

# Inflation and Unemployment in the Long Run\*

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## **Abstract**

We study the long-run relation between money, measured by inflation or interest rates, and unemployment. We first discuss data, documenting a strong positive relation between the variables at low frequencies. We then develop a framework where both money and unemployment are modeled using explicit microfoundations, integrating and extending recent work in macro and monetary economics, and providing a unified theory to analyze labor and goods markets. We calibrate the model, to ask how monetary factors account quantitatively for low-frequency labor market behavior. The answer depends on two key parameters: the elasticity of money demand, which translates monetary policy to real balances and profits; and the value of leisure, which affects the transmission from profits to entry and employment. For conservative parameterizations, money accounts for some but not that much of trend unemployment – by one measure, about 1/5 of the increase during the stagflation episode of the 70s can be explained by monetary policy alone. For less conservative but still reasonable parameters, money accounts for almost all low-frequency movement in unemployment over the last half century.

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# 1 Introduction

Since it is very relevant for what we do in this project, we begin by reviewing an exercise in Lucas (1980). He was interested in two fundamental propositions from monetary economics: the quantity equation, which can be interpreted as saying that (other things equal) inflation moves one-for-one with the growth rate in the money supply; and the Fisher equation, which can be interpreted as saying that (other things equal) the nominal interest rate moves one-for-one with inflation.<sup>1</sup> These relations are derived from elementary economic principles, and are almost ‘model free’ in the sense e.g. that the quantity equation emerges from a variety of formalizations, and the Fisher equation is basically a no-arbitrage condition. This does not mean they are consistent with data. Indeed, as Lucas emphasized, one ought not expect them to hold at each point in time since there may be a lot going on to complicate matters in the short run; yet they may still be useful ideas if they are consistent with longer-run observations.

To investigate this, Lucas plotted inflation vs. the growth rate of  $M1$ , using annual data, from 1955-1975, which we reproduce in the upper left panel of Figure 1.1, except using quarterly data, and extended to 2005.<sup>2</sup> Although the simple regression line slopes upwards, it is not that easy to see the quantity equation in the picture – but, again, there may be a lot going on at high frequencies to obscure the relation. So Lucas filtered the data, using progressively stronger filters to remove more and more of the short-run ‘noise.’ We do the same in the other panels of Figure 1.1, using HP filters with a parameter varying

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<sup>1</sup>Lucas actually looked not at the Fisher equation per se, but the relation between money growth and nominal rates. If the quantity equation is correct, this amounts to the same thing, but in any case we look at both.

<sup>2</sup>All figures are at the end of the paper. Also, we actually put together data for all variables discussed below going back to 1948, but focus on the sample starting in 1955 for three reasons: this is where Lucas started; it gives us exactly a half century of data, which is a nice round number; and certain series like inflation seem especially erratic in the late 40s and early 50s. But we are not trying to hide anything – results for the full data set are at <http://www.wz.wz.unibas.ch/witheo/aleks/BMWII/BMWII.html>.

from 0 to 160,000 as indicated on each panel (Lucas used moving average filters, but nothing hinges on this detail). As one can plainly see, with progressively stronger filtering, a distinct pattern emerges, and eventually it appears that the quantity equation looks really quite good.

This finding is robust on several dimensions. One can e.g. look at five-year averages, a different way to filter the data, shown in the final panel of Figure 1.1, and the message is the same. Or one can measure variables in different ways, as we do in Figures 1.2 and 1.3, e.g., where we replace  $M1$  by  $M2$  and by  $M0$ , and the picture is similar. One can also redo the exercise in levels (looking at  $p$  vs.  $M$  rather than growth in  $p$  and  $M$ ) and the results are similar.<sup>3</sup> In terms of the Fisher equation, Figure 1.4 plots inflation vs. the nominal interest rate using Aaa corporate bonds to define the nominal rate (the conclusions are similar using e.g. the T-Bill rate). After we filter out the ‘noise’ the Fisher equation also looks very good. Figures 1.5 makes a similar point when we replace inflation by  $M1$  growth (results for  $M2$  and  $M0$  are similar). Just as Lucas concluded from his exercise, we conclude from this that the ideas represented by the quantity and Fisher equation hold up quite well in long-run data.

