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Inflation and real activity with firm-level productivity shocks*

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Abstract

In the last ten years there has been an explosion of empirical work examining price setting behavior at the micro level. The work has in turn challenged existing macro models that attempt to explain monetary nonneutrality, because these models are generally at odds with much of the micro price data. In response, economists have developed a second generation of sticky-price models that are state dependent and that include both fixed costs of price adjustment and idiosyncratic shocks. Nonetheless, some ambiguity remains about the extent of monetary nonneutrality that can be attributed to costly price adjustment. Our paper takes a step toward eliminating that ambiguity.

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1 Introduction

In the last ten years there has been an explosion of empirical work examining price setting behavior at the micro level. This work has been carried out on price data for many countries and has uncovered several common features of those data. The work has in turn challenged existing macro models that attempt to explain monetary nonneutrality, because these models are generally at odds with much of the micro price data. In response, economists have developed a second generation of sticky-price models that are state dependent and that include both fixed costs of price adjustment and idiosyncratic shocks.[1] Nonetheless, ambiguity remains about the extent of monetary nonneutrality that can be attributed to costly price adjustment. Our paper takes a step toward eliminating that ambiguity. We trace the degree of nonneutrality to subtle aspects of the idiosyncratic productivity process, the distribution of fixed costs, and the extent to which price stickiness varies across firms. In turn, these findings suggest that empirical research measuring these objects would help in refining our models of price setting.

We believe that there are a number of key features of the micro price data that a theoretical model should seek to match if it is to be viewed as broadly consistent with those data: (1) there are both large positive and negative price changes that are intermixed with many small price changes, (2) average price changes are an order of magnitude larger than needed to keep up with inflation, (3) many prices are set infrequently, with changes occurring less frequently than once a year, while some prices appear to be completely flexible, (4) the frequency of positive price changes is positively correlated with inflation and the frequency of negative price changes is negatively correlated with inflation, leading to little correlation of the overall frequency of price adjustment changes with inflation, (5) aggregate hazards are relatively flat, (6) the idiosyncratic part of price changes is more volatile and less persistent than that due to aggregate factors, and (7) the size of a price change is not related to the time since the last price change. In their survey of the microeconomic evidence on price setting, these features led Klenow and Malin (2011) to conclude that idiosyncratic forces are dominant in accounting for actual pricing behavior and are largely consistent with a state-dependent approach to price setting.

Our analysis is related to a number of recent papers, most notably Golosov and Lucas (2007), Midrigan (2011), Nakamura and Steinsson (2008), and Gertler and Leahy (2008). As we detail below, our contribution in part involves a more comprehensive match between our model and the facts about microeconomic price behavior from the U.S. CPI data. To that

[1] For example, see Golosov and Lucas (2007), Burstein and Hellwig (2007), Midrigan (2009), Gertler and Leahy (2008), Nakamura and Steinsson (2008) and Vavra (2013) to name a few.

end, we rely extensively on the empirical work of Klenow and Kryvtsov (2008). Our models do generate significant nonneutrality, and as Midrigan (2011) argues, the nonneutrality is related in part to the existence of many small price changes. The failure to account for small price changes is the primary reason that the Golosov and Lucas (2007) model obtains a Caplin and Spulber- type result of significant rigidities at the micro level along with almost complete flexibility at the macro level. Nakamura and Steinsson (2008) analyze the effects of including multiple sectors and intermediate inputs into a menu-cost model. They find that sectoral heterogeneity is important for producing monetary nonneutralities. Costain and Nakov (2011a,b), using a broadly complementary methodology, also investigate the consequences that matching a large number of the features of micro-pricing has on monetary nonneutrality. They apply their approach to scanner-level data rather than broad consumer price data. Gertler and Leahy (2008) is another notable recent contribution to this literature, but unlike the previously cited papers, they do not attempt to match many of the moments associated with price changes at the product level.

Capturing the rich distribution of prices requires us to embed significant heterogeneity across firms. The way we incorporate heterogeneity is standard in the literature, and assumes that firms face idiosyncratic shocks to their productivity or marginal cost. We also incorporate a fraction of firms who set prices flexibly, which helps the model account for the many small price changes in the data. We calibrate the model to match the distribution of price changes as well as the median duration of prices documented in Klenow and Kryvtsov (2008). It matches these facts with firms incurring relatively small menu costs. An outgrowth of this calibration is that the model is consistent with many of the other pricing facts listed above. We are also able to identify features of the productivity process that are essential to finding flat aggregate hazard functions and to producing monetary nonneutality. Notably, we find that the micro pricing features are consistent with both flat hazards and a significant amount of nonneutrality, although the degree of nonneutrality is somewhat less than that found in many empirical studies.

An important aspect of our work is the finding that more than one specification of our model can match the steady-state distribution of price changes and the duration of prices as estimated by Klenow and Kryvtsov(2008). The two parameterizations illustrate that in matching the steady-state distribution, there is a trade-off between the fraction of sticky-price firms and the menu costs incurred by those firms. While the two parameterizations generate similar steady-state behavior, their implications for the nonneutrality of money are quite different: there is no simple relationship between the degree of overall steady-state stickiness and the extent of nonneutrality. This theoretical point is familiar from Caplin and Spulber (1987) and Caballero and Engel (2007), but our work shows that it can also be important in a quantitative model.

We begin in section 2 with a description of the model, paying particular attention to the details of the individual firm's pricing decisions. In section 3, we present our steady-state results, which match the data on price changes documented by Klenow and Kryvtsov (2008). The distribution of price changes as well as the median duration of price changes can be matched with either a relatively small fraction of flexible-price firms or a somewhat larger fraction of flexible-price firms. Section 4 contains an analysis of the role that idiosyncratic productivity shocks play in producing the model's aggregate dynamics. We do this by following up on work by Klenow and Kryvtsov (2008), Caballero and Engel (2007) and Costain and Nakov (2011a), who present various decompositions of the dynamics of the price level in response to a monetary shock. Here we find that the state-dependent elements of pricing are indeed important for the model's behavior. In section 5, we investigate the mechanisms of our model in more detail with the idea of isolating the key features necessary for producing nonneutralities. We find that a more parsimonious model does a reasonable job at matching some, but not all, of the micro data while at the same time producing dynamics that are similar to those of our more flexible benchmark model. In Section 6 we examine the model's ability to generate flat or downward-sloping aggregate hazards even though the state-specific hazards are upward sloping. Section 7 concludes.

2 The Model

The basic elements of the model are drawn from the state-dependent pricing framework of Dotsey, King and Wolman (1999), or DKW for short, with the addition of stochastic variation in productivity at the firm level. We refer to the different productivity levels as microstates. Firms are heterogeneous with respect to productivity realizations but share a common stochastic process.[2] As in the DKW framework, some firms are heterogeneous with respect to the realization of fixed costs, but we also allow for a simple form of heterogeneity in the distribution of fixed costs: a fraction of firms are flexible-price firms, facing zero adjustment costs. After analyzing the stochastic adjustment framework, we describe the standard features of the rest of the model.

[2] The framework can easily incorporate firm-specific demand distrubances. However, we focus on productivity shocks because they are the main focus of the related literature and because studies such as Boivin, Giannoni, and Mihov (2009) indicate that much of the idiosyncraticvariation can be attributed to supply-type shocks.

2.1 A stochastic adjustment framework

To study the effect of serially correlated firm-level shocks along with heterogeneity in adjustment costs, we extend the DKW framework while retaining much of the tractable general equilibrium analysis.

Heterogeneous productivity: The relative productivity level, e_t, of a firm is its microstate. We assume that there are K different levels of the microstate that may occur, $\underline{e}_k$, $k = 1, 2, ...K$, so that a firm of type k at date t has total factor productivity

$$a_{kt} = a_t \underline{e}_k \tag{1}$$

where a_t is a common productivity shock. We assume that the relative productivity levels are ordered so that $\underline{e}_1 < \underline{e}_2 < ...\underline{e}_K$.

Transitions between microstates are governed by a state transition matrix, Q, where

$$q_{kf} = prob(e_{t+1} = \underline{e}_f | e_t = \underline{e}_k) \tag{2}$$

We also use the notation $q(e'|e)$ to denote the conditional probability of state e' occurring next period if the current microstate is e. We assume that these transitions are independent across firms and that there is a continuum of firms, so that the law of large numbers applies. The stationary distribution of firms across microstates is then given by a vector Φ such that

$$\Phi = Q^T * \Phi$$

That is, the k^{th} element of Φ (denoted ϕ_k) gives the fraction of firms in the k^{th} microstate (these firms have productivity level $\underline{e}_k$).[3]

Heterogeneous adjustment costs: A fraction f of firms face zero adjustment costs and thus have perfectly flexible prices. In each period, the remaining fraction $1 - f$ of firms draw from a nondegenerate distribution $G()$ of fixed costs. Because the fraction and the identity of flexible-price firms are fixed over time, the only iteraction between the two types arises from general equilibrium considerations. We focus on the details of optimal pricing and adjustment for the firms that do face adjustment costs and then briefly discuss the flexible-price firms.

[3] The stationary probability vector can be calculated as the eigenvector associated with the unit eigenvector of the transpose of Q. See, for example, Kemeny and Snell (1976).

2.1.1 Optimal pricing and adjustment

Consider a firm that faces a demand $d(p,s)y(s)$ for its output if it is charging a relative price p in aggregate state s. Following DKW, we assume that this firm faces a fixed labor cost of adjustment ξ, which is drawn from a continuous distribution with support $[0,\mathcal{B}]$.

It is convenient to describe the optimal adjustment using three value functions: $\underline{v}$, the value of the firm if it does not adjust; v^o, its value if it has a current adjustment cost realization of 0; and v, its maximized value given its actual adjustment cost ξ.

In terms of these value functions, the market value of a firm is governed by

$$v(p,e,s,\xi) = \max\left\{\underline{v}(p,e,s), [v^o(e,s) - \lambda(s)w(s)\xi]\right\} \tag{3}$$

if it has a relative price of p, is in microstate e, aggregate state s, and draws a stochastic adjustment cost of ξ. In (3), $\lambda(s)$ is the marginal value of state contingent cash flow and $w(s)$ is the real wage in state s.[4] The market value, v, is then the maximum of two components, one being the value conditional on adjustment ($[v^o(e,s)-\lambda(s)w(s)\xi]$) and the other the value of nonadjustment ($\underline{v}(p,e,s)$).

Defining the real flow of profits as $z(p,e,s)$, the nonadjustment value $\underline{v}$ – the value of continuing with the current relative price p for at least one more period – obeys the Bellman equation,

$$\underline{v}(p,e,s) = [\lambda(s)z(p,e,s) + \beta E v(p',e',s',\xi')|(p,e,s)] \tag{4}$$

with $p' = p/\pi(s')$ and $\pi(s')$ denoting the gross inflation rate in the next period. Therefore, the value $\underline{v}$ depends on the current relative price p, marginal utility $\lambda(s)$, the flow of real profits $z(p,e,s)$, and the discounted expected future value $\beta E v(p',e',s',\xi')$.[5]

The "costly adjustment value" is given by the value of the firm if it is free to adjust, $v^o(e,s)$, less the cost of adjustment, which depends on the macro state through $\lambda(s)w(s)$ and also on the realization of the random adjustment cost ξ. In turn, the "free adjustment value" v^o obeys

$$v^o(e,s) = \max_{p^*}\left\{\lambda(s)z(p^*,e,s) + \beta E v(p',e',s',\xi')|p^*,e,s)\right\} \tag{5}$$

with $p' = p^*/\pi(s')$.

Notice that there are asymmetries in the determinants of these values. The nonadjust-

[4] The aggregate state, s, will be determined as part of the general equilibrium but can be left unspecified at present. Note that the values are in marginal utility units, which can be converted into commodity units by dividing through by marginal utility.

[5] In forming conditional expectations it is not necessary to condition on ξ because it is iid.

ment value $\underline{v}(p, e, s)$ depends on the relative price, the microstate and the macro-state but not on the adjustment cost ξ because this is not paid if adjustment does not take place. The free adjustment value $v^o(e, s)$ does not depend on the relative price (since the firm is free to choose a new price) or the adjustment cost ξ (since the adjustment decision involves no fixed cost).

As in other generalized partial adjustment models, such as the prior DKW analysis, the firm adjusts if

$$[v^o(e, s) - \lambda(s)w(s)\xi] > \underline{v}(p, e, s).$$

Accordingly, there is a threshold value of the adjustment cost, such that

$$\underline{\xi}(p, e, s) = \frac{v^o(e, s) - \underline{v}(p, e, s)}{\lambda(s)w(s)} \tag{6}$$

Firms facing a lower adjustment cost adjust. Those with a higher cost charge the same nominal price as they did last period, implying that their relative price falls.

The fraction of (non-flexible-price) firms that adjusts, then, is

$$\alpha(p, e, s) = G(\underline{\xi}(p, e, s)) \tag{7}$$

where G is the cumulative distribution of adjustment costs. The adjustment decision depends on the state of the economy, but it also depends on the distribution of adjustment costs in two ways. First as highlighted by (7), the adjustment cost distribution governs the fraction of adjusting firms given the threshold. Second, stochastic adjustment costs provide the firm with an incentive to wait for a low adjustment cost realization: the adjustment cost distribution affects this incentive and thus the adjustment threshold.

