New Trade Models, Same Old Gains?*

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Abstract

The theory of international trade is rich in reasons why countries may gain from trade. In this paper we investigate whether new sources of gains from trade necessarily lead to larger gains from trade. The main contribution of our paper is to show that for a particular but important class of models, independently of the number of channels through which countries may gain from trade, the total size of the gains from trade is pinned down by the value of two sufficient statistics: (*i*) the share of expenditure on domestic goods; and (*ii*) a gravity-based estimate of the elasticity of imports with respect to variable trade costs. Accordingly, the mapping from observed trade data to the total size of the gains from trade is independent of the details of the model we use. Although it may be tempting to conclude that richer trade models necessarily entail larger gains from trade, our theoretical analysis demonstrates that this is not the case.

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1 Introduction

The theory of international trade is rich in reasons why countries may gain from trade. In a neoclassical world, opening up to trade acts as an expansion of the production possibilities frontier and may lead to both consumption and production gains; see e.g. Samuelson (1939). In a "new" trade model, international trade may also increase the number of available varieties in each country; see e.g. Krugman (1980). Finally, in more recent trade models, international trade may lead to aggregate productivity gains through intra-industry reallocation; see e.g. Melitz (2003). In this paper we investigate whether new sources of gains from trade necessarily lead to larger gains from trade.

The main contribution of our paper is to show that for a particular but important class of trade models, new sources of gains from trade may change the *composition* of these gains, but they have no effect on their *total size:* conditional on observed trade data, the gains from trade predicted by all these models are the same. Although it may be tempting to conclude that richer trade models necessarily entail larger gains from trade, our theoretical analysis demonstrates that this is not the case.

We focus on models featuring one factor of production, complete specialization, iceberg trade costs, a CES import demand system, and a gravity equation.[1] Examples of trade models satisfying these restrictions include, among others, Krugman (1980), Eaton and Kortum (2002), Anderson and van Wincoop (2003), and multiple variations and extensions of Melitz (2003).[2] Within that class of models, we show that under either perfect competition or monopolistic competition à la Krugman (1980) or Melitz (2003), there exists a common estimator of the gains from trade. This estimator only depends on the value of two aggregate statistics: (i) the share of expenditure on domestic goods, λ, which is equal to one minus the import penetration ratio; and (ii) a gravity-based estimator $\overline{\varepsilon}$ of the elasticity of imports with respect to variable trade costs, which we refer to as the "trade elasticity." Irrespective of the number of channels through which countries may gain from trade, these two aggregate statistics are sufficient for welfare analysis. In other words, the mapping from observed trade data, λ and $\overline{\varepsilon}$, to the total size of the gains from trade is independent of the model we use.

Our approach can be sketched as follows. We start by showing that the percentage change

[1] A CES import demand system is conceptually distinct from CES preferences; it entails restrictions on the interplay between domestic demand and supply. Country j's import demand system is CES if the elasticity of substitution of j's import demand from country i (relative to the demand for domestic goods) with respect to the trade cost from i' to j is zero for $i \neq i'$ and is common across $i \neq j$.

[2] Notable extensions of Melitz (2003) satisfying the restrictions above include Chaney (2008), Arkolakis (2008), and Eaton, Kortum, and Kramarz (2008).

in the ideal price index associated with any small change in trade costs is equal to $-\widehat{\lambda}\big/\varepsilon$, where $\widehat{\lambda}$ is the percentage change in the share of expenditure devoted to domestic goods caused by the change in trade costs and ε is the true value of the trade elasticity. For $\varepsilon < 0$, which is the empirically relevant case, being more open, $\widehat{\lambda} < 0$, implies a welfare gain. We then use our assumption that ε is constant across equilibria in order to integrate small changes in real income between the initial trade equilibrium and the autarky equilibrium. This allows us to establish that the total size of the gains from trade, i.e. the percentage change in real income necessary to compensate a representative consumer for going to autarky, is equal to $\lambda^{1/\varepsilon} - 1$. Finally, assuming that the true trade elasticity ε can be consistently estimated by $\overline{\varepsilon}$ using a gravity equation, we conclude that the gains from trade can be consistently estimated by $\lambda^{1/\overline{\varepsilon}} - 1$.

This last formula offers a very convenient way to measure gains from trade in practice. For example, the import penetration ratios for the U.S. and Belgium for the year 2000 were 7% and 27%, respectively.[3] This implies that $\lambda_{US} = 0.93$ and $\lambda_{BEL} = 0.73$. Anderson and Van Wincoop (2004) review studies that offer gravity-based estimates for the trade elasticity all within the range of -5 and -10. Thus, the total size of the gains from trade range from 0.7% to 1.5% for the U.S. and from 3.2% to 6.5% for Belgium, whatever the composition of these gains may be.

The common features of the trade models for which we derive these formulas are described in Section 2. As previously mentioned, these features include: (*i*) one factor of production; (*ii*) complete specialization; (*iii*) iceberg trade costs; (*iv*) a CES import demand system; and (*v*) a gravity equation. Given the importance of these trade models in the existing literature, and without any pretense of generality, we refer to them as "standard" trade models.

Section 3 focuses on the case of standard trade models with perfect competition. In this situation, the logic behind our welfare formula is fairly intuitive. In a neoclassical environment, a change in trade costs affects welfare through changes in terms-of-trade. Since there is only one factor of production, changes in terms-of-trade only depend on changes in relative wages and trade costs. Under complete specialization and a CES import demand system, these changes can be directly inferred from changes in the relative demand for domestic goods using an estimate of the trade elasticity, which the gravity equation provides.

A direct corollary of our analysis under perfect competition is that two very well-known

[3]Import penetration ratios are calculated from the *OECD Input-Output Database: 2006 Edition* as imports over gross output (rather than GDP), so that they can be interpreted as a share of (gross) total expenditures allocated to imports (see Norihiko and Ahmad (2006)).

neoclassical trade models, Anderson (1979) and Eaton and Kortum (2002), have identical welfare implications. In Anderson (1979), like in any other "Armington" model, there are only consumption gains from trade, whereas there are both consumption and production gains from trade in Eaton and Kortum (2002). Nevertheless, both models are "standard" trade models. As a result, conditional on the values of the share of domestic expenditure and the trade elasticity, the magnitude of the gains from trade predicted by these two models must be the same. In other words, as we go from Anderson (1979) to Eaton and Kortum (2002), the appearance of production gains is exactly compensated by a decline in consumption gains from trade.

Section 4 turns to the case of standard trade models with monopolistic competition. In this situation, the intuition behind our welfare formula is more subtle. In addition to their effects on relative wages, changes in trade costs now have implications for firms' entry decisions as well as their selection into exports. Both effects lead to changes in the set of goods available in each country, which we must also take into account in our welfare analysis. A CES import demand system, however, greatly simplifies this analysis. On the one hand, it guarantees that the number of entrants must remain constant under free entry. On the other hand, it guarantees that any welfare change not caused by changes in the number of entrants—whether it affects relative wages or the set of goods available in a given country—can still be inferred from changes in the share of domestic expenditure using the trade elasticity. Our welfare formula directly follows from these two observations.

Section 5 offers several extensions of our results. We first explore how our simple welfare formula may generalize to other standard trade models. Following the recent literature on trade and firm heterogeneity, we consider models with endogenous marketing costs, as in Arkolakis (2008), and models with multi-product firms, as in Bernard, Redding, and Schott (2007) and Arkolakis and Muendler (2007). In these extensions, our simple welfare formula remains unchanged. Finally, we demonstrate how to generalize our welfare formula to non-standard trade models, including models with multiple sectors or factors, as in Costinot and Komunjer (2007), Chor (2009), and Donaldson (2008), and tradable intermediate goods, as in Eaton and Kortum (2002), Alvarez and Lucas (2007), and Di Giovanni and Levchenko (2009).

Our paper is related to the recent literature in public finance trying to isolate robust insights for welfare analysis across different models; see e.g. Chetty (2009). Our paper demonstrates that for many standard trade models, the share of expenditures devoted to domestic goods, λ, and an estimate of the trade elasticity, $\bar{\varepsilon}$, are sufficient statistics for the computation of the gains from trade. This "sufficient statistics approach" allows us to make predictions about counterfactual outcomes and welfare without having to solve for all endogenous variables in our model. In a field such as international trade where general equilibrium effects are numerous, this represents a significant benefit.

In the trade literature, our paper is most closely related to Arkolakis, Demidova, Klenow, and Rodriguez-Clare (2008). The main difference between our paper and theirs is in terms of scope and generality. The authors used closed forms to compute the real wage as a function of λ and ε in a Melitz-type model with CES preferences, monopolistic competition, free or restricted entry, and heterogenous firms with a Pareto distribution of productivity. Noting that a similar expression had been derived by Eaton and Kortum (2002)—and could have been derived by Krugman (1980)—Arkolakis, Demidova, Klenow, and Rodriguez-Clare (2008) argued that the gains from trade in these models were the same.[4] Our analysis goes beyond their original claim by formally deriving, without the use of any closed form solution, general conditions under which λ and $\bar{\varepsilon}$ are sufficient statistics for the estimation of the gains from trade. This generality allows us to offer a unifying perspective on the welfare implications of standard trade models.

Another related paper is Atkeson and Burstein (2009), which focuses on the welfare gains from trade liberalization through its effects on entry and exit of firms and their incentives to innovate in a monopolistically competitive environment with symmetric countries (as in Melitz (2003)). At the theoretical level, they show that small symmetric changes in trade costs have the same welfare effects as in Krugman (1980). While the spirit of their results is similar to ours, our paper differs from theirs in that we focus on a different class of models, we consider arbitrary changes in trade costs (including movements to autarky), and we offer sufficient statistics for the computation of the gains from trade.[5]

[4]In a recent paper, Feenstra (2009) uses duality theory to revisit, among other things, the results of Arkolakis, Demidova, Klenow, and Rodriguez-Clare (2008). Under the same functional form assumptions, he shows how the gains from trade in the Melitz-type model computed by Arkolakis, Demidova, Klenow, and Rodriguez-Clare (2008) can be interpreted as "production gains" from trade, whereas the gains from trade in Krugman (1980) can be interpreted as "consumption gains." However, he does not discuss the fact that conditional on trade data, the total size of the gains from trade predicted by these two models is the same. This is our main focus.

[5]Our analysis is also related, though less closely, to Feenstra (1994), Klenow and Rodriguez-Clare (1997), Broda and Weinstein (2006), Feenstra and Kee (2008), Goldberg, Khandelwal, Pavcnik, and Topalova (2009), and Feenstra and Weinstein (2009) who investigate how to measure particular sources of gains from trade under monopolistic competition. By contrast, the goal of our paper is to stress that in a fairly large class of models, whatever the composition of the gains from trade may be, the total size of the gains from trade can always be computed using the same sufficient statistics.

2 Standard Trade Models

The objective of this section is to clarify the scope of our analysis by describing some of the main features of the trade models considered in the next two sections.

The Basic Environment. Throughout this paper, we consider a world economy comprising $i = 1, ..., n$ countries; one factor of production, labor; and multiple goods indexed by $\omega \in \Omega$. The number of goods in Ω may be continuous or discrete. Each country is populated by a continuum of workers with identical homothetic preferences. P_i denotes the ideal price index in country i. We assume that workers are immobile across countries. L_i and w_i denote the total endowment of labor and the wage in country i, respectively. Finally, we assume that technology is such that all cost functions are linear in output.

Bilateral Imports. We denote by X_{ij} the value of country $j's$ imports from country i, by $Y_j \equiv \sum_{i'=1}^{n} X_{i'j}$ the total expenditure in country j, and by $\lambda_{ij} \equiv X_{ij}/Y_j$ the share of country $j's$ total expenditure that is devoted to goods from country i. In any trade equilibrium, we assume complete specialization in the sense that almost all goods are bought from only one source, though this source may vary across countries. Formally, if we denote by $\Omega_{ij} \subset \Omega$ the set of goods that country j buys from country i, complete specialization requires the measure of goods in $\Omega_{ij} \cap \Omega_{i'j}$ to be equal to zero for all i, $i' \neq i$, and j.[6] Accordingly, bilateral imports can be expressed as

$$X_{ij} = \int_{\omega \in \Omega_{ij}} p_j(\omega) q_j(\omega) d\omega, \tag{1}$$

where $p_j(\omega)$ and $q_j(\omega)$ denote the price and quantity consumed of good ω in country j.[7]

Bilateral Trade Costs. Trade flows are subject to variable trade costs of the standard iceberg form: in order to sell one unit in country j, firms from country i must ship $\tau_{ij} \geq 1$ units. We assume that the matrix of variable trade costs $\boldsymbol{\tau} \equiv \{\tau_{ij}\}$ is such that $\tau_{ii} = 1$ for all i and $\tau_{il}\tau_{lj} \geq \tau_{ij}$ for all i, l, j. Depending on the market structure, trade flows may also be subject to fixed costs (Section 4).