Lucas warns us, however, that the method is risky. Take any two series, he says, plot progressively stronger filtered versions, and one will see patterns emerge. To illustrate his point Lucas does the exercise for inflation and unemployment, two variables that he ‘knew’ were unrelated at low frequency, in the sense that he was persuaded by the arguments of Friedman (1967) and Phelps (1969) that the long-run Phillips curve must be vertical (although he does say this explicitly, it seems from related work such as Lucas 1973 he bought into the idea of a ‘natural rate’ independent of inflation). Lo and behold, with progressive filtering, a pattern between inflation and unemployment emerged when

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<sup>3</sup>In the interest of space we do not show all the figures here; go to the address in footnote 2 for additional figures and much more information, including details of the data sources, calibration and simulation programs, etc.

Lucas did it, and emerges even more obviously when we redo it with updated data. As Figure 1.6 shows, contrary to what Lucas thought he ‘knew’ from theory, inflation and unemployment are related in the long run, and positively.

While we like his method for extracting information about long-run relations, we do not agree with all of Lucas’ conclusions. In terms of method, we are persuaded that this filtering technique, while not perfect, is useful – in particular, while there is no guarantee that forces driving the short-run deviations are irrelevant for understanding the true long-run relation, the approach does have the virtue of allowing one to avoid taking a stand on exactly what the forces are behind the high frequencies. Where we think Lucas went wrong is his devotion to a vertical long-run Phillips curve. We find the evidence of a positive relation between inflation and unemployment about as clear as the evidence for the quantity or Fisher equation, and based on this data there seems little reason to deem one observation compelling and another statistical artifact; moreover, we will argue, a positive long-run relation between these variables is as much “an implication of a coherent economic theory” as Lucas said the other relations are.

We are not the first to suggest this, and Friedman (1977) himself was trenchant when he said the following: “There is a natural rate of unemployment at any time determined by real factors. This natural rate will tend to be attained when expectations are on average realized. The same real situation is consistent with any absolute level of prices or of price change, *provided allowance is made for the effect of price change on the real cost of holding money balances*” (emphasis added). He also noted that in the data he was examining at the time one could see emerging evidence of an upward slope to the long run Phillips curve (others have discussed similar points; see e.g. Beyer and Farmer 2007 and the references therein). Again, we will show here that basic economic theory predicts such a pattern just as clearly as the data depicts such a pattern.

Before proceeding we mention some more evidence. In principle, if the Fisher and quantity equations are valid, it does not matter if we examine the relationship between unemployment and either inflation, interest, or money growth rates. But even if the Fisher and quantity equations hold in the longer run, they do not hold exactly. In Figures 1.7 and 1.8 we redo the exercise replacing inflation with interest and  $M1$  growth rates ( $M2$  and  $M0$  give similar results). Also, in Figures 1.9 to 1.11 we redo the exercises using employment rather than unemployment.<sup>4</sup> Based on all of this, we think it is obvious there is a negative relation between monetary variables and labor market performance in the longer run, even if things may go the other way in the shorter run, including e.g. the 60s, where a downward sloping Phillips curve is evident. While we welcome more, and more sophisticated, econometric analyses, for the purpose of this paper we take this fact as given.

As a final application of the method, and because we will need it later, Figures 1.12 to 1.14 show the relation between the nominal rate and the inverse of velocity,  $M/pY$ , commonly interpreted as money demand. The different plots use  $M1$ ,  $M2$  and  $M0$ . As has been documented many times, the relationship is negative, although it is confounded by what looks like a structural shift that occurs some time in the late 80s or early 90s, depending on which panel one looks at. Similar results obtain when we replace the nominal interest rate by the inflation or money growth rates.<sup>5</sup> In any event, we will use some version of this money demand relation in the calibration below, as is done in most quantitative monetary economics (see e.g. Cooley-Hansen 1989, Lucas 2000, Lagos-Wright 2005, and the references therein).

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<sup>4</sup>This is complicated by a long-term trend in employment over the sample, presumably due to demographic and other factors. To control for this we filter the data twice: once to eliminate the very long-run trend, and again to eliminate very high-frequency fluctuations.

<sup>5</sup>That is, the results are similar except for one detail: while  $M0/pY$  and  $M1/pY$  behave as expected, a simple regression indicates  $M2/pY$  actually rises with inflation or money growth, although this may be likely due to the structural shift mentioned above.

