

# Talent, Labor Quality and Economic Development

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April 2013

## Abstract

We develop a quantitative theory of labor quality based on the division of the labor force between unskilled and skilled workers, and investments in the quality of skilled labor. We use the theory to quantify the importance labor quality and Total Factor Productivity for cross-country income differences. A central ingredient in our analysis are the observed achievement levels (talent) constructed from the *Program of International Student Assessment* (PISA) scores in a sample of 59 countries. Our findings imply that the cross-country differences in the quality of labor are almost twice as large as the conventional measures using Mincer returns. Thus, the implied disparities in TFP levels are smaller. The elasticity of output per worker with respect to TFP is about 2.1 -2.2, and in line with recent studies.

KEYWORDS: PISA, Labor Quality, TFP, Economic Development.

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\*Central Bank of Uruguay and Department of Economics FCS-UDELAR, Federal Reserve Bank of St. Louis, and Department of Economics, Arizona State University, respectively. We thank Juan Carlos Cordoba, Oksana Leukhina, Diego Restucia and Todd Schoellman for comments, and seminar participants at Central Bank of Uruguay, Concordia, Di Tella, Economic Development Conference at Toronto, FCS-Uruguay, Human Capital Conference at Arizona State, ITAM Summer Camp, LACEA Meetings, SED, UBC, USC, Texas A&M, and Yale. The usual disclaimer applies. for The views expressed in this article are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

# 1 Introduction

It is well known by now that the observed cross-country differences in output per worker are large. For example, the richest countries in the world economy are about 30-40 times more productive than their poorest counterparts; see Hall and Jones (1999), McGrattan and Schmitz (2000), Parente and Prescott (2000), among others.

One potential reason why countries differ in their output per worker might be differences in labor input per worker, or *labor quality*. Given the difficulties associated with directly measuring labor quality, the literature has inferred workers' skills by human capital levels constructed using Mincerian returns to schooling. Standard development accounting procedures impute cross-country differences in the quality of labor to differences in such human capital levels. These studies attribute roughly 10 percent of the output per worker differences to differences in the quality of labor across countries; see Caselli (2005) for a review.

In this paper, we take an alternative, more direct approach to the measurement of labor quality. We use the observed test scores from the Programme for International Student Assessment (PISA) in conjunction with a dynamic model to compute the labor quality in each country. The PISA is an internationally standardized assessment of student achievement that is organized and conducted by OECD. The tests are administered to 15-year old individuals in schools and provide a useful quantification of skills towards the end of the schooling period.

There are important advantages in the use of PISA scores for our purposes. One advantage is the uniform assessment of young people near the end of compulsory schooling. It captures students of the same age in each country independently of the structure of national school systems. By contrast, other studies focus on testing students in specific grades which may be distorted by the fact that countries may differ in their grade-entry ages and grade-repetition rules. Another advantage of PISA is that the tests are constructed to evaluate a range of relevant skills that capture how well young adults are prepared to meet future work demands, by being able to analyze, reason, and communicate their ideas effectively. While previous studies are curriculum-based, PISA tests the young adults' ability to "use their knowledge and skills in order meet real-life challenges" (OECD 2001, p. 16). By design, PISA provides a single, comparable measure of skills for each country that can be used to index the skills of individuals prior to their entry into the labor force.

We develop a parsimonious model that allow us to use PISA scores to measure efficiency

units of labor across countries. In our framework countries differ in two key dimensions – talent and Total Factor Productivity (TFP). We use the PISA achievement score and our model to construct a measure of labor quality, and quantify its role in accounting for cross-country differences in output per worker. Our main questions are: how large are the differences in implied labor quality across countries? What are the resulting magnitudes of TFP differences?

Our model has a representative household with a continuum of members. The members are heterogeneous and are born with some innate efficiency units of labor or *talent*. The household divides each cohort in two groups: *skilled* workers and *unskilled* workers. Converting a member of the household into a skilled worker is costly: it requires time (foregone earnings) and goods. As more goods are invested, the higher is the resulting quality of each skilled worker. Quality of an unskilled worker is just the talent he was born with. The production technology uses capital, unskilled labor and skilled labor as factors of production. Labor of both types includes the fraction of each worker type and the quality of each worker type. Thus, while the distribution of talent is exogenous in our model, labor quality is *endogenous*. When the elasticity of substitution between labor types is in the empirically plausible range, we show that an increase in TFP increases the fraction of skilled workers as well as investments in their quality. In this sense, differences in TFP not only have a direct effect on the output per worker, but also an *indirect* effect via labor quality.

In a sample of 59 countries, we measure talent by the observed PISA score. We calibrate two other critical parameters in the model – the importance of goods in the enhancement of the quality of skilled labor and the share of unskilled labor in the aggregate technology – using only the U.S. data to deliver the observed expenditures per tertiary student as a fraction of GDP per worker, and the fraction of the population with secondary education or less. We then examine the quantitative implications of the observed cross-country differences in PISA scores for the differences in output per worker and for the differences in the fraction of labor that is unskilled. We subsequently quantify the role of TFP, along with the PISA scores, in accounting for the output and the division of labor across countries.

Our findings indicate that the use of PISA scores leads to substantially *larger* differences in labor quality across countries than in standard analyses based on Mincerian returns. Using Mincerian returns, labor quality in the poorest 10 percent of the countries in our sample is

about 86 percent of the quality in the richest 10 percent of the countries. In our model, labor quality in the poorest 10 percent is only about 55 percent of the quality in the richest 10 percent when the model reproduces observed differences in output per worker, and about 45 percent when in addition it reproduces the observed division of labor between skilled and unskilled workers in each country. Our finding on labor quality differences is similar to that in Schoellman (2011) who uses data on immigrants in the U.S. to infer variation in labor quality, exploiting the fact that immigrants from richer countries earn higher returns to schooling than immigrants from poorer countries. He concludes that labor quality differences between poor and rich countries are twice as large as those under conventional measures using Mincer returns; e.g. Hall and Jones (1999), Hendricks (2002).

As a result of the larger implied differences in labor quality, TFP differences emerging from our model are smaller than those emerging from standard analyses. Put differently, output per worker reacts more to changes in TFP. In terms of elasticities, the resulting elasticity of output per worker with respect to TFP larger is around 2.1 – 2.2 in our model. The TFP elasticity is 1.5 in the standard growth model, and roughly 1.7 under a measure of labor quality based on Mincer returns, as this variable is correlated with output per worker.

**Related Literature** Our paper is closely related to Manuelli and Seshadri (2010) and Erosa, Koreshkova, and Restuccia (2010), who examine the role of human capital in explaining cross-country income differences. Manuelli and Seshadri (2010) develop a model of human capital acquisition in early childhood, via schooling and over the life cycle, where investments of goods in human capital formation play a central role. They argue that TFP differences are quantitatively small and that measured differences in output largely reflect differences in unmeasured labor quality. The TFP elasticity of output per worker in their model is roughly 6.5. Erosa et al. (2010) develop a model where heterogenous households invest in the quantity and quality of schooling. Using data on earnings inequality and its persistence across generations to calibrate their model, these authors find TFP differences that lead to a TFP elasticity of output per worker similar in magnitude to ours (around 2). Indeed, our results are quite close to those in Erosa et al. (2010), as a modest variation of the key parameter in their model would yield our elasticity estimates (see Erosa et al. (2010), Table 5). Overall, our findings contribute to estimates of potential variation of labor quality

across countries. *Observable* variation in talent via PISA scores, combined with our model of division of labor and investment in the quality of skilled workers, goes a long way towards capturing the variation of labor quality and its interplay with TFP differences.

In spite of the use of a different methodology, our work is also closely related to the approach initiated by Hanushek and Kimko (2000), surveyed by Hanushek and Woessmann (2008) and recently extended by Hanushek and Woessmann (2012). By focusing on growth rates, these authors find a strong and positive association between their measure of educational achievement derived from the international student achievement tests and economic growth.

Our paper is organized as follows. Section 2 presents in detail the data we use in the paper, with particular emphasis on data from the PISA program. Section 3 develops the theoretical framework. Section 4 discusses the parameterization of the model and its mapping to data. In section 5, we perform a number of hypothetical experiments to highlight how the model works. Section 6 presents the basic findings from our experiments. In section 7 we evaluate in detail the quantitative importance of TFP and how our findings compare with those emerging from standard analyses. In section 8 issues, we study the implications our framework for related issues of interest: skill premia and the possibility of skill-biased technology differences across countries. Finally, section 10 concludes.

## 2 Data

We summarize below the central aspects of data that pertain to our study. We concentrate on PISA scores, enrollment rates, the division of labor between unskilled and skilled workers, and corresponding observations on output per worker.

### 2.1 PISA

The PISA is an assessment that was jointly developed by participating economies. In this paper, we use the last assessments by PISA which were carried out in 2009 and include 65 economies. PISA tests abilities in *mathematics*, *reading* and *science*. The study is organized and conducted by the OECD, ensuring as much comparability among participants as

possible.<sup>1</sup>

The PISA sampled students between 15 years and 3 months and 16 years and 2 months old. Only students that are enrolled in an educational institution, regardless of the grade level or type of institution in which they were enrolled are sampled. The average age of OECD country students participating in PISA was 15 years and 8 months, varying by a maximum of only 2 months among the participating countries.<sup>2</sup>

The performance tests lasted for two hours and were taken using paper and pencil. They include both multiple-choice items and questions that require students to construct their own responses. The PISA aims to test not merely the mastery of the school curriculum, but also important knowledge and skills needed in adult life (OECD 2000, pp. 8). Each subject was tested at differing levels of difficulty in order to represent a coherent and comprehensive indicator of the continuum of students' abilities. Using item response theory, PISA mapped performance in each subject on a scale with an international mean of 500 test-score points across the OECD countries and an international standard deviation of 100 test-score points.

