

# International recessions\*

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## Abstract

## 1 Introduction

A distinguished feature of the recent financial crisis was the high degree of international synchronization where most of the developed countries experienced large macroeconomic contractions. This level of synchronization between the United States and the largest industrialized economies is especially strong in the recent recession and there is not evidence of such a co-movement in previous recessions. This observation raises two questions. First, what mechanism is responsible for the high degree of international synchronization? Second, why was the synchronization stronger in the recent recession?

The first question relates to the sources of international co-movement. There are two main reasons why countries may experience macroeconomic co-movement. It may be the result of synchronized disturbances (global or common shocks) or the result of country-specific shocks that spill to other countries (international transmission of country specific shocks). In this paper we focus on the second source of international co-movement. By doing so we

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do not exclude the possibility that certain shocks are internationally correlated. However, if common shocks play an important role in generating cross-country co-movement, we would like to understand why they are correlated, which ultimately requires us to investigate the international ‘transmission’ of these shocks.<sup>1</sup>

Although there are many shocks that affect individual countries, not all shocks generate positive co-movement. For example, in the international Real Business Cycle model of Backus, Kehoe and Kydland (1992), country-specific productivity shocks generate very weak cross-country co-movement, unless the shocks are internationally correlated. See Heathcote and Perri (2004). The international RBC model has been extended in various directions with the inclusion of nontradable sectors, nominal rigidities and other exogenous sources of business cycle fluctuations such as shocks to preferences, monetary policy and investment specific technology. However, it is not obvious whether these additional sources of business cycle can generate large cross-country co-movements, unless the shocks are internationally correlated.

A source of business cycle fluctuations that has not been fully explored in the international literature is the role of financial shocks, that is, changes in financial market conditions that affect the availability of credit for borrowers. The goal of this paper is to investigate the international transmission of these shocks. Using a two-country incomplete-markets model we show that, if countries are financially integrated, changes in the financial conditions of one country could have large macroeconomic effects on other countries. Furthermore, the macroeconomic response to positive and negative changes in market conditions are highly asymmetric. While financial improvements lead to gradual and long-lasting expansions, financial contractions lead to severe but short recessions.

The role of financial shocks for macroeconomic fluctuations have been recently studied in Jermann and Quadrini (2009) but in a closed economy. Furthermore, while in Jermann and Quadrini credit shocks are purely exogenous, in our paper we provide a micro foundation for these shocks which is based on self-fulfilling expectations. In this respect there are some similarities with the multiple equilibria property of the model studied in Kocherlakota (2009). In our model the multiplicity of equilibria derive from ‘occasionally binding’ enforcement constraints. This is another important difference between our paper and other studies that investigate

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<sup>1</sup>A well-known literature in international economics uses factor models to investigate the sources of international business cycles. See for example, Crucini, Kose and Otrok (2011). These studies identify three sources of business cycles: common factors, nation-specific factors and idiosyncratic components. International co-movement is induced by the ‘common factor’. However, because of the reduced form approach, these studies cannot disentangle whether the common factor derives from ‘common shocks’ or the ‘transmission’ of country-specific shocks.

the macroeconomic impact of financial shocks (for example, Christiano, Motto and Rostagno (2009) and Jermann and Quadrini (2009)). Most of these contributions limit the analysis to equilibria with always binding constraints and the quantitative properties are studied using linear approximation techniques. In our model, instead, borrowing constraints are only occasionally binding and this central to our analysis. Mendoza (2010) also studies an economy with occasionally binding constraints but does not investigate the importance of financial shocks. Furthermore, by focusing on a small open economy, the paper does not address the issue of international co-movement which is one of the central issues studied in our paper. Occasionally binding constraints are also central to Brunnermeier and Sannikov (2010). However, the analysis of this paper is limited to productivity shocks in a closed economy.

We now turn to the second question. If changes in financial markets conditions play an important role in business cycle fluctuations and can generate significant cross-country co-movement, why was the co-movement stronger in the most recent recession? Our theoretical framework provides two possible explanations. First, financial shocks generate international co-movements only if countries are financially integrated. Although the process of financial integration has progressed since the early 1980s, it has reached a new high in the 2000s. Therefore, one explanation for the strong international co-movement in the recent crisis is because financial markets were more integrated than in previous recessions. The second explanation relates to the asymmetric impact of changing credit markets conditions. While the macroeconomic impact of credit expansions are gradual with limited short-term effects, the impact of credit contractions is highly non-linear and the macroeconomic consequences are severe only when the credit contraction is big. Furthermore, the longer the credit expansion that precedes the credit contraction, the more severe the macroeconomic impact is. Thus, the second explanation for the higher cross-country co-movement observed in the most recent recession is that credit market conditions deteriorated sharply (large credit contraction) and the deterioration took place after an extended period of credit expansion (higher leverage).

In addition to the papers already mentioned above, our study is related to two recent contributions: Dedola & Lombardo (2010) and Devereux & Yetman (2010). Both studies investigate the international transmission of shocks in models with financial market frictions. They also show that shocks to the financial system can generate cross-country spillovers in macroeconomic variables. Also related is the study of Enders, Kollmann & Muller (2010). This paper introduces a banking sector in an international model and shows that shocks to this sector could have important effects on the global economy. The theoretical findings of

these papers are consistent with the empirical results of Helbling, Huidrom, Kose & Otrok (2010) according to which credit market shocks matter in explaining global business cycles, especially during the 2009 global recession.

## 2 Some features of the international business cycle

In this section we present some facts about the properties of the international business cycle with special focus on recessionary episodes.

### 2.1 International macroeconomic co-movement

Figure 1 plots the US GDP against the GDP of the other G7 countries during the recent recession, up to the second quarter of 2009. The numbers are percent deviations from the level of GDP in the quarter preceding the beginning of the recession identified by the NBER Business Cycle Dating Committee (fourth quarter of 2007). Four quarters before the official recession are also plotted. The figure reveals the strong co-movement in macroeconomic activity among the G7 countries.

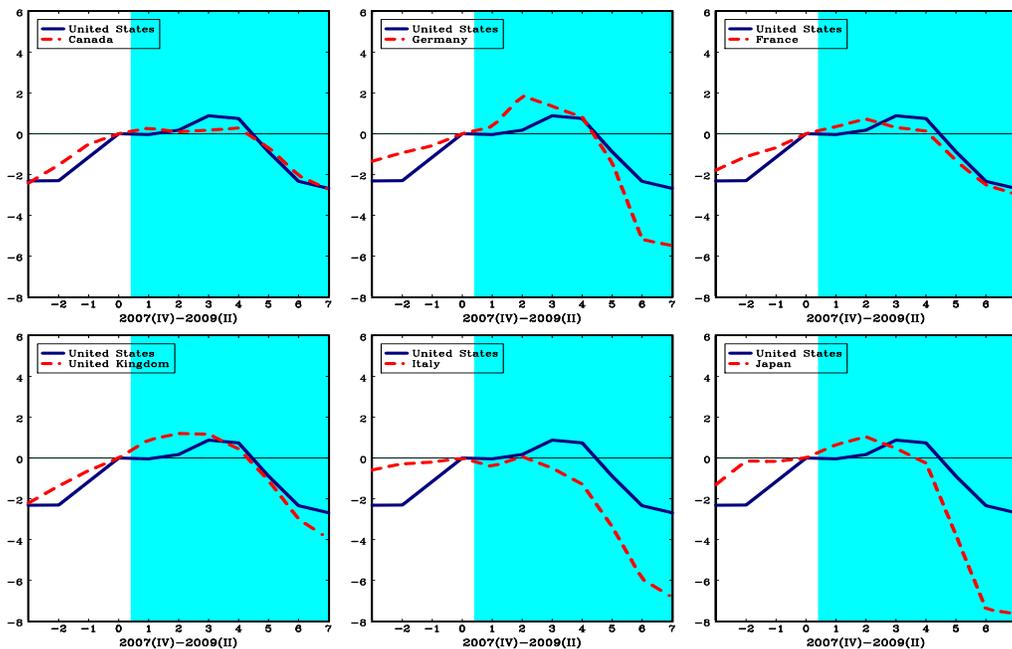


Figure 1: The dynamics of GDP during the 2008 recession: US vs. other G7 countries.

To examine whether the international synchronization of the recent recession differs from previous contractions, Figure 2 plots the GDP dynamics for the G7 countries in six of the most recent US recessionary episodes: one recession experienced in the first half of the 1970s, two in the first half of 1980s, one in the early 1990s and two in the 2000s. A quick glance at the figure shows that the macroeconomic synchronization of the US with other G7 countries has been significantly stronger in the recent recession. While the G7 countries experienced very different GDP dynamics during the previous US recessions, in the most recent contraction all countries moved in the same direction.

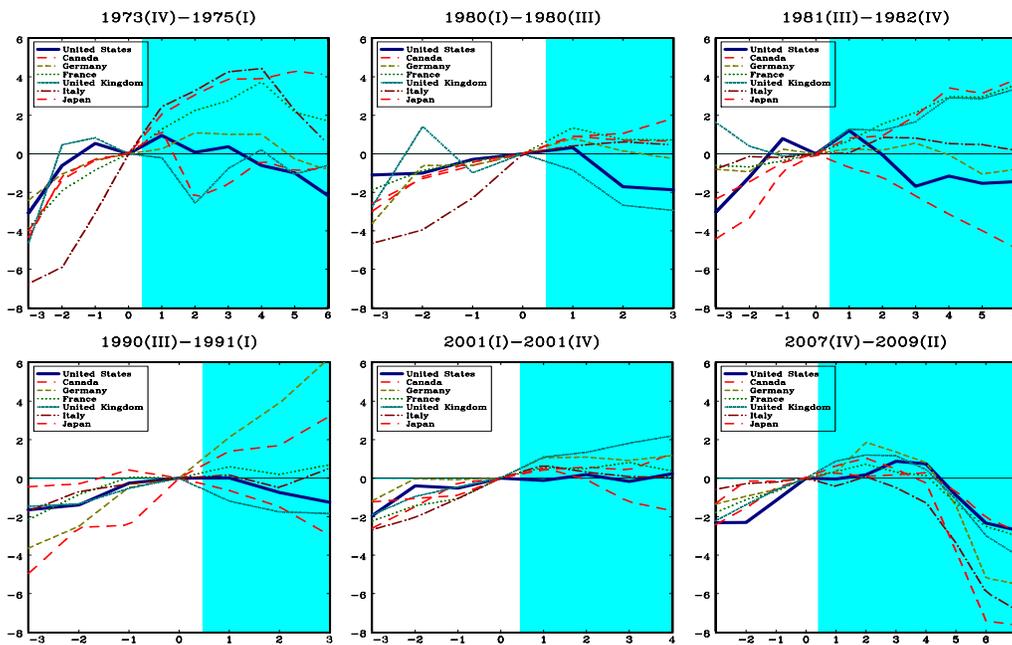


Figure 2: The dynamics of GDP during the six most recent recessions in the G7 countries.

The higher cross-country synchronization of the recent recession can also be seen in Figure 3 which plots the average correlation of US GDP with the GDP of each of the other G7 countries. The correlations are computed on rolling windows of 10 and 20 years. The dates in the graph correspond to the end points of the window used to compute the correlation. Although the figure shows that the increase in correlation can also be seen in previous recessions, the current contraction stands out as the one that marks an increase in correlation larger than in earlier periods. For a similar point see also Imbs (2010).

The dramatic increase in co-movement is also observed in other variables, in particular asset

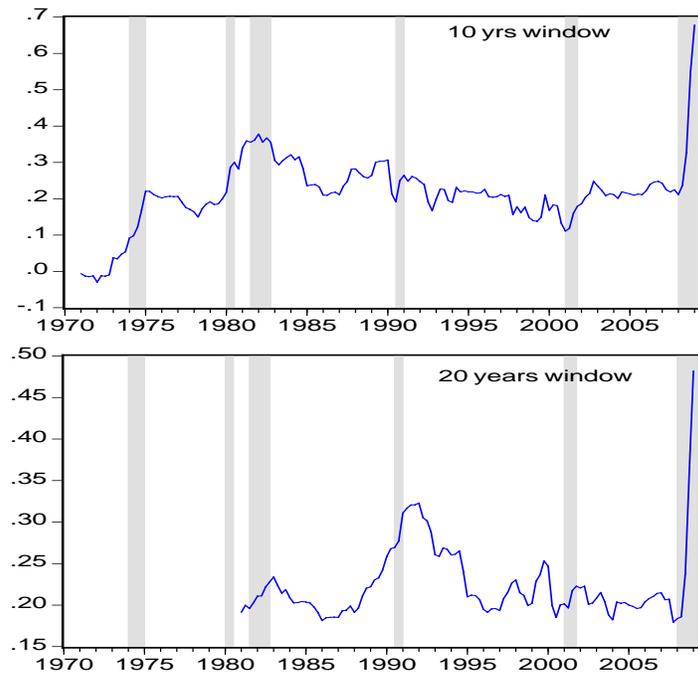


Figure 3: Average rolling correlations of US GDP with other G7 countries.

