

# Can the Government Roll Over Its Debt Forever?

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**I**n the past dozen years, the federal government has regularly run large deficits, usually well in excess of \$100 billion per year. The amount of federal government debt outstand-

ing has quadrupled during this time, from a value of \$908 billion at the end of fiscal year 1980 to a value of \$3,665 billion at the end of fiscal year 1991. Even after correcting for inflation, the amount of government debt has grown by a factor of 2.5 over this period. This apparent explosion in the amount of government debt has led to spirited and protracted public debate about federal tax policy and federal expenditures. Despite the widely professed desire to reduce the federal deficit and to limit the growth of federal government debt, a consensus about how to achieve these alleged goals has not yet emerged. Faced with continuing deficits, the

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government has resorted to rolling over its debt—that is, issuing new debt to pay the interest on existing debt and to pay off holders of maturing debt.

Is rolling over the debt the solution that we have been looking for? Can the government simply roll over its debt forever without having to take the politically costly steps of raising taxes or cutting expenditures in the future? This article discusses the feasibility of rolling over government debt forever. As we will see, this question is related to another important question about the future of the economy: Is the economy as a whole saving an appropriate amount for the future? In addition, both of these questions are related to the question of whether an entity can run a Ponzi game.

### THE SIMPLE ARITHMETIC OF GOVERNMENT DEBT ACCUMULATION

To address the question of whether the government can roll over its debt forever, we need to quantify the factors that contribute to the growth of government debt over time. We begin by specifying the relationship between government deficits and the growth rate of government debt. Then we examine whether the public would be willing to hold ever-increasing amounts of government debt, thereby permitting the government to roll over its debt forever.

**Primary and Total Deficits.** Although it is tempting to think of both “debt” and “deficits” as representing the “D word,” there is an important distinction between debt and deficits. Government debt is the liability of the government owed to holders of government bonds at any particular moment; it is measured in dollars as of a particular date, such as \$3,665 billion as of September 30, 1991. A government deficit is the excess of government expenditures over government receipts during a particular period. The government deficit equals the increase in the amount of government debt during a particular interval; it is measured in terms

of dollars per unit of time, such as \$320.9 billion per year during fiscal year 1991 (October 1, 1990 - September 30, 1991). In terms of familiar accounting concepts, government debt is a balance sheet concept, whereas the government deficit is an income statement concept.

Although the definition of the government deficit as the excess of government expenditures over government receipts during a particular period seems fairly unambiguous, actually two different deficit concepts are widely used. The difference between these two deficit concepts lies in whether interest payments on government debt are included as part of government expenditure. One deficit concept, known as the primary deficit, does not include interest payments on the government debt as part of government expenditure. Thus, the primary government deficit is calculated as all noninterest expenditure by the government minus government receipts. The primary government deficit was “only” \$34.9 billion in fiscal 1991 (Table 1).

The other deficit concept, known as the total deficit or simply the deficit, includes interest payments by the government as part of government expenditure. Thus the total deficit equals total government expenditure, including interest payments, minus government receipts. In fiscal 1991, interest payments by the government amounted to \$286.0 billion, so that the total government deficit of \$320.9 billion exceeded the primary government deficit by \$286.0 billion.

Why are there two different deficit concepts? The reason economists and policymakers look at both of these deficit concepts is that each concept provides the answer to a different question. Specifically, the primary deficit answers the question: Are current taxes sufficient to pay for spending on current government programs? More precisely, the primary deficit measures the extent to which spending on current programs exceeds the taxes currently collected. The total deficit answers a different

**TABLE 1**  
**Government Deficit**  
**Fiscal Year 1991**  
**(October 1, 1990 - September 30, 1991)**

Government Expenditures	
Noninterest expenditures <sup>a</sup>	\$795.3 billion
Interest payments by government <sup>b</sup>	\$286.0 billion
Total expenditures <sup>c</sup>	\$1,081.3 billion
Government Receipts <sup>c</sup>	\$760.4 billion
Primary Deficit = \$795.3 billion - \$760.4 billion = \$34.9 billion	
Total Deficit = \$1,081.3 billion - \$760.4 billion = \$320.9 billion	

<sup>a</sup>Source: calculated as total expenditures minus interest payments by government.

<sup>b</sup>Source: Treasury Bulletin, March 1992.

<sup>c</sup>Source: Economic Report of the President, 1992, Table B-75.

question: How much will the government have to borrow to pay for its expenditures? The total deficit during a year measures the increase in government debt during that year.

**The Debt-GNP Ratio.** How do we gauge whether a government's debt is too large? One way to gauge the size of a government's debt is by the government's ability to repay the debt. Governments that have access to larger tax bases would be able to support larger amounts of debt than governments with smaller tax bases. For the federal government, we can gauge the size of the tax base by some measure of national income, such as Gross National Product (GNP) or Gross Domestic Product (GDP). In this article, we will use GNP as the measure of national income, and thus we will use the ratio of government debt to GNP—known as the debt-GNP ratio—to gauge the size of government debt.