The Import Demand System. Let $\mathbf{X} \equiv \{X_{ij}\}$ denote the $n \times n$ matrix of bilateral

[6] Note that this definition of complete specialization allows multiple countries to produce the same good. It only rules out equilibria such that multiple countries sell the same good in the same country.

[7] Since the number of goods in Ω_{ij} may be continuous or discrete, one should think of X_{ij} as a Lebesgue integral. Thus when Ω_{ij} is a finite or countable set, $\int_{\omega \in \Omega_{ij}} p_j(\omega) q_j(\omega) d\omega$ is equivalent to $\sum_{\omega \in \Omega_{ij}} p_j(\omega) q_j(\omega)$.

imports and $\mathbf{E}$ denote the vector of country specific equilibrium variables in the economy. Under perfect competition (Section 3), $\mathbf{E}$ is equal to the vector of wages in each country, whereas under monopolistic competition $\mathbf{E}$ includes the vector of wages and number of entrants in each country (Section 4). We refer to the mapping from variable trade costs, $\boldsymbol{\tau}$, and equilibrium variables, $\mathbf{E}$, to bilateral imports, $\mathbf{X}$, as the *import demand system.* With a slight abuse of notation, we write $X_{ij} \equiv X_{ij}(\boldsymbol{\tau}, \mathbf{E})$. This mapping, of course, depends on the other primitives of the model: preferences, technology, and market structure. This simple formulation allows us to distinguish between the direct impact of variable trade costs and their indirect impact through general equilibrium effects.

Throughout this paper, we restrict ourselves to a class of models where the import demand system satisfies the two following properties.

Property (I): CES. Let $\varepsilon_j^{ii'} \equiv \partial \ln(X_{ij}/X_{jj}) / \partial \ln \tau_{i'j}$ denote the elasticity of relative imports with respect to variable trade costs and let $\boldsymbol{\varepsilon}_j \equiv \left\{\varepsilon_j^{ii'}\right\}_{i,i' \neq j}$ denote the associated $(n-1) \times (n-1)$ matrix. In any trade equilibrium, the import demand system is such that

$$\boldsymbol{\varepsilon}_j = \begin{pmatrix} \varepsilon & 0 & \ldots & 0 \\ 0 & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & 0 \\ 0 & \ldots & 0 & \varepsilon \end{pmatrix} \tag{2}$$

with $\varepsilon < 0$, which is the empirically relevant case. We refer to an import demand system such that Equation (2) is satisfied for all j as a "CES import demand system" and to ε as the "trade elasticity" of that system.[8]

Our choice of terminology derives from the fact that in the case of a CES *demand* system, changes in relative demand, C_k/C_l, for two goods k and l are such that $\partial \ln(C_k/C_l) / \partial \ln p_{k'} = 0$ if $k' \neq k, l$ and $\partial \ln(C_k/C_l) / \partial \ln p_k = \partial \ln(C_{k'}/C_l) / \partial \ln p_{k'} \neq 0$ for all $k, k' \neq l$. Nevertheless, it should be clear that the assumption of a CES import demand system is conceptually distinct from the assumption of CES preferences. While the import demand obviously depends on preferences, it also takes into account the supply side as this affects the allocation of expenditures to domestic production. In fact, CES preferences are neither necessary nor sufficient to obtain a CES import demand system.

[8] It should be clear that a CES import demand system imposes a lot of symmetry among countries. First, since all the diagonal terms are equal in Equation (2), any change in bilateral trade costs, τ_{ij}, must have the same impact on relative demand, X_{ij}/X_{jj}, for all $i \neq j$. Second, since all the off-diagonal terms are equal to zero, any change in a third country trade costs, $\tau_{i'j}$, must have the same impact on X_{ij} and X_{jj}.

Two additional comments are in order. First, the trade elasticity is a *partial derivative*: it captures the direct effect of changes in variable trade costs on $p_j(\omega)$ and Ω_{ij}, but not their indirect effect through changes in wages or the total number of entrants. Second, the trade elasticity is an *upper-level* elasticity: it tells us how changes in variable trade costs affect *aggregate* trade flows, whatever the particular margins of adjustment, $p_j(\omega)$ or Ω_{ij}, may be.

Property (II): Gravity. In any trade equilibrium, the import demand system satisfies "gravity" in the sense that bilateral imports can be decomposed into

$$\ln X_{ij}(\boldsymbol{\tau}, \mathbf{E}) = A_i(\boldsymbol{\tau}, \mathbf{E}) + B_j(\boldsymbol{\tau}, \mathbf{E}) + \varepsilon \ln \tau_{ij} + \nu_{ij}, \tag{3}$$

where $A_i(\cdot)$, $B_j(\cdot)$, ε and ν_{ij} all depend on preferences, technology, and market structure. Note that according to Equation (3), ν_{ij} is not a function of $\boldsymbol{\tau}$ and $\mathbf{E}$. This is an important restriction, which makes Equation (3) an assumption, rather than a mere definition of ν_{ij}.

Under standard orthogonality conditions, the previous gravity equation offers a simple way to obtain a consistent estimate, $\bar{\varepsilon}$, of the trade elasticity using data on bilateral imports, X_{ij}, and bilateral trade costs, τ_{ij}. For example, if the underlying distribution of τ_{ij} and ν_{ij} across countries satisfies $E(\nu_{ij} \ln \tau_{i'j'}) = 0$, for any i, i', j, and $j' = 1, ..., N$, then $\bar{\varepsilon}$ can be computed as a simple difference-in-difference estimator. In the rest of this paper, we will remain agnostic about the exact form of the orthogonality condition associated with Equation (3), but assume that the same orthogonality condition can be invoked in all models. Without any risk of confusion, we can therefore refer to $\bar{\varepsilon}$ as the "gravity-based" estimate of the trade elasticity, whatever the particular details of the model may be.[9]

It is worth emphasizing that the two previous properties, CES and gravity, are different in nature and will play distinct roles in our analysis. CES imposes restrictions on how changes in variable trade costs affect relative import demands *across* trade equilibria. By contrast, gravity imposes restrictions on the cross-sectional variation of bilateral imports *within* a given trade equilibrium. The former property will allow us to express gains from trade as a function of the share of expenditure on domestic goods and the true trade elasticity, whereas the latter will be important to obtain an estimate of the trade elasticity from observable trade data. Finally, note that both properties are easy to check since they do not require to solve for the endogenous equilibrium variables included in $\mathbf{E}$.

[9] Of course, the exact value of $\bar{\varepsilon}$ as a function of trade data depends on the choice of the orthogonality condition. The crucial assumption for our purposes, however, is that conditional on the choice of the orthogonality condition, the exact value of $\bar{\varepsilon}$ is the same in all models.

Asymptotic Behavior. For technical reasons, we also assume that for any pair of countries $i \neq j$, $\lim_{\tau_{ij} \to +\infty} (w_i \tau_{ij} / w_j) = +\infty$. This mild regularity condition guarantees that trade equilibria converge to the autarky equilibrium as variable trade costs $\boldsymbol{\tau}$ go to infinity.

To summarize, the main features of the trade models analyzed in our paper include: (*i*) one factor of production; (*ii*) complete specialization; (*iii*) iceberg trade costs; (*iv*) a CES import demand system; and (*v*) gravity. Although these assumptions are admittedly restrictive, it is easy to check that they are satisfied in many existing trade models including Anderson (1979), Krugman (1980), Eaton and Kortum (2002), Anderson and van Wincoop (2003), Bernard, Eaton, Jensen, and Kortum (2003), and multiple variations and extensions of Melitz (2003), such as Chaney (2008), Arkolakis (2008), Eaton, Kortum, and Kramarz (2008), Bernard, Redding, and Schott (2007) and Arkolakis and Muendler (2007).[10]

From now on, we refer to trade models satisfying the assumptions described in this section as "standard" trade models. The rest of our paper explores the welfare implications of this class of models under two distinct market structures: perfect and monopolistic competition. The theoretical question that we are interested in is the following. Consider a hypothetical change in variable trade costs from $\boldsymbol{\tau}$ to $\boldsymbol{\tau}'$, while keeping labor productivity and labor endowments fixed around the world.[11] What is the percentage change in real income needed to bring a representative worker from some country j back to her original utility level?

3 Gains from Trade (I): Perfect Competition

We start by assuming perfect competition. Given our assumptions on technology and the number of factors of production, standard trade models simplify into Ricardian models under perfect competition. In this case, the vector of country specific equilibrium variables is equal to the vector of wages, $\mathbf{E} = (w_1, ..., w_n)$. To simplify notations, we suppress the arguments $(\boldsymbol{\tau}, \mathbf{E})$ from our trade variables in the rest of this section.

[10] This being said, we wish to be very clear that our analysis does not apply to all variations and extensions of Melitz (2003). Helpman, Melitz, and Rubinstein (2008), for example, falls outside the scope of our paper. In their model bilateral trade flows go to zero for sufficiently large bilateral trade costs. This corresponds to a situation in which ν_{ij} in Equation (3) is a function of trade costs, thereby violating our gravity property. For similar reasons, the trade elasticity is not constant in this model, contradicting our CES property

[11] This is a non-trivial restriction. When measuring the gains from trade, we will implictly abstract from any direct channel through which changes in trade costs may affect labor productivity and labor endowments.

3.1 Equilibrium conditions

Perfect competition requires goods to be priced at marginal costs:

$$p_j(\omega) = \frac{w_i \tau_{ij}}{z_i(\omega)}, \text{ for all } \omega \in \Omega_{ij}, \tag{4}$$

where $z_i(\omega) > 0$ is the labor productivity for the production of good ω in country i. In addition, perfect competition requires each good to be produced in the country that minimizes costs of production and delivery. Hence, we have

$$\Omega_{ij} = \left\{ \omega \in \Omega | \frac{w_i \tau_{ij}}{z_i(\omega)} = \min_{1 \leq i' \leq n} \frac{w_{i'} \tau_{i'j}}{z_{i'}(\omega)} \right\}. \tag{5}$$

Finally, labor market clearing implies

$$Y_j = w_j L_j, \tag{6}$$

Equipped with these three equilibrium conditions, we now investigate how changes in variable trade costs affects welfare in each country.

3.2 Welfare analysis

Without loss of generality, we focus on a representative worker from country j and use labor in country j as our numeraire, $w_j = 1$. We start by considering a small change in trade costs from $\boldsymbol{\tau}$ to $\boldsymbol{\tau} + d\boldsymbol{\tau}$. Since the set of goods Ω is fixed under perfect competition, changes in the ideal price index satisfy

$$\widehat{P}_j = \int_{\Omega} \lambda_j(\omega) \widehat{p}_j(\omega) d\omega, \tag{7}$$

where $\widehat{x} \equiv dx/x$ denotes the relative change in a given variable x; and $\lambda_j(\omega)$ is the share of expenditure on good ω in country j. Using Equations (4) and (5) and the fact that there is complete specialization, we can rearrange Equation (7) as

$$\widehat{P}_j = \sum_{i=1}^{n} \lambda_{ij} \left(\widehat{w}_i + \widehat{\tau}_{ij} \right). \tag{8}$$

Equation (8) reminds us that in a neoclassical environment, all changes in welfare must be coming from changes in terms of trade. Since labor in country j is our numeraire, $\widehat{w}_j = 0$, these changes are exactly equal to $\widehat{w}_i + \widehat{\tau}_{ij}$. By the Envelope Theorem, changes in trade shares can only have a second order effect.

While the previous result is well-known, stronger welfare predictions can be derived in the case of a CES import demand system. The core of our analysis relies on the following lemma.

