default decision is driven by their expectations about enforcement probability P are those with $\theta(w)$ between 0

¹²Given the continuity of F , we can assume without loss that an indifferent agent always chooses to repay.

and 1. Accordingly, these are the agents who behave strategically because their incentives to default depend on their beliefs about the behavior of other agents (i.e., about the default rate).

Lemma 2. *The range of returns associated with $\theta(w) \in (0, 1)$ is $(\bar{w}, \bar{w}/(1 - \gamma\mu))$. Agents with returns outside this range either never default ($w \geq \bar{w}/(1 - \gamma\mu)$) or always default ($w < \bar{w}$).*

The enforcement probability P is endogenous and depends on both the aggregate default rate ψ , defined as

$$\psi := \int_{\{w:a=0\}} dF(w), \quad (5)$$

along with the economy-wide enforcement capacity X . Specifically,

$$P := \min \{X/\psi, 1\}. \quad (6)$$

The key implication of this *enforcement capacity constraint* is that agents' decisions to default are strategic complements. That is, when the constraint binds ($X < \psi$), P goes down as more agents default, making default more attractive to any individual agent. The definition of P assumes that under imperfect enforcement, i.e., when $P < 1$, the defaulted loans subject to enforcement are randomly selected from the pool of loans in default. This is justified by the fact that w is privately observed by agents. Alternatively, such random assignment of enforcement capacity could result from imposing a sequential servicing constraint in bankruptcy courts.¹³ It is worth pointing out that, as further discussed in Section 4, our approach can accommodate alternative functional forms of the relationship between default rates and enforcement efficacy.

The fraction of strategic agents in the population, which is equal to $F(\bar{w}/(1 - \gamma\mu)) - F(\bar{w})$ by Lemma 2, determines the strength of strategic complementarities and is a function of economic

¹³We also assume that agents cannot be profiled during enforcement based on payoff-irrelevant characteristics, such as the address, names, etc. Such possibility would violate nondiscrimination laws, so we do not consider them here. Segmented enforcement is beneficial in our environment and is known to unravel the complementarities. This has been shown by Carrasco and Salgado (2014).

fundamentals, namely, the loan contract (\bar{w}), agent heterogeneity (F) and the payoff from lack of enforcement ($\gamma\mu$). Regarding the latter, there is no need to build enforcement capacity when $\gamma = 0$, since an agent's payoff under default is unaffected by enforcement. It is only when γ is not too low that credit provision requires building enforcement capacity to prevent the deadweight loss from project liquidation associated with strategic default.

Finally, while we assume a continuous distribution of returns to ease exposition and provide better intuition, our proofs work with a discrete F .¹⁴ Accordingly, our results should be interpreted as the limit of an economy with a discrete distribution that is arbitrarily close to F . In addition, we assume that the distribution of propensities to default is single-peaked. This assumption is satisfied by the unimodal distributions commonly used in the literature of financial frictions, such as the lognormal or Pareto distributions. Nonetheless, our equilibrium characterization techniques cover the general case without such a restriction.

Assumption 1. 1) w has a continuous cdf F with density f and full support in $[0, \infty)$; 2) $\frac{F(w)}{wf(w)}$ is increasing and $\lim_{w \downarrow 0} \frac{F(w)}{wf(w)} < 1$; 3) $Ew > 1$.

Having described the enforcement game, we proceed to characterize the equilibrium behavior of entrepreneurs. As the first step, we show that under common knowledge of economic fundamentals strategic complementarities can lead to multiple equilibria. We then introduce idiosyncratic uncertainty about fundamentals to obtain uniqueness.

2.1.1 Multiplicity under common knowledge

To understand why multiplicity obtains under common knowledge of fundamentals, observe that if entrepreneurs perfectly anticipate the aggregate default rate ψ (and hence the enforcement probability P), self-fulfilling beliefs about P can potentially sustain in equilibrium both a high default rate (imperfect enforcement) or a low default rate (perfect enforcement). To pin down

¹⁴A discrete distribution is technically convenient to prove uniqueness in the global game version of the model.

these equilibria, we make use of the fact that propensities to default are decreasing in returns (Lemma 1). Given this, if all agents expect the same $E(P) = P$, the default rate equals the mass of agents with returns lower than those of the agent indifferent about defaulting or not, i.e., $\psi = F(\hat{w})$, where \hat{w} solves $P = \theta(\hat{w})$. Accordingly, equilibria are determined by the indifference condition $P = \theta(\hat{w})$, and the capacity constraint $P = \min\{X/F(\hat{w}), 1\}$, which in the case of $P = X/F(\hat{w})$ leads to the equilibrium condition

$$X = \theta(\hat{w})F(\hat{w}). \tag{7}$$

Under Assumption 1, $\theta(\hat{w})F(\hat{w})$ is single-peaked and (7) admits up to two solutions, leading to up to three equilibria. This result is formalized in Proposition 1 and illustrated in Figure 2.

Proposition 1. *Under common knowledge of X , if $\bar{w} < w_{\max} := \operatorname{argmax}_w \theta(w)F(w)$, equilibrium is unique iff $X < \underline{X} := F(\bar{w})$ or $X > \bar{X} := \theta(w_{\max})F(w_{\max})$. Otherwise, there are three equilibria in the case of $X \in (\underline{X}, \bar{X})$ and two equilibria in the case of $X = \underline{X}$ and $X = \bar{X}$. If $\bar{w} \geq w_{\max}$ equilibrium is always unique.*

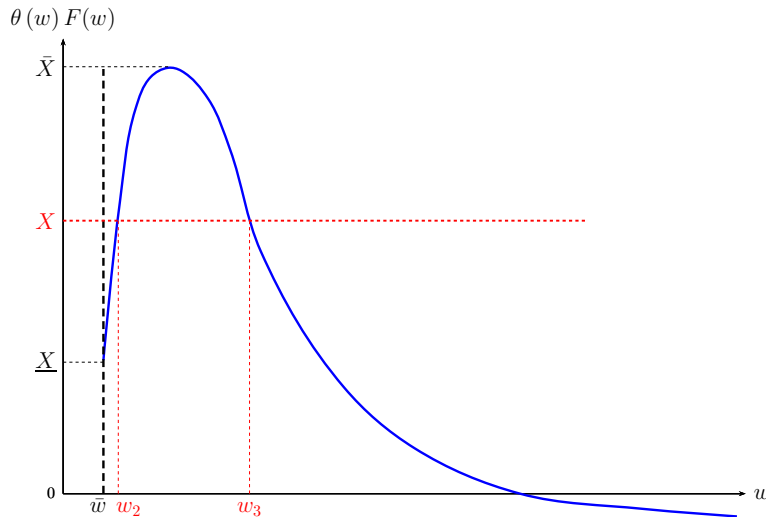


Figure 2: Multiplicity of Equilibria

Figure 2 shows that, when the enforcement capacity X is not too low, there is an equilibrium with perfect enforcement ($P = 1$), in which only insolvent agents default (the default rate is $F(\bar{w})$). However, there are also two additional equilibria with imperfect enforcement ($P < 1$) in which some agents default strategically, thus exhibiting higher deadweight losses from excessive project liquidation. Note that in these cases the default rates are $F(w_2)$ and $F(w_3)$, respectively.

2.1.2 Global games selection

Multiplicity questions the ability of the theory to make predictions about how outcomes depend on economic fundamentals. This, however, critically depends on the assumption of common knowledge of fundamentals, which translates into common knowledge of the equilibrium default rate ψ and enforcement probability P . In practice, borrowers may have a hard time knowing the fundamentals with exact precision or, more generally, they may not be able to perfectly infer the equilibrium default rate. Here we consider such a case by introducing idiosyncratic noise about enforcement capacity and show uniqueness of equilibrium when the noise vanishes. In terms of the specific setup, we follow closely [Frankel, Morris and Pauzner \(2003\)](#).

Formally, we assume that each agent receives a signal $x = X + \nu\eta$, where $\nu > 0$ is a scaling factor and η is an i.i.d. random variable with continuous distribution H that has full support on $[-1/2, 1/2]$. That is, agents' signals are correlated through X but their signal noise is i.i.d. The signal is the only source of information about X . In particular, we assume that agents' prior about X is uniformly distributed on $[0, 1]$,¹⁵ and the noise represents an agent's "uncertainty" or lack of confidence in their inference about X from the observed loan terms.¹⁶ We discuss in Section 4 alternative ways to introduce noise and justify this lack of common knowledge.

¹⁵Our results do not hinge upon the uniform prior assumption. As [Frankel, Morris and Pauzner \(2003\)](#) show, equilibrium selection arguments work in the limit as signal error goes to zero since any well-behaved prior will be approximately uniform over the small range of X that are possible given an agent's signal.

¹⁶If the information structure is endogenous, e.g., when signals come from prices or public signals, [Angeletos and Werning \(2006\)](#) and [Angeletos, Hellwig and Pavan \(2006\)](#) show that equilibrium may not be unique.

In this environment, Lemma 1 implies that an agent with signal x and return w will repay if $E(P|x) \geq \theta(w)$ and default otherwise. We first show that, as noise vanishes, there is a unique limit equilibrium given by cutoff strategies.

Proposition 2. *The enforcement game has a unique equilibrium as $\nu \rightarrow 0$. Equilibrium strategies are characterized by a signal cutoff $k(w)$ such that*

- *If $x \geq k(w)$, agents choose to repay ($a(x) = 1$)*
- *If $x < k(w)$, agents choose to default ($a(x) = 0$).*

The intuition behind the existence of a unique equilibrium is that small uncertainty about fundamentals induces large uncertainty about the actions of others, hindering the agents' ability to sustain multiple equilibria by coordinating their beliefs about the default rate ψ .^{17,18}

2.1.3 Equilibrium default rate and enforcement probability

We next characterize equilibrium default strategies. To do so, we need to pin down signal thresholds $k(w)$ in the limit as $\nu \rightarrow 0$, which satisfy the system of indifference conditions

$$E(P|x = k(w)) = \theta(w) \text{ for all } w \text{ with } \theta(w) \in (0, 1). \quad (8)$$

These conditions state that each agent with return w is indifferent between defaulting or repaying when she receives signal $x = k(w)$. Solving this system requires knowing the distribution of the

¹⁷The presence of strategic uncertainty even at infinitesimal noise levels rests on higher-order beliefs. When an agent observes signal x , she considers it possible that X is $\nu/2$ away from x . As a result, she also admits that other agents may get a signal as far as ν away from x , and thus that they admit a possibility that X is as far as $\frac{3}{2}\nu$ away from x , thus placing positive probability that other agents think that other agents observe signals as far as 2ν away from x . When this reasoning is repeated ad infinitum, infinite-order beliefs about X will fan out for any arbitrarily small ν . This divergence of higher-order beliefs translates into divergent beliefs about ψ .

¹⁸The formal proof uses the fact that games with strategic complementarities feature a smallest and largest Nash equilibrium, both in cutoff strategies (Milgrom and Roberts, 1990), and shows that both must coincide. Specifically, it shows that, given the noise structure, $E(P|x = k(w))$ goes up when $k(\cdot)$ is raised for all w , implying that there can be only one profile of cutoffs at which $E(P|x = k(w)) = \theta(w)$ for all $\theta(w) \in (0, 1)$, i.e., at which each strategic agent is indifferent between defaulting or not when she receives her cutoff signal.

enforcement probability $P = \min\{X/\psi, 1\}$ conditional on $x = k(w)$, which depends on agents' beliefs about capacity X and the default rate ψ . Beliefs about X conditional on $x = k(w)$ converge to $k(w)$ as $\nu \rightarrow 0$, since $X \in [x - \nu/2, x + \nu/2]$. However, given that the default rate ψ is given by the mass of agents with signals below their respective thresholds, beliefs about the default rate, in turn, may depend on the equilibrium thresholds $k(\cdot)$ of all types, making the problem of solving individual indifference conditions intractable.

Nonetheless, we are able to identify key properties of beliefs that allow us to pin down $k(w)$ for all w . To derive these properties we make the following observations. First, decreasing default propensities $\theta(\cdot)$ imply that thresholds $k(\cdot)$ must be (weakly) decreasing in returns. Second, an agent receiving signal $x = k(w)$ knows that other agents' signals fall in $[k(w) - \nu, k(w) + \nu]$ since all signals must lie in the interval $[X - \nu/2, X + \nu/2]$. In this context, there are two cases to consider. In the first case, all other signal thresholds $k(w')$ fall outside $[k(w) - \nu, k(w) + \nu]$. In the second case, some thresholds fall within $[k(w) - \nu, k(w) + \nu]$.

