WORKING PAPER NO. 13-44
POLARIZED BUSINESS CYCLES

Marina Azzimonti
Federal Reserve Bank of Philadelphia

Matthew Talbert
University of Texas – Austin

November 2013
Polarized Business Cycles *

Marina Azzimonti† and Matthew Talbert‡

Abstract

We are motivated by four stylized facts computed for emerging and developed economies: (i) business cycle movements are wider in emerging countries; (ii) economies in emerging countries experience greater economic policy uncertainty; (iii) emerging economies are more polarized and less politically stable; and (iv) economic policy uncertainty is positively related to political polarization. We show that a standard real business cycle (RBC) model augmented to incorporate political polarization, a ‘polarized business cycle’ (PBC) model, is consistent with these facts. Our main hypothesis is that fluctuations in economic variables are not only caused by innovations to productivity, as traditionally assumed in macroeconomic models, but also by shifts in political ideology. Switches between left-wing and right-wing governments generate uncertainty about the returns to private investment, and this affects real economic outcomes. Since emerging economies are more polarized than developed ones, the effects of political turnover are more pronounced. This translates into higher economic policy uncertainty and amplifies business cycles. We derive our results analytically by fully characterizing the long-run distribution of economic and fiscal variables. We then analyze the effect of a permanent increase in polarization on PBCs.

JEL Classification: E3, H3

Keywords: Polarized Business Cycle, Real Business Cycle, Economic Policy Uncertainty, Polarization, Political Stability, Political Economy, Fiscal Policy, Markov Equilibrium, Time Consistency.

*We would like to thank Paul Klein for an excellent discussion at the 2011 Vienna Macro Workshop. This paper also benefited from insightful conversations with Mark Aguiar, Pablo Neumeyer, Juan Pablo Nicolini, and seminar participants at the Federal Reserve Bank of Cleveland, the Federal Reserve Bank of Philadelphia, the Universidad Autonoma de Barcelona, the Universidad Torcuato di Tella, and the 2011 SED Meetings. Michael Chimowitz provided excellent research assistance for this project. An earlier version of this paper circulated under the title ‘Partisan cycles and the consumption volatility puzzle’. The corresponding author can be contacted at Marina.Azzimonti@gmail.com. The views expressed in this paper are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or the Federal Reserve System. This paper is available free of charge at www.philadelphiafed.org/research-and-data/publications/working-papers/

†FRB of Philadelphia
‡University of Texas - Austin
1 Introduction

We are motivated by a set of observations drawn from comparing stylized facts in emerging versus developed economies. First, aggregate economic variables (output and consumption) tend to be more volatile and less persistent in emerging countries. This is a well-known fact. Second, emerging economies experience greater economic policy uncertainty than developed ones. Figure 1 depicts the evolution of economic policy uncertainty, measured by a news search-based index in an emerging country (Mexico) and a developed one (Sweden). Consistent with our claim, Mexico’s uncertainty index has been on average 17% higher than Sweden’s index between 1990 and 2003.

![Figure 1: Economic policy uncertainty (news based) in Mexico and Sweden.](image)

Note: Economic policy uncertainty is proxied by a news-based index constructed by Brogaard and Detzel (2012), following the methodology in Baker, Bloom, and Davis (2012).

Third, political polarization is higher in emerging countries, and their political processes are more unstable. Finally, there is a positive correlation between economic policy uncertainty and political polarization. Figure 2 illustrates this, using two alternative policy-based measures of economic policy uncertainty: the volatility of government revenues as a percentage of output (left panel) and the relative volatility of government consumption to output (right panel). The correlation between polarization and the news-based index of economic policy uncertainty computed by Brogaard and Detzel (2012) is 0.24, in a sample of 19 countries.

Analyzing the evolution of political polarization and policy uncertainty in the U.S. suggests that this relationship may also hold true within a country over time. As pointed out by Baker, Bloom, and Davis (2012), the recent recession has been associated with greater-than-historical economic policy uncertainty (see Figure 3). Interestingly, 2006-2011 has also been a period of higher-than-historical political polarization, as seen in the figure.

The objective of this paper is to develop a theory that is consistent with these facts, centered on the interaction between political frictions and the real business cycle. Our main hypothesis is that fluctuations in economic variables are not only caused by innovations to productivity, as traditionally assumed in macroeconomic models, but also by shifts in political ideology. In our
Figure 2: Economic policy uncertainty (policy based) and polarization. (▲ Developed and ◦ Emerging).

Note: Economic policy uncertainty is proxied by two policy-based indexes. Left panel: coefficient of variation of revenues as a percentage of output in 1960-2003. Right panel: relative standard deviation of government consumption to output, series de-trended using a band-pass filter (2-20). Political polarization is obtained from Lindqvist and Ostling (2010).

Figure 3: Economic policy uncertainty and polarization in the U.S., 1985-2011.

Note: Economic policy uncertainty is proxied by the news-based index constructed by Baker, Bloom and Davis (2012). Political polarization is obtained from McCarty, Poole, and Rosenthal (2006), based on the voting patterns of members of the U.S. House of Representatives and Senate. The historical average is for the period 1900-2005.

model, disagreement between policymakers is rooted in their opposing views regarding the size and scope of the government. These are translated in preferred levels of spending, which depend on the incumbents’ ideology. Since public spending must be financed with distortionary instruments, wedges on investment are also affected by this ideology. Figure 4, which shows the evolution of the investment wedge (proxied by capital taxation) and the identity of the party in power (proxied by party affiliation of the President) in the U.S., provides suggestive evidence of this. Episodes where
the Republican party takes power are followed by sharp declines in the investment wedge.\footnote{This is robust to the introduction of TFP shocks and other control variables, such as composition of the legislature, in a regression where the dependent variable is the percentage growth in capital taxes (to eliminate the time trend). Details available upon request from the author.}

![Figure 4: Capital taxes in the U.S., 1958:Q1-2010:Q2 (line) and political ideology (shaded area=Democratic President).](image)

**Note:** Capital taxes are obtained from Fernández-Villaverde, Guerrón, Kuester, and Rubio-Ramírez’s (2012) estimations. Shaded areas represent periods where the President belongs to the Democratic Party.

Because parties alternate in power, political turnover in polarized societies induces economic policy uncertainty. This gives rise to polarized business cycles (PBC). The mechanism is the following: switches between a left and right government generate uncertainty about the returns to private investment, and this affects real economic outcomes, amplifying business cycles. Countries that are more polarized exhibit greater economic policy uncertainty (e.g., larger swings in the level of spending and revenue financing) and hence wider fluctuations in output and consumption.

We elaborate on this argument in a dynamic political economy model and provide intuition by looking at an example economy for which analytical solutions can be computed. We fully characterize the long-run distribution of fiscal and economic variables and study how second moments are affected by the degree of political polarization. We then calibrate a more general environment to analyze the quantitative impact of polarization on the PBC of the U.S. economy. In particular, we conduct a counter-factual experiment where the degree of polarization in the U.S. increased permanently to Mexico’s level (a rise of about 30%). The results from this experiment shed light on how the recent increase in political polarization between 2006-2011, depicted in Figure 3, may have affected the business cycle during the last economic crisis, through its impact on economic policy uncertainty.

**Connections to existing literature**

Our setup embeds Persson and Svensson’s (1989) political economy model of public goods provision in a standard stochastic neoclassical framework. Political parties that disagree on the size of government stochastically alternate in power. Left-wing parties place more weight on public
spending than right-wing parties and hence impose a higher burden on the public sector in order to finance a larger level of expenditures. This introduces an additional source of volatility for economic variables: economic policy uncertainty. We can interpret changes in government policy triggered by switches in ideology as political shocks. In contrast to total factor productivity (TFP) shocks, a political shock affects consumption immediately through changes in agents’ disposable income, while the response of output (caused by changes in investment) is delayed and muted. As a result, consumption volatility can be larger than output volatility in the presence of political shocks. We thus provide a novel potential explanation to the consumption volatility puzzle.\(^2\) Our mechanism is related to the earlier work of Dotsey (1990) and Baxter and King (1993) who study the effects of exogenous government expenditure shocks on macroeconomic activity. A main departure is that public policy is endogenous in our model. Additionally, we are able to generate the stylized fact that government spending is more volatile in emerging economies than in developed ones (the average volatility is three times as large, as shown in Table 8) and to provide a reasonable channel by which this happens.\(^3\)

We endogenize public spending by building on a growing literature on political macroeconomics. There are two important frictions borrowed from this literature—in addition to political disagreement—relative to a standard neoclassical economy that are key to our results. The first one is that the policymaker lacks commitment. Inefficiencies are introduced because neither party can credibly commit to follow a particular sequence of fiscal policy. This relates to the theories of political failure presented by Persson and Svensson (1989) and Besley and Coate (1998) in two-period models. We focus instead on an infinite horizon economy and characterize time-consistent outcomes as Markov-perfect equilibria following Klein, Krusell, and Rios-Rull (2008).

The second friction results from the outcome of future elections being uncertain at the time policy choices are taken under incomplete markets (a set of contingent claims that allows the current policymaker to insure against shocks does not exist). This generates additional inefficiencies because the incumbent party is more shortsighted than its constituency when subject to political uncertainty. This was first pointed out by Alesina and Tabellini (1990) and more recently studied in fully dynamic models by Aguiar and Amador (2011) and Azzimonti (2011). Similar effects have been studied in environments with debt instead of capital by Ales, Maziero, and Yared (2012), Barseghyan, Battaglini, and Coate (2011), Caballero and Yared (2008), Debortoli and Nunes (2010), and Ilzetzki (2011). While their environments are completely symmetric, the ideology of the policymaker may change over time in our model, so we analyze equilibria where policy functions are asymmetric. Persson and Svensson (1989) and Cukierman, Edwards, and Tabellini (1992) study asymmetric equilibria, but in two-period deterministic models where the emphasis is on manipulation of government policy. In our environment, the incumbent party also has incentives

\(^2\) Aguiar and Gopinath (2007) argue that the volatility of permanent income dominates the volatility of transitory shocks to income in emerging economies, which explains why their consumption volatility exceeds the volatility of output. Neumeyer and Perri’s (2005) explanation relies instead on the existence of financial frictions and is based on the financing of a firm’s working capital. Current interest rate shocks affect labor (and hence output) with a lag, while private savings (and hence consumption) adjust immediately. The emphasis in these models is on real shocks to the economy, either through total factor productivity (transitory vs. permanent) or interest rates. In this paper, the additional volatility of consumption arises due to ideological swings of policymakers, which can be interpreted as political shocks.

\(^3\) Woo (2005) and Bachmann and Bai (2013b) rely on preference shocks to the planner’s welfare to generate large levels of public consumption volatility but do not consider a voting game.
to use policy strategically because changes in the investment wedge, by affecting individual savings, modify the revenue base inherited by its successor. Milesi-Ferretti and Spolaore (1994) and more recently Song (2012) and Azzimonti (2013) study the effect of strategic manipulation on partisan cycles.

The interaction of these two frictions gives rise to economic policy uncertainty, which generates politically driven business cycles. We contribute to a recent but growing literature that tries to understand the effects of economic policy uncertainty on the aggregate economy (see Baker, Bloom, and Davis, 2012 or Fernández-Villaverde and Rubio-Ramírez, 2010). While most of this literature assumes that fiscal policy follows an exogenous process, we derive fluctuations in government policy endogenously from politico-economic fundamentals. The exceptions are Bachmann and Bai (2013a, 2013b) who quantify the impact of political factors on the volatility and cyclicality of government consumption. Like Bachmann and Bai (2013a), we find that the procyclicality of government consumption decreases in the presence of political frictions. Our channel is, however, quite different, since it relies on the degree of polarization rather than on the counter-cyclicality of wealth inequality. Bachmann and Bai (2013b) also analyze government consumption over the business cycle, but in a model where the policymaker is subject to taste shocks and implementation lags. Our specifications are related, as a change in ideology could be seen as a taste shock in a static model. Theoretically, the main difference lies in the form taken by the continuation utility: An incumbent must evaluate its constituency’s welfare under the opposition’s policy if the re-election was lost. This results in the values taken by political shocks being endogenous in our model: They depend on technological as well as institutional characteristics of the economy, such as polarization and political instability. Moreover, we show how changes in political polarization affect the dynamic behavior of PBCs. We conjecture that polarization shocks could be an important force driving variations in the innovation of observed fiscal processes, which were documented by Fernández-Villaverde, Guerrón, Kuester, and Rubio-Ramírez (2012).

Finally, a technical contribution relative to previous papers is the theoretical characterization of stochastic politico-economic equilibria in a standard real business cycle model. More important, our tractable model allows us to compute long-run moments of the distribution of economic and fiscal variables analytically and show how they are affected by changes in polarization.  

The paper is organized as follows. Section 2 describes a set of stylized facts that characterize the business cycle properties of emerging and developed economies for a panel of countries. The main assumptions of the model are summarized in Section 3, where the stochastic politico-economic equilibrium is defined. Intuition on how this model helps explain the mechanisms of the model is presented in Section 4, where we find tractable analytical expressions and decompose the cyclicality and volatility of relevant variables between TFP and political shocks. We calibrate the model to the U.S. economy in Section 5 and perform our main experiment by varying the degree of political polarization across countries in Section 5.2. The main business cycle moments for our artificial economy are computed and contrasted with the ones analyzed in Section 2. Conclusions and extensions are contained in Section 6. All proofs are relegated to the Appendix.

