Courts and Contractual Innovation: A Preliminary Analysis

Mitchell Berlin and Yaron Leitner*
Federal Reserve Bank of Philadelphia

October 2005

Abstract

We explore a model in which agents enter into a contract but are uncertain about how a judge will enforce it. The judge can consider a wide range of evidence, or instead, use a rule-based method of judgment that relies on limited information. We focus on the following tradeoff: Considering a wide range of evidence increases the likelihood of a correct ruling in the case at hand but undermines the formation of precedents that resolve legal uncertainty for subsequent agents.

In a model of contractual innovation, we show that the use of evidence increases the likelihood of innovation in any period, while rule-driven judgments increase the rate of diffusion of the innovation. When courts can use a mixture of evidence and rules, the minimum amount of evidence that induces adoption is (weakly) decreasing over time. We also examine the breadth of precedents. Overlapping jurisdictions reduce the optimal breadth of precedents because broad precedents are more likely to introduce conflict. Accordingly, overlapping jurisdictions increase the value of using evidence. We use our model to interpret differences between the legal systems in the U.S. and England.

*The views expressed here are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or of the Federal Reserve System. The authors thank Philip Bond, Ronel Elul, Leonard Nakamura, and David Skeel for helpful discussions. The most recent version of this paper is available at www.philadelphiafed.org/econ/wps/index.html.
1 Introduction

Courts play a crucial role in enforcing contractual agreements. We usually assume that agents write the best contract for themselves and that courts enforce their agreement accurately; and in routine transactions this may be a reasonable approximation. But in novel or complicated contractual situations legal risk arises because the agents’ intentions must be interpreted by the court or because the agreement raises broader legal or social issues. In these cases, the courts play a central role in resolving legal risk; some recent examples include court decisions concerning ATM fees, the poison pill and other defensive mechanisms against takeovers, and the enforcement of credit swaps in the event of sovereign defaults. While it is difficult to quantify the overall effect of legal uncertainty on economic decision making, individual cases suggest that the effects can be large. For example, Kamma, Weintrop, and Weir (1988) find evidence of significant investor losses associated with the Delaware Supreme Court’s decision to uphold Unocal’s poison pill amendment; these losses occurred to shareholders of other Delaware firms that appeared to be targets of hostile takeover attempts at the time. Kamma et al. interpret these losses as investors’ (negative) valuation of the precedent established by the court’s ruling.

Legal systems differ in their rules governing judicial interpretation, notably: (i) The extent to which precedents are binding; and (ii) The range of evidence a court can (or must) consider in resolving a contractual dispute. For example, it is widely held that judges view precedents as more binding in England than in the U.S. (see, for example, Atiyah and Summers, 1987). Eric Posner (1998) has argued that within the U.S. some states have stricter rules limiting the admissibility of evidence outside the “four corners” of the formal contractual agreement. Our focus is on the ways in which the rules of interpretation affect

---

1The legal issues in these examples concerned whether the Comptroller of the Currency could preempt state and local laws limiting ATM fees, whether defensive mechanisms that entrenched current management were consistent with boards of directors’ fiduciary responsibilities, and the conditions in which the buyer of a credit-default swap can make a claim for restitution against the seller of the swap.

2The term rules of interpretation should be interpreted broadly to include system-wide standards that are not necessarily mandatory, in addition to mandatory standards enforced by precedents from higher courts.
the resolution of legal uncertainty in common law legal systems, although our analysis may be more broadly applicable.

We explore a theoretical model in which agents enter into a contract but are uncertain about how a judge will interpret and enforce it in the event of dispute. The judge can use two different methods for resolving a dispute, which introduces our main trade-off. On the one hand, the judge may consider a wide range of evidence—for example, the agents’ discussions leading up to the writing of a contract or the agents’ prior actions under the contractual agreement before their dispute led them to court. Considering a wide range of evidence may increase the likelihood that the judge makes a correct ruling in the case at hand, but the use of evidence comes at a cost. Specifically, using a wide array of evidence may undermine the formation of precedents that resolve legal uncertainty for subsequent agents. Alternatively, the judge can use a rule-based method of judgment that relies on a more limited information set. This method is less likely to lead to a correct judgment in any one period but speeds the dynamic resolution of legal uncertainty. We discuss the relationship between these different methods and the rules of interpretation under the Uniform Commercial Code.\(^3\)

We examine some of the implications of this tradeoff in a number of applications. In a model of contractual innovation in which courts can either use evidence or rule-driven judgments, we show that the use of evidence increases the likelihood of innovation in any period, while rule-driven judgments increase the rate of diffusion of the innovation. We also explore a model in which courts can use a mixture of evidence and rules. In this application we show that the minimum amount of evidence that is necessary to induce agents to adopt the innovation is (weakly) decreasing over time.

We also examine the breadth of precedents. If precedents were fully binding, our basic

\(^3\)We consider a number of interpretations along the way. One interpretation emphasizes two judicial approaches to contract interpretation in the common law tradition. The use of evidence corresponds to the subjectivist approach, which seeks to uncover the contracting agents’ true intentions, while rule-based judgment corresponds to the objectivist approach, which seeks to determine the intentions of reasonable agents in comparable situations. See, for example, Chapter 7 of Farnsworth (1999). In another interpretation, the use of evidence corresponds to a substantive orientation—in which judges seek substantive justice in the case at hand—while the rule-based decision corresponds to a more formalist approach. Atiyah and Summers (1987) use this distinction to contrast the U.S. and English legal systems. Note, we use the term formalism without the pejorative connotation of Djankov, La Porta, Lopez-de-Silanes, and Shleifer (2003).
tradeoff says that the broadest possible precedents would be more desirable because they reduce legal risk to the greatest possible extent. However, overlapping jurisdictions or multiple sources of law—a characteristic of the US legal system, with its 50 state court systems and a federal court system—create an offsetting cost for the use of broad precedents. A precedent created in one location can undermine a precedent from another location, thereby increasing rather than reducing legal uncertainty. Accordingly, overlapping jurisdictions increase the value of using evidence. This finding is broadly consistent with the observation that precedents have less binding force in the U.S. than in England and also with the observation that U.S. courts are typically less formalist than English courts.

Our main contribution is to provide a theoretical analysis of how different legal systems resolve legal risk, with special attention to the process of contractual innovation. Understanding the legal mechanisms for handling innovations should provide insights into the relationship between the legal system and economic performance, a matter that has received intensive study in the (mainly empirical) law and finance literature in recent years.

