Labor Supply, Frictions, and the Business Cycle

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Background/Motivation

Two frameworks currently serve as benchmarks for thinking about labor market outcomes in an aggregate setting:

- *frictionless models*– some version of the growth model extended to include a labor/leisure tradeoff, with Hansen (1985) as one prototype.
- frictional models– some version of a worker search problem extended to an equilibrium setting, with Shimer (2005) as one prototype.

The Need of an Integrated Model

Both assumptions are relevant in the real world:

- for some individuals the margin of working vs not working is very relevant
- for some individuals this margin is effectively irrelevant.

Objectives of the project

First objective is to build a simple yet *empirically reasonable* hybrid model.

- Our current criterion for *empirically reasonable* is to match observed labor market flows across E, U, N states.

Second objective is to use the model to examine several questions of interest in aggregate labor market analysis

- Steady-state impact of changes in frictions.
- Steady-state impact of labor tax and transfer programs.
- Business cycle fluctuations in the labor market.

Some Related Work

- Frictions in RBC model:

  Merz (1994), Andolfatto (1996), Alvarez and Veracierto (1999), Gomes, Greenwood and Rebelo (2002), Ljungqvist and Sargent (2008).

- Three-State Models of Labor Market Dynamics:

  Burdett, Keifer, Mortensen, Neumann (1984), Andolfatto, Gomme and Storer (1998), Alvarez and Veracierto (1999), Garibaldi and Wasmer (2006), Tripier (2003), Veracierto (2008).

- Labor Supply with Incomplete Markets/Frictions:

  Domeij and Floden (2005), Chang and Kim (2006, 2007), Pijoan-Mas (2006), Low, Meghir, and Pistaferri (2008).

Model: Workers

- Utility:
$$E\left[\sum_{t=0}^{\infty}\beta^t[\log(c_t)-\alpha e_t)]\right].$$
Indivisible labor: $e_t \in \{0,1\}$.
- We focus on the steady state.
- Constraints:
$$c_t + k_{t+1} = (1+r-\delta)k_t + (1-\tau)ws_t e_t + T$$
and
$$k_{t+1} \geq 0.$$
- Idiosyncratic productivity shocks (s) follow a stochastic process
$$\ln s_{t+1} = \rho \ln s_t + \varepsilon_{t+1}.$$
- Note: Only self-insurance is allowed.

Model: Firms and Government

Firms:

- Production function:

$$Y = K^{\theta} L^{1-\theta}, \text{ where } K = \int k_i di \text{ and } L = \int e_i s_i di.$$

- Competitive markets:

$$w = MPL, r = MPK.$$

Government:

- Budget constraint:

$$T = \tau w L.$$

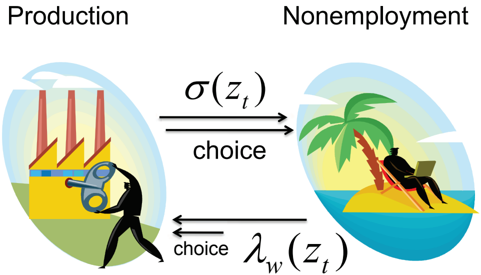
Frictions

Frictions captured by two exogenous parameters:

- σ: separation probability
- λ_w: employment opportunity arrival rate

Later on, we will consider (exogenous) fluctuations in σ and λ_w.

When they fluctuate, we let them comove one-for-one with z (as in the Pissarides model, where this occurs endogenously).



Model: Consumers

- A consumer's state consists of
  - her location
  - the level of asset holdings
  - her productivity.
- Individuals (potentially) make two choices:
  - consumption/saving
  - work/leisure.

Model: Consumers

- A consumer's state consists of her location at the time that the labor supply decision needs to be made, the level of asset holdings, and productivity.
- Individuals (potentially) make two choices: consumption/saving and work/leisure.
- Define

$$V(k, s) = max\{W(k, s), N(k, s)\}$$

  where
  - $W(k, s)$ is the maximum value for an individual who works;
  - $N(k, s)$ is the maximum value for an individual who does not work.

