WORKING PAPERS
RESEARCH DEPARTMENT

WORKING PAPER NO. 13-14
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First draft: May 2009
This draft: April 2013
Supersedes Working Paper No. 12-8

RESEARCH DEPARTMENT, FEDERAL RESERVE BANK OF PHILADELPHIA
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Abstract

We study trade between an informed seller and an uninformed buyer who have existing inventories of assets similar to those being traded. We show that these inventories may lead to prices that increase even absent changes in fundamentals (a "run-up"), but may also make trade impossible (a "freeze") and hamper information dissemination. Competition may amplify the run-up by inducing buyers to enter loss-making trades at high prices to prevent a competitor from purchasing at a lower price and releasing bad news about inventory values. Inventories also prevent seller competition from delivering the Bertrand outcome, in which prices match sellers' valuations. We discuss both empirical implications and implications for regulatory intervention in illiquid markets.

*We thank Jeremy Berkowitz, Mitchell Berlin, Alexander Bleck, Michal Kowalik, Andrew Postlewaite, Abraham Ravid, James Thompson, and Alexei Tchistyi for their helpful comments. We also thank seminar participants at the Federal Reserve Bank of Philadelphia and conference participants at the AFA meeting, the Finance Theory Group meeting, the FIRS meeting, the SFA meeting, the Eastern Finance Association meeting, the European Economic Association meeting, the European Finance Association meeting, the Tel Aviv Finance Conference, the Oxford Financial Intermediation Theory conference, and the Federal Reserve System Committee on Financial Structure and Regulation. An earlier draft circulated under the title "Why Do Markets Freeze?" Any remaining errors are our own. The views expressed here are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or of the Federal Reserve System. This paper is available free of charge at www.philadelphiafed.org/research-and-data/publications/working-papers/.

1 Introduction

Consider the sale of mortgages by a loan originator to a buyer. As widely noted, the originator has a natural information advantage and knows more about the quality of the underlying assets than other market participants. One consequence, which has been much discussed, is that he will attempt to sell only the worst mortgages.[1] However, a second important feature of this transaction has received much less attention. Both the buyer and the seller may hold significant inventories of mortgages similar to those being sold, and they may care about the market valuation of these inventories, which affects how much leverage they can take. Consequently, they may care about the dissemination of any information that affects market valuations of their inventories. In this paper, we analyze how inventories affect trade—in particular, prices and information dissemination. Our setting applies to the sale of mortgage-related products, but more broadly, to situations in which the seller has more information about the value of the asset being traded.

Our main result is that the effect of inventories on trade depends on the buyer's and seller's initial leverage, or more precisely, on how tight their capital constraints are. When capital constraints are moderately tight, concerns about the value of existing inventories lead to prices that increase even without any changes to fundamentals (a market "run-up"); but when capital constraints are very tight (i.e., initial leverage is very high), trade becomes impossible (a market "freeze"), and information dissemination ceases.

Our results cast light on several features of the market for structured financial products that have attracted much attention. First, it is widely believed that these products were overpriced in the period leading up to the financial crisis.[2] Second, this market collapsed

[1] See, for example, Ashcraft and Schuermann (2008), and Downing, Jaffee and Wallace (2009).

[2] Coval et al. (2009) provide formal evidence. Less direct evidence is provided by Faltin-Traeger et al. (2010), who show that the prices of asset-backed securities failed to reflect relevant and observable information about sponsor credit quality, and by Ashcraft et al. (2010) and Rajan et al. (2012), who show that subordination levels failed to reflect the riskiness of underlying cash flows. Finally, the claim that structured financial products were overpriced is closely related to the widely held view that these same products received excessively favorable credit ratings: see, for example, Griffin and Tang (2012) for evidence.

during the financial crisis. Third, the collapse of this market attracted concern not just because of the associated fall in potentially socially beneficial trade, but also because it severely hampered the dissemination of information (see Scott and Taylor, 2009).

To illustrate the intuition for our results, consider the case of one buyer and one seller, where the buyer makes a take-it-or-leave it offer to the informed seller. The motive for trade is that the buyer values the asset by $\Delta > 0$ more than the seller. This is our basic framework. In this case, whenever the seller agrees to sell at a price p, the market infers that the value of the asset is less than p. This leads to a reduction in the value of inventories and a potential violation of capital constraints. When capital constraints have sufficient slack, the buyer can prevent violation of the constraints by increasing the price, while still maintaining positive profits; hence, we obtain a run-up. However, when capital constraints are very tight, the buyer can no longer increase the price without losing money. At this point, the buyer prefers not to make any offer, and so trade completely breaks down. In particular, even sellers with the lowest possible valuations do not sell their assets.

(Of course, asymmetric information by itself can lead to a reduction in trade, even absent inventories and capital constraints. However, when there are strictly positive gains for even the lowest valuation seller, as there are in our setting, some trade still survives absent inventories and capital constraints. In particular, sellers with sufficiently low valuations still trade. In contrast, inventories and capital constraints can lead to a complete market breakdown.)

Our main results continue to hold under different trading environments/protocols, including the case in which competing uninformed buyers make offers to a single informed seller, and the case in which competing sellers, who are homogeneously informed, make offers to a single uniformed buyer. These cases also provide new insights.

First, when multiple competing buyers are present, a buyer may be forced to raise his bid not only because he is leveraged but also because a competing buyer is leveraged; moreover,

a buyer may be forced to acquire assets at a loss-making price, just to make sure that a competing buyer does not acquire them at a lower price. The key insight here is that a purchase by one buyer may lead to the release of information that causes a violation of the capital constraint of a competing buyer, and this may force the competing buyer to increase the price.

Second, the analysis of competing sellers shows that while competition may lead sellers with high valuations to sell the asset for its true value (i.e., the Bertrand outcome), tight capital constraints force sellers with low valuations to pool, leading to prices that are above fundamental values.

Our baseline results, which are obtained in a static setting with a single round of trade, are suggestive of a dynamic process in which buyers increase leverage and prices until the market eventually breaks down. In a dynamic extension of our basic framework, we model this process explicitly and show that high leverage may lead to a run-up in prices that is followed by a market freeze. This result is interesting because the run-up in prices and subsequent freeze occur even though (by assumption) the underlying asset fundamentals remain unchanged. In this sense, the run-up shares features of a "bubble." In our model, this result reflects the fact that because of a lemons problem when the buyer adds assets to his balance sheet, he reduces the market value of his existing assets, and so his capital constraint tightens. This forces him to bid a higher price in the next trade, or else not bid at all.

As noted above, one application of our analysis is to the market for structured financial products. Our analysis provides an explanation for prices being disconnected from fundamentals, for market breakdown, and for the lack of information revelation in the market breakdown. Related, our analysis predicts that tight capital constraints are associated both with a market breakdown, and high asset prices immediately before the breakdown. Also related, our analysis predicts short-run momentum followed by long-run reversal.

A separate application is to the effects of broker-dealer inventories on prices. By interpreting buyers in our model as market-makers, our model predicts that higher inventories may lead to higher prices, consistent with the empirical findings in Manaster and Mann (1996). In contrast, previous models of the effect of market-maker inventories on prices, such as Amihud and Mendelson (1980) and Ho and Stoll (1981, 1983), assume symmetric information and predict that as inventories increase prices fall, either because the dealer is risk averse and concerned about future price movements or because he is not allowed to carry too much inventory.

We also use our model to discuss implications for regulatory intervention in illiquid markets. On the buyer's side, our analysis highlights the potential role of a large investor unencumbered by existing inventories (the government, for example); one implication is that by purchasing assets, the government may impose a cost on potential buyers who choose not to trade. On the seller's side, our analysis suggests potential limitations to the standard prescription that sellers should retain a stake in the assets they sell; under some circumstances, this prescription may lead to a market breakdown.

1.1 Related literature

Our paper relates to the literature on trade under asymmetric information, in which the seller is better informed and the gains from trade are common knowledge (e.g., Akerlof, 1970; Samuelson, 1984). As noted earlier, as long as there are gains from trade between the buyer and the seller with the lowest possible valuation, this literature predicts a partial breakdown but not a complete freeze as in our setting.[3] Moreover, we show that adding inventories and capital constraints to this standard lemons problem not only leads to complete breakdowns but also leads to prices that increase in leverage, even absent changes in

[3] Akerlof (1970) provides an example in which the market breaks down completely. However, in his example, there are no gains from trade between the buyer and the seller with the lowest possible valuation.

the asset fundamentals.[4]

In our setting, trade is always efficient, and so increasing the price increases welfare, as the probability that the seller will accept the buyer's offer increases. In this sense, our paper differs from papers in which price manipulation creates distortions that are suboptimal from a social point of view.[5]

When there are multiple buyers, the combination of capital constraints and inventories generates a situation in which the fact that the seller trades with one buyer has externalities for other buyers. In contrast to existing auction-theoretic papers dealing with externalities (e.g., Jehiel, Moldovanu, and Stacchetti, 1996), the externality depends on the price paid, rather than on simply whether another buyer obtains the asset.

Two recent papers obtain periods of no trade in a dynamic lemons problem.[6] To do so, they add the assumption that some noisy information about the asset quality is revealed (exogenously). In Kremer and Skrzypacz (2007), information is revealed at some future point in time T and there exists $t < T$, such that trade ceases on the time interval $[t, T]$. In Daley and Green (2012), information is revealed gradually. Instead, we obtain a no trade result by adding inventories and capital constraints to a standard lemons problem.

Our paper also relates to the literature that explores the link between leverage and trade. For example, Shleifer and Vishny (1992) show that high leverage may force firms to sell assets at fire-sale prices, while Diamond and Rajan (2011) show that the prospect of fire sales may lead to a market freeze. Other papers explore feedback effects between asset prices and leverage: Low prices reduce borrowing capacity, and hence asset holdings and prices also; see, e.g., Kiyotaki and Moore (1997). In contrast, we model a situation in which firms can meet their financial needs by staying with the status quo. Therefore, there is no need for fire

[4] An alternative explanation for a complete market breakdown in situations in which there are gains from trade involves Knightian uncertainty; see, e.g., Easley and O'Hara (2010).

[5] Examples include Allen and Gale (1992); Brunnermeier and Pedersen (2005); Goldstein and Guembel (2008).

[6] Also related, in a recent paper, Glode, Green, and Lowery (2012) endogenize the extent of adverse selection in a *static* standard lemons problem, showing that firms may overinvest in financial expertise. The outcome of this is that if uncertainty increases, the probability of efficient trade is reduced.

sales or "cash in the market" pricing, as in Allen and Gale (1994). Moreover, in our setting trade affects posterior beliefs about the value of existing inventories, and prices may increase even after valuations fall.

In a contemporaneous paper, Milbradt (2012) shows that a trader who is subject to a capital constraint that is based on mark-to-market accounting may suspend trade so that losses are not reflected on its balance sheet. While the general idea relates to us, the two models are very different. The main difference is that in Milbradt (2012), transaction prices are exogenous and always reflect the true asset value, whereas in our setting transaction prices are endogenous and may depart from fundamentals. This delivers new predictions for the relation of prices to fundamentals, and also for expected holding returns. In addition, in our setting, a market freeze completely halts information dissemination, while in Milbradt (2012), trade suspension has no effect on information dissemination among market participants, who by assumption are symmetrically informed; while for non-market participants, trade suspension reduces the perceived value of the asset.

As noted earlier, our paper also relates to the market microstructure literature that links market-maker inventories to prices. Finally, our paper relates to the literature on equity issuance, in which the issuing firm cares about the market valuation of its remaining equity.[7] However, we do not focus on signaling. Instead, we show how leverage affects the bidding strategies of uninformed buyers.

1.2 Paper outline

Section 2 describes the model. Section 3 analyzes the simplest case, in which there is one buyer with inventories. Section 4 analyzes a two-period dynamic extension. Section 5 analyzes the effects of competition between multiple buyers, while Section 6 analyzes competing sellers. Section 7 discusses several other extensions. Section 8 summarizes empirical implications. Section 9 discusses policy implications. Section 10 concludes. The appendices contain

[7] See, for example, Allen and Faulhaber (1989), Grinblatt and Hwang (1989), and Welch (1989).

proofs and omitted details.

2 The model

The model is based on a simple variant of Akerlof's (1970) "lemons problem." The new feature is that the buyer has an inventory of the traded asset and is subject to a capital constraint.

In the basic model, an uninformed buyer makes a take-it-or-leave-it offer to buy one unit of an asset from an informed seller. The buyer and seller are risk neutral. The value of the asset is v to the seller and $v + \Delta$ to the buyer, where $\Delta > 0$ denotes the gains from trade. Since $\Delta > 0$, trade is always efficient. Both Δ and the distribution of v are common knowledge, and for simplicity, we assume that v is drawn from a uniform distribution on $[0, 1]$. However, the exact value of v is the seller's private information. Consequently, trade affects posterior beliefs about v and, hence, the market value of each unit of asset. The buyer's offer and seller's response are publicly observable.[8]

In one interpretation, the seller is a loan originator. The gains from trade may reflect the fact that the buyer has a lower cost than the seller of retaining risky assets on his balance sheet; for example, the buyer may face lower borrowing costs or less stringent regulation. Alternatively, the buyer might be a broker-dealer who helps with the matching process between the seller and other investors, who have higher valuations for the asset.

The buyer has an inventory of M units of the asset, which he acquired earlier.[9] The buyer also has cash and a short-term debt liability. The liability net of cash holdings is L, and so the buyer can roll over his liabilities only if the value of his noncash assets exceeds L. Assume, for simplicity, that the buyer holds only the traded asset and that the purchase of additional units is financed out of existing cash holdings and/or new short-term borrowing.

[8] We obtain similar results under alternative information assumptions, e.g., when the market observes the terms of accepted offers but does not observe the terms of rejected offers. See Section 7.3.

[9] If instead the buyer's inventory consists of assets whose values are correlated with the value of the asset being traded, we obtain qualitively similar results.