The historical behavior of the debt-GNP ratio over the last century in the United States is shown in Figure 1. Notice that the debt-GNP ratio rose sharply during World War I and World War II, and then fell gradually after these wars (and also fell gradually for about a half century after the Civil War). In addition to the increases in the debt-GNP ratio during wars, the debt-GNP ratio also rose sharply during the Great Depression of the 1930s and during the 1980s.

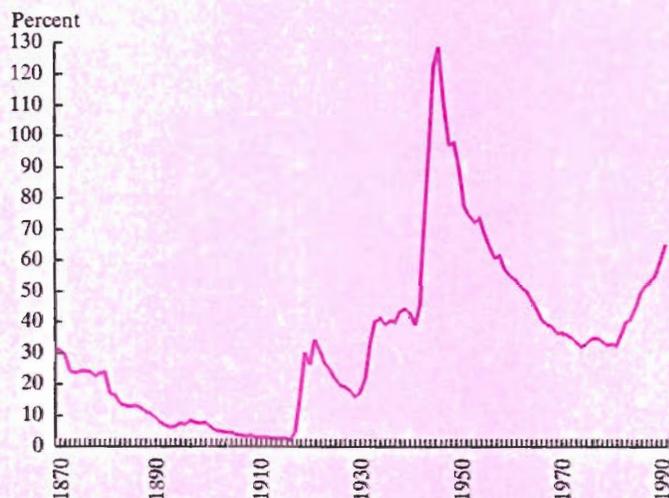
What causes the debt-GNP ratio to increase from one year to the next? Just as a matter of simple arithmetic, the debt-GNP ratio will rise whenever the growth rate of the numerator, i.e., the growth rate of government

debt, is higher than the growth rate of the denominator, i.e., the growth rate of GNP. As we have discussed earlier, the increase in government debt during a year equals the total deficit, which in turn equals the primary deficit plus interest payments by the government. Thus, the debt-GNP ratio tends to increase when (1) the primary government deficit is large; (2) interest payments by the government are large; and (3) the growth rate of GNP is small. The following equation, which is an approximation derived in Appendix A, captures the simple arithmetic of government debt accumulation:

$$(1) \text{ growth rate of debt-GNP ratio} = \text{primary deficit/debt} + \text{interest rate} - \text{growth rate of GNP}$$

Note that when the growth rate of the debt-

FIGURE 1  
Debt-GNP Ratio



Sources: Ratio of government debt to GNP. Source of government debt (end of fiscal year): 1869-1939 from *Historical Statistics of the United States*, series y338; 1940-1969 from *Banking and Monetary Statistics*, 1941-1970, Table 13.1, C; 1970-1979 from *Federal Reserve Board Annual Statistical Digest*, 1970-1979, Table 27; 1980-1989 from *Federal Reserve Board Annual Statistical Digest*, 1980-1989, Table 26; 1990-1991 from *Treasury Bulletin*, March 1992, Table FD-1. Source of GNP: 1869-1958, Balke, Nathan S. and Robert J. Gordon, Appendix B Historical Data, in *The American Business Cycle: Continuity and Change*, Robert J. Gordon (ed.), Chicago and London: The University of Chicago Press, 1986; 1959-1991 from Data Resources Incorporated (1960 GNP is 2 percent higher in DRI than in Balke and Gordon).

GNP ratio is positive, this ratio is growing, and when the growth rate of the debt-GNP ratio is negative, the debt-GNP ratio is falling.

The three components of the growth rate of the debt-GNP ratio on the right-hand side of equation (1) explain, in an arithmetic sense at least, the historical behavior of the debt-GNP ratio shown in Figure 1. The sharp increase in the debt-GNP ratio during both world wars resulted from sharp increases in the primary deficit (Figure 2). Of course, the increase in the primary deficit reflects the large increase in military expenditure during wartime. The rise

in the debt-GNP ratio during the Great Depression resulted from large declines in GNP during the early 1930s and from large primary deficits beginning in 1932. The decline in the debt-GNP ratio during the three-and-a-half decades following World War II resulted from a combination of factors: (1) a small—indeed usually negative—primary deficit; and (2) an interest rate that was usually smaller than the growth rate of GNP. However, during the 1980s the debt-GNP ratio departed from its typical pattern of peacetime behavior and began to rise. Arithmetically, the positive growth rate of the debt-GNP ratio was accounted for by a relatively large ratio of the primary deficit to government debt in the early 1980s and by the fact that

the interest rate exceeded the growth rate of GNP for most of the 1980s.