Model: Bellman Equations (Worker)

$$W(k,s) = \max_{c,k'}\{\log(c) - \alpha + \beta E_{s'}[(1-\sigma+\sigma\lambda_w)V(k',s') + \sigma(1-\lambda_w)N(k',s')]\}$$

subject to

$$c + k' = rk + (1-\tau)ws + (1-\delta)k + T$$

and

$$c \geq 0, k' \geq 0.$$

- Note that a worker who gets separated might get an employment opportunity in the same period.
- Recall that σ is the job separation rate and λ_w is the job arrival rate.

Model: Bellman Equations (Nonworker)

$$N(k,s) = \max_{c,k'} \{\log(c) + \beta E_{s'}[\lambda_w V(k',s') + (1-\lambda_w) N(k',s')]\}$$

subject to

$$c + k' = rk + (1-\delta)k + T$$

and

$$c \geq 0, k' \geq 0.$$

- Note that an individual who gets a job offer decides whether or not to work.

Unemployment in the Model

- We call a person *unemployed* if she likes to work if given the opportunity, i.e., she would like to work at the *going* wage rate but does not have the opportunity.
- We think this captures the essence of what economists have in mind when they talk about unemployment.
- For period 1994-2007 the average for this unemployment rate is 8.3% (versus 5.1%).
- We compute the flow data to reflect this notion of unemployment.

Matching the Flow Data

Question: Can a reasonable parametrization of this model account for both standard aggregate outcomes as well as the distribution and flows of workers across labor market states?

- Several parameters are standard: β, α, θ, δ, τ.
- Less standard parameters are the ones related to the
  - frictions: σ, λ_w
  - productivity process: ρ, σ_ε^2.

Calibration

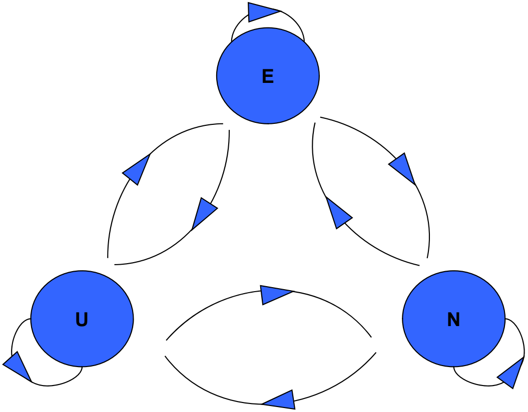
Set 1 period = 1 month.
Standard parameters:

- Set β, α, θ, δ, τ as is usually done.
- $\beta = 0.9967$, $\alpha = 0.557$, $\theta = 0.3$, $\delta = 0.0067$, (annual return to capital$= 0.04$, $E/P = 63.2\%$, $rK/Y = 0.3$, $I/Y = 0.2$), $\tau = 0.3$.

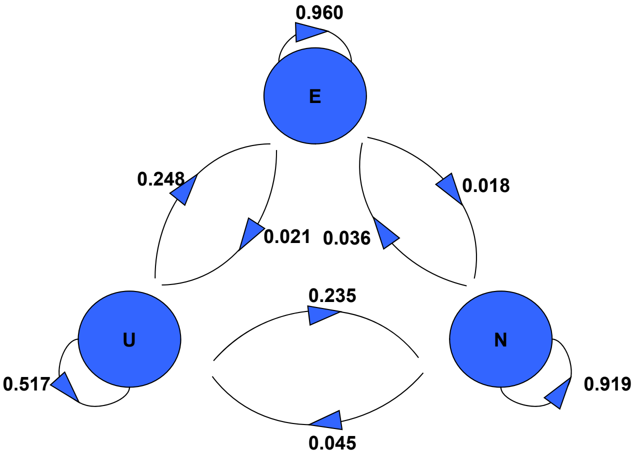
Less standard parameters:

- Set λ_w to match the unemployment rate.
- Set σ to match E$\rightarrow$U flow.
- We examine different values for ρ and σ_ε and the implied flows.
  - Today we show results for $\rho = 0.92$ and $\sigma_\varepsilon = 0.21$.
  - Results for flows quites similar as long as $\rho > 0.5$ and $\sigma_\varepsilon > 0.05$ (measured annually).

