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Supply-Side Policies and the Zero Lower Bound*

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Abstract

This paper examines how supply-side policies may play a role in fighting a low aggregate demand that traps an economy at the zero lower bound (ZLB) of nominal interest rates. Future increases in productivity or reductions in mark-ups triggered by supply-side policies generate a wealth effect that pulls current consumption and output up. Since the economy is at the ZLB, increases in the interest rates do not undo this wealth effect, as we will have in the case outside the ZLB. We illustrate this mechanism with a simple two-period New Keynesian model. We discuss possible objections to this set of policies and the relation of supply-side policies with more conventional monetary and fiscal policies.

Keywords: Zero lower bound, supply-side policies, New Keynesian models.

JEL classification numbers: E30, E50, E60.

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1. Introduction

Although the point of this paper is utterly unoriginal and well understood by many, it is, however, not fully appreciated by a wider audience: supply-side policies can play a role in economies trapped at the zero lower bound (ZLB) of the nominal interest rates. The essence of the argument is straightforward: any policy that raises future output (for instance, by improving productivity or by reducing mark-ups) generates a wealth effect that increases the desire to consume today and decreases the desire to save. Thus, supply-side measures address the core of the problem of the ZLB, the weakness of current aggregate demand. Supply-side policies are helpful precisely because there is an aggregate demand shortfall.

This point provides a formal support for proposals of structural reforms in countries that have suffered from the dire consequences of debt crises and the ZLB.[1] Far from being a call for "more of the same," supply-side policies, such as reforming labor market institutions, liberalizing service sectors to strengthen competition, or improving professional and vocational education, can be part of a coherent strategy to fight stagnation.

Our point is different from the more traditional "grow-out-of-debt" argument based on the idea that, as a country grows, its debt burden becomes proportionally smaller. While that argument is trivially true as an accounting proposition, its formulation usually fails at specifying how to get that growth going. Our insistence in the wealth effect illuminates, in comparison, which type of mechanism will work to deliver the desired result.

Obviously, the possibility of using supply-side policies to cure the maladies of the ZLB should not be read as an argument for inaction along other fronts. Fiscal and monetary policy can be used in a coordinated fashion. For instance, fiscal policy can be directed toward expenditures, such as investments on infrastructure or R&D, that, beyond pulling aggregate demand today, may raise future productivity. Our position is, more modestly, that supply-side policies should not be forgotten and that, in many economies, they may be one of the most powerful tools left around.

Think, for instance, about the cases of countries such as Portugal or Spain that are members of the Euro zone. Without their own currency, these countries cannot rely much on monetary policy. Similarly, policies such as exchange rate depreciation or tariffs, which may induce an increase in aggregate demand, are out of the question, at least while the currency union is maintained. At the same time, perhaps in an unfair fashion, fiscal policy is severely limited by a growing level of sovereign debt and the ever-increasing cost of servicing

[1] Technically, countries such as Portugal or Spain are not at the ZLB, since the nominal interest rates are slightly positive. However, the ECB will not let the short-term nominal interest rate cannot fall further. The rigidity of the nominal interest rate is all we need to deliver the results here. In fact, having a slightly positive nominal interest rate when the natural interest rate should be negative makes the situation even worse.

it. Financial markets are forcing peripheral countries to undertake a contractionary fiscal consolidation.[2] But even in the absence of debt crisis, the evidence in Ilzetzki, Mendoza, and Végh (2010) that fiscal multipliers in high-debt countries are zero hints at possible decreasing returns of fiscal policy as countries accumulate larger debts. With both monetary and fiscal policy off the table, supply-side policies are among the last men standing.

Fortunately, these countries also have a sufficient number of "low-hanging fruits" in terms of supply-side reforms that can be easily snatched. Anyone even vaguely familiar with the deep inadequacies of the Spanish labor market or with the surrealistic regulations in many sectors of its economy cannot but forecast considerable gains out of structural reforms. One interesting aspect of our argument is that it does not depend on a permanent change in the growth trend of the economy, something that after 25 years of endogenous growth theory is still a policy chimera, but only on the possibility of increases in the level of output. As long as we can generate a wealth effect that is significant, supply-side policies will play a positive role. Thus, we are much more sanguine about the role of supply-side policies in Eurozone countries than in the U.S. or the United Kingdom where, arguably, there are less productivity gains to be picked up.

We illustrate all the previous paragraphs with a simple two-period New Keynesian model. Prices are fixed in the first period but can be changed, at a cost, in the second period. This nominal rigidity makes output partially demand-determined. The representative household consumes, supplies labor, holds money, and saves. When the (gross) nominal interest rate is above 1, the household holds money to diminish transaction costs and saves in terms of an uncontingent nominal bond. When the nominal interest rate is 1 (the nominal rate of return of money net of the marginal reduction of transaction costs), the household is indifferent between holding money or bonds. Because of price rigidity, prices cannot adjust as fast as they should and the real interest rate is not low enough to induce a sufficient level of consumption in the current period.

Then, if we suddenly increase productivity in the second period (or, alternatively, we lower the market power of firms), future output and consumption will rise. Because of the Euler equation of consumption, higher future consumption is followed by either higher interest rates and/or higher consumption today. Since, at the ZLB, the nominal rates are stuck at zero, this wealth effect of higher future output is translated into higher consumption and hours worked today.

[2] The reasons why this happens, justified or not, are uninteresting here. While a better regulation of financial markets or of rating agencies could ease the limitations to activist fiscal policies in small open economies, there is disappointingly little each of these countries can do on its own for this reform to come about.

This argument is not particularly novel even if it is perhaps cast in unfamiliar terms. Already Krugman (1998), who started the modern literature on the ZLB, used a drop in future productivity as the reason for the economy to be stuck at a low level of output. Krugman conjectured that Japan's problems could come from the lower growth potential caused by demographic aging. In this paper, we are just reverting the direction of the change in future output and thinking about it as a policy option.[3] Our alternative mechanism of increased competition is, as far as we are aware, original in the literature, although it follows rather directly from the logic of the model.

A possible reason why this point is not discussed more often is that increments in productivity in the current period actually make the problem of the ZLB worse. Higher productivity today means that the current weak demand can be satisfied with even less inputs, thus further reducing the level of employment and, in many environments, aggravating the deflationary spiral created by the ZLB. That is why we focus on the importance of future productivity gains or reductions in mark-ups, which do not suffer from this problem. In any case, in practice, nearly all policies that increase productivity will have a considerable implementation lag. Hence, when we talk about supply-side policies, we are talking about future productivity increases (and more competition in the goods market has positive effects in the short run even if it was implemented in the first period).

Our argument of a wealth effect that, when the interest rate does not respond, increases current consumption and labor resembles the mechanism in the "news" literature (Jaimovich and Rebelo, 2009). Instead of using a more general class of preferences that control for the wealth effects, as in Jaimovich and Rebelo, we rely on the absence of changes in the interest rate to deliver the result. More generally, there is a common point that any positive wealth effect, regardless of where it comes from, helps demand today. For instance, fiscal policies that decrease future government consumption achieve the same objective of raising future (private) consumption.[4]

Finally, we highlight that the model we present is of interest in itself. It is a simple environment that allows us to easily find an exact solution and to characterize it. Also, it embodies all the classical results about the ZLB highlighted in the literature. Our quest for simplicity puts us close to the attractive model of Mankiw and Weinzierl (2011). Our emphasis and goals are, however, different. We incorporate an explicit labor supply decision,

[3] Rogoff (1998), in his discussion of Krugman's paper, makes en passant the same point that future productivity gains are a solution to the ZLB problem, but without explicitly linking it to a policy strategy.