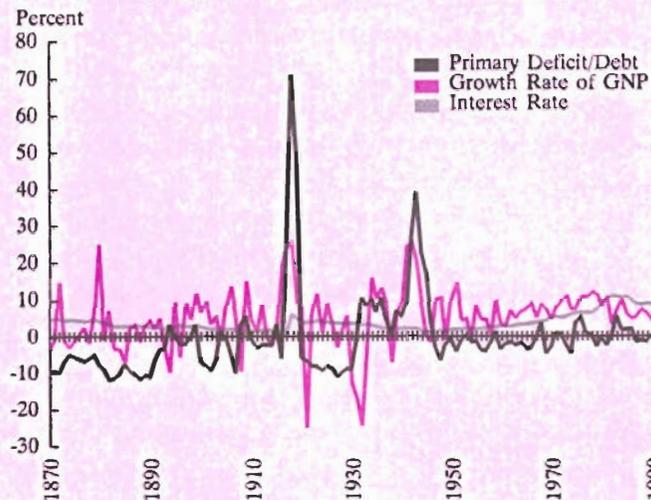
**Rolling Over Government Debt.** Our discussion of the debt-GNP ratio was motivated by the desire to gauge the size of government debt relative to the government's ability to repay that debt. What problems might be associated with a high value of the debt-GNP ratio? If the debt-GNP ratio were to become too large, the public might begin to suspect that one day the government would default on its debt, and this suspicion might make the public unwilling to buy additional government debt.

There are many ways the government could default on its debt. The government could simply renounce its liabilities and refuse to pay holders of government bonds. Alternatively, the government could heavily tax the principal and/or interest on government bonds, effectively defaulting on at least a fraction of its liabilities. More subtly, the government could print money and create inflation, which reduces the real purchasing power of its dollar liabilities represented by government bonds. Another problem with a very high debt-GNP ratio is that the interest payments on government debt become a very large fraction of GNP. If the debt-GNP ratio becomes extremely large, the increase in government debt needed to pay the interest on the outstanding government debt could become larger than all of GNP,<sup>1</sup> and the public would not be able to buy this debt.

The willingness or unwillingness of the public to buy additional government debt when the debt-GNP ratio gets large determines whether the government can roll over its debt forever. If a policy of rolling over government debt forever would cause the debt-GNP ratio to grow forever without bound, the public would be-

<sup>1</sup> If the debt-GNP ratio exceeds the reciprocal of the interest rate on government bonds, interest payments on government debt would exceed GNP.

FIGURE 2  
Components of  
Debt-GNP Growth Rate



Sources: Primary deficit calculated as total deficit minus interest payments by the government. Source of total deficit: 1869-1939 from Historical Statistics of the United States, series y337; 1940-1991 from Economic Report of the President, February 1992, Table B-74, on-budget. Source of interest payments: 1869-1969: from Historical Statistics of the United States, series y461; 1970-1991 from Treasury Bulletin, various issues, Table FFO-3. Interest rate calculated as interest payments in current fiscal year divided by government debt at end of previous fiscal year (see note to Figure 1 for source of data on government debt). Growth rate of GNP calculated from GNP data described in note to Figure 1.

come unwilling to buy the government debt offered for sale and the rollover policy would have to terminate. However, if the debt-GNP ratio falls forever when the government is pursuing a rollover policy, it would be possible to roll over government debt forever.

But how could the debt-GNP ratio fall forever while the government is rolling over its debt? To answer this question, we will first precisely define a policy of rolling over the debt in terms of the primary deficit, and then we will use equation (1) to see how the debt-GNP ratio changes over time under a policy of debt rollover.

Quite simply, a government is rolling over its debt if its primary deficit is zero, so that its total deficit equals its interest payments on government debt. In this case, the government sells additional government bonds (debt) to pay the interest on government debt and to pay off holders of maturing government debt. If the government can run a zero primary deficit forever, selling bonds to cover the total deficit, then it can roll over its debt forever. Whether the government is able to run a zero primary deficit forever depends on whether the debt-GNP ratio eventually becomes too large when the government runs a zero primary deficit year after year.

To see if a government can run a zero primary deficit forever, we simply set the primary deficit in equation (1) equal to zero and observe that in this case the growth rate of the debt-GNP ratio equals the interest rate minus the growth rate of GNP. If the interest rate is higher than the growth rate, the debt-GNP ratio grows forever without bound, and eventually the government would lose its ability to roll over its debt. However, if the interest rate is smaller than the growth rate of GNP, the growth rate of the debt-GNP ratio would be negative, and the government could roll over its debt forever. For instance, if the interest rate is 3 percent per year and the growth rate of GNP is 4 percent per year, interest payments amount to 3 percent of government debt. If the government sells new bonds to pay these interest payments, the supply of government debt will increase by 3 percent per year, which is less than the 4 percent annual growth rate of GNP. Thus, the debt-GNP ratio would decline.

For most of the last century in the United States, the interest rate on government debt has been lower than the growth rate of GNP (Figure 2). In fact, the average interest rate on government debt was 4.12 percent per year, and the average growth rate of GNP was 5.86 percent per year over the period 1869-1991. If this pattern with the average interest rate below the

average growth rate were to continue to hold forever, it would appear that the U.S. government could roll over its debt forever.

### WHAT HAPPENS WHEN THE INTEREST RATE IS LESS THAN THE GROWTH RATE OF GNP?

We have seen that over the last century the average interest rate on government debt was lower than the average growth rate of GNP. One important implication of having an interest rate lower than the growth rate of GNP is that the government can roll over its debt forever. In this section, we discuss two other important—and surprising—implications of having an interest rate lower than the economy's growth rate.

