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Comment on Cavalcanti and Nosal's "Counterfeiting as Private Money in Mechanism Design"*

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Abstract

In this comment, I extend Cavalcanti and Nosal's (2010) framework to include the case of perfectly divisible money and unrestricted money holdings. I show that when trade takes place in Walrasian markets, counterfeits circulate and the Friedman rule is still optimal.

1 Introduction

This comment could be titled "Disappointment." Let me explain why after I briefly summarize the elegant paper by Ricardo Cavalcanti and Ed Nosal. In their world, agents meet bilaterally to conduct their business. They also need only one piece of paper – money – to trade, and they cannot hold more than one piece of paper at a time. The difference between Kiyotaki and Wright (1989) is that they can all produce a fake piece of paper, although at a cost. Different agents have different abilities to produce fake money. Some are good at it – they have a low cost of producing fake money, and some are bad at it – their cost is high. All counterfeits, however, share the same property that they vanish after being used. Needless to say, this is annoying for somebody acquiring fake money. Cavalcanti and Nosal then describe the set of incentive feasible allocations, i.e. the allocations that could be

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implemented if we were to add a market structure onto this economy, and they characterize the allocation that maximizes a welfare function.

They have two main results. Proposition 2 describes conditions under which counterfeiting does not exist. This is interesting, but because space is limited, let me concentrate on their first proposition. Proposition 1 states that when some agents are very good at producing fake money, i.e. their cost is arbitrarily close to zero, then any optimum allocation has the property that counterfeits circulate and consumption is lower than its efficient level. The intuition is simple. If money buys too much, the value of money is high so that the return from counterfeiting it is also high. The planner can reduce the incentives to fake money by lowering the amount that one piece of paper can buy, i.e. lowering the value of money. However, the planner is unable to totally eliminate counterfeiting as some people are incredibly good at it. Cavalcanti and Nosal conclude that "the deviation from the first-best optimum quantity of money is biased toward inflation." Aha. I am growing hopeful!

Let me explain why I am hopeful. Why is it interesting to study counterfeiting? The reason why I care is that, in some previous research, I believed the threat of counterfeiting could be a reason why a monetary authority may want some inflation above the Friedman rule. The Friedman rule requires the rate of return of money to equal the rate of time preference, so that there is no intertemporal cost of holding money. After all, money should be the solution to trading frictions and should not be part of the problem. Unfortunately, the Friedman rule is very powerful, and some of us who belong to the school of new monetarist economics (see Williamson and Wright, forthcoming) have long been battling against the Friedman rule. As long as the Friedman rule is optimal, there is no role for monetary policy along the business cycle.[1] Also problematic is that the Friedman rule makes money so good that, in most models, it eliminates the coexistence between money and credit. To be fair, there are ways to deviate from the Friedman rule (see e.g. Sanches and Williamson, 2010), but counterfeiting is appealing because its existence relies on the very assumptions that

[1] Sticky price models have the same problem: Remove stickiness and deflation (or a zero nominal interest rate in their framework) is optimal, whatever shocks hit the economy.

give rise to the need for money itself. By stating that "the deviation from the first-best optimum quantity of money is biased toward inflation," Cavalcanti and Nosal actually give us, the new monetarists (or at least me), hope.

Disappointment is looming. So could inflation above the Friedman rule be optimal after all, for the very same reasons that explain the existence of money? It is tempting to draw this conclusion, but it is difficult to do so using a model in which agents' money holding is restricted to be either zero or one. Cavalcanti and Nosal never succumb to the temptation, and rightly so. However, this begs the question: What happens if we relax this assumption? When I do so below, I find that the Friedman rule is still optimal. Damn.

2 Nonvanishing Counterfeits

Consider a version of a Rocheteau and Wright (2005) environment with a measure one of buyers and a measure one of sellers. There are two goods, X and H. Buyers consume good X, while sellers produce good X using a linear technology. Both buyers and sellers consume good H and can produce good H according to a linear technology. Buyers' preferences over an allocation $(x, h) \in \mathbb{R}_+ \times \mathbb{R}$ are represented by the utility $U(x, h) = u(x) - h$, where $h < 0$ means that h units of good H are consumed. For any $(x, h) \in \mathbb{R}_+ \times \mathbb{R}$, sellers' utility is $U(x, h) = -x + h$.