[4] Similarly, there is also a somewhat more indirect connection with the literature on uncertainty shocks (see Bloom, 2009 or Fernández-Villaverde *et al.,* 2011). Reductions in future uncertainty increase the desire to consume today because they lower precautionary saving. In the absence of a response of the interest rate caused by the ZLB, those effects would be much bigger than in the standard case.

monopolistic competition, a role for money through transaction costs, and (partial) price rigidity in the second period. These features are relevant for the economics of the mechanism that we explore. For example, our specification of the transaction costs makes it transparent when the demand for money is satiated and the different forces that affect it.[5] Monopolistic competition is required to talk about changes in the market power of firms. On the other hand, we eliminate government expenditure and have a simpler set of policy tools, since our objective is not to assess fiscal or monetary policy. However, it would be straightforward to incorporate all these elements in the model at the cost of some extra notation.

The rest of the paper is organized as follows. In section 2 we present our model and in section 3 we discuss its equilibrium conditions. Section 4 outlines a parametric specification that leads us, in section 5, to generate some numerical results. In section 6 we discuss objections to our argument and we conclude in section 7. The appendix contains extra algebra.

2. Model

We fix a simple monetary environment with two periods, $t \in \{1, 2\}$. The presence of money is motivated by three mechanisms. First, money reduces, up to some level, the transaction costs required to reach a given level of consumption. This mechanism, introduced by Sims (1994), generates a demand for money both when interest rates are positive and when they are at the ZLB. Second, money can be a store of value between periods. When the ZLB binds, the household is indifferent between using nominal bonds or money as a saving vehicle: both assets yield the same return. Thus, when the ZLB binds, the household can hold more money in equilibrium than it would otherwise need to minimize its transaction costs. Third, money in period 2 appears in the utility function. This captures the idea that money is valuable in the long run. Also, it sets up a terminal condition to induce the household to hold money at the end of period 2 beyond the desire to reduce transaction costs.[6]

Nominal rigidities appear in two forms. First, prices in the first period are given and fixed. This rigidity is a form of inflation inertia where prices are predetermined (for instance, because firms set their prices before shocks in the economy are realized). Second, firms have

[5] Also, it introduces a new channel, which in the interest of space we do not explore, where changes in the transaction technology have an effect on when the economy is at the ZLB. We are unware of this point being mentioned before in the literature.

[6] This extra utility term makes the problem more symmetric between the two periods. When the household holds money in the first period, it gets the reduction in transaction cost and an asset that pays off in the second period (even if its real rate of return is negative). In the absence of money-in-the-utility, in the second period, money would yield only a reduction in transaction cost. The asymmetry between the two periods would induce a large movement in the price level that would hide the channel that we are interested in.

to pay a cost to change their prices in the second period. Nominal rigidities make output partially demand-determined and give the ZLB a real bite.

Our model abstracts from two important aspects. First, we do not have uncertainty to keep the model as analytically tractable as possible, and we generate a ZLB through a discount factor bigger than 1. We could also imagine that this high discount factor is a random variable and that the fixed price level p_1 comes about because firms must decide their prices before observing the realization of the factor.[7] Second, and for simplicity, fiscal policy is trivial.

2.1. Household

There is a representative household with preferences:

$$\frac{c_1^{1-\sigma}}{1-\sigma} - \frac{l_1^{\psi}}{\psi} + \beta\left(\frac{c_2^{1-\sigma}}{1-\sigma} - \frac{l_2^{\psi}}{\psi} + \gamma \log \frac{m_2}{p_2}\right)$$

where c_t is consumption at time t, l_t is labor supply, and $\frac{m_2}{p_2}$ are real balances (nominal money m_2 divided by the price level p_2). We will assume $\sigma > 1$, that is an elasticity of intertemporal substitution lower than 1 (as most of the empirical literature estimates), but, since we have finite periods, we do not assume that $\beta < 1$. Also, we do not bound labor supply by 1. This can be easily accomplished by the right choice of units.

The budget constraints for the household are:

$$(1 + s(v_1))\, p_1 c_1 + m_1 + \frac{b}{R} = p_1 w_1 l_1 + p_1 F_1 + p_1 T$$

and

$$(1 + s(v_2))\, p_2 c_2 + m_2 = p_2 w_2 l_2 + p_2 F_2 + m_1 + b$$

where b is an uncontingent nominal bond, R is the gross nominal interest rate, w_t is the wage in period t, F_t denotes profits from firms, and T denotes transfers.

The function $s(\cdot)$ parametrizes the transaction costs (in resource terms) of consuming c_t when the real balances of money used for transactions in the period are $\frac{m_t}{p_t}$ as a function of velocity $v_t = \frac{p_t c_t}{m_t}$. The transaction cost function is nonnegative, twice continuously differentiable, and there exists a level of velocity $\overline{v} > 0$, to which we refer as the satiation level, such that $s(\overline{v}) = s'(\overline{v}) = 0$, and $s'(\cdot) \geq 0$.

[7] The first draft of our model had that precise feature. We disregarded it as an unnecessary complication that did not add much economic insight.

The FOCs of the household's problem are:

$$c_1^{\sigma} l_1^{\psi-1} = \frac{w_1}{1 + s(v_1) + s'(v_1) v_1}$$
$$c_2^{\sigma} l_2^{\psi-1} = \frac{w_2}{1 + s(v_2) + s'(v_2) v_2}$$
$$\frac{R-1}{R} = s(v_1)' v_1^2$$
$$\gamma = c_2^{1-\sigma} \frac{\frac{1}{v_2} - s(v_2)' v_2}{1 + s(v_2) + s'(v_2) v_2}$$

and

$$\frac{1}{c_1^{\sigma}} = \beta \frac{1}{c_2^{\sigma}} R \frac{p_1}{p_2} \frac{1 + s(v_1) + s'(v_1) v_1}{1 + s(v_2) + s'(v_2) v_2}$$

five conditions that, together with the two budget constraints, determine the seven choices of household. The first two conditions are the static optimality conditions that equate the ratio of marginal utilities of leisure and consumption with their relative prices (wages over the marginal cost of consumption once we consider transaction costs). The third equation tells us that the household will hold cash until its marginal return (in terms of reduction of transaction costs) is equal to its opportunity cost given by R. The fourth equation is the demand for money in the second period. The final equation is the Euler equation for bond holdings (where we account for the marginal change in transaction costs induced by changing consumption).

When the ZLB binds, the third equation implies $0 = s(v_1)' v_1^2$, which indicates that the opportunity cost of holding money has been reduced to zero. Since $\frac{p_1 c_1}{m_1} \neq 0$, we must have that $s(v_1)' = 0$, that is, the household is satiated in its need for money in period 1 for transaction costs. Conversely, outside the ZLB, we have $s(v_1)' > 0$.

2.2. The Final Good Producer

There is one final goods produced using intermediate goods with the production function:

$$y_t = \left(\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{1}$$

where ε is the elasticity of substitution.

Final goods producers are perfectly competitive and maximize profits subject to the production function (1), taking as given all intermediate goods prices p_{ti} and the final goods

price p_t. Thus, the input demand functions are:

$$y_{it} = \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} y_t \qquad \forall i,$$

and the price level $p_t = \left(\int_0^1 p_{it}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$.

2.3. Intermediate Good Producers

Each intermediate firm produces differentiated goods out of labor with a technology $y_{it} = A_t l_{it}$, where l_{it} is the labor input rented by the firm and A_t is productivity. Therefore, the real marginal cost of all intermediate goods producers is $mc_t = \frac{w_t}{A_t}$.

The monopolistic firms face nominal rigidities. Prices in period 1, p_1, are fixed. At time 2, they reoptimize their prices to p_{i2}, but they pay an adjustment cost

$$AC_{i,2} = \frac{\phi}{2}\left[\frac{p_{i2}}{p_{i1}} - 1\right]^2.$$

per unit of goods sold. This Rotemberg setup introduces rigidities in the second period without the need to keep track of distributions (as would happen in a Calvo environment) or to solve a (discrete) menu cost problem.

