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BANKING PANICS AND PROTRACTED RECESSIONS

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Abstract

This paper develops a dynamic theory of money and banking that explains why banks need to hold an illiquid portfolio to provide socially optimal transaction and liquidity services, opening the door to the possibility of equilibrium banking panics. Following a widespread liquidation of banking assets in the event of a panic, the banking portfolio consistent with the optimal provision of transaction and liquidity services during normal times cannot be quickly reestablished, resulting in an unusual loss of wealth for all depositors. This negative wealth effect stemming from the liquid portion of the consumers' portfolio is strong enough to produce a protracted recession. A key element of the theory is the existence of a dynamic interaction between the ability of banks to offer transaction and liquidity services and the occurrence of panics.

Keywords: banking panics, medium of exchange, random matching, transaction services, liquidity insurance

JEL Classifications: E32, E42, G21

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1. INTRODUCTION

Economists usually refer to a sudden and apparently unexpected withdrawal of funds from banks as a banking panic. For example, Calomiris and Gorton (1991) define a banking panic as an event in which numerous depositors suddenly choose to exercise the option of converting their checkable deposits into currency from a significant number of banks in the banking system to such an extent that these banks suspend convertibility. It is a consensus among economists that banking panics are events usually associated with inefficient economic outcomes. For instance, Bordo, Eichengreen, Klingebiel, and Martinez-Peria (2001) have conducted an extensive cross-country study of the incidence and costs of banking crises spanning 120 years of financial history, concluding that output losses associated with problems in the banking system are substantial.

Although researchers usually agree that panics are costly events, they largely disagree with respect to the way in which problems in the banking system affect real economic activity. A fundamental issue in this debate is whether a particular channel of transmission studied in the literature is capable of explaining the usual severity and persistence of recessions associated with banking crises, as documented in Boyd, Kwak, and Smith (2005) and Reinhart and Rogoff (2009). Specifically, it remains a challenge to provide a theory that identifies the occurrence of a panic as an event capable of generating a severe and protracted recession.

A distinct transmission channel in the literature is that identified by Friedman and Schwartz (1963) in their seminal account of banking and monetary developments in the United States from 1867 to 1960. These authors have argued that the main implication of a banking panic is its tendency to generate a contraction of the balance sheet of the banking system which usually contributes to depress real economy activity. In particular, they have formulated the hypothesis that the collapse of the banking system starting in October 1930 and ending in March 1933 with the week-long national banking holiday significantly contributed to the severity and persistence of the contraction in output during the Great Depression. At the heart of their argument lies the view that widespread bank failures substantially affect the ability of banks to provide essential transaction and liquidity services to households and firms, so the Friedman-Schwartz transmission mechanism emphasizes the services provided on the liability side of banks' balance sheet.

In an influential article, Bernanke (1983) has argued that the Friedman-Schwartz transmission mechanism is not capable of explaining the persistence of the decline in output observed during the Great Depression. As a possible explanation for this phenomenon, Bernanke has proposed the so-called credit channel, emphasizing the services provided on the asset side of banks' balance sheet. This channel has been subsequently formalized in Bernanke and Gertler (1989), initiating a vast literature on the relevance of the credit channel as a source of aggregate fluctuations. In contrast to the credit channel, the Friedman-Schwartz transmission mechanism has not been successfully rationalized as a formal theory of money and banking, so it cannot be properly tested.

The goal of this paper is to formalize the view that the occurrence of a banking panic can be a prominent source of aggregate fluctuations through its effects on the ability of banks to provide transaction and liquidity services. In the theory of money and banking developed below, depositors do not need to withdraw their funds from the banking system for transaction purposes since one of the main functions of banks is to provide transaction services in the form of interest-bearing transferable deposits. However, depositors may sometimes need to withdraw their funds for other reasons, such as the possibility of relocation, so the withdrawal option remains a socially desirable characteristic of the deposit contract. In this sense, banks also provide liquidity insurance (or liquidity services) to depositors, in addition to transaction services.

I study the conditions under which the socially optimal provision of transaction and liquidity services requires the construction of an illiquid banking portfolio, opening the door to the possibility of equilibrium banking panics, and show that the collapse of the banking system due to systemwide bank failures necessarily results in a protracted recession. Following a widespread liquidation of banking assets in the event of a panic, the interest-bearing banking portfolio consistent with normal times cannot be quickly reestablished to offer socially optimal transaction and liquidity services, resulting in an unusual loss of wealth for all depositors that induces them to inefficiently reduce their current and future expenditures. Moreover, the theory explains why the occurrence of panics in consecutive periods, such as the series of systemic runs observed from 1930 to 1933 in the U.S., depresses overall economic activity in an unusual way.

The analysis in this paper builds on two apparently distinct strands of the literature on money and banking. The first focuses on the study of panics as an equilibrium outcome under rational expectations. The seminal contributions of Bryant (1980) and Diamond and Dybvig (1983) have initiated a vast literature on the real effects of panics.[1] However, the vast majority of papers in this literature does not account for the fact that bank liabilities are widely used as a medium of exchange. The second strand focuses precisely on the role of money and other assets as a medium of exchange, following the seminal contribution of Kiyotaki and Wright (1989). In an important contribution to this literature, Cavalcanti, Erosa, and Temzelides (1999) have modified the original Kiyotaki-Wright framework to study inside money creation (in the form of bank notes), with subsequent papers expanding their analysis. However, the connection between the ability of banks to provide transaction and liquidity services and the possibility of panics has not been established.

More recently, some researchers have taken a monetary approach to banking, explicitly accounting for the fact that bank liabilities serve as a medium of exchange. A prominent paper taking this approach is that of Gu, Mattesini, Monnet, and Wright (2013), who study inside money creation in the form of bank deposits that serve as a means of payment. However, there is nothing in their analysis that resembles a banking panic. In this paper, I build on their basic framework and introduce some other elements to create a socially useful role for a demand deposit contract, as in Diamond and Dybvig (1983). As should be expected, because these elements generate a socially beneficial role for the provision of liquidity insurance by the banking system, in addition to the provision of transaction services, they also open the door to the possibility of self-fulfilling panics.

Why is a demand deposit contract socially useful in my framework? In my version of

[1] Some prominent papers in this literature include Postlewaite and Vives (1987), Wallace (1988), Cooper and Ross (1998), Goldstein and Pauzner (2005), Ennis and Keister (2006), and Andolfatto and Nosal (2008).

Gu, Mattesini, Monnet, and Wright (2013), a typical agent may need to hold currency for a short period of time because of the possibility of being randomly relocated to a distant region, as in Champ, Smith, and Williamson (1996). The existence of an idiosyncratic relocation shock, combined with imperfect communication across distant regions, precludes the transfer of claims on the banking system in one region to the banking system in other regions, so an agent who needs to relocate to a different region has to withdraw currency from the banking system. Thus, some agents (movers) withdraw currency because it serves as a *temporary* store of value that allows them to transfer their wealth across distant regions. Thus, the withdrawal option in the deposit contract is socially useful because it provides insurance against the relocation risk.

It is important to keep in mind that, in my framework, a demand for currency does not arise from the need to make payments. In fact, agents do not need to withdraw currency from the banking system for transaction purposes because it is possible to transfer claims on bank accounts within the *same* geographical region to settle retail transactions. Unlike the original framework of Champ, Smith, and Williamson (1996), an agent who needs to relocate to a different region has the option of redepositing his balance in the banking system in that region (prior to engaging in retail transactions there) to benefit from a potentially higher rate of return on deposits. Because the total number of movers in each region is relatively small, these random relocations *per se* do not cause a panic as the desired level of reserves in the banking system can be easily reestablished in each region, provided that *only* movers are allowed to withdraw.

A banking system is essential in my analysis because it has the ability to supply a payment instrument with a higher purchasing power than currency (*transaction services*) and at the same time has the ability to provide insurance against the relocation risk (*liquidity services*). Because the liabilities of banks are partially backed by interest-bearing assets, it is possible to issue, in the case of perfect competition in the banking sector, an interest-bearing payment instrument widely accepted in transactions. The withdrawal option allows a depositor to transfer his wealth across regions in the event of relocation so that he or she can potentially benefit from the payment of interest on deposits.

The problem with this trading arrangement is that, under certain circumstances, the banking system needs to hold an illiquid portfolio to be able to provide socially useful transaction and liquidity services, which opens the door to the possibility of self-fulfilling costly banking panics when *nonmovers* rationally decide to withdraw for fear of widespread bank failures. Thus, if banks are unable to observe an agent's relocation status, they cannot differentiate the depositors who have a legitimate motive for exercising the withdrawal option from those who have a speculative motive. A key result in my analysis is that the occurrence of panics in equilibrium depends on the availability of productive projects. If the economy's productive capacity is sufficiently large, the banking system is able to provide socially optimal transaction and liquidity services by holding a liquid portfolio not subject to panics (i.e., a portfolio such that depositors who do not need to move do not have an incentive to withdraw as a result of self-fulfilling beliefs). In this case, a socially efficient allocation is the unique equilibrium outcome.

If the economy's productive capacity is relatively small, then the banking system is able to provide socially optimal transaction and liquidity services *only if* it holds an illiquid portfolio. In this case, there exists an equilibrium with the property that banking panics eventually occur and result in the destruction of wealth for all depositors, not only for those who suffer direct losses during the liquidation of the assets of the banking system. At a panic date, the banking system cannot offer valuable transaction services since it is unable to immediately rebuild its interest-bearing portfolio, so no consumer receives interest payments on his money holdings. The ensuing widespread wealth loss is strong enough to generate a recession as consumers immediately reduce their expenditures in retail transactions. Moreover, a panic episode generates a protracted recession because it takes time to rebuild the banking portfolio consistent with the optimal provision of transaction and liquidity services during normal times, so consumption does not quickly recover to the level consistent with normal times. Finally, I show that the occurrence of panics in consecutive periods depresses overall economic activity in an unusual way.

2. RELATED LITERATURE

My paper is certainly not the first in the literature to study panics in a dynamic framework. For instance, Ennis and Keister (2003) study the effects of bank runs on the levels of the capital stock and output in an endogenous growth model. More recently, Martin, Skeie, and von Thadden (2014a, 2014b) construct an infinite-horizon model of financial institutions that borrow short-term and invest in long-term assets, so they are subject to runs. However, in these papers, banks do not provide transaction services, and there is no clear mechanism capable of explaining a protracted recession caused by banking panics.

Chari and Phelan (2014) study the role of fractional reserve banking in providing useful transaction services to households and evaluate its social benefits and costs. Although their analysis is very interesting, Nosal (2014) has pointed out two important caveats. First, the demand deposit contract in the Chari-Phelan framework is not optimal (in fact, Nosal argues that it is not even an equilibrium contract) and bank runs can be easily avoided by designing an alternative contract. Second, the household's choice of payment instruments (currency versus bank deposits) is derived in an environment in which institutions are unable to respond to policy changes because of an exogenously imposed payments-in-advance constraint, which raises serious concerns about their policy recommendations.

In my analysis, I choose to explicitly model the frictions that make trade difficult, so inside money and banks arise endogenously to help mitigate trading frictions. This approach is consistent with the class of models referred to as New Monetarist Economics models; see, for instance, Nosal and Rocheteau (2011) and Williamson and Wright (2011).

3. MODEL

Time $t = 0, 1, 2, ...$ is discrete, and the horizon is infinite. Each period is divided into three subperiods or stages. There exist two symmetric regions that are identical with respect to all fundamentals. There is no communication between these regions. In each region, there are three types of agents, referred to as buyers, sellers, and bankers, who are infinitely lived. There is a $[0, 1]$ continuum of each type in each region.

Agents in each region interact as follows. In the first stage, the group of buyers and the group of bankers get together in a centralized meeting. In the second stage, each buyer is randomly and bilaterally matched with a seller with probability $\lambda \in \left(\frac{1}{2}, 1\right)$. In the third stage, the group of sellers and the group of bankers get together in a centralized meeting. Thus, each type is able to interact with the other two types at each date.

At each date, a fraction $\varepsilon \in [0,1]$ of buyers in one region is randomly relocated to the other region and vice versa. I refer to a buyer who is relocated as a mover and to a buyer who is not relocated as a nonmover. A buyer finds out whether he is going to be relocated at the end of the first stage, and the actual relocation occurs shortly after the idiosyncratic shock is realized. Unless otherwise explicitly stated, the relocation status of a buyer is privately observed.

There are two perfectly divisible commodities, referred to as good x and good y. A buyer is able to produce good x in the first subperiod. The available technology allows him to produce either zero units or one unit of good x. If good x is not properly stored in the subperiod it is produced, it will depreciate completely. All buyers have access to an *indivisible* storage technology for good x, which can be costlessly liquidated at any moment. In particular, a buyer can store either one unit or nothing. A seller is able to produce good y in the second subperiod. Good y is perishable and cannot be stored, so it must be consumed in the subperiod it is produced.

A banker is unable to produce either good but has access to a *divisible* technology that uses good x as input in the first subperiod and pays off at the beginning of the following date. Let $F(i)$ denote the payoff in terms of good x when $i \in \mathbb{R}_+$ is the amount invested. It follows that

$$F(i) = \begin{cases} (1+\rho)\, i \text{ if } 0 \leq i \leq \bar{\imath}, \\ (1+\rho)\, \bar{\imath} \text{ if } \bar{\imath} < i \leq 1, \end{cases}$$

with $\rho > 0$ and $\frac{1-\lambda}{1+\rho} \leq \bar{\imath} \leq 1-\lambda$. If prematurely liquidated, the technology returns $\delta < 1$. Assume $\delta + \rho > 1$. In addition, a banker has access to a perfectly *divisible* storage technology for good x, which can be costlessly liquidated at any moment. A banker is also able to access a technology that allows him to costlessly create (and destroy) an indivisible, durable, and portable object, referred to as bank money, that perfectly identifies him as an issuer. Figure 1 provides a timeline describing the sequence of events within a period.