Lemma 1 *In any standard trade model with perfect competition, percentage changes in the ideal price index satisfy*

$$\widehat{P}_j = -\widehat{\lambda}_{jj}\Big/\varepsilon. \tag{9}$$

The formal proof as well as all subsequent proofs can be found in the Appendix. The logic can be sketched as follows. Under perfect competition, for any exporter i', a one percent increase in $w_{i'}$ has the same effect on country j and other exporters as a one percent increase in $\tau_{i'j}$. By definition of $\varepsilon_j^{ii'}$, changes in bilateral imports must therefore satisfy

$$\widehat{X}_{ij} - \widehat{X}_{jj} = \textstyle\sum_{i' \neq j} \varepsilon_j^{ii'} \left(\widehat{w}_{i'} + \widehat{\tau}_{i'j}\right). \tag{10}$$

A direct implication of Equation (10) is that if all elasticities $\varepsilon_j^{ii'}$ are known, changes in terms of trade can be inferred from changes in relative imports. To do so, we simply need to invert a system of $(n-1) \times (n-1)$ equations. Assuming that $\boldsymbol{\varepsilon}_j$ is invertible, which will always be true in the case of a CES import demand system, Equations (8) and (10) imply

$$\widehat{P}_j = \boldsymbol{\lambda}'_j \boldsymbol{\varepsilon}_j^{-1} \widehat{\mathbf{X}}_j, \tag{11}$$

where

$$\widehat{\mathbf{X}}'_j = \left(\widehat{X}_{1j} - \widehat{X}_{jj}, ..., \widehat{X}_{(j-1)j} - \widehat{X}_{jj}, \widehat{X}_{(j+1)j} - \widehat{X}_{jj}, ...\widehat{X}_{nj} - \widehat{X}_{jj}\right);$$
$$\boldsymbol{\lambda}'_j = \left(\lambda_{1j}, ...\lambda_{(j-1)j}, \lambda_{(j+1)j}, ...\lambda_{nj}\right).$$

Equation (11) provides a general characterization of welfare changes as a function of initial trade shares, changes in trade flows, and upper level elasticities, whatever the particular characteristics of the import demand system may be. Lemma 1 simply derives from the fact that in the case of a CES import demand system we have $\boldsymbol{\lambda}'_j \boldsymbol{\varepsilon}_j^{-1} \widehat{\mathbf{X}}_j = -\widehat{\lambda}_{jj}\Big/\varepsilon$.

It is worth emphasizing that Lemma 1 is a local result that does not depend on the assumption that ε is the same across all trade equilibria or countries. If we were to relax the specification of the import demand system so that we had $\varepsilon_j^{ii} = \varepsilon_j(\boldsymbol{\tau}, \mathbf{E})$ and $\varepsilon_j^{ii'} = 0$ for all j and $i \neq i'$, then Lemma 1 would still hold. By contrast, our global results will heavily rely on the fact that ε is invariant to changes in trade costs.

We now consider the welfare impact of large changes in trade costs from $\boldsymbol{\tau}$ to $\boldsymbol{\tau}'$. Let P_j and P_j' denote the ideal price index in country j if trade costs are equal to $\boldsymbol{\tau}$ and $\boldsymbol{\tau}'$, respectively; and let $W_j \equiv \left(P_j'/P_j\right) - 1$ denote the (negative of) the percentage change in real income needed to bring a representative worker from country j back to her original utility level. Our first global result can be stated as follows.

Proposition 1 *In any standard trade model with perfect competition, W_j can be consistently estimated by $1 - \left(\lambda_{jj}/\lambda_{jj}'\right)^{1/\bar{\varepsilon}}$, where λ_{jj}' and λ_{jj} are evaluated at the new and initial trade equilibrium, respectively.*

Proposition 1 derives from two observations. On the one hand, the fact that ε is constant implies that we can integrate Equation (9) between $\boldsymbol{\tau}$ and $\boldsymbol{\tau}'$ to get $W_j = 1 - \left(\lambda_{jj}/\lambda_{jj}'\right)^{1/\varepsilon}$. On the other hand, the fact that $\bar{\varepsilon}$ is a consistent estimator of ε implies, by a standard continuity argument, that $1 - \left(\lambda_{jj}/\lambda_{jj}'\right)^{1/\bar{\varepsilon}}$ is a consistent estimator of $1 - \left(\lambda_{jj}/\lambda_{jj}'\right)^{1/\varepsilon}$.

The implication of this proposition is that the welfare effect of a change in trade costs in a standard trade model with perfect competition can be measured using only: (i) the initial and the new share of expenditure on domestic goods, λ_{jj} and λ_{jj}'; and (ii) the gravity-based estimate of the trade elasticity, $\bar{\varepsilon}$. This offers a parsimonious way to compute welfare changes resulting from changes in trade costs. In particular, one does not need to observe the way in which *all* prices change, as would be suggested by Equation (7); it is sufficient to have information about the trade elasticity, $\bar{\varepsilon}$, and the changes in trade flows as summarized by λ_{jj} and λ_{jj}'. Note also that since $\bar{\varepsilon}$ is negative in practice, see e.g. Anderson and Van Wincoop (2004), welfare increases, $W_j > 0$, whenever country j becomes more open, $\lambda_{jj}' < \lambda_{jj}$.

We define the *gains from trade in country* j, denoted by $\overline{W}_j$, as the percentage change in current income needed to bring country j's representative agent back to its original utility level after going to autarky, i.e. after increasing all τ_{ij}, $i \neq j$, to infinity.[12] Proposition 1 and the fact that $\lambda_{jj}' = 1$ under autarky immediately implies the following result.

Proposition 2 *In any standard trade model with perfect competition, $\overline{W}_j$ can be consistently estimated by $\left(\lambda_{jj}\right)^{1/\bar{\varepsilon}} - 1$, where λ_{jj} is evaluated at the initial equilibrium.*

This result implies that conditional on observed trade data, i.e. the values of λ_{jj} and $\bar{\varepsilon}$ in current trade equilibrium, the gains from trade predicted by all standard trade models under perfect competition must be the same. Within that class of models, new sources of gains from trade may affect the composition of the gains from trade, but not their total size.

[12] Formally, $\overline{W}_j$ is equal to $-W_j$ evaluated at the counterfactual equilibrium with $\boldsymbol{\tau} = +\infty$.

3.3 Anderson (1979) vs. Eaton and Kortum (2002)

To get a better understanding of the equivalence result emphasized in Proposition 2, we now compare two well-known trade models, Anderson (1979) and Eaton and Kortum (2002).

On the demand side, both models assume CES preferences. The main difference between the two models comes from the supply side. In Anderson (1979), countries cannot produce the goods produced by other countries: if $a_i(\omega) < +\infty$, then $a_{i'}(\omega) = +\infty$ for all $i' \neq i$.[13] By contrast, Eaton and Kortum (2002) assume that in each country, unit labor requirements are drawn from an extreme value distribution. From a qualitative standpoint, this is an important difference. It implies that there are both production and consumption gains from trade in Eaton and Kortum (2002), whereas there can only be consumption gains from trade in Anderson (1979).

Does that mean that the two models lead to different quantitative predictions about the size of the gains from trade? The answer is no. It is easy to check that both models fit the definition of standard trade models given in Section 2. In particular, the import demand system is such that

$$\frac{X_{ij}}{X_{jj}} = \left(\frac{T_i w_i \tau_{ij}}{T_j w_j}\right)^{\varepsilon}, \tag{12}$$

where T_i and T_j are country-specific technology parameters, $\varepsilon \equiv$"$1 - \sigma$" in Anderson (1979) and $\varepsilon \equiv$"$-\theta$" in Eaton and Kortum (2002). Because of alternative microtheoretical foundations, the (negative of the) trade elasticity is equal to the elasticity of substitution between goods (minus one) in Anderson (1979) and it is equal to the shape parameter of the productivity distribution in Eaton and Kortum (2002). In both models, however, Equation (12) implies that the import demand system is CES and satisfies gravity. We can therefore invoke Proposition 2 to conclude that conditional on two sufficient statistics, λ_{jj} and $\bar{\varepsilon}$, the gains from trade predicted by Anderson (1979) and Eaton and Kortum (2002) must be the same.

This equivalence illustrates one of the main points of our paper in a very clear manner. Since Eaton and Kortum (2002) allows countries to specialize according to comparative advantage whereas Anderson (1979) does not, one may think that the gains from trade predicted by Eaton and Kortum (2002) must be larger. Our analysis demonstrates that this is not the case. As we switch from Anderson (1979) to Eaton and Kortum (2002), the structural interpretation of the trade elasticity changes, reflecting the fact there is now one more margin, namely Ω_{ij}, for bilateral imports to adjust. However, conditional on the

[13] Under this assumption, endowment or "Armington" models such as Anderson (1979) can always be reinterpreted as particular Ricardian models; see Matsuyama (2007).

estimated value of the upper-level elasticity, $\bar{\varepsilon}$, more margins of adjustment can only affect the composition of the gains from trade.

4 Gains From Trade (II): Monopolistic Competition

We now turn to the case of monopolistic competition. In this environment, gains from trade may also derive from changes in the number of available varieties in each country, as in Krugman (1980), as well as from changes in aggregate productivity due to intra-industry reallocation, as in Melitz (2003).

To allow for these additional sources of gains from trade, we follow the previous literature and impose the following assumptions. On the demand side, we assume that workers in any country i have CES preferences represented by

$$U_i = \left(\int_{\omega \in \Omega} q_i (\omega)^{\frac{\sigma}{\sigma - 1}} d\omega \right)^{\frac{\sigma - 1}{\sigma}},$$

where $\sigma > 1$ is the elasticity of substitution between goods ω which we will refer to as "varieties" in this section.

On the supply side, we assume that there is an unbounded pool of potential entrants capable of producing differentiated varieties. In order to produce in country i, firms must incur a fixed entry cost, $f_e > 0$, in terms of domestic labor. M_i denotes the total measure of entrants in country i. Upon entry, these firms draw their productivity, $z(\omega)$, from a known distribution with density g_i. In order to sell their varieties to country j, firms from country i must then incur a fixed marketing cost, $f_{ij} \geq 0$, in terms of domestic labor. After marketing costs have been paid, trade flows are subject to iceberg trade costs $\tau_{ij} \geq 1$ as in the neoclassical case.

Throughout this section, we refer to standard trade models satisfying the previous assumptions as "standard trade models with monopolistic competition." In this case, the vector of country specific equilibrium variables is equal to the vector of wages and measures of entrants, $\mathbf{E} = (w_1, ..., w_n, M_1, ..., M_n)$. To simplify notation, we again suppress the arguments $(\boldsymbol{\tau}, \mathbf{E})$ from our trade variables in the rest of our analysis.

4.1 Equilibrium conditions

Because of CES preferences, monopolists charge a constant markup over marginal cost. In any country j, the price of a variety ω from country i is given by

$$p_j(\omega) = \frac{\sigma \tau_{ij} w_i}{(\sigma - 1)\, z(\omega)} \text{ for all } \omega \in \Omega_{ij}. \tag{13}$$

The associated profits, $\pi_{ij}(\omega)$, of a firm with productivity $z(\omega)$ operating in country i and selling in country j can be written as

$$\pi_{ij}(\omega) = \left[\frac{\sigma \tau_{ij} w_i}{(\sigma - 1)\, z(\omega)\, P_j}\right]^{1-\sigma} \frac{Y_j}{\sigma} - w_i f_{ij}, \tag{14}$$

where P_j is the ideal CES price index in country j,

$$P_j = \left[\sum_{i=1}^{n} \int_{\omega \in \Omega_{ij}} p_j^{1-\sigma}(\omega) d\omega\right]^{\frac{1}{1-\sigma}}. \tag{15}$$

The set of varieties from country i available in country j, Ω_{ij}, is determined by the following zero-profit condition

$$\Omega_{ij} = \{\omega \in \Omega | \pi_{ij}(\omega) \geq 0\}. \tag{16}$$

Equations (14) and (16) implicitly define a unique cut-off

$$z_{ij}^* = \left[\frac{\sigma \tau_{ij} w_i}{(\sigma - 1)\, P_j}\right] \left(\frac{w_i f_{ij} \sigma}{Y_j}\right)^{\frac{1}{\sigma-1}} \tag{17}$$

such that firms from country i sell variety ω in country j if and only if $z(\omega) \geq z_{ij}^*$. Finally, for any country $j = 1, ..., n$, free entry implies that total expected profits are equal to fixed entry costs,

$$\sum_{i=1}^{n} E\left[\pi_{ji}(\omega)\right] = w_j f_e, \tag{18}$$

and labor market clearing implies that total income is equal to the total wage bill

$$Y_j = w_j L_j. \tag{19}$$

4.2 Welfare analysis

Without loss of generality, we again focus on a representative worker from country j and use labor in country j as our numeraire, $w_j = 1$. Like in Section 3, we first consider small changes in trade costs from $\boldsymbol{\tau}$ to $\boldsymbol{\tau} + d\boldsymbol{\tau}$. Using Equations (13) and (15), and the definitions of M_i and z_{ij}^*, we can express changes in the ideal price index as

$$\widehat{P}_j = \sum_{i=1}^{n} \lambda_{ij} \left[(\widehat{w}_i + \widehat{\tau}_{ij}) - \frac{\widehat{M}_i}{\sigma - 1} + \frac{\gamma_{ij} \widehat{z_{ij}^*}}{\sigma - 1} \right], \tag{20}$$

where

$$\gamma_{ij} \equiv \left(z_{ij}^*\right)^{\sigma} g_i\left(z_{ij}^*\right) \Big/ \int_{z_{ij}^*}^{+\infty} z^{\sigma-1} g_i(z)\, dz. \tag{21}$$

Compared to welfare changes in the neoclassical case, Equation (8), there are two extra terms, $\widehat{M}_i / (\sigma - 1)$ and $\gamma_{ij} \widehat{z_{ij}^*} / (\sigma - 1)$. Under monopolistic competition, these two terms reflect the fact that changes in trade costs may affect the set of varieties available in each country, thereby creating new potential sources of gains from trade.