I depart from the mechanism design approach to monetary theory as envisaged by Wallace (forthcoming) by assuming that buyers and sellers trade good X in a Walrasian market in the morning and then trade good H in another Walrasian market in the afternoon. All agents are anonymous. Therefore, they need money to trade. Assuming that anonymous agents interact in a Walrasian market allows me to abstract from considering rather complex equilibrium concepts as in Li and Rocheteau (2009) or Nosal and Wallace (2007), while still retaining the essentiality of money.

At the start of a period, the stock of money is M. In the morning market, the price of good X is p, and to follow a now established convention, I denote as ϕ the real price of good H in terms of money in the afternoon market. I drop the time subscript for ease of notation.

To account for counterfeiting, i.e. the production of money by agents, I assume that buyers can produce any quantity of money at dawn. This is costly, however, and for each real unit of counterfeits ϕc a buyer produces, he has to suffer a cost ω. At dawn, each buyer draws a cost from the distribution $F(\omega)$. Each counterfeit is indistinguishable from one another – or legitimate money – and contrary to Cavalcanti and Nosal, counterfeits are perfectly durable. Durability implies that it is "easy" for counterfeits to circulate, since nothing "wrong" can happen when receiving a counterfeit. The next section deals with the case in which counterfeits are not durable. When $c(\omega)$ denotes the quantity of counterfeits produced by a buyer with a counterfeiting cost ω, the overall stock of money evolves according to the following law of motion:

$$M_{+} = M + \int c(\omega)\, dF(\omega) + T \tag{1}$$

where T is a monetary transfer by the monetary authority at the end of the afternoon market.

Absent any frictions, the efficient allocation is simply described as a level of consumption x^* such that $u'(x^*) = 1$ and no production of counterfeits, or $c^*(\omega) = 0$ for all ω.

In the text that follows I concentrate on stationary and symmetric equilibrium where the real stock of money, as measured in the afternoon, is constant, or $\phi_{+}M_{+} = \phi M$. If γ denotes the growth rate of money, then the inflation rate is $\phi/\phi_{+} = \gamma$.

It is quite clear that sellers have no need for money in the morning market. Therefore, they will not carry any from one period to the next. For each unit of money they acquire in the morning and bring to the afternoon market, they suffer the production cost $1/p$. However, they can use this unit of money to acquire ϕ units of good H in the afternoon. Since marginal cost has to equal marginal benefit, the seller's problem gives us the following price equation:

$$\phi p = 1. \tag{2}$$

I now describe the decision of buyers in more detail. If $V(m)$ denotes a buyer's expected utility of entering the morning's market with m units of money and $W(m)$ denotes his expected utility of entering the afternoon market with m units of money, then

$$\begin{aligned} V(m) &= \max_{x} u(x) + W(m - px) \\ & s.t. \;\; px \leq m \end{aligned} \tag{3}$$

If $C(m;\omega)$ denotes the expected utility of drawing counterfeiting cost ω at dawn, then

$$C(m;\omega) = \max_{c} V(m + c) - \phi c \omega \tag{4}$$

so that buyers can bring more money in the morning by counterfeiting it, and

$$\begin{aligned} W(m) &= \max_{h,m_+} -h + \beta \int C(m_+;\omega)\, dF(\omega) \\ & s.t. \;\; \phi m_+ \leq h + \phi m + \phi T \end{aligned}$$

In the afternoon, buyers choose to produce or consume good H and rebalance their money holding, subject to their budget constraint. If $m+T$ is higher than what they would like to bring into the next period, then they will consume some of good H. Otherwise, they will produce and sell good H. Notice that we can use the budget constraints to replace h in the above expression. Then, using the resulting expression for W in (3), we obtain

$$\begin{aligned} V(m) &= \max_{x,m_+} u(x) + \phi(m + T - px) - \phi m_+ + \beta \int C(m_+;\omega)\, dF(\omega) \\ & s.t. \;\; px \leq m \end{aligned} \tag{5}$$