[**Figure 1**]

Let me now describe preferences. A buyer is a consumer of good y, whereas a banker and a seller are consumers of good x. Let $x_t \in \{0, 1\}$ denote a buyer's production of good x at date t, and let $y_t \in \mathbb{R}_+$ denote his consumption of good y at date t. A buyer's preferences are represented by

$$-\gamma x_t + u(y_t),$$

where $\gamma \in \mathbb{R}_+$ and $u : \mathbb{R}_+ \to \mathbb{R}_+$ is continuously differentiable, increasing, and strictly concave, with $u(0) = 0$ and $u'(0) = \infty$. As previously mentioned, the production technology of good x allows a buyer to produce either zero units or one unit of good x at each date. But keep in mind that good x is perfectly divisible.

Let $y_t \in \mathbb{R}_+$ denote a seller's production of good y at date t, and let $x_t \in \mathbb{R}_+$ denote his consumption of good x at date t. A seller's preferences are represented by

$$v(x_t) - w(y_t),$$

where $v : \mathbb{R}_+ \to \mathbb{R}_+$ is continuously differentiable, strictly increasing, and concave, with $v(0) = 0$, and $w : \mathbb{R}_+ \to \mathbb{R}_+$ is continuously differentiable, strictly increasing, and convex, with $w(0) = 0$. Let $\beta \in (0, 1)$ denote the common discount factor for buyers and sellers. Assume $\beta(1 + \rho) > 1$.

A banker derives instantaneous utility x_t at date t if his consumption of good x at date t is given by $x_t \in \mathbb{R}_+$. Let $\hat{\beta} \in (0, 1)$ denote the banker's discount factor. Assume $\hat{\beta}(1 + \rho) \leq 1$.

4. EXCHANGE MECHANISM

To describe the exchange process in this economy, it is easier to start with the second stage. In this stage, a buyer is randomly matched with a seller with probability λ. A buyer wants good y but is unable to produce good x for a seller at that time. The use of personal credit is impossible because the pair will never meet again with a continuum of agents. The pair can trade if, for instance, the buyer has good x in storage. If a buyer wishes to hold good x in storage to trade with a seller, we can say that good x is used as commodity money. For simplicity, I refer to commodity money as currency. Is this trading arrangement socially desirable? Under a pure currency regime, a buyer needs to produce a commodity and hold it in inventory until he finds a trading partner. As a result, agents hold, at any point in time, an excessive amount of inventories for transaction purposes. These inventories could be either consumed or productively invested.

A superior monetary arrangement can be obtained if a group of bankers is willing to provide a medium of exchange that serves as an alternative to currency. This mechanism is essentially the same as that described in Gu, Mattesini, Monnet, and Wright (2013). Note that a banker is able to interact with the group of buyers in the first stage and with the group of sellers in the third stage. In the first stage, a buyer can produce one unit of good x and "deposit" it with a banker. In exchange for his deposit, the banker gives him a durable and indivisible object, referred to as bank money, that certifies the amount originally deposited plus any interest payment and entitles the bearer to receive this amount in the third stage. If a seller is willing to accept a privately issued claim in exchange for his output, then he is able to redeem this claim (i.e., convert a privately issued liability into a certain amount of good x) in the third stage, so we can think of this stage as the settlement stage. For simplicity, I will say that a buyer holds a unit of bank money when he chooses to deposit. When there is no risk of confusion between currency and bank money, I will simply say that a buyer holds a unit of money.

Throughout the analysis, I assume that an agent can carry, at most, one unit of money (either currency or bank money) at any moment, so individual money holdings are restricted to the set $\{0, 1\}$. As we shall see , this assumption ensures tractability without affecting results, as in Gu, Mattesini, Monnet, and Wright (2013). On the other hand, there is no exogenous restriction on the ability of an individual banker to create money, so the quantity of money issued by a banker belongs to the set $\{0, 1, 2, ...\}$.

What makes bank money equivalent to a demand deposit contract is the withdrawal option: If a depositor decides to withdraw, then he is entitled to receive at most the original deposit amount. Why is the withdrawal option socially desirable? The short answer is random relocations. Recall that a banker can access the productive technology only at the beginning of the period (before the realization of the idiosyncratic relocation shock). To be able to offer valuable transaction services to depositors, the members of the banking system need to receive deposits at the beginning of the period to make their portfolio decision.

Because of a lack of communication across regions, it is impossible to transfer a claim on the banking system in one region to the banking system in the other region. Consequently, a mover needs to hold his wealth in the form of currency prior to relocation, so he must be able to withdraw from the banking system. Thus, the relocation shock gives rise to a legitimate demand for withdrawals because the withdrawal option provides insurance against the relocation risk. As we shall see, a mover is willing to redeposit his wealth in the new region as long as he believes that the banking system there has the ability to pay a higher expected return on deposits than currency.

[**Figure 2**]

Figure 2 shows a typical sequence of transactions within a period. The symbol m indicates that bank money has been issued to a depositor. The symbols x and y indicate that good x and good y, respectively, have been transferred to other agents. A mover is able to withdraw currency from the banking system prior to his relocation and is willing to redeposit it in the banking system in the other region before engaging in retail transactions. As we shall see, this *expected* flow of resources across regions due to random relocations does not disrupt the investment plans of banks. Although a nonmover does not need to withdraw, I will show that he will be willing to withdraw if he believes that other depositors are also withdrawing, given that depositors are sequentially served in random order until the banking system runs out of assets. In this case, the payment mechanism will be severely disrupted.

5. SYMMETRIC INFORMATION

As a useful benchmark, it is helpful to start the analysis by assuming a depositor's relocation status is publicly observable. The members of the banking system offer a demand deposit contract specifying that, in exchange for one unit of good x, a depositor receives an *indivisible* unit of bank money, which is a transferable instrument that entitles the bearer to receive $\phi \in \mathbb{R}_+$ units of good x *in the settlement stage* (third stage). Thus, ϕ represents the gross return on bank money (or the gross return on deposits). Throughout the paper, I assume that there is perfect monitoring of the activities of bankers and that a demand deposit contract can be perfectly enforced.

If a depositor wishes to withdraw from the banking system after learning his relocation status, then he receives at most the original deposit amount (i.e., one unit of good x). As we shall see, allowing a depositor to withdraw one unit is a characteristic of an optimal contract. As in Diamond and Dybvig (1983), I assume that withdrawal orders are sequentially served in random order until the banking system runs out of assets. In other words, the demand deposit contract satisfies a sequential service constraint.

To make this assumption consistent with my physical environment, assume that the group of buyers and the group of bankers get together at the time deposit decisions are made (the beginning of the period) and that, shortly after their initial interaction, the group of buyers departs from this centralized location. Immediately after receiving an inflow of deposits, the members of the banking system make their portfolio decisions. Before entering the transaction stage, depositors have an opportunity to withdraw (after learning their relocation status). At this point, I assume that depositors are isolated and are able to sequentially contact the banking system in random order.

I assume, throughout the analysis, that a concerted suspension of convertibility is not an option. It is important to mention that this assumption does not generate an undue bias toward the existence of equilibrium bank runs given that Ennis and Keister (2009) have demonstrated that a concerted suspension of convertibility in the canonical Diamond-Dybvig model does not necessarily prevent a run when a banking authority is unable to commit to its actions.

When there is symmetric information, the members of the banking system are able to perfectly distinguish depositors who have a legitimate motive for exercising the withdrawal option (movers) from depositors who are not going to be relocated and do not need to withdraw (nonmovers). In this case, the banking system can condition the withdrawal option on the depositor's relocation status, so only movers are able to withdraw prior to relocation. As we shall see, there cannot be a banking panic under this type of contract, so an equilibrium allocation is expected to be stationary.

5.1. Distributions

To characterize an equilibrium allocation, it is helpful to start by describing the distributions of money holdings across different types of agents. These distributions can be summarized as follows. Let $m^1 \in [0,1]$ denote the invariant measure of buyers holding one unit of money at the end of the first stage, let $m^2 \in [0,1]$ denote the invariant measure of sellers holding one unit of money at the end of the second stage, and let $m^3 \in [0,1]$ denote the invariant volume of redemptions in the settlement stage.

If each buyer chooses to hold his wealth in the form of bank deposits, then a stationary equilibrium is consistent with the following invariant distributions:

$$m^1 = 1 \tag{1}$$

and

$$m^2 = m^3 = \lambda. \tag{2}$$

These distributions imply that each buyer enters the second stage (when bilateral transactions occur) holding one unit of bank money and that a measure λ of sellers enters the settlement stage holding one unit of bank money and chooses to redeem these claims. As we shall see, no buyer will choose to hold currency for transaction purposes in equilibrium (movers temporarily hold currency during their relocation but choose to deposit it in the banking system upon their arrival in the new region).

5.2. Buyers

Given these distributions, let me now describe the Bellman equation for a buyer. Let $V \in \mathbb{R}$ denote the expected utility of a buyer prior to the formation of bilateral matches in the second stage. The Bellman equation for a buyer is given by

$$V = \lambda \left[u\left(y\right) + \beta \left(-\gamma + V\right)\right] + \left(1 - \lambda\right) \beta V. \tag{3}$$

Here $y \in \mathbb{R}_+$ denotes the amount of good y that he will be able to purchase from the seller with whom he is matched in exchange for one unit of money.

With probability λ, a buyer will be matched with a seller and will be able to consume good y, entering the following period without money. Then, he will be able to rebalance his money holdings by producing one unit of good x and depositing it in the banking system. With probability $1 - \lambda$, a buyer will not find a trading partner and will enter the following period with the same amount of money. Thus, regardless of his trading history, a buyer enters the second stage holding one unit of money. If each buyer is willing to trade with a seller and is willing to produce to rebalance his money holdings, then the conjecture $m^1 = 1$ is consistent with individual behavior.

5.3. Sellers

Let $W \in \mathbb{R}$ denote the expected utility of a seller. The Bellman equation for a seller is given by

$$W = \lambda \left[-w\left(y\right) + v\left(\phi\right) + \beta W\right] + \left(1 - \lambda\right) \beta W. \tag{4}$$

Recall that a unit of money entitles the bearer to receive ϕ units of good x in the settlement stage. If each seller accepts to produce y in exchange for one unit of money, then the conjecture $m^2 = \lambda$ is consistent with individual behavior.

5.4. Bankers

Consider now the decisions of a typical banker. At any date, a banker has an opportunity to issue a unit of money with probability λ, according to the invariant distributions previously described. In equilibrium, some buyers start the current period without money because they had a trading opportunity in the previous period, so they need to rebalance their money holdings. Others start the period with one unit of money, so they do not need to rebalance their portfolios. Thus, from a banker's standpoint, he has an opportunity to issue one unit of money with probability λ.

When a banker issues a unit of money to a buyer, the latter will be able to spend it at the current date with probability λ, so a seller will claim its face value ϕ with the same probability. With probability $(1-\lambda)\lambda$, a seller will claim its face value at the following date. With probability $(1-\lambda)^2\lambda$, a seller will claim its face value two dates after issuance and so on. Because an individual banker faces idiosyncratic risk when issuing a unit of money (i.e., uncertainty regarding the date at which his claim will be redeemed), the members of the banking system have an incentive to engage in a risk-sharing scheme.

An effective arrangement can be constructed as follows. Suppose that all bankers agree that an individual banker who has an opportunity to issue a unit of money is supposed to save a fraction $s \in \mathbb{R}_+$ of the deposit amount. All bankers then decide how to invest all savings subject to the constraint that all claims presented for redemption in the settlement stage must be retired at face value ϕ. In other words, a banker is supposed to make a contribution s every time he has an opportunity to issue one unit of money in exchange for a disbursement ϕ on his behalf every time someone wants to retire a unit of money issued by him.

Let me now describe the investment decisions of the members of the banking system. Let $i^p \in \mathbb{R}_+$ denote the *per capita* amount invested in the productive technology, and let $i^s \in \mathbb{R}_+$ denote the *per capita* amount invested in storage, where *per capita* means per banker. In a stationary equilibrium, the *per capita* resource constraints for the members of the banking system are given by

$$i^p + i^s = F\left(i^p\right) + \lambda s + i^s - \lambda\phi \tag{5}$$

and

$$i^s \geq \lambda\phi. \tag{6}$$

At each date, a fraction λ of bankers is able to create a unit of money, so the *per capita* inflow of funds into the banking system is given by λs. The *per capita* disbursement due to redemptions is given by $\lambda\phi$. Constraint (6) reflects the fact that the productive technology pays off only at the beginning of the following period, so at least part of the amount invested in storage has to be liquidated to meet expected redemptions in the settlement stage. I have implicitly assumed that bankers do not want to prematurely liquidate the productive technology. As we shall see, this is consistent with equilibrium behavior.

Assume that, at the initial date, a fraction $1-\lambda$ of (randomly selected) buyers is endowed with one unit of bank money and that each member of the banking system is endowed with $F(i^p)$ units of good x to help cover these claims. Note that $i^s = \lambda\phi$ must hold at an optimum, so the *per capita* resource constraint can be rewritten as

$$i^p + \lambda\phi = F(i^p) + \lambda s. \tag{7}$$

Consider now the Bellman equation for a banker. Let $J \in \mathbb{R}$ denote the beginning-of-period expected utility of a banker. The Bellman equation for a banker is given by

$$J = \lambda\left(1 - s + \hat{\beta}J\right) + (1-\lambda)\hat{\beta}J. \tag{8}$$

A banker is able to consume $1-s$ every time he has an opportunity to issue one unit of money. Because $\hat{\beta}(1+\rho) \leq 1$, a banker is willing to immediately consume any profit. Note that the expected utility of a banker does not depend on the quantity of money he has previously issued because of the implementation of the risk-sharing scheme described above.

5.5. Terms of Trade and Participation Constraints

We now need to determine the terms of trade in the first and second stages. Start with the second stage. In each bilateral meeting, I assume the buyer makes a take-it-or-leave-it offer to the seller, so he will be able to capture all surplus from trade. A buyer is willing to trade provided $u(y) - \beta\gamma \geq 0$, and a seller is willing to trade provided $-w(y) + v(\phi) \geq 0$. The seller's participation constraint is binding when the buyer has all the bargaining power, so the amount of good y produced in exchange for a unit of money is given by

$$y = w^{-1} \left(v \left(\phi \right) \right). \tag{9}$$

Now I need to verify whether a buyer is willing to produce in the first stage to acquire a unit of bank money. The buyer's participation constraint in the first stage is given by

$$U \left(\phi \right) \geq \frac{\gamma \left(1 - \beta + \beta \lambda \right)}{\lambda}, \tag{10}$$

where the function $U : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is defined by

$$U \left(\phi \right) \equiv u \left(w^{-1} \left(v \left(\phi \right) \right) \right).$$

Note that $U \left(\phi \right)$ is increasing and strictly concave in ϕ, with $U \left(0 \right) = 0$. Because a buyer has the option of using currency as a medium of exchange, it follows that

$$\phi \geq 1, \tag{11}$$

which implies that the rate of return on bank money must be positive in equilibrium. In other words, bank money must command a higher purchasing power than currency to induce each buyer to become a depositor. If (11) is satisfied, then any buyer chooses to voluntarily deposit currency in the banking system.

