Common and Idiosyncratic Disturbances in Developed Small Open Economies*

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September 30, 2011

Abstract

Using an estimated dynamic stochastic general equilibrium model, this paper shows that shocks to a common international stochastic trend explain up to 16% of the variability of output in several small developed economies. These shocks explain roughly twice as much of the volatility of consumption growth as the volatility of output growth. Country-specific disturbances account for the bulk of the volatility in the data. Substantial heterogeneity in the estimated parameters and stochastic processes translates into a rich array of impulse responses across countries.

1 Introduction

This paper tackles a perennial question in international economics: is there a common international business cycle? To address this question, I build a dynamic general equilibrium (DSGE) model with country-specific features and disturbances that are both domestic and international. The model is estimated using Bayesian methods and data from several small developed economies. The estimated model is then used to gauge the impact that domestic

*I thank Roc Armenter, Luigi Bocola, Jesus Fernandez-Villaverde, Javier Garcia-Cicco, Keith Kuester, Jesper Linde, Frank Schorfheide, Robert Vigfusson, Kei-Mu Yi, and the seminar participants at Louisiana State University, University of Delaware, the Philadelphia Fed, the Federal Reserve System Committee on International Economic Analysis, the 2010 Winter Meetings of the Econometric Society, and the 2010 Midwest Macroeconomic Meetings for their helpful comments. Behzad Kianian and Joy Zhu provided superb research assistance. Parts of this manuscript were written while I was an assistant professor at NC State University. The views expressed here are those of the author and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or of the Federal Reserve System. E-mail: pguerron@gmail.com

and international shocks have on the countries in the sample and how the structure of these economies affect the propagation of those disturbances.

Two observations motivate this paper. To begin with, one apparent lesson from the so-called great recession is that some countries have been more vulnerable than others. This asymmetry could not be better exemplified by the diverging fortunes of two small open economies (SOEs) such as Canada and Spain. While the first country started its recovery in the first quarter of 2010, the former one was still coping with the worst of the recession. This empirical observation is, thus, a tale of heterogeneity: structural differences are responsible for heterogenous business cycles across countries. Hence, the inclusion of country-specific components in our formulation can potentially reveal whether distinct business cycles arise from country differentials in preferences, technology, shocks, or all of them.

The second equally intriguing observation is that most countries (developed and developing) moved into the great recession with surprising synchronization. For example, by the third quarter of 2009, 25 out of 30 OECD countries experienced negative growth for two or more consecutive quarters.[1] This synchronization tells us a story in which international shocks buffeting countries worldwide play a key role. Assessing the relative importance of this common disturbances is a goal of this paper.

There is substantial empirical evidence suggesting the existence of a world factor that drives international fluctuations. Examples of this research agenda include, among others, Kose et al. (2003), Stock and Watson (2005), and more recently Canova and Ciccarelli (2009).[2] All these manuscripts uncover the presence of a common (world) shock although its economic importance varies from study to study. The beauty behind these papers is that they are based on flexible reduced-form time-series formulations. This simplicity, however, blurs the economic interpretation of the uncovered disturbances.

But providing a structural analysis of the country-specific and common shocks is of special importance from a policy point of view. If, for instance, the common international disturbance explains a large proportion of the domestic fluctuations and arises from, say, technology changes, then efforts to mitigate domestic market imperfections may be of limited scope or even result in inefficient outcomes. In contrast, such policy efforts may be

[1] According to the International Finance Statistics database, out of 58 countries for which there is GDP information for 2009, 70% of them experienced a contraction for two or more consecutive periods.

[2] The estimation of world factors has also been addressed in important contributions by Stockman (1988), Norrbin and Schlagenhauf (1996), Gregory et al. (1997), Clark and Shin (2000), and Koren and Tenreyro (2007).

of some benefit if domestic volatility is mostly driven by country-specific disturbances.

To study international business cycles, this paper proposes and estimates a tractable open economy model rich enough to allow for country-specific disturbances as well as common international factors for several countries around the world. The model incorporates elements such as tradable and nontradable sectors, which help explain the dynamics of exchange rates found in the data. Methodologically, the paper shows how to express the likelihood function of an open economy model with both common factors and several idiosyncratic shocks. As will become clear, the proposed approach effectively blends ideas from two strands of the literature: dynamic factor analysis (Stock and Watson, 1993) and DSGE models (Christiano et al., 2005).

Tractability in the model hinges on three fundamental suppositions. To begin with, an essential premise is that co-movement among countries is due to a common stochastic productivity trend. Second, the estimation exercise considers seven developed economies, and each of them is treated as an small open economy (SOE).[3] The advantage behind this assumption is that trade flows of a given country with the rest of the world are summarized by the country's foreign asset position. This effectively reduces the dimensionality of the model. Finally, based on the empirical findings in Chang and Fernandez (2010), and Garcia-Cicco et al. (2009), the country-specific factors are identified as preference, interest rate, sectoral-specific factor productivity (TFP), and terms-of-trade disturbances. Furthermore, the models for the seven countries are jointly estimated using Bayesian methods.

An alternative to the proposed SOE framework would be to work with a multi-country DSGE model. Such an approach would allow us to fully characterize the multiple interactions among countries around the world. This specification, however, suffers from the curse of dimensionality, i.e., the size of the model grows with the number of countries. Estimating such a model with more than two countries is computationally challenging. It is for this reason that reduced-form models have been widely favored to extract international business cycles (Kose et al., 2008).

The substantive findings of this paper can be summarized as follows. First, the model successfully accounts for the heterogenous business cycles in the data, such as New Zealand's large fluctuations, the excess volatility of consumption in Norway, or the volatile patterns of real exchange rates. More interesting, this asymmetry in business cycles arises from differences in the countries' idiosyncratic disturbances as well as structural features, such

[3] The countries in the sample are Australia, Belgium, Canada, New Zealand, Norway, Spain, and Sweden.

as habit formation. The diversity in the parameter estimates in turn translates into a rich array of impulse responses across countries.

Second, the estimation exercise reveals the presence of a common stochastic productivity trend. Innovations to this trend are mildly persistent and explain about the same fraction of the variability of the growth rates of output and investment. Interestingly, those shocks explain roughly twice as much of the volatility of consumption growth as the volatility of output growth. For example, 16% of the output growth volatility and 37% of the variability of the growth rate of consumption in Australia are accounted for by the common trend. This estimated common factor closely tracks the average growth rates of output and (in particular) of consumption in the SOEs.

Third, country-specific preference and technology innovations account for a large fraction of the volatility in the data. For instance, these disturbances together explain 68% and 74% of the volatility of consumption in Spain and Sweden, respectively. Additionally, more than 40% of the dynamics of real exchange rates in all countries is captured by idiosyncratic term-of-trade shocks. Finally, I find that all these results are present even after introducing a country-specific trend or an international demand shock.

A key contribution of this paper is to underscore the heterogeneity among countries and how it affects the propagation of common shocks. In this sense, one could argue that (to the extend that) the recent financial crisis was common to all countries its impact is entirely specific to each economy. To see this point, note that the posterior estimates for habit formation substantially vary across countries. Spain, for example, displays a habit persistence almost twice as large as that present in Canada. Ultimately, this real friction induces inertia in the economy, which may help understand why the pace of the recovery in the later country has been significantly faster than in the former.

Because of its analysis of structural shocks on open economy models, this paper relates to the business cycle literature in SOEs (Aguiar and Gopinath, 2007; and Fernandez-Villaverde et al., 2009). The estimation of a structural model with temporal and cross-section data shares some commonalities with the panel data estimation literature (Woodridge, 2002). This paper is also close in spirit to the contributions of Boivin and Giannoni (2006), and Taylor (1993). Similar to Boivin and Giannoni, this paper estimates a DSGE model with multiple time series. The main difference is that while my study is concerned with extracting a factor common to several countries, they are interested in the effects of a large database on the estimation of closed economy DSGE models. Taylor (1993) estimates a reduced-form open economy model for the G7 countries. The key discrepancies between our studies are: 1) his approach is nonstructural and 2) unlike my approach, he estimates the model on a country-by-country basis, which rules out a common factor among the countries in his sample.

The rest of the paper is organized as follows. The next section provides some evidence on the presence of a common component in several developed small open economies. Section 3 describes the baseline model. The computation and estimation of the model are outlined in Sections 4 and 5. Finally, Sections 6 and 7 discuss the results and some extensions.

2 Trends and Cycles in Developed SOEs

To motivate the discussion to follow, the top panel in Figure 1 displays per-capita real GDP in several developed economies (see Section 5 for details on the data). For comparison purposes, the series are normalized to 0 in the first quarter of 1980 (the solid dark line corresponds to the average of the series). A visual inspection of this figure suggests that recessions and expansions tend to equally affect SOEs. For example, the recession of the early 1990s effected all countries, albeit in different degrees. Whereas Canada and New Zealand were severely beaten by that crisis, Norway emerged almost unscratched from the economic downturn. More important, SOEs seem to track each other over the years. For instance, although Spain was growing above the countries average in the early 1990s and Norway grew faster in the later part of that decade, their output trends tend to converge by the year 2010. Similarly, Sweden was growing below trend during most of the 1990s but its output quickly recovered at the onset of the new century. Together, these casual observations suggest that real GDP in developed SOEs may be co-integrated and consequently share common components.[4]

The HP-filtered business cycles (bottom left panel in Figure 1) confirm that output in the SOEs tend to closely co-move over the cycle. The figure also reveals some interesting differences across countries. For example, New Zealand, Norway, and Sweden are more volatile than the other economies in the sample. Furthermore, output in Norway seems countercyclical relative to the other countries in the last part of 1980s. The hp-filtered series also make clear that the countries were severely affected by the recent crisis. Indeed, all economies moved to a recession with surprising synchronization in 2009.

The final panel in Figure 1 portrays the quarterly growth rates of output found in

[4] More formally, Johansen's (1991) maximal eigenvalue statistic confirms that the GDP series in Figure 1 are indeed co-integrated (for further details see Guerron-Quintana, 2010).

the data. The new plot reinforces our previous finding of substantial co-movement among countries over the past two decades. This synchronization suggests that developed SOEs may share not only a stochastic trend but also a common factor that exclusively works at the business cycle frequencies. In sum, the data reveal three important patterns, which will play a fundamental role in shaping the rest of the paper. First, the developed economies in the sample exhibit similar trends over the past 30 years. Second, even at the business cycle frequencies, real GDP shows correlation across countries. Finally, there is heterogeneity among the SOEs studied in this paper.

3 Model

In the spirit of Gali (2008) and Gali and Monacelli (2005), I assume that there are N SOEs indexed by $j \in [1, N]$. Each of these countries is modeled à la Mendoza (1995) and Corsetti et al. (2008) with some modifications to improve the empirical fitting. The small open economy framework is convenient because the countries' interactions with the rest of the world are summarized by the level of foreign indebtedness of each country. Such an assumption simplifies the solution of the model since there is no need to model the specific interactions of country j with, say, country j'. Crucially, it is assumed that all countries share a common stochastic trend (this premise will be relaxed momentarily). There are, however, several shocks that are country-specific. These assumptions are intended to capture the salient features reported in the previous section while making the solution and estimation of the model feasible. In what follows, I will describe the government and the problems faced by households and firms in country j. For clarity, variables/parameters not indexed by j are common to all SOEs.

