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RISING INDEBTEDNESS AND TEMPTATION:  
A WELFARE ANALYSIS**

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# Rising Indebtedness and Temptation: A Welfare Analysis\*

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## Abstract

Is the observed large increase in consumer indebtedness since 1970 beneficial for U.S. consumers? This paper quantitatively investigates the macroeconomic and welfare implications of relaxing borrowing constraints using a model with preferences featuring temptation and self-control. The model can capture two contrasting views: the positive view, which links increased indebtedness to financial innovation and thus better consumption smoothing, and the negative view, which is associated with consumers' *over-borrowing*. I find that the latter is sizable: the calibrated model implies a social welfare *loss* equivalent to a 0.4 percent decrease in per-period consumption from the relaxed borrowing constraint consistent with the observed increase in indebtedness. The welfare implication is strikingly different from the standard model without temptation, which implies a welfare *gain* of 0.7 percent, even though the two models are observationally similar. Naturally, the optimal level of the borrowing limit is significantly tighter according to the temptation model, as a tighter borrowing limit helps consumers by preventing over-borrowing.

JEL Classification: D91, E21, E44, G18

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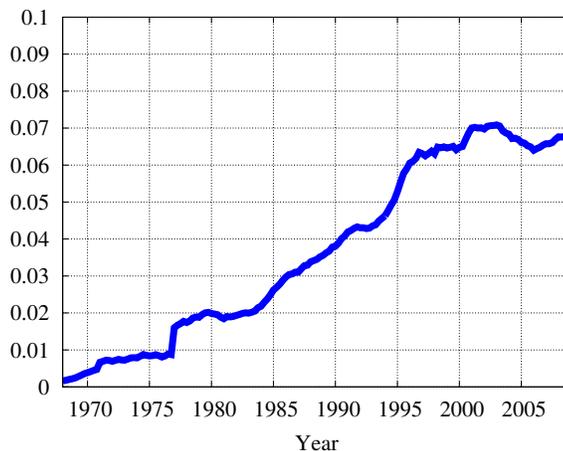


Figure 1: Total unsecured consumer debt over GDP

## 1 Introduction

Since the 1970s, there has been a substantial increase in the indebtedness of U.S. consumers, although that trend might reverse as a result of the recent downturn. Total household debt in the U.S. increased from 43 percent of GDP in 1982 to 62 percent in 2000.<sup>1</sup> Both unsecured and secured debt increased. Figure 1 shows the trend of unsecured consumer debt relative to GDP.<sup>2</sup> It was close to zero before 1970 but has gradually increased since then, and it has stabilized around 7 percent since 2000. While an increase in indebtedness is often seen as a result of an innovation in the financial sector and thus is linked to a gain in social welfare, there are two channels through which rising indebtedness is associated with a welfare loss. First is the general equilibrium effect; increased indebtedness might induce under-saving, which slows down capital accumulation. Second, there is a popular perception that consumers might be *over-borrowing* and *over-consuming*. While the first channel is studied, among others, by [Campbell and Hercowitz \(2009\)](#) and [Obiols-Homs \(2011\)](#), the second channel has not been studied, since it cannot be systematically captured by models with the standard exponential preferences.<sup>3</sup> This paper intends to fill the void.

In order to analyze over-borrowing and over-consuming, I introduce preferences featuring temptation and self-control, which is developed by [Gul and Pesendorfer \(2001, 2004a,b\)](#). Specifically, I use a version of the macroeconomic model with the temptation preferences, which is developed by [Krusell et al. \(2010\)](#). There are three benefits of using the framework for a macroe-

<sup>1</sup> [Smith \(2009\)](#).

<sup>2</sup> Unsecured consumer debt is measured as the revolving consumer credit in the G.19 series of the Federal Reserve Board (FRB). In the FRB data, total consumer credit consists of non-revolving and revolving credit. Revolving credit mainly consists of loans for automobiles, mobile homes, and boats but also includes some unsecured credit. [Livshits et al. \(2010\)](#) constructed an unsecured consumer credit data series that includes not only revolving credit but also a part of non-revolving credit. However, the difference between the revolving credit and the unsecured consumer credit they constructed is small (less than one percentage point as a percentage of disposable income) for the period for which more reliable data are available (after 1989).

<sup>3</sup> Both [Campbell and Hercowitz \(2009\)](#) and [Obiols-Homs \(2011\)](#) investigate a cross-section of the welfare consequences associated with rising debt in the U.S., but both use the standard exponential discounting preferences.

conomic analysis of over-borrowing. First, in the temptation model, consumers are *tempted* to borrow and consume more than they would choose if they could exert *self-control*. As a result, they and suffer from over-borrowing. Therefore, this framework is naturally suitable in studying over-borrowing in response to a relaxed borrowing constraint. Second, the temptation model developed by [Krusell et al. \(2010\)](#) includes the model with hyperbolic discounting consumers, which is originally developed by [Strotz \(1956\)](#) and introduced into the standard macroeconomic framework by [Laibson \(1997\)](#), as a special case.<sup>4 5</sup> As I employ this special case for the analysis, existing findings associated with the hyperbolic discounting model are valid with the temptation model used in this paper. Finally, not only is the temptation model more general than the hyperbolic discounting model, the temptation model enables more straightforward welfare analysis; the hyperbolic discounting model is time-inconsistent, in which a consumer consists of “multiple selves” as they have different preferences in each point of time. In this model, how to put weights to these multiple self is not a trivial problem. On the other hand, the temptation model is time-consistent and does not suffer such problem.

There are supporting evidences – based on both survey results and estimated structural models – that consumers face temptation and self-control problem, which supports the use of the temptation model for the analysis. At the same time, literature also found that models with and without temptation are *observationally similar* in macroeconomic aggregates. Most importantly, [Angeletos et al. \(2001\)](#) compare the macroeconomic implications of models with and without temptation and argue that the temptation model replicates various dimensions of consumption and savings behavior better than the standard model without temptation, although both models are observationally similar in terms of the average life-cycle profile of aggregate saving.<sup>6</sup> Furthermore, [Barro \(1999\)](#) shows the observational equivalence between the neoclassical growth models with and without temptation. My findings echo theirs: models with and without temptation have similar macroeconomic implications. But how about welfare implications? This is the key issue investigated in this paper.

Although the model with temptation potentially has welfare implications that are very different from those in the standard model without temptation, not many papers study the welfare implications of the macroeconomic model with temptation. There are few exceptions. [Krusell et al. \(2010\)](#) show that a savings subsidy (or negative capital income tax) is optimal in the

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<sup>4</sup> The actual discount factor function used in the “hyperbolic discounting” models is not precisely hyperbolic, but it is called by that name because originally a hyperbolic function was used. *Quasi-hyperbolic* or *quasi-geometric* discounting is a more precise description of the actual discount factor function typically used in the literature, although I use the terminology *hyperbolic discounting* since it is commonly used.

<sup>5</sup> [Angeletos et al. \(2001\)](#) provide a good summary of the literature. [Strotz \(1956\)](#) first formalized the hyperbolic discounting preferences. [Phelps and Pollak \(1968\)](#) use the quasi-hyperbolic discounting function in the context of intergenerational time preferences. [Laibson \(1997\)](#) studies the role of an illiquid asset like housing as commitment device for time-inconsistent consumers. [Laibson et al. \(2003\)](#) use a hyperbolic discounting model to explain the credit card puzzle. [Laibson et al. \(2007\)](#) use a simulated method of moments to jointly estimate key parameters associated with the hyperbolic discounting model. [İmrohoroğlu et al. \(2003\)](#) study the macroeconomic and welfare effects of having an unfunded Social Security program in the model with hyperbolic discounting consumers. [Tobacman \(2009\)](#) investigates the wealth distribution of such a model.

<sup>6</sup> While both [Angeletos et al. \(2001\)](#) and [Barro \(1999\)](#) conduct their analysis based on the hyperbolic discounting model, the hyperbolic discounting model is a special case of the temptation model used in this paper. Therefore, their findings are applicable for the temptation model used in this paper.

neoclassical growth model with temptation, while it is optimal not to tax capital income in the model without temptation. [İmrohoroğlu et al. \(2003\)](#) study welfare implications of social security reforms. [Malin \(2008\)](#) studies mandatory savings floors.

The model developed in this paper is built on a life-cycle general equilibrium model with incomplete markets initially developed by [Huggett \(1996\)](#) and [Aiyagari \(1994\)](#). This paper introduces preferences featuring temptation and self-control into the standard general equilibrium model with market incompleteness. The model is closest to the one in [İmrohoroğlu et al. \(2003\)](#), but they do not focus on intra-generational heterogeneity. In addition, this paper is the first one to solve for the equilibrium transition dynamics between steady states in a model with temptation.

There are three main findings. First, not only are the models with and without temptation observationally similar in the steady-state equilibrium, as shown in [Angeletos et al. \(2001\)](#), the aggregate response associated with an increased indebtedness, which is induced by a relaxed borrowing limit, is both qualitatively and quantitatively similar between the two models. Second, however, the two models have strikingly different welfare implications. According to the calibrated model, while a relaxed borrowing limit is associated with a social welfare gain equivalent to 0.7 percent of flow consumption in the model without temptation, the temptation model implies a welfare loss of 0.4 percent. The difference is due to the over-borrowing by consumers in response to a relaxed borrowing limit. The problem is serious from a policy perspective because the models with and without temptation are hard to distinguish but have contrasting welfare implications. [Barro \(1999\)](#) argues that we can largely keep relying on the neoclassical growth model with exponential discounting consumers as the workhorse framework, even though there is some evidence in favor of temptation, because the growth models with the two different preference specifications are observationally equivalent. The case I study in this paper shows that one needs to be careful even if the temptation model is observationally similar to the standard model without temptation, because the two models could have very different welfare implications. Finally, the optimal level of the borrowing limit is lower, at about 7 percent of average income, in the model with temptation compared with the standard model without temptation, whose optimal borrowing limit is about 19 percent. Even in a standard no-temptation model like those in [Campbell and Hercowitz \(2009\)](#) and [Obiols-Homs \(2011\)](#), there is a level of the borrowing limit at which the gain from a relaxed borrowing limit (better consumption smoothing) is dominated by the negative general equilibrium effect (capital decumulation). The reason why the optimal borrowing limit is substantially lower in the model with temptation is over-borrowing. When consumers exhibit temptation preferences, there is a potential for extra welfare gain from restricting borrowing of consumers.

Although the model used for analysis is rich in features, there are limitations. First, the model abstracts from aggregate shocks. Second, I assume that all consumers have the same preferences. For example, in the temptation models, all consumers share the same parameter values associated with temptation. Third, I do not allow any commitment device for consumers. As [Laibson \(1997\)](#) showed, consumers with temptation would optimally try to use commitment devices, if available, to restrain themselves from over-consuming in the future. Examples are durable goods (such as housing) or retirement saving instruments (such as individual retirement

accounts (IRAs)).<sup>7</sup> Finally, I consider only unsecured debt. I leave these issues for future research.

The rest of the paper is organized as follows. Section 2 presents the model. At the end of the section, I will argue that the model with the Strotz-Laibson hyperbolic discounting preferences is a special case of the temptation model developed by Krusell et al. (2010) and employed in this paper. Section 3 describes how the model is calibrated for quantitative exercises. Section 4 gives an overview of the computational algorithm with which the model is solved. Section 5 presents the main results of the paper, using steady-state analysis. Section 6 conducts an analysis explicitly taking into account the equilibrium transition path from an initial steady state to a new one. Section 7 addresses the sensitivity of the main results. Section 8 concludes. The separate Supplementary Appendix includes more details of the calibration and the computational algorithm.

## 2 Model

The model is based on the general equilibrium life-cycle model of Huggett (1996), with the version of the Gul-Pesendorfer preferences featuring temptation and self-control that is developed by Krusell et al. (2010). After completing the description of the model, in Section 2.9, I will provide an alternative formulation of the consumer’s problem based on the Strotz-Laibson hyperbolic discounting preferences, and argue that, when the strength of temptation is taken to infinity, the two formulations generate identical optimal decision rules. Therefore, all existing macroeconomic implications under the Strotz-Laibson hyperbolic discounting preferences are valid under the temptation preferences. The reason I use the temptation preferences is that the temptation preferences allow straightforward welfare analysis compared with the hyperbolic discounting preferences, which features multiple-selves within a consumer.

### 2.1 Demographics

Time is discrete and starts from 0. In each period, the economy is populated by  $I$  overlapping generations of consumers. In period  $t$ , a measure  $(1 + \nu)^t$  of consumers are born.  $\nu$  is the constant population growth rate. Each generation is populated by a mass of consumers. Consumers are born at age 1 and could live up to age  $I$ . An age- $i$  consumer survives to age- $i + 1$  with probability  $s_i$ . With probability  $(1 - s_i)$ , the consumer dies.  $I$  is the maximum possible age, which implies  $s_I = 0$ . Consumers retire at a fixed age  $I_R < I$ . Consumers with age  $i < I_R$  are called workers, and those with age  $i \geq I_R$  are called retirees.