---

4This computation is non-trivial due to the persistence of both TFP and political ideology. If shocks were iid instead, variables would evolve according to an AR(1) process. Under persistent Markov shocks, the dynamics cannot be reduced to well-known standard AR or MA processes.
2 Stylized Facts

The countries under study are summarized in Table 7, Appendix 7.1. Following Aguiar and Gopinath (2007), we use the S&P classification for developed markets for our developed economies and classify all other countries as emerging.\footnote{Two primary criteria used in defining a country as a developed market are (i) it is located in a high-income country as defined by the World Bank and (ii) its capital markets are highly developed and transparent with large market capitalization.}

The data are obtained from Kaminsky, Reinhart, and Végh (2004), who compile a comprehensive cross-country panel for our main variables of interest from the IMF World Economic Outlook (WEO) and IMF Government Financial Statistics (GFS) data sets. Because most series are non-stationary, we compute business cycle moments (volatilities, autocorrelations, and correlations) on filtered data. Following Comin and Gertler (2006), natural logarithms of each (GDP deflated) variable are de-trended using a band-pass filter. Because this filter is a two-sided moving average filter, the advantage of following this approach relative to using the HP-filter is that we can isolate data for different frequencies. This is particularly important when analyzing the effect of political variables on economic outcomes, because the frequency of political ideology (turnover between parties) is smaller and, hence, more persistent than that of the business cycle.

We focus on the ‘medium-term cycle’ where frequencies range between 2 and 20 years. The high frequency component, between 2 and 8 years, coincides with the standard definition for the real business cycle. The medium term cycle 2-20 reflects cycles of eight years to a decade in the time domain, more in line with political cycles, which are the focus of our analysis. The resulting statistics for the medium-term cycle (2 to 20) are similar to an HP filter with parameter $w = 100$ (results using the HP-filter are available in Appendix 7.1). Since data availability is not consistent across the four variables for individual countries, the period studied in each case reflects the longest time span for which we have complete data for that country. Sample lengths for each country are reported in Table 7, Appendix 7.1.

Fact 1: Business cycle movements are wider in emerging countries.

Output, consumption, and investment are more volatile and less persistent in emerging countries. The volatility of output in emerging economies is about twice the volatility in developed ones, as seen in Table 8. This has previously been documented by Aguiar and Gopinath (2007) for quarterly data. They also point to the striking difference in the volatility of consumption relative to output between the two groups. Consumption is as volatile as output for developed economies ($\sigma(c)/\sigma(y) = 1.00$), but it is 25% more volatile than output in emerging ones. There are some exceptions, as can be seen in the first column of Tables 9 and 10 in Appendix 7.1, which reports the individual values for each country. While a traditional neoclassical economy would not generate such patterns, Aguiar and Gopinath (2007) show that an RBC model augmented by trend shocks can generate some of the observed variability. While the authors do not explicitly consider the causes underneath the divergence in the shock processes of emerging and developed economies, they point to government policy as a potential explanation. In this paper, we take their conjecture one step further by actually considering how political factors affect the business cycle. This is motivated by our second stylized fact.
Table 1: Volatility of economic variables

<table>
<thead>
<tr>
<th>Moment</th>
<th>Developed Economies</th>
<th>Emerging Economies</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(y)$</td>
<td>2.51e-2</td>
<td>4.95e-2</td>
</tr>
<tr>
<td>$\sigma(I)/\sigma(y)$</td>
<td>3.80</td>
<td>3.86</td>
</tr>
<tr>
<td>$\sigma(c)/\sigma(y)$</td>
<td>1.00</td>
<td>1.25</td>
</tr>
</tbody>
</table>

Autocorrelations

| $\rho(y)$                  | 0.65                | 0.62               |
| $\rho(I)$                  | 0.59                | 0.5                |
| $\rho(c)$                  | 0.60                | 0.49               |

Cyclicality

| $\rho(y, I)$               | 0.84                | 0.68               |
| $\rho(y, c)$               | 0.80                | 0.64               |
| $\rho(y, g_c)$             | 0.24                | 0.25               |

Note: This table contains the average value of moments computed for a set of emerging and developed economies. Relative volatility measures for each country can be found in Tables 9 and 10 in Appendix 7.1. Data are obtained from Kaminsky, Reinhart, and Végh (2004). The natural logarithms of deflated variables are de-trended using a band-pass filter 2-20.

Fact 2: Emerging economies experience greater economic policy uncertainty.

There are several ways of computing economic policy uncertainty. A standard measure involves the volatility of policy variables themselves. In this paper, we consider two alternatives: the volatility of government consumption and the variation in the revenue-to-output ratio. We find that general government consumption is more volatile than output by a factor of three ($\sigma(g_c)/\sigma(y) = 3.37$) in emerging countries, twice the value observed in developed ones, $\sigma(g_c)/\sigma(y) = 1.47$. The coefficient of variation in revenues (as a percentage of output) is also significantly larger in emerging countries, where it takes a value of 19 versus 8.6 in developed economies.

Baker, Bloom, and Davis (2012) constructed a more direct measure for the degree of uncertainty related to economic policy in the U.S. based on news media. The index averages a set of components reflecting the frequency of references in the news to: (i) economic policy uncertainty, (ii) federal tax code provisions set to expire in the future, and (iii) forecaster disagreement over inflation and government purchases. Brogaard and Detzel (2012) applied this methodology to an extended set of countries. Table 2 shows that the average value of this index for emerging economies is larger than for developed economies. Unfortunately, we do not have access to data from a wider set of countries.

We thank the authors for kindly sharing their data with us. The developed countries included are Australia, Canada, France, Germany, Italy, Japan, the Netherlands, Spain, Sweden, Switzerland, the UK, and the U.S.. The emerging economies are Brazil, China, Hong Kong, India, Korea, Malaysia, Mexico, Russia, and South Africa.
countries (as in our economic data set) for a longer interval of time, but the evidence is in line with our finding for policy-based measures.

**Table 2: Economic Policy Uncertainty (EPU)**

<table>
<thead>
<tr>
<th>Moment</th>
<th>Developed Economies</th>
<th>Emerging Economies</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Policy-based:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(g)/\sigma(y)$</td>
<td>1.47</td>
<td>3.37</td>
</tr>
<tr>
<td>$CV(Rev/y)$</td>
<td>8.64</td>
<td>19.04</td>
</tr>
<tr>
<td><strong>News-based:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>nEPU</td>
<td>2.57</td>
<td>2.78</td>
</tr>
</tbody>
</table>

*Note: nEPU is obtained from Brogaard and Detzel (2012), for the longest span of data available between 1990 and 2003. $\sigma(g)/\sigma(y)$ refers to relative volatility of government consumption (natural logarithms of deflated variables, de-trended using a band-pass filter 2-20), and $CV(Rev/y)$ denotes the coefficient of variation of general government revenue as a ratio of GDP (in percentage terms). Data for Revenues, $g$ and $y$ are obtained from Kaminsky, Reinhart, and Végh (2004), for the longest span of data available between 1960 and 2003.*

Fact 3: Emerging countries are more polarized and their political processes are less stable.

Figure 5 depicts percentage deviations of each variable with respect to its median.

![Figure 5: Polarization and Stability, relative to their means.](image)

The plot shows that most emerging economies have higher polarization and lower political
stability indexes than developed economies, since they locate in the upper left quadrant of the
graph.

Differences are sizable, with variations up to 40% away from the median values in both variables. The correlation is, however, not perfect. This is important to keep in mind, because these two institutional variables affect the volatility of economic series in different ways, as explained in Section 4. A theory that abstracts from political stability or polarization will not be able to account for the cross-country differences in the data.

Fact 4: There is a positive correlation between economic policy uncertainty and political polarization.

Empirical studies using polarization offer reassuring but ultimately unsatisfying support for our hypothesis that polarization matters for business cycles. Starting with Easterly and Levine (1997) a large literature has developed attributing economic outcomes to ideological differences in the population. However, most of these papers use non-partisan heterogeneity in the population, for example, employing ethnic, religious, and linguistic divisions as their polarization measures deliver negative outcomes.7 Closer to our partisan, model Alt and Lassen (2000), Canes-rone and Ang Park (2011), and Lindqvist and Östling (2010) empirically link political polarization to economic performance using different and more relevant measures of partisan disagreement over the size of government. We adopt the measure from Lindqvist and Östling directly. They use self-reported political preferences from the 1999-2002 World Values Survey (a description of their method can be found in Appendix 7.1). Importantly, they find that political polarization is not endogenous to economic performance. Using this measure, we find our third stylized fact: Economic policy uncertainty is positively related to the degree of political polarization, as shown Figure 2.

Table 3 reports the coefficients of several specifications of a regression where the dependent variable is policy-based economic policy uncertainty (measured by the volatility of government spending or revenues to output ratios) and the independent variables are polarization and political stability. Robust standard errors are reported in parentheses. 8

The coefficient on polarization is positive and statistically significant at the 5% level, regardless of whether we consider the volatility in revenues to GDP or government consumption, indicating that more polarized countries experience higher economic policy uncertainty. This suggests that polarization amplifies business cycles.

Clearly, political stability affects this relationship, since polarization would be irrelevant in a country with no political turnover. Consistent with this intuition, the coefficient of political stability is negative in specifications (1) and (2), where it enters additively. We also considered an interaction term in specifications (3) and (4). The negative (and significant coefficient) suggests that the amplification effects of polarization are smaller in countries with low political turnover (e.g., high political stability). These regressions are not intended to capture causality, since there could potentially be endogeneity problems in their specification. Our objective is just to point out that the positive correlation between polarization and policy volatility is significant.

---

7 Alesina and Zhuravskaya (2008) provide a recent novel measure of population heterogeneity and its effect on the quality of government.
8 We are not reporting the results using the news-based measure of EPU because we have too few observations in the intersection between our polarization data set and Brogaard and Detzel’s (2012).
### Table 3: Economic policy uncertainty and polarization.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polarization</td>
<td>9.1***</td>
<td>1.82***</td>
<td>15.3***</td>
<td>2.38***</td>
</tr>
<tr>
<td></td>
<td>(2.09)</td>
<td>(0.49)</td>
<td>(3.95)</td>
<td>(0.70)</td>
</tr>
<tr>
<td></td>
<td>(7.05)</td>
<td>(1.55)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polarization × Pol Stability</td>
<td>n.a.</td>
<td>n.a.</td>
<td>-1.36***</td>
<td>-0.19**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.51)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>Observations</td>
<td>46</td>
<td>46</td>
<td>46</td>
<td>46</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.77</td>
<td>0.79</td>
<td>0.74</td>
<td>0.150</td>
</tr>
</tbody>
</table>

Notes: The dependent variable for columns (1) and (3) is the coefficient of variation in revenues to output \(\text{CV(Rev/y)}\) and for columns (2) and (4) is the relative volatility of public consumption \(\sigma(g)/\sigma(y)\). Robust standard error are in parentheses. Constant term excluded because it was statistically insignificant.

* Significant at 10%. ** Significant at 5%. *** Significant at 1%.

### 3 A theory of Polarized Business Cycles


#### 3.1 Economic environment

Technology is characterized by a constant returns-to-scale production function that uses capital \(k\) and labor \(l\) to produce a single consumption good, \(F(z, k, l)\). The variable \(z\) represents an aggregate productivity shock that follows a first-order Markov process. Capital depreciates at rate \(\delta\).

There is also a public good, denoted by \(g\), that can be produced from the consumption good, according to a linear technology. We normalize the time endowment in the economy to 1. Thus, the aggregate resource constraint reads

\[
c + g + k' = F(z, k, 1) + (1 - \delta)k.
\]

There are competitive labor and capital markets and competitive production of public goods. The relative price of private and public goods is one in equilibrium. The wage rate is denoted by \(w\) and the rental rate of capital by \(r\). Firms hire labor and capital in order to maximize profits after observing their productivity shock. Their decision problem is static and deterministic, implying

\[
w = F_l(z, k, l) \text{ and } r = F_k(z, k, l).
\] (1)

Citizens live forever and discount the future at rate \(\beta < 1\). They derive utility from the consumption of private and public goods. Political disagreement arises from heterogeneity in agents’ preferences regarding the overall size of the government. We assume that there are two types of agents indexed by \(i\), with \(i \in \{L, R\}\).
The instantaneous utility of a type $i$ agent is separable in private and public consumption

$$(1 - \lambda_i)u(c) + \lambda_i v(g)$$

where $u$ and $v$ are increasing and concave, and the weights on public consumption satisfy

$$\lambda_L = \bar{\lambda} + \xi \quad \text{and} \quad \lambda_R = \bar{\lambda} - \xi.$$ 

Since $\lambda_R \leq \lambda_L$, we can think of $R$ as right-wing (small government) and $L$ as left-wing (large government) individuals. The variance of $\lambda_i$ is determined by $\xi$, which can be interpreted as a measure of the degree of political polarization in society. If $\xi$ was equal to zero, agents would be completely homogeneous. As $\xi$ increases, views regarding the provision of $g$ become more conflicting. This parameter will be the key variable governing the volatility of government distortions in cross-country comparisons. Complementarity between private and public consumption would induce a direct co-movement between the two goods and additional volatility in consumption. By assuming separability, we are reducing this degree of freedom.

Citizens finance private consumption and investment with capital and labor income. The government raises revenues by taking a proportion $\tau$ of $\omega_i(k)$, which is a function of individual income. This representation allows for a wide range of commonly used financial instruments. Examples include capital income taxes, where $\omega_i(k) = [r + (1 - \delta)] k_i$; investment taxes or permits, where $\omega_i(k) = k_i' - (1 - \delta) k_i$; total income taxes, where $\omega_i(k) = w l_i + r k_i + (1 - \delta) k_i$; or other non-standard financing instruments such as expropriations and nationalizations (also known as ‘creative fiscal financing’ in some developing countries).

$$c_i = w l_i + r k_i + (1 - \delta) k_i - k_i' - \tau \omega_i(k).$$

Since leisure is not valued, the supply of labor is inelastic. The choice of investment $k_i'$ is intertemporal and depends on government policy. While the current investment wedge $\tau$ is known at the time of decision-making, citizens need to form expectations about future policy $\tau'$. This is the main source of economic policy uncertainty in our model. We postpone a description about how these expectations are formed until the next section. Note that because all agents face the same policy and because preferences are additively separable, investment decisions are independent of their type. As a result, we can focus the analysis on a representative agent that accumulates capital according to a standard Euler equation

$$u(c) = \beta \mathbb{E} \left[ R(\tau') u_c(c') \right],$$

where $R(\tau')$ denotes the net (and uncertain) return on investment. When the government expropriates or taxes capital or total income taxes, $R(\tau') = [r' + 1 - \delta](1 - \tau')$. If the government finances expenditures with investment taxes or permits, $R(\tau') = \frac{1}{1 - \tau'} [r' + (1 - \delta)(1 - \tau')]$. Regardless of the specification, we can see that $\tau$ distorts investment decisions, so we will refer to it as the ‘investment wedge.’ Because this is a closed economy and agents’ private decisions are identical, abstracting from private debt is without loss of generality in this environment. The assumption of a closed economy is made in order to focus on the effects of domestic policy (e.g., investment wedges) on allocations. That is, we specifically abstract from the effects of exchange rate shocks on the volatility of real variables, because this is well understood in the international finance literature—which
proposes such shocks as alternative explanations for the difference in second moments between developed countries and emerging economies—.