3.1 Households

Each small open economy is populated by a continuum of households. They choose on consumption of tradable goods (produced at home and abroad) and non-tradable goods, labor, investment, capital, and purchases of foreign bonds based on the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \upsilon_{j,t} \left[\log \left[C_{j,t} - b^{[j]} \overline{C}_{j,t} \right] - \frac{\psi^{[j]}}{2} h_{j,t}^2 \right]. \tag{1}$$

Note that the model displays external habit formation with parameter b and that intertemporal decisions are buffeted by the country-specific disturbance $\upsilon_{j,t}$. The consumption bundle, C_j, consists on a tradable bundle, $C_{j,T}$, and a non-tradable good, $C_{j,N}$. The tradable basket in turn is composed of goods produced at home, $C_{j,H}$, and abroad, $C_{j,F}$.

$$\begin{aligned} C_j &= \left(a_T^{1-\phi^{[j]}} C_{j,T}^{\phi^{[j]}} + a_N^{1-\phi^{[j]}} C_{j,N}^{\phi^{[j]}}\right)^{1/\phi^{[j]}}, \\ C_{j,T} &= \left(a_H^{1-\varrho^{[j]}} C_{j,H}^{\varrho^{[j]}} + a_F^{1-\varrho^{[j]}} C_{j,F}^{\varrho^{[j]}}\right)^{1/\varrho^{[j]}}. \end{aligned}$$

Here, a_H and a_F are the weights on the consumption of the domestically and foreign produced goods, respectively ($a_F = 1 - a_H$). Similarly, a_T and a_N are the shares of the traded bundle and the non-traded good in the consumption basket ($a_N = 1 - a_T$). The presence of non-tradable goods follows the growing evidence that such an assumption improves the model's predictions regarding the dynamics of exchange rates and consumption (Engel and Wang, 2011). Let P_H, P_F and P_N be the prices of the tradable good, the foreign good, and the nontradable good, respectively (where there is no risk of confusion and to improve readability, I will suppress the country index). Then the consumption-based price indices are

$$P = \left(a_T P_T^{\frac{\phi}{\phi-1}} + a_N P_N^{\frac{\phi}{\phi-1}}\right)^{\frac{\phi-1}{\phi}}, \text{ and } P_T = \left(a_H P_H^{\frac{\varrho}{\varrho-1}} + a_F P_F^{\frac{\varrho}{\varrho-1}}\right)^{\frac{\varrho-1}{\varrho}}.$$

The household's budget constraint is given by

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + P_{N,t}C_{N,t} + \widetilde{P}_{H,t}I_{H,t} + \frac{P_{F,t}B_{H,t+1}}{1+r_{j,t}} \leq W_t h_t + R_{k,t}K_t + P_{F,t}B_{H,t},$$

where $B_{H,t+1}$ is a bond that promises one unit of the foreign tradable good tomorrow; $R_{k,t}$ is the return on capital; W_t is the wage rate in country j. As in Corsetti et al. (2008), $\widetilde{P}_{H,t}$ is both the price of a unit of investment and the wholesale price of a unit of the home-made tradable good. The interest rate at which domestic residents borrow/save abroad is:

$$r_{j,t} = r^* + \varphi\left[\exp\left(-\left(\frac{\overline{B}_{j,H,t+1}}{p_{j,y}Y_{j,t}} - b_H^{[j]}\right)\right) - 1\right] + \exp\left(\mu_{j,t} - 1\right) + 1.$$

The second expression on the right hand side corresponds to a cost of adjusting the debt-to-output ratio from its steady state value, $b_H^{[j]}$ (this cost is needed to close the model; see Schmidtt-Grohe and Uribe, 2003). The price index $p_{j,y}$ is the steady state relative price of

GDP, $Y_{j,t}$ (see below). For convenience, it is assumed that households take as given the cost of borrowing from abroad, i.e., $\overline{B}_{j,H,t+1}$ is beyond their influence. Furthermore, domestic residents also face a premium, $\mu_{j,t}$, when borrowing from foreign markets. Capital evolves according to

$$K_{t+1} = (1-\delta)K_t + S\left(\frac{I_{H,t}}{I_{H,t-1}}\right) I_{H,t}.$$

Here the adjustment cost is given by $S\left(\frac{I_{H,t}}{I_{H,t-1}}\right) = \frac{\kappa^{[j]}}{2}\left(\frac{I_{H,t}}{I_{H,t-1}} - g^j\right)^2$ where g^j is the steady-state growth rate in country j. The functional form is based on Christiano et al.'s (2005) finding that this specification is well suited to match the dynamics of investment.

3.2 Firms

Domestic firms produce traded and non-traded goods. Tradable good producers use capital and labor to produce according to the technology

$$Y_{H,t}^j = A_{H,t}^j \left(X_t^j h_{H,t}^j\right)^{1-\alpha} \left(K_{H,t}^j\right)^{\alpha} \tag{2}$$

. They take wholesale prices as given and maximize profits $\widetilde{P}_{H,t} Y_{H,t} - W_t h_{H,t} - R_{k,t} K_{H,t}$. These traded goods are sold in competitive world markets. Non-tradable good producers takes wholesale prices as given and use technology

$$Y_{N,t}^j = A_{N,t}^j \left(X_t^j h_{N,t}^j\right)^{1-\alpha} \left(K_{N,t}^j\right)^{\alpha} \tag{3}$$

to maximize profits $\widetilde{P}_{N,t} Y_{N,t} - W_t h_{N,t} - R_{k,t} K_{N,t}$. Production at home is affected by the stationary technology shocks $A_{N,t}$ and $A_{H,t}$. In the baseline formulation, the nonstationary productivity disturbance X_t^j also buffets domestic production. This disturbance has a country-specific trend, $X_t^{d,j}$, and a second trend shared with other countries, X_t^w. As a result, country j evolves along the trend:

$$X_t^j = \left(X_t^{d,j}\right)^{\zeta_j} \left(X_t^w\right)^{1-\zeta_j}, \tag{4}$$

which implies that the growth rate in country j is $g_t^j \equiv \left(g_t^{d,j}\right)^{\zeta_j} \left(g_t^w\right)^{1-\zeta_j}$. Moreover, it is assumed that each country grows at the same rate in steady state: $g^j = g = g^w$, which implies $g^{d,j} = g$. Although this assumption is made for tractability reasons, it also captures the notion that countries grow along the same balanced growth path in the long run. Otherwise, one country — the one with the largest growth rate — would become infinitely rich at the expense of the other countries.

As in Corsetti et al. (2008), there are distribution costs, which induce a wedge between the retail price of tradable goods (P_H and P_F) and their wholesale price ($\widetilde{P}_H$ and $\widetilde{P}_F$). It is assumed that delivering one unit of tradable goods to consumers demands η units of the non-tradable goods. If we assume that the distribution sector is populated by competitive firms, then the retail and wholesale prices are given by

$$P_H = \widetilde{P}_H + \eta^{[j]} P_{N,t}, \text{ and } P_F = \widetilde{P}_F + \eta^{[j]} P_{N,t}.$$

Because of the distribution sector, the law of price price holds at the wholesale level but not at the consumer level.

The functional forms (2 and 3) capture the notion that technology progress at home results from a combination of domestic and external components. Several elements support the choice of such a production function. First and foremost, the productivity shock, X_t, is a rather parsimonious way to account for the presence of a common stochastic trend in SOEs as suggested by the empirical evidence in Section 2. Second, it follows an old tradition in macroeconomics starting with Kydland and Prescott (1982), who take productivity shocks as the main source of fluctuations in the economy. Furthermore, it is flexible enough to allow for short-run trend differentials, which in my formulation, arise from country-specific TFP shocks. Next, Glick and Rogoff (1995) assume that domestic productivity consists of a world component common to all economies and a country-specific term. Their world factor is estimated by first computing the Solow residuals for the countries in their sample and then taking a GNP-weighted average. The country-specific shock is obtained as the difference between the world process and the country's Solow residual. A final motivation comes from the evidence in Baxter and Crucini (1995) and Rabanal et al. (2009). These authors report the presence of co-integrated stochastic trends in several industrialized economies (the United States, European countries, and Canada).

3.3 Stochastic Processes

The growth rate of the common trend, g_t^w, follows

$$\log g_t^w = \left(1 - \rho_g\right) \log g + \rho_g \log g_{t-1}^w + \sigma_g \varepsilon_{g,t}. \tag{5}$$

For future reference, denote $g_{[\%]} \equiv 100 \log g$ as the average growth rate in percentage points. Similarly, the country-specific growth rate is

$$\log g_t^{d,j} = \left(1 - \rho_g^{[dj]}\right) \log g^{[dj]} + \rho_g^{[dj]} \log g_{t-1}^{d,j} + \sigma_g^{[dj]} \varepsilon_{g,t}^{d,j}.$$

The stochastic processes driving the remaining four idiosyncratic shocks are:

$$\begin{aligned} \log A_{H,t}^j &= \rho_h^{[j]} \log A_{H,t}^j + \sigma_h^{[j]} \varepsilon_{H,t}^j, \quad \log \mu_{j,t} = \rho_\mu^{[j]} \log \mu_{j,t} + \sigma_\mu^{[j]} \varepsilon_{\mu,t}^j, \\ \log A_{N,t}^j &= \rho_n^{[j]} \log A_{N,t}^j + \sigma_n^{[j]} \varepsilon_{N,t}^j, \quad \log \vartheta_{j,t} = \rho_\vartheta^{[j]} \log \vartheta_{j,t} + \sigma_\vartheta^{[j]} \varepsilon_{\vartheta,t}^{[j]}. \end{aligned} \tag{6}$$

The shocks $\varepsilon_{\cdot,t}$ are assumed to be independent normal distributed with mean 0 and variance 1.

3.4 Equilibrium

As in Mendoza (1995), it is assumed that 1) the foreign (importable) good is the numeraire, and 2) the relative price of the domestically produced traded good is exogenously determined in world markets. In particular, let $\widetilde{p}_{H,t} \equiv \widetilde{P}_{H,t}/\widetilde{P}_{F,t}$ follow an exogenous stochastic process. These normalizations in turn imply that the domestic household's budget constraint is

$$p_{H,t}C_{H,t} + p_{F,t}C_{F,t} + p_{N,t}C_{N,t} + \widetilde{p}_{H,t}I_{H,t} + \frac{B_{H,t+1}}{1 + r_{j,t}} \leq w_t h_t + r_{k,t}K_t + B_{H,t},$$

A lowercase price refers to the price of that good relative to the numeraire. For example, w_t and $r_{k,t}$ are the price of labor and capital in terms of the imported good. The presence of distribution costs implies that the relative price of the foreign commodity at the retail level ($p_{F,t} = P_{F,t}/\widetilde{P}_{F,t}$) is different from 1. With this convention, the real exchange rate in country j is $p = \left(a_T p_T^{\frac{\phi}{\phi-1}} + a_N p_N^{\frac{\phi}{\phi-1}}\right)^{\frac{\phi-1}{\phi}}$. Clearly, movements in the real exchange rate arise from variations in the terms of trade (through its effect on the price of tradables), and fluctuations in the markets for tradable goods produced at home and nontradable goods. These fluctuations may result from (country-specific or common) productivity shocks or demand related innovations. Finally, domestic GDP is defined as in Mendoza (1995): $Y_t = Y_{H,t}^{1/2} Y_{N,t}^{1/2}$. This definition captures the observation that the traded sector output in developed economies accounts for about 1/2 of total GDP.

An equilibrium is defined in the standard way: Given a set of prices $\{p_{H,t}, p_{F,t}, p_{N,t}, \widetilde{p}_{H,t}, r_{j,t}, w_t, r_{k,t}\}$, the allocations $\{C_{H,t}, C_{F,t}, C_{N,t}, I_{H,t}, B_{H,t+1}, h_t, h_{H,t}, h_{N,t}, K_t, K_{H,t}, K_{N,t}, Y_{H,t}, Y_{N,t}\}$ maximize the household's utility, maximize the firms' profit problem, and satisfy the economy's resource constraints.

Maximizing behavior by households and firms imposes the following first order conditions:

$$
\begin{gathered}
\frac{a_T^{1-\phi}\; a_H^{1-\varrho}}{C_t}\left(\frac{C_t}{C_{T,t}}\right)^{1-\phi}\left(\frac{C_{T,t}}{C_{H,t}}\right)^{1-\varrho}=\widetilde{\lambda}_t p_{H,t},\\
\frac{a_T^{1-\phi}\; a_F^{1-\varrho}}{C_t}\left(\frac{C_t}{C_{T,t}}\right)^{1-\phi}\left(\frac{C_{T,t}}{C_{F,t}}\right)^{1-\varrho}=\widetilde{\lambda}_t p_{F,t},\\
\frac{a_N^{1-\phi}}{C_t}\left(\frac{C_t}{C_{N,t}}\right)^{1-\phi}=\widetilde{\lambda}_t p_{N,t},\;\; \psi h_t=\widetilde{\lambda}_t w_t,\;\; \frac{\widetilde{\lambda}_t}{1+r_t}=\beta\mathbb{E}_t\widetilde{\lambda}_{t+1},\\
\widetilde{\gamma}_t=\beta\mathbb{E}_t\left(\widetilde{\lambda}_{t+1}r_{k,t+1}+(1-\delta)\,\widetilde{\gamma}_{t+1}\right),\\
\widetilde{\lambda}_t\widetilde{p}_{H,t}=\widetilde{\gamma}_t\left(S_t'\frac{I_{H,t}}{I_{H,t-1}}+S_t\right)-\beta\mathbb{E}_t\widetilde{\gamma}_t S_{t+1}'\left(\frac{I_{H,t+1}}{I_{H,t}}\right)^2,\\
\alpha\widetilde{p}_{H,t}\frac{Y_{H,t}}{K_{H,t}}=r_{k,t}\;,\;\;(1-\alpha)\,\widetilde{p}_{H,t}\frac{Y_{H,t}}{h_{H,t}}=w_t,\\
\alpha p_{N,t}\frac{Y_{N,t}}{K_{N,t}}=r_{k,t}\;,\;\;(1-\alpha)\,p_{N,t}\frac{Y_{N,t}}{h_{N,t}}=w_t.
\end{gathered}
$$

Here, the multipliers associated with the household budget constraint and the capital accumulation equation are $\widetilde{\lambda}_t$ and $\widetilde{\gamma}_t$, respectively.

As we will see momentarily, the benchmark model is sufficiently rich to account for the dynamics of the data. Indeed, adding other features such as a common demand shock or a common transitory technology disturbance has little impact on the main predictions from the benchmark model.