We study an environment with positive trend inflation. Because adjustment costs are bounded above and inflation continually erodes a firm's relative price if it does not adjust, there will exist a maximum number of periods over which a firm may choose not to adjust its price. Because historical prices depend on the state h that a firm was in when it last adjusted, as well as the state k that it is currently in, this maximal number, J_{hk}, will depend on both these states. Thus, the distribution that describes our economy takes on a finite but potentially large number of elements.

2.1.2 Flexible-price firms

For the fraction ω^f of firms with zero adjustment costs, things are much simpler. These flexible-price firms face the same productivity process as other firms and simply charge the price in state k that maximizes profits *in state k*. Thus, they set a price, p_k^f, that is a constant markup over their marginal cost, which depends on both their microstate and the aggregate state of the economy.

2.1.3 Dynamics and accounting

Because we want to study the effects of this joint distribution on macroeconomic activity, our framework requires that we track the distribution of firms over the set of historical prices and the evolving levels of micro-productivity. The core mechanics are as follows. We start with a joint distribution of relative prices and productivity that prevailed last period. This distribution is then influenced by the effects of microproductivity transitions (Q), the adjustment decisions of firms ($\alpha(p, e, s)$); and the effects of inflation on relative prices. The net effect is to produce a new distribution of relative prices prevailing in the economy. Table 1 summarizes some of the key notation and equations.

Table 1: Conceptual and accounting elements in microstate model

Concept	Symbol	Comment
past relative price	$p_{j-1,h,t-1}$	h = historical state when price set (at $t-j$)
past fraction	$\omega_{j-1,h,l,t-1}$	l = microstate at t-1
current fraction	$\theta_{j,h,k,t}$	$\theta_{j,h,k,t} = \sum_l q_{lk}\omega_{j-1,h,l,t-1}$
adjustment rate	$\alpha_{j,h,k,,t}$	depends on e, p, s
current relative price	$p_{j,h,t}$	$p_{jht} = p_{j-1,h,t-1}/\pi_t$
current fraction	$\omega_{j,h,k,,t}$	$\omega_{jhkt} = (1-\alpha_{jkht})\theta_{jkht}$
fraction flex-price firms	ω^f	ω^f is time-invariant

Initial conditions and sticky prices. Let $p_{j-1,h,t-1}$ be the previous period's relative price of a firm that last changed its price at date $t-1-(j-1) = t-j$ when it was in microstate h. Let $\omega_{j-1,h,l,t-1}$ be the fraction of (sticky-price) firms in this situation, that charged this price and also had productivity level l. Ranging over all the admissable j, h, l, this information gives the joint distribution of productivity and prices at date t-1.

If such a firm chooses not to adjust, its relative price evolves according to

$$p_{j,h,t} = p_{j-1,h,t-1}/\pi_t \tag{8}$$

where π_t is the current inflation rate (π_t is shorthand for $\pi(s_t)$ from above). That is, one effect on the date t distribution of relative prices comes from inflation.

Endogenous fractions: Two micro-disturbances hit a firm in our model, so that its ultimate decisions are conditioned on its productivity (e) and its adjustment cost (ξ). For the purpose of accounting and ease of presentation, it is convenient to specify that the productivity shock occurs first and the adjustment cost shock second, but doing so has no substantive implications.

As above, let $\omega_{j-1,h,l,t-1}$ be the fraction of (sticky-price) firms that charged the price $p_{j-1,h,t-1}$ when they were in microstate l last period. As a result of stochastic productivity transitions there will be a fraction

$$\theta_{j,h,k,t} = \sum_{l} q_{lk}\omega_{j-1,h,l,t-1}$$

of sticky-price firms that start period t with a j-period old nominal price set in microstate h and have a microstate k in the current period.

However, not all of these firms will continue to charge the nominal price that they set in the past. To be concrete, consider firms with a j period old price set in microstate h that are now in state k. Of these firms, the adjustment rate is $\alpha_{j,h,k,t}$. Then, the fraction of sticky-price firms choosing to continue charging the nominal price set j periods ago will be

$$\omega_{j,h,k,t} = (1 - \alpha_{j,h,k,t})\theta_{j,h,k,t} \tag{9}$$

Taking all of these features into account, we can see that transitions are governed by two mechanisms: the exogenous stochastic transitions of the microstates ($q_{l,k}$) and the endogenous adjustment decisions of firms ($\alpha_{j,h,k,t}$). As discussed, the adjustment decision depends on the firm's relative price, its microstate and the macroeconomic states.

Given that firms currently in microstate k adjust from a variety of historical states, it follows that the fraction of adjusting firms is given by

$$\omega_k^f + \omega_{0,k,k,t} = \omega^f \phi_k + \sum_{j}\sum_{h} \alpha_{j,h,k,t}\theta_{j,h,k,t}. \tag{10}$$

We use the redundant notation $\omega_{0,k,k,t}$ to denote the fraction of sticky-price firms that adjust in microstate k so that this is compatible with (9), and we use the notation ω_k^f to denote the fraction of firms that have flexible prices and are in state k. Since the distribution of microstates is assumed to be stationary, there is a constraint on the fractions,

$$\phi_k = \omega^f \phi_k + \left(\omega_{0,k,k,t} + \sum_h \sum_j \omega_{j,h,k,t} \right) \left(1 - \omega^f\right).$$

Equivalently, a fraction ϕ_k of both flexible- and sticky-price firms are in state k at the end of the period.

2.1.4 State variables suggested by the accounting

As suggested by the discussion above, there are two groups of natural endogenous state variables in the model. One is the vector of past relative prices $p_{j-1,h,t-1}$ for $h = 1, 2, ...K$ and $j = 1, 2, ...J_h$. The other is the fraction of sticky-price firms that enter the period with a particular past microstate (l) and a relative price that was set j periods ago in microstate h,

$$\omega_{j-1,h,l,t-1}. \tag{11}$$

Thus, the addition of microstates raises the dimension of the minimum state space introduced by the stochastic adjustment model structure from roughly $2 * J$ to roughly $J * K + J * K^2$, where J is the maximum number of periods of nonadjustment and K is the number of microstates. However, this is only an approximation because the maximum number of periods can differ across microstates: J_{kh} is the endpoint suitable for firms currently in microstate k that last adjusted in historical microstate h.

2.1.5 The adjustment process

Recall, that for each price lag $(j-1)$, microstate last period (l) and historical state (h), a fraction $\omega_{j-1,h,l,t-1}$ enters the period. Then, the microstate transition process leads to a fraction of sticky-price firms $\theta_{j,h,k,t} = \sum_l q_{l,k} \omega_{j-1,h,,l,t-1}$ having a price lag j, a current microstate k; and a historical state (h). Of these firms, a fraction $\alpha_{j,h,k,t}$ chooses to adjust while a fraction $\eta_{j,h,k,t} = 1 - \alpha_{j,h,k,t}$ chooses not to adjust, leaving $\omega_{j,h,k,t} = \eta_{j,h,k,t} \theta_{j,h,k,t}$ charging relative price $p_{j,h,t}$ and experiencing microstate k. One thing that is important to stress at this stage is that we allow for zero adjustment or for complete adjustment in various situations (particular j, k, h entries).

2.2 The DSGE model

We now embed this generalized partial adjustment apparatus into a particular DSGE model, which is designed to be simple on all dimensions other than pricing.

2.2.1 The Household

As is conventional, there are two parts of the specification of household behavior, aggregates and individual goods. We assume that there are many identical households that maximize

$$\max_{c_t,n_t} E_0\{\sum_t \beta^t[\frac{1}{1-\sigma}c_t^{1-\sigma} - \frac{\chi}{1+\gamma}n_t^{1+\gamma}]\}$$
$$\text{subject to: } c_t \leq w_t n_t + \left(1-\omega^f\right)\sum_j\sum_h\sum_k \omega_{j,h,k,t}z_{j,h,k,t} + \omega^f \sum_k \phi_k z_{k,t}^f,$$

where c_t and n_t are consumption and labor effort, respectively, $z_{j,h,k,t}$ is the profits remitted to the household by a type (j,h,k) firm, and $z_{k,t}^f$ is the profits of a flexible price firm in state k. The consumption aggregate, c, is given by the standard Dixit-Stiglitz demand aggregator. Thus, $c = \left(\int_0^1 (y(i))^{\frac{\epsilon-1}{\epsilon}}\,di\right)^{\frac{\epsilon}{\epsilon-1}}$. There is an economy-wide factor market for the sole input, labor, which earns a wage, w. The first-order condition determining labor supply is

$$\lambda_t w_t = \chi n_t^{\gamma}, \tag{12}$$

and, hence, γ^{-1} is the Frisch labor supply elasticity. The first order condition determining consumption is

$$c_t^{-\sigma} = \lambda_t \tag{13}$$

where λ_t is the multiplier on the household's budget constraint, which serves also to value the firms.

2.2.2 Firms

Two aspects of the firm's specification warrant discussion. First, we adopt a simple production structure, but we think of it as standing in for some of the elements in the "flexible supply side" model of Dotsey and King (2006). Thus, production is linear in labor, $y(i) = a(i)n(i)$, where $y(i)$ is the output of an individual firm, $a(i)$ is the level of its technology, and $n(i)$ is hours worked at a particular firm. Hence, real marginal cost, ψ_t, is given by $\psi_t = w_t/(a_t e_k)$ for a firm that is in microstate k at date t.

Second, for sticky-price firms, the optimal pricing condition given the structure of demand, productivity, and adjustment costs satisfies the first-order condition

$$0 = \lambda(s) z_p(p^*, e, s) + \beta\eta E[\underline{v}_p(p', e', s')], \tag{14}$$

with $p' = p^*/\pi(s')$ and the nonadjustment probability being $\eta(p, e, s) = 1 - \alpha(p', e', s')$.

The marginal value for a nonadjusting firm is

$$\underline{v}_p(p, e, s) = \lambda(s) z_p(p, e, s) + \beta E[\eta(p', e', s') \frac{1}{\pi(s')} \underline{v}_p(p', e', s')], \tag{15}$$

with $p' = p/\pi(s')$.[6]

For the fraction, ω^f, of firms with flexible prices, the profit maximization problem yields the static first-order condition:

$$z_p^f(p^{*,f}, e, s) = 0.$$

2.2.3 Money and equilibrium

To close the model, it is necessary to specify money supply and demand and to detail the conditions of macroeconomic equilibrium. We impose the money demand relationship $M_t/P_t = c_t$. Ultimately, the level of nominal aggregate demand is governed by this relationship along with the central bank's supply of money. The model is closed by assuming that nominal money supply growth follows an autoregressive process,

$$\Delta \log(M_t) = \rho \Delta \log(M_{t-1}) + x_{mt},$$

[6] Maximizing the "free adjustment value" (5) implies a first-order condition,

$$0 = \lambda(\varsigma) z_p(p^*, \upsilon, \varsigma) + \beta E[\frac{1}{\pi(\varsigma')}) v_p(\frac{p^*}{\pi(\varsigma')}, \upsilon', \varsigma', \xi')$$

The value function v takes the form

$$v(p, \upsilon, \varsigma, \xi) = \left\{ \begin{array}{ll} \underline{v}(p, \upsilon, \varsigma) & \text{if } \xi \geq \bar{\xi}(p, \upsilon, \varsigma) \\ [v^o(\upsilon, \varsigma) - \lambda(\varsigma) w(\varsigma) \xi] & \text{if } \xi \leq \bar{\xi}(p, \upsilon, \varsigma) \end{array} \right\}$$

so that

$$v_p(p, \upsilon, \varsigma, \xi) = \left\{ \begin{array}{ll} \underline{v}_p(p, \upsilon, \varsigma) & \text{if } \xi \geq \bar{\xi}(p, \upsilon, \varsigma) \\ 0 & \text{if } \xi \leq \bar{\xi}(p, \upsilon, \varsigma) \end{array} \right\}.$$

Since $\underline{v}_p$ does not depend on ξ, we can express the FOC as in the text. A similar line of reasoning leads to condition (15). We use this first-order approach as an element of producing candidate steady-state equilibria. We then confirm that the candidate is indeed an equilibrium, by checking that adjusting firms' behavior satisfies (5). John and Wolman (2008) discuss the possibility that solutions to the first order conditions may not satisfy (5).

where x_{mt} is a mean zero random variable.

There are three conditions for equilibrium. First, labor supply is equal to labor demand, which is a linear aggregate across all the production input requirements of firms plus labor for price adjustment:

$$n_t = \left(1-\omega^f\right)\sum_j\sum_h\sum_k \omega_{j,h,k,t} n_{j,h,k,t} + \omega^f \sum_k \phi_k n^f_{k,t} + \left(1-\omega^f\right)\sum_j\sum_h\sum_k \omega_{j,h,k,t} E\xi_{j,h,k,t}.$$

In this expression, $n_{j,h,k,t}$ is the labor used in production in period t by a sticky-price firm in state k that last adjusted its price in period $t-j$ when it was in state h; $n^f_{k,t}$ is the labor used in production in period t by a flexible-price firm in state k; and $E\xi_{j,h,k,t}$ is expected price adjustment costs for a firm in period t in state k that last adjusted its price in period $t-j$ when it was in state h:

$$E\xi_{j,h,k,t} = E\left(\xi|\xi < G^{-1}\left(\alpha_{j,h,k,t}\right)\right)$$

The second equilibrium condition is that consumption must equal output, and the third is that money demand must equal money supply.