The banker's participation constraint is given by $s \leq 1$. Throughout the analysis, I assume the terms of trade in the deposit market are such that each banker earns zero profit in equilibrium, so we must have

$$s = 1. \tag{12}$$

In addition, the investment plan implemented by the members of the banking system must be such that it maximizes the expected utility of depositors.

5.6. Equilibrium

Given these descriptions of individual behavior and feasibility conditions, it is now possible to provide a formal definition of equilibrium.

Definition 1 *A stationary equilibrium is a set of values* $\{V, W, J\}$*, an investment plan* $\{i^p, i^s, s, \phi\}$*, a production level* y*, and a set of invariant distributions* $\{m^1, m^2, m^3\}$ *such that* ***(i)*** *the invariant distributions* $\{m^1, m^2, m^3\}$ *satisfy (1)-(2);* ***(ii)*** *the value functions* $\{V, W, J\}$ *satisfy the Bellman equations (3)-(4) and (8);* ***(iii)*** *the investment plan* $\{i^p, i^s, s, \phi\}$ *satisfies (5)-(6) and (10)-(12) and is consistent with the maximization of the expected utility of depositors; and* ***(iv)*** *the quantity* y *is consistent with the bargaining protocol specified in (9).*

It is important to keep in mind that allowing a mover to withdraw one unit of the good is indeed a characteristic of an optimal contract. Unlike the original Diamond-Dybvig framework, a depositor who is a mover does not withdraw from the banking system for immediate consumption. As previously mentioned, a mover has an opportunity to redeposit his wealth in the other region prior to engaging in retail transactions there. If the return on deposits is the same in both regions, there is no loss in consumption as long as the banking system in either region is not liquidated. This means that allowing a depositor to withdraw less than one unit is clearly not optimal. On the other hand, if the deposit contract allows a depositor to withdraw more than one unit, it needlessly increases the amount of resources transferred across regions at each date, raising the likelihood of a premature liquidation of productive investments.

The next step toward the characterization of equilibrium is to explicitly derive an investment plan consistent with the maximization of the expected utility of depositors. The following lemma describes the optimal investment plan.

Lemma 2 *Consider the following investment plan:* $i^p = \bar{\iota}$*,* $i^s = \lambda + \rho\bar{\iota}$*,* $s = 1$*, and* $\phi = 1 + \frac{\rho\bar{\iota}}{\lambda}$*. This plan is the unique solution consistent with the maximization of the expected utility of depositors.*

An important property of the optimal investment plan described above refers to the state of the banking system at the time withdrawal requests can be made. Given that $i^p = \bar{\iota}$ and $i^s = \lambda + \rho\bar{\iota}$, the *per capita* liquidation value of the assets of the banking system at the time withdrawal requests can be made is given by $\lambda + \bar{\imath}\,(\rho + \delta)$. Suppose that $\lambda + \bar{\imath}\,(\rho + \delta) \geq 1$. In this case, the banking system is able to make good on the withdrawal option even if all depositors choose to exercise it at the same time. However, a nonmover is better off if he does not exercise the withdrawal option (I will make this point very clear in Section 6.6). Thus, the banking system is not subject to panics if $\lambda + \bar{\imath}\,(\rho + \delta) \geq 1$, so we can say that the investment plan consistent with the optimal provision of transaction services implies a liquid banking system.

Suppose now that $\lambda + \bar{\imath}\,(\rho + \delta) < 1$. In this case, it is impossible to meet the demand for withdrawals if, for some reason, all depositors choose to exercise the withdrawal option. Thus, we can say that the banking system is illiquid and subject to panics. When an agent's relocation status is publicly observable, the fact that the optimal investment plan implies an illiquid banking system is not a problem. Because the members of the banking system can perfectly differentiate movers from nonmovers, it is possible to deny a withdrawal order made by any nonmover to preserve the investment plan previously described, so the fact that the banking system is illiquid has no consequence for the equilibrium allocation provided that the total number of movers in each region is not too large. The following assumption guarantees that the total number of withdrawals due to random relocations can be met with the optimal level of reserves.

Assumption 1 Assume $\varepsilon < 1 - \bar{\imath} < \lambda + \rho\bar{\imath}$.

Note that movers, who temporarily hold currency as a store of value, are willing to redeposit their balances upon arrival in the new region, so the investment plan previously described is not disrupted. To formally show the existence of equilibrium, I need to make an additional assumption to guarantee that the buyer's participation constraint is satisfied.

Assumption 2 Assume $\lambda \left[U\left(1 + \frac{\rho\bar{\imath} + (1-\lambda)}{\lambda}\right) - U\left(1 + \frac{\rho\bar{\imath}}{\lambda}\right)\right] \leq \gamma \leq \frac{\lambda U(1)}{1 - \beta + \beta\lambda}$.

Given these assumptions, I can now formally establish existence and uniqueness. Throughout the analysis, I ignore the possibility of autarky as an equilibrium, so by existence I mean the existence of interior (non-autarkic) equilibria and by uniqueness I mean the existence of a unique interior equilibrium. In addition to existence and uniqueness, I also derive some welfare properties of the equilibrium allocation.

Proposition 3 *There exists a unique stationary equilibrium that is Pareto optimal.*

In this equilibrium, a buyer consumes $w^{-1}\left(v\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)\right)$ units of good y when he has an opportunity to trade with a seller and produces one unit of good x when he needs to rebalance his money holdings, and a seller produces $w^{-1}\left(v\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)\right)$ units of good y and consumes $1+\frac{\rho\bar{\iota}}{\lambda}$ units of good x when he has an opportunity to trade with a buyer. Finally, note that the equilibrium return on bank deposits is given by $\phi = 1+\frac{\rho\bar{\iota}}{\lambda}$.

Note that the assumed indivisibility of money holdings, combined with the unity upper bound, is not restrictive in the sense that it is possible to implement an efficient allocation as an equilibrium outcome even though money holdings are restricted to the set $\{0,1\}$. In what follows, I shall use this efficient allocation as a legitimate benchmark to study the welfare consequences of banking panics.

In this section, I have demonstrated that a banking system has a social value because it is able to provide perfect insurance against the relocation risk by allowing movers to withdraw their balances prior to relocation, as in Diamond and Dybvig (1983), and at the same time is able to raise the purchasing power of money by issuing a payment instrument backed by interest-bearing assets, as in Williamson (2012) and Gu, Mattesini, Monnet, and Wright (2013). This socially beneficial role of a banking system has been demonstrated by assuming that a depositor's relocation status is publicly observable. As we shall see, this assumption is far from being innocuous.

6. ASYMMETRIC INFORMATION

In this section, I consider the case in which a depositor's relocation status is privately observable. To be clear, this is the only deviation from the analysis in the previous section. Thus, the members of the banking system can no longer condition the withdrawal option on a depositor's relocation status. One possibility is to simply remove the withdrawal option for all depositors, which is socially undesirable because a mover, who has a legitimate demand for withdrawals, will be unable to transfer his wealth across regions. This would certainly influence his *ex ante* deposit decision. Thus, allowing depositors to withdraw upon their requests is a socially desirable characteristic of the deposit contract. However, the withdrawal option also opens the door to the possibility of a purely speculative demand for withdrawals that can lead to socially undesirable outcomes, as in the Diamond-Dybvig theory of banking panics.

I follow the standard approach in the literature and allow agents to coordinate their actions based on the realization of a sunspot variable. See, for instance, Cooper and Ross (1998), Peck and Shell (2003), and Allen and Gale (2007). Suppose now that there is an identically and independently distributed stochastic process $\{S_t\}_{t=0}^{\infty}$ with no effects on fundamentals but potentially with an effect on behavior due to expectations. The random variable S_t is publicly observable and can take on two values, either n or r. The realization of S_t occurs *shortly after* the relocation status of each buyer is privately revealed.

As we shall see, in equilibrium, all buyers voluntarily choose to hold their wealth in the form of deposits. After investment decisions have been made, a fraction ε of these depositors is going to be randomly relocated and so chooses to exercise the withdrawal option. Nonmovers choose whether to withdraw depending on the realization of the sunspot variable *and* the state of the banking system. Specifically, nonmovers choose to withdraw when they observe $S_t = r$ *and* the banking system is illiquid and choose not to withdraw otherwise. Thus, the realization $S_t = r$ does not trigger a run if the banking portfolio is liquid, so the choice of the banking portfolio is crucial for the occurrence of a panic in equilibrium.

Before I formally characterize individual behavior, let me provide a verbal description of an equilibrium allocation. At any given date, the banking system receives an inflow of funds in the form of new deposits, and its members make investment decisions to maximize the expected utility of depositors. After investment decisions have been made, a depositor decides whether to withdraw from the banking system after learning his relocation status. If $S_t = r$ and the investment plan of banks is such that the banking portfolio is illiquid, then a panic occurs as both movers and nonmovers choose to withdraw. In this case, the assets of the banking system are completely liquidated and currency is temporarily used as a medium of exchange. See Figure 3 that follows. In the following period, the banking system is reestablished, receiving an inflow of funds in the form of deposits. Following a panic, the banking system has no productive investment coming to fruition because all assets have been liquidated. Thus, the reestablishment of the desired banking portfolio can take several periods.

[**Figure 3**]

If *either* $S_t = n$ *or* $S_t = r$ and the banking system is liquid, then nonmovers do not withdraw, so the banking system is able to meet the demands for withdrawals due to random relocations. Movers, who temporarily hold currency as a store of value, are willing to redeposit their balances upon arrival in the new region, so the investment plan of banks is not interrupted. When the banking system is not liquidated, all currency is eventually held as bank reserves in each region. The sequence of typical transactions when there is no panic continues to be represented by Figure 2.

6.1. Distributions

Let $S^t = (S_0, ..., S_t) \in \{n, r\} \times ... \times \{n, r\}$ denote a partial history of realizations of the publicly observable sunspot variable. As in the previous section, it is helpful to start by describing the distributions of money holdings across different types of agents. Let $m_t^1(S^t) \in [0, 1]$ denote the (state-contingent) measure of buyers holding one unit of money (either currency or bank money) prior to the formation of bilateral matches, let $m_t^2(S^t) \in [0, 1]$ denote the measure of sellers holding one unit of money (either currency or bank money) shortly after bilateral matches are dissolved, and let $m_t^3(S^t) \in [0, 1]$ denote the volume of redemptions in the settlement stage. These distributions crucially depend on whether or not a depositor who has lost the full value of his deposit in the event of a panic is able to produce again before entering the transaction stage (second stage).

In what follows, I consider two possibilities. In this section, I assume that any depositor who ends up losing the full value of his deposit in the event of a panic is able to produce again before engaging in bilateral transactions. In this case, a panic will not affect the extensive margin in the transaction stage. In the following section, I assume that any depositor who ends up losing the full value of his deposit needs to wait until the following period to produce again. In this case, a panic will affect the extensive margin in the transaction stage. I shall argue that the latter case is the version of the model that I consider the most relevant to understand the role of the banking panics of the Great Depression.

If each buyer chooses to hold his wealth in the form of bank deposits, then an equilibrium allocation is consistent with the following distributions of money holdings:

$$m_t^1\left(S^t\right) = 1, \tag{13}$$

$$m_t^2\left(S^t\right) = \lambda, \tag{14}$$

$$m_t^3\left(S^t\right) = \lambda \hat{I}_t\left(S^t\right), \tag{15}$$

for all $S^t \in \{n, r\} \times ... \times \{n, r\}$, where $\hat{I}\left(S^t\right)$ represents an indicator function defined by

$$\hat{I}_t\left(S^t\right) = \begin{cases} 0 \text{ if } i_t^s\left(S^{t-1}\right) + \delta i_t^p\left(S^{t-1}\right) < 1 \text{ and } S_t = r, \\ 1 \text{ otherwise.} \end{cases}$$

Here, $i_t^p\left(S^{t-1}\right) \in \mathbb{R}_+$ represents the *per capita* investment in the productive technology, and $i_t^s\left(S^{t-1}\right) \in \mathbb{R}_+$ represents the *per capita* investment in storage. Recall that the investment decisions at date t must be made prior to the realization of S_t. The *per capita* liquidation value of the assets of the banking system at the time withdrawal requests can be made is given by $i_t^s\left(S^{t-1}\right) + \delta i_t^p\left(S^{t-1}\right)$, so the banking portfolio is illiquid if $i_t^s\left(S^{t-1}\right) + \delta i_t^p\left(S^{t-1}\right) < 1$.

When there is no panic, the nonbank public is able to trade using bank money as a means of payment, so the volume of redemptions in the settlement stage is given by λ. When there is a panic, the banking system is liquidated, so the nonbank public temporarily reverts to currency to settle bilateral transactions. In this case, a seller is able to consume one unit of good x shortly after trading with a buyer, so nothing happens in the settlement stage.

