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# Working Papers

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**AGGREGATE EMPLOYMENT GROWTH AND THE  
DECONCENTRATION OF METROPOLITAN EMPLOYMENT**

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# Aggregate Metropolitan Employment Growth and the Decentralization of Metropolitan Employment

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## A B S T R A C T

In this paper we document that the disparity in employment densities across U S metropolitan areas has lessened substantially over the postwar period. To account for this deconcentration of metropolitan employment, we develop a system-of-cities model in which an increase in aggregate metropolitan employment causes congestion costs to increase faster for the more dense metro areas. A calibrated version of the model reveals that the (roughly) two and a half-fold increase in postwar aggregate metropolitan employment implies, by itself, more deconcentration than actually observed. Thus, rising aggregate metropolitan employment appears to be a powerful force favoring deconcentration, although some benefit of greater employment density appears to have partially offset the effects of rising congestion costs for the more dense metro areas.

## I. Introduction

The issue of convergence in income levels between regions of a country, or between countries, has received a great deal of attention in recent years. The focus of this research has been on determining whether “there are automatic forces that lead to convergence over time in the levels of per capita income and product” (Barro and Sala-i-Martin (1992, p. 223). For the United States, the evidence suggests that there is a tendency for per capita incomes to converge at the state and regional levels. In this paper we provide an alternative perspective on the phenomenon of convergence. We examine the evolution of employment densities across U.S. metropolitan areas over the post-World War II period. Thus, in contrast to existing studies on convergence, our focus is on the density of economic activity (rather than per capita income or product) and how these densities have evolved over time across metropolitan areas rather than states or regions.

We accomplish two tasks in this paper. First, we document that there has been a pronounced trend toward less disparity in employment densities across U.S. metropolitan areas over the post-World War II period, a trend we label *deconcentration of metropolitan employment*. Second, we argue that an important reason for this trend is that the initially more dense metropolitan areas couldn't absorb the postwar increase in aggregate metropolitan employment as cheaply as the initially less dense metro areas. Because dense metro areas are nearer to using the full capacity of local resources, adding jobs and people in these areas leads to relatively faster increases in traffic congestion, pollution, and the cost of living. Using a calibrated version of a system-of-cities model we show that rising overall metropolitan employment coupled with faster increases in congestion costs for dense metro

areas constituted a powerful force favoring employment deconcentration.

Our ...nding of employment deconcentration complements recent studies on population growth in urban areas. Glaeser, Scheinkman and Shleifer (1995) examine population growth in a cross-section of U.S. cities between 1960 and 1990. For the most part, Glaeser et al. focus on cities rather than MSA's, but their results for MSA's indicate that MSA's in the Northeast grew more slowly than MSA's elsewhere. Glaeser et al.'s ...nding is broadly consistent with ours, since the Northeast contains a relatively large share of the nation's dense metropolitan areas.

More recently, Black and Henderson (1998) have examined the evolution of urban population and the number of metropolitan areas in the United States between 1900 and 1990. They characterize their ...nding as one of "parallel growth" in the number of metropolitan areas of different types, where the type of an MSA is determined by its relative population size (relative to average MSA population). Also, Eaton and Eckstein (1997) examined population growth in urban areas in France and Japan and found that cities grew at the same rate regardless of population size. In contrast to these studies, we distinguish MSA's by their employment densities (not their absolute or relative population size) and ...nd that the initially more dense MSA's grew less rapidly than the initially less dense MSA's.

On the theoretical level, there has also been a revival of interest in the urban underpinnings of economic growth (Lucas (1988)). Black and Henderson (forthcoming) and Eaton and Eckstein (1997) have extended some versions of endogenous growth models to include an urban dimension. These authors have tended to focus on steady-state growth in order to make explicit the connection between cities and the rate of human capital accumulation.

In contrast, the purpose of the theory developed in this paper is to assess the quantitative importance of aggregate metropolitan employment growth and rising congestion costs for postwar deconcentration of metropolitan employment.

Our focus on aggregate metropolitan employment growth as a cause for employment deconcentration is in the spirit of the macroeconomic literature on income convergence. As noted in the opening line, quoted above, of the Barro and Sala-i-Martin (1992) article, the issue of macroeconomic interest is whether there are automatic forces that lead to convergence. To the extent employment growth is inherent to an economic system, we too identify an automatic force favoring a more uniformly dense spatial distribution of employment.

## II. The Deconcentration of Metropolitan Employment

### Data

We use County Business Patterns (CBP) data for the years 1951, 1959, 1968, 1979, 1989, and 1994. The data consist of full- and part-time employees covered by the Federal Insurance Contributions Act (FICA).<sup>1</sup> Generally, employees of establishments exempt from FICA, such as most government employees, self-employed persons, and railroad employees, are excluded

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<sup>1</sup> County Business Patterns data reflect employees on the payrolls of covered firms during the first quarter of the year. With the exception of 1951 and 1994, the first quarter for all other years in our sample occurred about one year before business cycle peaks. The first quarter of 1951 occurred two years before a business cycle peak. At this writing, the expansionary phase of the business cycle that began in the second quarter of 1991 has not yet reached its peak. Nonetheless, five of the six periods between 1951 and 1994 occurred at about roughly the same phase of business cycle expansions, and all six periods occurred during an expansionary phase of the cycle.

from County Business Patterns. Our data set consists of 2,488 of the 3,137 counties and county equivalents (boroughs, independent cities, parishes, etc.) that make up the United States.<sup>2</sup> Data on variables other than employment (population and land area of counties) were taken from The City and County Data Book.

Although counties represent a finer level of geographical detail, we chose metropolitan statistical areas (MSAs) as the geographical unit for our analysis. An MSA typically consists of a central city of at least 50,000 people, as well as any contiguous counties that are metropolitan in character, as determined by the percentage of its nonagricultural labor force and by the amount of commuting between the counties and the central city.

For each of the six years, we constructed a common set of MSAs by combining counties according to the 1983 classification of metropolitan counties. This procedure gave us a sample consisting of employment and other data for 297 MSAs. Although some of these MSAs would not have qualified as MSAs in earlier years (according to the MSA definition implicit in the 1983 classification of metro areas), we chose the 1983 classification of counties as metro or nonmetro as opposed to a classification from some other year because our intent is to use employment density as an indicator of congestion levels. This requires that we measure metropolitan land area as the area of the region around a central city in which

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<sup>2</sup>We have less than complete coverage of counties for a variety of reasons. Some counties that were separately identified in later years in our sample were combined with other counties in 1951. Rather than exclude these counties from our data set, the counties that were combined in 1951 were combined for all periods in our dataset. In addition, the independent cities in Virginia and the independent cities of St. Louis, MO, and Washington, DC, which are treated by the Census Bureau as separate counties, were dropped from our analysis.

people may live and find it practical to commute into the city. Congestion levels would rise as this region gains employment and population. Unfortunately, there is no direct way to measure the area of such a region for each MSA.<sup>3</sup> The best we can do is determine which counties were classified as metropolitan toward the end of our sample period and then assume that it was practical to live in those counties and commute to the central city in earlier years as well. For this reason, we chose the 1983 classification of metropolitan counties.

As noted previously, the employment coverage of CBP is not complete. To get some indication of coverage of metropolitan employment in our data set, we compared the total number of workers in our MSAs to total nonfarm payroll employment in each of the six years. By this measure, our MSAs contain anywhere between 63 to 68 percent of total nonfarm workers. Since the number of nonfarm workers employed in nonmetro areas is likely to exceed the number of farm workers employed in metro areas, this measure gives a lower bound on the true coverage.

## Facts

We use our sample to document the disparity in employment densities across U.S. metropolitan areas. We do this by constructing Lorenz curves for MSA employment for each of the years for which we have data. In these Lorenz curves, we treat each square mile of metropolitan land as we would treat each household in an income distribution. Just as in an income distribution each household is associated with its income, in our analysis each square mile

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<sup>3</sup> Reported measures of MSA land area are "activity-based." A county close to a central city will not be classified as metropolitan until it becomes sufficiently urban in character. In contrast, a measure of MSA land area based on distance from central city would always include such a county. This latter concept of MSA land area is the relevant one for assessing congestion levels.

of metropolitan land is associated with its level of employment. Thus, the Lorenz curve for MSA employment plots the cumulative share of MSA employment against the cumulative share of MSA land area, where MSA land areas (square miles) have been ranked according to decreasing employment density (we assume that employment in each MSA is spread uniformly over the land area of that MSA). If each MSA were equally dense, the Lorenz curve would be the diagonal line. But if employment density varies across MSAs, the Lorenz curve will be bowed above the diagonal line.

Figure 1 plots the Lorenz curves for each of the years while Table 1 reports the values of the Lorenz curves at selected points. Two facts stand out. First, it is evident that there are huge disparities in employment density across U.S. metropolitan areas. For instance, in 1994, the 10 percent of most dense metropolitan land area accounted for 41 percent of total metropolitan employment.<sup>4</sup> Second, it is also evident that the disparity in employment densities was greater in earlier years. For instance, the 10 percent most dense metropolitan land area accounted for 57 percent of total metropolitan employment in 1951. As Figure 1 shows, the Lorenz curves have been shifting over time toward the equal density line. The extent of disparity, as measured by the Gini inequality index, fell more or less steadily from 0.66 in 1951 to 0.53 in 1994.<sup>5</sup>

Probably many factors contributed to this deconcentration trend. Our aim is to evaluate

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<sup>4</sup>For total metropolitan employment in a given year, we used total employment (as reported in the CBP for that year) for our sample of constructed MSAs.

<sup>5</sup>If the spatial distribution of metropolitan workers not covered by CBP is similar to that of covered metropolitan workers, these shares should be a reasonable estimate of the spatial distribution of all metropolitan workers.

the role that increasing metropolitan employment and rising congestion costs may have played. Aggregate metropolitan employment rose 2.4 times between 1951 and 1994. If (as we believe) rapidly rising congestion held back employment growth in the initially dense metropolitan areas, the share of employment accounted for by the initially dense metro areas should fall and those accounted for by the initially less dense metro areas should rise. Table 2 shows the employment share of MSA's grouped by deciles and ranked by their 1951 employment density (i.e., their initial density).<sup>6</sup> It is clear from Table 2 that employment shares have indeed shifted in favor of the initially less dense metro areas. The 30 densest MSA's accounted for 54 percent of total metropolitan employment in 1951 but only 33 percent in 1994 while the collective employment share of the third through the tenth deciles rose from 37 percent to 57 percent. There is also a clear tendency for employment shares to rise faster for lower density MSA's.<sup>7</sup>

Of course, the mere fact that dense MSA's lost employment share to less dense MSA's does not prove that increasing employment and rising congestion costs were important factors in that development. Other factors have contributed to the inverse relationship between initial density and subsequent employment growth as well. For instance, the advent of air-conditioning may have led to faster growth of the initially less dense "sunbelt cities" by making them more pleasant places to live and work (O'Leary, 1997); miniaturization and the development of lightweight materials may have reduced firms' incentives to locate in the densest metropolitan areas in order to lower transportation costs (Garrison and Renshaw (1980)); the advent of the interstate highway network may have accelerated employment

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<sup>6</sup>The first nine groups have 30 MSA's each and the final group has 27.

<sup>7</sup>A similar tendency is discernible in ungrouped data as well.

growth in previously remote and poorly connected low density MSA's (Loven (1978) and Coleman (1978)); a change in people's preferences in favor of less urbanized living may have made low density MSA's more attractive (Beale (1977, 1982)). Thus, to argue that increasing employment and rising congestion costs was an important factor in the deconcentration of metropolitan employment, it is necessary to demonstrate that the postwar increase in metropolitan employment is capable of generating declines in the Gini index of inequality of the magnitude shown in Figure 1. The objective of the quantitative general equilibrium model developed in this paper is to provide that demonstration.

Before proceeding further we need to address a potential concern regarding our finding that employment share of the less dense metro areas rose over the postwar period. The concern is that western counties tend to contain more land area than counties elsewhere, which implies that employment densities of western MSA's tend to be lower than employment densities of MSA's elsewhere. Because population and employment in the postwar period have grown rapidly in the West, the regional bias in the measurement of employment density may partly account for the rapid growth of (apparently) low density MSA's.<sup>8</sup>

While there is some truth to this, it must be remembered that many western MSA's were developed to more fully take advantage of modern transportation technologies and may, in truth, be bigger in land area than MSA's elsewhere. Still, to ensure that the pattern in Table 2 is not driven by exceptionally large MSA's, we excluded MSA's whose land areas went beyond two standard deviations of the mean MSA land area. Eight MSA's fell into this group, six in the West.<sup>9</sup> This gave us a sample of 289 MSA's.

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<sup>8</sup> We thank Joe Gyourko for bringing this to our attention.

<sup>9</sup> The eight excluded MSA's were Bakersfield, CA; Duluth-Superior, MN/WI; Fort Worth-Arlington, TX;

Table 3 shows the employment shares by deciles for the sample of 289 MSA's. The findings reported in Table 3 are, for the most part, very close to those reported in Table 2. The employment share of the first decile declined from 54 percent to 35 percent, while the collective employment share of the third through the tenth deciles rose from 36 percent to 55 percent. However, because we have eliminated some very low density MSA's that grew rapidly, the gain in employment share in the lower deciles is somewhat muted. In what follows, we use these 289 MSA's as our basic data set.