When $\phi\lambda$ denotes the real Lagrangian multiplier on the budget constraint, the first-order and envelope conditions for problem (5) are, using (2),

$$u'(x) = 1 + \lambda \tag{6}$$

$$\beta \int C'(m_+;\omega)\, dF(\omega) = \phi \tag{7}$$

$$V'(m) = \phi(1 + \lambda) \tag{8}$$

As is now well known, the quasi-linearity assumption on the utility for good H implies that buyers have no wealth effects. As a consequence, all buyers chose the same money holding m_+ when exiting the afternoon market, as shown by (7). The first-order and envelope conditions for problem (4) are:

$$V'(m+c) \leq \phi\omega \; (= \text{ if } c > 0) \tag{9}$$

$$C'(m;\omega) = V'(m+c) \tag{10}$$

Using (6) to replace λ in (8), and then using (8) to replace V' in (10), and finally using (10) to replace the expression for C' in (7), we obtain the following equilibrium conditions

$$\beta \int \phi_+ u'(x)\, dF(\omega) = \phi. \tag{11}$$

It remains to characterize the buyers' counterfeiting decision. I guess the following: Buyers who are drawing a high counterfeiting cost will not produce any counterfeit as it is too costly. So all buyers with $\omega \geq \bar{\omega}$, for some level $\bar{\omega}$, consume $x = \phi m$ and choose $c(\omega) = 0$. Since the marginal buyer with $\bar{\omega}$ is in this class, I obtain that for all buyers with $\omega \geq \bar{\omega}$,

$$u'(\phi m) = \bar{\omega}. \tag{12}$$

Then, all buyers who draw a low cost, $\omega < \bar{\omega}$, will produce counterfeits, until the marginal benefits of doing so equals the marginal cost, or

$$u'(\phi m + \phi c(\omega)) = \omega. \tag{13}$$

In particular, if $\omega > 1$, they will not be able to consume the efficient amount, and if $\omega < 1$, they will consume more than the efficient amount. Using these regions, I can rewrite (11) as

$$\beta \left[\int_0^{\bar{\omega}} \omega dF(\omega) + \bar{\omega}\left[1 - F(\bar{\omega})\right] \right] = \gamma. \tag{14}$$

where I have used stationarity and the fact that $\gamma = \phi/\phi_+$. What does (14) tell us? In equilibrium, the discounted marginal benefit of money has to equal its marginal cost. Money has a much lower benefit for a buyer with a low counterfeiting cost than for a buyer with a high cost, and (14) accounts for this difference.

I can now define a symmetric and stationary equilibrium as a counterfeiting cost $\bar{\omega}$ that, given γ, solves (14). Indeed, the equilibrium allocation can then be derived from $\bar{\omega}$ going backward through the equations above.

Before I analyze the equilibrium, it is instructive to study the law of motion for money (1). In particular, notice that the monetary authority can undo any increase of the money stock due to counterfeiting by taxing money holdings. Therefore, the monetary authority can truly pick the desired γ, although it naturally influences the buyers' counterfeiting decision. Now going back to (14), notice that the expected marginal value of money, on the left-hand side, is actually increasing in $\bar{\omega}$. But the measure of counterfeiters is given by $F(\bar{\omega})$. This implies that as the monetary authority increases γ, more buyers produce counterfeits!

The usual belief is that by raising γ, money (and therefore counterfeits) becomes less valuable, thus reducing the incentives to fake money. So, the result that $\bar{\omega}$ rises with γ seems surprising at first, but the intuition is actually simple. When inflation is low, it is relatively inexpensive to hold money across periods. Hence, there are less incentives to produce counterfeits than when inflation is high, *given* buyers have the opportunity to acquire money in the afternoon. So the optimal monetary policy is to set γ to β, i.e., the Friedman rule. Then (1) implies that $\bar{\omega} = 1$, and only a measure of incompressible counterfeiters remain. In other words, the Friedman rule minimizes the production of counterfeits: Although it will not achieve the first-best allocation, it is still the best policy possible.