4 Likelihood with One Common Factor

This section shows how the likelihood of the model with one common factor can be evaluated via the Kalman Filter. Let us start by noticing that the solution to the log-linearized version of the stationary model for country j can be represented as

$$\begin{aligned}\overline{S}_{j,t} &= \Pi^{[j]}\overline{S}_{j,t-1}+\eta^{[j]}\overline{\xi}_{j,t},\\ \mathbb{Y}_{j,t} &= \Phi^{[j]}\overline{S}_{j,t},\end{aligned}$$

where $\overline{S}_{j,t}=[S'_{j,t},\widehat{g}_t^w]'$, $\mathbb{Y}_{j,t}$, and $\overline{\xi}_{j,t}=[\xi'_{j,t},\varepsilon_{g,t}]'$ denote the vectors of states, controls, and structural shocks, respectively. Here, $S_{j,t}=[\widehat{y}_{j,t-1},\widehat{c}_{j,t-1},\widehat{i}_{j,H,t-1},\widehat{p}_{j,t},\widehat{k}_{j,t},\widehat{b}_{j,H,t},\widehat{\overline{p}}_{j,H,t},\widehat{\mu}_{j,t},$ $\widehat{\upsilon}_{j,t},\widehat{a}_{j,H,t},\widehat{a}_{j,N,t},\widehat{g}^d_{j,t}]'$, and $\xi_{j,t}=[\varepsilon^j_{\upsilon,t},\varepsilon^j_{H,t},\varepsilon^j_{N,t},\varepsilon^j_{\mu,t},\varepsilon^{d,j}_{g,t}]'$. A lowercase variable corresponds to a de-trended (stationary) variable, $c_t=C_t/X_t$, while a hat indicates log-deviations from steady state. The matrices Π, Φ, and η depend on the structural parameters of each country. The structure behind $\overline{\xi}_{j,t}$ allows us to disentangle the fluctuations in the small open economy j due to the common factor — represented here by the shock $\varepsilon_{g,t}$ — from those due to country-specific conditions represented by the shocks $\varepsilon^j_{\cdot,t}$.

Define the expanded vector of states as $\mathbb{S}_t=[\mathbb{Y}'_{1,t},\mathbb{Y}'_{2,t},\cdots,\mathbb{Y}'_{N,t},S'_{1,t},S'_{2,t},\cdots,S'_{N,t},\widehat{g}_t]'$, and the expanded vector of structural shocks as $\xi_t=[\xi'_{1,t},\xi'_{2,t},\cdots,\xi'_{N,t},\varepsilon_{g,t}]'$. Here, $\dim(\mathbb{S}_t)=[N\times N_c+N\times(N_s-1)+1]$ and $\dim(\xi_t)=[(N_{sh}-1)\times N+1]$, where N is the number of countries in the sample, $N_c=\dim(\mathbb{Y}_{j,t})$, $N_s=\dim\left(\overline{S}_{j,t}\right)$, and $N_{sh}=\dim\left(\xi_{j,t}\right)$. Then the state space representation of the model with N countries is

$$\begin{aligned}\mathbb{S}_t &= \mathbb{F}\,\mathbb{S}_{t-1}+\eta\xi_t,\\ \mathbb{Y}_t^{Data} &= g_{[\%]}+\mathbb{H}\,\mathbb{S}_t.\end{aligned}\tag{7}$$

With this formulation in hand, the Kalman filter can be used to evaluate the likelihood. Since there are three observables per country (growth rates of output, consumption, and investment, interest rates, and the growth rate of exchange rates) and four country-specific shocks, the state space model does not require measurement errors (the technical appendix shows the forms of the matrices $\mathbb{F}$ $\mathbb{H}$ and η).

The state space representation (7) suffers from the curse of dimensionality due to three sources. The first source comes from the number of countries, N. Indeed, adding an extra country requires N_c+N_s elements in the expanded state vector. Second, the complexity of the DSGE model under study determines N_c, N_s, and N_{sh}. An additional state, control, or structural shock increases the size of $\mathbb{S}_t$ by N. Finally, the number of observables in the measurement equation, $\dim\left(\mathbb{Y}_t^{Data}\right)$, grows with the number of countries and observables to be explained. All these sources amount to increasing the dimensions of $\mathbb{F}$, $\mathbb{H}$, and η, which is problematic since the Kalman filter requires multiplying and inverting objects that depend on those matrices. The larger those matrices are, the more expensive the evaluation of the likelihood is.

Assuming a small open economy framework effectively controls the dimensionality problem by limiting N_c, N_s and N_{sh}, i.e., containing the model's complexity. However, such an assumption does not preclude the first and third sources of dimensionality from happening. To control for those sources, we restrict our study to explain output, consumption, and investment in seven developed SOEs. Even after these restrictions, the expanded state vector, $\mathbb{S}_t$, has dimension 120; an extra country increases the number of states in $\mathbb{S}_t$ by 17 while an extra disturbance increase its size by 7.

Let Θ^j denote the set of all structural parameters in country j. Operationally, the following steps are used to evaluate the likelihood of the system (7):

1. For country j, solve the DSGE model using Θ^j as the relevant parameters.

2. Using the model's solution build the matrices $\Pi^{[j]}$, $\Phi^{[j]}$, $\eta^{[j]}$, and $\sum^{[j]}$ as shown in the technical appendix.

3. Repeat steps 1 and 2 for all countries in the sample.

4. Finally, compute the state representation (7) and build its likelihood, $\mathcal{L}$, using the Kalman Filter.

Although this section shows how to evaluate the likelihood with only one common factor, which corresponds to the stochastic trend in Section 6, it can be easily extended to allow for multiple common factors (Section 7.3).

Before proceeding to the next sections, it is important to state some potential shortcomings associated with the proposed methodology. One concern regarding the model arises from the lack of explicit trade flows of the SOEs with one another and with large economies. This lack of terms-of-trade effects implies that all cross-dynamics are attributed to the common shock. To the extent that spillovers are present in the data, the estimated common shock is most likely upward biased. This is of some concern since our sample includes Australia and Canada, which have strong trade links with Japan and the United States, respectively, and European countries, which trade significantly with one another.

Technology is hardly the only driver of international business cycles. Other potential disturbances include, for example, demand factors. In general, these misspecifications may result in biased parameters and processes. To shed light on these issues, Section 7 discusses in some detail extensions to the benchmark model.

The approach proposed here has some shortcomings but so do reduced form methods. Stock and Watson (2005) argue that there is no flawless way to extract international common factors. For example, Kose et al.'s (2008) approach shares with the proposal here that the common shock reflects a mixture of purely international common and country-specific elements transmitted through trade (spillovers). Stock and Watson's (2002) principal component framework puts heavy weights on large countries, resulting in estimating a common component when in fact there is none.

5 Estimation

This section describes the data as well as the econometric approach used to estimate the multi-country model proposed in the previous sections. For each country in the sample, the parameter space is divided in non-estimated parameters $\Theta_1^j = \left[\alpha, \beta, \varphi, \delta, b_H^{[j]}, \psi^{[j]}\right]$ and estimated parameters $\Theta_2^j = [\rho_h^{[j]}, \rho_n^{[j]}, \rho_\mu^{[j]}, \rho_\vartheta^{[j]}, \rho_g^{[dj]}, \sigma_h^{[j]}, \sigma_n^{[j]}, \sigma_\mu^{[j]}, \sigma_\vartheta^{[j]}, \sigma_g^{[dj]}, b^{[j]}, \kappa^{[j]}, \phi^{[j]}, \varrho^{[j]}, g_{[\%]}, \rho_g, \sigma_g]$. The lack of indexation in the last three parameters reflects the assumption that there is a common shock buffeting all SOEs. The parameter $b_H^{[j]}$ is set to match the ratio of net exports to output observed in the data for each country. Without loss of generality, the steady state of labor is normalized to 1. This assumption in turn pins down the value for $\psi^{[j]}$. r^* is calibrated to match the average real interest rate in each country, which in turn determines the value of the discount factor β. The remaining parameters in Θ_1^j are set to $\alpha = 0.32$, $\delta = 0.025$, and $\varphi = 0.001$, which are standard choices in the literature (Schmitt-Grohe and Uribe, 2003; Neumeyer and Perri, 2005). In total, there are 133 country-specific parameters (19 per country) plus three common parameters to be estimated.

The data, which are described in the appendix, consist of real interest rates, the quarterly growth rates of per-capita real output, consumption, investment, and the growth rate of exchange rates for the following developed economies: Australia, Belgium, Canada, New Zealand, Norway, Spain, and Sweden. As argued in Section 2, the assortment of countries in our sample share a common stochastic trend. Furthermore, they are geographically located in different continents and export different goods. For example, Australia and

Norway are big commodity exporters while Spain relies on tourism and financial services.[5] Ultimately, one expects that these countries be buffeted by shocks that are not necessarily correlated across them. Altogether, these features should provide enough information to identify the international stochastic trend in the model, g_t^w, as well as the country-specific terms. Finally, Australia, Canada, and New Zealand have been studied elsewhere (Lubik and Schorfheide, 2007; Justiniano and Preston, 2008) so their inclusion facilitates comparison with the related literature.

Why do we not include additional countries in the exercise? One reason is computational costs. As argued above, adding a country increases the size of the state space by 15%. More important, most of the remaining developed SOEs are located in Europe. Therefore, by including them in the estimation we risk recovering a regional shock common to all countries in the European area. The correlation of this disturbance with the remaining countries in the sample is most likely weak, which may lead us to conclude (incorrectly) that the factor is unimportant to, say, Canada and Australia. But this is problematic because we are interested in recovering a factor that is meaningful for countries in different geographical areas. Alternatively, we could incorporate emerging economies into the analysis. Yet Aguiar and Gopinath (2007) forcefully argue that these economies display markedly different business cycles, which makes their inclusion unsuitable for our purposes.

Following the recent literature, e.g. Schorfheide (2000) and Smets and Wouters (2007), the log-linearized version of the model is estimated using Bayesian methods. Let $p(\Theta_2)$ denote the prior distribution on the parameters of interest and $\mathcal{L}$ the likelihood of the model. The Bayes theorem in turn implies that the posterior of the structural parameters, $p\left(\Theta_2|\mathbb{Y}^{Data}\right)$, is proportional to $p(\Theta_2)\mathcal{L}$, where the likelihood can be evaluated following the algorithm outlined in the previous section. Then $p\left(\Theta_2|\mathbb{Y}^{Data}\right)$ is characterized using the random walk Metropolis-Hasting (MH) procedure (for details, see the excellent survey of An and Schorfheide, 2007). The results are based on 400,000 draws from the posterior simulator after an initial burn-in phase of 200,000 iterations. The acceptance rate for the MH algorithm was set to approximately 0.23 as suggested by Casella and Roberts (2004).

The priors imposed during estimation are reported in Table 1. The prior mean $g_{[\%]}$

[5] Norway's inclusion is of special interest because of its oil producer status. This feature implies that Norway's economic data can provide valuable information about oil shocks. To the extent that such shocks affect economic activity (Hamilton, 2008), I consider that including Norway helps to better characterize the dynamic properties of the common shocks buffeting the countries in our sample. The alternative is to directly use oil prices. This approach, however, requires modeling the oil sector in some detail, which adds some unnecessary complexities to an already elaborated formulation.

is set to the average quarterly growth rate in percentage points across all countries and observables in the sample. The priors for the persistence parameters reflect the view that the structural shocks display some autocorrelation. The relatively large standard deviation helps to account simultaneously for processes with low and high persistence. Following Justiniano and Preston (2008) and Garcia-Cicco et al. (2009), it is assumed that there is some habit formation present in the data. Since there is little information about the adjustment cost of investment in open economies, I chose a very wide uniform prior. Finally, the priors for the volatilities allow for a wide range of values with a 95 percentile credible set given by $[0.08, 0.45]$.

6 Results

In this section, the implications of the common factor are explored using a variance decomposition exercise and impulse responses.

6.1 A First Pass

Given the considerable attention that productivity has received in the recent small open economy literature, Table 1 presents the posterior distributions for the country-specific productivity processes (upper panel) and the common productivity process, g_t^w (bottom panel). For completeness, the table also reports the estimates for some country-specific parameter such as habit formation, the cost of adjusting investment, and the parameters controlling the elasticities of substitution in the consumption bundles. A first pass at the results reveals that the growth rate of the common factor displays some mild persistence and that its volatility is bounded away from zero. This is an encouraging finding because it suggests that the factor is statistically relevant. The average growth rate median is 0.24%, which is somewhat below the average quarterly growth rate across countries and observables in the sample.

