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Exogenous vs. Endogenous Separation*

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Abstract

This paper assesses how various approaches to modeling the separation margin affect the quantitative ability of the Mortensen-Pissarides labor matching model. The model with a constant separation rate fails to produce realistic volatility and productivity responsiveness of the separation rate and worker flows. The specification with endogenous separation succeeds along these dimensions. Allowing for on-the-job search enables the model to replicate the Beveridge curve. All specifications, however, fail to generate sufficient volatility of the job finding rate. While adopting the Hagedorn-Manovskii calibration remedies this problem, the volume of job-to-job transitions in the on-the-job search specification becomes essentially zero.

JEL codes: E24, J63
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1 Introduction

In its complete form, the Mortensen-Pissarides job matching model (henceforth MP model) endogenously determines both the match creation and separation margins.[1] While researchers agree that match creation is appropriately viewed as endogenous, there is little consensus as to the proper treatment of the separation margin. In this paper, we assess how these various approaches to modeling the separation margin affect the ability of the MP model to explain key facts about unemployment, transition rates, worker flows and other variables.

For this purpose, we use textbook job matching models exposited by Pissarides (2000) that differ only with respect to how the separation margin is modeled. Specifically, match separation is parameterized in three ways: (i) exogenous (and constant) separation rate; (ii) endogenous separation into unemployment; and (iii) endogenous separation with on-the-job search (OJS).[2] This approach allows us to examine transparently the role of the separation margin in a unified framework. Furthermore, our evaluation of the model considers the full dynamic stochastic equilibrium, solved for via a nonlinear method.

We calibrate the model following the standard practice used in the literature.[3] Statistics calculated from simulated data for the three specifications are then compared to corresponding statistics from the empirical data, including transition rates and worker flows, constructed by Fujita and Ramey (2006). The results show, first of all, that the model with a constant separation rate fares poorly in accounting for the volatility of key labor market variables. It does not, of course, explain the substantial variability of the separation rate observed in the data, nor does it generate anywhere near the empirical volatility of unemployment.[4] In addition, the cyclical behavior of gross worker flows in this version of the model is clearly counterfactual: in the data, both unemployment-to-employment (UE) and employment-to-unemployment (EU) worker flows are countercyclical, whereas they are both procyclical in the model. This counterfactual implication arises due to the omission of the cyclical variations of the separation rate.

On the other hand, the two specifications with endogenous determination of separation rates each generate substantially greater volatility of unemployment and worker flows.[5] In the model with OJS, for example, the standard deviation of unemployment equals 60 percent of its empirical value. Moreover, the three specifications match closely the standard deviations of UE and EU flows. Introducing realistic variability at the separation margin

[1] Throughout this paper, the terms "separation" and "job finding" denote movements of workers between employment and unemployment.

[2] In the earlier version of this paper (Fujita and Ramey (2011)) we also consider a case in which separation rates are determined by an exogenous stochastic process.

[3] In particular, the flow value of unemployment, which is known to be crucial for equilibrium volatility, is set to 70 percent of average output.

[4] The latter point has been stressed by Shimer (2005) and Costain and Reiter (2008).

[5] Note that these three specifications are calibrated to match the empirical volatility of the separation rate, so the paper does not directly assess the performance of the model along this dimension. However, there are numerous statistical relationships between the separation rate and other variables that are not used for calibration, and these are available for model evaluation.

thus substantially improves the performance of the model in accounting for unemployment and worker flow variability.

In the data, the separation rate and the two worker flow variables exhibit substantial negative correlations with productivity. The exogenous separation specification fails to replicate this pattern. The two versions with endogenous separation, however, exhibit realistic responsiveness of these variables to productivity. Thus endogeneity of the separation rate appears central to explaining the cyclical properties of the separation rate and worker flows.

The two endogenous separation specifications differ in their ability to account for the Beveridge curve relationship, wherein unemployment and vacancies display a strong negative correlation. In the absence of OJS, the model produces a counterfactually positive unemployment-vacancy correlation, due to the "echo" effect that higher unemployment during downturns makes it easier to find workers, stimulating vacancy posting. With OJS, however, downturns also imply a fall in the number of employed searchers, militating against the rise in unemployment. The unemployment-vacancy correlation becomes strongly negative in this case, matching closely the empirical value. Endogenous separation is therefore consistent with the Beveridge curve relationship when OJS is added to the model.

In summary, the endogenous separation specification with OJS implies empirically reasonable volatility and productivity responsiveness of unemployment, the separation rate and worker flows, together with realistic Beveridge curve and transition rate correlations. Each of the remaining three specifications fails decisively along one or more of these dimensions. This provides strong support for the OJS model as the most valid specification.

The results also show, however, that the MP model under the standard calibration does not produce realistic volatility of the job finding rate, irrespective of how the separation margin is modeled. The empirical standard deviation of the job finding rate is nearly five times the simulated value for each of the three specifications, and the comparison is similar for the productivity elasticity. The two specifications without OJS also deliver an insufficient productivity responsiveness of market tightness (i.e., the vacancy-unemployment ratio). In the OJS specification, however, market tightness is more responsive to productivity, with a productivity elasticity equal to roughly 50 percent of the empirical value. In the OJS specification, the substantial variation in the unemployment rate, together with vacancies that are negatively correlated with unemployment, lead market tightness to be relatively responsive to productivity.

Hagedorn and Manovskii (2008) propose an alternative calibration strategy, drawing on empirical information on wages and profits, which raises the volatility of unemployment, market tightness and other variables in the exogenous separation version of the MP model. To investigate the robustness of the above findings to this alternative, the exogenous separation and OJS versions are suitably recalibrated. In line with Hagedorn and Manovskii's findings, this procedure yields much more realistic volatility of unemployment, the job finding rate, vacancies and market tightness. It does not, however, remedy the key failings of the model with a constant separation rate; in particular, the separation rate and worker flows continue to display unrealistic variability and productivity comovement. Moreover, in the OJS specification, the volume of job-to-job transitions becomes essentially zero. This is

because the worker's bargaining weight is very low under the alternative calibration, making OJS unattractive in nearly all circumstances.

The previous literature contains numerous papers that evaluate dynamic stochastic equilibria of various versions of the MP model.[6] Notably, Mortensen (1994) carries out a calibration-simulation analysis of an OJS specification in continuous time, and stresses the model's ability to explain facts about job creation and destruction in manufacturing. That paper also shows that the model delivers countercyclical worker flows and a negative Beveridge correlation, consistent with the results obtained in the current paper.

Several previous papers have analyzed business cycle properties of the MP model by means of comparative statics analysis of steady states.[7] In particular, Mortensen and Nagypál (2007b) and Mortensen and Nagypál (2007a) may be viewed as a unified treatment of the exogenous and endogenous separation specifications, respectively, within the steady-state paradigm. The former paper allows the separation rate to be either constant or exogenously time-varying, and argues that unemployment volatility is increased in the time-varying case. The latter paper stresses that the effects of endogenous separation depend on how match-specific productivity varies with match duration.

The paper proceeds as follows. Section 2 introduces the three specifications of the MP model and constructs theoretical measures that correspond to the empirical data series. The calibration procedure and numerical solution method are discussed in Section 3, and results are presented in Section 4. In Section 5, the dynamic interrelationships between labor market variables are considered. Section 6 investigates the implications of the Hagedorn-Manovskii calibration approach, and Section 7 concludes.

2 MP Model

There is a unit mass of atomistic workers and an infinite mass of atomistic firms. Time periods are weekly. In any week t, a worker may be either matched with a firm or unemployed, while a firm may be matched with a worker, unmatched and posting a vacancy, or inactive. Unemployed workers receive a flow benefit of b per week, representing the total value of leisure, home production and unemployment insurance payments. Firms that post vacancies pay a posting cost of c per week. Let u_t and v_t denote the number of unemployed workers and posted vacancies, respectively, in week t. In the case of no OJS, the number of new matches formed in week t is determined by a matching function $m(u_t, v_t)$, having a Cobb-Douglas:

[6] Dynamic stochastic equilibria of the MP model with a constant separation rate are evaluated in Merz (1995), Andolfatto (1996), Hall (2005), Nason and Slotsve (2005), Shimer (2005), Yashiv (2006), Fujita and Ramey (2007), Hagedorn and Manovskii (2008) and Shimer (2010); Mortensen and Pissarides (1994), Cooley and Quadrini (1999), Cole and Rogerson (1999), Den Haan et al. (2000), and Walsh (2005) analyze dynamic stochastic equilibria of specifications with an endogenous separation rate, while OJS is assessed in Mortensen (1994), Pissarides (1994), Krause and Lubik (2006), Nagypál (2005) and Tasci (2006). Recently, Menzio and Shi (2011) analyze unemployment and worker transitions using an OJS model that uses an alternative "directed search" approach.