Related Literature

Apart from its general connections to the broader literature on law and finance, our paper is closely related to a number of works in the economics and legal literatures that emphasize the effects of legal risk on contracting practices. In the economics literature, Franks and Sussman (2005) examine the dynamics of contractual innovation in a world with legal risk. In their model, legal risk leads to inefficient contractual innovations because agents know that judges are likely to make errors, and so write contracts that are only optimal in light of the judge's likely mistakes. These contracts then become inefficient standards for subsequent agents. In their model, legal risk can also lead to dynamic traps, as agents avoid risky innovations to seek the certainty of standardized contracts. Gennaioli

4 Although our focus is on common law systems like the U.S. and England, it may also apply to a broader range of legal systems. For cross-country evidence on the use of precedent, see MacCormick and Summers (1997). For a comparative discussion of judicial interpretation in different legal systems, see Chapter 30 in Zweigert and Kotz (1998).

5 There is now a large literature relating legal systems to financial development and growth. The two seminal works are La Porta, Lopez-de-Silanes, Shleifer, and Vishny (1997, 1998). Djankov, et al. (2003) explicitly consider the role of courts. Pistor et al. (2002) provide cross-national evidence concerning the rate of legal innovation. Levine (forthcoming) contains a useful review of much of the literature.
Gennaioli (2003) shows that uncertainty about the judge’s bias can lead agents to forgo optimally state-contingent contracts in favor of rigid contracts that constrain judicial discretion. In a world where judges may either be biased or efficient, Gennaioli and Shleifer (2005) formally examine Richard Posner’s (2005, 6th ed.) conjecture that precedents tend to evolve toward efficient rules. Chatterji and Filipovich (2002) present a model in which contractual ambiguity induces agents to write incomplete contracts as a hedge against legal risk. None of these papers formally examine different methods of judicial interpretation as an element of the legal system or how interpretation affects the dynamic resolution of legal risk.\(^6\)

In the legal literature, Goetz and Scott (1985) argue that boilerplate contractual terms are a potentially efficient mechanism for overcoming legal risk and that formalist styles of contractual interpretation can induce agents to bear the risks of introducing new language into a contract, thereby reducing legal risk.\(^7\) The cost of contractual standardization is that boilerplate language becomes stale. The first part of their argument has clear connections to our work. However, for Goetz and Scott (1985) and in Scott’s subsequent work, the court’s use of case-specific evidence is an unmitigated bad, both because it induces sloppy contracting (thereby reducing the production of useful boilerplate) and because it undermines contractual enforcement. In turn, the main tradeoff in our paper—and the results that follow—differ from those in Goetz and Scott (1985). Eric Posner (1998) discusses how legal risk and judicial methods of interpretation affect contract form. He shows that judges’ willingness to consider a wide array of noncontractual evidence leads agents to write more incomplete contracts.

There are also a few papers that discuss various aspects of judicial interpretation in the absence of legal risk or dynamic considerations, the main elements of our model. Shavell

---

\(^6\)Franks and Sussmann (2005) do contrast systems in which legislation is the primary mechanism for innovation versus systems in which judicial rulings are the primary mechanism. They also have a discussion of passive versus active judges in their account of the different judicial approaches to the introduction of the floating charge in England and the U.S.

\(^7\)This is an argument that Scott has followed up in a number of subsequent works pressing for a renewed formalism in contract interpretation. See, for example, Scott (2000a, 2000b).
(2003) presents optimal rules of interpretation for judges who are fully informed about the agents’ intentions and the optimal contract when it is costly for agents to include explicit contractual terms. Anderlini, Felli, and Postlewaite (2003a, 2003b) discuss conditions in which it is optimal for an asymmetrically informed judge to override contractual terms.

The rest of the paper is organized as follows: In Section 2, we illustrate the main tradeoff through an example. In Section 3, we present the model. In Section 4, we compare two systems: one in which judges rule based on facts that are common across agents, and one in which judges rule based on idiosyncratic evidence. In Section 5, we apply this tradeoff in a model of contractual innovation, and in Section 6, we allow for conflicting precedents and examine the optimal breadth of a precedent. We conclude in Section 7.

2 An example

There are two states $s_1$ and $s_2$, and two projects. A pair of agents can select at most one project. Each project yields two units of a consumption good. If the agents choose the first project, the two units go to agent 1 in state $s_1$ and to agent 2 in state $s_2$. If they choose the second project, the two units go to agent 2 in state $s_1$ and to agent 1 in state $s_2$.

The two agents can make sure that each agent ends up with one unit by entering a bilateral contract that says that the agent with two units transfers one unit to the other one; this may be the preferred outcome if agents are risk averse. The specific contract depends on the project chosen. If they choose the first project, they enter a contract that says that agent 1 transfers one unit to agent 2 in state 1, and agent 2 transfers one unit in state 2. Similarly, if they choose the second project, they enter a contract that says that agent 2 transfers one unit in state 1, and agent 1 transfers in state 2. In addition to the units from the project, each agent has one unit that can be seized; thus, the judge can enforce a contract even if he cannot observe who has the two units.

There are two types of judges. The first enforces every contract as if it was the first contract; the second enforces every contract as if it was the second. If the agents knew the judge’s type, they could choose the appropriate project and contract and obtain the highest
possible utility; for example, if they knew the judge is type 1, they could choose the first project and enter the first contract.

Legal uncertainty stems from the fact that the agents do not know the judge’s type. Ex-ante the judge is equally likely to be of either type. Therefore, no matter what project and contract the agents choose, the judge rules incorrectly with probability 1/2. However, after the judge rules in the first case, his type becomes known, and agents can adjust their agreement (project plus contract) accordingly so that it is correctly enforced.

Now suppose that instead of making a ruling based on his type, the judge looks at some evidence that indicates which project the agents selected; with probability 0.9, the evidence is correct and points to the right project, and with probability 0.1, the evidence is misleading and points to the wrong project. When the judge considers this type of evidence, he is more likely to rule correctly in the specific case. The downside is that agents cannot learn his type and adjust their agreement. In other words, legal uncertainty is not reduced for other pairs of agents who face the same choice problem.