**The Economy Has Too Much Capital.** The most important factor determining the standard of living of future generations is the long-run rate of economic growth. One of the primary ways that an economy can help promote economic growth is to save for the future by increasing the capital stock of productive equipment and structures. This process of capital accumulation combines a present sacrifice in the form of reduced present consumption with a future benefit in the form of increased future output and consumption. At various times in recent history, policymakers have made the judgment that the future gain is worth the present sacrifice, and national economic policy focused directly on stimulating capital formation by providing tax incentives in the form of accelerated depreciation allowances and the investment tax credit.

Is it possible for an economy to overdo it? More precisely, is it possible for an economy to accumulate and maintain a level of capital that is unambiguously too high? Surprisingly, the answer is yes. An economy can accumulate so much capital that the current sacrifice associated with current investment actually leads to a future sacrifice in the form of reduced future consumption. In this situation, the present

sacrifice associated with capital formation is clearly not worth undertaking. An interest rate smaller than the growth rate of the economy signals that such a situation exists.

To see how it would be possible to have too much capital, suppose a piece of capital requires \$5 worth of resources every year to maintain it in working order, but the capital contributes additional output worth only \$4 per year. The economy would be suffering a net loss of \$1 per year and would be better off without the capital.<sup>2</sup> At the level of the national economy, we can say that an economy has too much capital if in every year the amount of resources devoted to creating new capital and maintaining old capital is greater than the contribution to total output of the total capital stock. To put this condition in the language of national income accounting, an economy has too much capital if in every year gross investment (the amount of resources devoted to new capital formation and replacement of depreciated capital) exceeds gross capital income (which measures the contribution of capital to total output). We write this condition as:

$$(2) \text{ too much capital if:} \\ \text{gross investment} > \text{gross capital income} \\ \text{in every year.}$$

Now we can relate the condition for too much capital to the relationship between the interest rate and the growth rate. This relationship is clearest for an economy growing at a

constant rate year after year, so let's suppose that the economy is growing at constant rate  $g$  every year. Thus, for example, GNP is growing at the rate  $g$  and the total capital stock,  $K$ , is also growing at the rate  $g$ . With the capital stock growing at the rate  $g$  per year, the amount of net capital formation during a year is  $gK$ . In addition, some resources are devoted to replacing capital that depreciates during the year. Letting  $d$  be the fraction of the capital stock that depreciates during a year, the total amount of depreciation during a year that must be offset by capital formation is  $dK$ . Gross investment is the sum of net capital formation and depreciation:

$$(3) \text{ gross investment} = gK + dK = (g + d)K$$

The contribution of capital to total output is measured by gross capital income. Letting  $R$  denote the gross rate of return on capital, we have:

$$(4) \text{ gross capital income} = RK$$

Comparing gross investment in equation (3) with gross capital income in equation (4), we see that the economy has too much capital if  $(g + d)K > RK$  in every year, or equivalently:

$$(5) \text{ too much capital if:} \\ g + d > R \\ \text{in every year}$$

To see the role of the interest rate in this condition, we observe that in an economy in which there is no uncertainty, the interest rate  $r$  would equal the net rate of return on capital, which is the gross rate of return  $R$  minus the rate of depreciation. In symbols we have:

$$(6) \quad r = R - d \\ \text{(interest rate) (net rate of return on capital)}$$

Finally, we obtain the condition for too much capital in terms of the interest rate and the growth rate by subtracting the depreciation rate  $d$  from both sides of equation (5) and using the fact that  $r = R - d$  to obtain:

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<sup>2</sup>In this numerical example, net investment is zero, but the same principle applies when there is positive net investment. For example, consider a firm that operates a factory with a work force that grows by 2 percent per year. If the firm maintains a constant ratio of capital to labor, the firm's capital stock would grow by 2 percent per year. However, if the contribution to total output of each unit of capital is only 1 percent of the value of the capital stock, then the firm would be pouring more resources into the factory than it gets out of the factory, and it would be better off closing that factory.

- (7) too much capital if:  $g > r$   
in every year.

Thus, we can see that in the absence of uncertainty, an economy growing at a constant rate has too much capital if the interest rate is less than the growth rate. An economy in this situation could realize both a present gain and a future gain by permanently reducing the amount of investment. Present consumption would increase as the economy's current resources shifted from investment to consumption. Future consumption would increase as fewer resources were, on net, poured into the formation and maintenance of capital. As a result of the reduction in investment, the capital stock would fall, and as capital became less abundant, the rate of return on capital would increase. When the rate of investment has fallen enough, the net rate of return on capital and the interest rate will rise above the growth rate of the economy, so that the symptom of too much capital will disappear.

Recall that during the period 1869-1991 the average interest rate in the United States was smaller than the average growth rate. Thus, equation (7) would seem to suggest that the United States has too much capital. We will take another look at this provocative implication later in this article.

**Ponzi Games.** In the early 20th century, Charles Ponzi promised investors the opportunity to double their money in 90 days by investing in international postal coupons. Over the course of eight months, Ponzi acquired about \$15,000,000 from 40,000 investors. Not surprisingly, Ponzi's promises proved to be too good to be true, and Ponzi was arrested in August 1920.<sup>3</sup> Economists now use the term "Ponzi game" to describe a situation in which an entity (a person, business, or government) sells securities to investors and never uses any of its own

money to pay dividends or interest or to repay the principal. Any subsequent payments (such as dividends, interest, or return of principal) to holders of these securities are financed by selling additional securities. Our discussion will focus on rational Ponzi games, which are Ponzi games in which there is no fraud or deceit on the part of the seller of securities and no lack of understanding or foresight on the part of buyers of these securities.