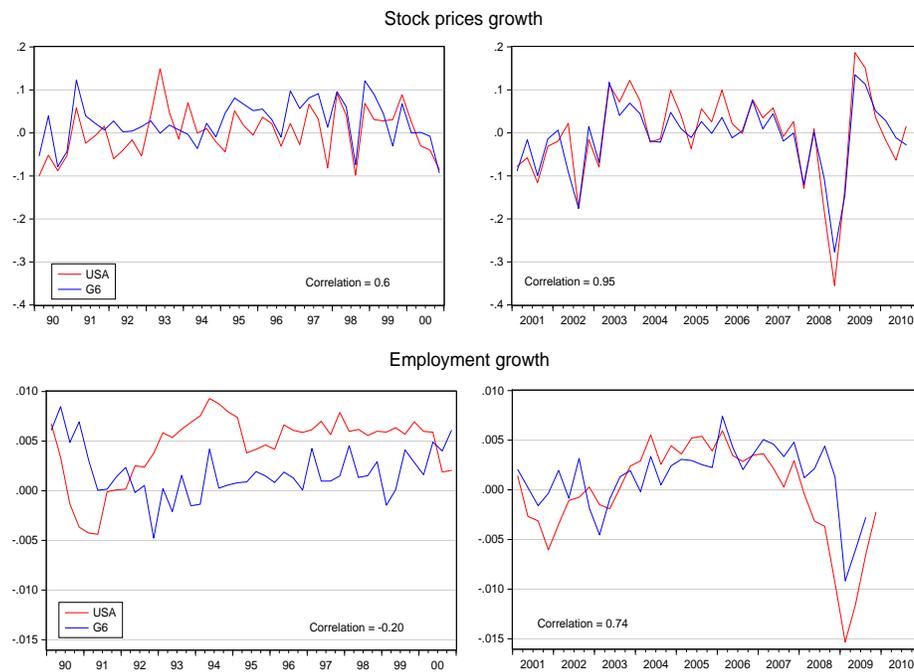


Figure 4: International co-movement in stock prices and employment

prices and employment. The top two panels of Figure 4 plot the growth rate of stock prices in the 1990s and in the 2000s.<sup>2</sup> The bottom two panels plot the growth rate of employment in the US and in the G6 during the same two subperiods. The figure shows quite clearly how the last recession (and more in general the entire last decade) represents a period of high international synchronization between US and the rest of the developed world.

## 2.2 International co-movement in financial markets

The most recent recession also displays a high degree of international co-movement in financial markets. In addition to the co-movement in asset prices shown above, the flows of business borrowing in industrialized countries have collapsed in unison during the most recent recession. Unfortunately, comparable data on the flows and stocks of borrowing is not available for all industrialized countries. We were able to collect data for the United States, the Euro area, Japan and Canada. For the US the data is from the Flows of Funds of the Federal Reserve Board and it is available for the whole business sector. For the Euro area, Japan and Canada, the data is limited to the corporate sector.

Figure 5 plots the growth rates in the real stock of debt for nonfinancial businesses (corporations only for Euro area, Japan and Canada). Because the flow and stock data is not seasonally adjusted, we report the year-on-year growth rates to avoid seasonal movements. The plotted series start in the first quarter of 2000 because the data for the Euro area is only available starting in the first quarter of 1999 when the Euro was introduced. The year-on-year growth then implies that the first available growth rate is for the first quarter of 2000.

Three debt indicators are shown. The top panel plots the growth rate in the gross stock of debt scaled by GDP. Since the tightness of financial constraints depends not only on the availability of gross debt but also on the availability of liquid assets, in the second and third panels we subtract to the stock of debt two measures of liquidity. The first measure is closer to a concept of cash (cash, checkable deposits and time deposits) while the second measure includes a broader set of liquid assets such as money market funds and shares of mutual funds.

Whatever the measure we focus on, it is evident that the growth rate of business liabilities have fallen simultaneously in all countries during the recent crisis. Simple correlation measures reported in Table 1 confirm the high international co-movement in the flows of debt financing.

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<sup>2</sup>The stock prices in US are the MSCI BARRA US stock market index, while stock prices in the G6 are computed using the MSCI BARRA EAFE + Canada index which is an average of stock prices in advanced economies except the US.

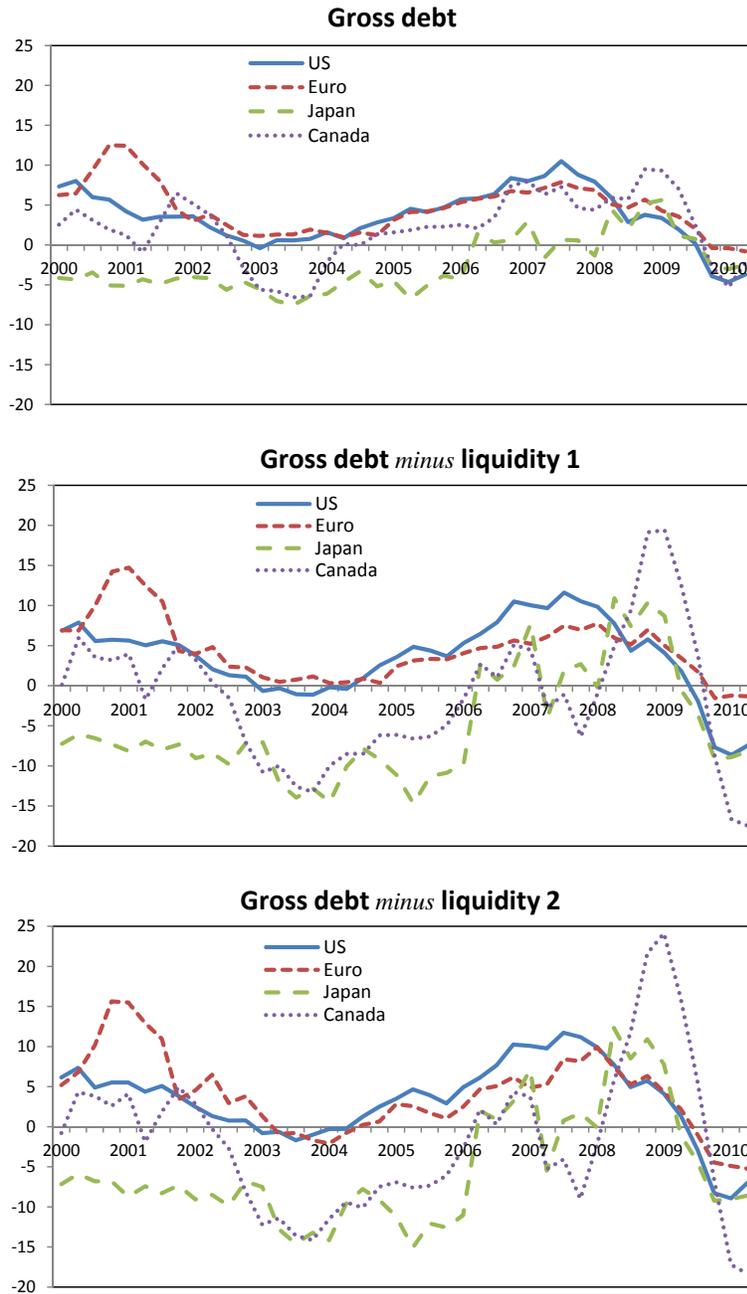


Figure 5: Business debt in major industrialized countries. For the US data is from the Flows of Funds Accounts and for the whole nonfinancial business sector. *Debt* is defined as credit markets instruments. *Liquidity 1* is the sum of foreign deposits, checkable deposits and currency, time and savings deposits. *Liquidity 2* is Liquidity 1 plus money market funds, securities RPs, commercial paper, treasury securities, agency and GSE backed securities, municipal securities, mutual fund shares. For other countries data is for nonfinancial corporations and the definition of variables is similar to the US.

Table 1: Cross-correlations in debt growth

<i>Gross debt</i>			
	Euro	Japan	Canada
US	0.74	0.32	0.68
Euro	-	0.19	0.50
Japan	-	-	0.70

<i>Gross debt-liquidity 1</i>			
	Euro	Japan	Canada
US	0.68	0.51	0.55
Euro	-	0.31	0.55
Japan	-	-	0.69

<i>Gross debt-liquidity 2</i>			
	Euro	Japan	Canada
US	0.73	0.52	0.43
Euro	-	0.35	0.49
Japan	-	-	0.69

### 2.3 Labor input and labor productivity

The contribution of Backus, Kehoe and Kydland (1992) to the study of international business cycle places productivity shocks at the central stage of macroeconomic fluctuations. Although the international RBC has been extended in several directions by adding various frictions and shocks, TFP shocks still play a central role in these models. An implication of having TFP shocks as a major source of business cycle fluctuations is that the productivity of labor is highly pro-cyclical. However, the correlation between labor productivity and measures of labor input is very weak in the data. Furthermore, unless TFP shocks are correlated across countries, it is difficult to generate cross-country co-movements with TFP shocks. Since empirical measurements of country-specific TFP based on Solow residuals are weakly correlated, the hypothesis that the cross-country co-movement can be explained with internationally correlated TFP shocks is empirically implausible.

Figure 6 plots working hours and labor productivity (output per hour) in the private non farm sector of the US economy for the six most recent recessions. The last panel shows that in the recent recession labor productivity has actually increased for most of the period. This pattern can also be seen in the 2001 recession. By contrast, in the first four recessions, labor productivity has slowed down markedly and its level at the end of the recession was not higher than before the recession.

The weak association between productivity and labor is a general property not only in the US but also in other G7 countries. Figure 7 plots the ten years rolling correlation of growth

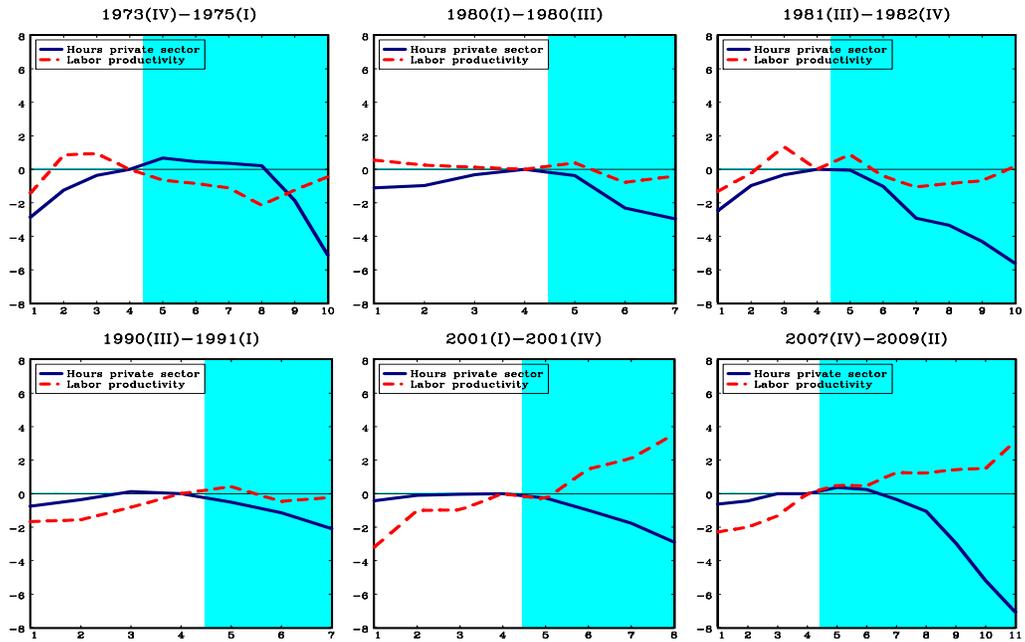


Figure 6: Labor productivity (output per hour) and hours in recessions

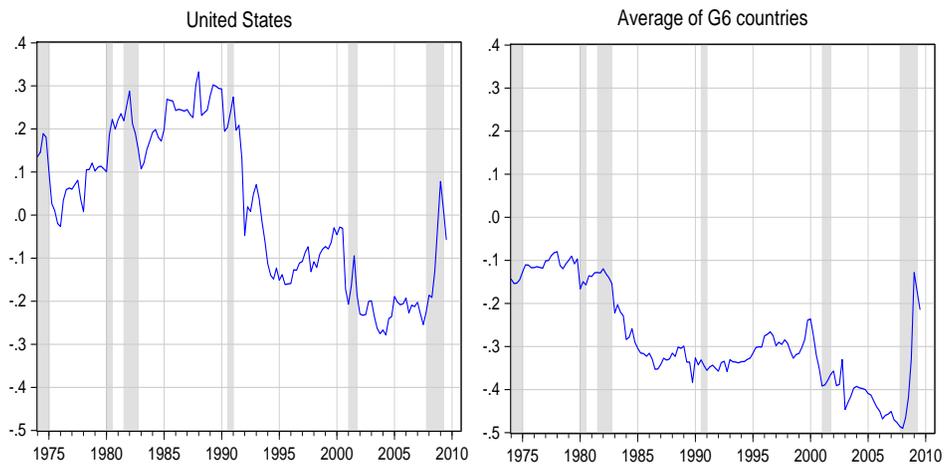


Figure 7: Ten years rolling correlations of GDP per worker and employment growth.

in GDP per worker and growth in employment for the US and for the remaining G6 countries. Since comparable data on hours are not available for all G6 countries we use simply GDP per worker as a measure of productivity and employment as a measure of labor. For the US the average correlation over the period is close to zero. It is positive in the early period but negative in the most recent period. A pattern also documented in Gali and Gambetti (2009). But even in the early period the correlation is not that high even if positive. For the G6 countries the correlation is negative throughout the whole period. It is also interesting to observe that the correlation numbers are close to their minimum values at the beginning of the latest recession. In the fourth quarter of 2007 the US correlation was -0.2 and in the G6 countries it was -0.5.