2.3 Calibration of macroeconomic parameters

Typically, we will be calibrating at a monthly frequency. Our benchmark settings for the macroeconomic parameters are as follows: $\beta = .97^{1/n}$ where n is the number of periods in a year. The steady-state annualized inflation is 2.5%. The coefficient of relative risk aversion, $\sigma = .25$, labor supply elasticity γ^{-1} is 20, and the demand elasticity ε is 5. The potential for any type of endogenous propagation in this type of model is closely related to the elasticity of marginal cost with respect to output. Thus, low labor supply elasticities or coefficients of relative risk aversion greater than one severely compromise persistence in this simple and stark setting.[7] Dotsey and King (2005, 2006) explore many features of more sophisticated models that are capable of generating low marginal cost responses to output. We view our parameter settings as stand-ins for the more realistic persistence-generating mechanisms that are present in larger models. For robustness, we will also look at how setting $\sigma = 1$ or 2 affects some of our results.

[7] Combining the household's first-order conditions and approximating the wage by marginal cost and labor and consumption by output yields

$$d\ln mc/d\ln y = \gamma + \sigma,$$

which indicates that values of $\sigma > 1$ preclude low elasticities of marginal cost with respect to output.

3 Benchmark Parameterizations and Properties of our Steady-State Price Distribution

We choose the parameters for our benchmark cases so that the steady-state distribution of price changes from the model is consistent with the facts reported by Klenow and Kryvtsov (2008). The critical parameters are those governing the distributions of fixed costs and firm-level productivity.

3.1 Choosing parameters

Klenow and Kryvtsov (2008), referred to as KK below, report on the distribution of price changes from individual price data underlying the U.S. CPI from 1988 to 2004. Figure 1, reproduced from KK, displays a histogram of regular, nonzero price changes, where the set of regular price changes is meant to exclude sales.[8] As Midrigan (2011) and Guimaraes and Sheedy (2011) argue, the existence of sales has little implication for the nonneutrality of money, implying that for our purposes the emphasis in terms of matching the micro-price date should be placed on matching regular price changes. Other relevant price change statistics reported by Klenow and Kryvtsov (2008) are listed in the first column of Table 2. In choosing parameters, we target the histogram in Figure 1 and the median adjustment probability of 0.139 from Table 2. Note that while the histogram represents a direct measurement of price changes, in calculating adjustment probabilities KK impose a Calvo pricing structure on the data. For each category of goods and services they assume that there is a fixed adjustment probability. They then use the price data to compute maximum likelihood estimates of the adjustment probabilities for each category. The median adjustment probability across categories is 0.139.

[8] All prices and changes in prices are expressed in logs and log deviations.

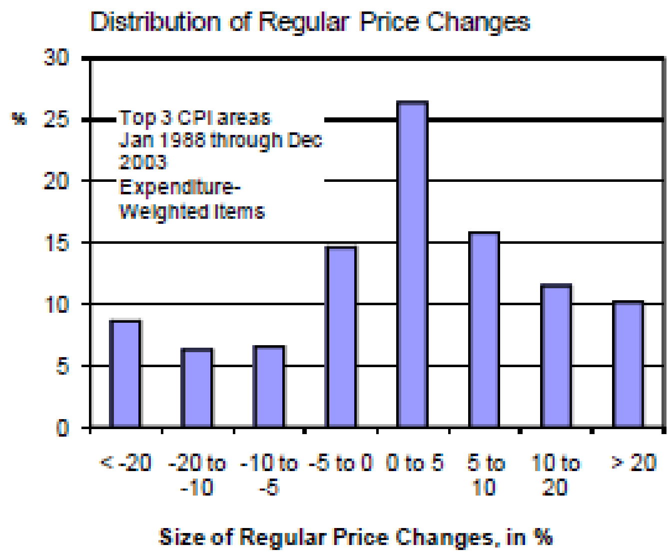


Figure 1. Price Change Distribution from Klenow and Kryvtsov

To choose parameters, we minimize a weighted sum of the squared deviations of the data "moments" (histogram elements and median duration) from the moments of our model's steady-state distribution. The parameters we allow to vary are (1) the elements of the productivity transition matrix Q, (2) the productivity realizations e_k, (3) the fraction of firms that are flexible-price firms ω^f, and (4) the upper bound of the fixed-cost distribution $G\left(\right)$. We fix the number of productivity realizations at seven, forcing the interior elements of that distribution to be equally spaced, but allowing the minimum and maximum productivities to be chosen freely.[9,10]

Regarding the cost of adjusting prices, as in DKW we employ a flexible distribution based on the tangent function. The distribution is governed by three parameters, the upper bound on the fixed cost, B, and the curvature parameters cc and dd :

$$G\left(\xi\right) = \frac{1}{cc}\left\{\tan\left(\frac{\xi - k_2}{k_1}\right) + dd \cdot \pi\right\}, \tag{16}$$

[9] To find parameters such that the model's steady-state distribution matches the data, we use a simulated annealing algorithm developed by William Goffe (1994,1992).

[10] We initially tried to use a Tauchen-type AR(1) process, but the restrictions it imposed on the transition matrix were inconsistent with the distribution of price changes.

with

$$k_1 = \frac{B}{[\arctan(cc - dd \cdot \pi) + \arctan(dd \cdot \pi)]} \quad (17)$$

$$k_2 = \arctan(dd \cdot \pi) \cdot k_1. \quad (18)$$

This function can produce a wide range of distributions depending on the values of cc and dd. For example, it can approximate a single fixed cost of adjustment, adjustment probabilities that are relatively flat over a wide range, or the nearly quadratic adjustment such as has been used by Cabellero and Engel. Apart from the sensitivity analysis in Section 4.6.1 , we fix the form of the distribution of fixed costs to be approximately quadratic ($cc = 4$ and $dd = 0$ in (16)), but allow B, the upper bound of the support, to be a free parameter. As indicated above, we also allow for a fraction, ω^f, of firms to adjust their prices flexibly. The presence of these firms is important for generating small price changes in the distribution.[11]

3.2 Effect of parameters on pricing statistics

Each parameter– or set of parameters– plays an important role in determining the steady-state distribution of price changes. Here we will provide a basic discussion of each parameter's role, proceeding as though all other parameters were fixed. In matching moments from the data we will implicitly be varying all parameters simultaneously, but this discussion should provide some intuition.

The upper bound on fixed costs has an obvious effect on the length of time firms hold their prices fixed: higher B means greater duration of prices, and therefore larger average price changes for the sticky-price firms. Increasing the fraction of flexible price firms ω^f leads to a larger number of small price changes, as well as raising the overall frequency of price adjustment, because those flexible price firms whose state doesn't change end up changing their price by a small amount due to the low steady state inflation in the model. Greater dispersion of idiosyncratic productivity shocks tends to raise the dispersion of relative prices and thus to spread out the distribution of price changes. This reasoning is exact for flexible price firms, but also occurs for a given persistence of prices in the sticky price sector, because optimal prices will vary by more across states.

The productivity transition matrix also affects price behavior. For a given median ad-

[11] Midrigan (2011) and Costain and Nakov (2011b) emphasize the importance of generating small price changes for the model's ability to also generate nonneutralities. Midrigan explores two methods distinctly different from ours for doing so, namely, complementarity in price adjustment and the probability of drawing a zero menu cost. He finds that his model's nonneutrality is not significantly affected by the way in which small price changes are introduced.

justment probability, greater persistence in the productivity process tends to increase the dispersion of prices. If productivity is entirely transitory (i.i.d.) then expected future productivity is invariant to current productivity. In this case, with prices adjusting infrequently an adjusting firm's optimal price will not vary much with its current state. In contrast, a highly persistent productivity process makes expected future productivity vary closely with current productivity, and for a given degree of price stickiness optimal prices will also be sensitive to current productivity. This reasoning does not apply exactly, because the degree of price stickiness varies with the productivity process. Greater persistence in the productivity process will tend to increase the degree of price stickiness; greater persistence means a lower probability of a productivity change and thus a lower probability of change in the firm's optimal price. Note that all of the reasoning in this paragraph abstracts from the fact that nonzero steady-state inflation interacts with the productivity process in affecting the incentives for price adjustment.

3.3 Benchmark parameter values

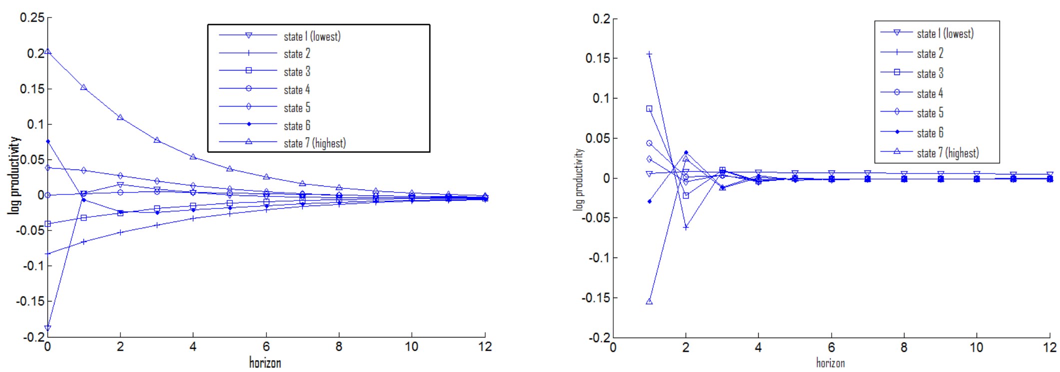
We have found two sets of parameters that enable the model's steady state to closely match the chosen moments. The next subsection will provide more details on the closeness of that match and will discuss in detail the properties of the steady-state equilibrium for both cases. Here we simply provide information on the parameter values.

One case will be called the low-flexibility benchmark because it has a relatively low proportion of flexible-price firms. The other will be called the high-flexibility benchmark because it has a higher fraction of flexible price firms. The low-flexibility benchmark has the smallest fraction of flexible-price firms that allowed us to match the desired moments. The high-flexibility benchmark has roughly four times as many flexible-price firms and matched the data moments equally well. In each case, the estimated idiosyncratic shock process required a large dispersion of productivity for the extreme states in order to match the fat tails of the empirical price change distribution. However, the two cases required very different upper bounds on the cost distribution as well as very different transition matrices.

For the low-flexibility benchmark, the fraction of firms with flexible prices is $\omega^f = 0.027$. The maximal fixed cost of changing prices is $B = 0.012$, which corresponds to 6.0 percent of labor hours. The productivity levels are given by $e_k \in \{0.83, 0.92, 0.96, 1.0, 1.04, 1.08, 1.22\}$. The dispersion in productivity appears at first glance to be rather large. Note, however, that while there is no direct mapping between the products in our model and plants in the literature on the relative productivity of plants, productivity displays considerably less dispersion than that found when looking at plants. For example, Foster, Haltiwanger, and

Krizan (2006) find interquartile differences in productivity of .57 for the U.S. retail trade sector[12].

For the high-flexibility benchmark, the fraction of firms with flexible prices is naturally higher: $\omega^f = 0.102$. Given that both cases match the distribution of price changes and the median price duration, sticky-price firms need to face higher costs of price adjustment: the upper bound on the cost distribution is very large, $B = 0.21$. This does not imply that high costs of price adjustment are actually incurred, as we will see below. While the productivity levels are quite similar to the low-flexibility case, the productivity transition matrix differs greatly. Figure 2 plots the paths of expected productivity conditional on each productivity state. In the low-flexibility benchmark, productivity is relatively persistent, whereas in the high-flexibility benchmark productivity is relatively transitory.



a. low-flexibility benchmark   b. high-flexibility benchmark

Figure 2. Expected productivity state by state.

3.4 Steady-state properties

In both of the benchmark cases, the hypothetical Calvo adjustment probability calculated from the model's steady-state equilibrium matches KK's estimate exactly. KK calculate price duration as the inverse of the Calvo parameter, so our benchmark cases imply identical price durations to KK's median sector.[13] Panel A of Figures 3 and 4 shows that the histograms of price adjustment match the data quite closely. Across these two important dimensions of observable data then, the high- and low-flexibility benchmarks are nearly identical. Of

[12] See also Syverson (2004) and Foster, Haltiwanger, and Syverson (2008).

[13]
$$\lambda_{KK} = -\ln\left(\left(1-\omega^f\right)\cdot\left(1-\sum_h \omega_{0,k}\right)\right)$$

course, we know that in other ways, the two parameterizations are not identical. In this subsection, we examine the implications of the two benchmark cases for other aspects of price adjustment behavior. In the next section we turn to the implications for monetary nonneutrality.

