Banking, Liquidity and Inflation

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Abstract

This paper develops a search-theoretic model to study the interaction between banking and monetary policy and how this interaction affects the allocation and welfare. In decentralized monetary economies, uncertainty regarding opportunities to trade typically implies an inefficiency in allocation due to liquidity constraints. This paper studies how banking arises endogenously in a monetary economy to improve the allocation by reallocating liquidities across agents. We show that banking can always improve allocation in the decentralized market, but the existence and welfare implication of banking depend on the monetary policy. For low money growth rates, banking does not exist. For high money growth rates, banking exists and is welfare-improving. In particular, banking is needed to support a monetary equilibrium. For moderate money growth rates, banking exists and is welfare-reducing. Owing to general equilibrium feedback, banking can be supported in equilibrium even though welfare is higher without banking. One implication is that, due to the non-linear effect of inflation on the welfare, measuring the welfare cost of inflation by extrapolating historical data may underestimate the actual cost.

J.E.L. Classification Numbers: E40, E50
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1 Introduction

This paper develops a search-theoretic model of money and banking to study the roles of banking and liquidity in a monetary economy. Silveira and Wright (2007) show that due to the liquidity constraint of entrepreneurs, good production projects may not be implemented

by the best operator, leading to welfare loss. Our paper studies how intermediation arises endogenously in this environment to improve the allocation. We introduce competitive financial intermediaries to channel liquidity among entrepreneurs in the decentralized market for ideas. Possessing a record keeping technology, each financial intermediary takes deposits and make loans at a competitive interest rates. Also, borrowing from financial intermediaries incurs a fixed intermediation cost. We use the model to study the interaction between financial intermediation and monetary policy and how this interaction affects the allocation and welfare.

First, we show that the use of financial intermediation can always improve the allocation. Second, the welfare effect of intermediation depends on the monetary policy. For low inflation rates, intermediation is not used. For moderate inflation rates, intermediation is used but is welfare-reducing. For high money growth rates, intermediation is used and is welfare improving. In particular, for sufficiently high inflation, intermediation is needed to support monetary equilibrium. Third, in the presence of intermediation, inflation is less harmful.

Let us briefly describe our model and give the basic intuition of our findings. In this paper, banking is introduced to facilitate decentralized trading of intermediate inputs for production. In particular, we borrow the setup in Silveira and Wright (2007) to study the roles of banking in the market for production projects ("ideas") which are used as an input for production. Owing to the anonymity in the decentralized market for ideas, entrepreneurs need to bring liquidity (e.g. money) to this market to purchase ideas. Since innovators (i.e. sellers of ideas) have different random reservation prices with respect to their ideas, some entrepreneurs may end up with too much liquidity while others may end up with too little liquidity. The degree of this liquidity constraint depends on the real value of money, which in turn depends on the inflation rate. Inflation reduces the real value of money, and thus makes the liquidity constraint more binding. This problem can be resolved by having a financial intermediary which channels the funds from entrepreneurs with excess liquidity to those lacking liquidity. However, the use of intermediation involves resource costs, in particular in enforcing the repayment from the borrowers. Naturally, costly intermediation

is used when inflation is relatively high (liquidity problem is severe) and is not used when inflation is relatively low (liquidity problem is mild).

An interesting finding is that, in an economy with moderate inflation, intermediation is used even though it is welfare-reducing. The intuition is that, when an entrepreneur chooses to borrow from a financial intermediary, he considers only his own net *private* gain from borrowing, ignoring the general equilibrium effect. However, borrowing will also lower his demand for money in the money market, and thus reduce the equilibrium value of money. A lower value of money is going to tighten other entrepreneurs' liquidity constraints, pushing more entrepreneurs to (costly) borrow. This will lead to deadweight welfare loss to society.

Apparently, by studying the market for ideas, our paper is closely related to Silveira and Wright (2007). Note that while we choose to study banking in the market for ideas, we do expect that the main findings of our paper can be generalized and applied to other decentralized trading. The way banking is modeled in this paper is related to Berentsen, Camera and Waller (2006). There are two key differences. First, Berentsen, Camera and Waller study environment in which enforcement of repayment by borrowers is either costless or infinitely costly. In our paper, there is prefect enforcement but is subject to a finite fixed cost. Second, the fractions of borrowers and lenders are fixed in their paper, but in our environment, it is endogenous and depends on monetary policy. These differences generate some interesting new implications in our model. Another related paper is Bencivenga and Camera (2007) who also study the relationship between inflation and costly banking. We focus on the inefficiency of banking due to the competitive nature of the banking sector. This type of inefficiency is ruled out in their paper because a bank is modeled as an optimal contract among a coalition of agents. He, Huang, and Wright (2005) also studies banking in the Lagos and Wright (2005) environment, but they focus on the safekeeping function of banking.

2 Environment

Time is discrete and denoted $t = 0, 1, 2, ...$ There are two types of immortal agents: measure one of innovators (who are good at coming up with ideas), and measure one of entrepreneurs (who are better at implementing ideas). There are two markets: centralized market, denoted CM, and a decentralized market, denoted DM.

Each period is divided into two sub-periods. In the first sub-period, agents perform production and consumption and money holding adjustment in the CM. In the second sub-period, agents meet bilaterally and trade ideas which are implemented in the next CM. When the DM opens, each innovator comes up with a new idea that has value $R_i \geq 0$ if he implements it himself in the next CM. R_i is randomly distributed with a uniform (0,1) distribution, and its value is known when one enters the DM. If an innovator meets an entrepreneur in the DM, the former has an idea which has value R_i to him, and has value $R_e = 1$ to the latter.[1] The discount factor between one DM and the next CM is β. An idea is assumed to be indivisible and rivalry. The price at which an idea is traded is in terms of money, by which mean some liquid assets the entrepreneur has on hand. The sequence of events is illustrated in figure (1).

2.1 The CM

In the CM, agents produce, consume and adjust their money holding. In a typical period, the utility of an agent is given by

$$U(X) - H,$$

where $U : \mathbb{R}_+ \to \mathbb{R}_+$ denotes the utility of consuming $X \geq 0$ units of the consumption good, and $H \in \mathbb{R}_+$ denotes the effort on production. We assume that $U(.)$ is twice continuously differentiable, strictly increasing, strictly concave, and satisfies $U(0) = 0$, $U'(\bar{X}) = 1$

[1] Here, both R_e and the upper bound of R_i being 1 are normalization. Also, Silveira and Wright (2007) consider a more general case in which both innovators and entrepreneurs have random valuations.

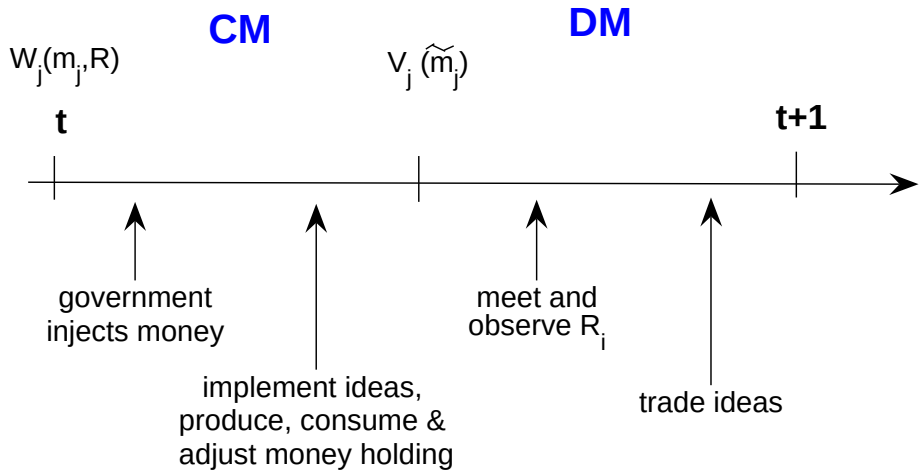

Figure 1: Timeline (No Banking)

for some $\bar{X} > 0$. For simplicity, we assume that the output, Y, of production is given by the sum of the effort and the value of the idea:

$$Y(H, R) = H + R$$

Agents can hold any non-negative amount of money $\hat{m} \in \mathbb{R}_+$. The total money stock at the beginning of the CM is M. The gross growth rate is $\mu = \frac{M}{M_{-1}}$, where M_{-1} denotes the money stock in the previous period. Agents receive monetary transfers at the entrance of the CM. In what follows we express an agent's money holding as a fraction of the beginning of the period money supply: $\frac{\hat{m}}{M}$. Let m and $\tilde{m}$ denote the individual money holdings at the beginning of the CM and the DM respectively.

Let $\phi \in \mathbb{R}_+$ be the price of money balance in terms of the consumption good in the CM. We will focus on stationary equilibrium in which the money growth rate, μ, is constant and the price of money is also constant over time. Let $W_j(m_j, R)$ be the value function for entrepreneurs ($j = e$) and innovators ($j = i$) entering the CM with m_j money holding and

an idea with value R in hand. Then the budget constraint of agents in the CM is

$$Y(H,R) + \phi(m_j + \Delta m) \geq X + \phi \tilde{m}_j,$$

where $\tilde{m}_j$ is money balance taken out of the CM, and $\Delta m = \frac{M - M_{-1}}{M} = 1 - \frac{1}{\mu}$ is the money transfer from the government. For $j = i, e$, the CM problem is

$$\begin{aligned} W_j(m_j, R) &= \max_{X,H,\tilde{m}_j \geq 0} U(X) - H + V_j(\tilde{m}_j) \\ s.t.\ \ X &= H + R + \phi(m_j - \tilde{m}_j + \Delta m) \end{aligned}$$

where $V_j(\tilde{m}_j)$ is the value function for entrepreneurs and innovators entering the DM with $\tilde{m}_j$, before meetings occur. From now on, we will assume that the utility function U is such that $H > 0$ even for the richest agents, so that we can focus on interior solution. Under this assumption, the budget constraint can be used to eliminate H in the objective function, simplifying the W_j to

$$\begin{aligned} W_j(m_j, R) &= \phi m_j + \phi \Delta m + R + \max_X \{U(X) - X\} + \max_{\tilde{m}_j} \{-\phi \tilde{m}_j + V_j(\tilde{m}_j)\} \qquad (1) \\ &= W_j(0,0) + \phi m_j + R, \end{aligned}$$

where $W_j(0,0) = \phi \Delta m + U(\bar{X}) - \bar{X} + \max_{\tilde{m}_j}\{-\phi \tilde{m}_j + V_j(\tilde{m}_j)\}$. Therefore, $W_j(m_j, R)$ is linear in both m_j and R. We will use this result to derive the bargaining solution below.

2.2 The DM

When an entrepreneur and an innovator meet, the value of R_i is observed by both agents. Since $R_i \leq R_e = 1$, the entrepreneur can always implement the idea at least as good as the

innovator. Efficiency requires that all ideas be implemented by the entrepreneurs. Owing to liquidity constraint in the market for ideas, this efficient allocation may not be supported. Let $p \in \mathbb{R}_+$ denote the price they would agree if there were no issues of liquidity. If $\tilde{m}_e \geq p$, then they trade immediately. The liquidity constraint requires that $\tilde{m}_e \geq p$. For simplicity, we assume that the price is determined by take-it-or-leave-it offers from the entrepreneur.[2]

3 No Banking

Innovator in DM

We first examine the case without banking. Consider an innovator bringing money holding $\tilde{m}_i$ and idea R_i into the DM. If an innovator keeps her idea, her payoff is

$$\beta W_i(\frac{\tilde{m}_i}{\mu}, R_i) = \beta W_i(0,0) + \phi\beta\frac{\tilde{m}_i}{\mu} + \beta R_i.$$

Here, the innovator does not spend her money balance and brings it forward to the next CM. The real value of this money balance (in terms of CM good) in the next CM is $\frac{\phi\tilde{m}_i}{\mu}$. Note that the next period money balance is re-scaled by the money growth rate because we normalize the money balance by the total stock of money. Also, we have made use of the result in (1) to evaluate the value in the next CM. If the innovator sells her idea at a price p, her payoff is

$$\beta W_i(\frac{\tilde{m}_i + p}{\mu}, 0) = \beta W_i(0,0) + \phi\beta\frac{\tilde{m}_i + p}{\mu}.$$

Here, the innovator's real money balance in the next CM is increased by $\phi\frac{p}{\mu}$. Therefore, the innovator has a reservation price $\bar{p}(R_i) = \frac{R_i\mu}{\phi}$ for an idea R_i.