But does the common factor's statistical significance translate into economic relevance? To answer this question, Figure 2 displays the filtered common factor, $\widehat{g}_t^w$, against several time series (the posterior modes are used to compute the implied factor). The upper left panel plots the factor (dotted line) and the unweighted average of the output growth rates in the sample (solid line). To facilitate comparison, the variables are demeaned and normalized so that their highest value is 1. Clearly, the factor does a good job tracking the major movements in the average growth rate, e.g., the contraction in the early 1990s, the boom in the middle and last part of the 1990s, or the small contraction in the early 2000s. Indeed, the correlation between $\widehat{g}_t^w$ and the average growth rate is 0.77 (a strong confirmation of the relationship between those two variables). Put differently, the model indicates that average growth in the data is in a large part driven by the common technology disturbance.

The second panel shows the common factor against the average consumption growth rate. The co-movement is stronger than that with the average growth rate of output. The correlation is even higher at 0.92, which suggests that the common trend plays a very important role in accounting for the dynamics of average consumption in the sample. This result is expected once we recall that consumption is highly responsive to trend shocks (Aguiar and Gopinath, 2007). The dynamics of the common factor and the average growth rate of investment are portrayed in the left lower panel. Although the factor tracks parts of the dynamics of average investment, it also misses the persistent decline in the early 1990s or the slow rebound during the great recession. This last observation is not that surprising once we recall that financial frictions were a main player in the recent crisis.

The final panel presents the common factor versus the average of the growth rates of exchange rates in the sample. Direct observation indicates that the factor is substantially less correlated with exchange rates (the correlation is 0.34). For example, the common element fails to track the sustained depreciation during the first half of the 1980s. As will become clear momentarily, shocks to the price of tradable goods accounts for a large fraction of the variability of exchange rates, which explains the low correlation. In sum, this simple exercise reveals that the model provides an adequate description of the data. Furthermore, the estimation approach indeed recovers a component that is common to all countries and, more important, this factor correlates well with economic activity.

On a closer inspection, Figure 2 indicates that the common factor also captures the crisis/recovery of the last few years. At face value, this finding suggests that the recent global turmoil was triggered by a technology disturbance. This observation, however, needs to be qualified because most economic observers would agree that financial considerations played an important role in the crisis. Hence it is plausible that the role of the common shock in recent years may be exacerbated by the lack of a fully-fledged financial sector in the model. In my view, the key insight from Figure 2 is that a shock common to all countries contributed to the recession in 2008 and 2009. Whether this disturbance was driven by technology, financial frictions or both is a matter for future research.[6]

To further illustrate my previous point, Figure 3 displays the growth rates in percentage points for the actual variables (solid lines) and those implied by the common factor (dashed lines).[7] If we look at the output series, it is clear that most of the contraction in Belgium, Canada, and Sweden may be attributed to the common technology disturbance. In contrast, the shock overpredicts the recessions experienced by Australia and New Zealand. For these countries, it must be the case that idiosyncratic disturbances dampened the impact of the global shock. The investment series (last row) nicely exemplifies that the same shock can be asymmetrically amplified in each economy. For example, whereas the decline of investment in Canada was predicted to be -7.2% at the trough of the recent recession, the estimated contraction in Australia was -9.4%.

When we turn to the country-specific productivities, two apparent patterns emerge from Table 1: a) The tradable sector productivities are, on average, more volatile than those in the nontradable sector; and b) the tradable productivities display substantial persistence. As explained in the next sections, this large persistence and volatility implies that the country-specific productivities explain a large fraction of the variability of output in the sample. Interestingly, the estimated productivities for the tradable sector are consistent with previous studies. Aguiar and Gopinath (2007), for example, use a partial information approach to estimate a production function similar to the one in the tradable sector. They find that the persistence of productivity to be 0.97 for Canada. Justiniano and Preston (2008) and Garcia-Cicco et al. (2009) report very persistent productivity processes for Canada and Argentina, respectively. Similarly, Glick and Rogoff (1995) find that productivity in the G7 countries has a persistence close to one. (Section 6.3 provides intuition about why our approach delivers such a persistent productivity process.)

The estimated parameters also indicate that habit formation and costly investment are present in the countries, albeit to different degrees. For example, while Norway has the smallest adjustment cost of investment (0.21), the largest habit formation is present in Belgium (0.50). Norway's small cost is necessary to capture the large volatility present in its real variables. Similarly, the large habit persistence in Belgium is required to predict its smooth consumption profile (more on this in the next sections). When we turn to

[6] In principle, one could incorporate a financial sector a la Bernanke-Gertler in the model. However, the model has already many interesting dimensions so a financial block would clutter some valuable insight revealed by the benchmark model.

[7] To obtain these paths, the model was simulated using the filtered common shock as the only disturbance buffeting each country.

the other parameters in Table 1, we note that delivering one tradable good requires more nontradable goods in Australia than in any other country ($\eta = 1.35$). If we recall that Australia is a big country and far away from its trade partners, then one expects a large distribution cost. In contrast, Belgium (a smaller country with close trade partners) has a significantly smaller cost. Finally, there is small variation in the elasticities of substitution between home and foreign-traded goods ($1/1 - \varrho^{[j]}$) and between traded and nontraded goods ($1/1 - \phi^{[j]}$). Nevertheless these estimated values are consistent with those reported in Corsetti et al. (2008).

6.2 Common Shocks or Idiosyncratic Disturbances? Variance Decomposition

The first panel in Table 2 presents the second moments predicted by the model as well as those found in the data (numbers in square brackets).[8] The first three rows correspond to the standard deviation of output growth and the volatilities of the growth rates of consumption and investment relative to that of output. The fourth and fifth rows display the standard deviations of the real interest rate and the exchange rate, respectively. The empirical moments display the usual patterns found in small developed economies: 1) Output is more volatile than consumption but substantially less volatile than investment; and 2) real exchange rates are more volatile than output. There are, though, other features worth stressing. Output in New Zealand is more volatile than output in the other countries. Additionally, in Norway, consumption is more volatile than output.

The theoretical moments reveal that the model replicates the salient features found in the data. To begin, the model accounts to some extent for the variability of output in all countries. Indeed, the model correctly ranks the countries in terms of their output fluctuations. While New Zealand is the more volatile economy, Belgium and Canada have the lowest output volatility. Similarly, the model predicts the excessive volatility of investment relative to output. The proposed model also accounts for the almost equal volatile series of output and consumption in Canada. More important, it is capable of replicating the excess variability of consumption found in Norway. But this is a remarkable feature of the model given that it can simultaneously account for the less volatile pattern of consumption in the remaining countries. Aguiar and Gopinath (2007) attribute the excessive consumption volatility to the growth productivity shocks. Here, however, the stochastic trend is

[8] The results in this and the next sections are based on the posterior modes of the parameters.

common to all countries so it cannot alone account for the large volatility of consumption in only one country.

The combination of low habit formation and volatile preference shocks, $\upsilon_{j,t}$, in Norway induces the excess volatility.[9] From the households' first order condition, more volatile preference innovations distort the intertemporal substitution of consumption, inducing households to consume more up-front or to defer consumption for the future. In either case, the growth rate of consumption displays more volatility. In addition, if habit formation is sufficiently low, consumption volatility can be larger than that of output. To further illustrate this argument, I recomputed the second moments for Norway but set its habit formation to 0.5 and its preference volatility, σ_{ϑ}, to half its estimated value. Under this counterfactual scenario, the volatility of output drops to 1.63 while the ratio σ_c/σ_y is now 0.76. Clearly, the model predicts a smoother profile for consumption.

The second panel in Table 2 displays the fraction of the volatilities attributed to each structural shock. For example, Australia's productivity shock in the tradable sector, A_H, explains 35% and 83% of the variability of output and investment, respectively. The row labeled g^w reports the contribution of the common shock to the fluctuations in the model. For Canada, the common innovation accounts for 11%, 22%, and 12% of output, consumption, and investment, respectively.

Rather than further discussing individual outcomes, I find it more illustrative to highlight general patterns, resulting from the variance decomposition exercise. To begin with, the common productivity shock, g_t, contributes more to the fluctuations of consumption than to those of output and investment (The effect on these last two variables is remarkably small in New Zealand and Norway.) Since the common productivity shock is permanent, households anticipate output to increase permanently in the future. This anticipation generates a wealth effect that makes consumption contemporaneously rise by more than the initial response of output (Garcia-Cicco et al., 2009). It is precisely this initial spike in consumption that drives its large volatility following the common technology disturbance. In addition, the long lasting feature of the productivity shock implies that the return to capital will be high in the future. Households feel no urgency to boost capital today and, as a consequence, investment is not very volatile in response to the shock.

Interestingly, the degree of trade openness has no impact on how the common factor

[9] Neumeyer and Perri (2005), Fernandez-Villaverde et al. (2009), and Garcia-Cicco et al. (2009) provide a comprehensive discussion on the role of premium shocks in generating volatile consumption series in emerging economies.

influences a given economy. Belgium, a relatively open economy, is mildly affected by shocks to the common trend. In contrast, the same trend shocks have a far larger impact on Australia, which is comparatively less open to trade.[10] Understanding this behavior is beyond the scope of this paper but one may venture that Australia's dependence on commodities makes it more sensitive to worldwide fluctuations.

A second feature in Table 2 is that the common innovation is less important in those economies that are relatively more volatile, namely, New Zealand and Norway. Moreover, shocks to the common trend play a small role in accounting for the dynamics of output and investment in those countries. Third, interest rates and exchange rates are barely influenced by the common disturbance. Note, for example, that the shock explains only 2% of the movement in interest rates in Canada. Turning to the country-specific trend, $g^{d,j}$, we note that is only relevant for New Zealand's output, consumption, and investment. Indeed, a back-of-the-envelope calculation reveals that this trend is required to generate the excessive volatility found in New Zealand's business cycles.

The numbers in Table 2 indicate that the productivity innovation in the tradable sector, A_H^j, is the only shock that explains a nontrivial fraction of the fluctuations in the observables. Furthermore, this shock accounts for a large fraction of the volatility of investment. This result is expected if one recalls that investment exclusively uses tradable goods. In terms of exchange rates, note that shocks to the wholesale price of tradables, $\widetilde{p}_{H,t}$, accounts for the lion's share of the volatility of exchange rates. Indeed, those shocks explain between 40% and 86% of the real exchange rate fluctuations in Norway and Australia, respectively. Interestingly, this finding echoes the empirical results in other papers such as Engel (1999).[11] The tradable and nontradable technology disturbances also contribute to the dynamics of exchanges rates but to a lesser degree than shocks to the price of tradables. At face value, this finding suggests that supply-side innovations are important drivers behind exchange rates (a result consistent with the claims in Corsetti et al., 2007). Yet recall that $\widetilde{p}_{H,t}$ is exogenous so it may well be that this shock captures non-modeled demand-side innovations inside the small open economy and abroad. Unfortunately, the model is silent about the fundamental forces (beyond disturbances to the price of tradables) driving exchange rates in the data.

[10] During the period $1980-2010$, the trade openess indices for Australia and Belgium were 0.36 and 1.37, respectively. The index is defined as the ratio of exports plus imports to gross domestic product.

[11] Indeed, his decomposition exercise shows that more than 90% of the real exchange rate movement between, for example, Canada and the U.S. can be accounted by the relative price of traded goods.

In sum, we observe that all shocks play to some degree an important role in the business cycles of the SOEs. Yet country-specific disturbances explain a substantially larger fraction of the countries' volatilities than the common productivity shock does. This pattern raises two relevant questions: 1) What accounts for the small explanatory power of the common shock in the model? and 2) How do the results from my approach compare with the related literature?

Intuitively, the small role of the common shock can be explained as follows. Figure 1 reveals that the data display co-movement but also substantial asymmetry at the business cycle frequencies. Hence, if the model has a chance to match the data, it must allow the country-specific shocks to account for the bulk of the fluctuations in the data. To this end, the estimated high persistence of the (stationary) tradable productivity disturbance, A_H^j, implies that its unconditional variance is substantially large. With the stationary and nonstationary productivity disturbances fighting to explain the variability of output (they enter the model only via the production function), the large volatility of the former shock leaves little room for the common trend shock to account for the fluctuations found in the data. In contrast, less persistence and hence less volatility of A_H^j (or A_N^j) would assign more importance to the nonstationary productivity process, which would result in two counterfactual predictions: 1) large co-movement across countries; and 2) excessive volatility in consumption.