We now proceed to theory. Since we are primarily for this paper interested in the longer-run relation between monetary variables and unemployment, we abstract from factors commonly believed to matter in the short run, including information problems or other forms of real-nominal confusion, as well as stickiness in wages or prices. Instead we focus on Friedman’s suggestion that to understand the effect of monetary variables on the “natural rate” allowance really must be made for “the effect of price change on the real cost of holding money balances.” To this end, it seems obvious that it would be good to have a theory where the cost of holding money balances can be made precise, which suggests to us a theory where the benefits of holding money balances are made explicit. Additionally, it would seem good to have a theory of unemployment that has proven successful in other contexts.

In recent years much progress has been made studying monetary economics and unemployment using theories that incorporate frictions – in the case of unemployment, search and matching frictions; and in the case of money, some sort of double coincidence problem due to specialization and spatial separation, combined with information problems like imperfect record keeping.<sup>6</sup> It is not surprising that models with frictions are useful for understanding dynamic labor markets and hence unemployment, as well as for understanding the role of money and hence inflation. However, existing models along these lines analyze either unemployment *or* inflation in isolation. We integrate these models into a unified framework that allows one to analyze unemployment *and* money together using logically consistent microfoundations. This theory predicts that inflation and unemployment should move together.

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<sup>6</sup>In terms of unemployment, we have in mind Mortensen and Pissarides (1994), as well as Diamond (1981,1982), Mortensen (1982), Pissarides (1985,1990), Merz (1995), Andolfatto (1996), and recent contributions by Shimer (2005), Hall (2005), Hagedorn and Manovskii (2007) and others. In terms of money, we have in mind models along the lines of Lagos and Wright (2005), as well as Kiyotaki and Wright (1989,1993), Aiyagari and Wallace (1991), Matsuyama et al. (1993), Shi (1995,1997), Trejos and Wright (1995), Kochelakota (1998), Wallace (2001), Williamson (2006), Molico (2006) and others.

We then consider the issue quantitatively. To this end we calibrate the model to standard observations, including money demand, and ask how it accounts for long-run labor market observations over the last half century when we (counterfactually) assume monetary policy is the (only) impulse. This is a common method in modern macro, as epitomized e.g. when one asks of the Kydland and Prescott (1982) model how well it accounts for output fluctuations when the only impulse is a shock to productivity. As in that application, the target here is not 100%; we just want to know ‘how much?’ Although there are details to discuss, one way to summarize the findings is to ask the following question: For reasonable parameter values, how much of an increase in unemployment does the model predict from a run up in inflation or nominal interest rates like we saw during the stagflation of the 70s? The answer, we show, depends on two key parameters: the value of leisure and money demand elasticity.

For a conservatively low estimate of the money demand elasticity, if we set the value of leisure so that a real version of the model generates realistic unemployment fluctuations in response to productivity shocks, we account for only 20% of the increase in the raw unemployment series, and around 13% of filtered unemployment, during stagflation. This is nothing to scoff at, but obviously does leave plenty of room for other factors, including productivity, demographics, fiscal policy etc. However, if we set the value of leisure slightly higher, the model can account for virtually all of trend unemployment during the period, although of course it then generates excessive unemployment fluctuations in response to real shocks (about double the data). For a bigger money demand elasticity, the basic message is similar, although the model accounts for more of the data with a low value of leisure, and does not generate as excessive unemployment fluctuations in response to real shocks.

We conclude that while conservative parameter estimates imply monetary factors account for some but not the majority of trend unemployment, one does

not have to stretch parameters too far to account for much more. It should be no surprise that some parameters matter a lot for the issues at hand. That the value of leisure can make a big difference in search-based models of the labor market is very well known; see e.g. the discussion of Shimer (2005) in Hagedorn and Manovskii (2007).<sup>7</sup> That the elasticity of money demand matters a lot for the effects of inflation is equally well known; see e.g. Lucas (2000). It is to be expected therefore that both matter in the integrated model. While our results do depend on these parameters, they suggest that monetary factors may be important for labor market outcomes, not only theoretically but also quantitatively.

The rest of the paper is organized as follows. In Section 2 we describe the basic model. In Section 3 we show how to solve for equilibrium in the labor market taking the goods market as given, and vice-versa, then put things together to get general equilibrium. In the presentation in Section 3 we focus on steady states, and relegate the dynamic-stochastic case to the Appendix. Also, in Section 3 we use Nash bargaining in both goods and labor markets, because it is easy and well understood, but in Section 4 we consider different pricing mechanisms, including price taking and posting, because as we discuss there are reasons to find these alternatives appealing. In Section 5 we present the quantitative analysis. Section 6 concludes.<sup>8</sup>

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<sup>7</sup>The macro-labor literature has not yet converged on the best way to extend the baseline search model to generate realistic unemployment responses to real shocks, but everyone agrees that a high value of leisure gets the job done. It is likely that some other ‘tricks’ to generate realistic unemployment responses to real shocks would also work for us. One way to summarize this is our robust finding that an increase in inflation from 0 to 10% will have the same impact as a drop in labor productivity of between 2/3 and 3/2 of 1%, independent of how we specify the labor market parameters, whose only role is to determine how this impulse is propagated to unemployment.