3 The model

There is an infinite number of periods $t = 1, 2, \ldots$. Each period has two stages. In the first stage, a pair of agents selects a project $p \in P$. In the second stage, a state $s \in S$ is realized and the two agents go to court. The judge rules by selecting an outcome $a \in A$. The agents have a preferred outcome. If the judge chooses the preferred outcome, each agent obtains a utility $u$; otherwise, each agent obtains $v < u$. In the first case, we say the judge rules correctly; in the second case that he rules incorrectly. Formally, let $\phi : (p, s) \to A$ denote the agents’ preferred outcome given $p$ and $s$. The utility for each agent given $p, s,$ and $a$ is

$$U(p, s, a) = \begin{cases} 
    u & \text{if } \phi(p, s) = a \\
    v & \text{otherwise.}
\end{cases}$$

(1)

*Why do agents go to court?* The two agents agree on the preferred outcome in each state when they choose the project, but they disagree at a later stage. Formally, there is a random variable $\tilde{e}$ whose realization becomes known after $s$ is realized; the random variable takes
the values $\varepsilon$ and $-\varepsilon$ with equal probabilities. Agent's 1 utility is

$$U_1(p, s, a) = \begin{cases} u & \text{if } \phi(p, s) = a \\ v + \varepsilon & \text{otherwise,} \end{cases} \quad (2)$$

and agent's 2 utility is

$$U_2(p, s, a) = \begin{cases} u & \text{if } \phi(p, s) = a \\ v - \varepsilon & \text{otherwise.} \end{cases} \quad (3)$$

Assume $v + \varepsilon > u$; then once the agents observe $\vec{e}$, one of them prefers that the judge rules $\phi(p, s)$, while the other prefers that the judge rules differently.

The legal system. Denote by $p_t$ the project chosen by pair $t$, and by $s_t$ the state realized in period $t$. The judge rules according to some function $\gamma_t(p_t, s_t, \phi)$. Agents do not know what $\gamma_t$ is; their beliefs regarding $\gamma_t$ are given by $\Pr(\gamma_t(p, s, \phi) = a|I_t)$, where $I_t$ is the public information available at the beginning of period $t$.

Agents’ problem. Denote by $\Pr(s)$ the probability that state $s$ will be realized; assume that each state is realized with a positive probability. The agents in period $t$ choose $p \in P$ to maximize their expected utility

$$\sum_{s \in S} \sum_{a \in A} \Pr(s) \Pr(\gamma_t(p, s, \phi) = a|I_t) U(p, s, a). \quad (4)$$

Denote the probability that the judge will rule correctly in state $s$ by

$$m_t(p, s) \equiv \Pr(\gamma_t(p, s, \phi) = \phi(p, s)|I_t), \quad (5)$$

and note that (4) can be rewritten as

$$\begin{align*}
(4) &= \sum_{s \in S} \Pr(s) \left[ \sum_{a : a = \phi(p, s)} \Pr(\gamma_t(p, s, \phi) = a|I_t)u + \sum_{a : a \neq \phi(p, s)} \Pr(\gamma_t(p, s, \phi) = a|I_t)v \right] \\
&= \sum_{s \in S} \Pr(s) [m_t(p, s)u + (1 - m_t(p, s))v] \\
&= v + (u - v) \sum_{s \in S} \Pr(s) m_t(p, s). \quad (6)
\end{align*}$$

Thus, maximizing (4) is the same as maximizing the expected probability of a correct judgment,

$$b_t(p) \equiv \sum_{s \in S} \Pr(s)m_t(p, s). \quad (7)$$
Denote $p^* = \arg \max_{p \in P} b_t(p)$ and $b_t^* = b_t(p^*)$.

For simplicity, we focus on the case where there are only two possible outcomes, each is equally likely to be chosen by the judge. In this case,

$$m_1(p, s) = \frac{1}{2}$$

for every pair $(p, s)$, and the choice of project in the first period does not matter; all projects provide the same expected utility. In addition, $b_0^* = \frac{1}{2}$; a priori judges are equally likely to choose the correct or the incorrect outcome.

4 Looking at evidence vs. creating precedents

In this section we compare two special legal systems: one in which the court decides based on facts that are common across agents, and one in which the court decides based on facts that are idiosyncratic to the case at hand. In the first system precedents are created; in the second system they are not.

4.1 Creating precedents

The first legal system is as follows: There is a set of common elements $F$ (with individual element $f$) and a function $h : S \rightarrow F$ that specifies a common element for every state; this function defines a partition of $S$. The judge in period $t$ observes $h(s_t)$ and rules according to some function $g : F \rightarrow A$, as follows:

$$\gamma_t(p, s, \phi) = g(h(s)).$$

One interpretation is that upon observing a single state ($s$) the judge draws out the essential features that he believes to be important to the case at hand. These essential features are what we call common elements. Crucially, common elements are comparable across agents, who can read about the court’s judgment in the public record. Formally, the public record contains the judge’s decision and the basis for the ruling, i.e., the facts he observed. The judge makes no announcement about possible rulings in states he has not observed,
as we discuss below. In the beginning of period $t$, the record is $I_t = [h(s_{t'}), g(h(s_{t'}))]_{t'=1}^{-1}$, where $s_{t'}$ denotes the state that was realized in period $t'$.

The judge does not observe $p_t$ and he does not observe $\phi$. If we think of $\phi$ as representing a contract between the two agents, we can interpret the fact that the judge does not observe $\phi$ in two different ways. In one interpretation, the contract is not clear about the agents’ intentions and the judge must interpret the contract according to some standard, for example, he can use the common law’s “reasonable person” as a guide for construing the agents’ contractual goals. Another possible interpretation is that the judge is not bound by the agents’ intentions when he makes a ruling. In practice, this may happen if the judge has a different objective than enforcing the parties’ will; for example, he may take into account third parties who are affected by the bilateral agreement.\footnote{A recent court case provides an interesting example of legal uncertainty and the use of rules. Eternity Global Master Fund, a hedge fund, had purchased a credit default swap from Morgan Guaranty Trust to hedge Argentine bonds. When Argentina announced a “voluntary” rescheduling of its debt, Eternity sought to unwind its positions. Morgan refused claiming that Eternity had exchanged its bonds “voluntarily” and that the contract limited Morgan’s obligation to “mandatory” exchanges. Eternity countered that the exchange had been economically coercive and, therefore, effectively mandatory. The judge first ruled that it was irrelevant whether the exchange was mandatory, but then reversed himself in a second decision. The second opinion explicitly rejects consideration of the economic context of the exchange and refers to the dictionary meaning of the word mandatory. Pointing to the dictionary meaning of a contract term is common when judges use the plain meaning rule for interpreting disputed terms. See Eternity Global Master Fund Limited, plaintiff against Morgan Guaranty Trust Company of N.Y. and JP Morgan Chase Bank, Defendants, United States District Court for the Southern District of N.Y., Oct. 29, 2002, and June 5, 2003.}