### III. The Model

We need a model to assess the extent to which increase in aggregate metropolitan employment was responsible for postwar employment deconcentration. We adapted the system-of-cities model described in Henderson (1987) for this purpose. There are two conceptual differences between Henderson's model and ours: (i) we take the number of locations in the economy as exogenously given and fixed in land area and (ii) we allow a location's employment density to affect congestion costs.<sup>10</sup>

There are  $M$  locations indexed by  $i = 1; 2; 3; \dots; M$  and a large number of individuals who live and work in these locations. The technological opportunities available to these people, their preferences and endowments, and market equilibrium conditions are described below.

#### Technology

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Las Vegas, NV; Phoenix/Mesa, AZ; Reno, NV; Riverside/San Bernardino, CA; and Tucson, AZ.

<sup>10</sup>In all other respects, our model is considerably simpler than Henderson's. The simplicity is forced on us because we use estimates of agglomeration and congestion parameters available in micro studies to render the model numerical. This "calibration" step imposes limitations on the sophistication of the model.

There is one internationally traded good and  $M$  local goods. The  $m$ -level production function for producing the internationally traded good in location  $i$  is

$$Y = \lambda_i^{-1} \bar{A}_i^{-1} (N_i)^{\alpha} n^{\beta} k^{1-\alpha-\beta}, \quad 0 < \alpha < 1 \quad (1)$$

where  $n$  and  $k$  are the labor and capital used by any  $m$  in location  $i$ ;  $\lambda_i$  is an economy-wide technology index, and  $\bar{A}_i$  is an index that captures the impact of location on city  $i$ 's production capabilities. For instance, the production advantages conferred by being a port would be captured by a high  $\bar{A}_i$ ; as would proximity to output and input markets. Finally,  $\bar{A}_i^{-1} (N_i)^{\alpha}$  is a function of local employment, denoted  $N_i$ , that takes into account the production advantages of agglomeration:

$$\bar{A}_i^{-1} (N_i)^{\alpha} = \max\{N_{\min}^{\alpha}, N_i^{\alpha}\} g; \quad N_{\min} > 0; \alpha > 0 \quad (2)$$

This specification of the agglomeration function has two important features. First, it restricts agglomeration economies to be a constant below some "threshold" level of employment,  $N_{\min}$ . Second, it asserts that in the range where agglomeration benefits are increasing in local employment, the elasticity of agglomeration benefits with respect to change in employment is a constant.

Each location also produces a local good that can be consumed only by local residents. The local good in location  $i$  is produced using a technology that is linear in the traded good:

$$G_i = (\mu_i) i^{-1} Y \quad (3)$$

Here  $\mu_i$  is an index specific to location  $i$  and is meant to capture the effect of location on the production of the local good. For instance, housing being an important local good,  $\mu_i$  could

reflect the costs of producing housing in location  $i$ : The other factor  $j_i$  is a function that models the costs imposed by employment density on the production of the local good

$$j_i = e^{\alpha(N_i/A_i)}; \alpha > 0 \quad (4)$$

where  $A_i$  is land area of location  $i$ : Thus, according to (3) and (4), higher employment density makes the production of the local good less efficient. An important property of the  $j_i$  function is that its elasticity with respect to employment density is increasing in employment density:

$$\frac{d(\ln j_i)}{d(\ln(N_i/A_i))} = \alpha(N_i/A_i) \quad (5)$$

#### Endowments and Preferences

There are two types of individuals: those who own capital and those who do not. Both types have one unit of labor that they inelastically supply to firms in their location of residence. We assume (for tractability) that owners of capital are immobile and their location exogenously given. There is a measure  $N_i^F \geq 0$  of owners in location  $i$ : The total measure of all individuals is given by  $N$ :

Individuals who do not own capital locate to maximize utility. The utility that a mobile individual gets from living in location  $i$  is given by:

$$U_i = \frac{1}{\mu} (N_i) g_i \mu c_i^{1-\mu}; \quad 0 < \mu < 1; \quad (6)$$

where  $g_i$  and  $c_i$  are the individual's consumption of the local and traded good, respectively. The function  $\frac{1}{\mu} (N_i)$  is an amenity index that takes account of agglomeration benefits for consumers:

$$v_i(N_i) = \max_{\hat{g}; N_i \hat{g}} \{N_{\min} \hat{g}; N_{\min} > 0; \hat{g} > 0\} \quad (7)$$

This specification parallels the specification of agglomeration benefits in the production of the traded good and has similar properties. For simplicity, we assume that the employment level beyond which agglomeration benefits for consumers begin to increase with local employment is the same level as that for producers.

### Equilibrium Conditions

Let the traded good be the numeraire. Let the price of the local good in location  $i$  be  $p_i$ ; the wage rate  $w_i$ ; and let the world rental rate on capital be  $r$ . We will focus on equilibria in which there is a positive measure of mobile individuals residing in each location. Formally, we require

$$N_i > N_i^F \quad (8)$$

Utility maximization implies that a mobile individual in location  $i$  chooses  $g_i = \mu(w_i = p_i)$  and  $c = (1 - \mu)w_i$ . Thus, the indirect utility of a mobile individual residing in location  $i$  is

$$V_i = v_i(N_i) \mu^\mu (1 - \mu)^{(1-\mu)} p_i^{1-\mu} w_i$$

Given (8), in equilibrium workers migrate until utility across locations is equalized. This implies

$$v_i(N_i) \mu^\mu (1 - \mu)^{(1-\mu)} p_i^{1-\mu} w_i = \bar{V} \quad (9)$$

We assume that all local goods are supplied competitively. The producers of these goods take the employment density in each location as given. Therefore, the price of the local good

in location  $i$  will equal its marginal cost

$$p_i = \alpha_i e^{\theta(N_i - A_i)} \quad (10)$$

Turning to the traded good, a firm that locates in location  $i$  takes the level of local employment and the product wage in that location as given. It also takes  $r$  as given. In equilibrium, the product wage in each location must be such that the profit from producing the traded good is zero in all locations. These zero profit conditions are

$$w_i = [\alpha_i (1 - \mu)^{\theta} A_i^{-1} (N_i)]^{\frac{1}{\theta}} r^{\frac{1 - \theta}{\theta}} \quad (11)$$

Finally, the sum of labor supply across all locations must equal the exogenously given total supply of labor in the economy:

$$\sum_{i=1}^M N_i = N \quad (12)$$

Denote  $\mu^{\theta} (1 - \mu)^{\theta} A_i^{-1} (N_i)^{\frac{1 - \theta}{\theta}}$  by  $H(\mu; \mu; \alpha_i)$  and the product of location-specific factors,  $\alpha_i^{\frac{1 - \theta}{\theta}}$ ; by  $S_i$ . Then, substituting equations (10) and (11) into equation (9) yields:

$$H(\mu; \mu; \alpha_i) S_i (N_i)^{\frac{1}{\theta}} \Phi(N_i) e^{\theta \mu \frac{N_i}{A_i}} \Phi^{\frac{1 - \theta}{\theta}} = \bar{V} \quad (13)$$

In what follows, the  $M$  equations in (13) and equation (12) are used to solve for the  $M + 1$  unknowns,  $N_i$ ;  $i = 1; 2; 3; \dots; M$ ; and  $\bar{V}$ . This procedure assumes that the unobserved distribution of immobile individuals can always be specified to satisfy the inequalities in equation (8) for the levels of  $N_i$  calculated using the  $M$  equations in (13) and equation (12). It is possible to proceed this way because we assume that these cities can import or export the traded good and the capital stock from each other or the rest of the world. If

the open-economy assumption is dropped, it would be necessary to impose economywide resource balance conditions for the traded good and the capital stock. The distribution of immobile workers will then matter for the determination of equilibrium.<sup>11</sup>

#### IV. Properties of the Model

The material in this section provides the background for the numerical section to follow.

##### Equilibrium Employment Density for a Single Location

Let  $e^{\circ}$  be denoted by  $\bar{e}$ ;  $\mu^{\circ}$  by  $\bar{\mu}$ ; and  $(N_{\min} = A_i)$  by  $\bar{D}_i$ . Then, using (2) and (7), the l.h.s. of equation (13) may be written as a function of density  $D$ :

$$V_i(D) = H(\bar{e}; \bar{\mu}; \bar{s}_i) \Phi S_i \Phi A_i^{1+\gamma} \Phi \max\{D^{1+\gamma}; \bar{D}_i^{1+\gamma}\} g_i \Phi e^{i \pm D} \Phi e^{-i(\bar{e})}$$

It is convenient to work with the logarithmic transform of  $V_i(D)$ : Let  $\ln(D)$  be denoted by  $d$ ;  $\ln(\bar{D}_i)$  by  $\bar{d}_i$ ; and  $\ln(H(\bar{e}; \bar{\mu}; \bar{s}_i) \Phi S_i \Phi A_i^{1+\gamma} \Phi e^{-i(\bar{e})})$  by  $s_i$ . Then:

$$\ln(V_i(e^d)) \equiv v_i(d) = s_i + (1 + \gamma) \Phi \max\{d; \bar{d}_i\} g_i \pm \Phi d \quad (14)$$

The function  $v_i(d)$  is continuous over the entire range  $(j-1; j+1)$  and differentiable everywhere except  $\bar{d}_i$ . In the  $(j-1; \bar{d}_i)$  range, where agglomeration economies are insensitive to changes in local employment, the function is strictly decreasing and strictly concave:

$$\partial v_i / \partial d = \partial^2 v_i / \partial d^2 = -\gamma \Phi e^d < 0 \quad (15)$$

<sup>11</sup> If each immobile worker in city  $i$  owns  $K_i^F$  units of capital and his utility function has the same form as that of mobile individuals, the total demand for the traded good in city  $i$  would be  $Y_i^D = [(1 - \mu) + \mu e^{i \pm A_i}] [w_i N_i + r(N_i - N_i^F) K_i^F]$ . The supply of the traded good in city  $i$  is  $Y_i = [(1 - \bar{e}) - r]^{(1 - \bar{e})} [\bar{s}_i \bar{A}_i - (N_i)]^{1 - \bar{e}} N_i$ . For a closed economy,  $S_i Y_i^D$  must equal  $S_i Y_i$ ; which means that knowledge of  $N_i^F$  and  $K_i^F$  would be needed to determine the equilibrium employment levels.

In the  $(\bar{d}_i; + 1)$  range, where agglomeration economies increase with local employment, the behavior of  $v_i$  reflects the interaction of congestion costs and agglomeration economies. The first and second derivatives with respect to  $d$  over this range are

$$\frac{\partial v_i}{\partial d} = (1 + \alpha) \beta_i \pm \Phi e^d \quad (16)$$

$$\frac{\partial^2 v_i}{\partial d^2} = \beta_i \pm \Phi e^d < 0 \quad (17)$$

Thus,  $v_i(d)$  continues to be strictly concave but with regard to the sign of the first derivative two possibilities exist: (i)  $\bar{D}_i \leq (1 + \alpha) \beta_i$  or, equivalently,  $d_i^1 \leq \ln[(1 + \alpha) \beta_i]$ ; in this case (16) implies that the  $v_i(d)$  function is strictly decreasing over the range  $(\bar{d}_i; + 1)$ . (ii)  $d_i^1 < \ln[(1 + \alpha) \beta_i]$ ; in this case (16) implies that  $v_i(d)$  initially increases, reaching a local maximum at  $d = \ln[(1 + \alpha) \beta_i]$ ; and then declines.

Figures 2(a) and 2(b) illustrate these two cases. These cases arise because agglomeration economies are related to employment while congestion costs are related to employment density. Thus, it is possible for a compact location to become quite dense before it attains the employment level beyond which agglomeration economies increase with size. In this case, which is the first case noted above, agglomeration economies can slow down the decline in utility that occurs with increasing density but they cannot reverse it (Figure 2(a)). In contrast, a location that is large in land area might attain  $N_{min}$  before it gets too dense. In this case, which is the second case noted above, increasing agglomeration economies will overcome the utility-depressing effect of increasing density for some range of employment density (Figure 2(b)).

The shape of the  $v_i(d)$  function bears directly on the possibility of multiple equilibrium employment densities for a single location. The densities corresponding to the points where

the  $v_i(d)$  function intersects the horizontal "utility at other locations" line are all equilibrium density levels. In Figure 2(a), where the  $v_i(d)$  function is monotonically declining there is only one equilibrium density level. In contrast, there are three equilibrium density levels in Figure 2(b). As usual, the middle equilibrium,  $d_i^M$ ; is unstable: a small increase or decrease in density, by raising or lowering the utility level above or below what mobile workers can get in other locations, will induce further increases or decreases in density. The other two equilibria  $d_i^L$  and  $d_i^H$ ; are stable.

### Equilibrium Employment Densities for the System of Cities

The purpose of this section is to describe how equilibrium employment densities respond to an increase in total employment. We focus attention only on stable equilibria<sup>12</sup>. This restriction is formalized in the following definition of a system of cities equilibrium.