6.2. Bankers

Let me now characterize individual behavior. Start with the group of bankers. Let $s_t\left(S^{t-1}\right) \in \mathbb{R}_+$ denote the individual contribution a banker is supposed to make to the common pool of assets when he has an opportunity to issue a unit of money and let $\phi_t\left(S^t\right) \in \mathbb{R}_+$ denote the value of a unit of money (either currency or bank money). An investment plan $\left\{i_t^p\left(S^{t-1}\right), i_t^s\left(S^{t-1}\right), s_t\left(S^{t-1}\right), \phi_t\left(S^t\right)\right\}_{t=0}^{\infty}$ must satisfy the following law of motion:

$$\begin{aligned} i_t^p\left(S^{t-1}\right)+i_t^s\left(S^{t-1}\right) &= F\left(i_{t-1}^p\left(S^{t-2}\right)\right)\hat{I}_{t-1}\left(S^{t-1}\right)+ \\ &\quad +(1-\lambda)\, s_t\left(S^{t-1}\right)\left[1-\hat{I}_{t-1}\left(S^{t-1}\right)\right]+\lambda s_t\left(S^{t-1}\right) \qquad (16) \end{aligned}$$

for all $S^{t-1} \in \{n, r\} \times ... \times \{n, r\}$. The initial conditions $i_{-1}^p \in \mathbb{R}_+$ and $S_{-1} \in \{n, r\}$ are taken as given. In addition, the value of money $\phi_t\left(S^t\right)$ must satisfy $\left[\lambda\phi_t\left(S^t\right) - i_t^s\left(S^{t-1}\right)\right]\hat{I}_t\left(S^t\right) \leq 0$ for all $S^t \in \{n, r\} \times ... \times \{n, r\}$. When there is no panic, the value of money is the same as the gross return on deposits, so it must respect the feasibility condition imposed by the available technologies. When there is a panic, the value of money is the same as the purchasing power of currency. Thus, the value of a unit of money is given by

$$\phi_t\left(S^t\right) = \begin{cases} 1 \text{ if } i_t^s\left(S^{t-1}\right)+\delta i_t^p\left(S^{t-1}\right) < 1 \text{ and } S_t = r, \\ \lambda^{-1} i_t^s\left(S^{t-1}\right) \text{ otherwise,} \end{cases} \qquad (17)$$

for all $S^{t-1} \in \{n, r\} \times ... \times \{n, r\}$.

Let me now describe the value function of a typical banker. Let $J_t\left(S^{t-1}\right) \in \mathbb{R}$ denote the beginning-of-period expected utility of a banker following the partial history $S^{t-1} \in \{n, r\} \times ... \times \{n, r\}$. The sequence $\left\{J_t\left(S^{t-1}\right)\right\}_{t=0}^{\infty}$ can be recursively defined as follows:

$$\begin{aligned} J_t\left(S^{t-1}\right) &= \lambda\left[1-s_t\left(S^{t-1}\right)\right]\hat{I}_{t-1}\left(S^{t-1}\right)+ \\ &\quad +\left[1-s_t\left(S^{t-1}\right)\right]\left[1-\hat{I}_{t-1}\left(S^{t-1}\right)\right]+\hat{\beta}E\left[J_{t+1}\left(S^t\right)\right], \qquad (18) \end{aligned}$$

where $E(\cdot)$ represents the expectation with respect to the sunspot variable. If a panic did not occur at the previous date, then a banker is able to issue a unit of money with probability λ. If a panic occurred at the previous date, each banker is able to issue a unit of money because no one is a depositor at the beginning of the period.

6.3. Buyers

Let $V_t\left(S^t\right) \in \mathbb{R}$ denote the postdeposit expected utility of a buyer following the partial history $S^t \in \{n, r\} \times ... \times \{n, r\}$. The sequence $\left\{V_t\left(S^t\right)\right\}_{t=0}^{\infty}$ can be recursively defined as follows:

$$V_t\left(S^t\right) = -p_t\left(S^{t-1}\right)\gamma\left[1 - \hat{I}_t\left(S^t\right)\right] + \lambda\left[U\left(\phi_t\left(S^t\right)\right) - \beta\gamma\right] + \beta E\left[V_{t+1}\left(S^{t+1}\right)\right], \qquad (19)$$

where $p_t\left(S^{t-1}\right) \in [0, 1]$ represents the probability of loss in the event of a panic, which must satisfy

$$p_t\left(S^{t-1}\right) = \max\left\{0, 1 - i_t^s\left(S^{t-1}\right) - \delta i_t^p\left(S^{t-1}\right)\right\} \qquad (20)$$

for all $S^{t-1} \in \{n, r\} \times ... \times \{n, r\}$. Note that a panic does not occur when the banking portfolio is liquid. When $S_t = r$ and $i_t^s\left(S^{t-1}\right) + \delta i_t^p\left(S^{t-1}\right) < 1$, a panic occurs and the banking system in each region is liquidated. Because depositors are sequentially served in random order, a depositor is able to withdraw one unit with probability $i_t^s\left(S^{t-1}\right) + \delta i_t^p\left(S^{t-1}\right) < 1$. If a depositor loses all his savings in the event of a panic, then he needs to produce again to acquire purchasing power (currency) in order to trade with a seller. Thus, the conjecture $m_t^1\left(S^t\right) = 1$ for all $S^t \in \{n, r\} \times ... \times \{n, r\}$ is consistent with individual behavior. In the next section, I consider the case in which a depositor is unable to produce after withdrawal orders are sequentially served, so he has to wait until the following period.

So far I have implicitly assumed that each buyer is willing to deposit in the banking system even though a panic can occur with some positive probability. Now I need to verify whether it is individually rational for a buyer to deposit in the banking system. In particular, a buyer is willing to deposit provided that

$$-\pi p_t\left(S^{t-1}\right)\gamma + \pi\lambda U\left(\phi_t\left(S^{t-1}, r\right)\right) + (1 - \pi)\lambda U\left(\phi_t\left(S^{t-1}, n\right)\right) \geq \lambda U(1) \qquad (21)$$

for all $S^{t-1} \in \{n, r\} \times ... \times \{n, r\}$. Note that bank money commands a higher purchasing power than currency when a panic does not occur, but a buyer who chooses to hold currency is not subject to loss if a panic occurs. Thus, a buyer is willing to hold bank money provided that the rate of return on deposits (conditional on not having a panic) is sufficiently large to compensate him for the possibility of suffering a loss in the event of a panic.

6.4. Sellers

Let $W_t\left(S^t\right) \in \mathbb{R}$ denote the expected utility of a seller following the partial history $S^t \in \{n,r\} \times ... \times \{n,r\}$. The sequence $\left\{W_t\left(S^t\right)\right\}_{t=0}^{\infty}$ can be recursively defined as follows:

$$W_t\left(S^t\right) = \lambda\left[-w\left(y_t\left(S^t\right)\right) + v\left(\phi_t\left(S^t\right)\right)\right] + \beta E\left[W_{t+1}\left(S^{t+1}\right)\right]. \tag{22}$$

A seller is willing to produce for a buyer in exchange for a unit of money provided that the value of money is sufficiently large to compensate him for the disutility of production. As previously mentioned, the value of money depends on the investment decisions of the members of the banking system and the depositors' withdrawal decisions.

6.5. Terms of Trade and Participation Constraints

As in the previous section, the terms of trade in each bilateral meeting are such that the buyer extracts all surplus from the seller. Thus, it follows that

$$y_t\left(S^t\right) = w^{-1}\left(v\left(\phi_t\left(S^t\right)\right)\right) \tag{23}$$

for all $S^t \in \{n,r\} \times ... \times \{n,r\}$. When banking panics can occur in equilibrium, the buyer's participation constraint is given by

$$-\pi p_t\left(S^{t-1}\right)\gamma + \pi\lambda U\left(\phi_t\left(S^{t-1},r\right)\right) + (1-\pi)\lambda U\left(\phi_t\left(S^{t-1},n\right)\right) \geq (1-\beta+\lambda\beta)\gamma \tag{24}$$

for all $S^{t-1} \in \{n,r\} \times ... \times \{n,r\}$. Note that condition (21) implies that (24) is necessarily satisfied because Assumption 2 ensures $\lambda U(1) \geq (1-\beta+\lambda\beta)\gamma$.

In equilibrium, each banker earns zero profit, so we must have

$$s_t\left(S^{t-1}\right) = 1 \tag{25}$$

for all $S^{t-1} \in \{n,r\} \times ... \times \{n,r\}$. In addition, the investment plan implemented by the members of the banking system must be such that it maximizes the expected utility of depositors.

6.6. Postdeposit Coordination Game

Consider now the postdeposit coordination game. All depositors play this game *after* each one of them privately learns his relocation status. It is clear that a mover always chooses to withdraw from the banking system prior to relocation. A nonmover decides whether to withdraw based on his beliefs regarding the actions of other depositors.

It is a best response for a nonmover to withdraw if the banking system is illiquid and he believes all other nonmovers are withdrawing. It is a best response for a nonmover not to withdraw otherwise. Thus, widespread withdrawals are a pure-strategy Nash equilibrium of the coordination game when the banking system is illiquid. In addition, there exists a second pure-strategy Nash equilibrium with the property that movers withdraw and nonmovers do not withdraw.

To understand why a nonmover is better off if he does not withdraw when the banking system is liquid, consider the following argument. Assume that $i_t^s\left(S^{t-1}\right)+\delta i_t^p\left(S^{t-1}\right)\geq 1$, which means that the banking system is liquid. If a fraction $\alpha\in(0,1)$ of depositors decides to withdraw, then the banking system has to liquidate a fraction α of its portfolio, leaving *at least* $(1-\alpha)\left[i_t^s\left(S^{t-1}\right)+\delta i_t^p\left(S^{t-1}\right)\right]$ to be paid to the remaining depositors. This means that a depositor who chooses not to exercise the withdrawal option is entitled to receive at least

$$\frac{(1-\alpha)\left[i_t^s\left(S^{t-1}\right)+\delta i_t^p\left(S^{t-1}\right)\right]}{(1-\alpha)}=i_t^s\left(S^{t-1}\right)+\delta i_t^p\left(S^{t-1}\right)\geq 1.$$

So a nonmover is better off if he leaves the money in the bank because he will be holding a claim that is worth at least one unit.

6.7. Equilibrium

Given these descriptions of individual behavior and feasibility conditions, it is now possible to provide a formal definition of equilibrium under asymmetric information.

Definition 4 *An equilibrium is a sequence of values*

$$\left\{V_t\left(S^t\right),W_t\left(S^t\right),J_t\left(S^{t-1}\right)\right\}_{t=0}^{\infty},$$

an investment plan

$$\left\{i_t^p\left(S^{t-1}\right), i_t^s\left(S^{t-1}\right), s_t\left(S^{t-1}\right), \phi_t\left(S^t\right)\right\}_{t=0}^{\infty},$$

a sequence $\left\{y_t\left(S^t\right)\right\}_{t=0}^{\infty}$ *specifying production of good* y*, a sequence* $\left\{p_t\left(S^{t-1}\right)\right\}_{t=0}^{\infty}$ *specifying the probability of loss in the event of a panic, and distributions*

$$\left\{m_t^1\left(S^t\right), m_t^2\left(S^t\right), m_t^3\left(S^t\right)\right\}_{t=0}^{\infty}$$

such that ***(i)*** *the distributions* $\left\{m_t^1\left(S^t\right), m_t^2\left(S^t\right), m_t^3\left(S^t\right)\right\}_{t=0}^{\infty}$ *satisfy (13)-(15);* ***(ii)*** *the values* $\left\{V_t\left(S^t\right), W_t\left(S^t\right), J_t\left(S^{t-1}\right)\right\}_{t=0}^{\infty}$ *satisfy (18)-(19) and (22);* ***(iii)*** *the investment plan* $\left\{i_t^p\left(S^{t-1}\right), i_t^s\left(S^{t-1}\right), s_t\left(S^{t-1}\right), \phi_t\left(S^t\right)\right\}_{t=0}^{\infty}$ *satisfies (16)-(17) and (24)-(25) and is consistent with the maximization of the expected utility of depositors;* ***(iv)*** $\left\{y_t\left(S^t\right)\right\}_{t=0}^{\infty}$ *is consistent with the bargaining protocol specified in (23); and* ***(v)*** *the probability of loss in the event of a panic* $\left\{p_t\left(S^{t-1}\right)\right\}_{t=0}^{\infty}$ *satisfies (20).*

Let me now divide the set of equilibrium allocations into two mutually exclusive categories. If an equilibrium exists and $i_t^s\left(S^{t-1}\right)+\delta i_t^p\left(S^{t-1}\right) \geq 1$ holds in all periods and states, then I shall refer to it as an equilibrium with a liquid banking system. In this case, the banking system is able to provide transaction and liquidity services without giving nonmovers a reason to withdraw due to self-fulfilling beliefs, so panics do not occur in equilibrium. If there exists an equilibrium for which the aforementioned condition does not hold, then I shall refer to it as an equilibrium with an illiquid banking system.

6.8. Liquid Banking System

Let me start by showing that the efficient allocation described in Proposition 3 is the *unique* equilibrium under asymmetric information when the technology parameter $\bar{\iota}$ lies in the range $\frac{1-\lambda}{\rho+\delta} \leq \bar{\iota} \leq 1-\lambda$.

Proposition 5 *Suppose that* $\frac{1-\lambda}{\rho+\delta} \leq \bar{\iota} \leq 1-\lambda$*. The efficient equilibrium described in Proposition 3 is the unique equilibrium.*

When productive projects are relatively abundant in the economy, the optimal provision of transaction and liquidity services in the form of bank deposits is not accompanied by the possibility of a panic. Because bank money is partially backed by sufficiently high-yielding assets, it is possible to induce the members of the banking system to pay interest on deposits under a competitive regime without giving depositors a reason to run on the banking system, so bank money commands a higher purchasing power than currency in *any* state of the world. In this case, the provision of socially optimal transaction and liquidity services does not open the door to the possibility of panics. As we shall see, the situation is very different when the economy's productive capacity is relatively small.