As a simple example of a rational Ponzi game, consider an entity that sells \$100 million of long-term bonds, promising to pay an interest rate of 4 percent per year. At the end of one year, when it is time to pay investors \$4 million in interest, the entity sells an additional \$4 million of bonds to investors, bringing total bonds outstanding to \$104 million. Then at the end of two years, when \$4.16 million of interest (4 percent of \$104 million) is due, the entity sells an additional \$4.16 million of bonds, and so on. The amount of bonds outstanding grows at the rate of interest, which is 4 percent per year in this example. For this Ponzi game to be feasible, the public must be willing to hold the ever-increasing amount of bonds issued. If investors' wealth is growing at, say, 5 percent per year, there would be sufficient demand by the public for newly issued bonds, and thus the entity would be able to sell additional bonds to pay the interest on its debt without having to use any of its own resources.

In the Ponzi game described above, suppose that the entity selling the bonds is the government. Then the Ponzi game amounts to rolling over government debt forever. The Ponzi game will be feasible, that is, the government will be able to roll over its debt forever, provided that the growth rate of aggregate wealth exceeds the interest rate. The growth rate of aggregate wealth is not readily measured, but in the absence of a trend in the ratio of wealth to GNP, the growth rate of aggregate wealth can be proxied by the growth rate of GNP. Thus, the government will be able to roll over its debt

<sup>3</sup>See O'Connell and Zeldes (1992).

forever if the growth rate of GNP exceeds the interest rate.<sup>4</sup>

To summarize, if the interest rate is lower than the growth rate of GNP, (1) the economy has too much capital; (2) entities can run rational Ponzi games; and (3) in particular, the government can roll over its debt forever. As we have seen, over the last century in the United States, the average interest rate has been lower than the average growth rate of GNP. Thus, it might seem that the United States has too much capital, that entities can run rational Ponzi games, and that the government can roll over its debt forever. However, these three results do not strike most observers as plausible descriptions of the U.S. economy. The implausibility of these results stimulated new research into these questions in the past several years. A point of departure for much of this research is the fact that the results presented above were derived under the assumption of a constant interest rate and a constant growth rate, but, as is evident in Figure 2, the interest rate, and especially the growth rate, have displayed substantial variability in the United States. Recent research has focused on uncertainty as the source of variation in the interest rate and the growth rate and has found that the results summarized above need to be substantially altered when uncertainty is incorporated into the analysis.

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<sup>4</sup>The discussion in this article ignores distortions arising from taxes or from externalities. In a recent paper, Ian King (1992) has argued that with endogenous growth arising from externalities in the stock of knowledge, it is possible for Ponzi games to be feasible even though the economy does not suffer from overaccumulation of capital. This result arises because the private and social returns to capital differ in the presence of externalities. Capital overaccumulation occurs if the social rate of return to capital is lower than the growth rate of the economy, and Ponzi games are feasible if the private rate of return to capital is lower than the growth rate of the economy. In King's model, the social rate of return can be higher than the growth rate, which can be higher than the private rate of return.

## THE IMPORTANCE OF UNCERTAINTY

Recent research into the questions of whether an economy has too much capital and whether a government can roll over its debt forever has shown that simply comparing the average interest rate and the average growth rate of the economy can produce misleading answers to these questions. Much of this research is ongoing and many important questions remain unanswered, but this research has yielded some important insights.

**Another Look at Whether an Economy Has Too Much Capital.** In a world without uncertainty, we can compare the interest rate and the growth rate of the economy to determine whether the economy has too much capital. In deriving equation (7) we used the fact [equation (6)] that in the absence of uncertainty, the net rate of return on capital,  $R - d$ , equals the interest rate,  $r$ , on government debt. However, in the presence of uncertainty, the rates of return on different assets, in particular the rates of return on capital and on government bonds, can in general differ. Thus, the comparison of the interest rate and the growth rate in equation (7) is no longer appropriate for assessing whether an economy has too much capital.

In the presence of uncertainty, the appropriate criterion for determining whether an economy has too much capital is equation (2): If gross investment exceeds gross capital income in every year, the economy has too much capital. If gross investment is less than gross capital income in every year, we conclude that the economy is not plagued by too much capital. A recent study<sup>5</sup> has examined gross investment and gross capital income in the United States for the period 1929-1985 and found that

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<sup>5</sup> Andrew B. Abel, N. Gregory Mankiw, Lawrence H. Summers, and Richard J. Zeckhauser, "Assessing Dynamic Efficiency: Theory and Evidence," *Review of Economic Studies*, 56 (January 1989), pp. 1-20.

in every year, including the Great Depression of the 1930s, gross investment was less than gross capital income. Thus, despite the fact that the average interest rate was less than the average growth rate of the economy, we can conclude that the United States was not afflicted with too much capital.<sup>6</sup> This study also examined six other countries, including Japan, which is often cited as a country with high rates of saving and investment. For all of these countries, including high-investing Japan, gross investment was always less than gross capital income, and hence, none of these countries had too much capital.