<sup>8</sup>Some recent attempts to bring monetary issues to bear on search-based labor models include Farmer (2005), Blanchard and Gali (2005), and Gertler and Trigari (2006), but they take a different tact by assuming nominal rigidities. We generate interesting effects without nominal wage or price stickiness, as which seems distinctly preferable given we are interested in intermediate- to long-run phenomena. Lehmann (2006) is more in line with our approach, although details are different. Shi (1998,1999) and Shi and Wang (2006) are also worth mentioning. Rocheteau et al. (2006) and Dong (2007) integrate modern monetary

## 2 The Basic Model

Time is discrete and continues forever. Each period, there are three distinct locations where economic activity takes place: a labor market, in the spirit of Mortensen-Pissarides; a goods market, in the spirit of Kiyotaki-Wright; and a general market, in the spirit of Arrow-Debreu. For brevity we call these the MP, KW and AD markets. While it does not matter for the results, for concreteness we assume these markets convene sequentially, as shown in Figure 2. Also, without loss of generality we assume that agents discount at rate  $\beta$  between one AD market and the next MP market, but not between the other markets. There are two types of private agents, firms and households, indexed by  $f$  and  $h$ . The set of households is  $[0, 1]$ ; the set of firms has arbitrarily large measure, although not all will be active at any point in time. Households work, consume, and enjoy utility; firms simply maximize profits and pay out dividends to households.

As is standard in modern theories of unemployment, a household and a firm can combine to create a job that produces output  $y$ . Let  $e$  index employment status:  $e = 1$  indicates that a household (firm) is matched with a firm (household);  $e = 0$  indicates otherwise. For now, it is easiest to think of agents matching bilaterally in the MP and KW markets and multilaterally in AD, although we also discuss other interpretations below. As indicated in Figure 2, there are three value functions for the three markets,  $U_e^i$ ,  $V_e^i$  and  $W_e^i$ , which generally depend on type  $i \in \{h, f\}$ , employment status  $e \in \{0, 1\}$ , and possibly other state variables. Also, note  $\hat{U}_f^i$  in the Figure is the MP value function next

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economics into an alternative theory of unemployment – Rogerson’s (1988) indivisible labor model – and while that approach leads to some interesting results, there are reasons to prefer Mortensen-Pissarides. Earlier, Cooley and Hansen (1989) stuck a cash-in-advance constraint into Rogerson, as Cooley and Quadrini (2004) and Andofatto et al. (2003) did to Mortensen-Pissarides. Our framework actually nests as special case something that looks like a standard cash-in-advance model, as well as a money-in-the-utility-function model. We prefer to lay out the role of money explicitly, however, because the additional generality is useful, and also because we find it easier than having to decide based on implicit theorizing when cash-in-advance applies, or how money enters utility.

period, since a “hat” indicates the value of any variable next period.

In the benchmark model discussed in the text, we assume policy and productivity are constant, and focus on steady states; in this case, the only state variable for agents that we need to track, other than  $i$  and  $e$ , is real balances  $z$ .<sup>9</sup> We adopt the following convention for measuring real balances, which facilitates presentation of the dynamic-stochastic model (discussed in the Appendix). When an agent brings in  $m$  dollars to the AD market we let  $z = m/p$ , where  $p$  is the current price level, denote his real balances. He then takes  $\hat{z} = \hat{m}/p$  out of this market and into next period’s MP market, still deflated by  $p$ . If he were to bring  $\hat{z}$  through the next KW market and into the next AD market, its real value is then given by  $\hat{z}\hat{p}$ , where  $\hat{p} = p/\hat{p}$  converts  $\hat{z}$  into the units of  $x$  in that AD market. Notice  $\hat{p} = 1/(1 + \pi)$ , where  $\pi$  is the inflation rate between this and the next AD market.