In addition to the Calvo parameter and implied median duration, Table 2 lists several other statistics reported by KK describing the behavior of prices. For each of those statistics, the low-flex and high-flex benchmarks are quite close to each other. Compared to the data, the model produces an average price duration that is somewhat low (7.7 months vs. 8.6 months in the data), and it produces a fraction of unchanged prices that is somewhat high (0.88 vs. 0.73 in the data). Overall however, for both benchmark cases the steady-state price adjustment behavior implied by the model is quite close to that measured in the data.

In terms of matching the data on price changes in the CPI, our model does so in much more detail than most of the existing literature. Golosov and Lucas (2007) match only three moments: the fraction of prices that don't change, the mean of positive price changes, and the standard deviation of positive price changes. In restricting their attention to these three moments, their estimated model drastically underestimates the fraction of small price changes and as a result generates no nonneutralities. Midrigan (2011), using both grocery store data and the CPI, concentrates on matching both the degree of small price changes and the kurtosis found in the distribution of price changes. Costain and Nakov (2011a,b), also using scanner data and using a flexible adjustment cost structure somewhat different from ours, are able to reasonably match the behavior of price changes with their model. Unlike in our model, the shape of the state-contingent hazards is insensitive to value function losses associated with not changing prices. Thus, their approach yields a more Calvoesque model with the distribution of price changes being approximately invariant to inflation. Nakamura and Steinsson (2008) match the mean frequency and absolute size of price changes across varying numbers of sectors and investigate the role of sectoral heterogeneity in the dynamic responses of their economy to monetary shocks. We view all of these papers as complementary to our work.

Table 2				
	data	**low-flex benchmark**	**high-flex benchmark**	**approximately fixed costs**
fraction flexible		0.027	0.102	0.05
hypothetical Calvo parameter	0.139	0.139	0.139	0.136
median duration	7.2	7.2	7.2	7.2
average duration	8.6	7.71	7.70	7.85
Pr(same)	0.73	0.87	0.87	0.87
Pr(small)*	0.43	0.40	0.38	0.47
mean abs(dlnp)	0.11	0.098	0.091	0.12

$*$ Pr(small) is the fraction of price changes less than 5%.

Having established that both the benchmark model and the model with more flexible-price firms are broadly consistent with the microeconomic data on price adjustment, we now describe additional features of the model's stationary distribution of prices and price adjustment. This discussion will refer to Figures 3 and 4. We have already discussed the close correspondence between the histogram of nonzero price changes in Panel A of Figure 3 and the corresponding histogram in Klenow and Kryvtsov (2008). Panel B of Figures 3 and 4 plots the distribution of prices by age. In the low-flex benchmark, 13.0% of prices are newly set in any period; 49% of prices are less than six months old; and 76% of prices are no more than twelve months old. The mean and median ages of prices are 8.0 and 6.0, respectively. In contrast, there is a much larger fraction of long-lived prices in the high-flex case; the fraction of newly set prices differs little, at 12.9%, but 26% of prices are less than six months old; and only 44% of prices are no more than twelve months old. The mean and median ages are 18.5 and 15.0.

Panel C of both figures displays the aggregate hazard function with respect to time, by which we mean the probability of price adjustment conditional on the time since the last adjustment. These figures share two notable features. The first is the relative flatness over the 40 months plotted in the figure. The second is the sharp decline in the hazard from one to two months. In section 5, we discuss in detail how the properties of the idiosyncratic productivity process determine the shape of the aggregate hazard function. At this juncture we will just make two relatively simple points. First, the sharp decline is due to all the flex-price firms changing prices and leaving only sticky-price firms over the remainder of the hazard distribution. This is the reason that there is a greater initial drop for the model with a relatively high fraction of flexible-price firms. Second, the relatively flat hazard results from an interaction between positive average inflation and the nature of the productivity process. To see this, refer to Panel D of Figure 3, which plots adjustment probabilities (hazards) as a function of relative price (price charged divided by price level) for each level of productivity. If a nominal price is not adjusted, the corresponding relative price decreases because of positive inflation. If productivity is unchanged, such a move to the left in Panel D corresponds to a higher adjustment probability – a rising hazard with respect to time. However, the productivity process often involves changes in productivity, and Panel D shows that increases in productivity can correspond to very large decreases in adjustment probability. This property is at the root of the flat aggregate hazard displayed in Panel C, which will be discussed in more detail in section 5. Of further interest is that Campbell and Eden (2010) display empirical hazards with a bowl shape similar to that displayed in panel D.

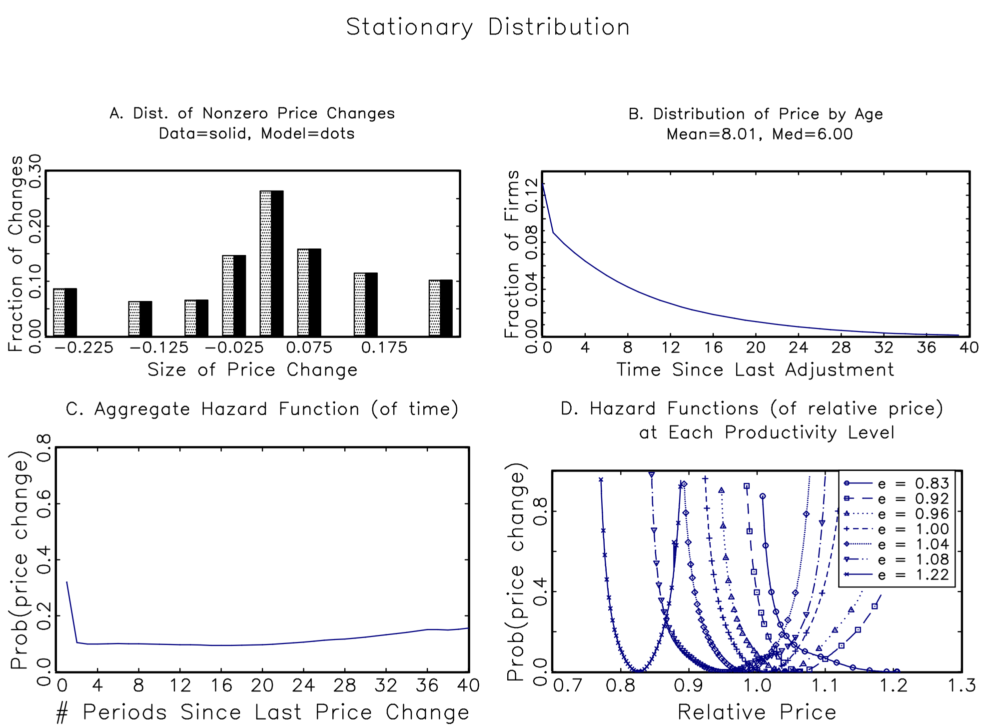


Figure 3. Low-flex benchmark steady state.

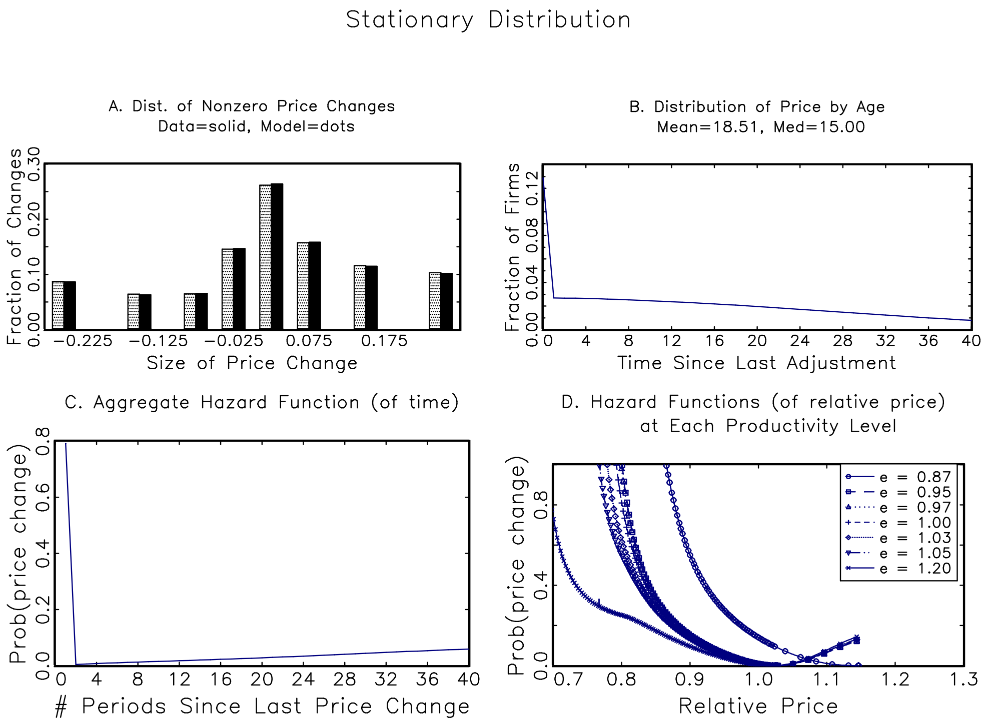
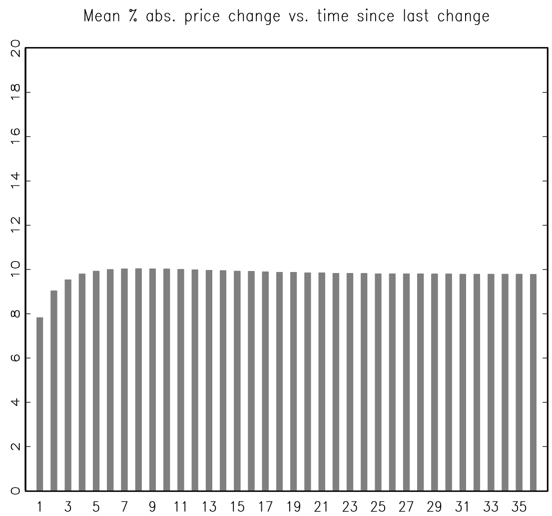


Figure 4. High-flex benchmark steady state.

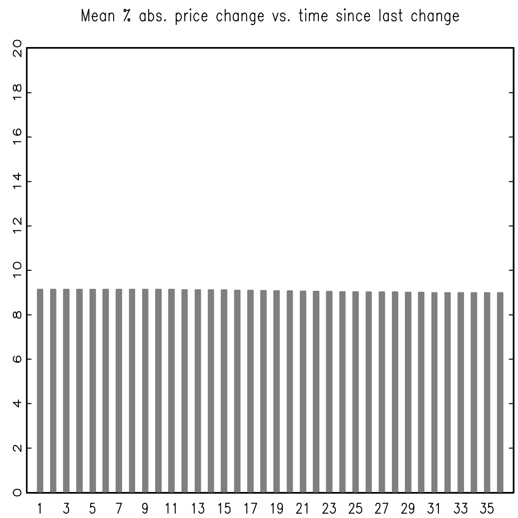
While we have been stressing the commonalities across the two benchmark cases, there are important differences across the two cases in the parameters governing both fixed costs and productivity transitions. The low-flexibility benchmark has low fixed costs for the sticky-price firms and few firms (2.7%) with flexible prices, whereas the high-flexibility benchmark has high fixed costs for the sticky-price firms and many firms (10.7%) with flexible prices. In addition, the productivity process is more persistent in the low-flexibility benchmark. While those differences offset each other in some dimensions – for example, in determining the distribution of nonzero price changes – they are not entirely offsetting. Comparing Figures 3 and 4, the most striking differences between the low-flex and the high-flex benchmarks are in the mean age of a price (reported in panel B) and the hazard functions at each productivity level (panel D). The fact that the high-flexibility benchmark has such high fixed costs makes the sticky-price firms keep their price fixed for quite a long time: the median age of a price is 15 months in the high-flexibility benchmark, compared to six months in the low-flexibility benchmark (Panel B). Although firms face the potential for much higher fixed costs in the high-flexibility benchmark, in the steady-state equilibrium the main effect of these high fixed costs is to sharply reduce the frequency of price adjustment– resources used for price adjustment increase only modestly: 0.30% of labor is used in final goods production in the low-flexibility benchmark and 0.36% in the high-flexibility benchmark.

In Panel D, we observe that in the high-flexibility benchmark the hazard functions are flat over a wider range. This follows from the high fixed costs of price adjustment: firms are willing to tolerate larger deviations from their optimal price, given the prospect of paying relatively large fixed costs. Another feature of Panel D in the two figures is that the vector of steady-state relative prices is more concentrated in the high-flexibility benchmark: $\{1.144, 1.027, 1.027,. 1.026, 1.025, 1.024, 1.018\}$ as opposed to $\{1.21, 1.06,.1.04, 1.01, .98, .95, .83\}$ in the low-flexibility benchmark. With a relatively transitory productivity process in the high-flexibility benchmark, optimal prices for the sticky-price firms do not vary much across productivity states. Therefore, high costs of adjustment lead firms to adjust infrequently. In contrast, sticky-price firms in the low-flexibility case see infrequent changes in their optimal prices, but the changes that do occur are large, and in combination with low costs of price adjustment this leads to an equilibrium in which the sticky-price firms adjust their prices frequently.