[7] See Hornstein et al. (2005), Shimer (2005), Mortensen and Nagypál (2007b), Mortensen and Nagypál (2007a) and Pissarides (2009).

form:

$$m(u_t, v_t) = Au_t^{\alpha} v_t^{1-\alpha}.$$

Thus, an unemployed worker's probability of obtaining a match in week t, denoted by $f(\theta_t)$, equals $A\theta_t^{1-\alpha}$, where the variable $\theta_t = v_t/u_t$ indicates market tightness. The job filling rate for a vacancy, denoted by $q(\theta_t)$, equals $A\theta_t^{-\alpha}$. The value of v_t in each week is determined by free entry.

A worker-firm match can produce an output level of $z_t x$ during week t, where z_t and x are aggregate and match-specific productivity factors, respectively. The aggregate factor is determined according to the following exogenous process:

$$\ln z_t = \rho_z \ln z_{t-1} + \varepsilon_t^z, \tag{1}$$

where ε_t^z is an i.i.d. normal disturbance with mean zero and standard deviation σ_z. Determination of x is discussed below.

Before engaging in production in week t, the worker and firm negotiate a contract that divides match surplus according to the Nash bargaining solution, where π gives the worker's bargaining weight and the disagreement point is severance of the match. Let $S_t(x)$ indicate the value of match surplus in week t for given x, and let U_t and V_t be the values received by an unemployed worker and a vacancy-posting firm, respectively. The worker and firm will agree to continue the match if $S_t(x) > 0$, while they will separate if separation is jointly optimal, in which case $S_t(x) = 0$. As the outcome of bargaining, the worker and firm receive payoffs of $\pi S_t(x) + U_t$ and $(1-\pi)S_t(x) + V_t$, respectively. Let x^h denote the value of the match-specific productivity in a new match. The unemployment value satisfies:

$$U_t = b + \beta \mathbb{E}_t \Big[f(\theta_t)\pi S_{t+1}(x^h) + U_{t+1} \Big], \tag{2}$$

where $\mathbb{E}_t$ represents the expectation operator with respect to the aggregate state in t and β is the discount factor. The value of posting a vacancy is written as:

$$V_t = -c + \beta \mathbb{E}_t \Big[q(\theta_t)(1-\pi) S_{t+1}(x^h) + V_{t+1} \Big]. \tag{3}$$

In free entry equilibrium, $V_t = 0$ for all t, implying that:

$$\beta q(\theta_t)(1-\pi)\mathbb{E}_t S_{t+1}(x^h) = c. \tag{4}$$

This condition determines θ_t in every period.

2.1 Exogenous Separation

In the exogenous separation version of the MP model, $x = x^h$ is assumed to hold at all times and for all matches. At the end of each week, matches face a risk of exogenous separation. The probability that any existing match separates at the end of week t is given by s.

Let $M_t(x)$ denote the value of a match in week t when the match-specific factor is x.

Since the worker and firm seek to maximize match value as part of Nash bargaining, $M_t(x^h)$ must satisfy the following Bellman equation:

$$M_t(x^h) = \max\Big\{M_t^c(x^h), U_t + V_t\Big\},$$

where $M_t^c(x^h)$ represents the value of the match when the continuation is chosen, and is written as:

$$M_t^c(x^h) = z_t x^h + \beta\mathbb{E}_t\Big[(1-s)M_{t+1}(x^h) + s(U_{t+1} + V_{t+1})\Big].$$

Match surplus then can be expressed as:

$$\begin{aligned} S_t(x^h) &= M_t(x^h) - U_t - V_t \\ &= \max\Big\{S_t^c(x^h), 0\Big\}, \end{aligned} \tag{5}$$

where $S_t^c(x)$ represents match surplus when continuation is chosen. Substituting for U_t from (2) and setting $V_t = 0$ for all t, we can express this term as follows:

$$S_t^c(x^h) = z_t x^h - b + \beta\big(1 - s - f(\theta)\pi\big)\mathbb{E}_t S_{t+1}(x^h).$$

Equations (4) and (5) determine the equilibrium paths of θ_t and $S_t(x^h)$ for given realizations of the z_t process.

2.2 Endogenous Separation

In the endogenous separation version (without OJS), x follows a Markov process. All new matches start at $x = x^h$, but the value of x may switch in subsequent weeks. At the end of each week t, a switch occurs with probability λ. In the latter event, the value of x for week $t+1$ is drawn randomly according to the c.d.f. $G(x)$, taken to be truncated lognormal with parameters μ_x and σ_x for $x < x^h$, and $G(x^h) = 1$. With probability $1-\lambda$, x maintains its week t value into week $t+1$.[8]

When OJS is not allowed, match value satisfies:

$$M_t(x) = \max\Big\{M_t^c(x), U_t + V_t\Big\}$$

where $M_t^c(x)$ again represents the value of the match when continuation of the match is chosen, and is expressed as follows:

$$M_t^c(x) = z_t x + \beta\mathbb{E}_t\left[(1-s)\left(\lambda\int_0^{x^h} M_{t+1}(y)dG(y) + (1-\lambda)M_{t+1}(x)\right) + s(U_{t+1} + V_{t+1})\right],$$

[8]The literature sometimes assumes that match-specific productivity follows an i.i.d. process (e.g., Den Haan et al. (2000) and Krause and Lubik (2007)). However, the degree of persistence is not an innocuous consideration with respect to the cyclicality of the aggregate separation rate. As discussed below, the parameter λ is calibrated by matching the persistence of the aggregate separation rate.

Note that s captures an exogenous component of the separation rate. The Bellman equation for match surplus is

$$S_t(x) = \max\left\{S_t^c(x), 0\right\}, \tag{6}$$

where $S_t^c(x)$ represents the value of match surplus after continuation of the match is chosen:

$$S_t^c(x) = z_t x - b + \beta\mathbb{E}_t\left[(1-s)\left(\lambda\int_0^{x^h} S_{t+1}(y)dG(y) + (1-\lambda)S_{t+1}(x)\right) - f(\theta_t)\pi S_{t+1}(x^h)\right].$$

Equations (4) and (6) determine the equilibrium paths of θ_t and $S_t(x)$ for given realizations of the z_t process.

2.3 OJS

The OJS version of the MP model extends the endogenous separation version by allowing matched workers to search at a cost of a. The worker search pool expands to $u_t + \phi_t$, where ϕ_t indicates the number of matched workers who search in week t. Total match formation in week t is now equal to $m(u_t + \phi_t, v_t)$. The expressions for matching probabilities stay the same as before, with a suitable redefinition of the market tightness variable; that is, $\theta_t \equiv v_t/(u_t + \phi_t)$.