Entrepreneur in DM

Consider an entrepreneur with money holding $\tilde{m}_e$ meeting an innovator with idea R_i.

[2]Silveira and Wright (2007) consider a more general case in which the price is determined by generalized Nash bargaining. Also, in their model, the innovator and the entrepreneur have an option to meet again in the next CM where the entrepreneur can raise more money. We abstract from these interesting extensions to focus on the effects of banking on the market for ideas in the simplest possible case.

The bargaining solution implies that, if $\tilde{m}_e \geq \bar{p}(R_i)$, then the entrepreneur can afford to buy the idea and get

$$V_e^1(\tilde{m}_e, R_i) = \beta W_e(0,0) + \beta R_e + \beta\phi \frac{\tilde{m}_e - \bar{p}(R_i)}{\mu}. \tag{2}$$

Here, the entrepreneur's real money balance in the next CM is reduced by $\phi\frac{p}{\mu}$. If $\tilde{m}_e < \bar{p}(R_i)$, then the entrepreneur cannot afford to purchase the idea and gets

$$V_e^0(\tilde{m}_e, R_i) = \beta W_e(0,0) + \beta\phi \frac{\tilde{m}_e}{\mu}. \tag{3}$$

Whether the innovator trades or not, he gets $\beta(W_i(0,0) + R_i) + \phi\beta\frac{\tilde{m}_i}{\mu}$: the innovator receives no trade surplus because she has no bargaining power.

<u>Demand for money in CM</u>

The value function of an innovator entering the DM is thus

$$\begin{aligned} V_i(\tilde{m}_i) &= \int_0^1 \beta(W_i(0,0) + R_i)dR_i + \phi\beta\frac{\tilde{m}_i}{\mu} \\ &= \beta W_i(0,0) + \frac{\beta}{2} + \phi\beta\frac{\tilde{m}_i}{\mu} \end{aligned} \tag{4}$$

An innovator's optimal choice of money balance taken to the DM (i.e. $\tilde{m}_i$ in (1)) is the solution to $\max_{\tilde{m}_i}[-\phi\tilde{m}_i + V_i(\tilde{m}_i)]$ and is given by

$$\tilde{m}_i \begin{cases} = 0, & \text{if } \mu > \beta \\ \in [0, \infty), & \text{if } \mu = \beta \\ = +\infty, & \text{if } \mu < \beta \end{cases} \tag{5}$$

That is, an innovator chooses not to bring any money to the DM if the money growth

rate is higher than β, indifferent between any amount of money if equal to β, and to bring an infinite amount if lower than β. We will focus on cases with $\mu \geq \beta$ and assume that when innovators are indifferent they choose $\tilde{m}_i = 0$. The value function of an entrepreneur entering the DM is

$$\begin{aligned} V_e(\tilde{m}_e) &= \int_0^{\tilde{m}_e \frac{\phi}{\mu}} V_e^1(\tilde{m}_e, R_i) dR_i + \int_{\tilde{m}_e \frac{\phi}{\mu}}^1 V_e^0(\tilde{m}_e, R_i) dR_i \\ &= \beta W_e(0,0) + 2\beta\phi \frac{\tilde{m}_e}{\mu} - \frac{\beta(\phi \tilde{m}_e)^2}{2\mu^2} \end{aligned}$$

The two terms on the right hand side of the first equality capture the case when $\tilde{m}_e \geq \bar{p}(R_i)$ and $\tilde{m}_e \leq \bar{p}(R_i)$. The second equality is derived by using (2) and (3). An entrepreneur's optimal choice of money balance taken to the DM (i.e. $\tilde{m}_e$ in (1)) is the solution to $\max_{\tilde{m}_e}[-\phi \tilde{m}_e + V_e(\tilde{m}_e)]$. This implies that, if $\tilde{m}_e > 0$, then

$$-\phi + 2\beta \frac{\phi}{\mu} - \beta \frac{\phi^2 \tilde{m}_e}{\mu^2} = 0,$$

or

$$\tilde{m}_e = \frac{2\mu\beta - \mu^2}{\beta\phi}. \tag{6}$$

<u>Equilibrium</u>

The money market equilibrium in the CM requires

$$\tilde{m}_e + \tilde{m}_i = 1. \tag{7}$$

Denote the equilibrium price of money (with no banking) as ϕ^{NB}. Under the simplifying assumption that $\tilde{m}_i = 0$ for $\mu \geq \beta$, we define the equilibrium as follows.

Definition 1. *A stationary monetary equilibrium without banking is given by ϕ^{NB} satisfying (6) and (7) with $\phi^{NB} > 0$.*

Proposition 1. *(Existence of equilibrium without banking) For any $\mu \in [\beta, 2\beta]$, there exists a stationary monetary equilibrium without banking.*

If $\mu > \beta$, then $\tilde{m}_i = 0$ and $\tilde{m}_e = 1$. (6) then implies $\phi^{NB} = 2\mu - \mu^2/\beta$ which is non-negative for $\mu \leq 2\beta$[3]. When $\mu \geq 2\beta$, money has no value and there is no monetary equilibrium (i.e. no ideas are traded). Let $\bar{R}_i^{NB} \in [0, 1]$ be the cut-off value of R_i such that an entrepreneur's liquidity constraint is just binding: $\tilde{m}_e = \bar{p}(R_i)$. In equilibrium, $\tilde{m}_e = 1$ and this cut-off is pinned down by the condition

$$\bar{R}_i^{NB}\mu = \phi^{NB}$$

The left-hand-side of the equation is the real reservation price of the marginal entrepreneur in terms of the *current* period good while the right-hand side is the maximum real price an entrepreneur is able to pay (i.e. the real money balance $\phi^{NB}\tilde{m}_e = \phi^{NB}$).[4] In figure 2, the left-hand-side is represented by the upward sloping line and the right-hand-side is represented by the horizontal line.[5] So the equilibrium amount of trade is given by:

$$\bar{R}_i^{NB} = \frac{\phi^{NB}}{\mu} = 2 - \frac{\mu}{\beta}$$

Note that there exists a unique stationary monetary equilibrium for $\mu \in [\beta, 2\beta]$ and that money growth always reduces trade. In figure 2, an entrepreneur with $R_i \leq \bar{R}_i^{NB} = \frac{\phi^{NB}}{\mu}$ buys the idea at $p = R_i\mu$. After trade, these entrepreneurs still have money left over. The total real money surplus is $\frac{(\phi^{NB})^2}{2\mu}$. For the rest of the entrepreneurs, they need extra funding to purchase the idea. The total money shortage is $(1 - \frac{\phi^{NB}}{\mu})^2\mu/2$. Therefore, there is a

[3]The upper bound being 2β is a result of the assumptions of $R_e = 1$ and $R_i \sim$uniform$(0, 1)$.

[4]Since the nominal price (in terms of a fraction of the current money stock) is $p = \frac{R_i\mu}{\phi}$, the real price (in terms of the current period goods) is given by $p\phi = R_i\mu$.

[5]One may interpret the figure as a supply-demand diagram which determines the equilibrium given a "demand curve" (ϕ) and a "supply curve" ($R_i\mu$).

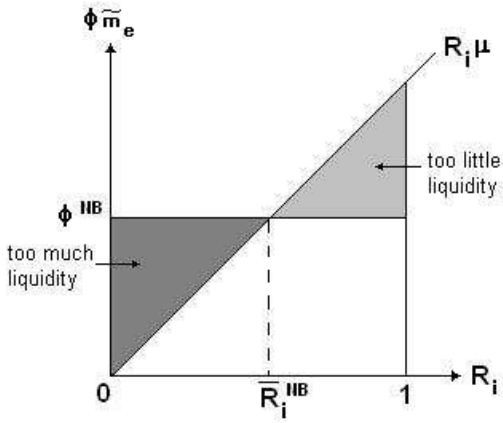

Figure 2: Potential gain from trading liquidity

potential role for borrowing and lending between entrepreneurs whenever $\mu > \beta$. When $\mu = \beta$, the opportunity cost of holding money is zero. As a result, no entrepreneurs are liquidity constrained and all ideas are traded (i.e. $\bar{R}_i^{NB} = 1$).

4 Costless Banking

Banking

Suppose in the DM there are competitive banks taking deposits at an interest rate r^D and making loans at an interest rate r^L. Each bank has a record keeping technology allowing it to keep financial record of entrepreneurs. Suppose entrepreneurs can commit to repay the bank in the CM and banks can commit to repay depositors in the CM. Free entry implies zero profit for banks and thus $r^D = r^L = r$ for some $r \geq 0$. Figure 3 illustrates the flow of funds in the CM and DM. Figure 4 shows the time line. In the DM, after meeting and observing the realization of R_i, an entrepreneur can choose to lend money to or borrow money from a bank. In the next CM, deposits and loans will be repaid. In general, an entrepreneur meeting an innovator with low R_i has excess liquidity and would like to lead his surplus money holding to a bank after trade to earn interest income. An entrepreneur meeting an innovator with high R_i may find the surplus from trade smaller than the return from deposit and chooses to lend all his money holding to the bank. An entrepreneur with

intermediate level of R_i is liquidity constrained and will choose to borrow from the bank to finance the trade. Anonymity of entrepreneurs in the market for ideas implies that money is still needed as a medium of exchange.

A representative bank in the competitive loan market takes r^L and r^D as given, and chooses the amount of loans (l) and deposit (d) to maximize its profit (π):

$$\begin{aligned} \max_{l,d} \pi &= r^L l - r^D d, \\ \text{s.t.} \quad & d \geq l \end{aligned}$$

Here, there is a feasibility constraint restricting that the amount of loans lent out has to be no more than the amount of deposits taken in. When $r^L > r^D$, the problem is not defined since banks will choose $l = d = +\infty$, implying $\pi = +\infty$. When $r^L < r^D$, banks choose $l = d = 0$ to earn $\pi = 0$. This cannot clear the loan market when entrepreneurs choose to save a positive amount. So whenever there is positive saving, we must have $r^L = r^D$. Banks' optimization problem then implies

$$\begin{cases} d = l & if\ r > 0 \\ d \geq l, & if\ r = 0 \end{cases} \tag{8}$$

In both cases, profits of the banks are zero.