### 2.2 Preferences

The preferences of consumers are time separable and characterized by an instantaneous utility function, two discount factors,  $\delta$  and  $\beta$ , and another parameter  $\gamma$ . The instantaneous utility

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<sup>7</sup> Although it is likely that if such commitment devices are available, consumers in the model will use them more extensively if the borrowing constraint is relaxed, notice that I calibrate the borrowing constraint to match the observed level of indebtedness. If these commitment devices are available for consumers in the model, the degree of relaxation of the borrowing constraint must be calibrated to be even larger. Therefore, introducing commitment devices that allow consumers to restrain themselves from over-consuming and over-borrowing will not necessarily weaken the main results of the paper. Exactly how the main results of the paper will be affected by the introduction of the commitment devices is left for future research.

function  $u(c)$  is standard: it is strictly increasing and strictly concave in  $c$ . Consumers do not value leisure; there is no labor supply decision. In Section 7, I relax the assumption and introduce a labor-leisure decision as a sensitivity analysis.

$\delta$  and  $\beta$  are called *self-control discount factor* and *temptation discount factor*, respectively.  $\gamma$  represents *strength of temptation*.  $\delta$  is the only discount factor if the consumer can exert perfect self-control and thus is not affected by temptation. In other words, in a special case where the temptation is nonexistent (strength of temptation  $\gamma$  is zero), the model with temptation and self-control preferences reverts to a standard exponential discounting model with  $\delta$  as the only discount factor.  $\beta < 1$  is the additional discount factor that a consumer is tempted to discount future utility when making consumption-savings decision. Formal formulation of the consumers' problem is presented in Section 2.7.

### 2.3 Technology

There is a representative firm that has access to the constant returns to scale production technology in the form of  $Y = ZF(K, L)$ .  $Y$  is output,  $Z$  is the level of total factor productivity,  $K$  is capital stock, and  $L$  is labor supply. Capital depreciates at a constant rate  $\kappa$  per period.

### 2.4 Endowment

Consumers are born with zero assets. Each consumer is endowed with one unit of time each period. Time is inelastically supplied for work, since leisure is not valued. Labor productivity of a consumer is characterized by  $e(i, p)$ , where  $i$  captures the life-cycle profile of labor productivity, and  $p$  is the idiosyncratic shock to labor productivity.  $p$  is assumed to have finite support:  $p \in \{p_1, p_2, \dots, p_N\}$ . Each newborn consumer draws its initial  $p$  from an i.i.d. distribution where  $\pi_p^0$  is the probability attached to each  $p$ . After the initial  $p$  is drawn,  $p$  follows a first-order Markov process with  $\pi_{p,p'}$  as the transition probability from  $p$  to  $p'$ .

### 2.5 Market Arrangements

Capital and labor are traded competitively. Consumers are not allowed to trade state-contingent securities but can borrow or save using asset  $a$ , subject to a borrowing limit  $\underline{a}_t$ .

### 2.6 Government

The government has three roles in the model: (i) running the Social Security program, (ii) collecting a proportional income tax, and (iii) collecting accidental bequests using estate taxes and redistributing the proceeds with a lump-sum transfer.

The government runs a simple pay-as-you-go Social Security program. The government imposes a flat payroll tax with the tax rate of  $\tau_S$  on all workers and uses the proceeds to finance the Social Security benefits  $b_{t,i}$  of current retirees. It is assumed that all retirees receive the same amount ( $\bar{b}_t$ ) of benefits regardless of their age or contribution, and the government budget associated with the Social Security program balances each period. Formally,  $b_{t,i} = 0$  for  $i < I_R$  and  $b_{t,i} = \bar{b}_t$  for  $i \geq I_R$ .

The government collects a proportional general income tax with tax rate  $\tau_I$ . Both capital and labor income are taxed at the same rate. The proceeds are not redistributed or valued by consumers.

Because of the stochastic death, there are accidental bequests in the model. I assume that

the government collects all of the accidental bequests using estate taxes and redistributes the proceeds equally to the surviving consumers every period.  $d_t$  denotes the lump-sum transfer under the program in period  $t$ .

## 2.7 Consumer's Problem

The problem of an age- $i$  consumer with the current productivity shock  $p$  and asset position  $a$  in period  $t$  can be characterized recursively as follows:

$$V_t(i, p, a) = \max_{a' \geq \underline{a}_t} \left[ U_t(i, p, a, a') + \gamma \left( W_t(i, p, a, a') - \max_{\tilde{a}' \geq \underline{a}_t} W_t(i, p, a, \tilde{a}') \right) \right] \quad (1)$$

where

$$U_t(i, p, a, a') = u(c) + \delta s_i \sum_{p'} \pi_{p,p'} V_{t+1}(i+1, p', a') \quad (2)$$

$$W_t(i, p, a, a') = u(c) + \beta \delta s_i \sum_{p'} \pi_{p,p'} V_{t+1}(i+1, p', a') \quad (3)$$

$$c + a' = (a + d_t)(1 + r_t(1 - \tau_I)) + e(i, p)(1 - \tau_I - \tau_S)w_t + b_{t,i} \quad (4)$$

Equation (1) is the Bellman equation. Equations (2) and (3) define *self-control utility* and *temptation utility*, respectively. The only difference between the two is, while future utility is discounted by  $\delta$  in the former, it is discounted by  $\beta\delta$  in the latter. Naturally, when  $\beta < 1$ , the consumer is tempted to consume more this period when the consumer is maximizing the temptation utility rather than self-control utility. Equation (4) is the standard budget constraint, with consumption ( $c$ ) and next-period assets ( $a'$ ) on the left-hand side, and current-period assets ( $a$ ), transfers ( $d_t$ ), after-tax interest income ( $(a + d_t)r_t(1 - \tau_I)$ ), after-tax labor income ( $e(i, p)(1 - \tau_I - \tau_S)w_t$ ), and Social Security benefits ( $b_{t,i}$ ) on the right-hand side. The maximand of the Bellman equation consists of two parts – the self-control utility, and the part that contains the temptation utility.  $\gamma$  determines the relative strength of the latter.  $a' = g_t^a(i, p, a)$  is the optimal decision rule associated with the Bellman equation above.

In order to understand the non-standard Bellman equation, let's consider the two extreme cases first. In an extreme case where  $\gamma = 0$ , the temptation part of the problem drops out completely, and the consumer's problem reverts back to the one with the standard exponential discounting preferences with the discount factor  $\delta$ . This is interpreted as the case when the consumer has a perfect self-control and thus is not affected by temptation to consume more today rather than in the future. In the other extreme case where  $\gamma \rightarrow \infty$ , the utility-maximizing consumer wants to choose  $a'$  that maximizes the temptation utility, as the relative importance of the self-control utility becomes zero. Notice, however, that since the difference between  $W_t(i, p, a, a')$  and  $\max_{\tilde{a}' \geq \underline{a}_t} W_t(i, p, a, \tilde{a}')$  becomes zero, the value updated in the Bellman equation is based on the self-control utility, but with  $a'$  that maximizes the temptation utility. The intuition is, although the consumer wants to choose  $a'$  to maximize the self-control utility, the consumer succumbs to the temptation and chooses  $a'$  that maximizes the temptation utility. In an intermediate case where  $\gamma \in (0, \infty)$ , the consumer chooses  $a'$  to balance the two forces; on the one hand, the consumer wants to choose  $a'$  that maximizes the self-control utility, which is associated

with the discount factor  $\delta$ . On the other hand, the consumer also wants to consume more, to maximize the temptation utility with discount factor  $\beta\delta$ .  $\gamma$  determines the relative strength of the latter.

## 2.8 Equilibrium

I will first define the recursive competitive equilibrium where the demographic structure is stationary, even though the size of the population is growing at a constant rate  $\nu$ . Then I will move on to define the steady-state recursive competitive equilibrium, where prices  $\{r_t, w_t\}_{t=0}^\infty$  and government policy variables  $\{b_{t,i}, d_t\}_{t=0}^\infty$  are constant over time, although the aggregate variables are growing at the population growth rate.

Let  $\mathbf{M}$  be the space of an individual state, i.e.,  $(i, p, a) \in \mathbf{M}$ . Let  $\mathcal{M}$  be the Borel  $\sigma$ -algebra generated by  $\mathbf{M}$ , and  $\mu$  the probability measure defined over  $\mathcal{M}$ . I will use a probability space  $(\mathbf{M}, \mathcal{M}, \mu)$  to represent a type distribution of consumers.

**Definition 1 (Recursive competitive equilibrium)** *Given a sequence of total factor productivity  $\{Z_t\}_{t=0}^\infty$ , a sequence of borrowing limits  $\{a_t\}_{t=0}^\infty$ , and the initial type distribution of consumers  $\mu_0$ , a recursive competitive equilibrium is a sequence of prices  $\{r_t, w_t\}_{t=0}^\infty$ , government policy variables  $\{b_{t,i}, d_t\}_{t=0}^\infty$ , aggregate capital stock and labor supply  $\{K_t, L_t\}_{t=0}^\infty$ , value functions  $\{V_t(i, p, a)\}_{t=0}^\infty$ , optimal decision rules  $\{g_t^a(i, p, a)\}_{t=0}^\infty$ , and the measure after normalization with respect to population growth,  $\{\mu_t\}_{t=0}^\infty$ , such that:*

1. *In each period  $t$ , given the prices and policy variables,  $V_t(i, p, a)$  is a solution to the consumer's optimization problem defined in Section 2.7, and  $g_t^a(i, p, a)$  is the associated optimal decision rule.*
2. *The prices  $\{r_t, w_t\}_{t=0}^\infty$  are determined competitively, i.e.,*

$$r_t = Z_t F_K(K_t, L_t) - \kappa \tag{5}$$

$$w_t = Z_t F_L(K_t, L_t) \tag{6}$$

where

$$K_{t+1} = \frac{1}{1 + \nu} \int_{\mathbf{M}} g_t^a(i, p, a) d\mu_t \tag{7}$$

$$L_t = \int_{\mathbf{M}} e(i, p) d\mu_t \tag{8}$$

3. *Given the initial measure  $\mu_0$ , the sequence of the measure of consumers  $\{\mu_t\}_{t=0}^\infty$  is consistent with the demographic transition, the stochastic process of shocks, and the optimal decision rules, after normalization with respect to population growth in each period  $t$ .*
4. *Government satisfies the period-by-period budget balance with respect to the Social Security program in each period  $t$ , i.e.,*

$$\int_{\mathbf{M}} b_{t,i} d\mu_t = \int_{\mathbf{M}} e(i, p) w_t \tau_S d\mu_t \tag{9}$$

5. Government satisfies the period-by-period budget constraint with respect to the estate tax and lump-sum transfer in each period  $t$ , i.e.,

$$\int_{\mathbf{M}} d_{t+1} d\mu_{t+1} = \frac{1}{1+\nu} \int_{\mathbf{M}} (1-s_i) g_t^a(i, p, a) d\mu_t \quad (10)$$

**Definition 2 (Steady-state recursive competitive equilibrium)** A steady-state recursive competitive equilibrium is a recursive competitive equilibrium where total factor productivity, borrowing limits, type distribution, prices, government policy variables, aggregate capital stock and labor supply, value functions, and optimal decision rules are constant over time, after normalizing the type distribution of consumers by the population growth rate.

Notice that although I use the word *steady state*, the model is on a balanced growth path with a constant population growth rate and the type distribution (after normalization) of heterogeneous consumers is stationary. The measure of consumers is normalized to be a probability measure (total measure is one) each period, which makes all the aggregate variables constant over time instead of growing at the population growth rate.

## 2.9 Alternative Formulation of the Consumer's Problem with Hyperbolic Discounting

I will provide an alternative formulation of the consumer's problem defined in Section 2.7, based on the Strotz-Laibson hyperbolic discounting preferences. After showing the recursive formulation of the consumer's problem, I will argue that the hyperbolic discounting preferences are the special case of the temptation preferences; the resulting optimal decision rules are the same as in the problem based on the preferences featuring temptation and self-control with  $\gamma \rightarrow \infty$ . Finally, I discuss the difference of the two specifications in welfare implications.