When an employed searching worker makes a new match in week t, the worker must renounce the option of keeping his current job before bargaining with the new firm at the start of week $t+1$. As a consequence, the worker receives a payoff of $\pi S_{t+1}(x^h) + U_{t+1}$ from the new match. Since the worker's payoff from the current job cannot exceed this value, it is optimal for the worker always to accept a new match.[9]

In the OJS version of the model, the match continuation decision is characterized by:

$$M_t(x) = \max\left\{M_t^{cs}(x), M_t^{cn}(x), U_t + V_t\right\},$$

where $M_t^{cn}(x)$ and $M_t^{cs}(x)$represent the value of continuation of the match with no OJS and with OJS, respectively. These two terms are expressed as follows:

$$M_t^{cn}(x) = z_t x + \beta\mathbb{E}_t\left[(1-s)\left(\lambda\int_0^{x^h} M_{t+1}(y)dG(y) + (1-\lambda)M_{t+1}(x)\right) + s(U_{t+1} + V_{t+1})\right],$$

[9]The assumption that all new matches start at the highest productivity level greatly simplifies the analysis of OJS, in that it implies all job offers are accepted (see Mortensen (1994) and Pissarides (2000)). Without it, solving for dynamic stochastic equilibria may require more explicit consideration of the distribution of match productivities. Moreover, the assumption leads to counterfactual implications concerning how wages and separation rates depend on match duration. Examining the micro- and macro-level implications of this modeling issue is beyond the scope of the paper. See Mortensen and Nagypál (2007a) for analysis of the issue within the steady-state paradigm.

$$M_t^{cs}(x) = z_t x - a + \beta \mathbb{E}_t \Bigg[f(\theta_t) \Big(\pi S_{t+1}(x^h) + U_{t+1} + V_{t+1} \Big)$$
$$+ (1 - f(\theta_t)) \Bigg\{ (1-s) \Bigg(\lambda \int_0^{x^h} M_{t+1}(y) dG(y) + (1-\lambda) M_{t+1}(x) \Bigg) + s(U_{t+1} + V_{t+1}) \Bigg\} \Bigg].$$

Assuming that the worker's search decision is contractible, the Bellman equation for match surplus is written as:

$$S_t(x) = \max \Big\{ S_t^{cs}(x), S_t^{cn}(x), 0 \Big\}, \tag{7}$$

where $S_t^{cs}(x)$ and $S_t^{cn}(x)$ represent match surplus with and without OJS. Using the equation for U_t and setting $V_t = 0$, these two terms may be expressed as:

$$S_t^{cs}(x) = z_t x - a - b + \beta(1 - f(\theta_t))(1-s) \mathbb{E}_t \Bigg[\lambda \int_0^{x^h} S_{t+1}(y) dG(y) + (1-\lambda) S_{t+1}(x) \Bigg], \tag{8}$$

$$S_t^{cn}(x) = z_t x - b$$
$$+ \beta \mathbb{E}_t \Bigg[(1-s) \Bigg(\lambda \int_0^{x^h} S_{t+1}(y) dG(y) + (1-\lambda) S_{t+1}(x) \Bigg) - f(\theta_t) \pi S_{t+1}(x^h) \Bigg]. \tag{9}$$

Equilibrium θ_t and $S_t(x)$ are determined by (4) and (7) in this case.

2.4 Measurement

Equilibrium worker transition rates and flows are measured as follows. A worker who is unemployed in week t becomes employed in week $t+1$ with probability $f(\theta_t) = A\theta_t^{1-\alpha}$. Thus, for all specifications the measured job finding rate and number of UE flows for week $t+1$ are

$$JFR_{t+1} = A\theta_t^{1-\alpha}, \qquad UE_{t+1} = A\theta_t^{1-\alpha} u_t.$$

Moreover, in the specification with a constant separation rate, a worker who is employed in week t becomes unemployed in week $t+1$ with probability s, giving the following measured separation rate and the number of EU flows:

$$SR_{t+1} = s, \qquad EU_{t+1} = s(1 - u_t).$$

The latter equation shows that EU (separation) flows in the model with a constant separation rate vary procyclically, insofar as unemployment moves countercyclically. We show below that this is counterfactual.

Separation rates and EU flows in the two endogenous separation versions of the model depend on the distribution of x across existing matches. Let $e_t(x)$ denote the number of matches in week t having match-specific factors less than or equal to x; note that $e_t(x^h)$ gives

total employment. Since $S_t(x)$ is strictly increasing in x wherever $S_t(x) > 0$, there exists a value R_t such that $S_t(x) = 0$ if and only if $x \leq R_t$. Thus, separation into unemployment occurs at the start of week $t+1$ whenever $x \leq R_{t+1}$.[10] In equilibrium, $e_{t+1}(x) = 0$ for $x \leq R_{t+1}$. The employment distribution differs depending on whether or not OJS is allowed.

Endogenous separation without OJS. In the absence of OJS, the employment distribution evolves according to:

$$e_{t+1}(x) = (1-s)\Big(\lambda\big[G(x) - G(R_{t+1})\big]e_t(x^h) + (1-\lambda)\big[e_t(x) - e_t(R_{t+1})\big]\Big),$$

for $x \in (R_{t+1}, x^h)$. For $x = x^h$, it evolves according to:

$$e_{t+1}(x^h) = (1-s)\Big(\lambda\big[1 - G(R_{t+1})\big]e_t(x^h) + (1-\lambda)\big[e_t(x^h) - e_t(R_{t+1})\big]\Big) + f(\theta_t)u_t, \quad (10)$$

which gives the evolution of the stock of employment. Next, total EU flows and the separation rate are, respectively, given by:

$$EU_{t+1} = se_t(x^h) + (1-s)\Big(\lambda G(R_{t+1})e_t(x^h) + (1-\lambda)e_t(R_{t+1})\Big), \quad (11)$$

$$SR_{t+1} = \frac{EU_{t+1}}{e_t(x^h)}. \quad (12)$$

The implied law of motion for unemployment is:

$$u_{t+1} = u_t + EU_{t+1} - UE_{t+1}. \quad (13)$$

Lastly, vacancies are determined simply by:

$$v_t = \theta_t u_t.$$

Endogenous separation with OJS. Allowing for the possibility of OJS somewhat complicates the evolution of labor market variables. First, it can be shown that there exists a value R_t^s such that the match surplus from OJS exceeds the surplus from continuing the match with no OJS if and only if $x < R_t^s$. In other words, OJS is chosen whenever $R_t < R_t^s$ and $x \in (R_t, R_t^s)$. Therefore, for $x \in (R_t, R_t^s)$:

$$\begin{aligned} e_{t+1}(x) =& (1-s)\Big(\lambda\big[G(x) - G(R_{t+1})\big]\big[e_t(x^h) - e_t(R_t^s) + (1-f(\theta_t))e_t(R_t^s)\big] \\ &+ (1-\lambda)(1-f(\theta_t))\big[e_t(x) - e_t(R_{t+1})\big]\Big), \end{aligned}$$

[10] When $x = R_{t+1}$, the firm and worker could also choose to continue their match, as a matter of indifference. It is slightly more convenient for notational purposes to specify that separation occurs at the R_{t+1} margin.

while, for $x \in [R_t^s, x^h)$:

$$\begin{aligned} e_{t+1}(x) =& (1-s)\Big(\lambda\big[G(x) - G(R_{t+1})\big]\big[e_t(x^h) - e_t(R_t^s) + (1-f(\theta_t))e_t(R_t^s)\big] \\ &+ (1-\lambda)\big[e_t(x) - e_t(R_t^s) + (1-f(\theta_t))(e_t(R_t^s) - e_t(R_{t+1}))\big]\Big). \end{aligned}$$

In essence, the presence of OJS alters the evolution of the employment distribution in that OJS makes it possible for those on-the-job searchers to avoid endogenous separation into unemployment (when the search is successful) given that their match-specific factor starts at the highest level x^h in the following period. The law of motion for the total employment stock is written as:

$$\begin{aligned} e_{t+1}(x^h) =& (1-s)\Big(\lambda\big[1 - G(R_{t+1})\big]\big[e_t(x^h) - e_t(R_t^s) + (1-f(\theta_t))e_t(R_t^s)\big] \\ &+ (1-\lambda)\big[e_t(x^h) - e_t(R_t^s) + (1-f(\theta_t))(e_t(R_t^s) - e_t(R_{t+1}))\big]\Big) \\ &+ f(\theta_t)(u_t + e_t(R_t^s)). \end{aligned} \tag{14}$$

Note that (14) differs from the corresponding equation for the version without OJS (equation (10)), even though job-to-job transitions simply reshuffle workers within the employment pool. This property comes from the fact that when on-the-job searchers find a new job, they essentially avoid endogenous separation into unemployment. Accordingly, EU flows are measured differently in the model with OJS, relative to those in the model without OJS:

$$\begin{aligned} EU_{t+1} =& se_t(x^h) + (1-s)\Big(\lambda G(R_{t+1})\big[e_t(x^h) - e_t(R_t^s) + (1-f(\theta_t))e_t(R_t^s)\big] \\ &+ (1-\lambda)(1-f(\theta_t))e_t(R_{t+1})\Big). \end{aligned} \tag{15}$$

The expressions for the separation rate and the law of motion for unemployment remain the same as (12) and (13), respectively. Lastly, vacancies are determined by:

$$v_t = \theta_t(u_t + e_t(R_t^s)).$$

3 Simulation

Before examining the quantitative properties of the different versions discussed above, this section presents the calibration procedure. We then lay out the method to compute the stochastic dynamic equilibrium of the model and summarize the procedure to evaluate various quantitative aspects of the model.