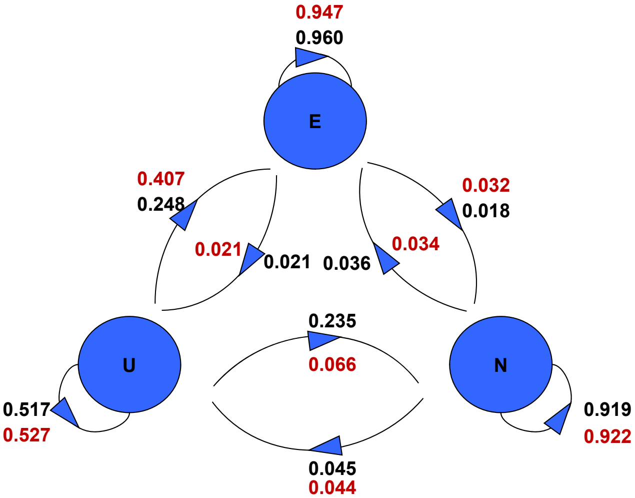
Labor Market States



Adjusted Labor Market Flows in the CPS data, 1994-2007



Labor Market Flows in the Model



The Model's Performance

- The model predicts that
  - the U to N flow is too low
  - the U to E flow is too large.
- Empirical evidence suggests that transitory transitions and measurement error could create significant biases in the measurement of these flows.

Overall, the model does a reasonable job of accounting for the flow rates observed in the data.
$\longrightarrow$ *Persistent productivity shocks* are key to matching the persistence of E and N states.

Changes in Frictions: Steady-State Implications

- One of the defining features of the Pissarides-style matching models is that the level of frictions plays a key role in determining:
  - not only the level of unemployment,
  - but also the level of aggregate employment.
- Intuitively, labor supply considerations will *attenuate* the impact of changes in frictions on aggregate employment.
- If it becomes harder to find a job then workers will be more willing to continue to work once they find it.
- We explore the quantitative importance of these effects.

The Impact of a Change in λ_w in the Pissarides-style Search Models

- In the Pissarides-style search model

$$u_{t+1} = (1-\lambda_w)u_t + \sigma(1-\lambda_w)(1-u_t).$$

At steady-state

$$\bar{u} = \frac{\sigma(1-\lambda_w)}{\sigma(1-\lambda_w)+\lambda_w}.$$

Note that individuals who separate from their jobs get an employment opportunity within the same period.

- Set $\sigma = 0.039$.
- Calibrate λ_w such that $\bar{u} = 0.083$, which gives $\lambda_w = 0.30$.
- Change λ_w proportionally in both models.

The Impact of a Change in λ_w on E/P and Unemployment

Our model			Pissarides model		
	E/P	u		E/P	u
$\lambda_w = 0.6$	63.5%	4.6%	$\lambda_w = 0.41$	94.8%	5.2%
Benchmark	63.2%	8.3%	$\lambda_w = 0.30$	91.7%	8.3%
$\lambda_w = 0.4$	63.0%	9.3%	$\lambda_w = 0.28$	90.7%	9.2%
$\lambda_w = 0.2$	61.0%	18.8%	$\lambda_w = 0.14$	80.4%	19.6%

- Similar effects on unemployment in both models.
- Much smaller effects on employment in our model: there is considerable *substitution* between *voluntary nonemployment* spells and *involuntary unemployment* spells.