Interestingly, the importance of the trend shocks uncovered from my approach is consistent with the results in Norrbin and Schlagenhauf (1996) and Aguiar and Gopinath (2007). Using a dynamic factor model, the first authors report that 40% and 11% of the variance of the forecast error in Canada can be attributed to country-specific and international (common) factors, respectively. Aguiar and Gopinath find that such trend shocks account for a small fraction of the fluctuations in developed economies.[12]

A more appropriate way to contrast my results with those in the literature is to take one of those procedures and apply it to my sample. To that end, I follow Kose et al.'s methodology (2008). From all of the available procedures in the literature, their method is the closest to mine in the sense that our approaches assign all cross-dynamics (country-specific disturbances spread through spillovers) to the common factor. In addition, we rely on Bayesian methods, which make comparisons more transparent. In particular, Kose et al.'s technique decomposes each observable i in the sample into common and country-

[12] Using a different approach, Koren and Tenreyro (2007) find that global sectoral shocks account for a small fraction of industrial production volatility in Canada.

specific components: $y_{i,t} = \omega_i + \omega_i^{world} f_t^{world} + \omega_i^{country} f_{j,t}^{country} + \varepsilon_{i,t}$. Here, $\mathbb{E}\varepsilon_{i,t}\varepsilon_{k,t} = 0$ for $i \neq k$, $y_{i,t}$ is the growth rate of either output, consumption, or investment; f^{wolrd} is a factor common to all countries and observables; $f_{j,t}^{country}$ is a country-specific factor; $\varepsilon_{i,t}$ is an observable-specific disturbance, and the ω_i's are the factor loadings. These loadings capture the degree to which fluctuations in $y_{i,t}$ can be accounted for by each factor.[13]

The contribution of each factor to the volatility of the observables is reported in the last panel of Table 2. According to this decomposition, the world factor (f^{wolrd}) explains, for example, about 33% , 37%, and 15% of the volatility of Australia's output, consumption, and investment, respectively. By contrasting the results from Kose et al.'s method with mine, we can argue that about half of that 33% explanatory power for Australian output can be attributed to shocks to the stochastic trend.

For the remaining countries (but Belgium and New Zealand), we see that labor-augmenting nonstationary technology shocks can account for at least one-fourth of Kose et al.'s world factor. According to both methodologies, international disturbances explain a relatively small fraction of the fluctuations in Norway's output. Kose et al.'s dynamic factor analysis indicates that country and idiosyncratic shocks account for 85% of Norwegian output. My approach interprets that fraction as being mostly driven by domestic technology shocks and preference disturbances.

Broadly speaking, the approach proposed in this paper and Kose et al.'s methodology nicely square in several dimensions. The two methods predict that the world shock explains a larger fraction of the volatility of consumption than the variability of investment. Similarly, our techniques downplay the relevance of the international common factor to explain the volatility of real interest rates and exchange rates. Indeed, these variables are mostly driven by country-specific innovations. Finally, according to the two decomposition schemes, the common (international) factor accounts for a smaller fraction of the business cycles of the most volatile countries, namely, New Zealand and Norway.

6.3 Common Shocks or Idiosyncratic Disturbances? Impulse Responses

To further clarify the importance of the common shock, this section reports the dynamic paths from the estimated model after a positive shock to the structural disturbances. All responses are expressed as percentage deviations from their trends (exchange rates are

[13] For additional details on the estimation, such as priors and identification assumptions, the interested reader can consult Kose et al. (2003 and 2008).

plotted as percentage changes from steady state). Figure 4 presents the impulse responses following a one-standard deviation shock to the common stochastic trend in the model ($\varepsilon_{g,t}$). Because households anticipate the productivity shock to be permanent, they immediately raise their consumption by more than the initial response of output. Furthermore, households feel they will be wealthier in the future so leisure increases contemporaneously at the expense of lowering labor. For example, while consumption is initially 0.75% above its steady state in Australia, its output level only rises 0.5%. In the long term, output, consumption, and investment increase by 1.25%, which is consistent with the permanent effect of a common productivity growth shock: $(1-\zeta_j)\sigma_g/\left(1-\rho_g\right)$.

Figure 4 also reveals that there is substantial heterogeneity in the countries' responses even though the estimated stochastic process (equation 5) is the same for all the economies. This observation is accounted for by the different estimates of habit formation, the cost of adjustment in investment and the weight of the common trend, ζ_j, reported in Table 1. It is precisely different weights in the trend functions (equation 4) that account for the different long term impact of the common productivity shock. More to the point, recall from the previous section that the parameters $d^{[j]}$ and $\varkappa^{[j]}$ are calibrated to match relevant data in each country. This asymmetry in some of the structural parameters necessarily changes the propagation mechanism of the trend shocks inside each country, which in turn explains the heterogeneous impulse responses. For example, the large initial response of investment in Norway is likely a consequence of the fairly low estimate for its adjustment cost function.

For completeness, Figure 5 present the dynamic effects of the other shocks in the model. Output is more reactive than consumption to the stationary tradable and nontradable technology innovations. As explained above, the temporary nature of the shocks is behind this finding. Interestingly, tradable productivity induces a highly persistent response in all variables, in particular consumption. This persistence results from the large estimates for $\rho_h^{[j]}$ recovered by the econometric approach. When we turn to the domestic trend shock, we note that it has a disproportionately larger impact on New Zealand, which is necessary to explain its more volatile business cycles.

Consistent with the results in Table 2, real exchange rates are driven by shocks to the terms of trade, and innovations to the tradable and nontradable productivities. More important, while a tradable productivity shock appreciates the real exchange rates in all countries, a nontradable productivity innovation has the opposite effect. The appreciation following the productivity increase in the tradable sector is consistent with the findings in

Corsetti et al. (2009).[14] Finally, the responses to the premium and preference innovations are standard. For brevity, they are not discussed here but the interested reader can consult the working version paper.

7 Extensions

A clear message from Section 2 is that even after removing trends in the data the SOEs tend to closely co-move over the cycle (see the hp-filtered series in Figure 1). This observation suggests that the countries may be also sharing a stationary shock. For example, one can easily think of a common demand shock resulting from a worldwide decline in confidence. To explore this possibility, I introduce an additional common (and stationary) disturbance either in the production sector or in the demand side of the model.[15]

The second moments in Table 1 indicate that Norway and New Zealand are substantially more volatile than the other economies. Hence the second extension assesses the impact that absence of those countries have on the estimated common factor. There is a concrete message from these alternative scenarios: whereas common factors can account for up to one-third of the business cycles in the developed SOEs, a large part of their fluctuations is captured by domestically brewed elements.

[14] For the estimated parameters, the model displays a strong wealth effect after a technology shock to tradables. As a consequence, the demand for domestically produced goods rises, which induces a real apprecaition of the exchange rate (See Corsetti et al., 2007).

[15] Adding an additional common disturbance is consistent with Stock and Watson's (2005) finding that FAVARs favor a formulation with two rather than one common international shock.

7.1 Common Stationary Productivity

To further study the impact of productivity, let us assume that the production functions are given by

$$Y_{H,t}^{j} = A_t^w A_{H,t}^{j} \left(X_t^j h_{H,t}^{j} \right)^{1-\alpha} \left(K_{H,t}^{j} \right)^{\alpha},$$

and

$$Y_{N,t}^{j} = A_t^w A_{N,t}^{j} \left(X_t^j h_{N,t}^{j} \right)^{1-\alpha} \left(K_{N,t}^{j} \right)^{\alpha},$$

for the tradable and nontradable sectors, respectively. Note that the SOEs now share a common trend, X, as well as a stationary innovation, A^w. Moreover, let $\log(A_t^w)$ follow an AR(1) process whose persistence and volatility are given by $\rho_{A,w}$ and $\sigma_{A,w}$, respectively. Table 3 shows the variance decomposition under the new specification.[16] Since the stationary common shock enters the economy solely through the production functions, it is hardly surprising that the new shock mostly contributes to the volatility of output. Together the two international factors (g^w and A^w) explain about 25% and 31% of the fluctuations of output in Australia and Canada, respectively. Two-thirds of the world shock recovered from the Kose et al.'s approach corresponds to what my method identifies as technology-related disturbances. More important, in accordance with the benchmark model, country-specific innovations explain the big chunk of the business cycles (at least 70%). Furthermore, the common factors continue to play a relatively small role accounting for the volatility of Norway and New Zealand.

7.2 Common Demand Innovation

Suppose that the preference shock is composed of a country-specific part, $\upsilon_{j,t}$ as in the benchmark model, and a common preference disturbance, υ_t^w. The utility function has the following functional form:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\upsilon_{j,t} \upsilon_t^w \right) \left[\log \left[C_{j,t} - b\overline{C}_{j,t} \right] - \frac{\psi^{[j]}}{2} h_{j,t}^2 \right].$$

As with the other shocks, $\log(\upsilon_t^w)$ is assumed to follow an AR(1) process with persistence and volatility parameters given by $\rho_{\upsilon,w}$ and $\sigma_{\upsilon,w}$, respectively. The idea behind this new specification is to factor in a common demand shock such as a global decline in confidence

[16] The mode of the estimated parameters governing the common factors are $\rho_g = 0.22$ and $\sigma_g = 1.36$ for the trend process and $\rho_{A,w} = 0.93$ and $\sigma_{A,w} = 0.41$.

or a worldwide limited access to consumer credit. Ultimately, these events should manifest themselves as a contraction in demand at home.

Table 4 reveals that the common demand shock accounts for a limited fraction of the business cycles.[17] As one may expect, the shock is relevant for output and consumption but not for investment. Overall, the main conclusions from the benchmark model remain almost intact. The only noticeable difference is that country-specific disturbances are more important now for Spain than in the baseline formulation. From a variance decomposition perspective, the model with two common technology innovations seems to be the more relevant specification towards understanding the business cycles in the developed SOEs.

7.3 Five Countries

Business cycles in New Zealand and Norway are clearly more volatile than in the other countries in the sample (see Figure 1 and Table 2). This observation suggests that the presence of those countries may dampen the role of the common factor while raising the relevance of country-specific shocks. The explore this possibility, I re-estimate the benchmark model but exclude New Zealand and Norway from the estimation. Two clear pictures emerge from the new exercise (Table 5). For Belgium, Canada and Sweden the international trend explains a fraction of their fluctuations that is slightly larger than in the baseline model. The first observation accords our intuition that the common trend shock should become more important once we remove the more volatile economies.

In contrast, the common factor accounts for a smaller fraction of their business cycles than in the benchmark specification for Australia and Spain. Since New Zealand is a major trade partner of Australia, the reduced influence of the trend shock on this economy indicates that trade flows between these countries are partially captured in the model through the common factor.

8 Concluding Remarks

This paper uncovers the presence of common factors driving fluctuations in developed SOEs. A novelty in the paper is that these factors are recovered using a DSGE model and Bayesian methods. When the data are expressed in growth rates, the common factor is

[17] The mode of the estimated parameters governing the common factors are $\rho_g = 0.14$ and $\sigma_g = 2.18$ for the trend process and $\rho_{\upsilon,w} = 0.94$ and $\sigma_{\upsilon,w} = 0.38$ for the preference disturbance.

identified as a stochastic trend in the production function. Shocks to this trend explain up to 16% of the volatility of the growth rates of output and investment. Interestingly, those shocks explain roughly twice as much of the volatility of consumption growth as the volatility of output growth. In contrast, the common factor plays a small role in describing the dynamics of exchange rates or interest rates. TFP, preference, and terms-of-trade shocks explain the bulk of the volatility of output, consumption, investment, interest rates, and exchange rates.

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1 Technical Appendix

1.1 Likelihood with One Common Factor

This section shows the structure of the matrices required for the state-space representation (Section 4). A 0 denotes a conformable matrix of zeros. Let $\sum^{[j]}$ be a diagonal matrix composed of the scalars $\left\{\sigma_{\vartheta}^{[j]}, \sigma_{h}^{[j]}, \sigma_{n}^{[j]}, \sigma_{\mu}^{[j]}, \sigma_{a}^{[j]}, \sigma_{g}^{[dj]}\right\}$ and partition the matrices $\Pi^{[j]}$, $\Phi^{[j]}$, and $\eta^{[j]}$ as follows

$$\begin{aligned} \Pi^{[j]} &= \begin{bmatrix} \Pi_s^{[j]} & \Pi_{sg}^{[j]} \\ \Pi_{gs}^{[j]} & \Pi_g \end{bmatrix}, \ \Phi^{[j]} = \begin{bmatrix} \Phi_s^{[j]} & \Phi_g^{[j]} \end{bmatrix}, \\ \eta^{[j]} &= \begin{bmatrix} 0 & 0 \\ \sum^{[j]} & 0 \\ 0 & \sigma_g \end{bmatrix}, \ \sum{}^{[j]} = \begin{bmatrix} \sigma_{\vartheta}^{[j]} & 0 & 0 & 0 \\ 0 & \sigma_{\varkappa}^{[j]} & 0 & 0 \\ 0 & 0 & \sigma_{\mu}^{[j]} & 0 \\ 0 & 0 & 0 & \sigma_a^{[j]} \end{bmatrix}. \end{aligned}$$

Here, the matrices $\Pi_s^{[j]}$, $\Pi_{sg}^{[j]}$, $\Pi_{gs}^{[j]}$, Π_g, $\Phi_s^{[j]}$, and $\Phi_g^{[j]}$ have dimensions $(N_s-1)\times(N_s-1)$, $(N_s-1)\times 1$, $1\times(N_s-1)$, 1×1, $N_c\times(N_s-1)$, and $N_c\times 1$, respectively. Then for the case of two countries, the matrices $\mathbb{F}$ and η take the following forms

$$\mathbb{F}= \begin{bmatrix} 0 & 0 & G_s^{[1]} & 0 & G_g^{[1]} \\ 0 & 0 & 0 & G_s^{[2]} & G_g^{[2]} \\ 0 & 0 & \Pi_s^{[1]} & 0 & \Pi_{sg}^{[1]} \\ 0 & 0 & 0 & \Pi_s^{[2]} & \Pi_{sg}^{[2]} \\ 0 & 0 & \Pi_{gs}^{[1]} & \Pi_{gs}^{[2]} & \Pi_g \end{bmatrix}, \ \eta = \begin{bmatrix} \Phi_s^{[1]}\begin{bmatrix} 0 \\ \sum^{[1]} \end{bmatrix} & 0 & \Phi_g^{[1]}\sigma_g \\ 0 & \Phi_s^{[2]}\begin{bmatrix} 0 \\ \sum^{[2]} \end{bmatrix} & \Phi_g^{[2]}\sigma_g \\ \begin{bmatrix} 0 \\ \sum^{[1]} \end{bmatrix} & 0 & 0 \\ 0 & \begin{bmatrix} 0 \\ \sum^{[2]} \end{bmatrix} & 0 \\ 0 & 0 & \sigma_g \end{bmatrix},$$

where, $G_s^{[j]} \equiv \Phi_s^{[j]}\Pi_s^{[j]} + \Phi_g^{[j]}\Pi_{gs}^{[j]}$, $G_g^{[j]} \equiv \Phi_s^{[j]}\Pi_{sg}^{[j]} + \Phi_g^{[j]}\Pi_g$. The element Π_g has not been indexed because it is common for both countries. $\mathbb{H}$ is a matrix of ones and zeros that selects the appropriate elements of $\mathbb{S}_t$ needed to build the model's equivalent to the observables found in the data. This analysis can be easily extended when two or more countries are in the data.