3.1 Calibration

There are three specifications of the model to calibrate. Parameter choices for the these three cases are given in Table 1.

The parameters b, α and π are set to the standard values, as discussed by Mortensen and Nagypál (2007b). First, the flow value of unemployment is set to 0.7, which amounts to 70 percent of average output per worker, given that productivity is normalized to unity. We will later consider the calibration proposed by Hagedorn and Manovskii (2008), in which b is set to a higher value. The elasticity parameter of the matching function α and the bargaining weight of workers π are both set to 0.7. This is close to the values of a and π (0.72) used in Shimer (2005). Mortensen and Nagypál (2007b) argue that 0.72 is empirically too high and estimate it at 0.45. However, a more recent paper by Brügemann (2008) reconciles the two estimates and proposes estimates between 0.54 and 0.63. In this paper, we use the value originally estimated by Shimer (2005) as a benchmark value and then later examine the robustness of our results when the alternative values $\alpha = \pi = 0.5$ are used.

Calibration of the vacancy posting cost c draws on survey evidence on employer recruitment behavior. Survey results cited in Barron et al. (1997) point to an average vacancy duration of roughly three weeks. Moreover, Barron and Bishop (1985) find an average of about nine applicants for each vacancy filled, with two hours of work time required to process each application. These figures suggest an average investment of 20 hours per vacancy filled, or 6.7 hours per week the vacancy is posted. This amounts to 17 percent of a 40-hour workweek; thus, it is reasonable to assign this value to c, given that weekly productivity is normalized to unity.

Next, to ensure comparability across different versions of the model, the highest value of match-specific productivity x^h is adjusted to generate mean match productivity of unity in all cases. The cost of searching on the job a in the OJS specification is chosen so that the mean monthly job-to-job transition rate in the simulated data matches the value of 3.2 percent calculated by Moscarini and Thompson (2007) using the CPS data.

The parameters for the aggregate productivity process ρ_z and σ_z are set to the values proposed by Hagedorn and Manovskii (2008).[11] The value of the weekly discount factor β is consistent with an annual interest rate of four percent.

Selection of the remaining parameters relies on monthly job finding and separation rate data from Fujita and Ramey (2006). These data derive from the CPS for the 1976-2005 period and are adjusted for margin error and time aggregation error. In all cases, the parameters A and s are chosen to ensure that the simulated data generate mean monthly job finding and separation rates of 34 percent and two percent, respectively, consistent with the Fujita-Ramey evidence.

For endogenous separation versions of the model, the arrival rate of the match-specific productivity shock λ and its standard deviation σ_x are selected to match the standard deviation and first-order autocorrelation of the simulated separation rate series, aggregated to quarterly, logged and HP filtered (with smoothing parameter 1,600), to the empirical values of these moments in the Fujita-Ramey data. More specifically, σ_x is used to achieve the standard deviation of the cyclical component of the empirical separation rate series. We

[11] In the earlier version of the paper (Fujita and Ramey (2011)), we set these two parameters based on the VAR evidence in Fujita and Ramey (2007), in which case these two parameters are set to 0.99 and 0.027, respectively, instead of the values in Table 1. We find that the results are robust with respect to this alternative calibration.

Table 1: Parameter Values for Benchmark Calibration

Parameter	Constant	Endogenous without OJS	Endogenous with OJS
b	0.7	0.7	0.7
c	0.17	0.17	0.17
a	–	–	0.13
A	0.095	0.094	0.096
α	0.7	0.7	0.7
π	0.7	0.7	0.7
x_h	1	1.15	1.1
s	0.005	0.0034	0.0042
λ	–	0.085	0.085
σ_x	–	0.16	0.214
ρ_z	0.9895	0.9895	0.9895
σ_z	0.0034	0.0034	0.0034
β	0.9992	0.9992	0.9992

Notes. b: unemployment payoff; c: vacancy posting cost; a: OJS cost; A: scale parameter of the matching function; α: elasticity parameter of the matching function; π: worker bargaining weight; x^h: highest value of match-specific productivity; s: exogenous separation rate; λ: arrival rate of the match-specific productivity shock; σ_x: S.D. of the match-specific productivity shock; ρ_z: persistence of the aggregate productivity process; σ_z: S.D. of the aggregate productivity shock; β: discount factor.

adjust λ to match the first-order autocorrelation coefficient of the series. The chosen value 0.085 implies a mean waiting time of three months between switches of the match-specific productivity factor. The persistence of the separation rate is useful in identifying the arrival rate, in that more (less) frequent arrival of the shock tends to raise (lower) the persistence. We discuss the intuition behind this property of the model in Section 5, where we examine alternative parameterizations.

3.2 Solution Method

The model consists of the free entry condition (4), the surplus equation (5), (6) or (7), and the driving process (1). To solve the model, let the stochastic elements be represented on grids. The method of Tauchen (1986) is used to represent the process z_t as a Markov chain having a state space $\{z_1, ..., z_I\}$ and the transition matrix $\Delta^z = [\delta^z_{ij}]$, where $\delta^z_{ij} = \Pr\{z_{t+1} = z_j | z_t = z_i\}$. $G(x)$ is approximated by a discrete distribution with support $\{x_1, ..., x_M\}$, satisfying $x_1 = 1/M$, $x_m - x_{m-1} = x^h/M$ and $x_M = x^h$. The associated probabilities $\{\gamma_1, ..., \gamma_M\}$ are $\gamma_m = g(x_m)/M$ for $m = 1, ..., M-1$, where $g(x)$ is the lognormal density,

and $\gamma_M = 1 - \gamma_1 - ... - \gamma_{M-1}$. Market tightness and match surplus may be represented as:

$$\theta_t = \theta(z_i), \quad S_t(x_m) = S(z_i, x_m),$$

where z_i is the aggregate state prevailing in period t. Equations (4), (5) and (6) take the forms, for $i = 1, ..., I$, $m = 1, ..., M$:

$$\beta A \theta(z_i)^{-\alpha}(1-\pi) \sum_j \delta_{ij}^z S(z_j, x^h) = c, \tag{16}$$

$$S(z_i, x^h) = \max\Big\{ z_i x^h - b + \beta(1 - s - \beta A \theta(z_i)^{1-\alpha}\pi) \sum_j \delta_{ij}^z S(z_j, x^h), 0 \Big\}, \tag{17}$$

$$\begin{aligned} S(z_i, x_m) = &\max\Big\{ z_i x_m - b + \beta(1-s)\Big(\lambda \sum_{j,n} \delta_{ij}^z \gamma_n S(z_j, x_n) + \beta(1-\lambda) \sum_j \delta_{ij}^z S(z_j, x_m)\Big) \\ &- \beta A \theta(z_i, s_k)^{1-\alpha} \pi \sum_j \delta_{ij}^z S(z_j, x^h), 0 \Big\}, \end{aligned} \tag{18}$$

and similarly for (7). Numerical solutions are obtained via backward substitution. For example, let $\theta^T(z_i)$ and $S^T(z_i, x^h)$ be the functions obtained after T iterations of (16) and (17). At iteration $T+1$, these functions are updated to

$$S^{T+1}(z_i, x^h) = \max\Big\{ z_i x^h - b + \beta(1 - s - \beta A \theta^T(z_i)^{1-\alpha}\pi) \sum_j \delta_{ij}^z S^T(z_j, x^h), 0 \Big\}$$

$$\theta^{T+1}(z_i) = \left(\frac{\beta A(1-\pi)}{c} \sum_j \delta_{ij}^z S^{T+1}(z_j, x^h) \right)^{\frac{1}{\alpha}}.$$

Convergence follows as a consequence of the saddlepoint stability property of the matching model, which makes for stability in the backward dynamics.[12]

3.3 Evaluation Procedure

The empirical data series used for purposes of model evaluation are constructed as follows. Job finding and separation rates, and UE and EU flows, are quarterly averages of the monthly series from Fujita and Ramey (2006), covering 1976Q1-2005Q4. Employment and the unemployment rate are quarterly averages of the CPS official monthly series covering the same period. The productivity series is obtained by dividing quarterly GDP by the employment series in the CPS. Vacancies are measured as quarterly averages of the monthly composite Help-Wanted index constructed by Barnichon (2010). All quarterly series are logged and HP

[12] In solving the model, $I = 13$ and $M = 200$ are chosen. The tolerance for point-wise convergence of $\theta(z_i)$ and $S(z_i, x_m)$ is 10^{-8}.

filtered, with a smoothing parameter of 1,600.