**Debt Rollover When the Average Interest Rate Is Lower Than the Average Growth Rate.**

We have just seen that the introduction of uncertainty invalidates the comparison of the average interest rate and the average growth rate for the purpose of determining whether an economy has too much capital. Now we will see that the introduction of uncertainty also invalidates the comparison of the average interest rate and the average growth rate for the purpose of determining whether a Ponzi game is feasible. We focus this discussion on a particular Ponzi game, namely rolling over government debt forever. This section presents a numerical example with the following surprising feature: despite the fact that the interest rate on government debt is lower than the average growth rate of GNP, the expected value of the debt-GNP ratio grows without bound. Eventually, the government would become unable to roll over its debt.

Before presenting this example it is useful to calculate an exact expression for the growth rate of the debt-GNP ratio when the government is following a rollover policy. (Equation

(1) is an approximate expression.) Remember that a rollover policy means that the primary deficit is zero in every year. If the current amount of government debt is  $B$  and if the government has a zero primary deficit, its total deficit is  $rB$ , where  $r$  is the interest rate. Thus, the government must sell an additional  $rB$  bonds, and the amount of bonds next year rises to  $(1+r)B$ . If the current level of GNP is  $Y$  and if the growth rate of GNP over the next year is  $g$ , the level of GNP next year is  $(1+g)Y$ . Thus, the value of the debt-GNP ratio next year is  $[(1+r)/(1+g)][B/Y]$ , which is  $(1+r)/(1+g)$  times as large as the current debt-GNP ratio,  $B/Y$ . Thus, if  $r$  is larger than  $g$ , so that  $(1+r)/(1+g)$  is larger than one, the debt-GNP ratio grows between this year and next year. Alternatively, if  $r$  is smaller than  $g$ , so that  $(1+r)/(1+g)$  is smaller than one, the debt-GNP ratio falls between this year and next year. These results are consistent with the approximation in equation (1).<sup>7</sup>

Now we can discuss the numerical example presented in Table 2, which has the following features: the interest rate  $r$  is constant and is smaller than the average value of  $g$ , the growth rate of GNP. However,  $g$  varies in such a way that the average value of  $(1+r)/(1+g)$  is greater than 1, so that the expected value of the debt-GNP ratio in the next period is always greater than the current value of the debt-GNP ratio. In this example, the uncertainty comes from the fact that GNP growth is unpredictable from one period to the next. To make the example simple, suppose that GNP growth is determined by the flip of a fair coin each period. If the coin comes up heads, GNP grows by 60 percent during the next period, and if the coin

<sup>6</sup> This conclusion is based on the implicit assumption that the fact that gross investment has always been smaller than gross capital income will continue forever.

<sup>7</sup> The approximation involved in equation (1) is that the growth rate of a ratio is approximately equal to the growth rate of the numerator minus the growth rate of the denominator. (See Appendix A, Derivation of the Growth Rate of the Debt-GNP Ratio.)

**TABLE 2**  
**A Growing Debt-GNP Ratio**  
**with the Interest Rate**  
**Below the Average Growth Rate**

period	1	2	3
debt	\$100	\$104.70	\$109.62
GNP	\$1000	\$600 (50%) \$1600 (50%)	\$360 (25%) \$960 (25%) \$960 (25%) \$2560 (25%)
expected GNP	\$1000	\$1100	\$1210
debt/GNP	0.10	0.1745 (50%) 0.0654 (50%)	0.3045 (25%) 0.1142 (25%) 0.1142 (25%) 0.0428 (25%)
expected debt/GNP	0.1000	0.1200	0.1439

comes up tails, GNP falls by 40 percent.<sup>8</sup> Thus, if GNP is currently \$1000, there is a 50 percent chance that next period's GNP will be \$1600 and a 50 percent chance that next period's GNP will be \$600. Thus, the average, or expected, value of next period's GNP is \$1100 ( (\$1600+\$600)/2 ), which represents a 10 percent expected growth rate.

Now suppose that the interest rate on government debt is always 4.7 percent per period, which is less than the average growth rate of the economy, and let's see how the debt-GNP ratio behaves in this economy. Suppose that in period 1 the amount of government debt is \$100. Thus, the debt-GNP ratio is \$100/\$1000 = 0.10.

The first panel of numbers in Table 2 shows the evolution of government debt over time. With a 4.7 percent interest rate, the

<sup>8</sup> These large changes in GNP in this example were chosen to make the effects very apparent. To make the example seem more realistic, think of a period as being a decade rather than a year. Notice that between 1929 and 1933 in the United States real GNP fell by 30 percent and nominal GNP fell by 46 percent, so a 40 percent drop in GNP during a decade is not inconceivable. However, the probability of such a bad decade is almost surely much less than the value of 50 percent assumed in this example.

amount of government debt grows at the rate of 4.7 percent per period. Thus, government debt equals \$104.70 in period 2 and \$109.62 in period 3.