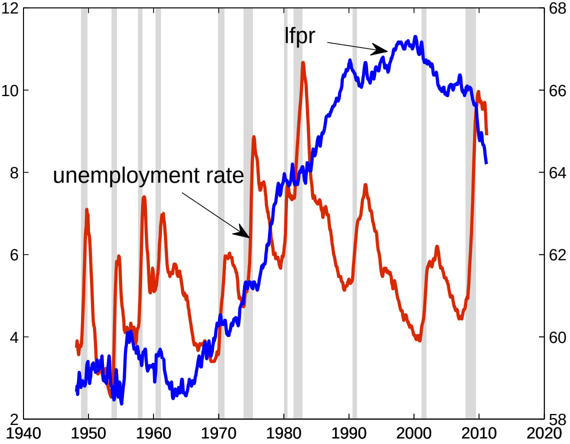
Business Cycle Analysis

- We built a model which is empirically reasonable and captures both frictions and the labor supply channel.
- We can now use it to examine how various shocks affect the cyclical properties of labor market variables:
  - the distribution of workers across employment (E), unemployment (U), and not in the labor force (N)
  - the associated flows between these three states.

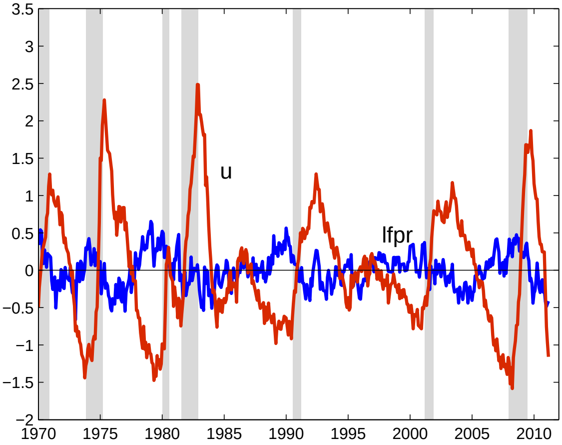
Is the Participation Margin Relevant for Business Cycle Analysis?

- Based on the cyclical behavior of labor market stocks it is tempting to conclude that movements in and out of the labor force have a negligible impact.
- Widespread belief that generating fluctuations in the job-finding rate is the key for a successful model of labor market fluctuations.

The Unemployment and Labor Force Participation Rates

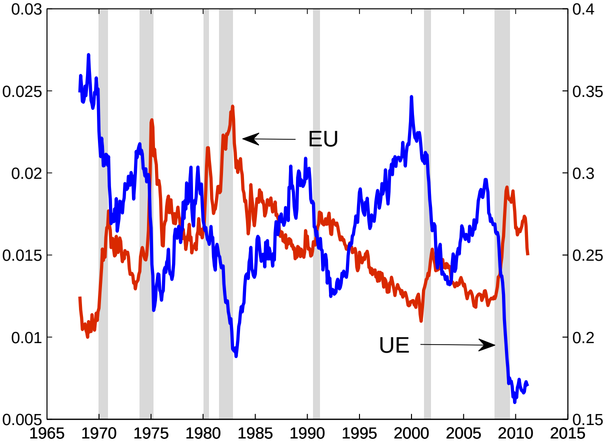


Cyclical Components



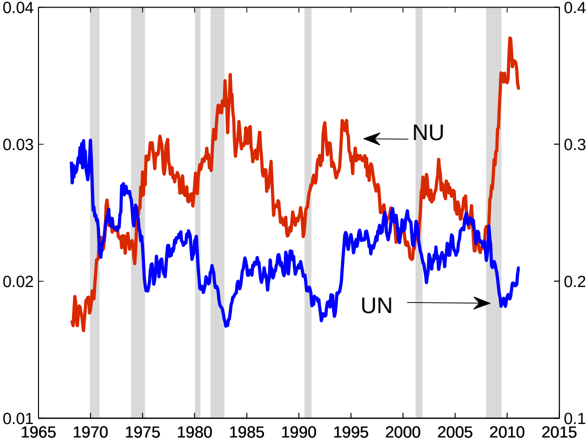
- Unemployment is strongly countercyclical.
- Labor force participation rate is mildly procyclical.