6.9. Illiquid Banking System

Suppose now that the technology parameter $\bar{\imath}$ lies in the range $\frac{1-\lambda}{1+\rho} \leq \bar{\imath} < \frac{1-\lambda}{\rho+\delta}$. In this case, I will show that an equilibrium with an illiquid banking system exists. In this equilibrium, the degree of history dependence is such that only the partial history $(S_{t-1}, S_t) \in \{n, r\} \times \{n, r\}$ matters for the characterization of endogenous variables. As a result, it follows that $V_t\left(S^t\right) = V\left(S_{t-1}, S_t\right)$, $W_t\left(S^t\right) = W\left(S_{t-1}, S_t\right)$, $J_t\left(S^{t-1}\right) = J\left(S_{t-1}\right)$, $i_t^p\left(S^{t-1}\right) = i^p\left(S_{t-1}\right)$, $i_t^s\left(S^{t-1}\right) = i^s\left(S_{t-1}\right)$, $s_t\left(S^{t-1}\right) = s\left(S_{t-1}\right)$, $\phi_t\left(S^t\right) = \phi\left(S_{t-1}, S_t\right)$, $p_t\left(S^{t-1}\right) = p\left(S_{t-1}\right)$, and $y_t\left(S^t\right) = y\left(S_{t-1}, S_t\right)$ for all $S^t \in \{n, r\} \times ... \times \{n, r\}$. In addition, the distributions of money holdings across different types of agents are given by

$$m_t^1\left(S^t\right) = 1, \tag{26}$$

$$m_t^2\left(S^t\right) = \lambda, \tag{27}$$

$$m_t^3\left(S^t\right) = \lambda I\left(S_t\right), \tag{28}$$

for all $S^t \in \{n, r\} \times ... \times \{n, r\}$, where the indicator function $I\left(S_t\right)$ is such that $I\left(S_t\right) = 1$ if $S_t = n$ and $I\left(S_t\right) = 0$ if $S_t = r$.

Conjecture that the choice $i^p\left(S_{t-1}\right) = \bar{\imath}$ for each $S_{t-1} \in \{n, r\}$ is consistent with the maximization of the expected utility of depositors. This means that the members of the banking system optimally choose the same level of productive investment regardless of history. Then, it follows from (16)-(17) that

$$i^p(n) = \bar{\iota}, \tag{29}$$

$$i^s(n) = \lambda + \rho\bar{\iota}, \tag{30}$$

$$\phi(n, n) = 1 + \frac{\rho\bar{\iota}}{\lambda}, \tag{31}$$

$$i^p(r) = \bar{\iota}, \tag{32}$$

$$i^s(r) = 1 - \bar{\iota}, \tag{33}$$

$$\phi(r, n) = \frac{1 - \bar{\iota}}{\lambda}. \tag{34}$$

Because $1 - (1 - \delta)\bar{\iota} \leq \lambda + (\delta + \rho)\bar{\iota} < 1$, this investment plan implies that, regardless of history, the banking system is illiquid at the time withdrawal requests can be made. This means that the realization of the sunspot signal r triggers a banking panic as both movers and nonmovers will optimally choose to withdraw from the banking system. As a result, we must have

$$\phi(r, r) = \phi(n, r) = 1. \tag{35}$$

This indicates that agents temporarily revert to currency in the event of a panic, which occurs with probability π.

Note that the optimal level of reserves at any date preceded by a panic is smaller than that observed at any date not preceded by a panic. As previously mentioned, a panic results in the premature liquidation of the assets of the banking system, which means that the feasible set for the members of the banking system at any date preceded by a panic is smaller because no productive investment is coming to fruition at the beginning of the period. To reestablish the desired level of investment in the productive technology, the members of the banking system choose a level of reserves that is necessarily smaller than that chosen at any date not preceded by a panic (when the feasible set is larger). As a result, the rate of return on deposits is lower during the recovery of the banking system from a panic. Note that $\phi(r, n) = \frac{1-\bar{\iota}}{\lambda} < 1 + \frac{\rho\bar{\iota}}{\lambda} = \phi(n, n)$. Because each depositor's wealth is smaller during the recovery, this necessarily implies a temporary reduction in individual consumption.

Given the investment plan previously described, the state-contingent probability of loss in the event of a panic must satisfy the following conditions:

$$p(n) = 1 - \lambda - (\delta + \rho)\,\bar{\iota} \tag{36}$$

and

$$p(r) = (1 - \delta)\,\bar{\iota}. \tag{37}$$

The counterpart of the desired reestablishment of productive investment following a panic is a higher probability of loss in the event of a panic at any date preceded by a panic. Note that the probability of a panic is given by π at any date, but the probability of loss in the event of a panic is higher today if a panic occurred in the previous period. This property of equilibrium clearly illustrates the dynamic interaction between the ability of banks to provide socially useful transaction and liquidity services and the occurrence of panics.

The next step toward the characterization of an equilibrium allocation is to demonstrate that the investment plan previously described maximizes the expected utility of depositors. To formally establish this result, it is helpful to impose a condition on the parameter δ governing the liquidation value of the productive technology. First, note that $\frac{1-\bar{\iota}}{\lambda} = 1 + \frac{\rho\bar{\iota}}{\lambda}$ if and only if $\bar{\iota} = \frac{1-\lambda}{1+\rho}$, Thus, there exists a value $\bar{\iota}^* > \frac{1-\lambda}{1+\rho}$ such that $U'\left(\frac{1-\bar{\iota}}{\lambda}\right) \leq \beta\,(1+\rho)\,U'\left(1 + \frac{\rho\bar{\iota}}{\lambda}\right)$ if and only if $\bar{\iota} \in \left[\frac{1-\lambda}{1+\rho}, \bar{\iota}^*\right]$. As we raise the parameter $\bar{\iota}$ from $\frac{1-\lambda}{1+\rho}$ to $\bar{\iota}^*$, the dispersion in state-contingent consumption increases because a higher $\bar{\iota}$ means that, following a banking panic, the required reduction in reserves (and, consequently, the rate of return on deposits) to maintain the desired level of productive investment has to be larger, holding everything else constant. In what follows, it is useful to make the following assumption.

Assumption 3 Assume $\bar{\iota}^* \geq \frac{1-\lambda}{\rho+\delta}$.

This assumption can be viewed as a restriction on the parameter δ governing the liquidation value of the productive technology. For instance, it cannot be too small. Given this additional assumption, let me now formally establish the optimality of the investment plan previously described.

Lemma 6 *Suppose that* $\frac{1-\lambda}{1+\rho} \leq \bar{\iota} < \frac{1-\lambda}{\rho+\delta}$. *There exists* $\bar{\pi} > 0$ *such that the investment plan (29)-(35) maximizes the expected utility of depositors provided* $\pi \in (0, \bar{\pi})$, *with the threshold* $\bar{\pi}$ *satisfying*

$$\bar{\pi} \leq 1 - \frac{U'\left(\frac{1-\bar{\iota}}{\lambda}\right)}{\beta\left(1+\rho\right)U'\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)}.$$

To verify the optimality of a particular investment plan in a dynamic framework, one must pay attention to an intertemporal tradeoff regarding the amount of liquidity in the banking system that is absent in one-shot models of banking panics. For instance, in a dynamic framework, a reduction in the level of productive investment today certainly reduces the probability of loss in the event of a panic today, as in the Diamond-Dybvig model, but it also raises the probability of loss in the event of a panic tomorrow because less productive investment will be coming to fruition tomorrow (restricting the feasible choices tomorrow). This intertemporal tradeoff arises only in a dynamic framework and is more or less important depending on how effective the productive technology is.

The next step is to verify whether a buyer is willing to deposit in the banking system knowing that a panic occurs with probability π. A buyer is willing to deposit at any date provided

$$\left(1-\pi\right)\lambda\left[U\left(\frac{1-\bar{\iota}}{\lambda}\right) - U\left(1\right)\right] \geq \pi\left(1-\delta\right)\bar{\iota}\gamma \tag{38}$$

and

$$\left(1-\pi\right)\lambda\left[U\left(1+\frac{\rho\bar{\iota}}{\lambda}\right) - U\left(1\right)\right] \geq \pi\left[1-\lambda-\left(\delta+\rho\right)\bar{\iota}\right]\gamma. \tag{39}$$

These two conditions indicate that a sufficiently low probability of a panic ensures that a buyer is willing to deposit in the banking system to benefit from a higher expected return on deposits. To precisely characterize this threshold for the probability π associated with the realization r, define the values $U_r \equiv U\left(\frac{1-\bar{\iota}}{\lambda}\right) - U\left(1\right)$ and $U_n \equiv U\left(1+\frac{\rho\bar{\iota}}{\lambda}\right) - U\left(1\right)$. If the probability π associated with the realization r is such that

$$0 < \pi \leq \min\left\{\frac{\lambda U_n}{\left[1-\lambda-\left(\delta+\rho\right)\bar{\iota}\right]\gamma+\lambda U_n}, \frac{\lambda U_r}{\left(1-\delta\right)\bar{\iota}\gamma+\lambda U_r}\right\} \equiv \hat{\pi},$$

then conditions (38) and (39) are simultaneously satisfied, so each buyer is willing to deposit in the banking system even though a panic occurs with probability π.

Now I need to specify the value functions for each type of agent. Start with the value functions for a buyer. These are given by

$$\begin{aligned} V(S_{t-1}, S_t) &= -p(S_{t-1})\gamma[1 - I(S_t)] + \lambda[U(\phi(S_{t-1}, S_t)) - \beta\gamma] + \\ &\quad +\beta[\pi V(S_t, r) + (1-\pi)V(S_t, n)] \end{aligned} \tag{40}$$

for any $(S_{t-1}, S_t) \in \{n, r\} \times \{n, r\}$. Because these are four linear equations in four unknowns, it is straightforward to analytically solve for the values $V(n, n)$, $V(r, n)$, $V(n, r)$, and $V(r, r)$.

In a similar fashion, we can define the value functions for a seller. For each $(S_{t-1}, S_t) \in \{n, r\} \times \{n, r\}$, it follows that

$$\begin{aligned} W(S_{t-1}, S_t) &= \lambda[-w(y(S_{t-1}, S_t)) + v(\phi(S_{t-1}, S_t))] + \\ &\quad +\beta[\pi W(S_t, r) + (1-\pi)W(S_t, n)]. \end{aligned}$$

Because the bargaining protocol implies $w(y(S_{t-1}, S_t)) = v(\phi(S_{t-1}, S_t))$ for all $(S_{t-1}, S_t) \in \{n, r\} \times \{n, r\}$, it follows that

$$W(n, n) = W(n, r) = W(r, n) = W(r, r) = 0. \tag{41}$$

The value functions for a banker satisfy

$$J(r) = [1 - s(r)] + \hat{\beta}[\pi J(r) + (1-\pi)J(n)]$$

and

$$J(n) = \lambda[1 - s(n)] + \hat{\beta}[\pi J(r) + (1-\pi)J(n)].$$

Because $s(r) = s(n) = 1$, it follows that

$$J(r) = J(n) = 0. \tag{42}$$

Finally, the level of production in each bilateral meeting is given by

$$y(S_{t-1}, S_t) = w^{-1}(v(\phi(S_{t-1}, S_t))) \tag{43}$$

for each $(S_{t-1}, S_t) \in \{n, r\} \times \{n, r\}$. Let me now formally establish existence.

Proposition 7 *Suppose that* $\frac{1-\lambda}{1+\rho} \leq \bar{\iota} < \frac{1-\lambda}{\rho+\delta}$*. There exists an equilibrium with an illiquid banking system in which the distributions of money holdings are given by (26)-(28), the value functions are given by (40)-(42), the investment plan is given by (29)-(35), the probability of loss in the event of a panic is given by (36)-(37), and the production of good* y *is given by (43) provided* $\pi \in (0, \pi^*)$*, with* $\pi^* = \min\{\bar{\pi}, \hat{\pi}\}$*.*

It is important to keep in mind that a panic results in a loss of wealth for all depositors, substantially affecting the retail sector of the economy. As we have seen, some depositors are able to withdraw one unit before the banking system runs out of assets, whereas others lose the full value of their deposits. Because bankers can no longer access the productive technology after the liquidation of assets, it is not possible to provide useful transaction services at a panic date. Thus, each buyer necessarily holds an asset with a lower purchasing power than bank deposits, which means that a banking panic produces a negative wealth effect. As a result, each seller is willing to produce and sell a smaller amount, so the economy falls into recession.

Another important property of the equilibrium allocation previously described is that the recovery from a panic is not immediate. Table 1 below summarizes the consumption of each type as a function of the partial history $(S_{t-1}, S_t) \in \{n, r\} \times \{n, r\}$.

Table 1: State-Contingent Consumption by Type of Agent

	(n, n)	(r, n)	(r, r) or (n, r)
buyer	$w^{-1}\left(v\left(1 + \frac{\rho\bar{\iota}}{\lambda}\right)\right)$	$w^{-1}\left(v\left(\frac{1-\bar{\iota}}{\lambda}\right)\right)$	$w^{-1}\left(v\left(1\right)\right)$
seller	$1 + \frac{\rho\bar{\iota}}{\lambda}$	$\frac{1-\bar{\iota}}{\lambda}$	1

Note that $1 + \frac{\rho\bar{\iota}}{\lambda} > \frac{1-\bar{\iota}}{\lambda} > 1$, which implies $w^{-1}\left(v\left(1 + \frac{\rho\bar{\iota}}{\lambda}\right)\right) > w^{-1}\left(v\left(\frac{1-\bar{\iota}}{\lambda}\right)\right) > w^{-1}\left(v\left(1\right)\right)$. This means that the nonbank public's consumption reaches its lowest level when a panic occurs. In this case, there is an abrupt decline in the amount traded in each bilateral transaction as a result of the destruction of inside money and associated wealth effect. Note also that, at any nonpanic date preceded by a panic, consumption remains below the level consistent with any nonpanic date not preceded by a panic (i.e., "normal times"). As a result, we can say that the disruption of the payment mechanism following a widespread liquidation of the banking system generates a *protracted* recession because consumption does not immediately recover to the level consistent with normal times.

So far, I have demonstrated that an equilibrium with an illiquid banking system exists. Let me now show that the efficient allocation can be an equilibrium outcome under asymmetric information only if depositors completely ignore the sunspot variable.

Proposition 8 *Suppose that* $\frac{1-\lambda}{1+\rho} \leq \bar{\iota} < \frac{1-\lambda}{\rho+\delta}$. *Then, the efficient allocation described in Proposition 3 can be implemented as an equilibrium outcome only if it is common knowledge that each depositor completely ignores the sunspot variable.*

If the economy's productive capacity is relatively small, the efficient allocation can be implemented only if each depositor who finds out he is a nonmover firmly believes that other nonmovers are never going to withdraw regardless of the realization of the sunspot variable. As a result, there exist at least two equilibria when the technology parameter lies in the range $\frac{1-\lambda}{1+\rho} \leq \bar{\iota} < \frac{1-\lambda}{\rho+\delta}$. In both equilibria, the banking system is illiquid. The equilibrium with banking panics is socially inefficient and occurs when depositors rationally choose to coordinate their actions based on the realization of the sunspot variable.