To conform with the empirical series, the weekly data from the model are averaged to quarterly frequency, logged and HP filtered using smoothing parameter 1,600. Each simulated quarterly series consists of 320 observations, of which the last 120 are used to calculate the reported statistics. For each of the three specifications, 1,000 replications are run and averages of the statistics across the replications are presented.

4 Main Results

This section discusses the main results of the paper. Table 2 presents various second moment properties of the three specifications of the MP model as well as those of the observed data. In each panel of the table, we present the standard deviation, the elasticity with respect to labor productivity, the correlation coefficient with labor productivity, and the autocorrelation coefficient for the seven variables listed across the first row of the table. Later, we also consider cross correlations between unemployment and vacancies, i.e., the Beveridge correlations, and between the separation rate and the job finding rate.

4.1 Unemployment and Worker Transition Rates

The first three columns of Table 2 compare the empirical moments of unemployment and worker transition rates with the values obtained from the three specifications of the model. The empirical standard deviation of unemployment, equalling 9.6 percent, is roughly nine times greater than the value of 1.1 percent generated by the exogenous separation specification. This conforms to the observation of Costain and Reiter (2008) and Shimer (2005) that the MP model with a constant separation rate produces far too little unemployment volatility.

However, the empirical separation rate is not in fact constant, as it has a standard deviation of 5.8 percent. The other two versions of the MP model, which allow for fluctuations in the separation rate, are calibrated to match the standard deviation of the empirical series. These two specifications yield significantly greater unemployment volatility. The standard deviation of unemployment in the OJS specification, for example, is 5.9 percent, or over 60 percent of its empirical value. Thus, incorporating variability at the separation margin, under either of the two specifications, greatly enhances the ability of the MP model to produce realistic unemployment volatility.

At the same time, all three specifications of the MP model yield highly unrealistic volatility of the job finding rate, with the empirical standard deviation being almost six times the simulated value in each specification. Improving the model's performance at the separation margin does not mitigate its problems at the job finding margin.

With respect to contemporaneous correlations with productivity, all three specifications produce strong negative comovement between unemployment and productivity. Similarly, all three specifications give rise to strong positive productivity comovement for the job finding rate. Actually, the positive correlation in the model is much stronger than the empirical

Table 2: Second Moment Properties: Benchmark Calibration

X_t	u_t	JFR_t	SR_t	UE_t	EU_t	v_t	v_t/u_t
			(a) Data				
σ_X	0.096	0.077	0.058	0.042	0.052	0.126	0.218
$cor(p_t, X_t)$	−0.460	0.369	−0.535	−0.337	−0.521	0.564	0.527
$cov(p_t, X_t)/\sigma_p$	−5.914	3.786	−4.157	−1.879	−3.644	9.524	15.437
$cor(X_t, X_{t-1})$	0.926	0.804	0.631	0.416	0.560	0.920	0.930
			(b) Constant separation				
σ_X	0.011	0.013	–	0.006	0.001	0.034	0.043
$cor(p_t, X_t)$	−0.884	0.996	–	0.572	0.860	0.988	0.999
$cov(p_t, X_t)/\sigma_p$	−0.720	0.959	–	0.258	0.041	2.486	3.205
$cor(X_t, X_{t-1})$	0.859	0.764	–	0.386	0.859	0.701	0.764
			(c) Endogenous separation without OJS				
σ_X	0.056	0.013	0.057	0.044	0.054	0.021	0.042
$cor(p_t, X_t)$	−0.925	0.994	−0.908	−0.865	−0.897	−0.437	0.998
$cov(p_t, X_t)/\sigma_p$	−4.116	1.004	−4.110	−3.041	−3.876	−0.757	3.358
$cor(X_t, X_{t-1})$	0.826	0.764	0.608	0.825	0.585	0.648	0.764
			(d) Endogenous separation with OJS				
σ_X	0.059	0.014	0.058	0.047	0.054	0.042	0.096
$cor(p_t, X_t)$	−0.886	0.995	−0.904	−0.797	−0.899	0.969	0.998
$cov(p_t, X_t)/\sigma_p$	−3.936	1.020	−3.944	−2.839	−3.704	3.063	7.180
$cor(X_t, X_{t-1})$	0.847	0.764	0.700	0.846	0.682	0.674	0.764

Notes. σ_X: standard deviation of the variable X; $cor(p_t, X_t)$: correlation between labor productivity p_t and X_t; $cov(p_t, X_t)/\sigma_p$: elasticity of X_t with respect to p_t; $cor(X_t, X_{t-1})$: correlation between X_t and X_{t-1}. Data sources: u_t: quarterly average of monthly official BLS unemployment rate; transition rates and worker flows: quarterly average of the monthly series constructed by Fujita and Ramey (2006); v_t: quarterly average of monthly composite Help-Wanted index constructed by Barnichon (2010); p_t: quarterly output per worker constructed by dividing real GDP by CPS employment series; sample period: 1976Q1-2005Q4. All series are logged and HP filtered, with smoothing parameter 1,600. See subsection 2.4 for measurement in the model. Simulated data are quarterly averages of weekly series, logged and HP filtered, with smoothing parameter 1,600. Each replication computes simulated statistics from a sample of 120 quarterly observations. Reported statistics are averages over 1,000 replications.

value. This is because the model does not adequately replicate the sluggishness of the labor market, as pointed out by Fujita and Ramey (2007), who introduce a one-time job creation cost in the version with a constant separation rate to address this problem. The exogenous separation specification fails to replicate the negative correlation between productivity and the separation rate that is a robust feature of the data. The two endogenous separation specifications succeed in capturing this negative correlation.

Elasticities of the variables with respect to productivity are shown in the next row.[13]

[13]These productivity elasticities are computed as follows. Let p_t denote productivity in quarter t, and let X_t be any series. Then the productivity elasticity is $cov(p_t, X_t)/\sigma(p_t)$.

The productivity elasticities offer somewhat cleaner measures of comovement, insofar as they reflect the effects of variations in productivity in isolation from other disturbances; see Mortensen and Nagypál (2007b). The elasticities may also be interpreted as rough measures of responsiveness to productivity shocks. For unemployment, the empirical productivity elasticity of -5.9 is roughly eight times greater in magnitude than the elasticities produced by the exogenous separation version of the model. However, when the separation margin is endogenized, whether OJS is allowed or not, the elasticity increases considerably to a level not far from the empirical counterpart.

Findings are similar for the separation rate elasticities, where the exogenous separation specification provides highly unrealistic values, while those of the endogenous separation specifications are empirically plausible. Across all three specifications, however, the productivity elasticities of the job finding rate are far too low: the empirical value is 3.8, while the simulated values are always around 1.

In summary, introducing endogenous determination of the separation rate greatly magnifies the degree of unemployment volatility generated by the MP model. Moreover, when the separation rate is endogenous, the model generates realistic responsiveness of unemployment and the separation rate to productivity shocks. However, in all three specifications, the simulated job finding rate is deficient in both its volatility and its responsiveness to productivity.