<u>Entrepreneur's decision in DM</u>

After meeting in the DM, an entrepreneur with $\tilde{m}_e$ and R_i chooses the amount of saving (lending if positive and borrowing if negative ($s \in \mathbb{R}$)), money brought to the next CM ($m_e \in \mathbb{R}_+$, as a fraction of *next* period money stock) and whether or not to buy the idea ($y \in \{0, 1\}$) to maximize the expected payoff:

$$\max_{s,y,m_e} \beta W_e(0,0) + \beta y + \beta\phi(1+r)\frac{s}{\mu} + \beta\phi m_e$$

subject to $m_e\mu = \tilde{m}_e - y\frac{R_i\mu}{\phi} - s \geq 0$. The budget constraint says that the amount of money

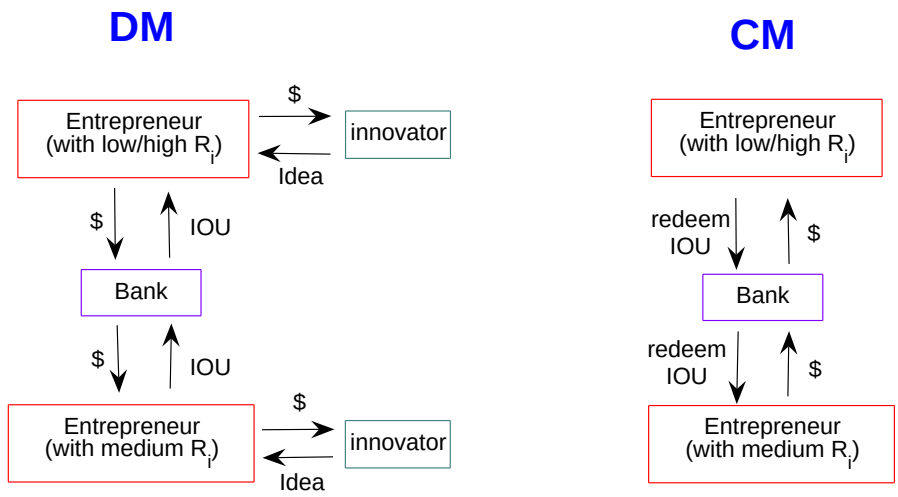

Figure 3: Flow of Funds

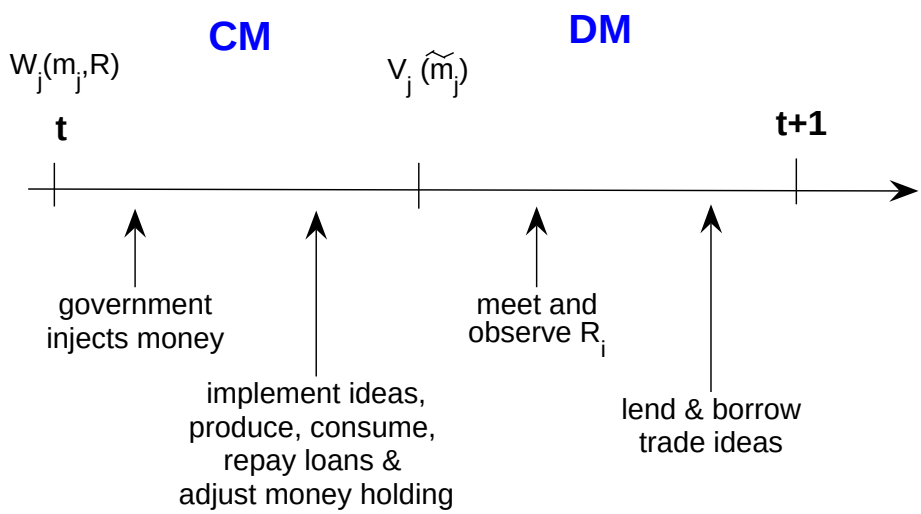

Figure 4: Timeline

brought to next period is equal to the initial money holding minuses the expenditure on purchasing idea and saving. We need to adjust the left-hand-side by the money growth rate because the two sides are normalized by money stocks in two different periods. Substituting this budget constraint into the objective function, we have

$$\max_{y,m_e} \beta W_e(0,0) + \beta y + \beta \frac{\phi}{\mu}(1+r)(\tilde{m}_e - \mu m_e - y\frac{R_i \mu}{\phi}) + \beta \phi m_e$$

Note that optimization implies $m_e = 0$ if $r > 0$ and $m_e \in \mathbb{R}_+$ if $r = 0$. Then, an entrepreneur's problem becomes

$$\begin{aligned} & \beta W_e(0,0) + \beta \max\{1 + \frac{\phi}{\mu}(1+r)(\tilde{m}_e - \frac{R_i \mu}{\phi}), \frac{\phi}{\mu}(1+r)\tilde{m}_e\} \qquad (9) \\ = \; & \beta W_e(0,0) + \beta \frac{\phi}{\mu}(1+r)\tilde{m}_e + \beta \max\{1-(1+r)R_i, 0\}. \end{aligned}$$

The last term captures an entrepreneur's comparison between the value of the idea ($R_e = 1$) and the opportunity cost (including interest) of buying the idea ($(1+r)R_i$). Therefore, the value function of an entrepreneur entering the DM is

$$V_e(\tilde{m}_e) = \int_0^1 \left[\beta W_e(0,0) + \beta \frac{\phi}{\mu}(1+r)\tilde{m}_e + \beta \max\{1-(1+r)R_i, 0\} \right] dR_i$$

Denote the optimal saving of an entrepreneur with (m, R_i) by $s(\tilde{m}_e, R_i)$. (9) implies that the cut-off value $\bar{R}_i(r)$ that makes an entrepreneur indifferent between trading and no trading is given by

$$\bar{R}_i(r) = \frac{1}{1+r}$$

As a result, the value function of an entrepreneur entering the DM can be simplified to:

$$V_e(\tilde{m}_e) \quad = \quad \beta W_e(0,0) + \beta\frac{\phi}{\mu}(1+r)\tilde{m}_e + \beta\int_0^{\frac{1}{1+r}}(1-(1+r)R_i)dR_i$$

Therefore, $V_e'(\tilde{m}_e) = \beta\frac{\phi}{\mu}(1+r) > 0$ and thus the value function is linear. The market clearing condition in the CM requires that

$$\arg\max V_e(\tilde{m}_e) - \phi\tilde{m}_e = 1.$$

So, in equilibrium, the optimal money demand is characterized by the first order condition of the above problem:

$$\begin{aligned} \beta\frac{\phi}{\mu}(1+r) &= \phi, \\ \text{or } r &= \frac{\mu}{\beta} - 1. \end{aligned} \tag{10}$$

Basically, the use of banking relaxes entrepreneur's liquidity constraint in purchasing ideas in the DM. Therefore, when choosing the optimal amount of money brought to the DM, an entrepreneur simply looks at whether the real rate of return of money is higher than the subjective discount rate (i.e. whether $\frac{\phi}{\mu}(1+r) - \frac{1}{\beta} > 0$) across two CM's. He will demand $\tilde{m}_e = 0$ when the real rate of return is lower than the subjective discount rate. He will demand $\tilde{m}_e = +\infty$ when the real rate of return higher than the subjective discount rate, and will demand any $\tilde{m}_e \in \mathbb{R}_+$ when the rate of return is equal to the subjective discount rate. To clear the money market in CM, the nominal interest rate, r, has to exactly compensate for the inflation and discounting (i.e. zero real return rate).

<u>Equilibrium</u>

The cut-off value of idea is thus $\bar{R}_i^B = \bar{R}_i(r) = \frac{\beta}{\mu}$. Entrepreneurs' optimal choices of (y, s)

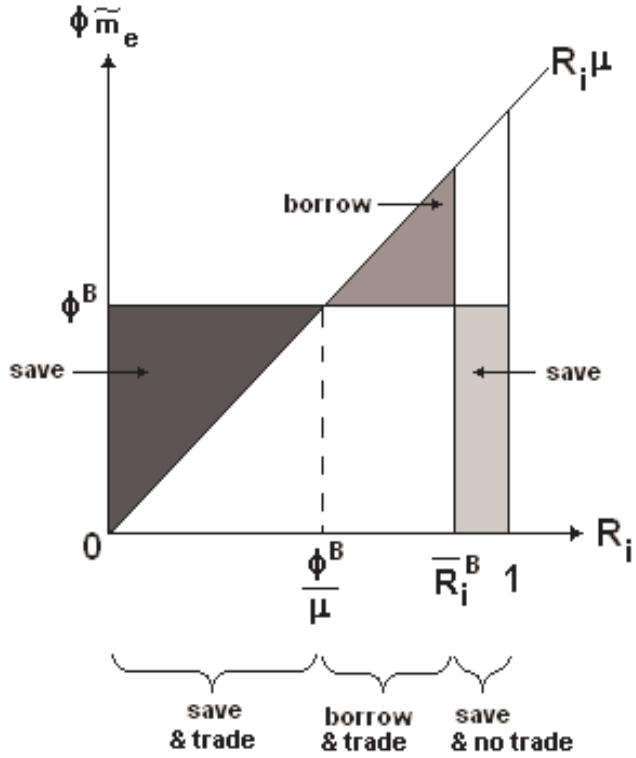

Figure 5: Saving and Borrowing

as a function of R_i is illustrated by Figure (5)[6]:

$$\begin{cases} y = 1, s = \tilde{m}_e - \frac{R_i\mu}{\phi} \geq 0 & \text{, if } R_i \in [0, \frac{\phi}{\mu}] \\ y = 1, s = \tilde{m}_e - \frac{R_i\mu}{\phi} < 0 & \text{, if } R_i \in (\frac{\phi}{\mu}, \bar{R}_i^B] \\ y = 0, s = \tilde{m}_e \geq 0 & \text{, if } R_i \in (\bar{R}_i^B, 1] \end{cases} \tag{11}$$

As discussed earlier, the entrepreneurs with low and high R_i's will save, and the entrepreneurs with medium R_i will borrow. Only the entrepreneurs with low and medium R_i's will trade. The loan market clearing condition in the DM requires that the aggregate saving from the entrepreneurs is equal to the total deposit minus the total loans:

$$\int_0^1 s(\tilde{m}_e, R_i) dR_i = d - l.$$

[6]Here we assume that, when entrepreneurs are indifferent between saving and not saving (which happens when $r = 0$), they choose to save.

Then condition (8) from the bank's optimization implies that

$$\begin{cases} \int_0^1 s(\tilde{m}_e, R_i)dR_i = 0 & if \;\; r > 0 \\ \int_0^1 s(\tilde{m}_e, R_i)dR_i \geq 0 & if \;\; r = 0 \end{cases}$$

Substituting in the saving functions from (4), we simplify the left-hand-side to $\tilde{m}_e - \frac{\beta^2}{2\mu\phi}$. Imposing the money market clearing condition in the CM (i.e. $\tilde{m}_e = 1$), we have

$$\begin{cases} \phi = \frac{\beta^2}{2\mu} & if \;\; r > 0 \\ \phi \geq \frac{\beta^2}{2\mu} & if \;\; r = 0 \end{cases} \tag{12}$$

Definition 2. *A stationary monetary equilibrium with costless banking is a pair* (ϕ^B, r) *satisfying (10), (12) with* $\phi^B > 0$, $r \geq 0$.

When $\mu > \beta$, there exists a unique stationary monetary equilibrium with costless banking where $\phi^B = \frac{\beta^2}{2\mu}$, $r = \frac{\mu}{\beta} - 1 > 0$, $\tilde{m}_e = 1$, $\tilde{m}_i = 0$ and $\bar{R}_i^B = \frac{\beta}{\mu}$. Fraction $\bar{R}^B - \frac{\phi^B}{\mu} = \frac{\beta}{\mu}(1 - \frac{\beta}{2\mu})$ of entrepreneurs are borrowers and the rest are lenders. Since the interest rate in the loan market is positive, the excess supply of loans is zero.

When $\mu = \beta$, we have multiple equilibria: any $\phi^B \in [\frac{\beta}{2}, \infty)$, $r = 0$, $\tilde{m}_e = 1$, $\tilde{m}_i = 0$ and $\bar{R}_i^B = 1$. Fraction $\max\{1 - \frac{\phi^B}{\mu}, 0\}$ of entrepreneurs are borrowers and the rest are lenders. All these equilibria are equivalent in terms of real allocations and payoffs. They differ only in terms of the real value of money and the borrowing-lending decision in the DM. At the lower bound where $\phi^B = \frac{\beta}{2}$, half of the set of entrepreneurs are liquidity constrained and need to borrow. The excess supply of loans is zero. As the value of money (ϕ^B) goes up, fewer entrepreneurs are liquidity constrained and there are fewer borrowers. There is excess supply of loans in the loan market, but it is consistent with the interest rate being zero. For $\phi^B \geq \beta$, no entrepreneurs are liquidity constrained and there are no borrowers. Again, there is excess supply of loans. Also, at the Friedman rule, a banking equilibrium with $\phi^B = \beta$ is

identical to an equilibrium without banking.