Flows Between Employment (E) and Unemployment (U)



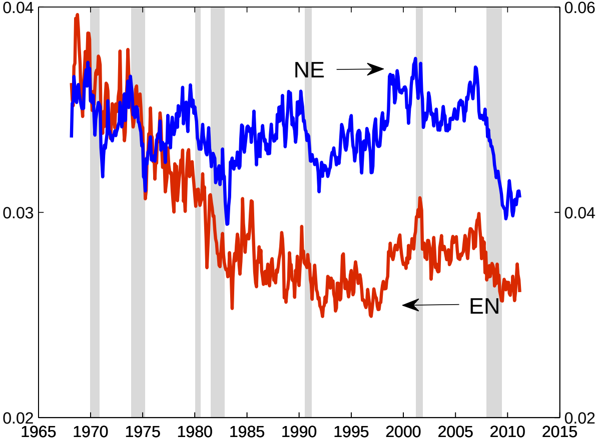
- U-to-E is strongly procyclical.
- E-to-U is countercyclical.

Flows Between Nonparticipation (N) and Unemployment



- U-to-N is procyclical.
- N-to-U is countercyclical.

Flows Between Nonparticipation (N) and Employment



- E-to-N and N-to-E are both mildly procyclical.

The Role of the Participation Margin: Flows

Work by Elsby, Hobijn and Şahin:

- They decompose the time-series variation in each of the labor market states into components accounted for by each of the associated worker flow hazards.
- Preliminary results from the decomposition suggest that flows between U and N account for a substantial fraction of variation in the unemployment rate in the U.S., as much as 40 percent.
- One possibility is that they are capturing transitions related to misclassifications rather than actual changes in labor market states.
- Even when the flows data are purged from suspicious transition (de-NUN-ification), they still find an important role for the flows between U and N.

Summary

- While labor market stocks show little cyclical impact of the participation margin, the flow data tell quite a different story.
- The mild procylicality of participation does not mean that participation margin is irrelevant.
- For some individuals the margin of working vs not working is very relevant while for some individuals this margin could be effectively irrelevant.
- Few analyses have all three states. Veracierto (2008) is the most ambitious example. He found radically counterfactual predictions for the behavior of labor market stocks.
  - Can we account for the joint behavior of employment, unemployment and labor force participation?
  - Can we account for cyclical movements in worker flows across all three states?

Business Cycle Calibration

z **shock:**

- We assume that z_t is a two-point Markov process: $z_t \in \{z_b, z_g\}$.
- Quarterly log TFP during 1968-2009 has the estimated AR(1) persistence of 0.935 and the standard deviation of the residual 0.0056 (after taking out the linear trend).
- To match these, we set $\{z_b, z_g\} = \{0.984, 1.016\}$ and $\pi_{gg} = \pi_{bb} = 0.9839$.

Frictions:
We first assume that λ_w and σ are constant.

Computation

We apply Krusell and Smith's (1998) "limited information" approach.

1. Reduce Ω to some limited information. Here, we choose the current aggregate capital stock K and the aggregate capital-labor ratio at the previous period, $M_{-1} \equiv K_{-1}/L_{-1}$.
2. The consumers have to forecast K' and also have to calculate $M = K/L$. We use the forecasting rules:

   $$\log(K') = a_0 + a_1 \log(K) + a_2 \log(z) + a_3 \log(M_{-1})$$

   and

   $$\log(M) = b_0 + b_1 \log(K) + b_2 \log(z) + b_3 \log(M_{-1}).$$

3. Obtain r and w from z and the forecasted M. Obtain T from w, K, and the forecasted M. Perform the optimization.
4. Simulate the economy. Check the law of motion for K' and the forecasting rule for M. Modify the coefficients and repeat.

Computation

- Converged forecasting rules (laws of motion):

$$\log(K') = 0.648 + 0.990\log(K) + 0.0276\log(z) - 0.00269\log(M_{-1}),$$

$$R^2 = 1.0000,$$

and

$$\log(M) = -0.765 + 0.944\log(K) - 0.290\log(z) + 0.189\log(M_{-1}),$$

$$R^2 = 0.9986.$$

- Consumers can predict K' and M accurately with limited information.