**Definition:** The collection  $\{f_i^a, v_i^a, s_i, d_i^1; N\}$  is a system of cities equilibrium if it satisfies the following conditions:

$$v_i^a = s_i + (1 + \tau) \Phi \max\{f_i^a, \bar{d}_i g_i\} \pm \Phi d_i^a \quad \forall i = 1, 2, \dots, M \quad (18)$$

$$\sum_{i=1}^M A_i \Phi d_i^a = N \quad (19)$$

$$d_i^a \geq (i-1) d_i^1 [(\ln[(1 + \tau) \pm 1] + 1)] \quad \forall i = 1, 2, \dots, M \quad (20)$$

Conditions (18) and (19) are the equal utility and aggregate labor resource balance conditions, respectively. These correspond to equations (13) and (12) of the previous section. The stability requirement is incorporated in condition (20). For the case where  $d_i^1 < \ln[(1 + \tau) \pm 1]$ ;

<sup>12</sup>This is justifiable on the ground that unstable equilibria (i.e., equilibria in which some location is in an unstable equilibrium) are "razor's edge" cases.

this condition prohibits  $d_i^2$  from lying in the closed interval  $[d_i^1; \ln[(1 + \lambda) = \pm]]$ ; as shown in Figure 2(b), this interval corresponds to the domain of  $d$  for which the  $v_i(d)$  function is upward sloping. For the case where  $d_i^1 > \ln[(1 + \lambda) = \pm]$ ; (20) does not impose any restriction at all, since  $d_i^2$  can then lie anywhere on the real line. No restriction is needed because in this case the  $v_i(d)$  function is decreasing in  $d$  over the entire real line.<sup>13</sup>

The possibility of multiple stable equilibria complicates comparative statics. These complications are addressed in the numerical section. For clarity, the comparative statics result is presented only for the case where the  $v_i(d)$  function is downward sloping for every location.

**Proposition (Comparative Statics):** Let  $(d_i^1; v_i^1; s_i; d_i^1; N^1)$  be the initial system-of-cities equilibrium and suppose that  $(d_i^2; v_i^2; s_i; d_i^1; N^2)$  is the new system-of-cities equilibrium for  $N^2 > N^1$ . Let  $\lambda_i = (N_i^2 - N_i^1) / N_i^1$  denote the (gross) growth rate of employment in location  $i$ . If  $d_i^1 > \ln[(1 + \lambda) = \pm]$  for all  $i$ , then  $\lambda_i > 1$  for all  $i$  and  $d_i^1 > d_j^1$  implies  $\lambda_i < \lambda_j$  (see Appendix for proof).

This inverse relationship between employment density and employment growth can be intuitively explained as follows. If in each location agglomeration benefits are sensitive to local employment (i.e.,  $d_i^1 > \ln[(1 + \lambda) = \pm]$  for all  $i$ ), then a 1 percent increase in employment in each location would raise agglomeration benefits by  $(1 + \lambda)$  percent in each location. On the other hand, since a 1 percent increase in a location's employment implies a 1 percent increase in

<sup>13</sup>The presence of the stability restriction implies that employment density of a location may change discontinuously in response to a change in an environmental factor, such as aggregate employment. Consequently, such a system-of-cities equilibrium may fail to exist for some parameter values. However, because of the large number of diverse locations involved, this difficulty is not encountered in any of the computational experiments conducted in Section 6.

its density (because the area of each location is being held fixed), the cost of living would rise by (approximately)  $\pm \frac{1}{\epsilon} \frac{dD_i}{D_i}$  in each location. Thus, the cost of living will rise relatively more in dense areas. If utility of a mobile worker is a decreasing function of density in each location (i.e.,  $\frac{dU_i}{dD_i} < 0$  for all  $i$ ), then a 1 percent increase in total employment must, in equilibrium, result in a less than 1 percent increase in employment in dense locations and a more than 1 percent increase in employment in less dense ones.

## V. Parameter Selection and Calibration

The numerical specification of the model described by (18) - (20) involves choosing values for three groups of parameters: (i) the agglomeration-related parameters,  $\alpha$ ;  $\beta$ ; and  $\frac{1}{\epsilon}$ , (ii) the congestion-related parameter  $\gamma$ ; and (iii) the location-specific factors  $s_i$ .<sup>14</sup> We use existing microstudies to guide our selection of the agglomeration- and congestion-related parameters. The location-specific factors  $s_i$  are determined by calibrating the model to reproduce the actual 1951 employment densities for the 289 MSA's in our data set.

However, before we can proceed with parameter selection and calibration, there is an important preliminary step. The available estimates of agglomeration and congestion parameters give the strength of these effects in relation to changes in local population rather than local employment. Therefore, to use these estimates we need to know how employment and population are related in our locations.<sup>15</sup> We used the employment data for 1979 and

<sup>14</sup>Each  $s_i$  is a sum of both location-specific as well as economywide variables. However, since the  $s_i$ 's differ across locations only because of differences in the location-specific variables, we refer to the  $s_i$ 's as location-specific factors.

<sup>15</sup>Note that it is not advisable to use the (easily available) aggregate employment to population ratio for this purpose. For one thing, the demographics of large and small MSA's tend to be systematically different,

the population data for 1980 to gauge the relationship between employment and population for our full set of 297 MSAs.<sup>16</sup> We regressed the logarithm of location employment in 1979 against a constant and the logarithm of location population in 1980. The slope was estimated to be 1.086. Also, the median employment to population ratio was 0.32. We use these relationships below.

### Estimates of Agglomeration-Related Parameters

Recall that  $\alpha = \alpha^*$  and  $d_i^1 = \ln(N_{min} = A_i)$ . Since we have observations on the land area of each MSA ( $A_i$ ); we need estimates of  $\alpha^*$ ;  $\alpha$ ;  $N_{min}$  and  $\beta$  only.

Let's begin with  $\alpha^*$ ; the exponent to labor input in the production function for the traded good. Under perfect competition, the equality of wages and marginal product of labor implies that the share of value added absorbed by compensation to workers is  $\alpha^*$ .<sup>17</sup> Average  $\alpha^*$ , as measured by labor's share in US GDP, has been about two thirds for the post-WWII period. Since this estimate is relatively precise, we set  $\alpha^* = 0.67$ .

To bound  $\alpha$  and  $N_{min}$ , we turn to studies that attempt to estimate the degree of agglomeration economies for US cities. As discussed in Monras (1981), there are essentially two ways of obtaining such an estimate. In the first method, the zero profit condition for firms is used to deliver a relationship between a location's nominal wage and such characteristics so there are systematic differences in the employment to population ratio across MSAs of different sizes. Furthermore, as noted earlier, the County Business Pattern data do not cover all workers, so that the employment to population ratio for any location in our data set is most likely lower than the employment to population ratios reported for aggregate data.

<sup>16</sup>Since land area is not an issue here, we used the full set of MSAs rather than the restricted set.

<sup>17</sup>Since the local good is linearly produced using the traded good, the share of labor's compensation in the local good is  $\alpha^*$  as well.

as its population size, industry mix, etc. In this approach, an estimate of the coefficient on population size is an estimate of the strength of agglomeration economies. In the second method, the production function is estimated directly using data on value added, employment, capital stock, population size, industry mix, etc. Again, the coefficient on population size provides an estimate of the strength of agglomeration economies.

Turning first to estimates obtained using the zero profit condition, note that the zero profit condition (11) in conjunction with agglomeration function (2) implies:

$$\ln w_i = \text{constant} + \alpha_i^{-1} \ln \hat{A}_i + \beta \alpha_i^{-1} n_i \hat{A}_i + \gamma \alpha_i^{-1} n_{\min} (1 - \hat{A}_i)$$

where  $n_i$  is the natural log of  $N_i$ ,  $n_{\min}$  is the natural log of  $N_{\min}$  and  $\hat{A}_i$  is an indicator function that takes on the value 1 when  $n_i$  exceeds  $n_{\min}$  and 0 otherwise. Leo Sveikauskas (1975) estimated a relationship of this form for each of 14 two digit manufacturing industries. He used SMSA population rather than employment as a regressor and ignored the possibility of thresholds (i.e., assumed that  $\hat{A}$  is 1 for every observation). He obtained estimates of  $\beta \alpha_i^{-1}$  that range from 0.0116 to 0.0855 with a median value of around 0.048 (Table IV, p. 404).<sup>18</sup> Using the estimated relationship between log employment and log population for our MSAs, and a labor share of two thirds, Sveikauskas' estimates imply a median estimate of  $\beta$  of about 0.03.<sup>19</sup>

However, Sveikauskas' estimates of  $\beta \alpha_i^{-1}$  suffer from (at least) two opposite biases.

<sup>18</sup> The data pertain to all SMSAs in 1967.

<sup>19</sup> Moomav (1981) adjusted Sveikauskas' estimates of  $\beta \alpha_i^{-1}$  for the observed labor share in each industry and reported estimates of  $\beta$  that range from 0.006 to 0.0485 with a median value of 0.0266. However, as noted by Ciccone and Hall (1996), the Census of Manufactures data overstate value added per worker in larger cities and hence underestimate the worker's share of value added for those cities.

First, he used only a limited number of variables to control for location-specific factors  $\hat{A}_i$ . Because there is positive dependence between  $\hat{A}_i$  and  $n_i$  in equilibrium, the omission of relevant location-specific factors will bias the estimates of  $\phi_i^{-1}$  upward. On the other hand, Sveikauskas did not consider the possibility that agglomeration economies may be insensitive to changes in population up to a certain level (i.e., the possibility of a threshold like  $N_{min}$ ), which may have biased his estimate of  $\phi_i^{-1}$  downward.<sup>20</sup> Still, the extent of the downward bias is much less certain than the upward bias that almost certainly exists and is likely to be quite significant. Therefore, it is reasonable to conclude that 0.03 is an upper bound estimate of  $\phi_i^{-1}$ . We proceed on the assumption that Sveikauskas' estimates suggest a value of  $\phi_i^{-1}$  between, say, 0.015 and 0.03. This implies a value of  $\phi_i$  between 0.0225 and 0.045.

Turning to production function estimates, observe that the location-specific production function (1) in conjunction with the agglomeration function (2) implies the following relationship:

$$y_i = \text{constant} + \ln \hat{A}_i + \phi_i n_i + \phi_i n_{min} \Phi(1 - \hat{A}_i) + (1 - \phi_i) k_i \quad (21)$$

where  $y_i$  is the log of location  $i$ 's average labor productivity and  $k_i$  is the log of its capital per worker ratio. As before,  $\hat{A}_i$  is an indicator variable that takes on the value 1 if  $n_i$  exceeds  $n_{min}$  and zero otherwise. David Segal (1976) estimated a relationship of this form on data from 58 SMSAs for 1967. He used SMSA population rather than employment as the size variable and used a term like  $\phi_i \hat{A}_i$ ; instead of  $\phi_i n_i + \phi_i n_{min} \Phi(1 - \hat{A}_i)$ ; to take into

<sup>20</sup>The reason for the downward bias is as follows. If a population "threshold" exists,  $y_i$  will be less sensitive to differences in population size. A estimation strategy that ignored "thresholds" (and used the full variation in population size) would assign a smaller coefficient on population to "..." the relative insensitivity of  $y_i$  to population size.

account differences in productivity stemming from differences in population size. He found measurable productivity differences for SMSAs with populations above and below 2 million and estimated that difference to be about 8 percent in favor of SMSAs with a population over 2 million.

Because Segal used broad population categories to measure SMSA size, his estimate of an 8 percent productivity differential in favor of large SMSAs cannot be used to determine  $\rho$ . On the other hand, his finding that productivity differences are discernible between the group of SMSAs with populations greater than 2 million and the group of remaining SMSAs may be interpreted as evidence that  $N_{min}$  cannot be any greater than 750,000 workers.<sup>21</sup> We proceed on the assumption that a plausible value of  $N_{min}$  cannot exceed 750,000.

We did not find any usable estimates of  $\rho$ . For want of something better, we set its value equal to 0.01:

### Estimates of Congestion-Related Parameters

Recall that  $\pm = \mu \Phi$ ; where  $\mu$  is the share of local goods in the household budget and  $\Phi$  is the percentage change in the price of the local good due to a unit change in employment density.

In our model, the relationship between the price of the local good and employment density is given by equation (10). This equation implies the following relationship:

$$\ln p_i = \ln \pi_i + \rho \Phi_i$$

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<sup>21</sup>We used the estimated relationship between log employment and log population for our SMSAs to determine the employment level corresponding to a population of 2 million.

Rdback (1982) estimated a relationship of this form using data from 73 SMSAs for 1973. She used the logarithm of the average residential site price as the dependent variable and various SMSA-specific factors and SMSA population density as regressors. The coefficient on the density variable in her regression is  $2.0 \times 10^{-4}$  (Table 3, p. 1272). Since the median MSA employment to population ratio for our locations is 0.32, Rdback's estimate of the density coefficient implies a  $\rho$  value of (approximately)  $6.0 \times 10^{-4}$ :

To get a perspective on this estimate, consider an MSA with employment density of 4900 workers per square mile. This figure is about the upper limit of the employment densities observed in our data set. For such an MSA, a 1 percent increase in employment density would mean an additional 49 workers per square mile and an increase in the site price of residential land of approximately 0.0294 ( $= 49 \times 6.0 \times 10^{-4}$ ) percent.