When all other thresholds fall outside $[k(w) - \nu, k(w) + \nu]$, since thresholds are decreasing in w , the agent is certain that only those with $w' < w$ will default, i.e., $\psi = F(w)$, and her indifference condition in the limit boils down to

$$E(P|x = k(w)) = \frac{k(w)}{F(w)} = \theta(w),$$

leading to limit threshold $k(w) = F(w)\theta(w)$.

However, when some thresholds fall within $[k(w) - \nu, k(w) + \nu]$, the agent deems the default rate ψ as random, since some agents with thresholds within the noise range of her $k(w)$ will default and some will not, depending on their signal realization. In this case, individual beliefs about ψ will be a complicated function of signal threshold $k(w)$.¹⁹

¹⁹To understand how heterogeneity makes this approach intractable, consider the case of agents using a single threshold $k(w) = k$. In this case, the default rate of strategic agents (those with θ between 0 and 1) is given by

To overcome this problem, we proceed as follows. First, we adapt the result by [Sakovics and Steiner \(2012\)](#) to show that agents with thresholds within the range of noise, conditional on receiving signals equal to their respective thresholds ($x = k(w)$), on average believe that the default rate ψ is uniformly distributed. That is, if we average agents' beliefs about ψ when each receives their threshold signal $x = k(w)$, weighted by their presence in the population ($f(w)$), we obtain the uniform distribution. In addition, we show that the cluster of heterogeneous thresholds that fall into $[k(w) - \nu, k(w) + \nu]$ converge to the same limit threshold $k(w)$. We combine this fact with the above property of average beliefs to circumvent the need to solve individual indifference conditions. We instead *average* the indifference conditions of agents in any potential cluster, thereby allowing us to work with average rather than individual beliefs to characterize its limit threshold.²⁰ Specifically, we use the average condition

$$\int_{w \in W'} E(P|x = k(w))dF(w) = \int_{w \in W'} \theta(w)dF(w), \quad (9)$$

where W' is a subset of returns of agents that may cluster on the same limit threshold $k(w)$ as $\nu \rightarrow 0$, and replace the average belief about ψ by the uniform distribution. Since $X = k(w)$ in the limit when $x = k(w)$, we can characterize the average expectation in (9) and pin down $k(w)$ for any cluster W' . We then use the fact that signal thresholds $k(\cdot)$ must be (weakly) decreasing to characterize which clusters, if any, arise in equilibrium.²¹

the fraction of those agents receiving signals below k . This implies that, since signal noise is i.i.d., an agent whose signal $x = X + \mu\eta$ is equal to k believes that the strategic default rate is given by $H(\eta)$ (i.e., by the mass of agents with signal noise lower than hers), which is distributed uniformly in $[0, 1]$ — the cdf of a random variable is uniformly distributed. This feature of beliefs is known as the *Laplacian* property ([Morris and Shin, 2003](#)).

²⁰[Sakovics and Steiner \(2012\)](#) call this property the *belief constraint*. It is driven by the fact that the beliefs in higher default rates conditional on $x = k(w)$ held by agents with high $k(w)$ are offset by the low- $k(w)$ agents' beliefs in lower default rates. Although they derive the belief constraint by averaging the beliefs of the whole population about the average action to characterize the unique threshold in games with symmetric equilibria, it generalizes to any subset of agent types in the game. As a consequence, it can be used in environments that yield asymmetric equilibria, as it is not necessary to know a priori which agents are going to cluster in the limit.

²¹Decreasing $k(\cdot)$ implies that a cluster must be an interval of returns. Thus, we can use the indifference conditions of agents just outside the interval, given by $k(w) = F(w)\theta(w)$, to identify its boundaries.

The above analysis allows a closed-form characterization of equilibrium cutoffs, formalized in Proposition 3 below. The proposition fully pins down entrepreneur default choices as a function of fundamentals. While we prove this proposition in the context of our model, our characterization technique goes beyond the specific framework considered here and generalizes the existing results established by [Sakovics and Steiner \(2012\)](#). While [Sakovics and Steiner \(2012\)](#) restrict agent preferences to induce clustering of equilibrium strategies exogenously, we characterize the endogenous emergence of decision clusters as a function of agent heterogeneity.²²

Proposition 3. *In the limit, as $\nu \rightarrow 0$, there exists $w^* \geq \bar{w}$, such that*

$$k(w) = \begin{cases} \theta(w)F(w) & \text{for all } w > w^* \\ \theta(w^*)F(w^*) & \text{for all } w \in [\bar{w}, w^*], \end{cases}$$

where $w^* = \bar{w}$ if $\bar{w} \geq w_{\max}$ and, if $\bar{w} < w_{\max}$, w^* is the unique solution in (w_{\max}, ∞) to

$$\theta(w^*)F(w^*)(1 - \log \theta(w^*)) - F(\bar{w}) = \int_{\bar{w}}^{w^*} \theta(w)dF(w). \quad (10)$$

Proposition 3 is the main result of our paper. Its key economic implication is that, despite agents being heterogeneous, a subset of them cluster on the same equilibrium strategy, making default and enforcement fragile in response to changes in economic fundamentals. Figure 3 illustrates the equilibrium default threshold $k(w)$ characterized in Proposition 3. As is clear from the figure, agents whose returns are between \bar{w} and w^* use the exact same signal cutoff, despite having different propensities to default. This result is quite stark and shows that among the equilibria of the game under common knowledge of fundamentals, the selected equilibrium corresponds to the one with the lowest default rate if $X \geq \theta(w^*)F(w^*)$ but, when $X < \theta(w^*)F(w^*)$

²²[Sakovics and Steiner \(2012\)](#) work with games in which restrictions on agent preferences guarantee the emergence of a single cutoff $k(w) = k$ in the limit.

it is equal to the equilibrium featuring the highest default rate and the lowest enforcement probability. Consequently, the default rate discontinuously jumps when X falls below $\theta(w^*)F(w^*)$, leading to inefficiently high losses from liquidation as enforcement fundamentals deteriorate.

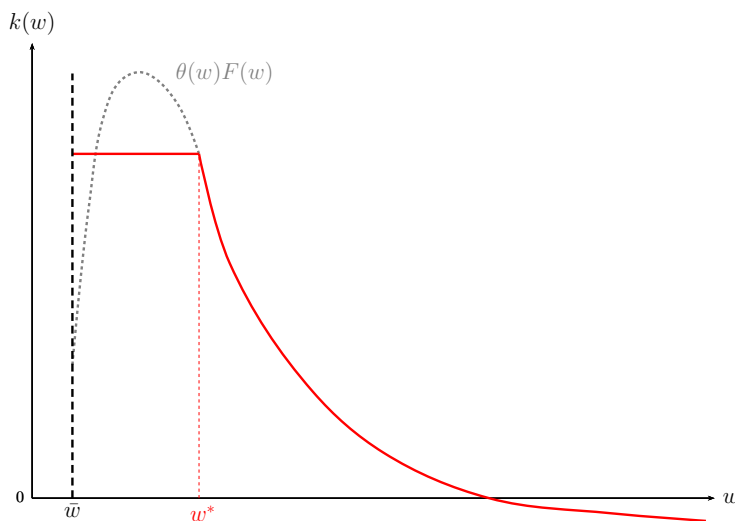


Figure 3: Equilibrium Default Cutoffs

The partial clustering of defaults is driven by differences in beliefs (conditional on $x = k(w)$) regarding the equilibrium default rate, which persist even at infinitesimal noise levels. To gain some intuition behind this result, assume by way of contradiction that $k(w)$ is strictly decreasing so that there is no clustering. If signal thresholds $k(w)$ are strictly decreasing, an agent with returns w knows that, whenever he finds default optimal, i.e., her signal is below $k(w)$, so must all agents with $w' < w$. This is because they have strictly higher propensities to default $\theta(w')$ and their signals are equal to x in the limit. In this context, consider how the equilibrium indifference condition $E(P|x = k(w)) = \theta(w)$ changes as returns increase. An infinitesimal increase in returns from w to $w + dw$ causes the default rate to go up by $f(w)$, pushing down expected enforcement probability $E(P|x = k(w))$, while also lowering the propensity to default $\theta(w)$. Hence, whether an agent with returns $w + dw$ wants to default at the same time as an

agent with returns w , i.e., uses the same signal threshold $k(w)$, depends on whether the increase in the default rate is large enough to compensate for the drop in propensity to default. This is actually the case at returns levels where the mass of returns is concentrated, that is, around the peak of the distribution of returns. Moreover, this effect “snowballs” down the distribution of returns until subsequent agents have a weaker presence in the population so that higher default rates no longer compensate for their lower intrinsic propensity to default $\theta(w)$. At that point, $k(w)$ becomes strictly decreasing, as Figure 3 shows.

The above logic also explains why a single cluster obtains. Clustering is associated with intervals of the return distribution where the mass of returns are concentrated. For unimodal distributions satisfying Assumption 2, this happens in a single interval around their peak. In contrast, multiple clusters could arise under multimodal distributions. Our characterization result in the Appendix (Theorem 2) allows for such a possibility.

The coordination of strategic default choices has important policy implications. Default by the cluster could be avoided by preventing the agents who are most prone to defaulting from doing so. As a result, despite the potentially high cost of disincentivizing default by the agents who are most prone to defaulting, this may turn out to be the best strategy as it may avert the aforementioned snowball effect. [Sakovics and Steiner \(2012\)](#) derive this result formally in the context of their model, which naturally carries forward to our model of endogenous clustering.

2.2 Provision of credit

Having characterized the equilibrium in the enforcement game, we next turn to the analysis of our model’s implications for credit provision by laying out the lender’s problem and analyzing the equilibrium of the lending market, after X has been fixed.

Lender's profits from a loan (b, \bar{w}) , given project return w , are

$$\pi(w, m) := \begin{cases} (y + b)\bar{w} - b & k(w) \leq X \\ (m + (1 - m)(1 - \gamma))\mu(y + b)w - b & k(w) > X. \end{cases} \quad (11)$$

Bertrand competition implies that the equilibrium contract (b, \bar{w}) maximizes entrepreneurs' expected payoffs subject to a non-negative profit condition and to the repayment behavior as determined by capacity X and thresholds $k(w)$.²³ That is, lenders solve the following problem

$$\max_{b, \bar{w}} \left[\int_{\{w: k(w) \leq X\}} (y + b)(w - \bar{w})dF + (1 - P)\gamma \int_{\{w: k(w) > X\}} \mu(y + b)wdF \right], \quad (12)$$

subject to non-negative expected profits

$$b \leq \int_{\{w: k(w) \leq X\}} (y + b)\bar{w}dF + (P + (1 - P)(1 - \gamma)) \int_{\{w: k(w) > X\}} \mu(y + b)wdF, \quad (13)$$

and the enforcement capacity constraint $P = \min \{X/\psi, 1\}$.

The above maximization problem has an interior solution under the assumption that the deadweight loss from defaulting $(1 - \mu)$ is high enough so that arbitrarily large loans, which lead to high default propensities, cannot be recouped through liquidation.

Assumption 2. $\int_{\bar{w}}^{\infty} \bar{w}dF(w) + \mu \int_0^{\bar{w}} wdF(w) < 1$ for all $\bar{w} \geq 0$.²⁴

The next proposition characterizes the relationship between enforcement capacity and credit.

It formally establishes that the minimum capacity needed to sustain the low default equilibrium

²³Payoff maximization subject to zero profits follows from the usual arguments. Since agents are ex-ante identical, there is no profitable deviating contract for a lender: either such contract is less attractive to consumers or yields negative profits. In addition, if the equilibrium contract does not maximize agent payoffs, a lender can make positive profits by offering a contract arbitrarily close to the payoff-maximizing one and attract all agents.

²⁴The assumption guarantees that, as b goes to infinity and hence $b/(y + b)$ goes to one, there is no repayment cutoff \bar{w} that would make lenders earn non-negative profits, even when $P = 1$.

is increasing in the amount of credit provided by lenders.

Proposition 4. $X^*(b) := \theta(w^*)F(w^*)$ is increasing in b .