Another attribute of price adjustment displayed by our two benchmark steady states is the relationship between the size of price changes and the length of time since a price was last adjusted. An interesting feature of the data is that there is basically no relationship between these two elements. Figures 5.a and 5.b indicate that this is true of our benchmark economies as well. The lower-flexibility benchmark is graphed in the left panel and the higher flexibility benchmark is graphed in the right panel. The time since the last price change is on the horizontal axis and the average size of price changes made by firms that last changed their price is on the vertical axis. The figures show that the size of price changes is nearly invariant to the time elapsed since the last price change.



a. Low-flex case.

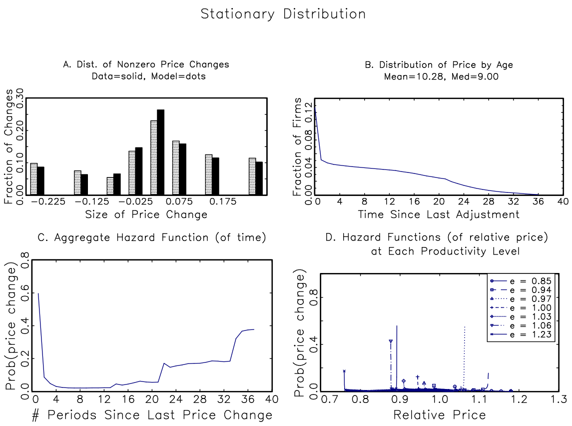


b. High-flex case.

Figure 5

3.5 A nearly standard (S,s) example

Here we present the steady state for a parameterization in which the distribution of price adjustment costs is nearly degenerate – common fixed costs across firms. This is important both as a general robustness check and because a number of papers in the literature, most notably Golosov and Lucas (2007) and Midrigan (2011) adopt such a specification. The basic features of that steady state are displayed in Figure 6.a and 6.b.



a. Steady state with common fixed costs.

b. Size of Δp, duration, common fixed costs.

Figure 6

As in the benchmark cases this specification fits the price change distribution as shown in Panel A of Figure 6.a. The maximum fixed cost is .007 and 0.26% of labor is used in price setting. Because there are no small costs of adjustment, there need to be more flexible-price firms in this economy than in the low-flex benchmark (0.05 as opposed to 0.027) in order to match the number of small price changes. The 0.05 fraction of flexible-price firms places this example between our two benchmarks, though closer to the low-flex case. The productivity levels are given by $e_k \in \{0.85, 0.94, 0.97, 1.0, 1.03, 1.06, 1.22\}$,which is similar to that of our low-flexibility benchmark. The vector of relative prices $\{1.18, 1.030, 1.022, 1.020, 1.014, 1.017, .835\}$, however, shares features of both benchmark steady states. The prices associated with the extreme productivity levels are approximately the inverse of productivity as in the low-flex benchmark, but the prices associated with the other productivity levels are closely bunched as in the high-flex benchmark. The hazard function is also approximately flat for the first two years and the mean age of a price change is 10.28 years, fairly close to that of the low-flex benchmark. As shown in Panel D of Figure 6.a the state-contingent hazards are no longer bowl shaped but have the U shape normally associated with a standard (S,s) model. Finally, there is little relationship between the time since the last price change and the size of that price change as depicted in Figure 6.b. Thus, we can match the micro-price data with a fixed cost example.

4 Dynamics

Next we examine and compare the local linear dynamics generated by our two benchmark specifications. First, we examine each model's response to a random walk shock. Because the shock has no dynamics, this case is interesting for studying the model's internal dynamics. Subsequently we analyze the case where the monetary shock is persistent. One aspect of state-dependent pricing models that has received much attention is the decomposition of price changes into components associated with fixed and varying adjustment probabilities. Because such a decomposition will appear in our discussion of the model's dynamics, that decomposition needs to be made clear before proceeding to the dynamics.

4.1 Decomposing price changes

In order to better understand the role that state dependence has on the model's dynamics, we decompose price level changes into four fundamental components, along the lines of Klenow and Kryvtsov (2008), Caballero and Engel (2007) and Costain and Nakov (2011a). Starting from the price index equation:

$$P_t^{1-\varepsilon} = \sum_{h=1}^{S}\sum_{k=1}^{S}\theta_{J,h,k,t}P_{0,k,t}^{1-\varepsilon} + \sum_{j=1}^{J-1}\sum_{h=1}^{S}\sum_{k=1}^{S}\alpha_{j,h,k,t}\theta_{j,h,k,t}P_{0,k,t}^{1-\varepsilon} + \sum_{j=1}^{J-1}\sum_{h=1}^{S}\sum_{k=1}^{S}(1-\alpha_{j,h,k,t})\theta_{j,h,k,t}P_{0,h,t-j}^{1-\varepsilon},$$

we decompose the *log-linearized* detrended price index ($\hat{P}_t$) into four components: (i) a time-dependent component $\mathfrak{T}_t$, which reflects changes in reset prices ($\hat{p}_{0,k,t} + \hat{P}_t$) holding constant the distribution of firms and their adjustment probabilities:[14]

$$\begin{aligned}\mathfrak{T}_t \;=\; & \sum_{j=1}^{J}\sum_{h=1}^{S}\sum_{k=1}^{S}\alpha_{jhk}\theta_{jhk}p_{0,k}^{1-\varepsilon}\cdot\left(\hat{p}_{0,k,t} + \hat{P}_t\right) + \\ & \sum_{j=1}^{J-1}\sum_{h=1}^{S}\sum_{k=1}^{S}(1-\alpha_{jhk})\theta_{jhk}p_{jhk}^{1-\varepsilon}\cdot\left(\hat{p}_{0,h,t-j} + \hat{P}_{t-j}\right);\end{aligned}$$

(ii) an extensive-margin component $\mathcal{E}_t$, which reflects changes in the average adjustment probability applied to a representative firm:

$$\mathcal{E}_t = (1-\varepsilon)^{-1}\,\bar{p}\cdot\bar{\alpha}_t,$$

[14] In these expressions, $\widehat{x}_t$ means percent deviation from steady state of x, and dx_t means level deviation from steady state of x.

where $\bar{p}$ can be thought of as representing the average desired price change (though note that $\bar{p}$ is negative if the price change is positive):

$$\bar{p} \equiv \sum_{j=1}^{J} \sum_{h=1}^{S} \sum_{k=1}^{S} \theta_{jhk} \left(p_{0,k}^{1-\varepsilon} - p_{jhk}^{1-\varepsilon} \right),$$

and $\bar{\alpha}_t$ is the mean deviation from steady state of adjustment probabilities:

$$\bar{\alpha}_t \equiv \sum_{j=1}^{J-1} \sum_{j=1}^{S} \sum_{k=1}^{S} \theta_{jhk} \cdot d\alpha_{j,h,k,t};$$

(iii) a selection-effect component $\mathcal{S}_t$, which reflects the fact that adjustment decisions may be related to the magnitude of desired price changes:

$$\mathcal{S}_t = \left(\frac{1}{1-\varepsilon} \right) \left(\sum_{j=1}^{J-1} \sum_{h=1}^{S} \sum_{k=1}^{S} \theta_{jhk} \left(p_{0,k}^{1-\varepsilon} - p_{jhk}^{1-\varepsilon} \right) \cdot d\alpha_{j,h,k,t} - \bar{p} \cdot \bar{\alpha}_t \right);$$

and finally, (iv) a shifting distribution effect $\mathcal{D}_t$, which quantifies the contribution to the price level of changes in the distribution of firms, holding fixed the prices they charge and their adjustment behavior:[15]

$$\begin{aligned} \mathcal{D}_t = & \left(\frac{1}{1-\varepsilon} \right) \sum_{j=1}^{J-1} \sum_{h=1}^{S} \sum_{k=1}^{S} \left(p_{0,k}^{1-\varepsilon} \alpha_{jhk} + p_{jhk}^{1-\varepsilon} \left(1 - \alpha_{jhk}\right) \right) \cdot d\theta_{j,h,k,t} + \\ & \left(\frac{1}{1-\varepsilon} \right) \sum_{h=1}^{S} \sum_{k=1}^{S} p_{0,k}^{1-\varepsilon} \cdot d\theta_{J,h,k,t}. \end{aligned} \tag{19}$$

In the long run, because adjustment probabilities and the distribution of firms return to steady state, the change in the price level must be entirely accounted for by the time-dependent component. Initially, however, there can be substantial divergence between those paths. Caballero and Engel (2007) refer to the extensive margin effect, which corresponds to our selection effect, as a critical "weathervane" for determining the importance of state dependence.

[15] Costain and Nakov (2011a) decompose the behavior of inflation in their model into intensive margin ($\mathcal{I}_t$), extensive margin and selection components. Our time-dependent, $\mathfrak{T}_t$, component is closely related to their intensive margin, but not identical. Our $\mathfrak{T}_t$ is simply the effect on the price level of changes in reset prices, holding fixed all distributions and adjustment probabilities. In contrast, their $\mathcal{I}_t$ "leaves out" some of that effect if changes in reset prices are correlated with (steady-state) adjustment probabilities, putting it instead into the selection effect.

4.2 Low-flexibility benchmark: Random walk money shock

The shock is a permanent 1% increase in M in period zero. Money is the solid line in Panel A of Figure 7. In response to the shock, the price level rises on impact by only about 0.10% (Panel A). The half life of output (and the price level) is about eight months (Panel B), which is slightly larger than the median price duration in the steady state. However, output only gradually returns to steady state and it takes roughly 30 months for the effects of the shock to fully dissipate. While the overall response is less than half as big as that found in the empirical literature, it is much larger than the effect found by Golosov and Lucas (2007) and similar to that in Costain and Nakov (2011b). The frequency of price adjustment responds very little to the monetary shock (Panel D), which is suggestive of the findings in Boivin, Giannoni and Mihov (2009) and Mackowiak, Moench and Wiederholt (2009) that idiosyncratic or sectoral shocks are more important than aggregate shocks in determining firms' price adjustment behavior.

Panel C presents the price level decomposition described above. The state-dependent part of price changing is indeed important: in the impact period, approximately half of the response of the price level is accounted for by the selection effect, meaning that firms whose adjustment decision changes in response to the shock have a different distribution of desired price changes than the unconditional distribution. The selection effect dies out smoothly over time, but from approximately periods five to fifteen, shifts in the distribution of firms contribute nonnegligibly to the behavior of the price level. The extensive margin contributes almost nothing to the behavior of the price level. As discussed in the derivation of our decomposition, in the long run all price changes are due to the time-dependent component.

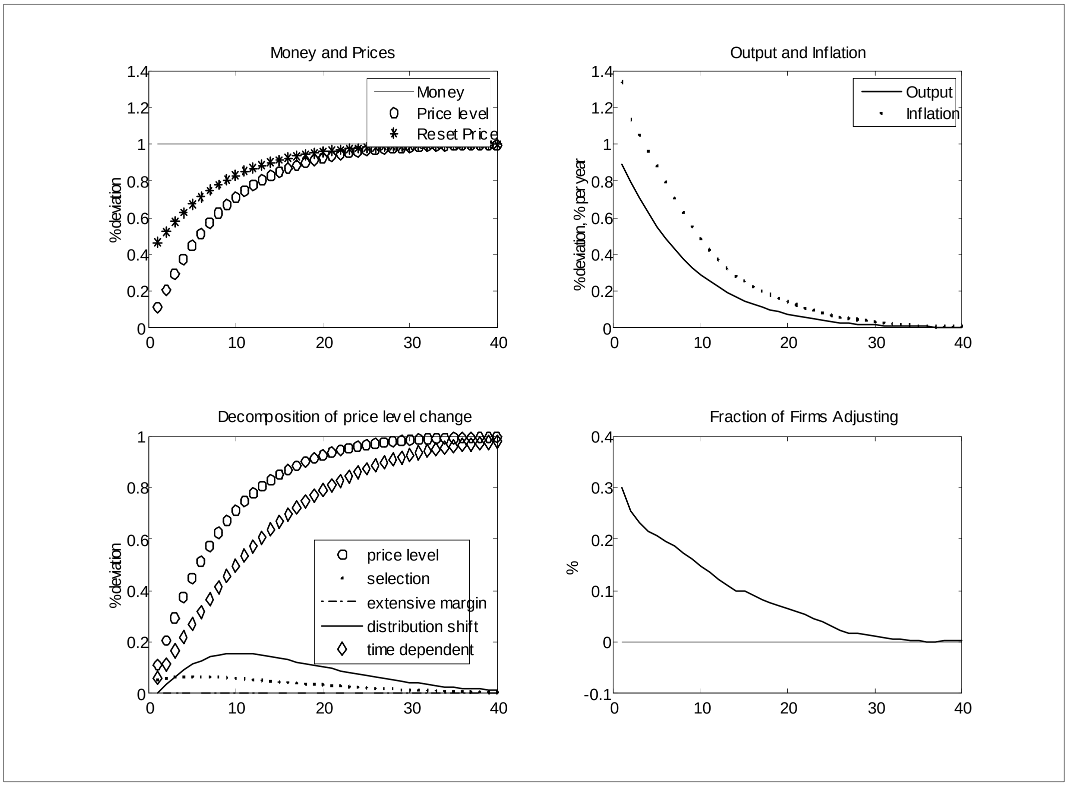


Figure 7. IRF to random walk money shock, low- flex case.