The fact that the productivity of labor is not pro-cyclical suggests that other shocks besides TFP may play an important role in generating business cycle fluctuations. In this paper we investigate the role of credit shocks, that is, perturbations that affect the credit capacity of borrowers. There are other sources of business cycle fluctuations that could weaken the correlation between productivity and labor. Our focus on credit shocks is motivated by two considerations. The first consideration is that the flows of credit are highly pro-cyclical (see previous subsection). The second consideration is that credit shocks can generate sizable cross-country co-movements as we will show in the rest of this paper. Although there are other shocks that generate a weak or even negative correlation between productivity and labor, it is not easy to generate cross-country co-movement in real and financial flows with alternative types of shocks.

### **3 The model without capital accumulation**

We start with a simple model without capital accumulation. This allows us to provide the intuitions for some of the key results of the paper analytically. After the presentation of the simple model it will be easy to extend it with capital accumulation.

There are two types of atomistic agents, investors and workers. A key difference between these two types of agents is the availability of different investment opportunities. Due to the assumption of markets segmentation only investors have access to the ownership of firms while workers can only save in the form of bonds. Investors discount the future at rate  $\beta$  while the discount factor of workers is  $\delta > \beta$ . The different discounting between the owners of firms (investors) and workers implies that firms borrow from workers subject to the enforcement constraints as we will describe below.

To facilitate the presentation we first describe the closed-economy version of the model. Once we have characterized the autarkic equilibrium, it will be trivial to extend it to the environment with international mobility of capital.

### 3.1 Investors and firms

Investors have lifetime utility  $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$ . They are the owners of firms and derive income only from dividends. Denoting by  $d_t$  the dividends paid by firms, the effective discount factor for investors is  $m_{t+1} = \beta u_c(d_{t+1})/u_c(d_t)$ . This is also the discount factor used by firms since they maximize shareholders' wealth. As we will see, fluctuations in the effective discount rate play a central role in the analysis of this paper.

Firms operate the production function  $F(h_t) = \bar{k} h_t^\nu$ , where  $\bar{k}$  is a fixed input of capital and  $h_t$  is the variable input of labor. The parameter  $\nu$  is smaller than 1 implying decreasing returns to scale in the variable input. The input of capital is fixed and does not depreciate. Therefore, in this version of the model we can think of  $\bar{k}$  as a normalizing constant.

Firms start the period with intertemporal debt  $b_t$ . Before producing they choose the labor input  $h_t$ , the dividends  $d_t$ , and the next period debt  $b_{t+1}$ . The budget constraint is

$$b_t + w_t h_t + d_t = F(h_t) + \frac{b_{t+1}}{R_t},$$

where  $R_t$  is the gross interest rate.

The payments of wages,  $w_t h_t$ , dividends,  $d_t$ , and current debt net of the new issue,  $b_t - b_{t+1}/R_t$ , are made before the realization of revenues. This implies that the firm faces a cash flow mismatch during the period. The cash needed at the beginning of the period is  $w_t h_t + d_t + b_t - b_{t+1}/R_t$ . From the budget constraint we can verify that this is equal to the revenue  $F(h_t)$ . To cover the cash flow mismatch, the firm contracts an intra-period loan which is equal to the liquidity need  $l_t = w_t h_t + d_t + b_t - b_{t+1}/R_t$ . This loan is repaid at the end of the period, after the realization of revenues.

Debt contracts are not perfectly enforceable. At the end of the period the firm can divert the liquidity  $l_t$  and default. Default gives the lender the right to liquidate the firm's capital. Suppose that the liquidation value is  $\xi_t \bar{k}$ , where  $\xi_t$  is stochastic. This is the value that guarantees the firm's liabilities. Since default arises at the end of the period, the total liabilities of the firm are  $l_t + b_{t+1}/R_t$ . To ensure that the firm does not default, the total liabilities are

subject to the enforcement constraint<sup>3</sup>

$$\xi_t \bar{k} \geq l_t + \frac{b_{t+1}}{R_t}.$$

Fluctuations in  $\xi_t$  affect the ability to borrow and, as we will see, they generate pro-cyclical movements in real and financial variables.<sup>4</sup> Our goal is to derive the variable  $\xi_t$  endogenously from liquidity considerations. As we will describe below, fluctuations in this variable are induced by self-fulfilling expectations leading to multiple equilibria. For the moment, however, we treat  $\xi_t$  as a purely exogenous stochastic variable. Once we have characterized the equilibrium with an exogenous  $\xi_t$ , we will make  $\xi_t$  endogenous.

To illustrate the role played by fluctuations in  $\xi_t$ , consider a pre-shock equilibrium in which the enforcement constraint is binding. Starting from this equilibrium, suppose that  $\xi_t$  decreases. In response to the decline in  $\xi_t$  the firm is forced to reduce either the dividends and/or the input of labor. To see this, let's start with the case in which the firm is unwilling to change the input of labor. This implies that the intra-period loan  $l_t = F(h_t)$  also does not change. Thus, the only way to satisfy the enforcement constraint is by reducing the intertemporal debt  $b_{t+1}$ . We can then see from the budget constraint,  $w_t h_t + d_t + b_t = b_{t+1}/R_t + F(h_t)$ , that the reduction in  $b_{t+1}$  requires a reduction in dividends. Thus, the firm is forced to substitute debt with equities.

Alternatively the firm could keep the dividend payments unchanged and reduce intra-period loan  $l_t = F(h_t)$ . This will also ensure that the enforcement constraint is satisfied but it requires the reduction in the input of labor. Therefore, after a negative shock to  $\xi_t$ , the firm faces a trade-off: paying lower dividends or cutting employment. As we will see, the optimal choice depends on the relative cost of changing these two margins which in turn depends on the stochastic discount factor for investors  $m_{t+1} = \beta u_c(d_{t+1})/u_c(d_t)$ .

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<sup>3</sup>Here we adopt a similar approach to Hart and Moore. After defaulting the firm bargains the repayment with the lender. Under the assumption that the firm has all the bargaining power, the lender would recover only the threat value  $\xi_t \bar{k}$ . In anticipation of this, the lender will never lend more than  $\xi_t \bar{k}$ .

<sup>4</sup>Eisfeldt and Rampini (2006) provide some evidence that the liquidity of capital  $\xi_t$  must be procyclical to match the amount of capital reallocation observed in the data.

**Firm's problem:** The optimization problem of the firm can be written recursively as

$$V(\mathbf{s}; b) = \max_{d, h, b'} \left\{ d + Em'V(\mathbf{s}'; b') \right\} \quad (1)$$

subject to:

$$b + d = F(h) - wh + \frac{b'}{R} \quad (2)$$

$$\xi \bar{k} \geq F(h) + \frac{b_{t+1}}{R_t}, \quad (3)$$

where  $\mathbf{s}$  are the aggregate states, including the shock  $\xi$ , and the prime denotes the next period variable. The enforcement constraint takes into account that the intra-period loan is equal to the firm's output, that is,  $l_t = w_t h_t + d_t + b_t - b_{t+1}/R_t = F(h_t)$ .

In solving this problem the firm takes as given all prices and the first order conditions are

$$F_l(h) = \frac{w}{1 - \mu}, \quad (4)$$

$$REm' = 1 - \mu, \quad (5)$$

where  $\mu$  is the Lagrange multiplier for the enforcement constraint. These conditions are derived under the assumption that dividends are always positive, which will be the case if the investors' utility satisfies  $u_c(0) = \infty$ . The detailed derivation is in Appendix A.

We can see from condition (4) that limited enforcement imposes a wedge in the demand for labor if the enforcement constraint is binding, and therefore,  $\mu > 0$ . This derives from the fact that the input of labor needs to be financed and, because of the agency problem, part of the financing has to come from equity (through lower payment of dividends). As long as the cost of equity ( $1/Em'$ ) is greater than the cost of debt (the interest rate  $R$ ), expanding the input of labor is costly in the margin because the firm needs to substitute debt with equity. It is then the equity premium  $1/Em' - R$  that determines the labor wedge as can be seen from condition (5).<sup>5</sup> This wedge is strictly increasing in  $\mu$  and disappears when  $\mu = 0$ , that is, when

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<sup>5</sup>Notice that we are using the term 'equity premium' to denote the differential between the expected shareholders' return and the interest rate on bonds. Since shareholders and bondholders are different agents, the equity premium is not only determined by the cost of risk (risk premium).

the enforcement constraint is not binding.

**Some partial equilibrium properties:** The characterization of the firm's problem in partial equilibrium provides helpful insights about the property of the model once extended to a general equilibrium set-up. For partial equilibrium we mean the allocation achieved when the interest rate and the wage rate are both exogenously given and constant.

Under these conditions, equation (5) shows that  $\mu$  decreases with the expected discount factor  $Em'$ . A decrease in  $\xi$ , that is, a negative credit shock, makes the enforcement constraint tighter. Because firms reduce the payment of dividends, the investors's consumption has to decrease. This induces a decline in the discount factor  $m' = \beta u_c(d')/u_c(d)$  and an increase in the multiplier  $\mu$  (condition (5)). Condition (4) then shows that the demand for labor declines.

Intuitively, when the credit conditions become tighter, firms need to rely more on equity financing and less on debt. However, it is costly to increase equity in the short-term since investors must cut consumption and their utility is concave. Because of this, the firm does not find optimal in the short-term to raise enough equity to keep the pre-shock production scale and it cuts employment. If investors' utility were linear (risk-neutrality), the discount factor would be equal to  $Em' = \beta$  and the credit shock would not affect employment. This also requires that the interest rate does not change, which is the case in the partial equilibrium considered here. In the general equilibrium, of course, prices also change. In particular, movements in the demand of credit and labor affect the interest rate  $R$  and the wage rate  $w$ . To derive the aggregate effects we need to close the model and characterize the general equilibrium.

### 3.2 Closing the model and general equilibrium

There is a representative household/worker with lifetime utility  $E_0 \sum_{t=0}^{\infty} \delta^t U(c_t, h_t)$ , where  $c_t$  is consumption,  $h_t$  is labor and  $\delta$  is the intertemporal discount factor. It will be convenient to assume that the period-utility takes the form

$$U(c_t, h_t) = \log(c_t) - \alpha \frac{h_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}.$$

The work's budget constraint is

$$w_t h_t + b_t = c_t + \frac{b_{t+1}}{R_t},$$

and the first order conditions for labor,  $h_t$ , and next period bonds,  $b_{t+1}$ , are

$$U_h(c_t, h_t) + w_t U_c(c_t, h_t) = 0, \quad (6)$$

$$\delta R_t E_t \left\{ \frac{U_c(c_{t+1}, h_{t+1})}{U_c(c_t, h_t)} \right\} = 1. \quad (7)$$

We can now define a competitive equilibrium. The aggregate states  $\mathbf{s}$  are given by the credit conditions  $\xi$  and the aggregate stock of bonds  $B$ .