The intuition behind Proposition 4 is as follows. First, the positive relation between loan size b and the lowest capacity guaranteeing the low default equilibrium $X^*(b)$ is driven by the fact that a higher b requires a higher repayment cutoff \bar{w} . Given this, an increase in \bar{w} has two effects on the cluster signal threshold $\theta(w^*)F(w^*)$: a *direct effect* by increasing the default propensities of all agents, and a *strategic effect* by affecting the pool of strategic agents in the population (recall that the range of strategic agents is given by $(\bar{w}, \bar{w}/(1 - \gamma\mu))$ by Lemma 2).

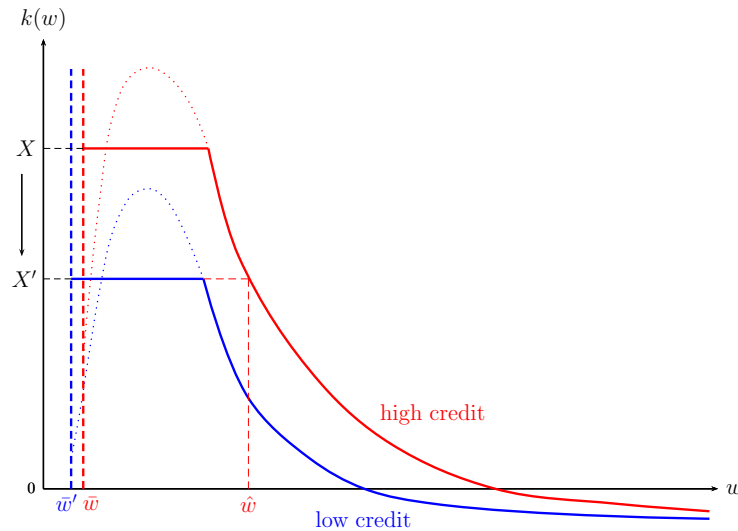


Figure 4: Equilibrium Strategy in the Global Game

The main economic implication of Proposition 4 is the presence of an endogenous borrowing constraint brought about by limited enforcement capacity. In particular, a smaller enforcement capacity leads to a contraction of credit, as long as the liquidation losses from allowing the cluster to default are not negligible.²⁵ Figure 4 illustrates this basic idea by considering a fall

²⁵If X is very low it is possible that the loan size b at which $X^*(b) = X$ is so small that the cluster contains only a tiny mass of entrepreneurs. In such a case, it may be optimal to increase b and let the cluster default.

in enforcement capacity from X to X' . Initially, b is high because there is enough capacity to sustain a high repayment cutoff \bar{w} while enforcing a low default rate $F(\bar{w})$. At X' , however, the previous default rate $F(\bar{w})$ becomes unsustainable for the same loan size b because agents with returns between \bar{w} and \hat{w} would default strategically, raising the default rate to $F(\hat{w})$. Since liquidation is socially wasteful, lenders have no alternative but to lower b to avert it.

The above result implies that the efficient equilibrium is fragile to shocks that affect enforcement, inducing the credit market to absorb the shock through contraction of the credit supply. Credit rationing leads to a fall in investment and output, propagating the shock underlying the decline in enforcement capacity. It remains to be shown whether preventing the cluster from defaulting is indeed optimal for lenders. As we show in the next section, this is indeed the case under reasonable parameterizations of the model.

2.3 Optimal enforcement capacity

We close the model by looking at the planner's problem of choosing the optimal capacity. Formally, we assume that the planner chooses X_o to maximize expected entrepreneur payoffs net of capacity costs $c(X_o)$. That is, she solves

$$\max_{X_o} [E(V(X_o - s)) - c(X_o)], \tag{14}$$

where $V(X)$ denotes the expected payoff of agents from the equilibrium contract associated with X , i.e., the loan that solves (12), and the expectation is taken over the distribution of shocks.

The optimal choice of X , and hence the sensitivity of credit to shocks, depends on the cost function $c(X_o)$. Specifically, as long as capacity costs are not too low, the borrowing constraint implied by $X \geq X^*(b)$ binds, implying that shocks generate credit volatility. Since X_o is determined by the specific functional forms of F and $c(X_o)$, as well as the distribution of shock s ,

we parameterize the model to provide a quantitative assessment of the propagation mechanism.

3 Quantitative Analysis

To illustrate the implications of our theoretical results, we consider a simple numerical example that matches several features of the U.S. credit market. While our model is too simplistic to fully speak to the data, our goal is to show that the mechanism in question is of potential quantitative importance. We also highlight how heterogeneity critically affects the results.

Specifically, we consider a binary shock that hits with probability $p \in [0, 1]$ and consider two cases. In the first case, the shock is large, simulating a financial crisis; whereas, in the second case, we look at the effects of a smaller shock that reflects average fluctuations in default rates. Accordingly, we set the size of the *crisis* shock to $s = 0.018$ to capture the magnitude of the 2007-10 financial crisis in the U.S., given that the default rate on commercial loans went up from an average of about 2.3% in the pre-crisis period 2000-07 to 4.1% in 2008-09.²⁶ We set the *average* business-cycle shock to 0.4% ($s = 0.004$), which is half the average absolute deviation of the default rate on commercial loans between 2000 and 2007. We chose the remaining parameters to ensure that, when $p = 0$, the model's static implications are consistent with the U.S. data. The idea is to capture in a simple way how the rise in the default rate on existing loans at the onset of the crisis might affect the new loans market by exhausting existing enforcement capacity.

3.1 Parameterization

We parameterize the model in two steps. First, we independently select the value of average returns Ew , liquidation value μ , and the shape of return distribution F following the quantitative business-cycle literature that focuses on enforcement frictions (we also normalize equity y to

²⁶Source: Federal Reserve Bank of St. Louis.

one). Second, we jointly choose the volatility of returns σ , linear capacity costs $c(X_o) = c_o X_o$, and private benefit from lack of enforcement γ to match the average debt-to-equity ratio in the U.S., the average delinquency rate on commercial loans, and to make the equilibrium size of the default cluster equal to the size of the crisis shock. We choose the latter target so that the model could reproduce a surge in defaults similar to the one observed in the data as a consequence of an unanticipated shock, such as a negative shock to project returns, that triggers a switch to the high default equilibria. Table 1 presents the parameter values, the model equilibrium statistics and the corresponding data targets.

The model matches well the three U.S. data targets: an average debt-to-equity ratio of 0.75 in the period 1971-2012; the 2000-07 average delinquency rate of 2.3% on commercial and industrial loans; and the 1.8% increase in the delinquency rate experienced in 2008-09. Table 1 also presents additional equilibrium properties in the parameterized economy. Capacity costs represent 200 basis points of aggregate debt, which are in line with existing estimates on legal enforcement costs as we argue below. In equilibrium, enforcement is almost at full capacity (95%), which is consistent with the fact that enforcement is quite sensitive to the number of new default cases, as illustrated in Figure 1. Finally, because of substantial return heterogeneity, the pool of strategic agents only includes 6.8% of the population. Despite such a low number, the induced economic fragility leads to a credit supply that is 20% smaller than the equilibrium credit level in the absence of strategic incentives to default, i.e., when $\gamma = 0$. This is because the endogenous borrowing constraint $X = X^*(b)$ is binding, since lenders want to avoid a spike in the default rate of at least 1.7%, caused by the cluster defaulting.

These results are driven by our specific choice of parameters. The values of the independently selected parameters are drawn from [Bernanke, Gertler and Gilchrist \(1999\)](#) and [Christiano, Motto and Rostagno \(2014\)](#), two of the workhorse models of the business cycle literature on

Table 1: Parameters and Statistics

Parameter	Value	Source
<i>Independently Selected</i>		
y	1	
Ew	1.02	Bernanke, Gertler and Gilchrist (1999)
μ	0.88	Bernanke, Gertler and Gilchrist (1999)
F	Lognormal	Bernanke, Gertler and Gilchrist (1999), Christiano, Motto and Rostagno (2014)
<i>Jointly Calibrated</i>		
σ	3/8	
$c(X)$	0.088X	
γ	0.25	
Statistic	Model	Data
<i>Targeted</i>		
Debt-to-equity ($\frac{b}{y}$)	0.8	0.75 ^a
Default rate (ψ)	2.3%	2.3% ^b
Cluster size ($F(w^*) - F(\bar{w})$)	1.7%	1.8 ^c
<i>Other</i>		
Capacity costs/debt ($c(X_o)/b$)	0.2%	
Capacity utilization ($\frac{\psi}{X}$)	95%	
% strategic agents ($\theta(w) \in (0, 1)$)	6.8%	

^a Gomes, Jermann and Schmid (2016) estimate that the average leverage ratio $\frac{b}{k}$ in the US between 1971 and 2012 is 0.42, which corresponds in our model to a debt-to-equity ratio of 0.75. The debt-to-equity ratio is 0.52 in Christiano, Motto and Rostagno (2014) and 1 in Bernanke, Gertler and Gilchrist (1999).

^b2000-2007 average delinquency rate on commercial and industrial loans. Source: Federal Reserve Bank of St. Louis.

^cDifference between the 2008-09 and the 2000-07 average delinquency rates on business loans.

financial frictions. We next discuss the jointly calibrated parameters and argue that their values are consistent with the literature and with existing evidence.

Heterogeneity of returns (σ). Since the implied agent heterogeneity is a crucial parameter in our model, it is important to note that heterogeneity is also essential here to deliver the targeted default rate and leverage. To illustrate the necessity of having heterogeneous returns,

Figure 5 depicts the relationship between X and both b (left) and ψ (right), assuming different distributions of returns. As is clear from the figure, the default rate hardly rises above zero for $\sigma = \frac{1}{6}$. This is because more concentrated return distributions have a thinner lower tail; hence, as b and \bar{w} go up with capacity, the default rate barely changes. Having a concentrated distribution of returns ($\sigma = \frac{1}{16}$ or $\frac{1}{8}$) also leads to counterfactually high debt-to-equity ratios (b/y) and to low enforcement capacity utilization (ψ/X), which is at odds with the observed variation in enforcement delays. In contrast, as returns become more heterogeneous, the model can deliver equilibrium levels of borrowing, default and capacity utilization in line with the data. Furthermore, these variables exhibit significant variation as a consequence of changes in X . For instance, at $\sigma = \frac{3}{8}$, as X ranges between 0 and 0.05, b/y varies between 0.5 and 1, ψ between 0.2% and 4.8%, and ψ/X between 82% and 98%. As a comparison, for $\sigma = \frac{1}{16}$, b/y varies too much, from 2.7 to 4.7, while ψ barely moves (between 0% and 0.3%) and utilization rates never go above 3%.

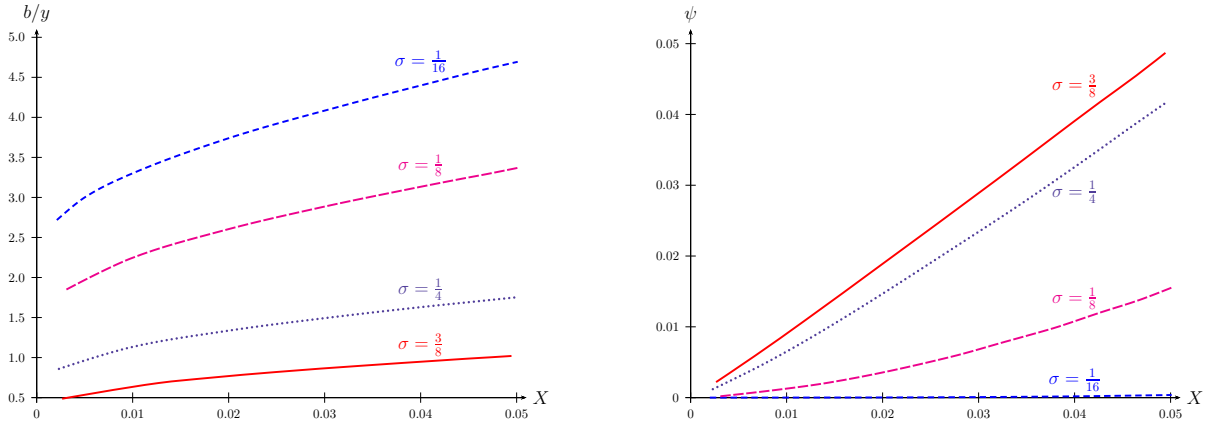


Figure 5: Enforcement Capacity, Credit and Default Rates

Independently, heterogeneity of returns affects equilibrium outcomes in our model by mod-

ulating the degree of strategic complementarity. As returns become more concentrated, so do agents' propensities to default, leading to a large default cluster. This leads to unrealistically large default waves if capacity falls below the cluster threshold. For instance, when $X = 0.05$, the cluster size is 1.7% for $\sigma = \frac{3}{8}$, while it is 59% for $\sigma = \frac{1}{16}$.