Agents know $h$, but they do not know $g$. They believe that $g$, which we refer to as the judge’s type, is drawn from some set $G = \{g_1, g_2, \ldots, g_n\}$ according to some probability distribution $\Pr(g_i)$; thus, $\Pr(g(f) = a) = \sum_{g_i \in G: g_i(f) = a} \Pr(g_i)$. This is one way of formalizing the view that the same evidence may be interpreted differently by different judges, depending on the legal principles the judge brings to bear on the case, or perhaps, depending on the judge’s personal prejudices. The assumption that agents know $h$, that is, that all judges share a common view of the essential features of the case is for simplicity alone. We could perform a similar analysis if different judges classified states according to different conceptual schemes, that is, if we allowed them to use different partitions.

Assume that for two common elements $f \neq f'$, knowing $g(f)$ does not change the agents’
priors regarding \( g(f') \); that is, if \( f \neq f' \),

\[
Pr(g(f') = a' | g(f) = a) = Pr(g(f') = a').
\]

(10)

It then follows that agents update their beliefs regarding court ruling as follows:

\[
Pr(\gamma_t(p, s, \phi)) = a | I_{t+1}) = \begin{cases} 
1 & \text{if } h(s) = h(s_t) \text{ and } g(h(s_t)) = a \\
0 & \text{if } h(s) = h(s_t) \text{ and } g(h(s_t)) \neq a \\
Pr(\gamma_t(p, s, \phi)) = a | I_t) & \text{if } h(s) \neq h(s_t)
\end{cases}
\]

(11)

This is our way of modeling precedents, which has two main features: (i) Seeing how the judge rules when he has considered fact \( f \) resolves all uncertainty about how future courts will rule when they face the same fact. (ii) However, observing the judge’s ruling for fact \( f \) adds no information as to how he will rule if he considers a different fact \( f' \).

The first part follows from the assumption that all judges are constrained to use the same function \( g \) (equation (9)). Alternatively, one can assume that each judge has his own function \( g_t \), but judges must follow precedents. A binding precedent means that subsequent judges must rule the same way for essentially similar cases. Here, if the judge rules based on a fact that was used in a previous case, he must be consistent with the prior decision, although each judge can rule according to his own interpretation of the law for facts that have not been considered previously. The assumption that precedents are perfectly binding is a polar case that captures one essential role of precedent, the resolution of legal uncertainty. We relax this assumption later in the paper.\(^9\)

The second part, that a court’s ruling for a given fact is completely uninformative about the way courts will rule when they observe a different fact, is mainly a technical simplification; this is another polar case. However, the underlying idea, that agents do not update their beliefs about future judgments in situations far removed from the case at hand can be interpreted as representing the common law view that judgments must be rooted

\(^9\)For those readers who do not believe that precedents actually have binding force, consider the following quote from Summers, in his chapter on precedent in the United States, specifically N.Y. State: “The tendency of courts to follow precedents in contract, torts, and property is so pronounced that N.Y. appellate courts routinely remark that, although they may not agree with an established precedent, they nonetheless felt constrained to follow it.” (MacCormick and Summers, 1997, p. 372). Most scholars note that the binding force of precedent is quite powerful in commercial law, although it is less powerful in statute law and in constitutional law.
in the facts of the particular case at hand. According to this view, it is inappropriate for judges to speculate about how they would judge were the facts significantly different.\textsuperscript{10}

\textit{Costless adjustment.} We assume that agents can adjust their project costlessly so that its ideal outcome is consistent with prior rulings. This allows us to focus on one aspect of the role of precedent in isolation, the resolution of legal uncertainty. Effectively, we assume that as long as agents can predict a judge’s ruling in a particular state, they can adjust their contract to achieve their desired ends; thus, there are no good or bad precedents.\textsuperscript{11} We recognize that precedents may also inefficiently constrain agents’ contractual choices, but we abstract from these issues in the present paper.

More formally, we assume that

**Assumption 1** \textit{For every vector of outcomes} \((a_s)_{s\in S}\), \textit{there exists a unique project} \(p \in P\), \textit{such that} \(\phi(p, s) = a_s\) \textit{for every} \(s \in S\).

Then the solution to the agents’ problem is as follows: Consider the agents in period \(t\). Suppose the states \(s_1, s_2, \ldots s_{t-1}\) were realized in the previous periods, and let

\[
S_t' = \{s \in S : \text{there exists } t'<t, \text{such that } h(s) = h(s_{t'})\}.
\]  

(12)

The set \(S_t'\) includes the states for which a precedent was created before period \(t\). Given equations (8) and (11), it follows that \(m_t(p, s) = 1/2\) for every pair \((p, s)\) such that \(s \notin S_t'\); in these states the choice of project does not matter. However, Assumption 1 implies that there exists a project \(p \in P\) whose ideal outcome is consistent with the judge’s rulings in the other states \(s \in S_t'\). This project is the agents’ optimal choice, and is denoted by \(p^*\). It follows that

\[
b_t^* = \sum_{s \notin S_t'} \Pr(s) \times \frac{1}{2} + \sum_{s \in S_t'} \Pr(s) \times 1.
\]  

(13)

\textsuperscript{10}Note that we abstract from the hierarchical dimension of precedent, i.e., that lower courts are formally bound by the decisions of higher courts. This would be important in a model that focuses either on the enforceability of precedents or the process of correcting mistaken or obsolete precedents, interesting issues that we do not address.

\textsuperscript{11}Eternity v. Morgan provides a concrete illustration of our costless adjustment assumption. Assume that the best outcome for two firms is that a coercive exchange be treated as an involuntary exchanges. In light of the judge’s ruling, we effectively assume that future agents can direct the judge to consider the economic conditions surrounding the exchange.
Denote $\mu_t \equiv \sum_{s \in S'_t} \Pr(s)$; this expression represents the amount of legal uncertainty resolved up to period $t$. It then follows that

$$b^*_t = \mu_t + \frac{1}{2}(1 - \mu_t).$$

(14)

Since $S'_{t+1} \supset S'_t$, it follows that $\mu_{t+1} \geq \mu_t$. Note that $\mu_1 = 0$. In addition, since every state is realized with a positive probability, $\lim_{t \to \infty} \mu_t = 1$; eventually, every state is realized at least once and all uncertainty is resolved. It follows that $b^*_t = \frac{1}{2}$, $b^*_t$ increases in $t$, and $\lim_{t \to \infty} b^*_t = 1$. This is true for every realization of $\{s_t\}_{t=1}^\infty$.