This property of the model establishes a connection between the availability of productive projects and the ability of banks to provide socially useful transaction and liquidity services without introducing the possibility of costly panics. Indeed, an important prediction is that panics are expected to occur when the economy's productive capacity is relatively small. In this respect, my analysis is related to the view that the occurrence of panics is associated with business-cycle conditions, as advocated in Gorton (1988).

7. EXTENSIVE MARGIN EFFECTS

Suppose now that a depositor who ends up losing the full value of his deposit in the event of a panic needs to wait until the following period to produce again to rebalance his portfolio. In this case, an equilibrium allocation is consistent with the following distributions of money holdings:

$$m_t^1\left(S^t\right) = \left[1 - \hat{I}_t\left(S^t\right)\right]\left[i_t^s\left(S^{t-1}\right) + \delta i_t^p\left(S^{t-1}\right)\right] + \hat{I}_t\left(S^t\right), \tag{44}$$

$$m_t^2\left(S^t\right)=\lambda m_t^1\left(S^t\right), \tag{45}$$

$$m_t^3\left(S^t\right)=m_t^2\left(S^t\right)\hat{I}_t\left(S^t\right) \tag{46}$$

for all $S^t \in \{n,r\} \times ... \times \{n,r\}$. If there is no panic, then the distributions are the same as those described in the previous section. If there is a panic, then only a fraction $i_t^s\left(S^{t-1}\right)+\delta i_t^p\left(S^{t-1}\right)<1$ of buyers enters the transaction stage holding a unit of currency. This means that the number of trade meetings is given by $\lambda\left[i_t^s\left(S^{t-1}\right)+\delta i_t^p\left(S^{t-1}\right)\right]<1$. A panic affects both the quantity traded in each bilateral meeting (intensive margin) and the total number of trade meetings (extensive margin), so the fall in aggregate consumption associated with a panic is amplified when a depositor is unable to produce after withdrawal orders are sequentially served.

In this case, the expected utility of a buyer can be recursively defined by

$$\begin{aligned} V_t\left(S^t\right) &= \left\{-p_t\left(S^{t-1}\right)\beta\gamma+\left[1-p_t\left(S^{t-1}\right)\right]\lambda\left[U\left(\phi_t\left(S^t\right)\right)-\gamma\beta\right]\right\}\left[1-\hat{I}_t\left(S^t\right)\right] \\ &\quad +\lambda\left[U\left(\phi_t\left(S^t\right)\right)-\gamma\beta\right]\hat{I}_t\left(S^t\right)+\beta E\left[V_{t+1}\left(S^{t+1}\right)\right]. \end{aligned} \tag{47}$$

The impossibility of production after withdrawal orders are served significantly influences a buyer's decision to deposit in the banking system. In particular, a buyer is willing to deposit in the banking system provided

$$\begin{aligned} &\pi\left[1-p_t\left(S^{t-1}\right)\right]\lambda U\left(\phi_t\left(S^{t-1},r\right)\right)+(1-\pi)\lambda U\left(\phi_t\left(S^{t-1},n\right)\right) \\ \geq\ &\lambda U(1)+\pi p_t\left(S^{t-1}\right)(1-\lambda)\beta\gamma \end{aligned} \tag{48}$$

for all $S^{t-1} \in \{n,r\} \times ... \times \{n,r\}$. Note that it may be harder to induce a buyer to deposit in the banking system when those who end up losing their savings in a banking panic need to wait until the following period to rebalance their portfolios.

The expected utility of a seller is now given by

$$W_t\left(S^t\right)=\lambda m_t^1\left(S^t\right)\left[-w\left(y_t\left(S^t\right)\right)+v\left(\phi_t\left(S^t\right)\right)\right]+\beta E\left[W_{t+1}\left(S^{t+1}\right)\right]. \tag{49}$$

In the event of a panic, the probability of a trade meeting for a seller is smaller when depositors are unable to produce after withdrawing from the banking system. Because the bargaining protocol is such that $w\left(y_t\left(S^t\right)\right) = v\left(\phi_t\left(S^t\right)\right)$ for all $S^t \in \{n,r\} \times ... \times \{n,r\}$, it follows that $W_t\left(S^t\right) = 0$ for all $S^t \in \{n,r\} \times ... \times \{n,r\}$.

Given these changes relative to the version of the model described in Section 6, an equilibrium is defined in the same fashion by simply replacing the distributions (44)-(46), the value functions (47)-(49), and the participation constraint (48).

Now I want to show that there exists an equilibrium in which the investment plan and the production level in each bilateral meeting are the same as those described in Proposition 7. Suppose $\frac{1-\lambda}{1+\rho} \leq \bar{\iota} < \frac{1-\lambda}{\rho+\delta}$. Then, the following distributions are consistent with an equilibrium outcome:

$$m^1(n,n) = m^1(r,n) = 1, \tag{50}$$

$$m^1(n,r) = \lambda + (\delta+\rho)\,\bar{\iota}, \tag{51}$$

$$m^1(r,r) = 1 - (1-\delta)\,\bar{\iota}, \tag{52}$$

$$m^2(n,n) = m^2(r,n) = \lambda, \tag{53}$$

$$m^2(n,r) = \lambda^2 + \lambda(\delta+\rho)\,\bar{\iota}, \tag{54}$$

$$m^2(r,r) = \lambda - \lambda(1-\delta)\,\bar{\iota}, \tag{55}$$

$$m^3(n,n) = m^3(r,n) = \lambda, \tag{56}$$

$$m^3(n,r) = m^3(r,r) = 0. \tag{57}$$

From (48), it follows that a buyer is willing to deposit in the banking system at any date and state provided

$$(1-\pi)\,\lambda\left[U\left(\frac{1-\bar{\iota}}{\lambda}\right) - U(1)\right] \geq \pi(1-\delta)\,\bar{\iota}\sigma \tag{58}$$

and

$$(1-\pi)\,\lambda\left[U\left(1+\frac{\rho\bar{\iota}}{\lambda}\right) - U(1)\right] \geq \pi\left[1-\lambda-(\delta+\rho)\,\bar{\iota}\right]\sigma, \tag{59}$$

where $\sigma \equiv \beta\gamma(1-\lambda) + \lambda U(1)$. If the probability π associated with the realization r is such that

$$0 < \pi \leq \min\left\{\frac{\lambda U_n}{\left[1-\lambda-(\delta+\rho)\,\bar{\iota}\right]\sigma + \lambda U_n}, \frac{\lambda U_r}{(1-\delta)\,\bar{\iota}\sigma + \lambda U_r}\right\} \equiv \tilde{\pi},$$

then conditions (58) and (59) are simultaneously satisfied, so each buyer is willing to deposit in the banking system even though a panic occurs with probability π. Let me now formally establish the existence of an equilibrium with an illiquid banking system.

Proposition 9 *Suppose that* $\frac{1-\lambda}{1+\rho} \leq \bar{\iota} < \frac{1-\lambda}{\rho+\delta}$. *There exists* $\pi^{**} \in (0,1)$ *such that, for any* $\pi \in (0,\pi^{**})$, *there exists an equilibrium with an illiquid banking system in which the distributions of money holdings are given by (50)-(57), the value functions are given by (41)-(42) and (47), the investment plan is given by (29)-(35), the probability of loss in the event of a panic is given by (36)-(37), and the production of good* y *is given by (43).*

The state-contingent levels of *individual* consumption are the same as those described in Table 1, so the effects of a panic on the intensive margin remain the same. The main difference from the analysis in the previous section is the contraction of the extensive margin in the event of a panic. Note that this endogenous reduction in the number of trade meetings has an important dynamic effect on aggregate consumption. Specifically, the contraction of the extensive margin in the event of a panic is larger at any panic date preceded by a panic. In other words, the fact that a panic occurred in the previous period amplifies the negative effect of a panic in the current period, so the existence of extensive margin effects matters for aggregate consumption.

The effect of a systemic run on the extensive margin can explain why a sequence of banking panics, such as that observed during the Great Depression, tends to depress real economic activity in an unusual way, giving support to the alleged potency of the Friedman-Schwartz transmission mechanism. To understand this point, consider a sample path in which the unusual event of having banking panics at two consecutive dates actually occurs at dates T and $T+1$. In the absence of extensive margin effects, aggregate consumption falls to the minimum level (i.e., λ for good x and $\lambda w^{-1}(v(1))$ for good y) at both dates. This means that, at any panic date preceded by a panic, aggregate consumption remains depressed but does not fall further.

Consider now the presence of extensive margin effects due to the impossibility of production after withdrawal orders are sequentially served. Table 2 below summarizes aggregate consumption, with each row describing the level of aggregate consumption in each state $(S_{t-1}, S_t) \in \{n, r\} \times \{n, r\}$. Each column is in decreasing order.

Table 2: State-Contingent Aggregate Consumption

	good x	**good** y
(n,n)	$\lambda + \rho\bar{\iota}$	$\lambda w^{-1}\left(v\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)\right)$
(r,n)	$1-\bar{\iota}$	$\lambda w^{-1}\left(v\left(\frac{1-\bar{\iota}}{\lambda}\right)\right)$
(n,r)	$\lambda^2 + \lambda(\delta+\rho)\bar{\iota}$	$\left[\lambda^2 + \lambda(\delta+\rho)\bar{\iota}\right] w^{-1}(v(1))$
(r,r)	$\lambda - \lambda(1-\delta)\bar{\iota}$	$\left[\lambda - \lambda(1-\delta)\bar{\iota}\right] w^{-1}(v(1))$

In the presence of extensive margin effects, aggregate consumption falls to $\lambda^2 + \lambda(\delta+\rho)\bar{\iota}$ for good x and to $\left[\lambda^2 + \lambda(\delta+\rho)\bar{\iota}\right] w^{-1}(v(1))$ for good y at date T and falls to $\lambda - \lambda(1-\delta)\bar{\iota}$ for good x and to $\left[\lambda - \lambda(1-\delta)\bar{\iota}\right] w^{-1}(v(1))$ for good y at date $T+1$. Note that $\lambda > \lambda^2 + \lambda(\delta+\rho)\bar{\iota} > \lambda - \lambda(1-\delta)\bar{\iota}$, which means that a panic further depresses aggregate consumption if it occurs at a date preceded by a panic.

This property of the model is consistent with the Friedman-Schwartz hypothesis that the series of widespread bank failures starting in October 1930 and ending in March 1933 with the week-long national banking holiday significantly contributed to the severity and persistence of the contraction in output during the Great Depression.

8. CONCLUSIONS

This paper formalizes the view that the occurrence of banking panics can be a prominent source of aggregate fluctuations through its effects on the ability of banks to offer transaction and liquidity services in the form of interest-bearing deposits with the withdrawal option. In particular, the framework developed above explains why panic-induced recessions tend to be protracted events and why the occurrence of panics in consecutive periods, such as the series of systemic runs observed from 1930 to 1933 in the U.S., can depress real economic activity in an unusual way. The dynamic interaction between the ability of banks to provide transaction and liquidity services and the occurrence of panics is a key element of the theory. I have shown that the banking system is able to offer socially optimal transaction and liquidity services by holding a liquid portfolio not subject to runs only if productive projects are relatively abundant in the economy. Otherwise, the banking system is able to provide socially optimal transaction and liquidity services only if it holds an illiquid portfolio, opening the door to the possibility of self-fulfilling systemic runs. As a result, there exists an equilibrium with the property that banking panics eventually occur and significantly depress real economic activity.

A banking panic disrupts the investment plans of banks in such a way that the recovery of the desired banking portfolio consistent with normal times is not immediate, so panic-induced recessions are protracted events. As we have seen, the inability of banks to quickly reestablish the optimal provision of transaction and liquidity services following a panic results in an unusual loss of wealth for depositors that is capable of producing a prolonged recession, so the fact that bank liabilities function as a medium of exchange is crucial for the relevance of the transmission mechanism described above. This is an important contribution to the literature given that it is extremely difficult to construct an environment in which a demand deposit contract is *essential* (and not simply imposed to obtain panics in equilibrium) and the accumulation of assets is affected by panics in a persistent way.

An interesting extension of the model is to consider other specifications of the production function. The particular functional form used above has allowed me to derive several interesting analytical results. Although it may not be possible to obtain a closed-form solution under alternative functional forms, other specifications can imply a longer recovery period following a panic-induced recession if one is willing to simulate the model.

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APPENDIX

A.1. Proof of Lemma 2

If we substitute $i^p = \bar{\iota}$ and $s = 1$ into (7), we find $\phi = 1 + \frac{\rho\bar{\iota}}{\lambda}$. Because $i^s = \lambda\phi$, it follows that $i^s = \lambda + \rho\bar{\iota}$. To establish the optimality of this investment plan, consider the following variational argument. Given the investment decision at the previous date, the promised return on deposits at the current date is given by

$$1 + \frac{(1+\rho)\,\bar{\iota} - i^p}{\lambda}.$$

Given the investment decision at the following date, the promised return on deposits at the following date is given by

$$1 + \frac{F\left(i^p\right) - \bar{\iota}}{\lambda}.$$

Now define the relevant payoff function $\Gamma : \mathbb{R}_+ \rightarrow \mathbb{R}$ as follows:

$$\Gamma\left(i^p\right) \equiv \lambda U\left(1 + \frac{(1+\rho)\,\bar{\iota} - i^p}{\lambda}\right) + \beta\lambda U\left(1 + \frac{F\left(i^p\right) - \bar{\iota}}{\lambda}\right).$$

For any $i^p < \bar{\iota}$, the slope of Γ is given by

$$-U'\left(1 + \frac{(1+\rho)\,\bar{\iota} - i^p}{\lambda}\right) + \beta\left(1+\rho\right)U'\left(1 + \frac{(1+\rho)\,i^p - \bar{\iota}}{\lambda}\right).$$

Note that

$$U'\left(1 + \frac{(1+\rho)\,i^p - \bar{\iota}}{\lambda}\right) > U'\left(1 + \frac{(1+\rho)\,\bar{\iota} - i^p}{\lambda}\right)$$

for any $i^p < \bar{\iota}$. Because $\beta\left(1+\rho\right) > 1$, the slope of Γ is such that

$$-U'\left(1 + \frac{(1+\rho)\,\bar{\iota} - i^p}{\lambda}\right) + \beta\left(1+\rho\right)U'\left(1 + \frac{(1+\rho)\,i^p - \bar{\iota}}{\lambda}\right) > 0$$

for any $i^p < \bar{\iota}$. Because the productive technology pays off nothing for anything invested above $\bar{\iota}$, it follows that $i^p = \bar{\iota}$ is consistent with the maximization of the expected utility of depositors. This argument also proves that $i^p = \bar{\iota}$ is the unique solution. **Q.E.D.**

A.2. Proof of Proposition 3

In Lemma 2, I have already established that the investment plan $i^p = \bar{\iota}$ is the unique solution consistent with the maximization of the expected utility of depositors. As we have seen, the choice $i^p = \bar{\iota}$ implies $i^s = \lambda + \rho\bar{\iota}$ and $\phi = 1 + \frac{\rho\bar{\iota}}{\lambda}$. Because $\phi = 1 + \frac{\rho\bar{\iota}}{\lambda} > 1$, each buyer is willing to deposit in the banking system. Because $U(\phi) > U(1) \geq \frac{\gamma(1-\beta+\beta\lambda)}{\lambda}$, the buyer's participation constraint is satisfied. In addition, we have $J = W = 0$ and $V = (1-\beta)^{-1}\lambda\left[u(y) - \beta\gamma\right]$, with $y = w^{-1}\left(v\left(1 + \frac{\rho\bar{\iota}}{\lambda}\right)\right)$.