## 2.1 Households

We now consider the different markets in turn, starting with AD. Household  $h$  with employment status  $e$  and real balances  $z$  solves

$$W_e^h(z) = \max_{x, \hat{z}} \left\{ x + \beta \hat{U}_e^h(\hat{z}) \right\} \tag{1}$$

$$\text{st } x = ew + (1 - e)(b + \ell) + \Delta - T + z - \hat{z},$$

where  $x$  is consumption,  $w$  the wage,  $b$  UI benefits,  $\ell$  production of  $x$  by the unemployed,  $\Delta$  dividend income, and  $T$  a lump sum tax. Employment status  $e$  is carried out of AD into MP next period. Notice  $w$  is paid in AD even though matching and bargaining occur in MP (this is not important, but it makes some things more transparent, as discussed below). Also, as in most of the literature

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<sup>9</sup>For matched agents, in principle, the wage  $w$  is a state, since it is set in MP and carried forward to KW and AD, although it can be renegotiated next MP. To reduce clutter in the text,  $w$  is subsumed in the notation; in the Appendix we present the general case where policy and productivity follow stochastic processes and unemployment varies endogenously over time, and these variables as well as wages are explicit state variables.

using MP models, utility is linear in  $x$ , although we have other goods traded in the KW market where agents have general utility.<sup>10</sup>

It is useful to provide a few results concerning AD before discussing the rest of the model. Substituting  $x$  from the budget equation into the objective function in (1), we get

$$W_e^h(z) = I_e + z + \max_{\hat{z}} \left\{ -\hat{z} + \beta \hat{U}_e^h(\hat{z}) \right\} \quad (2)$$

where  $I_e = ew + (1 - e)(b + \ell) + \Delta - T$  is income. Notice  $W_e^h$  is linear in  $z$  and  $I_e$ . Moreover, the choice of  $\hat{z}$  is independent of  $z$  and  $I_e$ , although it could depend on  $e$  through  $\hat{U}_e^h$ . However, we assume below that the KW utility function is independent of  $e$ , which turns out to make  $\partial \hat{U}_e^h / \partial \hat{z}$  and hence  $\hat{z}$  independent of  $e$ . This gives the convenient result that every  $h$  exits the AD market with the same  $\hat{z}$ , as long as we have an interior solution for  $x$ , which we can guarantee by assuming e.g. that  $\ell$  is not too small.

Now consider the KW market, where a different good  $q$  that gives utility  $v(\cdot)$  is traded. In this market households are anonymous, and this generates an essential role for a medium of exchange. To convey the idea, suppose  $h$  asks  $f$  for  $q$  now and promises to pay later – say, in the next AD market. If  $f$  does now know who  $h$  is, the latter can renege on such promises without fear of repercussion, so the former insists on quid pro quo. If  $x$  is not storable by  $h$ , money steps into the role of medium of exchange.<sup>11</sup> Of course, to make money essential we need only *some* anonymous trade – we need not rule out *all* barter, credit, etc. A nonmonetary version of the model with perfect credit is

<sup>10</sup>All we really need for tractability is quasi-linearity: everything goes through if we assume AD utility is  $x + \Upsilon_e(\mathbf{x})$ , where  $\mathbf{x}$  is a vector of AD goods other than  $x$ . To reduce notation we assume a single AD good in the text and discuss the general case in footnotes.

<sup>11</sup>Kocherlakota (1997), Wallace (2001), Corbae et al. (2003), Araujo (2004), and Aliprantis et al. (2006) provide formal discussions. Monetary theory generally goes into considerable detail about specialization and other features of the environment, in addition to anonymity, that give rise to a role for media of exchange, and we see no need to repeat all of that here, although we should say something about why money plays this role and not other (real) claims. One answer is to invoke additional information frictions, by assuming agents may not recognize counterfeit claims but always recognize currency, as in Lester et al. (2007).

of interest in its own right, embedding as it does a retail sector into MP. But to study unemployment and nominal variables, we focus on the monetary economy (actually, the perfect credit version is a special case when we run the Friedman rule  $i = 0$ , since this makes cash and credit equivalent).