The breadth of precedents. A precedent is broader if it applies to more cases. Formally, the breadth of a precedent created in state $s$ is

$$\mu(s) \equiv \sum_{s' : h(s') = h(s)} \Pr(s').$$

(15)

Different functions $h$ induce different breadths; in particular, if $h_1$ defines a broader partition of $S$ than $h_2$, then $h_1$ induces broader precedents. It follows from equation (12) that if $h_1$ induces broader precedents than $h_2$, then $\mu_t(h_1) \geq \mu_t(h_2)$ with a strict inequality for some $t$; thus, broad precedents reduce uncertainty faster. In Section 6 we extend the model to allow for conflicting precedents and show that broad precedents no longer imply a faster resolution of uncertainty.

4.2 Looking at evidence

The second legal system is as follows: The judge in period $t$ observes a piece of evidence $\tilde{e}_t$ which is a random variable:

$$\tilde{e}_t(p_t, s_t) = \begin{cases} \phi(p_t, s_t) & \text{with probability } m \\ a \neq \phi(p_t, s_t) & \text{with probability } 1 - m. \end{cases}$$

(16)

In other words, with probability $m < 1$, the judge observes the agents’ preferred outcome and with probability $1 - m$, he observes a different outcome. The judge rules according to

$$\gamma_t(p, s, \phi) = \tilde{e}_t(p, s).$$

(17)

If we interpret $\phi$ as a contract, then evidence refers to a range of interactions that may be highly informative about the agents’ true intentions but lie outside the contract proper.
This piece of evidence can represent, for example, evidence on pre-contractual negotiations, interactions between the agents under prior contractual agreements, oral communications, etc.\textsuperscript{12} It is assumed that \( m > \frac{1}{2} \); therefore, using evidence is better than using a rule if the goal is to determine the agents’ intentions. The rationale for this assumption is that the judge bases his decision on more information. In our model, the use of evidence would be strictly dominated if this were not true. (See Scott (2000a, 2000b) for the alternative view that a judge considering such evidence is likely to misread the agents’ intentions.)

The record contains the evidence and the ruling. It is assumed that \( \bar{\epsilon}_t \) are iid; thus, a ruling in one case does not provide any information regarding rulings in other cases.

We obtain \( m_t(p, s) = m \) for every pair \((p, s)\). In addition, \( b_t^* = m \) for every \( t \); thus, legal uncertainty is not reduced through time.

\subsection*{4.3 Interpretation}

Our two stylized legal systems have connections to real world legal systems. For example, Atiyah and Summers (1987) have drawn the distinction between the formalist approach of the English legal system and the substantive approach of the U.S. legal system. They argue that English judges are more likely to read contracts literally and narrowly, a particular type of rule, while U.S. judges are more likely to consider a broad range of evidence, including noncontractual evidence, so as to achieve a “just” outcome. Indeed, confronted with a contract that is not clear about the agents’ intentions, the Uniform Commercial Code, which has been adopted in part or in whole in all fifty United States, directs the judge to consider: (i) interactions between agents under the existing contract (the \textit{course of performance})—which may vary substantially from the explicit contractual terms; (ii) interactions between the agents under agreements prior to the current one (the \textit{course of dealing}); and (iii) common business practices (\textit{usage of trade}).\textsuperscript{13}

A second connection relates to the historical development of U.S. legal interpretation

\textsuperscript{12}\textsuperscript{Note that it is assumed here that the only way agents affect the realization of this random variable is through their choice of project. Thus, we do not analyze the interesting possibility that the availability of evidence may be a contracting choice.}

\textsuperscript{13}\textsuperscript{Under English law, course of performance is not accepted as evidence of contracting agents’ intentions (Farnsworth, p. 490).}
in the twentieth century. The first *Restatement of Contracts*, an influential codification of legal thinking about contracts, reflects the *objectivist* view of contract interpretation, in which contractual ambiguity leads the judge to inquire what a reasonable person in a similar situation to the contracting agents would have understood the contract to mean. This need not correspond to the understanding of the contracting agents themselves. The *Restatement (Second) of Contracts* is a later codification that adopts the *subjectivist* view that the primary role of the court is to determine the true intentions of the contracting agents. In practice, the two codifications adopt very different approaches to the admissibility of *parol evidence*, evidence of negotiations prior to the final contract, with the *Restatement Second* adopting a much more liberal approach in its willingness to consider such evidence. According to the most authoritative current treatise on contracts, Farnsworth’s *Contracts* (3rd edition, 1999), the more liberal approach to the use of parol evidence of the *Restatement Second* has increasingly gained the upper hand among jurists. In our model, this would represent a movement from a more rule-oriented system to a more evidence-oriented system in the latter half of the twentieth century.\(^{14}\)

### 4.4 The tradeoff

Suppose we want to maximize a weighted sum of the agents’ utilities across all periods. This is the same as maximizing \(E \sum_{t=0}^{\infty} w_t b_t^r\), where the expectation is with respect to the information before period 1 begins, when we do not know the sequence of states that will be realized. The next proposition implies that if we put a lot of weight on the first periods, the evidence system is preferred; otherwise, the rule-driven system is preferred. In addition, the rule-based system becomes more attractive when precedents are broad. Formally, assume that the breadth of every precedent created is \(\rho \in (0, 1)\), that is \(\mu(s) = \rho\) for every \(s \in S\). Then:

\[\text{Proposition 1} \ (i) \text{ Under the rule-driven system } E(b_t^r) = 1 - \frac{1}{2}(1 - \rho)^{t-1}, \text{ and under the}\]

\(^{14}\)In one sense, the objectivist/subjectivist distinction is a special case of the formalist/substantive distinction. Katz (2004) defines the degree of formalism by the extent to which courts rule on the basis of a less information.
evidence system $E(b_t^*) = m$. (ii) When $t > 1$, the difference $1 - \frac{1}{2}(1 - \rho)^{t-1} - m$ is strictly increasing in $t$ as well as in $\rho$; the difference is negative when $t = 1$ and positive when $t$ is large enough.