Specifically, suppose the buyer purchases $q \in \{0, 1\}$ additional units at a price per unit p, and let h denote the "market value" of the asset, defined as the expected value of $v + \Delta$, conditional on the trading outcome, using Bayes' rule. Then the buyer can roll over his debt if

$$h(M + q) \geq L + pq, \tag{1}$$

where $M + q$ is the buyer's total inventory of assets net of trade, and $L + pq$ is the buyer's total liabilities, net of trade. We refer to equation (1) as the buyer's *capital constraint*.

(Implicit here is that the threat of losing the asset induces the buyer to pay his obligations, and so the capital constraint is based on the value of the asset to the buyer. The nature of the results remains under a more general formulation in which h is the expected value of $\alpha(v + \gamma\Delta)$, where $\alpha \leq 1$ reflects constraints on the buyer's ability to pledge future cash flows, and $\gamma \in [0, 1]$ allows the capital constraint to be based on the value of the asset to creditors, who may have lower valuations than the buyer.[10])

If the buyer violates his capital constraint, he defaults and incurs a cost, which represents lost growth opportunities due to bankruptcy or closure by a regulator. Alternatively, one can think of a situation in which the buyer must raise L dollars to invest in a profitable opportunity with cash flows that cannot be promised to others (e.g., because of nonobservability). In this case, the cost of violating the capital constraint arises from the fact that the buyer cannot take advantage of the investment opportunity.

We focus on the case in which the capital constraint is satisfied before trading begins (i.e., when $q = 0$ and $h = \frac{1}{2} + \Delta$, so assets are evaluated at the prior). This assumption allows us to focus on the question of how the buyer changes his behavior to avoid violating the capital constraint, rather than on the much-studied fire sales that follow when the constraints are violated. We also assume that the cost of violating the constraint is sufficiently high so that the buyer's first priority is to satisfy his constraint.[11] Consequently, the buyer maximizes

[10] An earlier draft, available upon request, contains a full analysis of this general formulation.

[11] For example, for the results in Section 3, it is enough to assume that the cost of violating the constraint

expected profits subject to the constraint that his offer satisfies the capital constraint.[12]

We focus throughout on the case in which the gains from trade are not too high, $\Delta < \frac{1}{2}$. Our results on price run-ups extend to the case $\Delta \geq \frac{1}{2}$, but the market freeze result does not. In particular, when gains from trade are high, $\Delta \geq \frac{1}{2}$, the outcome in which the seller sells at price 1 satisfies the seller's participation constraint for each seller type, and it also gives the buyer positive profits, since the buyer acquires an asset with an expected value of $\frac{1}{2} + \Delta$ for a price of 1. (Note also that if all seller types sell, trade does not release any information, and so the market value of the asset remains at the prior.)

Finally, we assume that the quantity of the asset available for trade is smaller than the buyer's existing asset holdings. Specifically:

Assumption 1 $M > 1$

Assumption 1 implies the buyer's capital constraint is tightened when the seller accepts his offer (see discussion after (5) below). It also implies that to satisfy the capital constraint the offer price must be sufficiently high (see discussion after (4) below).

The assumption that the buyer can purchase only one unit is made for simplicity. The nature of the results remains if the seller has more than one unit for sale, and the buyer can choose a quantity in addition to a price.[13]

Although we focus on the case in which the buyer is subject to a capital constraint, we obtain similar results for the parallel case in which the seller is subject to a capital constraint; see Section 7.1.

As noted, we start by analyzing the case in which the uninformed buyer is a monopolist who makes a take-it-or-leave-it offer. In subsequent sections, we study different trading protocols, including the case in which competing uninformed buyers make offers, and the

is at least $1 + \Delta$.

[12] From the law of iterated expectations, the value of inventoried assets equals its prior and hence does not enter the objective function.

[13] An earlier draft, available upon request, contains a full analysis of the case in which the buyer can offer to buy any quantity $q \in [0, 1]$.

case in which competing informed sellers make offers. We also study a dynamic extension of our basic model.

3 A monopolist buyer

Much of the economics behind our main results is present in the simplest case of a monopolist buyer. We start with the benchmark case $M = 0$, in which the buyer has no inventories. Then we analyze the main case with inventories, $M > 0$.

3.1 Benchmark: Buyer does not have inventories

Absent inventories, the buyer offers to purchase the asset at a price p, which maximizes his expected profits. The seller accepts the offer if and only if $v \leq p$, which happens with probability p, since v is uniform on $[0, 1]$.[14] Conditional on the seller accepting the offer, the expected value of the asset to the buyer is $\frac{1}{2}p + \Delta$. Since the buyer pays p, his expected profit per unit bought is $\Delta - \frac{1}{2}p$. Taking into account the probability of trade, the buyer's expected profit is $\pi(p) \equiv p(\Delta - \frac{1}{2}p)$. The buyer's profit-maximizing bid is $p = \Delta$.

Proposition 1 *In the benchmark case of no inventories, the buyer offers to buy the asset at a price Δ. The seller accepts this offer if and only if $v \leq \Delta$.*

3.2 Buyer cares about the value of his inventory

When a seller accepts an offer, the market infers that v is below p. Hence, the market value of existing inventories falls from the prior of $(\frac{1}{2} + \Delta)M$ to $(\frac{1}{2}p + \Delta)M$. To ensure that the capital constraint continues to hold after the offer is accepted, the offer p must satisfy

$$(\frac{1}{2}p + \Delta)(M + 1) \geq L + p, \tag{2}$$

which follows from equation (1) with $h = \frac{1}{2}p + \Delta$ and $q = 1$.

[14] If $p > 1$, the acceptance probability is simply 1. However, since $\Delta \leq \frac{1}{2}$, offers $p > 1$ generate negative profits, since the buyer pays $p > 1$ for an asset with an expected value of $\Delta + \frac{1}{2} < 1$.

Define $\delta \equiv \frac{L}{(\frac{1}{2}+\Delta)M}$, a measure of the buyer's initial "leverage," or, more precisely, a measure of how tight the capital constraint is before trade begins; the assumption that the capital constraint is initially satisfied is equivalent to $\delta \leq 1$. Also let

$$P(\delta) \equiv \frac{\delta M(1+2\Delta) - 2\Delta(M+1)}{M-1}. \tag{3}$$

Then equation (2) simplifies to

$$p \geq P(\delta). \tag{4}$$

In other words, the capital constraint puts a lower bound on the price. Intuitively, a higher price helps ensure that the capital constraint is satisfied because it ensures that the value of inventories does not fall too much after a seller accepts an offer. While a higher price also reduces profits, which tightens the capital constraint, Assumption 1 guarantees that the inventory value effect is the dominant one.

If instead the seller rejects the offer, the market infers that v is above p, and the capital constraint is relaxed. Hence, the buyer faces a constrained optimization problem, namely, choose a price p to maximize expected profits $\pi(p)$ subject to the constraint that either he makes no offer, $p = 0$, or else the offer satisfies equation (4). Observe that the lower bound on the price $P(\delta)$ is increasing in the buyer's initial leverage δ.

Figure 1 illustrates the optimal solution to the buyer's constrained optimization problem. The parabola represents the profits function $\pi(p)$, and the vertical lines represent the lower bound $P(\delta)$ for three different values of δ. If leverage is low, i.e., $P(\delta) \leq \Delta$, the capital constraint is not binding, and the buyer can continue to make his benchmark offer Δ. If leverage is intermediate, $P(\delta) \in (\Delta, 2\Delta)$, the capital constraint binds, and it is optimal to offer $p = P(\delta)$, since a higher price reduces profits. Finally, if leverage is high, $P(\delta) > 2\Delta$, the buyer would lose money if he offers $p \geq P(\delta)$. In this case, the buyer prefers not to trade. In other words, the market "freezes."

The condition $P(\delta) \leq 2\Delta$ reduces to $\delta \leq \frac{4\Delta}{1+2\Delta}$. Hence, we obtain the following:

Proposition 2 *In the basic model of a monopolist buyer who cares about the value of his inventories, trade can happen if and only if leverage is not too high, i.e.,* $\delta \leq \frac{4\Delta}{1+2\Delta}$. *In this case, the buyer offers to purchase the asset at a price* $\max\{\Delta, P(\delta)\}$, *which increases in the buyer's initial leverage.*

Proposition 2 says that when the capital constraint is moderately tight (i.e., leverage is intermediate), the price, and hence the probability of trade, is increasing in leverage and is above the benchmark bid Δ. However, when the capital constraint is tight (i.e., leverage is high), trade completely breaks down.

An immediate corollary to Proposition 2 concerns the effect of high leverage and the corresponding market breakdown on the revelation of the seller's information about asset values:

Corollary 1 *If initial leverage is high,* $\delta > \frac{4\Delta}{1+2\Delta}$, *market participants learn nothing about the value* v *of the asset.*

3.3 Discussion

Although simple, the monopolist buyer case already delivers many of the main implications. First, the market price is not determined solely by the buyer's beliefs about the asset's value ("fundamentals") but is also affected by the tightness of the buyer's capital constraint. In particular, the price increases in the tightness of this constraint. Second, and related, the buyer's expected return from holding the asset is decreasing in the tightness of his capital constraint. Third, very tight capital constraints lead to market breakdown and prevent information dissemination.

4 Dynamic run-ups and breakdowns

The static model is suggestive of a dynamic process in which the buyer increases leverage and prices until the market breaks down eventually. To model this explicitly, we extend our

single-period model to a two-period model in which the monopolist buyer trades sequentially with two potential sellers. Each seller sells a different asset, and the values of the two assets are assumed to be independent.[15] Hence, one cannot infer anything about the value of one asset by observing trade in the other asset. This allows us to focus only on the effect of leverage, which changes endogenously: The outcome of trade with the first seller affects the buyer's leverage before he trades with the second seller. One of our results is that a sufficiently tight capital constraint leads both to a market freeze and to high prices before the freeze, even though, by assumption, there is no change in fundamentals.

Specifically, seller i ($i = 1, 2$) can sell one unit of asset i and can trade only in period i. The value (per unit) of asset i is v_i to the seller and $v_i + \Delta$ to the buyer, where v_1, v_2 are independent random variables drawn from a uniform distribution on [0,1]. Before trading begins, the buyer has inventories of M units of asset 1 and M units of asset 2. In the first period, the buyer makes an offer p_1 to the first seller, who can either accept or reject the offer, and in the second period, the buyer makes an offer p_2 to the second seller, who can also either accept or reject it. We normalize the discount rate to unity.

Since the parameters in each period are the same, it is suboptimal to delay offers; if it is suboptimal to make an offer in the first period, it is also suboptimal to make an offer in the second period. Thus, a bidding strategy can be summarized by (p_1, p_a, p_r), where p_1 denotes the offer to the first seller, and p_a, p_r denote the offer to the second seller, given that the first seller accepted or rejected the offer, respectively. Since the first seller accepts the buyer's offer with probability p_1, the buyer's expected profits are

$$\pi(p_1) + p_1\pi(p_a) + (1 - p_1)\pi(p_r). \tag{5}$$

As in Section 3.2, when an offer is accepted, the market value of inventories falls, and the capital constraint is tightened; in particular, by Assumption 1 the fall in the value of

[15] One can think of the two assets as idiosyncratic components of the same class of assets, e.g., mortgage-backed securities.

inventories cannot be offset by potential profits from buying the asset.[16] Hence, if an offer is accepted in the first period, the buyer faces a tighter capital constraint (i.e., a higher leverage) before he makes an offer in the second period. In contrast, rejected offers relax the constraint because the market value of inventories rises. The potentially binding constraints are as follows:

The capital constraint must be satisfied if the first seller accepts the buyer's offer. For the case in which the buyer does not make a second offer (i.e., $p_a = 0$), we obtain

$$(\frac{1}{2}p_1 + \Delta)(M+1) + (\frac{1}{2} + \Delta)M \geq L + p_1, \tag{6}$$

and for the case in which the buyer makes a second offer (i.e., $p_a > 0$) and this second offer is accepted, we obtain

$$(\frac{1}{2}p_1 + \Delta)(M+1) + (\frac{1}{2}p_a + \Delta)(M+1) \geq L + p_1 + p_a. \tag{7}$$

In both cases, the market value of the first asset is $\frac{1}{2}p_1 + \Delta$. In the first case, the market value of the second asset remains at its prior $(\frac{1}{2} + \Delta)$, while in the second case the market value of the second asset drops to $\frac{1}{2}p_a + \Delta$. Note that since constraint (7) implies constraint (6), requiring that the capital constraint be satisfied at the end of the second period is equivalent to requiring that it be satisfied after every period.

The capital constraint must also be satisfied if the first seller rejects the offer. In this case, the capital constraint is loosened after the first period, and the potentially binding constraint is when the buyer makes a second offer and this offer is accepted. Hence, the capital constraint is

$$(\frac{1}{2}(1 + p_1) + \Delta)(M) + (\frac{1}{2}p_r + \Delta)(M+1) \geq L + p_r. \tag{8}$$

The problem reduces to finding a bidding strategy that maximizes the buyer's expected

[16] Specifically, when a seller accepts an offer to sell at a price p, the value of the buyer's existing assets falls by $(\frac{1}{2} + \Delta)M - (\frac{1}{2}p + \Delta)M = \frac{1}{2}(1-p)M > \frac{1}{2}(1-p)$. But the buyer's potential profits are at most $\Delta - \frac{1}{2}p \leq \frac{1}{2}(1-p)$, since $\Delta < \frac{1}{2}$.

profits such that if $p_1 > 0$ and $p_a = 0$, constraints (6) and (8) are satisfied, and if $p_a > 0$, constraints (7) and (8) are satisfied.

It turns out that whenever the buyer's first-period offer p_1 is rejected, his capital constraint becomes sufficiently slack that he can make his unconstrained optimal bid of Δ in the second period. The intuition is that the first-period offer p_1 can satisfy constraint (6) only if p_1 is high, or the capital constraint is very slack to begin with. In either case, the buyer's capital constraint has a lot of slack if the first offer is rejected; therefore, the buyer can set his second offer equal to the benchmark offer Δ.