The government is subject to a period-by-period balanced budget constraint

\[ \tau \omega(k) = g. \]

Clearly, governments would like to use debt in this environment to smooth out the distortionary costs of financing expenditures that result from productivity shocks. However, since incumbents disagree on spending levels in our economy, they would also want to use public debt to strategically manipulate their successor’s policy. Persson and Svensson (1989) study this manipulation in a two-period model that abstracts from productivity shocks. The effects of introducing public debt in our environment, which features both types of shocks, are not obvious: We could be overestimating or underestimating the effects of polarization on business cycles. The characterization of PBCs under non-balanced budgets is a really interesting but challenging problem in itself, so we defer its analysis to future research.

3.2 Political environment

There are two political parties \( L \) and \( R \) representing each group in the population. The incumbent party is chosen at the beginning of a period and sets policy in order to maximize the utility of its constituency. Agents and firms then choose allocations, taking as given current policy and expectations of future policy. Parties alternate in power following an exogenous Markov process, where \( p \) denotes the type-independent probability of retaining office in the next period, with

\[ p \in [0.5, 1]. \]

Despite the fact that parties are symmetric in terms of re-election prospects, we allow for the probability of re-election to be greater than 0.5, reflecting incumbency advantage effects. The micro-foundations of this specification come from a probabilistic voting model as shown in Azzimonti (2011).

A key feature of the environment is the government’s lack of commitment; revenue and spending policy promises are not credible unless they are ex-post efficient. The party in power plays a game against the opposition, taking its opponent’s policy as given. Alternate realizations of history (defined by the sequence of policies and realizations of productivity shocks up to time \( t \)) may result in different current policies. In principle, this dynamic game allows for multiple subgame perfect equilibria that can be constructed using reputation mechanisms. We will rule out such mechanisms and focus instead on Markov perfect equilibria (MPE), defined as a set of strategies that depend only on the current payoff-relevant states of the economy: \( k \) and \( z \). Because parties have different objectives, their policy choices differ in equilibrium, so strategies are also functions of their type.

The two key equilibrium objects are the spending rule of incumbent \( i, G_i(z,k) \) and the investment decision of our representative citizen \( H_i(z,k) \). Note that the latter is a function of the identity of the party in power due to the fact that the investment wedge \( \tau \), which is party-specific, affects private savings. The investment wedge rule \( T_i(z,k) \) is trivially determined from the government’s budget constraint. The value function of a citizen type \( i \) when his group is in power will be denoted by \( V_i(z,k) \) and when his group is out of power by \( W_i(z,k) \). This distinction is important when
incumbents are ‘partisan,’ as assumed in this paper, because they take into account citizen’s welfare even when out of office (in contrast to model where politicians are office motivated).

3.3 Political equilibrium

An incumbent party chooses the provision of public good \( g \) knowing that it might be replaced by a different policymaker with probability \( p \). Suppose that a left-wing government \( L \) is elected. Given the stock of public capital \( k \) and the current state of the economy \( z \), the incumbent’s objective function today is given by

\[
\max_g \ (1 - \lambda_L)u(c) + \lambda_Lv(g) + \beta E_z[pV_L(z', k') + (1 - p)W_L(z', k')]
\]

where the consumption of its constituency satisfies

\[
c = f(z, k) + (1 - \delta)k - g - k' \equiv C(z, k, k', g).
\]

Private savings \( k' \) given current spending \( g \) satisfies the Euler equation

\[
u_c(c) = \beta E_{zL} \left[ R(\tau'_j)u_c(C_j(z', k')) \right] \tag{2}\]

where \( C_j(z', k') = C(z', k', H_j(z', k'), G_j(z', k')) \) and the future investment wedge satisfies the government budget constraint \( \tau'_j = \frac{G_j(z', k')}{\omega(k')} \). Expectations \( E_{zL} \) are taken with respect to productivity \( z' \) and the identity of tomorrow’s incumbent \( j \), given that \( L \) is currently in power and the current realization of TFP is \( z \).

The functional equation (2) determines future capital as a function of current capital, productivity, and public spending under an \( L \)-type government, \( k' = H_L(z, k, g) \). It summarizes an agent’s optimal reaction to a one-period deviation of \( g \) from the equilibrium rule that an incumbent would follow in the Markov-perfect equilibrium, \( G_L(z, k) \). Agents expect tomorrow’s incumbent of type \( j \) to follow the equilibrium strategy \( g'_j = G_j(z', k') \), and capital to satisfy \( k''_j = H_j(z', k') \) under such policy. Consistency requires that \( H_i(z, k) = H_i(z, k, G_i(z, k)) \) for all \( i \).

The description of the problem is completed by defining the functions \( V_L(z, k) \) and \( W_L(z, k) \):

\[
V_L(z, k) = (1 - \lambda_L)u(C_L(z, k)) + \lambda_Lv(G_L(z, k)) + \beta E_z[pV_L(z', k') + (1 - p)W_L(z', k')] \tag{3}
\]

and

\[
W_L(z, k) = (1 - \lambda_L)u(C_R(z, k)) + \lambda_Lv(G_R(z, k)) + \beta E_z[(1 - p)V_L(z', k') + pW_L(z', k')] \tag{4}
\]

where, as before, \( C_i(z, k) = C(z, k, H_i(z, k), G_i(z, k)) \). The main difference between equations (3) and (4) is that spending levels in the second equation are chosen by a right-wing party and hence do not maximize the objective of incumbent \( L \). A second difference comes from the expectation over political ideology since \( p \) denotes the probability of retaining power for a given incumbent.

The political uncertainty, combined with the conflict over the provision of public goods, creates incentives to act strategically. This becomes clear when analyzing incumbent \( L \)’s first-order condition,

\[
(1 - \lambda_L)u_c(-1 - H_g) + \lambda_Lv_g + \beta E_z[pV'_{k,L} + (1 - p)W'_{k,L}]H_g = 0.
\]
When choosing \( g \), the decision-maker trades off the current benefit of larger government expenditures given by the increase in \( v(g) \) to the current cost of financing this increase, which lowers today’s consumption \( c \). In addition, it takes into account the dynamic effects of this policy change, since larger investment wedges reduce current savings by \( H_g \). This affects continuation utilities \( V \) and \( W \) directly by reducing future income and indirectly by lowering future spending of incumbent \( j \). By controlling the level of investment—via changes in the investment wedge—an incumbent party can affect the spending level of future policymakers through changes to tomorrow’s revenue base. This form of manipulation has been extensively studied in the political economy literature in the context of optimal debt management pioneered by Persson and Svensson (1989) but received less attention in economies where private rather than public savings are affected.

**Definition 3.1 (MPE)** A Markov-perfect equilibrium satisfies

i. Given current policy and expectations on future policy, agent’s and firm’s decisions are a competitive equilibrium.

ii. Given equilibrium allocations and expectations on future policy, current policy solves incumbent \( i \)’s problem.

iii. The incumbent party’s choices are consistent with private expectations,

\[ g = G_i(z, k). \]

This definition imposes consistency between citizens’ and government’s decisions. Additionally it implies that private expectations are correct and that no incumbent has an incentive to deviate from the MPE. A theoretical characterization of the MPE is non-trivial in general, but under some restrictive assumptions on the primitives of the economy, it is possible to find an analytical solution to the model.

**4 Theoretical characterization**

Uncertainty about economic policy affects the economy differently than uncertainty about total factor productivity (TFP). For example, consumption and investment react instantaneously to a change in ideology, while output changes with a one-period lag. Keeping TFP constant, this results in consumption and investment volatility being larger than output volatility. It also affects the comovement between economic variables and their persistence. To illustrate this further, it is useful to analyze an example economy.

**Assumption 4.1** Suppose that (i) preferences over private and public consumption, \( u \) and \( v \), are logarithmic, (ii) productivity innovations follow a two-state Markov process: \( z_s \) with \( s \in \{H,L\} \) and symmetric transition matrix, where \( \pi = P(z' = z_i/z = z_i) \geq 0.5 \), (iii) the production function is Cobb-Douglas \( F(z, k, l) = e^z k^\alpha l^{1-\alpha} \), (iv) there is full depreciation \( \delta = 1 \), and (v) the revenue base is given by \( \omega(k) = wl + rk \).
Under these assumptions, we can show that private investment is proportional to output \( y = e^z k^\alpha \) and decreasing in public spending, \( H(z, k, g) = \alpha \beta y - g \). Because private consumption is also linear in output, we guess that government spending follows a linear and type-dependent rule. This guess is verified in the following proposition.

**Proposition 4.1** Under Assumption 1, the MPE satisfies

\[
T_i = \lambda_i \eta, \quad H_i(z, k) = \alpha \beta (1 - \lambda_i \eta) e^z k^\alpha, \quad G_i(z, k) = \lambda_i \eta e^z k^\alpha \quad \text{and} \quad C_i(z, k) = (1 - \alpha \beta) (1 - \lambda_i \eta) e^z k^\alpha \quad \text{where} \quad \eta = \frac{1 - 2 \alpha \beta p - \alpha^2 \beta^2 (1 - 2 p)}{1 + \alpha \beta (1 - 2 p)}.
\]

**Proof 4.1** See Appendix 7.2

The investment wedge \( T_i = \lambda_i \eta \) is independent of the stock of capital and productivity but depends on the identity of the incumbent in power. Since \( \lambda_R < \lambda_L \), distortions are smaller under right-wing governments than under left-wing incumbents. This results in a smaller desired government size, since \( \lambda_R \eta \) also represents \( R \)'s marginal propensity to spend under logarithmic utility. Since disposable income is larger, individuals choose investment and consume more than under a left-wing party. Given that political turnover follows a first-order Markov process, with \( p = Pr(\lambda_i = \lambda_i / \lambda_{i-1} = \lambda_i) \) for \( i \in \{L, R\} \), we can re-interpret the stochastic investment wedge as a ‘political shock’: a random variable that follows a Markov-chain and introduces economic policy uncertainty. This has interesting implications regarding the underlying dynamics of the model.

### 4.1 Transitional dynamics in polarized societies

To make the exposition simpler, ignore TFP shocks for the moment, focusing only on the political dimension. From Proposition 4.1, we know that \( k_{t+1} = \alpha \beta (1 - \lambda_t \eta) e^z k_t^\alpha \), where \( \lambda_t \in \{\lambda_L, \lambda_R\} \) and \( \eta \) are defined in eq. (12). Elections take place every period, determining the identity of the incumbent in power. When the left-wing party wins, the investment wedge is given by \( \lambda_L \eta \). Individuals save a proportion \( \alpha \beta (1 - \lambda_L \eta) \) of their total income when \( L \) is in power. When the right-wing party wins, their propensity to save is \( \alpha \beta (1 - \lambda_R \eta) \) instead.

Figure 6 depicts private investment as a function of the current stock of capital. Keeping \( z = z_H \), the line \( H_L(z_H, k) \) represents tomorrow’s value of capital assuming that \( L \) is currently in power. If there was no political turnover (that is, \( L \) was in power forever) capital would eventually converge to \( k_{LH}^* \). If instead a right-wing party were in power forever, steady-state capital, \( k_{RH}^* \), would be larger. Moreover, the speed of convergence for a given value of \( z \), defined as

\[
\gamma_{iz} = \alpha \beta (1 - \lambda_i \eta) e^z k^\alpha - 1,
\]

is higher under \( i = R \). That is, growth is faster and the economy converges to a larger steady state level of capital under governments that have a smaller weight on public consumption. The intuition relies on the fact that a left-wing party prefers a larger share of output to be devoted to public goods provision. Because financing this good is costly under our assumptions, inefficiencies are more pronounced than under an \( R \)-type government and have long-term distortionary consequences in
the economy. The speed of convergence slows down when realizations of the TFP shocks are low, as seen from the fact that the dotted lines (corresponding to $z = z_L$) lie below the solid ones (corresponding to $z = z_H$) under either party.

Starting from a low level of capital, the series exhibits an increasing trend until it reaches the ‘ergodic set’ at which point it fluctuates around a constant mean. The lower bound of this interval is determined by $\mathcal{H}_L(z_L, k) = k$; that is, where the lowest dotted line intersects the 45° line. Its upper bound is defined by $\mathcal{H}_H(z_H, k) = k$. Moreover, this process is stationary.

4.2 Polarized Business Cycles (PBCs)

Once the economy reaches its ergodic set, fluctuations will be driven by TFP shocks (as in a standard RBC model) as well as political shocks (e.g., changes in the investment wedge). We will refer to economic cycles in the ergodic set as polarized business cycles (PBC).