Hence, prices p_{2t} are chosen to maximize

$$\max_{p_{2t}} \left(\frac{p_{i2}}{p_2} - mc_2 - \frac{\phi}{2}\left[\frac{p_{i2}}{p_{i1}} - 1\right]^2\right) y_{i2}$$
$$\text{s.t. } y_{it} = \left(\frac{p_{i2}}{p_2}\right)^{-\varepsilon} y_2.$$

The solution of that problem leads to an aggregate pricing condition:

$$1 - \varepsilon + \varepsilon mc_2 - \phi\frac{p_2}{p_1}\left[\frac{p_2}{p_1} - 1\right] + \varepsilon\frac{\phi}{2}\left[\frac{p_2}{p_1} - 1\right]^2 = 0.$$

2.4. Government

The government policy is extremely simple. It issues m units of currency in the first period, which it rebates back to the representative household as transfers. Then, the budget constraint of the government is $m = T$. Also, by clearing in the money market, $m = m_2 = m_1$.

2.5. Aggregation

Standard algebra and symmetry in the firm's behavior gives us

$$(1+s(v_1))\,c_1 = A_1 l_1$$

and

$$(1+s(v_2))\,c_2 = \left(1-\frac{\phi}{2}\left[\frac{p_2}{p_1}-1\right]^2\right)A_2 l_2$$

Also, using the consumption-labor optimality condition of the household, we get:

$$mc_t = \frac{w_t}{A_t} = \frac{c_t^{\sigma} l_t^{\psi-1}\left(1+s(v_t)+s'(v_t)\,v_t\right)}{A_t}$$

3. Equilibrium

Given a feasible policy sequence determined by m and T and an initial price level p_1, an equilibrium is an allocation and prices c_1, l_1, v_1, R, c_2, l_2, p_2, and v_2 that solve:

$$1-\varepsilon+\varepsilon\frac{c_2^{\sigma} l_2^{\psi-1}\left(1+s(v_2)+s'(v_2)\,v_2\right)}{A_2}-\phi\frac{p_2}{p_1}\left[\frac{p_2}{p_1}-1\right]+\varepsilon\frac{\phi}{2}\left[\frac{p_2}{p_1}-1\right]^2=0$$

$$(1+s(v_1))\,c_1 = A_1 l_1$$

$$(1+s(v_2))\,c_2 = \left(1-\frac{\phi}{2}\left[\frac{p_2}{p_1}-1\right]^2\right)A_2 l_2$$

$$\frac{1}{c_1^{\sigma}} = \beta\frac{1}{c_2^{\sigma}}R\frac{p_1}{p_2}\frac{1+s(v_1)+s'(v_1)\,v_1}{1+s(v_2)+s'(v_2)\,v_2}$$

$$s(v_1)'\,v_1^2 = \begin{cases} \frac{R-1}{R} \text{ if } R>1 \\ 0 \text{ if } R=1 \end{cases}$$

and

$$\gamma = c_2^{1-\sigma}\frac{\frac{1}{v_2}-s(v_2)'\,v_2}{1+s(v_2)+s'(v_2)\,v_2}$$

plus the definition:

$$v_t = \frac{p_t c_t}{m},\ t\in\{1,2\}.$$

The previous equilibrium conditions display a convenient recursive structure. Given p_1,

we can use the block:

$$1-\varepsilon+\varepsilon\frac{c_2^{\sigma}l_2^{\psi-1}\left(1+s\left(v_2\right)+s'\left(v_2\right)v_2\right)}{A_2}-\phi\frac{p_2}{p_1}\left[\frac{p_2}{p_1}-1\right]+\varepsilon\frac{\phi}{2}\left[\frac{p_2}{p_1}-1\right]^2=0$$

$$\left(1+s\left(v_2\right)\right)c_2=\left(1-\frac{\phi}{2}\left[\frac{p_2}{p_1}-1\right]^2\right)A_2l_2$$

$$\gamma=c_2^{1-\sigma}\frac{\frac{1}{v_2}-s\left(v_2\right)'v_2}{1+s\left(v_2\right)+s'\left(v_2\right)v_2}$$

to find p_2, c_2, and l_2. Even within this first block, we can find two sub-blocks. First:

$$l_2=\frac{1+s\left(v_2\right)}{1-\frac{\phi}{2}\left[\frac{p_2}{p_1}-1\right]^2}\frac{c_2}{A_2} \tag{2}$$

that determines l_2 as function of p_2 and c_2, and, second:

$$1-\varepsilon+\varepsilon\Omega\left(p_2,p_1,c_2,m\right)\frac{c_2^{\sigma+\psi-1}}{A_2^{\psi}}-\phi\frac{p_2}{p_1}\left[\frac{p_2}{p_1}-1\right]+\varepsilon\frac{\phi}{2}\left[\frac{p_2}{p_1}-1\right]^2=0 \tag{3}$$

$$\gamma=c_2^{1-\sigma}\frac{\frac{1}{v_2}-s\left(v_2\right)'v_2}{1+s\left(v_2\right)+s'\left(v_2\right)v_2} \tag{4}$$

where

$$\Omega\left(p_2,p_1,c_2,m\right)=\left(1+s\left(v_2\right)+s'\left(v_2\right)v_2\right)\left(\frac{1+s\left(v_2\right)}{1-\frac{\phi}{2}\left[\frac{p_2}{p_1}-1\right]^2}\right)^{\psi-1}.$$

that determines p_2 and c_2.

The recursive structure of the equilibrium is derived from the fact that, beyond p_1 and m, we do not have any state variable in the model. Thus, prices and quantities in the second period can be derived without having to know any variable determined in period 1. In particular, they are determined independently of whether the ZLB binds or not.

If prices are flexible, $\phi=0$, and there are no transaction costs in the second period,[8] we just have $\Omega\left(p_2,p_1,c_2,m\right)=1$. Then $l_2=\frac{c_2}{A_2}$, $p_2=\frac{m}{\gamma c_2^{\sigma}}$, and

$$c_2=A_2^{\frac{\psi}{\sigma+\psi-1}}\left(\frac{\varepsilon-1}{\varepsilon}\right)^{\frac{1}{\sigma+\psi-1}}.$$

While in the calibrated model below we will assume that $\phi>0$ and that $s\left(v_2\right)>0$, both

[8] Since money enters into the utility function, there will still be a demand for money in the second period.

frictions will be small enough that the expressions above will (nearly) hold. These three equations embody several simple messages:

1. In $t = 2$, the economy presents a classical dichotomy: quantities are determined by preferences and technology, and the price level by money supply and consumption.

2. The price level p_2 is proportional to m conditional on c_2.

3. Increases in A_2 raise consumption and lower labor supply and velocity: $\frac{p_2 c_2}{m} = \frac{1}{\gamma c_2^{\sigma-1}}$.

The intuition is simple: both consumption and leisure are normal goods and, hence, when A_2 is high, we observe more consumption and less labor supply in the second period. Velocity is lower because prices fall less than consumption rises. With nominal rigidities, this effect becomes stronger.

With the variables in the second period, we can go back to the first period and solve:

$$\begin{aligned} (1 + s(v_1))\, c_1 &= A_1 l_1 \\ \frac{1}{c_1^\sigma} &= \beta \frac{1}{c_2^\sigma} R \frac{p_1}{p_2} \frac{1 + s(v_1) + s'(v_1)\, v_1}{1 + s(v_2) + s'(v_2)\, v_2} \\ \frac{R-1}{R} &= s(v_1)'\, v_1^2 \end{aligned}$$

for c_1, l_1, and R. If $R > 1$, we are done. Otherwise, we fix $R = 1$ and solve:

$$\begin{aligned} (1 + s(v_1))\, c_1 &= A_1 l_1 \\ \frac{1}{c_1^\sigma} &= \beta \frac{1}{c_2^\sigma} \frac{p_1}{p_2} \frac{1 + s(v_1)}{1 + s(v_2) + s'(v_2)\, v_2} \end{aligned}$$

for c_1 and l_1 or:

$$c_1 = \left(\frac{1}{\beta} \frac{p_2}{p_1} \frac{1 + s(v_2) + s'(v_2)\, v_2}{1 + s(v_1)} \right)^{\frac{1}{\sigma}} c_2 \tag{5}$$

and $l_1 = \frac{c_1}{A_1}$.