Lemma 1 *If $p_1 > 0$, then $p_r = \Delta$.*

It remains to characterize p_1 and p_a. Our main result in this section is:

Proposition 3 *Holding M and Δ fixed,*

(A) When the buyer's initial leverage is low, the buyer makes the benchmark bid Δ in both periods.

(B) When the buyer's initial leverage is intermediate, the buyer offers to pay strictly more than the benchmark in the first period, and if the first offer is accepted, the buyer offers to pay even more in the second period (i.e., $p_a > p_1 > \Delta$).

(C) When the buyer's initial leverage is high, the buyer withdraws from the market in the second period (i.e., $p_a = 0$) if his first offer is accepted. The buyer's initial bid (p_1) is increasing in leverage, and when initial leverage is sufficiently high, the buyer initially bids more than the benchmark, and he may even bid more than his valuation; that is, the market freeze is preceded by high prices.

(D) When the buyer's initial leverage is very high, trade completely breaks downs.

Proposition 3 captures a few aspects of a dynamic behavior. If initial leverage is relatively moderate (Part (B)), the buyer has enough slack in his capital constraint to make two rounds of offers. But unless leverage is very low, the buyer still needs to consider his capital

constraint, and this leads him to bid more than the benchmark price in the first period. If his first bid is accepted, his capital constraint is tightened, forcing him to bid even more in the second period. Hence, prices increase even though, by assumption, there is no change in fundamentals.

If instead initial leverage is sufficiently high (Part (C)), there are two consequences. First, the buyer's capital constraint has too little slack for two profitable bids to be accepted. So in particular, if the buyer's first offer is accepted, then there is no trade in the second period, i.e., the market freezes. Second, the tight capital constraint pushes the initial bid high, for the same reasons as in the benchmark case. Consequently, high leverage leads to a market freeze that is preceded by high prices (a "run-up").[17]

Finally, note that while the buyer never bids above his valuation in the second period, it might be optimal for him to do so in the first period, even though he loses money if his offer is accepted. The advantage of engaging in such a loss making offer is that if the offer is rejected, the market valuation of the buyer's inventory rises, thereby relaxing the buyer's capital constraint. This allows the buyer to make a profitable offer in the second period. Note, however, that the buyer will find it optimal to bid above his valuation only if his initial leverage is sufficiently high, so that without making such an offer, he will not be able to trade at all.

[17] For some parameter values, Part (C) includes an interval of leverage levels in which the price before the freeze is below the benchmark level of Δ. The reason is that when the buyer's capital constraint is tight enough that he wants to have only one offer accepted, the sellers in periods 1 and 2 are effectively in competition. This competition effect puts downward pressure on the price. So if the buyer's capital constraint is both sufficiently tight to be in this case, but sufficiently slack that a bid below Δ is feasible, then the buyer's initial bid is below Δ. As noted, this interval does not always exist: in particular, it does not exist when the gains-from-trade parameter Δ is sufficiently high (but still below $\frac{1}{2}$, as assumed throughout). Moreover, this interval would not exist in perturbations of the environment that weaken the competition effect. Two examples are introducing more buyers to restore "competitive balance" between the two sides of the market, and changing the dynamic model to one in which there are two different buyers, with each buyer only active in one period, again restoring competitive balance (note that if both buyers have inventories of the same assets, trade in the first period tightens the capital constraint of the second buyer).

4.1 Discussion

The results above extend the implications of the static model. When capital constraints are moderately tight, prices rise over time, even without any change in fundamentals. Related, prices exhibit short-run momentum, but long-run reversal (assuming the true value of the asset eventually becomes public information). When capital constraints are sufficiently tight, there is a market breakdown that is preceded by a run-up in prices. In this case, assets bought shortly before a market breakdown have low expected returns.

5 Competing buyers

Until now, we have focused on the case of a single buyer. As we have shown, concerns about preserving the market value of existing asset inventories affect prices and information dissemination. When the buyer is very leveraged and his capital constraint has little slack, such concerns lead to a trade breakdown and prevent the dissemination of information about asset quality. However, if the buyer is only moderately leveraged, these same concerns drive up prices even though there is no change in fundamentals.

A natural question is how these results are affected by the presence of multiple competing buyers. One might conjecture that when multiple buyers are present, it is hard for any individual buyer to prevent the dissemination of information about asset values. In this section, we show that this conjecture is only partially correct. When all competing buyers are very leveraged, concerns about the market value of inventories again lead to a trade breakdown and prevent the dissemination of information about asset quality. However, under some circumstances in which one buyer is more leveraged than another, competition does indeed force trade and price dissemination to occur, even though the most leveraged buyer expects to lose money if his offer is accepted. In this sense, competition actually strengthens our previous finding that inventories may drive up prices: now, inventories of one buyer drive up the price offered by a second buyer and may even force a buyer to make

a loss-making offer.

Put slightly differently, the combination of capital constraints and inventories generates a situation where one buyer's bid has externalities for other buyers. Moreover, and in contrast to existing auction-theoretic papers dealing with externalities,[18] the externality depends on the price paid, rather than on simply whether another buyer obtains the asset.

To simplify the exposition, we focus on the case of two buyers. Buyer i has an inventory of M_i units of the asset and a debt liability L_i. The gain from trade with buyer i is Δ_i, and buyer i's initial leverage is $\delta_i = \frac{L_i}{(\frac{1}{2}+\Delta_i)M_i}$. Everything is common knowledge, except for the true value of the asset (v), which is private information to the seller. As before, $\Delta_i \in (0, \frac{1}{2})$, $M_i > 1$, the capital constraint for each buyer is initially satisfied (so $\delta_i \leq 1$), and the cost for violating it is large.

Both buyers make offers simultaneously, and we denote buyer i's offer by p_i. The seller has one unit for sale and can accept at most one offer.[19] Hence, if $v > \max\{p_1, p_2\}$, the seller rejects both offers and trade does not take place. Otherwise, the seller accepts the offer with the highest price, and if prices coincide, $p_1 = p_2$, the seller chooses one buyer randomly.

As in the monopolist buyer case, when the seller accepts the offer of buyer i, one can infer that $v \leq p_i$. Hence, the market value of buyer i's inventories falls, and to ensure that buyer i's capital constraint remains satisfied, the offer p_i must satisfy $p_i \geq P_i(\delta_i)$, where $P_i(\delta_i)$ is defined parallel to equation (3); that is,

$$P_i(\delta_i) \equiv \frac{\delta_i M_i(1 + 2\Delta_i) - 2\Delta_i(M_i + 1)}{M_i - 1}. \tag{9}$$

However, now the acceptance of offer p_i may also lead to a violation of the capital constraint of buyer $-i$, who did not purchase the asset, since the market value of his inventories falls as well. Specifically, buyer -i's capital constraint is violated when the seller accepts

[18] See, for example, Jehiel, Moldovanu, and Stacchetti (1996).

[19] An earlier draft, available upon request, contains a full analysis of the case in which the asset is divisible and trade is nonexclusive in the sense that the seller can choose quantities $q_i \in [0, 1]$ to sell to each buyer, such that $q_1 + q_2 \leq 1$. The nature of the results remains.

buyer i's offer if

$$(\frac{1}{2}p_i + \Delta_{-i})M_{-i} < L_{-i}. \tag{10}$$

Defining

$$P_i^0(\delta_i) \equiv \delta_i(1 + 2\Delta_i) - 2\Delta_i, \tag{11}$$

equation (10) reduces to $p_i < P_{-i}^0(\delta_{-i})$; that is, buyer -$i$'s capital constraint is violated when the seller accepts buyer i's offer if $p_i < P_{-i}^0(\delta_{-i})$, and is satisfied otherwise.

Observe that both $P_i(\delta_i)$ and $P_i^0(\delta_i)$ increase in buyer $i's$ initial leverage. For use below, we omit the variable δ_i and simply write P_i and P_i^0. To avoid technical issues associated with continuous-action games, we also assume that the price space is finite, and the values $\{P_i, P_i \pm \varepsilon, \Delta_i, \Delta_i \pm \varepsilon, 2\Delta_i, 2\Delta_i \pm \varepsilon\}_{i\in\{1,2\}}$ lie within this space. The "tick" size ε is assumed to be close to zero, and for clarity, we exclude it from the statements of the results.

Because of the externalities generated by each buyer's bid on other buyers, there are typically Nash equilibria in which buyer i makes a bid that violates buyer $-i$'s capital constraint, if accepted, and forces buyer $-i$ to make a higher bid himself. However, not all equilibria of this type are robust, in the sense that there is no good reason for buyer i to make such a bid in the first place. Accordingly, we focus on equilibria that are robust in the sense of not entailing dominated strategies. Specifically, we characterize equilibria that survive the following iterated process of elimination of weakly dominated strategies. In the first stage, we eliminate all strategies that are weakly dominated. In the second stage, we consider the game remaining after the first stage and eliminate strategies that are weakly dominated in this new game. And so on. Lemma 2 characterizes offers that survive the first elimination round.

Lemma 2 *(A) If $P_i < 2\Delta_i$, the offer p_i survives the first round of elimination of weakly dominated strategies if and only if* $\max\{\Delta_i, P_i\} \leq p_i < 2\Delta_i$.

(B) If $P_i = 2\Delta_i$, the unique offer to survive the first round of elimination of weakly dominated strategies is $p_i = 2\Delta_i$.

(C) If $P_i \in (2\Delta_i, 1]$, the offers $p_i = 0$ and $p_i = P_i$ survive the first round of elimination of weakly dominated strategies. In contrast, any offer $p_i \neq P_i$ such that $p_i \geq 2\Delta_i$ is eliminated.

(D) If $P_i > 1$, the only offer that survives the first round of elimination of weakly dominated strategies is the offer $p_i = 0$.

Part (A) says that when a buyer has a profitable trade, he always tries to exploit it by making an offer that yields positive profits and does not violate his capital constraint. This behavior is similar to the single-buyer case previously analyzed. The result in part (C) that the loss-making offer $P_i \in (2\Delta_i, 1]$ is undominated reflects the fact that, with competition, a buyer may wish to make a "preemptive" bid to ensure that his capital constraint is not violated should the other buyer make an offer at a low price.[20]

5.1 Equilibrium outcomes

We start with the case in which both buyers have sufficiently low leverage that each would offer to purchase the asset if he were the only buyer; formally, $P_1 \leq 2\Delta_1$ and $P_2 \leq 2\Delta_2$. From Lemma 2, we know that buyer i offers to purchase the asset at a price that is between his monopoly offer $\max\{\Delta_i, P_i\}$ and his zero-profits offer $2\Delta_i$. If the two buyers have very different valuations (i.e., $\max\{\Delta_1, P_1\} > 2\Delta_2$ or $\max\{\Delta_2, P_2\} > 2\Delta_1$), the buyer with the highest valuation can continue to make his monopoly offer, so competition has no effect on the equilibrium price. Otherwise, competition drives the equilibrium price to $\min\{2\Delta_1, 2\Delta_2\}$. This is a standard outcome for settings with public buyer valuations: The buyer with the highest valuation acquires the asset at a price determined by the zero-profit condition of the buyer with the second-highest valuation.[21]

Proposition 4 *If both buyers have low leverage, i.e., $P_1 \leq 2\Delta_1$ and $P_2 \leq 2\Delta_2$, then the only*

[20] Part (D) reflects the fact that such a preemptive bid is possible only if the buyer has sufficient slack in his capital constraint. The reason is that while a preemptive bid can ensure that the value of the buyer's existing inventories remains at the prior, the purchase of an additional unit at a loss tightens the constraint and can lead to its violation.

[21] See, for example, Ho and Stoll (1983).

equilibrium outcome that survives iterated elimination of weakly dominated strategies is such that whenever the seller agrees to sell, he sells the asset to the buyer with the higher valuation for price equal to the maximum of the two monopoly prices, $\max\{\Delta_1, P_1\}$ *and* $\max\{\Delta_2, P_2\}$*, and the zero-leverage competition price,* $\min\{2\Delta_1, 2\Delta_2\}$*.*

Next, we consider the case in which buyer 1 has low leverage, but buyer 2 has sufficiently high leverage that he would not offer to purchase the asset if he were the only buyer; formally, $P_1 \leq 2\Delta_1$ and $P_2 > 2\Delta_2$. The key observation is that if buyer 1 offers to purchase the asset at a price $p_1 < P_2^0$, and the seller accepts buyer 1's offer, the expected value of v drops to $\frac{p_1}{2}$, and this causes buyer 2's capital constraint to be violated. Consequently, when buyer 2 is highly leveraged, so P_2^0 is high, buyer 2 may bid more aggressively to ensure that the seller does not accept a lower bid from buyer 1.[22]

Proposition 5 *If buyer 1 has low leverage and buyer 2 has high leverage (i.e.,* $P_1 \leq 2\Delta_1$ *and* $P_2 \in (2\Delta_2, 1]$*), then the only equilibrium outcome that survives iterated elimination of weakly dominated strategies is as follows:*

(A) If buyer 2's leverage is not too high, i.e., $P_2^0 \leq \max\{\Delta_1, P_1\}$*, then whenever the seller agrees to sell, he sells the asset to buyer 1 for a price* $\max\{\Delta_1, P_1\}$*.*

(B) If buyer 2's leverage is higher, i.e., $P_2^0 > \max\{\Delta_1, P_1\}$*, then whenever the seller agrees to sell, he sells the asset for a price* P_2*. If* $P_2 < 2\Delta_1$*, the seller sells to buyer* 1*, and if* $P_2 \geq 2\Delta_1$*, the seller sells to buyer* 2*, who makes negative profits.*

Proposition 5 shows that the buyers' capital constraints continue to affect prices even when there are multiple competing buyers. Part (A) reflects the simple intuition that if buyer 2 prefers not to trade, then buyer 1 can act as a monopolist. In this case, there is basically no interaction between the buyers.