Rdback's population density coefficient measures the proportionate increase in residential site price given a unit increase in population density holding all the other factors in her regression constant. In reality, an increase in MSA population density is likely to be accompanied by increases in other nonhousing-related costs. Since we use  $\rho$  as the parameter to capture all of the costs associated with increases in congestion, we view her estimate of the coefficient on the population density variable as providing a lower bound for our model  $\rho$ :

To estimate  $\mu$ , we used the 1972-73 consumption expenditure shares of urban wage earners and clerical workers reported in Jacobs and Shipp (1990).<sup>22</sup> We summed the expenditure shares on food, shelter, utilities (including fuels and public services), public transportation, entertainment, and sundries. These categories amounted to 56.8 percent of total household

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<sup>22</sup> Expenditure shares tend to change over time. Since our estimate of  $\rho$  is derived from a study based on 1973 data, we used the expenditure shares for the closest available year.

expenditures (Table 2, p. 22). Since some of these components are not entirely local, we estimate  $\mu$  to be 0.50. Since we believe this estimate to be relatively precise, we set  $\mu$  to 0.50.

Taken together, these estimates imply a value of  $\pm$  no smaller than  $3.0 \times 10^4$ . We proceed on the assumption that a plausible value of  $\pm$  is between  $3.0 \times 10^4$  to  $6.0 \times 10^4$ .

### Calibration of MSA-Specific Factors

The next step is to determine the MSA-specific factors. This is done by choosing agglomeration and congestion parameters within the ranges noted above and calibrating the model to match employment densities observed in 1951 for each of our 289 locations.

This step would be straightforward but for the fact that the observed employment density for each location must satisfy the stability restriction noted in the definition of system-of-cities equilibrium (equation (20)). For each location, this restriction depends on its land area,  $A_i$ ; and on the values of the agglomeration- and congestion-related parameters. In particular, it does not depend on the location-specific factor. Therefore, once the agglomeration- and congestion-related parameters are chosen, these choices (along with  $A_i$ ) determine the density zone that conflicts with stability. The difficulty is that there is no assurance that actual employment densities in 1951 will lie outside this "instability zone."

Figure 3 illustrates the problem for a value of  $N_{\min} = 100,000$ ,  $\beta = 0.045$  and  $\pm = 3.0 \times 10^4$ . It gives a scatter plot of all 289  $(d_i^1; d_i)$  pairs for 1951. The vertical line is erected at  $\ln[(1 + \beta)\pm]$  and the diagonal line is the  $45^\circ$  line. For any scatter point lying below the  $45^\circ$ -line and to the left of the vertical line,  $d_i^1 < d_i < \ln[(1 + \beta)\pm]$ . Thus, any point in the cross-hatched triangular area violates the stability restriction. For these parameter choices, quite a few locations in 1951 had employment densities that fell in the "instability zone."

To proceed with the calibration step, we need to empty out the “instability zone.” This can be done by either lowering  $\beta$ ; or raising  $\alpha$  (both of which cause the triangular cross-hatched area to shrink by a leftward move of its vertical side) or by raising  $N_{\min}$  (which causes the scatterplot to migrate upward). Since we wanted to retain the ability to vary  $\beta$  and  $\alpha$  within the ranges noted earlier, we met the stability requirement by setting  $N_{\min}$  high enough so that with  $\beta$  at its top value and  $\alpha$  at its bottom value there are no observations in the “instability zone” in 1951. We found that an  $N_{\min}$  value of 400,000 was sufficient to accomplish this.

For our baseline model, we set  $N_{\min} = 400,000$  workers. We set  $\beta = 0.034$  and  $\alpha = 4.5 \times 10^{-4}$ , which are the midpoints of the respective ranges of  $\beta$  and  $\alpha$ .

The calibration step was then performed as follows. We first normalized the location-specific factor for the densest location with more than  $N_{\min}$  workers in 1951 to 1. Then, using this location’s area and its actual employment density in 1951 (and the selected values for  $N_{\min}$ ,  $\beta$ , and  $\alpha$ ), we determined from equation (18) the implied value of log utility in 1951 (denoted  $v_{51}^a$ ). For the remaining locations, we used their areas, their actual employment densities in 1951, and the computed value of  $v_{51}^a$  to solve for the unique location-specific factors from corresponding equations in (18).

## VI. Findings

In this section we determine the equilibrium MSA employment densities implied by our model when the level of total employment in the model is set equal to the level of aggregate metropolitan employment in 1994. The equilibrium was found by constructing a mapping

from  $v$  ( $\cdot v_{51}^a$ ) to equilibrium log employment density for each MSA. For MSAs in which the 1951 employment density was in excess of  $d_1^1$ ; these mappings traced out the  $v_i(d)$  lying below  $v_{51}^a$ . For other MSAs, the mapping may be discontinuous and may trace out only portions of the  $v_i(d)$  function lying below  $v_{51}^a$ .<sup>23</sup> In any case, these individual mappings were aggregated to generate a mapping from  $v$  to aggregate metropolitan employment. Like individual mappings, the aggregate mapping is downward sloping with (possibly) points of discontinuity. The equilibrium was found by lowering  $v$  until employment implied by the aggregate mapping rose to within 1 percent of actual 1994 aggregate metropolitan employment.

Figure 4 plots the actual (and predicted) Lorenz curve for 1951 and the actual and predicted Lorenz curves for 1994. Table 4 reports the values of these Lorenz curves at selected points.<sup>24</sup> It is clear that the model predicts more employment deconcentration than actually observed: the predicted Gini index of inequality for 1994 is 0.32 as compared with the actual Gini index of 0.53. Why is the predicted Gini coefficient for 1994 lower than actual? Table 5 shows the actual and predicted employment shares of MSAs (not MSA land areas) grouped by deciles. The model's prediction for the first decile is the same as actual, and its prediction for the third through the tenth deciles are close to actual. However, the model underpredicts the employment share of MSAs in the second decile by 7 percentage points. This large discrepancy between the actual and predicted employment shares of

<sup>23</sup>For instance, if the 1951 employment density is at a level like  $d_1^1$  in Figure 2(b), the mapping will trace out the  $v_i(d)$  function until  $v$  reaches the level of the kink (in the  $v_i(d)$  function) and then jump to the point that's on the same level as the kink but to the right of  $d_1^1$ ; lower values of  $v$  will trace out the  $v_i(d)$  function lying below this point.

<sup>24</sup>The values for 1951 and 1994 in Table 4 may be slightly different from those reported in Table 1 because Table 4 is based on a sample of 289 MSAs whereas Table 1 is based on a sample of 297 MSAs.

relatively dense MSA's accounts for the much lower predicted  $G$ ini coefficient. Still, except for this important discrepancy, the model does reasonably well at accounting for the postwar deconcentration of metropolitan employment.

Table 6 presents the model's prediction for MSA's ranked by their initial, i.e., 1951, employment density (the results presented in Table 5 allow specific MSA's to change deciles over time). The 29 most dense MSA's in 1951 (the top decile) accounted for 54 percent of total metropolitan employment. By 1994 their share had fallen to 35 percent, but the model predicts that their share would fall to 29 percent. Looking at the opposite end of the density scale, we find that the 57 least dense MSA's in 1951 (the bottom two deciles) accounted for 2 percent of total metropolitan employment. By 1994, their share had risen to 5 percent, but the model predicted that their share would jump to 15 percent.

These discrepancies for the initially most and least dense MSA's suggest that other forces, such as technological change, may have increased relatively more in the most dense MSA's. Gaspar and Glaeser (1998) suggest that improvements in telecommunications technology may favor employment growth in cities and other dense locations. They argue that while improvements in telecommunication substitute for face-to-face contacts, they also increase the need for all types of interactions and so favor job growth in dense areas. Recall that following the vast majority of micro studies, we treat agglomeration economies as a function of MSA employment size. Recently, Ciccone and Hall (1996) have suggested that higher densities (as opposed to greater numbers) of people and jobs promote innovation and technological change and therefore growth. In reality, these positive effects may tend to somewhat dampen the increases in congestion costs associated with increases in density and provide a

possible explanation for these discrepancies.

Finally, the two panels in Figure 5 show model predictions at the level of individual MSA's. In these panels MSA's are always ranked (in descending order) by their 1951 employment density. Figure 5(a) plots the actual employment densities for our locations in 1951 and alongside the predicted employment densities for 1994. All locations are predicted to have higher employment density in 1994 than in 1951. As one would expect, there is a clear tendency for predicted density to rise proportionately more for less dense MSA's (the spikes in predicted employment density will be discussed shortly). Figure 5(b) plots the actual employment densities for 1994 along with those predicted by our model. The predicted density plot is not as jagged as the actual one; that is, many more MSA's changed their rankings (relative to their rankings in 1951) in the data than in the model. In terms of our model, in which location-specific factors are calibrated to match 1951 employment densities, this feature underscores the importance of changes in location-specific factors in accounting for individual MSA employment densities in 1994.

Nevertheless, the fact that changes in rankings occur in the model at all (as evidenced by the spikes in predicted employment densities) is noteworthy. These spikes occur because of the feedback effects of agglomeration economies when employment in a location increases beyond  $N_{min}$  and that location's land area is large enough for  $d^1$  to be less than  $\ln[(1 + \epsilon)^{\pm}]$ : Figure 6 illustrates this feedback effect for the Fresno metropolitan area, a location that experienced an employment spike in our model. The top horizontal line is the computed  $v^{\alpha}$  in 1951 with  $d_{Fresno}^{51}$  as Fresno's observed employment density in that year. The bottom horizontal line is the predicted  $v^{\alpha}$  in 1994 and  $d_{Fresno}^{94}$  is the predicted employment density for

Fresno. Because the  $V_{\text{Fresno}}(d)$  function has an increasing segment, the equilibrium density in 1994 is considerably higher than it would be if that segment were absent.

## VII. Sensitivity Analysis

In this section, we examine the sensitivity of the predictions of the baseline model to changes in  $\pm$ ,  $N_{\text{min}}$ , and location-specific factors. In presenting these results, we always ranked MSAs by their 1951 employment densities.

Figure 7 plots the predictions of the model when all agglomeration parameters are kept at their baseline values but the value of  $\pm$  is varied. In this plot, the middle bar for each group is the baseline prediction while the first bar is the model's prediction when  $\pm = 3.0 \times 10^{-4}$  and the third bar is its prediction when  $\pm = 6.0 \times 10^{-4}$ . To interpret this plot it is important to recognize that  $\pm$  is not the only parameter that changes across the three simulations. Since  $\pm$  is used in the calibration step, a change in  $\pm$  also changes the settings of the location-specific factors. Thus, the plot reflects differences in both  $\pm$  and induced differences in location-specific factors.

To sort out the effects of these simultaneous changes, it is useful to consider the expression for the first round (or impact) effect on the utility levels of mobile workers in 1994 of a change in  $\pm$ : Using equation (18), this change can be written as:

$$v_i^0(1994) - v_i^{\pm}(1994) = (S_i^0 - S_i) \left( \pm^0 - \pm \right) \Phi_i^{d_i^{\pm}(1994)}$$

where  $v_i^0(1994)$  is the utility level that would prevail in location  $i$  if the employment density of location  $i$  is held fixed at the value predicted for it in 1994 by the baseline model (we denote this predicted equilibrium density by  $d_i^{\pm}(1994)$ ) and  $S_i^0$  is the  $i$ th location-specific factor when

the congestion parameter is set to  $\pm^0$ .

This expression for change in utility has two parts: The first part is the effect of the induced change in the location-specific factor and the second is the direct effect of a change in  $\pm$ . Figure 8 (a) plots the direct impact effect of a decrease in  $\pm$  from the baseline value of  $4.5 \times 10^{-4}$  to  $3.0 \times 10^{-4}$ : As we would expect, there is a substantial increase in the utility level of mobile workers in the very dense locations relative to other, less dense ones. If this were the only effect in operation, our model would generate an increase in the employment share of the most dense location along with (quite possibly substantial) decreases in employment shares of less dense locations.

Turning to the induced change in location-specific factors, our calibration step implies:

$$s_i^0 \quad s_i = s_i (\pm^0 \quad \pm) \left( e^{d_i(1951)} \quad e^{d_i(1951)} \right)$$

Figure 8 (b) plots the induced changes in location-specific factors. The effect of induced changes in location-specific factors is roughly opposite to the direct effects plotted in Figure 8 (a). This is intuitive: A decrease in  $\pm$  lowers congestion costs and dense locations gain more from that reduction than less dense cities. By itself, this would imply a greater concentration of workers in more dense locations. However, since the calibration step forces the model to reproduce the (unchanged) employment density for 1951, this increased attractive force of lower congestion cost must be countered by making dense locations less attractive relative to less dense ones. Hence, the calibration step increases the location-specific factors of less dense locations relative to the more dense ones. Figure 8 (c) shows that the overall impact effect of a reduction in  $\pm$  on the utility level of mobile workers is small. Consequently, the changes in employment share required to reach a new equilibrium is small.

Figure 9 (a) compares the predictions of the baseline model to models in which the  $\beta^1$  is 0.023 and 0.045. Evidently variations in  $\beta^1$  do not affect employment shares very much. The same result is apparent in Figure 9 (b), which compares the predictions of the baseline model to one in which  $N_{min}$  is raised to 750,000. We also investigated whether the predictions for 1994 are sensitive to changes in the way the location-specific factors are calibrated. Figure 9 (c) compares the predictions of the baseline model to those of a model in which the location-specific factors are calibrated to match observed employment densities in 1959.<sup>25</sup> The results of these sensitivity analyses are summarized in Table 7. As is evident, these changes have very little effect on the model predictions for 1994.