**Definition 3.1 (Recursive equilibrium)** *A recursive competitive equilibrium is defined by a set of functions for (i) workers' policies  $h(\mathbf{s})$ ,  $c(\mathbf{s})$ ,  $b(\mathbf{s})$ ; (ii) firms' policies  $h(\mathbf{s}; b)$ ,  $d(\mathbf{s}; b)$  and  $b(\mathbf{s}; b)$ ; (iii) firms' value  $V(\mathbf{s}; b)$ ; (iv) aggregate prices  $w(\mathbf{s})$ ,  $R(\mathbf{s})$  and  $m(\mathbf{s}')$ ; (v) law of motion for the aggregate states  $\mathbf{s}' = \Psi(\mathbf{s})$ . Such that: (i) household's policies satisfy the optimality conditions (6)-(7); (ii) firms' policies are optimal and  $V(\mathbf{s}; b)$  satisfies the Bellman's equation (1); (iii) the wage and interest rates are the clearing prices in the markets for labor and bonds, and the discount factor for firms is  $m(\mathbf{s}') = \beta u_c(d_{t+1})/u_c(d_t)$ ; (iv) the law of motion  $\Psi(\mathbf{s})$  is consistent with the aggregation of individual decisions and the stochastic processes for  $z$  and  $\xi$ .*

To illustrate the main properties of the model, we look at some special cases. Consider first the economy without shocks. In this economy the enforcement constraint binds in a steady state equilibrium. To see this, consider the first order condition for the bond, equation (7), which in a steady state becomes  $\delta R = 1$ . Using this condition to eliminate  $R$  in (5) and taking into account that in a steady state  $Em' = \beta$ , we get  $\beta/\delta = 1 - \mu$ . Because  $\delta > \beta$  by assumption, the lagrange multiplier  $\mu$  is greater than zero. Firms want to borrow as much as possible because the cost of borrowing—the interest rate—is smaller than their discount rate.

In a model with uncertainty, however, the constraint may not be always binding. However, they will become binding after a large and unexpected decline in  $\xi$ . In this case firms will be forced to cut dividends and this affects the discount factor  $Em'$ . Furthermore, the change in the demand for credit impacts on the equilibrium interest rate. Using condition (5) we can see that these changes affect the multiplier  $\mu$ , which in turn impacts on the demand for labor (see equation (4)). On the other hand, a sufficiently large increase in  $\xi$  may make the enforcement constraint non-binding. This implies that the response to a positive credit shock is bounded since the multiplier  $\mu$  cannot be negative. Therefore, the responses of the economy to credit shocks could be asymmetric: negative shocks induce large falls in employment and output

while the impacts of positive shocks is moderate since  $\mu$  cannot be negative.

### 3.3 Capital mobility

Let's consider now two countries, domestic and foreign, with the same size, preferences and technology as described in the previous section. Although we characterize here only the case with two symmetric countries, the model can be easily extended to any number of countries and with different degrees of heterogeneity. For the moment we continue to assume that  $\xi_t$  is an exogenous stochastic variable that is specific to each country. Once we allow for cross-country capital mobility, we have to specify what agents can do in an integrated financial market. We continue to assume that there is market segmentation in the ownership of firms, that is, workers are unable to purchase shares of firms. However, in addition to domestic bonds they can purchase foreign financial assets as specified below. Furthermore, investors are now able to purchase shares of foreign firms.

**Investors/firms:** Because firms are subject to country specific financial shocks, investors would gain from diversifying the cross-country ownership of shares. Therefore, in an economy that is financially integrated, investors choose to own the worldwide portfolio of shares and we have a representative 'worldwide' investor. Because domestic and foreign firms are owned by the same representative shareholder, they will use the same discount factor  $m_{t+1} = \beta u_c(d_{t+1} + d_{t+1}^*)/u_c(d_t + d_t^*)$ , where investors' consumption is the sum of dividends paid by domestic firms,  $d_t$ , plus the dividends paid by foreign firms,  $d_t^*$ .<sup>6</sup> From now on we will use the star superscript to denote variables pertaining to the foreign country.

Besides the common discount factor, firms continue to solve problem (1) and the first order conditions are given by equations (4) and (5). Let's focus on condition (5), which we rewrite here for both countries,

$$\begin{aligned} R_t E m_{t+1} &= 1 - \mu_t, \\ R_t^* E m_{t+1}^* &= 1 - \mu_t^*. \end{aligned}$$

The first condition is for firms located in the domestic country and the second for firms located in the foreign country. Since the discount factor is common to domestic and foreign

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<sup>6</sup>A perfect diversification of portfolios is optimal because investors' utility depends only on consumption. If investors derived utility also from leisure, a perfect diversification will not be necessarily optimal.

firms, that is,  $Em_{t+1} = Em_{t+1}^*$ , and the interest rate is equalized across countries,  $R_t = R_t^*$ , the above conditions imply that the lagrange multiplier will also be equalized, that is,  $\mu_t = \mu_t^*$ . Therefore, independently of which country is hit by a shock, if the enforcement constraint is binding in one country, it will also be binding in the other. This also implies that the labor wedges are equalized across countries. In fact, condition (4) is still the optimality condition for the choice of labor in both countries, that is,

$$\begin{aligned} F_h(h_t) &= w_t \left( \frac{1}{1 - \mu_t} \right), \\ F_h(h_t^*) &= w_t^* \left( \frac{1}{1 - \mu_t^*} \right). \end{aligned}$$

This property is crucial for understanding the cross-country impact of a financial shock as we will describe below.

**Households/workers:** Although workers are still prevented from accessing the market for the ownership of firms, with capital mobility they can engage in international financial transactions with foreign workers. More precisely, in addition to holding bonds issued by domestic firms, domestic workers can buy state-contingent claims from foreign workers. We still assume that firms borrow from domestic workers but they cannot sign state contingent contracts with workers. The assumption that firms borrow only from domestic workers is without loss of generality: whether they borrow domestically or abroad is irrelevant in an integrated capital market. The unavailability of state-contingent claims between firms and workers is essential to retain market incompleteness.

Denote by  $n_{t+1}(\mathbf{s}_{t+1})$  the units of consumption goods received at time  $t + 1$  by domestic workers if the aggregate states are  $\mathbf{s}_{t+1}$ . These are worldwide states, and therefore, they include the aggregates states of both countries, as will be made precise below. Of course, in equilibrium, the consumption units received by workers in the domestic country must be equal to the consumption units paid by workers in the foreign country, that is,  $n_{t+1}(\mathbf{s}_{t+1}) + n_{t+1}^*(\mathbf{s}_{t+1}) = 0$ . This must be satisfied for any possible realization of the aggregate states  $\mathbf{s}_{t+1}$ .

The budget constraint of a worker in the domestic country is

$$w_t h_t + b_t + n_t = c_t + \frac{b_{t+1}}{R_t} + \int_{\mathbf{s}_{t+1}} n_{t+1}(\mathbf{s}_{t+1}) q(\mathbf{s}_{t+1}) / R_t,$$

where  $q_t(\mathbf{s}_{t+1})/R_t$  is the unit price of the contingent claims.

Given the specification of the utility function, the first order conditions for the choice of labor,  $h_t$ , next period bonds,  $b_{t+1}$ , and foreign claims,  $n_{t+1}(\mathbf{s}_{t+1})$ , are

$$\alpha h_t^\gamma c_t = w_t, \tag{8}$$

$$\delta R_t E_t \left( \frac{c_t}{c_{t+1}} \right) = 1, \tag{9}$$

$$\delta R_t \left( \frac{c_t}{c_{t+1}(\mathbf{s}_{t+1})} \right) p(\mathbf{s}_{t+1}) = q(\mathbf{s}_{t+1}), \quad \text{for all } \mathbf{s}_{t+1}, \tag{10}$$

where  $p(\mathbf{s}_{t+1})$  is the probability (or probability density) of the aggregate states in the next period for the world economy.

Since in equilibrium the prices and probabilities of the contingencies are the same for domestic and foreign workers, condition (10) implies that

$$\frac{c_t}{c_t^*} = \frac{c_{t+1}(\mathbf{s}_{t+1})}{c_{t+1}^*(\mathbf{s}_{t+1})} = \chi. \tag{11}$$

Therefore, the ratio of consumption of domestic and foreign workers remains constant over time. This is a well known property of environments with a full set of state-contingent claims. In our environment the constancy of the consumption ratio is among workers (and among investors) but not between workers and investors because of the assumption of market segmentation.

Before continuing we would like to clarify that the assumption of contingent claims among workers is not essential for the results of the paper. We could simply assume that workers can engage in international non-contingent lending and borrowing only. Or equivalently, that firms can engage in international borrowing. However, the availability of contingent claims greatly simplifies the characterization of the equilibrium because it allows us to reduce the number of ‘sufficient’ state variables. This property will be convenient once we extend the model with capital accumulation.

**Aggregate states and equilibrium:** We can now define the equilibrium for the open-economy version of the economy. The aggregate states  $\mathbf{s}$  are given by the liquidation variables  $\xi$  and  $\xi^*$ , the financial liabilities of firms,  $B_t$  and  $B_t^*$ , and the net foreign asset position of domestic firms,  $N_t$ . Since in equilibrium the net foreign asset position of domestic firms is the negative of the foreign position, once we know  $B_t$ ,  $B_t^*$  and  $N_t$  we also know the total wealth of

domestic workers,  $B_t + N_t$ , and foreign workers,  $B_t^* - N_t$ . Therefore,  $\mathbf{s}_t = (\xi, \xi^*, B_t, B_t^*, N_t)$ .

**Definition 3.2 (Recursive equilibrium)** *A recursive competitive equilibrium is defined by a set of functions for: (i) households' policies  $h(\mathbf{s})$ ,  $c(\mathbf{s})$ ,  $b(\mathbf{s})$ ,  $n(\mathbf{s}; \mathbf{s}')$ ,  $h^*(\mathbf{s})$ ,  $c^*(\mathbf{s})$ ,  $b^*(\mathbf{s})$ ,  $n^*(\mathbf{s}; \mathbf{s}')$ ; (ii) firms' policies  $h(\mathbf{s}; b)$ ,  $d(\mathbf{s}; b)$ ,  $b(\mathbf{s}; b)$ ,  $h^*(\mathbf{s}; b)$ ,  $d^*(\mathbf{s}; b)$ ,  $b^*(\mathbf{s}; b)$ ; (iii) firms' values  $V(\mathbf{s}; b)$  and  $V^*(\mathbf{s}; b)$ ; (iv) aggregate prices  $w(\mathbf{s})$ ,  $w^*(\mathbf{s})$ ,  $R(\mathbf{s})$ ,  $m(\mathbf{s}, \mathbf{s}')$ ,  $q(\mathbf{s}; \mathbf{s}')$ ; (v) law of motion for the aggregate states  $\mathbf{s}' = \Psi(\mathbf{s})$ . Such that: (i) household's policies satisfy the optimality conditions (6)-(10); (ii) firms' policies are optimal and satisfy the Bellman's equation (1) for both countries; (iii) the wages clear the labor markets; the interest rates and the price for contingent claims clear the worldwide financial markets; the discount rate used by firms satisfies  $m(\mathbf{s}, \mathbf{s}') = \beta u_c(d_{t+1} + d_{t+1}^*)/u_c(d_t + d_t^*)$ ; (iv) the law of motion  $\Psi(\mathbf{s})$  is consistent with the aggregation of individual decisions and the stochastic process for  $\xi$  and  $\xi^*$ .*

The only difference with respect to the equilibrium in the closed economy is that there is the additional market for foreign claims and the discount factor for firms is given by the worldwide representative investor. The market clearing condition for the foreign claims is  $N(\mathbf{s}') + N^*(\mathbf{s}') = 0$ . This is in addition to the clearing conditions for the domestic bond markets (lending to firms).

Although the general definition of the recursive equilibrium is based on the set of state variables  $\mathbf{s}_t = (\xi, \xi^*, B_t, B_t^*, N_t)$ , we can use some of the properties derived above and characterize the equilibrium with a smaller set of states. Let  $W_t = B_t + B_t^*$  be the worldwide wealth of households/workers. This is the sum of bonds issued by domestic firms,  $B_t$ , and foreign firms,  $B_t^*$ . Then using the fact that the consumption ratio of domestic and foreign workers is constant at  $\chi$  and the employment policy of firms does not depend on the individual debt, the recursive equilibrium can be characterized using the state variables  $\mathbf{s}_t = (\xi, \xi^*, W_t)$ . Essentially, the assumption of cross-country risk-sharing among workers and investors (but not between workers and investors) allows us to reduce the number of 'endogenous' states to only one variable.