[22] From Lemma 2, we know that such a preemptive bid is possible only if $P_2 \leq 1$. If instead, $P_2 > 1$, buyer 2 bids $p_2 = 0$, and buyer 1's unique best response is to act as if he were a monopolist; that is $p_1 = \max\{\Delta_1, P_1\}$. Accordingly, Proposition 5 focuses on the case $P_2 \leq 1$.

In part (B), in contrast, the buyers interact in a nondegenerate way. If buyer 2's leverage is relatively high so that $P_2 \in (2\Delta_2, 2\Delta_1)$, buyer 2's capital constraint leads him to compete more aggressively with buyer 1, and consequently buyer 1 ends up paying an amount P_2 that is determined by buyer 2's capital constraint. If $P_2 \geq \max\{2\Delta_1, 2\Delta_2\}$, buyer 1 can no longer compete; therefore, whenever trade occurs, buyer 2 acquires the asset at a price P_2. In the latter case, buyer 2 makes negative profits, even though he would not bid at all if he were the only buyer. Buyer 2 is forced to make this bid, since otherwise the seller trades with buyer 1 and buyer 2's capital constraint is violated.

It is worth contrasting this last result, in which competition induces buyer 2 to bid when he would otherwise have exited the market, with the existing literature on nonexclusive contracting. In this literature,[23] latent offers deter entry. In contrast, in our setting, latent offers induce entry: buyer 2 enters precisely because of buyer 1's latent offer.

Finally, consider the case in which both buyers are so leveraged that, if bidding individually, trade collapses in the sense that no one makes an offer. Clearly, no trade is an equilibrium that survives iterated elimination of weakly dominated strategies, since given that one buyer is unwilling to make an offer, the unique best response for the other buyer is also not to make an offer. Moreover, no trade is the only outcome to survive iterated elimination of weakly dominated strategies when $P_1 \neq P_2$.[24]

Proposition 6 *If both buyers are highly leveraged (i.e., $P_i > 2\Delta_i$ for $i \in \{1, 2\}$), then a no-trade equilibrium survives iterated elimination of weakly dominated strategies. When $P_1 \neq P_2$, this is the unique equilibrium that survives iterated elimination.*

[23] See, for example, Bisin and Guaitoli (2004), who study a moral hazard environment, and Attar, Mariotti, and Salanié (2011), and Ales and Maziero (2011), who study an adverse selection environment.

[24] In the nongeneric case $P_1 = P_2 > \max\{P_1^0, P_2^0\}$, we cannot rule out other equilibria in which one buyer makes a latent offer, knowing that it will not be accepted in equilibrium, and the second buyer makes a loss-making offer to rule out a situation in which the seller trades with the first buyer and the capital constraint of the second buyer is violated.

Proposition 6 shows that when both buyers have tight capital constraints, the conclusions of the single-buyer case continue to hold: trade collapses, and price dissemination stops. Indeed, the condition $P_i > 2\Delta_i$ in Proposition 6 is equivalent to the condition for no trade ($\delta_i > \frac{4\Delta_i}{1+2\Delta_i}$) in Proposition 2.

Note that in the special case in which buyers differ only in their inventory positions, but not in their valuations of the asset, Propositions 4-6 can be summarized as:

Corollary 2 *If $\Delta_1 = \Delta_2 = \Delta$, then: (i) if $\max\{P_1, P_2\} \leq 2\Delta$, the price is 2Δ; (ii) if $P_1 \leq 2\Delta < P_2 \leq 1$, the price is P_2, and buyer 2 purchases the asset at a loss; and (iii) if $\min\{P_1, P_2\} > 2\Delta$, trade completely breaks down.*

5.2 Discussion

As noted, many of the implications of introducing competition among buyers are consistent with our previous analysis of the monopoly buyer case. The primary new implication is that prices and trade depend on the distribution of leverage among potential buyers. In particular, both the minimum and maximum leverage of potential buyers matter, as illustrated by Corollary 2.

6 Seller competition

The previous section explored the effects of buyer competition and showed that even when multiple potential buyers compete for an asset, individual capital constraints still affect prices and trade. In this section, we turn to seller competition. In this case, a natural conjecture might be that seller competition drives sellers to the Bertrand outcome in which each seller offers to sell the asset at its true value v. However, we show that the Bertrand outcome arises only when the buyer's capital constraint is very slack. In contrast, when the buyer's capital constraint is tighter, the implications of the simple single-buyer-single-seller case continue to hold. A market freeze is the unique equilibrium outcome when leverage is

high, and within an important class of equilibria,[25] prices are increasing in leverage—i.e., a price run-up—when leverage is intermediate.

The driving force behind these results is that sellers understand that the buyer cannot afford to accept a low price offer, because doing so would reveal to the market that the asset is of low value, thereby violating the buyer's capital constraint. When the true value of the asset is low, this effect stops sellers from competing to offer to sell the asset at a low price.

Formally, we extend our basic model to include two competing sellers.[26] Each seller sells the same type of asset, and each seller knows the asset value v. As before, each seller has one indivisible unit for sale. We focus on perfect Bayesian equilibria of the following game: Both sellers offer prices simultaneously. For comparability with our prior analysis, we assume the buyer is only interested in acquiring one unit of the asset; consequently, the buyer can select at most one offer.[27] We denote a seller's strategy (i.e., an asking price as a function of v) by $f(v)$ and restrict attention to pure strategy symmetric equilibria (i.e., both sellers use the same strategy) in which $f(v)$ is nondecreasing in v. As in Section 5, we assume a finite price space and exclude from the expressions below the tick size, which is assumed to be close to zero.

We add the following two ingredients to the trading game: First, before sellers make offers, the buyer makes a publicly observable decision whether to participate in the market. If the buyer decides not to participate, the game ends with the sellers making no offers, and the market value of the asset remains at the prior. This initial step rules out equilibria in which the buyer's capital constraint is violated along the equilibrium path. We refer to this

[25] Because sellers know the value of the asset, the trading game in which sellers make offers is a signalling game, and for many parameter values multiple equilibria exist.

[26] The results below extend to the case in which there are $n > 2$ sellers.

[27] The nature of our results remains even if the buyer can select both offers. However, allowing the buyer to purchase from both sellers mutes the force of competition, and consequently, there is never full revelation of the true value—even if the buyer's initial leverage is low. In particular, the price in Proposition 7 becomes $f(v) = \max\{\Delta, v\}$. The price in Proposition 8 becomes $f(v) = \max\{\Delta, P(\delta), v\}$. (The last statement follows from the logic leading to Proposition 8 and the observation that if the buyer can select both offers, a seller cannot gain by offering Δ instead of the pooling price; hence, the pooling price is based only on how leveraged the buyer is.)

requirement as the buyer's ex-ante participation constraint.

Second, we assume that there is a small probability $\rho > 0$ that only one of the sellers is present and that when a seller makes an offer, he does not know whether the other seller is present. We focus on the case in which ρ is infinitesimally small. This assumption rules out equilibria with the following unrealistic feature: low-type sellers do not trade, but high-type sellers do, and low-type sellers are indifferent between (a) not making an offer, and (b) mimicking the high-type seller offer but being rejected anyway because the buyer receives different offers from the two sellers and assigns negative out-of-equilibrium beliefs.[28] More formally, this assumption straightforwardly implies:

Lemma 3 *In any equilibrium, there exists a cutoff $\bar{v} \in [0,1]$, which depends on the equilibrium, such that trade occurs if $v < \bar{v}$ but does not occur if $v > \bar{v}$.*

Our first result is to confirm that, whenever the buyer's capital constraint is sufficiently slack, the Bertrand outcome in which both sellers offer to sell for the true value v is an equilibrium outcome.

Of course, given the signalling nature of the trading game, there are also other equilibria in which the sellers offer to sell at higher prices and do not deviate to lower prices because of out-of-equilibrium beliefs.[29] However, the Bertrand equilibrium outcome is the one that maximizes both social surplus and buyer welfare.

Proposition 7 *If the buyer's initial leverage is low, $\delta \leq \frac{2\Delta(M+1)}{(1+2\Delta)M}$, then in the equilibrium that is most preferred by the buyer, each seller offers a price $f(v) = v$, and trade always occurs along the equilibrium path.*

Our central observation in this section is that, as leverage increases, full revelation of

[28] Another way to rule out such equilibria is to assume that there is a small probability that a seller does not observe the true value v but instead observes a value that is drawn from a uniform distribution on $[0,1]$. More broadly, this assumption as well as our original assumption relate to the notion of trembling-hand equilibrium, assuming that sellers can tremble but buyers cannot.

[29] For example, the equilibrium in Proposition 8 below is also an equilibrium when the buyer's initial leverage is low.

the price is impossible, as it leads to a violation of the buyer's capital constraint. In this case, Lemma 3 and the buyer's ex-ante participation constraint imply that for trade to be possible, sellers with low valuations must pool with sellers with higher valuations, so that low values are not revealed. The fact that $f(v)$ is nondecreasing then implies that there exists some $\tilde{v} > 0$ such that sellers with values on the interval $(0, \tilde{v})$ offer the same price, which we denote by $\tilde{p}$. In this case, the Bertrand outcome cannot be obtained when the true value of the asset is low. (In contrast, it may still be obtained for asset values above $\tilde{v}$.)

In fact, it is straightforward to see that as $\rho \to 0$, the only possibility for the pooling interval $\tilde{v}$ and pooling price $\tilde{p}$ is that $\tilde{v} = \tilde{p} = 2\Delta$, as follows. First, the buyer buys the asset only if its expected value exceeds the price, $\frac{1}{2}\tilde{v} + \Delta \geq \tilde{p}$. Second, a seller with an asset worth 0 would deviate and offer the price Δ unless $\tilde{p} \geq 2\Delta$, since the lower offer Δ is always accepted (regardless of beliefs), whereas in equilibrium the buyer accepts the pooling offer from an individual seller only half the time. Third, a seller with an asset worth $\tilde{v}$ only makes the offer $\tilde{p}$ if $\tilde{p} \geq \tilde{v}$. The first and second of these inequalities yield $\tilde{v} \geq 2\Delta$, while the first and third yield $2\Delta \geq \tilde{v}$. Hence $\tilde{v} = 2\Delta$, and the first and third inequalities imply $\tilde{p} = 2\Delta$ also.

When the buyer's capital constraint is only moderately tight, there is an equilibrium in which sellers with values in $[0, 2\Delta]$ pool and offer a price 2Δ. However, if the buyer's capital constraint is instead tight enough that it is violated when the buyer purchases the asset for 2Δ and the market's expectation of v is Δ, then no equilibrium with trade exists, i.e., the market freezes. These observations are formally established by:

Proposition 8 *(A) If the buyer's initial leverage is intermediate,* $\delta \in (\frac{2\Delta(M+1)}{(1+2\Delta)M}, \frac{4\Delta}{1+2\Delta}]$,[30] *then in the equilibrium that is most preferred by the buyer, each seller offers a price* $f(v) = \max\{v, 2\Delta\}$, *and trade always occurs along the equilibrium path. (B) If the buyer's initial*

[30] Note that the condition $\delta > \frac{2\Delta(M+1)}{(1+2\Delta)M}$ is equivalent to saying that $(v+\Delta)(M+1) - v < L$ as v approaches 0; that is, the capital constraint is violated as the value of the asset approaches 0 and the price fully reveals the true value.

leverage is high, $\delta > \frac{4\Delta}{1+2\Delta}$, trade cannot occur.

Combining Proposition 7 and Proposition 8, we obtain that as the buyer's initial leverage increases from low to intermediate, the distribution of prices increases in a first-order stochastic dominance (FOSD) sense. In particular, sellers with low valuations offer price 2Δ instead of $v < 2\Delta$. Hence, we continue to have some notion of a price run-up.[31]

(As a partial caveat, note that the conclusion that prices increase in leverage is dependent on the equilibrium selection rule. For example, as noted in footnote 29, the equilibrium characterized in Proposition 8 is an equilibrium for all leverage levels $\delta \leq \frac{4\Delta}{1+2\Delta}$; so if this equilibrium is always played, prices are independent of leverage.)

6.1 Discussion

The above results imply that the conclusions of the one-buyer-one-seller case continue to hold even when there are multiple competing sellers. In addition, this section delivers the new implication that the extent to which competition among sellers leads to revelation of the asset value depends on the tightness of the buyer's capital constraint. One testable implication of this is that the variance of holding returns for the buyer should be lower when the buyer's capital constraint is slack, but higher when the buyer's capital constraint is tight, since in the latter case the purchase price is less closely related to the true asset value.

7 Other Extensions

7.1 Seller is capital constrained

In the analysis so far, we assumed that the buyer is capital constrained, but the seller is not. A similar intuition applies when the seller is capital constrained and can sell only a fraction of his assets. In particular, a seller close to his capital constraint may not accept an offer $p \geq v$ because accepting the offer reduces the market value of the units that he retains.

[31] Note that now the price is a random variable; in the previous sections, the price was deterministic.

The seller will accept the offer only if p is sufficiently high, so that conditional on accepting the offer, his capital constraint continues to be satisfied. The analysis is similar to the basic model, but now the lower bound on p depends on the seller's initial leverage rather than the buyer's. Appendix B contains more details.

7.2 Marking to market

In the analysis so far, we assumed that the market value of the asset is derived from Bayes' rule, so market values, which determine borrowing capacity, are updated both after an offer is accepted and after an offer is rejected. Alternatively, we can assume that the capital constraint is based on asset book values and that book values are updated only when an offer is accepted. In particular, under marking-to-market accounting, the asset book value is equal to the most recent transaction price, and so rejected offers do not affect book values. As we explain below, this alternative assumption leads to qualitatively similar predictions. Hence, our model is consistent with the interpretation that marking-to-market accounting can cause many of the phenomena we discussed earlier, but it also predicts that one would see qualitatively the same phenomena even without marking to market.