We consider fifty nine countries participating in the PISA 2009 study. These are: Albania\*, Argentina\*, Australia, Austria, Belgium, Brazil\*, Bulgaria\*, Canada, Chile, Colombia\*, Croatia\*, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hong Kong-China\*, Hungary, Iceland, Indonesia\*, Ireland, Israel, Italy, Japan, Jordan\*, Kazakhstan\*, Korea, Kyrgyz Republic\*, Latvia\*, Lithuania\*, Luxembourg, Macao-China\*, Mexico, The Netherlands, New Zealand, Norway, Panama\*, Peru\*, Poland, Portugal, Romania\*, Russian Federation\*, Republic of Serbia\*, Singapore\*, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Thailand\*, Trinidad and Tobago\*, Tunisia\*, Turkey, UAE-Dubai\*, United Kingdom, United States, Uruguay\*. Those marked with a "\*" are non-OECD countries.<sup>3</sup> We report results using the Math test score. We note, however, that the correlation

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<sup>1</sup>For detailed information on the PISA study and its database, see the PISA homepage at [www.pisa.oecd.org](http://www.pisa.oecd.org).

<sup>2</sup>The PISA sampling procedure ensured that a representative sample of the target population was tested in each country. Most PISA countries employed a two-stage stratified sampling technique. The first step consists on drawing a random sample of schools, and in the second stage students are randomly sampled in each of these schools making sure that each 15-year-old student in a school has equal probability of selection. This procedure typically yield a sample of between 4,500 and 10,000 students per country.

<sup>3</sup>We exclude from the total list of PISA participating countries China-Shanghai and China-Taipei, given the focus on only one urban area. We also exclude Liechtenstein, Montenegro and Azerbaijan, as we lack data for these countries on the composition of the labor force. Finally, we also exclude Qatar, due to its outlier character in terms of output per worker.

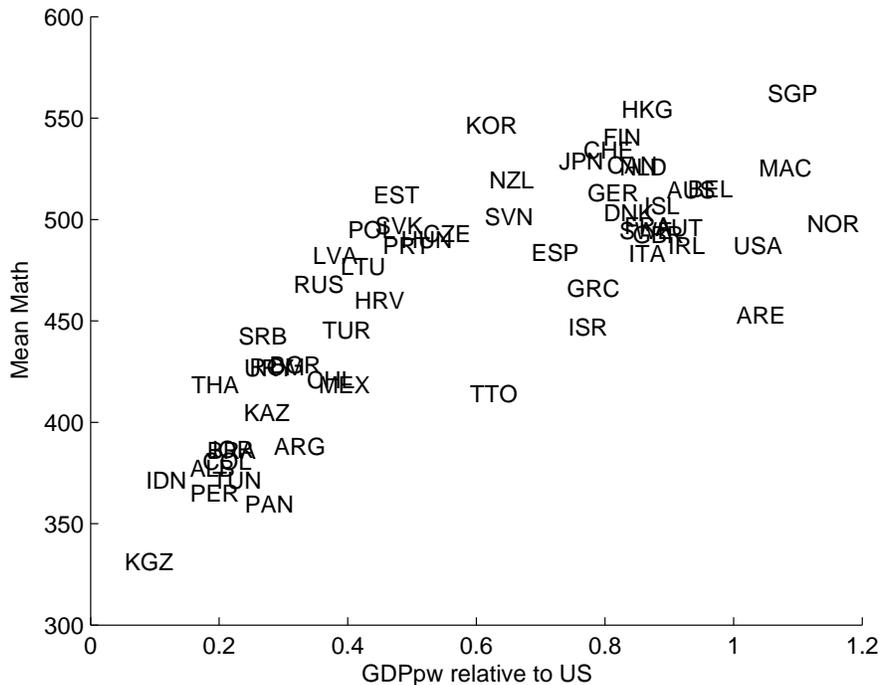


Figure 1: Mean PISA Scores (Math) and GDP per worker.

of student performance between the three subjects is substantial; 0.95 between reading and math, 0.98 between reading and science, and 0.97 between math and science.

## 2.2 Related Data

To characterize facts on cross-country relationship between test scores and economic development, we also use country level data on the countries' GDP per worker in 2007 (PPP) taken from the Penn World Tables 7.0. Since the tests are administered to 15 year-old individuals, and in some countries a non-trivial fraction of them are not at school, we also document and subsequently use the enrollment rates at school at that age provided by the PISA study.

Our framework has central implications for the division of the labor force between *skilled* and *unskilled* workers. For the data counterpart of these concepts, we also consider data from Barro and Lee (2010). We define skilled individuals as those with *strictly more than high school education*, whereas unskilled people are their complement. We consider the fraction of individuals in each group, out of the population 15 years and older.

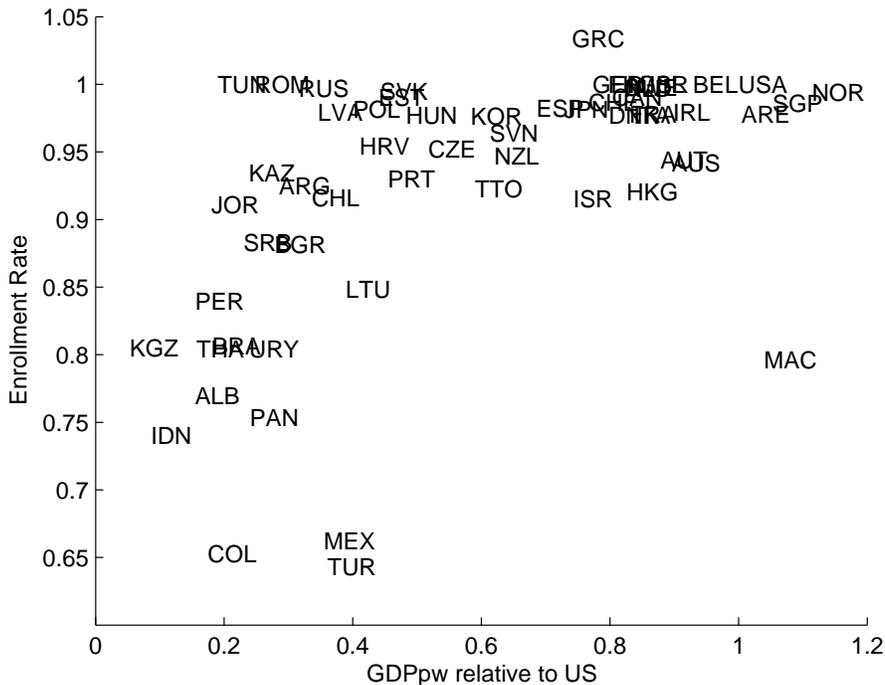


Figure 2: Enrollment Rates (15 years old) and GDP per worker.

### 2.3 Facts Summary

Table 1 and Figures 1, 2 and 3 summarize main features from the data. We note first that there are substantial differences in PISA scores across countries. The ratio of the 90th percentile to the 10th percentile is about 1.4, with a maximum gap of a factor of about 1.7, between Hong Kong and the Kyrgyz Republic. The gap between the United States and the Kyrgyz Republic is a factor of nearly 1.5.

Second, there is a positive relationship between enrollment rates and output per worker, with a correlation in the sample of about 0.50. About 80 percent of individuals at 15 are at school in Uruguay, whereas all of them are in school in Finland and the U.S. At the bottom of the distribution, only about 64 percent of 15-year old individuals are at school in Turkey.

Third, there is a strong and positive relationship between PISA scores and output per worker. Richer countries have are, on average, high achievement scores. As Table 1 indicates, the correlation between mean math score and output per worker is almost 0.8. Differences in output per worker, however, dwarf observed differences in PISA scores. For instance, the

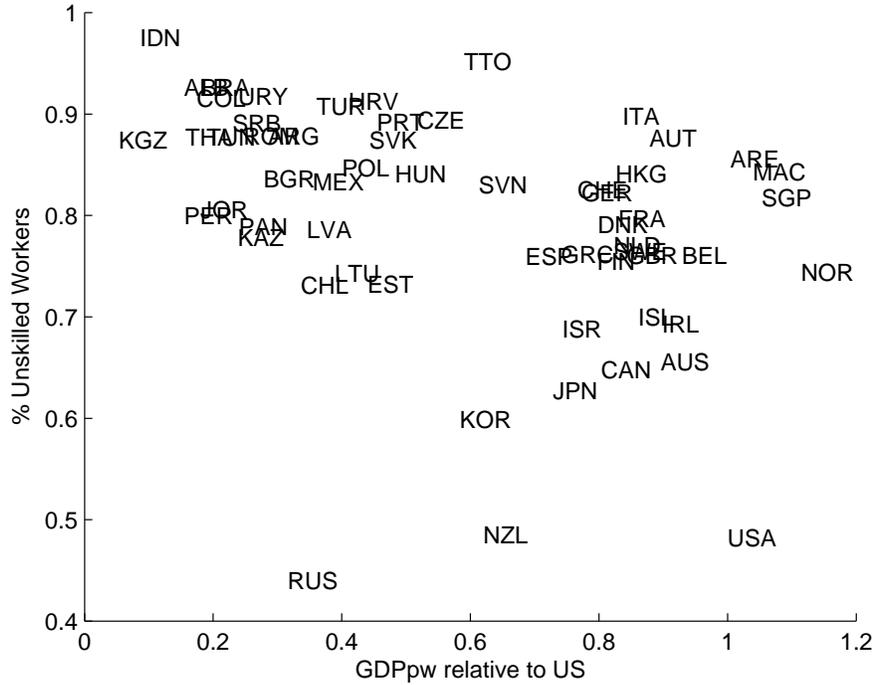


Figure 3: Fraction of unskilled workers and GDP per worker.

gap between the U.S. and Argentina is a factor of nearly 1.3 in PISA scores, whereas the gap in output per worker is about 3.5.

Finally, the data reveals a negative relationship between output per worker and the fraction of unskilled individuals. While about 98 percent of workers in Indonesia (the lowest observation in our sample) are unskilled at an output per worker level of about 9 percent of the U.S., only 48.2 percent of workers are unskilled in the U.S. That is, more than 50 percent of U.S. workers have strictly more than high school education. The correlation between the fraction of unskilled workers and output per worker is about -0.33.

### 3 Theoretical Framework

There is a single representative household in the economy. The household comprises at time  $t$  a continuum of members of total size  $L_t$ , who value only consumption. The size of the household (population) grows at the constant rate ( $g_L$ ). The household is infinitely lived and maximizes

$$\sum_{t=0}^{\infty} \beta^t L_t \log(C_t/L_t), \quad (1)$$

where  $\beta \in (0, 1)$  and  $C_t$  denotes total household consumption at date  $t$ .