For an age- $i$  consumer, the expected lifetime utility,  $U_i$ , can be defined as follows:

$$U_i = u(c_i) + \beta \mathbb{E} \sum_{j=i+1}^I \delta^{j-i} u(c_j) \quad (11)$$

According to the Strotz-Laibson set-up, in period  $t$ , instantaneous utility in period  $t, t+1, t+2, t+3, \dots$ , is discounted by  $1, \beta\delta, \beta\delta^2, \dots$ . Since  $\beta$  is used only to discount utility from the current period and the next, while  $\delta$  is used to discount future utility every period,  $\beta$  and  $\delta$  are called *short-term* and *long-term* discount factors, respectively. Notice that the standard exponential discounting is a special case with  $\beta = 1$ : in this case, future utility is discounted exponentially at the constant discount factor  $\delta$ . The important feature of this class of preferences is that the preferences exhibit time inconsistency; the discount factor applied between period  $t+1$  and  $t+2$  in period  $t$  is  $\delta$ , while the discount factor between the same periods changes to  $\beta\delta$  in period  $t+1$ . With  $\beta \in (0, 1)$ , the preferences imply a present bias: if there is no constraint or commitment device, consumers over-borrow and over-consume from the perspective in previous periods.

The problem of an age- $i$  consumer with the current productivity shock  $p$  and asset position  $a$  in period  $t$  can be characterized by the following Bellman equation:

$$\tilde{V}_t(i, p, a) = \max_{a' \geq a_t} \left[ u(c) + \beta \delta s_i \sum_{p'} \pi_{p,p'} V_{t+1}(i+1, p', a') \right] \quad (12)$$

subject to the budget constraint (4).  $a' = g_t^a(i, p, a)$  is the optimal decision rule associated with the Bellman equation above. Notice that the value function on the left-hand side,  $\tilde{V}_t(i, p, a)$ , is different from the one on the right-hand side, which is  $V_t(i, p, a)$ . The value function is updated with the following equation:

$$V_t(i, p, a) = \left[ u(c) + \delta s_i \sum_{p'} \pi_{p,p'} V_{t+1}(i + 1, p', a') \right] \quad (13)$$

where  $a' = g_t^a(i, p, a)$  and subject to (4).

Mechanically, the consumer chooses the optimal asset level  $a'$  with the discounting factor  $\beta\delta$  but the actual value is evaluated with the discount factor  $\delta$ .<sup>8</sup> This is exactly the problem described in Section 2.7 with  $\gamma \rightarrow \infty$ . In other words, the optimal decision rule  $g_t^a(i, p, a)$  obtained from (1) are equivalent to the optimal decision rule obtained from (12). Krusell et al. (2010) formally prove the equivalence in the neoclassical growth model.<sup>9</sup>

Although the solution of the consumer's problem is the same between the formulation based on the temptation preferences and on the hyperbolic discounting, there is an important difference in terms of how to conduct welfare analysis. In the case of the hyperbolic discounting preferences, since the utility of the consumer changes over time, the same consumer at different point of time are interpreted as different "selves," and the consumer's problem is understood as the dynamic game among multiple selves. Under such circumstances, unless one uses Pareto criteria, it is necessary to assign welfare weights to different selves to conduct welfare analysis. On the other hand, the preferences featuring temptation and self-control, which is developed by Gul and Pesendorfer (2001) and employed by Krusell et al. (2010), there is no time-inconsistency problem, although their approach still allows to study preference reversals which leads to over-borrowing and over-consuming. Since there is no multiple selves within a consumer, it is straightforward to implement welfare analysis using their approach.

### 3 Calibration

This section describes how the steady-state model is calibrated.<sup>10</sup> Consequently, the time script  $t$  is dropped throughout the section. Each of the subsections below corresponds to those in Section 2.

#### 3.1 Demographics

One period is set as one year in the model. Age 1 in the model corresponds to the actual age of 20.  $I$  is set at 81, meaning that the maximum actual age is 100.  $I_R$  is set at 45, implying that consumers retire at the actual age of 65. The population growth rate,  $\nu$ , is set at 1.2 percent

<sup>8</sup> İmrohoroğlu et al. (2003) distinguish the two cases in terms of what hyperbolic discounting consumers expect about their own future decisions. According to their classification, a *naive* consumer wrongly thinks that future selves make decisions in a time-consistent manner (using only the discount factor  $\delta$ ). On the other hand, a *sophisticated* consumer thinks that future selves are time-inconsistent (using both  $\beta$  and  $\delta$ ). I use the sophisticated consumers, as in Laibson (1996) and Laibson et al. (2007). Angeletos et al. (2001) find that naive and sophisticated hyperbolic discounting consumers behave similarly in a life-cycle model.

<sup>9</sup> See Proposition 6.

<sup>10</sup> More details can be found in the Supplementary Appendix.

annually. This is the average annual population growth rate of the U.S. over the last 50 years. The survival probabilities  $\{s_i\}_{i=1}^I$  are taken from the life table in [Social Security Administration \(2007\)](#).<sup>11</sup>

### 3.2 Preferences

First of all, I assume  $\gamma \rightarrow \infty$ . As I discussed in Section 2.9, the assumption makes the temptation model and the hyperbolic discounting model equivalent in terms of allocation. For the period utility function, the following constant relative risk aversion (CRRA) functional form is used:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{14}$$

$\sigma$  is set at 1.5, which is the commonly used value. It is also the point estimate of [Laibson et al. \(2007\)](#). Sensitivity of the main results with respect to the value of  $\sigma$  is investigated in Section 7.

Discount factors  $\beta$  and  $\delta$  are calibrated to be different for different model economies, but the calibration strategy is common. For all economies, I set the temptation discount factor  $\beta$  at first and calibrate the self-control discount factor  $\delta$  so that the capital-output ratio of the economy in the baseline steady state is 3.0, which is the historical average value of the U.S. economy. In other words, different model economies have different values of discount factors  $\beta$  and  $\delta$ , but they have the same aggregate capital stock in equilibrium.<sup>12</sup>

In the model without temptation,  $\beta = 1$  by assumption. I found that with  $\delta = 0.9698$  the steady-state equilibrium of the model generates a capital-output ratio of 3.0. For the model with temptation, I use  $\beta = 0.70$  as the baseline value of the temptation discount factor and calibrate  $\delta$ . The temptation discount factor of 0.70 is the one-year discount factor typically obtained from laboratory experiments.<sup>13</sup> Moreover, the benchmark point estimate of [Laibson et al. \(2007\)](#) is  $\beta = 0.703$ , or the annual discount rate of about 40 percent. The same calibration strategy generates  $\delta = 0.9852$ . The calibrated value of  $\delta$  is higher than 0.958, which is the value that [Laibson et al. \(2007\)](#) estimate jointly with  $\beta$ . A large part of the difference is due to the existence of the mortality shock in the current model, which [Laibson et al. \(2007\)](#) do not have. If  $\delta$  is adjusted by being multiplied by the average survival probability (0.9828), the resulting *effective*  $\delta$  is 0.968. I also investigate the case when the discount rate is 80 percent annually, which is twice as high as in the baseline temptation model. An 80 percent annual discount rate implies a temptation discount factor  $\delta = 0.56$ . Using the same calibration strategy, the economy with  $\delta = 0.56$  yields  $\delta = 0.9930$ .

<sup>11</sup> Table 4.C6 of [Social Security Administration \(2007\)](#). An average of the survival probabilities of males and females is used.

<sup>12</sup> Both [Angeletos et al. \(2001\)](#) and [Tobacman \(2009\)](#) calibrate  $\delta$  for the model without temptation (i.e., exponential discounting model) such that the average wealth holding at age 63 (the age just before retirement) is the same as in the model with temptation (i.e., hyperbolic discounting model) where  $\beta$  and  $\delta$  are jointly estimated from data. Since the life-cycle profile of asset holdings is similar in the two models, their strategy is similar to the strategy adopted in this paper.

<sup>13</sup> Although most existing literature estimate parameters with hyperbolic discounting model in mind, the model is equivalent to the temptation model in terms allocation.

### 3.3 Technology

The following standard Cobb-Douglas production function is assumed:

$$Y = ZF(K, L) = ZK^\theta L^{1-\theta} \quad (15)$$

$Z$  is normalized such that, in the baseline steady state, the equilibrium wage is one. The procedure implies  $Z = 0.896$ .  $\theta$  is set at 0.36, which corresponds to the average capital share of income of the U.S. economy. The depreciation rate of capital is set at  $\kappa = 0.06$  per year. [Huggett \(1996\)](#) calibrates  $\kappa = 0.06$  by matching the depreciation-output ratio of the model economy to its U.S. counterpart.

### 3.4 Endowment

I assume the following multiplicative form of individual productivity.

$$e(i, p) = e_i p \quad (16)$$

$e_i$  represents the average age-earnings profile and  $p$  is the individual productivity shock. Since retirement age is fixed at  $I_R$ ,  $e_i = 0$  for  $i \geq I_R$ . To calibrate  $\{e_i\}_{i=1}^{I_R-1}$ , I follow [Huggett \(1996\)](#) and use the data on the median earnings of male workers of different age groups from [Social Security Administration \(2007\)](#).<sup>14</sup> The median earnings data are multiplied by the employment to population ratio of males in each age group. The employment to population ratio for each age group is obtained from [McGrattan and Rogerson \(2004\)](#).<sup>15</sup> Finally, the resulting age-productivity profile is smoothed out by fitting the age profile of the product of median earnings and the employment to population ratio to a quadratic function of age. The resulting hump-shaped earnings profile can be seen in [Figure 2](#).

The stochastic process for  $p$  is calibrated by combining what I call the *bottom 99%*, whose earnings dynamics are captured by the stochastic process of household earnings estimated from the PSID (Panel Study of Income Dynamics) and the *top 1%*, which represent the upper tail of the earnings distribution and are added to replicate the substantial concentration of earnings and wealth in the U.S.<sup>16</sup> It is important that the model captures the observed concentration of earnings and wealth, in order to make sure that the strength of the partial and general equilibrium effects generated by the model is reasonable. As for the stochastic process associated with the *bottom 99%*, I follow the literature and assume that the logarithm of  $p$  is initially drawn from a normal distribution  $N(0, \sigma_0^2)$  and follows an AR(1) process with the persistence parameter  $\rho_p$  and the standard deviation of the innovation term  $\sigma_\epsilon$ . The triplet that characterizes the stochastic process is calibrated to  $(\rho_p, \sigma_0^2, \sigma_\epsilon^2) = (0.98, 0.30, 0.04)$ . The choice is in the middle of estimates in the literature. The persistence parameter  $\rho_p$  is estimated to be close to unity in the literature. For example, [Storesletten et al. \(2004\)](#) obtained  $\rho_p = 0.9989$ , while [Huggett \(1996\)](#) calibrates  $\rho_p = 0.96$ . The variance of initial earnings  $\sigma_0^2$  ranges from 0.2735 in [Storesletten et al. \(2004\)](#) to 0.38 in [Huggett \(1996\)](#).  $\sigma_\epsilon^2$  is set so that the life-cycle profile of earnings variances replicates its

<sup>14</sup> The earnings data are taken from Table 4.B6 of [Social Security Administration \(2007\)](#).

<sup>15</sup> Table 3, 4, and 5 of [McGrattan and Rogerson \(2004\)](#).

<sup>16</sup> See [Budría et al. \(2002\)](#).

empirical counterpart, for example, as shown in [Storesletten et al. \(2004\)](#). The AR(1) process obtained above is approximated using the algorithm of [Tauchen \(1986\)](#).<sup>17</sup>

The *top 1%* is added since the PSID, which is used to estimate the stochastic process of individual productivity shocks often used in the literature, is known to under-sample the top end of the U.S. earnings distribution. The approach employed here corrects such shortcomings by augmenting the estimated stochastic process of earnings with an additional state that captures the *top 1%* of the earnings distribution. In other words, the approach here is a combination of the literature that uses the estimated stochastic process to calibrate the earnings shock and the literature that directly calibrates the earnings shock to capture the high concentration of income and wealth independently from an empirically obtained stochastic process for earnings.<sup>18</sup> Specifically, the *top 1%* is characterized by an additional state of productivity shock,  $p_1$ , which is higher than the highest  $p$  of the *bottom 99%*. The parameters associated with the *top 1%* are calibrated to satisfy the following: (i) initially 1% of consumers draw  $p_1$ , (ii) the probability that a *bottom 99%* consumer becomes a *top 1%* is set such that the proportion of the *top 1%* among a cohort is always 1%, (iii) the probability of a *top 1%* remaining in the state is 0.92, (iv) when a *top 1%* falls to the *bottom 99%*, the new  $p$  is drawn from the ergodic distribution of  $p$  among the *bottom 99%*, (v) the level of  $p_1$  is calibrated such that the earnings Gini index of the baseline steady state is 0.61.<sup>19</sup> The probability of remaining a *top 1%* (0.92) is based on [Federal Reserve Bank of Dallas \(1995\)](#), which reports that 47.3 percent of households in the top 1% of income distribution in 1979 remained in the top 1% in 1988.

### 3.5 Market Arrangements

In the baseline steady state, the borrowing limit  $\underline{a}$  is set at zero, i.e., there is no borrowing. In experiments, I will relax the borrowing limit to the extent such that the aggregate amount of debt is the same between the model and the corresponding U.S. economy. In other words, I will back out the degree of relaxation of the borrowing constraint from the observed increase in indebtedness.