Specifically, in the static models, the capital constraint continues to put a lower bound on the price, and we can apply the logic we used earlier.[32] In the dynamic case, the capital constraint also puts lower bounds on bid prices, but whether the capital constraint is tightened after the first offer is accepted depends on the initial book value of the asset. This is an important difference from our model, in which acceptance of an offer always tightens the capital constraint—even if the sequence of prices is increasing.[33]

[32] In the seller competition case, the equilibrium price in Proposition 8 becomes $f(v) = \max\{\bar{P}(\delta), v)$, where $\bar{P}(\delta)$ is the minimum price required to satisfy the buyer's capital constraint under marking-to-market accounting.

[33] If the initial book value of each asset is sufficiently high (e.g., more than 2Δ), mark-to-market valuation produces similar predictions to our main model. In particular, whenever the buyer makes a profitable offer, $p < 2\Delta$, and his offer is accepted, the book value of the asset falls, and the capital constraint is tightened. As before, this may force the buyer to either increase his next bid or else stop bidding, even though there is no change in the fundamentals. However, since rejected offers have no effect on the capital constraint, the buyer will not make loss-making offers. (As discussed in an earlier draft, this last implication changes if the

7.3 Market does not observe terms of rejected offers

In the analysis so far, we assumed that the buyer's and seller's actions are publicly observable. In particular, the market observes the offer terms even if the offer is rejected. This assumption is appropriate for some cases, e.g., when a dealer publicly posts a bid price. However, in some cases (e.g., if the buyer and seller negotiate the offer terms privately), it might be more natural to assume that the market observes the offer terms only if the offer is accepted. Our main results continue to hold under this alternative assumption.

In particular, when buyers make offers, there is a positive probability that offers are rejected along the equilibrium path, and so inferences regarding the asset value are the same as in the case in which the market observes the terms of rejected offers; hence, the analysis remains unchanged. If instead, trade occurs along the equilibrium path with probability 1, as in Propositions 7 and 8, then the event that an offer is rejected is an out-of-equilibrium event, and we have more flexibility in assigning market beliefs regarding the asset value; this actually strengthens our results.[34]

8 Empirical implications

In this section, we collect our model's main empirical implications; most of these have already been noted, at least in passing:

Price movements unrelated to fundamentals: A basic implication of our analysis is that the price of an asset may respond to variables other than beliefs about its fundamental value. In particular, as the buyer's capital constraint tightens, the price increases, even without any change to fundamentals. In Section 4 we show how this force can lead to prices that increase over time. Although hard to definitively test, there is at least some evidence that prices

buyer can offer to purchase just a small quantity of the asset.)

[34] In particular, now it is possible to implement a pooling price and a pooling interval $\tilde{v} = \tilde{p} = P(\delta)$, which increases in the buyer's initial leverage, and the equilibrium price in part (A) of Proposition 8 becomes $f(v) = \max\{v, P(\delta)\}$ instead of $f(v) = \max\{v, 2\Delta\}$.

in the market for structured financial products were partially divorced from fundamentals in the run-up to the financial crisis (see references in footnote 2). This prediction is also consistent with anecdotal accounts of the same period; for example, Lewis (2010) suggests that prior to the crisis, prices increased in a way not supported by fundamentals.[35]

Tight capital constraints lead to market breakdown: Adrian and Shin (2010) document a sharp increase in dealers' leverage prior to the recent financial crisis. More anecdotally, many market observers expressed the view that concerns about the value of inventories induced firms not to sell their assets. For example, an analyst was quoted in *American Banker*[36] as saying that "Other [companies] may be wary of selling assets for fear of establishing a market-clearing price that could force them to mark down the carrying value of their nonperforming portfolio." Also related is the view expressed in Lewis (2010) that dealers who sold credit default swaps on subprime mortgage bonds did not make a market in these securities so that information is not revealed and their positions do not lose money (see footnote 35).

Market breakdown is associated with a loss of information: In our analysis, trade is a source of information, and so when the market freezes, information dissemination ceases. Loss of information was a primary concern expressed by observers during the financial crisis; for just one example, see Scott and Taylor (2009).

Expected holding returns and the tightness of capital constraints: Related to the prediction that the tightness of capital constraints affects prices, the tightness of capital constraints also affects a buyer's expected holding returns. When capital constraints are sufficiently tight, Proposition 3 implies both that the market freezes and that prices before the freeze are high.

[35] For example, on page 184, Lewis writes that "Burry [an investor who bought credit default swaps on subprime mortgage bonds] sent his list of credit default swaps to Goldman and Bank of America and Morgan Stanley with the idea that they would show it to possible buyers, so he might get some idea of the market price. That, after all, was the dealer's stated function: middleman. Market-makers. That is not the function they served, however. 'It seemed the dealers were just sitting on my lists and bidding extremely opportunistically themselves,' said Burry. The data from the mortgage servicers was worse every month...and yet the price of insuring those loans, they said, was falling." On page 185, he adds that "The firms always claimed that they had no position themselves...but their behavior told him otherwise."

[36] "Nonperformance Space: Risky Assets Find Market" (*American Banker*, August 19, 2009).

So when capital constraints are tight, one should see low average returns on assets purchased shortly before a market freeze. A second implication, which emerges from the dynamic model, is that prices may exhibit short-run momentum, increasing as capital constraints endogenously grow tighter; followed by long-run reversal, as the assets purchased last have lower expected returns. A third implication, delivered by our analysis of competition among sellers, is that the variance of expected returns on different assets increases as the buyer's capital constraint becomes tighter.

In broker-dealer markets, prices are increasing in dealer inventories: This is essentially a special case of the first implication above. As discussed in the introduction, this prediction is consistent with the empirical findings of Manaster and Mann (1996), which are not easily explained by previous models of market-maker inventories. More formally, this implication is a corollary of our Proposition 2. To see this, observe that the derivative of $P(\delta)$ with respect to inventories M has the same sign as $4\Delta - \delta(1 + 2\Delta)$, which by Proposition 2 is positive whenever trade occurs.[37]

9 Policy implications

Our analysis has implications for government attempts to defrost markets and for regulatory proposals aimed at improving market functioning.

9.1 Defrosting frozen markets

Consider the case in which trade has completely broken down because the buyer's capital constraint is too tight.[38] One option open to a government is to offer to buy the seller's assets. A central question is whether such government purchases can succeed without taxpayer

[37] Note that here we are characterizing only the direct effect of inventories, in the sense that we are holding leverage δ constant. If changes in inventory levels also affect leverage, there is an additional indirect effect on prices; this indirect effect reinforces the direct effect if higher inventories are associated with greater leverage (as seems likely).

[38] The discussion can easily be extended to the case of more than one buyer, as in Proposition 6.

subsidies (in expectation). Our model has two implications in this respect. First, if the government faces the same lemons problem that potential buyers do, a subsidy-free purchase scheme is possible *only if* the asset is worth more to the government than to the seller. Second, even if this condition holds, a government purchase may impose a cost on the original buyer. In particular, if the government is not the most efficient holder of the asset, then any subsidy-free purchase scheme either leads to a violation of the buyer's capital constraint, or else it forces him to purchase the asset at a loss. This is similar to the negative externality that one buyer imposes on another buyer when there are competing buyers.

Another option is to remove assets from the buyer's balance sheet; that is, to replace assets with cash. If the buyer can borrow against the full value of his assets, as assumed in our analysis so far, then again, purchasing assets from the buyer can relax his capital constraint only if the purchase involves a taxpayer subsidy. If instead the buyer has limited borrowing capacity, purchasing assets from the buyer might relax his capital constraint even if the purchase does not involve a taxpayer subsidy.

9.2 Should regulation mandate some retention of the asset by the seller?

A commonly voiced regulatory proposal is that sellers of assets subject to asymmetric information problems, such as issuers of asset-backed securities, should be required to retain some stake in the assets they sell.[39] Our analysis identifies a potential cost to this proposal, namely, that under some circumstances it leads to a market breakdown. Specifically, if the seller is required to retain a very large fraction of his assets on his balance sheet, and if the seller does not have sufficient slack in his capital constraint, then trade is impossible. The reason is that trade may reduce the market value of the assets that the seller is forced to retain and this may lead to a violation of his constraint.[40]

[39] See, for example, section 15G of the Investor Protection and Securities Reform Act of 2010.

[40] Formally, it follows from the analysis in Appendix B that trade is possible only if $P_s(\delta_s) \leq 2\Delta$. If $\gamma < 1$, this condition reduces to $x \geq \frac{\delta_s(\frac{1}{2}+\gamma\Delta)M_s-(1+\gamma)\Delta M_s}{(1-\gamma)\Delta.}$. Otherwise ($\gamma = 1$), the condition reduces to $\delta_s \leq \frac{2\Delta}{\frac{1}{2}+\Delta}$.

The goal that regulators appear to have in mind with this regulation is to reduce moral hazard on the part of asset sellers; for example, to discourage loan originators from making bad loans and/or shirking on monitoring later on. Our analysis does not speak to this issue, and it seems likely that the regulation will have its intended effect in this regard. Our point here is instead to draw attention to a potentially significant cost of this regulation, namely, that it can lead to the breakdown of socially efficient trade.

10 Summary

We analyze how existing stocks of assets—inventories—affect prices and information dissemination. When market participants are close to their maximal leverage, concerns about the revelation of bad news prevent socially beneficial trade and information dissemination. However, when market participants are further from their maximal leverage, inventories lead to higher equilibrium prices, even absent any changes in the asset fundamentals; this stimulates socially beneficial trade. Because purchasing assets increases both buyer inventories and buyer leverage, the predictions above imply that prices may first increase before trade completely breaks down; we show this formally in the dynamic extension of our basic model. We also show that our main results (run-ups and breakdowns) continue to hold not only when the buyer is a monopolist who makes a take-it-or-leave-it offer, as in our basic model, but also when competing (uninformed) buyers make offers or competing (homogeneously informed) sellers make offers. In particular, competition from other less leveraged buyers may lead to a buyer acquiring assets at a price so high he loses money; while inventories prevent seller competition from delivering the Bertrand outcome, in which the price matches seller valuations.

Our results are consistent with several features of the market for structured financial products and the recent financial crisis. First, prices may move independently from fundamentals, consistent with both anecdotal accounts and more formal evidence. Second, a

market collapse may be preceded by a period in which assets trade at above fundamental values. Third, a market collapse is associated with a breakdown in information dissemination. Separately, our model also predicts that asset prices are increasing in broker-dealer inventories, consistent with empirical evidence but different from the predictions of existing market microstructure models.

References

[1] Adrian, Tobias, and Hyun-Song Shin, 2010, Liquidity and leverage, *Journal of Financial Intermediation*, 19, 418-437.

[2] Akerlof, George A., 1970, The market for "lemons": Quality uncertainty and the market mechanism, *Quarterly Journal of Economics*, 84, 488-500.

[3] Ales, Laurence, and Pricila Maziero, 2011, Adverse selection and non-exclusive contracts, Unpublished manuscript, Carnegie Mellon University and University of Pennsylvania.

[4] Allen, Franklin, and Gerald R. Faulhaber, 1989, Signaling by underpricing in the IPO market, *Journal of Financial Economics*, 23, 303-323.

[5] Allen, Franklin, Douglas Gale, 1992, Stock price manipulation, *Review of Financial Studies*, 5, 503–529.

[6] Allen, Franklin, and Douglas Gale, 1994, Liquidity preference, market participation and asset price volatility, *American Economic Review*, 84, 933-955.

[7] Amihud, Yakov, and Haim Mendelson, 1980, Dealership market: Market making with inventory, *Journal of Financial Economics* 8, 31-53.

[8] Ashcraft, Adam, B., and Til Schuermann, 2008, Understanding the securitization of subprime mortgage credit, *Federal Reserve Bank of New York Staff Reports*, no. 318.

[9] Ashcraft, Adam, B., Paul Goldsmith-Pinkham and James Vickery, 2010, MBS ratings and the mortgage credit boom, *Federal Reserve Bank of New York Staff Reports*, no. 449.

[10] Attar, Andrea, Thomas Mariotti, and François Salanié, 2011, Nonexclusive competition in the market for lemons, *Econometrica*, 79, 1869–1918.

[11] Bisin, Alberto, and Danilo Guaitoli, 2004, Moral hazard and nonexclusive contracts, *RAND Journal of Economics,* 35, 306-328.

[12] Brunnermeier, Markus K., and Lasse H. Pedersen, 2005, Predatory trading, *Journal of Finance*, 60, 1825-1863.

[13] Coval, Joshua D., Jakub W. Jurek and Erik Stafford, 2009, Economic catastrophe bonds, *American Economic Review*, 99, 628-666.

[14] Daley, Brendan, and Brett Green, 2012, Waiting for news in the market for lemons, *Econometrica*, 80, 1433–1504.

[15] Diamond, Douglas, W., and Raghuram G. Rajan, 2011, Fear of fire sales, illiquidity seeking, and credit freezes, *Quarterly Journal of Economics*, 126, 557-591.

[16] Downing, Chris, Dwight Jaffee, and Nancy Wallace, 2009, Is the market for mortgage-backed securities a market for lemons?, *Review of Financial Studies*, 22, 2457-2494.

[17] Easley, David, and Maureen O'Hara, 2010, Liquidity and valuation in an uncertain world, *Journal of Financial Economics*, 97, 1-11.

[18] Faltin-Traeger, Oliver, Kathleen W. Johnson, and Christopher Mayer, 2010, Issuer credit quality and the price of asset backed securities, *American Economic Review*, 100, 501-05.

[19] Glode, Vincent, Richard C. Green, and Richard Lowery, 2012, Financial expertise as an arms race, *Journal of Finance*, 67, 1723-1759.

[20] Goldstein, Itay, and Alexander Guembel, 2008, Manipulation and the allocation role of prices, *Review of Economic Studies*, 75, 133-164.

[21] Griffin, John M. and Dragon Yongjun Tang, 2012, Did subjectivity play a role in CDO credit ratings? *Journal of Finance*, 67, 1293–1328.