4.3 High-flexibility benchmark: Random walk money shock

Figure 8 displays the same set of impulse response functions for the high-flexibility benchmark. There is a larger impact effect on output, and over time, there is significantly more nonneutrality in this case. The half life of output has doubled to roughly 16 months. Inflation is hump shaped, reaching its peak deviation from steady state after almost one and a half years. There is also mild overshooting. The greater nonneutrality arises because although the overall degree of stickiness is the same in both model economies, the sticky price sector in this specification is a good deal stickier. The average and median duration of the sticky-price sector is around 38 months in the more flexible benchmark as compared with 9.5 and 7.0 months, respectively, for the less flexible benchmark model. The results are suggestive of the findings of Carvalho (2006) who found that the relative stickiness among sectors is important for generating nonneutrality. Another interesting feature that distinguishes the two models is the length of time over which the change in distribution significantly contributes to price level changes. After 40 months, the distribution in the more flexible case has still not returned to steady state. This is due to the extreme stickiness found in the sticky price sector of this specification.

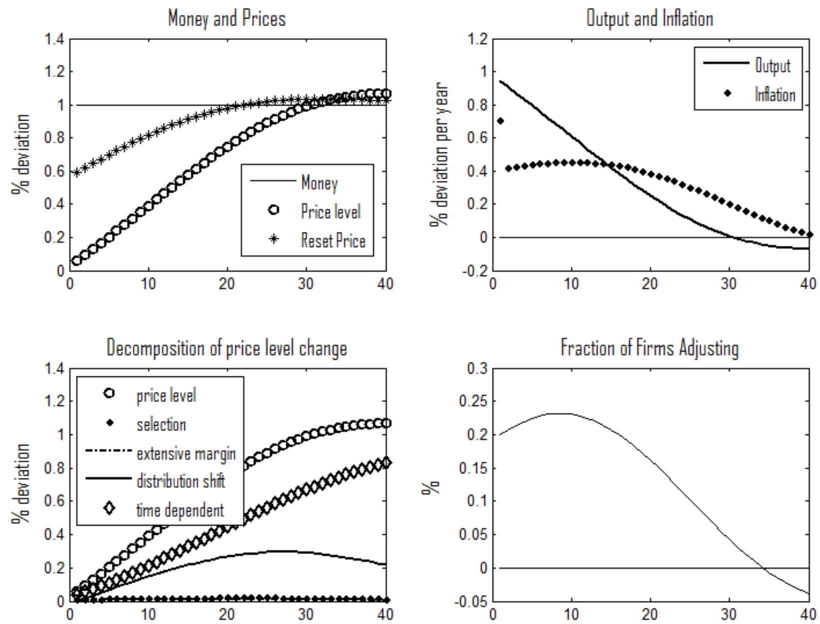


Figure 8. IRF high-flex random walk money.

4.4 Low-flexibility Benchmark: persistent money growth shock

In response to a persistent money growth shock (autocorrelation of .50 at a quarterly frequency), there are substantially greater effects on output (Figure 9). As in the random walk case, the price level does not respond much on impact and the response of output and inflation is now hump shaped with the peak response in inflation slightly leading that of output. It takes almost 3 years for both variables to return to steady state. The fraction of firms adjusting is somewhat larger than in the random walk case due to the persistent nature of the shock, which results in the price level rising by nearly 5 times as much as it does in the random walk case. This magnified effect on the price level induces more firms to adjust prices in response to the shock as their current price is now further away from where it will optimally be reset. As in the previous case, the price level decomposition indicates that state-dependent aspects are important for price setting.

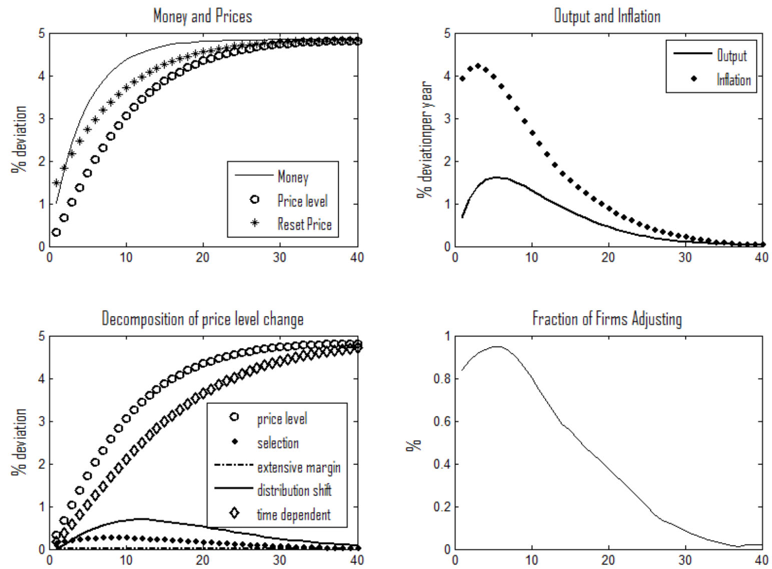


Figure 9. IRF low flex, persistent money shock.

4.5 High-flexibility benchmark: Persistent money growth shock

As in the random walk case, the more flexible specification displays an even greater degree of nonneutrality (Figure 10). The output response is almost twice as large and once again there is oscillatory behavior. Also, there is more movement in the price distribution in this specification. What is clear in comparing these two alternative specifications is that different ways of matching the economy's distribution of price changes have distinct implications for aggregate dynamics. There is no direct link between nonneutrality in the dynamics and many of the detailed statistics that inform us about stickiness in the steady state. This result argues for taking a sectoral approach when investigating the implications of steady-state price rigidities for economic behavior. It also implies that one may need to know a good deal more about the distribution of price changes at the micro level than Klenow and Kryvstov's histogram and the overall median duration of prices. Although those statistics are derived from a large amount of data, they are insufficient to pin down the parameters of our model. The extreme differences in the dynamics across the two benchmarks indicate the value of targeting an even richer set of facts when estimating the processes governing firm-level heterogeneity.

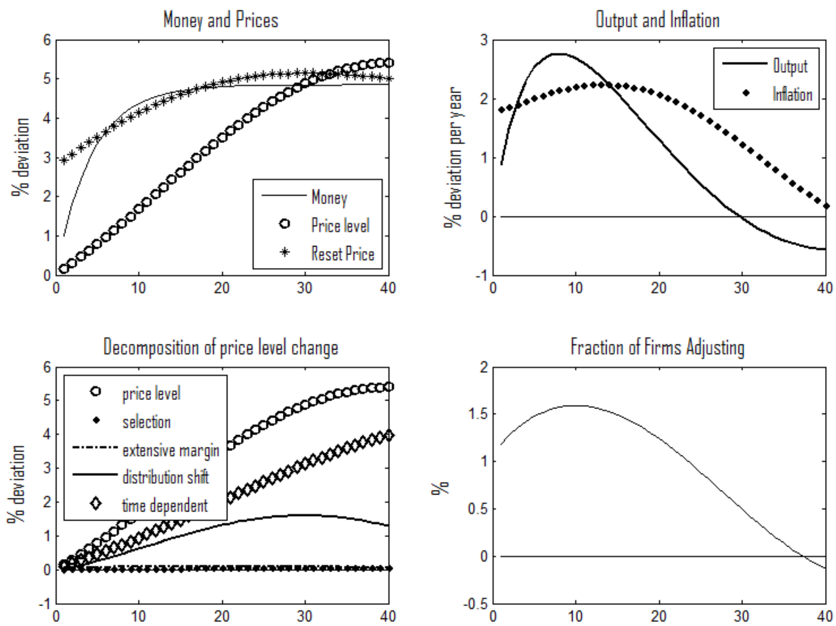


Figure 10. IRF high-flex, persistent money.

4.6 Sensitivity

In order to examine the sensitivity of our benchmark results, we vary the model along two independent dimensions. First, we change the specification of adjustment costs to be approximately zero variance, and then we examine how the dynamics are altered by changing the parameter that governs relative risk aversion.

4.6.1 Changing the specification on adjustment costs

There is a small effect on the less-flexible benchmark results if we calibrate adjustment costs so that they are nearly identical across firms and across time (Figure 11). Recall that this specification also matches the steady-state degree of stickiness in the KK data and the distribution of price changes. As mentioned, there are more flexible-price firms in this economy than in the low-flexibility benchmark. The dynamics in this economy are a bit more persistent and show some oscillatory behavior, which is similar to what occurred in the more-flexible benchmark model. This occurs because the sticky-price firms are somewhat stickier (they have an average duration of price equal to 12.3 months, whereas in the low-flex benchmark the average duration is 9.5 months). Apparently, greater stickiness leads to some oscillatory behavior. It is generally more costly to adjust prices in this specification, and as a result, firms wait longer before adjusting, implying that a relatively bigger mass of firms adjust. This leads to some overshooting in adjustment fractions and the distribution of firms displays oscillatory behavior as it converges back to steady state. Oscillations in inflation are a natural implication.

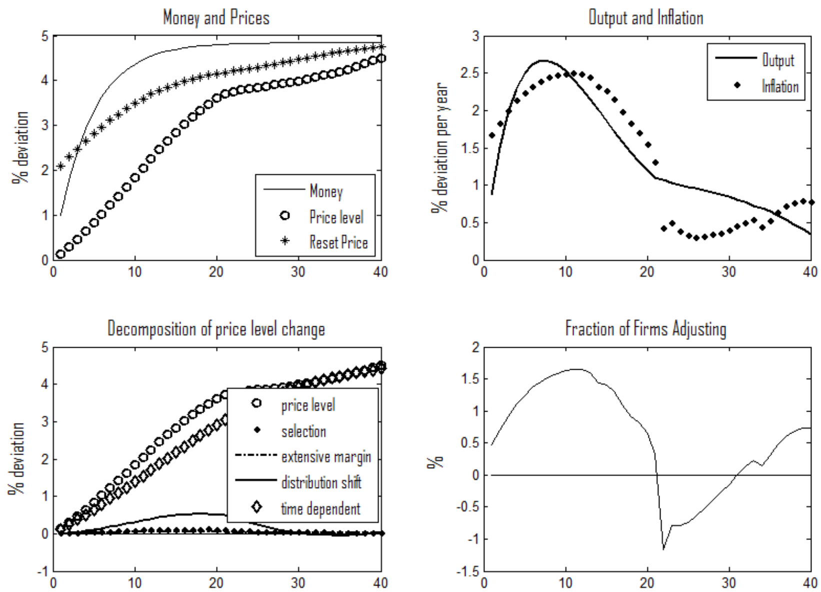
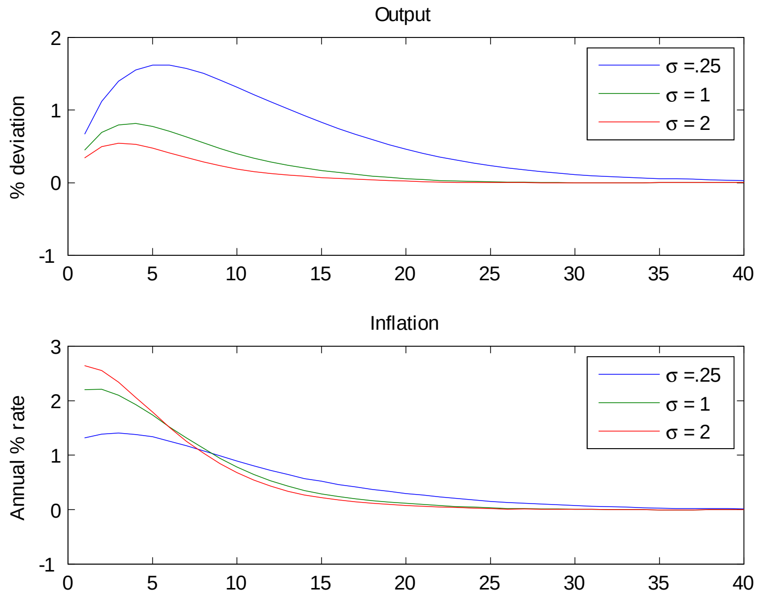


Figure 11. IRF common fixed cost persistent money.

4.6.2 Changing relative risk aversion

In figure 12, we examine the dynamics of the less flexible benchmark specification with coefficients of relative risk aversion of 1 and 2. This coefficient does not influence the steady state, so all other parameters are the same as the benchmark, which sets $\sigma = .25$. Recall that σ basically governs the elasticity of marginal cost with respect to output, and larger values of σ imply larger changes in marginal cost. Hence firms adjust prices more aggressively as σ increases and the response to a monetary shock becomes less persistent. Dotsey and King (2005, 2006) explored alternative mechanisms, such as nonconstant elasticity of demand, the use of intermediate inputs in production, and varying capacity utilization that produce low elasticities of marginal cost with respect to output. Elsewhere in the New Keynesian literature, researchers commonly adopt habit persistence in consumption and investment adjustment costs to augment endogenous persistence in the models. We view the choice of $\sigma = 0.25$ as a stand-in for these types of mechanisms.

sigma



$_1 5.pdf$

Figure 12. IRFs, different values of σ.