Inflation, Banking and Welfare

Note that the measure of trade ($\frac{\beta}{\mu}$) is decreasing in inflation. Maximum amount of trade ($\bar{R}_i^B = 1$) is achieved when $\mu = \beta$. Measuring the welfare by the average utility of agents, we have the welfare for $k = NB, B$ given by:

$$\begin{aligned} W^k &= 2(U(\bar{X}) - \bar{X}) + \int_0^{\bar{R}_i^k} 1 dR_i + \int_{\bar{R}_i^k}^1 R_i dR_i \\ &= 2(U(\bar{X}) - \bar{X}) + \bar{R}_i^k + \frac{1}{2} - \frac{(\bar{R}_i^k)^2}{2} \end{aligned}$$

The three terms on the right-hand-side of the first equality capture respectively the surplus in the CM, the value of ideas implemented by entrepreneurs and the value of ideas implemented by innovators.

Now, we will compare the allocation with and without banking. Note that, without banking, the cut-off value, $\bar{R}_i^{NB}$, is pinned down by the money demand decision. In equilibrium, the first order condition (6) implies

$$\begin{aligned} \frac{\phi}{\mu}(1 - \bar{R}_i^{NB}) &= \frac{\phi}{\mu}(\frac{\mu}{\beta} - 1) \\ 1 - \bar{R}_i^{NB} &= \frac{\mu}{\beta} - 1 \end{aligned} \tag{13}$$

The left- and right-hand sides capture respectively the benefit and cost of bringing the marginal dollar to the DM. Bringing one extra dollar to the DM relaxes the liquidity constraint and allows $\frac{\phi}{\mu}$ more extra trades, each of which generates payoff $1 - \bar{R}_i^{NB}$ in terms of next period utility. At the same time, bringing one extra dollar incurs a (net) opportunity cost of ($\frac{\mu}{\beta} - 1$) in terms of next period dollars. In terms of next period utility, the cost is $\frac{\phi}{\mu}(\frac{\mu}{\beta} - 1)$.

With banking, the cut-off value, $\bar{R}_i^B$, is pinned down by the borrowing decision. In equilibrium, condition (4) implies

$$1 - \bar{R}_i^B = (\frac{\mu}{\beta} - 1)\bar{R}_i^B \tag{14}$$

The left- and right-hand sides capture respectively the benefit and cost of borrowing for the marginal entrepreneur in the DM. Comparing the right-hand sides of (13) and (14), we can see that banking reduces entrepreneurs' cost of buying ideas: by lending out the money balance unused for trade, the excess balance is not subject to inflation and discounting.

Proposition 2. *(Inflation and welfare with costless banking)*

(i) When $\mu = \beta$, $\bar{R}_i^B = \bar{R}_i^{NB} = 1$ *and* $W^B = W^{NB}$.

(ii) When $\mu \in (\beta, 2\beta]$, $1 > \bar{R}_i^B > \bar{R}_i^{NB} > 0$, $W^B > W^{NB}$, $0 > \frac{d\bar{R}_i^B}{d\mu} > \frac{d\bar{R}_i^{NB}}{d\mu}$ *and* $0 > \frac{dW^B}{d\mu} > \frac{dW^{NB}}{d\mu}$.

(iii) When $\mu > 2\beta$, $\bar{R}_i^B > \bar{R}_i^{NB} = 0$ *and* $W^B > W^{NB}$.

When $\mu = \beta$, all ideas are traded and welfare is maximized with or without banking. In this case, banking is viable but cannot improve welfare.

When $\mu \in (\beta, 2\beta]$, banking allows more ideas to be traded and thus implies higher welfare. The marginal effect of inflation is larger in magnitude when there is no banking for two reasons. First, the marginal effect of inflation on the number of trades is larger without banking (i.e. $|\frac{d\bar{R}_i^{NB}}{d\mu}| > |\frac{d\bar{R}_i^B}{d\mu}|$). Condition (13) suggests that, without banking, higher inflation raises the opportunity cost of holding money, and thus less ideas are traded (i.e. lower $\bar{R}_i^{NB}$). Condition (14) suggests that, with banking, the impact of inflation on $\bar{R}_i^{NB}$ is smaller because less liquidity is needed.

Second, the gain from the marginal trade is higher without banking ($1 - \bar{R}_i^{NB} > 1 - \bar{R}_i^B$). This is because the marginal value of trades is diminishing and because the number of trades is higher with banking. Therefore, inflation is less harmful in the presence of banking.

When $\mu > 2\beta$, monetary equilibrium does not exist without banking, but exists with costless banking. Without banking, the only reason to bring money to the DM is to buy ideas. A very high inflation will make the cost of holding money so high so that there is no

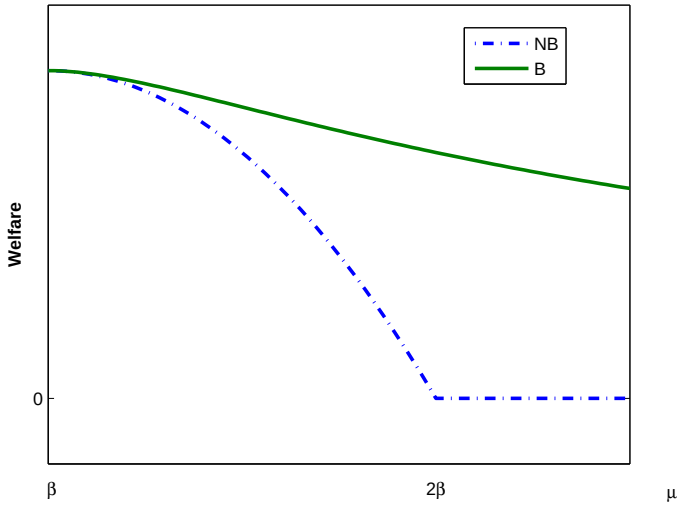

Figure 6: Welfare in No Banking and Costless Banking Equilibria

trades of ideas, implying zero value of money. With banking, there is an additional motive to bring money to the DM to lend to the banks. With high inflation, liquidity is relatively scarce in the DM (i.e. excess demand for loans if the price of money does not adjust). The scarcity of money in the DM induces entrepreneurs to demand more money in the CM, raising the price of money, ϕ. As a consequence of a higher ϕ, entrepreneurs' liquidity constraints in the DM are relaxed. So, when the money growth rate is higher than 2β, banking is needed to support a monetary equilibrium. (Figure 6)

Now, we derive the effects of banking on the price of money. At the Friedman rule, every entrepreneur is liquidity unconstrained and has excess liquidity after trades. Banking is not needed. Close to the Friedman rule, banking can improve the allocation of ideas. If we introduce banking but keep the price of money unchanged, there will be excess supply of loans. To clear the loan market, ϕ has to go down to induce a higher (net) demand for loans by making more entrepreneurs liquidity constrained. So banking lowers the value of money in a low inflation economy. In contrast, for a high inflation rate, most entrepreneurs are liquidity constrained. If we introduce banking but keep the price of money unchanged, there will be excess demand for loans. To clear the loan market, ϕ has to go up to induce a higher (net) supply of loans by making fewer entrepreneurs liquidity constrained. So banking raises

the value of money in a low inflation economy.

Mathematically, considering the price ϕ as a function of μ (i.e. $\phi^{NB}(\mu) = 2\mu - \frac{\mu^2}{\beta}$ and $\phi^B(\mu) = \frac{\beta^2}{2\mu}$), we have $\phi^{NB}(\beta) > \phi^B(\beta)$ and $\phi^B(2\beta) > \phi^{NB}(2\beta) = 0$. Since $\phi^B(.)$ and $\phi^{NB}(.)$ are strictly decreasing and continuous in μ, we have the following result:

Proposition 3. *(Value of money with costless banking) There exists a unique $\mu^* \in (\beta, 2\beta)$ such that $\phi^{NB}(\mu) \gtreqless \phi^B(\mu)$ for $\mu \lesseqgtr \mu^*$.*

To see the intuition, recall that when $\mu = \beta$, a banking equilibrium with $\phi^B = \beta$ is identical to an equilibrium without banking. In this equilibrium, $r = 0$ and there is excess supply of loans. So, if we raise μ by a small amount without adjusting ϕ^B, there should still be excess supply in the loan market, implying $r = 0$. This would mean that the real return rate of money in the CM is lower than the subjective discount rate (i.e. $\frac{\phi^B}{\mu}(1+r) - \frac{1}{\beta}$) and no one will choose to buy money in the CM. So the price of money needs to adjust. As ϕ^B goes down, more entrepreneurs in the DM become liquidity constrained and need to borrow. In equilibrium, ϕ^B has to drop so much so that there is no excess supply in the loan market, supporting a positive interest rate equals to $\frac{\mu}{\beta}$. In an economy with banking, moving μ above the Friedman rule can induce a *discontinuous* drop in the price of money.[7] In an economy without banking, the price of money drops continuously. But as the inflation rate increases further, the price of money in an economy without banking will converge to zero as μ goes to 2β, while it remains positive when there is banking.

5 Costly Banking

Entrepreneur's decision in DM

Now, we consider the case when entrepreneurs have to incur a fixed effort/utility cost $\eta > 0$ to borrow but no cost to deposit. One may interpret it as the borrower's cost of credibly committing to repay. An entrepreneur in the DM chooses saving (s), money brought to the

[7] Except for the equilibrium in which $\phi^B = \frac{\beta}{2}$ for $\mu = \beta$. In this equilibrium, an increase in the inflation will induce a continuous drop in ϕ^B.

CM (m_e) and whether or not to buy the idea ($y \in \{0,1\}$):

$$\max_{s,y,m_e} \beta W_e(0,0) + \beta y + \beta \frac{\phi}{\mu}(1+r)s - \iota(s)\eta + \beta \frac{\phi}{\mu} m_e$$

subject to $m_e\mu = \tilde{m}_e - y\frac{R_i\mu}{\phi} - s \geq 0$ and an indicator function

$$\iota(s) = \begin{cases} 1 & if \quad s < 0 \\ 0 & if \quad s \geq 0 \end{cases}$$

The objective function can be written as

$$\beta W_e(0,0) + \beta \max\{\max_{s,m_e}(1 + \frac{\phi}{\mu}(1+r)s - \iota(s)\frac{\eta}{\beta} + \frac{\phi}{\mu}m_e), \max_{s,m_e}(\frac{\phi}{\mu}(1+r)s - \iota(s)\frac{\eta}{\beta} + \frac{\phi}{\mu}m_e)\}$$

Again, the non-negativity constraint for m_e requires that $rm_e = 0$. Also, there is no reason to pay the fixed cost and borrow unless an entrepreneur is liquidity constrained. So, $\iota(s) = 0$ when $\tilde{m}_e - y\frac{R_i\mu}{\phi} \geq 0$ and thus the problem becomes

$$\begin{cases} \beta W_e(0,0) + \beta \max\{1 + \frac{\phi}{\mu}(1+r)(\tilde{m}_e - \frac{R_i\mu}{\phi}), \frac{\phi}{\mu}(1+r)\tilde{m}_e\} & \text{, if } \tilde{m}_e \geq \frac{R_i\mu}{\phi} \\ \beta W_e(0,0) + \beta \max\{1 + \frac{\phi}{\mu}(1+r)(\tilde{m}_e - \frac{R_i\mu}{\phi}) - \frac{\eta}{\beta}, \frac{\phi}{\mu}(1+r)\tilde{m}_e\} & \text{, if } \tilde{m}_e < \frac{R_i\mu}{\phi} \end{cases}$$

If $\tilde{m}_e \geq p = \frac{R_i\mu}{\phi}$, an entrepreneur is not liquidity constrained and will choose to save and trade if and only if

$$R_i \leq \bar{R}_1 \equiv \frac{1}{1+r}$$

If $\tilde{m}_e < p = \frac{R_i\mu}{\phi}$, an entrepreneur is liquidity constrained and will choose to borrow and trade if and only if

$$R_i \leq \bar{R}_2 \equiv \frac{1}{1+r} - \frac{\eta}{\beta(1+r)} \tag{15}$$

The optimal choice of an entrepreneur given any $(\tilde{m}_e, R_i)$ pair is shown in Figure 7. Above the upward-sloping line $\phi\tilde{m}_e = R_i\mu$, entrepreneurs are not liquidity constrained. In this case, they choose to trade whenever $R_i \leq \bar{R}_1$. Below the upward-sloping line, entrepreneur are liquidity constrained. In this case, they choose to trade whenever $R_i \leq \bar{R}_2$. Note that $\bar{R}_1 > \bar{R}_2$.