Now I want to show that this equilibrium allocation is Pareto optimal. A seller is willing to participate in any trading arrangement provided $W \geq 0$. Similarly, a banker is willing to participate in a trading arrangement provided $J \geq 0$. In the equilibrium allocation described above, both participation constraints hold with equality. It is clear that it is not possible to make either a seller or a banker better off without making a buyer worse off.

It remains to verify whether it is possible to achieve a higher level of expected utility for a buyer without making other agents worse off. There is one relevant feasible deviation that I need to check to conclude that the allocation is indeed Pareto optimal. Suppose that a buyer who holds a unit of money decides to produce a unit of good x and transfer it to a banker with the expectation that the banker can raise the purchasing power of *existing* deposits (i.e., no additional unit of money is issued). Note that it is infeasible to increase the level of investment in the productive technology given that the economywide productive capacity is fully utilized. Thus, these additional resources are necessarily invested in storage. In this case, it is feasible to implement the following return on deposits:

$$1 + \frac{\rho\bar{\iota}}{\lambda} + \frac{1-\lambda}{\lambda} = 1 + \frac{\rho\bar{\iota} + (1-\lambda)}{\lambda}.$$

Note that each banker remains indifferent and that the original investment plan is not altered in other periods. Now I need to verify whether a buyer holding a unit of money (i.e., a buyer who enters the period as a depositor) is willing to produce in order to increase the purchasing power of deposits in this way. A depositor is willing to produce provided that

$$-\gamma+\lambda U\left(1+\frac{\rho\bar{\imath}+(1-\lambda)}{\lambda}\right)>\lambda U\left(1+\frac{\rho\bar{\imath}}{\lambda}\right).$$

Rearranging this expression, we obtain the following condition:

$$\gamma<\lambda\left[U\left(1+\frac{\rho\bar{\imath}+(1-\lambda)}{\lambda}\right)-U\left(1+\frac{\rho\bar{\imath}}{\lambda}\right)\right].$$

If $\gamma \geq \lambda\left[U\left(1+\frac{\rho\bar{\imath}+(1-\lambda)}{\lambda}\right)-U\left(1+\frac{\rho\bar{\imath}}{\lambda}\right)\right]$, then a depositor is better off if he does not produce a unit of good x to raise the purchasing power of deposits. As a result, there is no feasible deviation that can increase the expected utility of buyers without making other agents worse off, which means that the aforementioned equilibrium allocation is indeed Pareto optimal. **Q.E.D.**

A.3. Proof of Proposition 5

Consider the portfolio choice $i_t^p\left(S^{t-1}\right)=\bar{\imath}$ and $i_t^s\left(S^{t-1}\right)=\lambda+\rho\bar{\imath}$ for any $S^{t-1}\in\{n,r\}\times ...\times\{n,r\}$. I have already established the optimality of this portfolio choice under the assumption that agents do not expect a banking panic to occur (see Lemma 2). Now it remains to verify whether each nonmover's decision to not withdraw in the postdeposit game is consistent with this investment plan. Because the liquidation value of the assets of the banking system at the time withdrawal requests can be made is such that $i_t^s\left(S^{t-1}\right)+\delta i_t^p\left(S^{t-1}\right)=\lambda+(\rho+\delta)\,\bar{\imath}\geq 1$ for all $S^{t-1}\in\{n,r\}\times ...\times\{n,r\}$, it is a dominant strategy for a nonmover *not* to withdraw from the banking system. The argument is basically the same as that described in Section 6.6. Thus, individual behavior is consistent with the choice of the aforementioned portfolio. Since all other equilibrium conditions are satisfied, it follows that the efficient allocation described in Proposition 3 is an equilibrium outcome even though the relocation status of each buyer is not publicly observable.

To establish uniqueness, I need to show that, given the expectation that a panic can be triggered by the realization of the sunspot signal r, the members of the banking system continue to optimally choose $i_t^p\left(S^{t-1}\right)=\bar{\imath}$ and $i_t^s\left(S^{t-1}\right)=\lambda+\rho\bar{\imath}$ for any $S^{t-1}\in\{n,r\}\times ...\times\{n,r\}$. Because this portfolio choice necessarily implies a liquid banking system, the expectation of a panic triggered by the realization of the sunspot signal r is not consistent with individual behavior because nonmovers will optimally choose not to withdraw. As a result, there cannot be an equilibrium with banking panics.

To verify the optimality of the aforementioned portfolio choice when agents contemplate the possibility of a panic, note that, for any contemporaneous choice of the level of productive investment $i^p \in (0, \bar{\iota})$, the liquidation value of the assets of the banking system at the current date is given by $(1+\rho)\bar{\iota} + \lambda - (1-\delta) i^p > \lambda + (\rho+\delta)\bar{\iota} \geq 1$, which means that the expectation of a panic triggered by the realization of the sunspot signal r at the current date is not consistent with individual behavior. But a panic can potentially occur at the following date if $i^p < \bar{\iota}$. To determine the optimal choice of the level of productive investment given the expectation that a panic can occur at the following date if the banking portfolio is illiquid, define the payoff function $\Gamma : \mathbb{R}_+ \to \mathbb{R}$ by

$$\begin{aligned}\Gamma(i^p) &= \lambda U\left(\frac{(1+\rho)\bar{\iota} - i^p}{\lambda} + 1\right) - \pi\left[1 - (1+\rho) i^p - \lambda + (1-\delta)\bar{\iota}\right]\beta\gamma \\ &\quad + \lambda(1-\pi)\beta U\left(\frac{(1+\rho) i^p - \bar{\iota}}{\lambda} + 1\right)\end{aligned}$$

if $0 \leq i^p < \frac{1-\lambda+(1-\delta)\bar{\iota}}{1+\rho}$ and

$$\Gamma(i^p) = \lambda U\left(\frac{(1+\rho)\bar{\iota} - i^p}{\lambda} + 1\right) + \lambda\beta U\left(\frac{(1+\rho) i^p - \bar{\iota}}{\lambda} + 1\right)$$

if $\frac{1-\lambda+(1-\delta)\bar{\iota}}{1+\rho} \leq i^p \leq \bar{\iota}$. This expression gives the relevant payoff for a depositor as a function of the contemporaneous level of productive investment i^p. Note that Γ is discontinuous at $i^p = \frac{1-\lambda+(1-\delta)\bar{\iota}}{1+\rho}$. This value implies the minimum level of investment in the productive technology consistent with no panic at the following date, so it corresponds to the best run-proof contract of Cooper and Ross (1998). As previously mentioned, there exists an intertemporal tradeoff regarding the amount of liquidity in the banking system. In particular, the decision to increase consumption today to such an extent that the level of productive investment falls below the threshold $\frac{1-\lambda+(1-\delta)\bar{\iota}}{1+\rho}$ necessarily increases the probability of loss in the event of a panic tomorrow (i.e., a run-proof contract will not be available tomorrow in this case).

Suppose $\pi = 0$. Then, the payoff function Γ is continuous and its slope is given by

$$-U'\left(1+\frac{(1+\rho)\,\bar{\imath}-i^p}{\lambda}\right)+\beta\,(1+\rho)\,U'\left(1+\frac{(1+\rho)\,i^p-\bar{\imath}}{\lambda}\right)>0$$

for any $0 < i^p < \bar{\imath}$. For any $\pi > 0$, the function Γ may not be monotonically increasing in the interval $\left(0,\frac{1-\lambda+(1-\delta)\bar{\imath}}{1+\rho}\right)$, but it must be the case that $\Gamma\left(\frac{1-\lambda+(1-\delta)\bar{\imath}}{1+\rho}\right) \geq \Gamma\left(i^p\right)$ for any $i^p \in \left(0,\frac{1-\lambda+(1-\delta)\bar{\imath}}{1+\rho}\right)$. Thus, the optimal level of productive investment is given by $i^p = \bar{\imath}$. Given this portfolio choice, it turns out that the expectation of a panic triggered by the realization of the sunspot signal r is not consistent with individual behavior, so it cannot be an equilibrium. As a result, the unique equilibrium is indeed the efficient equilibrium. **Q.E.D.**

A.4. Proof of Lemma 6

To verify this claim, consider the following variational argument. Define the payoff function $\Gamma^n : \mathbb{R}_+ \to \mathbb{R}$ by

$$\begin{aligned}\Gamma^n\left(i^p\right) &= \lambda U\left(\frac{(1+\rho)\,\bar{\imath}-i^p}{\lambda}+1\right)-\pi\left[1-(1+\rho)\,i^p-\lambda+(1-\delta)\,\bar{\imath}\right]\beta\gamma \\ &\quad +\lambda\,(1-\pi)\,\beta U\left(\frac{(1+\rho)\,i^p-\bar{\imath}}{\lambda}+1\right)\end{aligned}$$

if $0 \leq i^p \leq \frac{(1+\rho)\bar{\imath}-(1-\lambda)}{1-\delta}$ and

$$\begin{aligned}\Gamma^n\left(i^p\right) &= -\pi\left[1-(1+\rho)\,\bar{\imath}-\lambda+(1-\delta)\,i^p\right]\gamma+(1-\pi)\,\lambda U\left(\frac{(1+\rho)\,\bar{\imath}-i^p}{\lambda}+1\right) \\ &\quad -(1-\pi)\,\pi\left[1-(1+\rho)\,i^p-\lambda+(1-\delta)\,\bar{\imath}\right]\beta\gamma \\ &\quad +(1-\pi)^2\,\lambda\beta U\left(\frac{(1+\rho)\,i^p-\bar{\imath}}{\lambda}+1\right)\end{aligned}$$

if $\frac{(1+\rho)\bar{\imath}-(1-\lambda)}{1-\delta} < i^p \leq \bar{\imath}$. This expression gives the relevant payoff for a depositor as a function of the contemporaneous level of productive investment i^p, given that a panic did not occur in the previous period. Note that Γ^n is discontinuous at $i^p = \frac{(1+\rho)\bar{\imath}-(1-\lambda)}{1-\delta}$. This value implies the maximum level of investment in the productive technology consistent with no panic at the current date, so it corresponds to the best run-proof contract of Cooper and Ross (1998). This contract can rule out the possibility of a panic only at the current date. As previously mentioned, there exists an intertemporal tradeoff regarding the amount of liquidity in the banking system. In particular, a run-proof contract today necessarily increases the probability of loss in the event of a panic tomorrow.

In a similar fashion, define the payoff function $\Gamma^r : \mathbb{R}_+ \to \mathbb{R}$ by

$$\begin{aligned}\Gamma^r\left(i^p\right) &= -\pi\left(1-\delta\right)i^p\gamma + \left(1-\pi\right)\lambda U\left(\frac{1-i^p}{\lambda}\right) \\ &\quad - \left(1-\pi\right)\pi\left[1-\left(1+\rho\right)i^p - \lambda + \left(1-\delta\right)\bar{\iota}\right]\beta\gamma \\ &\quad + \left(1-\pi\right)^2\lambda\beta U\left(\frac{\left(1+\rho\right)i^p - \bar{\iota}}{\lambda} + 1\right).\end{aligned}$$

This expression gives the relevant payoff as a function of i^p, given that a panic occurred in the previous period. In this case, a run-proof contract is feasible if and only if $i^p = 0$. But this is a trivial portfolio choice that is equivalent to holding currency directly.

For any $\frac{(1+\rho)\bar{\iota}-(1-\lambda)}{1-\delta} < i^p < \bar{\iota}$, the slope of Γ^n is given by

$$-\pi\gamma\left(1-\delta\right) - \left(1-\pi\right)U'\left(\phi^n\right) + \left(1-\pi\right)\pi\left(1+\rho\right)\beta\gamma + \left(1-\pi\right)^2\left(1+\rho\right)\beta U'\left(\phi^n_+\right),$$

where $\phi^n = 1 + \frac{(1+\rho)\bar{\iota}-i^p}{\lambda}$ and $\phi^n_+ = 1 + \frac{(1+\rho)i^p-\bar{\iota}}{\lambda}$. We can rewrite this expression as

$$\left(1-\pi\right)\left[\left(1-\pi\right)\left(1+\rho\right)\beta U'\left(\phi^n_+\right) - U'\left(\phi^n\right)\right] + \pi\gamma\left[\left(1-\pi\right)\left(1+\rho\right)\beta - \left(1-\delta\right)\right].$$

Note that $\phi^n > \phi^n_+$ if and only if $i^p < \bar{\iota}$. Because $\pi \leq 1 - \frac{U'\left(\frac{1-\bar{\iota}}{\lambda}\right)}{\beta(1+\rho)U'\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)} < 1 - \frac{1}{\beta(1+\rho)}$, the slope of Γ^n is strictly positive for any $\frac{(1+\rho)\bar{\iota}-(1-\lambda)}{1-\delta} < i^p < \bar{\iota}$.