Then, for  $h$  with real balances  $z$  and employment status  $e$  in KW,

$$V_e^h(z) = \alpha_h v(q) + \alpha_h W_e^h [\rho(z - d)] + (1 - \alpha_h) W_e^h(\rho z), \quad (3)$$

where  $\alpha_h$  is the probability of trade,  $(q, d)$  represents the terms of trade, and we multiply any real balances taken out of KW by  $\rho$  to get their value in AD, as discussed above. Using the linearity of  $W_e^h$ , following from (2), we have

$$V_e^h(z) = \alpha_h [v(q) - \rho d] + W_e^h(\rho z). \quad (4)$$

The probability  $\alpha_h$  is given for now by a general matching function  $\alpha_h = \mathcal{M}(B, S)/B$ , where  $B$  and  $S$  are the measures of buyers and sellers in the market. Assuming  $\mathcal{M}$  satisfies the usual assumptions, including CRS,  $\alpha_h = \mathcal{M}(Q, 1)/Q$  where  $Q = B/S$  is the queue length or market tightness.

As long as the surplus for  $h$  in KW is positive, all households participate and  $B = 1$ ; since only firms with  $e = 1$  can participate, since they are the only ones with output for sale,  $S = 1 - u$  where  $u$  is the unemployment rate when KW convenes.<sup>12</sup> Thus,  $\alpha_h = \mathcal{M}(1, 1 - u)$ . This gives us our first spillover across markets: buyers in the goods market are better off when there are more sellers, which means less unemployment in the labor market. While the exact relation depends on details, the robust idea is that it is better to be a buyer when unemployment is low, because the probability of trade can be better, and also because in equilibrium the terms of trade can be better.

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<sup>12</sup>To be clear, let  $u$  be the unemployment rate starting the period. After the current MP market, it changes to  $\hat{u}$ , the rate starting next period, and it is  $\hat{u}$  rather than  $u$  that determines  $\alpha_h$  in the KW market. In steady state  $u = \hat{u}$ , and so we can ignore this for now, but we are more careful in the dynamic model presented in the Appendix.

For  $h$  in the MP market,

$$U_1^h(z) = V_1^h(z) + \delta [V_0^h(z) - V_1^h(z)] \quad (5)$$

$$U_0^h(z) = V_0^h(z) + \lambda_h [V_1^h(z) - V_0^h(z)], \quad (6)$$

where  $\delta$  is the exogenous rate at which matches are destroyed and  $\lambda_h$  the endogenous rate at which they are created. The latter is determined by another matching function,  $\lambda_h = \mathcal{N}(u, v)/u$ , where  $u$  is unemployment and  $v$  is the number of vacancies posted when MP convenes. By CRS,  $\lambda_h = \mathcal{N}(1, \tau)$ , where  $\tau = v/u$  is labor market tightness. Wages are determined when firms and households meet in MP, although they are paid in the next AD market, which is not important but is convenient because we do not have to worry about how  $w$  is paid – e.g. in dollars or goods. There is commitment to  $w$  within a period, but in ongoing matches it can be renegotiated next period when MP reconvenes (this is convenient but not especially important for results).

This completes the household problem. Before moving on, we show how to collapse the three markets into one handy equation. Substituting  $V_e^h(z)$  from (4) into (5) and using the linearity of  $W_e^h$ , we have

$$U_1^h(z) = \alpha_h [v(q) - \rho d] + \rho z + \delta W_0^h(0) + (1 - \delta)W_1^h(0)$$

Something similar can be done for  $U_0^h$ . Updating these to next period and inserting into (2), the AD problem becomes

$$W_e^h(z) = I_e + z + \max_{\hat{z}} \left\{ -\hat{z} + \beta \hat{\alpha}_h \left[ v(\hat{q}) - \hat{\rho} \hat{d} \right] + \beta \hat{\rho} \hat{z} \right\} + \beta \mathbb{E}_{\hat{e}} \hat{W}_{\hat{e}}^h(0) \quad (7)$$

where the expectation is wrt next period's employment status  $\hat{e}$  conditional on  $e$ . We will see that the terms of trade  $(\hat{q}, \hat{d})$  in the next KW market do not depend on  $\hat{e}$  – see (14) below – so therefore (7) implies  $\hat{z}$  is independent of  $e$ ,  $I_e$  and  $z$ . Indeed, the choice of  $\hat{z}$  is very simple here, almost a static problem.

## 2.2 Firms

Firms obviously carry no money out of AD. In MP,

$$U_1^f = \delta V_0^f + (1 - \delta)V_1^f \quad (8)$$

$$U_0^f = \lambda_f V_1^f + (1 - \lambda_f)V_0^f, \quad (9)$$

where  $\lambda_f = \mathcal{N}(u, v)/v = \mathcal{N}(1, \tau)/\tau$ . This is completely standard. Where we deviate from textbook MP theory is that, rather than having  $f$  and  $h$  each consume a share of their output, in our model,  $f$  takes  $y$  to the goods market, where they look trade with other agents. The uncontroversial idea is that people do not necessarily want to consume what they make each day at work. This generates a role for a separate goods, or retail, sector. Although it might be interesting to proceed differently, here we consolidate production and retail activity within the firm.