The second panel of numbers in Table 2, which shows GNP, requires a little additional explanation. As shown in the first column, GNP is \$1000 in period 1. The second column shows that there is a 50 percent chance that GNP in period 2 will be \$600 and a 50 percent chance that GNP in period 2 will be \$1600, so that the expected value of GNP in period 2 is  $(\$600 + \$1600)/2 = \$1100$ . The third column of numbers shows the possible values of GNP in period 3. If GNP in period 2 is \$600, there is a 50 percent chance it will fall by 40 percent, to \$360, in period 3, and a 50 percent chance it will rise by 60 percent, to \$960, in period 3. Alternatively, if GNP in period 2 is \$1600, there is a 50 percent chance it will fall by 40 percent, to \$960, in period 3, and a 50 percent chance it will rise by 60 percent, to \$2560, in period 3. Taking account of all of these possibilities for the value of GNP in period 3, there is a 25 percent chance it will be \$360, a 50 percent chance it will be \$960, and a 25 percent chance it will be \$2560. The average, or expected, value of GNP in period 3 is \$1210.

The third panel of numbers in Table 2 shows the possible values of the debt-GNP in each of the three periods. These numbers are calculated by dividing the value of debt in the first panel by the value of GNP in the second panel. For example, in period 2, debt will equal \$104.70. There is a 50 percent chance GNP will equal \$600, in which case the debt/GNP ratio will be  $\$104.70/\$600 = 0.1745$ , as reported in the third panel; there is a 50 percent chance GNP will equal \$1600, in which case the debt/GNP ratio will be  $\$104.70/\$1600 = 0.0654$ . The average, or expected, value of the debt-GNP ratio in period 2 is  $(0.1745 + 0.0654)/2 = 0.1200$ , which is higher than the debt-GNP ratio in period 1. Despite the fact that the interest rate is smaller than the average growth rate of GNP, the risk of a sharp

drop in GNP makes the expected value of the debt-GNP ratio in period 2 higher than the value of the debt-GNP ratio in period 1. As shown in the third column, the expected value of the debt-GNP ratio in period 3 is 0.1439. In fact, the expected value of the debt-GNP ratio will grow at a rate of approximately 20 percent per period forever. Eventually, the expected value of the debt-GNP ratio would become so large that the government would be unable to roll over its debt despite the fact that the interest rate on government debt is lower than the average growth rate of the economy.

#### WHAT CAN WE CONCLUDE ABOUT UNITED STATES FISCAL POLICY?

We have shown that in the presence of uncertainty it may be impossible for the government to roll over its debt forever, even though the average interest rate is lower than the average growth rate of GNP. So, how then do we empirically assess whether the government can roll over its debt forever? This question is at the frontier of economic research and has not yet been fully resolved. Nevertheless, recent research has yielded some insights and some speculation about future findings.

One important insight is that if an economy has too much capital, Ponzi games are possible and the government can roll over its debt forever. However, a recent study cited earlier<sup>9</sup> found that none of the countries studied, including the United States, is afflicted by too much capital.

Does the finding that an economy does not have too much capital imply that Ponzi games are not possible and, in particular, that the government cannot roll over its debt forever? In a world without uncertainty, the answer to this question would be "yes," as we illustrated earlier. Unfortunately, the answer is ambigu-

<sup>9</sup> Abel, Mankiw, Summers, and Zeckhauser (1989).

ous in the presence of uncertainty: in some economies that do not have too much capital, it is possible for the government to roll over its debt forever, while in other economies that do not have too much capital, it is impossible for the government to roll over its debt forever.<sup>10</sup>

The current state of economic research suggests that the crucial issue for determining whether a government can roll over its debt forever is whether there is a rich enough set of existing securities in the economy. If the set of existing securities is not rich enough in the relevant sense, government debt might be such a sufficiently different and attractive security that investors would welcome the opportunity to hold it in their portfolios and would allow the government to roll over its debt forever. However, if the set of existing securities is sufficiently rich, government debt may not be sufficiently different or attractive for investors to allow the government to roll its debt over forever.<sup>11</sup> Unfortunately, the current state of economic research does not allow a convincing empirical test to distinguish between these two cases, so we cannot yet test whether an actual government can roll over its debt forever.<sup>12</sup>

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<sup>10</sup>Technically, under certainty, capital overaccumulation is a necessary and sufficient condition for Ponzi games and for rolling over government debt forever. Under uncertainty, capital overaccumulation is a sufficient, but not necessary, condition for Ponzi games and for rolling over government debt forever.

<sup>11</sup>Blanchard and Weil (1992) present examples of economies that do not have too much capital. In some of these examples, the set of securities is not sufficiently rich, and the government can roll over its debt forever. In other examples, the set of securities is sufficiently rich, and the government cannot roll over its debt forever.