Proof: (i) Consider the rule-driven system. Given that precedents were created for a portion $\mu_t$ of the states, then in the next period with probability $\mu_t$ no new precedent is created, and with probability $1 - \mu_t$, a new precedent of breadth $\rho$ is created. Thus,

$$
\mu_{t+1}|\mu_t = \begin{cases} 
\mu_t & \text{with probability } \mu_t \\
\mu_t + \rho & \text{with probability } 1 - \mu_t.
\end{cases}
$$

(18)

It follows that

$$
E(\mu_{t+1}|\mu_t) = \mu_t\mu_t + (1 - \mu_t)(\mu_t + \rho)
$$

(19)

$$
= \mu_t + \rho(1 - \mu_t),
$$

and

$$
E(\mu_{t+1}) = E(E(\mu_{t+1}|\mu_t)) = E(\mu_t) + \rho(1 - E(\mu_t))
$$

(20)

$$
= \rho + (1 - \rho)E(\mu_t).
$$

Using the formula for the sum of a geometric series and the fact that $E(\mu_1) = 0$, it follows that

$$
E(\mu_t) = \rho \frac{1 - (1 - \rho)^{t-1}}{1 - (1 - \rho)}
$$

(21)

$$
= 1 - (1 - \rho)^{t-1}.
$$

Using equations (14) and (21), it follows that under the rule-based system

$$
E(b_t^*) = E(\mu_t) + \frac{1}{2}(1 - E(\mu_t))
$$

(22)

$$
= \frac{1}{2} + \frac{1}{2}E(\mu_t)
$$

$$
= \frac{1}{2} + \frac{1}{2}[1 - (1 - \rho)^{t-1}]
$$

$$
= 1 - \frac{1}{2}(1 - \rho)^{t-1}.
$$

The second part of (i) is immediate.
(ii) Since $\rho \in (0, 1)$ and $t > 1$, it follows that the difference $1 - \frac{1}{2}(1 - \rho)^{t-1} - m$ is strictly increasing in $t$ and in $\rho$. When $t = 1$, we obtain that the difference equals $\frac{1}{2} - m < 0$, and when $t$ is large enough, we obtain

$$\lim_{t \to \infty} [1 - \frac{1}{2}(1 - \rho)^{t-1} - m] = 1 - m > 0.$$  \hspace{1cm} (23)

Q.E.D.

5 An application: The speed of innovation.

Suppose that in addition to the projects in $P$, there is another project the agent can choose; denote this benchmark project by $p'$. We refer to the projects in $P$ as the new type of projects, and to $p'$ as the old type. While using the new type of project involves legal uncertainty, using the old type of project does not. In particular, if a pair $t$ chooses $p'$, they obtain a utility $v'_t$ that does not depend on the state or the judge’s ruling; in this case they do not go to court.

It is optimal for pair $t$ to adopt the new type of project if the judge is sufficiently likely to choose their preferred outcome, that is,

$$v + (u - v)b'_t \geq v'_t.$$ \hspace{1cm} (24)

Suppose that $v'_t$ are iid according to some distribution function with continuous support $[\underline{v}, \overline{v}]$. Assume that $u > \overline{v}$; thus without legal uncertainty, all agents adopt the new type of project. In addition, $\underline{v} < v + (u - v)\frac{1}{2} < \overline{v}$; thus with no prior resolution of legal uncertainty, those with a low reservation utility adopt, while those with high reservation utility do not.

Legal uncertainty is reduced only if a case is brought to court, that is if a pair adopts the new type of project, and then only if the court creates a precedent.

Denote by $T_i$ the time it takes until a new pair adopts the new type of project given that $i$ pairs have already adopted, and let $E(\cdot)$ denote the expectations operator. We use the letter “R” to denote the rule-based system and “E” to denote the evidence system. The following proposition states that a rule-driven legal system yields a quicker speed of
adoption (than does a system that uses evidence) only after some point in time; before this
happens, a system that uses evidence induces faster adoption.

**Proposition 2** (i) \( E(T^E_1) < E(T^R_1) \). (ii) There exists \( \tau > 1 \), such that \( E(T^E_i) > E(T^R_i) \) if and only if \( i \geq \tau \).

Proof. Denote by \( b_i \) the probability of a correct judgment given that \( i \) pairs have already
adopted the new type of project, and denote by \( H_i \) the probability that a pair will adopt
the new type of project given that \( i \) pairs have already adopted. Then \( H_i \equiv \Pr(v'_t < v + (u - v)b_i) \) and \( E(T_i) = 1/H_i \). In the rule-driven system, \( b_i^R = \frac{1}{2}, b_i^R \) is increasing in \( i \),
and \( \lim_{i \to \infty} b_i^R = 1 \). In the evidence system, \( b_i^E = m > 1/2 \). Therefore, there exists \( \tau > 1 \),
such that \( b_i^R > b_i^E \) if and only if \( i \geq \tau \). The result then follows because \( H_i \) is increasing in \( b_i \). Q.E.D.

Intuitively, in a rule-driven legal system, each court’s decision adds to the body of case
law and reduces uncertainty for subsequent entrants. Once a sufficient number of cases
have appeared before a judge, subsequent entry can become quite rapid because residual
legal uncertainty is low. In a system that uses evidence, there is no such time dependence
because each case is decided on its individual merits.

### 5.1 A mixed system

A special case of the analysis above is when \( \bar{v} > v + (u - v)\frac{1}{2} \). In this case, in the rule-driven
system, we obtain that in the proof of Proposition 2, \( H_0 = 0 \), and the innovation process
does not start at all. Suppose now that the judge in each period can use a mixture of
evidence and precedents. We can then ask: What is the minimum probability of looking
at evidence that is necessary to get the innovation process started, that is, to induce some
agents to adopt the new type of project?