4.2.1 Impulse-responses

A positive TFP shock has the same effects on economic variables as those found in a standard real business cycle (RBC) model. In particular, current output increases immediately as the economy becomes more productive. The positive income effect induces a concurrent increase in private consumption, which individuals smooth out over time by also raising investment. Under logarithmic utility it turns out that a 1% increase in output results in exactly 1% rise in both public and private consumption. This can be seen clearly in Panels (A) and (B) of Figure 7 where the size of the response to a productivity shock coincides for all variables (we are depicting percentage changes relative to steady state). Output is represented by a dotted line, while consumption is represented by a solid line.
Now consider the response of the economy to a political shock, starting from the steady state attained under a left-wing government. Suppose that party $R$ gains power for only one period and $L$ regains control of the government forever after. The impulse-response function of consumption and output is depicted in Panel (C) of Figure 7. The main difference in both variables lies in the timing of responses: Consumption reacts immediately, while output only jumps upward with a one-period lag. The reason is the following: The switch in political ideology generates a reduction in both public spending—as seen in Panel (D)—and the investment wedge, which triggers an increase in current consumption. Output remains unchanged due to the fact that capital is given and labor is inelastic.\footnote{The response under endogenous labor depends on how the labor supply depends on income and substitution effects. If only substitution effects are relevant, then a decrease in the investment wedge would increase net-wages, inducing an increase in labor and hence a contemporaneous increase in output.} Individuals also increase investment in response to the regime change, since lower wedges behave similarly to positive income shocks. The larger stock of capital in the second period increases production at that point. This, together with the fact that a left-wing government regains power, increases government spending in that period above its steady-state value. As time goes by, the effects of the shock dissipate slowly until the economy converges to the original steady state.

The previous discussion makes it clear that while positive TFP shocks (increases in $z$) and political shocks (power switches from $L$ to $R$) increase agents’ disposable income, they have very...
different implications for output dynamics. An immediate testable implication of the model is that consumption boosts that are observed leading GDP booms are associated with changes in ideological views of the government (i.e., on government spending), rather than with innovations in productivity. Traditional TFP shocks result in coincidental movements in private consumption instead.\footnote{This statement assumes that the wage-elasticity of the labor supply is small.}

### 4.2.2 Volatility and amplification in the PBC model

The first step to analyzing volatility and amplification under economic policy uncertainty is to characterize the long-run distribution of capital, since it determines the evolution of all other variables in the PBC model. Because policy functions are linear in the states when variables are expressed in natural logarithms, we will be able to characterize this distribution analytically. The first and second moments of (log) capital are summarized in Lemma 4.1.

**Lemma 4.1** Define \( \hat{x}_j = \ln(1 - \tau_j) \), where the political shock satisfies \( \tau_j = \lambda_j \eta \). Then \( \hat{k}' \) follows,

\[
\hat{k}' = q + \alpha \hat{k} + \hat{x} + z
\]

where \( q = \log(\alpha \beta) \), \( \hat{x} \in \{\hat{x}_L, \hat{x}_R\} \) and \( z \in \{z_H, z_L\} \). The shocks have unconditional means \( \bar{x} = 0.5(\hat{x}_R + \hat{x}_L) \) and \( \bar{z} = 0.5(z_H + z_L) \) and unconditional variances

\[
\sigma_x^2 = 0.5^2(\hat{x}_R - \hat{x}_L)^2 \quad \text{and} \quad \sigma_z^2 = 0.5^2(z_H - z_L)^2.
\]

The long-run distribution of \( \hat{k} \) has the following properties.

i. The mean is \( E(\hat{k}) = \frac{q + \bar{x} + \bar{z}}{1 - \alpha} \).

ii. The covariances are \( \text{Cov}(\hat{k}, \hat{x}) = \frac{(2p-1)\sigma_x^2}{1 - \alpha(2p-1)} > 0 \) and \( \text{Cov}(\hat{k}, z) = \frac{(2\pi-1)\sigma_z^2}{1 - \alpha(2\pi-1)} > 0 \).

iii. The variance is \( \text{Var}(\hat{k}) = \frac{\sigma_x^2 + \sigma_z^2}{1 - \alpha^2} + \frac{2\alpha}{1 - \alpha^2} \left[ \text{Cov}(\hat{k}, \hat{x}) + \text{Cov}(\hat{k}, z) \right] \equiv \sigma_k^2. \)

**Proof 4.2** See Appendix 7.3.

The ergodic distribution of capital is not trivial to characterize in our environment due to the persistence of both, political, \( \tau \), and productivity shocks, \( z \). If these shocks were iid instead (\( \pi = 0.5 \) and \( p = 0.5 \)), capital would simply follow an AR(1) process. Under persistent shocks, the process is autoregressive, but its errors are no longer white noise: They follow a discrete Markov process instead. Another important difference with a standard model is that while the process determining political turnover is exogenous, the value taken by the shock affecting the process, \( \hat{x} \), is determined within the model. More important, it depends on technological and institutional characteristics.
of the economy, since \( \eta \) is a function of the capital share \( \alpha \) and the probability of re-election \( p \).\footnote{We have assumed that the probability of re-election is independent of productivity shocks, so \( \lambda \) and hence \( \hat{x} \) are independent of \( z \). An interesting extension would consider the effect of such a correlation.} Finally, notice that while the capital stock is linear in TFP shocks, it is non-linear in political shocks, since \( \hat{x}_j = \ln(1 - \tau_j) \). This implies that innovations to the variance of political shocks, given by the degree of political polarization \( \xi \), directly affect decision rules (more on this later). This would not be the case for TFP shocks, since mean preserving increases in the spread would vanish.

Because output, consumption, and public spending are proportional to capital, their processes are also stationary and their evolution can be simply characterized from Lemma 4.1. Due to our full depreciation assumption, investment behaves exactly like capital in this model. Finally, since the investment wedge is independent of capital, its stochastic properties are inherited from the process determining \( \lambda \). Proposition 4.2 presents a decomposition of the long-run volatility of output, public and private consumption, and the investment wedge as a function of each type of shock.

**Proposition 4.2** Let \( \hat{c} = \ln c, \hat{y} = \ln y, \hat{\tau} = \ln \tau \) and \( \hat{x} = \ln(1 - \tau \eta) \), then:

1. The long-run variance of (log) output satisfies
   \[
   \text{Var}(\hat{y}) = \sigma_z^2 + \alpha^2 \text{Var}(\hat{k}) + 2\alpha \text{Cov}(\hat{k}, z). \tag{5}
   \]
2. The long-run variance of (log) private consumption satisfies
   \[
   \text{Var}(\hat{c}) = \text{Var}(\hat{y}) + \sigma_x^2 + 2\text{Cov}(\hat{y}, \hat{x}) \tag{6}
   \]
   where \( \text{Cov}(\hat{y}, \hat{x}) = \alpha \sigma_x^2 \frac{2p-1}{1-\alpha(2p-1)} \geq 0 \).
3. The long-run variance of (log) public consumption satisfies
   \[
   \text{Var}(\hat{g}) = \text{Var}(\hat{y}) + \sigma_{\hat{\tau}}^2 + 2\text{Cov}(\hat{y}, \hat{\tau}) \tag{7}
   \]
   where \( \sigma_{\hat{\tau}}^2 = 0.5^2(\hat{\tau}_R - \hat{\tau}_L)^2 \) and \( \text{Cov}(\hat{y}, \hat{\tau}) = \alpha 0.5^2(\hat{\tau}_R - \hat{\tau}_L)(\hat{x}_R - \hat{x}_L) \frac{2p-1}{1-\alpha(2p-1)} \leq 0 \).

**Proof 4.3** See Appendix 7.4.

This proposition shows that the presence of economic policy uncertainty amplifies real business cycles. The volatility of capital in a standard RBC model is smaller than in the PBC model, since in the absence of political shocks \( \sigma_x^2 = \text{Cov}(\hat{k}, \hat{x}) = 0 \). Since \( \text{Var}(\hat{k})^{PBC} > \text{Var}(\hat{k})^{RBC} \), output is more volatile in the PBC model than in the RBC model:

\[
\text{Var}(\hat{y})^{PBC} > \text{Var}(\hat{y})^{RBC}.
\]

This proposition also illustrates the consumption volatility puzzle that arises under the lens of a traditional neoclassical framework. Because political shocks are abstracted from, \( \sigma_x^2 = \text{Cov}(\hat{y}, \hat{x}) = 0 \), so consumption is predicted to be at most as volatile as output in the RBC model.
the model is augmented to include volatility in political ideology then $\text{Var}(\hat{c}) \geq \text{Var}(\hat{y})$ as the covariance between political and economic shocks is positive (recall that $p \geq 0.5$).

$$[\text{Var}(\hat{c}) - \text{Var}(\hat{y})]^{\text{PBC}} > [\text{Var}(\hat{c}) - \text{Var}(\hat{y})]^{\text{RBC}} = 0.$$ An obvious question is whether the model can generate $\text{Var}(\hat{c}) < \text{Var}(\hat{y})$ in the PBC under some specification. We address this in Section 5 by analyzing a more general environment where some of the restrictive assumptions in this example economy are relaxed. In particular, we show that $\text{Var}(\hat{c}) \leq \text{Var}(\hat{y})$ when assuming less than full depreciation for an economy with U.S. characteristics. The reason is that the response to TFP shocks is much smaller under $\delta < 1$.

Public consumption may be more or less volatile than output once political shocks are considered. The reason is that the covariance between output and the investment wedge is negative: When there is a shift from left to right, the investment wedge goes down, but since investment increases, future output goes up. The following lemma shows sufficient conditions for $\text{Var}(\hat{g}) \geq \text{Var}(\hat{y})$.

**Lemma 4.2** Suppose that $p \leq \frac{1 + 2\alpha}{6\alpha}$ and $\bar{\lambda} \leq \frac{1}{2\eta}$, with $\eta = \frac{1 - 2\alpha \beta p - \alpha^2 \beta^2 (1 - 2p)}{1 + \alpha \beta (1 - 2p)}$. Then

$$[\text{Var}(\hat{g}) - \text{Var}(\hat{y})]^{\text{PBC}} \geq 0.$$ 

**Proof 4.4** See Appendix 7.5.

This lemma shows that countries that are politically unstable (that is, where $p$ is low) will tend to exhibit excess volatility of public consumption relative to output. This provides a rational for our empirical finding, since emerging economies often have lower political stability than developed ones. Note that for empirically relevant values of the capital share $\alpha$, the upper bound does not violate incumbency advantage. For example, when $\alpha = 0.36$, the constraint requires that $p < 0.963$.

In our model, the investment wedge is always negatively correlated with output. This results from our balanced budget assumption. In the data, the correlation between the revenue to output ratio (a proxy for the investment wedge) and GDP takes positive as well as negative values, and it is on average close to zero for our sample; assuming a non-balanced budget constraint would mitigate but not necessarily overturn our theoretical result. If the government were able to borrow and lend, the tax-smoothing motive would create incentives to keep taxes low in bad times. This would increase the contemporaneous correlation between output and the investment wedge. However, the political shock would still push the correlation to take on negative values. Which effect dominates would depend on the variability and persistence of political versus TFP shocks. Relaxing the balanced budget assumption and computing this correlation quantitatively is thus an interesting extension to our paper.

### 4.2.3 The cyclicality of public spending in the PBC model

There exists a large and growing debate regarding the cyclicality of public spending. The disagreement in the empirical literature can be traced back to the proxy used for government spending: public consumption, public investment, or current government expenditures. Once the variability in the composition of total expenditures across countries is considered, different findings can be reconciled. From a theoretical standpoint, researchers have proposed theories trying to explain
some of these facts by, directly or indirectly, restricting focus to different components of government spending. Ilzetzki (2011) proposes a theory of the cyclicality of transfers. Azzimonti (2011), on the other hand, models government spending as productive public investment and finds it to be procyclical (both in the model and using U.S. state data). Bachmann and Bai (2013b) focus on government consumption and show that the correlation between government consumption and spending is close to one in a stochastic version of the model studied by Klein et al. (2008). When agents’ preferences over public goods are subject to preference shocks, public consumption becomes less procyclical. 12

In our model, the correlation between public consumption and output is also dampened by the existence of economic policy uncertainty. The following proposition illustrates this point.

**Proposition 4.3** Suppose that \( p \geq 0.5 \). Let \( \hat{g} = \ln g, \hat{y} = \ln y, \) and \( \hat{\tau} = \ln(\tau) \), then the correlation between public consumption and output satisfies

\[
\text{Corr}(\hat{g}, \hat{y})^{\text{PBC}} = \frac{\text{Var}(\hat{y}) + \text{Cov}(\hat{\tau}, \hat{y})}{\sqrt{\text{Var}(\hat{g}) \text{Var}(\hat{y})}} < \text{Corr}(\hat{g}, \hat{y})^{\text{RBC}} = 1, \tag{8}
\]

where \( \text{Cov}(\hat{\tau}, \hat{k}) = 0 \).

**Proof 4.5** See Appendix 7.6.

As previously shown in Panel A of Figure 7, a one percent increase in output results in a one percent increase in public consumption when political shocks are abstracted from the model. The excessive cyclicality of public consumption predicted by a standard model (one without political frictions) is also evident when inspecting equation 8 above. If political shocks are ignored, \( \text{Cov}(\hat{\tau}, \hat{y}) = 0 \), and the volatility of public consumption equals that of GDP, \( \text{Var}(\hat{g}) = \text{Var}(\hat{y}) \). Therefore, \( \text{Corr}(\hat{g}, \hat{y})^{\text{RBC}} = 1 \). Once we introduce political turnover between parties who disagree on the size of government, the cyclicality of public consumption goes down. To see this, consider the case with no incumbency advantage, \( p = 0.5 \). From the lemma above, we have that \( \text{Cov}(\hat{\tau}, \hat{k}) = 0 \). Since \( \text{Var}(\hat{g}) > \text{Var}(\hat{y}) \) (from Lemma 4.2), equation (8) implies that \( \text{Corr}(\hat{g}, \hat{y})^{\text{PBC}} < 1 \). In other words, once there is political turnover between polarized parties, the cyclicality of public consumption dampens. It is worth noting that political turnover per se (that is, \( p < 1 \)) is not enough to generate this result. Absent polarization we would have \( \hat{\tau}_R = \hat{\tau}_L \), in which case the correlation between public consumption and output would still be equal to 1.