**Endowments** Each household member is born with  $z$  units of talent. The talent is distributed with support in  $Z = [0, \bar{z}]$ , with cdf  $G(z)$  and density  $g(z)$ , which is invariant to population growth. Household members have one unit of time which he/she supplies inelastically. Depending upon his type, each household member can be a *skilled* or *unskilled* worker. We describe below this decision and the associated incomes in detail. The household is also endowed with an initial capital stock  $K_0 > 0$ .

**Technology** There is a representative firm that operates a constant returns to scale technology. This technology requires three inputs: capital  $K$ , and two types of labor services: skilled labor  $S$  and unskilled labor  $U$ . Output ( $Y$ ) is given by

$$Y = F(K, U, S; A) = A K^\alpha [\mu U^\rho + (1 - \mu) S^\rho]^{(1-\alpha)/\rho} \quad (2)$$

where  $A$  stands for a Total Factor Productivity parameter, and  $\rho \in (-\infty, 1)$ . The elasticity of substitution between skilled and unskilled labor is  $1/1 - \rho$ . Output per worker,  $y \equiv Y/L$ , is given by

$$y = A k^\alpha l^{1-\alpha}, \quad (3)$$

with  $l \equiv [\mu u^\rho + (1 - \mu) s^\rho]^{1/\rho}$ ,  $u \equiv U/L$  and  $s \equiv S/L$ . We subsequently refer to  $l$  as *labor input* or *labor quality*.

The representative firm behaves competitively and faces rental prices  $R, W_S, W_U$  for the use of capital, skilled and unskilled labor, respectively. Capital depreciates at the rate  $\delta$ .

**The Household Problem** We assume that the segregation of individuals by skill is costly. This segregation only applies to newborns, and cannot be changed once a household member has been assigned to either pool. Converting one newborn into a skilled worker requires goods, and implies foregone earnings for a period. If a newborn household member

is selected for the unskilled labor pool at  $t$ , her talent is contemporaneously transformed into efficiency units of unskilled labor, and her income is given by  $W_{U,t}z$ .

If she becomes part of the skilled pool instead, it takes one period to contribute to skilled labor. The household invests  $x_t$  consumption goods in the ‘quality’ of her efficiency units, an amount common to all household members. Investing  $x_t$  implies that her talent is amplified by the factor  $h_{t+1}$ , where  $h_{t+1} = x_t^\phi$ ,  $\phi \in (0, 1)$ . Her contribution to household’s income is then given at  $t + 1$  by  $W_{S,t+1}zh_{t+1}$ . It follows that only individuals with sufficiently high levels of talent become skilled. Given rental prices, there exists a unique threshold  $\hat{z}_t$  such that newborn household members with talent below this threshold become unskilled workers at  $t$ , and those with talent above it are skilled workers, from  $t + 1$  on.

If there are  $N_t \equiv g_L L_{t-1}$  newborns in each period, and  $N_t(1 - G(\hat{z}_t))$  newborns become skilled, the household invests a total of  $N_t x_t (1 - G(\hat{z}_t))$  goods at  $t$  to amplify their talent with quality  $h_{t+1}$ . Then, the aggregate quantities of unskilled and skilled labor evolve according to

$$U_t = U_{t-1} + \underbrace{N_t \int_0^{\hat{z}_t} z g(z) dz}_{\text{Additions to unskilled labor}} \quad (4)$$

$$S_t = S_{t-1} + \underbrace{N_{t-1} h_t \int_{\hat{z}_{t-1}}^{\bar{z}} z g(z) dz}_{\text{Additions to skilled labor}} \quad (5)$$

We also assume that the cost for the household of transforming one unit of consumption into investment, is potentially different from one. We represent these costs by an exogenous barrier  $p_t \geq 1$ .

The problem of the household is to choose sequences of consumption, the fractions of household members who are skilled and unskilled, the quantity of goods allocated to affect the productivity of each skilled member, and the amount of capital to carry over to the next period. Formally the household problem is to select  $\{C_t, I_t, \hat{z}_t, x_t\}_0^\infty$  to maximize (1) subject to (4), (5),

$$C_t + I_t + N_t(1 - G(\hat{z}_t))x_t = (W_{U,t}U_t + W_{S,t}S_t) + R_t K_t,$$

$$h_{t+1} = x_t^\phi,$$

$$K_{t+1} = (1 - \delta)K_t + \frac{I_t}{p_t}$$

with

$$N_0, S_0, U_{-1}, K_0 > 0.$$

The solution to the household problem is then characterized by the following First Order Conditions:

$$\frac{p_t}{C_t/L_t} = \beta \frac{[R_{t+1} + (1 - \delta)p_{t+1}]}{C_{t+1}/L_{t+1}} \quad (6)$$

$$\frac{W_{U,t} \hat{z}_t + x_t}{C_t/L_t} = \beta \frac{W_{S,t+1} \hat{z}_t x_t^\phi}{C_{t+1}/L_{t+1}} \quad (7)$$

$$\frac{1 - G(\hat{z}_t)}{C_t/L_t} = \beta \frac{W_{S,t+1} \left( \int_{\hat{z}_{t+1}}^{\bar{z}} z g(z) dz \right) \phi x_t^{\phi-1}}{C_{t+1}/L_{t+1}} \quad (8)$$

Condition (6) is the standard Euler equation for capital accumulation. Condition (7) states that the discounted compensation of the household member with marginal skill  $\hat{z}_t$  at  $t$ , weighted by the marginal utility of consumption at  $t + 1$ , must be equal to the compensation of an unskilled household member *plus* the cost of skill conversion,  $x_t$ , weighted by the marginal utility of consumption at  $t$ . Finally, condition (8) states that the marginal cost of investing one unit of the consumption good in the quality of a skilled worker must equal its discounted marginal benefit. This benefit depends on the rental price of skilled labor at  $t + 1$  and the ‘raw’ addition member to the pool of skilled labor at  $t + 1$ ,  $\int_{\hat{z}_t}^{\bar{z}} z g(z) dz$ .

**Equilibrium** In competitive equilibrium, the markets must clear and factor prices are paid their marginal products. Equilibrium in the markets for unskilled and skilled labor implies

$$U_t^* = U_{t-1}^* + N_t \int_0^{\hat{z}_t^*} z g(z) dz \quad (9)$$

$$S_t^* = S_{t-1}^* + N_{t-1} h_t^* \int_{\hat{z}_{t-1}^*}^{\bar{z}} z g(z) dz, \quad (10)$$

for all  $t = 0, 1, 2, \dots$ , where a ‘\*’ over a variable denotes its equilibrium value. Equilibrium in the goods market implies

$$C_t^* + I_t^* + N_t(1 - G(\hat{z}_t^*))x_t^* = Y_t^*,$$

Factor prices equal

$$W_{U,t}^* = \frac{\partial F(K_t^*, S_t^*, U_t^*)}{\partial U_t} \quad (11)$$

$$W_{S,t}^* = \frac{\partial F(K_t^*, S_t^*, U_t^*)}{\partial S_t} \quad (12)$$

$$R_t^* = \frac{\partial F(K_t^*, S_t^*, U_t^*)}{\partial K_t} \quad (13)$$

for all  $t = 0, 1, 2, \dots$ . It is now possible to define a competitive equilibrium. A competitive equilibrium is a collection of sequences  $\{C_t^*, K_t^*, \hat{z}_t^*, x_t^*, W_{U,t}^*, W_{S,t}^*, R_t^*\}_0^\infty$ , such that (i) given  $\{W_{U,t}^*, W_{S,t}^*, R_t^*\}_0^\infty$ , the sequences  $\{C_t^*, K_t^*, x_t^*, \hat{z}_t^*\}_0^\infty$  solve the household problem; (ii) factor prices are competitive for all  $t$ ; (iii) markets clear for all  $t$ .

### 3.1 Balanced Growth

Along a balanced growth path, aggregate investment of both types as well as output, consumption, skilled and unskilled labor grow at the constant population growth rate  $g_L$ . Factor prices, capital per worker ( $k \equiv K/L$ ), investment per new skilled worker  $x$ , and the threshold  $\hat{z}$  are constant.

Note that the laws of motion for unskilled labor imply that unskilled labor per worker ( $u \equiv U/L$ ) equals  $\int_0^{\hat{z}^*} z g(z) dz$  along the balanced growth path. Similarly, skilled labor per worker, ( $s \equiv S/N$ ), equals

$$s^* = \frac{h^* \int_{\hat{z}^*}^{\bar{z}} z g(z) dz}{1 + g_L}$$

Given the properties of the aggregate technology, we can write  $Y = Lf(k, s, u)$ . Hence, along the balanced growth path, (6) and competitive factor prices imply

$$\frac{\partial f(k^*, s^*, u^*)}{\partial k} = p \left( \frac{1}{\beta} - (1 - \delta) \right). \quad (14)$$

Similarly, condition (7) and competitive prices imply

$$\frac{\partial f(k^*, s^*, u^*)}{\partial u} \hat{z} + x^* = \beta \frac{\partial f(k^*, s^*, u^*)}{\partial s} \hat{z}^* x^{*\phi} \quad (15)$$

Likewise,

$$(1 - G(\hat{z}^*)) = \beta \frac{\partial f(k^*, s^*, u^*)}{\partial s} \left[ \int_{\hat{z}}^{\bar{z}} z g(z) dz \right] \phi x^{*\phi-1} \quad (16)$$

Hence, (14), (15) and (16) can be used to solve for a steady-state equilibrium. They determine the steady-state capital stocks per worker of both types, the threshold value  $\hat{z}^*$  and the investment in labor quality  $x^*$ .

How changes in TFP and investment barriers affect the allocation of talent and then, the segregation of individuals between skilled and unskilled workers? We show below that under empirically plausible conditions, economies with low TFP and high investment barriers are characterized by a lower fraction of skilled individuals than their counterparts.