Thus the predictions of the baseline model concerning employment deconcentration appear to be robust. Given how hard it is to estimate these key model parameters, the robustness is very gratifying. We should emphasize that it is the calibration step that delivers this robustness: the requirement that the model match the 1951 employment densities for any selection of parameter values constrains how much the predicted employment shares for 1994 can vary in response to these selections.

### VIII. Implications and Conclusion

This article looked at how US metropolitan areas of different densities absorbed the increase in aggregate metropolitan employment over the postwar period. Our findings show that disparities in employment density across US metro areas has lessened substantially over the postwar period. Employment has grown faster in the initially less dense metro areas compared to the initially more dense metro areas.

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<sup>25</sup> In this case the value of  $N_{min}$  was also raised to 500,000 to satisfy the stability requirement.

We argued that the postwar increase in aggregate metropolitan employment was an important force underlying this deconcentration of metropolitan employment. Increase in aggregate metropolitan employment leads to deconcentration because congestion costs rise faster for the initially most dense metro areas than for the initially less dense ones. Using a calibrated version of the static system-of-cities model, we showed that the 2.4 fold increase in aggregate metropolitan employment between 1951 and 1994 was indeed a powerful force favoring employment deconcentration. Our baseline model predicts that the increase in aggregate metropolitan employment, by itself, implies more employment deconcentration than actually observed. The predicted Gini coefficient of inequality in employment densities for 1994 is 0.32 compared to the actual Gini for 1994 of 0.53. We also demonstrated that the predictions of our baseline model are very robust to changes in underlying parameter values.

What are the wider implications of our modeling? In the past, economists attempted to relate the spatial dispersal of employment to changes in technology, preferences, or transportation policies. They focused on these factors because they overlooked the effects of growth in aggregate metropolitan employment on employment deconcentration. We show that absent any change in technology, preferences, or policies, employment in the US would be more deconcentrated than it is now. Thus, our modeling leads us to believe that changes in technology, preferences, and transportation policies may have acted, on net, to increase employment concentration. For instance, it is possible that the improvements in transportation infrastructure (such as those resulting from the federal highway program) reduced congestion costs for large metro areas and so was a factor leading to greater concentration as well. Thus, by determining what would have happened to the distribution of metropolitan employment

if all fundamentals except aggregate metropolitan employment had remained unchanged, our study potentially illuminates the true effects of postwar changes in technology and transportation policies on the distribution of metropolitan employment.

We conclude this study by referring to the macroeconomic theme with which we began, namely, convergence. As noted earlier, macroeconomists have focused on convergence in per capita income or product across regions. We wish to bring to their attention the phenomenon of convergence in employment densities. We believe these two phenomena are closely related, but existing models (including our own) do not make a connection between them. Following Barro and Sala-i-Martin (1992), macroeconomists have analyzed convergence in regional per capita income as a collection of closed regional economies transiting to the same, or possibly different, steady states; convergence in employment densities is not a necessary consequence of these models. In contrast, we assumed perfect inter-urban factor mobility and costless transportation of the traded good; adjusted for amenities and the cost of living there is never any difference in our model in real per capita earnings of workers across MSAs. Thus, our model does not display convergence in the sense meant by Barro and Sala-i-Martin. Of course, the assumption of perfect factor mobility is a convenient abstraction, as is the assumption of closed regional economies. Presumably, a more complete understanding of regional convergence (in per capita earnings and employment densities) would require consideration of the imperfect mobility of factors and goods.

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## A P P E N D I X

### P r o o f o f P r o p o s i t i o n (C o m p a r a t i v e S t a t i c s)

To begin with, note that  $d_i^1 > \ln[(1 + \alpha)^\pm]$  implies that the  $v_i(d)$  function is strictly decreasing in  $d$  for all  $i$  so that there is a unique equilibrium density for each location in both the initial and the new equilibrium.

(i) Now observe that with no change in any  $A_i$ , an increase in total employment implies that equilibrium employment density must increase in at least one location. For specificity, suppose it rises for location  $i^0$ . It follows that  $v_{i^0}^2 < v_{i^0}^1$ . Then, equation (18) (in the definition of system-of-cities equilibrium) implies that  $v_i^2 < v_i^1$  for all  $i$  and hence  $d_i^2 > d_i^1$  for all  $i$  as well. Since  $d_i^2 = \ln \frac{1}{2} + d_i^1$ ;  $\frac{1}{2} > 1$  for all  $i$ :

(ii) Next, observe that for any pair of locations  $i$  and  $j$ ; equation (18) implies:

$$v_i^2 - v_i^1 = v_j^2 - v_j^1 \quad (22)$$

Since  $d_i > d_i^1$  for all  $i$ ; substituting (18) into (22) yields:

$$i \pm (e^{d_i^2} - e^{d_i^1}) + (1 + \alpha)(d_i^2 - d_i^1) = j \pm (e^{d_j^2} - e^{d_j^1}) + (1 + \alpha)(d_j^2 - d_j^1)$$

Using the fact that  $d_i^2 = \ln \frac{1}{2} + d_i^1$  then gives:

$$i \pm e^{d_i^1} (\frac{1}{2} - 1) + (1 + \alpha) \ln \frac{1}{2} = j \pm e^{d_j^1} (\frac{1}{2} - 1) + (1 + \alpha) \ln \frac{1}{2} \quad (23)$$

Now suppose that  $d_i^1 > d_j^1$ . Then, for any common value of  $\frac{1}{2}$  the l.h.s. of (23) is less than its r.h.s. Furthermore, since  $d_i^1 > \ln[(1 + \alpha)^\pm]$  for all  $i$ ; the l.h.s. and the r.h.s. of (23) are strictly decreasing for  $\frac{1}{2} < 1$  and  $\frac{1}{2} < 1$ ; respectively. It follows then that (23) implies  $\frac{1}{2}_i < \frac{1}{2}_j$ : ■

Table 1

Selected Values of the Lorenz Curves for MSA Employment, 1951 - 1994

Cum. Share of MSA Land Area	Cum. Share of Metropolitan Employment					
	1951	1959	1968	1979	1989	1994
Top 1 % Most Dense	0.21	0.19	0.17	0.14	0.14	0.11
Top 5 % Most Dense	0.44	0.41	0.39	0.34	0.34	0.29
Top 10 % Most Dense	0.57	0.55	0.52	0.47	0.45	0.41
Top 20 % Most Dense	0.71	0.68	0.66	0.62	0.61	0.58
Top 30 % Most Dense	0.80	0.78	0.77	0.73	0.72	0.70
Top 40 % Most Dense	0.88	0.86	0.85	0.82	0.81	0.80
Top 50 % Most Dense	0.93	0.91	0.90	0.88	0.88	0.86
Top 70 % Most Dense	0.98	0.97	0.97	0.96	0.96	0.95
Top 90 % Most Dense	0.99	0.99	0.99	0.99	0.99	0.99

Table 2

Shares of Metropolitan Employment by MSA Deciles, 1951-94\*

Deciles	1951	1959	1968	1979	1989	1994
1st	0.54	0.51	0.47	0.41	0.37	0.33
2nd	0.10	0.10	0.10	0.10	0.10	0.10
3rd	0.11	0.11	0.12	0.12	0.12	0.13
4th	0.08	0.08	0.10	0.11	0.12	0.13
5th	0.05	0.05	0.05	0.06	0.06	0.06
6th	0.05	0.05	0.05	0.06	0.06	0.07
7th	0.03	0.03	0.03	0.04	0.04	0.04
8th	0.02	0.02	0.03	0.04	0.05	0.05
9th	0.02	0.02	0.03	0.04	0.04	0.05
10th	0.01	0.02	0.02	0.03	0.03	0.04

\* MSA's ranked by 1951 employment density. The sample contains 297 MSA's.

Table 3

Shares of Metropolitan Employment by MSA Deciles, 1951-94\*

Excluding MSAs with Very Large Areas						
Deciles	1951	1959	1968	1979	1989	1994
1st	0.54	0.52	0.48	0.42	0.39	0.35
2nd	0.10	0.10	0.10	0.10	0.10	0.10
3rd	0.11	0.11	0.12	0.12	0.12	0.12
4th	0.07	0.08	0.09	0.11	0.12	0.12
5th	0.05	0.06	0.06	0.06	0.06	0.07
6th	0.05	0.05	0.05	0.06	0.07	0.07
7th	0.03	0.03	0.04	0.04	0.05	0.05
8th	0.02	0.03	0.03	0.04	0.05	0.06
9th	0.01	0.02	0.02	0.03	0.03	0.03
10th	0.01	0.01	0.01	0.02	0.02	0.02

\* MSAs ranked by 1951 employment density. The sample contains 289 MSAs.

Table 4

Selected Values of Lorenz Curves for MSA Employment

1951, 1994, and 1994 Predicted

Cum Share of MSA L and Area	Cum. Share of Metropolitan Employment		
	1951	1994 Actual	1994 Predicted
Top 1 % Most Dense	0.20	0.10	0.09
Top 5 % Most Dense	0.41	0.28	0.20
Top 10 % Most Dense	0.55	0.40	0.30
Top 20 % Most Dense	0.70	0.56	0.43
Top 30 % Most Dense	0.78	0.63	0.54
Top 40 % Most Dense	0.86	0.78	0.63
Top 50 % Most Dense	0.91	0.85	0.71
Top 70 % Most Dense	0.97	0.94	0.84
Top 90 % Most Dense	0.99	0.99	0.95

Table 5

Actual and Predicted Metro Employment Shares by MSA Deciles, 1994\*

Deciles	Actual for 1951	Actual for 1994	Predicted for 1994
1st	0.54	0.39	0.39
2nd	0.10	0.20	0.13
3rd	0.11	0.11	0.12
4th	0.07	0.08	0.08
5th	0.05	0.05	0.06
6th	0.05	0.06	0.06
7th	0.03	0.04	0.05
8th	0.02	0.03	0.05
9th	0.01	0.03	0.03
10th	0.01	0.02	0.02

\* For Each Column MSAs Ranked by Decreasing Employment Density

Table 6

Actual and Predicted Metro Employment Shares by MSA Deciles, 1994\*

Deciles	Actual for 1951	Actual for 1994	Predicted for 1994
1st	0.54	0.35	0.29
2nd	0.10	0.10	0.07
3rd	0.11	0.12	0.12
4th	0.07	0.12	0.09
5th	0.05	0.07	0.07
6th	0.05	0.07	0.07
7th	0.03	0.05	0.06
8th	0.02	0.06	0.06
9th	0.01	0.03	0.07
10th	0.01	0.02	0.08

\* MSAs ranked by decreasing 1951 Employment Density

Table 7

## Shares of Metropolitan Employment by MSA Deciles, 1951-94\*

Model/Deciles	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Baseline	0.29	0.07	0.12	0.09	0.07	0.07	0.06	0.06	0.07	0.08
High±	0.29	0.08	0.12	0.09	0.07	0.08	0.06	0.06	0.07	0.09
Low±	0.29	0.07	0.13	0.10	0.07	0.06	0.06	0.06	0.07	0.07
High <sup>1</sup>	0.29	0.07	0.13	0.10	0.07	0.07	0.07	0.06	0.07	0.07
Low <sup>1</sup>	0.29	0.08	0.12	0.09	0.07	0.08	0.06	0.06	0.07	0.09
N <sub>min</sub> = 75,000	0.29	0.07	0.11	0.08	0.07	0.08	0.06	0.06	0.07	0.09
Calibrated to 1959	0.30	0.07	0.12	0.09	0.07	0.08	0.06	0.06	0.07	0.08