[22] Grinblatt, Mark, and Chuan Yang Hwang, 1989, Signalling and the pricing of new issues, *Journal of Finance,* 44, 393-420.

[23] Ho, Thomas, and Hans R. Stoll, 1981, Optimal dealer pricing under transactions and return uncertainty, *Journal of Financial Economics*, 9, 47-73.

[24] Ho, Thomas, and Hans R. Stoll, 1983, The dynamics of dealer markets under competition, *Journal of Finance*, 38, 1053-1074.

[25] Jehiel, Philippe, Benny Moldovanu, and Ennio Stacchetti, 1996, How (not) to sell nuclear weapons, *American Economic Review*, 86, 814-829.

[26] Kiyotaki, Nobuhiro, and John Moore, 1997, Credit cycles, *Journal of Political Economy*, 105, 211-248.

[27] Kremer, Ilan, and Andrzej Skrzypacz, 2007, Dynamic signaling and market breakdown, *Journal of Economic Theory*, 133, 58-82.

[28] Lewis, Michael, 2010, *The Big Short: Inside the Doomsday Machine* (W. W. Norton & Company, New York, London).

[29] Manaster, Steven, and Steven C. Mann, 1996, Life in the pits: Competitive market making and inventory control, *Review of Financial Studies*, 9, 953-975.

[30] Milbradt, Konstantin, 2012, Level 3 assets: Booking profits and concealing losses, *Review of Financial Studies*, 25, 55-95.

[31] Rajan, Uday, Amit Seru, and Vikrant Vig, 2012, The failure of models that predict failure: Distance, incentives and defaults, *Journal of Financial Economics,* forthcoming.

[32] Samuelson, William, 1984, Bargaining under asymmetric information, *Econometrica*, 52, 995-1005.

[33] Scott, Kenneth E., and John B. Taylor, 2009, Why Toxic Assets Are So Hard to Clean Up, *Wall Street Journal*, July 21.

[34] Shleifer, Andrei, and Robert W. Vishny, 1992, Liquidation values and debt capacity: A market equilibrium approach, *Journal of Finance*, 47, 1343-1366.

[35] Welch, Ivo, 1989, Seasoned offerings, imitation costs, and the underpricing of initial public offerings, *Journal of Finance*, 44, 421-449.

Appendix A: Proofs

Proof of Lemma 1. Since choosing $p_r = \Delta$ maximizes second-period profits, it is enough to show that the capital constraint is not violated after choosing this offer; that is, we need to show that $p_r = \Delta$ satisfies equation (8): $(\frac{1}{2} + \frac{1}{2}p_1 + \Delta)M + (\frac{3}{2}\Delta)(M+1) \geq L + \Delta$, which can be rewritten as $(\frac{1}{2}p_1 + \Delta)(M+1) + (\frac{1}{2} + \Delta)M + \frac{1}{2}\Delta(M-1) + \frac{1}{2}p_1 \geq L + p_1$. The last equation follows since the offer p_1 satisfies equation (6), and since $M > 1$. Q.E.D.

Proof of Proposition 3. In the dynamic case, the total buyer inventory is $2M$, so leverage is $\delta = \frac{L}{(\frac{1}{2}+\Delta)2M}$. Since the capital constraint is initially satisfied, $\delta \leq 1$. Also define $\sigma \equiv \frac{\frac{1}{2}M-\Delta}{\frac{1}{2}(M-1)}$ and $H \equiv \frac{L-2\Delta(M+1)}{\frac{1}{2}(M-1)}$. Observe that $\sigma > 1 > 2\Delta$, and H is a monotone transformation of the buyer's initial leverage; in particular, $H = \frac{(\frac{1}{2}+\Delta)2M\delta-2\Delta(M+1)}{\frac{1}{2}(M-1)}$.

By Lemma 1, $p_r = \Delta$. Hence, for strategies in which $p_a > 0$, the optimal (p_1, p_a) maximizes expected profits $V(p_1, p_a) \equiv \pi(p_1) + p_1\pi(p_a) + (1-p_1)\pi(\Delta)$, subject to the capital constraint (7), which reduces to $p_1 + p_a \geq H$. For strategies in which $p_1 > 0$ and $p_a = 0$, the optimal p_1 maximizes $V(p_1, 0)$, subject to the capital constraint (6), which reduces to $p_1 \geq H - \sigma$.

Part (A) follows because if $H \leq 2\Delta$ then the benchmark solution $p_1 = p_a = \Delta$ is achievable.

In contrast, when $H > 4\Delta$, it is optimal to choose $p_a = 0$, as follows. Suppose to the contrary that $p_a \neq 0$. Then either $p_a > 2\Delta$, and the strategy (p_1, p_a) is strictly dominated by the strategy $(p_1, 0)$ (here we are using the observation from the main text that constraint (7) implies constraint (6)); or $p_1 > 2\Delta \geq p_a$, which is strictly dominated by the alternate strategy $(\tilde{p}_1, \tilde{p}_a) = (p_a, 0)$, since $V(p_a, 0) \geq V(p_1, p_a)$, and the fact that $\sigma > 1 \geq p_1$ implies that $\tilde{p}_1 = p_a \geq H - p_1 \geq H - \sigma$.

By continuity, there exist $H_1, H_2 \in (2\Delta, 4\Delta)$, such that whenever $H \in (2\Delta, H_1)$ (i.e., intermediate leverage), it is optimal to choose $p_a > 0$ and whenever $H > H_2$ (i.e., higher leverage), it is optimal to choose $p_a = 0$.

To finish the proof of part (B), note that whenever $p_a > 0$ and $H > 2\Delta$, the capital constraint (7) is binding because at least one of p_1 and p_a exceeds Δ, and $\frac{\partial}{\partial p_a} V(p_1, p_a) = p_1\pi'(p_a)$ while $\frac{\partial}{\partial p_1} V(p_1, p_a) \leq \pi'(p_1)$. Hence if $H > 2\Delta$ and $p_a > 0$, then p_1 maximizes $V(p, H-p)$, which is a cubic in p with a negative coefficient on the cubic term. From above, we know $H < 4\Delta$. To show that $p_a > p_1 > \Delta$, we need to show that $p_1 \in (\Delta, \frac{H}{2})$. This result follows if $\frac{d}{dp_1} V(p_1, H-p_1)\Big|_{p_1=\Delta} > 0 > \frac{d}{dp_1} V(p_1, H-p_1)\Big|_{p_1=H/2}$. Evaluating, $\frac{d}{dp_1} V(p_1, H-p_1) = \pi'(p_1) + \pi(H-p_1) - p_1\pi'(H-p_1) - \pi(\Delta)$. Since π is a quadratic with its maximum at Δ, for any p, $\pi(p) = \pi(\Delta) + \frac{1}{2}(p-\Delta)\pi'(p)$. Given this, $\frac{d}{dp_1} V(p_1, H-p_1)\Big|_{p_1=\Delta} = \pi(H-\Delta) - \Delta\pi'(H-\Delta) - \pi(\Delta) = \left(\frac{1}{2}(H-\Delta-\Delta) - \Delta\right)\pi'(H-\Delta)$, which is positive. Similarly, $\frac{d}{dp_1} V(p_1, H-p_1)\Big|_{p_1=H/2} = (1-\frac{H}{2})\pi'(\frac{H}{2}) + \pi(\frac{H}{2}) - \pi(\Delta) = (1 - \frac{H}{2} + \frac{1}{2}(\frac{H}{2} -$

$\Delta))\pi'(\frac{H}{2}) = (1 - H/4 - \Delta/2)\pi'(H/2)$, which is negative.

To finish the proof of part (C), note that whenever $p_a = 0$, the objective function is quadratic and if we impose $p_1 > 0$, the optimal solution is $\tilde{p}_1 = \max\{\Delta - \frac{1}{2}\Delta^2, H - \sigma\}$, which is increasing in H (and hence, in leverage). Since $V(\tilde{p}_1, 0) > 0$ whenever $\tilde{p}_1 \leq 2\Delta$, the buyer's expected profits are positive if he chooses $\tilde{p}_1$ when $H \leq 2\Delta + \sigma$. Hence, by continuity, there exists $H_3 > 2\Delta + \sigma > 4\Delta$, such that if $H \in (H_2, H_3)$, it is indeed optimal to choose $p_1 > 0$. Moreover, if $H \in (2\Delta + \sigma, H_3)$, the initial bid is more than 2Δ, so the buyer expects to lose money in the first period. (To see why footnote 17 holds, observe that if $3\Delta < 1$, then $\sigma > 3\Delta$, and so, $H - \sigma < \Delta$ when H is sufficiently close to 4Δ. Hence, it is possible that the initial bid $\tilde{p}_1$ is below the benchmark Δ.)

To finish part (D), observe that as $\delta \to 1$, i.e., the capital constraint becomes very tight, then H approaches

$$H \to \frac{M - 2\Delta}{M - 1} + \sigma > 1 + \sigma > 2\Delta + \sigma > 4\Delta.$$

So by observations above, the buyer's best non-degenerate offer is $(p_1, p_a) = (H - \sigma, 0)$. But because $H - \sigma > 1$, this offer leads to negative expected profits (over the two periods). Hence, the buyer prefers not to bid at all.

Lemma A-1 *For every $i \in \{1, 2\}$, one of the following is true: (i) $P_i = P_i^0 = 2\Delta_i$; (ii) $P_i > P_i^0 > 2\Delta_i$; or (iii) $P_i < P_i^0 < 2\Delta_i$.*

Proof of Lemma A-1. From equations (9) and (11), $P_i = \frac{M_i P_i^0 - 2\Delta_i}{M_i - 1}$. Hence, the result follows. Q.E.D.

Proof of Lemma 2. *Part (A):* From Lemma A-1, $P_i < P_i^0 < 2\Delta_i$. The offer $p_i \in [\max\{\Delta_i, P_i\}, 2\Delta_i)$ survives the first stage of elimination, since it is a unique best response for buyer i when buyer $-i$ bids $p_{-i} = p_i - \varepsilon$. Any offer with $p_i < \Delta_i$ is weakly dominated by raising the offer to Δ_i. Finally, any offer with $p_i \geq 2\Delta_i$ or $p_i < P_i$ is weakly dominated

by the offer P_i^0 as follows. The offer P_i^0 produces positive profits whenever it is accepted (e.g., if $p_{-i} = 0$) and guarantees that buyer i's capital constraint is satisfied, regardless of $-i$'s offer. In contrast, offering $p_i \geq 2\Delta_i$ never leads to positive profits, and offering $p_i < P_i$ violates i's capital constraint if it is accepted (e.g., if $p_{-i} = 0$).

Part (B): From Lemma A-1, $P_i = P_i^0 = 2\Delta_i$. The offer $p_i = 2\Delta_i$ weakly dominates any other, as follows: The offer $p_i = 2\Delta_i$ produces zero profits and guarantees that buyer i's capital constraint is satisfied, regardless of $-i$'s offer. In contrast, the offer $p_i > 2\Delta_i$ produces negative profits whenever it is accepted (e.g., if $p_{-i} = 0$), and the offer $p_i < 2\Delta_i$ violates i's capital constraint if accepted (e.g., $p_{-i} = 0$).

Part (C): From Lemma A-1, $P_i > P_i^0 > 2\Delta_i$. The offer $p_i = 0$ survives the first elimination round, since it is the unique best response for buyer i when buyer $-i$ bids $p_{-i} = 0$. Specifically, if buyer i bids $p_i = 0$, he obtains zero profits and his capital constraint is satisfied; if he bids $p_i > 0$ and his offer is accepted, he either makes negative profits or his capital constraint is violated.

The offer $p_i = P_i$ survives the first elimination round, since it is a unique best response when buyer $-i$ bids $p_{-i} \in (0, P_i^0 - \varepsilon)$. Specifically, the offer $p_i = P_i$ guarantees that buyer i's capital constraint is satisfied, while if he bids $p_i \in [P_i^0, P_i)$ his capital constraint is violated whenever the seller accepts; if he bids $p_i \in [0, P_i^0 - \varepsilon]$ his capital constraint is violated whenever the seller accepts buyer $-i$'s offer; and if he bids $p_i > P_i$, his expected profits are reduced.

Any offer $p_i > P_i$ is weakly dominated by the alternate offer P_i. The alternate offer weakly increases buyer i's profits, with a strict increase whenever $-i$'s offer is lower, and does not affect whether the capital constraint is satisfied.

Any offer $p_i \in [2\Delta, P_i)$ is weakly dominated by $\tilde{p}_i = 0$, as follows. If $p_{-i} > p_i$, the outcome is the same under both offers, since the seller accepts the other buyer's offer. If $p_{-i} = p_i$, the offer $\tilde{p}_i = 0$ is at least as good since under both offers the capital constraint

of buyer i is violated with probability p_{-i}, but the offer p_i leads to zero or negative profits while $\tilde{p}_i = 0$ leads to zero profits. If $p_{-i} < p_i$, the offer $\tilde{p}_i$ is strictly preferred since under $\tilde{p}_i = 0$, the capital constraint of buyer i is violated with probability 0 or p_{-i}, while under p_i, it is violated with probability p_i.[41]

Part (D): Any offer $p_i > 0$ is weakly dominated by $\tilde{p}_i = 0$, as follows. If the offer $p_i < p_{-i}$, buyer i's utility is the same under the alternate offer $\tilde{p}_i = 0$. If instead $p_i \geq p_{-i}$, buyer i's capital constraint is violated (since $P_i > 1$) whenever the seller accepts offer p_i, whereas the offer $\tilde{p}_i = 0$ satisfies the capital constraint if $p_{-i} = 0$. Q.E.D.