Intuitively, by knowing  $W_t$ , we know the worldwide liability of firms, but not the distribution between domestic and foreign firms. However, to characterize the firms' policies, we only need to know the worldwide debt, which is equal to  $W_t$ . Since investors own an internationally diversified portfolio of shares, effectively there is only one representative global investor. It is as if there is a representative firm with two productive units: one unit located in the domestic country and the other in the foreign country. Since both units have a common owner, it does

not matter how the debt is distributed between the two units. What matters from the perspective of the investor, is the total debt and the total payment of dividends. This has some similarity with the problem solved by a multinational firms that faces demand uncertainty in different countries as studied in Goldberg and Kolstad (1995). There is also some similarity with the problem faced by multinational banks that own subsidiaries in different countries. Cettorelli and Goldberg (2010) provide evidence that multinational banks do reallocate financial resources internally in response to country specific shocks.

Total workers' wealth is also a sufficient statistics for the characterization of the workers' policies since the consumption ratio between domestic and foreign households remains constant at  $\chi$ . Therefore, once we solve for the aggregate worldwide consumption, country-specific consumption can be determined by  $\chi$ . This property limits the computational complexity of the model, making feasible the use of non-linear approximation methods. We will come back to this point after the description of the general model with capital accumulation.

We are not ready to prove the following proposition about the impact of a financial shock.

**Proposition 3.1** *An unexpected change in  $\xi_t$  (domestic credit shock) has the same impact on employment and output of domestic and foreign countries.*

**Proof 3.1** *We have already shown that the Lagrange multiplier  $\mu_t$  is common for domestic and foreign firms. If the wage ratio in the two countries does not change, the first order conditions imply that all firms choose the same employment and investment. To complete the proof we have to show that the cross-country wage ratio stays constant. Because firms in both countries have the same demand for labor and the ratio of workers' consumption remains constant, the first order condition for the supply of labor from workers implies that the wage ratio between the two countries does not change.*

Therefore, independently of whether a credit shock hits the domestic or foreign markets, both countries experience the same macroeconomic consequences. This is one of the channels generating cross-country co-movement. Once we make fluctuations in  $\xi_t$  endogenous, we will see that there is a second channel of international co-movement. Before this, however, we would like to point out another feature of the model with exogenous credit shocks. Although a country-specific credit shock can generate macroeconomic co-movement, the financial flows tend to move in opposite directions in the two countries.

To understand this, consider an initial equilibrium in which the enforcement constraints are not binding in either countries. Starting from this equilibrium suppose that the domestic economy is hit by a credit contraction (reduction in  $\xi_t$ ) inducing binding enforcement constraints in both countries. Since  $\xi_t$  is lower only in the domestic country, the outstanding debt of domestic firms contracts but the debt of foreign firms does not contract or even increases. Therefore, the model with ‘exogenous’ credit shocks does not generate cross-country co-movement in the flows of financing. This may be seen as a counterfactual feature of the model since recent data shows a high degree of cross-country co-movement also in the flows of financing. In particular, the growth rate of corporate liabilities contracted sharply during the recent crisis not only in the US but also in Canada, Japan and the Euro area, the countries for which comparable data is available. However, as we will see in the next section, once we make fluctuations in  $\xi_t$  and  $\xi_t^*$  endogenous, the model also generates a high degree of co-movement in financial flows, introducing a second source of real macroeconomic synchronization.

## 4 Endogenous credit shocks

After illustrating how a credit shock propagates to the real sector of the economy, we now provide a micro foundation for endogenous fluctuations in  $\xi_t$ . We proceed first with the closed-economy model and then we extend it to a two-country set-up.

**Financial autarky:** Suppose that in case of liquidation, the capital of the firm can be sold either to households or firms. In the first case one unit of capital will be transformed in  $\underline{\xi}$  units of consumption. Alternatively, the capital of the liquidated firm can be sold to other firms for productive purposes. In this case one unit of capital can be transformed in  $\bar{\xi}$  units of reinstalled capital. The reallocation in other firms is more efficient, that is,  $\underline{\xi} < \bar{\xi} \leq 1$ . However, in order for non-defaulting firms to buy additional capital, they need liquid funds. In this sense our model shares some features of the model studied in Kiyotaki and Moore (2008). Since all firms face the enforcement constraint

$$\xi_t \bar{k} \geq \frac{b_{t+1}}{R_t} + y_t, \tag{12}$$

a non-defaulting firm can buy additional capital only if the firm has previously chosen not to borrow up to the limit, that is, the production and dividend choices are such that the above constraint is not binding. Therefore, if at the beginning of the period firms choose not

to borrow up to the limit, ex-post there will be firms that are able to buy the capital of a defaulting firm. In this case the market price of the liquidated capital is  $\bar{\xi}$ . On the other hand, if at the beginning of the period all firms choose to borrow up to the limit, ex-post there will not be any firm with liquidity required to buy the capital of a liquidated firm. In this case the capital of a defaulting firm can only be sold to households and the market price is  $\underline{\xi}$ .

Since the value of liquidated capital depends on the financial choices of firms, which in turn depends on the expected liquidation value, the model could generate multiple (self-fulfilling) equilibria. Suppose that the market expects that in case of liquidation  $\xi_t = \underline{\xi}$ . The low price of liquidated capital makes the enforcement constraint (12) tighter which may induce firms to borrow up to the limit in order to contain the cut in dividends and/or production. Then, if all firms borrow up to the limit, there will not be ex-post any firm that has liquidity to buy the capital of a defaulting firm. Thus, the ex-post liquidation price is  $\underline{\xi}$ , fulfilling the market expectation.

Now suppose that the expected liquidation price is  $\bar{\xi}$ . Because the enforcement constraint (12) is not tight in the current period but could become tighter in the future, firms may choose not to borrow up to the limit. But then, in case of liquidation, there will be firms capable of purchasing the liquidated capital and the market price is  $\bar{\xi}$ . So also in this case we have that the expectation of a high liquidation value is fulfilled by the firms' choice.

Whether multiple equilibria could arise depends on the particular states of the economy. Three cases are possible:

1. The liquidation price is  $\underline{\xi}$  with probability 1. This arises if we are in a state in which firms choose to borrow up to the limit independently of the expectation over  $\xi_t$ . Therefore, the liquidation price  $\bar{\xi}$  cannot arise.
2. The liquidation price is  $\bar{\xi}$  with probability 1. This arises if we are in a state in which firms do not borrow up to the limit independently of the expectation about  $\xi_t$ . Therefore, the liquidation price  $\underline{\xi}$  cannot arise.
3. The liquidation price is  $\underline{\xi}$  with some probability  $p \in (0, 1)$ . This arises if we are in a state in which firms choose to borrow up to the limit when the expectation for the liquidation value is  $\xi_t = \underline{\xi}$  but they do not borrow up to the limit when the expectation for the liquidation price is  $\xi_t = \bar{\xi}$ . Therefore, we have multiple equilibria.

The third case is the most interesting because it generates multiple sunspot equilibria, and therefore, potential fluctuations in  $\xi_t$ . In this case the low liquidation price  $\underline{\xi}$  could arise with

any probability  $p$ . In general we can denote by  $p_t(\mathbf{s}_t)$  the probability of  $\xi_t = \underline{\xi}$ . Besides the fact that the probability distribution of  $\xi_t$  could be time variant, the properties of the model characterized in previous section do not change.

**Financial integration:** As in the case of a closed economy, different values of  $\xi_t$  are associated to self-fulfilling expectations. Although each country could have different liquidation values of capital, we want to show that the value of  $\xi_t$  cannot be different from  $\xi_t^*$  once the two countries become financially integrated.

As we have seen in the previous section, if the enforcement constraint is binding in one country, it must also be binding in the other country, that is,  $\mu_t = \mu_t^* > 0$ . This excludes the possibility of equilibria with  $\xi_t = \underline{\xi}$  and  $\xi_t^* = \bar{\xi}$ . This is because  $\xi_t = \underline{\xi}$  requires  $\mu_t > 0$  but  $\xi_t^* = \bar{\xi}$  requires  $\mu_t^* = 0$ . Therefore, financial integration implies perfect cross-country co-movement in  $\xi_t$ , which introduces a second channel of real macroeconomic synchronization: not only a change in one country  $\xi$  affects the real sector of the other country but movements in  $\xi$  become perfectly correlated.

Also in the case of financial integration the probability of  $\xi_t = \underline{\xi}$  can be expressed as a function of the aggregate states, that is,  $p(\mathbf{s}_t)$ . Now, however, one of the two equilibrium can be induced by expectations in one of the two countries. For simplicity, suppose that in states with multiple equilibria, the domestic country expects the  $\xi_t = \underline{\xi}$  with probability  $\underline{p}$ . The same for the foreign country. Based on this assumption we have that  $p(\mathbf{s}_t) \in \{0, 2\bar{p}(1 - \bar{p}) + \bar{p}^2, \bar{p}^2, 1\}$ . The probability is zero when firms choose not to borrow up to the limit ( $\mu_t = \mu_t^* = 0$ ) even if the expectation is for  $\xi_t = \xi_t^* = \underline{\xi}$ . The probability is  $2\bar{p}(1 - \bar{p}) + \bar{p}^2$  if firms choose to borrow up to the limit ( $\mu_t = \mu_t^* > 0$ ) when either  $\xi_t$  or  $\xi_t^*$  are equal to  $\underline{\xi}$ . The probability is  $\bar{p}^2$  if firms choose to borrow up to the limit ( $\mu_t = \mu_t^* > 0$ ) only if both  $\xi_t$  or  $\xi_t^*$  are equal to  $\underline{\xi}$ . Finally, the probability is 1 if firms choose to borrow up to the limit ( $\mu_t = \mu_t^* > 0$ ) independently of the values of  $\xi_t$  and  $\xi_t^*$ .

The general definition of equilibrium is analogous to the definition provided for the model with exogenous  $\xi_t$ . We simply need to add the probability function  $p(\mathbf{s}_t)$  which must be consistent with the optimal decisions of firms as described above.

## 5 Model with capital accumulation

We now relax the assumption that the input of capital is fixed. This introduces additional state variables that increases the computational complexity of the model. Since the enforcement constraint is only occasionally binding, we need to use global approximation techniques. Unfortunately, these techniques are computationally intensive and become quickly impractical when we have a large numbers of state variables. Therefore, in order to contain the sufficient set of state variables, we will make some special assumptions about the production technology.

**Investors-firms:** The production function takes the form

$$y_t = (K_t + K_t^*)^{1-\theta} k_t^\theta h_t^\nu \equiv F(K_t + K_t^*, k_t, h_t),$$

where  $K_t$  is the ‘aggregate’ capital in the domestic country and  $K_t^*$  in the foreign country,  $k_t$  is the ‘individual’ input of capital and  $h_t$  is the ‘individual’ input of labor. We assume that  $\theta + \nu < 1$ .

The dependence of the production function from the worldwide stock of capital,  $K_t + K_t^*$ , captures positive externalities. The purpose of the externalities is to have constant returns in the reproducible factor (AK technology), without losing the competitive structure of the model, that is, each producer runs a production technology with non-increasing returns.

Given  $i_t$  the flow of investment, the stock of capital evolves according to

$$k_{t+1} = (1 - \tau)k_t + \Upsilon\left(\frac{i_t}{k_t}\right)k_t,$$

where  $\tau$  is the depreciation rate and the function  $\Upsilon(\cdot)$  is strictly decreasing and concave, capturing adjustment costs in investment. The adjustment cost is a common assumption in international macro models and it is made to prevent excessive volatility of investments.

The budget constraint of the firm is

$$b_t + d_t + i_t = F(K_t, k_t, h_t) - w_t h_t + \frac{b_{t+1}}{R_t},$$

and the enforcement constraint

$$\xi_t k_{t+1} \geq F(k_t, h_t) + \frac{b_{t+1}}{R_t}.$$

We will now take advantage of the AK structure of the production function and normalize the model by the worldwide stock of capital  $K_t + K_t^*$ . Using the tilde sign to denote normalized variables, we can rewrite the budget constraint, law of motion for capital and enforcement constraint as

$$\tilde{b}_t + \tilde{d}_t + \tilde{i}_t = F(\tilde{k}_t, h_t) - \tilde{w}_t h_t + \frac{g_t \tilde{b}_{t+1}}{R_t}, \quad (13)$$

$$g_t \tilde{k}_{t+1} = (1 - \tau) \tilde{k}_t + \Upsilon \left( \frac{\tilde{i}_t}{\tilde{k}_t} \right) \tilde{k}_t, \quad (14)$$

$$\xi_t g_t \tilde{k}_{t+1} \geq F(\tilde{k}_t, h_t) + \frac{g_t \tilde{b}_{t+1}}{R_t}. \quad (15)$$

The variable  $g_t = (K_{t+1} + K_{t+1}^*) / (K_t + K_t^*)$  is the gross growth rate of worldwide capital and  $\tilde{k}_t = k_t / (K_t + K_t^*)$  is the normalized individual capital. We will denote by  $s_t = K_t / (K_t + K_t^*)$  the aggregate share of capital owned by domestic firms ( $s_t^* = 1 - s_t$  is the share of foreign firms). Since in equilibrium  $k_t = K_t$ , we also have that  $\tilde{k}_t = s_t$ .