When will we be at the ZLB? Given a p_1, equation (5) reveals that when $p_2 c_2^\sigma$ is too small. In this situation, and from the Euler equation, the only way in which the household can satisfy intertemporal optimality is by reducing c_1, which translates, directly, into less labor. When prices are fully flexible, any problem caused by a too low $p_2 c_2^\sigma$ can be easily undone with reductions in p_1. Hence, our model nicely illustrates how the ZLB is a problem because prices today are not fully flexible. As mentioned above, even at the ZLB, our equilibrium has a recursive structure and, hence, p_2 and c_2 are given by (2)-(4). Furthermore, we also have the paradox of thrift at work: a higher β lowers c_1 and, with it, output.

We can see in equation (5) the main mechanism at work: as a response to $p_2 c_2^{\sigma}$ that is too small and with a fixed price level p_1, we can either lower c_1 or we can increase p_2 or c_2. In this paper, we argue that increases in c_2 are a possibility that has often been overlooked.

4. A Parametric Specification

To make further progress, we specify the transaction cost function in terms of the velocity v_t:

$$s(v_t) = \begin{cases} 0 \text{ if } 0 < v_t < \sqrt{\frac{\alpha_1}{\alpha_0}} \\ \alpha_0 v_t + \alpha_1 \frac{1}{v_t} - 2\sqrt{\alpha_0 \alpha_1} \text{ if } v_t \geq \sqrt{\frac{\alpha_1}{\alpha_0}} \end{cases}$$

This function, continuous and differentiable, has two parts. For velocities sufficiently small, it is zero, as the demand for money has been satiated. But, when we reach a threshold ($\sqrt{\frac{\alpha_1}{\alpha_0}}$), the transaction cost grows in a convex fashion.

The interpretation is simple. A low velocity means that there is a large quantity of money in relation to the nominal price of consumption. Hence, the transaction cost is zero and cannot be reduced further (we could translate the whole function by a constant α_3 if we want to keep some positive minimum level of transaction costs). After the threshold, there is little money in relative terms, and the household is required to use resources in executing transactions. Convexity is a natural assumption. The functional form is the same as in Schmitt-Grohé and Uribe (2011), except for the flat part before the satiation point.

Now we can go back to the equilibrium equation:

$$\frac{R-1}{R} = s(v_1)' v_1^2$$

and since $s'(v_t) = \alpha_0 - \alpha_1 \frac{1}{v_t^2}$ and $s'(v_t) v_t^2 = \alpha_0 v_t^2 - \alpha_1$, we get the demand for money:

$$\frac{m}{p_1} = \frac{c_1}{\sqrt{\frac{\alpha_1}{\alpha_0} + \frac{1}{\alpha_0}\frac{R-1}{R}}}$$

that shows that money holdings increase with consumption and decrease with the nominal interest rate (the opportunity cost). It is also the case that if $R > 1$, $v_1 > \sqrt{\frac{\alpha_1}{\alpha_0}}$.

At the ZLB, any holding of money that satisfies:

$$\frac{m}{p_1} \geq \frac{c_1}{\sqrt{\frac{\alpha_1}{\alpha_0}}}$$

is compatible with an equilibrium because, at the margin, the household is also holding money

just as a store of value. We assume that the actual holdings of money are determined by clearing in the money market:

$$\frac{m}{p_1} = \frac{c_1}{\sqrt{\frac{\alpha_1}{\alpha_0}}}.$$

Also, at the ZLB, $s(v_1) = s(v_1)' = 0$, that is $v_1 = \sqrt{\frac{\alpha_1}{\alpha_0}}$.

We can come back to equation (5), and with some algebra, rewrite it as:

$$c_1 = \left(\frac{1}{\beta}\frac{1}{p_1}\frac{m}{\gamma}\left(1 - \alpha_0 (v_2)^2 + \alpha_1\right)\right)^{\frac{1}{\sigma}}.$$

This expression tells us that anything that reduces money velocity in period 2 will increase consumption in period 1. As mentioned above, this means that anything that increases c_2 will also increase c_1.

5. Some Numerical Results

In this section we offer some numerical results that illustrate the forces at work in our model. We do not see this as a calibration exercise (we are not aiming to match any moment of the data), but just as a quantitative exercise to better understand the economic mechanism.

We will proceed as follows. First, we will calibrate our model. Second, we will introduce three variations of the benchmark model that we just presented. These variations will be helpful to interpret the results of our numerical investigation. Third, we will present a case where technology and market power are constant over time, which will tell us how the economy behaves in the absence of policy changes. Fourth, we will implement different exercises where we show how increases in future consumption (either through increases in productivity or reductions in market power) increase consumption today. Finally, we will close with some extra experiments that demonstrate the usefulness of our model to revisit some of the classical results in the literature.

We start, then, by setting up a simple numerical version of our model with a calibrated utility function:

$$-c_1^{-1} - \frac{l_1^2}{2} + 1.2\left(-c_2^{-1} - \frac{l_2^2}{2} + \log\frac{m_2}{p_2}\right)$$

that is, we fix $\sigma = 1$, $\psi = 2$, $\beta = 1.2$, and $\gamma = 1$. The values of σ and ψ are standard in the business cycle literature, γ is just a normalization of the units of currency, and β is a large number to induce the ZLB to bind.

The transaction cost function of money is:

$$s(v_t) = \begin{cases} 0 \text{ if } 0 < v_t < \sqrt{0.75} \\ 0.4v_t + 0.3\frac{1}{v_t} - 2\sqrt{0.12} \text{ if } v_t \geq \sqrt{0.75} \end{cases}$$

where we pick the parameters to generate a small transaction cost (for instance, in the second period of our example, with less than 0.25 percent of output, in the first period the ZLB binds and hence the costs are zero). The parameter controlling the elasticity of substitution among goods is $\varepsilon = 10$ (again, a conventional value), and the price adjustment cost $\phi = 1$ (which implies an adjustment cost of 0.44 percent of the second period output). Finally, we set $m = 1.1$ (this generates a p_2 slightly bigger than 1) and $p_1 = 1$, around 7.6 percent higher than in a flexible prices equilibrium.[9] With these parameter choices, we find a unique equilibrium in our numerical exercises.

To facilitate the interpretation of the results, we compare the model presented in the previous section with three alternative versions (that we derive in the appendix) that are nested within it when we set some parameters to zero. First, we eliminate money ($m = 0$), monopolistic competition ($\varepsilon = \infty$), and all forms of rigidities ($\phi = 0$). This is a simple neoclassical environment with analytical solution and that helps us to think in the right order of magnitude for each variable. This model is also Pareto efficient, so it can also be read as the social planner's problem. We call this version model I. Second, we reintroduce monopolistic competition ($\varepsilon = 10$), but without money ($m = 0$) or rigidities ($\phi = 0$). We still have a simple solution and, in addition, the presence of a mark-up gets us much closer to the quantitative results of the model presented in the previous section. We call this version model II. Third, we reintroduce money ($m = 1.1$) and the ZLB, but prices are still flexible ($\phi = 0$). We call this version model III.[10] For reference, we call the model presented in the previous section model IV.