Proof of Proposition 4. First, consider the case $\Delta_1 \neq \Delta_2$, and assume, without loss of generality, that $\Delta_1 > \Delta_2$. If $P_2 = 2\Delta_2$ and $P_1 = 2\Delta_1$, the result follows immediately from Lemma 2. If $P_2 = 2\Delta_2$ and $P_1 < 2\Delta_1$, we know from Lemma 2 that buyer 2 bids $p_2 = 2\Delta_2$; therefore, if $\max\{\Delta_1, P_1\} > 2\Delta_2$, buyer 1's best response is to bid $p_1 = \max\{\Delta_1, P_1\}$, and if $\max\{\Delta_1, P_1\} \leq 2\Delta_2$, buyer 1's best response is to bid $\min\{2\Delta_2 + \varepsilon, 2\Delta_1 - \varepsilon\}$. If $P_2 < 2\Delta_2$, then from Lemma 2, $p_2 < 2\Delta_2$ therefore, if $\max\{\Delta_1, P_1\} \geq 2\Delta_2$, buyer 1's best response is to bid $p_1 = \max\{\Delta_1, P_1\}$, and if $\max\{\Delta_1, P_1\} < 2\Delta_2$, standard competition arguments imply that buyer 2 bids $p_2 = 2\Delta_2 - \varepsilon$ and buyer 1 bids $p_1 = 2\Delta_2$.

Next, consider the case $\Delta_1 = \Delta_2 = \Delta$. If $\max\{P_1, P_2\} = 2\Delta$, then by Lemma 2, the equilibrium price is 2Δ and is offered by the buyer with the highest P_i. If $\max\{P_1, P_2\} < 2\Delta$, Lemma A-1 and standard competition arguments imply that both buyers offer the price $2\Delta - \varepsilon$. Q.E.D.

Proof of Proposition 5: From the first elimination round (Lemma 2), we know that $p_1 \in [\max\{\Delta_1, P_1\}, 2\Delta_1]$ and $p_2 \in [0, 2\Delta_2) \cup \{P_2\}$. From Lemma A-1, $P_2 > P_2^0 > 2\Delta_2$.

Part (A): $P_2^0 \leq \max\{\Delta_1, P_1\}$. Hence, any offer $p_2 \geq \max\{\Delta_1, P_1\}$ is above $2\Delta_2$ and so

[41] Note that an offer $p_i \in (0, 2\Delta_i)$ is not eliminated, as follows: Offering $p_i \in (0, 2\Delta_i)_i$ is strictly preferred to offering 0 if the other buyer offers $p_{-i} = p_i$, since the capital constraint is violated in both cases, but p_i leads to positive profits. Offering $p_i \in (0, 2\Delta_i)$ is strictly preferred to P_i, if the other buyer offers $p_{-i} = 0$, since the capital constraint is satisfied in both cases, but p_i strictly increases profits.

gives negative profits if accepted (e.g., if $p_1 = \max\{\Delta_1, P_1\}$), whereas the offer $p_2 = 0$ gives zero profits and guarantees that the capital constraint of buyer 2 is satisfied given any p_1 that survives the first round. Hence, any offer that survives the second elimination round must satisfy $p_2 < \max\{\Delta_1, P_1\}$. Buyer 1's unique best response to any such offer is to bid $p_1 = \max\{\Delta_1, P_1\}$.

Part (B): $P_2^0 > \max\{\Delta_1, P_1\}$. There is no equilibrium in which buyer 1 bids $p_1 < P_2^0$ and buyer 2 bids $p_2 < 2\Delta_2$ because in such an equilibrium buyer 2's capital constraint is violated; but then buyer 2 would deviate to offer $p_2 = P_2$, which wins since $P_2 > P_2^0$. Nor is there an equilibrium in which buyer 1 bids $p_1 \geq P_2^0$ and buyer 2 bids $p_2 < 2\Delta_2$ because buyer 1 can increase his profits by deviating to $p_1 = P_2^0 - \varepsilon \geq 2\Delta_2 > p_2$. Consequently, in any candidate equilibrium $p_2 = P_2$. Hence, if $P_1 = 2\Delta_1$, the unique equilibrium is that buyer 2 bids $p_2 = P_2$ and buyer 1 bids $p_1 = 2\Delta_1$; and if $P_1 < 2\Delta_1$ and $P_2 < 2\Delta_1$, the unique equilibrium is that buyer 2 bids $p_2 = P_2$ and buyer 1 bids $p_1 = \min\{P_2 + \varepsilon, 2\Delta_1 - \varepsilon\}$. If instead, $P_1 < 2\Delta_1$ and $P_2 \geq 2\Delta_1$, the unique equilibrium outcome is that buyer 2 bids $p_2 = P_2$, buyer 1 bids a lower price $p_1 < 2\Delta_1 \leq P_2$, and the equilibrium price is P_2. Q.E.D

Proof of Proposition 6. From Lemma A-1, $P_i > P_i^0 > 2\Delta_i$ for $i \in \{1, 2\}$. For use throughout the proof, note that this means that neither buyer can make an offer that is accepted with positive probability, makes non-negative profits, and satisfies the buyer's capital constraint upon acceptance.

From the first elimination round (Lemma 2), $p_i \in [0, 2\Delta_i) \cup P_i$, for $i \in \{1, 2\}$. In addition, we know that for $i \in \{1, 2\}$, the offers $p_i = 0$ and $p_i = P_i$ survive the first round. In fact, the offers $p_1 = 0$ and $p_2 = 0$ cannot be eliminated in any round, since each one is a unique best response against the other. Hence, the no trade equilibrium survives the elimination process. The remainder of the proof establishes that, provided $P_1 \neq P_2$, this is the unique equilibrium to survive the elimination process.

Start with the case $P_1^0 > P_2$. The offer $p_1 = P_1$ survives the second elimination round,

since it is a unique best response if buyer 2 bids $p_2 = P_2$. However, any offer $p_1 \in (0, 2\Delta_1)$ is eliminated, since it violates buyer 1's capital constraint with a positive probability, while the offer $p_1 = P_1$ never violates the constraint.[42] Hence, buyer 1 bids either $p_1 = 0$ or $p_1 = P_1$. In the third elimination round, the offer $p_2 = 0$ weakly dominates any other remaining offer for buyer 2, since $p_2 = 0$ gives buyer 2 zero profits and guarantees that his capital constraint is satisfied, while any other remaining offer gives zero profits if $p_1 = P_1$ and leads to negative profits and/or violates buyer 2's capital constraint, if $p_1 = 0$. Hence, the unique equilibrium that survives the third elimination round is that both buyers bid nothing. The case $P_2^0 > P_1$ is similar.

The rest of the proof deals with the case in which both $P_1^0 \leq P_2$ and $P_2^0 \leq P_1$; and without loss, assume $P_1 < P_2$.

Note first that if every offer $p_i \in (0, P_{-i}^0)$ is eliminated in the first elimination round, then (from the second elimination round) the other buyer $-i$ must offer $p_{-i} = 0$, and the unique equilibrium is no trade, and the proof is complete. The rest of the proof deals with the case in which for both buyers some offer $\bar{p}_i \in (0, P_{-i}^0)$ survives the first round. Consequently, for each buyer i the offer P_i survives the second round, since it is a unique best response when the buyer $-i$ bids $p_{-i} = \bar{p}_{-i}$.

Next, we show that if $P_1^0 < 2\Delta_2$, no offer $p_2 \in [P_1^0, 2\Delta_2)$ survives the second elimination round. In particular, any such offer is weakly dominated by $\tilde{p}_2 = 0$, as follows. If $p_1 = P_1$, buyer 2 is indifferent between $\tilde{p}_2 = 0$ and p_2, since $2\Delta_2 \leq P_2^0 \leq P_1$ and so in both cases he obtains zero profits and his capital constraint is satisfied. If $p_1 = 0$, buyer 2 strictly prefers $\tilde{p}_2 = 0$. It remains to show that buyer 2 weakly prefers $\tilde{p}_2 = 0$ to p_2 given all other potential offers from buyer 1 that survive the first round; any such offer would have $p_1 \in (0, 2\Delta_1)$. Given such offer, if buyer 2 offers $\tilde{p}_2 = 0$, his capital constraint is violated with probability of at most p_1, but if he chooses $p_2 \in [P_1^0, 2\Delta_2)$, his capital constraint is violated with a higher

[42] If $p_2 < p_1$, the offer p_1 violates with probability p_1. If $p_2 \geq p_1$, we know from the first round that $p_2 \leq P_2 < P_1^0$, and so the offer p_1 violates the constraint with probability of at least p_1.

probability, namely p_2.

In the third elimination round, any offer $p_1 \in (0, 2\Delta_1)$ is eliminated, since it is weakly dominated by the offer $\tilde{p}_1 = P_1$, as follows. If $p_2 = P_2$, both offers provide the same utility, since $P_2 > P_1 > 2\Delta_1$. If instead, $p_2 < P_1^0$, then buyer 1's capital constraint is violated with a positive probability under the offer p_1 but is never violated under the offer $\tilde{p}_1$. Hence, buyer 1 bids either $p_1 = 0$ or $p_1 = P_1$. If $p_1 = P_1$ is eliminated in the third round, we are done and the unique equilibrium is no trade; otherwise, since $P_2^0 \leq P_1$, the only offer for buyer 2 that survives the fourth round of elimination is $p_2 = 0$, and the unique equilibrium is no trade. Q.E.D.

Proof of Corollary 2. Parts (i) and (iii) follow immediately. For part (ii), observe that $2\Delta < P_2^0 < P_2$ (by Lemma A-1), and so part (B) in Proposition 5 applies. Q.E.D.

Proof of Lemma 3. The proof is by contradiction. Consider an equilibrium in which each seller follows strategy $f(\cdot)$, and suppose the claim in the lemma is not true. Then there exists $0 \leq v' < v'' \leq 1$, such that trade occurs if the value of the asset is v'' but does not occur if the value of the asset is v'. Hence, when the two sellers offer $f(v'')$, the buyer accepts one of the offers, but when the two sellers offer $f(v')$, the buyer rejects both offers. It must also be the case that if only one seller is present and this seller offers $f(v'')$, the buyer accepts the offer. The reason is that this event is along the equilibrium path and the Bayesian inferences regarding the value of the asset are the same as in the event that both sellers offer $f(v'')$.

We obtain a contradiction by showing that a seller with valuation v' can strictly gain by offering $f(v'')$ instead of $f(v')$. Specifically, if he offers $f(v')$, his offer is rejected and his profits are zero. If he offers $f(v'')$, his profits are clearly nonnegative, and with probability ρ, when he is the only seller, his profits are strictly positive, namely, $f(v'') - v' > f(v'') - v'' \geq 0$, where the last inequality follows from the participation constraint of a seller with valuation v''. Q.E.D.

Proof of Proposition 7. Fix a tick size $\varepsilon < 2\Delta$, and consider any value of $\rho > 0$ that is small enough, such that:

$$\varepsilon > \left(\frac{2\rho}{1+\rho} + \varepsilon\right)\frac{1-\rho}{2} \tag{A-1}$$

and

$$\Delta > \frac{1}{1-\frac{2\rho}{1+\rho}}\left(\frac{\varepsilon}{2} + \frac{2\rho}{1+\rho}\right). \tag{A-2}$$

Let $Z(v)$ be the smallest element in the set $\{\varepsilon, 2\varepsilon, ...\}$ that is strictly above $v + \frac{2\rho}{1+\rho}(1-v)$. Observe that

$$\frac{2\rho}{1+\rho}(1-v) < Z(v) - v \leq \frac{2\rho}{1+\rho} + \varepsilon. \tag{A-3}$$

We show that if leverage is low, i.e.,

$$\left(\Delta + \frac{1}{1-\frac{2\rho}{1+\rho}}\left(\varepsilon - \frac{\varepsilon}{2} - \frac{2\rho}{1+\rho}\right)\right)(M+1) - \varepsilon \geq L, \tag{A-4}$$

then the following is an equilibrium: Each seller offers a price $f(v) = Z(v)$; the buyer accepts exactly one offer (in the probability $1-\rho$ event that he receives two offers, he accepts each offer with probability $1/2$); and out-of-equilibrium beliefs are that $v = 0$ if the two sellers make different offers or if a seller offers to sell at a price 0, or at a price that is strictly above 1.

Seller offers are best responses: Equilibrium seller profits are $\left(\frac{1-\rho}{2} + \rho\right)(f(v) - v)$, and by equation (A-3) are at least $\rho(1-v) \geq 0$. Deviating and offering a price above 1 leads to a rejection (because of out-of-equilibrium beliefs), and hence zero profits. Deviating and offering a price $p \in (f(v), 1]$ gives profits of at most $\rho(p-v)$, which by above is less than equilibrium profits. Deviating and offering a price $p \in [\varepsilon, f(v))$ gives profits of at most $f(v) - \varepsilon - v = \left(\frac{1-\rho}{2} + \rho\right)(f(v) - v) - \varepsilon + (f(v) - v)\frac{1-\rho}{2}$, which are below equilibrium profits, using equation (A-3) and equation (A-1). Finally, deviating and offering a price $p = 0$ trivially reduces profits relative to the equilibrium level.

Acceptance by the buyer satisfies his capital constraint: Conditional on observing an offer $Z(v)$, the market believes that $v + \frac{2\rho}{1+\rho}(1-v)$ is distributed uniformly over $[Z(v) - \varepsilon, Z(v))$;

hence, the expected value of v is $\frac{1}{1-\frac{2\rho}{1+\rho}}\left(Z(v)-\frac{\varepsilon}{2}-\frac{2\rho}{1+\rho}\right)$. Hence, we need to show that

$$\left(\Delta+\frac{1}{1-\frac{2\rho}{1+\rho}}\left(Z(v)-\frac{\varepsilon}{2}-\frac{2\rho}{1+\rho}\right)\right)(M+1)-Z(v)\geq L. \tag{A-5}$$

Since the coefficient on $Z(v)$ is positive, the result follows from equation (A-4).