**Assumption 1** *The density  $g(\cdot)$  is log-concave.*

**Proposition 1** *Let  $\rho \in [0, 1)$ . Assume that assumption 1 holds. Then,*

1. *There is a unique steady state with  $\hat{z}^* > 0$  and  $x^* > 0$ .*
2. *An increase in TFP (a reduction in investment barrier) implies:*
  - *a reduction in  $\hat{z}^*$ , if  $\rho \in (0, 1)$ ;*
  - *no effects on  $\hat{z}^*$ , if  $\rho = 0$ .*  
*(Cobb Douglas case)*
  - *No effects on  $\hat{z}^*$  if  $\phi \rightarrow 0$ .*

**Proof.** See Appendix. ■

The above proposition is important in many respects and some comments are in order. First, it provide conditions under which a unique steady state exists. This is naturally important for the quantitative analysis that we conduct later. These conditions are sufficient, and easily satisfied. Many widely used distributions satisfy the log-concavity requirement; this family includes the uniform, the exponential, the normal and the gamma distributions. Second, the proposition shows that the relative fraction of skilled workers and investment in their quality *always* increases with an increase in TFP (reduction of investment barrier) when the parameter restrictions are satisfied. Thus, whether or not these forces account for the magnitudes of changes empirically observed is a quantitative issue. It is worth emphasizing here that a host of empirical estimates of the elasticity of substitution between skilled and labor fall in the range required. For the limiting case of the Cobb-Douglas production function, changes in TFP or relative investment prices have no effects on the skill segregation. Finally, investments in goods are *essential* for any steady-state effects of TFP or relative investment prices on skill segregation. In the absence of such investments, changes in these parameters are neutral.

## 4 Parameter Values

We start by setting the model period equal to *four* years, a compromise regarding the time it takes to become a skilled worker. To calibrate parameter values we use the U.S. as a benchmark. We later explain what aspects of our parameterization change for the cross-country analysis.

**Technology** We set the capital share ( $\alpha$ ) in the aggregate production function to standard value of 0.33. Empirical studies indicate an elasticity of substitution between skilled and unskilled labor of around 1.5 (Katz and Murphy (1992), Heckman, Lochner, and Taber (1998)). Then, we select the parameter  $\rho$  in the aggregate production technology to  $\rho = 1/3$ . The depreciation rate is set to 26.6% (7.4% annual level), so the model is consistent with an investment to output and capital to output ratios in data. Using NIPA data for the period 1960-2010 and a broad notion of capital, we calculate an average investment to output ratio of 0.27 and a related capital to output ratio of 0.8 at the four year frequency (3.2 at the

annual level). We normalize the Total Factor Productivity level ( $A$ ) to 1.0.

From the technology side, the curvature parameter in the production of skills ( $\phi$ ) and the share parameter in the production function ( $\mu$ ) remain to be set. We choose these parameters so that in stationary equilibrium the model reproduces two empirical targets. The first target is the fraction of unskilled workers in the U.S. from Barro and Lee (2010). This amounts to about 48%, and corresponds to the fraction of the population with completed secondary education or less, aged 15 years or older. The second target are expenditures per tertiary student as a fraction of GDP per worker (at PPP values). This fraction amounted to 28.7% in 2004, and is taken from the UNESCO 2007 World Education Indicators report.

**Preferences and Demographics** We choose the discount factor so that the steady-state capital to output ratio amounts to 0.8, or about 3.2 at the yearly frequency. This implies a discount factor ( $\beta$ ) equal to 0.869, or 0.966 at the yearly frequency. This choice dictates an interest rate at the annual level of about 3.6%. We set the growth rate in population equal to the annual value of about 0.9%, calculated using the population figures in the Penn World Tables 7.0.

**Talent and Individual Earnings** We calibrate the distribution of talent using PISA data. We assume that the distribution of talent is a Gamma distribution with parameters  $\kappa$  and  $\theta$ . We choose  $\kappa$  and  $\theta$  in order to reproduce the observed *mean* and *coefficient of variation* in the PISA test for Mathematics for the U.S. We assume that talent units from PISA are mapped into individual earnings (efficiency units) for unskilled workers as  $W_U^* z^\epsilon$ , and for skilled workers as  $W_S^* z^\epsilon h$ . This amounts to a choice of units for our model, that we discipline with cross sectional data. We pick  $\epsilon$ , in conjunction with the rest of parameter values, so that the model reproduces in stationary equilibrium the relevant skill premium for males in U.S. data. From the U.S. Current Population Survey, we calculate that average earnings (per worker) of skilled to unskilled workers averaged 1.88 for the period 2001-2010.<sup>4</sup>

Table 2 and 3 below shows the resulting parameter values. Our parameterization implies  $\phi = 0.34$ ,  $\mu = 0.323$  and  $\epsilon = 0.26$ . It is not problematic for the model to reproduce the data, as the Table illustrates. The table also shows that our parametric Gamma approximation

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<sup>4</sup>See section 8 for a discussion of the notion of skill premia in the context of the model.

to the distribution is very good, as it generates multiple percentiles of the distribution quite well.

## 5 Model Mechanics

We explore in this section the long-run effects of a number of changes in the calibrated parameters of the benchmark economy. In doing so, we highlight the role of different forces at work, and help the reader understand the main results in subsequent sections.

Specifically, we consider the following empirically-motivated departures from the benchmark case. We consider a reduction in the mean talent, an increase in the dispersion of talent, as well as a reduction in TFP. We first entertain all these changes in isolation. We then combine all these changes.

The motivation for the experiments that follow is straightforward. We documented in section 2 substantial differences in PISA scores across countries and the correlation between the PISA scores and output per worker. In addition, since there are within-country variation in test scores that may matter in context of our environment, we also assess the effects of changes in dispersion. Finally, the variation in TFP, in conjunction with changes in the distribution of PISA scores, contributes to assessing the relative importance of TFP for output variation and the the division of labor across countries.

**A Reduction in Mean Talent** In our first experiment, we consider a reduction in the mean math score to the level in the 10th percentile country in the cross-country distribution of PISA scores. Results are summarized in Table 4. Intuitively, all else remaining the same, a reduction in the mean talent leads to a reduction in the value of investing in skilled workers (see equations 8 or 16). Hence, as Table 4 shows, this reduction leads to a reduction in the augmented skills of skilled workers ( $h$ ) and to an increase in the threshold  $\hat{z}$  across steady states, which in turn implies an increase in the fraction of unskilled workers. Given these changes, the wage premium ( $W_S^*/W_U^*$ ) increases as the ratio of skilled to unskilled labor declines.

Quantitatively, the effects of the reduction in mean talent on output per worker is not large. Reducing mean talent from the benchmark case to the bottom 10th percentile implies a reduction of about 18%. The corresponding decline in output across steady states is about

8.1%.

**An Increase in Dispersion of Talent** Within-country dispersion in PISA scores varies inversely with the level of economic development. Hence, in our second experiment we consider an increase in the *coefficient of variation* of talent to the level in the 90th percentile country in the cross-country distribution of this variable. That is, we make the distribution of talent more unequal by increasing the coefficient of variation from the benchmark value of 0.19 to 0.23. Table 4 shows that this change leads to an increase in the number of unskilled individuals across steady states. This reduction also results in an *increase* in the investments on skilled workers. This is intuitive: an increase in dispersion, all else equal, implies an increase in the talent of those at top tail of the talent distribution; i.e. skilled workers. Quantitatively, as Table 4 shows, the most significant changes are in the fraction of unskilled workers, that goes up by about 1%. Given our parameterization, a more unequal distribution of talent results in higher output. However, output increases mildly by about 0.4%.

**A Reduction in Total Factor Productivity** Reducing the value of TFP leads to the results discussed in Proposition 1. As goods are an input for the quality of skills and since the elasticity of substitution between labor inputs is higher than 1, the reduction in TFP leads to an increase in the threshold  $\hat{z}$  and to a reduction in the quality of skilled labor. This translates into an increase in the steady-state value of unskilled labor, and an increase in the wage premium.

Quantitatively, the magnitude of the effects in Table 4 are substantial. Reducing the value of TFP by 50 percent implies an increase in the fraction of unskilled workers of about 11.4%, a reduction in augmented skills of about 36% and a reduction in output of nearly 74%. As we elaborate later, reductions of TFP of these magnitudes are indeed required to account for the output per worker gaps in our sample.

It is important to note that the effects of TFP differences on output in our model are significantly larger than those emerging in the standard model under the same capital share. Halving TFP in the growth model leads to a reduction in output about 65 percent in the growth model (versus a 74 percent reduction in our model). The additional *amplification* effect in our setup stems from the fact that the labor input is endogenous. In addition to the

standard effects on capital formation, a reduction in TFP leads to a different composition of the labor force (higher fraction of unskilled workers) and to lower investments in skilled labor.

**All Together Now** The last row in Table 4 shows the combined effects of all changes simultaneously. As a poor country in our sample is similar to this hypothetical scenario, the combined effects are illustrative of the quantitative implications of our model. The table shows that when all changes are combined, the fraction of unskilled labor increases by 13.1%, the augmented skills drop by more than a third, and output drops by about 76%. The table also demonstrates that the changes in TFP capture the bulk of the effects on output and the composition of the labor force.

## 6 Findings

We investigate in this section the extent to which the forces in our model can account for the observed differences in the division of labor in our sample of countries, and the corresponding differences in output per worker and the quality of labor.

Our analysis explicitly takes into account the international heterogeneity in enrollment rates that we documented earlier. Only a fraction of  $\gamma \in (0, 1)$  of the unskilled pool in the model can be converted into skilled workers – those who are at school at age 15 in the data. We assign to the rest of the cohort (fraction  $1 - \gamma$ ) a common set of skills,  $z_{min}$ . This procedure implies that the stationary values of unskilled and skilled labor per worker obey, respectively,

$$u^* = (1 - \gamma)z_{min}^\epsilon + \gamma \int_{z_{min}}^{\hat{z}^*} z^\epsilon g(z) dz,$$

$$s^* = \frac{\gamma}{1 + g_L} h^* \int_{\hat{z}^*}^{\infty} z^\epsilon g(z) dz.$$

For each country,  $\gamma$  is the fraction of the population that is 15 years old and is enrolled in the educational system, as reported in PISA data. We set  $z_{min}$  for each country equal to the 10th percentile of the PISA Math test score for that country. We subsequently investigate the sensitivity of our results to these choices.

The values of the country-specific population growth rates are those reported in the Penn World Tables 7.0.