\*MSAs Ranked by Decreasing 1951 Employment Density

**Figure 1**  
**Lorenz Curves for MSA Employment: 1951-1994**

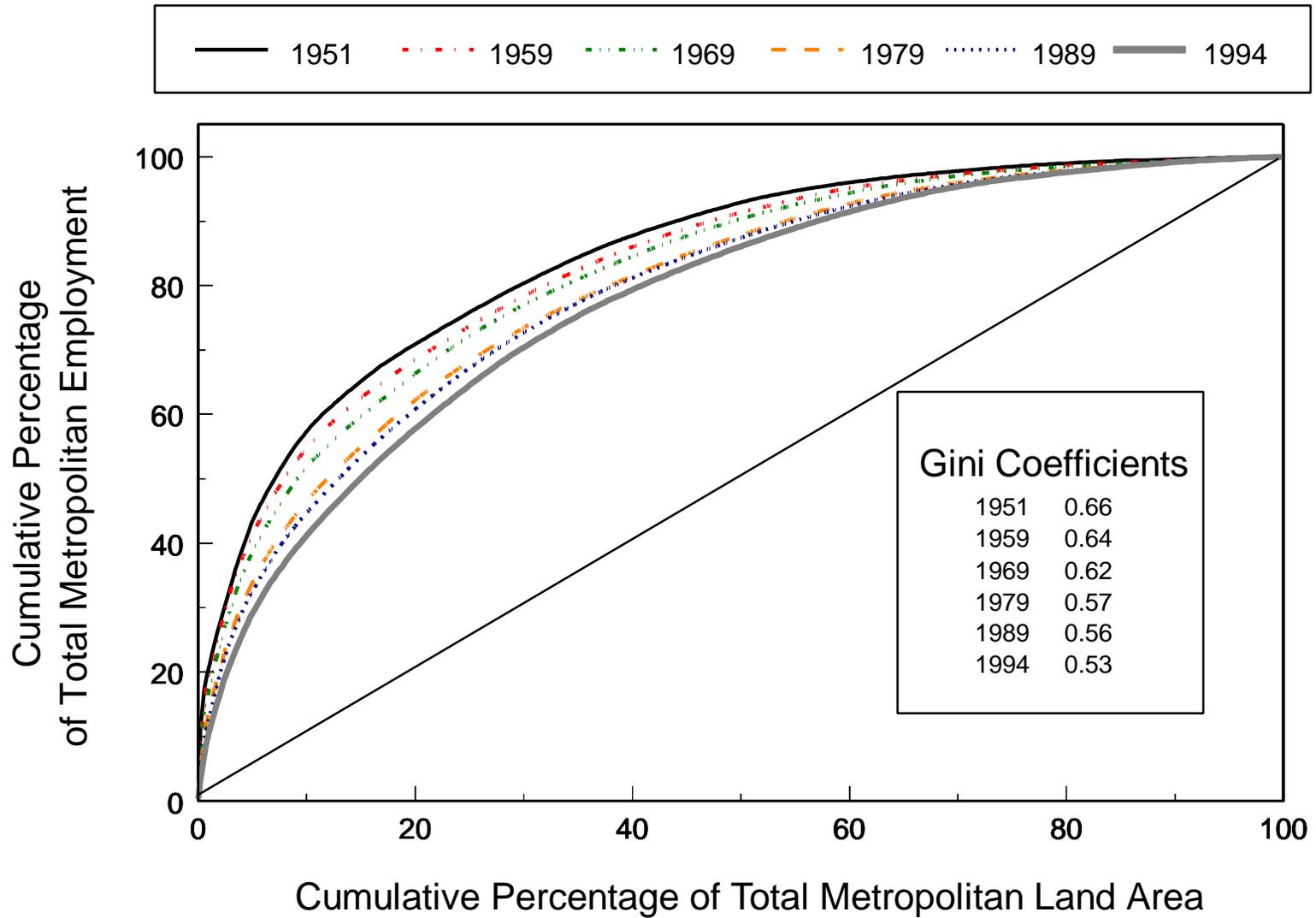


Figure 2(a)  
**v(d) Function For a Compact Metro Area**

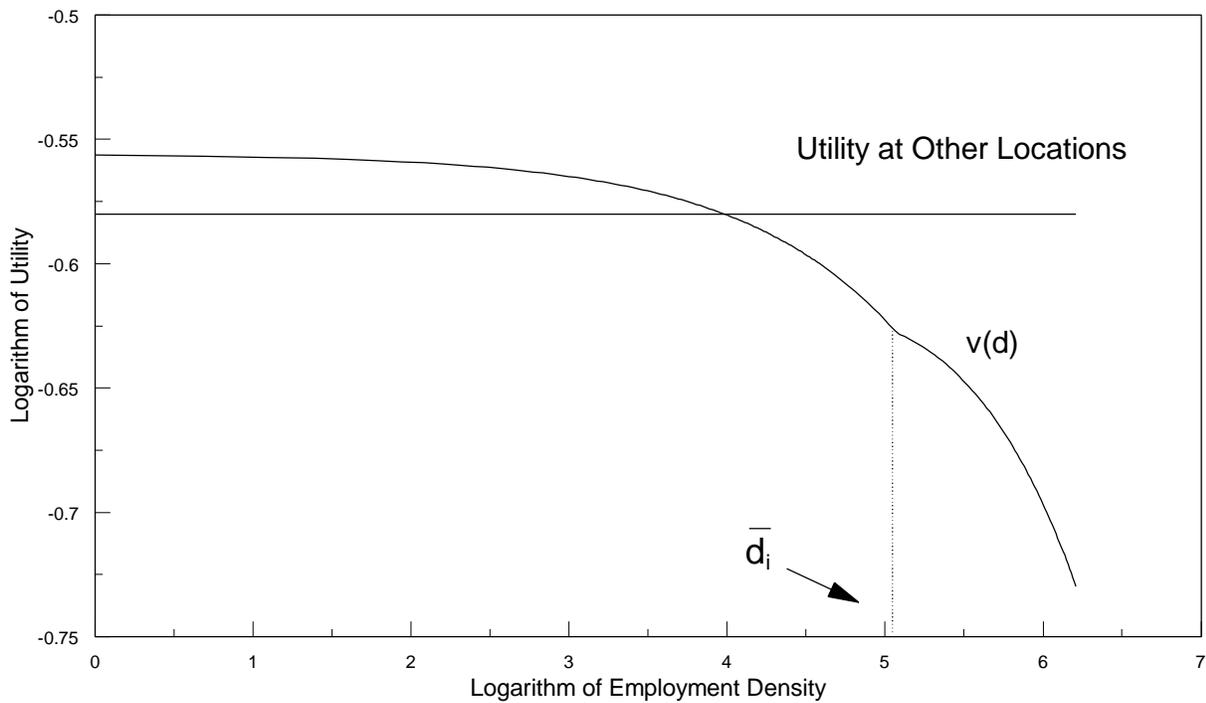


Figure 2(b)  
**v(d) Function For a Spread-Out Metro Area**

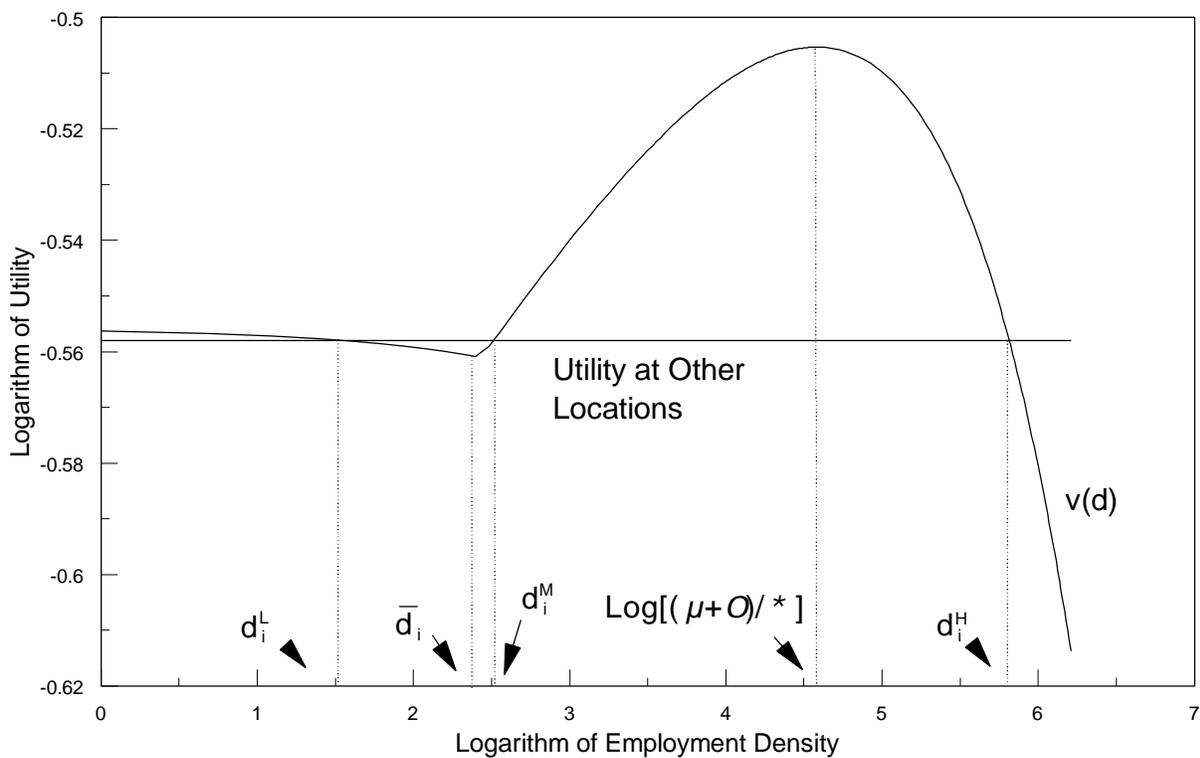
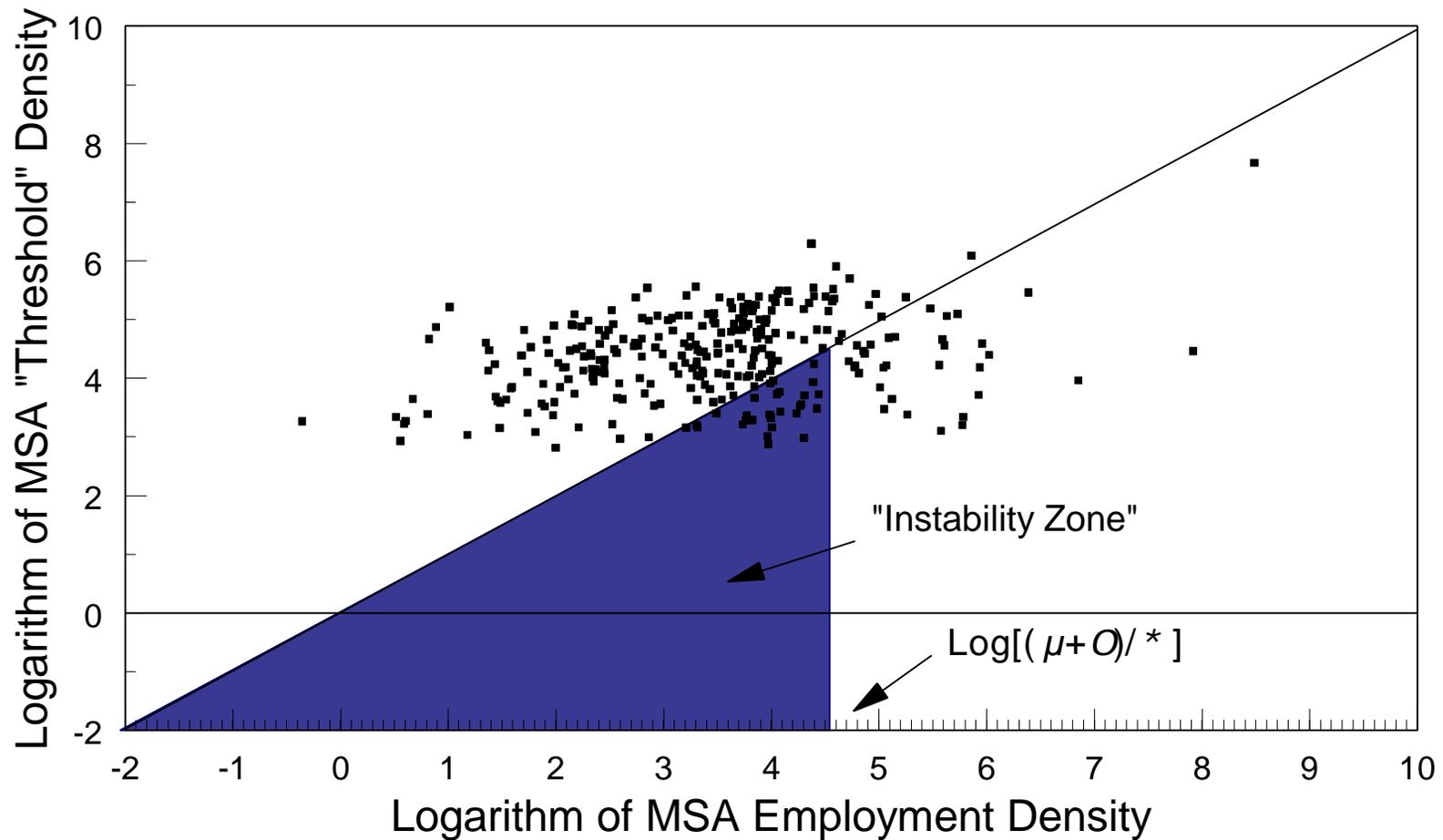


Figure 3  
**Restrictions Implied by Stability When  
 $N_{\min} = 100,000$  Workers**