First, trade costs may affect firms' entry decisions. If a change in trade costs raises the number of entrants in country i, $\widehat{M}_i > 0$, the total number of varieties in country j will increase and its CES price index will decrease by $\lambda_{ij} \widehat{M}_i / (\sigma - 1) > 0$. Second, trade costs may affect firms' selection into exports. If a small change in trade costs lowers the cut-off productivity level, $\widehat{z_{ij}^*} < 0$, the total number of varieties in country j will also increase. But, unlike changes in M_i, changes in z_{ij}^* will affect the composition of varieties from country i in country j, as new exporters are less productive than existing ones. This argument explains why the CES price index will decrease by $-\lambda_{ij} \gamma_{ij} \widehat{z_{ij}^*} / (\sigma - 1) > 0$ with the coefficient γ_{ij} adjusting for changes in the number and composition of varieties available in country j.

Again, the question that we want to ask is: Does the introduction of new sources of gains from trade lead to larger gains from trade? Using Lemma 2, we will demonstrate that in the case of a standard trade model with monopolistic competition, the answer is still no.

Lemma 2 *In any standard trade model with monopolistic competition, percentage changes in the ideal price index satisfy*

$$\widehat{P}_j = -\widehat{\lambda}_{jj} \Big/ \varepsilon. \tag{22}$$

Equation (22) shows that conditional on $\hat{\lambda}_{jj}$ and ε, the welfare impact of changes in trade costs is the same as under perfect competition. The proof of Lemma 2 can be sketched as follows. Using Equation (17) and the fact that $\widehat{Y}_j = \widehat{w}_j = 0$ by our choice of numeraire, we can express changes in the productivity cut-off, $\widehat{z^*_{ij}}$, as

$$\widehat{z^*_{ij}} = \widehat{w}_i + \widehat{\tau}_{ij} + \left(\frac{1}{\sigma - 1}\right)\widehat{w}_i - \widehat{P}_j. \tag{23}$$

Combining Equations (20) and (23), we then obtain

$$\widehat{P}_j = \sum_{i=1}^n \lambda_{ij}\left[\frac{\left(\sigma - 1 + \gamma_{ij}\right)\left(\widehat{w}_i + \widehat{\tau}_{ij}\right)}{\sigma - 1 + \gamma_j} - \frac{\widehat{M}_i}{\sigma - 1 + \gamma_j} - \frac{\gamma_{ij}\widehat{w}_i}{\left(\sigma - 1 + \gamma_j\right)(1 - \sigma)}\right] \tag{24}$$

where $\gamma_j \equiv \sum_{i=1}^n \lambda_{ij}\gamma_{ij}$. Equation (24) illustrates two potential differences between perfect and monopolistic competition. The first one, which we have already mentioned, is that the set of varieties available in country j is not fixed. As a result, changes in terms of trade, $\widehat{w}_i + \widehat{\tau}_{ij}$, may not be sufficient to compute welfare changes; in principle, one may also need to keep track of changes in the number and composition of varieties, as captured by $-\widehat{M}_i\big/\left(\sigma - 1 + \gamma_j\right) + \gamma_{ij}\widehat{w}_i\big/\left(\sigma - 1 + \gamma_j\right)(1 - \sigma)$. The second difference, which is more subtle, is related to the impact of terms of trade changes, $\widehat{w}_i + \widehat{\tau}_{ij}$. Even in the absence of changes in the set of available varieties, Equation (24) shows that changes at the extensive margin, i.e. changes in Ω_{ij}, may directly affect the mapping between $\widehat{P}_j$ and $\widehat{w}_i + \widehat{\tau}_{ij}$. Because changes in terms of trade may lead to the creation and destruction of varieties with different prices in different countries, their impact may vary across countries, hence the correction term $\left(\sigma - 1 + \gamma_{ij}\right)\big/\left(\sigma - 1 + \gamma_j\right)$. Under perfect competition, new varieties never have a different impact since the price of a good no longer produced by one country is equal to its price in the new producing country.

The rest of our proof relies on the properties of a CES import demand system. We first show that under a CES import demand system, the second of these two differences necessarily is absent. Symmetry across countries implies $\gamma_{ij} = \gamma_j = 1 - \sigma - \varepsilon$ for all i and j, which means that the impact of changes in terms of trade must be the same for all exporters in any importing country. As a result, we can use the same strategy as under perfect competition to infer changes in terms of trade from changes in relative imports. After simple rearrangements, Equation (24) can be expressed as

$$\widehat{P}_j = -\frac{\widehat{\lambda}_{jj} - \widehat{M}_j}{\varepsilon}. \tag{25}$$

According to Equation (25), welfare changes in country j only depends on changes in two domestic variables: the share of expenditure on domestic goods, $\widehat{\lambda}_{jj}$; and the number of entrants, $\widehat{M}_j$.[14] To conclude the proof of Lemma 2, we show that, although the number of varieties consumed in country j may vary, we necessarily have $\widehat{M}_j = 0$. The formal argument uses the fact that under a CES import demand system, aggregate revenues are proportional to aggregate profits. As a result, the free entry condition completely pins down the number of entrants, independently of the value of variable trade costs.

Lemma 2 can again be used to analyze the impact of a change in variable trade costs. The exact same logic as in Section 3 leads to the two following propositions.

Proposition 3 *In any standard trade model with monopolistic competition, W_j can be consistently estimated by $1 - \left(\lambda_{jj} / \lambda'_{jj}\right)^{1/\overline{\varepsilon}}$, where λ'_{jj} and λ_{jj} are evaluated at the new and initial trade equilibrium, respectively.*

Proposition 4 *In any standard trade model with monopolistic competition, $\overline{W}_j$ can be consistently estimated by $(\lambda_{jj})^{1/\overline{\varepsilon}} - 1$, where λ_{jj} is evaluated at the initial equilibrium.*

A direct implication of Propositions 2 and 4 is that conditional on two sufficient trade statistics, λ_{jj} and $\overline{\varepsilon}$, the gains from trade predicted by standard trade models with perfect and monopolistic competition must be the same. Within the class of standard trade models, as we switch from perfect to monopolistic competition, the composition of the gains from trade changes, but their total size does not.

Notwithstanding the importance of standard trade models with monopolistic competition in the existing literature, it is obvious that the strong equivalence between these models and standard trade models with perfect competition heavily relies on the fact that $\widehat{M}_j = 0$. With this in mind, we focus in the next subsection on the equivalence between two standard trade models with monopolistic competition, Krugman (1980) and Melitz (2003). As we will see in Section 5, the equivalence between these two models can also be generalized to situations in which $\widehat{M}_j \neq 0$.

4.3 Krugman (1980) vs. Melitz (2003)

In line with our analysis under perfect competition, we conclude this section by comparing two well-known standard trade models with monopolistic competition. The first one corre-

[14] In the absence of a CES import demand system, one can show that changes in the price index still take a very simple form, namely $\widehat{P}_j = -\left(\hat{\lambda}_{jj} - \hat{M}_j\right) \Big/ \left(1 - \sigma - \gamma_{jj}\right)$. In general, however, there is no simple mapping between trade elastisticities, ε_j, and the relevant elasticity for welfare computations, $1 - \sigma - \gamma_{jj}$.

sponds to the case in which g_i is a degenerate density function with all the mass at some single productivity level z_i that may differ across countries, and $f_{ij} = 0$ for all i, j. Under these assumptions, there is no firm heterogeneity and the model described in this section reduces to Krugman (1980). The second model corresponds to the case in which g_i is the density function associated with a Pareto distribution, $g_i(z) \equiv \theta b^{\theta} z^{-\theta-1}$ for $z \geq b$, as in most extensions and variations of Melitz (2003).[15] For expositional purposes, we will simply refer to this model as Melitz (2003), though it should be clear that we implicitly mean "Melitz (2003) with Pareto" as other distributional assumptions may not satisfy the properties of standard trade models described in Section 2; see e.g. Helpman, Melitz, and Rubinstein (2008).

In Krugman (1980), the absence of fixed exporting costs entails $z^*_{ij} = 0$ and $\gamma_{ij} = 0$ for all i, j. Since entry also is invariant to trade barriers, $\widehat{M}_i = 0$, this model therefore only features consumption gains from trade, just like the Armington model presented in Section 3. By contrast, in Melitz (2003), although entry remains invariant to trade barriers, $\widehat{M}_i = 0$, changes in trade costs affect the productivity cutoffs z^*_{ij}. These changes at the extensive margin may lead to changes in the number and composition of consumed varieties as well as changes in aggregate productivity.

Since trade leads to the exit of the least efficient firms in this richer model, it may be tempting to conclude that the gains from trade are larger. Our theoretical analysis contradicts this intuition. In both Krugman (1980) and Melitz (2003), it is easy to check that the import demand system satisfies

$$\frac{X_{ij}}{X_{jj}} = \left(\frac{M_i}{M_j}\right) \cdot \left(\frac{w_i f_{ij}}{w_j f_{jj}}\right)^{1+\frac{\varepsilon}{\sigma-1}} \cdot \left(\frac{T_i w_i \tau_{ij}}{T_j w_j}\right)^{\varepsilon}. \tag{26}$$

where T_i and T_j are country-specific technology parameters; $\varepsilon \equiv 1 - \sigma$ in Krugman (1980) and $\varepsilon \equiv -\theta$ in Melitz (2003). Like in Section 3, the introduction of a new margin of adjustment changes the structural interpretation of the trade elasticity, from a preference to a technological parameter. Yet, because Equation (26) implies that the import demand system is CES and satisfies gravity, the two models are both standard trade models with monopolistic competition. We can therefore invoke Proposition 4 to conclude that conditional on $\overline{\varepsilon}$ and λ_{jj}, the gains from trade are the same in the two models.[16]

[15] See e.g. Antras and Helpman (2004), Helpman, Melitz, and Yeaple (2004), Ghironi and Melitz (2005), Bernard, Redding, and Schott (2007), Chaney (2008), Arkolakis (2008), and Eaton, Kortum, and Kramarz (2008).

[16] As a careful reader may have already noticed, our assumption that there exists a common orthogonality

5 Extensions

The objective of this section is twofold. First, we wish to establish the robustness of our simple welfare formula by offering additional examples of standard trade models, not considered in Sections 3 and 4, in which the gains from trade can be consistently estimated by $\lambda^{1/\bar{\varepsilon}} - 1$. Our choice of examples is motivated by recent developments in the literature on firm heterogeneity and trade. In line with this literature, we focus on variations and extensions of Melitz (2003) including: (*i*) endogenous marketing costs; and (*ii*) multi-product firms.[17] Although such extensions are crucial to explain micro-level facts, we show that they leave our simple formula unchanged. In a standard trade model with endogenous marketing costs or multi-product firms, the share of domestic expenditure and the trade elasticity remain sufficient statistics for welfare analysis.

Second, we wish to illustrate how our simple welfare formula may generalize to non-standard trade models. Motivated again by the existing trade literature, we consider generalizations of standard trade models featuring: (*i*) multiple sectors and factors; and (*ii*) tradable intermediate goods. While our simple welfare formula no longer holds in these richer environments, we demonstrate that generalized versions can easily be derived using the same logic as in Sections 3 and 4. For all extensions, formal proofs can be found in the Appendix.

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condition such that ε can be estimated using a gravity equation is somewhat stronger under monopolistic than perfect competition. In the case of Anderson (1979) and Eaton and Kortum (2002), we had $\nu_{ij} \equiv 0$ so that, for example, $E(\nu_{ij} \ln \tau_{i'j'}) = 0$ was trivially satisfied in both models. While the same is true in Krugman (1980), this is not the case in Melitz (2003) where $\nu_{ij} \equiv (f_{ij})^{1+\frac{\varepsilon}{\sigma-1}}$. For a difference-in-difference estimator to be a consistent estimator, we would therefore need additional assumptions about the joint distribution of fixed and variable trade costs. In our view, this issue is similar to the problem one would face under perfect competition if part of the variable trade costs were not observable. Again one would need observable and unobservable component of trade costs to be uncorrelated, or an instrumental variable, in order to avoid omitted variable bias. The fact that this unobserved component is fixed rather than variable does not change what we view primarily as an econometric issue, which we have little to contribute to. As we pointed out in footnote 10, this issue is different from the economic issue raised by Helpman, Melitz, and Rubinstein (2008): according to their model, ν_{ij} is a function of τ_{ij}.