4.2 Worker Flows

The fourth and fifth columns consider gross flows of workers between unemployment and employment. As panel (b) of Table 2 indicates, the exogenous separation specification produces almost no volatility in UE and EU flows. This is contrary to the data, where the standard deviations for both flows are roughly half of the standard deviation of unemployment. The two specifications with endogenous separation rates, in contrast, do a good job of matching the empirical standard deviations of both UE and EU flows. Thus, endogeneity at the separation margin is crucial for producing realistic variability in worker flows.

In terms of correlation with productivity, the exogenous separation specification gives rise to a counterfactual pattern whereby worker flows exhibit a strong positive correlation with productivity. This contradicts the substantial negative correlation seen in the data. In the exogenous separation specification, worker flows are driven principally by procyclical movements in the job finding rate, allowing little scope for explaining their observed countercyclical movements. The two endogenous separation specifications, on the other hand, produce strong negative correlations between productivity and worker flows.

Results on productivity elasticity indicate that worker flows are almost entirely unresponsive to productivity in the exogenous separation specification, whereas they exhibit strong negative responses in the two endogenous separation specifications.

Note that the exogenous separation specification produces procyclical separation (EU) flows because employment is procyclical and thus the number of workers who separate is procyclical. For a symmetric reason, hiring (UE) flows would be countercyclical if the job finding rate were to be treated as constant. The countercyclicality of both EU and UE flows

is a salient feature of the data that indicates the importance of variations in the separation rate for understanding labor market dynamics, as emphasized by Fujita and Ramey (2006) and Fujita (2011).

4.3 Vacancies and Market Tightness

Vacancies and market tightness are considered in the last two columns of Table 2. First, all three specifications produce insufficient volatility of both vacancies and market tightness, consistent with the low volatility of the job finding rate discussed earlier. Observe that market tightness is significantly more volatile in the OJS version than in the other specifications, however. In particular, the standard deviation of market tightness in the OJS specification is more than twice that of the endogenous separation specification without OJS, even though the differences in the standard deviations of unemployment and vacancies are small. This comes from the fact that the two variables are strongly positively correlated in the version without OJS, while they are strongly negatively correlated in the version with OJS, as discussed below. The latter negative correlation serves to increase the variability of market tightness.

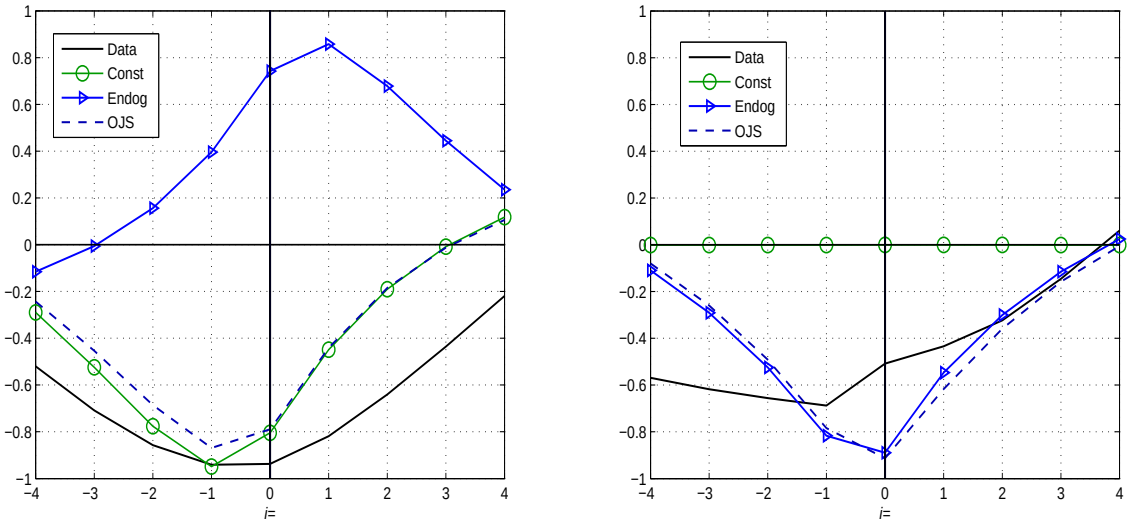
The exogenous separation model replicates the procyclical movements of vacancies seen in the data, whereas the endogenous separation model without OJS yields countercyclical movements. The latter finding reflects conflicting effects on the incentive to post vacancies. Following a negative productivity shock, the returns to forming a new match are relatively low, reducing vacancy posting incentives. This effect drives vacancies downward in the exogenous separation version of the model. In the endogenous separation version without OJS, however, the separation rate rises in response to the productivity shock, pushing up the number of unemployed workers. This raises the vacancy matching probability and enhances the incentive to post vacancies. On balance, the latter effect dominates, and vacancies become negatively correlated with productivity. Since unemployment is also negatively correlated with productivity, vacancies and unemployment become positively correlated in this version of the model.

The OJS model, on the other hand, produces a strong positive correlation between vacancies and productivity, despite the fact that the separation rate is determined endogenously. With OJS, a negative productivity shock induces a fall in the number of employed searchers, which partially offsets the rise in unemployment. Thus, endogenous separation is consistent with realistic vacancy comovement once OJS is incorporated. Moreover, procyclical vacancy adjustment leads to negative correlation between vacancies and unemployment.[14] Note finally that all three specifications yield positive productivity comovement for market tightness, consistent with the data.

The empirical productivity elasticity of vacancies far exceeds the elasticities obtained from all three specifications, in line with the comparison in terms of the standard deviations.

[14]The mechanism discussed here explains why endogenizing the separation margin without OJS reduces the volatility of vacancies relative to the exogenous separation specification and introducing OJS restores the level of the volatility.

Figure 1: Cross Correlations



(a) Correlation between v_{t+i} and u_t

(b) Correlation between SR_{t+i} and JFR_t

For market tightness, however, the OJS version of the model performs noticeably better than the other two versions, generating productivity elasticity that is roughly one-half of its empirical value.

In summary, the OJS version of the model performs the best among the versions considered. It matches all correlation patterns. In particular, the OJS version overcomes the deficiency of the endogenous separation model (without OJS) that vacancies become countercyclical. Even with OJS, however, the MP model fails to match the volatility of vacancies and thereby the job finding rate.

4.4 Cross Correlations

Next, we examine whether the three versions of the MP model can replicate the dynamic relationship between the key labor market variables. Specifically, we consider cross correlations between unemployment and vacancies (i.e., the Beveridge curve) and between the separation rate and the job finding rate.

Panel (a) of Figure 1 presents the Beveridge correlations, where the current-period unemployment rate is associated with future and lagged values of vacancies up to four quarters. First observe that a large value of contemporaneous correlation between unemployment and vacancies observed in the data is reasonably well matched by the value generated by the exogenous separation specification. The endogenous separation specification, in contrast, produces a highly counterfactual value of 0.75. In this version of the model, a negative productivity shock produces a large inflow into unemployment, making workers easier to find and raising the incentive to post vacancies. For the OJS model, the unemployment-vacancy

contemporaneous correlation amounts to -0.79, which is reasonably close to the empirical value. Here, procyclical movements in the number of employed searchers lead to procyclical changes in vacancy posting incentives, giving rise to a realistic Beveridge correlation.

With respect to the lead-lag relationship, the observed data suggest some tendency for vacancies to lead unemployment.[15] Qualitatively, this pattern is captured well in the constant separation rate version and the OJS version of the model. This reflects the mechanics of the model, wherein the search friction produces some lagged response in unemployment after the response in vacancy posting. In the endogenous separation version without OJS, however, the feedback from the movement of the separation rate into vacancy posting as discussed above erases this feature, generating the tendency that unemployment leads vacancies.

Panel (b) presents cross correlations between the two transition rates, where the current-period job finding rate is associated with future and lagged values of the separation rate. In the data, job finding and separation rates exhibit strong negative correlation contemporaneously. The correlations are zero under the assumption of the constant separation rate. The two endogenous separation specifications, on the other hand, produce strong negative contemporaneous correlations, on the order of -0.9. The latter specifications achieve the correct transition rate comovement chiefly because the two rates themselves respond realistically to the common underlying productivity process.