Table 1: Prior Distributions

	$b^{[j]}$	$\kappa^{[j]}$	$\frac{1}{1-\varrho^{[j]}}$	$\frac{1}{1-\phi^{[j]}}$	$\eta^{[j]}$	$\zeta^{[j]}$	$\rho_h^{[j]}$	$\sigma_h^{[j]}$	$\rho_n^{[j]}$	$\sigma_n^{[j]}$	$\rho_g^{[dj]}$	$\sigma_g^{[dj]}$
	$\mathcal{G}(0.5, 0.2)$	$\mathcal{U}(0, 100)$	$\mathcal{G}(0.85, 0.2)$	$\mathcal{G}(0.74, 0.2)$	$\mathcal{G}(1.0, 0.2)$	$\mathcal{B}(0.5, 0.2)$	$\mathcal{B}(0.5, 0.2)$	$\mathcal{IG}(6, 1)$	$\mathcal{B}(0.5, 0.2)$	$\mathcal{IG}(6, 1)$	$\mathcal{B}(0.5, 0.2)$	$\mathcal{IG}(6, 1)$
Posterior Distributions												
Australia	0.21 [0.15,0.27]	0.83 [0.67,1.07]	0.80 [0.60,1.23]	0.61 [0.44,0.89]	1.35 [1.06,1.66]	0.41 [0.19,0.53]	0.99 [0.97,0.99]	1.95 [1.69,2.36]	0.49 [0.32,0.89]	1.13 [1.01,1.36]	0.63 [0.34,0.88]	0.14 [0.09,0.24]
Belgium	0.50 [0.45,0.57]	0.65 [0.55,0.84]	0.81 [0.59,1.33]	0.66 [0.44,0.97]	0.72 [0.59,1.03]	0.72 [0.54,0.81]	0.99 [0.98,0.99]	1.00 [0.91,1.17]	0.96 [0.91,0.98]	1.18 [1.05,1.39]	0.66 [0.49,0.79]	0.13 [0.11,0.25]
Canada	0.18 [0.14,0.28]	0.89 [0.78,1.15]	0.80 [0.58,1.11]	0.64 [0.44,0.95]	1.31 [0.98,1.74]	0.62 [0.41,0.70]	0.99 [0.94,0.99]	1.41 [1.26,1.68]	0.90 [0.84,0.94]	1.23 [1.10,1.47]	0.50 [0.29,0.67]	0.14 [0.10,0.24]
New Zealand	0.18 [0.13,0.26]	0.45 [0.37,0.59]	0.80 [0.52,1.22]	0.61 [0.39,0.89]	1.00 [1.00,1.22]	0.59 [0.40,0.75]	0.99 [0.98,0.99]	2.50 [2.22,3.08]	0.36 [0.26,0.74]	1.78 [1.64,2.32]	0.54 [0.43,0.80]	0.86 [0.26,1.09]
Norway	0.22 [0.15,0.30]	0.21 [0.17,0.25]	0.80 [0.61,1.23]	0.66 [0.46,0.91]	0.91 [0.70,1.23]	0.52 [0.36,0.66]	0.98 [0.97,0.99]	1.53 [1.38,1.74]	0.99 [0.97,0.99]	1.91 [1.73,2.24]	0.47 [0.26,0.64]	0.14 [0.08,0.17]
Spain	0.34 [0.28,0.43]	0.74 [0.60,0.92]	0.80 [0.50,1.06]	0.58 [0.40,0.91]	0.96 [0.65,1.25]	0.59 [0.33,0.68]	0.99 [0.96,0.99]	1.37 [1.19,1.58]	0.96 [0.92,0.98]	1.11 [1,1.31]	0.57 [0.27,0.75]	0.16 [0.09,0.23]
Sweden	0.21 [0.14,0.32]	0.80 [0.67,1.05]	0.80 [0.48,1.08]	0.67 [0.44,0.98]	1.16 [0.91,1.64]	0.57 [0.32,0.68]	0.99 [0.98,0.99]	1.63 [1.40,1.95]	0.94 [0.88,0.96]	1.51 [1.30,1.71]	0.57 [0.36,0.78]	0.15 [0.12,0.48]

Common Factor

	$g_{[\%]}$	ρ_g	σ_g
Prior	$\mathcal{B}(0.45, 0.1)$	$\mathcal{B}(0.5, 0.2)$	$\mathcal{IG}(6, 1)$
Posterior	0.24 [0.48,0.68]	0.19 [0.82,0.99]	1.70 [0.83,1.12]

For the Gamma, G, and Beta, B, distributions, the values in parenthesis are the mean and standard deviation. For the inverse gamma, IG, the values are the share and scale parameters. For the uniform distribution, U, the values are the lower and upper bounds.

The 95 percentile probability interval is in square brackets.

Table 2: Second Moments

	Australia	Belgium	Canada	New Zeland
σ_y	1.49 [1.03]	1.11 [0.78]	1.11 [0.87]	2.23 [1.95]
σ_c/σ_y	0.84 [0.95]	0.74 [0.72]	0.99 [0.97]	0.81 [0.80]
σ_i/σ_y	3.82 [2.93]	3.46 [3.04]	3.36 [2.85]	4.21 [2.60]
σ_R	4.09 [2.58]	2.90 [2.64]	4.01 [2.47]	6.22 [2.79]
σ_p	4.64 [4.38]	1.38 [1.38]	2.72 [2.51]	5.17 [3.84]

Variance Decomposition

	y	c	i	R	p	y	c	i	R	p	y	c	i	R	p	y	c	i	R	p
g^w	0.16	0.37	0.16	0.03	0.00	0.06	0.11	0.08	0.02	0.00	0.11	0.22	0.12	0.02	0.00	0.03	0.09	0.05	0.00	0.00
g^d	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.02	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.05	0.14	0.07	0.01	0.00
A_H	0.35	0.17	0.83	0.38	0.09	0.21	0.07	0.51	0.41	0.22	0.29	0.17	0.28	0.28	0.14	0.36	0.11	0.85	0.30	0.11
A_N	0.35	0.33	0.00	0.00	0.04	0.34	0.30	0.00	0.00	0.32	0.45	0.47	0.00	0.00	0.11	0.36	0.41	0.00	0.00	0.08
$\widetilde{p}_H$	0.01	0.00	0.00	0.47	0.86	0.16	0.18	0.36	0.12	0.45	0.01	0.00	0.46	0.46	0.75	0.05	0.06	0.02	0.62	0.80
μ	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.01	0.09	0.00	0.02	0.02	0.10	0.10	0.00	0.00	0.00	0.00	0.02	0.00
υ	0.13	0.11	0.00	0.08	0.00	0.20	0.31	0.02	0.34	0.00	0.11	0.11	0.13	0.13	0.00	0.14	0.19	0.01	0.04	0.00

Variance Decomposition Reduced Form

World	0.33	0.37	0.15	0.00	0.02	0.36	0.22	0.06	0.00	0.00	0.42	0.40	0.34	0.01	0.07	0.19	0.21	0.11	0.00	0.04
Country	0.25	0.08	0.33	0.03	0.02	0.19	0.18	0.09	0.03	0.07	0.17	0.13	0.06	0.03	0.06	0.41	0.33	0.19	0.01	0.06
Idiosyn	0.42	0.55	0.52	0.97	0.96	0.45	0.60	0.85	0.97	0.93	0.41	0.47	0.60	0.96	0.85	0.40	0.46	0.70	0.99	0.89

Table 2 (Continued): Second Moments

	Norway	Spain	Sweden
σ_y	1.84 [1.35]	1.16 [0.88]	1.45 [1.25]
σ_c/σ_y	1.12 [1.06]	0.78 [0.96]	0.99 [0.91]
σ_i/σ_y	5.99 [4.57]	3.44 [2.76]	3.15 [2.23]
σ_R	4.24 [2.70]	4.25 [3.47]	6.28 [2.58]
σ_p	2.16 [2.04]	2.45 [1.94]	3.32 [2.86]

Variance Decomposition

	y	c	i	R	p	y	c	i	R	p	y	c	i	R	p
g^w	0.06	0.11	0.07	0.01	0.00	0.14	0.26	0.15	0.03	0.00	0.08	0.17	0.11	0.01	0.00
g^d	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.01	0.01	0.00	0.00
A_H	0.20	0.04	0.39	0.39	0.23	0.33	0.15	0.82	0.29	0.15	0.29	0.10	0.80	0.24	0.12
A_N	0.29	0.28	0.00	0.00	0.36	0.35	0.35	0.00	0.00	0.10	0.34	0.37	0.00	0.00	0.11
$\widetilde{p}_H$	0.18	0.17	0.49	0.05	0.40	0.03	0.03	0.01	0.50	0.75	0.07	0.06	0.06	0.65	0.77
μ	0.01	0.01	0.00	0.11	0.00	0.01	0.01	0.01	0.06	0.00	0.01	0.01	0.01	0.05	0.00
υ	0.27	0.39	0.04	0.45	0.00	0.14	0.18	0.01	0.12	0.00	0.21	0.27	0.01	0.04	0.00

Variance Decomposition Reduced Form

World	0.13	0.10	0.03	0.04	0.01	0.30	0.37	0.21	0.00	0.02	0.36	0.25	0.28	0.00	0.09
Country	0.04	0.07	0.02	0.08	0.06	0.33	0.21	0.37	0.01	0.02	0.08	0.08	0.06	0.10	0.11
Idiosyn	0.81	0.83	0.95	0.88	0.93	0.36	0.41	0.41	0.99	0.96	0.55	0.66	0.66	0.90	0.80

Table 3: Second Moments - Common Stationary Productivity

	Australia					Belgium					Canada					New Zeland				
	y	c	i	R	p	y	c	i	R	p	y	c	i	R	p	y	c	i	R	p
g^w	0.05	0.19	0.06	0.02	0.00	0.02	0.05	0.04	0.00	0.00	0.03	0.11	0.05	0.01	0.00	0.00	0.02	0.01	0.00	0.00
A^w	0.20	0.03	0.02	0.02	0.00	0.25	0.04	0.10	0.04	0.00	0.28	0.04	0.05	0.03	0.00	0.08	0.01	0.02	0.01	0.00
g^d	0.00	0.01	0.00	0.00	0.00	0.09	0.28	0.09	0.04	0.00	0.01	0.02	0.01	0.00	0.00	0.04	0.13	0.06	0.01	0.00
A_H	0.36	0.21	0.91	0.50	0.10	0.28	0.01	0.65	0.28	0.11	0.26	0.19	0.88	0.40	0.14	0.36	0.13	0.90	0.32	0.11
A_N	0.28	0.45	0.00	0.00	0.04	0.33	0.56	0.00	0.00	0.18	0.33	0.54	0.00	0.00	0.09	0.34	0.48	0.00	0.00	0.08
$\widetilde{p}_H$	0.01	0.00	0.00	0.41	0.87	0.01	0.02	0.02	0.53	0.71	0.00	0.00	0.00	0.42	0.77	0.05	0.05	0.01	0.62	0.81
μ	0.01	0.00	0.00	0.04	0.00	0.01	0.02	0.01	0.10	0.00	0.01	0.02	0.01	0.10	0.00	0.00	0.00	0.00	0.02	0.00
υ	0.10	0.10	0.00	0.02	0.00	0.01	0.01	0.00	0.00	0.00	0.06	0.08	0.00	0.03	0.00	0.13	0.19	0.00	0.02	0.00