Our first step is to compute case I, where $A_1 = A_2 = 1$ and $\varepsilon_1 = \varepsilon_2 = 10$. The results are in table 1, where $1 + r$ is the real interest rate (defined as the rate of return of a real bond). The second column shows the results for model I. There we see the convenience of our parameterization: consumption and labor are equal to 1 in both periods and the real interest is just the inverse of the discount factor ($0.833 = 1/1.2$). In the absence of money,

[9] By setting p_1 "too high," we ensure that output is below what it would be given preferences, technology, and flexible prices. In the old disequilibrium models of the 1970s, there was an alternative case when p_1 was too low, often called the "repressed inflation case." This case will resurface later in this section.

[10] Models II and III also consider monopolistic competitors in period 1. If nothing is said otherwise we will assume that the elasticiy of substitution is constant across periods. When analyzing cases where the elasticity of substitution changes across periods, we will call ε_1 the elasticity of period 1 and ε_2 the elasticiy of period 2.

price levels and the nominal interest rate are not defined. In the third row, we move to model II. Market power works as a consumption tax that decreases consumption and labor in both periods, here by 3.5 percent. The real interest rate is unchanged.

Table 1: Case I, $A_1 = A_2 = 1$ and $\varepsilon_1 = \varepsilon_2 = 10$

	Model I	Model II	Model III	Model IV
c_1	1	0.965	0.965	0.931
l_1	1	0.965	0.965	0.931
p_1	-	-	0.929	1
c_2	1	0.965	0.946	0.947
l_2	1	0.965	0.948	0.953
p_2	-	-	1.095	1.094
$1+r$	0.833	0.833	0.848	0.914
R	-	-	1	1

The fourth row is model III, where we introduce money but prices are fully flexible. Here, and with our parameters, the ZLB is binding ($R = 1$). However, the ZLB does not matter for allocations because prices are fully flexible and p_1 and p_2 adjust to deliver the "right" real interest rate. Since the transaction costs are zero in the first period, the allocation in that period is the same as in model II ($c_1 = l_1 = 0.965$). In the second period, the transaction costs are not zero, and they induce a reduction in consumption (by 2.0 percent) and labor (by 1.8 percent). While the transaction costs actually paid (0.2 percent) are rather small, they create a wedge that lowers consumption. The price levels, 0.929 and 1.095, do the job of adjusting the real interest rate to (nearly) the level of the case without money, 0.848. The slight difference comes from the lower consumption in period 2, which increases the marginal utility in that period, and the transaction costs, also in period 2, that appear in the Euler equation.

Finally, in the fifth row, we have the complete model (model IV). Consumption in period 1 is now more than 3 percent lower than in model III. With p_1 fixed, the real interest rate can go down only to 0.914 and households want to save more. The real interest rate is too high when compared with model III. The only way in which the savings market can clear is by a reduction in consumption in period 1, which is achieved by a fall in demand that, given the nominal rigidity, lowers production. Consumption and labor in the second period are higher than in model III. The reason is that the presence of a positive price adjustment cost ϕ makes p_2 rise a bit less than in model III, to 1.094. A lower price level induces more consumption and labor. Labor also rises to pay for the adjustment cost. As a final point,

note that even if we are at the ZLB, the economy still experiences inflation, just not enough to lower the real interest rate sufficiently.

Table 2: Case II, A_1= 1, A_2 = 1.05 and $\varepsilon_1 = \varepsilon_2 = 10$

	Model I	Model II	Model III	Model IV
c_1	1	0.965	0.965	0.936
l_1	1	0.965	0.965	0.936
p_1	-	-	0.938	1
c_2	1.033	0.997	0.981	0.982
l_2	0.984	0.950	0.936	0.944
p_2	-	-	1.041	1.039
$1+r$	0.889	0.889	0.901	0.962
R	-	-	1	1

We move now to the case II where supply-side policies have been able to increase A_2 by 5 percent, to 1.05. In model I, c_2 goes up and l_2 goes down with respect to case I. As shown in the appendix, in model I, labor is a decreasing function of technology. Hence, as A_2 goes up, l_2 falls. The contrary is true for c_2. However, in the first period, the allocation is the same since we are not more productive at time 1. The real interest rate increases to 0.889 to induce the household to save enough to clear the asset market. The results for model II are quite similar. There is an increase in c_2 and a reduction in l_2 with respect to case I. Model III does not change much with respect to case I. The economy is at the ZLB, but allocations are not affected when compared with model II (except for the fact that inflation and, therefore, transaction costs in period 2 are now different, which changes c_2 and l_2). Prices behave, though, differently: p_1 is somewhat higher and p_2 is slightly lower than in case I. A higher productivity implies a lower marginal cost and, with it, a lower optimal price of monopolistic producers in period 2. Since inflation adjusts the real interest rate to (nearly) the value it takes in model II, p_1 also changes.

Finally, model IV shows us the main mechanism in this paper: the effect of increases in future productivity on consumption in period 1. Consumption increases in the second period to 0.982 while labor falls to 0.944 with respect to case I. Similarly, p_2 is now only 1.039 when in case I it was 1.094. Next, the most important implication: higher consumption in the second period increases consumption in period 1 by 0.6 percent. The effect is not large because of the increase in the real interest rate induced by a lower p_2. However, this just a simple numerical example to illustrate our argument. We would need a fully-fleshed business

cycle model to evaluate the quantitative size of this mechanism and how big of an increase in productivity we would require to get a sizable impact.

Table 3: Case III, A_1= 1, $A_2 = 1.30$ and $\varepsilon_1 = \varepsilon_2 = 10$

	Model I	Model II	Model III	Model IV
c_1	1	0.965	0.965	0.954
l_1	1	0.965	0.965	0.954
p_1	-	-	0.982	1
c_2	1.191	1.150	1.150	1.133
l_2	0.916	0.885	0.885	0.882
p_2	-	-	0.830	0.848
$1+r$	1.182	1.182	1.183	1.179
R	-	-	1	1.001

In case III (table 3), we increase A_2 to 1.30 to show that, with a sufficiently large increase in future productivity, we actually get out of the ZLB (although barely so, in our numerical example). Although this may seem like a large number, again, we are just dealing with a numerical example (and there are cases where structural reforms took place, as in Spain in 1959, where increases in productivity of 30 percent did occur).

In table 4, we report case IV where, instead of affecting productivity per se, supply-side policies increase the level of competition in the economy and reduce the mark-ups (perhaps, with a more forceful enforcement of antitrust law). We model such policy as changing ε_1 (the parameter controlling market power) from 10 in the first period to $\varepsilon_2 = 100$ in the second. This rather reduced-form approach is justified because, for our argument, we do not really need to be particularly explicit about the mechanism that generates market power.[11] Since there was no monopolistic competition to begin with, model I is unaffected with respect to case I. In model II, the second period allocation gets much closer to the first best (the one in model I). To induce the right level of consumption in the first period, the interest rate goes up to 0.888. In model III we have a similar result: consumption and labor grow in the second period and prices move to induce the real interest rate that ensures that markets clear in the first period. Finally, in model IV, since we are at the ZLB and p_1 is fixed, as c_2 increases, the Euler equation implies that c_1 also increases by 0.5 percent. This expansionary effect of increased competition works even if ε_1 also increases to 100 (we omit the corresponding table

[11] This is the same experiment as in Blanchard and Giavazzi (2003) when they explore the effects of goods market de-regulation in Europe.

for concision). This result is interesting because in the case of improvements in productivity, increases in A_1 may actually reduce l_1. This "paradox of productivity" does not apply, then, to reductions in the market power of firms.[12]

Table 4: Case IV, A_1= 1, A_2 = 1 and ε_1= 10, ε_2 = 100

	Model I	Model II	Model III	Model IV
c_1	1	0.965	0.965	0.936
l_1	1	0.965	0.965	0.936
p_1	-	-	0.938	1
c_2	1	0.997	0.981	0.980
l_2	1	0.997	0.982	0.982
p_2	-	-	1.041	1.043
$1+r$	0.833	0.888	0.901	0.959
R	-	-	1	1

We close this section with two additional experiments that revisit two classical topics and prove the usefulness of our model. First, we divide adjustment costs by 10: ϕ is now only 0.1. The results in table 5 confirm the old argument by DeLong and Summers (1986), more recently revisited by Werning (2011), that increasing price flexibility (but short of reaching full price flexibility) may not help. Due to higher price flexibility, consumption and labor are lower in the first period. The concrete mechanism in our paper, though, is different: with more price flexibility, prices rise too fast, not fall too fast as in DeLong and Summers. More pointedly, with $\phi = 0.1$, p_2 can respond more to demand conditions in period 2. A slightly higher p_2 lowers c_2 and with it, c_1 (although in our calibration the effect is minimal as we already start with a low ϕ: labor in period 1 goes from 0.93089 to 0.93079, because of rounding this small drop does not show up in table 5). Or, in other words, more flexible prices in the second period lower demand and with it output, generating less consumption and less output in the first period. Welfare implications are more nuanced because bigger price flexibility also implies an allocation closer to first best and less wasted resources in adjustment costs. In our example, welfare goes up when $\phi = 0.1$.