In more detail, consider a third legal system that is a combination of the first two. In
each period, the judge observes two facts: \( h(s_t) \) and \( \tilde{e}_t(p_t, s_t) \). He then chooses one fact as
the basis for his ruling. With probability \( q \), he chooses \( \tilde{e}_t(p_t, s_t) \), and with probability \( 1 - q \),
he chooses \( h(s_t) \). If he chooses \( h(s_t) \), he rules according to the function \( g \); otherwise, he
rules $\tilde{e}_t$. Then
\[
\gamma_t(p,s,\phi) = \begin{cases} 
  h(g(s)) & \text{with probability } 1 - q \\
  \tilde{e}_t(p,s) & \text{with probability } q.
\end{cases}
\] (25)

The probability $q$ is a choice variable determined by the designer of the legal system. The record contains the two facts observed, the fact chosen, and the ruling. In our formulation, it does not matter if the record contains the two facts, or just one because: (i) if the judge rules based on $\tilde{e}_t$, agents learn nothing about $g$ even if $h(s_t)$ is in the record; and (ii) if the judge rules based on $h(s_t)$, agents learn $g(h(s_t))$ even if $\tilde{e}_t$ is in the record.

A key assumption is that the judge cannot adopt the best of both methods of interpretation. He cannot rule in the case at hand based on the idiosyncratic evidence, while creating a precedent that holds for subsequent cases. If it was possible to rule on the basis of evidence and also to announce a hypothetical ruling based on a common element, the judge would both increase the probability of a correct judgment in the current case and reduce uncertainty for all subsequent agents, clearly a first best.

In practice, the first best is often infeasible. Making a general ruling to create a precedent while making an exception for the case at hand based on special considerations creates problems. The most fundamental problem is that this contradicts the legal principle that similar cases should be treated the same, a principle that underlies the rationale for binding precedents.\textsuperscript{15} A judge’s finding that certain facts are truly essential is undermined if he makes an exception for the case at hand. Another problem is signal extraction; subsequent agents have a harder time disentangling the logic of the judge’s opinion and, thus, have a harder time determining what precedent has actually been set. A third problem is legitimacy. The judge’s willingness to actually rule on the basis of his own reasoning provides agents with greater assurance that the judge hasn’t ruled arbitrarily or corruptly.\textsuperscript{16} In light of these reasons, we examine legal systems that are second best.\textsuperscript{17}

\textsuperscript{15}According to Eisenberg (1988): “[A] court should reason by articulating and applying rules that it is ready to apply in the future to all persons who are situated like the disputants.” (p. 9)

\textsuperscript{16}According to Eisenberg (1988): “Retroactivity also serves to ensure that the rule a court announces is sufficiently well considered that the court is willing to apply the rule to individuals who stand before it.” (p. 127)

\textsuperscript{17}Judges sometimes engage in a practice called prospective overruling; they decide the case at hand on the basis of an existing precedent but announce a new precedent to be used for subsequent cases. Judges use
The next proposition shows that once more uncertainty has been resolved, it is less necessary to look at evidence. Thus, the minimum probability necessary is decreasing through time.

Consistent with previous notation, suppose uncertainty was resolved for the states in $S'$ and denote $\mu = \sum_{s \in S'} \Pr(s)$. Denote by $q_{\min}(\mu)$ the minimum probability needed to have the innovation process continue.

**Proposition 3** If $\mu_1 < \mu_2$, then either $q_{\min}(\mu_1) = q_{\min}(\mu_2) = 0$ or $q_{\min}(\mu_1) > q_{\min}(\mu_2)$.

Proof: Denote $H(b) \equiv \Pr(v' \leq v + (u-v)b)$; this is the probability that a pair will adopt the new type of projects if they believe that the judge will rule correctly with probability $b$. With probability $\mu$, the agents observe $s \in S'$; so they do not face legal uncertainty. Otherwise, if evidence is used (probability $q$), the judge rules correctly with probability $m$, and if evidence is not used (probability $1-q$), the judge rules correctly with probability $1/2$. Denote

$$
d(q) = mq + \frac{1}{2}(1-q). \tag{26}
$$

The ex-ante probability of ruling right is $b = \mu + (1-\mu)d(q)$. The innovation process continues if and only if $H(b) > 0$. This happens if and only if $v + (u-v)b \geq \overline{v}$, which is equivalent to $b \geq \overline{b}$, where $\overline{b} \equiv \frac{v-u}{u-v}$. Note that $b \geq \overline{b}$ is equivalent to $\mu + (1-\mu)d(q) > \overline{b}$, which is equivalent to

$$
d(q) > \frac{\overline{b} - \mu}{1-\mu}. \tag{27}
$$

Since $d(0) = \frac{1}{2}$, it follows that if $\frac{\overline{b} - \mu}{1-\mu} < \frac{1}{2}$, then $q_{\min}(\phi) = 0$; in this case agents innovate even if evidence is not used. Otherwise, $q_{\min}(\phi)$ solves $d(q) = \frac{\overline{b} - \mu}{1-\mu}$, and we obtain,

$$
q_{\min}(\phi) = \frac{\overline{b} - \mu - 1/2}{m - 1/2}. \tag{28}
$$

Since $\overline{b} < 1$, it follows that when $\mu$ is higher, $q_{\min}(\mu)$ is lower. Q.E.D.

---

this practice when they view the existing precedent as wrong but recognize that agents have made significant investments believing that the existing precedent was binding. On the one hand, this is quite different from announcing the essential facts of a case and then ruling on the basis of a different set of facts. That said, legal scholars have argued that prospective overruling creates tensions for precisely the reasons we discuss. See Eisenberg (1988), Chapter 7, and Atiyah and Summers (1987), Chapter 5.
6 Conflicting precedents

We now extend the model to allow for two locations. This permits us to examine the optimal breadth of precedents. In a single location model, precedents always resolve uncertainty and broader precedents resolve more uncertainty. However, in a model with multiple locations (or jurisdictions) broader precedents may lead to conflicts.

The idea of multiple locations can be interpreted in two different ways. The first is literal. In the United States, there are 50 state court systems, as well as the federal court system. And within individual state systems there are often multiple departments; for example, there are four administrative departments in N.Y. State. Thus, the resolution of a case often raises issues of conflicting precedents from different jurisdictions. A second interpretation is that different lines of precedents may develop in two series of cases, whose similarities are not initially recognized. At some point, a clever lawyer will recognize the relevance of another line of precedent because it benefits his client in a dispute.

Judges in each location tend to follow precedents from their own location; however, there is a positive probability that a judge will rule based on a precedent created in the other location. Thus, \( \gamma_t \) depends on the record created in each location.