Turning to the lead-lag relationship, the data imply that the separation rate leads the job finding rate, which is indicated by the fact that larger negative correlations are achieved when lagged values of the separation rate are associated with the current-period job finding rate. While the correlations for the two endogenous separation versions exhibit a slight negative phase shift, they fail to adequately capture the overall dynamic pattern. Of course, all of these dynamic correlations are zero under the assumption of a constant separation rate.

5 Alternative Parameterizations

5.1 Robustness of Calibration

This section evaluates the robustness of our results to setting the matching function elasticity α and the worker bargaining weight π at a lower value of 0.5.[16] The model is re-calibrated to achieve the same moment conditions discussed above.

Calibrated parameter values in the exogenous separation version and the two endogenous separation versions are presented in Table 3. The results are presented in Table 4. The results are very similar to those under the benchmark calibration. That is, the endogenous

[15] For this, observe that correlations between lagged values of vacancies and current unemployment tend to be larger (in absolute value) than those between future values of vacancies and current unemployment.

[16] As mentioned before, our benchmark calibration of these two parameters is based on Shimer (2005), who estimates the elasticity with respect to unemployment at 0.72. Mortensen and Nagypál (2007b) estimate the elasticity parameter at 0.45 using a different method. Brügemann (2008) then reconciles the difference between these two estimates and proposes a value between 0.54 and 0.63. Our choice of 0.5 is thus a conservative value for checking the robustness of our results.

Table 3: Parameter Values for Alternative Calibration: $\alpha = \pi = 0.5$

Parameter	Constant	Endogenous without OJS	Endogenous with OJS
b	0.7	0.7	0.7
c	0.17	0.17	0.17
a	–	–	0.08
A	0.068	0.067	0.071
α	0.5	0.5	0.5
π	0.5	0.5	0.5
x_h	1	1.18	1.125
s	0.005	0.0019	0.0038
λ	–	0.085	0.085
σ_x	–	0.27	0.326
ρ_z	0.9895	0.9895	0.9895
σ_z	0.0034	0.0034	0.0034
β	0.9992	0.9992	0.9992

Notes. See Table 1 for variable definitions.

separation version without OJS performs much better than the exogenous separation version in terms of volatility of the separation rate and unemployment, as well as productivity correlation of the separation rate; all versions face the same difficulty of generating enough volatility of vacancies and the job finding rate; and the OJS version remedies the counterfactual behavior of vacancies in the endogenous separation version without OJS and generates a relatively large volatility of market tightness. Evaluations based on cross correlations also yield the same conclusions as in the benchmark calibration.

5.2 Effect of Shock Arrival Rate

This section considers the effect of raising the arrival rate λ of the match-specific productivity shock in the endogenous separation versions of the model. For this purpose we change only this parameter while keeping other parameters at their earlier values.[17]

Recall that in our benchmark calibration, we used the arrival rate parameter to match the persistence of the separation rate. This moment is useful in identifying the arrival rate because, in the model, more frequent shock arrival tends to raise the persistence of the separation rate. To see this point, suppose that the match-specific productivity shock is i.i.d. over time. This property implies that the separation rate is calculated simply as the mass of employment relationships whose new productivity draws come below the threshold level R_t, and these draws are made every period by every match. On the other hand, when the match-specific productivity shock is highly persistent, increases in the separation rate in

[17]The purpose of this second experiment is to highlight the effect of this parameter change, particularly on the persistence of the separation rate, rather than to examine the robustness of the results. Therefore, we did not recalibrate the model in this instance.

Table 4: Second Moment Properties: Calibration with $\alpha = \pi = 0.5$

X_t	u_t	JFR_t	SR_t	UE_t	EU_t	v_t	v_t/u_t
			(a) Constant separation				
σ_X	0.019	0.022	–	0.010	0.001	0.029	0.045
$cor(p_t, X_t)$	−0.883	0.996	–	0.574	0.859	0.953	0.999
$cov(p_t, X_t)/\sigma_p$	−1.239	1.655	–	0.448	0.072	2.079	3.319
$cor(X_t, X_{t-1})$	0.859	0.764	–	0.387	0.859	0.630	0.764
			(b) Endogenous separation without OJS				
σ_X	0.064	0.023	0.057	0.043	0.054	0.026	0.045
$cor(p_t, X_t)$	−0.920	0.992	−0.913	−0.813	−0.901	−0.525	0.997
$cov(p_t, X_t)/\sigma_p$	−4.799	1.825	−4.250	−2.881	−3.972	−1.131	3.666
$cor(X_t, X_{t-1})$	0.834	0.764	0.608	0.829	0.580	0.702	0.764
			(c) Endogenous separation with OJS				
σ_X	0.067	0.024	0.056	0.045	0.052	0.032	0.093
$cor(p_t, X_t)$	−0.906	0.995	−0.941	−0.767	−0.934	0.963	0.998
$cov(p_t, X_t)/\sigma_p$	−4.582	1.844	−3.990	−2.644	−3.718	2.339	7.086
$cor(X_t, X_{t-1})$	0.852	0.764	0.705	0.842	0.683	0.643	0.764

Notes. See notes to Table 2 for details on data construction and simulation. See panel (a) of Table 2 for empirical moments. Parameter values are presented in Table 3.

the face of a recessionary shock are concentrated in the impact period, and few separations occur in the following periods, even though underlying aggregate productivity is persistently low. In other words, once the matches that have become unviable due to the negative shock are destroyed on impact, only those that experience a switch of productivity can potentially be destroyed in the ensuing periods. Adjustments in the separation rate are therefore less persistent.

Tables 5 and 6 present the parameter values and simulation results, respectively when the arrival rate of the match-specific productivity shock is raised from 0.085 to 0.125. This corresponds to changing the mean arrival time of the shock from 3 months to 2 months. Again, the main results that we have already discussed so far remain the same. The main difference can be observed in the persistence of the separation rate, especially in the version without OJS, in which the first order autocorrelation coefficient increases from 0.61 to 0.67. Accordingly, persistence of EU flows also increases. When OJS is allowed, the effect is relatively minor. This is because the job finding rate directly affects the separation rate, thus reducing the impact of the persistence effect mentioned in the preceding paragraph on the behavior of the separation rate (see (11) and (15)).

Table 5: Parameter Values for Alternative Calibration: $\lambda = 0.125$

Parameter	Endogenous without OJS	Endogenous with OJS
b	0.7	0.7
c	0.17	0.17
a	–	0.13
A	0.093	0.096
α	0.7	0.7
π	0.7	0.7
x_h	1.23	1.16
s	0.0037	0.0043
λ	0.125	0.125
σ_x	0.215	0.271
ρ_z	0.9895	0.9895
σ_z	0.0034	0.0034
β	0.9992	0.9992

Notes. See Table 1 for variable definitions.

Table 6: Second Moment Properties: Calibration with More Frequent Shock Arrival

X_t	u_t	JFR_t	SR_t	UE_t	EU_t	v_t	v_t/u_t
(a) Endogenous separation without OJS							
σ_X	0.058	0.013	0.058	0.046	0.055	0.025	0.042
$cor(p_t, X_t)$	−0.900	0.993	−0.921	−0.827	−0.914	−0.376	0.997
$cov(p_t, X_t)/\sigma_p$	−4.187	1.011	−4.318	−3.093	−4.071	−0.803	3.383
$cor(X_t, X_{t-1})$	0.841	0.764	0.670	0.842	0.650	0.688	0.764
(b) Endogenous separation with OJS							
σ_X	0.058	0.014	0.056	0.046	0.053	0.044	0.096
$cor(p_t, X_t)$	−0.867	0.995	−0.889	−0.764	−0.885	0.974	0.997
$cov(p_t, X_t)/\sigma_p$	−3.752	1.016	−3.750	−2.659	−3.522	3.197	7.134
$cor(X_t, X_{t-1})$	0.849	0.764	0.715	0.847	0.698	0.679	0.764

Notes. See notes to Table 2 for details on data construction and simulation. Parameter values are presented in Table 5

6 Hagedorn-Manovskii Calibration

Lastly, we consider the implications of the calibration strategy proposed by Hagedorn and Manovskii (2008, henceforth HM) within our setup. This approach is a natural choice given our finding so far that insufficient volatility of the job finding rate remains a weakness of the model regardless of how the separation margin is modeled. For brevity, only the constant and OJS specifications are considered. We ask whether the HM calibration can raise the

volatility of the variables of interest without adversely influencing other desirable features of the OJS version of the model.