As we said above,  $f$  participates in KW iff  $e = 1$ . When  $f$  makes a sale of  $q$  in this market, the rest of the output  $y - q$  is transformed into  $x = \zeta(y - q)$  units of the AD good later that period, with  $\zeta' \geq 0$  and  $\zeta'' \leq 0$  (there is a constraint  $q \leq y$ , but it is easy to give conditions making this slack). We could also simply assume unsold output vanishes between the KW and AD markets, but we like the idea of giving  $f$  an opportunity cost of KW trade.<sup>13</sup> It is useful to write the opportunity cost as  $c(q) = \zeta(y) - \zeta(y - q)$ . Unless otherwise stated,  $\zeta$  is linear so that  $x = y - q$  and  $c(q) = q$  (in Section 4.2 we need the general case). With  $\zeta$  linear, we can interpret  $x$  and  $q$  as one good that  $f$  can store across markets, but since  $h$  generally values it differently in KW and AD,  $f$  wants to sell at least some of it in the former.

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<sup>13</sup>One could alternatively assume  $y - q$  is carried forward to the next KW market, but then we would need to track the inventory distribution across firms. Having them liquidate inventories in the AD market allows us to have an opportunity cost of trade while avoiding this technical problem, just like the AD market allows us to avoid tracking a distribution of money holdings across households in the KW market.

In KW,

$$V_1^f = \alpha_f W_1^f(y - q, \rho d) + (1 - \alpha_f) W_1^f(y, 0) \quad (10)$$

where  $\alpha_f = \mathcal{M}(B, S)/S$ . The AD value of  $f$  with  $x$  in inventory and  $z$  in cash receipts is

$$W_1^f(x, z) = x + z - w + \beta \hat{U}_1^f, \quad (11)$$

given wage commitment  $w$ . Simplifying, we get

$$V_1^f = R - w + \beta \left[ \delta \hat{V}_0^f + (1 - \delta) \hat{V}_1^f \right], \quad (12)$$

where  $R = y + \alpha_f(\rho d - q)$  is expected revenue in units of the AD good. This is our second spillover effect: the terms of trade in the goods market ( $q, d$ ) affects  $R$ , and in equilibrium this affects entry and ultimately employment. Again the exact relation depends on details, but the robust idea is that as long as firms are deriving at least some of their profits from cash transactions, monetary factors affect their decisions.

To model entry, as is standard, we assume any  $f$  with  $e = 0$  has no current revenue or wage obligations, but can pay  $k$  in units of  $x$  in any AD market to enter the next MP market with a vacancy, which allows a probability of matching. Thus

$$W_0^f = \max \left\{ 0, -k + \beta \lambda_f \hat{V}_1^f + \beta (1 - \lambda_f) \hat{V}_0^f \right\},$$

where  $\hat{V}_0^f = \hat{W}_0^f = 0$  by free entry. In steady state  $k = \beta \lambda_f V_1^f$ , which by (12) can be written

$$k = \frac{\beta \lambda_f (R - w)}{1 - \beta(1 - \delta)}. \quad (13)$$

Average profit across all firms in a period is  $(1 - u)(R - w) - vk$ . As we said, firms pay out profit as dividends. If we assume the representative  $h$  holds the representative portfolio – say, shares in a mutual fund – this gives the equilibrium dividend  $\Delta$ .

## 2.3 Government

The government consumes  $G$ , pays UI benefit  $b$ , levies tax  $T$ , and prints money at rate  $\pi$ , so that  $\hat{M} = (1 + \pi)M$ , where in steady state  $\pi$  is inflation. Hence, their period budget constraint is  $G + bu = T + \pi M/p$ , which we assume holds at every date (without loss of generality, since Ricardian equivalence holds). For steady state analysis, we can equivalently describe monetary policy in terms of setting the nominal interest rate  $i$  or the growth rate of money  $\pi$ , by virtue of the Fisher equation  $1 + i = (1 + \pi)/\beta$ . In the stochastic model in the Appendix we specify policy in terms of interest rate rules. We always assume  $i > 0$ , although we can take the limit as  $i \rightarrow 0$ , which is the Friedman rule.