Results: Standard Aggregates

		$std(x)/std(Y)$			$corrcoef(x, Y)$		
	$std(Y)$	C	I	E	C	I	E
Data	.016	.81	4.7	.68	.87	.92	.84
Model	.010	.29	3.4	.60	.89	.99	.93
Hansen Model	.019	.22	4.5	.85	.76	.99	.99

- Our model behaves somewhat similar to the basic RBC model.
- Output and employment fluctuate less.

Results: Labor Market Variables

	Volatilities: $std(x)/std(Y)$							
	u	lfpr	f_{EU}	f_{EN}	f_{UE}	f_{UN}	f_{NE}	f_{NU}
Data	7.6	.21	5.4	2.0	4.9	3.8	2.7	4.0
Model	1.6	.60	1.8	3.2	0.9	3.8	2.2	2.2

	Correlations: $corrcoef(x, Y)$							
	u	lfpr	f_{EU}	f_{EN}	f_{UE}	f_{UN}	f_{NE}	f_{NU}
Data	$-.87$	.46	$-.82$	.33	.78	.78	.64	$-.70$
Model	$-.51$	.91	.00	$-.26$	.09	$-.07$	$-.24$	$-.25$

Labor market statistics, except for E, are off:

1. Unemployment does not exhibit much cyclicality.
2. Labor force participation rate varies too much and is strongly procyclical.
3. Flows are at odds with data.

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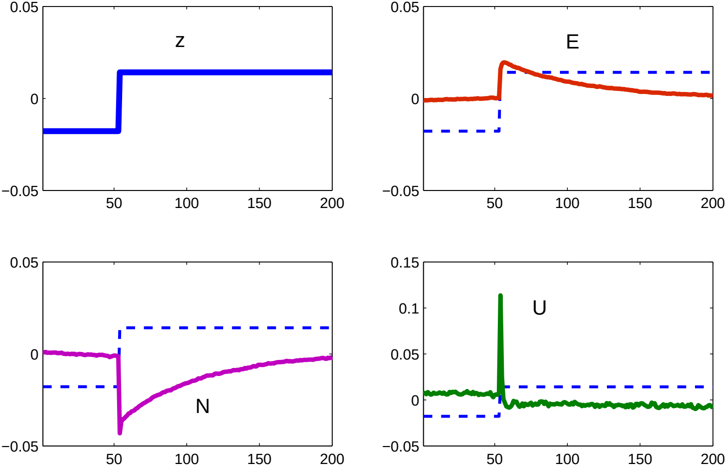
The Behavior of Labor Market Stocks



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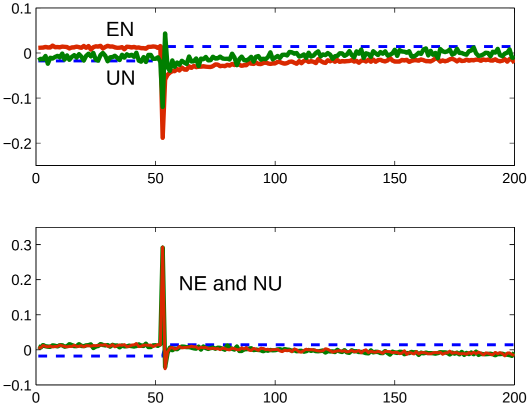
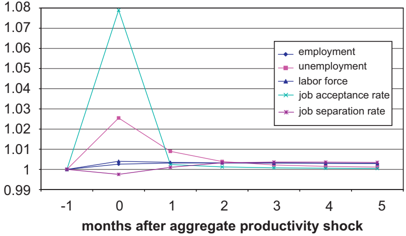


Veracierto (2008)



Veracierto (2008) adds the "third state (not in the labor force)" to a version of the Lucas-Prescott island model. He finds that

- the unemployment rate $(U/(E+U))$ becomes procyclical, while it is strongly countercyclical in the data,
- the labor force participation rate $(E+U)$ becomes strongly procyclical, while it is only mildly procyclical in the data.