To solve for the equilibrium, we consider two different cases separately: $\phi\tilde{m}_e \in [0, \bar{R}_2\mu]$ and $\phi\tilde{m}_e > \bar{R}_2\mu$. In the first case, the real value of money is so low that some entrepreneurs are liquidity constrained and there is positive borrowing, implying $r \geq 0$. In the second case, the real value of money is so high that all entrepreneurs are not liquidity constrained and thus there is no borrowing, implying $r = 0$.

Case 1: $\phi\tilde{m}_e \in [0, \bar{R}_2\mu]$

Note that an equilibrium with $r > 0$ can only exist when $s(1, R_i) < 0$ for some R_i (i.e. some entrepreneurs choose to borrow). As figure 7 suggests, this requires $\phi < \bar{R}_2\mu$. Now, let us first characterize this equilibrium and then we can derive the condition for the existence of this equilibrium. In this equilibrium, an entrepreneur brings $\tilde{m}_e = 1$ to the DM and an entrepreneur chooses to trade and save if $R_i \in [0, \frac{\phi}{\mu}]$, chooses to trade and borrow if $R_i \in (\frac{\phi}{\mu}, \bar{R}_2]$ and chooses to save and not trade if $R_i \in (\bar{R}_2, 1]$. As shown in the Appendix, the value function over the relevant region (which is $[0, \frac{\bar{R}_2\mu}{\phi}]$) is given by

$$V_e(\tilde{m}_e) \;=\; \beta(W_e(0,0)+1) - \frac{\beta(1+r)\bar{R}_2^2}{2} - \beta\bar{R}_2(1+r)(1-\bar{R}_2) - \eta + \frac{\phi}{\mu}[\beta(1+r)+\eta]\tilde{m}_e$$

Therefore, $V_e'(\tilde{m}_e) = \frac{\phi}{\mu}[\beta(1+r)+\eta]$. Here, the marginal value of bring an extra dollar to the DM consists of two parts. The first part is the real return of money $(\beta\frac{\phi}{\mu}(1+r))$. The second part is that it helps to reduce the likelihood of being liquidity constrained and thus

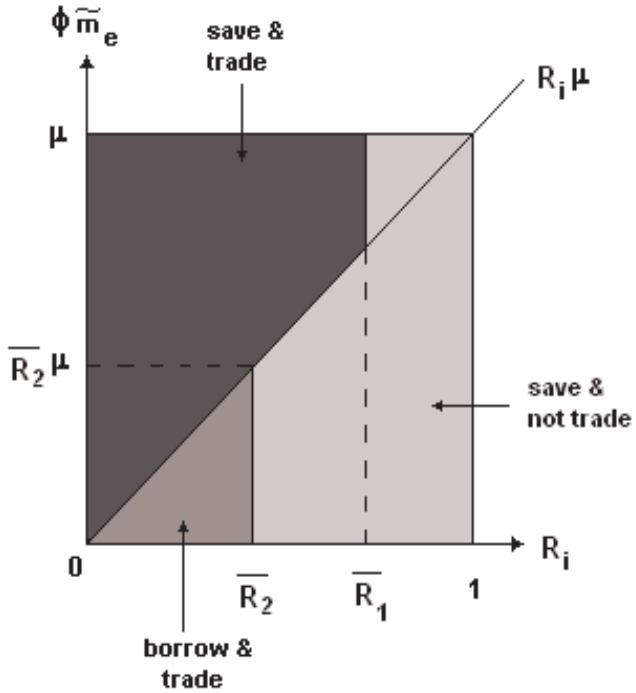

Figure 7: Optimal Choice of Entrepreneur

avoiding the expected fixed cost of borrowing a loan ($\frac{\phi}{\mu}\eta$). Again, V_e is linear in $\tilde{m}_e$ in this region. In equilibrium, the money market clearing condition, $\tilde{m}_e = 1$, implies that the first order condition $V_e'(\tilde{m}_e) = \phi$) is satisfied. Therefore,

$$r = \frac{\mu - \eta}{\beta} - 1. \tag{16}$$

When (16) is satisfied, entrepreneurs are indifferent between any $\tilde{m}_e \in [0, \frac{\bar{R}_2\mu}{\phi}]$. The equilibrium condition $\tilde{m}_e = 1$ then requires that

$$\phi \leq \bar{R}_2\mu. \tag{17}$$

Let $\bar{R}_i^{CB}$ denote the cut-off value of idea such that an entrepreneur is indifferent between trading or not. Condition (15) implies $\bar{R}_i^{CB} = \bar{R}_2 = \frac{\beta-\eta}{\mu-\eta}$.

As in condition (12), we can use the banks' optimal decision to derive equilibrium con-

ditions regarding the excess supply of loans and the interest rate. The loan market clearing condition in the DM implies

$$\begin{cases} \tilde{m}_e - \frac{\mu(\beta-\eta)^2}{2\phi(\mu-\eta)^2} = 0 & \text{, if} \quad r > 0 \\ \tilde{m}_e - \frac{\mu(\beta-\eta)^2}{2\phi(\mu-\eta)^2} \geq 0 & \text{, if} \quad r = 0 \end{cases}$$

Imposing the money market clearing condition in the CM ($\tilde{m}_e = 1$), we have

$$\begin{cases} \phi = \frac{\mu(\beta-\eta)^2}{2(\mu-\eta)^2} & \text{, if} \quad r > 0 \\ \phi \geq \frac{\mu(\beta-\eta)^2}{2(\mu-\eta)^2} & \text{, if} \quad r = 0 \end{cases} \tag{18}$$

<u>Equilibrium</u>

Definition 3. *A stationary monetary equilibrium with costly banking is a pair* (ϕ^{CB}, r) *satisfying (16), (17) , (18) with* $\phi^{CB} > 0$ *,* $r \geq 0$*.*

Firstly, consider the case with $r > 0$. Condition (16) implies that $\mu - \eta - \beta > 0$. Condition (18) then implies $\phi^{CB} = \frac{\mu(\beta-\eta)^2}{2(\mu-\eta)^2}$. Then condition (17) is satisfied if and only if

$$\begin{aligned} & \phi^{CB} && \leq \bar{R}_2\mu \\ \Leftrightarrow \quad & \tfrac{\mu(\beta-\eta)^2}{2(\mu-\eta)^2} && \leq \frac{\beta-\eta}{\mu-\eta}\mu \\ \Leftrightarrow \quad & (\beta-\eta)^2 && \leq 2(\beta-\eta)(\mu-\eta) \\ \Leftrightarrow \quad & (\beta-\eta)(\beta+\eta-2\mu) && \leq 0 \\ \Leftrightarrow \quad & \eta && \leq \beta. \end{aligned}$$

Now, consider the case with $r = 0$. Condition (16) implies that $\mu - \eta - \beta = 0$. Condition (18) then implies $\phi^{CB} \geq \frac{\mu(\beta-\eta)^2}{2(\mu-\eta)^2}$. Then condition (17) is satisfied if and only if

$$
\begin{aligned}
& \phi^{CB} && \leq \bar{R}_2 \mu \\
\Leftrightarrow \quad & \phi^{CB} && \leq \frac{(\beta-\eta)(\beta+\eta)}{\beta} \\
\Rightarrow \quad & \eta && \leq \beta, \text{ if } \phi^{CB} \geq 0
\end{aligned}
$$

So, we have the following result:

Proposition 4. *(Existence of equilibrium with costly banking) If $\eta \leq \min\{\beta, \mu-\beta\}$, there exists an equilibrium with costly banking.*

When $\eta < \mu - \beta$, we have a unique equilibrium with $\phi^{CB} = \frac{\mu(\beta-\eta)^2}{2(\mu-\eta)^2}$, $r = \frac{\mu-\eta}{\beta} - 1 > 0$, and $\bar{R}_i^{CB} = \frac{\beta-\eta}{\mu-\eta}$. Fraction $\frac{(\beta-\eta)(2\mu-\beta-\eta)}{2(\mu-\eta)^2} > 0$ of entrepreneurs are borrowers.

When $\mu = \beta+\eta$, we have a continuum of equilibria with any $\phi^{CB} \in [\frac{(\beta+\eta)(\beta-\eta)^2}{2\beta^2}, \frac{(\beta-\eta)(\beta+\eta)}{\beta}]$, $r = 0$, and $\bar{R}_i^{CB} = 1 - \frac{\eta}{\beta}$. Corresponding to these equilibria, the equilibrium fraction of borrowers is $\max\{\bar{R}^{CB} - \frac{\phi^{CB}}{\beta}, 0\} \in [0, \frac{(\beta-\eta)(\beta+\eta)}{2\beta^2}]$. These equilibria are equivalent in terms of the allocation of ideas but are not payoff equivalent due to the fixed cost of borrowing. With the highest value of money (i.e. $\phi^{CB} = \frac{(\beta-\eta)(\beta+\eta)}{\beta}$), no entrepreneurs are liquidity constrained and thus no fixed cost is incurred. As the value of money goes down, more and more entrepreneurs are liquidity constrained and higher fixed cost is incurred.

Now, we take a look at the conditions for the existence of an equilibrium with costly banking (i.e. $\eta \leq \min\{\beta, \mu-\beta\}$). Firstly, if $\beta < \eta$, the payoff of getting an idea ($\beta R_e = \beta$) is lower than the fixed cost (η), and thus no entrepreneurs want to borrow even when the price of the idea is zero. Secondly, note that the *net* real rate of return of buying money in the CM is

$$
\begin{aligned}
& \frac{\beta(1+r)+\eta}{\mu} - 1 \\
= \quad & \frac{\beta r + (\beta+\eta-\mu)}{\mu}.
\end{aligned}
$$

Since $r \geq 0$, if $\eta > \mu - \beta$, then the real rate of return is always positive, implying entrepreneurs would demand an infinite amount of money in the CM.

Case 2: $\phi\tilde{m}_e \in (\bar{R}_2\mu, \infty)$

As shown in figure (7), there is no borrowing in this equilibrium, implying $r = 0$, and accordingly $\bar{R}_1 = 1$. The equilibrium allocation is exactly the same as an equilibrium without banking: entrepreneurs with $R_i \leq \frac{\phi}{\mu}$ will trade, others will save in the bank at a zero interest rate. In this case, the equilibrium price of money is ϕ^{NB} which is derived in section 3. No entrepreneur has an incentive to borrow at a zero rate when the surplus from trade cannot cover the fixed cost of borrowing for the entrepreneur drawing $R_i = \frac{\phi^{NB}}{\mu}$. As shown in the Appendix, this equilibrium exists when

$$(\eta + \beta - \mu)(\beta - \eta) > 0 \tag{19}$$

<u>Inflation, Banking and Welfare</u>

Proposition 5. *(Banking and Trading) If $\eta \leq min\{\beta, \mu - \beta\}$, then $\bar{R}_i^{NB} \leq \bar{R}_i^{CB} \leq \bar{R}_i^{B}$.*

Less ideas are traded with costly banking than with costless banking. More ideas are traded in an equilibrium with banking than in an equilibrium without banking.