### 3.6 Government

The payroll tax rate for the Social Security contribution  $\tau_S$  is set at 0.10, which is the average contribution to the Social Security program as a fraction of labor income in the U.S. The proportional income tax rate of  $\tau_I = 0.2378$  is set to match the U.S. historical average of the ratio of total (federal, state, and local) government consumption over total income (0.195).

<sup>17</sup>  $n_p = 17$  abscissas are used. The abscissas are equally spaced between  $-\zeta\sigma_p$  and  $\zeta\sigma_p$ , where  $\sigma_p$  is the standard deviation of the ergodic distribution of  $p$ . [Tauchen \(1986\)](#) chooses  $\zeta = 3$  while [Huggett \(1996\)](#) uses  $\zeta = 4$ . I set  $\zeta = 2.1$  so that the life-cycle profile of earnings variances implied by the obtained Markov stochastic process is close to the one implied by the original AR(1) process. In general, for a small  $n_p$ , properties of the Markov process obtained using [Tauchen's \(1986\)](#) method vary with the choice of  $\zeta$ .

<sup>18</sup> A leading example of the latter approach is [Castañeda et al. \(2003\)](#).

<sup>19</sup> The earnings Gini is reported in [Budría et al. \(2002\)](#).

## 4 Computation

Since there is no analytical solution to the model, the model is solved numerically.<sup>20</sup> The space of asset holdings is discretized, and the choice with respect to assets is also constrained by the discretized state space. The equilibrium prices (wage and interest rate) and the government policy variables (transfer and Social Security benefits) are found using iteration.

## 5 Results: Steady-State Analysis

This section presents the main results, based on steady-state comparison. The starting point of the analysis is the economy without debt (i.e.,  $\underline{a} = 0$ ), which is calibrated in Section 3. Since this economy mimics the U.S. economy in 1970 at which time unsecured consumer debt was almost nonexistent, I call the economy the *1970 economy*. Next, in order to replicate the increased indebtedness between the 1970s and the 2000s with the model, I assume that the increased indebtedness is due to a relaxed borrowing limit that consumers face. Relaxing the borrowing limit is a parsimonious way to capture various types of innovation in the consumer credit market that happened over the last three decades. The borrowing limit is calibrated such that the aggregate debt is 7 percent of output in the steady state of the model. Since this level of debt is observed in the U.S. economy in the 2000s, I call it the *2000 economy*. I implement this procedure separately for models with varying degree of temptation. The primary interest is how different the macroeconomic implications are among the models.

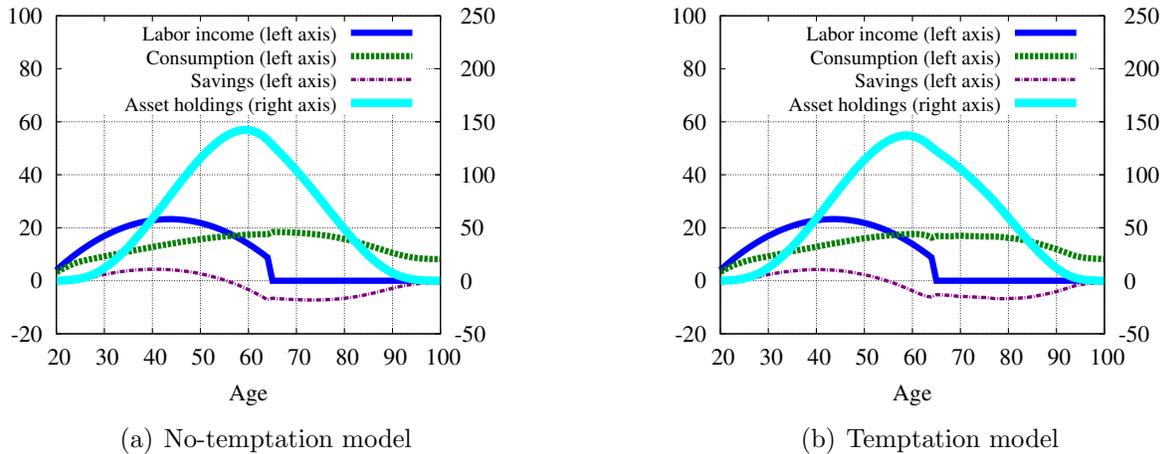
Section 5.1 compares the 1970 economy with and without temptation. I show that the macroeconomic implications are very similar between the models. This observational similarity result is the reconfirmation of Angeletos et al. (2001). In Section 5.2, the macroeconomic implications of increased indebtedness due to the relaxed borrowing limit are investigated, by comparing the 1970 economy and the 2000 economy. I will show that models with and without temptation again exhibit similar responses to the relaxed borrowing limit. Section 5.3 analyzes the welfare implications. The focus is on the difference in the implications between models with and without temptation. If, in addition, welfare implications of increased indebtedness are also similar between the two models, there is no need to use the non-standard preferences for an analysis of increased indebtedness. What I will show is that this is not the case: although the macroeconomic implications are similar, the welfare implications are substantially different between the models with and without temptation. Finally, in Section 5.4, I investigate the difference in the optimal borrowing limit among models varying degree of temptation.

### 5.1 Macroeconomic Implications: The 1970 Economy

Figure 2 compares the average life-cycle profiles of the 1970 model economies without temptation on the left and with temptation (with the temptation discount factor  $\beta = 0.70$ ) on the right. What is most striking is that there is little difference between the two model economies in terms of the average life-cycle profile. In both economies, the average consumption profile is smoother than the income profile. Consumers save during the working period and dissave during the retirement period. As a result, asset holdings increase until retirement age and decrease after that in both models. Although the temptation model features the temptation discount factor

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<sup>20</sup> Details of the numerical procedure are found in the Supplementary Appendix.



**Figure 2: Comparison between models with and without temptation: average life-cycle profiles**

( $\beta$ ), which, ceteris paribus, reduces savings and shifts consumption forward, when the model with temptation is calibrated to generate the same capital-output ratio as in the model without temptation, the self-control discount factor ( $\delta$ ) is calibrated higher in the temptation model. As a result, the effect of the temptation discount factor on the average life-cycle profile is negated.

The models with temptation and self-control exhibit a slightly higher concentration of wealth, as more consumers are consuming all of their income and saving nothing. The wealth Gini index is 0.786 for the model without temptation, while it is 0.806 for the temptation model with  $\beta = 0.70$ . The wealth Gini for both economies is not far from 0.803, which is the wealth Gini of the U.S. economy, reported by [Budría et al. \(2002\)](#). For the temptation model with  $\beta = 0.56$ , the wealth Gini index is 0.820. [Tobacman \(2009\)](#) also compares the wealth inequality implied by the models with and without temptation. In the baseline case with both liquid and illiquid assets, the model with temptation exhibits a Gini coefficient of 0.508, which is slightly higher than the value for the model without temptation (0.488). The magnitude of the difference is comparable to what is obtained here.

## 5.2 Macroeconomic Implications: Increased Indebtedness

Table 1 summarizes the macroeconomic implications of rising aggregate debt from 1970 to 2000. The first panel (the first four rows) summarizes the results of the standard model without temptation (i.e., exponential discounting model). The first row in each panel shows the levels in the 1970 economy, without debt. The second row is associated with the 2000 steady-state economy. Notice that the general equilibrium (GE) effect is taken into account when the new steady-state equilibrium is obtained. The last row is capturing only the partial equilibrium (PE) effect; the prices (interest rate and wage) are fixed at the 1970 level, but the borrowing limit is relaxed to the 2000 level. By comparing the second (GE) and the third (PE) rows, one can see the strength of the general equilibrium effect in the steady-state economy. In the second panel, the baseline temptation model ( $\beta = 0.70$ ) is employed but the borrowing limit of the no-temptation model is

**Table 1: Macroeconomic effect of rising indebtedness**

Economy <sup>1</sup>	GE <sup>2</sup>	$\underline{a}$ <sup>3</sup>	D/Y	K <sup>4</sup>	Y <sup>4</sup>	C <sup>4</sup>	r%	wage	Var(c) <sup>5</sup>
<b>No-temptation model (<math>\beta = 1.00</math>)</b>									
1970	–	0.000	0.000	1.000	1.000	1.000	6.00	1.000	0.592
2000	GE	–0.570	–0.070	0.957	0.984	0.991	6.34	0.984	0.580
2000	PE	–0.570	–0.082	0.914	0.968	0.991	6.00	1.000	0.562
<b>Temptation model (<math>\beta = 0.70</math>) with <math>\underline{a}</math> of no-temptation model</b>									
1970	–	0.000	0.000	1.000	1.000	1.000	6.00	1.000	0.588
2000	GE	–0.570	–0.085	0.958	0.986	0.990	6.33	0.985	0.577
2000	PE	–0.570	–0.098	0.915	0.969	0.990	6.00	1.000	0.562
<b>Temptation model (<math>\beta = 0.70</math>)</b>									
1970	–	0.000	0.000	1.000	1.000	1.000	6.00	1.000	0.588
2000	GE	–0.376	–0.070	0.963	0.986	0.992	6.29	0.986	0.577
2000	PE	–0.376	–0.076	0.928	0.973	0.991	6.00	1.000	0.565
<b>Temptation model (<math>\beta = 0.56</math>)</b>									
1970	–	0.000	0.000	1.000	1.000	1.000	5.98	1.000	0.594
2000	GE	–0.297	–0.070	0.964	0.987	0.992	6.26	0.988	0.589
2000	PE	–0.297	–0.074	0.934	0.976	0.991	5.98	1.000	0.580

<sup>1</sup> 1970: Economy with no borrowing. 2000: Economy calibrated to debt-to-output ratio of 7 percent.

<sup>2</sup> GE: general equilibrium. PE: partial equilibrium with prices fixed at the 1970 level.

<sup>3</sup> Borrowing limit relative to total income.

<sup>4</sup> Level in the 1970 (no-debt) economy normalized to one.

<sup>5</sup> Average of cross-sectional variances of consumption for all age groups.

applied. This panel is intended to highlight the difference in the responses of the two economies when the borrowing limit is relaxed to the same extent. The third panel is associated with the baseline temptation model ( $\beta = 0.70$ ). Notice that the borrowing limit  $\underline{a}$  is calibrated to be different from the no-temptation model, but the debt-to-output ratio, which is the calibration target, is the same at 7 percent. In the last panel, results from the temptation model with a lower temptation discount factor ( $\beta = 0.56$ ), when the same calibration strategy as in the first and the third panels is employed, are shown.

As shown in second row of the first panel, the borrowing limit of 57 percent of the average income is needed to generate the aggregate amount of debt as large as 7 percent of output in the model without temptation. In the 2000 economy, the equilibrium capital stock is 4.3 percent lower than in the 1970 economy without borrowing. Since labor is inelastically supplied, the decline in the capital stock generates a decline in output; output and aggregate consumption in the 2000 economy are 1.6 and 0.9 percent lower than in the 1970 economy, respectively. The equilibrium interest rate goes up from 6.00 percent in 1970 to 6.34 in 2000 as capital becomes more scarce, and wage declines by 1.6 percent. Since a relaxed borrowing limit implies better consumption smoothing, the cross-sectional consumption variance declines as the borrowing con-

straint is relaxed: consumption variance drops from 0.592 in the 1970 economy to 0.580 in the 2000 economy.

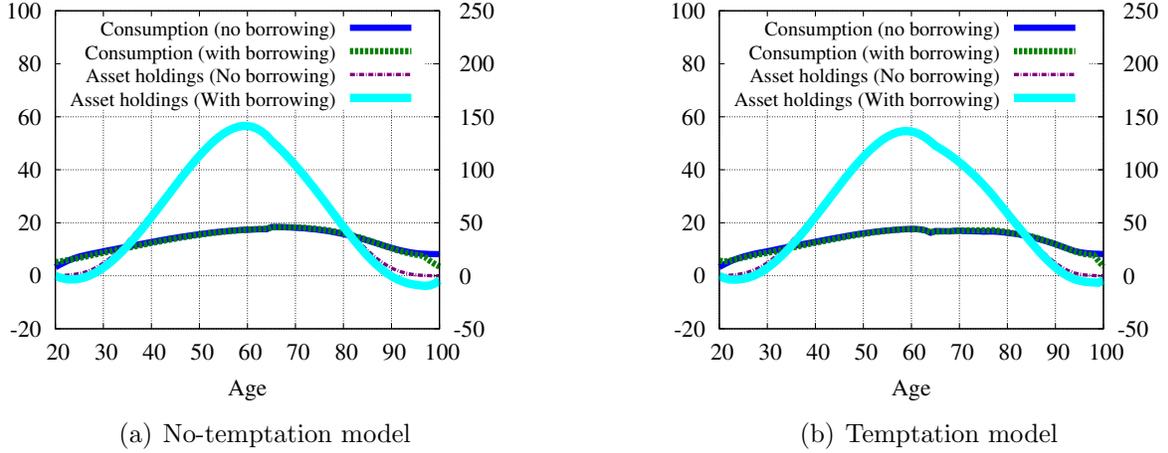
What is the role of general equilibrium in shaping the macroeconomic implications discussed above? By comparing the second and third rows, it is clear that, without the general equilibrium effect, macroeconomic responses are stronger. In other words, the general equilibrium effect partly attenuates the macroeconomic responses to the relaxed borrowing limit. Without the general equilibrium effect, both capital stock and output decrease even more, debt increases more, and the log-consumption inequality declines to a larger extent, too.