3 Vanishing Counterfeits

I now modify the environment slightly. I assume that buyers are of several types as a function of ω. In other words, rather than drawing a counterfeiting cost each period, they are endowed with a counterfeiting technology that is more or less expensive to operate. I still assume the distribution of counterfeit cost is $F(\omega)$, and I now refer to an agent with a cost ω as a type ω agent. Also, for simplicity, I assume that counterfeits vanish with probability $\pi = 1$ at lunchtime, just before the afternoon market opens. Hence, the overall stock of money as measured at night is

$$M_+ = M + T \tag{15}$$

where T is the monetary transfer at the end of the afternoon market.

In what follows, I still concentrate on stationary and symmetric equilibrium where $\phi_+ M_+ = \phi M$. I use ψ to denote the measure of counterfeits in the stock of legitimate money. Then

$$\psi = \frac{\int c(\omega)\, dF(\omega)}{M}. \tag{16}$$

It is quite clear that sellers have no need for cash in the morning market. Therefore, they will not carry any from one period to the next. Also, for each unit of money they acquire in the morning market, they suffer the production cost $1/p$. However, if this unit of money does not vanish, they can use it to acquire ϕ units of good H in the afternoon. Since marginal cost has to equal marginal benefit, the seller's problem gives us the following price equation

$$(1 - \psi)\, \phi p = 1, \tag{17}$$

where ψ is the probability that the unit of money vanishes at midday. Notice that here I use the assumption that buyers and sellers interact in a Walrasian market. The way I see things is that the auctioneer collects payment from all buyers, mixes all notes together, and pays the sellers. So the probability that a seller receives a fake note is just the measure of counterfeits in the economy, ψ. Sellers anticipate that they will receive counterfeits, and they adjust their price accordingly.

I now describe the decision of buyers in more detail. I guess the following: Buyers with $\omega < \bar{\omega}$ will not access the afternoon market, as they do not need to accumulate money.

Rather, these buyers will produce counterfeits. Buyers with $\omega \geq \bar{\omega}$ access the afternoon market and do not produce any counterfeits. Because the equilibrium is stationary, if a buyer finds it profitable to produce counterfeits once, then he will always find it profitable. First, let me consider the decision of counterfeiters. If a buyer decides to counterfeit, I assume that he produces only the amount of counterfeits he needs,[2] and his problem is then simply

$$\begin{aligned} & \max_{c,x} u(x) - \phi\omega c \\ s.t. \quad & px \leq c \end{aligned}$$

then $px = c$ and the first-order condition gives

$$u'(x) = \frac{\omega}{1-\psi}$$

where I have used (17) to replace for ϕp. Notice that counterfeiters do not care about inflation when deciding how many fakes to produce: Given the price *level* ϕ, they just produce what they need. Then, the lifetime expected payoff of a counterfeiter is V^c such that

$$(1-\beta) V^c = u(x^c) - \frac{\omega}{1-\psi} x^c \tag{18}$$

I now analyze the decisions of a buyer when he participates in the afternoon market. If $V(m)$ denotes a buyer's expected utility of entering the morning's market with m units of money and $W(m)$ is his expected utility of entering the afternoon market with m units of money, then

$$\begin{aligned} V(m) &= \max_{x} u(x) + W(m - px) \\ & \quad s.t. \;\; px \leq m \end{aligned} \tag{19}$$

[2] A buyer may want to produce more counterfeits today and store them for later use. All I care about here is his true expenditure px, as this is the amount of counterfeit that goes into circulation.

and

$$\begin{aligned} W(m) &= \max_{h,m_+} -h + \beta V(m_+) \\ & s.t.\ \ \phi m_+ \leq h + \phi m + \phi T \end{aligned}$$

Notice that we can use the budget constraints to replace h in the above expression. Then, using the resulting expression for W in (19), we obtain

$$\begin{aligned} V(m) &= \max_{x,m_+} u(x) + \phi(m + T - px) - \phi m_+ + \beta V(m_+) \\ & s.t.\ \ px \leq m \end{aligned} \tag{20}$$