[17] Another type of standard trade model entails heterogeneous quality, as in Baldwin and Harrigan (2007) and Johnson (2009). While the introduction of quality considerations are crucial to explain the variation in the distribution of prices across firms and countries, it is isomorphic to a change in the units of account, which must again leave our welfare predictions unchanged. Quality considerations may, of course, have important distributional consequences in environments with multiple factors of production; see e.g. Verhoogen (2008) and Kugler and Verhoogen (2008). They may also matter in the presence of minimum quality requirements, which we are also abstracting from; see e.g. Hallak and Sivadasan (2009).

5.1 Other standard trade models

Throughout this subsection, we assume that all assumptions introduced in Section 2 hold and that we have monopolistic competition and CES preferences as in Section 4. Compared to Section 4, however, we relax some of our supply-side assumptions to allow for endogenous marketing costs and multi-product firms.

Endogenous marketing costs. We start by considering the case in which marketing costs are endogenous, as in Arkolakis (2008) and Eaton, Kortum, and Kramarz (2008). In order to reach consumers with probability x in country j, a firm from country i must now pay a fixed cost equal to

$$f_{ij}(x) = f_{ij} \times \left[\frac{1 - (1-x)^{1-\mu}}{1-\mu} \right].$$

The model considered in Section 4 corresponds to the particular case in which $\mu = 0$. In this situation, the marginal cost of reaching an additional consumer is constant and firms find it optimal to reach every potential consumer or none at all.

While the introduction of endogenous marketing costs is important to explain variations in the distribution of firm size, it is easy to show that it has no effect on our welfare formula. The introduction of a new margin, the share x of consumer that a firm wants to reach, again affects the structural interpretation of the trade elasticity, but nothing else. The share of domestic expenditure and the trade elasticity remain sufficient statistics for the computation of the gains form trade.

Multi-product firms. In Section 4, all firms can only produce one good. In the spirit of Bernard, Redding, and Schott (2007) and Arkolakis and Muendler (2007) we assume here that each firm can produce up to N goods, which we will refer to as products. Since the is a continuum of firms and a discrete number of products per firm, there are no cannibalization effects: sales of one product do not affect the sales of other products sold by the same firm. We allow productivity levels to be correlated across different products within the same firm, and assume that firms incur in the same marketing costs for each product.

The introduction of multi-product firms has the same type of implications as the introduction of endogenous marketing costs. It matters crucially for micro-level phenomena, such as the impact of trade liberalization on firm-level productivity, but it has no effect on the magnitude of the gains from trade. As far as our welfare formula is concerned, the only thing that multi-product firms change is the structural interpretation of the trade elasticity, which now includes adjustments in the number of products within each firm.[18]

5.2 Non-standard trade models

This final subsection relaxes some of the assumptions of standard trade models introduced in Section 2. We start by introducing multiple sectors and factors and conclude with tradable intermediate goods.

Multiple sectors. Suppose that goods $\omega \in \Omega$ are separated into $s = 1, ..., S$ sectors and that consumers in country j spend a constant share η_j^s of their income on goods from sector s. The key difference between Section 2 and the present section is that CES and gravity now refer to properties of the import demand system at the *sector level*. Formally, CES now implies that bilateral imports from country i to country j in sector s, X_{ij}^s, satisfy

$$\partial \ln \left(X_{ij}^s / X_{jj}^s \right) / \partial \ln \tau_{i'j}^{s'} = \varepsilon^s \text{ if } s' = s \text{ and } i' = i;$$
$$\partial \ln \left(X_{ij}^s / X_{jj}^s \right) / \partial \ln \tau_{i'j}^{s'} = 0 \text{ otherwise.}$$

Similarly, gravity now implies that bilateral imports can be decomposed into

$$\ln X_{ij}^s (\boldsymbol{\tau}, \mathbf{E}) = A_i^s (\boldsymbol{\tau}, \mathbf{E}) + B_j^s (\boldsymbol{\tau}, \mathbf{E}) + \varepsilon^s \ln \tau_{ij}^s + \nu_{ij}^s,$$

with τ_{ij}^s the iceberg trade cost between i and j in sector s, as in Costinot and Komunjer (2007) and Chor (2009). All other properties of standard trade models are unchanged.

Our results in the multi-sector case can be summarized as follows. Under perfect competition, our welfare formula generalizes to $\prod_{s=1}^{S} \left(\lambda_{jj}^s \right)^{\eta_j^s / \overline{\varepsilon}^s} - 1$, where λ_{jj}^s represents the share of expenditure in sector s that goes to domestic goods in the initial equilibrium. This formula is similar to the one derived by Donaldson (2008) using a multi-sector extension of Eaton and Kortum (2002). For $S = 1$, this formula reduces to the one derived in Section 3. For $S > 1$, however, we see that more aggregate statistics are necessary to estimate the gains from trade: elasticities and shares of expenditure at the sector level, but also data on the share of expenditure across sectors. This should not be too surprising: the less symmetry we assume across goods, the more information we need to estimate the gains from trade.

[18] This basic point would remain true if we did not have a CES import demand system. In that case, we would need to estimate more elasticities, i.e., the entire matrix $\boldsymbol{\varepsilon}_j$. But conditional on *upper-level* elasticities, predictions about the gains from trade would have to be the same with or without multi-product firms.

By contrast, our estimator of the gains from trade under monopolistic competition becomes $\prod_{s=1}^{S}\left(\lambda_{jj}^{s}\eta_{j}^{s}/\delta_{j}^{s}\right)^{\eta_{j}^{s}/\overline{\varepsilon}^{s}}-1$, where δ_{j}^{s} is the share of employment in sector s in the initial equilibrium. Compared to the one-sector case, we see that the mapping between data and welfare is no longer the same under perfect and monopolistic competition. The reason is simple. The equivalence between these two market structures in Sectors 3 and 4 relied on the fact that there was no change in entry. In the multi-sector case, however, changes in employment across sectors lead to changes in entry, which must be reflected in the computation of the gains from trade. This explains the correction term, $\left(\eta_{j}^{s}/\delta_{j}^{s}\right)^{\left(\eta_{j}^{s}/\overline{\varepsilon}^{s}\right)}$, in our new formula.

In a related paper, Balistreri, Hillberry, and Rutherford (2009) have developed variations of the Armington and Melitz (2003) models with a non-tradeable sector to illustrate the same idea: if changes in trade costs lead to changes in entry, then models with perfect and monopolistic competition no longer have the same welfare implications. This being said, while the equivalence between standard trade models is admittedly weaker in the multi-sector case, it is worth emphasizing that the core insights of Propositions 2 and 4 still hold. In line with Section 3's results, multi-sector extensions of Eaton and Kortum (2002) and Anderson (1979) must therefore have the same welfare implications. The same is true about multi-sector extensions of Krugman (1980) and Melitz (2003), in line with Section 4's results. Conditional on a given market structure, either perfect or monopolistic competition, there still exists aggregate sufficient statistics for welfare analysis.

Multiple factors. Although the standard trade models presented in Section 2 only feature one factor of production, labor, it is trivial to extend our results to situations in which there are $f=1,...,F$ factors, but all goods $\omega\in\Omega$ use these factors in the same proportions. In this situation, all our results go through with a "composite input" playing the same role as labor in Sections 3 and 4. The situation in which goods may vary in factor intensity is, of course, more complex.

One way to introduce differences in factor intensity is to assume that: (i) there are multiple sectors, as in the previous extension; (ii) all goods from the same sector have the same factor intensity; but (iii) factor intensity differs across sectors. Under these assumptions, it is easy to check that changes in relative factor prices now need to be included in our welfare analysis.[19] This is an important observation, which illustrates that the one factor

[19]Under perfect competition, for example, W_j is equal to $\prod_{s=1}^{S}\left(\lambda_{jj}^{s}/\lambda_{jj}^{s'}\right)^{\eta_{j}^{s}/\varepsilon^{s}}\left(c_{j}^{s}/c_{j}^{s'}\right)^{\eta_{j}^{s}}-1$, where c_{j}^{s} and $c_{j}^{s'}$ are average unit costs of production in sector s in the initial and the new trade equilibrium,

assumption imposed in Section 2 is a non-trivial aspect of our sufficient statistics approach. Having one factor of production allows us to create a one-to-one mapping between changes in terms of trade and changes in the share of expenditure on domestic goods, which would not exist if all costs of production in country j were not proportional to w_j (or the price of a composite good).

Tradable intermediate goods. In Section 2, all goods were final goods. We now investigate how our welfare formula would generalize to environments in which goods $\omega \in \Omega$ are intermediate goods which can either be used to produce a unique non-tradeable final good or other intermediate goods, as in Eaton and Kortum (2002), Alvarez and Lucas (2007), Atkeson and Burstein (2009), and Di Giovanni and Levchenko (2009). Formally, we assume that after fixed costs have been paid (if any), the unit cost of production of good ω in country i, $c_i(\omega)$, can be written as

$$c_i(\omega) = \frac{w_i^{\beta_i} P_i^{1-\beta_i}}{z(\omega)}, \tag{27}$$

where $1-\beta_i$ represents the share of other intermediate goods in the production of good ω. Similarly, we assume that fixed costs under monopolistic competition are such that firms from country i must incur: (i) a fixed entry cost equal to $w_i^{\alpha_i} P_i^{1-\alpha_i} f_e$ in order to produce in country i, where $1-\alpha_i$ represents the share of intermediate goods in entry costs; and (ii) a fixed marketing cost equal to $w_i^{\beta_i} P_i^{1-\beta_i} f_{ij}$, in order to sell their varieties to country j. The models considered in Sections 3 and 4 correspond to the special case with $\alpha_i = \beta_i = 1$.

Under perfect competition, the introduction of intermediate goods amplifies the gains from trade as follows. Conditional on the observed values of the share of expenditure and the trade elasticity, the estimator of the gains from trade becomes $(\lambda_{jj})^{1/(\beta_j \bar{\varepsilon})} - 1$. This expression is similar to the one derived in Eaton and Kortum (2002) and Alvarez and Lucas (2007). Jones (2009) convincingly argues that β_j is on average equal to 1/2, hence a country like Belgium with $\lambda_{BEL} = 0.73$ experiences gains from trade (using $\bar{\varepsilon} = -5$) of 13% rather than 6%. Intuitively, a given decrease in λ_{jj} is now associated with bigger welfare gains in country j since it also captures the lower costs of intermediate goods. The larger the share β_j of intermediate goods in the production of other intermediate goods, the larger the amplification effect caused by this input-output loop.

Under monopolistic competition, we can use the same logic to show that the estimator of the gains from trade is $(\lambda_{jj})^{\frac{1}{\beta_j \bar{\varepsilon} + 1 - \alpha_j}} - 1$. For $\alpha_j = 1$, our welfare formula is therefore

respectively. By our choice of numeraire, we must have $c_j^s / c_j^{s'} = 1$ if sectors are of the same factor intensity or if there is only one sector, but this may not be true otherwise.

the same under both monopolistic and perfect competition. By contrast, for $\alpha_j \neq 1$, we see that conditional on trade data, λ_{jj} and $\bar{\varepsilon}$, the gains from trade predicted by models with monopolistic competition are larger, reflecting the increase in the number of entrants associated with the decrease in country j's ideal price index. If we assume that intermediate goods are just as important in entry costs as in marketing and production costs (i.e., $\alpha_j = \beta_j$), then we can use our modified formula to compute Belgium's gains from trade. Using again $\bar{\varepsilon} = -5$ and $\beta_j = 1/2$, these gains would now be 17% rather than 13%. Of course, if $\alpha_j > \beta_j$ then trade leads to a lower expansion of entry and lower gains from trade (relative to the case with $\alpha_j = \beta_j$).

The broad implications of this last extension are very similar to those we reached in our multi-sector extension: unless the introduction of intermediate goods does not lead to changes in the number of entrants, which is the case for $\alpha_j = 1$, the welfare implications of models with perfect and monopolistic competition are no longer the same. Nevertheless, within both classes of model, there still exist aggregate sufficient statistics for welfare analysis.

Although this section was not meant as an exhaustive analysis of all possible variations and generalizations of standard trade models, we wish to conclude by pointing out one class of extensions which we view as particularly important. Throughout this section, we have relaxed various supply-side assumptions, but we have maintained the assumption of CES preferences under monopolistic competition. Allowing for quasi-linear or translog preferences as in Melitz and Ottaviano (2008) and Feenstra and Weinstein (2009) would introduce variations in mark-ups, which may further enrich our welfare formula. We leave a formal analysis of these extensions for future work.