We conduct two sets of experiments below. First, we assign to each country the distribution of talent parameterized from PISA data, and compute stationary equilibria. The results and their relation to data are reported in Table 5. Second, we add variation in relative investment prices and TFP across countries. We use the investment prices reported in the Penn World Tables 7.0, and find the relative TFP levels such that the output per worker in the model is the same as in the data. The second experiment shows the consequences of *all* of the driving forces in our model and the results are reported in Table 6. For both experiments, we organize our results according to the observed distribution of output per worker; e.g. poorest 10% countries versus richest 10% countries.

**Variation in PISA Scores** Our results indicate that adding *only* the variation in PISA scores leads to lower investments in the quality of skilled labor and a higher fraction of unskilled labor in poorer countries. In turn, these changes lead to lower values of aggregate capital in poorer economies, and given the effects on labor inputs, result in lower values of output.

Quantitatively, variation in the PISA scores implies that augmented skills ( $h$ ) in the poorest 10% of countries are about 96.5% of that in the richest 10% (see Table 5). Similarly, the fraction of unskilled labor in the poorest 10% is about 3.1% higher than that in the the richest 10%. Output per worker, as a result, is about 90% in the poorest 10% relative to the richest 10%.

As Table 5 demonstrates, the model generates less than 20% of the gap in the fraction of unskilled workers between the poorest 10% and the richest 10%. In terms of output per worker, the productivity in the richest 10% of the countries is about 11% higher than in the poorest 10%. In the data, the output gap amounts to a factor of more than 8. As in the previous literature, there is still an ample role for TFP differences across countries to account for observed income differences.

**Variation in Technology and PISA Scores** When we add technology variation (i.e., TFP variation and observed variation in relative price of investment) to the observed variation in PISA scores, we find that our framework can now account for a substantial

fraction of the differences in the division of labor across countries (see Table 6). With TFP levels that match the output per worker levels *exactly*, the model captures more than 100% of the differences in the fraction of unskilled workers between the poorest 10% and the richest 10%. Thus, with technology differences and their interplay with the PISA score differences, the model can go a long way toward understanding the cross-country differences in the division of the labor.

Table 6 illustrates how large TFP differences need to be in order to account of the output per worker differences. TFP at the poorest 10% is about 37% of that at the richest 10%. Not surprisingly, augmented skills are now substantially lower in poorer countries (only 49% relative to richer countries) since TFP has an indirect effect on the division of labor and the investment in skilled workers. These changes in quality and division of labor, in conjunction and in response to lower levels in talent, result in smaller differences in TFP than those that emerge from standard analyses. We elaborate on this issue in the next section.

## 7 TFP and Economic Development

We now use our model to assess the relative importance of Total Factor Productivity vis-a-vis other factors in accounting for the differences in output per worker across countries. We also provide a comparison of the findings from our model with standard findings in the development literature.

**The Relative Importance of TFP** Recall that differences in talent, relative prices and TFP are not enough to reproduce differences in the division of the labor force. We now force the model to reproduce *exactly* both the levels of output per worker and the fraction of unskilled workers. Specifically, we introduce an implicit tax (*wedge*) in the model, so that in conjunction with TFP variation and PISA scores, the model can jointly reproduce output per worker differences in our sample and the division of the labor force by skill in stationary equilibrium, country by country. The tax wedge affects the conversion of talent into skilled labor: if the household transfers an individual with talent  $z$  to the skilled pool at  $t$ , the contribution to household income at  $t + 1$  is now

$$W_{S,t+1} z^\epsilon h_{t+1} (1 - \tau),$$

where  $\tau$  is the tax wedge. Tax collections are returned to the household in a lump-sum manner.

We present the results in Table 7. As the table shows, differences in TFP are smaller relative to those in Table 6. The model requires *higher* tax wedges in poorer countries than in rich countries. These country-specific wedges lead to wider differences between rich and poor countries in the values of the aggregate labor input. These larger cross-country differences in aggregate labor, in turn, lead to TFP differences across countries that are smaller. In Table 7, TFP at the poorest 10% countries is about 41% of the richest 10% countries whereas in Table 6 the corresponding number is only 37% when the model was not forced to reproduce the cross-country variation in the division of labor.

Tables 5, 6 and 7 show the progression of the magnitude of the labor input per worker (labor quality) across experiments. The first experiment ('variation in PISA scores only') indicates that the labor input at the poorest 10% countries is about 89% of the labor input at the richest 10% countries. When variation in technology is added in the second experiment, and TFP is chosen to match output per worker differences, the corresponding figure for labor input is about 55%; matching also the division of labor by skill in the third experiment leads to a labor input of 45%. In our model, more than half of the rich to poor ratio of labor input per worker is delivered by (measured) differences in achievement as of age 15 (i.e., PISA scores) and the rest is due to the combined effects of TFP, relative prices, tax wedges and their interplay with achievement scores.

Summing up, matching the division of labor within countries leads to smaller TFP differences between rich and poor countries. Quantitatively, however, the bulk of the resulting TFP differences are already captured when we force the model to reproduce output per worker across countries. Matching the division of labor exactly contributes only modestly to reducing the implied TFP gaps across countries.

**Comparison with Standard Exercises** We now compare the predictions of our model with those resulting from the one-sector growth model with differences in labor quality across economies. These differences in labor quality are in turn driven by differences in years of schooling via Mincerian returns.

Recall that in our model, output per worker ( $y$ ) can be written as

$$y = Ak^\alpha l^{1-\alpha},$$

where  $k$  is capital per worker and  $l$  is labor per worker. This provides the basis for a comparison with findings from the standard one-sector model; the central difference between our model and the standard one is in the notion of labor per worker.

As in Hall and Jones (1999), Caselli (2005) and others, suppose that labor per worker in each country is given by

$$l = \exp \psi(s),$$

where  $\psi$  is a function of years of schooling ( $s$ ), determined by rates of return that vary with average years of schooling in consistency with empirical estimates; Psacharopoulos (2004). Specifically, we set  $\psi(s) = 0.134 s$  for  $s \in [0, 4]$ ,  $\psi(s) = 0.134 \cdot 4 + 0.101 (s - 4)$  for  $s \in (4, 8]$ , and  $\psi(s) = 0.134 \times 4 + 0.101 \times 4 + 0.068 \times (8 - s)$  for  $s > 8$ . We use the average years of schooling in each country as reported by Barro and Lee (2010). All other parameters in the model are set as previously in Section 4. We input into the one-sector growth model the relative prices of investment from Penn World Tables 7.0 and find the levels of TFP that reproduce the observed relative levels of output per worker.

Results are presented in Table 8, where we also present for comparison the corresponding results from our model when we match the division of labor by skill as well as when we do not. As the table indicates, the implied differences in TFP are much *larger* in the one-sector growth model with labor quality measured via Mincerian returns. In the one-sector growth model, TFP at the poorest 10% countries is about 28% of the richest 10% countries (Panel A); to contrast, the ratio in our model is 41.1% when we match the division of labor by skill (Panel C). Clearly, the larger TFP differences are due to the differences in the value of labor quality in the two cases. In the one-sector growth model, labor quality is about 86% at the poorest 10% relative to the richest 10% whereas our model generates differences 45.4%. Figure 4 illustrates how labor quality varies with development across all income levels, by comparing labor quality in our model with labor quality under the conventional Mincer measure. As the figure shows, the slope of the predicted relationship is much larger in our model than under the conventional notion of labor quality.

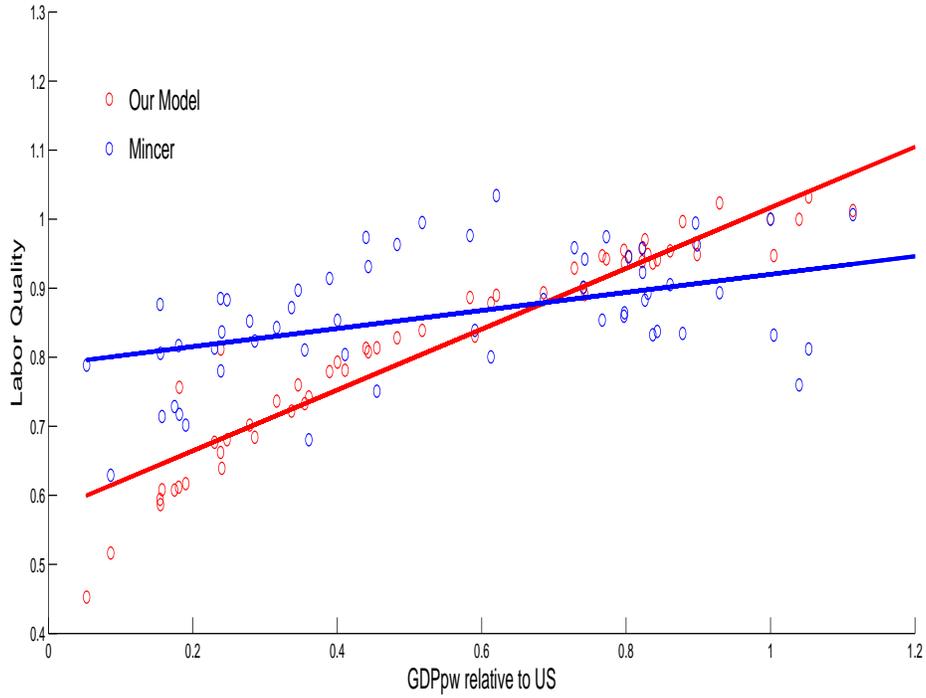


Figure 4: Labor Quality and GDP per worker. Model versus Mincerian measure

**Elasticities** A related and very convenient way to summarize our findings is to calculate the elasticity of output per worker across countries with respect to TFP. As it is well known, the one-sector growth model without labor quality adjustments implies an elasticity of 1.5. With labor quality measured via Mincerian returns, we estimate an elasticity of around 1.7 since labor quality is correlated with output per worker. Our model implies elasticities that are *larger*, of about 2.1-2.2; the elasticity amounts to 2.08 when we do not match the division of labor in each country, and about 2.17 when we do.

**Summary** In our model, TFP not only has the standard effect on output but also an indirect effect on our measure of labor input through the division of labor by skill and the augmenting of skilled labor. In other words, the labor input in our setup is a function of TFP. This important feature is reflected in our finding of an elasticity of output per worker with respect to TFP that is larger than the one obtained using the standard growth model. We conclude from our findings that the explicit consideration of achievement scores in the context of our model considerably reduces the importance of TFP in our understanding of

cross-country income differences. Nonetheless, it is worth emphasizing that our findings still attribute a central role to TFP differences. As Table 8 shows, TFP differences between the richest 10% and the poorest 10% countries in our sample are in excess of a factor of two.