It is worth mentioning that our selection of $\sigma = 3/8$ represents the midpoint between the value of $\sigma = 1/4$ in [Christiano, Motto and Rostagno \(2014\)](#) and $\sigma = 1/2$ in [Bernanke, Gertler and Gilchrist \(1999\)](#).

Capacity costs (c_o). Our choice of unit capacity costs $c_o = 0.088$ represents 12.5% of the average pre-liquidation value of the projects subject to liquidation in equilibrium. To put this number into perspective, [Bris, Welch and Zhu \(2006\)](#) use bankruptcy data from Arizona and New York State and estimate that direct bankruptcy costs represent 8.1% of the firm's pre-bankruptcy value in Chapter 7 bankruptcies (liquidation) and 16.9% in Chapter 11 (restructuring). [Djankov et al. \(2008\)](#) estimate that legal enforcement costs in the U.S. are 7% of the pre-bankruptcy value of a generic firm filing for bankruptcy. These include court and attorney fees, as well as other fees such as bankruptcy administrator, notification and accountant fees. These studies do not include indirect costs such as government funding of the bankruptcy court system not covered by court fees, potentially underestimating overall enforcement costs.

Rents from lack of enforcement (γ). The parameter γ is set equal to 25%. Although it is hard to map γ to the data, it determines in the model the recovery rates on defaulted loans when X falls below the cluster threshold, i.e., when enforcement is imperfect ($P < 1$). We find that the equilibrium drop in recovery rates when we keep b constant and unanticipatedly reduce X below $X^*(b)$ is consistent with the international evidence regarding the relationship between loan recovery rates and enforcement delays. Specifically, we use the cross-country database on credit enforcement constructed by [Djankov et al. \(2008\)](#) to estimate the drop in recovery rates

associated with an increase in enforcement delays associated with the increase in court caseload illustrated in Figure 1. To do so, we focus on the sub-sample of 20 countries with English legal origin in their dataset and run an OLS regression of the average recovery rate (*arecov* in the dataset) on the time lag in years between default and receiving a payment (*atimepay*) and GDP per capita, which we include as a control for the level of economic development.²⁷ The regression predicts a 3.7 percentage point drop in recovery rates (from 85% to 81%) associated with the 38% increase in bankruptcy timelines experienced in the U.S. between 2008 and 2015.²⁸ In the model, recovery rates drop between 2 and 5 percentage points (from a value of 77%) as X falls below $X^*(b)$, depending on the magnitude of the reduction in capacity.

3.2 Credit response to enforcement shocks

Figure 6 shows the impact of the shock on credit. It plots loan levels for each realization of the shock as a function of shock probability p . When p is very low, the crisis shock implies a contraction of credit by 30%. Under the average shock, credit still goes down by a sizable 5%. Crucially, although the gap $b(s = 0) - b(s > 0)$ shrinks as the shock becomes more likely, it is still substantial at all shock probabilities. The reason is that higher shock probabilities induce the planner to accumulate precautionary capacity X_o to dampen the effect of the shock on credit provision, but this extra capacity leads to a higher supply of credit in normal times by relaxing the borrowing constraint $X = X^*(b)$. Accordingly, the model can endogenously generate large credit fluctuations as a result of variations in enforcement.

²⁷We obtain a coefficient on *atimepay* equal to -9.42 , significant at the 1% level, with an adjusted R^2 of 0.84. The estimate is robust to introducing additional controls.

²⁸Enforcement delays are calculated using the following formula, proposed by the Italian Ministry of Justice: $Delay_t = 365 \times \frac{Pending_{t+1} + Pending_t}{New_t + Solved_t}$, where t represents the year, and the variables are the stock of pending cases at the beginning of the year, and new and solved cases during the year.

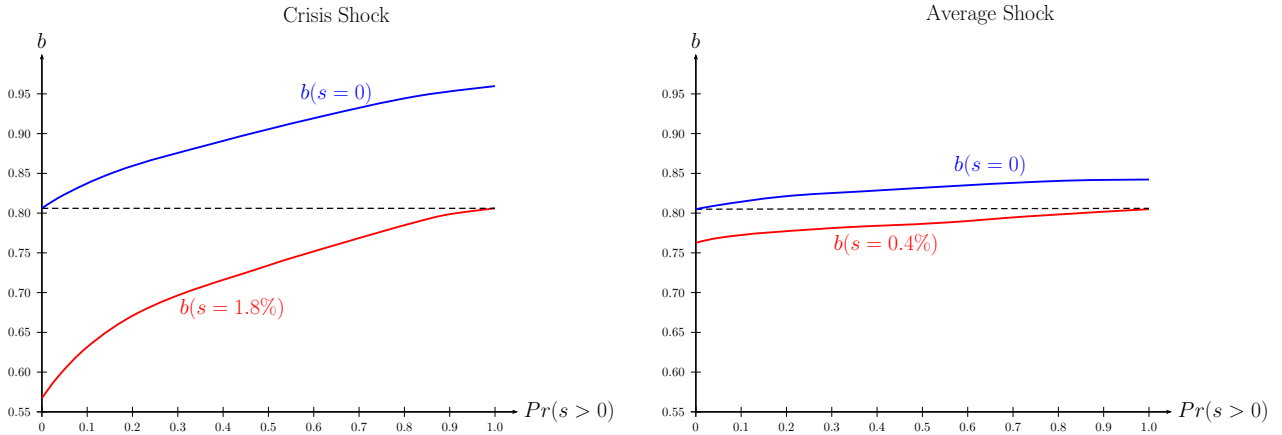


Figure 6: Enforcement Capacity and Shock Propagation

4 Limitations and Concluding Remarks

We conclude by discussing several limitations of our analysis and some of the modeling shortcuts we have adopted.

- *Enforcement technology.* We impose a particular structure on enforcement technology. These assumptions, while specific, are important only in that they deliver an intrinsic propensity to default $\theta(\cdot)$ that is strictly decreasing in returns. Our characterization technique can be applied to any alternative model in which default propensities are decreasing, and the presence of a single default cluster relies on the distribution of default propensities being single peaked.²⁹ We find decreasing propensities a natural feature of default choices: the better the financial health of a borrower, the lower the incentives to default are (*ceteris paribus*).
- *Debt contracts.* We do not consider the issue of optimality of debt contracts and restrict lenders to the use of non-contingent defaultable debt. The reason for such a restriction is that, while existing studies on debt contract design focus on the bilateral case of one

²⁹In an earlier version of the model, we worked directly with a generic distribution of default propensities.

borrower and one lender with perfect enforceability, our environment features multilateral contracting with strategic complementarities, endogenous contract enforceability and a competitive lending market. In this context, it is not clear whether the problem is tractable, and some restrictions on the contract space might still be necessary, such as whether contracts can be contingent on aggregate default rates, enforcement capacity or other economic aggregates—implicitly assuming that they are verifiable in court. In addition, we would have to make assumptions about the enforcement technology regarding the conditions under which some contract clauses are enforceable, given that enforcement levels are endogenous. Finally, lender competition may become non-trivial, since lenders can introduce contingencies that depend on the competitors' offers.

However, on positive grounds, debt contracts are ubiquitous, which motivates our approach. In addition, an extensive finance literature highlights reasons why debt contracts may prevail.³⁰

- *Public signals.* In our model borrowers have idiosyncratic information about fundamentals. This is what breaks common knowledge of the mapping between enforcement and default rates, which is at the heart of equilibrium multiplicity. However, if agents have access to a public signal, such as market prices, they may regain their ability to coordinate via self-fulfilling beliefs and the global game might exhibit multiple equilibria as well ([Angeletos and Werning, 2006](#); [Angeletos et al., 2006](#)). The lack of public signals, in our view, is quite natural in this context. In actual lending markets borrowers sign contracts with at most a few lenders and thus there is not a single market price that is perfectly observed by everyone. In addition, aggregate statistics, or foreclosure signs in a neighborhood, are

³⁰In addition to [Townsend \(1979\)](#) and [Gale and Hellwig \(1985\)](#), the papers by [Mookherjee and Png \(1989\)](#) and [Krasa and Villamil \(2000\)](#) show that non-contingent debt contracts are optimal or close to optimal in bilateral contracting, while the corporate finance literature provides examples of optimality of debt in the presence of informational asymmetries.

likely to be observed by each borrower with an idiosyncratic noise of what these signals may imply for the strength of enforcement. The presence of such an idiosyncratic noise on top of any public information is sufficient to break common knowledge.

- *Information spillovers through contract terms.* As a modeling shortcut, we have assumed that borrowers do not perfectly infer the equilibrium default rate from the debt contract they receive. A way of addressing it, for example, would be to assume that lenders are heterogeneous in their cost of funds, or that they face some noise, while borrowers meet lenders at random. This approach would prevent borrowers from regaining common knowledge. As a related point, while agents receive signals about enforcement capacity, we could alternatively introduce noise about other aggregate parameters, such as expected returns (Frankel, 2012) or the benefits from lack of enforcement (Frankel, 2017), since what matters for the selection is that agents do not have common knowledge about the mapping between aggregate default rates and enforcement probability.
- *Ex-ante heterogeneity.* We have not explicitly considered ex-ante heterogeneity among borrowers. For example, borrowers could differ in their equity y before they enter the market. Such heterogeneity is ubiquitous in credit markets and plays an important role in shaping individual incentives to default. In this regard, it is worthwhile to emphasize that the techniques we use apply to such an environment. While debt contracts offered by lenders would likely be heterogeneous, to the extent that contract terms leave residual ex-post heterogeneity in default propensities, the analysis would proceed unchanged.

Our analysis leaves out other issues, such as the possibility of sequential learning and dynamic spillovers associated with borrowers observing how default and enforcement unfold over time, or the study of a dynamic economy with long-lived entrepreneurs. These are natural directions for future research.

A Appendix: Omitted Proofs

We first prove equilibrium multiplicity under common knowledge and then present the proofs of the global games version of the model. The proofs of Lemmas 1 and 2 are trivial and therefore omitted.

A.1 Equilibrium multiplicity under common knowledge

To prove Proposition 1 we first show that $\theta(w)F(w)$ is single-peaked.

Lemma 3. *If Assumption 1 holds then $\theta(w)F(w)$ is single-peaked and it is increasing at 0.*

Proof. Note that the derivative of $\theta(w)F(w)$ is given by

$$\left(1 - \frac{1}{\mu\gamma} \left(1 - \frac{\bar{w}}{w}\right)\right) f(w) - \frac{\bar{w}}{w^2} \frac{1}{\mu\gamma} F(w),$$

which has the same sign as

$$1 - \frac{w}{\bar{w}}(1 - \mu\gamma) - \frac{F(w)}{wf(w)}.$$

If $\frac{F(w)}{wf(w)}$ is increasing the expression is strictly decreasing. Now, since the second term is zero at $w = 0$ and $\frac{F(w)}{wf(w)}$ is increasing and $\lim_{w \downarrow 0} \frac{F(w)}{wf(w)} < 1$, the expression—and hence the slope of $\theta(w)F(w)$ —is initially positive and eventually negative for high enough w . That is, $\theta(w)F(w)$ is single peaked. \square

Proof of Proposition 1. First consider the case of $X < \underline{X}$. If $\bar{w} < w_{max}$, it must be that (10) has only solutions, $w_1 < \bar{w}$ and $w_2 > w_{max}$. The former cannot be an equilibrium since it would require $\psi = F(w_1) < F(\bar{w})$, i.e., some agents that strictly prefer to default would choose to repay, and hence equilibrium is unique. The same argument applies when $\bar{w} \geq w_{max}$. If $X = \underline{X}$ then $w_1 = \bar{w}$ is also an equilibrium.