Apparently, $\phi^B > \phi^{CB}$ because $\frac{d\phi^{CB}}{d\eta} < 0$. Considering the price ϕ as a function of μ (i.e. $\phi^{NB}(\mu) = 2\mu - \frac{\mu^2}{\beta}$ and $\phi^{CB}(\mu) = \frac{\mu(\beta-\eta)^2}{2(\mu-\eta)^2}$), we have $\phi^{NB}(\beta + \eta) > \phi^B(\beta + \eta)$ and $\phi^B(2\beta) > \phi^{NB}(2\beta) = 0$. Since $\phi^{CB}(.)$ and $\phi^{NB}(.)$ are strictly decreasing and continuous in μ, we have the following result:

Proposition 6. *(Value of money with costly banking) There exists a unique $\mu^* \in (\beta+\eta, 2\beta)$ such that $\phi^{NB}(\mu) \gtreqless \phi^{CB}(\mu)$ for $\mu \lesseqgtr \mu^*$.*

While banking can increase trades, it also incurs the fixed cost. We can measure the welfare by the average utility of entrepreneurs and innovators. As before, when there is no banking, the welfare is

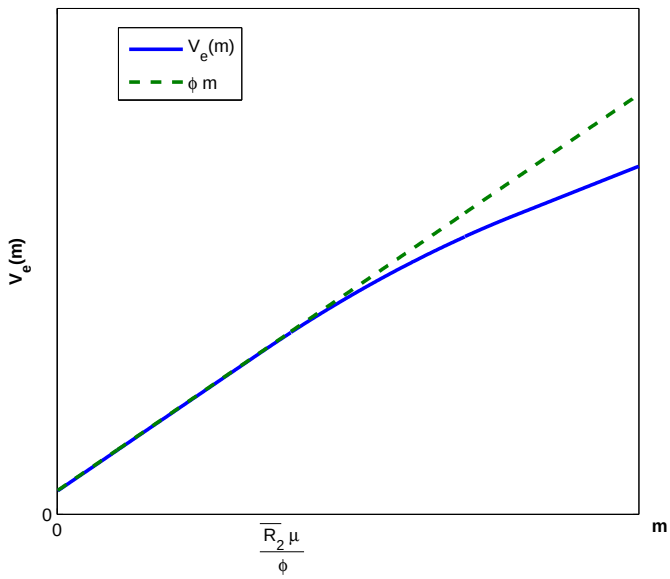

Figure 8: Value Function of an Entrepreneur in DM

$$W^{NB}(\mu) = 2(U(\bar{X}) - \bar{X}) + 1 - \frac{(1 - \bar{R}_i^{NB})^2}{2}$$

When there is costly banking, the welfare is

$$W^{CB}(\mu) = 2(U(\bar{X}) - \bar{X}) + 1 - \frac{(1 - \bar{R}_i^{CB})^2}{2} - \eta(\bar{R}_i^{CB} - \frac{\phi^{CB}}{\mu})$$

The last term captures the total fixed cost which is the product of fixed cost (η) and the number of borrowers ($\bar{R}_i^{CB} - \frac{\phi^{CB}}{\mu}$).

Now, we compare the steady state welfare between economies with different money growth rates, μ. We have shown that, for $\mu \in [\beta, \beta + \eta)$, banking is not viable and thus the welfare is given by W^{NB}. As discussed above, when $\mu = \beta + \eta$, there is a continuum of banking equilibria with different welfare levels. All these equilibria support the same amounts of trades ($\bar{R}^{CB} = \bar{R}^{NB}$) but incur different amounts of fixed cost. There is a "high welfare equilibrium" associated with a high value of money and zero fixed cost. There is also a "low welfare equilibrium" associated with a low value of money and positive fixed cost. The welfare level of the "low welfare equilibrium" is given by $W^{CB}(\beta + \eta)$. It is easy to show

that $W^{CB}(\beta+\eta) < W^{NB}(\beta+\eta)$. By the continuity of W^{NB} and W^{CB}, for sufficiently small $\Delta > 0$, we still have $W^{CB}(\beta+\eta+\Delta) < W^{NB}(\beta+\eta+\Delta)$. Therefore, for moderate inflation, even though banking is viable, it is not efficient. An economy without banking can achieve a higher welfare.

The situation is different when the inflation rate is high. In particular, when $\mu = 2\beta$, $\bar{R}_i^{NB} = 0$ and $W^{NB}(2\beta) = 2(U(\bar{X}) - \bar{X}) + \frac{1}{2}$. The welfare level in a banking equilibrium is

$$\begin{aligned}
W^{CB}(2\beta) &= 2(U(\bar{X}) - \bar{X}) + 1 - \frac{(1-\bar{R}_i^{CB})^2}{2} - \eta(\bar{R}_i^{CB} - \frac{\phi^{CB}}{\mu}) \\
&= 2(U(\bar{X}) - \bar{X}) + 1 - \frac{\beta^2}{2(2\beta-\eta)^2} - \eta\frac{(\beta-\eta)(2\beta-\eta)}{2(2\beta-\eta)^2} \\
&> 2(U(\bar{X}) - \bar{X}) + 1 - \frac{\beta^2}{2(2\beta-\eta)^2} - \frac{(\beta-\eta)(2\beta-\eta)}{2(2\beta-\eta)^2} \\
&= 2(U(\bar{X}) - \bar{X}) + 1 - \frac{\beta^2}{2(2\beta-\eta)^2} - \frac{3\beta^2 - 4\beta\eta + \eta^2}{2(2\beta-\eta)^2} \\
&= 2(U(\bar{X}) - \bar{X}) + \frac{1}{2} \\
&= W^{CB}(2\beta).
\end{aligned}$$

Therefore, we have the following result:

Proposition 7. *(Inflation and Welfare) For any $\eta \in (0, \min\{\beta, \mu-\beta\})$, there exists $\overline{\Delta}_1, \overline{\Delta}_2 > 0$ such that*

(i) $W^{CB} < W^{NB}$ for any $\mu \in [\beta+\eta, \beta+\eta+\overline{\Delta}_1]$,

(ii) $W^{CB} > W^{NB}$, for any $\mu \in [2\beta - \overline{\Delta}_2, +\infty]$.

Banking has both positive and negative effects on welfare. On the negative side, use of banking incurs a fixed cost which is a deadweight loss to society. On the positive side, banking improves the allocation of ideas. Note that the improvement of welfare depends on the inflation rate. When the inflation is low, most of the ideas are efficiently allocated even without banking, thus the gain from trading those remaining ideas is small. In this case, the welfare improvement from better allocation of ideas is outweighed by the deadweight loss.

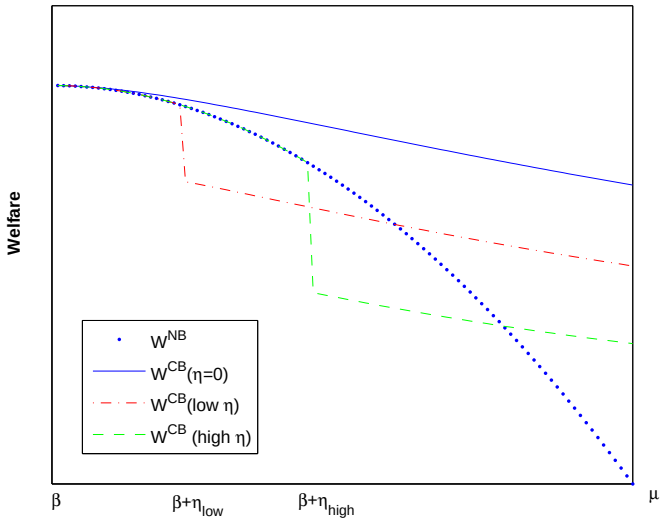

Figure 9: Welfare With and Without Costly Banking

When the inflation is high, most of the ideas are not traded without banking. In this case, the welfare improvement from better allocation of ideas outweighs the deadweight loss. This is illustrated in figure (9).

Now, we study the welfare effect of changing the fixed cost.

Proposition 8. *(Fixed Cost and Welfare) For any $\mu \in (\beta, 2\beta)$, there exists $\widetilde{\Delta}_1, \widetilde{\Delta}_2 > 0$ such that*

(i) $W^{CB} > W^{NB}$, for any $\eta \in [0, \widetilde{\Delta}_1]$,

(ii) $W^{CB} < W^{NB}$, for any $\eta \in [\mu - \beta - \widetilde{\Delta}_2, \mu - \beta]$.

(iii) For $\mu = 1$, there exists an $\bar{\eta} \in (0, 1 - \beta)$ such that,

$$\begin{cases} W^{CB} > W^{NB} & if\ \eta < \bar{\eta} \\ W^{CB} = W^{NB} & if\ \eta = \bar{\eta} \\ W^{CB} < W^{NB} & if\ \eta > \bar{\eta} \end{cases}$$

Proposition 2 implies that, when the fixed cost is zero, banking can always improve welfare. By continuity, banking can improve welfare for small fixed costs. Moreover, for

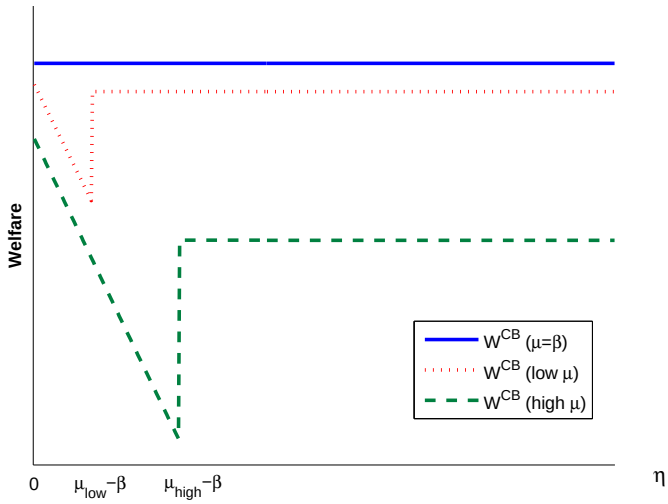

Figure 10: Effect of Fixed Cost on Welfare

a fixed cost sufficiently large relative to the inflation rate, the deadweight loss of banking outweighs the gain from a better allocation of ideas.

Figure (9) plots W^{NB} and various W^{CB} for different money growth rates. Figure (10) plots W^{NB} and various W^{CB} for different sizes of fixed cost. Figure (11) shows the distribution of outcomes for different combinations of the fixed cost and money growth rate. In figure (11), we can see that banking is used only when two conditions are satisfied: (i) $\eta < \beta$ (indicated by the vertical line) and (ii) $\eta < \mu - \beta$ (indicated by the upward-sloping straight line). Also, the curve indicating $W^B = W^{NB}$ is concave because, when there is no banking, the marginal distortion of inflation is increasing in money growth rate. It is interesting to compare this result with Bencivenga and Camera (2007). In their model, banking potentially can also reduce welfare, but this suboptimal outcome cannot be supported in equilibrium because banking is modeled as an optimal contract.

Figure (12)-(14) show the equilibrium effect of money growth on the interest rate, the price of money, the amount of ideas traded, the total fixed cost and the welfare.

Proposition 9. *(Banking and Welfare Cost of Inflation) For any* $\eta \in (0, \min\{\beta, \mu - \beta\})$, $|\frac{d}{d\mu}W^{CB}| < |\frac{d}{d\mu}W^{NB}|$.