We study a two-period model and focus on judges in a pure rule-based system. In the first period, there is a different pair in each location, and judges in location \( i \) rule based on \( g_i \) and \( h_i \). Consider now the second period, and focus (without loss of generality) on location 1. Denote by \( S_i^1 \) the set of states for which a precedent was created in location \( i \) in the first period. (Here, the subscript refers to a location; in the previous sections the subscript \( t \) referred to a period.) We assume that in the second period the judge in location 1 rules according to

\[
\gamma(p, s) = \begin{cases} 
  g_1(h_1(s)) & \text{if } s \notin S_2^1 \\
  \beta g_2(h_2(s)) + (1 - \beta)g_1(h_1(s)) & \text{otherwise,}
\end{cases} \tag{29}
\]

where

\[
\beta = \begin{cases} 
  1 & \text{with probability } \beta \\
  0 & \text{with probability } 1 - \beta.
\end{cases} \tag{30}
\]
If the other location (location 2) created a precedent for the case in hand, there is a probability $\beta$ that location 1 will adopt the other precedent. Otherwise, the court in location 1 follows its own precedents or if there is none, it creates a new one. Assume for simplicity that if both locations create a precedent for the same case, then the precedents contradict one another; that is, $g_1(s) \neq g_2(s)$ for every $s$. These assumptions above can be relaxed. In particular, our results are not sensitive to the details of how conflicting precedents are resolved, as long as there is some probability that the local precedent will not be followed.\footnote{It is important that agents cannot instruct the court to use the precedent of their preferred jurisdiction. While real-world agents do include such clauses in contracts, these clauses are imperfectly enforceable. If we interpret the locations as states, another state’s court may (successfully) claim that its own interests in the case override the interests of the state cited in the agents’ contract. See Siegal and Borchers (2005). If we interpret the conflict as one in which two different areas of law are merged, agents will typically be unable to write enforceable clauses to avoid such novel legal developments.}

For simplicity, we focus on a particular specification of the information available to the contracting agents: When agents choose their project, they know the record created in their location, but not in the other location; therefore, they can adjust their project only to the precedents created in their own location.

Denote $\rho = \sum_{s \in S_1'} \Pr(s)$ and $\rho' = \sum_{s \in S_2'} \Pr(s)$; the breadth of the precedent created in location 1 is $\rho$, and the breath of the precedent in location 2 is $\rho'$. Assume that agents know the functions $h_i$, so they know $\rho$ and $\rho'$. Let $b_i$ denote the probability of a correct ruling for the second pair in location $i$. Then

$$b_1 = \rho + \frac{1}{2}(1 - \rho) - \beta \rho \rho'.$$  \hspace{1cm} (31)

The first two terms are the same as in the single location case. The last term represents the case in which the judge rules based on a precedent from the other location. This happens when the state that is realized in period 2 belongs to $S_1' \cap S_2'$ and $\tilde{\beta} = 1$. In this case the resolution of uncertainty is zero instead of one because agents adjusted their project to $g_1$ but not to $g_2$. The overall effect is $1 \times \Pr(\tilde{\beta} = 1) \times \Pr(s \in S_1' \cap S_2') = \beta \rho \rho'$. Note that equation (31) assumes that the agents adjust their project according to the precedent created in their own location. Alternatively, the agents could choose a project at random and obtain $b_1 = \frac{1}{2}$. The first choice is preferred when $\rho + \frac{1}{2}(1 - \rho) - \beta \rho \rho' > \frac{1}{2}$. Since $\rho > 0,$
this is equivalent to $\beta \rho' < \frac{1}{2}$.

Using equations similar to (29) and (31), we obtain that the probability of a correct ruling in location 2 is

$$b_2 = \rho' + \frac{1}{2}(1 - \rho') - \beta \rho' \rho.$$  

(32)

6.1 Optimal breadth

We solve for the breadth that leads to the maximal reduction in legal uncertainty in both locations. The objective function is to choose $\rho$ and $\rho'$ that maximize $b_1 + b_2$. It follows from (31) and (32) that

$$b_1 + b_2 = 1 + \frac{1}{2}(\rho + \rho') - 2\beta \rho' \rho$$

(33)

Differentiating with respect to $\rho$, we obtain

$$\frac{1}{2} - 2\beta \rho' = 0,$$

(34)

and differentiating with respect to $\rho'$, we obtain

$$\frac{1}{2} - 2\beta \rho = 0.$$

The optimal solution is

$$\rho^* = \rho'^* = \frac{1}{4\beta}.$$  

(35)

(Note that the condition $\beta \rho' < \frac{1}{2}$ that assures that adjusting the project to the precedent in one’s location is satisfied.) As one may expect, broad precedents are more useful when $\beta$ is low, that is, when the likelihood of conflict is low.

6.2 Interpretation

We showed that when there is a large chance of conflict it is better to create narrower precedents; thus, the advantage of using rules versus using evidence is reduced. This suggests a connection between multiple jurisdictions, the extent to which precedents are binding, and the use of evidence. Our model is consistent with Atiyah and Summers’ (1987) observation that precedents are less binding in the U.S. than in England, a fact they ascribe (partly)
to the greater number of overlapping jurisdictions and, thus, the greater prevalence of conflicting precedents in the U.S. Our results also suggest one reason why U.S. courts may adopt a less formalist approach than do English courts. Precedents that are more likely to generate conflicts are less able to resolve legal uncertainty. In turn, we expect U.S. courts to use evidence more often.

This interpretation is less consistent with another of Atiyah and Summers’ (1987) observations that in England precedents tend to be more narrowly drawn than precedents in the U.S. According to our simple model, the possibility of conflict would induce U.S. courts not only to use evidence more often but also to draw relatively narrow precedents. We are currently working on an extension of the model that may resolve this issue and bring our theoretical model closer to the empirical evidence.

7 Conclusion

Considering a wide range of evidence may increase the likelihood that the judge makes a correct ruling in the case at hand but it undermines the formation of precedents that resolve legal uncertainty for subsequent agents. In a model of contractual innovation, this tradeoff implies that the use of evidence increases the likelihood of innovation in any period, while rule-driven judgments increase the rate of diffusion of the innovation. When courts can use a mixture of evidence and rules, the minimum amount of evidence that is necessary to induce agents to adopt the innovation is (weakly) decreasing over time. If precedents are fully binding, our tradeoff says that the broadest possible precedents are more desirable. But if there are overlapping jurisdictions or multiple sources of law, the optimal breadth of precedents is reduced because broad precedents are more likely to conflict with one another. Accordingly, overlapping jurisdictions increase the value of using evidence.
References