As in the simpler model without capital accumulation, investors hold an internationally diversified portfolio of shares, and firms use the common discount factor  $m_{t+1} = \beta [(d_{t+1} + d_{t+1}^*) / (d_t + d_t^*)]^{-\sigma}$ . In terms of variables normalized by the worldwide capital, the discount factor can be rewritten as

$$m_{t+1} = g_t^{-\sigma} \beta \left( \frac{\tilde{d}_{t+1} + \tilde{d}_{t+1}^*}{\tilde{d}_t + \tilde{d}_t^*} \right)^{-\sigma} = g_t^{-\sigma} \tilde{m}_{t+1}.$$

The optimization problem solved by an individual firm can be rewritten as

$$\tilde{V}(\tilde{\mathbf{s}}; \tilde{k}, \tilde{b}) = \max_{\tilde{d}, \tilde{h}, \tilde{i}, \tilde{b}'} \left\{ \tilde{d} + g^{1-\sigma} E \tilde{m}' \tilde{V}(\tilde{\mathbf{s}}'; \tilde{k}', \tilde{b}') \right\} \quad (16)$$

subject to (13), (14), (15),

where  $\tilde{V}$  is the firm's value normalized by aggregate worldwide capital  $K + K^*$ , and  $\tilde{\mathbf{s}}$  denotes the normalized aggregate states as specified below.

We can now see the analytical convenience of having the capital externality. Thanks to the AK structure, we can write the firm's value function as  $V_t = (K_t + K_t^*) \cdot \tilde{V}_t$  and rescale the

problem of the firm by worldwide capital. By doing so, we do not need to keep track of the aggregate stock of capital as a state variable. Of course, because we are looking at a general equilibrium, we also need to make sure that the supply of labor does not grow over time. This will be the case with the worker's utility specified earlier.

Appendix B derives the first order conditions for the firm's problem. After imposing the equilibrium conditions  $k_t = K_t$  and  $\tilde{k}_t = s_t$ , the first order conditions can be written as

$$F_h(s_t, h_t) = \frac{\tilde{w}_t}{1 - \mu_t}, \quad (17)$$

$$g_t^{-\sigma} R_t E \tilde{m}_{t+1} = 1 - \mu_t, \quad (18)$$

$$Q_t \Upsilon'(\tilde{i}_t) = 1, \quad (19)$$

$$Q_t = \xi_t \mu_t + \bar{g}_t^{-\sigma} E \tilde{m}_{t+1} \left\{ (1 - \mu_{t+1}) F_k(s_{t+1}, h_{t+1}) - \tilde{i}_{t+1} + [1 - \tau + \Upsilon(\tilde{i}_{t+1})] Q_{t+1} \right\}. \quad (20)$$

Here  $\mu_t$  is the Lagrange multiplier associated with the enforcement constraint and  $Q_t$  is the Lagrange multiplier associated with the law of motion for the stock of capital (Tobin's  $q$ ). We can verify that there is no capital that enters these equations. This confirms that we can ignore the stock of capital when we solve for the normalized equilibrium.

Notice that the property established in the simpler model for which the Lagrange multiplier is common across domestic and foreign firms, also applies to this extended model. In fact, from condition (18) we can see that the common discount factor and the equalization of the interest rates across countries imply  $\mu_t = \mu_t^*$ . Therefore, if the enforcement constraint is binding in one country, it must also be binding in the other. The labor wedge in the demand of labor,  $1/(1 - \mu_t)$ , is also equalized across countries.

**Aggregate states and equilibrium:** Denote by  $\tilde{W}_t = \tilde{B}_t s_t + \tilde{B}_t^* (1 - s_t)$  the normalized worldwide wealth of households/workers. Thanks to the AK technology and the normalization described above, we only need to keep track of two 'endogenous' state variables:  $\tilde{W}_t$  and  $s_t$ . Therefore, compared to the simpler model considered earlier, the introduction of capital accumulation adds only one state variable, that is, the share of worldwide capital owned

by domestic firms,  $s_t$ . This additional state is necessary because of the adjustment cost in investment. In absence of adjustment costs, we could also ignore  $s_t$ .

In addition to  $\tilde{W}_t$  and  $s_t$ , we also have the state  $\xi_t$ . However, this variable takes only two values. Therefore, having only two continuous states variables, it becomes manageable to solve the model numerically using global approximation methods. Appendix C reports the list of equilibrium conditions and describes the computational procedure.

We are not ready to prove the following proposition about the impact of a credit shock.

**Proposition 5.1** *A change in domestic credit markets conditions (change in  $\xi_t$ ) has the same impact on domestic and foreign employment, output, investment and asset prices.*

**Proof 5.1** *We have already shown that the Lagrange multiplier  $\mu_t$  is common for firms of both countries which in turn implies that  $\xi_t^* = \xi_t$ . Since countries are symmetric, the responses of real and financial variables must be the same.*

## 5.1 Extension with productivity shocks

Since a large body of literature in international macroeconomics has developed from the International Real Business Cycle, it would be informative to investigate how our model performs with productivity shocks. This allows us to compare quantitatively the business cycle properties induced by fluctuations in credit market conditions with those induced by the typical TFP shocks.

To this end we specify the production function as

$$y_t = z_t(K_t + K_t^*)^{1-\theta} k_t^\theta h_t^\nu \equiv F(z_t, K_t + K_t^*, k_t, h_t),$$

where  $z_t$  denotes the stochastic level of productivity. The variable  $z_t$  is country-specific and follows a first order Markov process.

In addition to the state variables  $\tilde{W}_t$ ,  $s_t$  and  $\xi_t$ , we now have  $z_t$  and  $z_t^*$ . With the approximation of the processes for the exogenous productivities with parsimonious discrete Markov chains, the computational complexity of the model remains feasible.

## 6 Quantitative analysis

This section studies the properties of the model quantitatively after the calibration of the parameters. The model is solved numerically using the procedure described in Appendix C.

We think of country 1 as the US and country 2 as the other countries in the group of the seven largest industrialized economies, that is, Canada, Japan, France, Germany, Italy, UK. We refer to this group as G6 countries. The discount factor for workers,  $\delta$ , and the discount factor for investors,  $\beta$ , are set to target an average interest rate of 1.6 percent and an average return on equity of 7 percent. In the deterministic steady state the interest rate is equal to  $1/\delta - 1$  and the return on equity is equal to  $1/\beta - 1$ . In the stochastic economy the relations between the intertemporal discount factors and the average returns are more complex. Therefore, in order to choose  $\delta$  and  $\beta$  we have to follow an iterative procedure where we fix these two parameters as well as all other parameters, solve the model and check whether the average returns meet the targets. The required values are  $\delta = 0.996$  and  $\beta = 0.984$ . Therefore, there is a 1 percent difference between the two discount factors. We will see that in equilibrium the differential between the interest rate and expected return on equity is bigger than 1 percent.

The utility function takes the form  $U(c, h) = \ln(c) - \alpha h^{1+1/\eta}/(1+1/\eta)$  where  $\eta$  is the Frisch elasticity of labor supply. We set the elasticity to 0.75 which is between the micro and macro estimates. The parameter  $\alpha$  is set so that working hours are 0.3 on average.

Next we parameterize the production function. The parameter  $\nu$  is chosen to have a steady state labor income share of 0.7. Without uncertainty, the fraction of output going to workers in the form of wages is equal to  $\nu\beta/\delta$ .<sup>7</sup> Given the values of  $\delta$  and  $\beta$ , we choose  $\nu$  so that this fraction is equal to 0.7. Of course, in the stochastic economy the average labor share is not exactly 0.7 but the difference is small. Next we impose the return to scale of an individual firm to  $\theta + \nu = 0.9$ . Given the value of  $\nu$  chosen above, this pins down the value of  $\theta = 0.9 - \nu$ .

The stock of capital evolves according to  $k' = (1 - \tau)k + \Upsilon(i/k)k$ , with the function  $\Upsilon$  taking the form

$$\Upsilon\left(\frac{i}{k}\right) = \frac{\phi_1}{1 - \zeta} \left(\frac{i}{k}\right)^{1 - \zeta} + \phi_2.$$

This functional form is widely used in the literature (see, for example, Jermann (1998)). The

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<sup>7</sup>From the first order condition of labor, equation (4), we derive  $wh/F(z, k, h) = \nu(1 - \mu)$ , which provides an expression for the labor share. To derive an expression for  $\mu$  we use condition (5) evaluated in the version of the model without uncertainty. Taking into account that in a deterministic steady state  $m' = \beta$  and  $R = 1/\delta$ , this condition becomes  $\beta/\delta = 1 - \mu$ . Substituting in the labor share  $\nu(1 - \mu)$ , we get the expression reported in the main text.

parameters  $\phi_1$  and  $\phi_2$  are chosen so that in the deterministic steady state  $Q = 1$  and  $I = \tau K$ . This requires  $\phi_1 = \tau^\zeta$  and  $\phi_2 = -\zeta\tau/(1 - \zeta)$ . Therefore, we need to choose two parameters,  $\tau$  and  $\zeta$ . The first is the depreciation rate which we set to  $\tau = 0.02$ . The second determines the sensitivity of the adjustment cost which we set to  $\zeta = 0.5$ .

At this point we are left with the parameters for the stochastic properties of the shocks. Let's start with the productivity series. We construct Solow residuals series for the US and for the aggregate of the remaining G6 countries. Using these series we estimate a two state autoregressive process,

$$\begin{pmatrix} \log(z_{t+1}^{US}) \\ \log(z_{t+1}^{G6}) \end{pmatrix} = \begin{bmatrix} \rho_z & \psi_z \\ \psi_z & \rho_z \end{bmatrix} \begin{pmatrix} \log(z_t^{US}) \\ \log(z_t^{G6}) \end{pmatrix} + \begin{pmatrix} \epsilon_{t+1}^{US} \\ \epsilon_{t+1}^{G6} \end{pmatrix},$$

where the log series are linearly detrended and  $\epsilon_{t+1}^{US}$  and  $\epsilon_{t+1}^{G6}$  are mean zero white noises with standard deviation  $\sigma_z^{US}$  and  $\sigma_z^{G6}$  respectively. The estimation returns  $\rho_z = 0.98$ ,  $\psi_z = -0.008$ ,  $\sigma_z^{US} = 0.0059$ ,  $\sigma_z^{G6} = 0.0065$ . Finally the correlation between residuals is 0.15.

The estimation shows that there is very low cross-country comovement between the Solow residuals of the two countries and they have very similar standard deviations. Therefore, the process for the productivity variables can be well approximated by symmetric and independent first order autoregressive processes with autocorrelation parameter  $\rho = 0.98$  and standard deviation of residuals  $\sigma = 0.0062$ .

Let's turn now to the financial shocks. The variable  $\xi$  takes only two values. In addition to the choice of these two values we have to pin down  $\bar{p}$ , that is, the probability with which the equilibrium with  $\xi = \underline{\xi}$  arises in states with multiple equilibria. We choose  $\underline{\xi}$ ,  $\bar{\xi}$  and  $\bar{p}$  to match three targets: (i) the average leverage (debt over capital), (ii) the standard deviation of debt, (iii) the frequencies of crisis.<sup>8</sup> The full list of parameter values are reported in Table 2.

Appendix C describes the computational procedure based on the discretization of the state space. In particular, the exogenous states  $z_t$  and  $z_t^*$  are each approximated with a three-state Markov chain using Tauchen (1986). The endogenous states  $\tilde{b}_t$  and  $s_t$  are each discretized on a grid containing eleven points. Values of the endogenous outside the grids are determined through bi-linear interpolation.

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<sup>8</sup>Although the three parameters need to be chosen jointly, we can identify the primary parameter that affect each of the three targets. The average leverage is mostly determined by the average value of  $\xi$ . The standard deviation of debt is mostly determined by the standard deviation of  $\xi$ . The frequency of crisis is mostly determined by  $\bar{p}$ .