Even though political shocks induce additional volatility in government consumption, generating \( \text{Var}(\hat{g}) > \text{Var}(\hat{y}) \), the correlation between \( \hat{y} \) and \( \hat{g} \) can become negative. Theoretically, government consumption can be procyclical or countercyclical in our model, as it is in the data (see Tables 9 and 10 in Appendix 7.1). Which force dominates depends primarily on political turnover and the size of political shocks. The latter is determined by the degree of polarization. In Section 5 we will investigate this further using a numerical example.

12Our specifications are related, as a change in ideology could be seen as a taste shock in a static model. The main difference lies in the continuation utility of the government in a dynamic model: The current incumbent needs to evaluate the welfare of its constituency when out of power, and this induces him to behave differently than in the case where he would be facing a constituency with fluctuating preferences.
4.3 The effects of polarization in the PBC model

The purpose of this section is to show that our model can be consistent with the four stylized facts presented in the introduction. Clearly, there are potentially many forces that could drive differences in real business cycles and fiscal policies across countries. Real exchange rate shocks and the degree of development of financial systems, among others, have been proposed as potential explanations in the literature. Our theory centers on the interaction between productivity shocks and switches in political ideology instead. Our main experiment consists of increasing the degree of polarization, \( \xi \) and analyzing its affects on PBCs.

The first result establishes that the volatility of the investment wedge increases with political polarization. This follows from the fact that increases in \( \xi \) widen the gap between \( \lambda_L \) and \( \lambda_R \).

**Corollary 4.1 (to Proposition 4.1)** The volatility of the investment wedge increases with polarization

\[
\frac{\partial \sigma^2}{\partial \xi} = \eta(\tau_L - \tau_R) > 0.
\]

**Hence, there is a positive correlation between polarization and economic policy uncertainty.**

This prediction is consistent with stylized Fact 4, when EPU is measured by the volatility of the revenue to output ratio. In addition, since emerging economies tend to be more polarized—as established by Fact 3—we should expect them to experience greater economic policy uncertainty. This provides a model-based rationale for Fact 2.

![Figure 8: Increase in polarization (– benchmark \( \xi \), - - high polarization \( \xi' \)).](image)

The additional volatility in economic policy spills over onto the real economy through its effects on private investment. The solid (blue) lines in Figure 8 depict the investment functions \( \mathcal{H}_i(z, k; \xi) \)
under each party, $i \in \{L, R\}$ for a given productivity level $z$. As before, $H_R(z, k; \xi)$ lies above $H_L(z, k; \xi)$ because individuals invest more under a type-$R$ government that sets a lower wedge. When polarization increases from $\xi$ to $\xi'$, the curve $H_R(z, k; \xi)$ moves upward while the curve $H_L(z, k; \xi)$ moves downward, represented by the broken (red) lines in the figure (the result is obtained by differentiating the policy functions computed in Proposition 4.1).

An immediate result is that the ergodic set will expand, as seen by the fact that the range taken by capital under a low $\xi$ in the long run is smaller than the one under a high polarization level $\xi'$ (Figure 8). As a consequence, the volatility of investment goes up. This can also be derived analytically from Lemma 4.1.

**Corollary 4.2** *(to Lemma 4.1 and Proposition 4.2)* The volatility of investment increases with polarization,

$$\frac{\partial \text{Var}(\hat{k})}{\partial \xi} > 0.$$ 

Therefore,

$$\frac{\partial \text{Var}(\hat{y})}{\partial \xi} > 0 \quad \text{and} \quad \frac{\partial \text{Var}(\hat{c})}{\partial \xi} > 0.$$ 

An increase in $\xi$ reduces the value taken by $\hat{x}_L$ while raising $\hat{x}_R$, causing the volatility $\sigma^2_{\hat{x}}$ to go up. Because the variance of investment is increasing in $\sigma^2_{\hat{x}}$ (both directly and indirectly through the covariance term), then $\text{Var}(\hat{k})$ rises as well. From Proposition 4.2, we can see that this results in higher volatility of output and consumption. Moreover, it is straightforward to show that the excess volatility of consumption, $[\text{Var}(\hat{c}) - \text{Var}(\hat{y})]^{PBC}$, increases as well. These results, together with Fact 3, are consistent with Fact 1: business cycle movements are wider in emerging countries.

Finally, notice that we have considered a mean-preserving increase in polarization, which leaves the average long-run value of investment unchanged: $\frac{\partial E\hat{k}}{\partial \xi} = 0$. In our model, an increase in polarization looks very much like a mean-preserving increase in the innovation of fiscal shocks, traditionally studied in the DSGE literature (see for example Fernández-Villaverde, Guerrón, Kuester and Rubio-Ramírez, 2012). The exception is that fiscal shocks are micro-founded by political frictions in our PBC model. In other words, higher polarization causes EPU to rise in our model, which amplifies business cycles.

## 5 Quantitative Analysis

In this section, we calibrate our benchmark model and test whether its quantitative implications are in line with stylized facts from the U.S. economy. We then conduct the following counterfactual experiment: Suppose that the U.S. became as polarized and politically unstable as Mexico, how should we expect this to affect relevant PBC moments? Qualitatively, the effects have been discussed in previous sections, the aim of this section is to quantify those effects in a less restrictive model (in particular, one where there is less than full depreciation). The numerical procedure used to solve the model involves finding a fixed point in equilibrium policy rules. Details of this approach are described Appendix 7.7. Computation is non-trivial because it is necessary to guess four functions: The savings rules for individuals under a left-wing and right-wing government $H_L(z, k)$ and
\( \mathcal{H}_R(z, k) \), and the spending rules of each party \( \mathcal{G}_L(z, k) \) and \( \mathcal{G}_R(z, k) \). In addition, we need to solve for the savings function under a one-period deviation \( H(z, k, g) \).

As standard in the RBC literature, we will augment the model to include capital adjustment costs to investment \( \Phi(k, k') \)

\[
\Phi(k, k') = \phi \left( \frac{k'}{k} - 1 \right)^2 k
\]

as modeled by Greenwood, Hercowitz, and Krusell (2000). The introduction of this feature is motivated by the observation that while consumption and public spending volatilities are larger in emerging economies than in developed ones, the relative volatility of investment is roughly the same \((\sigma(I)/\sigma(y) = 3.80 \text{ and } \sigma(I)/\sigma(y) = 3.86 \text{ respectively, see Table 8})\), suggesting the presence of such costs. The qualitative characteristics of the model are unchanged (we also simulated the model without such costs), but the quantitative results differ once the volatility of investment becomes a calibration target. Otherwise, as in any RBC model without political shocks, this volatility is overestimated.

### 5.1 Calibration

Because we are building on the neoclassical framework, many of the parameters are standard. A time period represents a year, so the discount factor is \( \beta = 0.95 \). The share of capital \( \alpha \) is set to 0.36 and the depreciation rate \( \delta \) is 0.1. Preferences are logarithmic. Productivity, specified as \( z' = \rho z + \epsilon' \), is discretized using a two-state Markov process with an autocorrelation coefficient of \( \rho = 0.91 \) and a conditional standard deviation of \( 3.62e^{-2} \). The latter is chosen to match the annual standard deviation of de-trended output observed in the U.S. over the sample period 1960-2003, of 2.2\%. Adjustment costs \( \phi \) are chosen so that model-generated investment volatility is equal to the value observed in the U.S. for the same period, of 7.3\%.  \(^{13}\)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{vol}(\epsilon) )</td>
<td>0.0362</td>
<td>( \text{vol}(y) = 2.2e^{-2} )</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.625</td>
<td>( \text{vol}(I) = 7.3e^{-2} )</td>
</tr>
<tr>
<td>( \xi )</td>
<td>0.013</td>
<td>( \text{vol}(g) = 2.92e^{-2} )</td>
</tr>
<tr>
<td>( \bar{\lambda} )</td>
<td>0.36</td>
<td>mean(( gExp/y )) = 0.24</td>
</tr>
</tbody>
</table>

**Table 4: Calibrated parameters**

Note: Cyclical moments computed by de-trending (using a band-pass filter 2-20) the natural logs variables over the sample 1960-2003. \( I \) and \( y \) are obtained from Kaminsky, Reinhart, and Végh (2004), and \( gExp \) is consolidated government expenditures net of transfers, from NIPA Table 3.1 Government Current Receipts and Expenditures.

\(^{13}\)The model implied moments are obtained by simulating the political equilibrium for 11,000 periods where the first 1,000 are discarded to eliminate the effects of initial conditions.
The probability of re-election $p$ is obtained from data on political stability, assigned by the Political Risk Services Group’s (PRS) International Country Risk Guide. The data set is described in Appendix 7.1. The value of $p = 0.83$ used in the model corresponds to the average value observed in developed countries and generates an average tenure in power of about 10 years, in line with political turnover in the U.S.. The mean value of $\lambda_i$ is chosen to match the average ratio of public spending (net of transfers) to output $gExp/y$, which is a good proxy of the level of distortions generated by government spending. The variable determining polarization, $\xi$, is set so that the volatility of government consumption obtained from the model is equal to the one observed in the U.S. for the same time period, 2.92%. Table 4 summarizes the values of the parameters obtained from the calibration along with the target moments.

Table 5 reports the fit of the model for a broader set of business cycle moments (those marked with asterisks are matched as part of our calibration strategy). The first thing to note is that the introduction of political frictions to an otherwise standard neoclassical framework does not undermine the fit of the model to relevant economic variables. For example, the cyclical behavior of consumption and investment is remarkably close to its empirical counterpart, as seen by comparing our predicted measures of $\rho(y, c)$ and $\rho(y, I)$ to the U.S. values. Moreover, the correlation between private consumption and investment is also close to its data counterpart.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(y)$</td>
<td>2.2e-2</td>
<td>2.2e-2*</td>
</tr>
<tr>
<td>$\sigma(I)/\sigma(y)$</td>
<td>3.31</td>
<td>3.31*</td>
</tr>
<tr>
<td>$\sigma(c)/\sigma(y)$</td>
<td>0.85</td>
<td>0.94</td>
</tr>
<tr>
<td>$\sigma(g)/\sigma(y)$</td>
<td>1.33</td>
<td>1.33*</td>
</tr>
<tr>
<td>$\rho(y)$</td>
<td>0.60</td>
<td>0.55</td>
</tr>
<tr>
<td>$\rho(g)$</td>
<td>0.84</td>
<td>0.43</td>
</tr>
<tr>
<td>$\rho(y, I)$</td>
<td>0.87</td>
<td>0.85</td>
</tr>
<tr>
<td>$\rho(y, c)$</td>
<td>0.91</td>
<td>0.97</td>
</tr>
<tr>
<td>$\rho(y, g)$</td>
<td>0.35</td>
<td>0.20</td>
</tr>
</tbody>
</table>

* calibrated moments.

Even though it was not a target of our calibration, the relative volatility of consumption to output $\sigma(c)/\sigma(y)$ is in line with the observed value for the U.S.. In contrast with our analytical example, this relative volatility is smaller than one. This follows from relaxing the full depreciation assumption, which decouples the response of consumption and output to changes in TFP. The model predicts that government spending is pro-cyclical, though the degree of cyclical is slightly below the one observed in the U.S.. The persistence of public spending is too low in our calibrated economy, since $\rho(g) = 0.43$, while it takes the value 0.84 in the data. This results from abstracting from adjustment costs associated to changing government financing and assumes no implementation lags in enacting fiscal policy. Bachmann and Bai (2013b) discuss how these two features affect the
persistence of government purchases in a related model.

5.2 Counterfactual experiment: from U.S. to Mexico

In this section, we analyze the effects that a permanent increase in polarization and political instability (to Mexico’s levels) have on the U.S. economy. We first study how the economic response to political and productivity shocks change in a highly polarized environment, and then quantify the impact of polarization on long-run moments of the U.S.’ PBC.

The empirical estimate of Mexico’s polarization is about 30% higher than the U.S. recorded level (see Tables 9 and 10 in Appendix 7.1). Hence, we will increase the value of $\xi$ from 0.013 in the benchmark calibration to $\xi' = 0.017$. The impulse-responses of consumption and output to political versus TFP shocks are depicted in Figure 9 for the benchmark (solid lines), as well as the high-polarization scenario (dotted lines). In both cases, the shocks are assumed to last 10 periods, the average tenure in power of a political party. The right panel shows that consumption responds slightly less than output to a TFP shock in this calibrated economy. This differs from the response computed for our closed form solution example, where both variables reacted identically to an innovation in productivity. The difference arises from the fact that there is less than full depreciation in the benchmark calibration. Notice that the degree of polarization does not have a large impact on these responses, as can be seen from the fact that the solid and dotted lines are close to each other. This implies that any additional volatility in economic variables arises from the resulting increase in economic policy uncertainty, rather than on the response to productivity fluctuations when polarization levels are large.

![Figure 9: Response to 10-period political and TFP shocks.](image)

The left panel of Figure 9 shows that the responses of output and consumption are indeed much larger in the high-polarization scenario. This illustrates the amplification effect of an endogenous rise in economic policy uncertainty induced by greater political polarization. Because the difference in investment wedges is larger in the high-polarization scenario, the response to a switch
from a left-wing government to a right-wing government widens. As in the closed-form example, consumption reacts immediately while output increases with a lag. The persistence of the shock is larger, again due to the less-than-full depreciation assumption. The figure also indicates that the relative volatility of consumption $\sigma(c)/\sigma(y)$ must increase when polarization rises. The response of consumption is not only larger than that of output when there is a switch in political ideology, but the difference between them is wider when $\xi$ is high. According to this model, the recent increase in polarization in the U.S. should be accompanied by a spike in the relative volatility of consumption after 2006.