The second panel in Table 1 summarizes the results for the baseline temptation model ( $\beta = 0.70$ ), but with the borrowing limit obtained for the no-temptation model (0.57 of average income). Most changes are quite similar between the first and second panels. But there is one important difference: the response of aggregate debt is stronger in the temptation model. When the borrowing limit is relaxed such that debt increased by 7 percent of output in the model without temptation, the aggregate debt over output ratio increased by 8.5 percent in the temptation model.

In the third panel, I implement the same procedure as in the first panel for the baseline temptation model ( $\beta = 0.70$ ). Since the response of aggregate debt to a relaxed borrowing limit is stronger in the temptation model, the borrowing limit that generates the aggregate debt of 7 percent of output is more strict than in the no-temptation model. Indeed, the borrowing limit is calibrated to be 37.6 percent of the average income in the temptation model. The macroeconomic responses to a relaxed borrowing limit in the temptation are only slightly weaker than in the no-temptation model. Capital stock, output, and consumption decline by 3.7, 1.4, and 0.8 percent, respectively, in the temptation model, while the drops are 4.3, 1.6, and 0.9 percent in the no-temptation model. Cross-sectional consumption variances decline by 1.1 percentage points in the temptation model, compared to a 1.2 percentage point decline in the model without temptation. As in the no-temptation model, the general equilibrium effect partly offsets the macroeconomic responses to the relaxed borrowing limit.

Figure 3 compares the 1970 (no borrowing) and 2000 (7 percent debt-to-output ratio) economies with and without temptation. The left panel corresponds to the no-temptation model, and the right panel is associated with the temptation model. Each panel shows how the average life-cycle profiles of consumption and asset holdings react when the borrowing limit is relaxed. The main finding is that the response between the two models is almost indistinguishable.

The last panel in Table 1 summarizes the macroeconomic implications of an increased indebtedness for the temptation model with a lower temptation discount factor ( $\beta = 0.56$ ). As in the case for the baseline temptation model ( $\beta = 0.70$ ), the borrowing limits are calibrated so that the amount of aggregate debt is 7 percent of output in the new steady state. As the column labeled  $\underline{a}$  shows, the borrowing limit has to be even tighter (29.7 percent) than in the baseline temptation model (37.6 percent) because of the stronger response of debt to a relaxation of the borrowing constraint. The size of the response of macroeconomic aggregates is slightly weaker than in the temptation model with a higher (lower) discount factor (rate). For example, capital stock, output, and consumption decline by 3.6, 1.3, and 0.8 percent, respectively, in the model with  $\beta = 0.56$ , while the drops are 3.7, 1.4, and 0.8 percent in the baseline temptation model with  $\beta = 0.70$ .



**Figure 3: Comparison between models with and without temptation: response of consumption (left axis) and asset holdings (right axis) to a relaxed borrowing limit**

### 5.3 Welfare Implications: Increased Indebtedness

In this section, I will investigate the welfare implications of a relaxed borrowing limit. Before starting the analysis, an issues related to the welfare analysis in the current environment need to be addressed. Since the model used here features a heterogeneous agent model with life-cycle and uninsured idiosyncratic shocks, there is no obvious way to define social welfare. I investigate social welfare in multiple ways. First, I use the ex-ante expected lifetime utility in the steady-state equilibrium as social welfare. The virtue of this welfare criterion is that this naturally takes into account both the welfare gain or loss from changes in aggregate consumption (efficiency effect) and the welfare gain or loss due to changes in the degree of insurance (insurance effect). For this reason, the social welfare function is widely used together with incomplete market models with finitely lived consumers; for example, [Conesa et al. \(2009\)](#) use it to investigate the optimal capital income taxation. Formally, social welfare is defined as follows:

$$\mathbb{E}V = \sum_p \pi_p^0 V(1, p, 0) \tag{17}$$

where  $V(\cdot)$  is defined as in equation (1).

Second, I also look at cross-sectional welfare implications. Because of the rich heterogeneity of the model, it is also important to look at the heterogeneity of the welfare effect for different types of consumers. To that end, I also investigate the expected lifetime utility in the steady-state equilibrium for consumers with different initial productivity  $p$ . Since the productivity shock is highly persistent, looking at the welfare implications for consumers with different initial  $p$  roughly corresponds to studying the heterogeneous effects on consumers with different productivity potentials.

Finally, in Section 6, I will investigate the welfare effect associated with the rising indebtedness taking the equilibrium transition path into account. The analysis enables us to study the heterogeneous welfare effect on consumers in different cohorts along the transition path.

**Table 2: Welfare implications of rising indebtedness**

GE <sup>2</sup>	Welfare gain <sup>1</sup>			
	EV	Initial productivity <sup>3</sup>		
		Low	Medium	High
<b>No-temptation model (<math>\beta = 1.00</math>)</b>				
PE	+2.69	+6.63	+2.50	+0.16
GE	+0.65	+3.02	+0.52	+0.31
<b>Temptation model (<math>\beta = 0.70</math>)</b>				
PE	+0.61	+1.88	+0.18	+0.13
GE	-0.39	+0.44	-0.31	+0.96
<b>Temptation model (<math>\beta = 0.56</math>)</b>				
PE	-0.46	+0.81	-0.43	-0.61
GE	-1.09	-0.67	-1.13	+1.60

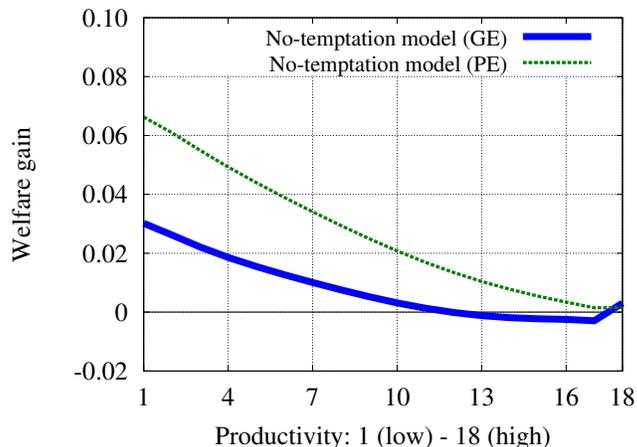
<sup>1</sup> Measured by the percentage increase in per-period consumption at all ages and states associated with the move from the 1970 steady-state economy (without debt) to the 2000 economy (with debt-to-output ratio of 7 percent).

<sup>2</sup> GE: general equilibrium. PE: partial equilibrium with prices fixed at the 1970 level.

<sup>3</sup> Low: consumers with the lowest initial  $p$ . Medium: consumers with the median initial  $p$ . High: consumers with highest initial  $p$ .

Notice that the important consideration in the current paper is the welfare loss due to the relaxed borrowing constraint and induced over-consumption. The preferences featuring temptation and self-control capture such a welfare loss naturally, because consumers succumb to the temptation of choosing consumption by discounting future value by  $\beta\delta$ , while the actual value is based on the discount factor  $\delta$ . In other words, consumers choose consumption (savings) that is higher (lower) than the level associated with the highest welfare. The over-consumption (under-saving) problem is substantial. For example, if consumers have perfect self-control in the model with  $\beta = 0.70$ , consumers make consumption-savings decisions based only on  $\delta$  and the resulting capital-output ratio in the steady state is 22.5 percent higher than in the baseline temptation model.

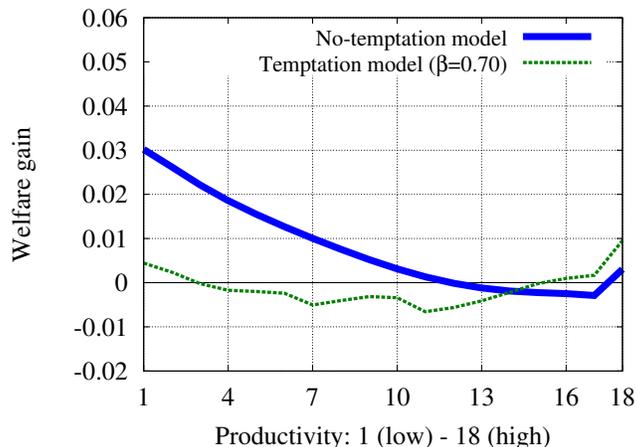
Table 2 summarizes the welfare implications of rising indebtedness in the models with and without temptation. The three panels correspond to the no-temptation model, the baseline temptation model with  $\beta = 0.70$ , and the temptation model with a lower temptation discount factor  $\beta = 0.56$ . For each model, two cases are shown. *GE* denotes the case where the 2000 economy, which is a steady-state equilibrium, is compared to the 1970 economy. *PE* denotes to the case where the prices are fixed at the 1970 levels, but the borrowing limit of the 2000 economy is used. The column marked as *EV* in Table 2 shows the changes in social welfare associated with the increased indebtedness, induced by a relaxed borrowing limit, expressed as a percentage change in per-period consumption. The comparison between the no-temptation model (the first panel) and the baseline temptation model (the second panel) shows that the welfare implications



**Figure 4: Heterogeneity of welfare gain: Partial and general equilibrium**

of a relaxed borrowing limit are very different, although, as shown in the previous section, the macroeconomic implications are similar. In the case where the prices are fixed at 1970 levels (partial equilibrium, or PE in the table), while the no-temptation model implies a welfare gain of 2.7 percent by moving from the 1970 economy (no borrowing) to the 2000 economy (with a debt-to-output ratio of 7 percent), the temptation model implies a substantially smaller welfare gain of 0.6 percent. The model with a higher degree of temptation (the third panel) implies a welfare loss of 0.5 percent. The general equilibrium effect offsets part of the gain or increases the loss, through lower output associated with lower capital stock. If the general equilibrium effect is taken into account, while the no-temptation model implies a welfare gain of 0.7 percent, the baseline temptation model implies a welfare loss of 0.4 percent. The temptation model with  $\beta = 0.56$  implies an even larger welfare loss of 1.1 percent.

Figure 4 compares the heterogeneous welfare effects with and without a general equilibrium effect in the model without temptation. In the figure, each line represents the welfare gain of moving from the 1970 economy to the 2000 economy for consumers with different initial productivity  $p$  (1 is the lowest and 18 is the highest). The last three columns of Table 2 present welfare gains of consumers with low, medium, and high productivity initially. With respect to the welfare effects on consumers with different productivity potentials, three groups with different initial productivity are affected differently in the model without temptation. First, those with initial low productivity benefit most from the relaxed borrowing limit. The consumers with the lowest initial productivity gain as much as 6.6 percent of per-period consumption in the partial equilibrium case, and 3.0 percent with the general equilibrium effects considered. This is because the likelihood that they are constrained by the borrowing limit is highest for this group of consumers. However, they experience a welfare loss from the general equilibrium effect. In Figure 4, the line representing the welfare effect with the general equilibrium effect considered, is located below the line representing the welfare effect without the general equilibrium effect, for consumers with low initial productivity. The reason is a lower equilibrium wage and a higher equilibrium interest rate, caused by a lower capital stock. Since the low-productivity consumers tend to borrow more often, and the main source of their income is labor income, both price



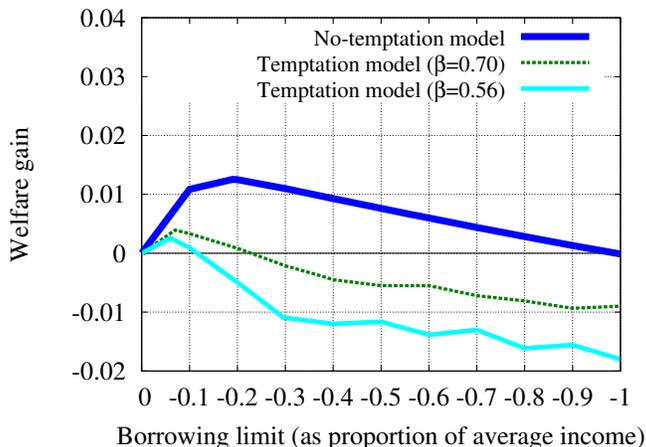
**Figure 5: Heterogeneity of welfare gain: Models with and without temptation**

effects hit the consumers negatively.