Table 2: List of parameters

Discount factor for households/workers, $\delta$	0.996
Discount factor for entrepreneurs, $\beta$	0.986
Utility parameter, $\alpha$	16.293
Production technology, $\theta$	0.200
Production technology, $\nu$	0.700
Depreciation rate, $\tau$	0.020
Capital adjustment cost, $\zeta$	0.050
Productivity persistence, $\rho_z$	0.980
Productivity volatility, $\sigma_z$	0.006
Low liquidation value, $\underline{\xi}$	0.550
High liquidation value, $\bar{\xi}$	0.650
Frequency of low liquidation value, $\bar{p}$	0.200

## 6.1 Results

In presenting the results we outline four main properties: (i) the asymmetric response to shocks; (ii) the countercyclicality of labor productivity in response to credit shocks; (iii) the severity of crisis after long periods of credit and macroeconomic booms; (iv) the importance of credit shocks for the volatility of labor and asset prices.

**Asymmetry:** Figure 8 plots the impulse responses to a credit expansion and to a credit contraction. Since the impact of changing credit market conditions are equal in the two countries, we report only the responses for one country.

A credit expansion is generated starting from the limiting equilibrium in which the economy converges after a long series of draws  $\xi_t = \underline{\xi}$ . From this equilibrium we consider a sequence of draws  $\xi_t = \bar{\xi}$  starting at  $t = 1$ . Therefore, a credit expansion is generated by a permanent switch from  $\underline{\xi}$  to  $\bar{\xi}$ . Similarly, the impulse responses to a credit contraction are generated starting from the limiting equilibrium in which the economy converges after a long series of draws  $\xi_t = \bar{\xi}$ . Starting at  $t = 1$  the economy experiences a sequence of draws  $\xi_t = \underline{\xi}$ . The draws of productivity are assumed to be  $z_t = \bar{z}$ , that is, the mean value. It is important to point out that agents do not know in advance the actual draws of  $\xi_t$  and  $z_t$ , and therefore, they make their choices taking into account the uncertainty induced by the stochastic distribution of  $\xi_t$  and  $z_t$ .

In response to the credit expansion we see a gradual increase in the stock of debt and a persistent expansion in labor, output. The magnitude of the macroeconomic expansion, however, is not large at impact. The responses to a credit contraction display a different pattern. The stock of debt declines more quickly and the responses of labor, output and

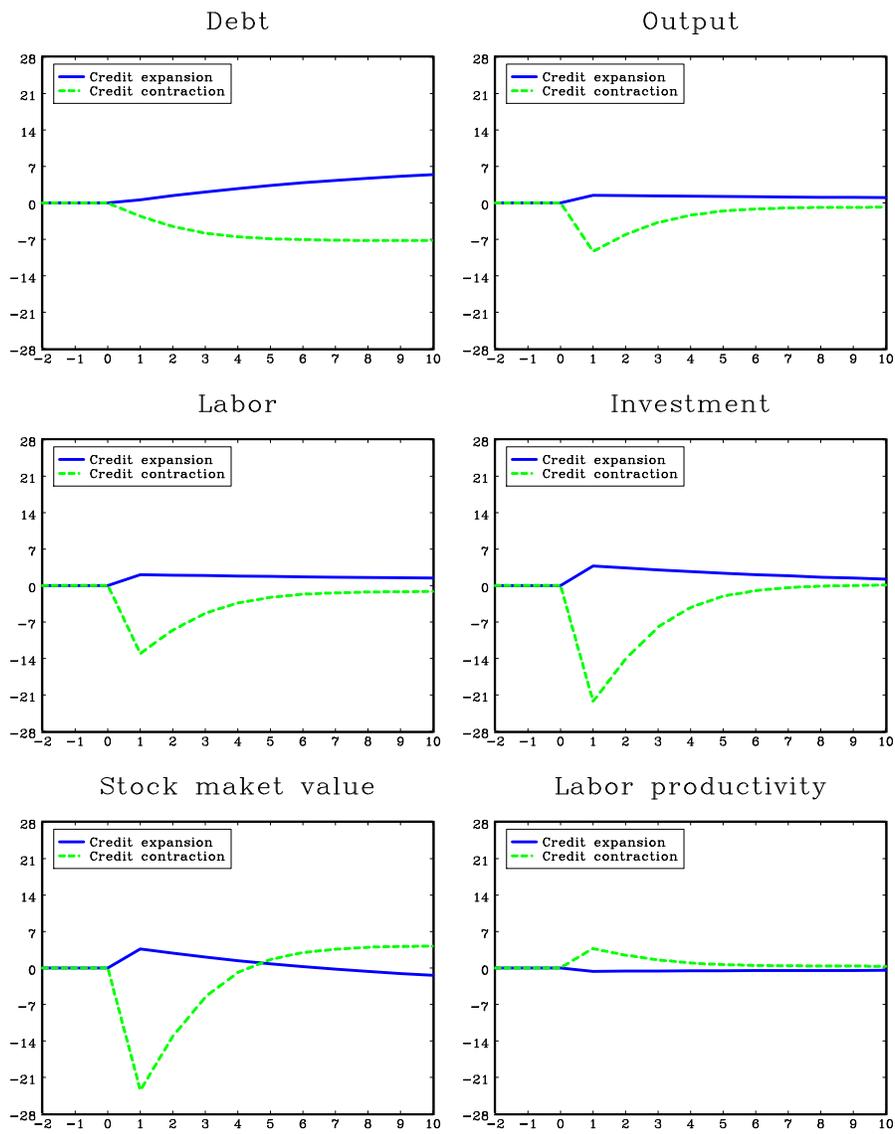


Figure 8: Impulse responses to credit expansions and contractions.

investment are much larger at impact. However, the initial declines are not very persistent. Therefore, there is a strong asymmetry in the responses to credit expansions and contractions.

**Countercyclicality in labor productivity:** The last panel of Figure 8 plots the impulse responses of labor productivity generated using the approach described above. As in the previous figure we see an asymmetry between credit expansions and credit contractions. More importantly, a credit expansion generates a decline in labor productivity while a credit contraction generates an increase. This is important for capturing one of the facts outlined in the first section of the paper, that is, the weak or even negative correlation between labor productivity and employment.

**Credit booms and severity of recessions:** Figure 9 plots the impulse responses to a credit expansion that later reverts back to the initial level. A credit expansion is generated as described above. Starting from an equilibrium to which the economy converges after a long series of  $\xi_t = \underline{\xi}$ , we assume that at time 1 the economy experiences a switch to  $\xi_t = \bar{\xi}$  (credit expansion). The value of  $\xi$  stays at the higher value for several periods and then it reverts back to  $\underline{\xi}$  permanently. We consider credit booms with duration of 4 quarters (left panels) and 20 quarters (right panels).

The key finding is that the severity of the credit contraction increases with the duration of the credit expansion. After a protracted credit boom, the economy accumulates large leverages. When the credit reversal arrives, the required de-leveraging is more severe generating a stronger macroeconomic contraction. In this way the model can explain why recessions that arise after long periods of financial and macroeconomic expansions tend to be more damaging for the real economy.

**Volatility of labor and asset prices:** Table 3 reports the standard deviations of various variables. Three versions of the economy are considered: the economy with productivity shocks only; the economy with credit shocks only; and the economy with both shocks. The statistics are computed after detrending the simulated series with a band-pass filter that preserves cycles of 1.5-8 years (Baxter and King (1999)).

Two properties are especially noticeable. First, the model with credit shocks can generate much higher volatility of labor, bringing the model closer to the data. Second, credit shocks also generate a high volatility of asset prices. In particular, in the version of the model with only credit shocks, the stock market value (equity value of firms) is almost three times more

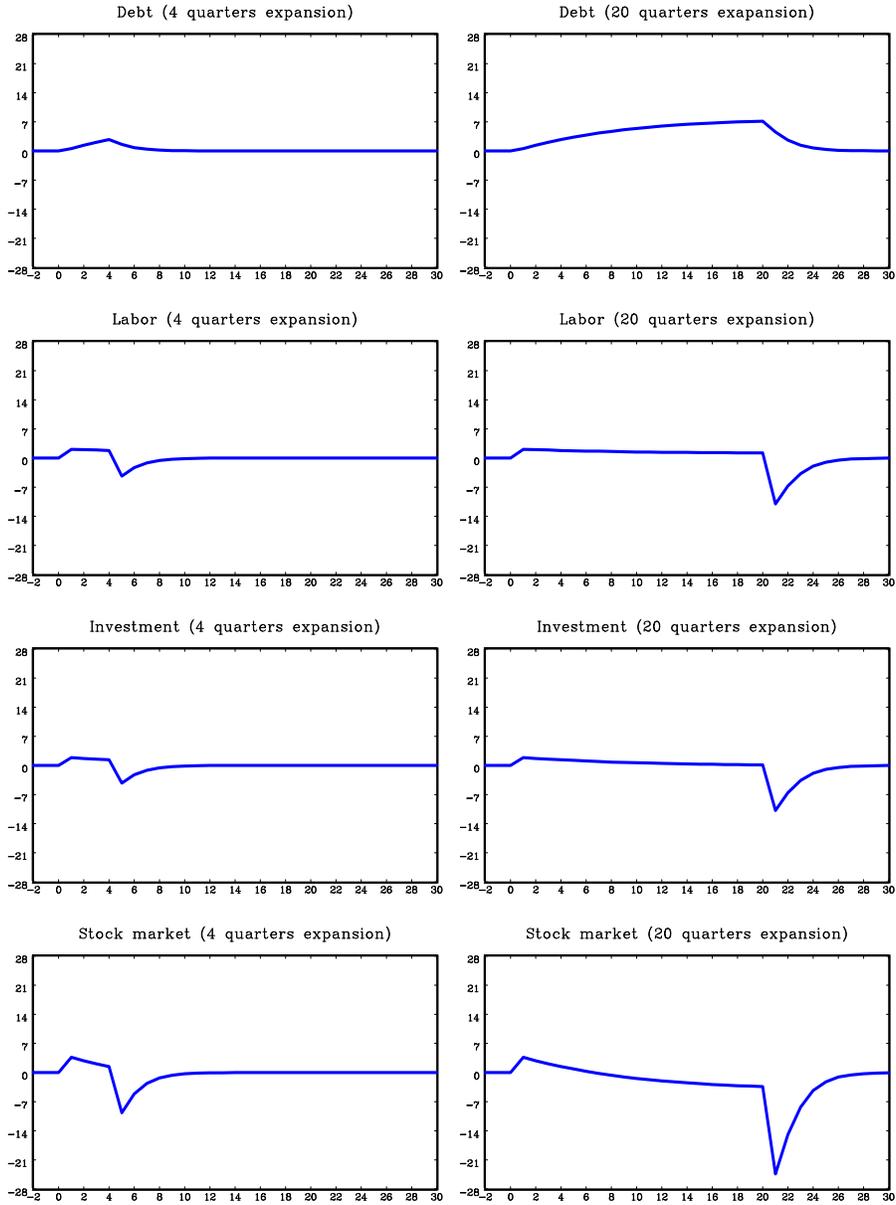


Figure 9: Duration of credit expansions and severity of contractions.

Table 3: Business cycle statistics of key variables from detrended simulated series.

	Productivity shocks only	Credit shocks only	Both shocks
<i>Standard deviations</i>			
Output	0.76	0.88	1.16
Consumption	0.44	0.68	0.77
Labor	0.26	1.26	1.26
Investment	0.77	2.27	2.36
Tobin's q	0.38	1.14	1.18
Stock market value	0.54	2.46	2.45
Interest rate	0.25	0.48	0.48
Return on equity	0.37	5.82	5.82
<i>Expected returns (% annualized)</i>			
Interest rate	1.56	1.40	1.40
Return on equity	5.62	6.96	6.96
Equity risk premium	0.06	1.74	1.74
Nonbinding constraints, %	99.99	96.44	96.04

volatile than output. This can also be seen in the bottom panel of Figure 8 plotting the impulse responses of the market value of equity to a credit expansion and contraction. In contrast, with only productivity shocks, the value of the firm is less volatile than output.

As a result of the higher volatility of asset prices and of the discount factor for investors, the model can also generate a non-negligible equity risk premium. Here we should be careful in defining the equity risk-premium. Since bond holders (workers) have a higher discount factor than equity holders (investors), the difference between the expected return on equity (for investors) and the return on bonds (for workers) is not the risk premium. In fact, even in absence of risk, the return on equity will be higher than the return on bonds. In fact, given the calibration of  $\delta = 0.996$  and  $\beta = 0.986$ , the return differential in absence of risk would be 1 percent. Given this, we define the 'equity risk premium' as the difference between the return differential in the economy with risk and in the economy without risk. This is about 1.74 percent yearly and it mainly generated by credit shocks. We also observe that the volatility of equity returns is quite high in the model but the volatility of the interest rate is quite small.<sup>9</sup>

<sup>9</sup>Notice that the standard deviations for the returns are calculated on undetrended data.