Second, let $X > \bar{X}$. In this case, $\theta(w)F(w)$ lies below X , implying that, for any given P such that all agents with default propensity less than P default, there is enough capacity so that the enforcement probability is higher than P . Thus, equilibrium is unique and involves $\psi = F(\bar{w})$. If $X = \bar{X}$ then w_{max} is a solution of (10), representing a second equilibrium.

Finally, if $X \in (\underline{X}, \bar{X})$ Lemma 3 implies that there are three equilibria, given by the two solutions in $[\bar{w}, \infty)$ to (10) and by another equilibrium with $\psi = F(\bar{w})$. since $F(\bar{w}) < X$, and thus, the planner can credibly sustain $P = 1$ in equilibrium. \square

A.2 Equilibrium of the global game

To prove the results, we proceed as follows. First, we present equilibrium existence, selection, and characterization results for the model with a generic discrete distribution of returns. Then,

we provide the proofs of the results in the paper by deriving the implications of the results for a discrete distribution when it is assumed to be arbitrarily close to some continuous distribution F that obeys Assumption 1.

A.2.1 General discrete distribution of returns

In this economy, $\mathcal{W} \subset [0, \infty)$ is a finite set of possible returns, each with a positive mass, that are distributed according to the commonly known discrete distribution F , with probability mass function f .

Given contract (b, \bar{w}) , we make the following assumption about agent payoffs.

Assumption 3. For all $w \in \mathcal{W}$,

(i) $u(0, w, 0) \neq u(1, w, 0)$; and

(ii) $u(0, w, 1) \neq u(1, w, 1)$.

Condition (i) says that no agent is indifferent between paying back the loan and defaulting when $P = 0$, i.e., there is no agent with $\theta(w) = 0$. Similarly, (ii) says that no agent is indifferent at $P = 1$, that is, there is no agent with $\theta(w) = 1$. This assumption is made for technical convenience, since it simplifies the proof of uniqueness by implying the existence of dominance regions for ν sufficiently small. It is not needed as F approximates a continuous distribution, since the mass of agents for which it is violated becomes arbitrarily small.

Note that agents with $w < \bar{w}$ ($\theta(w) > 1$) and those with $\theta(w) < 0$ behave in a non-strategic fashion: The former agents always choose $a = 0$, and the latter choose $a = 1$, regardless of P . Hence, our focus is on pinning down the behavior of types in the set $\mathcal{W}^* := \{w \in \mathcal{W} : \theta(w) \in (0, 1)\}$, with its lowest and highest elements respectively denoted w_l and w_h .

We assume that the noise scale factor satisfies $0 < \nu < \bar{\nu} := \min \{\theta(w_h)F(\bar{w}), 1 - \theta(w_l)\}$.³¹

We first establish that there exists a unique equilibrium of the game with finite types, featuring cutoff strategies.

Theorem 1. *The game has an essentially unique equilibrium.³² Equilibrium strategies are characterized by cutoffs $k(w)$ on signal x , such that all agents of type $w \in \mathcal{W}^*$ choose action $a = 1$ if $x \geq k(w)$ and $a = 0$ otherwise.*

Proof. The proof logic is as follows. First, we argue that the set of equilibrium strategy profiles has a largest and a smallest element, each involving monotone (cutoff) strategies. Second, we show that there is at most one equilibrium in monotone strategies (up to differences in behavior at cutoff signals). But this implies that the equilibrium is essentially unique.

The existence of a smallest and a largest equilibrium profile in monotone strategies follows from existing results on supermodular games by [Milgrom and Roberts \(1990\)](#) and [Vives \(1990\)](#).

³¹This upper bound on ν is helpful to show uniqueness of equilibrium by ensuring that boundary issues associated with signals close to 0 or 1 only arise when capacity is such that all agents have a dominant strategy.

³²In the sense that equilibrium strategies may differ in zero probability events.

Consider the game in which we fix the profile \mathbf{x} of signal realizations and agents choose actions $\{0, 1\}$ after observing their own signals. It is straightforward to check that the game satisfies the conditions of Theorem 5 in [Milgrom and Roberts \(1990\)](#), which states that the game has a smallest and a largest equilibrium. That is, there exist two equilibrium strategy profiles, $\underline{\mathbf{a}}(\mathbf{x})$ and $\bar{\mathbf{a}}(\mathbf{x})$ such that any equilibrium profile $\mathbf{a}(\mathbf{x})$ satisfies $\underline{\mathbf{a}}(\mathbf{x}) \leq \mathbf{a}(\mathbf{x}) \leq \bar{\mathbf{a}}(\mathbf{x})$. Moreover, if we fix the action profile of all agents, the difference in expected payoff from choosing $a = 0$ versus $a = 1$ for any given agent is increasing in \mathbf{x} since default rates are the same across signal profiles while X is higher in expectation the higher is the signal profile, thus implying a higher expected enforcement probability. That is, expected payoffs exhibit increasing differences w.r.t. \mathbf{x} , and Theorem 6 in [Milgrom and Roberts \(1990\)](#) applies: $\underline{\mathbf{a}}(\mathbf{x})$ and $\bar{\mathbf{a}}(\mathbf{x})$ are nondecreasing functions of \mathbf{x} . But, because an agent's strategy can only depend on her own signal, all agents must be following cutoff strategies.

To show that there is at most one equilibrium in monotone strategies, we make use of the next two lemmas. The first lemma shows that equilibrium cutoffs are bounded away from zero and one. The second lemma uses these bounds to establish the following translation result: When all cutoffs are shifted by the same amount Δ , expected enforcement probabilities go up. Equipped with such results we will show that enforcement probabilities go up as we move from the smallest to the largest equilibria, implying that there must be a unique profile of cutoffs at which indifference conditions (15) are satisfied.

Let $\mathbf{k} + \Delta = (k(w) + \Delta)_{w \in \mathcal{W}^*}$, while $\underline{\mathbf{k}}$ and $\bar{\mathbf{k}}$ represent the profile of cutoffs associated with the smallest and largest equilibrium, respectively. Abusing notation, let $E[P|\mathbf{k}; x]$ represent the expected enforcement probability of an agent receiving signal x when agents use cutoff profile \mathbf{k} .

Lemma 4. *If \mathbf{k} is a profile of equilibrium cutoffs then $k(w) \in [\theta(w)F(\bar{w}) - \nu/2, \theta(w) + \nu/2]$ for all $w \in \mathcal{W}^*$.*

Proof. Note that k is an equilibrium if it solves the following set of indifference conditions:

$$E[P|\mathbf{k}; k(w)] = \theta(w) \quad \forall w \in \mathcal{W}^*. \quad (15)$$

Note also that the value of X conditional on x is at least $x - \nu/2$. Given this and the fact that enforcement probability is given by (6), we have that

$$E[P|\mathbf{k}; k(w)] \geq E[X|\mathbf{k}; k(w)] \geq k(w) - \nu/2.$$

But this implies that $E[P|\mathbf{k}; k(w)] > \theta(w)$ when $k(w) > \theta(w) + \nu/2$, a contradiction. Likewise, the value of X conditional on x is at most $x + \nu/2$. Accordingly,

$$E[P|\mathbf{k}; k(w)] \leq E\left[\frac{X}{F(\bar{w})} \mid \mathbf{k}; k(w)\right] \leq \frac{k(w) + \nu/2}{F(\bar{w})},$$

which yields the above lower bound on $k(w)$ when we replace $E[P|\mathbf{k}; k(w)]$ with $\theta(w)$. \square

Lemma 5. *If \mathbf{k} is a profile of equilibrium cutoffs then $E[P|\mathbf{k}; k(w)] < E[P|\mathbf{k} + \Delta; k(w) + \Delta]$ for all $\Delta > 0$ and all $w \in \mathcal{W}^*$ such that $k(w) + \Delta \leq \bar{k}(w)$.*

Proof. First, note that the density of X conditional on an agent receiving signal $x \in [\nu/2, 1-\nu/2]$ is given by $h\left(\frac{x-X}{\nu}\right)$. Also notice that an agent of type w defaults if she receives a signal $x < k(w)$, and thus, that the fraction of type- w agents defaulting when capacity is X is given by $H\left(\frac{k(w)-X}{\nu}\right)$. Since $\nu \leq \theta(w_h)F(\bar{w}) \leq \theta(w)F(\bar{w})$ and, by Lemma 4, $k(w) \geq \theta(w)F(\bar{w}) - \nu/2$, we have that $k(w) \geq \nu/2$. Likewise, $k(w) + \Delta \leq \bar{k}(w) \leq 1 - \nu/2$ by Lemma 4 and the fact that $\nu \leq 1 - \theta(w_l)$. Hence, we can obtain the following inequality by a well-defined change of variables:

$$\begin{aligned}
E[P(X)|\mathbf{k}; k(w)] &= \\
&\int_{-1/2}^{1/2} \min \left\{ \frac{X}{F(\bar{w}) + \sum_{w'} H\left(\frac{k(w')-X}{\nu}\right) f(w')}, 1 \right\} h\left(\frac{k(w)-X}{\nu}\right) dX \\
&< \int_{-1/2}^{1/2} \min \left\{ \frac{X + \Delta}{F(\bar{w}) + \sum_{w'} H\left(\frac{k(w')-X}{\nu}\right) f(w')}, 1 \right\} h\left(\frac{k(w)-X}{\nu}\right) dX \\
&= \int_{-1/2}^{1/2} \min \left\{ \frac{X'}{F(\bar{w}) + \sum_{w'} H\left(\frac{k(w')+\Delta-X'}{\nu}\right) f(w')}, 1 \right\} h\left(\frac{k(w)+\Delta-X'}{\nu}\right) dX' \\
&= E[P|\mathbf{k} + \Delta; k(w) + \Delta].
\end{aligned}$$

The inequality is strict because \mathbf{k} being an equilibrium profile means that $E[P|\mathbf{k}; k(w)] = \theta(w) < 1$ for all $w \in \mathcal{W}^*$. Accordingly, enforcement probabilities, conditional on $x = k(w)$, are less than 1 for a positive measure of $X \in [x - \nu/2, x + \nu/2]$, and hence, expected enforcement probabilities go up strictly when capacity increases by Δ . \square

Equipped with Lemma 5, we can argue that $\underline{\mathbf{k}} = \bar{\mathbf{k}}$. Assume, by way of contradiction, that $\underline{k}(w) < \bar{k}(w)$ for some $w \in \mathcal{W}^*$. Denote $\hat{w} = \arg \max_{w \in \mathcal{W}^*} (\bar{k}(w) - \underline{k}(w))$ and $\hat{\Delta} = \bar{k}(\hat{w}) - \underline{k}(\hat{w})$. By Lemma 5, we have that

$$\theta(\hat{w}) = E[P|\underline{\mathbf{k}}; \underline{k}(\hat{w})] < E[P|\underline{\mathbf{k}} + \hat{\Delta}; \bar{k}(\hat{w})] \leq E[P|\bar{\mathbf{k}}; \bar{k}(\hat{w})] = \theta(\hat{w}),$$

where the last inequality comes from the fact that default rates at $\bar{\mathbf{k}}$ are lower than at $\underline{\mathbf{k}} + \hat{\Delta} \geq \bar{\mathbf{k}}$, and thus, the expected enforcement probability conditional on $x = \bar{k}(\hat{w})$ is higher. \square

Next, we proceed to characterize the limit equilibrium as $\nu \rightarrow 0$. First, note that for threshold profile $k(\cdot)$ to be an equilibrium profile, it must satisfy the set of indifference conditions (15).

To solve this system of equations, we need to pin down agent beliefs when they receive their threshold signals. To do so, we make use of a generalized version of the original *belief constraint* (Sakovics and Steiner, 2012): On average, conditional on $x = k(w)$, agents with types in a subset $W' \subseteq \mathcal{W}^*$ believe that the default rate of agents in W' is uniformly distributed in $[0, 1]$. The default rate in W' when capacity is X is given by

$$\psi(X, W') = \frac{1}{\sum_{W'} f(w)} \sum_{W'} H\left(\frac{k(w) - X}{\nu}\right) f(w). \quad (16)$$

Lemma 6 (belief constraint). *For any subset $W' \subseteq \mathcal{W}^*$ and any $z \in [0, 1]$,*

$$\frac{1}{\sum_{W'} f(w)} \sum_{W'} P(\psi(X, W') \leq z | x = k(w)) f(w) = z, \quad (17)$$

where $P(\cdot | x = k(w))$ is the probability assessment of $\psi(W', X)$ by an agent receiving $x = k(w)$.