5 Dissecting the Benchmark Calibrations

The benchmark calibrations are quite successful in matching facts about individual price adjustment. However, the resulting model is large and complex, with several thousand dynamic equations and over a thousand state variables. The idiosyncratic productivity process is the chief reason for the model's size. Here we discuss the role these idiosyncratic shocks play in the model and the extent to which some basic properties of the data can be captured with a smaller model – that is, a specification that uses fewer states for the firm-level productivity process.

5.1 The role of (no) idiosyncratic shocks

Figure 13 displays the same set of steady-state graphs presented in Figure 3, for a parameterization without idiosyncratic productivity shocks. All other parameters are held fixed at their values in the benchmark calibration. It is obvious that the one-state model cannot replicate the basic features of the data. All of the price changes are quite small, there are no price decreases, and the aggregate hazard rate is upward-sloping (panel C). One perspective on the upward sloping aggregate hazard comes from panel D, which plots the price adjustment probability as a function of a firm's relative price. Because there are no idiosyncratic shocks, positive inflation means a firm's highest relative price always occurs in the period of adjustment. Also, note that without productivity changes at the level of the firm, one very important reason for price adjustment is missing. Thus, it is not surprising that prices become less flexible relative to the benchmark case: the fraction of firms adjusting falls from 0.12 to less than 0.08, the average duration between price changes increases from 7.6 months to 13 months, and the median age of a price rises from six months to seven months.

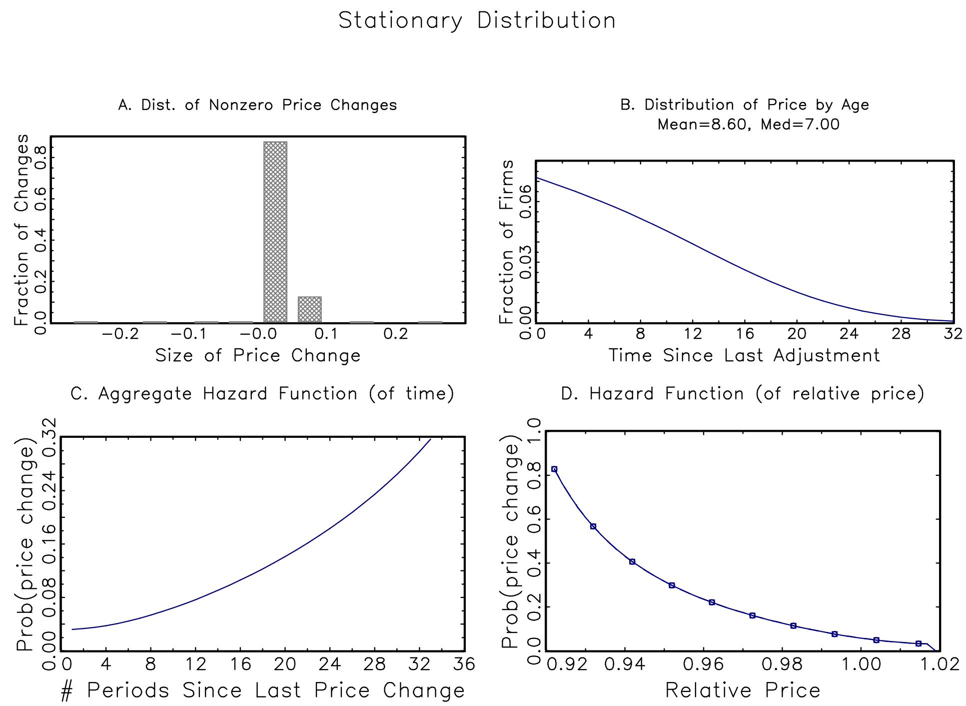


Figure 13. Klenow-Kryvtsov parameterization without heterogeneity.

Features such as the increasing hazard and lack of price decreases rendered the initial DKW model inconsistent with microeconomic data on price changes. Next, we show that taking a small step away from the one-state model allows us to take a large step toward matching those data.

5.2 How many states are needed?

Now we consider a model with a three-state productivity process. We fix the fraction of flexible-price firms to be 0.05, in between our two benchmarks. With so few states, we cannot exactly match the price change distribution or the duration of price setting, but our claim is that three states can go quite a long way toward replicating actual price adjustment behavior. We choose the parameters of the model in exactly the same manner that we chose the benchmark parameters. The three estimated technology levels are 0.86, 1.0 and 1.16, and the estimated transition probabilities are asymmetric and given by the following matrix:

$$G = \begin{bmatrix} 0.64 & 0.09 & 0.27 \\ 0.00 & 0.53 & 0.47 \\ 0.30 & 0.50 & 0.20 \end{bmatrix}.$$

The transition matrix indicates that the low technology state is more persistent than the other states. The optimal reset prices are $\{1.04, 1.0, 0.94\}$.

The distribution of price adjustment costs has the same functional form as in the benchmark cases. However, the upper bound on adjustment costs is .029, which is somewhat higher than in the low-flexibility benchmark, but a good deal lower than in the high-flexibility case.

Although the distribution of price changes is noticeably different from the data, it does contain significant numbers of both small and large price changes, which is a distinguishing feature of the data. Likewise, the three-state model produces both price increases and price decreases. The aggregate hazard shares the same general shape as the seven-state model (see Figure 14). As measured by the distribution of price ages, the degree of price stickiness is in between the two benchmark models.

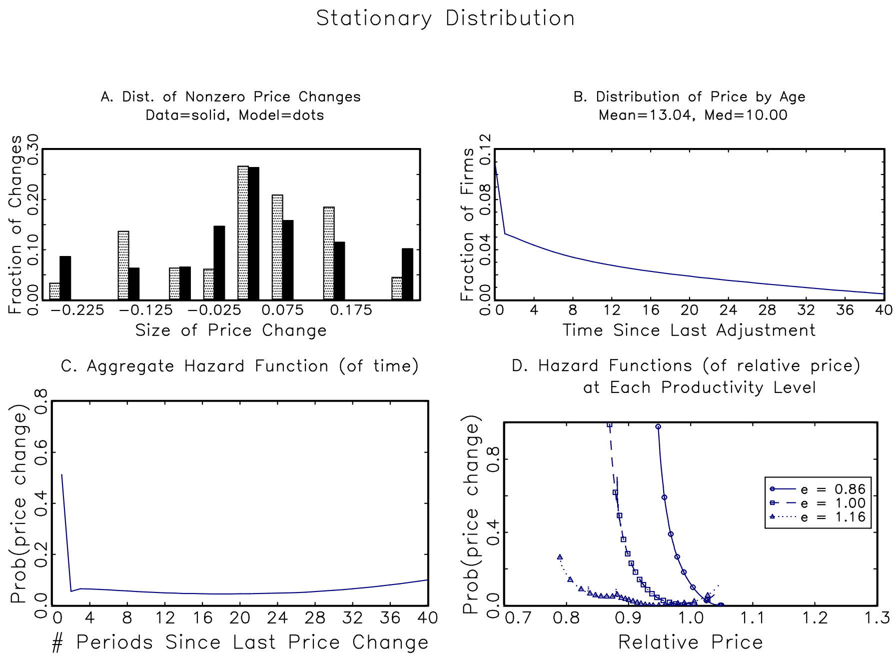


Figure 14. 3-state steady state

For the same three-state parameterization, Figure 15 displays responses to the persistent money supply shock. Just as in the benchmark cases, the increase in the money supply is initially soaked up almost entirely by output. The persistence of nonneutrality is somewhat in between the two benchmark cases and more strongly resembles the more flexible benchmark case. The price level decomposition is quite similar with there being a significant contribution on impact from the selection effect and a medium-run contribution from shifts in the distribution. There is also evidence of some oscillatory behavior in the fraction of firms adjusting, which translates into mild oscillations in output.

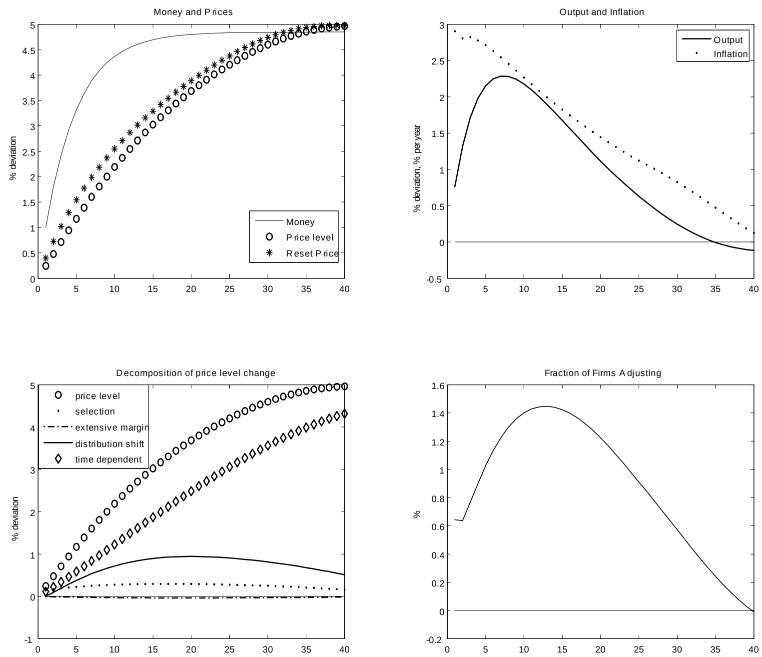


Figure 15. IRF, 3-state steady state

6 Aggregation and Upward- and Downward- Sloping Hazard Functions

A notable feature of both our benchmark case and the three-state parameterization is the wide range over which the aggregate hazard is relatively flat or even downward-sloping, even though conditional on remaining in the same state all hazards are upward sloping. This feature of our model is consistent with hazard functions estimated on micro-data underlying the Japanese CPI (see Ikeda and Nishioka (2007)). Along any pricing spell, the hazards for most goods and services in Japan are increasing and none are decreasing. Appropriately aggregating these various hazards produces a downward sloping hazard for goods and a relatively flat hazard for services. In order for our model to replicate the broad features of hazards in the U.S. and Japanese data, there must be many firms with a low hazard, and the fraction of firms with a low hazard must be increasing as we consider older and older prices (i.e as we move out along the hazard function). The reasoning is analogous to, but more involved than that which explains a downward-sloping aggregate hazard in a model with two types of firms with different, constant adjustment probabilities as in Calvo. Aggregating across these two types of firms gives a downward-sloping hazard even though the hazard rates are constant for each type. As the age of a price increases, the fraction of firms that have not adjusted is increasingly dominated by those with the lower hazard – implying a downward-sloping aggregate hazard rate. In the limit, the aggregate hazard approaches that of the low hazard type firms. In this example, the proportion of firms with a low hazard is increasing in age over a significant range and aggregation implies a downward-sloping hazard.

One can see this by examining the two left panels of Figure 16, where we graph the state-specific hazards *as functions of relative prices* and the weights placed on firms with those particular hazards in a model with only two microstates. First, examine the upper left figure, which gives the hazard rates for the various types of firms. The age of a price is represented by the number of symbols one must count to the right in order to reach the firm's reset price (at which point the hazard is zero). For example, the downward-sloping line with circles depicts the *upward*-sloping hazard (upward in age, that is) of low-productivity firms that remain low productivity firms, and the fourth circle to the left of the reset price is the relative price of a firm who reset its price while in the low productivity state four periods ago and is still in the low productivity state at that time. The upward sloping squares indicate the downward sloping (age) hazards of firms who were low productivity but switch to being high productivity. This hazard is downward sloping because over time inflation erodes the relative price and makes it closer to the optimal price of resetting high productivity firms. If there are enough of these types of firms, then the aggregate hazard will be downward sloping.

Thus, one needs to know the evolution of the fraction of firms over time. This evolution is given in the bottom left panel, where again the time elapsed since prices were reset is depicted by the distance from the reset price. As shown in the bottom left panel, the fraction of firms that transit from the low- to high-productivity state $\theta_{j,1,2}$ is increasing over time, eventually becoming more than half the firms. Because these are the firms that experience a falling hazard rate and their share is rising, the aggregate hazard is downward sloping.

When the idiosyncratic productivity shocks are i.i.d., the downward-sloping portion of the hazard function for the low to high transiting firms is smaller. The distribution of optimal prices is much narrower, so the fraction of firms that are on the downward-sloping portion never gets large enough to offset the upward-sloping hazards faced by most firms. Thus, a combination of relatively large persistence and dispersion in productivity shocks is required in order to generate a downward-sloping aggregate hazard.