[12] As we will see in our next experiment, reductions in market power may also be a better policy tool than reductions in price stickiness (for example, by changing commercial and labor law to allow for more frequent contract or collective bargaining agreement renegotiations), which can often deliver negative results.

Table 5: $\phi = 0.1$

	Model I	Model II	Model III	Model IV
c_1	1	0.965	0.965	0.931
l_1	1	0.965	0.965	0.931
p_1	-	-	0.929	1
c_2	1	0.965	0.946	0.946
l_2	1	0.965	0.948	0.949
p_2	-	-	1.095	1.095
$1+r$	0.833	0.833	0.848	0.913
R	-	-	1	1

Second, we increase m by 9 percent from 1.1 to 1.2 (see table 6). In our model, this is a permanent increase in the monetary base of the economy. Prices in model III also increase around 9 percent in both periods, but, since we have price flexibility, the allocations remain unchanged. More interesting is the response of model IV. Now p_1 is below the level it would be under price flexibility (model III). Hence, even if we stay at the ZLB, c_1 goes all the way up to 0.971. This experiment demonstrates both the importance of having prices fixed in the first period at too high a level for the ZLB to be really damaging and how permanent increases in money can ease the problems involved by the bound (Auberbach and Obstfeld, 2005).

Table 6: $m = 1.2$

	Model I	Model II	Model III	Model IV
c_1	1	0.965	0.965	0.971
l_1	1	0.965	0.965	0.971
p_1	-	-	1.014	1
c_2	1	0.965	0.946	0.940
l_2	1	0.965	0.948	0.963
p_2	-	-	1.195	1.206
$1+r$	0.833	0.833	0.848	0.829
R	-	-	1	1

We could perform other experiments to show classical results in the ZLB literature such as that an increase in A_1 lowers l_1 or that an increase in β lowers c_1. After all our previous explanations, though, we can skip the details.

6. Possible Objections

There are four main objections to our argument: the first two that we do not think are important and the second two that we think are.

The first objection is to ask why we want to embark in supply-side reforms, whose outcomes are uncertain and perhaps exceedingly small, when we have at hand monetary and fiscal policies. The ZLB comes about because future nominal output is too low. Monetary policy can fix that problem by increasing p_2, either through a commitment to temporarily higher inflation (Krugman, 1998, Eggertsson and Woodford, 2003) or through lump-sum transfers of cash (Auberbach and Obstfeld, 2005). Similarly, as shown by Correia *et al.* (2010), fiscal policy can neutralize the effects of the ZLB and achieve first best by using taxes to replicate the optimal path for the price level.[13]

However, monetary and fiscal policies may not be at hand after all. Monetary policy can be offline, either because of political pressures (for instance, a central bank reluctant to engage in expansionary monetary policy for some motive beyond our model) or institutional arrangements (a monetary union such as the Eurozone). Fiscal policy often has few degrees of freedom (partisan divisions within a polity, high debt-to-output ratios that cause large country spreads, constitutional limits, etc.). Therefore, supply-side policies become a second line of defense that we should not overlook (or, in the case where monetary and fiscal policy still work, a complementary one). Furthermore, supply-side policies may help alleviate the negative consequences of monetary or fiscal policies designed to fight the ZLB today. For example, they may generate higher future tax revenues that help to pay down the debt incurred by expansionary fiscal policy.

The second objection is to ask why we emphasize the importance of increases on future output when we are at the ZLB. Should not a government want to increase future output in all situations regardless of whether or not we are at the ZLB? Yes, it should if these increases are free. However, these increases are usually costly, either in pure economic terms (we need to build a new bridge or learn a new technology, more competition may reduce incentives to R&D in a model of endogenous growth) or politically (the reforms that yield higher productivity decrease the rents of some groups). But when we are at the ZLB, these structural reforms have a higher than normal rate of return. Not only do we obtain more consumption tomorrow, we also fight the demand problems today. Outside the ZLB, increases in future productivity are undone, in terms of consumption today, by an increase in the interest rate that ensures market clearing in the current period. At the ZLB, that effect disappears and hence consumption today also rises. Thus, reforms that would be too expensive either economically or politically

[13] See also Woodford (2011) for an analytic investigation of the effects of fiscal policy at the ZLB.

in normal times can become desirable at the ZLB.

The third objection (and the first important one) is that increases in current productivity may deteriorate the economic situation. As we saw before, we have an Euler equation that pins down c_1 as a function of future variables and l_1 is whatever quantity we need to produce c_1. Thus, a higher productivity today just lowers employment without any further benefit. We do not find this objection too compelling. First, most increases in productivity today are permanent (or least they have high persistence) and hence the contractionary effect today has to be compensated against the wealth effect on c_1.[14] Second, and more important, most structural reforms, such as reorganizing labor markets, take some time before having an impact. Thus, any policy action today is unlikely to have much effect on current productivity.[15] Finally, as we saw in our experiment 4, increases in the level of competition in the economy do not suffer from this problem.

The last, and most serious, objection is whether increases in future productivity are a feasible policy instrument. After more than two decades of endogenous growth literature, we do not hold a magic wand to miraculously beget higher output. It may well be the case that increases in future productivity are just not part of the feasible set of actions for a government or that the increases that a government can induce are too small to make much of a difference (more concretely, the wealth effect generated is insignificant). We see two counter-arguments to this objection. First, we are not after permanent increases in the growth trend of an economy. This is probably well beyond the reach of most governments. A wealth effect works even if we just generate a one-shot increase in the level of productivity over its original path. That goal is much more realistic. Second, the economies of countries such Spain have so many areas of inefficiency (the labor market being the paradigmatic case) that increases in productivity after some reforms are much more likely than in the U.S. or the United Kingdom. Similarly, increases in the level of competition of some European economies, which have many service sectors shielded from market forces, are quite possible.

Summarizing:

1. Monetary and fiscal policy can be used to fight the ZLB. Supply-side policies are just an additional tool that can be handy in some circumstances.

2. The ZLB is a situation where the rate of return of policies that increase productivity

[14] Also, in our model, even if labor goes down, welfare is increasing. In a more realistic environment, for example with heterogeneous agents and liquidity constraints where unemployment can be painful, welfare changes can go in either direction depending on the details of the economy.

[15] Given that we are dealing with a simple two-period model, we are vague about the length of a period and about whether or not productivity reforms will come online sufficiently fast. A more fleshed out quantitative model would be required for that task.

or the competition level of the economy is high.

3. Structural reforms are unlikely to increase productivity much today and hence aggravate the ZLB in comparison with the wealth effect of future increases in productivity. Reductions in the market power of firms in the economy do not suffer from this problem.