Our Benchmark: Frictions Comove with z

- The model does not perform well with z shocks only. Now we add shocks to frictions.
- It turns out that adding fluctuations in λ_w brings us very close to the data. We will use this as our benchmark.
- We assume that λ_w move in a perfectly correlated manner with z.
  - λ_w in each state is set so that the standard deviation of HP-filtered log unemployment relative to HP-filtered log output (7.6 in the data) matches the data. We set $\{\lambda_w(z_b), \lambda_w(z_g)\} = \{0.4869, 0.5831\}$.
  - This implies that the separation probability $\sigma(1 - \lambda_w)$ is random and takes on two possible values: $\{0.0180, 0.0146\}$. σ itself is constant.

Forecasting Rules

- Converged forecasting rules (laws of motion):

$$\log(K') = 0.605 + 0.992 \log(K) + 0.0299 \log(z) - 0.00468 \log(M_{-1}),$$

$$R^2 = 1.0000,$$

and

$$\log(M) = -0.595 + 0.827 \log(K) - 0.448 \log(z) + 0.275 \log(M_{-1}),$$

$$R^2 = 0.9998.$$

- Again, the forecasting is accurate.

Results: Standard Aggregates

		$std(x)/std(Y)$			$corrcoef(x, Y)$		
	$std(Y)$	C	I	E	C	I	E
Data	.016	.81	4.7	.68	.87	.92	.84
Model	.011	.28	3.4	.56	.90	.99	.96
Hansen Model	.019	.22	4.5	.85	.76	.99	.99

- Again, in terms of the fluctuations in Y, C, I, and E, the model looks similar (and similar to the model with z fluctuations only!).

Results: Labor Market Variables

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Model	$-.98$	.29	$-.89$	.45	.92	.33	.90	$-.90$

- Now the cyclicality of unemployment rate and the labor force participation rate are in line with data.
- The flows are too.
- The Veracierto problem does not appear!

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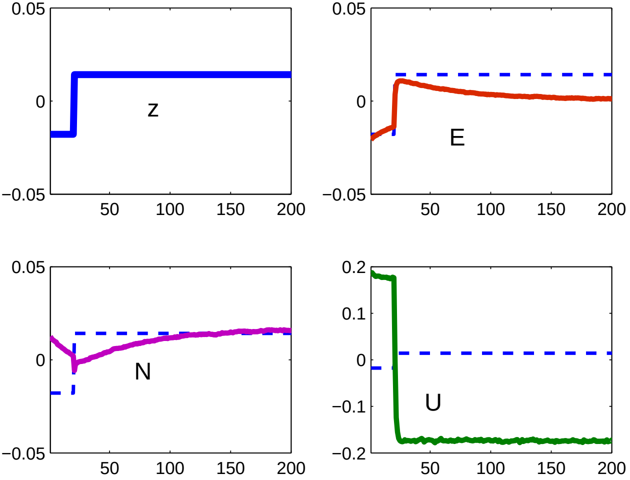
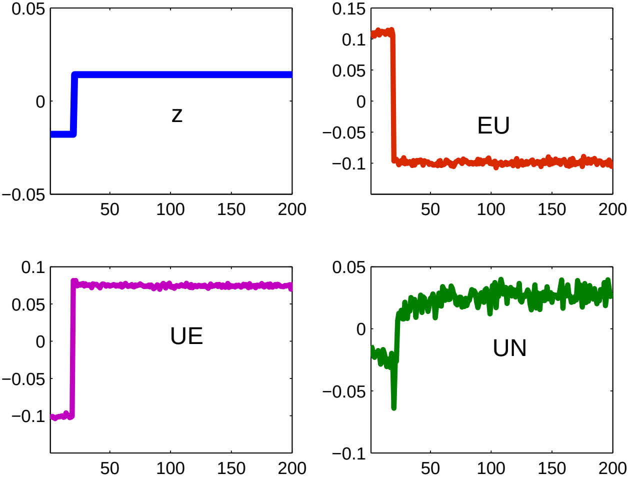
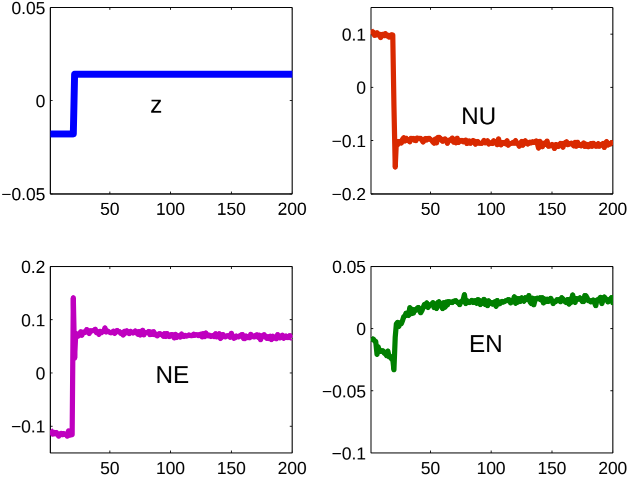