Table 6 (column 2) shows how a permanent increase in political polarization affects long-run policy and economic outcomes, once political as well as productivity shocks are considered. Results are presented as percentage changes from the benchmark economy.

The largest increase arises in the relative volatility of public policy, which increases by 32% as polarization rises. This is consistent with the fact that developing economies, which are in general more polarized, exhibit larger fluctuations in public consumption relative to output fluctuations. Moreover, our theory suggests a possible cause underlying the observed increase in U.S. economic policy uncertainty over the high-polarization period from 2006-2011, presented in Figure 3. Because the investment wedge fluctuates more when political disagreement worsens (e.g., when polarization is high), individuals face higher uncertainty regarding public policy.

The experiment also shows how this additional volatility spills over into the real economy: Output volatility rises 0.6%, while the relative volatility of consumption increases 2.2%. Output becomes more persistent (the autocorrelation rises 0.4%), and less correlated to investment and consumption. The cyclicality of government consumption goes down by more than 18%.

### Table 6: Counterfactual Experiment

<table>
<thead>
<tr>
<th>Moment</th>
<th>Benchmark</th>
<th>High $\xi' = 0.017$</th>
<th>Low $p = 0.744$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(y)$</td>
<td>2.2e-2</td>
<td>0.6%</td>
<td>0.4%</td>
</tr>
<tr>
<td>$\sigma(I)/\sigma(y)$</td>
<td>3.31</td>
<td>6.3%</td>
<td>1.5%</td>
</tr>
<tr>
<td>$\sigma(c)/\sigma(y)$</td>
<td>0.94</td>
<td>2.2%</td>
<td>4.1%</td>
</tr>
<tr>
<td>$\sigma(g)/\sigma(y)$</td>
<td>1.33</td>
<td>32.7%</td>
<td>40.4%</td>
</tr>
<tr>
<td>$\rho(y)$</td>
<td>0.55</td>
<td>0.4%</td>
<td>-0.6%</td>
</tr>
<tr>
<td>$\rho(y, I)$</td>
<td>0.85</td>
<td>-7.2%</td>
<td>-9.2%</td>
</tr>
<tr>
<td>$\rho(y, c)$</td>
<td>0.97</td>
<td>-1.9%</td>
<td>-3%</td>
</tr>
<tr>
<td>$\rho(y, g)$</td>
<td>0.2</td>
<td>-18.5%</td>
<td>-1.5%</td>
</tr>
</tbody>
</table>

Columns (2) and (4) are expressed as percentage changes relative to the benchmark economy.

The last column of Table 6 presents the percentage changes relative to the benchmark economy of a joint increase in polarization and political instability. In addition to rising $\xi$, we decrease $p$ from its benchmark value of 0.83 to Mexico’s level of 0.744 in this experiment. Political instability dampens the effect of polarization on the volatility of output (which increases by 0.4% versus 0.6% before) but intensifies its effect on the relative volatility of public and private consumption.
Interestingly, the reduction in the cyclicality of government consumption is lower when both variables are considered. This is consistent with the observation that emerging economies, which are usually more polarized and less politically stable than developed ones, have similar cyclicality of government consumption (see Table 8).

6 Conclusion and Extensions

We presented a model where political parties that disagree on the size of the government alternate in power. This introduces an additional source of volatility for economic variables, triggered by changes in government policy that can be interpreted as political shocks. We showed analytically that a standard real business cycle model augmented to incorporate political polarization, a polarized business cycle model (PBC), is consistent with a set of stylized facts computed for emerging and developed economies. We then calibrated the model to the U.S. economy and analyzed the effects on the PBC of a permanent increase in polarization. We showed that higher polarization induces greater economic policy uncertainty and amplifies fluctuations in output, private consumption, and investment. Our results suggest that the observed increase in economic policy uncertainty observed over the period 2006-2011 (see Figure 3) may have been caused by rising political polarization. Moreover, our theory sheds some light on the impact of this increase in polarization on the size of economic fluctuations during the last economic crisis.

While our theory made some progress in understanding the effects of political frictions on economic fluctuations, there is still scope for further research in this area. The most interesting extension consists of relaxing the balanced budget assumption, since its effects on the political equilibrium are non-trivial. On the one hand, debt would allow parties to smooth productivity reducing fluctuations on the investment wedge. On the other hand, debt would introduce an additional channel for manipulation, since it restricts future spending. The combined effects of productivity and political shocks on deficits are therefore unclear.

References


7 Appendix

7.1 Data

The data are obtained from Kaminsky, Reinhart, and Végh (2004), who compiled a comprehensive cross-country panel for our main variables of interest from the IMF World Economic outlook (WEO) and IMF Government Financial Statistics (GFS) data sets. Output $y$ is ‘gross domestic product.’ Consumption $c$ is ‘private consumption,’ which combines household consumption and non profit institutions. Investment $I$ is ‘national gross fixed capital formation.’ Public consumption $g_c$ is ‘consolidated general government consumption.’ The series are deflated using the GDP deflator. We compute business cycle moments (volatilities, autocorrelations and correlations) on band-pass filtered (2-20) natural logs of each GDP deflated variable, on the available time series for each country.

‘Political Polarization’ is based on interviews with respondents in 81 countries, compiled at the World Values Survey (Lindqvist and Östling, 2010). We use answers to the following question: ‘How would you place your views on this scale?’, where 1 means agree completely with the left (people should take more responsibility to provide for themselves) and 10 means agree completely with the right (the government should take more responsibility). Our measure of polarization is the standard deviation of responses per country.

‘Political Stability’ is obtained from Political Risk Services’ (PRS) International Country Risk Guide data set, and the variable name is ‘Government Stability.’ We use the 1980-1990s average for this variable. Countries are assigned ‘government stability’ points based on an assessment of the government’s ability to carry out its declared programs as well as its ability to stay in office using PRS’ proprietary methodology.

Since data availability is not consistent across the four economic variables for individual countries, the period studied in each case reflects the longest time span for which we have complete data for that country. Sample lengths for each country are reported in Table 7. We also dropped from the sample countries for which we had no Polarization data.
<table>
<thead>
<tr>
<th>Developed Economies</th>
<th>Period</th>
<th>Emerging Economies</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>1965-2003</td>
<td>Argentina</td>
<td>1985-2003</td>
</tr>
<tr>
<td>Denmark</td>
<td>1978-2003</td>
<td>Chile</td>
<td>1966-2003</td>
</tr>
<tr>
<td>Italy</td>
<td>1965-2003</td>
<td>Indonesia</td>
<td>1969-2003</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Tanzania</td>
<td>1969-2003</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Turkey</td>
<td>1969-2003</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Venezuela</td>
<td>1963-2003</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Zimbabwe</td>
<td>1969-2003</td>
</tr>
</tbody>
</table>
Table 8: Business cycle moments using H- filtered data

<table>
<thead>
<tr>
<th>Moment</th>
<th>Developed Economies</th>
<th>Emerging Economies</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Volatilities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma(y) )</td>
<td>2.37e-2</td>
<td>4.32e-2</td>
</tr>
<tr>
<td>( \sigma(I)/\sigma(y) )</td>
<td>3.79</td>
<td>3.83</td>
</tr>
<tr>
<td>( \sigma(c)/\sigma(y) )</td>
<td>0.97</td>
<td>1.21</td>
</tr>
<tr>
<td>( \sigma(g_c)/\sigma(y) )</td>
<td>1.39</td>
<td>3.47</td>
</tr>
<tr>
<td><strong>Autocorrelations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho(y) )</td>
<td>0.61</td>
<td>0.55</td>
</tr>
<tr>
<td>( \rho(c) )</td>
<td>0.54</td>
<td>0.42</td>
</tr>
<tr>
<td>( \rho(g_c) )</td>
<td>0.59</td>
<td>0.35</td>
</tr>
<tr>
<td><strong>Cyclicality</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho(y, I) )</td>
<td>0.85</td>
<td>0.65</td>
</tr>
<tr>
<td>( \rho(y, c) )</td>
<td>0.80</td>
<td>0.70</td>
</tr>
<tr>
<td>( \rho(y, g_c) )</td>
<td>0.23</td>
<td>0.24</td>
</tr>
<tr>
<td>( \rho(c, g_c) )</td>
<td>0.34</td>
<td>0.15</td>
</tr>
</tbody>
</table>

**Note:** This table contains the average value of moments computed for a set of emerging and developed economies. Relative volatility measures for each country can be found in Tables 9 and 10 in Appendix 7.1. Cyclical moments (autocorrelations and volatilities) are computed by HP-filtering \((w = 100)\) the natural logarithms of variables over the sample period 1960-2003. Data are obtained from Kaminsky, Reinhart, and Végh (2004).
<table>
<thead>
<tr>
<th>Country</th>
<th>$\sigma(c)/\sigma(y)$</th>
<th>$\sigma(I)/\sigma(y)$</th>
<th>$\sigma(g_c)/\sigma(y)$</th>
<th>$\rho(g_c,y)$</th>
<th>Polarization</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.67</td>
<td>3.66</td>
<td>1.24</td>
<td>0.08</td>
<td>2.62</td>
<td>8.21</td>
</tr>
<tr>
<td>Austria</td>
<td>0.76</td>
<td>3.34</td>
<td>1.39</td>
<td>0.21</td>
<td>2.57</td>
<td>8.44</td>
</tr>
<tr>
<td>Belgium</td>
<td>1.29</td>
<td>3.66</td>
<td>2.43</td>
<td>0.81</td>
<td>2.73</td>
<td>8.01</td>
</tr>
<tr>
<td>Canada</td>
<td>0.85</td>
<td>3.80</td>
<td>1.34</td>
<td>-0.27</td>
<td>2.56</td>
<td>8.10</td>
</tr>
<tr>
<td>Denmark</td>
<td>1.33</td>
<td>5.50</td>
<td>1.42</td>
<td>-0.24</td>
<td>2.15</td>
<td>7.97</td>
</tr>
<tr>
<td>Finland</td>
<td>0.69</td>
<td>3.66</td>
<td>0.74</td>
<td>0.31</td>
<td>2.46</td>
<td>8.87</td>
</tr>
<tr>
<td>France</td>
<td>0.71</td>
<td>4.45</td>
<td>0.97</td>
<td>-0.22</td>
<td>2.51</td>
<td>8.05</td>
</tr>
<tr>
<td>Germany</td>
<td>1.21</td>
<td>2.22</td>
<td>1.42</td>
<td>0.87</td>
<td>2.70</td>
<td>7.89</td>
</tr>
<tr>
<td>Greece</td>
<td>1.07</td>
<td>4.17</td>
<td>1.63</td>
<td>0.11</td>
<td>2.58</td>
<td>7.38</td>
</tr>
<tr>
<td>Ireland</td>
<td>1.10</td>
<td>4.00</td>
<td>1.45</td>
<td>0.17</td>
<td>2.54</td>
<td>8.86</td>
</tr>
<tr>
<td>Italy</td>
<td>0.93</td>
<td>3.41</td>
<td>1.90</td>
<td>0.43</td>
<td>2.68</td>
<td>7.11</td>
</tr>
<tr>
<td>Japan</td>
<td>0.64</td>
<td>2.59</td>
<td>2.03</td>
<td>2.59</td>
<td>0.35</td>
<td>7.97</td>
</tr>
<tr>
<td>Netherlands</td>
<td>1.42</td>
<td>4.09</td>
<td>1.92</td>
<td>0.16</td>
<td>2.11</td>
<td>8.62</td>
</tr>
<tr>
<td>New Zealand</td>
<td>1.21</td>
<td>3.58</td>
<td>1.73</td>
<td>0.62</td>
<td>2.20</td>
<td>7.82</td>
</tr>
<tr>
<td>Norway</td>
<td>1.80</td>
<td>4.82</td>
<td>1.69</td>
<td>0.32</td>
<td>2.34</td>
<td>8.09</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.96</td>
<td>4.63</td>
<td>1.75</td>
<td>0.5</td>
<td>2.73</td>
<td>8.04</td>
</tr>
<tr>
<td>Spain</td>
<td>1.04</td>
<td>3.25</td>
<td>1.02</td>
<td>0.41</td>
<td>2.51</td>
<td>8.26</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.83</td>
<td>4.53</td>
<td>1.01</td>
<td>-0.17</td>
<td>2.22</td>
<td>8.18</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.53</td>
<td>3.99</td>
<td>1.17</td>
<td>0.48</td>
<td>2.72</td>
<td>9.79</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1.18</td>
<td>4.39</td>
<td>1.37</td>
<td>-0.06</td>
<td>2.39</td>
<td>8.23</td>
</tr>
<tr>
<td>United States</td>
<td>0.85</td>
<td>3.31</td>
<td>1.33</td>
<td>0.35</td>
<td>2.70</td>
<td>8.78</td>
</tr>
<tr>
<td>Mean</td>
<td>1.00</td>
<td>3.85</td>
<td>1.47</td>
<td>0.25</td>
<td>2.53</td>
<td>8.22</td>
</tr>
</tbody>
</table>

**Note:** Cyclical moments (autocorrelations and volatilities) are computed by de-trending—using a band-pass filter 2-20—the natural logarithms of variables over the sample period 1960-2003 of data obtained from Kaminsky, Reinhart, and Végh (2004). Polarization is computed in Lindqvist and Östling (2010). Political Stability comes from Political Risk Services (PRS) government stability indicator.
Table 10: Relative volatility of consumption, investment, and public spending, *Emerging Economies*