Acceptance by the buyer generates non-negative profits: Conditional on purchasing at a price $Z(v)$, the value of the asset to the buyer is $\frac{1}{1-\frac{2\rho}{1+\rho}}\left(Z(v)-\frac{\varepsilon}{2}-\frac{2\rho}{1+\rho}\right)+\Delta$, so the buyer's expected profits are

$$\Delta+Z(v)\left(\frac{1}{1-\frac{2\rho}{1+\rho}}-1\right)-\frac{1}{1-\frac{2\rho}{1+\rho}}\left(\frac{\varepsilon}{2}+\frac{2\rho}{1+\rho}\right)>0, \tag{A-6}$$

where the inequality follows from condition (A-2).

Limit as $\varepsilon\to 0$: Taking the limit as $\varepsilon\to 0$ (so that $\rho\to 0$ also), $f(v)$ converges to v, and condition (A-4) collapses to $L\leq\Delta(M+1)$, which is equivalent to $\delta<\frac{2\Delta(M+1)}{(1+2\Delta)M}$. Since the buyer purchases the asset at the seller's reservation price, the limit equilibrium is the one that is most preferred by the buyer. Q.E.D.

Proof of Proposition 8.

Part (A): Fix a tick size $\varepsilon<\frac{1}{2}\Delta$, and consider any value of ρ that is small enough that $\frac{1}{1-\frac{2\rho}{1+\rho}}\frac{1}{2}<1$ and $\frac{2(\Delta-\varepsilon)}{1+\rho}\in(\Delta,2\Delta]$, conditions (A-1) and (A-2) are satisfied, and the following conditions are also satisfied:

$$\left(\frac{1}{1-\frac{2\rho}{1+\rho}}\frac{1}{2}\right)(M+1)\geq 1 \tag{A-7}$$

$$\Delta+\frac{1}{1-\frac{2\rho}{1+\rho}}\left(\frac{1}{2}\left(\frac{2(\Delta-\varepsilon)}{1+\rho}+\varepsilon\right)-\frac{\rho}{1+\rho}\right)\geq\frac{2(\Delta-\varepsilon)}{1+\rho}+\varepsilon. \tag{A-8}$$

(Observe that as $\rho\to 0$, condition (A-7) becomes $M>1$, and condition (A-8) becomes $\frac{1}{2}\varepsilon\geq 0$.)

Let v^* be the smallest $v\geq 0$ that satisfies $Z(v)\geq\frac{2(\Delta-\varepsilon)}{1+\rho}$, where $Z(v)$ is as defined in Proposition 7.

We show that if condition (A-4) is violated, but the following condition holds,

$$\left(\Delta+\frac{1}{1-\frac{2\rho}{1+\rho}}\left(\frac{1}{2}\frac{2(\Delta-\varepsilon)}{1+\rho}-\frac{\rho}{1+\rho}\right)\right)(M+1)-\frac{2(\Delta-\varepsilon)}{1+\rho}\geq L, \tag{A-9}$$

then the following is an equilibrium: Each seller offers a price $f(v)=\max\{Z(v^*),Z(v)\}$; the buyer accepts exactly one offer (in the probability $1-\rho$ event that he receives two offers, he accepts each offer with probability 1/2); out-of-equilibrium beliefs are that $v=0$ if the two sellers make different offers, or if a seller offers to sell at a price below $Z(v^*)$, or at a price that is strictly above 1.

Seller offers are best responses: For a seller with valuation $v\geq v^*$, offering $f(v)=Z(v)$ is best response, as in the proof of Proposition 7. For a seller with valuation $v<v^*$, equilibrium profits are $\left(\frac{1-\rho}{2}+\rho\right)(Z(v^*)-v)\geq\left(\frac{1-\rho}{2}+\rho\right)(Z(v)-v)\geq\rho(1-v)$. Deviating and offering a price $p>Z(v^*)$ is suboptimal as in Proposition 7. Deviating and offering a price $p\in[\Delta,Z(v^*)$ is suboptimal because the buyer will reject the offer (out-of-equilibrium beliefs), and the seller will make zero profits. Deviating and offering a price $p\leq\Delta-\varepsilon$ is suboptimal because, by the definition of v^*, equilibrium profits are at least $\left(\frac{1-\rho}{2}+\rho\right)\left(\frac{2(\Delta-\varepsilon)}{1+\rho}-v\right)=\Delta-\varepsilon-\frac{1+\rho}{2}v$, which exceed the maximal profits from this deviation, $\Delta-\varepsilon-v$.

Acceptance by the buyer satisfies his capital constraint: Conditional upon observing the offer $Z(v^*)$, the market believes that $v+\frac{2\rho}{1+\rho}(1-v)$ is distributed uniformly over $[\frac{2\rho}{1+\rho},Z(v^*))$. Hence, the expected value of v is $\frac{1}{1-\frac{2\rho}{1+\rho}}\left(\frac{1}{2}Z(v^*)-\frac{\rho}{1+\rho}\right)$, and we need to show that

$$\left(\Delta+\frac{1}{1-\frac{2\rho}{1+\rho}}\left(\frac{1}{2}Z(v^*)-\frac{\rho}{1+\rho}\right)\right)(M+1)-Z(v^*)\geq L. \tag{A-10}$$

This follows from conditions (A-7) and (A-9) and from the definition of v^*.

For offers $Z(v)>Z(v^*)$, the market value of the asset after acceptance is derived from the belief that $v+\frac{2\rho}{1+\rho}(1-v)$ is distributed uniformly over $(Z(v)-\varepsilon,Z(v)]$. Hence, as in Proposition 7, we need to show that equation (A-5) holds for every $Z(v)>Z(v^*)$. This is indeed the case since we know (A-10) holds, and the market value upon observing

$Z(v) > Z(v^*)$ is greater than the market value associated with the belief that $v + \frac{2\rho}{1+\rho}(1-v)$ is distributed uniformly over $[\frac{2\rho}{1+\rho}, Z(v))$.

Acceptance by the buyer generates non-negative profits: For acceptance of the offer $Z(v) > Z(v^*)$, expected profits are positive as in Proposition 7. For acceptance of $Z(v^*)$, it follows from above that expected profits are

$$\Delta + \frac{1}{1 - \frac{2\rho}{1+\rho}} \left(\frac{1}{2} Z(v^*) - \frac{\rho}{1+\rho} \right) - Z(v^*). \tag{A-11}$$

Since $Z(v^*) \leq \frac{2(\Delta-\varepsilon)}{1+\rho} + \varepsilon$ (from definition of v^*) and $\frac{1}{1-\frac{2\rho}{1+\rho}} \frac{1}{2} < 1$, condition (A-8) implies that expected profits are nonnegative.

Limit as $\varepsilon \to 0$: Taking the limit as $\varepsilon \to 0$ (so that $\rho \to 0$ also), v^* converges to 2Δ, $f(v)$ converges to $\max\{2\Delta, v\}$, and condition (A-9) collapses to $L \leq 2\Delta M$, i.e., $\delta \leq \frac{4\Delta}{1+2\Delta}$. The fact that condition (A-4) is violated collapses to $\delta > \frac{2\Delta(M+1)}{(1+2\Delta)M}$, and we know from the proof of Proposition 7 that a fully revealing equilibrium is impossible (alternatively, it follows from footnote 30). Hence, it follows from the main text that when $\varepsilon \to 0$, any equilibrium with trade must have $f(v) = 2\Delta$ whenever $v \in (0, 2\Delta)$. So in any equilibrium $f(v) \geq \max\{2\Delta, v\}$. Since the limit equilibrium achieves this lower bound on the price, it is the most preferred by the buyer.

Part (B): From above, we know that in any equilibrium with trade, seller on the interval $(0, 2\Delta)$ pool at a price 2Δ, and that conditional on observing the pooling price, the expected value of the asset to the buyer is $\Delta + \frac{1}{2}(2\Delta) = 2\Delta$. Hence, if the buyer purchases the asset at the pooling price, the value of his assets, net of trade, becomes $2\Delta(M+1) - 2\Delta = 2\Delta M$. Since $\delta > \frac{4\Delta}{1+2\Delta}$, it follows that $2\Delta M < L$, so the buyer's capital constraint is violated if he purchases the asset at the pooling price. Hence, from the buyer's ex-ante participation constraint, we cannot have an equilibrium with trade. Q.E.D.

Appendix B: Seller is capital constrained

In this appendix, we extend our basic model to the case in which the seller is capital constrained. For ease of exposition, we analyze the case in which the seller is constrained and the buyer is not.

The seller has M_s units of the asset but can sell only $x < M_s$. The market value of each unit is assumed to be the expected value of $v + \gamma\Delta$, for some $\gamma \in [0, 1]$. Thus, if the seller agrees to sell x units at a price per unit p, the market value of his remaining asset falls to $(\frac{1}{2}p + \gamma\Delta)(M_s - x)$. To insure that the value of remaining assets, net of revenue from selling x units, does not fall below L_s (which denotes the seller's liabilities), the offer p must satisfy:

$$(\frac{1}{2}p + \gamma\Delta)(M_s - x) + px \geq L_s. \tag{B-1}$$

Define $\delta_s \equiv \frac{L_s}{(\frac{1}{2}+\gamma\Delta)M_s}$, which is a measure of the seller's initial leverage, i.e., how tight his capital constraint is ($\delta_s \leq 1$). Then, we obtain a lower bound on the price, and the lower bound increases in δ_s:

$$p \geq P_s(\delta_s) = \frac{\delta_s(\frac{1}{2} + \gamma\Delta)M_s - \gamma\Delta(M_s - x)}{\frac{1}{2}(M_s + x)}. \tag{B-2}$$

The buyer's problem reduces to choosing an offer p such that equation (B-2) is satisfied. As in the basic model, if $P_s(\delta_s) \leq \Delta$, i.e., the seller's initial leverage is low, the buyer makes his benchmark bid, Δ. If leverage is intermediate, $P_s(\delta_s) \in (\Delta, 2\Delta)$, the buyer increases the bid to $P_s(\delta_s)$. If leverage is too high, $P_s(\delta_s) > 2\Delta$, trade cannot occur because any offer that gives the buyer positive profits will violate the seller's capital constraint, if accepted.

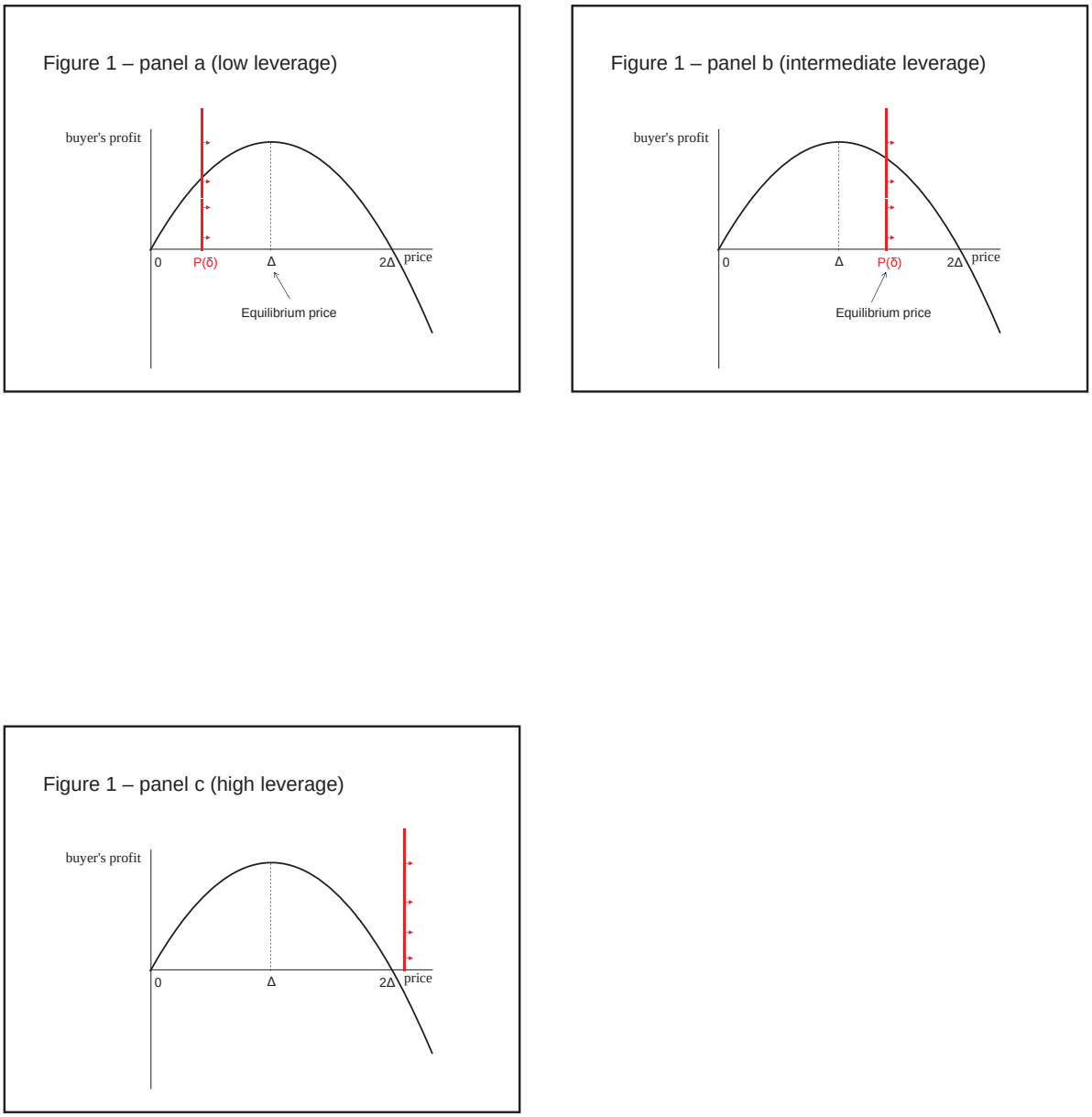

Figure 1: Each panel shows the buyer profit function as a function of the buyer's bid price. The vertical line represents the capital constraint. In panel a, the capital constraint is not binding; in panel b the capital constraint binds; and in panel c,the capital constraint is so tight that the buyer prefers not to bid, and the market freezes.