The Behavior of Labor Market Flows



The Behavior of Labor Market Flows



The Roles of Different Shocks

- Benchmark model (Benc): Three things move around exogenously: aggregate productivity z, the job finding rate λ_w, and the separation rate $\sigma(1 - \lambda_w)$.
- Experiment 1 (z): z shock only.
- Experiment 2 (Fric): Friction shocks only (λ_w fluctuates): both the job finding rate λ_w and the separation rate $\sigma(1 - \lambda_w)$ fluctuate.

Comparisons

	Volatilities: $std(x)$				Correlations: $corrcoef(x, Y)$		
	Y	u	lfpr	E	u	lfpr	E
Data	.016	.12	.003	.011	$-.87$	.46	.84
Benc	.011	.09	.002	.006	$-.98$	.29	.96
z	.010	.02	.005	.006	$-.51$	.91	.93
Fric	.002	.08	.004	.002	$-.95$	$-.85$	.58

- The Y fluctuations are largely from z shocks.
- The unemployment rate does not fluctuate much with z shocks only.
- E is not sufficiently cyclical under friction shocks only ("attenuation result," our *QE* 2010 paper).
- "z shocks" and "friction shocks" have opposite effects on lfpr.
  - Positive "z shock" (boom): $E \uparrow$ and $U \rightarrow \Rightarrow (E + U) \uparrow$.
  - Negative "friction shock" (boom): $E \rightarrow$ and $U \downarrow \Rightarrow (E + U) \downarrow$.
  - These two effects offset and generate a weakly procyclical lfpr. In Veracierto (2008), both E and U are procyclical.

Comparisons: $E \longleftrightarrow U$ Flows

	$std(x)$		$corrcoef(x, Y)$	
	f_{EU}	f_{UE}	f_{EU}	f_{UE}
Data	.085	.077	$-.82$	.78
Benc	.054	.044	$-.89$	.92
z	.018	.009	.00	.09
Fric	.054	.044	$-.79$	.82

- With z shock only, flows between E and U do not exhibit much cyclicality.
- Friction shocks are essential in accounting for behavior of the flows.
- Fluctuations in the job finding rate are important for $U \rightarrow E$ flow. Fluctuations in the separation rate are important for $E \rightarrow U$ flow.

Friction Shocks Only

- We have seen that adding friction shocks to the model with z shocks can successfully replicate key business cycle statistics and labor market dynamics. Can the model behave well if we only have friction shocks?
- We set z constant, and make λ_w fluctuate so that the standard deviation of the unemployment rate becomes as large as in the data (12%).
- Results:

		$std(x)/std(Y)$			$corrcoef(x, Y)$		
	$std(Y)$	C	I	E	C	I	E
Data	.016	.81	4.7	.68	.87	.92	.84
Model	.002	.43	3.6	.95	.57	.95	.73

- Y fluctuates too little compared to the data. Despite the large fluctuations in U, E fluctuation is dampened by the labor supply response ("attenuation").

Conclusions

- We develop an empirically reasonable model of the aggregate labor market that features a role for both labor supply and frictions.
- We use this model to revisit many issues of interest.
- Current practice of focusing on models that abstract from labor supply and focus on only frictions is potentially misguided.