Second, the group with high initial productivity does not gain much from the relaxed borrowing limit. For those with the highest initial productivity in the no-temptation model, the welfare gain without the general equilibrium effect is a mere 0.2 percent increase in per-period consumption. Since it is not likely that they are constrained by the borrowing limit, they do not gain much from a relaxed borrowing limit. However, they gain from the general equilibrium effect. This is because they most likely remain savers throughout their lives, and they benefit from a higher interest rate, although part of the gain is offset by a lower wage. The welfare gain with the general equilibrium effect is equivalent to a 0.3 percent increase in per-period consumption. This contrasting general equilibrium effect for high and low productivity consumers is exactly what [Campbell and Hercowitz \(2009\)](#) emphasize in a different but closely related environment. In the model by [Campbell and Hercowitz \(2009\)](#) with secured credit, high discount rate consumers who remain borrowers lose from the general equilibrium effect, and low discount rate consumers gain from the general equilibrium effect. In their setup, the general equilibrium effect is strong enough to incur a welfare loss for consumers with low initial productivity (in their case, high discount rate consumers), but the effect is not strong enough to overturn the welfare gain from the partial equilibrium effect here.

Finally, interestingly, consumers with medium initial productivity either enjoy a small welfare gain or suffer a welfare loss from the relaxed borrowing limit. In [Figure 4](#), consumers with initial productivity of 12-17 in the no-temptation model suffer by moving from the 1970 economy to the 2000 economy. This is due to the combination of the weak welfare gain from the relaxed borrowing limit and the stronger negative welfare loss from a lower wage and a higher interest rate. As a result, the solid line in [Figure 4](#), which represents the welfare effect for heterogeneous consumers, exhibits a U-shape. This is the same property that [Obiols-Homs \(2011\)](#) found in a similar environment.

[Figure 5](#) compares the heterogeneity of welfare effects across consumers with different initial productivity in the models with and without temptation. The general equilibrium effects are considered in the figure. [Table 2](#) also contains the cross-sectional welfare effects for consumers



**Figure 6: Level of borrowing limit and social welfare**

with different levels of initial productivity. Although the U-shape is also observed in the model with temptation in Figure 5, there are significant differences. The difference is especially striking for consumers with low initial productivity. Their gain from having a relaxed borrowing limit is significantly smaller in the case of the baseline temptation model. The key reason is the negative welfare effect of over-borrowing. Those who are close to the borrowing limit benefit from having a less strict borrowing limit, which facilitates consumption smoothing across age and states, but suffer from borrowing more than the level associated with the highest utility.

#### 5.4 Optimal Borrowing Limit with Temptation

The discussion in the previous section implies that the *optimal* level of the borrowing limit differs, potentially substantially, across models with varying degree of temptation. Here I define *optimal* as the level of the uniform borrowing limit that is associated with the highest social welfare defined as the ex-ante expected lifetime utility. Figure 6 exhibits social welfare, expressed as the increase in per-period consumption over the initial steady-state economy without debt, under different levels of the borrowing limit in the models with varying degree of temptation. The general equilibrium effect is taken into account. Three things are worth pointing out. First, the line for the model without temptation is located above the other lines, which are associated with the temptation models; the welfare gain is always higher in the no-temptation model, conditional on the same level of the borrowing limit. Second, all lines are hump shaped, because the negative general equilibrium effect from a lower capital stock dominates at some point for all economies, as the borrowing limit becomes relaxed. Third, the optimal level of the borrowing limit, which is associated with the highest point of each line in Figure 6, is decreasing in the degree of temptation. This is mainly because preferences featuring temptation and self-control imply a smaller (or negative) welfare gain from the relaxed borrowing limit for low and medium productivity consumers.

For the no-temptation model, the level of the uniform borrowing limit that maximizes social welfare is 19 percent of average income. This optimal level is lower than the level calibrated for the 2000 economy (57 percent). In other words, the model without temptation implies that

the borrowing limit is too lax in the 2000 U.S. economy. At the optimal borrowing limit, the social welfare gain is 1.25 percent, which is close to the double of the welfare gain in the 2000 economy (0.65 percent). Cross-sectionally, consumers with the lowest initial productivity gain by 3.5 percent instead of 3.0 percent in terms of flow consumption if the economy is at the optimal borrowing limit instead of the 2000 economy. Consumers with medium productivity gain by 1.1 percent instead of 0.5 percent. On the other hand, the gain enjoyed by consumers with the highest productivity declines from 0.3 percent to 0.2 percent. In the baseline temptation model ( $\beta = 0.70$ ), the optimal borrowing limit is 7 percent of average income. This optimal level is substantially lower than in the no-temptation model (19 percent) because of the welfare loss from over-borrowing. Furthermore, as in the no-temptation model, the optimal level is substantially lower than 37.6 percent, which is the borrowing limit corresponding to the 2000 economy. Indeed, the social welfare gain is positive (0.39 percent) if the optimal borrowing limit is implemented, compared to the welfare loss of 0.39 percent in the 2000 economy. In the case of the temptation model with a lower temptation discount factor ( $\beta = 0.56$ ), the optimal borrowing limit declines further to 6 percent of average income. The social welfare gain is again positive (0.26 percent), compared to the 1.1 percent welfare loss in the 2000 economy.

In sum, when the general equilibrium effect is strong and causes social welfare to decline, implementing a tighter borrowing limit generates a welfare gain, in economies both with and without temptation. The difference between the models with and without temptation is that social welfare starts to decline with a relatively low level of the borrowing limit in the temptation model, due to the over-consuming induced by the relaxed borrowing limit. Therefore, just as commitment by using an illiquid asset is valued in [Laibson \(1997\)](#) and forced saving might be welfare-improving in [Malin \(2008\)](#), tightening the borrowing limit in the temptation model can improve welfare as the tight borrowing limit prevents consumers from over-consuming, in addition to limiting the general equilibrium effect.

## 6 Results: Transition Analysis

This section presents the results of the analysis with the equilibrium transition path. In constructing the transition path between the initial steady state and the final one, I assume that the initial steady state is characterized by no borrowing ( $\underline{a} = 0$ ). The initial steady state corresponds to the 1970 economy in [Section 5](#). The final steady state is characterized by the borrowing limit associated with a debt-to-output ratio of 7 percent. This state corresponds to the 2000 economy in the previous section. Notice that the borrowing limit in the final steady state is different depending on the model, but all models generate the observed amount of debt in the 2000s. I assume that the borrowing limit relaxes linearly between period 0 (corresponds to 1970) and period 30 (corresponds to 2000).<sup>21</sup> After period 30, the borrowing limit stays at the level in the 2000 economy, while the economy converges to the 2000 steady state. In what follows, I first present the transition path of macroeconomic aggregates generated by the models ([Section 6.1](#)). The welfare analysis that explicitly takes into account the transition to the new steady state

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<sup>21</sup> An alternative assumption is to let the borrowing limit jump to the level in the 2000 economy from the beginning of the transition (1971). However, it turns out that this alternative scenario generates a counterfactual transition path of the debt-to-output ratio: the debt-to-output ratio increases immediately in the 1970s, while the debt-to-output ratio gradually increases in the U.S. economy ([Figure 1](#)).

follows (Section 6.2).<sup>22</sup>

### 6.1 Macroeconomic Aggregates

Figure 7 compares the macroeconomic aggregates between 1970 and 2010, for the models with and without temptation. The results with the no-temptation model are on the left side, while those of the baseline temptation model ( $\beta = 0.70$ ) are on the right side of the figure. Panels (a) and (b) compare the path of the debt-to-output ratio of the models and of the data (same as in Figure 1). It is clear that both models capture the dynamics of the debt-to-output ratio in the data quite well. In both models, the debt-to-output ratio gradually increases from the initial level of zero in 1970 and reaches about 7 percent around 2000. Panels (c) and (d) compare the transition path of the capital stock and output. Although there are some non-monotonic dynamics in the model with temptation, the long-run trend is the decline in the capital stock over time, as consumers borrow more or save less over time. As a result, output also continues to decrease over time. This is the source of the negative general equilibrium effect on welfare. Since labor is supplied inelastically, a declining capital stock yields a declining trend of wage and the increasing trend of the interest rate in the economy. These trends are present in both models, as shown in panels (e) and (f).

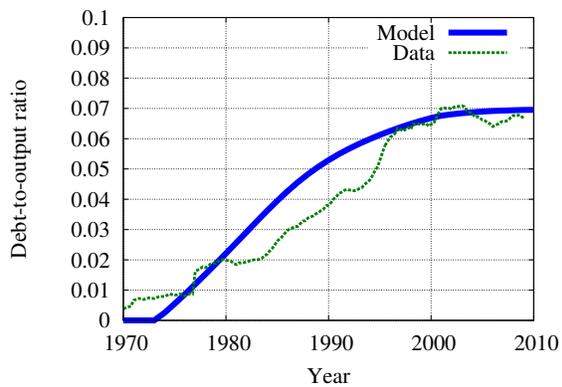
### 6.2 Welfare Implications

Similarly to what is shown in Section 5 using steady-state comparison, welfare implications are strikingly different between models with and without temptation along the transition path, although the dynamics of macroeconomic aggregates are similar. Figure 8 compares the welfare implications along the transition path in two models. Figures for the no-temptation model are on the left while those for the temptation model are on the right. First, panels (a) and (b) compare the ex-ante expected lifetime utility of newborns (age-20 consumers) in different years along the transition path. Welfare is measured as the uniform increase in per-period consumption against the initial steady state. For example, in panel (a), the welfare gain is approximately 0.65 percent in 2000; this means that an age-20 consumer in 2000 along the transition path is better off than if he had been born in 1970 (the initial steady state), by an increase in per-period consumption equivalent to 0.65 percent. Two things can be learned from comparing panels (a) and (b). First, the welfare gain from a relaxed borrowing limit is substantially higher in the model without temptation throughout the transition path. Second, while the welfare effect is positive throughout the transition path in the no-temptation model, the welfare effect is negative for all generations in the temptation model. Third, in the model without temptation, the welfare effect is non-monotone; consumers born after 1975 are worse off than those born in 1975 as the general equilibrium effect associated with the decumulating capital stock intensifies along the transition path. However, the negative general equilibrium effect is quantitatively small compared with the large gain during the transition between 1970 and 2000 in the no-temptation model.

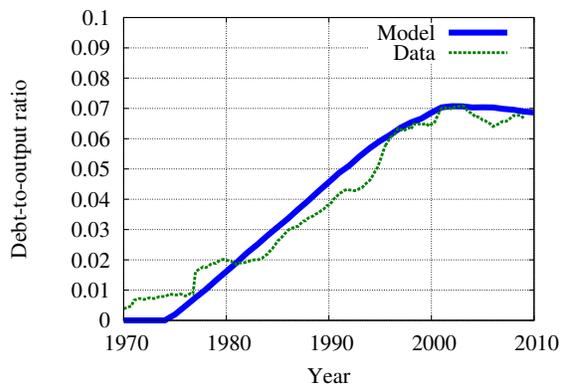
Panels (c) and (d) exhibit the heterogeneity of the welfare effects on newborns (age-20 consumers) with different initial productivity levels along the transition path in the models with and without temptation. The difference is striking. Consumers with initial low productivity gain substantially less in the temptation model. Whereas low-productivity consumers gain by

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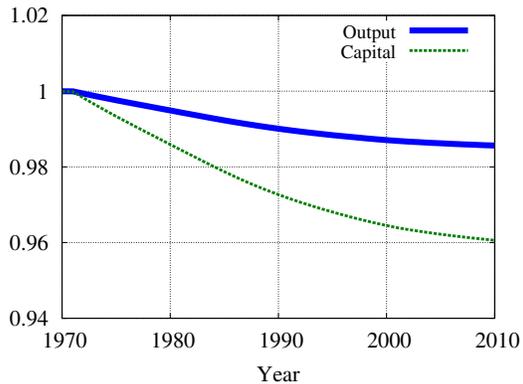
<sup>22</sup> The Supplementary Appendix describes the algorithm to compute the equilibrium transition path.