<table>
<thead>
<tr>
<th>Country</th>
<th>$\sigma(c)/\sigma(y)$</th>
<th>$\sigma(I)/\sigma(y)$</th>
<th>$\sigma(g_c)/\sigma(y)$</th>
<th>$\rho(g_c,y)$</th>
<th>Polarization</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algeria</td>
<td>2.03</td>
<td>3.52</td>
<td>1.55</td>
<td>-0.18</td>
<td>3.01</td>
<td>8.03</td>
</tr>
<tr>
<td>Argentina</td>
<td>1.34</td>
<td>2.76</td>
<td>1.22</td>
<td>0.83</td>
<td>3.15</td>
<td>6.6</td>
</tr>
<tr>
<td>Bangladesh</td>
<td>1.01</td>
<td>1.54</td>
<td>3.43</td>
<td>-0.17</td>
<td>3.43</td>
<td>5.67</td>
</tr>
<tr>
<td>Brazil</td>
<td>1.18</td>
<td>3.34</td>
<td>2.58</td>
<td>0.33</td>
<td>3.44</td>
<td>6.64</td>
</tr>
<tr>
<td>Chile</td>
<td>1.23</td>
<td>5.79</td>
<td>1.81</td>
<td>0.14</td>
<td>2.67</td>
<td>7.25</td>
</tr>
<tr>
<td>China</td>
<td>1.25</td>
<td>2.19</td>
<td>1.09</td>
<td>0.21</td>
<td>3.22</td>
<td>8.25</td>
</tr>
<tr>
<td>Colombia</td>
<td>1.27</td>
<td>4.38</td>
<td>6.08</td>
<td>0.02</td>
<td>3.14</td>
<td>6.67</td>
</tr>
<tr>
<td>Dominican Republic</td>
<td>1.40</td>
<td>3.58</td>
<td>8.97</td>
<td>-0.09</td>
<td>3.52</td>
<td>6.73</td>
</tr>
<tr>
<td>Egypt</td>
<td>1.35</td>
<td>4.66</td>
<td>2.72</td>
<td>0.33</td>
<td>2.78</td>
<td>8.17</td>
</tr>
<tr>
<td>El Salvador</td>
<td>0.38</td>
<td>2.58</td>
<td>1.80</td>
<td>0.34</td>
<td>3.70</td>
<td>6.09</td>
</tr>
<tr>
<td>Indonesia</td>
<td>0.88</td>
<td>3.53</td>
<td>4.82</td>
<td>-0.01</td>
<td>3.10</td>
<td>7.40</td>
</tr>
<tr>
<td>Iran</td>
<td>1.07</td>
<td>2.65</td>
<td>1.66</td>
<td>0.02</td>
<td>2.70</td>
<td>7.18</td>
</tr>
<tr>
<td>Jordan</td>
<td>1.12</td>
<td>2.60</td>
<td>3.58</td>
<td>-0.23</td>
<td>2.82</td>
<td>8.07</td>
</tr>
<tr>
<td>Korea</td>
<td>0.92</td>
<td>4.08</td>
<td>2.34</td>
<td>0.37</td>
<td>2.27</td>
<td>6.23</td>
</tr>
<tr>
<td>Mexico</td>
<td>1.14</td>
<td>2.84</td>
<td>2.59</td>
<td>0.24</td>
<td>3.55</td>
<td>7.49</td>
</tr>
<tr>
<td>Morocco</td>
<td>1.03</td>
<td>3.89</td>
<td>2.30</td>
<td>0.2</td>
<td>3.26</td>
<td>8.69</td>
</tr>
<tr>
<td>Nigeria</td>
<td>1.49</td>
<td>2.36</td>
<td>3.17</td>
<td>-0.19</td>
<td>2.87</td>
<td>6.61</td>
</tr>
<tr>
<td>Pakistan</td>
<td>1.82</td>
<td>4.83</td>
<td>2.62</td>
<td>0.36</td>
<td>1.92</td>
<td>7.09</td>
</tr>
<tr>
<td>Peru</td>
<td>0.93</td>
<td>3.15</td>
<td>2.67</td>
<td>0.53</td>
<td>3.22</td>
<td>6.09</td>
</tr>
<tr>
<td>Philippines</td>
<td>0.77</td>
<td>4.40</td>
<td>2.89</td>
<td>0.74</td>
<td>2.92</td>
<td>6.32</td>
</tr>
<tr>
<td>South Africa</td>
<td>2.07</td>
<td>4.72</td>
<td>4.26</td>
<td>0.54</td>
<td>3.11</td>
<td>7.60</td>
</tr>
<tr>
<td>Tanzania</td>
<td>1.51</td>
<td>5.12</td>
<td>7.50</td>
<td>0.57</td>
<td>3.32</td>
<td>7.46</td>
</tr>
<tr>
<td>Turkey</td>
<td>1.01</td>
<td>3.21</td>
<td>8.39</td>
<td>0.37</td>
<td>3.27</td>
<td>7.14</td>
</tr>
<tr>
<td>Venezuela</td>
<td>1.70</td>
<td>4.78</td>
<td>2.24</td>
<td>0.79</td>
<td>3.46</td>
<td>7.03</td>
</tr>
<tr>
<td>Zimbabwe</td>
<td>1.28</td>
<td>8.60</td>
<td>2.12</td>
<td>-0.03</td>
<td>3.32</td>
<td>6.70</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td><strong>1.25</strong></td>
<td><strong>3.80</strong></td>
<td><strong>3.37</strong></td>
<td><strong>0.24</strong></td>
<td><strong>3.09</strong></td>
<td><strong>7.09</strong></td>
</tr>
</tbody>
</table>

*Note:* Cyclical moments (autocorrelations and volatilities) are computed by de-trending—using a band-pass filter 2-20—the natural logarithms of variables over the sample period 1960-2003 of data obtained from Kaminsky, Reinhart, and Végh (2004). Polarization is computed in Lindqvist and Östling (2010). Political Stability comes from Political Risk Services (PRS) government stability indicator.
7.2 Proof of Proposition 4.1

With logarithmic utility, agent optimization implies

\[
\frac{1}{c} = \beta \mathbb{E} \left[ \frac{r'(1 - \tau')}{c'} \right]
\]

where the expectation is over both realization of TFP and party in power. Competitive firm behavior gives

\[
r = \frac{\alpha y}{k}, \quad y = rk + w.
\]

Using the guess \( c = s(1 - \tau)y \) (which from agent budget constraint implies \( k' = (1 - s)(1 - \tau)y \)), we have

\[
\frac{1}{s(1 - \tau)y} = \beta \mathbb{E} \left[ \frac{\alpha y'(1 - \tau')}{s(1 - \tau')y'} \right].
\]

Simple inspection reveals \( s = 1 - \alpha \beta \). Since \( g = \tau y \), substitution of \( s \) and \( \tau \) give

\[
c = (1 - \alpha \beta)(y - g), \quad (9)
\]

\[
k' = \alpha \beta(y - g). \quad (10)
\]

Given agent optimization, the current government’s problem is

\[
\max_{g_i} (1 - \lambda_i) ln(c) + \lambda_i ln(g) + \beta \mathbb{E}_z[pV_i(z', k') + (1 - p)W_i(z', k')]
\]

subject to (9), (10). Here the expectation is taken only over the realization of the TFP shock \( z \). \( V_i \) and \( W_i \) are defined as in the text, they are the equilibrium continuation values of remaining in power and losing power, respectively, for a given party of type \( i \). The government’s first-order condition can be written as

\[
-\frac{(1 - \lambda_i)}{y - g_i} + \frac{\lambda_i}{g_i} = \alpha \beta^2 \mathbb{E}_z[pV'_{k,i} + (1 - p)W'_{k,i}]. \quad (11)
\]

The equilibrium continuation values use equilibrium policy for \( g_i \). Assume (as is later confirmed) that the equilibrium policy for \( g_i \) is \( g_i = \lambda_i \eta y \). The derivative of the \( V_i(z, k) \) is then

\[
V_{k,i} = \frac{(1 - \lambda_i)}{c} (1 - \alpha \beta) \frac{\alpha y}{k} (1 - \lambda_i \eta) + \frac{\lambda_i}{g} \frac{\alpha y}{k} \lambda_i \eta + \beta \mathbb{E}_z[pV_{k,i} + (1 - p)W_{k,i}] \alpha \beta \frac{\alpha y}{k} (1 - \lambda_i \eta).
\]

We use the government’s first-order condition (government optimality must hold in equilibrium) to eliminate the \( \mathbb{E}_z[pV'_{k,i} + (1 - p)W'_{k,i}] \) term. After canceling terms and then updating one period, we have

\[
V_{k,i} = \alpha \frac{1}{k \eta} \Rightarrow \mathbb{E}_z[V'_{k,i}] = \alpha \frac{1}{k \eta}.
\]

The derivative of \( W \) is slightly more complex because the government’s FOC cannot be used to directly cancel the value function derivatives next period. It is

\[
W_{k,i} = \frac{(1 - \lambda_i)}{c} (1 - \alpha \beta) \frac{\alpha y}{k} (1 - \lambda_j \eta) + \frac{\lambda_i}{g} \frac{\alpha y}{k} \lambda_j \eta + \beta \mathbb{E}_z[(1 - p)V_{k,i} + pW_{k,i}] \alpha \beta \frac{\alpha y}{k} (1 - \lambda_j \eta)
\]
where \( g_j = \lambda_j \eta \) is the policy of the opposing party \( j \). We know the value of \( E_z[V_{k,i}]' \) and we can solve for \( E_z[W_{k,i}]' \) from (11). It is

\[
E_z[W_{k,i}]' = \left[ \frac{1}{\alpha \beta^2} \left[ \frac{(1 - \lambda_i)}{y' (1 - \lambda_i \eta)} + \frac{1}{\eta y'} - p\mathbb{E}_z[V_{k,i}] \right] - \frac{1}{1 - p} \right].
\]

After some algebra this becomes

\[
E_z[W_{k,i}]' = \frac{1}{k'} \left[ \frac{1 - \eta - p \alpha \beta}{\beta} \right].
\]

Returning to \( W_{k,i} \) and simplifying the first two terms we can write

\[
W_{k,i} = \frac{\alpha}{k} + \beta \left[ \frac{1}{k' (1 - p)} \left[ \frac{1 - \eta - p \alpha \beta}{\beta} \right] + \frac{1}{k' (1 - p) \eta} \right] \frac{\alpha \beta}{k} (1 - \lambda_i \eta).
\]

We have a closed-form expression for \( k' \) when party \( j \) is in power, \( k' = \alpha \beta (1 - \lambda_i \eta) y \). Inserting this into the above equation allows us to simplify \( W_{k,i} \) to

\[
W_{k,i} = \frac{\alpha}{k} \left[ 1 + \alpha \beta (1 - 2p) + \frac{p}{1 - p} \frac{1 - \eta}{\eta} \right].
\]

Just like \( V_{k,i} \), this can be updated by simply replacing \( k \) with \( k' \); no expectation operator is necessary. The government’s first-order condition (11) using the equilibrium policy rule can then be written as

\[
- \frac{(1 - \lambda_i)}{(1 - \lambda_i \eta)} + \frac{\lambda_i}{\lambda_i \eta y} = \alpha \beta^2 \left[ p \frac{\alpha}{\alpha \beta (1 - \lambda_i \eta) \eta y} + (1 - p) \frac{\alpha}{\alpha \beta (1 - \lambda_i \eta) y} \left[ 1 + \frac{\alpha \beta (1 - 2p)}{(1 - p) \eta} + \frac{p}{1 - p} \frac{(1 - \eta)}{\eta} \right] \right].
\]

Notice everything cancels except the primitives \( \alpha, \beta, \) and \( p \). After some more brief algebra, we arrive at

\[
\eta = \frac{1 - 2 \alpha \beta - \alpha^2 \beta^2 (1 - 2p)}{1 - \alpha \beta (1 - 2p)}.
\]

\( Q.E.D. \)

### 7.3 Proof of Lemma 4.1

We will work with \( t \) subscripts to make the exposition clearer. Taking logs to the savings rule, and defining \( \hat{k_t} = \ln k_t \) and \( \hat{x_t} = \ln (1 - \lambda_t \eta) \), we obtain

\[
\hat{k_{t+1}} = q + \alpha \hat{k_t} + \hat{x_t} + z_t
\]

where \( q = \alpha \beta \). Our objective is to compute the long-run moments of this variable. The TFP shock \( z_t \) follows a standard symmetric Markov process with transition probability \( \pi \), long-run mean \( \bar{z} = 0.5(z_H + z_L) \) and variance \( \sigma^2 = 0.5^2(z_H - z_L)^2 \). Since \( \hat{x_t} = \ln (1 - \lambda_t \eta) \) is just a transformation
of $\lambda_t$, it inherits its statistical properties. In other words, it also follows a symmetric Markov process with transition probability $\text{Prob}(\hat{x}_t = \hat{x}_i/\lambda_{t-1} = \hat{x}_i) = \text{Prob}(\hat{\lambda}_t = \lambda_i/\lambda_{t-1} = \lambda_i) = p$. Because of symmetry, the long-run probability of each state is 0.5, so the long-run mean and variance satisfy

$$\hat{x} = 0.5[\hat{x}_L + \hat{x}_R] \text{ and } \sigma^2_{\hat{x}} = 0.5^2(\hat{x}_R - \hat{x}_L)^2$$

We can write

$$\hat{k}_{t+1} = \alpha^{t+1}\hat{k}_0 + \sum_{s=0}^{t} \alpha^s[q + z_{t-s} + \hat{x}_{t-s}],$$

(i) Taking the period-zero expectation of the expression above we obtain

$$\mathbb{E}_0(\hat{k}_{t+1}) = \alpha^{t+1}\hat{k}_0 + \sum_{s=0}^{t} \alpha^s[q + \mathbb{E}_0(z_{t-s}) + \mathbb{E}_0(\hat{x}_{t-s})]$$

Taking limits,

$$\mathbb{E}(\hat{k}) = \lim_{t \to \infty} \mathbb{E}_0(\hat{k}_{t+1}) = \sum_{s=0}^{\infty} \alpha^s[q + \bar{z} + \bar{x}] = \frac{q + \bar{z} + \bar{x}}{1 - \alpha}.$$  