# Figure 4

## Lorenz Curves for MSA Employment

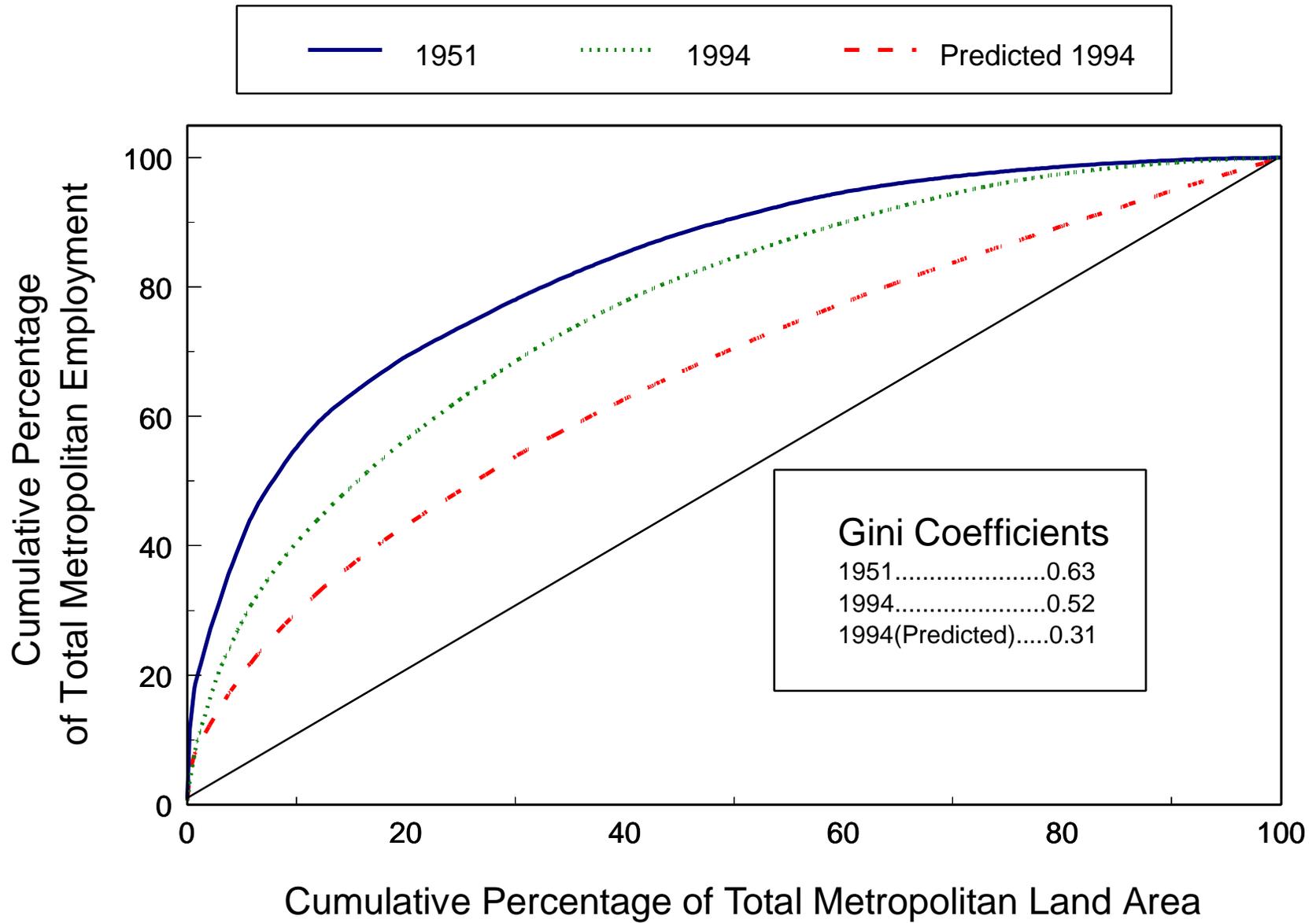


Figure 5(a)  
**Actual Density in 1951 and Predicted Density in 1994**

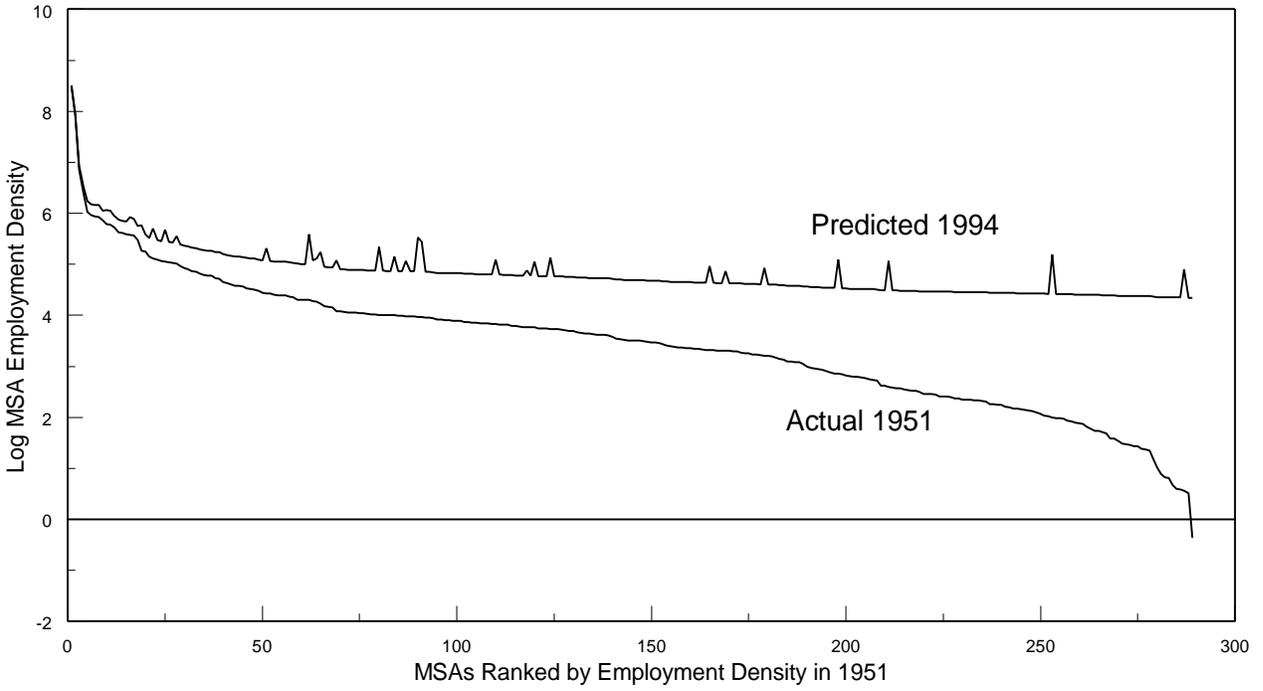


Figure 5(b)  
**Actual and Predicted Density in 1994**

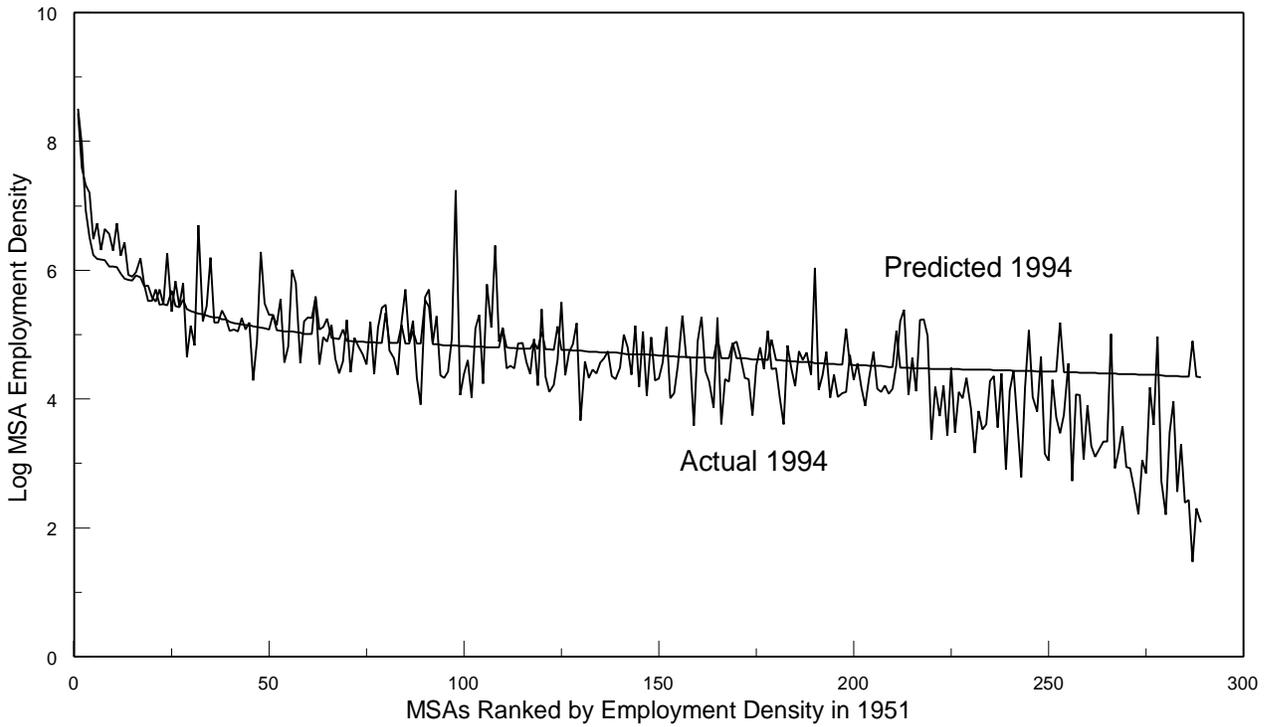


Figure 6

# $v(d)$ Function For Fresno Metro Area

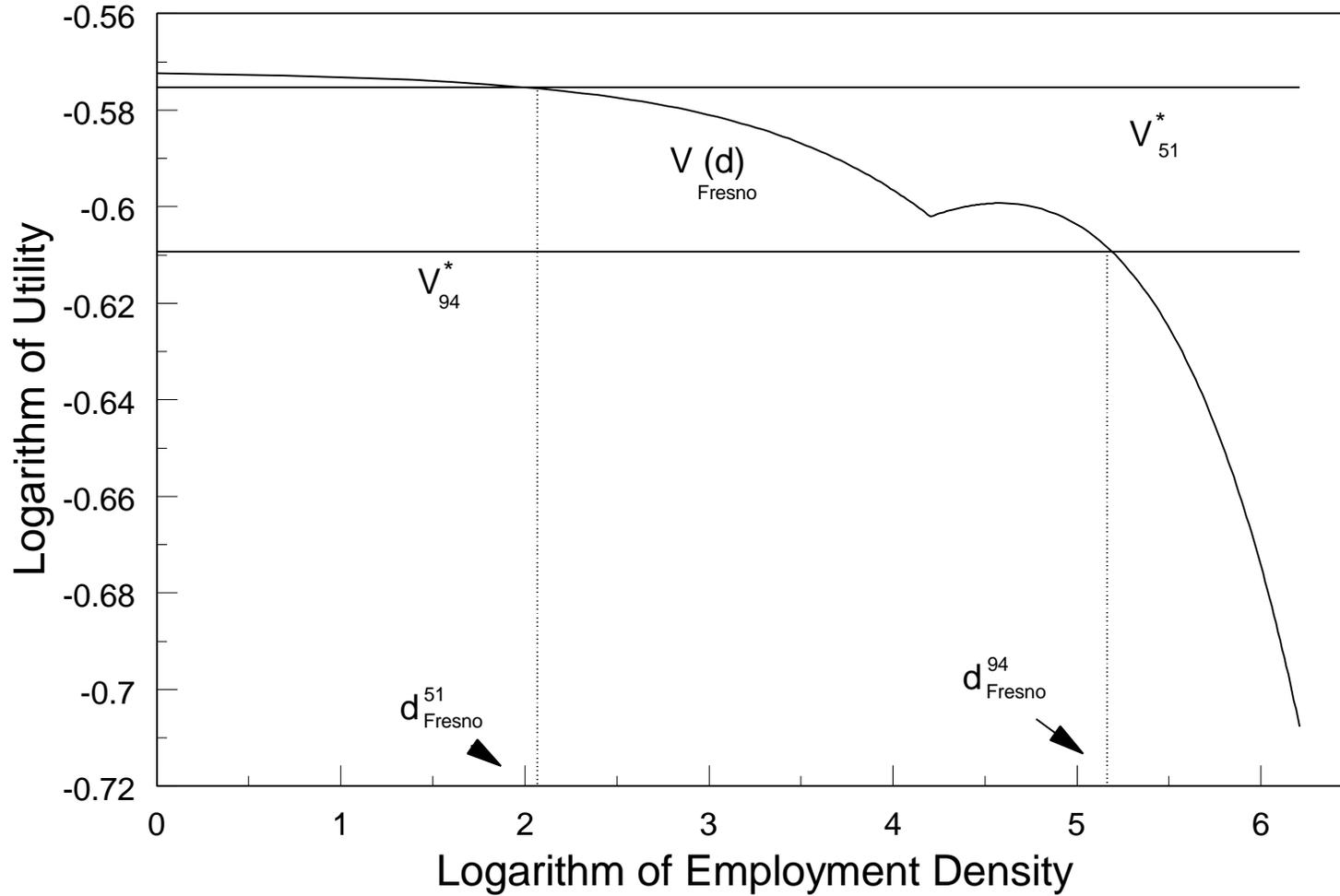


Figure 7

## Predictions of the Baseline Model Compared to Models with Low and High Values of Congestion Parameter

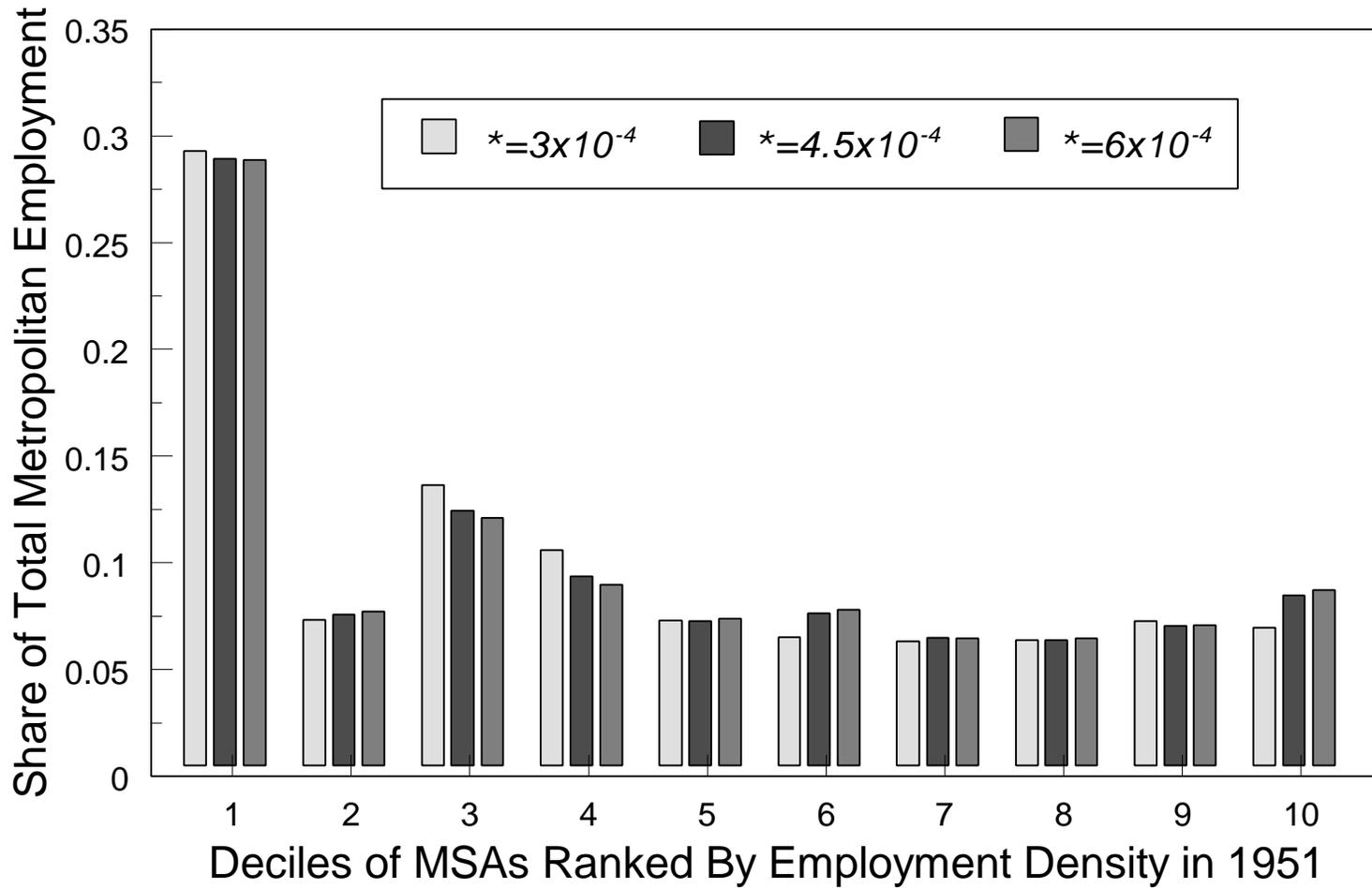


Figure 8(a)  
**Impact Effect on Log 1994 Utility of Lowering Delta to  $3 \times 10^{-4}$  Holding  $S_i$  Fixed**

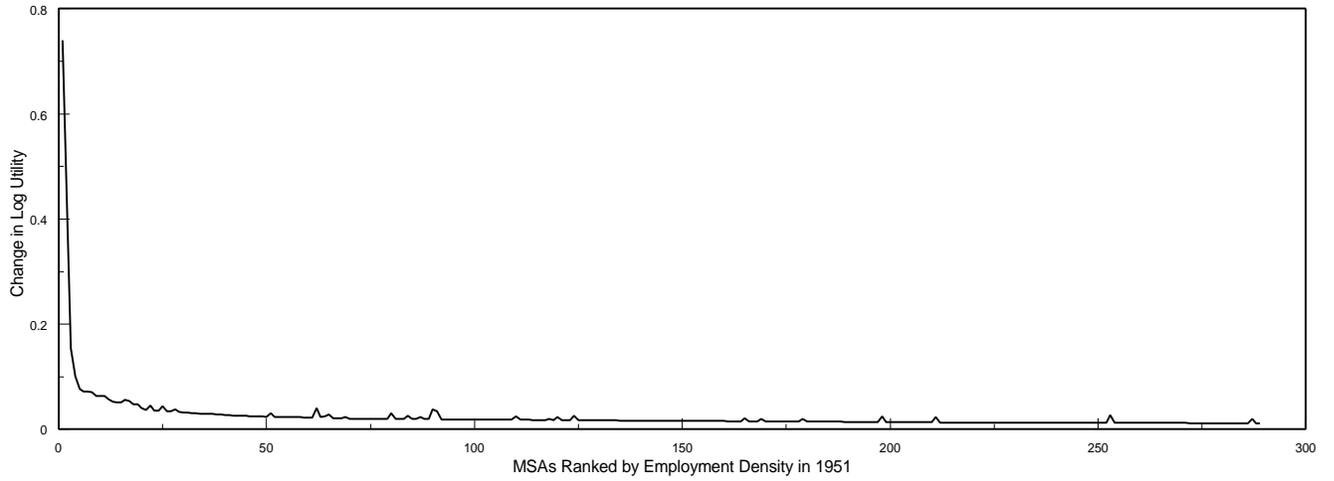