4. Supply-side policies can be hard to implement and too small in their effects, but in some countries there might be sufficient scope for them to work.

7. Conclusions

In this paper we have argued that supply-side policies can play a role in fighting situations where an economy is stacked at a ZLB. While we do not want to overemphasize the power of these policies, we should not forget about them either. Our results suggest the need for more detailed middle-size business cycle models in the style of Christiano, Eichenbaum, and Evans (2005), or Smets and Wouters (2003) modified to incorporate an explicit ZLB to measure how big the potential effects from these policies are and how they can complement more traditional monetary and fiscal policies.

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8. Appendix

8.1. Algebraic Derivations for Model I: No Money, Perfect Competition

The FOCs of the household are:

$$c_t^{\sigma} l_t^{\psi-1} = w_t$$
$$\frac{1}{c_1^{\sigma}} = \beta \frac{R}{c_2^{\sigma}}$$

where R is now a real interest rate.

The problem of the firms is just $w_t = A_t$ and market clearing $c_t = A_t l_t$. Then:

$$(A_t l_t)^{\sigma} l_t^{\psi-1} = A_t \Rightarrow l_t = A_t^{\frac{1-\sigma}{\sigma+\psi-1}}$$

and

$$R = \frac{1}{\beta}\left(\frac{c_2}{c_1}\right)^{\sigma} = \frac{1}{\beta}\left(\frac{A_2}{A_1}\right)^{\frac{\psi\sigma}{\sigma+\psi-1}}$$

8.2. Algebraic Derivations for Model II: No Money, Monopolistic Competition

The FOCs of the household are still:

$$c_t^{\sigma} l_t^{\psi-1} = w_t$$
$$\frac{1}{c_1^{\sigma}} = \beta \frac{R}{c_2^{\sigma}}$$

but now the problem of the firm is $mc_1 = mc_2 = \frac{\varepsilon-1}{\varepsilon}$ and since:

$$mc_t = \frac{w_t}{A_t}$$

we get:

$$w_t = \frac{\varepsilon-1}{\varepsilon} A_t$$

By market clearing, $c_t = A_t l_t$. Then:

$$(A_t l_t)^{\sigma} l_t^{\psi-1} = \frac{\varepsilon-1}{\varepsilon} A_t \Rightarrow l_t = \left(\frac{\varepsilon-1}{\varepsilon}\right)^{\frac{1}{\sigma+\psi-1}} A_t^{\frac{1-\sigma}{\sigma+\psi-1}}$$

and

$$R = \frac{1}{\beta}\left(\frac{c_2}{c_1}\right)^{\sigma} = \frac{1}{\beta}\left(\frac{A_2}{A_1}\right)^{\frac{\psi\sigma}{\sigma+\psi-1}}$$

8.3. Algebraic Derivations for Model III: Flex Prices

Now we introduce money and transaction costs, but prices are fully flexible. The FOCs of the household (see next subsection for the Lagrangian of the household):

$$c_1^{\sigma} l_1^{\psi-1} = \frac{w_1}{1 + s(v_1) + s'(v_1) v_1}$$
$$c_2^{\sigma} l_2^{\psi-1} = \frac{w_2}{1 + s(v_2) + s'(v_2) v_2}$$
$$\frac{R-1}{R} = s(v_1)' v_1^2$$
$$\gamma = c_2^{1-\sigma} \frac{\frac{1}{v_2} - s(v_2)' v_2}{1 + s(v_2) + s'(v_2) v_2}$$
$$\frac{1}{p_1 c_1^{\sigma}} = \beta \frac{R}{p_2 c_2^{\sigma}} \frac{1 + s(v_1) + s'(v_1) v_1}{1 + s(v_2) + s'(v_2) v_2}$$

The problem of the producers is still $mc_1 = mc_2 = \frac{\varepsilon-1}{\varepsilon}$ and we get:

$$mc_t = \frac{w_t}{A_t} = \frac{c_t^{\sigma} l_t^{\psi-1} (1 + s(v_t) + s'(v_t) v_t)}{A_t}$$

Therefore:

$$\frac{c_1^{\sigma} l_1^{\psi-1} (1 + s(v_1) + s'(v_1) v_1)}{A_1} = \frac{\varepsilon - 1}{\varepsilon}$$
$$(1 + s(v_1)) c_1 = A_1 l_1$$
$$\frac{c_2^{\sigma} l_2^{\psi-1} (1 + s(v_2) + s'(v_2) v_2)}{A_2} = \frac{\varepsilon - 1}{\varepsilon}$$
$$(1 + s(v_2)) c_2 = A_2 l_2$$
$$\frac{1}{c_1^{\sigma}} = \beta \frac{1}{c_2^{\sigma}} R \frac{p_1}{p_2} \frac{1 + s(v_1) + s'(v_1) v_1}{1 + s(v_2) + s'(v_2) v_2}$$
$$s(v_1)' v_1^2 = \begin{cases} \frac{R-1}{R} \text{ if } R > 1 \\ 0 \text{ if } R = 1 \end{cases}, R = 1 \text{ otherwise}$$
$$\gamma = c_2^{1-\sigma} \frac{\frac{1}{v_2} - s(v_2)' v_2}{1 + s(v_2) + s'(v_2) v_2}.$$

Note that we also have a recursive structure, with a second period block

$$\frac{c_2^{\sigma} l_2^{\psi-1}\left(1+s\left(v_2\right)+s'\left(v_2\right)v_2\right)}{A_2}=\frac{\varepsilon-1}{\varepsilon}$$
$$\left(1+s\left(v_2\right)\right)c_2=A_2 l_2$$
$$\gamma=c_2^{1-\sigma}\frac{\frac{1}{v_2}-s\left(v_2\right)' v_2}{1+s\left(v_2\right)+s'\left(v_2\right)v_2}$$

and a first period one:

$$\frac{c_1^{\sigma} l_1^{\psi-1}\left(1+s\left(v_1\right)+s'\left(v_1\right)v_1\right)}{A_1}=\frac{\varepsilon-1}{\varepsilon}$$
$$\left(1+s\left(v_1\right)\right)c_1=A_1 l_1$$
$$\frac{1}{c_1^{\sigma}}=\beta\frac{1}{c_2^{\sigma}}R\frac{p_1}{p_2}\frac{1+s\left(v_1\right)+s'\left(v_1\right)v_1}{1+s\left(v_2\right)+s'\left(v_2\right)v_2}$$
$$s\left(v_1\right)' v_1^2=\begin{cases}\frac{R-1}{R}\text{ if } R>1\\ 0\text{ if } R=1\end{cases},\ R=1\text{ otherwise}$$

8.4. Algebraic Derivations for Model IV: Nominal Rigidities

The Lagrangian associated with the problem of the household is:

$$\frac{c_1^{1-\sigma}}{1-\sigma}-\frac{l_1^{\psi}}{\psi}+\beta\left(\frac{c_2^{1-\sigma}}{1-\sigma}-\frac{l_2^{\psi}}{\psi}+\gamma\log\frac{m_2}{p_2}\right)$$
$$+\lambda_1\left(w_1 l_1+F_1+T-\left(1+s\left(v_1\right)\right)c_1-\frac{m_1}{p_1}-\frac{1}{p_1}\frac{b}{R}\right)$$
$$+\beta\lambda_2\left(w_2 l_2+F_2+\frac{m_1}{p_2}+\frac{b}{p_2}-\left(1+s\left(v_2\right)\right)c_2-\frac{m_2}{p_2}\right)$$