(ii) The covariance between $\hat{k}_t$ and $\hat{x}_t$ satisfies

$$\text{Cov}(\hat{k}_t, \hat{x}_t) = \text{Cov} \left( \alpha^{t}\hat{k}_0 + \sum_{s=1}^{t} \alpha^{s-1}[q + z_{t-s} + \hat{x}_{t-s}], \hat{x}_t \right) = \sum_{s=1}^{t} \alpha^{s-1}\text{Cov}(\hat{x}_{t-s}, \hat{x}_t),$$

because $z_t$ and $\hat{x}_t$ are uncorrelated. We can write

$$\text{Cov}(\hat{x}_t, \hat{x}_{t-s}) = \mathbb{E}(\hat{x}_t\hat{x}_{t-s}) - \mathbb{E}(\hat{x}_t)\mathbb{E}(\hat{x}_{t-s}).$$

The expectation of $\hat{x}_t\hat{x}_{t-s}$ depends on the path that $\hat{x}$ followed from period $t-s$ to period $t$,

$$\mathbb{E}(\hat{x}_t\hat{x}_{t-s}) = \sum_{i} \sum_{j} \text{Prob}(\hat{x}_{t-s} = \hat{x}_j)\text{Prob}(\hat{x}_t = \hat{x}_i|\hat{x}_{t-s} = \hat{x}_j)\hat{x}_j\hat{x}_i.$$ 

Defining $P$ as the transition matrix for $\hat{x}$, this expression becomes

$$\mathbb{E}(\hat{x}_t\hat{x}_{t-s}) = \sum_{i} \sum_{j} 0.5 \ P_{i,j}^s \ \hat{x}_j\hat{x}_i,$$

Where $P_{i,j}^s$ is the $\{i, j\}$ element of the transition matrix to the $s$th power. With reelection probability $p$, the transition matrix can be written (following Hamilton, 1994) as

$$P = \begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix} \Rightarrow \ P^s = \frac{1}{2} \begin{pmatrix} 1 + (2p - 1)^s & 1 - (2p - 1)^s \\ 1 - (2p - 1)^s & 1 + (2p - 1)^s \end{pmatrix},$$

Thus,

$$\mathbb{E}(\hat{x}_t\hat{x}_{t-s}) = 0.5(\hat{x}_H^2 + \hat{x}_L^2)0.5[1 + (2p - 1)^s] - \hat{x}_H\hat{x}_L[(1 - (2p - 1)^s)[0.5.$
The unconditional expectation of $\hat{x}_t$ implies

$$E(\hat{x}_t) = E(\hat{x}_{t-s}) = 0.5\hat{x}_L + 0.5\hat{x}_H.$$ 

Rearranging terms,

$$Cov(\hat{x}_t, \hat{x}_{t-s}) = 0.5\hat{x}_L + 0.5\hat{x}_H.$$

Hence,

$$\sum_{s=1}^{t} \alpha^{s-1} Cov(\hat{x}_{t-s}, \hat{x}_t) = \sigma_x^2 \sum_{s=1}^{t} \alpha^{s-1}(2p-1)^s,$$

which implies

$$Cov(\hat{k}_t, \hat{x}_t) = \sigma_x^2 (2p-1) \sum_{s=0}^{t-1} \alpha^s (2p-1)^s. \quad (15)$$

In the limit,

$$Cov(\hat{k}, \hat{x}) = \lim_{t \to \infty} Cov(\hat{k}_t, \hat{x}_t) = \sigma_x^2 \frac{(2p-1)}{1 - \alpha(2p-1)}.$$

The computation of $Cov(\hat{k}, z)$ is analogous.

(iii) The variance of $\hat{k}_{t+1}$ satisfies

$$Var(\hat{k}_{t+1}) = Var \left( q + \alpha \hat{k}_t + \hat{x}_t + z_t \right)$$

After some algebra, and re-arranging terms

$$Var(\hat{k}_{t+1}) = Var(\hat{x}_t) + Var(z_t) + \alpha^2 Var(\hat{k}_t) + 2 \left[ \alpha Cov(\hat{x}_t, \hat{k}_t) + \alpha Cov(z_t, \hat{k}_t) + Cov(\hat{x}_t, z_t) \right]$$

The last term is zero by assumption, $Cov(\hat{x}_t, z_t) = 0$. Taking limits as $t \to \infty$

$$Var(\hat{k}) = \sigma_x^2 + \sigma_z^2 + \alpha^2 Var(\hat{k}) + 2\alpha \left[ Cov(\hat{x}, \hat{k}) + Cov(z, \hat{k}) \right]$$

Re-arranging this, we obtain the expression in the lemma.

$$Q.E.D.$$  

7.4 Proof of Proposition 4.2

(i) From Proposition 4.1, we know that

$$c_t = (1 - \alpha \beta)(1 - \lambda_t \eta)y_t.$$ 

Taking logs $\hat{c}_t = \ln(1 - \alpha \beta) + \hat{x}_t + \hat{y}_t$. Then

$$Var(\hat{c}_t) = Var(\hat{y}_t) + Var(\hat{x}_t) + 2Cov(\hat{x}_t, \hat{y}_t).$$
Since $\hat{y}_t = z_t + \alpha \hat{k}_t$, $\text{Cov}(\hat{x}_t, \hat{y}_t) = \alpha \text{Cov}(\hat{x}_t, \hat{k}_t)$ with $\text{Cov}(\hat{x}_t, \hat{k}_t)$ defined in eq. (15). Taking limits,

$$\text{Var}(\hat{c}) = \text{Var}(\hat{y}) + \sigma_z^2 + 2\alpha \sigma_x^2 \frac{(2p - 1)}{1 - \alpha(2p - 1)},$$

where the long-run variance of $\hat{y}$ satisfies eq. 5. The variables $\text{Cov}((\hat{k}, z)$ and $\text{Var}(\hat{k})$ are defined in conditions (ii) and (iii) of Lemma 4.1.

(ii) From Proposition 4.1, we know that $g_t = \lambda_t \eta y_t$. Taking logs,

$$\hat{g}_t = \ln \eta + \hat{\lambda}_t + \hat{y}_t.$$  

Then, $\text{Var}(\hat{g}_t) = \text{Var}(\hat{\lambda}_t) + \text{Var}(\hat{y}_t) + 2\text{Cov}(\hat{\lambda}_t, \hat{y}_t)$. The limit of this expression delivers eq. (7) in the proposition. Given the long-run distribution of shocks, we have that

$$\text{Var}(\hat{\lambda}_t) = \mathbb{E}(\hat{\lambda}_t^2) - (\mathbb{E}(\hat{\lambda}_t))^2 = 0.5^2(\hat{\lambda}_L - \hat{\lambda}_R)^2 \equiv \sigma^2_\lambda.$$

To compute $\text{Cov}(\hat{y}, \hat{\lambda})$ note that $\text{Cov}((\hat{y}_t, \hat{\lambda}_t) = \alpha \text{Cov}(\hat{k}_t, \hat{\lambda}_t)$. Using eq. (13), we obtain

$$\text{Cov}(\hat{k}_t, \hat{\lambda}_t) = \text{Cov}\left(\alpha^t \hat{k}_0 + \sum_{s=1}^{t} \alpha^s - 1[q + z_{t-s} + \hat{x}_{t-s}, \hat{\lambda}_t]\right) = \sum_{s=1}^{t} \alpha^s - 1 \text{Cov}(\hat{x}_{t-s}, \hat{\lambda}_t).$$

We can write

$$\text{Cov}(\hat{x}_{t-s}, \hat{\lambda}_t) = \mathbb{E}(\hat{x}_{t-s}\hat{\lambda}_t) - \mathbb{E}(\hat{x}_{t-s})\mathbb{E}(\hat{\lambda}_t),$$

and given that both $\hat{\lambda}_t$ and $\hat{x}_t$ follow the same Markov process, we have

$$\mathbb{E}(\hat{x}_{t-s}\hat{\lambda}_t) = \sum_{i} \sum_{j} \text{Prob}(\lambda_t = \lambda_i) \text{Prob}(\lambda_t = \lambda_j | \lambda_{t-s} = \lambda_j) \hat{x}_j \hat{\lambda}_i.$$

$$\mathbb{E}(\hat{x}_{t-s}\hat{\lambda}_t) = \sum_{i} \sum_{j} 0.5 \ P_{i,j}^s \ \hat{x}_j \hat{\lambda}_i,$$

where $P_{i,j}^s$ is the element $i, j$ of the matrix $P^s$, defined in eq. (14). Replacing this and performing some algebraic manipulations, we obtain

$$\text{Cov}(\hat{x}_{t-s}, \hat{\lambda}_t) = 0.5^2(2p - 1)^s(\hat{\lambda}_R - \hat{\lambda}_L)(\hat{x}_R - \hat{x}_L) \leq 0$$

since $\hat{\lambda}_R \leq \hat{\lambda}_L$ and $\hat{x}_R \geq \hat{x}_L$. Finally,

$$\text{Cov}(\hat{k}_t, \hat{\lambda}_t) = \sum_{s=1}^{t} \alpha^s - 1[0.5^2(2p - 1)^s(\hat{\lambda}_R - \hat{\lambda}_L)(\hat{x}_R - \hat{x}_L)],$$

implying

$$\text{Cov}(\hat{k}, \hat{\lambda}) = \lim_{t \to \infty} \text{Cov}(\hat{k}_t, \hat{\lambda}_t) = 0.5^2(\hat{\lambda}_R - \hat{\lambda}_L)(\hat{x}_R - \hat{x}_L) \frac{2p - 1}{1 - \alpha(2p - 1)}.$$

$$Q.E.D.$$
7.5 Proof of Lemma 4.2

From Proposition 4.2 we know that

\[
\text{Var}(\hat{g}_t) - \text{Var}(\hat{y}) = \sigma^2 + 2\alpha \text{Cov}(\hat{\lambda}, \hat{k}) + 0.5^2(\hat{\lambda}_L - \hat{\lambda}_R)^2 \left[ 1 - 2\alpha \frac{2p - 1}{\lambda_L - \lambda_R} \right].
\]

Clearly

\[
2\alpha \frac{2p - 1}{1 - \alpha(2p - 1)} \leq 1 \iff p \leq \frac{1 + 3\alpha}{6\alpha}, \text{ and}
\]

\[
\frac{\hat{x}_R - \hat{x}_L}{\lambda_L - \lambda_R} \leq 1 \iff \hat{\lambda} \leq \frac{1}{2\eta},
\]

where the second sufficient condition uses the fact that \(\lambda_L = \hat{\lambda} + \xi\) and \(\lambda_R = \hat{\lambda} - \xi\).

Q.E.D.

7.6 Proof of Lemma 4.3

The long-run correlation between \(\hat{g}\) and \(\hat{y}\) satisfies

\[
\text{Corr}(\hat{g}_t, \hat{y}) = \frac{\text{Cov}(\hat{g}, \hat{y})}{\text{Var}(\hat{g})^{1/2}}\text{Var}(\hat{y})^{1/2},
\]

where

\[
\text{Cov}(\hat{g}, \hat{y}) = \text{Cov}(\ln \eta + \hat{\lambda} + \hat{y}, \hat{y}) = \text{Cov}(\hat{\lambda}, \hat{y}) + \text{Var}(\hat{y})
\]

From the definition of \(\hat{y}\), we know that \(\text{Cov}(\hat{\lambda}, \hat{y}) = \alpha \text{Cov}(\hat{\lambda}, \hat{k})\). The covariance on the right-hand side satisfies eq. (17).

Q.E.D.

7.7 Algorithm

The numerical implementation consists of finding a fixed-point in the two equilibrium policy rules: \(G_i(z, k)\) and \(H_i(z, k)\). Because of asymmetric preferences, stochastic productivity, and the importance of the transitional dynamics in determining the simulated moments of the model, it is not enough to solve at steady states as in Klein, Krusell and Rios-Rull (2008). We require a global solution for the equilibrium rules. To accomplish this, we make guesses at these rules and iterate on them using the agents’ and government’s first-order conditions until convergence. The algorithm proceeds as follows:

1. Start with a sufficiently large grid for \(k\) and \(g\).

2. Make “good” guesses for the functions \(G_i^N(z, k)\) and \(H_i^N(z, k)\) for each of the points on the grid of \(k\), the realization of the shock \(z\), and the party in power \(i\). We use cubic spline interpolation to determine policy for off-grid values of \(k\) and also the derivatives of the policy functions that appear in the government’s optimality condition.

3. Using these guesses and the current state, solve the agent’s Euler equation for \(k'\) at each state, \(\{k, z, i\}\) as well as the level of \(g\). This gives the function \(H_i(z, k, g) = k'\).
4. The government takes agent optimization $H_i$ in the current period as given. The next step is to solve the government’s first-order condition for $g$ given future policy $G^N$ and $H^N$ and current agent optimization $H_i$. The solution to this problem gives the updated guess for $g$ policy, $G_i^{N+1}(z,k) = g$.

5. Update the guess at equilibrium savings policy: $H_i^{N+1}(z,k) = H_i(z,k,G_i^{N+1}(z,k))$.

6. Repeat this process until $\max\{|G_i^{N+1}(z,k) - G_i^N(z,k)|, |H_i^{N+1}(z,k) - H_i^N(z,k)|\}$ is small enough.

Time-consistency introduces particular challenges to computation of the equilibrium. The government’s problem is not in general a contraction and has significant non-convexities. To overcome the lack of contraction in a similar framework, Ilzetzki (2011) solves the finite horizon problem for a long horizon. We do not need to resort to this; in practice our program converges relatively quickly given our “good” initial guesses.

The advantage we have in solving the problem is the “good” initial guess of having a closed form solution given Assumption 1. Starting with an exact solution for the policy functions, the parameters can be slowly adjusted to the desired calibration. This adjustment is done at times extremely slowly to maintain a contraction in the government’s problem. For a grid of 60 points for $k$ and 30 points for $g$ we consistently achieve convergence of 1e-7.