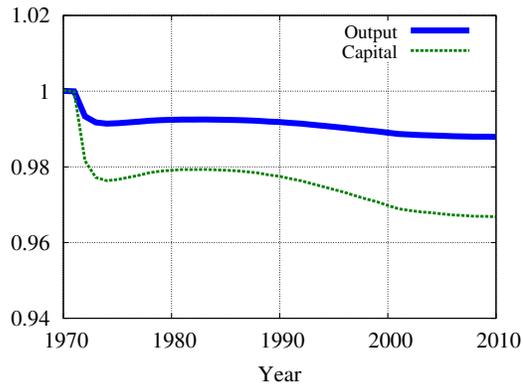
(a) No-temptation: debt/output



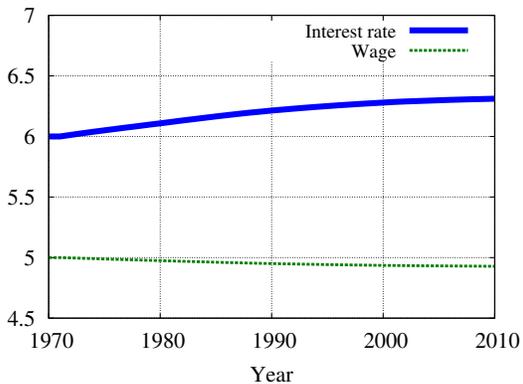
(b) Temptation: debt/output



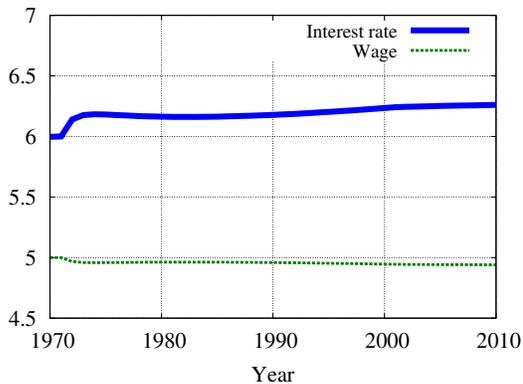
(c) No-temptation: capital and output



(d) Temptation: capital and output

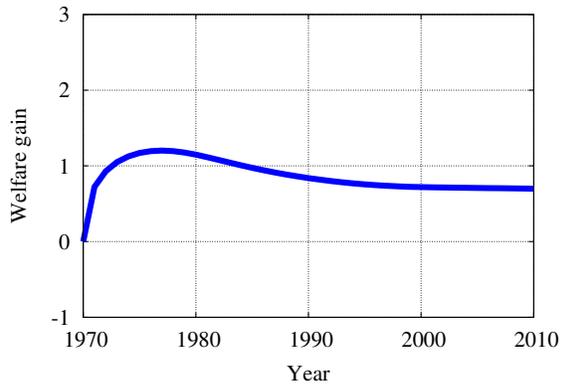


(e) No-temptation: prices

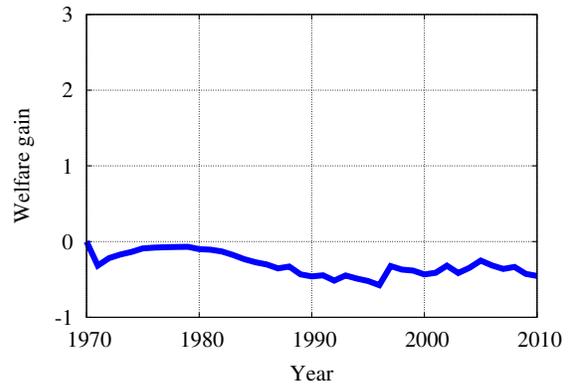


(f) Temptation: prices

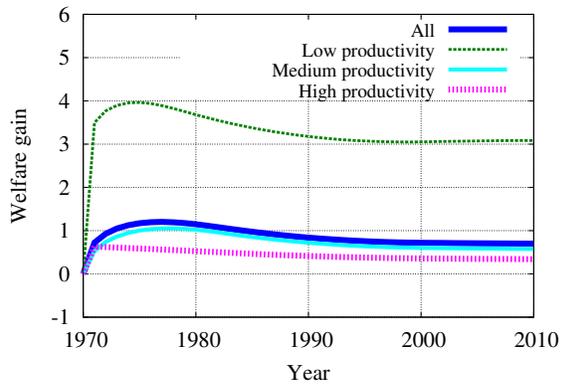
Figure 7: Comparison of macroeconomic aggregates along the transition path



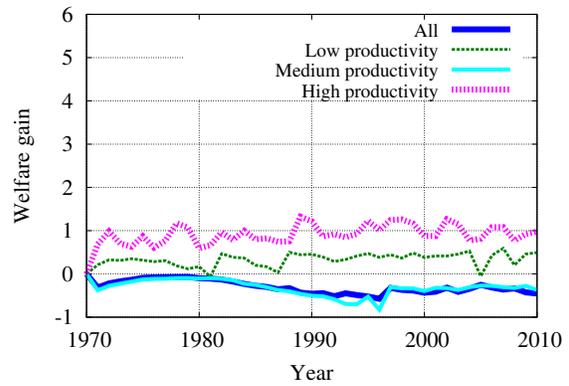
(a) No-temptation: welfare of newborns



(b) Temptation: welfare of newborns



(c) No-temptation: heterogeneity of welfare

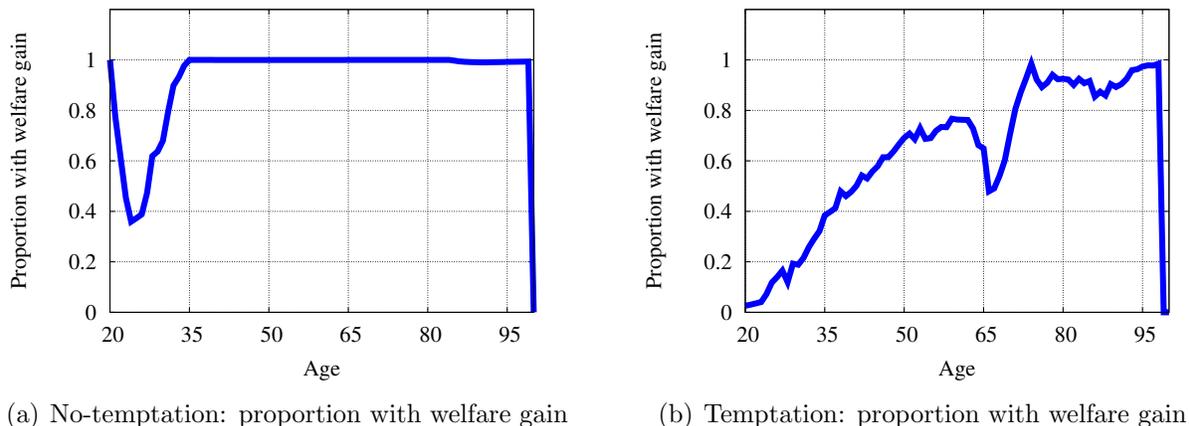


(d) Temptation: heterogeneity of welfare

**Figure 8: Comparison of welfare effects along the transition path**

3-4 percent in the no-temptation model, the welfare gain is about 0.5 percent in the temptation model. In the temptation model, the relaxed borrowing constraint induces both a welfare gain (better consumption smoothing) and a welfare loss (over-consuming). Consumers with initial medium productivity, who basically determine the average welfare gain or loss of the cohorts, gain by about 1 percent in the model without temptation, while they suffer a welfare loss throughout the transition path in the temptation model. Indeed, those who gain the most along the transition path in the temptation model are the high-productivity consumers who gain mainly from the general equilibrium effect.

In Figure 9, the proportion of consumers who gain from the transition to the 2000 economy among each age group in 1970 is shown, for both the model without temptation (panel (a)) and the temptation model (panel (b)). We can see that a very large proportion of consumers in 1970 gain from the switch to the transition path in the model without temptation. In total, 89 percent of consumers in 1970 (initial steady state) gain from the switch. On the other hand, in the temptation model, many consumers, especially the young ones, suffer from the transition. In total, only 49 percent of consumers in 1970 gain from switching to the transition path.



**Figure 9: Comparison of welfare effects on the initial consumers**

In sum, the transition analysis confirms the findings of the steady-state comparison: although the macroeconomic implications are similar between the models with different preference assumptions, the welfare implications are strikingly different if the welfare loss from over-borrowing is taken into account in the model with temptation and self-control. The ex-ante expected lifetime utility of newborns quickly rises above 1 percent (measured by per-period consumption growth) and slowly stabilizes at around 0.65 percent in 2000 in the no-temptation model, while the welfare measures stays negative and stabilizes at  $-0.4$  percent in 2000 in the temptation model.

## 7 Sensitivity Analysis

This section provides sensitivity analysis with respect to the risk aversion parameter  $\sigma$  and endogenous labor supply. First, I implement the same steady-state analysis as in Section 5 with  $\sigma = 3.0$  instead of the baseline value of  $\sigma = 1.5$ . The results are summarized in the top two panels of Table 3. As in the baseline experiments, the temptation model requires a more strict borrowing limit (47 percent of average income) than in the model without temptation (61 percent). As in the baseline experiments, the macroeconomic implications of moving from the 1970 economy to the 2000 economy are similar across the models with and without temptation. On the other hand, the welfare gain of relaxing the borrowing limit is very different; it is equivalent to a 5.1 percent increase in flow consumption in the no-temptation model, while it is a 2.3 percent increase in the temptation model. Although both economies imply a social welfare gain from the relaxed borrowing limit as the gain from better consumption smoothing is larger with a higher risk aversion, the size of the welfare gain is very different between the two economies.

The bottom two panels of Table 3 summarize the macroeconomic and welfare implications using the models with labor-leisure decisions. I assume the following non-separable utility function between consumption and leisure.

$$u(c, 1 - \ell) = \frac{(c^\mu(1 - \ell)^{1-\mu})^{1-\sigma}}{1 - \sigma} \quad (18)$$

I follow the same calibration strategy as for the baseline experiments, except for  $\mu$ . I calibrate

**Table 3: Macroeconomic and welfare implications: sensitivity analysis**

Economy <sup>1</sup>	GE <sup>2</sup>	$\underline{a}$ <sup>3</sup>	D/Y	K <sup>4</sup>	Y <sup>4</sup>	r%	Var(c) <sup>5</sup>	EV <sup>6</sup>
<b><math>\sigma = 3.0</math>, no-temptation model</b>								
1970	–	0.000	0.000	1.000	1.000	6.00	0.575	–
2000	PE	–0.610	–0.079	0.917	0.969	6.00	0.548	+9.17
2000	GE	–0.610	–0.070	0.937	0.977	6.51	0.569	+5.31
<b><math>\sigma = 3.0</math>, temptation model</b>								
1970	–	0.000	0.000	1.000	1.000	6.00	0.571	–
2000	PE	–0.470	–0.079	0.924	0.972	6.00	0.547	+5.30
2000	GE	–0.470	–0.070	0.940	0.978	6.49	0.566	+2.34
<b>Endogenous labor, no-temptation model</b>								
1970	–	0.000	0.000	1.000	1.000	6.00	0.472	–
2000	PE	–1.014	–0.077	0.928	0.972	6.00	0.470	+1.08
2000	GE	–1.014	–0.070	0.968	0.987	6.24	0.480	–1.31
<b>Endogenous labor, temptation model</b>								
1970	–	0.000	0.000	1.000	1.000	6.00	0.474	–
2000	PE	–0.815	–0.073	0.956	0.982	6.00	0.473	+0.52
2000	GE	–0.815	–0.070	0.977	0.991	6.16	0.480	–1.00

<sup>1</sup> 1970: Economy with no borrowing. 2000: Economy calibrated to debt-to-output ratio of 7 percent.

<sup>2</sup> GE: general equilibrium. PE: partial equilibrium with prices fixed at the 1970 level.

<sup>3</sup> Borrowing limit relative to total income.

<sup>4</sup> Level in the 1970 (no-debt) economy normalized to one.

<sup>5</sup> Average of cross-sectional variances of consumption for all age groups.

<sup>6</sup> Change in social welfare from the 1970 economy, measured as the percentage change in per-period consumption.

$\mu$  such that the average time spent working is 33 percent of the disposable time (which is one). As for the partial equilibrium effect, as in the baseline results, the temptation model implies a lower welfare gain from the relaxed borrowing limit (equivalent to per-period consumption growth of 0.5 percent) than the model without temptation (1.1 percent). However, if the general equilibrium effect is taken into account, both models imply a welfare *loss* associated with increased indebtedness, and the loss is larger for the no-temptation model. Why? As studied by Pijoan-Mas (2006), when consumers can use labor-leisure decisions, consumers can smooth consumption substantially through this channel. Therefore, the borrowing constraint is less important for consumption smoothing. This can be seen as the relatively small welfare gain in the partial equilibrium experiments. On the other hand, a weaker need for consumption smoothing means that the borrowing limit must be relaxed substantially to generate a debt-to-output ratio of 7 percent. Notice that the calibrated borrowing limits for the 2000 economies are very high for both models in Table 3. Besides, as in the baseline case, the no-temptation model implies a more lax borrowing limit. Under these circumstances, the stronger negative general equilibrium

effect in the model without temptation dominates for the total welfare effect. This experiment implies that the endogenous labor supply decision, and the consumption smoothing due to it, is crucially important in determining the welfare implications of a relaxed borrowing limit.