The effect that price dispersion and persistence have on the shape of the aggregate hazard is shown in three dimensions in Figure 17. Look first at the right-most slice of the right-hand panel; it represents the aggregate hazard function for a model with zero dispersion of the idiosyncratic shocks – that is, a model without idiosyncratic shocks. The hazard function is everywhere upward sloping, as are the hazards displayed in DKW for a model without idiosyncratic shocks. With zero dispersion all firms have the same optimal price. Constant nonzero inflation means that *age* moves all firms uniformly away from their optimal price, raising the adjustment probability for all firms. As the dispersion of idiosyncratic shocks increases (moving left along the axis labeled "dispersion"), the successive slices become flatter, and for dispersion around 0.7, the hazards begin to have downward-sloping portions. It is in this region that there is a large fraction of firms transiting from low to high productivity and then letting their relative price depreciate as described above. For very high degrees of dispersion, the hazards again become upward sloping. In this region reset prices vary greatly across productivity levels, so changes in productivity inevitably involve changes in price

The left-hand plot in Figure 17 illustrates how the aggregate hazard function varies with the idiosyncratic shock's persistence. Without persistence, the right-most slice of the figure, the hazard is everywhere upward sloping. In this case, reset prices are relatively insensitive to productivity because firms' future productivity levels are independent of productivity at the time they set their prices. As persistence increases, the reset prices spread apart, creating the condition described above under which hazards may slope downward. For very high degrees of persistence, transitions across states are rare, so that although they can lead to decreasing hazards for certain productivity trajectories, there are too few firms with such trajectories for the aggregate hazard to slope downward.

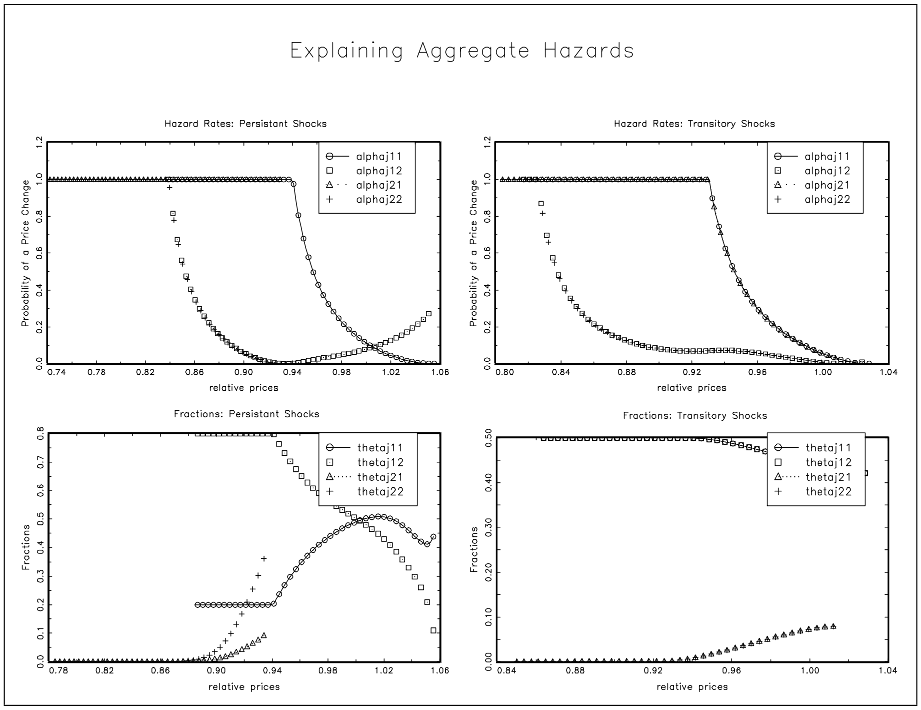


Figure 16. Explaining the Aggregate Hazard

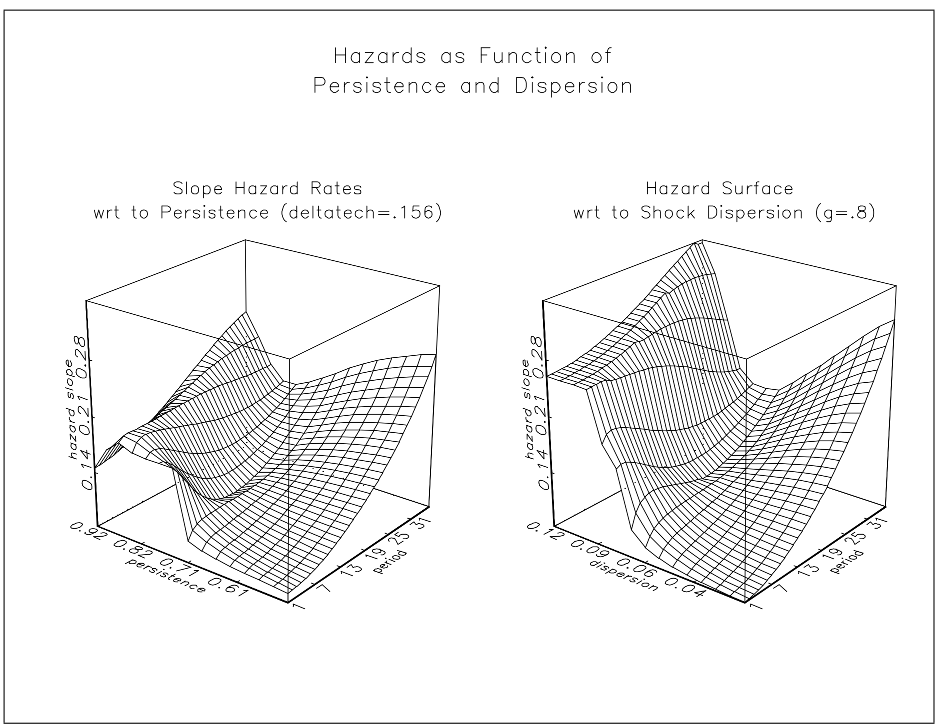


Figure 17. Hazard surfaces.

7 Summary and Conclusions

In this paper, we construct a state-dependent pricing model with idiosyncratic productivity variation. With only small menu costs, two parameterizations of the model are capable of matching many of the facts that have recently been uncovered concerning firms' pricing behavior. In particular, not only do they match the distribution of price changes, they also match the moderate degree of stickiness, measured by average price duration. Further, the dispersion in productivity across firms needed by the models to account for the size and dispersion of price changes in the data does not seem overly large when looked at through the lens of the plant productivity literature. Also, the aggregate hazard functions generated by the benchmark models are rather flat, which is consistent with the data. This result occurs despite the fact that conditional on productivity, all hazards are upward sloping, a feature that appears to be consistent with micro-hazard data from the Japanese CPI and U.S. scanner data. We are able to trace out the way that aggregation works in our model and show that flat hazards are a feature of the dispersion and persistence of idiosyncratic productivity shocks. Additionally, all of our seven-state models are consistent with the feature that the size of prices changes and the time since the last price change are uncorrelated.

Despite the relatively high degree of steady-state price flexibility in our model, there is moderate nonneutrality of monetary disturbances in the less flexible benchmark and significant nonneutrality in the more flexible benchmark. There is also substantial persistence in response to shocks, despite the lack of any internal propagation mechanisms, such as kinked demand, intermediate inputs, variable utilization, or habit persistence. From our pricing decompositions, it is clear that state-dependence plays an important role in the way monetary shocks propagate in the model. Our analysis also reinforces Cabellero and Engel's (2007) suggestion that steady-state stickiness and dynamic persistence are not as closely related in state-dependent models as is time-dependent models. Finally,we show that a three-state model may be sufficient for analyzing many of the interesting questions explored by our benchmark model. This is an important finding because the benchmark model is extremely large and computationally intensive to solve.

8 References

References

[1] Boivin, Jean, Marc P. Gainnoni, and Ilian Mihov (2009), "Sticky Prices and Monetary Policy: Evidence from Disaggregated U.S. Data," *American Economic Review,* 99:1,

March 2009, pp 350-384.

[2] Burstein, Ariel and Christian Hellwig (2007),"Prices and Market Shares in a Menu Cost Model," manuscript September 2007.

[3] Caballero Ricardo J. and Eduardo M. R. A. Engel (2007), "Price stickiness in Ss models: New interpretations of old results," *Journal of Monetary Economics 54,* pp 100-121.

[4] Calvo Guillermo (1983). "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics 12*, pp. 383-398.

[5] Campbell, Jeffrey R. and Benjamin Eden (2010), "Rigid Prices: Evidence from U.S. Scanner Data," Federal Reserve Bank of Chicago Working Paper 2005-08.

[6] Caplin, Andrew S. and Daniel F. Spulber (1987), "Menu costs and the neutrality of money," *Quarterly Journal of Economics* 102 (4), pp. 703-726.

[7] Carvalho, Carlos (2006) "Heterogeneity in Price Stickiness and the Real Effects of Monetary Shocks," *The B.E. Journal of Macroeconomics*: Vol. 2 : Iss. 1 (Frontiers).

[8] Costain, James and Anton Nakov (2011a), "Distributional Dynamics under Smoothly State-Dependent Pricing," *Journal of Monetary Economics,* 58, November, 646-665.

[9] Costain, James and Anton Nakov (2011b), "Price Adjustment in a General Model of State-Dependent Pricing," *Journal of Money, Credit and Banking*, 43(2-3), March-April, 385-406.

[10] Dixit, A.K., and Joseph E. Stiglitz (1976). "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67, 297-308.

[11] Dotsey, M., King, R.G., Wolman, A. L., (1999). State-dependent pricing and the general equilibrium dynamics of money and output. *Quarterly Journal of Economics* 114 (3), pp. 655–90.

[12] Dotsey, Michael, and Robert G. King (2005), "Implications of State-Dependent Pricing for Dynamic Macroeconomic Models," *Journal of Monetary Economics 52 (1)* , January, pp 213-242. .

[13] Dotsey, Michael, and Robert G. King, (2006) "Production, Pricing, and Persistence," *Journal of European Economic Association*, September, 893-928.

[14] Foster, Lucia, John Haltiwanger, and C. J. Krizan (2006), "Market Selection, Reallocation, and Restructuring in the U.S. Retail Trade Sector in the 1990's," *Review of Economics and Statistics,* November 88(4), pp 748-758.

[15] Foster, Lucia, John Haltiwanger, and Chad Syverson (2008), "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability," *American Economic Review,* 98:1, pp 394-425.

[16] Gertler, Mark and John Leahy, (2008), A Phillips Curve with an Ss Foundation," *Journal of Political Economy, 116 (3), 533-572.*

[17] Goffe, William L., Gary D. Ferrier, and John Rogers (1994) "Global Optimization of Statistical Functions with Simulated Annealing," *Journal of Econometrics*, 60, pp 65-99.

[18] Goffe, William L.,Gary D. Ferrier, and John Rogers (1992). "Simulated Annealing: An Initial Application in Econometrics," *Computer Science in Economics & Management*, Society for Computational Economics, vol. 5(2), pages 133-46, May.

[19] Golosov, M. and R.E. Lucas, (2007). "Menu costs and Phillips curves ". *Journal of Political Economy, 115(2)*, pp 171-199.

[20] Guimaraes, Bernardo and Kevin Sheedy (2011), "Sales and Monetary Policy," *American Economic Review, 101(2),* April 844-76.

[21] Ikeda, Daisuke, and Shinichi Nishioka (2007), "Price Setting Behavior and Hazard Functions: Evidence from Japanese CPI Micro Data," manuscript.

[22] John, A. Andrew and Alexander L. Wolman (2008), "Steady-State Equilibrium with State-Dependent Pricing", *Journal of Monetary Economics,* 55(1), March, 383-405.

[23] Kemeny, J. G. and J. L.Snell (1976),. *Finite Markov Chains.* New York: Springer-Verlag.

[24] Klenow, Peter J. and Oleksiy Kryvtsov (2008), "State-Dependent or Time-Dependent Pricing: Does It Matter for Recent U.S. Inflation?" *Quarterly Journal of Economics*, August , pp 863-904.

[25] Klenow, Peter J. and Benjamin A. Malin (2011), "Microeconomic Evidence on Price-Setting", In *Handbook of Monetary Economics*, Volume 3A, Elsevier B.V.

[26] Levy, Daniel, Mark Bergen, Shantanu Dutta, and Robert Venable, (1997). "The Magnitude of Menu Costs: Direct Evidence from Large US Supermarket Chains," *Quarterly Journal of Economics* 112, 791–825.

[27] Mackowiak, Bartosz, Emanuel Moench, and Mirko Wiederholt (2009), "Sectoral Price Data and Models of Price Setting," manuscript.

[28] Midrigan, Virgiliu (2011). "Menu Costs, Multi-Product Firms, and Aggregate Fluctuations," *Econometrica,* 79, July, 1139-1180.

[29] Nakamura, Emi and Jon Steinsson, (2008), "Five Facts About Prices: A Reevaluation of Menu Cost Models," *Quarterly Journal of Economics,* November, 1415-1464.

[30] Syverson, Chad (2004), "Product Substitutability and Productivity Dispersion," *Review of Economics and Statistics,* May 86(2), pp 534-550.

[31] Vavra, Joseph (2013), "Inflation Dynamics and Time-varying Volatility: New Evidence and an S,S Interpretation." manuscript.