6.1 Calibration Strategy

HM propose an approach to calibrating the MP model that draws on wage and profit data. In all three specifications, the wage rate determined by Nash bargaining may be expressed as:

$$w_t(x) = (1 - \pi)b + \pi(z_t x + \theta_t c),$$

where x is identically equal to x^h in the exogenous separation specification. HM point out that under standard calibrations, the empirical productivity elasticity of wages is much lower than the elasticity generated by the model. They propose an alternative calibration strategy that aims to match this elasticity, along with the empirical relationship between mean wage and profit levels.

To assess the implications of the HM calibration, this paper follows Hornstein et al. (2005) in varying the calibrated values of b and π in order to match the productivity elasticity of wages and the steady-state wage-productivity ratio to the values 0.5 and 0.97, respectively.

The new calibrations are reported in Table 7. As noted by Hornstein et al. (2005), matching the empirical statistics requires large increases in the b parameter and large decreases in the π parameter. For the exogenous separation specification, the A parameter is adjusted to match the mean job finding rate, while for the OJS model the parameters x^h, s and σ_x are also adjusted to normalize mean productivity and match the mean and standard deviation of the separation rate. We fix the shock arrival rate at 0.085 as in the benchmark calibration. Importantly, under the HM calibration the volume of job-to-job transitions is essentially zero, even when the search cost parameter a is set to zero; we cannot match the evidence from Moscarini and Thompson (2007) used in the other calibrations. The model is solved and simulated according to the procedures discussed earlier.

6.2 Results

Results are presented in Table 8. One can immediately see that the HM calibration produces much more realistic volatility of unemployment and the job finding rate for both the constant and OJS specifications. Moreover, the job finding rate becomes highly responsive to productivity. The responsiveness of the separation rate in the OJS model declines considerably, however. This reflects the fact that, following a negative productivity shock, strong downward movement in the job finding rate reduces separation incentives by worsening workers' outside option.

The HM calibration enhances the volatility of UE flows in the constant separation rate model, but it does not appreciably raise the volatility of EU flows, nor does it mitigate the counterfactual procyclicality of worker flows implied by this specification. In the OJS version of the model, worker flows become less responsive to productivity. For UE flows, in particular, strong procyclical movements in the job finding rate serve to neutralize the countercyclical

Table 7: Parameter Values for Hagedorn-Manovskii Calibration

Parameter	Constant	Endogenous with OJS
b	0.93	0.934
c	0.17	0.17
a	–	0
A	0.065	0.061
α	0.7	0.7
π	0.082	0.063
x_h	1	1.13
s	0.005	0.0048
λ	–	0.085
σ_x	–	0.124
ρ_z	0.9895	0.9895
σ_z	0.0034	0.0034
β	0.9992	0.9992

Notes. See Table 1 for variable definitions.

movements in the separation rate, leaving only small responsiveness to productivity. Recall that in the other calibrations, the OJS version successfully matches the countercyclicality of worker flows (see the discussion in subsection 4.2). However, this feature of the model is lost in the HM calibration. The HM calibration greatly improves the performance of both specifications in matching the empirical features of vacancies and market tightness. Finally, the Beveridge and transition rate correlations are essentially unaffected for the version with a constant separation rate, while they become somewhat smaller in magnitude for the OJS version.[18] Although fluctuations in the number of employed searchers play virtually no role in this case, the correct Beveridge correlation emerges because vacancies become much more responsive to productivity fluctuations.

6.3 HM Calibration and Incentives for OJS

Incentives for OJS are linked to the size of the worker's bargaining weight. Using (8) and (9), the net gain in match surplus from searching on the job versus not searching may be expressed as:

$$\begin{aligned}\text{Net gain from OJS } &= -a + f(\theta_t)\beta\mathbb{E}_t\pi S_{t+1}(x^h) \\ -f(\theta_t)(1-s)\beta\mathbb{E}_t&\left[\lambda\int_0^{x^h} S_{t+1}(y)dG(y) + (1-\lambda)S_{t+1}(x)\right].\end{aligned}$$

[18] Cross correlations for the HM calibration are similar to those shown in Figure 1; a corresponding figure for the HM case is available upon request.

Table 8: Second Moment Properties: Hagedorn-Manovskii Calibration

X	u_t	JFR_t	SR_t	UE_t	EU_t	v_t	v_t/u_t
			(a) Data				
σ_X	0.096	0.077	0.058	0.042	0.052	0.126	0.218
$cor(p_t, X_t)$	−0.460	0.369	−0.535	−0.337	−0.521	0.564	0.527
$cov(p_t, X_t)/\sigma_p$	−5.914	3.786	−4.157	−1.879	−3.644	9.524	15.437
$cor(X_t, X_{t-1})$	0.926	0.804	0.631	0.416	0.560	0.920	0.930
			(b) Constant separation				
σ_X	0.042	0.051	–	0.024	0.003	0.133	0.168
$cor(p_t, X_t)$	−0.870	0.980	–	0.564	0.834	0.969	0.983
$cov(p_t, X_t)/\sigma_p$	−2.721	3.665	–	1.015	0.160	9.508	12.227
$cor(X_t, X_{t-1})$	0.860	0.761	–	0.394	0.859	0.698	0.761
			(c) Endogenous separation with OJS				
σ_X	0.076	0.048	0.058	0.049	0.054	0.111	0.159
$cor(p_t, X_t)$	−0.719	0.969	−0.361	0.084	−0.206	0.935	0.972
$cov(p_t, X_t)/\sigma_p$	−3.919	3.535	−1.755	−0.274	−1.500	7.912	11.812
$cor(X_t, X_{t-1})$	0.827	0.762	0.419	0.601	0.408	0.666	0.762

Notes. See notes to Table 2 for data construction and simulation. Parameter values are presented in Table 7.

Observe that the benefit of OJS derives from the prospect of starting a new match at the highest level of surplus, $S_{t+1}(x^h)$. The current worker-firm match obtains only proportion π of this surplus, however. Thus, at very low values of π, such as that associated with the HM calibration, worker-firm matches receive a very small share of the surplus from new matches, so incentives for OJS are low.

7 Conclusion

This paper considers three specifications of the standard MP model that differ in how they treat the separation margin. The specifications are calibrated at weekly frequency and solved using a nonlinear method. Allowing for endogenous determination of the separation rate greatly increases the volatility of unemployment in the simulated data. In the specification with OJS, for example, the standard deviation of unemployment equals 60 percent of its empirical value. Thus, moving beyond the assumption of a constant separation rate goes a long way toward redressing the problem of insufficient unemployment volatility in the MP model.

The specification with a constant separation rate fails to reproduce the empirical volatility and productivity responsiveness of the separation rate and worker flows. The endogenous separation specifications, in contrast, yield empirically reasonable behavior along these dimensions, and the specification with OJS also generates a realistic Beveridge curve corre-

lation. Furthermore, the endogenous separation specifications imply more realistic dynamic interrelationships in comparison to the specification with a constant separation rate.

Two broad conclusions emerge from this analysis. First, the endogenous separation specification with OJS dominates the specification with a constant separation rate along all dimensions considered. From the empirical standpoint, there seems to be no justification for assuming a constant separation rate when modeling the separation margin.

Second, the OJS version of the MP model, as articulated in Pissarides (2000), does a remarkable job of matching labor market facts even under the standard calibration, although the model still generates insufficient volatility of the job finding rate and related variables. Adopting the HM calibration largely resolves the latter failings with some costs. In particular, the HM calibration implies virtually no job-to-job transitions in the OJS specification. It also makes the cyclicality of worker flows counterfactual. Exploring possible remedies for these issues appears to be an important topic for future research.

Lastly, the inability of the MP model to generate sluggish dynamics suggests that it does not deal adequately with key structural features of the labor market. Fujita and Ramey (2007) argue that fixed costs of vacancy creation may be salient in practice, and they show that introducing these costs into the MP model with a constant separation rates leads to substantial improvements in its dynamic performance. Further investigation in this direction might be useful for deepening our understanding of labor market dynamics.

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