## 8 Skill Premia

In this section, we investigate and discuss the implications of our model for the relative earnings of skilled to unskilled workers, or *skill premia*. This is important, as it reflects on the importance of the economic mechanisms that our paper highlights for data that we do not explicitly target.

We take measures of skill premia from Fernandez, Guner, and Knowles (2005). The data used by the authors are calculated directly from national surveys from 1990 to 1997. The countries included are Argentina, Australia, Belgium, Bolivia, Brazil, Canada, Chile, Colombia, Costa Rica, Czech Republic, Denmark, Ecuador, Finland, France, Germany, Hungary, Israel, Italy, Luxembourg, Mexico, Netherlands, Norway, Panama, Paraguay, Peru, Poland, Slovakia, Spain, Sweden, Taiwan, United Kingdom, Uruguay, United States and Venezuela. Strictly, this is a subset of the broad set of countries of our benchmark group. The skill premium is defined as the ratio of *earnings (labor income) per worker*, for skilled male workers to unskilled ones.<sup>5</sup> A skilled worker is an individual who has more years of education than those required to complete secondary school. Therefore, this definition is consistent with the data we use on educational attainment. Figure 5 plots skill premia (relative to the U.S. value) against output per worker for each country (relative to the U.S.). As the figure shows, there is a *negative* correlation between these two variables, with a correlation coefficient of  $-0.75$ . The skill premium in the poorer countries of this subsample is substantially higher than in richer countries. Table 9 indicates that the skill premium in the poorest ten percent of countries is about 2.4 times the skill premium in the richest ten percent.

We now construct a notion of skill premia within our model that is consistent with the data we focus on. Recall that earnings of an unskilled worker are given by  $W_U^* z^\epsilon$ , whereas the earnings of a skilled worker are given by  $W_S^* z^\epsilon h^*$ . Then, the skill premium (SP) defined as the ratio of per-worker earnings of skilled workers to per-worker earnings of unskilled workers, is given by

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<sup>5</sup>In their sample, workers are *husbands* of age between 36 and 45 years old.

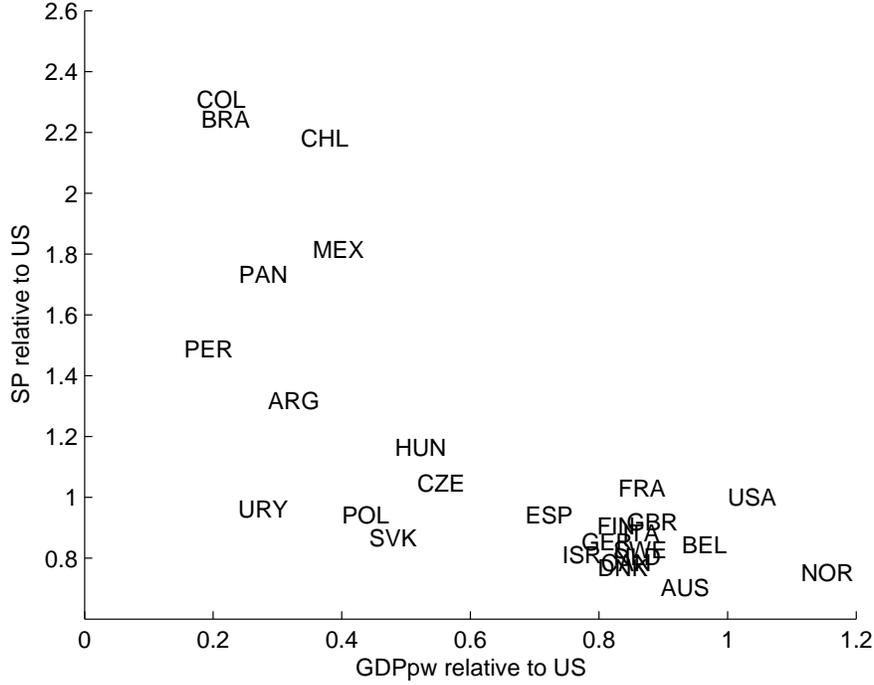


Figure 5: Skill Premia (relative to U.S.) and GDP per worker.

$$SP = \frac{(W_S^* h^* \int_{\hat{z}^*}^{\bar{z}} z^\epsilon g(z) dz) / (1 - G(\hat{z}^*))}{(W_U^* \int_0^{\hat{z}^*} z^\epsilon g(z) dz) / G(\hat{z}^*)}$$

We can rewrite the above as an expression that involves *relative wages*, augmented skills ( $h$ ), and conditional expectations of talent:

$$SP = \left( \frac{W_S^*}{W_U^*} \right) h^* \left( \frac{E(z^\epsilon | z \geq \hat{z}^*)}{E(z^\epsilon | z < \hat{z}^*)} \right)$$

We note that the notion of skill premia consistent with the data is *not* just relative wages. Hence, changes in the division of the labor force by skill will generate movements in all components components above. Notice that an increase in the fraction of unskilled workers (i.e. an increase in  $\hat{z}^*$ ) reduces the ratio of the conditional expectations of talent, but increases relative wages. Henceforth, changes in skill premia implied by the model are the result of countervailing forces.

**Findings** In line with our benchmark analysis, we conduct two types of experiments using only data from the restricted sample of countries. First, along with differences in PISA scores, we consider technology variation, forcing the model to reproduce differences in output per worker via TFP differences. In our second experiment, in addition to technology variation, we introduce tax wedges in order to reproduce the division of labor by skill.

Table 9 shows the performance of the model in terms of skill premia and its components. When we consider only technology variation within our restricted sample, the model does not perform well in terms of the observed differences in the division of the labor force by skill. As Table 9 demonstrates, this performance is effectively worse than in our benchmark case. In terms of skill premia, the model does not generate substantial differences across countries. In the data, skill premia at the top 10% poorest countries is about 144% higher than at the richest 10%; in the model it is just 2.7%.

When the model is forced to reproduce the division of labor by skill along output per worker via productivity differences, it generates a higher skill premia at the top 10% poorest countries that is about 115% higher than the richest 10%, and about 59% higher at the richest 20% versus the bottom 20%. We conclude that forcing the model to reproduce the division of labor by skill goes a long way towards accounting for skill premia. As we discussed earlier, accounting for the observed division of labor is not critical in quantifying the importance of productivity for income differences in our model. Nevertheless, reproducing the division of labor appears key in accounting for variation in skill premia.

## 9 Discussion

We provide below a few calculations in order to highlight the importance of aspects of the data and our environment. We investigate the potential importance of large changes in the distribution of talent, and the role of the components of the labor input as defined in this paper. For the latter case, we conduct separately three different experiments; we investigate the consequences of considering labor quality just as average PISA scores, the importance of investments in the quality of skilled workers as an amplification mechanism, and the role of a larger elasticity of substitution between skilled and unskilled labor.

## 9.1 How Important are Differences in Talent?

A natural way to assess the impact of differences in talent across countries is to conduct a counterfactual exercise where we assign to the poorest country in the sample the PISA distribution of a rich one, the United States.

The poorest country in our sample is the Kirgыз Republic (Kyrgyzstan), which incidentally has the lowest mean value of PISA (math) in the sample. This country has an output per-worker of just about 5.6 % of the United States in our data (the U.S. is richer by a factor of nearly 17). Our model implies that assigning Kyrgyzstan the PISA distribution of the U.S., leaving all else (e.g. TFP) the same, would lead to a level of output per worker that is about 14.2% higher than in the benchmark situation; a minor change given the large observed income disparities. Since the capital to output ratio is unchanged across steady-states, the experiment implies a change in the labor input of Kyrgyzstan of also about 14.2%, which stems from an increase in both the unskilled and skilled labor inputs. The fraction of unskilled workers drops by about 2%.

This exercise illustrates a central implication of the findings of our paper. While differences in talent and the endogeneity of labor quality in our model reduce non trivially the TFP differences required to reproduce income differences across countries, TFP differences are still the dominant force in accounting for income differences.

## 9.2 PISA as Labor Quality

Suppose that instead of using our theory of labor quality, we just simply use the *mean* levels of PISA scores as a measure of each country's labor input. Thus, there is no division of workers by skill and no investments in the quality of skilled labor. What would be the consequences for relative TFP's, etc, in this case?

To answer this question, we assign the mean levels of PISA scores as each country's labor input, and then proceed to find the level of TFP in each country that reproduces the empirical level of output per worker. Results are displayed in Table 10. The elasticity of output with respect to TFP is now of around 1.8. Hence, the resulting TFP differences are substantially larger than in the benchmark case, and quite similar to those emerging in the one-sector model with labor quality differences defined by Mincerian returns.

### 9.3 Amplification Effects: No Investments in Quality

What are the implications in terms of the amplifying role of TFP differences associated to investments in the quality of skilled workers? To answer this question, we set the parameter  $\phi$  to zero, eliminating as a result any investment incentive for the representative household in each economy. We then proceed to find the relative levels of TFP that reproduce the empirical levels of output per worker. Results are displayed in Table 10. TFP differences between poor and rich countries become significantly larger. With no investments in skilled workers, TFP levels in the poorest 10% of countries are just 27% of TFP levels in the richest 10%. This results in a corresponding elasticity of output with respect to TFP of around 1.65, quite similar to the elasticity observed under the conventional notion of labor quality.

We conclude from this exercise that investments in the quality of skilled workers are of first-order importance. Without them, the resulting TFP differences become quite similar to the growth model with conventional measures of labor quality.

### 9.4 High Elasticity of Substitution

Recently, authors such as Gancia, Mueller, and Zilibotti (2011) and others, have argued that the elasticity of substitution between skilled and unskilled labor is higher than the conventional value that we use. They estimate elasticities in the range of 1.4-2.5. We ask: what are the consequences for our findings of assuming a high value for this elasticity? We proceed to set such elasticity to the highest value in the range mentioned (2.5), which corresponds to a value of  $\rho = 0.6$ . We recalibrate the model under this assumption, and proceed to find the relative levels of TFP that reproduce the empirical levels of output per worker. Results are displayed in Table 10. TFP differences are very similar to the benchmark case and as a result, the elasticity of output with respect to TFP now slightly increases to about 2.2 – it is 2.1 in the benchmark case. It is clear from these findings our main findings do *not* depend critically on the value of the substitution elasticity.