	Norway					Spain					Sweden				
	y	c	i	R	p	y	c	i	R	p	y	c	i	R	p
g^w	0.04	0.09	0.06	0.01	0.00	0.04	0.05	0.07	0.01	0.00	0.05	0.07	0.12	0.01	0.00
A^w	0.08	0.01	0.02	0.02	0.00	0.18	0.01	0.06	0.02	0.00	0.12	0.01	0.05	0.01	0.00
g^d	0.00	0.00	0.00	0.00	0.00	0.29	0.49	0.79	0.10	0.00	0.24	0.43	0.71	0.04	0.00
A_H	0.19	0.05	0.40	0.39	0.24	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
A_N	0.27	0.28	0.00	0.00	0.37	0.29	0.19	0.00	0.00	0.16	0.37	0.24	0.00	0.00	0.15
$\widetilde{p}_H$	0.17	0.17	0.47	0.04	0.39	0.02	0.02	0.02	0.36	0.84	0.02	0.01	0.03	0.49	0.85
μ	0.01	0.01	0.00	0.11	0.00	0.00	0.00	0.01	0.04	0.00	0.01	0.01	0.01	0.05	0.00
υ	0.25	0.40	0.04	0.43	0.00	0.18	0.24	0.06	0.48	0.00	0.19	0.23	0.08	0.41	0.00

Table 4: Second Moments - Common Demand

	Australia					Belgium					Canada					New Zeland				
	y	c	i	R	p	y	c	i	R	p	y	c	i	R	p	y	c	i	R	p
g^w	0.18	0.39	0.15	0.04	0.00	0.03	0.07	0.05	0.01	0.00	0.11	0.22	0.11	0.02	0.00	0.05	0.13	0.07	0.01	0.00
υ^w	0.02	0.02	0.00	0.01	0.00	0.02	0.04	0.00	0.03	0.00	0.03	0.02	0.00	0.02	0.00	0.01	0.01	0.00	0.00	0.00
g^d	0.01	0.01	0.00	0.00	0.00	0.13	0.24	0.17	0.05	0.00	0.01	0.01	0.01	0.00	0.00	0.06	0.15	0.08	0.01	0.00
A_H	0.36	0.17	0.84	0.42	0.10	0.32	0.02	0.73	0.29	0.11	0.30	0.17	0.86	0.31	0.15	0.34	0.10	0.83	0.30	0.11
A_N	0.32	0.31	0.00	0.00	0.04	0.45	0.57	0.00	0.00	0.19	0.44	0.47	0.00	0.00	0.11	0.35	0.39	0.00	0.00	0.08
$\widetilde{p}_H$	0.01	0.00	0.00	0.45	0.86	0.02	0.03	0.02	0.55	0.70	0.01	0.00	0.00	0.47	0.74	0.05	0.05	0.02	0.63	0.81
μ	0.00	0.00	0.00	0.03	0.00	0.01	0.01	0.02	0.09	0.00	0.02	0.02	0.01	0.10	0.00	0.00	0.00	0.00	0.02	0.00
υ_j	0.11	0.09	0.00	0.06	0.00	0.01	0.02	0.00	0.00	0.00	0.09	0.09	0.01	0.08	0.00	0.14	0.18	0.00	0.03	0.00

	Norway					Spain					Sweden				
	y	c	i	R	p	y	c	i	R	p	y	c	i	R	p
g^w	0.06	0.10	0.07	0.01	0.00	0.07	0.06	0.10	0.01	0.00	0.09	0.09	0.15	0.01	0.00
υ^w	0.00	0.01	0.00	0.01	0.00	0.01	0.01	0.00	0.01	0.00	0.01	0.01	0.00	0.01	0.00
g^d	0.00	0.00	0.00	0.00	0.00	0.34	0.49	0.82	0.10	0.00	0.26	0.43	0.72	0.04	0.00
A_H	0.20	0.04	0.40	0.39	0.24	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
A_N	0.28	0.28	0.00	0.00	0.36	0.36	0.20	0.00	0.00	0.17	0.40	0.23	0.00	0.00	0.16
$\widetilde{p}_H$	0.18	0.17	0.48	0.05	0.40	0.02	0.02	0.02	0.39	0.83	0.02	0.02	0.04	0.54	0.84
μ	0.01	0.01	0.00	0.11	0.00	0.00	0.00	0.01	0.04	0.00	0.01	0.01	0.01	0.05	0.00
υ_j	0.26	0.39	0.04	0.44	0.00	0.18	0.21	0.05	0.43	0.00	0.20	0.21	0.07	0.35	0.00

Table 5: Variance Decomposition - Five Countries

	Australia					Belgium					Canada				
	y	c	i	R	p	y	c	i	R	p	y	c	i	R	p
g^w	0.09	0.25	0.11	0.02	0.00	0.06	0.10	0.07	0.01	0.00	0.13	0.29	0.24	0.02	0.00
g^d	0.14	0.13	0.02	0.28	0.00	0.01	0.02	0.01	0.00	0.00	0.14	0.14	0.02	0.32	0.00
A_H	0.38	0.14	0.87	0.34	0.08	0.21	0.07	0.51	0.40	0.21	0.25	0.04	0.70	0.16	0.08
A_N	0.37	0.47	0.00	0.00	0.05	0.36	0.32	0.00	0.00	0.33	0.43	0.47	0.00	0.00	0.13
$\widetilde{p}_H$	0.01	0.00	0.00	0.34	0.87	0.16	0.18	0.37	0.12	0.45	0.03	0.03	0.02	0.44	0.79
μ	0.00	0.00	0.00	0.03	0.00	0.00	0.01	0.01	0.09	0.00	0.01	0.02	0.01	0.06	0.00
υ	0.01	0.01	0.00	0.00	0.00	0.20	0.31	0.03	0.37	0.00	0.00	0.01	0.00	0.00	0.00

	Spain					Sweden				
	y	c	i	R	p	y	c	i	R	p
g^w	0.06	0.06	0.10	0.01	0.00	0.08	0.10	0.17	0.01	0.00
g^d	0.34	0.48	0.81	0.09	0.00	0.25	0.40	0.70	0.04	0.00
A_H	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
A_N	0.37	0.21	0.00	0.00	0.18	0.42	0.24	0.00	0.00	0.16
$\widetilde{p}_H$	0.02	0.02	0.02	0.36	0.82	0.02	0.02	0.05	0.55	0.84
μ	0.00	0.00	0.01	0.04	0.00	0.01	0.01	0.01	0.05	0.00
υ	0.20	0.23	0.06	0.49	0.00	0.21	0.23	0.07	0.35	0.00