6 Concluding Remarks

The theory of international trade is rich in reasons why countries may gain from trade. The first message of our paper is a cautionary one. Although it may be tempting to conclude that richer trade models necessarily entail larger gains from trade, our analysis demonstrates that this is not the case. Within the class of trade models considered in this paper, the number of sources of gains from trade varies, but the total size of the gains from trade does not. Put simply, since questions related to the magnitude of the gains from trade are by essence quantitative, they cannot be properly addressed by qualitative considerations.

The second message of our paper is a positive one. The flip side of our strong equivalence results is that within a particular but important class of trade models, there exist two sufficient statistics for welfare analysis: (*i*) the share of expenditure on domestic goods; and (*ii*) a gravity-based estimate of the trade elasticity. Hence only a very limited amount of macro data may be necessary to make robust welfare predictions, whatever the micro level details of a particular trade model may be.

A Proofs (I): Perfect Competition

Proof of Lemma 1. In the main text, we have already established that

$$\widehat{P}_j = \sum_{i=1}^{n} \lambda_{ij} \left(\widehat{w}_i + \widehat{\tau}_{ij}\right). \tag{28}$$

Under perfect competition we know that bilateral imports, X_{ij}, only depend on prices; and by Equation (4), we know that prices only depend on wages and variable trade costs through their product, $w_i \tau_{ij}$. Using these two observations and the definition of $\varepsilon^{j}_{ii'}$, we can express the percentage changes in relative imports as

$$\widehat{X}_{ij} - \widehat{X}_{jj} = \sum_{i' \neq j} \varepsilon_j^{ii'} \left(\widehat{w}_{i'} + \widehat{\tau}_{i'j}\right). \tag{29}$$

In the case of a CES import demand system, Equation (29) simplifies into

$$\widehat{X}_{ij} - \widehat{X}_{jj} = \varepsilon_j \left(\widehat{w}_{i'} + \widehat{\tau}_{i'j}\right). \tag{30}$$

Combining Equations (28) and (30), and noting that $\widehat{\lambda}_{ij} - \widehat{\lambda}_{jj} = \widehat{X}_{ij} - \widehat{X}_{jj}$, we obtain

$$\widehat{P}_j = \sum_{i=1}^{n} \lambda_{ij} \left(\frac{\widehat{\lambda}_{ij} - \widehat{\lambda}_{jj}}{\varepsilon_j}\right). \tag{31}$$

To conclude the proof of Lemma 1, we note that $\sum_{i=1}^{n} \lambda_{ij} = 1$ implies $\sum_{i=1}^{n} \lambda_{ij} \widehat{\lambda}_{ij} = 0$. Combining this observation with Equation (31), we get Equation (9). **QED.** ■

Proof of Proposition 1. By Lemma 1, we know that

$$d \ln P_j = -\frac{d \ln \lambda_{jj}}{\varepsilon}. \tag{32}$$

Let λ_{jj} and λ'_{jj} denotes the share of expenditure on domestic goods in the trade equilibria associated with $\boldsymbol{\tau}$ and $\boldsymbol{\tau}'$, respectively. Similarly, let P_j and P'_j denote the ideal price in country j in the two equilibria. Since ε is constant across all trade equilibria, we can integrate Equation (32) between $\boldsymbol{\tau}$ and $\boldsymbol{\tau}'$ to get

$$\frac{P'_j}{P_j} = \left(\frac{\lambda_{jj}}{\lambda'_{jj}}\right)^{1/\varepsilon} \tag{33}$$

By definition, we know that $W_j \equiv 1 - \left(P'_j / P_j \right)$. Thus, Equation (33) implies

$$W_j = 1 - \left(\frac{\lambda_{jj}}{\lambda'_{jj}} \right)^{1/\varepsilon} \tag{34}$$

Since $\overline{\varepsilon}$ is a consistent estimator of ε, by assumption, and W_j is a continuous function of ε, by Equation (34), we can invoke the continuous mapping theorem to conclude that $1 - \left(\frac{\lambda_{jj}}{\lambda'_{jj}} \right)^{1/\overline{\varepsilon}}$ is a consistent estimator of W_j. **QED**. ■

Proof of Proposition 2. By assumption, we know that for any $i \neq j$, $\lim_{\tau_{ij} \to +\infty} (w_i \tau_{ij}) = +\infty$. Thus, we must have $\lambda'_{jj} = 1$ at $\boldsymbol{\tau} = +\infty$. Proposition 2 directly follows from this observation, Proposition 1, and the definition of $\overline{W}_j \equiv - (W_j)_{\boldsymbol{\tau}=+\infty}$. **QED**. ■

B Proofs (II): Monopolistic Competition

Proof of Lemma 2. In order to establish Equation (22), we proceed in 6 steps. For expositional purposes, we again suppress the arguments $(\boldsymbol{\tau}, \mathbf{E}) \equiv (\boldsymbol{\tau}, w_1, ..., w_n, M_1, ..., M_n)$, but it should be clear that, like in the main text, all endogenous variables, P_j, z^*_{ij}, and X_{ij} are functions of $(\boldsymbol{\tau}, w_1, ..., w_n, M_1, ..., M_n)$.

Step 1: *Percentage changes in the CES price index are given by*

$$\widehat{P}_j = \sum_{i=1}^n \lambda_{ij} \left[\frac{\left(\sigma - 1 + \gamma_{ij}\right)\left(\widehat{w}_i + \widehat{\tau}_{ij}\right)}{\sigma - 1 + \gamma_j} - \frac{\widehat{M}_i}{\sigma - 1 + \gamma_j} - \frac{\gamma_{ij} \widehat{w}_i}{\left(\sigma - 1 + \gamma_j\right)(1 - \sigma)} \right], \tag{35}$$

where $\gamma_j \equiv \sum_{i'=1}^n \lambda_{i'j} \gamma_{i'j}$.

In the main text, we have already established that

$$\widehat{P}_j = \sum_{i=1}^n \lambda_{ij} \left[\widehat{w}_i + \widehat{\tau}_{ij} - \frac{\widehat{M}_i}{\sigma - 1} + \frac{\gamma_{ij} \widehat{z^*_{ij}}}{\sigma - 1} \right], \tag{36}$$

By differentiating Equation (17) and using the fact that $w_j = 1$, we know that

$$\widehat{z^*_{ij}} = \widehat{w}_i + \widehat{\tau}_{ij} + \left(\frac{1}{\sigma - 1} \right) \widehat{w}_i - \widehat{P}_j. \tag{37}$$

Combining Equations (36) and (37), we obtain Equation (35).

Step 2: *Percentage changes in the cut-off productivity levels are given by*

$$\begin{aligned}\widehat{z_{ij}^{*}} &= \widehat{w}_i + \widehat{\tau}_{ij} + \left(\frac{1}{\sigma-1}\right)\widehat{w}_i \\ &\quad - \textstyle\sum_{i'=1}^{n} \lambda_{i'j}\left[\left(\widehat{w}_{i'} + \widehat{\tau}_{i'j}\right)\left(\frac{1-\sigma-\gamma_{i'j}}{1-\sigma-\gamma_j}\right) + \frac{\gamma_{i'j}\widehat{w}_{i'}}{\left(1-\sigma-\gamma_j\right)\left(1-\sigma\right)} + \frac{\widehat{M}_{i'}}{1-\sigma-\gamma_j}\right].\end{aligned} \tag{38}$$

Equation (38) derives from Equations (35) and (37).

Step 3: *For any $i = 1, ..., n$, $j = 1, ..., n$, we must have $\gamma_{ij} = 1 - \sigma - \varepsilon_j$.*

Using Equations (1), (13), (16), (17), and the fact that $w_j = 1$, we can express bilateral imports by country j from country i as

$$X_{ij} = \left(\frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_i}{P_j}\right)^{1-\sigma} M_i \left[\int_{z_{ij}^{*}}^{+\infty} z^{\sigma-1} g_i\left(z\right) dz\right].$$

This implies

$$\widehat{X}_{ij} = \left(1-\sigma\right)\left(\widehat{w}_i + \widehat{\tau}_{ij}\right) - \left(1-\sigma\right)\widehat{P}_j + \widehat{M}_i - \gamma_{ij}\widehat{z_{ij}^{*}}. \tag{39}$$

Similarly, we have

$$\widehat{X}_{jj} = -\left(1-\sigma\right)\widehat{P}_j + \widehat{M}_j - \gamma_{jj}\widehat{z_{jj}^{*}}. \tag{40}$$

Combining Equations (39) and (40), we obtain

$$\widehat{X}_{ij} - \widehat{X}_{jj} = \left(1-\sigma\right)\left(\widehat{w}_i + \widehat{\tau}_{ij}\right) + \widehat{M}_i - \widehat{M}_j - \gamma_{ij}\widehat{z_{ij}^{*}} + \gamma_{jj}\widehat{z_{jj}^{*}},$$

which can be rearranged as

$$\begin{aligned}\widehat{X}_{ij} - \widehat{X}_{jj} &= \left(1-\sigma\right)\left(\widehat{w}_i + \widehat{\tau}_{ij}\right) + \widehat{M}_i - \widehat{M}_j \\ &\quad -\gamma_{ij}\textstyle\sum_{i'=1}^{n}\left[\left(\frac{\partial \ln z_{ij}^{*}}{\partial \ln w_{i'}}\right)\widehat{w}_{i'} + \left(\frac{\partial \ln z_{ij}^{*}}{\partial \ln \tau_{i'j}}\right)\widehat{\tau}_{i'j} + \left(\frac{\partial \ln z_{ij}^{*}}{\partial \ln M_{i'}}\right)\widehat{M}_{i'}\right] \\ &\quad +\gamma_{jj}\textstyle\sum_{i'=1}^{n}\left[\left(\frac{\partial \ln z_{jj}^{*}}{\partial \ln w_{i'}}\right)\widehat{w}_{i'} + \left(\frac{\partial \ln z_{jj}^{*}}{\partial \ln \tau_{i'j}}\right)\widehat{\tau}_{i'j} + \left(\frac{\partial \ln z_{jj}^{*}}{\partial \ln M_{i'}}\right)\widehat{M}_{i'}\right].\end{aligned} \tag{41}$$

By definition, we know that $\varepsilon_j^{ii'} = \frac{\partial \ln(X_{ij}/X_{jj})}{\partial \ln \tau_{i'j}}$. Thus Equation (41) implies

$$\varepsilon_j^{ii'} = \begin{cases} 1-\sigma-\gamma_{ij}\left(\frac{\partial \ln z_{ij}^{*}}{\partial \ln \tau_{ij}}\right) + \gamma_{jj}\left(\frac{\partial \ln z_{jj}^{*}}{\partial \ln \tau_{ij}}\right), & \text{if } i' = i; \\ -\gamma_{ij}\left(\frac{\partial \ln z_{ij}^{*}}{\partial \ln \tau_{i'j}}\right) + \gamma_{jj}\left(\frac{\partial \ln z_{jj}^{*}}{\partial \ln \tau_{i'j}}\right), & \text{otherwise.}\end{cases}$$

Using Equation (38), we can simplify the previous expression to

$$\varepsilon_j^{ii'} = \begin{cases} 1-\sigma-\gamma_{ij}+\lambda_{ij}\left(\frac{1-\sigma-\gamma_{ij}}{1-\sigma-\gamma_j}\right)\left(\gamma_{ij}-\gamma_{jj}\right), & \text{if } i'=i; \\ \lambda_{i'j}\left(\frac{1-\sigma-\gamma_{i'j}}{1-\sigma-\gamma_j}\right)\left(\gamma_{ij}-\gamma_{jj}\right), & \text{otherwise.} \end{cases} \tag{42}$$

In a CES import demand system, we know that $\varepsilon_j^{ii'} = \varepsilon_j$ if $i'=i$ and $\varepsilon_j^{ii'}=0$ otherwise. Combining this observation with Equation (42), we get $\gamma_{ij}=1-\sigma-\varepsilon_j$ for all i,j.

Step 4: *Percentage changes in relative imports are given by*

$$\widehat{X}_{ij}-\widehat{X}_{jj}=\varepsilon_j\left[\widehat{\tau}_{ij}+\widehat{w}_i+\left(\frac{1-\sigma-\varepsilon_j}{\varepsilon_j\left(1-\sigma\right)}\right)\widehat{w}_i\right]+\widehat{M}_i-\widehat{M}_j. \tag{43}$$

Equation (43) derives from Equations (38) and (41) and the fact that $\gamma_{ij}=1-\sigma-\varepsilon_j$ for all i,j.