Figure 8(b)  
**Effect on Location-Specific Factors for Lowering Delta to  $3 \times 10^{-4}$**

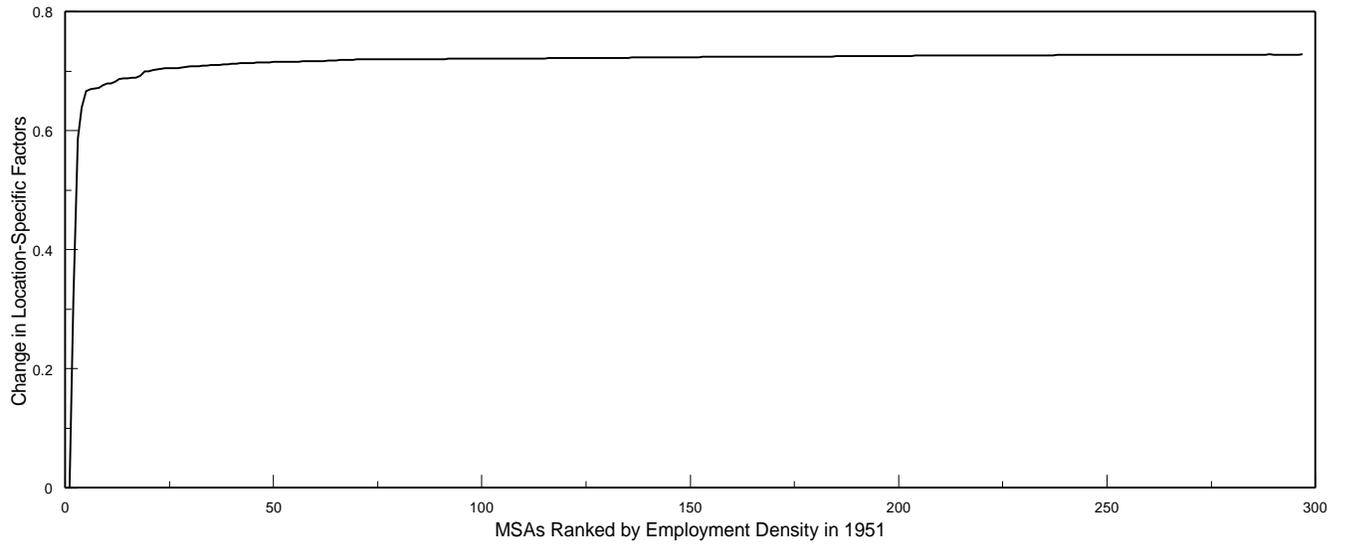


Figure 8(c)  
**Overall Impact on Log 1994 Utility of Lowering Delta to  $3 \times 10^{-4}$**

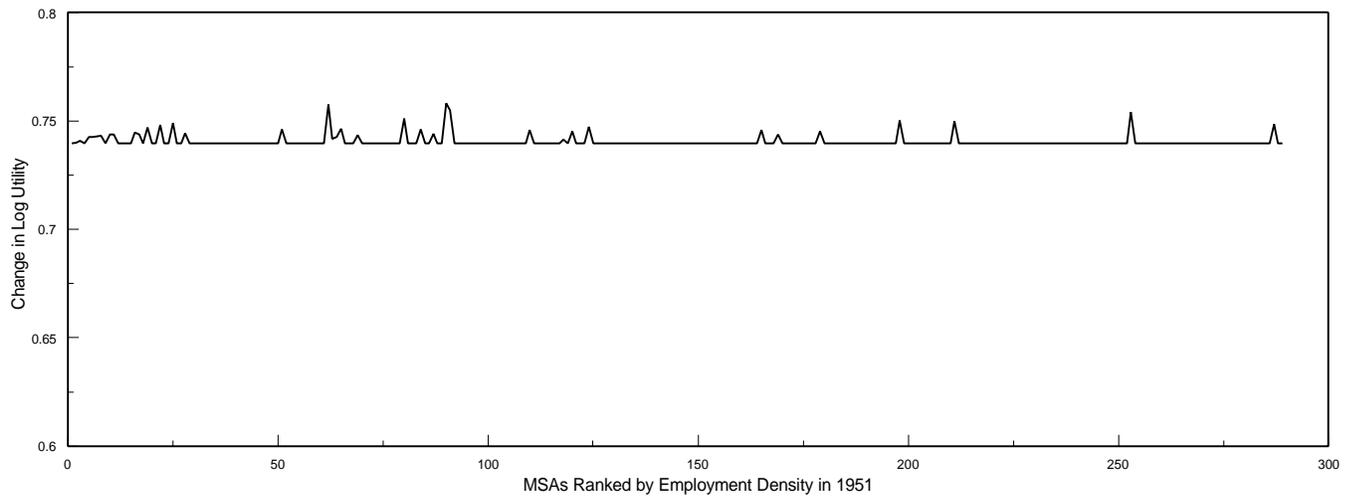


Figure 9(a)  
**Predictions of the Baseline Model Compared to Models with Low and High Values of Agglomeration Parameter ( $\mu$ )**

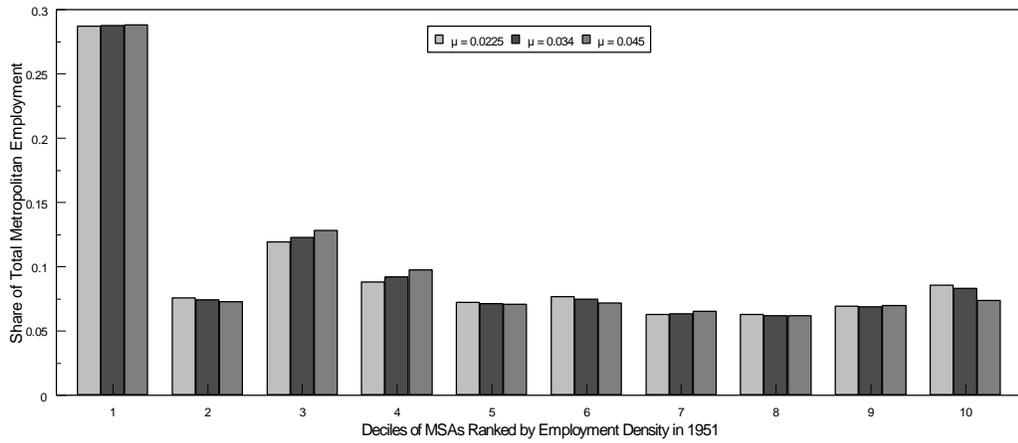


Figure 9(b)  
**Predictions of the Baseline Model Compared to the Model with High Threshold ( $N_{Min}$ )**

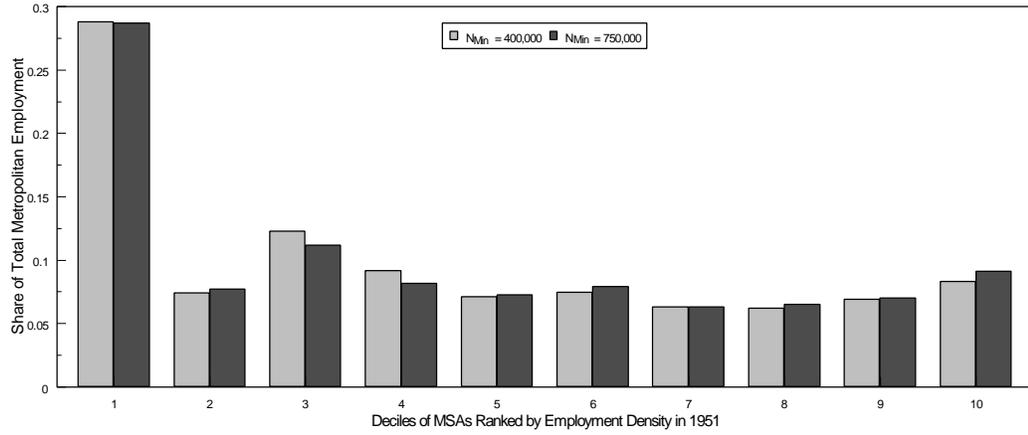


Figure 9(c)  
**Predictions of the Baseline Model Compared to Model with 1959 Location-Specific Factors**