This proposition suggests that, whenever banking is used, inflation is less harmful even

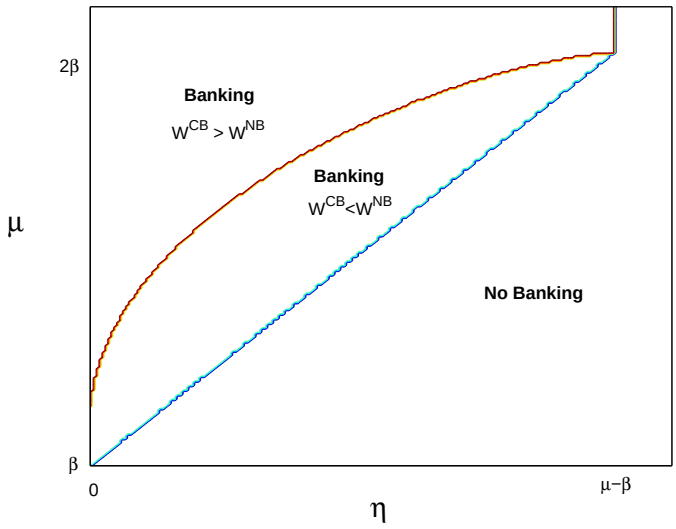

Figure 11: Distribution of Equilibrium

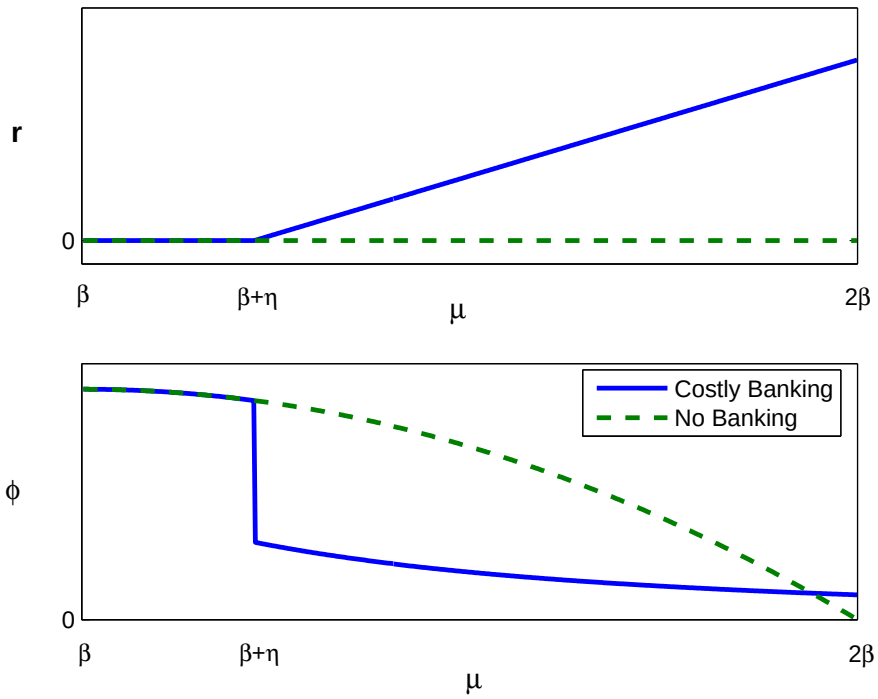

Figure 12: Effect of Money Growth on (i) r and (ii) ϕ

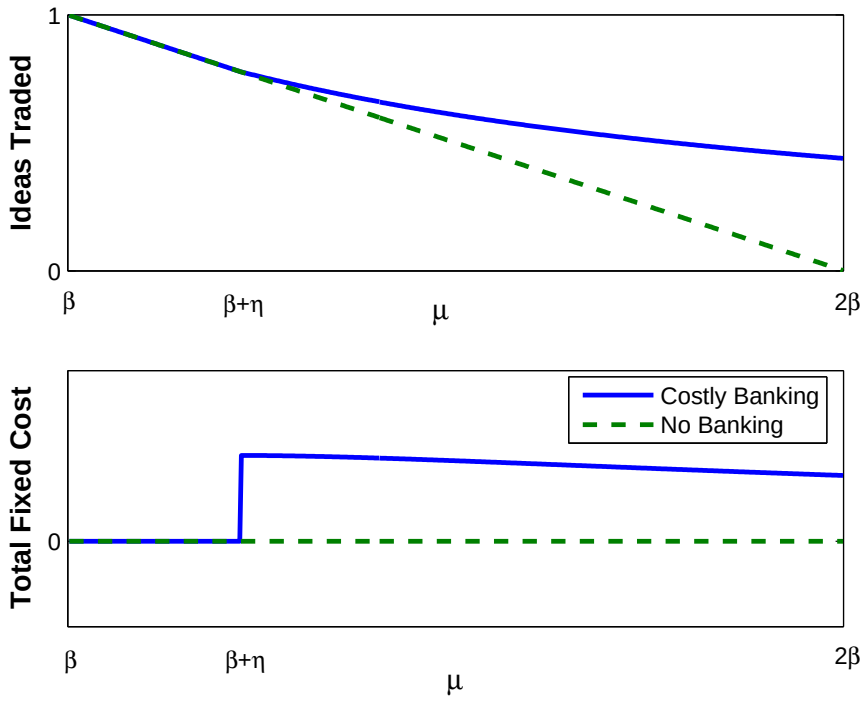

Figure 13: Effect of Money Growth on (i) Ideas traded and (ii) Total Fixed Cost

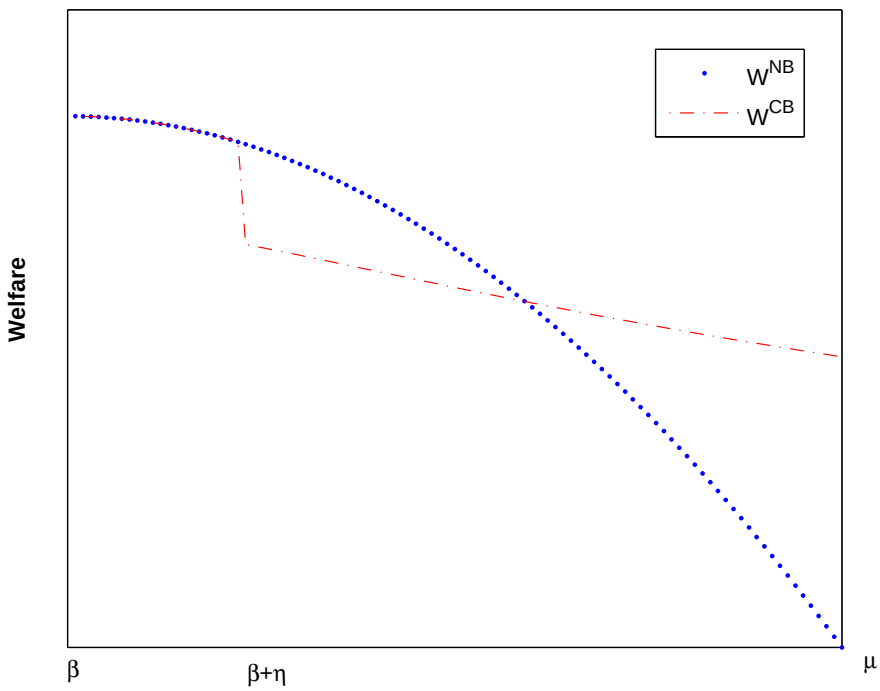

Figure 14: Effect of Money Growth on Welfare

when banking is costly. One implication of this finding is that, measuring the welfare cost by extrapolating observable data from high inflation region (far from the Friedman rule) to unobservable low inflation region (close to the Friedman rule), we may underestimate the actual welfare cost of inflation because this approach ignores the intermediation cost involved in using banking to solve liquidity problem.

6 Conclusion

This paper develops a search-theoretical model to study how money and banking interact to affect allocation and welfare. We highlight that banking and monetary models need to be studied together for properly assessing the welfare effect of banking and the welfare cost of inflation. An interesting implication of our model is that, due to general equilibrium feedback, banking can exist in equilibrium even when it is welfare-reducing. Moreover, the non-linear welfare effect of inflation implies that measuring welfare cost of inflation by extrapolating historical data may underestimate the actual cost.

References

Bencivenga, Valerie and Gabriele Camera. (2007). "Banks and Liquidity Insurance in a Matching Model of Money," Working Paper.

Berentsen,Aleksander, Gabriele Camera and Christopher Waller (2006), "Money, Credit and Banking," IEW - Working Papers iewwp219, Institute for Empirical Research in Economics - IEW.

He, Ping, Huang, Lixin and Wright, Randall D. (2005), "Money and Banking in Search Equilibrium," International Economic Review, Vol. 46, No. 2, pp. 637-670, May.

Silviera, Rafael and Randall Wright. (2007). "The Market for Ideas," Working Paper, University of Pennsylvania.

7 Appendix

Proof of Proposition 4

We first derive the value function V_e by evaluating its value over three regions: $m \in [0, \frac{\bar{R}_2\mu}{\phi}], (\frac{\bar{R}_2\mu}{\phi}, \frac{\bar{R}_1\mu}{\phi}]$ and $(\frac{\bar{R}_1\mu}{\phi}, \infty)$:

(1) For $m \in [0, \frac{\bar{R}_2\mu}{\phi}]$:

$$
\begin{aligned}
V_e^1(m) &= \int_0^1 V_e(m, R_i)dR_i \\
&= \beta W_e(0,0) + \beta \int_0^{\frac{\phi}{\mu}m} 1 + \frac{\phi}{\mu}(1+r)(m - \frac{R_i\mu}{\phi})dR_i + \beta \int_{\frac{\phi}{\mu}m}^{\bar{R}_2} 1 + \frac{\phi}{\mu}(1+r)(m - \frac{R_i\mu}{\phi}) - \frac{\eta}{\beta}dR_i \\
&\quad + \beta \int_{\bar{R}_2}^1 \frac{\phi}{\mu}(1+r)mdR_i \\
&= \beta W_e(0,0) + \beta \int_0^{\frac{\phi}{\mu}m} 1 + \frac{\phi}{\mu}(1+r)(m - \frac{R_i\mu}{\phi})dR_i + \beta \int_{\frac{\phi}{\mu}m}^{\bar{R}_2} 1 + \frac{\phi}{\mu}(1+r)(m - \frac{R_i\mu}{\phi}) - \frac{\eta}{\beta}dR_i \\
&\quad + \beta \int_{\bar{R}_2}^1 1 + \frac{\phi}{\mu}(1+r)(m - \frac{\bar{R}_2\mu}{\phi}) - \frac{\eta}{\beta}dR_i \\
&= \beta(W_e(0,0) + 1 + \frac{\phi}{\mu}(1+r)m) - \beta \int_0^{\bar{R}_2} (1+r)R_i dR_i - \beta \int_{\bar{R}_2}^1 (1+r)\bar{R}_2 dR_i - \int_{\frac{\phi}{\mu}m}^1 \eta dR_i \\
&= W_e(0,0) + \beta - \beta \int_0^{\bar{R}_2} (1+r)R_i dR_i - \beta \int_{\bar{R}_2}^1 (1+r)\bar{R}_2 dR_i - \eta(1 - \frac{\phi}{\mu}m) + \beta\frac{\phi}{\mu}(1+r)m \\
&= \beta W_e(0,0) + \beta - \frac{\beta(1+r)\bar{R}_2^2}{2} - \beta\bar{R}_2(1+r)(1-\bar{R}_2) - \eta + [\beta\frac{\phi}{\mu}(1+r) + \eta\frac{\phi}{\mu}]m
\end{aligned}
$$