Proof. The result follows directly from the proof of Lemma 1 in [Sakovics and Steiner \(2012\)](#). To understand why, first note that Lemma 4 guarantees that threshold signals, and thus the “virtual signals” defined in their proof, fall in $[\nu/2, 1 - \nu/2]$, which is needed for the belief constraint to hold. Second, it is straightforward to check that all the arguments and results in their proof hold unmodified if we condition all the probability distributions used in the proof on the event $w \in W'$ and focus on the aggregate action of agents with types in W' rather than the aggregate action in the population.³³ \square

The previous result is instrumental in characterizing equilibrium thresholds as ν goes to zero. In particular, it allows us to derive closed-form solutions for the above indifference conditions from which we can obtain \mathbf{k} . In stating this result, we refer to a partition $\Phi = \{W_1, \dots, W_I\}$ of \mathcal{W}^* as being monotone if $\max W_i < \min W_{i+1}$, $i = 1, \dots, I - 1$, and denote the lowest and highest elements of W_i by \underline{w}_i and \bar{w}_i , respectively. Also, let $F^-(w) = \sum_{w' < w} f(w')$.

Theorem 2. *In the limit, as $\nu \rightarrow 0$, the equilibrium cutoff strategies are given by a unique monotone partition $\Phi = \{W_1, \dots, W_I\}$ and a unique vector (k_1, \dots, k_I) satisfying the following conditions:*

- (i) $k(w) = k(w') = k_i$ for all $w, w' \in W_i$.
- (ii) $k_i > k_{i+1}$ for all $i = 1, \dots, I - 1$.
- (iii) $\theta(\underline{w}_i)F^-(\underline{w}_i) \leq k_i \leq \theta(\bar{w}_i)F^-(\bar{w}_i)$ for all $i = 1, \dots, I$.
- (iv) $\int_{F^-(\underline{w}_i)}^{F^-(\bar{w}_i)} \min\left\{\frac{k_i}{z}, 1\right\} dz = \sum_{W_i} \theta(w)f(w)$ for all $i = 1, \dots, I$.

Proof. From Theorem 1 we know that for each $\nu > 0$, there exists essentially a unique equilibrium, which is in monotone strategies. Let $k^\nu(w)$ represent the equilibrium threshold of type- w agents associated with $\nu > 0$, with \mathbf{k}^ν denoting the equilibrium cutoff profile. The first step of

³³When thresholds do not fall within ν of each other, the distribution \tilde{F} of virtual errors $\tilde{\eta}$ need not be strictly increasing, and thus, its inverse may not be well defined. Defining $\tilde{F}^{-1}(u) = \inf\{\tilde{\eta} : \tilde{F}(\tilde{\eta}) \geq u\}$ takes care of this issue and ensures that the proof of Lemma 2 in [Sakovics and Steiner \(2012\)](#) applies to the general case.

the proof is to show that \mathbf{k}^ν uniformly converges as $\nu \rightarrow 0$, and to identify the set of indifference conditions that pin down the limit equilibrium. Let

$$A_w(z|\mathbf{k}^\nu, W') := P(\psi(X, W') \leq z | x = k^\nu(w))$$

denote the strategic belief of an agent of type $w \in W'$ when she receives her threshold signal $x = k^\nu(w)$.

Lemma 7. *There exist a unique partition $\{W_1, \dots, W_I\}$ and a set of thresholds $k_1 > k_2 > \dots > k_I$ such that, as $\nu \rightarrow 0$, for all $w \in W_i$, $i = 1, \dots, I$, $k^\nu(w)$ uniformly converges to k_i . Moreover, thresholds $\mathbf{k} = (k_1, \dots, k_I)$ solve the system of limit indifference conditions*

$$\int_0^1 \min \left\{ \frac{k_i}{F(\bar{w}) + \sum_{\cup_{j<i} W_j} f(w') + z \sum_{W_i} f(w')}, 1 \right\} dA_w(z|\mathbf{k}, W_i) = \theta(w), \quad \forall w \in W_i, \forall i, \quad (18)$$

where $A_w(z|\mathbf{k}, W_i)$ represents the strategic beliefs of type- w agents in the limit and satisfies the belief constraint (17).

See proof below.

Equipped with this set of indifference conditions, we next prove that the partition of types is monotone and that thresholds satisfy (iii) and (iv) in the theorem.

We show that the partition of types must be monotone by way of contradiction. Assume that there are two types $w > \hat{w}$ such that $w \in W_i$ and $\hat{w} \in W_m$ with $m > i$. First note that the LHS in (18) is bounded below by $\frac{k_i}{F(\bar{w}) + \sum_{\cup_{j \leq i} W_j} f(w')}$ and bounded above by $\min \left\{ \frac{k_i}{F(\bar{w}) + \sum_{\cup_{j < i} W_j} f(w')}, 1 \right\}$. Given this, since $\theta(\hat{w}) < 1$ the enforcement probability when all agents with types in W_m default is strictly less than 1, i.e., $\frac{k_m}{F(\bar{w}) + \sum_{\cup_{j \leq m} W_j} f(w')} < 1$. Otherwise, (18) would be violated. In addition, $m > i$ implies that $k_m < k_i$ by the above lemma and that $\sum_{\cup_{j < i} W_j} f(w') < \sum_{\cup_{j \leq m} W_j} f(w')$.

Combining all this, we arrive at the following contradiction

$$\theta(w) \geq \min \left\{ \frac{k_i}{F(\bar{w}) + \sum_{\cup_{j < i} W_j} f(w')}, 1 \right\} > \frac{k_m}{F(\bar{w}) + \sum_{\cup_{j \leq m} W_j} f(w')} \geq \theta(\hat{w}).$$

The monotonicity of the type partition implies that $F(\bar{w}) + \sum_{\cup_{j \leq i} W_j} f(w') = F(\bar{w}_i)$ and that $F(\bar{w}) + \sum_{\cup_{j < i} W_j} f(w') = F^-(\underline{w}_i)$. Given this, it is straightforward to check that the above bounds on the LHS in (18) lead to condition (iii) in the theorem.

Finally, to obtain condition (iv) from (18), we make use of the belief constraint in the limit,

which can be written as

$$\frac{1}{\sum_{W_i} f(w)} \sum_{W_i} A_w(z|\mathbf{k}, W_i) f(w) = z. \quad (19)$$

Multiplying both sides of (18) by $\frac{f(w)}{\sum_{W_i} f(w)}$ and summing over $w \in W_i$ we get

$$\int_0^1 \min \left\{ \frac{k_i}{F^-(\underline{w}_i) + z \sum_{W_i} f(w')}, 1 \right\} d \left(\frac{1}{\sum_{W_i} f(w)} \sum_{W_i} A_w(z|\mathbf{k}, W_i) f(w) \right) = \frac{\sum_{W_i} \theta(w) f(w)}{\sum_{W_i} f(w)}.$$

Using the belief constraint (19) to substitute for the last term in the LHS, we obtain

$$\int_0^1 \min \left\{ \frac{k_i}{F^-(\underline{w}_i) + z \sum_{W_i} f(w')}, 1 \right\} dz = \frac{\sum_{W_i} \theta(w) f(w)}{\sum_{W_i} f(w)}. \quad (20)$$

Note that $F^-(\underline{w}_i) + z \sum_{W_i} f(w') \sim U[F^-(\underline{w}_i), F^-(\bar{w}_i)]$ with density $\frac{1}{\sum_{W_i} f(w)}$ since $z \sim U[0, 1]$. Hence, we can rewrite (20) as

$$\frac{1}{\sum_{W_i} f(w)} \int_{F^-(\underline{w}_i)}^{F^-(\bar{w}_i)} \min \left\{ \frac{k_i}{z}, 1 \right\} dz = \frac{\sum_{W_i} \theta(w) f(w)}{\sum_{W_i} f(w)},$$

yielding condition (iv). \square

Proof of Lemma 7. To prove convergence, we first partition the set of types into subsets W_i of types for sufficiently small ν as follows: (i) if we order the signal thresholds of all types, any adjacent thresholds that are within ν of each other belong to the same subset, and (ii) $j > i$ implies that the thresholds associated with types in W_j are lower than those associated with W_i by at least ν . Also, let $Q_w^\nu(\chi|\mathbf{k}^\nu, z) := P(X \leq \chi | x = k^\nu(w), \psi(X, W_i) = z)$ represent the beliefs about capacity X of an agent of type $w \in W_i$ conditional on receiving her threshold signal $k^\nu(w)$ and on the event that the default rate in W_i is equal to z .

Note that a type- w agent receiving signal $x = k^\nu(w)$ knows that all agents with types in W_j are defaulting if $j < i$ and repaying if $j > i$. Also, the support of $Q_w^\nu(\cdot|\mathbf{k}^\nu, z)$ must lie within $[k^\nu(w) - \nu/2, k^\nu(w) + \nu/2]$. Given this, by the law of iterated expectations, her expected enforcement probability conditional on $x = k^\nu(w)$ can be written in terms of her strategic belief as follows:

$$E(P|\mathbf{k}^\nu; k^\nu(w)) = \int_0^1 \int_{k^\nu(w) - \nu/2}^{k^\nu(w) + \nu/2} \min \left\{ \frac{\chi}{F^-(\bar{w}) + \sum_{\cup_{j < i} W_j} f(w') + z \sum_{W_i} f(w')}, 1 \right\} dQ_w^\nu(\chi|\mathbf{k}^\nu, z) dA_w(z|\mathbf{k}^\nu, W_i). \quad (21)$$

In addition, notice that we can always express $E(P|\mathbf{k}^\nu; k^\nu(w))$ in terms of the threshold signal $k^\nu(w)$ and relative threshold differences $\Delta_{w'} = (k^\nu(w') - k^\nu(w))/\nu$. Importantly, as [Sakovics and Steiner \(2012\)](#) emphasize, strategic beliefs depend on the relative distance between thresholds $\Delta_{W_i} = \{\Delta_{w'}\}_{w' \in W_i}$ rather than on their absolute distance. That is, keeping Δ_{W_i} fixed, $A_w(z|\mathbf{k}^\nu, W_i)$ does not change with ν .³⁴ This implies that strategic beliefs satisfy the belief constraint when $\nu = 0$.

Fix $k^\nu(w) = k_i$ for some $w \in W_i$ and fix Δ_{W_i} , for all $i = 1, \dots, I$ and all ν sufficiently small. By fixing relative differences, the partition $\{W_i\}_{i=1}^I$ still satisfies the above definition and thus, does not change as $\nu \rightarrow 0$. We are going to show that indifference condition $E(P|\mathbf{k}^\nu; k^\nu(w)) = \theta(w)$ is approximated by the limit condition in the lemma for ν sufficiently small.

Note that the inner integral in (21) is bounded below by $\min \left\{ \frac{k^\nu(w) - \nu/2}{F(\bar{w}) + \sum_{\cup_{j < i} W_j} f(w') + z \sum_{W_i} f(w')}, 1 \right\}$

and above by $\min \left\{ \frac{k^\nu(w) + \nu/2}{F(\bar{w}) + \sum_{\cup_{j < i} W_j} f(w') + z \sum_{W_i} f(w')}, 1 \right\}$. Hence,

$$\begin{aligned} \int_0^1 \min \left\{ \frac{k_i - \nu/2}{F(\bar{w}) + \sum_{\cup_{j < i} W_j} f(w') + z \sum_{W_i} f(w')}, 1 \right\} dA_w(z|\mathbf{k}^\nu, W_i) &\leq E(P|\mathbf{k}^\nu; k^\nu(w)) \\ &\leq \int_0^1 \min \left\{ \frac{k_i + \nu/2}{F(\bar{w}) + \sum_{\cup_{j < i} W_j} f(w') + z \sum_{W_i} f(w')}, 1 \right\} dA_w(z|\mathbf{k}^\nu, W_i). \end{aligned} \quad (22)$$

The first term inside these integrals is Lipschitz continuous. In addition, the next lemma shows that $dA_w(z|\mathbf{k}^\nu, k^\nu(w))$ is bounded for all ν .

Lemma 8. $0 \leq \frac{\partial A_w(z|\mathbf{k}^\nu, k^\nu(w))}{\partial z} \leq \frac{\sum_{W_i} f(w')}{f(w)}$ for all $w \in W_i$ and all z in the support of $A_w(\cdot|\mathbf{k}^\nu, k^\nu(w))$.