## 10 Concluding Remarks

We developed a parsimonious model of the division of labor between skilled and unskilled labor, and investments in the quality of skilled labor. We used this model and PISA achieve-

ment scores to draw inferences on labor quality differences across countries, and the relative importance of TFP in economic development.

A central finding of our paper are that labor quality differences, between the richest and the poorest countries in our sample, are roughly twice as large as those emerging under conventional (Mincer) notions of labor quality. As a result, the resulting TFP differences are smaller than in the growth model with the conventional notion of labor quality. In other words, output reacts more strongly to TFP changes in our setup than in the growth model, as labor quality is endogenous to variation in PISA scores and TFP. We find that the TFP-elasticity of output per worker is close to recent estimates from Erosa et al. (2010), and of about 2.1-2.2. These findings are robust to uncertainty on the magnitude of the substitution elasticity, and depend critically on the importance of investments in the quality of skilled labor.

# 11 Appendix

## Proof of Proposition 1

We start by defining the marginal products for our specification of the production technology. Output per worker is given by

$$y = F(K, U, S; A)/L = A k^\alpha l^{1-\alpha}$$

with  $l \equiv [\mu u^\rho + (1 - \mu)s^\rho]^{1/\rho}$ .

Hence, the marginal products of capital (MPK), skilled labor (MPS) and unskilled labor (MPU) are, respectively:

$$MPK = A\alpha\tilde{k}^{\alpha-1}, \tag{17}$$

$$MPS = A(1 - \alpha)\tilde{k}^\alpha[\mu + (1 - \mu)\tilde{s}^\rho]^{\frac{1-\rho}{\rho}} \tilde{s}^{\rho-1}(1 - \mu), \tag{18}$$

$$MPU = A(1 - \alpha)\tilde{k}^\alpha[\mu + (1 - \mu)\tilde{s}^\rho]^{\frac{1-\rho}{\rho}} \mu, \tag{19}$$

with  $\tilde{k} \equiv k/l$  and  $\tilde{s} \equiv s/u$ .

The model implies in steady state (balanced growth path):<sup>6</sup>

$$MPK = p \left( \frac{1}{\beta} - (1 - \delta) \right). \tag{20}$$

$$MPU \hat{z} + x = \beta MPS \hat{z} x^\phi \tag{21}$$

$$1 = \beta MPS E(z|z \geq \hat{z}) \phi x^{\phi-1} \tag{22}$$

Notice that equation (20) sets the stationary capital to labor ratio as a function of preference and technology parameters, as well as TFP and relative prices. Hence,  $\tilde{k} \equiv B(A, p)$ . Hence, we can rewrite (21) and (22) as the marginal products of skilled and unskilled labor in steady state as

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<sup>6</sup>To simplify notation, we omit from now on the (\*) notation for equilibrium values.

$$MPS = T(A, P)[\mu + (1 - \mu)\tilde{s}^\rho]^{\frac{1-\rho}{\rho}} \tilde{s}^{\rho-1}(1 - \mu), \quad (23)$$

$$MPU = T(a, p)[\mu + (1 - \mu)\tilde{s}^\rho]^{\frac{1-\rho}{\rho}} \mu, \quad (24)$$

with  $T(A, p) \equiv A(1 - \alpha)B(A, p)^\alpha$ . Clearly,  $T(A, p)$  is strictly decreasing in  $p$  and strictly increasing in  $A$ .

Using equations (21) and (22), after manipulations we can write:

$$1 = \beta x^\phi \left( \frac{MPS}{MPU} \right) \left[ 1 - \phi \frac{E(z|z \geq \hat{z})}{\hat{z}} \right] \quad (25)$$

Given the CES technology, we have:

$$\frac{MPS}{MPU} = \frac{1 - \mu}{\mu} [f(\hat{z})]^{\rho-1} x^{\phi(\rho-1)} \quad (26)$$

where  $f(\hat{z})$  is defined as

$$f(\hat{z}) \equiv \frac{\int_{\hat{z}}^{\bar{z}} z dG(z)}{\int_0^{\hat{z}} z dG(z)}$$

Note that the function  $f(\hat{z})$  satisfies important properties:  $f' < 0$ ,  $\lim_{\hat{z} \rightarrow 0} f(\hat{z}) = +\infty$ , and  $\lim_{\hat{z} \rightarrow \infty} f(\hat{z}) = 0$ .

Using (26) and (22) in equation (25), after manipulation, we get

$$[f(\hat{z})]^{1-\rho} = \frac{\beta(1 - \mu)}{\mu} T(A, p)^{\frac{\rho\phi}{1-\phi}} D(\hat{z})^{\rho\phi} \left[ 1 - \phi \frac{E(z|z \geq \hat{z})}{\hat{z}} \right] \quad (27)$$

where  $D(\cdot)$  is a strictly increasing function of  $\hat{z}$ .

Equation (27) allows us to establish the uniqueness and properties of the threshold  $\hat{z}$ . First, for  $\rho \in [0, 1)$  and from the properties of  $f(\hat{z})$ , the left-hand side is a non increasing function of  $\hat{z}$ ; for  $\rho \in (0, 1)$  is strictly decreasing, approaching 0 as  $\hat{z} \rightarrow \infty$ , and  $\infty$  as  $\hat{z} \rightarrow 0$ . The right-hand side of (27) is monotonically increasing and eventually positive if two conditions are met. First,  $\frac{E(z|z \geq \hat{z})}{\hat{z}}$  must be a decreasing function of  $\hat{z}$ . Lemma 1 below shows that this is indeed the case under log-concavity of the density  $g(\cdot)$ . Second, the right-hand side is eventually positive if

$$\lim_{\hat{z} \rightarrow \bar{z}} \frac{E(z|z \geq \hat{z})}{\hat{z}} < \frac{1}{\phi},$$

which is satisfied as the limit above equals 1. Hence, by the intermediate value theorem, there is a unique value  $\hat{z} > 0$  that solves (27). The fact that  $x$  is strictly positive follows from this, and the fact that marginal products are positive, using either (21) or (22). Figure A1 depicts the solution for the threshold  $\hat{z}$ .

Three central properties of the solution follow. First, from (27), in the limiting case of  $\rho = 0$  (Cobb-Douglas case), the threshold  $\hat{z}$  is independent of TFP and relative investment prices. Second, as  $\rho \in (0, 1)$ , and increase in TFP (a reduction in  $p$ ) implies a reduction in the threshold  $\hat{z}$  and viceversa. Figure A2 provides an illustration of this result. Finally, the threshold is independent of TFP and investment prices in the limiting case of  $\phi = 0$ . ■

**Lemma 1:**  $\frac{E(z|z \geq \hat{z})}{\hat{z}}$  is decreasing in  $\hat{z}$  if the density  $g(\cdot)$  is log-concave.

**Proof.** Let

$$\psi(\hat{z}) = \frac{E(z|z \geq \hat{z})}{\hat{z}}$$

Then,  $\psi'(\hat{z}) < 0$  iff

$$\frac{\partial E(z|z \geq \hat{z})}{\partial \hat{z}} < \frac{E(z|z \geq \hat{z})}{\hat{z}}$$

The derivative above is less or equal than one by Proposition 2 in Burdett (1996) if and only if

$$\int_{\hat{z}} (1 - G(\hat{z})) dz$$

is log-concave. Log-concavity of the density  $g(\cdot)$  ensures this. Furthermore,  $\frac{E(z|z \geq \hat{z})}{\hat{z}} > 1$ .

Hence,

$$\frac{\partial E(z|z \geq \hat{z})}{\partial \hat{z}} \leq 1 < \frac{E(z|z \geq \hat{z})}{\hat{z}}$$

and the result follows.

■

## References

- Barro, R. J., & Lee, J.-W. (2010). A New Data Set of Educational Attainment in the World, 1950-2010. Nber working paper no. 15902.
- Burdett, K. (1996). Truncated Means and Variances. *Economics Letters*, 52(3), 263–267.
- Caselli, F. (2005). Accounting for Cross-Country Income Differences. In Aghion, P., & Durlauf, S. (Eds.), *Handbook of Economic Growth*, Vol. 1, chap. 9, pp. 679–741. Elsevier.
- Erosa, A., Koreshkova, T., & Restuccia, D. (2010). How Important is Human Capital: A Quantitative Theory Assessment of World Income Inequality. *Review of Economic Studies*, 77, 1421–1449.
- Fernandez, R., Guner, N., & Knowles, J. (2005). Love and Money: A Theoretical and Empirical Analysis of Household Sorting and Inequality. *Quarterly Journal of Economics*, 120(1), 273–344.
- Gancia, G., Mueller, A., & Zilibotti, F. (2011). Structural Development Accounting. manuscript.
- Hall, R., & Jones, C. (1999). Why Do Some Countries Produce So Much More Output per Worker than Others?. *The Quarterly Journal of Economics*, 114, 83–16.
- Hanushek, E., & Kimko, D. (2000). Schooling, labor force quality, and the growth of nations. *American Economic Review*, 90(5), 1184–1208.
- Hanushek, E., & Woessmann, L. (2008). The Role of Cognitive Skills in Economic Development. *Journal of Economic Literature*, 46(3), 607–668.
- Hanushek, E., & Woessmann, L. (2012). Do Better Schools Lead to More Growth? Cognitive Skills, Economic Outcomes, and Causation. *Journal of Economic Growth*.
- Heckman, J., Lochner, L., & Taber, C. (1998). Explaining Rising Wage Inequality: Explanations With A Dynamic General Equilibrium Model of Labor Earnings With Heterogeneous Agents. *Review of Economic Dynamics*, 1(1), 1–58.