<sup>12</sup>A related—and also unresolved—question is why the average interest rate on government debt is so much lower than the average rate of return on capital. One potential explanation is that there is a very rich set of securities available but investors are very risk averse and essentially

Although we cannot yet empirically test whether an economy can roll over its debt forever, we are not left entirely in the dark about the future course of U.S. fiscal policy. Recently, Henning Bohn (1991a) has developed and implemented a test of whether a government is following a sustainable policy. This is not a test of whether a zero primary deficit accompanied by rolling over the debt is permanently sustainable. Rather it is a test of whether the historical tax and expenditure policies of the government can be permanently maintained without a major shift in the conduct of policy. Applying this test to data on U.S. fiscal policy, Bohn finds that this policy is sustainable. An important component of this conclusion is the finding that, on average, U.S. fiscal policy produces a smaller primary deficit (or a larger primary surplus) when the debt-GNP ratio becomes larger. This tendency of the government to run smaller (or even negative) primary deficits as the debt-GNP ratio gets larger is a means of keeping the debt-GNP ratio from growing too large.

While Bohn's result that U.S. fiscal policy is sustainable may appear comforting, this finding focuses attention on potentially painful choices. If the United States is to follow its historical pattern of reducing primary deficits when the debt-GNP ratio rises, the increase in the debt-GNP ratio over the past dozen years would seem to require a reduction in the primary deficit. Such a reduction in the primary deficit would require an increase in tax revenues and/or a cut in government expenditure, neither of which will be universally popular.

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pay a large premium for the opportunity to hold safe government debt. In this case, the government would not be able to roll over its debt forever. Another potential explanation is that the set of securities is not sufficiently rich and that investors find government debt sufficiently different and attractive that they willingly hold it at a low interest rate. In this case, the government might be able to roll over its debt forever. See Bohn (1991b).

## Derivation of the Growth Rate of the Debt-GNP Ratio

Let  $B$  be the amount of government bonds outstanding, and let  $Y$  be the measure of national income, such as GNP. Thus the debt-GNP ratio is  $B/Y$ . The growth rate of any ratio is approximately equal to the growth rate of the numerator minus the growth rate of the denominator so that

$$(A1) \quad \frac{\Delta(B/Y)}{B/Y} = \frac{\Delta B}{B} - \frac{\Delta Y}{Y}$$

where the symbol  $\Delta$  denotes the change from one period to the next. The change in government bonds,  $\Delta B$ , equals the total deficit, which equals the primary deficit plus interest payments:

$$(A2) \quad \Delta B = \text{primary deficit} + rB$$

where  $r$  is the interest rate on government bonds, so that  $rB$  is the amount of interest payments by the government. Now divide both sides of (A2) by the amount of government bonds  $B$  to obtain

$$(A3) \quad \Delta B/B = \text{primary deficit}/B + r$$

Now let  $g$  denote the growth rate of income so that

$$(A4) \quad \Delta Y/Y = g$$

Substituting (A3) and (A4) into (A1) yields

$$(A5) \quad \frac{\Delta(B/Y)}{B/Y} = \text{primary deficit}/B + r - g$$

which is equation (1) in the text of the article.

## An Economic Model of the Interest Rate and the Growth Rate

This appendix presents a general equilibrium model underlying the example presented in Table 2. Suppose that consumption equals output in every period as in the widely used Lucas (1978) asset pricing model. The standard condition determining the riskless interest rate  $r$  in a representative consumer economy is

$$(B1) \quad (1+r)\beta E_t\{u'(c_{t+1})/u'(c_t)\} = 1$$

where  $E_t\{\cdot\}$  is the expectation conditional on information at time  $t$ ,  $c_t$  is consumption per capita at time  $t$ ,  $u'(c_t)$  is the marginal utility of consumption at time  $t$ , and  $\beta > 0$  is the time preference discount factor (so that  $\beta^{-1}-1$  is the rate of time preference). Assume that the utility function is logarithmic so that  $u'(c_t) = 1/c_t$ . In this case, equation (B1) becomes

$$(B2) \quad 1+r = [\beta E_t\{(c_t/c_{t+1})\}]^{-1}$$

Now let  $g_{t+1} = (c_{t+1}/c_t) - 1$  be the growth rate of consumption and output between time  $t$  and time  $t+1$ , and assume that  $g_{t+1}$  is i.i.d. over time. Under this assumption we have

$$(B3) \quad 1+r = [\beta E\{1/(1+g_{t+1})\}]^{-1}$$

The ratio of the debt-GNP ratio in period  $t+1$  to the debt-GNP ratio in period  $t$  is  $(1+r)/(1+g_{t+1})$  and the expected value of this ratio is

$$(B4) \quad E\{(1+r)/(1+g_{t+1})\} = E\{1/(1+g_{t+1})\} [\beta E\{1/(1+g_{t+1})\}]^{-1} = 1/\beta$$

Notice that if  $\beta < 1$ , then  $1/\beta > 1$  and the expected value of the debt-GNP ratio grows over time. The example in Table 2 is based on the following assumptions:  $\beta = 0.8333$ ; and  $\Pr\{1+g_{t+1} = 0.6\} = \Pr\{1+g_{t+1} = 1.6\} = 0.5$ . These assumptions imply that  $1+r = 1.0473$ ,  $E\{1+g_{t+1}\} = 1.1$ , and  $E\{(1+r)/(1+g_{t+1})\} = 1/\beta = 1.2$ .

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