Suppose now $0 < i^p < \frac{(1+\rho)\bar{\iota}-(1-\lambda)}{1-\delta}$. In this case, the slope of Γ^n is given by

$$-U'\left(\phi^n\right) + \left(1-\pi\right)\left(1+\rho\right)\beta U'\left(\phi^n_+\right) + \pi\left(1+\rho\right)\beta\gamma.$$

Following the same steps, we conclude that the slope of Γ^n is also strictly positive for any $0 < i^p < \frac{(1+\rho)\bar{\iota}-(1-\lambda)}{1-\delta}$. Thus, we have established that Γ^n is strictly increasing in the intervals $\left(0, \frac{(1+\rho)\bar{\iota}-(1-\lambda)}{1-\delta}\right)$ and $\left(\frac{(1+\rho)\bar{\iota}-(1-\lambda)}{1-\delta}, \bar{\iota}\right)$. However, Γ^n is discontinuous at $i^p = \frac{(1+\rho)\bar{\iota}-(1-\lambda)}{1-\delta}$. In particular, it follows that $\lim_{i^p \to \frac{(1+\rho)\bar{\iota}-(1-\lambda)}{1-\delta}^-} \Gamma^n\left(i^p\right) > \lim_{i^p \to \frac{(1+\rho)\bar{\iota}-(1-\lambda)}{1-\delta}^+} \Gamma^n\left(i^p\right)$ when $\pi > 0$, so I need to verify whether $\Gamma^n\left(\bar{\iota}\right) > \Gamma^n\left(\frac{(1+\rho)\bar{\iota}-(1-\lambda)}{1-\delta}\right)$. Note that there exists $\pi' \in (0,1)$ sufficiently small such that $\Gamma^n\left(\bar{\iota}\right) > \Gamma^n\left(\frac{(1+\rho)\bar{\iota}-(1-\lambda)}{1-\delta}\right)$ for any $\pi \in (0, \pi')$. As a result, $i^p\left(n\right) = \bar{\iota}$ must hold at the optimum provided $0 < \pi < \bar{\pi} \equiv \min\left\{\pi', 1 - \frac{U'\left(\frac{1-\bar{\iota}}{\lambda}\right)}{\beta(1+\rho)U'\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)}\right\}$.

Consider now the payoff function Γ^r. For any $0 < i^p < \bar{\imath}$, the slope of Γ^r is given by

$$-\pi\gamma(1-\delta) - (1-\pi) U'(\phi^r) + (1-\pi)\pi(1+\rho)\beta\gamma + (1-\pi)^2 (1+\rho)\beta U'\left(\phi_+^n\right),$$

where $\phi^r = \frac{1-i^p}{\lambda}$. We can rewrite this expression as

$$[(1-\pi)(1+\rho)\beta - (1-\delta)]\pi\gamma + (1-\pi)\left[-U'(\phi^r) + (1-\pi)(1+\rho)\beta U'\left(\phi_+^n\right)\right].$$

Because $\bar{\imath} < \bar{\imath}^*$, it follows that $-U'\left(\frac{1-\bar{\imath}}{\lambda}\right) + (1-\pi)(1+\rho)\beta U'\left(1+\frac{\rho\bar{\imath}}{\lambda}\right) > 0$. Thus, the slope of Γ^r is strictly positive for any $i^p < \bar{\imath}$. As a result, $i^p(r) = \bar{\imath}$ must hold at the optimum. Thus, the conjecture $i^p(n) = i^p(r) = \bar{\imath}$ is indeed consistent with the maximization of the expected utility of depositors provided $\pi \in (0, \bar{\pi})$. **Q.E.D.**

A.5. Proof of Proposition 7

Lemma 6 has already established the optimality of the investment plan (29)-(35) when $\pi \in (0, \bar{\pi})$. We have also seen that a buyer is willing to deposit in the banking system provided $\pi \in (0, \hat{\pi})$. Thus, the allocation described in Proposition 7 is consistent with the equilibrium conditions provided $0 < \pi < \min\{\bar{\pi}, \hat{\pi}\}$. **Q.E.D.**

A.6. Proof of Proposition 8

Consider the investment plan associated with the efficient equilibrium described in Proposition 3. As we have seen, it follows that $i_t^p\left(S^{t-1}\right) = \bar{\imath}$ and $i_t^s\left(S^{t-1}\right) = \lambda + \rho\bar{\imath}$ for all $S^{t-1} \in \{n, r\} \times ... \times \{n, r\}$. Because the liquidation value of the portfolio of the banking system is such that $\lambda + (\delta + \rho)\bar{\imath} < 1$, it is a best response for a nonmover to withdraw if he believes that other nonmovers are also withdrawing. Thus, it is possible to implement the efficient allocation as an equilibrium outcome only if nonmovers *always* believe that other nonmovers are not going to withdraw regardless of the realization of the sunspot variable. **Q.E.D.**

A.7. Proof of Proposition 9

To establish the optimality of the investment plan (29)-(35), let $\hat{\Gamma}^n : \mathbb{R}_+ \rightarrow \mathbb{R}$ denote the relevant payoff function when a panic did not occur at the previous date, and let $\hat{\Gamma}^r : \mathbb{R}_+ \rightarrow \mathbb{R}$ denote the relevant payoff function when a panic occurred at the previous date. Note that $\hat{\Gamma}^n$ is given by

$$\begin{aligned}\hat{\Gamma}^n\left(i^p\right) &= \lambda U\left(\frac{(1+\rho)\,\bar{\iota}-i^p}{\lambda}+1\right)-\pi\left[1-(1+\rho)\,i^p-\lambda+(1-\delta)\,\bar{\iota}\right]\beta\hat{\gamma} \\ &\quad +\lambda\beta U\left(\frac{(1+\rho)\,i^p-\bar{\iota}}{\lambda}+1\right)\end{aligned}$$

if $0 \leq i^p \leq \frac{(1+\rho)\bar{\iota}-(1-\lambda)}{1-\delta}$ and

$$\begin{aligned}\hat{\Gamma}^n\left(i^p\right) &= -\pi\left[1-(1+\rho)\,\bar{\iota}-\lambda+(1-\delta)\,i^p\right]\hat{\gamma}+(1-\pi)\,\lambda U\left(\frac{(1+\rho)\,\bar{\iota}-i^p}{\lambda}+1\right) \\ &\quad -(1-\pi)\,\pi\left[1-(1+\rho)\,i^p-\lambda+(1-\delta)\,\bar{\iota}\right]\beta\hat{\gamma} \\ &\quad +(1-\pi)^2\,\lambda\beta U\left(\frac{(1+\rho)\,i^p-\bar{\iota}}{\lambda}+1\right)\end{aligned}$$

if $\frac{(1+\rho)\bar{\iota}-(1-\lambda)}{1-\delta} < i^p \leq \bar{\iota}$, where $\hat{\gamma} \equiv \beta\gamma+\lambda U(1)$. Note also that $\hat{\Gamma}^r$ is given by

$$\begin{aligned}\hat{\Gamma}^r\left(i^p\right) &= -\pi\,(1-\delta)\,i^p\hat{\gamma}+(1-\pi)\,\lambda U\left(\frac{1-i^p}{\lambda}\right) \\ &\quad -(1-\pi)\,\pi\left[1-(1+\rho)\,i^p-\lambda+(1-\delta)\,\bar{\iota}\right]\beta\hat{\gamma} \\ &\quad +(1-\pi)^2\,\lambda\beta U\left(\frac{(1+\rho)\,i^p-\bar{\iota}}{\lambda}+1\right).\end{aligned}$$

Thus, the same argument used in the proof of Lemma 6 applies here to show the optimality of the investment plan (29)-(35) when $\hat{\Gamma}^n : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ and $\hat{\Gamma}^r : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ are the relevant payoff functions. In particular, it is possible to show that there exists $\pi'' \in (0,1)$ such that the investment plan (29)-(35) maximizes the expected utility of depositors provided $\pi \in (0, \pi'')$.

I have already shown that a buyer is willing to deposit in the banking system provided $\pi \in (0, \tilde{\pi})$. Thus, the allocation previously described is consistent with the equilibrium conditions provided $0 < \pi < \pi^{**} \equiv \min\{\pi'', \tilde{\pi}\}$. **Q.E.D.**

Figure 1: Timeline

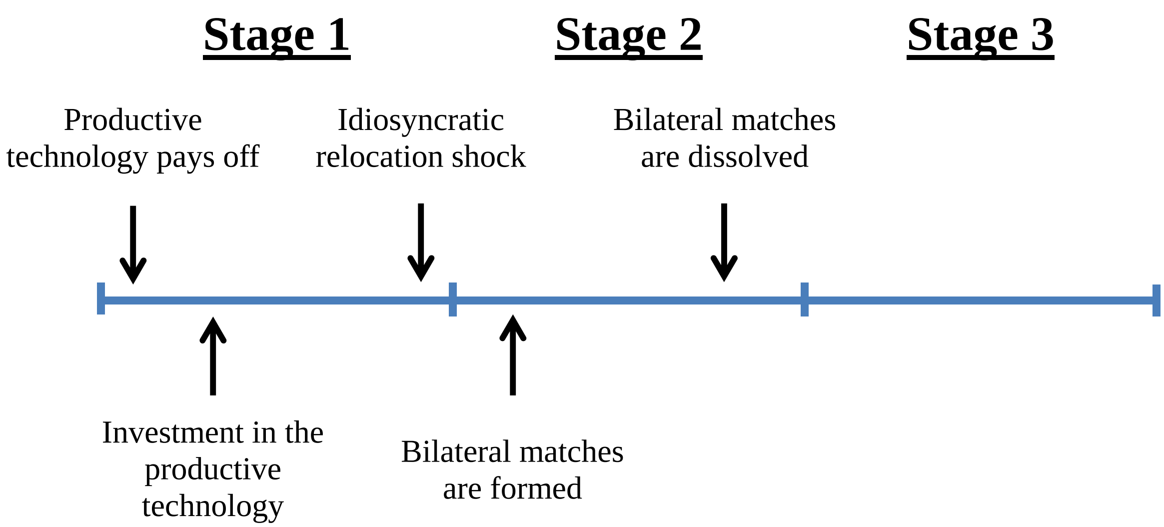

Figure 2: No Banking Panic

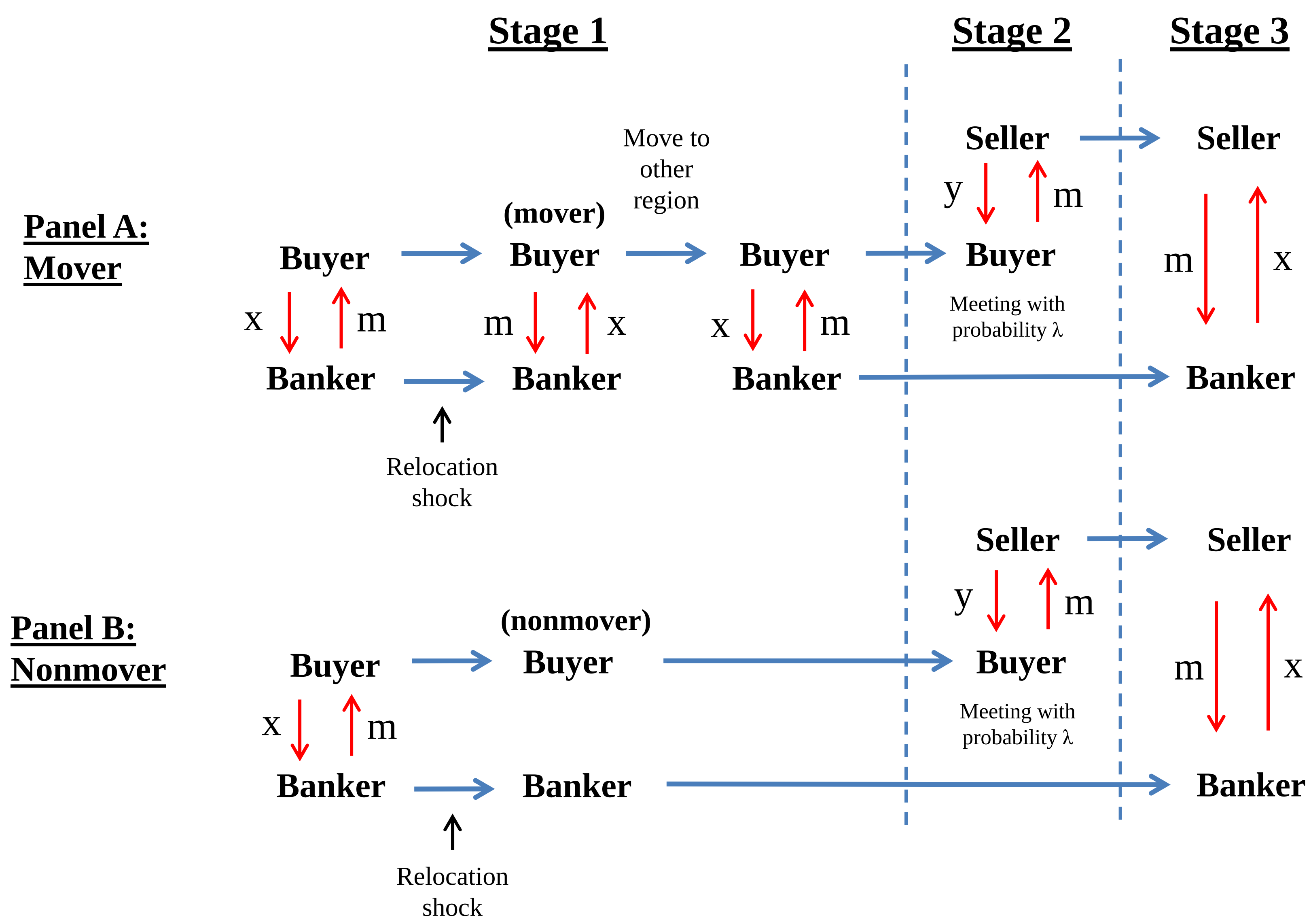

Figure 3: Banking Panic