See proof below.

Hence, the LHS and the RHS of (22) uniformly converge to each other as $\nu \rightarrow 0$, leading to limit indifference conditions (18). Note also that $k^\nu(w) \in [-\bar{\nu}/2, 1 + \bar{\nu}/2]$ and, keeping $\{W_i\}_{i=1}^I$ fixed, $\Delta_{w'} \in [-1, 1]$ for all $w' \in W_i$. That is, the solution to the system of indifference conditions $E(P|\mathbf{k}^\nu; k^\nu(w)) = \theta(w)$ lies in a compact set.³⁵ Accordingly, we can find $\hat{\nu}$ so that indifference conditions are within ε of the limit condition for all $\nu < \hat{\nu}$, leading to their solutions being in a neighborhood of the solution \mathbf{k} of limit indifference conditions (18). \square

³⁴This is straightforward to check. First, if we substitute $X = k^\nu(w) - \nu\eta$ (since agents with type w get her threshold signal) and $k(w') = \nu\Delta_{w'} + k^\nu(w)$ into (6), we find that $\psi(X, W_i)$ only depends on Δ_{W_i} and $k^\nu(w)$. But this means that $A_w(z|\mathbf{k}^\nu, W_i)$ only depends on Δ_{W_i} and $k^\nu(w)$ because h is independent of ν .

³⁵If $\{W_i\}_{i=1}^I$ is not kept fixed then when ν is very small, $E(P|\mathbf{k}^\nu; k^\nu(w))$ would be discontinuous at some ν , implying a violation of the indifference condition for some $w \in \mathcal{W}^*$.

Proof of Lemma 8. Let $\psi^{-1}(z, W_i)$ be the inverse function of $\psi(X, W_i)$ w.r.t. X . The latter function is decreasing in X as long as $0 < \psi(X, W_i) < 1$, implying that ψ^{-1} is well defined and decreasing in such a range of capacities. Since the signal of an agent of type w satisfies $x = X + \nu\eta$, we can express her strategic belief as

$$A_w(z|\mathbf{k}^\nu, W_i) = \mathbb{P}(\psi^{-1}(z, W_i) \leq k^\nu(w) - \nu\eta) = H\left(\frac{k^\nu(w) - \psi^{-1}(z, \mathbf{k}^\nu, W_i)}{\nu}\right).$$

Differentiating w.r.t. z yields

$$\begin{aligned} \frac{\partial A_w(z|\mathbf{k}^\nu, W_i)}{\partial z} &= \frac{1}{\nu} h\left(\frac{k^\nu(w) - \psi^{-1}(z, W_i)}{\nu}\right) \left(-\frac{\partial \psi^{-1}(z, W_i)}{\partial z}\right) \\ &= \frac{h\left(\frac{k^\nu(w) - \psi^{-1}(z, W_i)}{\nu}\right)}{\frac{1}{\sum_{W_i} f(w')} \sum_{W_i} H\left(\frac{k^\nu(w') - \psi^{-1}(z, W_i)}{\nu}\right) f(w')}. \end{aligned}$$

For all $z \in (0, 1)$, we must have $h\left(\frac{k^\nu(w) - \psi^{-1}(z, W_i)}{\nu}\right) > 0$ because h is bounded away from zero in its support. Hence, the last term is positive and weakly lower than $\frac{\sum_{W_i} f(w')}{f(w)}$. \square

A.2.2 Continuous distribution of returns

We now state our results under the assumptions introduced in text. Namely, we assume that F is an arbitrarily fine approximation of some continuous distribution satisfying Assumption 1. We do so by expressing equilibrium conditions from the previous section in terms of a continuous distribution F and solving them.

Proof of Proposition 2. As stated in the text, uniqueness should be interpreted as the existence of a unique equilibrium in nearby discrete- F economies (Theorem 1). \square

Proof of Proposition 3. To obtain the characterization of equilibrium threshold in our model, we proceed as follows. First, we express Theorem 2 in terms of continuous distributions. Second, we argue that the single peakedness of $\theta(w)F(w)$ implies the existence of a unique interval of types (\bar{w}, w^*) such that $k(w) = \theta(w^*)F(w^*)$ for types in the interval and $k(w) = \theta(w)F(w)$ for $w > w^*$. Finally, we use the conditions in the theorem to pin down w^* .

The version of Theorem 2 for continuous F implies the existence of a unique partition of types with propensity to default between 0 and 1 into intervals $\{(\underline{w}_j, \bar{w}_j)\}_{j=1}^J$ such that

- (a) if $k(w)$ is strictly decreasing in an interval i then it is constant in intervals $j - 1$ and $j + 1$ and vice versa;
- (b) if $k(w)$ is strictly decreasing in interval j then $k(w) = \theta(w)F(w)$ for all $w \in (\underline{w}_j, \bar{w}_j]$;

(c) if $\theta(w)F(w)$ is not strictly decreasing in $(\underline{w}_j, \bar{w}_j]$ then $k(w) = k_j$ for all $w \in (\underline{w}_j, \bar{w}_j]$ with k_j satisfying $k_j = \theta(\bar{w}_j)F(\bar{w}_j) \geq \theta(\underline{w}_j)F(\underline{w}_j)$ (with equality if $\underline{w}_j > \bar{w}$) and

$$\int_{F(\underline{w}_j)}^{F(\bar{w}_j)} \min \left\{ \frac{k_j}{z}, 1 \right\} dz = \int_{\underline{w}_j}^{\bar{w}_j} \theta(w)f(w)dw. \quad (23)$$

Part (a) follows from conditions (i)-(ii) in Theorem 2, meaning that k is decreasing, so we can partition the space of types into a collection of successive intervals in which k alternates between being strictly decreasing and constant. Part (b) follows from (ii)-(iii): a strictly decreasing k in a given interval of types is approximated by a (growing) collection of consecutive, singleton W_i in the discrete economy. But then, as the mass associated with each of these singletons goes to zero, F^- approximates F , and condition (iii) implies that k converges to $\theta(w)F(w)$.

Part (c) follows from parts (a) and (b) and conditions (iii) and (iv). Since $\theta(w)F(w)$ is continuous, parts (a) and (b) imply that $k(\bar{w}) = \theta(w)F(w)$ at the boundaries of an interval in which k is constant, except possibly when $\underline{w}_i = \bar{w}$, in which case condition (iii) requires that $k_j \geq \theta(\underline{w}_j)F(\underline{w}_j)$. Expression (23) is the continuous counterpart of (iv).

We now argue that the single peakedness of $\theta(w)F(w)$ (Lemma 3 in Appendix I) leads to a partition consisting of two intervals, the first one in which k is constant and the second one in which it is strictly decreasing.

First, notice that k decreasing implies that there must be at least one pooling threshold because $\theta(w)F(w)$ is initially increasing. To show why there is only one, we use the fact that condition (c) requires that $k_1 = \theta(\bar{w}_1)F(\bar{w}_1) \geq \theta(\underline{w}_1)F(\underline{w}_1)$. Given the single peakedness of $\theta(w)F(w)$ and $k(w)$ being decreasing, we must have that $\theta(w)F(w)$ is increasing at \underline{w}_1 and decreasing at \bar{w}_1 . Otherwise, either $\theta(w)F(w)$ is decreasing at \underline{w}_1 or $\theta(w)F(w)$ is increasing in $(\underline{w}_1, \bar{w}_1)$. The former case implies that $\theta(\bar{w}_1)F(\bar{w}_1) < \theta(\underline{w}_1)F(\underline{w}_1)$, violating (c). The latter case implies that $k_1 = \max_{w \in [\underline{w}_1, \bar{w}_1]} \theta(w)F(w)$, which implies that the LHS of (23) is greater than the RHS.³⁶

Accordingly, by single peakedness, if $\theta(w)F(w)$ is increasing at \underline{w}_1 and decreasing at \bar{w}_1 we cannot find another interval satisfying the same condition that does not intersect with $[\underline{w}_i, \bar{w}_i]$. Thus, there must be a unique interval of returns at which k is constant. Finally, since $\theta(w)F(w)$ is increasing in $[\bar{w}, \underline{w}_1]$, the monotonicity of $k(w)$ requires that $\underline{w}_1 = \bar{w}$.

We finish the characterization of equilibrium thresholds by showing that $\bar{w}_1 = w^*$, where w^* is the unique solution to (10) in (\bar{w}, ∞) when $\bar{w} < w_{max}$.

Condition (c) implies that $k_1 \geq F(\bar{w})$. Hence, solving the integral and substituting for

³⁶Since $k_1 = \max_{w \in [\underline{w}_1, \bar{w}_1]} \theta(w)F(w)$ we have that

$$\int_{F(\underline{w}_1)}^{F(\bar{w}_1)} \min \left\{ \frac{k_1}{z}, 1 \right\} dz > \int_{F(\underline{w}_1)}^{F(\bar{w}_1)} \min \left\{ \frac{\theta(F^{-1}(z))z}{z}, 1 \right\} dz = \int_{\underline{w}_j}^{\bar{w}_j} \theta(w)f(w)dw,$$

where the last equality comes from the change in variable $w = F^{-1}(z)$ ($dz = f(w)dw$).

$k_1 = \theta(w^*)F(w^*)$ and $\underline{w}_1 = \bar{w}$, we can express the LHS of (23) as

$$\int_{k_i}^{F(w^*)} \frac{k_i}{z} dz + \int_{F(\bar{w})}^{k_i} dz = k_i \log \left(\frac{F(w^*)}{k_i} \right) + k_i - F(\bar{w}) = \theta(w^*)F(w^*)(1 - \log \theta(w^*)) - F(\bar{w}).$$

Equating the RHS of the last expression to the RHS of (23) yields (10). To show that it has a unique solution in (\bar{w}, ∞) , we express it as

$$\theta(w^*)F(w^*)(1 - \log \theta(w^*)) - \int_{\bar{w}}^{w^*} \theta(w)f(w)dw = F(\bar{w}), \quad (24)$$

and differentiate the LHS w.r.t. w^* , which yields $(-\log \theta(w^*)) (\theta'(w^*)F(w^*) + \theta(w^*)f(w^*))$. The first term in this expression is positive, while the second term is the slope of $\theta(w^*)F(w^*)$, which is first positive and then negative in $[\bar{w}, \infty)$ when $\bar{w} < w_{max}$. That is, the LHS is first increasing and then decreasing in $[\bar{w}, \infty)$. Hence, since the RHS is constant, the above expression has at most two solutions in $[\bar{w}, \infty)$. But notice that \bar{w} is always a solution and that the LHS is increasing at \bar{w} if $\bar{w} < w_{max}$. This, combined with the fact that the LHS approaches zero as w grows while the RHS is strictly positive, implies that there exists a unique solution in (\bar{w}, ∞) .

Obviously, if $\bar{w} \geq w_{max}$ then $\theta(w)F(w)$ is strictly decreasing for $w \geq \bar{w}$, and conditions (a)-(c) lead to $k(w) = \theta(w)F(w)$, i.e., to $w^* = \bar{w}$. \square

Before proving Proposition 4, we need to establish that the lenders' problem has an interior solution.

Lemma 9. *The solution to (12) involves finite b for all $X \in [0, 1]$.*

Proof. To prove that lenders always set $b < \infty$, first note that Proposition 3 implies the existence of a type $\hat{w} \geq \bar{w}$ such that $a = 0$ if $w < \hat{w}$ and $a = 1$ otherwise. Therefore, we can express the budget constraint (13) as follows:

$$\frac{b}{y+b} \leq \int_{\hat{w}}^{\infty} \bar{w} dF(w) + \mu (P + (1-\gamma)(1-P)) \int_0^{\hat{w}} w dF(w). \quad (25)$$

The LHS converges to one as $b \rightarrow \infty$. Hence, we need to show that the RHS is strictly less than 1 for all \bar{w} . Since $\hat{w} \geq \bar{w}$ and the RHS is increasing in P for fixed \hat{w} , the RHS is bounded above by

$$\int_{\hat{w}}^{\infty} \hat{w} dF(w) + \mu \int_0^{\hat{w}} w dF(w), \quad (26)$$

which is strictly less than one for all \hat{w} by Assumption 2. \square

Proof of Proposition 4. To prove that $X^*(b) = \theta(w^*)F(w^*)$ is increasing in b , we first show that $\theta(w^*)F(w^*)$ is increasing in \bar{w} . Note that the propensity to default θ goes up with \bar{w} and that $\theta(w)F(w)$ is decreasing at w^* . Given this, if we can show that w^* goes down after an increase

in \bar{w} then we would have proven that $\theta(w^*)F(w^*)$ increases with \bar{w} . We do so by implicitly differentiating (24):

$$\frac{\partial LHS}{\partial w^*} \frac{dw^*}{d\bar{w}} = \int_{\bar{w}}^{w^*} \frac{\partial \theta(w)}{\partial \bar{w}} f(w) dw.$$

From the previous argument, we know that $\frac{\partial LHS}{\partial w^*} < 0$, while the RHS of the last expression is positive since $\frac{\partial \theta(w)}{\partial \bar{w}} > 0$. Hence, it must be that $\frac{dw^*}{d\bar{w}} < 0$.