Step 5: *Percentage changes in the CES price index satisfy*

$$\widehat{P}_j=-\frac{\widehat{\lambda}_{jj}-\widehat{M}_j}{\varepsilon_j}. \tag{44}$$

Since $\gamma_{ij}=1-\sigma-\varepsilon_j$ for all i,j, we can rearrange Equation (35) as

$$\widehat{P}_j=\sum_{i=1}^n\lambda_{ij}\left[\left(\widehat{w}_i+\widehat{\tau}_{ij}\right)+\left(\frac{1-\sigma-\varepsilon_j}{\varepsilon_j\left(1-\sigma\right)}\right)\widehat{w}_i+\frac{\widehat{M}_i}{\varepsilon_j}\right],$$

Combining the previous expression with Equation (43), we get

$$\widehat{P}_j=\sum_{i=1}^n\lambda_{ij}\left[\frac{\widehat{X}_{ij}-\widehat{X}_{jj}}{\varepsilon_j}\right]+\frac{\widehat{M}_j}{\varepsilon_j}.$$

Using the same logic as in Lemma 1, we then obtain Equation (44).

Step 6: *There are no changes in the measure of entrants,* $\widehat{M}_j=0$.

Equations (14) and (18) imply

$$\sum_{i=1}^n\frac{f_{ji}}{\left(z_{ji}^*\right)^{\sigma-1}}\int_{z_{ji}^*}^{+\infty}z^{\sigma-1}g_j\left(z\right)dz-\sum_{i=1}^n f_{ji}\int_{z_{ji}^*}^{+\infty}g_j\left(z\right)dz=f_e$$

Differentiating the previous expression, we obtain

$$\sum_{i=1}^{n} \theta_{ji} \widehat{z}_{ji}^{*} = 0, \tag{45}$$

where θ_{ji} is the share of total revenues in country j associated with sales in country i,

$$\theta_{ji} = \frac{\frac{f_{ji}}{\left(z_{ji}^{*}\right)^{\sigma-1}} \int_{z_{ji}^{*}}^{+\infty} z^{\sigma-1} g_j(z)\, dz}{\sum_{i'=1}^{n} \frac{f_{ji'}}{\left(z_{ji'}^{*}\right)^{\sigma-1}} \int_{z_{ji'}^{*}}^{+\infty} z^{\sigma-1} g_j(z)\, dz}.$$

Equations (18) and (19) further imply that

$$M_j \cdot \sum_{i=1}^{n} \frac{\sigma f_{ji}}{\left(z_{ji}^{*}\right)^{\sigma-1}} \int_{z_{ji}^{*}}^{+\infty} z^{\sigma-1} g_j(z)\, dz = L_j.$$

Differentiating the previous expression, we obtain

$$\widehat{M}_j + \sum_{i=1}^{n} \theta_{ji} \left(1 - \sigma - \gamma_{ij}\right) \widehat{z}_{ji}^{*} = 0. \tag{46}$$

Using the fact that $\gamma_{ij} = 1 - \sigma - \varepsilon_j$, Equations (45) and (46) imply

$$\widehat{M}_j = 0.$$

Combining the previous expression with Equation (44), we get Equation (22). **QED**. ■

C Proofs (III): Other Standard Trade Models

Endogenous marketing costs. In the main text, we have argued that in a model with endogenous marketing costs, the gains from trade can still be consistently estimated by $(\lambda_{jj})^{1/\bar{\varepsilon}} - 1$. To see this, note that the profit-maximization program of a firm with productivity z is now given by

$$\pi_{ij}(z) = \max_{x} \left\{ x \left[\frac{\sigma \tau_{ij} w_i}{(\sigma - 1) z P_j} \right]^{1-\sigma} \frac{Y_j}{\sigma} - w_i f_{ij} \left[\frac{1 - (1 - x)^{1-\mu}}{1 - \mu} \right] \right\},$$

where the optimal pricing rule is as in (13). The first-order condition of that program associated with x implies

$$x_{ij}(z) = 1 - \left(\frac{z_{ij}^*}{z}\right)^{\frac{\sigma-1}{\mu}}, \text{ for all } z \geq z_{ij}^*,$$

where $x_{ij}(z)$ represents the fraction of consumers from country j reached by a firm from country i with productivity z; and the productivity cut-off z_{ij}^* is still given by Equation (23). Since the price of a given good is infinite for a consumer who is not reached by a firm, the price index of a representative consumer in country j is now equal to

$$P_j^{1-\sigma} = \sum_{i=1}^n M_i \int_{z_{ij}^*}^{+\infty} \left[1 - \left(z_{ij}^* / z\right)^{\frac{\sigma-1}{\mu}}\right] \cdot \left(z w_i \tau_{ij}\right)^{1-\sigma} \cdot g_i(z)\, dz \; .$$

Differentiating the previous expression, we obtain

$$\widehat{P}_j = \sum_{i=1}^n \lambda_{ij} \left[(\widehat{w}_i + \widehat{\tau}_{ij}) - \frac{\widehat{M}_i}{\sigma - 1} + \frac{\widetilde{\gamma}_{ij} \widehat{z_{ij}^*}}{\sigma - 1}\right], \tag{47}$$

where $\widetilde{\gamma}_{ij}$ is given by

$$\widetilde{\gamma}_{ij} \equiv \frac{(\sigma-1)}{\mu} \frac{\int_{z_{ij}^*}^{+\infty} z^{\sigma-1} \left(\frac{z_{ij}^*}{z}\right)^{\frac{\sigma-1}{\mu}} g(z)\, dz}{\int_{z_{ij}^*}^{+\infty} z^{\sigma-1} \left(1 - \left(\frac{z_{ij}^*}{z}\right)^{\frac{\sigma-1}{\mu}}\right) g(z)\, dz}.$$

Starting from Equation (47), we can then follow the exact same steps as in Lemma 2, Propositions 3 and 4. The only difference is that $\widetilde{\gamma}_{ij}$ now plays the role of γ_{ij}. **QED**.

Multi-product firms. In the main text, we have argued that in a model with multi-product firms, the gains from trade can still be consistently estimated by $(\lambda_{jj})^{1/\bar{\varepsilon}} - 1$. Before showing this formally, let us introduce the following notation. We denote by z_k the productivity of a firm in producing its k-th product for $k = 1, ..., K$. Without loss of generality, we order products for each firm such that $z_1 \geq ... \geq z_K$ and denote by $g_i(z_1, ..., z_K)$ the density function from which productivity levels are randomly drawn across firms. With a slight abuse of notation we denote by $g_i(z_1)$ the marginal density of the highest productivity level, and similarly, we denote by $g_i(z_k | z_{k-1}, ..., z_1)$ the associated conditional densities for $k = 2, ..., K$.

Using the above notation, we can now express the CES price index of a representative agent in country j as

$$P_j^{1-\sigma} = \sum_{i=1}^n M_i \int_{z_{ij}^*}^{+\infty} S\left(z_1, z_{ij}^*\right) \cdot \left(w_i \tau_{ij}\right)^{1-\sigma} \cdot g_i\left(z_1\right) dz_1,$$

where the productivity cut-off z_{ij}^* is still given by Equation (23) and $S\left(z_1, z_{ij}^*\right)$ is constructed recursively as follows. For $k = K$, we set

$$S^K\left(z_1, ..., z_{K-1}, z_{ij}^*\right) \equiv \int_{z_{ij}^*}^{z_{K-1}} z_K^{\sigma-1} \cdot g_i\left(z_K | z_{K-1}, ..., z_1\right) dz_K \ .$$

Then for any $K > k \geq 2$, we set

$$S^k\left(z_1, ..., z_{k-1}, z_{ij}^*\right) \equiv \int_{z_{ij}^*}^{z_{k-1}} \left[z_k^{\sigma-1} + S^{k+1}(z_k, ..., z_1)\right] \cdot g_i\left(z_k | z_{k-1}, ..., z_1\right) dz_k,$$

Finally, we set

$$S\left(z_1, z_{ij}^*\right) \equiv z_1^{1-\sigma} + S^2\left(z_1, z_{ij}^*\right).$$

Differentiating the ideal price index we obtain

$$\widehat{P}_j = \sum_{i=1}^n \lambda_{ij} \left[\left(\widehat{w}_i + \widehat{\tau}_{ij}\right) - \frac{\widehat{M}_i}{\sigma - 1} + \frac{\widetilde{\widetilde{\gamma}}_{ij} \widehat{z_{ij}^*}}{\sigma - 1}\right], \tag{48}$$

where $\widetilde{\widetilde{\gamma}}_{ij}$ is now given by

$$\widetilde{\widetilde{\gamma}}_{ij} \equiv \frac{\left(z_{ij}^*\right)^{\sigma-1} \frac{g_i\left(z_{ij}^*\right)}{z_{ij}^*} - \int_{z_{ij}^*}^{+\infty} \frac{S_2\left(z_1, z_{ij}^*\right)}{z_{ij}^*} g_i\left(z_1\right) dz_1}{\int_{z_{ij}^*}^{+\infty} S\left(z_1, z_{ij}^*\right) g_i\left(z_1\right) dz_1},$$

where S_2 refers to the derivative of S with respect to its second argument. Starting from Equation (47), we can then follow the exact same steps as in Lemma 2, Propositions 3 and 4. The only difference is that $\widetilde{\widetilde{\gamma}}_{ij}$ plays the role of γ_{ij}. **QED**.

D Proofs (IV): Non-Standard Trade Models

Multiple sectors. In the main text, we have argued that in the multi-sector case, the gains from trade can be consistently estimated by $\prod_{s=1}^{S}\left(\lambda_{jj}^{s}\right)^{\eta_j^s/\bar{\varepsilon}^s}-1$, under perfect competition, and $\prod_{s=1}^{S}\left(\lambda_{jj}^{s}\eta_j^s/\delta_j^s\right)^{\eta_j^s/\bar{\varepsilon}^s}-1$, under monopolistic competition. We now demonstrate these two results formally.

Consider first the case of perfect competition. The same arguments as in Lemma 1 directly imply that

$$\widehat{X}_{ij}^{s}-\widehat{X}_{jj}^{s}=\varepsilon^{s}\left(\widehat{\tau}_{ij}^{s}+\widehat{w}_{i}\right), \tag{49}$$

and that

$$\widehat{P}_j=\sum_{s=1}^{S}\eta_j^s\sum_{i=1}^{n}\lambda_{ij}^{s}\left(\widehat{\tau}_{ij}^{s}+\widehat{w}_{i}\right), \tag{50}$$

where λ_{ij}^{s} is the share of expenditure on goods from country i in country j and sector s. Combining Equations (50) and (49) we obtain after simplifications

$$\widehat{P}_j=-\sum_{s=1}^{S}\eta_j^s\left(\frac{\widehat{\lambda}_{jj}^{s}}{\varepsilon^s}\right).$$

Integrating the previous expression as in the proof of Proposition 1 and using the definition of W_j, we get

$$W_j=1-\prod_{s=1}^{S}\left(\frac{\lambda_{jj}^{s}}{\lambda_{jj}^{s'}}\right)^{\eta_j^s/\varepsilon^s}. \tag{51}$$

Our estimator for the gains from trade under perfect competition derives from Equation (51) and the same argument as in the proof of Proposition 2.

Now consider the case of monopolistic competition. Using the same arguments as in Lemma 2, it is easy to show that

$$\widehat{P}_j=-\sum_{s=1}^{S}\eta_j^s\left(\frac{\widehat{\lambda}_{jj}^{s}-\widehat{M}_j^s}{\varepsilon^s}\right), \tag{52}$$

where M_j^s in the number of entrants in country j and sector s. In order to compute the changes in the number of entrants, we can adopt the same strategy as in Step 6 of the proof of Lemma 2. By free entry, for all $s = 1, ..., S$, we must have

$$\sum_{i=1}^{n} \frac{f_{ji}^{s}}{\left(z_{ji}^{s*}\right)^{\sigma^{s}-1}} \int_{z_{ji}^{s*}}^{+\infty} z^{\sigma^{s}-1} g_{j}^{s}(z)\, dz - \sum_{i=1}^{n} f_{ji}^{s} \int_{z_{ji}^{s*}}^{+\infty} g_{j}^{s}(z)\, dz = f_{e}^{s},$$

where the s-superscripts reflect that all variables, parameters and functions may now vary at the sector level. Differentiating the previous expression, we obtain

$$\sum_{i=1}^{n} \theta_{ji}^{s} \widehat{z}_{ji}^{s*} = 0, \tag{53}$$

with θ_{ji}^{s} the share of total revenues in country j and sector s associated with sales in country i. Free entry further implies that

$$M_{j}^{s} \cdot \sum_{i=1}^{n} \frac{\sigma^{s} f_{ji}^{s}}{\left(z_{ji}^{s*}\right)^{\sigma^{s}-1}} \int_{z_{ji}^{s*}}^{+\infty} z^{\sigma^{s}-1} g_{j}^{s}(z)\, dz = L_{j}^{s},$$

where L_{j}^{s} is the endogenous amount of labor in sector s in country j. Differentiating the previous expression and using Equation (53), we obtain $\widehat{M}_{j}^{s} = \widehat{L}_{j}^{s}$. Together with Equation (52), this implies

$$\widehat{P}_{j} = -\sum_{s=1}^{S} \eta_{j}^{s} \left(\frac{\widehat{\lambda}_{jj}^{s} - \widehat{L}_{j}^{s}}{\varepsilon^{s}} \right)$$

Like in the case of perfect competition, we can then integrate the previous expression and use the definition of W_j to get

$$W_{j} = 1 - \prod_{s=1}^{S} \left(\frac{\lambda_{jj}^{s} L_{j}^{s'}}{\lambda_{jj}^{s'} L_{j}^{s}} \right)^{\eta_{j}^{s}/\varepsilon^{s}}. \tag{54}$$

Our estimator for the gains from trade under monopolistic competition derives from Equation (54) and the fact that with Cobb-Douglas preferences, the share of employment in sector s under autarky must be equal to the share of expenditure η_{j}^{s}. **QED**.