## 7 Conclusion

The recent financial crisis has been characterized by an historically high degree of international synchronization in real and financial variables. We have proposed a theoretical environment in which endogenous fluctuations in credit market conditions is one of the driving forces of the business cycle. These fluctuations affect the real sector of the economy through a credit channel: booms enhance the borrowing capacity of firms and in the general equilibrium they lead to higher employment, production but lower productivity of labor. The opposite arises after a credit contraction. Interestingly, business cycle fluctuations induced by movements in credit markets are highly asymmetric, *i.e.*, contractions are sharper than expansions and they generate large fluctuations in asset prices.

When countries are financially integrated, movements in credit markets also generate large spillovers to the real and financial sectors of other countries. There are two channels of international transmission. The first is through the cost of capital which in an integrated financial market is equalized across countries. The second channel is based on the endogenous nature of credit market conditions. These conditions change when the economy switches from one self-fulfilling equilibrium to another self-fulfilling equilibrium. But in an integrated world market the switch in one country can only arise if the switch takes place also in the other. Therefore, changing financial market conditions are perfectly synchronized when financial markets are internationally integrated.

On the contrary, country-specific productivity shocks do not generate large cross-country co-movement in real macroeconomic variables unless the shocks are internationally correlated. However, if productivity shocks are correlated across countries and they are a major source of business cycle fluctuations, it is difficult to reconcile the fact that the correlation of labor productivity with measures of labor inputs is low and it has further declined in recent years.

Other sources of business cycle fluctuations besides the ones considered in this paper could also generate a weak or negative correlation between productivity and labor. However, it is not obvious whether the most common shocks studied in the literature (for example those considered by Smets and Wouters (2007)) can generate international co-movement in real and financial variables, unless the shocks are internationally correlated. Our interest in changing credit market conditions as a source of business cycle fluctuations is motivated by their ability to generate large cross-country co-movements in the real sector of the economy together with the large international co-movements in the flows of financing.

## Appendix

### A First order conditions

Consider the optimization problem (1) and let  $\lambda$  and  $\mu$  be the Lagrange multipliers associate with the two constraints. Taking derivatives we get:

$$\begin{aligned} d : \quad & 1 - \lambda = 0 \\ h : \quad & \lambda[F_h(z, h) - w] - \mu F_h(z, h) = 0 \\ b' : \quad & (1 + \phi\mu)Em'V_{b'}(\mathbf{s}'; b') + \frac{\lambda}{R} = 0 \end{aligned}$$

The envelope condition is:

$$V_b(\mathbf{s}; b) = -\lambda$$

The above conditions can be re-arranged as in (4) and (5).

### B First order conditions for the model with capital

Differentiating the firm's problem (16) with respect to  $h_t, \tilde{b}_{t+1}, \tilde{i}_t, \tilde{k}_{t+1}$ , we get:

$$F_h(z_t, \tilde{k}_t, h_t) = \frac{\tilde{w}_t}{1 - \mu_t} \tag{21}$$

$$\frac{1 - \mu_t}{R_t} + g_t^{-\sigma} E\tilde{m}_{t+1}\tilde{V}_b(\tilde{\mathbf{s}}_{t+1}; \tilde{k}_{t+1}, \tilde{b}_{t+1}) = 0 \tag{22}$$

$$Q_t \Upsilon' \left( \frac{\tilde{i}_t}{\tilde{k}_t} \right) = 1 \tag{23}$$

$$Q_t = \xi_t \mu_t + g_t^{-\sigma} E\tilde{m}_{t+1}\tilde{V}_k(\tilde{\mathbf{s}}_{t+1}; \tilde{k}_{t+1}, \tilde{b}_{t+1}) \tag{24}$$

where  $\mu_t$  is the lagrange multiplier associated with the enforcement constraint and  $Q_t$  (Tobin's q) is the lagrange multiplier associated with the law of motion of capital. The multiplier associated with the budget constraint is 1. For the foreign country we have the same conditions but with country specific variables denoted with the start superscript.

The envelope conditions are:

$$\tilde{V}_b(\tilde{\mathbf{s}}_t; \tilde{k}_t, \tilde{b}_t) = -1 \quad (25)$$

$$\tilde{V}_k = (1 - \mu_t)F_k(z_t, \tilde{k}_t, h_t) + \left[ 1 - \tau + \Upsilon \left( \frac{\tilde{i}_t}{\tilde{k}_t} \right) - \Upsilon' \left( \frac{\tilde{i}_t}{\tilde{k}_t} \right) \frac{\tilde{i}_t}{\tilde{k}_t} \right] Q_t \quad (26)$$

Substituting the envelope conditions and imposing the equilibrium conditions  $k_t = K_t$  and  $\tilde{k}_t = s_t$ , we obtain (17)-(20).

## C Dynamic system and solution approach

We will use the bar sign to denote aggregate worldwide variables normalized by the worldwide stock of capital. For example, the  $\bar{d}_t$  is the normalized worldwide dividend, defined as:

$$\bar{d}_t = \frac{d_t + d_t^*}{K_t + K_t^*} \equiv \tilde{d}_t + \tilde{d}_t^*.$$

The full list of equilibrium conditions are:

$$1 = \delta g_t^{-1} R_t E_t \left( \frac{\bar{c}_{t+1}}{\bar{c}_t} \right)^{-1} \quad (27)$$

$$\tilde{c}_t^* = \chi \tilde{c}_t \quad (28)$$

$$\tilde{w}_t h_t + \tilde{w}_t^* h_t^* + \bar{b}_t = \bar{c}_t + \frac{g_t \bar{b}_{t+1}}{R_t} \quad (29)$$

$$\bar{b}_t + \bar{d}_t + \bar{i}_t = F(z_t, s_t, h_t) + F(z_t^*, s_t^*, h_t^*) - \tilde{w}_t h_t - \tilde{w}_t^* h_t^* + \frac{\bar{g}_t \bar{b}_{t+1}}{R_t} \quad (30)$$

$$g_t (\xi_t s_{t+1} + \xi_t^* s_{t+1}^*) \geq \frac{g_t \bar{b}_{t+1}}{R_t} + F(z_t, s_t, h_t) + F(z_t^*, s_t^*, h_t^*) \quad (31)$$

$$(1 - \mu_t) \bar{d}_t^{-\sigma} = \beta g_t^{-\sigma} R_t E_t \bar{d}_{t+1}^{-\sigma} \quad (32)$$

$$\alpha h_t^\gamma = \frac{\tilde{w}_t}{\tilde{c}_t} \quad (33)$$

$$\alpha (h_t^*)^\gamma = \frac{\tilde{w}_t^*}{\tilde{c}_t^*} \quad (34)$$

$$g_t s_{t+1} = (1 - \tau) s_t + \Upsilon \left( \frac{\tilde{i}_t}{s_t} \right) s_t \quad (35)$$

$$g_t s_{t+1}^* = (1 - \tau) s_t^* + \Upsilon \left( \frac{\tilde{i}_t^*}{s_t^*} \right) s_t^* \quad (36)$$

$$F_h(z_t, s_t, h_t) = \frac{\tilde{w}_t}{1 - \mu_t} \quad (37)$$

$$F_h(z_t^*, s_t^*, h_t^*) = \frac{\tilde{w}_t^*}{1 - \mu_t} \quad (38)$$

$$Q_t \Upsilon' \left( \frac{\tilde{i}_t}{s_t} \right) = 1 \quad (39)$$

$$Q_t^* \Upsilon' \left( \frac{\tilde{i}_t^*}{s_t^*} \right) = 1 \quad (40)$$

$$Q_t = \xi_t \mu_t + \beta g_t^{-\sigma} E \left( \frac{\bar{d}_{t+1}}{\bar{d}_t} \right)^{-\sigma} \left\{ (1 - \mu_{t+1}) F_k(z_{t+1}, s_{t+1}, h_{t+1}) - \frac{\tilde{i}_{t+1}}{s_{t+1}} + \left[ 1 - \tau + \Upsilon \left( \frac{\tilde{i}_{t+1}}{s_{t+1}} \right) \right] Q_{t+1} \right\} \quad (41)$$

$$Q_t^* = \xi_t^* \mu_t + \beta g_t^{-\sigma} E \left( \frac{\bar{d}_{t+1}}{\bar{d}_t} \right)^{-\sigma} \left\{ (1 - \mu_{t+1}) F_k(z_{t+1}^*, s_{t+1}^*, h_{t+1}^*) - \frac{\tilde{i}_{t+1}^*}{s_{t+1}^*} + \left[ 1 - \tau + \Upsilon \left( \frac{\tilde{i}_{t+1}^*}{s_{t+1}^*} \right) \right] Q_{t+1}^* \right\} \quad (42)$$

Equations (27)-(42) form a dynamic system composed of 16 equations. Given the states  $z_t, \xi_t, z_t^*, \xi_t^*, \bar{b}_t, s_t$ , the unknown variables are  $h_t, h_t^*, c_t, c_t^*, w_t, w_t^*, i_t, i_t^*, Q_t, Q_t^*, g_t, \mu_t, R_t, \bar{d}_t, \bar{b}_{t+1}, s_{t+1}$ . Therefore, we have a dynamic system of 16 equations in 16 unknowns.

The computational procedure is based on the approximation of four functions:

$$\Gamma_1(\mathbf{s}_{t+1}) = \bar{c}_{t+1}^{-1}$$

$$\Gamma_2(\mathbf{s}_{t+1}) = \bar{d}_{t+1}^{-\sigma}$$

$$\Gamma_3(\mathbf{s}_{t+1}) = \bar{d}_{t+1}^{-\sigma} \left\{ (1 - \mu_{t+1}) F_k(z_{t+1}, s_{t+1}, h_{t+1}) - \frac{\tilde{i}_{t+1}}{s_{t+1}} + \left[ 1 - \tau + \Upsilon \left( \frac{\tilde{i}_{t+1}}{s_{t+1}} \right) \right] Q_{t+1} \right\}$$

$$\Gamma_4(\mathbf{s}_{t+1}) = \bar{d}_{t+1}^{-\sigma} \left\{ (1 - \mu_{t+1}) F_k(z_{t+1}^*, s_{t+1}^*, h_{t+1}^*) - \frac{\tilde{i}_{t+1}^*}{s_{t+1}^*} + \left[ 1 - \tau + \Upsilon \left( \frac{\tilde{i}_{t+1}^*}{s_{t+1}^*} \right) \right] Q_{t+1}^* \right\}.$$

In addition to these four function, we need to guess the function  $p(\mathbf{s}_{t+1})$ , that is, the probability of  $\xi_{t+1} = \underline{\xi}$ . This is necessary to compute the next period expectation.

The procedure starts with a guess for the values of the approximated functions  $\Gamma_1(\mathbf{s}_{t+1}), \Gamma_2(\mathbf{s}_{t+1}), \Gamma_3(\mathbf{s}_{t+1})$  and  $\Gamma_4(\mathbf{s}_{t+1})$ . We first form a two dimensional grid for the endogenous states  $\bar{b}$  and  $s$ . Then for each realization of the exogenous shocks— $z_t, \xi_t, z_t^*, \xi_t^*$ —we guess the values taken by the above functions over the grid points. Values outside the grid are obtained through bi-linear interpolation. Next we guess  $p(\mathbf{s}_{t+1})$  for each grid point. Once we know the approximated functions and probabilities for  $\xi_{t+1}$ , we can solve for the 18 unknowns of the system (27)-(42) at each grid point and for each realization of the shocks. In finding the solutions we check whether the enforcement constraint is binding ( $\mu_t > 0$ ) or not binding ( $\mu_t = 0$ ). We then use the solutions found at each grip point to update the guesses for the four functions and the probabilities  $p(\mathbf{s}_{t+1})$ . To update these probabilities we need to check whether multiple equilibria are possible at each grid point for the states. We keep iterating until the guesses for  $\Gamma_1(\mathbf{s}_{t+1}), \Gamma_2(\mathbf{s}_{t+1}), \Gamma_3(\mathbf{s}_{t+1}), \Gamma_4(\mathbf{s}_{t+1})$  and  $p(\mathbf{s}_{t+1})$  at each grid point are equal to the values obtained by solving the dynamic system.

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