The FOCs are:

$$c_1:\ c_1^{-\sigma}=\lambda_1\left(1+s\left(v_1\right)+s'\left(v_1\right)v_1\right)$$
$$l_1:\ l_1^{\psi-1}=\lambda_1 w_1$$
$$c_2:\ c_2^{-\sigma}=\lambda_2\left(1+s\left(v_2\right)+s'\left(v_2\right)v_2\right)$$
$$l_2:\ l_2^{\psi-1}=\lambda_2 w_2$$
$$m_1:\ \lambda_1\frac{1}{p_1}\left(1-s\left(v_1\right)' v_1^2\right)=\beta\lambda_2\frac{1}{p_2}$$
$$m_2:\ \frac{\gamma}{m_2}=\lambda_2\frac{1}{p_2}\left(1-s\left(v_2\right)' v_2^2\right)$$

and

$$R: \ \lambda_1 \frac{1}{p_1} = \beta \lambda_2 \frac{R}{p_2}$$

We can combine the first four conditions:

$$c_1^{\sigma} l_1^{\psi-1} = \frac{w_1}{1 + s(v_1) + s'(v_1) v_1}$$
$$c_2^{\sigma} l_2^{\psi-1} = \frac{w_2}{1 + s(v_2) + s'(v_2) v_2}$$

Then, with the last fifth and the seventh:

$$\left(1 - s(v_1)' v_1^2\right) = \beta \frac{\lambda_2}{\lambda_1} \frac{p_1}{p_2} = \frac{1}{R} \Rightarrow$$
$$\frac{R-1}{R} = s(v_1)' v_1^2$$

and

$$\frac{\gamma}{m_2} = \lambda_2 \frac{1}{p_2} \left(1 - s(v_2)' v_2^2\right) \Rightarrow$$
$$\gamma = \frac{c_2^{-\sigma}}{1 + s(v_2) + s'(v_2) v_2} \frac{m_2}{p_2} \left(1 - s(v_2)' v_2^2\right) \Rightarrow$$
$$\gamma = c_2^{1-\sigma} \frac{\frac{1}{v_2} - s(v_2)' v_2}{1 + s(v_2) + s'(v_2) v_2}$$

Finally:

$$\frac{1}{c_1^{\sigma}} = \beta \frac{1}{c_2^{\sigma}} R \frac{p_1}{p_2} \frac{1 + s(v_1) + s'(v_1) v_1}{1 + s(v_2) + s'(v_2) v_2}$$

In the case where the ZLB binds, we have the same conditions except that now $0 = s(v_1)' v_1^2$.

Now, we exploit the recursive structure of the previous equations and solve for the second period choices:

$$1 - \varepsilon + \varepsilon \frac{c_2^{\sigma} l_2^{\psi-1} (1 + s(v_2) + s'(v_2) v_2)}{A_2} - \phi \frac{p_2}{p_1} \left[\frac{p_2}{p_1} - 1\right] + \varepsilon \frac{\phi}{2} \left[\frac{p_2}{p_1} - 1\right]^2 = 0$$
$$(1 + s(v_2)) c_2 = \left(1 - \frac{\phi}{2} \left[\frac{p_2}{p_1} - 1\right]^2\right) A_2 l_2$$
$$\gamma = c_2^{1-\sigma} \frac{\frac{1}{v_2} - s(v_2)' v_2}{1 + s(v_2) + s'(v_2) v_2}$$

Note

$$l_2 = \left(\frac{1 + s(v_2)}{1 - \frac{\phi}{2}\left[\frac{p_2}{p_1} - 1\right]^2} \right) \frac{c_2}{A_2}$$

$$1 + s(v_2) + s'(v_2) v_2 = \frac{c_2^{1-\sigma}}{\gamma} \left(\frac{1}{v_2} - s(v_2)' v_2 \right)$$

Then:

$$s(v_2)' v_2^2 \frac{m}{p_2} = \frac{m}{p_2} \left[\alpha_0 v_2^2 - \alpha_1 \right]$$

and:

$$\frac{m}{p_2} - s(v_2)' v_2 c_2 = \frac{m}{p_2} \left(1 - \alpha_0 v_2^2 + \alpha_1 \right) = \frac{c_2}{v_2} \left(1 - \alpha_0 v_2^2 + \alpha_1 \right)$$

Also:

$$\begin{gathered} 1 + s(v_2) + s'(v_2) v_2 = \\ 1 + \alpha_0 v_t + \alpha_1 \frac{1}{v_t} - 2\sqrt{\alpha_0 \alpha_1} + \alpha_0 v_t - \alpha_1 \frac{1}{v_t} = \\ 1 + 2\alpha_0 v_t - 2\sqrt{\alpha_0 \alpha_1} \end{gathered}$$

and then

$$1 + 2\alpha_0 v_t - 2\sqrt{\alpha_0 \alpha_1} = \frac{c_2^{1-\sigma}}{\gamma} \frac{1}{v_t} \left(1 - \alpha_0 v_t^2 + \alpha_1 \right) \Rightarrow c_2 = \left(\gamma \frac{v_t + 2\alpha_0 v_t^2 - 2\sqrt{\alpha_0 \alpha_1} v_t}{1 - \alpha_0 v_t^2 + \alpha_1} \right)^{\frac{1}{1-\sigma}}$$

Finally, at the ZLB

$$\gamma = c_2^{1-\sigma} \frac{\frac{1}{v_2} - s(v_2)' v_2}{1 + s(v_2) + s'(v_2) v_2} \Rightarrow 1 + s(v_2) + s'(v_2) v_2 = \frac{c_2^{1-\sigma}}{\gamma} \left(\frac{1}{v_2} - s(v_2)' v_2 \right)$$

and then

$$\begin{aligned} c_1 &= \left(\frac{1}{\beta} \frac{p_2}{p_1} \left(1 + s(v_2) + s'(v_2) v_2 \right) \right)^{\frac{1}{\sigma}} c_2 \\ &= \left(\frac{1}{\beta} \frac{p_2}{p_1} \frac{c_2^{1-\sigma}}{\gamma} \left(\frac{1}{v_2} - s(v_2)' v_2 \right) \right)^{\frac{1}{\sigma}} c_2 \\ &= \left(\frac{1}{\beta} \frac{1}{p_1} \frac{m}{\gamma} \left(1 - s(v_2)' v_2^2 \right) \right)^{\frac{1}{\sigma}} \\ &= \left(\frac{1}{\beta} \frac{1}{p_1} \frac{m}{\gamma} \left(1 - \alpha_0 v_2^2 + \alpha_1 \right) \right)^{\frac{1}{\sigma}} \end{aligned}$$

We move now to the problem of the firms. Note that an equivalent problem for the intermediate good producer is:

$$\max_{p_{2t}} \left(\frac{p_{i2}}{p_2}\right)^{1-\varepsilon} - \left(\frac{p_{i2}}{p_2}\right)^{-\varepsilon} mc_2 - \frac{\phi}{2}\left[\frac{p_{i2}}{p_{i1}} - 1\right]^2 \left(\frac{p_{i2}}{p_2}\right)^{-\varepsilon}$$

with FOC:

$$(1-\varepsilon)\left(\frac{p_{i2}}{p_2}\right)^{1-\varepsilon}\frac{1}{p_{i2}} + \varepsilon\left(\frac{p_{i2}}{p_2}\right)^{-\varepsilon}\frac{1}{p_{i2}}mc_2 - \phi\frac{1}{p_{i1}}\left[\frac{p_{i2}}{p_{i1}} - 1\right]\left(\frac{p_{i2}}{p_2}\right)^{-\varepsilon} + \varepsilon\frac{\phi}{2}\left[\frac{p_{i2}}{p_{i1}} - 1\right]^2\left(\frac{p_{i2}}{p_2}\right)^{-\varepsilon}\frac{1}{p_{i2}} = 0$$

Then, we can apply the symmetry of all firms ($p_{i2} = p_2$) to get:

$$1 - \varepsilon + \varepsilon mc_2 - \phi\frac{p_2}{p_1}\left[\frac{p_2}{p_1} - 1\right] + \varepsilon\frac{\phi}{2}\left[\frac{p_2}{p_1} - 1\right]^2 = 0.$$