Tradable intermediate goods. Consider first the case of perfect competition. We assume that intermediate goods are aggregated into a non-tradable good that can be either consumed or combined with labor to produce a "composite input" that will be used, in turn, in the production of intermediate goods. Formally, if we denote by K_i the quantity of the non-tradable aggregate good allocated to the production of the composite input in country i, then the quantity produced of this input is $Q_i = \beta_i^{\beta_i}(1-\beta_i)^{1-\beta_i} L_i^{\beta_i} K_i^{1-\beta_i}$. Since the price of the non-tradable good is equal to the ideal price index in country i, the unit cost of Q_i is given by $c_i = w_i^{\beta_i} P_i^{1-\beta_i}$, justifying Equation (27) in the main text.

Under these assumptions, the equilibrium conditions remain given by Equations (4)-(6), but with c_j and Q_j substituting for w_j and L_j. Using the composite input in country j as our numeraire, $c_j = 1$, we can therefore follow the same logic as in Lemma 1 to show that changes in the ideal price index satisfy $\widehat{P}_j = -\widehat{\lambda}_{jj}/\varepsilon$. Since w_j is no longer our numeraire, we however need to take changes in into account in our welfare computations. Formally, we have $W_j \equiv 1 - \left(w_j P'_j / w'_j P_j \right)$, where w_j and w'_j are the wages in the initial and the new equilibrium, respectively. This implies

$$\widehat{1 - W}_j = \widehat{P}_j - \widehat{w}_j = -\widehat{\lambda}_{jj}/\varepsilon\beta_j, \tag{55}$$

where the second equality comes from the fact that

$$\beta_j \widehat{w}_j + (1 - \beta_j)\widehat{P}_j = 0, \tag{56}$$

by our choice of numeraire. Starting from Equation (55), we can then use the same arguments as in Propositions 1 and 2 to conclude that our estimator of the gains from trade is now given by $(\lambda_{jj})^{1/\left(\beta_j \bar{\varepsilon}\right)} - 1$.

Consider now the case of monopolistic competition. We maintain the assumption that labor and the aggregate non-tradable good are used to produce a common input with unit cost $c_i = w_i^{\beta_i} P_i^{1-\beta_i}$. Compared to the case of perfect competition, we assume that this common input is used both for the production of intermediate goods and the payment of fixed marketing costs, now equal to $c_i f_{ij}$. In addition, we assume that labor and the non-tradable aggregate good can be combined for the payment of fixed entry costs. In order to produce in country i, a firm must pay $c_i^e \equiv w_i^{\alpha_i} P_i^{1-\alpha_i} f_e$.

Under these assumptions, Equations (13)-(17) and (19) still hold in equilibrium, but with c_j and Q_j substituting for w_j and L_j. Given that $c_j Q_j = w_j L_j + P_j K_j$, then we now have

$$Y_j = w_j L_j + P_j K_j. \tag{57}$$

By contrast, using the composite good in country j as our numeraire, $c_j = 1$, Equation (18) becomes

$$\textstyle\sum_{i=1}^n \frac{f_{ji}}{\left(z_{ji}^*\right)^{\sigma-1}} \int_{z_{ji}^*}^{+\infty} z^{\sigma-1} g_j\left(z\right) dz - \sum_{i=1}^n f_{ji} \int_{z_{ji}^*}^{+\infty} g_j\left(z\right) dz = c_j^e. \tag{58}$$

Using Equations (13)-(17) and the same logic as in Lemma 2—Steps 1 through 5—it easy to show that changes in the ideal price index still satisfy

$$\widehat{P}_j = -\left(\widehat{\lambda}_{jj} - \widehat{M}_j\right)/\varepsilon. \tag{59}$$

We now build on Equation (58) and the logic of Step 6 in Lemma 2 to show that $\widehat{M}_j = -(\frac{1-\alpha_j}{\beta_j})\widehat{P}_j$. We use the following notations. We denote by L_j^Q and K_j^Q the amount of labor and the composite good used for production and marketing costs; and similarly, we denote by L_j^E and K_j^E the amounts of labor and the composite input used for entry. Our formal argument proceeds in three steps.

Step 1: *Percentage changes in the number of entrants satisfy*

$$\widehat{M}_j = \widehat{L}_j^E - (\frac{1-\alpha_j}{\beta_j})\widehat{P}_j. \tag{60}$$

Given our Cobb-Douglas aggregator, we know that

$$M_j f_e = \alpha_j^{-\alpha_j}(1-\alpha_j)^{\alpha_j - 1}\left(L_j^E\right)^{\alpha_j}\left(K_j^E\right)^{1-\alpha_j}, \tag{61}$$

$$\frac{K_j^E}{L_j^E} = \left(\frac{1-\alpha_j}{\alpha_j}\right)\left(\frac{w_j}{P_j}\right), \tag{62}$$

Differentiating Equations (61) and (62), we obtain after rearrangements

$$\widehat{M}_j = \widehat{L}_j^E + (1-\alpha_j)\left(\widehat{w}_j - \widehat{P}_j\right)$$

Combining the previous expression with Equation (56), which still holds by our choice of numeraire, we obtain Equation (60).

Step 2: *Percentage changes in the amount of the composite good satisfy*

$$\widehat{K}_j = \psi_j \widehat{L}_j^E - \left(\frac{1}{\beta_j}\right)\widehat{P}_j, \tag{63}$$

where $\psi_j \equiv \left(\frac{\beta_j - \alpha_j}{\alpha_j \beta_j}\right)\left(\frac{w_j L_j^E}{P_j K_j}\right)$.

Given our CES aggregator, we know that

$$\frac{K_j^Q}{L_j^Q} = \left(\frac{1-\beta_j}{\beta_j}\right)\left(\frac{w_j}{P_j}\right). \tag{64}$$

By definition, we also know that $K_j = K_j^Q + K_j^E$ and $L_j = L_j^Q + L_j^E$. Using Equations (62) and (64), we can rearrange the previous expression as

$$K_j = \left[L_j\left(\frac{1-\beta_j}{\beta_j}\right) + L_j^E\left(\frac{\beta_j-\alpha_j}{\alpha_j\beta_j}\right)\right]\left(\frac{w_j}{P_j}\right).$$

Differentiating the previous expression, we obtain

$$\widehat{K}_j = \left(\frac{\beta_j-\alpha_j}{\alpha_j\beta_j}\right)\left(\frac{w_jL_j^E}{P_jK_j}\right)\widehat{L}_j^E + \left(\widehat{w}_j - \widehat{P}_j\right). \tag{65}$$

Equation (63) directly derives from Equations (56) and (65).

Step 3: *The amount of labor used for entry does not vary with trade costs:* $\widehat{L}_j^E = 0$.

Differentiating Equation (58), we get

$$(1-\sigma)\sum_{i=1}^n \theta_{ji}\widehat{z}_{ji}^* = \frac{\left[\alpha_j\widehat{w}_j + (1-\alpha_j)\widehat{P}_j\right]c_j^e}{\sum_{i=1}^n f_{ji}\left(z_{ji}^*\right)^{1-\sigma}\int_{z_{ji}^*}^{+\infty} z^{\sigma-1}g_j(z)\,dz}, \tag{66}$$

where θ_{ji} the share of total revenues in country j associated with sales in country i. Equations (57) and (58) further imply that

$$M_j \cdot \sum_{i=1}^n \frac{\sigma f_{ji}}{\left(z_{ji}^*\right)^{\sigma-1}}\int_{z_{ji}^*}^{+\infty} z^{\sigma-1}g_j(z)\,dz = w_jL_j + P_jK_j.$$

Differentiating the previous expression and combining it with Equation (66), we obtain

$$\widehat{M}_j + \left(\frac{1-\sigma-\gamma}{1-\sigma}\right)\frac{\left[\alpha_j\widehat{w}_j + (1-\alpha_j)\widehat{P}_j\right]c_j^e}{\sum_{i=1}^n f_{ji}\left(z_{ji}^*\right)^{1-\sigma}\int_{z_{ji}^*}^{+\infty} z^{\sigma-1}g_j(z)\,dz} = (1-\kappa_j)\,\widehat{w}_j + \kappa_j\left(\widehat{P}_j + \widehat{K}_j\right),$$

where $\kappa_j \equiv P_jK_j/\left(w_jL_j + P_jK_j\right)$. Combining the previous expression with Equations (56), (60), and (63), we obtain

$$\widehat{L}_j^E = \frac{\widehat{w}_j\left(\alpha_j-\beta_j\right)}{\left(1-\beta_j\right)\left(1-\kappa_j\psi_j\right)}\left[1-\frac{\left(1-\sigma-\gamma\right)c_j^e}{\left(1-\sigma\right)\sum_{i=1}^n f_{ji}\left(z_{ji}^*\right)^{1-\sigma}\int_{z_{ji}^*}^{+\infty} z^{\sigma-1}g_j\left(z\right)dz}\right]. \tag{67}$$

Equations (67) and (58) imply

$$\widehat{L}_j^E = \frac{\widehat{w}_j\left(\alpha_j-\beta_j\right)}{\left(1-\beta_j\right)\left(1-\kappa_j\psi_j\right)}\frac{\sum_{i=1}^n f_{ji}H_j\left(z_{ji}^*\right)}{\left(1-\sigma\right)\sum_{i=1}^n f_{ji}\left(z_{ji}^*\right)^{1-\sigma}\int_{z_{ji}^*}^{+\infty} z^{\sigma-1}g_j\left(z\right)dz}, \tag{68}$$

where $H_j\left(z_{ji}^*\right)\equiv\gamma\left(z_{ji}^*\right)^{1-\sigma}\int_{z_{ji}^*}^{+\infty} z^{\sigma-1}g_j\left(z\right)dz+\left(1-\sigma-\gamma\right)\int_{z_{ji}^*}^{+\infty} g_j\left(z\right)dz$. Notice that

$$H_j'\left(z_{ji}^*\right) = \left(1-\sigma\right)\left[\gamma\left(z_{ji}^*\right)^{-\sigma}\int_{z_{ji}^*}^{+\infty} z^{\sigma-1}g_j\left(z\right)dz - g_j\left(z_{ji}^*\right)\right]=0, \tag{69}$$

where the second equality comes from the fact that $\gamma=\gamma_{ij}=\left(z_{ij}^*\right)^{\sigma}g_i\left(z_{ij}^*\right)/\int_{z_{ij}^*}^{+\infty} z^{\sigma-1}g_i\left(z\right)dz$. Since $\lim_{z_{ji}^*\to+\infty}H_j\left(z_{ji}^*\right)=0$, Equation (69) implies $H_j\left(z_{ji}^*\right)=0$ for all z_{ji}^*. Combining this observation with Equation (68), we obtain $\widehat{L}_j^E=0$.

To conclude, note that Steps 1 and 3 imply $\widehat{M}_j=-(\frac{1-\alpha_j}{\beta_j})\widehat{P}_j$. Together with Equation (59), this implies $\widehat{P}_j=-\beta_j\widehat{\lambda}_{jj}/\left(\beta_j\varepsilon+1-\alpha_j\right)$. Using Equation (56) and the fact that $\widehat{1-W_j}=\widehat{P}_j-\widehat{w}_j$, we obtain $\widehat{1-W_j}=-\widehat{\lambda}_{jj}/\left(\beta_j\varepsilon+1-\alpha_j\right)$. The rest of the proof is the same as under perfect competition. **QED**.

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