When $\phi\lambda$ denotes the Langrangian multiplier on the budget constraint, the first-order and envelope conditions for problem (20) are, using (17),

$$u'(x)(1-\psi) = 1 + \lambda \tag{21}$$

$$\beta V'(m_+) = \phi \tag{22}$$

$$V'(m) = \phi(1+\lambda) \tag{23}$$

Once again, the quasi-linearity assumption on the utility for good H implies that buyers have no wealth effects. As a consequence, all buyers choose the same money holding m_+ when exiting the afternoon market. Using (21) to replace λ in (23), and then using (23) to replace V' in (22), we obtain the following equilibrium conditions

$$\beta u'(x)(1-\psi) = \gamma. \tag{24}$$

where I have used the expression for $\gamma = \phi/\phi_+$. (24) gives us the consumption level $x^n = x(\gamma, \psi)$, if a buyer decides to behave legally.

It remains to characterize the buyer's counterfeiting decision. The lifetime payoff of choosing to produce counterfeits is (18), while the lifetime payoff of not counterfeiting is V, such that[3]

$$(1-\beta)V = u(x^n) - \frac{x^n}{1-\psi}. \tag{25}$$

A buyer who contemplates going rogue will compare (18) and (25). Therefore, he will opt for illegality whenever

$$u(x^c) - \omega\frac{x^c}{1-\psi} > u(x^n) - \frac{x^n}{1-\psi} \tag{26}$$

where

$$u'(x^c) = \omega\frac{1}{1-\psi} \text{ and } u'(x^n) = \frac{\gamma}{\beta}\frac{1}{1-\psi}. \tag{27}$$

Notice that the left-hand side of (26) is decreasing in ω. Hence, (26) and (27) define a level $\bar{\omega}$, such that all buyers with $\omega < \bar{\omega}$ go rogue. To complete the characterization of the equilibrium, I need to find ψ. Using the budget constraint of counterfeiters, I get $x(\omega)/(1-\psi) = \phi c(\omega)$. And replacing the resulting expression for $c(\omega)$ in (16), I obtain

$$\psi(1-\psi) = \frac{\int_0^{\bar{\omega}} x(\omega)\, dF(\omega)}{\phi M}$$

Since the equilibrium is stationary, ϕM is a constant and so is ψ. This completes the characterization of an equilibrium.

Now, let us suppose that the Friedman rule prevails, so that $\gamma = \beta$. By simply staring at (26) and (27), it is easy to see that buyers go rogue whenever $\omega < 1$ and stay legal otherwise. From (27), observe that an increase in γ from the Friedman rule implies that x^n declines. As a result, even those buyers with ω higher but close enough to 1 will consider going rogue. Doing so would increase ψ and give buyers even more incentives to go rogue. Hence, increasing inflation encourages counterfeiters. So the Friedman rule is still optimal: It decreases the cost of acquiring and holding money and therefore minimizes the amount

[3] Here are the details. We know that

$$V(m) = u(x) + \phi(m + T - px) - \phi m_+ + \beta V(m_+)$$

Since buyers who participate in the afternoon market hold the overall stock of (legitimate) money available in the economy, we have $m = M$ and $m_+ = M_+$. Using (15), $V(m)$ is simplified to

$$V(M) = u(x) - \phi px + \beta V(M_+)$$

Stationarity implies that $V(M) = V(M_+) = V$, and using (17), I obtain (25).

of counterfeiting in the economy.

4 Conclusion

The Friedman rule is very resilient. I modified Cavalcanti and Nosal's environment in more than one way. First, I relaxed the restrictions that money holdings are either zero or one. Second, I removed the assumption that meetings are bilateral. Third, I imposed an equilibrium concept rather than finding the best incentive feasible allocations. The result is mixed: I described two simple environments where counterfeits circulate but where the Friedman rule is optimal. Li and Rocheteau (2009) described an environment similar to the one above, except that they considered pairwise matching with bargaining. They find that although counterfeits will not circulate in equilibrium, the existence of the counterfeiting technology limits the size of the trades. The Friedman rule is also optimal in their framework.

This does not mean that the Friedman rule will always be optimal even in equilibrium with counterfeits. However, it means that we will have to work harder and add more restrictions to go beyond the Friedman rule.

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