Since $X^*(b)$ is increasing in \bar{w} , we only need to show that the equilibrium \bar{w} is strictly increasing in b when $X = X^*(b)$. To do so, we first note that the budget constraint (25) must hold with equality when $X = X^*(b)$. To see why, notice that the propensity to default $\theta(\cdot)$ and hence $k(\cdot)$ does not depend on b by Proposition 1. Accordingly, for any given \bar{w} , the objective function (12) is strictly increasing in b since P , $\{w : a = 0\}$ and $\{w : a = 1\}$ are constant for all b , whereas payoffs under both repayment and default are strictly increasing in b . Hence, since the LHS of budget constraint (25) is strictly increasing in b , for any given \bar{w} , lenders choose b so that the budget constraint binds.

Given that the budget constraint holds with equality and the LHS is increasing in b , we only need to show that the RHS of (25) is increasing at the equilibrium \bar{w} when $X = X^*(b)$. We do so by contradiction. Assume that the RHS is strictly decreasing in \bar{w} at the equilibrium contract associated with X . Since $X = X^*(b)$, we must have that all agents with $w < \bar{w}$ default and those with $w \geq \bar{w}$ repay, implying that $P = \min\{1, X/F(\bar{w})\}$ (see point (c) in the proof of Proposition 3). In this context, lowering \bar{w} while increasing b is feasible, since lenders' revenue goes up after a reduction in \bar{w} (the RHS is strictly decreasing by assumption) while the cluster threshold goes down, so $X \geq \theta(w^*)F(w^*)$ is still satisfied at the new contract. But notice that such an alternative contract strictly increases agents' expected payoffs since it increases gross returns while reducing the deadweight loss of defaults, which are reverted back to agents' payoffs given that the zero profit condition binds. Accordingly a lender can deviate and offer a contract close to this alternative contract and make positive profits, contradicting that the original contract is an equilibrium.³⁷ Hence, b must be increasing in \bar{w} at the equilibrium contract associated with X . \square

³⁷Formally, agents' payoffs are given by $\int_{\bar{w}}^{\infty} (y+b)(w-\bar{w})dF$ when $P = 1$. Since the budget constraint holds with equality, we can express it as

$$\int_{\bar{w}}^{\infty} (y+b)\bar{w}dF + \mu \int_0^{\bar{w}} (y+b)wdF.$$

Hence, agents' payoffs can be written as

$$b + (y+b) \left[\int_{\bar{w}}^{\infty} wdF - \mu \int_0^{\bar{w}} wdF \right],$$

which strictly go up if we increase b and lower \bar{w} .

References

- Agarwal, Sumit, Gene Amromin, Itzhak Ben-David, Souphala Chomsisengphet, Tomasz Piskorski, and Amit Seru, “Policy Intervention in Debt Renegotiation: Evidence from the Home Affordable Modification Program,” *Journal of Political Economy*, 2017, 125 (3), 654–712.
- Angeletos, George-Marios and Iván Werning, “Crises and Prices: Information Aggregation, Multiplicity, and Volatility,” *American Economic Review*, 2006, pp. 1720–1736.
- , Christian Hellwig, and Alessandro Pavan, “Signaling in a Global Game: Coordination and Policy Traps,” *Journal of Political Economy*, 2006, 114, 452–484.
- Arellano, Cristina and Narayana Kocherlakota, “Internal Debt Crises and Sovereign Defaults,” *Journal of Monetary Economics*, 2014, 68, S68–S80.
- Bae, Hee-Hong and Vidhan K Goyal, “Creditor Rights, Enforcement, and Bank Loans,” *Journal of Finance*, 2009, 64 (2), 823–860.
- Bank of Italy, “Financial Stability Report,” Technical Report 5 April 2013.
- Bassetto, Marco and Christopher Phelan, “Tax Riots,” *Review of Economic Studies*, 2008, 75 (3), 649–669.
- Bernanke, Ben S., Mark Gertler, and Simon Gilchrist, “The Financial Accelerator in a Quantitative Business Cycle Framework,” *Handbook of Macroeconomics*, 1999, 1, 1341–1393.
- Bond, Philip and Ashok S. Rai, “Borrower Runs,” *Journal of Development Economics*, 2009, 88 (2), 185–191.
- and Kathleen Hagerty, “Preventing Crime Waves,” *American Economic Journal: Microeconomics*, 2010, 2 (3), 138–159.

- Bris, Arturo, Ivo Welch, and Ning Zhu**, “The Costs of Bankruptcy: Chapter 7 Liquidation versus Chapter 11 Reorganization,” *Journal of Finance*, 2006, 61 (3), 1253–1303.
- Carpinelli, Luisa, Giuseppe Cascarino, Silvia Giacomelli, and Valerio Paolo Vacca**, “The Management of Non-Performing Loans: A Survey Among The Main Italian Banks,” *Questioni di Economia e Finanza*, February 2016, (311).
- Carrasco, Vinicius and Pablo Salgado**, “Coordinated Strategic Defaults and Financial Fragility in a Costly State Verification model,” *Journal of Financial Intermediation*, 2014, 23 (1), 129–139.
- Chan, Sewin, Andrew Haughwout, Andrew Hayashi, and Wilbert Van der Klaauw**, “Determinants of Mortgage Default and Consumer Credit Use: The Effects of Foreclosure Laws and Foreclosure Delays,” *Journal of Money, Credit and Banking*, 2016, 48 (2-3), 393–413.
- Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno**, “Risk Shocks,” *American Economic Review*, 2014, 104 (1), 27–65.
- Cordell, Larry, Liang Geng, Laurie Goodman, and Lidan Yang**, “The Cost of Foreclosure Delay,” *Real Estate Economics*, 2015, 43 (4), 916–956.
- Dávila, Eduardo and Anton Korinek**, “Pecuniary Externalities in Economies with Financial Frictions,” *Review of Economic Studies*, 2017, *Forthcoming*.
- Djankov, Simeon, Caralee McLiesh, and Andrei Shleifer**, “Private Credit in 129 Countries,” *Journal of Financial Economics*, 2007, 84 (2), 299–329.
- , **Oliver Hart, Caralee McLiesh, and Andrei Shleifer**, “Debt Enforcement Around the World,” *Journal of Political Economy*, 2008, 116 (6), 1105–1149.

- Favara, Giovanni, Enrique Schroth, and Philip Valta**, “Strategic Default and Equity Risk Across Countries,” *Journal of Finance*, 2012, *67* (6), 2051–2095.
- Frankel, David M.**, “Recurrent Crises in Global Games,” *Journal of Mathematical Economics*, 2012, *48*, 309–321.
- , “Efficient Ex-Ante Stabilization of Firms,” *Journal of Economic Theory*, 2017, *170*, 112–144.
- , **Stephen Morris, and Ady Pauzner**, “Equilibrium Selection in Global Games with Strategic Complementarities,” *Journal of Economic Theory*, 2003, *108* (1), 1–44.
- Gale, Douglas and Martin Hellwig**, “Incentive-Compatible Debt Contracts: The One-Period Problem,” *Review of Economic Studies*, 1985, *52* (4), 647–663.
- Gomes, Joao F, Urban J Jermann, and Lukas Schmid**, “Sticky Leverage,” *American Economic Review*, 2016, *106* (12), 3800–3828.
- Goodstein, Ryan, Paul Hanouna, Carlos D. Ramirez, and Christof W. Stahel**, “Contagion Effects in Strategic Mortgage Defaults,” *Journal of Financial Intermediation*, 2017, *30*, 50 – 60.
- Gupta, Arpit**, “Foreclosure Contagion and the Neighborhood Spillover Effects of Mortgage Defaults,” *Working Paper*, 2016.
- Herkenhoff, Kyle F. and Lee Ohanian**, “The Impact of Foreclosure Delay on U.S. Unemployment,” *NBER Working Paper No. 21532*, 2015.
- Ippolito, Filippo, José-Luis Peydro, Andrea Polo, and Enrico Sette**, “Double Bank Runs and Liquidity Risk Management,” *Journal of Financial Economics*, 2016, *122* (1), 135–154.

- Iverson, Benjamin**, “Get in Line: Chapter 11 Restructuring in Crowded Bankruptcy Courts,” *Management Science*, 2017, *Forthcoming*.
- Jappelli, Tullio, Marco Pagano, and Magda Bianco**, “Courts and Banks: Effects of Judicial Enforcement on Credit Markets,” *Journal of Money, Credit, and Banking*, 2005, *37* (2), 223–244.
- Krasa, Stefan and Anne P. Villamil**, “Optimal Contracts when Enforcement Is a Decision Variable,” *Econometrica*, 2000, *68* (1), 119–134.
- La Porta, Rafael, Florencio Lopez de Silanes, Andrei Shleifer, and Robert W Vishny**, “Law and Finance,” *Journal of Political Economy*, 1998, *106* (6), 1113–1155.
- Mayer, Christopher, Edward Morrison, Tomasz Piskorski, and Arpit Gupta**, “Mortgage Modification and Strategic Behavior: Evidence from a Legal Settlement with Country-wide,” *American Economic Review*, 2014, *104* (9), 2830–2857.
- Milgrom, Paul and John Roberts**, “Rationalizability, Learning, and Equilibrium in Games with Strategic Complementarities,” *Econometrica*, 1990, *58* (6), 1255–1277.
- Mookherjee, Dilip and Ivan Png**, “Optimal Auditing, Insurance, and Redistribution,” *The Quarterly Journal of Economics*, 1989, pp. 399–415.
- Morris, Stephen and Hyun Song Shin**, “Global Games: Theory and Applications,” in “Proceedings of the Eighth World Congress of the Econometric Society,” Cambridge University Press, 2003.
- Ponticelli, Jacopo**, “Court Enforcement, Bank Loans and Firm Investment: Evidence from a Bankruptcy Reform in Brazil,” *Quarterly Journal of Economics*, 2016, *131* (3), 1365–1413.

- Rodano, Giacomo, Nicolas Serrano-Velarde, and Emanuele Tarantino**, “Bankruptcy Law and Bank Financing,” *Journal of Financial Economics*, 2016, *120* (2), 363–382.
- Safavian, Mehnaz and Siddharth Sharma**, “When Do Creditor Rights Work?,” *Journal of Comparative Economics*, 2007, *35* (3), 484–508.
- Sakovics, Jozsef and Jakub Steiner**, “Who Matters in Coordination Problems?,” *American Economic Review*, 2012, *102* (7), 3439–3461.
- Schiantarelli, Fabio, Massimiliano Stacchini, and Philip E Strahan**, “Bank Quality, Judicial Efficiency and Borrower Runs: Loan Repayment Delays in Italy,” *NBER Working Paper 22034*, 2016.
- Shleifer, Andrei and Robert Vishny**, “Fire Sales in Finance and Macroeconomics,” *Journal of Economic Perspectives*, 2011, *25* (1), 29–48.
- Townsend, Robert**, “Optimal Contracts and Competitive Markets with Costly State Verification,” *Journal of Economic Theory*, 1979, *21* (2), 265–293.
- Vives, Xavier**, “Nash Equilibrium with Strategic Complementarities,” *Journal of Mathematical Economics*, 1990, *19* (3), 305–321.
- Zhu, Shuang and Kelley R Pace**, “The Influence of Foreclosure Delays on Borrowers’ Default Behavior,” *Journal of Money, Credit and Banking*, 2015, *47* (6), 1205–1222.