- Hendricks, L. (2002). How Important is Human Capital for Development? Evidence from Immigrant Earnings. *American Economic Review*, 92, 198–211.
- Katz, L. F., & Murphy, K. M. (1992). Changes in Relative Wages, 1963-1987: Supply and Demand Factors. *The Quarterly Journal of Economics*, 107(1), 35–78.
- Manuelli, R., & Seshadri, A. (2010). Human Capital and the Wealth of Nations. manuscript.
- McGrattan, E., & Schmitz, J. (2000). Explaining Cross-Country Income Differences. In Taylor, J., & Woodford, M. (Eds.), *Handbook of Macroeconomics*, chap. 3. Elsevier.
- Parente, S., & Prescott, E. (2000). *Barriers to Riches*. Cambridge: MIT Press.
- Psacharopoulos, G. (2004). Returns to Investment to Education: A Global Update. *World Development*, 22, 1325–1343.
- Schoellman, T. (2011). Education Quality and Development Accounting. *Review of Economic Studies*. *Forthcoming*.

Table 1: Summary Statistics

Statistic	PISA Score	Unskilled (%)	Enrollment (%)	Output Per-Worker
Mean	464	0.81	0.92	46,646
Median	483	0.82	0.98	42,988
St. Deviation	57	0.09	0.10	26,034
90th/10th Ratio	1.39	1.32	1.26	5.21
80th/20th Ratio	1.24	1.19	1.18	3.51
Corr ( $x$ , $Y/N$ )	0.79	-0.33	0.50	1.00

**Note:** Entries show summary statistics for the data we consider. For the correlation in the last row,  $x$  stands for the countries' mean PISA score, fraction of unskilled workers, enrollment rates and output per worker.

Table 2: Parameter Values

Parameter	Value
Discount Factor ( $\beta$ )	0.966
Population Growth Rate ( $g_L$ )	0.009
Substitution Elasticity ( $1/(1 - \rho)$ )	1.50
Capital Share ( $\alpha$ )	0.33
Depreciation Rate ( $\delta$ )	0.074
Share of Unskilled Labor ( $\mu$ )	0.32
Skill Curvature Parameter ( $\phi$ )	0.34
Talent Curvature Parameter ( $\epsilon$ )	0.26
Gamma Distribution ( $\theta$ )	17.6
Gamma Distribution ( $\kappa$ )	27.7

**Note:** Entries show the values of the calibrated parameters for our benchmark economy. Values for the discount factor and the depreciation rate are at the annual frequency. See text for details.

Table 3: Empirical Targets: Model and Data

Statistic	Model	Data
Capital to Output Ratio	0.48	0.48
Investment Rate	0.48	0.48
Fraction of Unskilled workers	0.48	0.48
Expenditure per tertiary student (% GDP per Worker)	29	29
Skill Premium	1.88	1.88
Mean Math score	487	487
Coeff. Variation Math score	0.19	0.19
<u>Talent Distribution</u>		
Median	481	488
10th percentile	373	368
25th percentile	422	425
75th percentile	546	551
90th percentile	607	609

**Note:** Entries in the top panel show the values of the statistics used as targets for our benchmark economy. Values for the capital to output ratio and the investment rate are at the annual frequency. Entries in the bottom panel show the data values for the talent distribution and the resulting values from its approximation via a Gamma distribution.

Table 4: Model Mechanics

	Fraction Unskilled	Quality ( $h$ )	Wage Premium ( $W_s/W_u$ )	Output
Benchmark	100.0	100.0	100.0	100.0
Reduction in Means (A)	100.7	97.2	102.9	91.9
Increase in Dispersion (B)	101.0	100.4	100.0	100.2
Reduction in TFP (C)	114.4	63.8	156.5	25.8
A-C Together	113.1	62.4	160.8	23.8

**Note:** Entries show the values of variables across steady states in response to a reduction in Mean Talent (A), an increase in Talent Dispersion (B), a reduction in TFP (C), and joint changes in all of them (A-C).

Table 5: Differences in PISA Scores

	Poor (Bottom 10%) vs. Rich (Top 10%)	Poor (Bottom 20%) vs. Rich (Top 20%)	Poor (Bottom 25%) vs. Rich (Top 25%)
<u>% Unskilled</u>			
Model	103.1	102.4	102.3
Data	117.0	115.2	112.9
<u>Output per worker</u>			
Model	89.7	91.1	90.9
Data	12.1	17.3	19.7
Augmented Skill ( $h$ )	96.5	97.1	97.1
Labor Input	89.1	91.1	90.9
Capital ( $K$ )	89.7	91.1	90.9
TFP	100.0	100.0	100.0

**Note:** Entries show the values of variables across steady states in response to changes in the distribution of talent according to PISA data. Data values for output per worker and the fraction of unskilled workers are shown by comparison. Entries pertain to ratios of averages of each variable within percentiles, at the top and bottom of the distribution of countries by output per worker. See text for details.

Table 6: Differences in Technology and PISA Scores

	Poor (Bottom 10%) vs. Rich (Top 10%)	Poor (Bottom 20%) vs. Rich (Top 20%)	Poor (Bottom 25%) vs. Rich (Top 25%)
<u>% Unskilled</u>			
Model	120.4	116.6	115.2
Data	117.0	113.6	112.9
<u>Output per worker</u>			
Model	12.1	17.3	19.7
Data	12.1	17.3	19.7
Augmented Skill ( $h$ )	49.0	55.4	57.8
Labor Input	55.5	63.4	64.8
Capital ( $K$ )	10.6	16.1	17.6
TFP	37.2	43.3	46.6

**Note:** Entries show the values of variables across steady states in response to changes in the distribution of talent according to PISA data, and technology (TFP and relative investment prices). TFP values are chosen to replicate exactly the level of output per worker in each country. Data values for output per worker and the fraction of unskilled workers are shown by comparison. Entries pertain to ratios of averages of each variable within percentiles, at the top and bottom of the distribution of countries by output per worker. See text for details.

Table 7: Matching the Division of Labor by Skill

	Poor (Bottom 10%) vs. Rich (Top 10%)	Poor (Bottom 20%) vs. Rich (Top 20%)	Poor (Bottom 25%) vs. Rich (Top 25%)
<u>% Unskilled</u>			
Model	117.0	116.2	112.9
Data	117.0	116.2	112.9
<u>Output per worker</u>			
Model	12.1	17.3	19.7
Data	12.1	17.3	19.7
Augmented Skill ( $h$ )	50.1	56.3	58.7
Labor Input	45.4	58.0	60.2
Capital ( $K$ )	10.6	16.1	17.6
TFP	41.1	47.0	49.6
Tax Wedge (Rich, %)	55	55	56
Tax Wedge (Poor, %)	71	69	67

**Note:** Entries show the values of variables across steady states in response to changes in the distribution of talent according to PISA data, technology (TFP and relative investment prices) and tax wedges. TFP values are chosen to replicate exactly the level of output per worker in each country. Tax wedges are selected to replicate exactly the fraction of unskilled workers in each country. Data values for output per worker and the fraction of unskilled workers are shown by comparison. Entries pertain to ratios of averages of each variable within percentiles, at the top and bottom of the distribution of countries by output per worker. See text for details.

Table 8: Comparison with One-Sector Growth Model

	Poor (Bottom 10%) vs. Rich (Top 10%)	Poor (Bottom 20%) vs. Rich (Top 20%)	Poor (Bottom 25%) vs. Rich (Top 25%)
<i>Panel A: Growth Model</i>			
Labor Input	85.9	86.9	88.6
Capital ( $K$ )	10.6	16.1	17.6
TFP	28.0	34.4	37.2
<i>Panel B: This paper (Matching <math>Y/L</math> only)</i>			
Labor Input	55.5	63.4	64.8
Capital ( $K$ )	10.6	16.1	17.6
TFP	37.2	43.3	46.6
<i>Panel C: This paper (Matching Division of Labor)</i>			
Labor Input	45.4	58.0	60.2
Capital ( $K$ )	10.6	16.1	17.6
TFP	41.1	47.0	49.6

**Note:** Entries compare the values of variables across steady states in the one-sector growth model, under the Mincerian notion of labor quality, with values emerging in our model. Entries repeat the values in previous tables for our model under two cases. See text for details.

Table 9: Implications for Skill Premia

	Poor (Bottom 10%) vs. Rich (Top 10%)	Poor (Bottom 20%) vs. Rich (Top 20%)	Poor (Bottom 25%) vs. Rich (Top 25%)
<i>Panel A: Data (Restricted Sample)</i>			
% Unskilled	133.3	121.3	116.0
Skill Premium	244.1	205.5	201.5
<i>Panel B: Matching Y/L only</i>			
% Unskilled	116.1	114.6	108.3
Augmented Skill ( $h$ )	56.1	59.8	68.9
Skill Premium	102.7	102.8	110.5
<i>Panel C: Matching Division of Labor</i>			
% Unskilled	133.3	121.3	116.0
Augmented Skill ( $h$ )	56.3	60.2	63.8
Skill Premium	215.4	158.8	144.3

**Note:** Entries compare the values of variables across steady states in our model under the scenarios considered previously when data values pertain to the restricted sample of countries considered. Panel B shows the implications when the talent distribution varies across countries and there are technology differences, and TFP is chosen to reproduce output per worker across countries. Panel C shows the implications when, in addition, tax wedges are chosen to reproduce the fraction of unskilled workers in each country. See text for details.

Table 10: Sensitivity

	Poor (Bottom 10%) vs. Rich (Top 10%)	Poor (Bottom 20%) vs. Rich (Top 20%)	Poor (Bottom 25%) vs. Rich (Top 25%)
<i>Panel A: This paper (Matching Y/L only)</i>			
Labor Input	55.5	63.4	64.7
TFP	37.2	43.3	46.6
<i>Panel B: PISA as Labor Input</i>			
Labor Input	74.2	77.6	77.5
TFP	30.8	37.1	40.7
<i>Panel C: <math>\phi = 0</math></i>			
Labor Input	95.2	93.0	92.7
TFP	27.0	33.2	36.5
<i>Panel D: High Substitution Elasticity</i>			
Labor Input	50.1	59.9	61.3
TFP	37.6	43.8	47.1

**Note:** Entries compare the values of variables across steady states under different cases in our model. Panel A shows the benchmark case of our model when TFP values are chosen to reproduce output per worker differences. Panel B shows the case when labor quality corresponds to average talent values in each country, and TFP values are chosen to reproduce output per worker differences. Panel C shows the case when there are no investments in skills ( $\phi = 0$ ), and TFP values are chosen to reproduce output per worker differences. Panel D shows the case when the elasticity of substitution between skilled and unskilled is higher than in the benchmark case (2.5 instead of 1.5), and TFP values are chosen to reproduce output per worker differences. See text for details.