(2) For $m \in (\frac{\bar{R}_2\mu}{\phi}, \frac{\bar{R}_1\mu}{\phi}]$:

$$
\begin{aligned}
V_e^2(m) &= \int_0^1 V_e(m, R_i)dR_i \\
&= \beta W_e(0,0) + \beta \int_0^{\frac{\phi}{\mu}m} 1 + \frac{\phi}{\mu}(1+r)(m - \frac{R_i\mu}{\phi})dR_i + \beta \int_{\frac{\phi}{\mu}m}^1 \frac{\phi}{\mu}(1+r)mdR_i \\
&= \beta W_e(0,0) + \beta\frac{\phi}{\mu}(1+r)m + \beta \int_0^{\frac{\phi}{\mu}m} 1 - (1+r)R_i dR_i \\
&= \beta W_e(0,0) + \beta\frac{\phi}{\mu}(1+r)m + \beta\frac{\phi}{\mu}m - \beta(1+r)\frac{(\frac{\phi}{\mu}m)^2}{2}
\end{aligned}
$$

(3) For $m \in (\frac{\bar{R}_1\mu}{\phi}, \infty)$:

$$
\begin{aligned}
V_e^3(m) &= \int_0^1 V_e(m, R_i)dR_i \\
&= \beta W_e(0,0) + \beta \int_0^{\bar{R}_1} 1 + \frac{\phi}{\mu}(1+r)(m - \frac{R_i}{\frac{\phi}{\mu}})dR_i + \beta \int_{\bar{R}_1}^1 \frac{\phi}{\mu}(1+r)mdR_i \\
&= \beta W_e(0,0) + \beta \int_0^{\bar{R}_1} 1 + \frac{\phi}{\mu}(1+r)(m - \frac{R_i}{\frac{\phi}{\mu}})dR_i + \beta \int_{\bar{R}_1}^1 1 + \frac{\phi}{\mu}(1+r)(m - \frac{\bar{R}_1}{\frac{\phi}{\mu}})dR_i \\
&= \beta W_e(0,0) + \beta + \beta\frac{\phi}{\mu}(1+r)m - \beta(1+r)\int_0^{\bar{R}_1} R_i dR_i - \beta(1+r)\bar{R}_1(1-\bar{R}_1) \\
&= \beta W_e(0,0) + \beta + \beta\frac{\phi}{\mu}(1+r)m - \beta(1+r)\frac{\bar{R}_1^2}{2} - \beta(1+r)\bar{R}_1(1-\bar{R}_1)
\end{aligned}
$$

We now show that, in the proposed equilibrium, the optimal money holding stays in the interval $[0, \frac{\bar{R}_2\mu}{\phi}]$. In particular, we will show:

(1) $V_e^1(m) - \phi m = k$ for $m \in [0, \frac{\bar{R}_2\mu}{\phi}]$ where k is a positive constant

(2) $V_e^2(m) - \phi m < k$ for $m \in (\frac{\bar{R}_2\mu}{\phi}, \frac{\bar{R}_1\mu}{\phi}]$

(3) $V_e^3(m) - \phi m < k$ for $m \in (\frac{\bar{R}_1\mu}{\phi}, \infty)$

First, consider the equilibrium with $r > 0$. Using the result that $\bar{R}_1 = \frac{\beta}{\mu-\eta}$, $\bar{R}_2 = \frac{\beta-\eta}{\mu-\eta}$, $\phi = \frac{\mu(\beta-\eta)^2}{2(\mu-\eta)^2}$, and $1 + r = \frac{\mu-\eta}{\beta}$, we can simplify the V_e derived above to get the followings:

(1) $V_e^1(m) - \phi m$

$$
\begin{aligned}
&= \beta(W_e(0,0)+1) - \frac{\beta(1+r)\bar{R}_2^2}{2} - \beta\bar{R}_2(1+r)(1-\bar{R}_2) - \eta + [\beta\frac{\phi}{\mu}(1+r) + \eta\frac{\phi}{\mu}]m - \phi m \\
&= \beta(W_e(0,0)+1) - \frac{\beta(1+r)\bar{R}_2^2}{2} - \beta\bar{R}_2(1+r)(1-\bar{R}_2) - \eta + \phi m - \phi m \\
&= \beta(W_e(0,0)+1) - \frac{\beta(1+r)\bar{R}_2^2}{2} - \beta\bar{R}_2(1+r)(1-\bar{R}_2) - \eta \\
&= \beta W_e(0,0) + \beta - \frac{\beta^2+\eta^2-2\beta\eta}{2(\mu-\eta)} - \frac{(\beta-\eta)(\mu-\beta)}{\mu-\eta} - \eta \\
&= \beta W_e(0,0) + \frac{(\beta-\eta)^2}{2(\mu-\eta)} = k > 0
\end{aligned}
$$

(2) Similarly, we get $V_e^2(m) - \phi m = \beta W_e(0,0) + \frac{(\beta-\eta)^2}{2(\mu-\eta)^2}\left[(\beta-\eta)m - \frac{(\beta-\eta)^2m^2}{4(\mu-\eta)}\right]$. We can show that $V_e^2 - \phi m$ is strictly concave and attains its maximum at $m = \frac{2(\mu-\eta)}{(\beta-\eta)}$ (which is the lower bound of region 2), with the maximum equals to k. Therefore, $V_e^2(m) - \phi m < k$ for all $m \in (\frac{\bar{R}_2\mu}{\phi}, \frac{\bar{R}_1\mu}{\phi}]$.

(3) First, note that $V_e^3(m)-\phi m$ is linear and strictly decreasing with $\frac{d}{dm}[V_e^3(m)-\phi m] = -\frac{\phi}{\mu}\eta < 0$. So, for any $m \in (\frac{\bar{R}_1\mu}{\phi}, \infty)$, $V_e^3(m) - \phi m$ is lower than $V_e^3(\frac{\bar{R}_1\mu}{\phi}) - \phi(\frac{\bar{R}_1\mu}{\phi}) = W_e(0,0) + \frac{-2\beta\eta+\beta^2}{2(\mu-\eta)}$ which is lower than k if $\frac{\eta^2}{2(\mu-\eta)} > 0$.

Now, we consider the equilibrium with $r = 0$. (16) implies that $\mu = \beta + \eta$. We can follow the same analysis as above to show that $V_e^2(m) - \phi m$ is maximized at $m = \frac{\bar{R}_2\mu}{\phi}$ which is equal to $V_e^1(m) - \phi m$ for any $m \in [0, \frac{\bar{R}_2\mu}{\phi}]$. Also, $r = 0$ implies $\bar{R}_1 = 1$, so the third region vanishes.

So, we have proved that $\arg max_m V_e(m) - \phi m \in [0, \frac{\bar{R}_2\mu}{\phi}]$, indeed in equilibrium an entrepreneur is indifferent between any m in this interval. Now we need to check that this is not an empty set, that is $\bar{R}_2 \geq 0$ which requires $\eta \leq \beta$. Finally, $r \geq 0$ requires $\eta \leq \mu - \beta$.

Proof of Condition (19)

We want to show that $\arg\max_m V_e(m) - \phi m > \frac{\bar{R}_2\mu}{\phi}$ when $r = 0$ and condition (19) is satisfied. First, it is easy to show that, when $r = 0$, $V_e^2(m) - \phi m$ in the previous proof attains its global maximum at $m = \frac{\bar{R}_2(\beta+\eta)}{\phi}$. Then, to show that choosing $m \leq \frac{\bar{R}_2\mu}{\phi}$ is not optimal (where V_e^1 is the corresponding value function), note that $V_e^2(\frac{\bar{R}_2\mu}{\phi}) - \phi\frac{\bar{R}_2\mu}{\phi} = V_e^1(m) - \phi m$ for all $m \leq \frac{\bar{R}_2\mu}{\phi}$. Therefore, we just need to show that $\frac{\bar{R}_2(\beta+\eta)}{\phi} > \frac{\bar{R}_2\mu}{\phi}$ which is equivalent to (19).

Proof of Proposition 5

$\bar{R}^B = \frac{\beta}{\mu} \geq \bar{R}^{CB} = \frac{\beta-\eta}{\mu-\eta}$ is obvious. Also, $\bar{R}^{CB} = \frac{\beta-\eta}{\mu-\eta} \geq \bar{R}^{NB} = 2 - \frac{\mu}{\beta}$ if $(\mu-\beta)(\beta-\mu+\eta) \leq 0$.

Proof of Proposition 8

First, $W^{NB}(\eta) < W^{CB}(\eta)$ for $\eta = 0$.

Second, $W^{NB}(\eta) > W^{CB}(\eta)$ for $\eta = \mu - \beta$

Finally, for $\mu = 1$, sign$(W^{CB} - W^{NB})$ =sign$(D(\eta))$ where $D(\eta) = (1-\beta)^2((1-\eta)^2 - \beta^2) - \eta(\beta-\eta)\beta^2(2-\eta-\beta)$. From above, we know already that $D(0) > 0$ and $D(1-\beta) < 0$. Also, we can show that $\frac{dD}{d\eta}(1-\beta) < 0$ and $\frac{d^2D}{d\eta^2} > 0$, implying that $\frac{dD}{d\eta} < 0$ for $\eta \in (0, 1-\beta)$. Therefore, there exists a cutoff $\bar{\eta}$ such that $W^{NB} = W^{CB}$.

Proof of Proposition 9

For any $\eta \in (0, \min\{\beta, \mu-\beta\})$, $|\frac{d}{d\mu}W^{CB}| < |\frac{d}{d\mu}W^{NB}|$.

The welfare effect of inflation when there is costly banking is given by

$$\frac{d}{d\mu}W^{CB}$$
$$= -\frac{d}{d\mu}\left[\frac{(1-\bar{R}_i^{CB})^2}{2}\right] - \frac{d}{d\mu}\left[\eta(\bar{R}_i^{CB} - \frac{\phi^{CB}}{\mu})\right]$$
$$= \frac{d\bar{R}_i^{CB}}{d\mu}(1-\bar{R}_i^{CB}) - \frac{d}{d\mu}\left[\eta\frac{(\beta-\eta)(2\mu-\beta-\eta)}{2(\mu-\eta)^2}\right]$$

$= \frac{d}{d\mu}\left[\frac{\beta-\eta}{\mu-\eta}\right](1-\frac{\beta-\eta}{\mu-\eta}) - \eta(\beta-\eta)\left[\frac{(\mu-\eta)-(2\mu-\beta-\eta)}{(\mu-\eta)^3}\right]$
$= -\frac{(\beta-\eta)(\mu-\beta)}{(\mu-\eta)^3} - \eta(\beta-\eta)\left[\frac{(\mu-\eta)-(2\mu-\beta-\eta)}{(\mu-\eta)^3}\right]$
$= -\frac{(1-\eta)(\beta-\eta)(\mu-\beta)}{(\mu-\eta)^3}$

The welfare effect of inflation when there is no banking is given by

$\frac{d}{d\mu}W^{NB}$
$= -\frac{d}{d\mu}\left[\frac{(1-\bar{R}_i^{NB})^2}{2}\right]$
$= \frac{d\bar{R}_i^{NB}}{d\mu}(1-\bar{R}_i^{NB})$
$= \frac{d}{d\mu}\left[2-\frac{\mu}{\beta}\right](1-\bar{R}_i^{NB})$
$= -\frac{\mu-\beta}{\beta^2}$

So the welfare effect of inflation is smaller with banking when

$|\frac{d}{d\mu}W^{CB}| < |\frac{d}{d\mu}W^{NB}|$
$\Leftrightarrow \frac{(1-\eta)(\beta-\eta)(\mu-\beta)}{(\mu-\eta)^3} < \frac{\mu-\beta}{\beta^2}$
$\Leftrightarrow \frac{(1-\eta)(\beta-\eta)}{(\mu-\eta)^3} < \frac{1}{\beta^2}$, since $\mu-\beta>0$
$\Leftrightarrow (1-\eta)(\beta-\eta) < \frac{(\mu-\eta)^2}{\beta^2}(\mu-\eta)$, since $\mu-\eta>\beta$

which is true since

$(1-\eta)(\beta-\eta) < \beta < \frac{(\mu-\eta)^2}{\beta^2}(\mu-\eta)$