Figure 1: Trend and Cycles in Developed SOEs

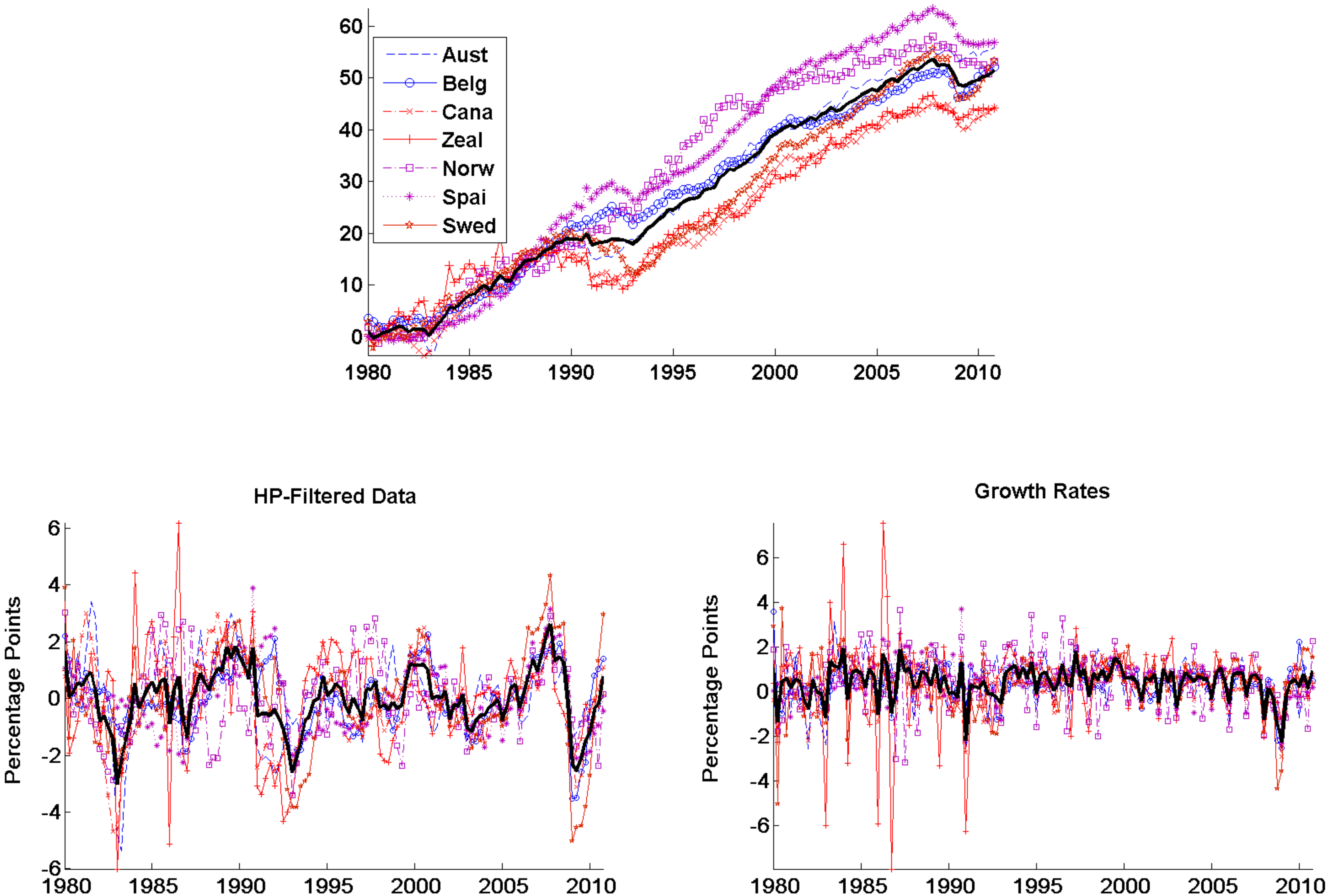

Figure 2: Filtered Common Factor

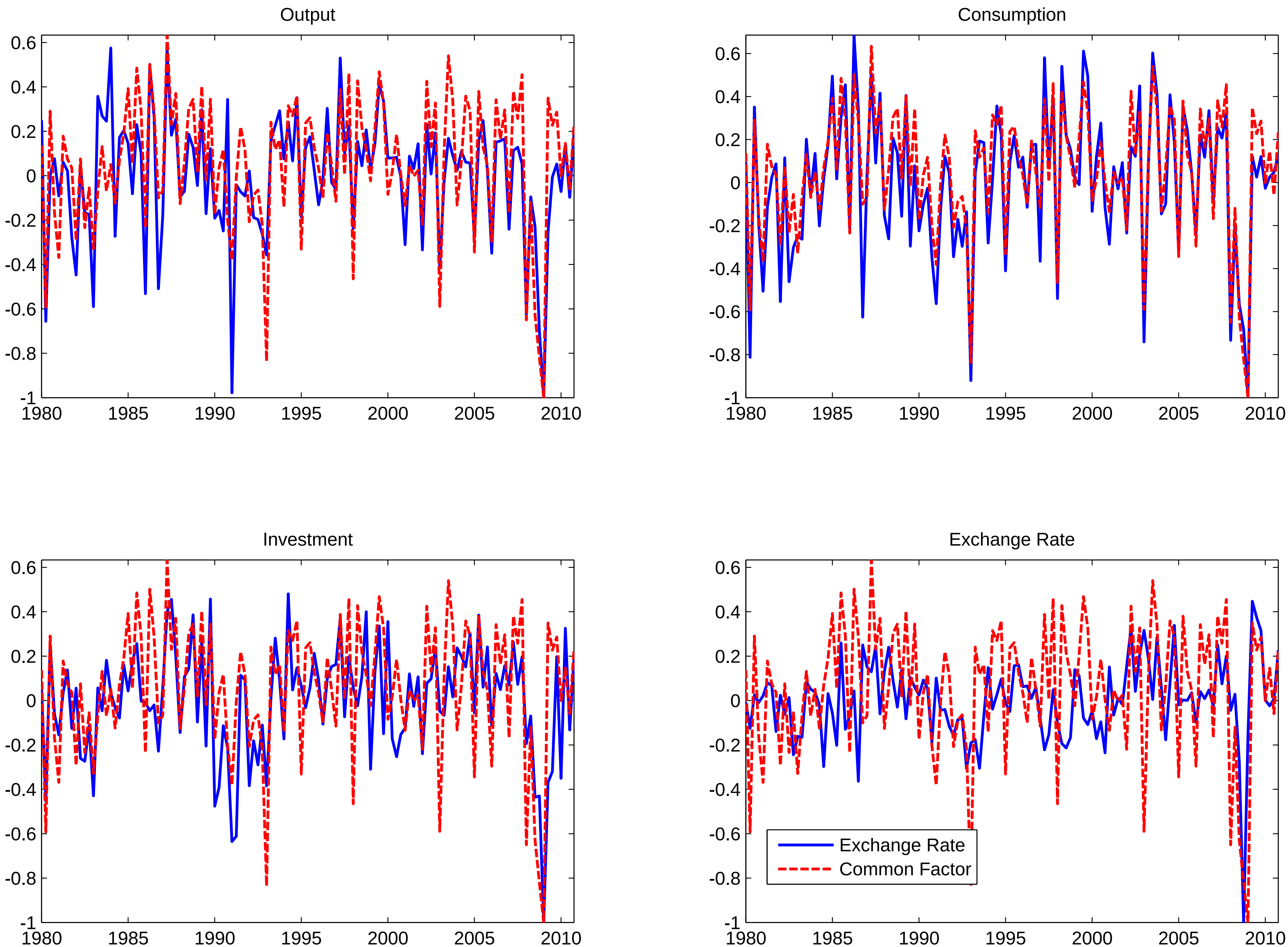

Figure 3: Recent Crisis

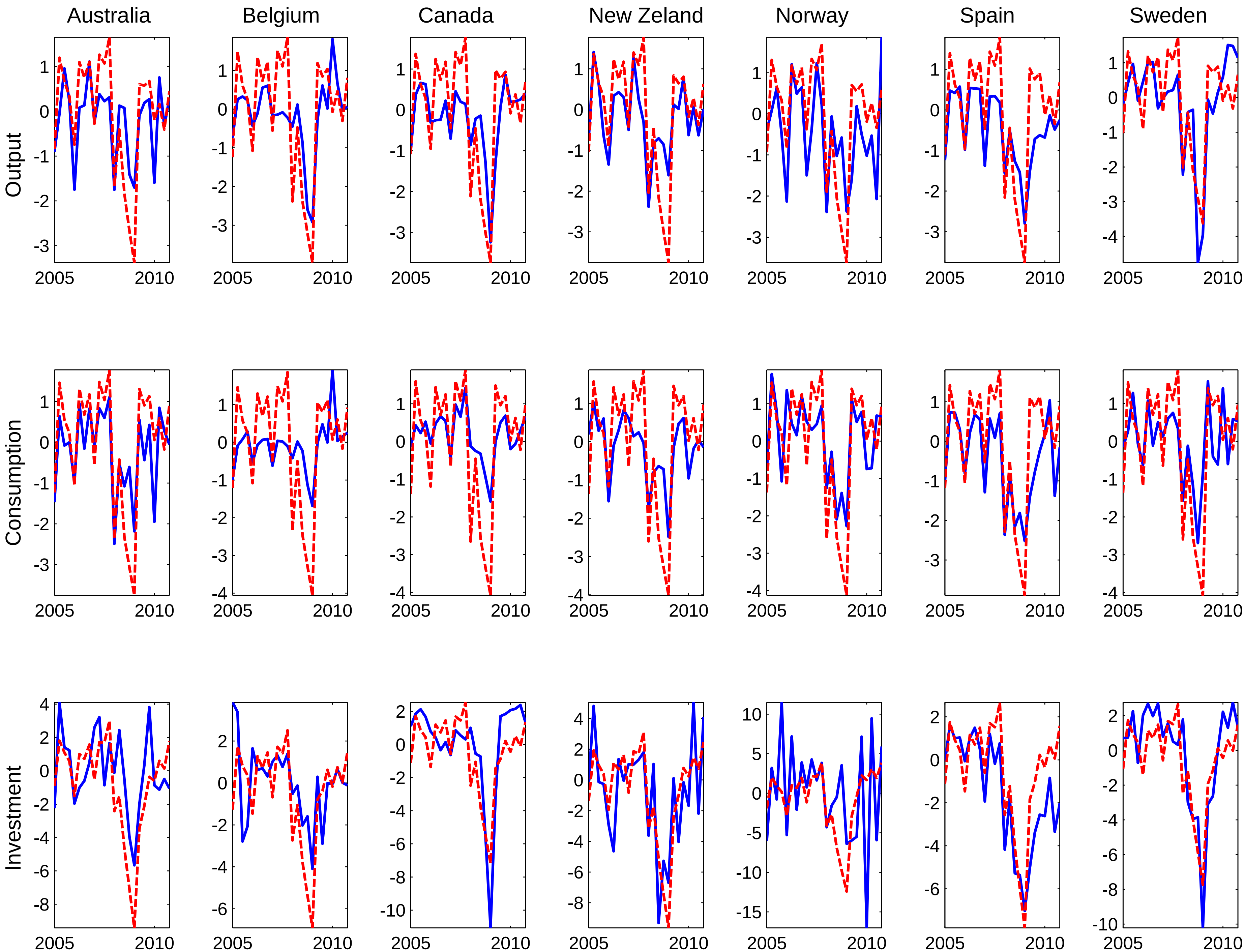

Figure 4: Common Trend Shock

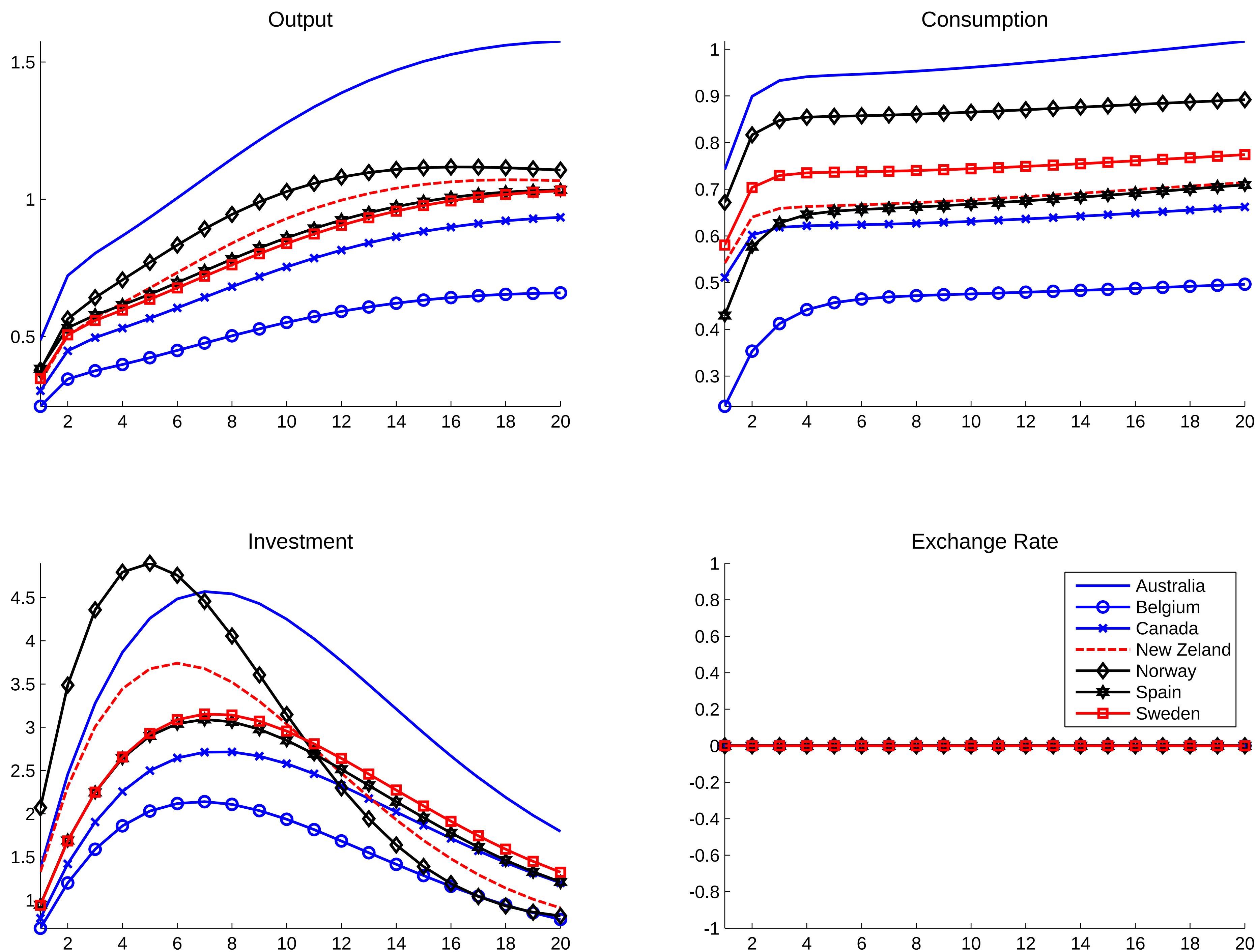

Figure 5: IRFs Country-Specific Innovations

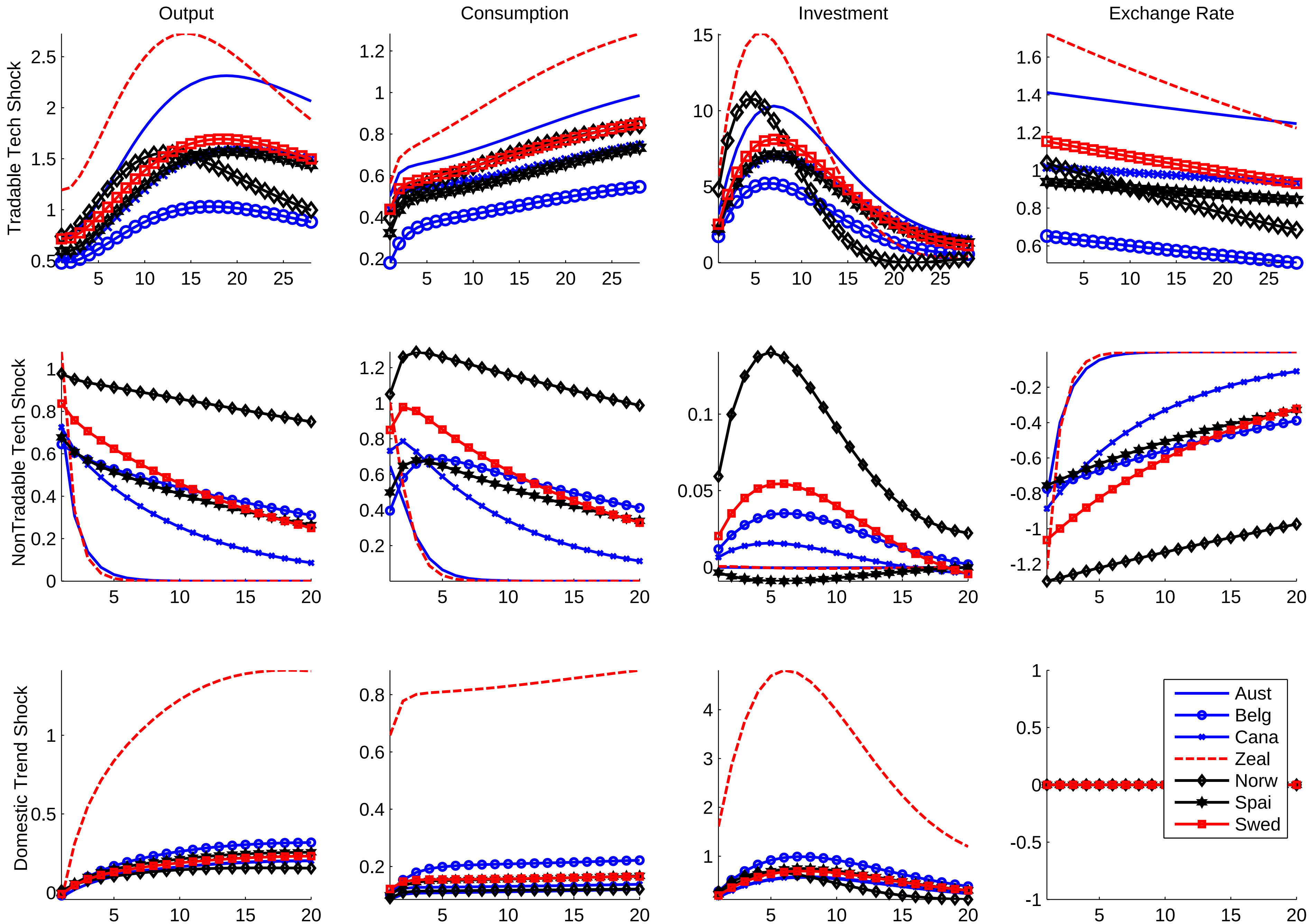

Figure 5 (Continued): IRFs Country-Specific Innovations