## 8 Conclusion

This paper investigates the macroeconomic and welfare implications of rising indebtedness in the U.S. using the model with preferences featuring temptation and self-control. The temptation model can capture the welfare loss associated with over-borrowing, together with the gain from better consumption smoothing and the general equilibrium effects. There are three main findings. First, not only are the models with and without temptation observationally similar in terms of macroeconomic aggregates in the steady state, but they also have similar predictions in terms of macroeconomic changes in response to a relaxed borrowing limit. Second, although the macroeconomic implications of the relaxed borrowing limit are similar between the two models, the welfare implications are very different; the temptation models imply significantly lower or even negative welfare effects associated with rising indebtedness. In particular, I find that when debt increases to the same extent as in the period 1970-2000, there is a loss of social welfare equivalent to a 0.39 percent decrease in per-period consumption in the temptation model. On the other hand, the standard model without temptation implies a welfare gain equivalent to a 0.65 percent increase in per-period consumption. Finally, the optimal borrowing limit becomes lower when the degree of temptation becomes higher. A tight restriction on borrowing could be welfare-improving according to the temptation model, not only because it restricts the negative general equilibrium effect, but also it helps consumers avoiding over-borrowing.

Even though the models with and without temptation are observationally similar along many dimensions, they have very different welfare implications. Therefore, from the normative perspective, it is important to find other and better ways to distinguish between the two models, although there is little need to use the non-standard preferences from the positive perspective. I list two potential ways to distinguish. First, if we can observe the borrowing constraint, consumers' response to changes in the borrowing constraint can be used to identify the degree of temptation. It would be even more helpful if the borrowing constraint for individual consumers can be observed. Second, it might be possible to combine the model implications and survey data to distinguish between the two models, if the survey data can be mapped into welfare implications. This issue is left for future research.

One interesting and important extension from the current paper is associated with consumer bankruptcy. The increase in consumer debt has been accompanied by a substantial increase in consumer bankruptcy filings. [White \(2007\)](#) argues that a high level of consumer bankruptcies can be better understood using hyperbolic discounting/temptation preferences. Recently, the consumer bankruptcy law was reformed to make bankruptcy more costly and not available to consumers with relatively high incomes in order to discourage abuse of the law. The standard equilibrium models of consumer bankruptcy imply that a tougher bankruptcy law might benefit consumers by allowing a stronger commitment to repay. But it is not clear if the intuition carries over when consumers suffer from over-borrowing. [Nakajima \(2009\)](#) investigates this issue.

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## Appendix A: Additional Details of Calibration

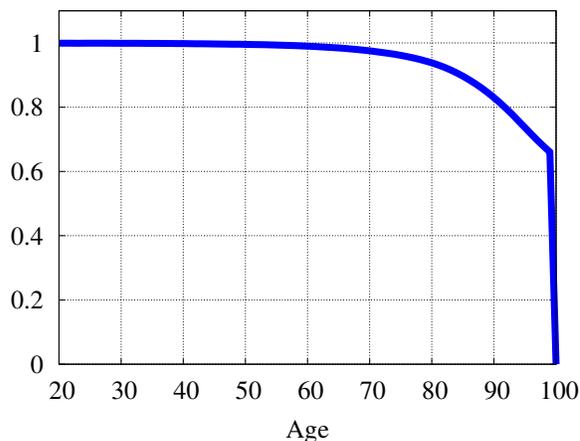


Figure 10: Conditional survival probabilities

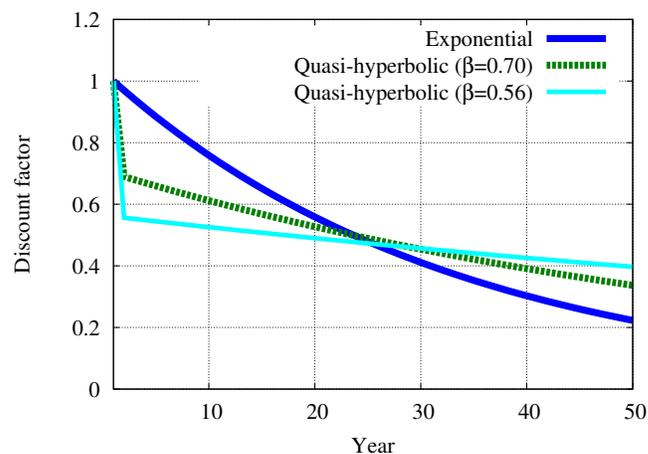


Figure 11: Comparison of discount factors

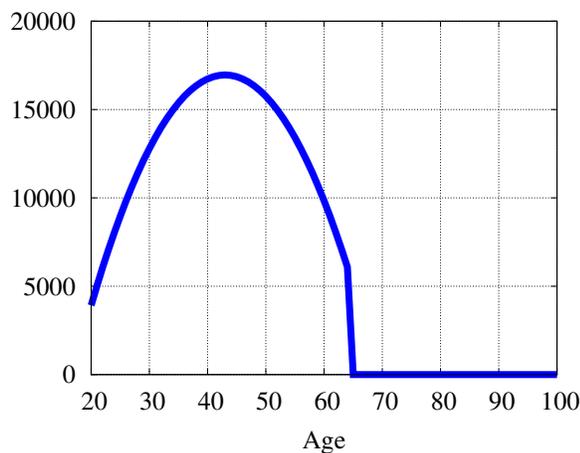


Figure 12: Average life-cycle profile of labor productivity

Figure 10 shows the survival probabilities conditional on age. This is used as  $\{s_i\}_{i=1}^I$  in the calibration. Figure 11 compares the discount factors of the standard exponential discounting and hyperbolic discounting models for periods (years) 1 to 50. The calibrated  $\beta$  and  $\delta$  are used. The figure shows that the discount factor function drops substantially more from period 1 to 2 in the case of the hyperbolic discounting preferences. On the other hand, the discount factor applied to utility in the distant future is higher for the hyperbolic discounting model. Laibson (1997) argues that housing, from which inhabitants can enjoy utility as long as they own it and live in it, has an extra value for hyperbolic discounting consumers, since the dividends can be enjoyed for a long period of time. Figure 12 shows the average life-cycle profile of the labor productivity. This is used as  $\{e_i\}_{i=1}^I$  in the calibration.

## Appendix B: Computation Algorithm

I will first describe below the computational algorithm to solve the steady-state equilibrium of the model with temptation. Since the focus is the steady-state equilibrium, I drop the time script in the algorithm. The solution method for the model without temptation (i.e., exponential discounting model) is straightforward. Adding the labor-leisure decision is also straightforward.

### Algorithm 1 (Computation algorithm for solving steady-state equilibrium)

1. *Set the initial guess of the aggregate capital  $K^0$  and per-consumer transfer  $d^0$ . Notice that aggregate labor supply  $\bar{L}$  can be computed independently from the model since there is no labor supply decision.*
2. *Given  $K^0$  and  $\bar{L}$ , compute the interest rate  $r$  and the wage  $w$ . The transfer used in the iteration is equal to the guess, i.e.,  $d = d^0$ . The Social Security benefit  $\bar{b}$  can be computed given  $w$  and the age distribution, which is exogenous to the model. Once  $\bar{b}$  is obtained,  $\{b_i\}_{i=1}^I$  can be set as  $b_i = \bar{b}$  for  $i \geq I_R$  and  $b_i = 0$  for  $i < I_R$ .*
3. *Given  $\{r, w, d, b_i\}$ , solve the consumer's optimization problem using backward induction.*
  - (a) *Set  $V(I + 1, p, a) = 0$  for all  $p$  and  $a$ .*
  - (b) *Solve the problem of the consumer of age- $I$ , using the Bellman equation (1) for all  $p$  and  $a$ . The optimal level  $a'$  is obtained by basically comparing values conditional on  $a'$  and choosing  $a'$  associated with the highest value. Notice that, since  $\gamma \rightarrow \infty$ , the optimal  $a'$  is chosen to maximize the temptation utility, while the value function is updated using self-control utility.*
  - (c) *With  $V(I, p, a)$  at hand, we can solve the problem of age- $I - 1$  consumers. Keep going back in the same way until the value function and the optimal decision rule for age 1 (initial age) consumers are obtained.*
4. *Using the obtained optimal decision rule  $g^a(i, p, a)$ , simulate the model.*
  - (a) *Set the type distribution for the newborns, which is exogenously given. In particular, all newborns have  $i = 1$  and  $a = 0$ . Initial  $p$  is distributed according to  $\pi_p^0$ .*
  - (b) *Update the type distribution using the stochastic process for  $p$  and the optimal decision rule  $g^a(I, p, a)$ .*
  - (c) *Keep updating until age  $I$  (last age).*
5. *Compute the aggregate capital stock  $K^1$  and total amount of accidental bequests implied by the simulated distribution. Notice that consumers survive according to the survival probability, and there is population growth, which makes the size of the younger population larger. Make these adjustments when computing the aggregate capital stock and accidental bequests.*

Specifically, when the measure of age 1 consumers is normalized to one, measures of age- $i$  consumers,  $\tilde{\mu}_i$  can be represented as follows:

$$\tilde{\mu}_i = \frac{1}{(1 + \nu)^{i-1}} \prod_{j=0}^{i-1} s_j \quad (19)$$

where  $s_0 = 1$ . Once the aggregate amount of accidental bequests is computed, we can compute the per-consumer lump-sum transfer  $d^1$ .

6. Compare  $\{K^0, d^0\}$  and  $\{K^1, d^1\}$ . If they are closer than a predetermined tolerance level, stop. Otherwise, update  $\{K^0, d^0\}$  and go back to step 2.

Next, I will describe the solution algorithm of an equilibrium that features the deterministic transition between two steady states. The first step is to obtain the two steady states using the Algorithm 1. Denote the initial and the new steady state by  $t = 0$  and  $t = \infty$ , respectively. Set the initial distribution along the transition path  $\mu_0$  as the type distribution of consumers in the initial steady state, and the value at the end of the transition  $V_\infty(i, p, a)$  as the value function in the new steady state. The only difference between the two steady states is the borrowing limit  $\underline{a}$ ; total factor productivity  $Z$  is assumed to be constant over time. I also assume that the transition is complete after  $T < \infty$  periods. Since the model economy converges to the new steady state only asymptotically, a large  $T$  is desirable for a good approximation. Now, in period 0 the economy is in the initial steady state, but in period 1, the transition, in particular the sequence of the borrowing limit  $\{\underline{a}_t\}_{t=0}^T$ , is revealed to consumers. Let  $\underline{a}_1 = \underline{a}_0 = 0$  and  $\underline{a}_t = \underline{a}_\infty$  for  $t = \tilde{T}, \tilde{T} + 1, \tilde{T} + 2, \dots, T$ , and  $\underline{a}_t$  gradually increases between period 1 and period  $\tilde{T} < T$ . I set  $\tilde{T} = 30$ . Since  $t = 0$  corresponds to 1970 (initial steady state without borrowing) and one period is a year,  $t = \tilde{T} = 30$  corresponds to 2000. After 2000, the borrowing limit is assumed to remain at the same level as in 2000.

### Algorithm 2 (Computation algorithm for solving equilibrium transition path)

1. Set the initial guess of sequences  $\{K_t^0, d_t^0\}_{t=0}^T$ . Notice that aggregate labor supply  $\{\bar{L}_t\}_{t=0}^T$  can be computed independently from the model.
2. Given  $\{K_t^0, d_t^0, \bar{L}_t\}_{t=0}^T$ , compute the sequence  $\{r_t, w_t, d_t, b_{t,i}\}_{t=0}^T$ .
3. Given  $\{r_t, w_t, d_t, b_{t,i}\}_{t=0}^T$ , solve the consumer's optimization problem using backward induction.
  - (a) Start from period  $T$ . Notice that we know the value function  $V_{T+1}(i, p, a) = V_\infty(i, p, a)$  for  $\forall(i, p, a)$  since the economy is assumed to have converged to the new steady state in period  $T$ .
  - (b) Solve the consumer's problem for  $\forall(i, p, a)$  in period  $T$ , given  $V_{T+1}(i, p, a)$ . The solution method for the model with temptation is the same as in the steady-state equilibrium described in Algorithm 1. The optimal decision rule  $g_T^a(i, p, a)$  and the value function  $V_T(i, p, a)$  are obtained. Notice that since the value function for the next period is given, there is no need to go back from age  $I$  as in Algorithm 1.

- (c) *Keep going back until  $t = 0$ .*
4. *Using the obtained optimal decision rule  $g_t^a(i, p, a)$ , simulate the model.*
- (a) *The type distribution in period 0 is given by  $\mu_0$ .*
- (b) *Update the type distribution using the stochastic process for  $p$  and the optimal decision rule  $g_t^a(i, p, a)$ . Make sure to normalize the population size each period.*
- (c) *Keep updating until period  $T$  (last period).*
5. *Compute  $\{K_t^1, d_t^1\}_{t=0}^T$  using the sequence of distribution  $\{\mu_t\}_{t=0}^T$  generated in the last step.*
6. *Compare  $\{K_t^0, d_t^0\}_{t=0}^T$  and  $\{K_t^1, d_t^1\}_{t=0}^T$ . If they are closer than a predetermined tolerance level, stop. Otherwise, update  $\{K_t^0, d_t^0\}_{t=0}^T$  and go back to step 2.*