## 3 Equilibrium

Various assumptions can be made concerning price determination in our different markets, including bargaining, price taking, and price posting with either directed or undirected search. We think the most reasonable scenario is the following: price taking in the AD market; wage bargaining in MP; and price posting with directed search in KW. We like price taking in the AD market because it is simple, and in any case the AD market is not our prime focus. In the MP market, bargaining seems realistic and is standard in the literature, although it is actually a simple reinterpretation here to alternatively say that our labor market has wage posting with directed search. The issues are less clear for the KW market, so we explicitly analyze several options: bargaining, price taking, and price posting with directed search.<sup>14</sup>

Posting with directed search – also known as competitive search equilibrium

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<sup>14</sup>It is important to point out that in the labor market competitive search is equivalent to Nash bargaining with a particular value for the bargaining power parameter (the Hosios 1990 condition), but this is *not* true for the goods market. This is due to a double holdup problem in the goods market, with ex ante investment in real balances by  $h$  and entry by  $f$ , which cannot be resolved for any bargaining power.

– is nice for a variety of reasons. First, it is fairly tractable after one incurs an initial set-up cost. Second, it has desirable efficiency properties (see e.g. Kircher 2007 for a recent discussion). Third, directed search should seem like a big step forward to those who criticize monetary theory with random matching for the assumption of randomness per se (Howitt 2005). It should also appease those who dismiss modern monetary economics simply because they “don’t like bargaining” (Phelan 2005). More seriously, posting models avoid the assumption that agents see each others’ money balances, usually made in bargaining models to avoid technical difficulties with private information. Finally, competitive search eliminates bargaining power as a free parameter, which is useful in calibration. Hence competitive search seems attractive, although we present bargaining first, mainly because it is easy and standard in literature.

In any case, we break the analysis into three parts. First, taking unemployment  $u$  as given, we determine the value of money in the goods market  $q$  as in Lagos-Wright (2005). Then, taking  $q$  as given, we determine  $u$  in the labor market as in Mortensen-Pissarides (1996). It is convenient to depict these results graphically in  $(u, q)$  space as the *LW curve* and *MP curve*. Their intersection determines the unemployment rate and the value of money, from which all other variables follow, in steady state. More generally, the dynamic-stochastic model is discussed in the Appendix.

### 3.1 The Goods Market

Imagine for now that in the KW market  $f$  and  $h$  meet and bargain bilaterally over  $(q, d)$ , subject to  $d \leq z$  and  $q \leq y$ , obviously, since neither party can trade more than they have. We use generalized Nash bargaining (Aruoba et al. 2007 study several other bargaining solutions in this kind of model). Let the threat points be given by continuation values and  $\theta \in (0, 1]$  the bargaining power of  $h$ . The surplus for  $h$  is  $v(q) + W_e^h[\rho(z - d)] - W_e^h(\rho z) = v(q) - \rho d$ . Similarly, the

surplus for  $f$  is  $\rho d - q$ . It is easy (see Lagos-Wright) to show  $d = z$  (intuitively, it is costly to carry cash when we are not at the Friedman rule). Given this, the first order condition from maximizing the Nash product wrt  $q$  can be written  $\rho z = g(q, \theta)$  where

$$g(q, \theta) \equiv \frac{\theta q v'(q) + (1 - \theta)v(q)}{\theta v'(q) + 1 - \theta}. \quad (14)$$

Now recall (7), which in terms of the choice of  $\hat{z}$  is summarized by

$$\max_{\hat{z}} \{-\hat{z} + \beta \hat{\alpha}_h v(\hat{q}) + \beta(1 - \hat{\alpha}_h) \hat{\rho} \hat{z}\},$$

where we inserted  $\hat{d} = \hat{z}$ , and it is understood that  $\hat{q}$  is a function of  $\hat{z}$  as given in (14). Taking the FOC for an interior solution, then inserting  $\partial \hat{q} / \partial \hat{z} = \hat{\rho} / g_1(\hat{q}, \theta)$ , by virtue of (14), we arrive at

$$\frac{1}{\beta \hat{\rho}} = \hat{\alpha}_h \frac{v'(\hat{q})}{g_1(\hat{q}, \theta)} + 1 - \hat{\alpha}_h.$$

To reduce this to one equation in  $(u, q)$  we do three things: (i) use the Fisher equation for the nominal interest rate to eliminate  $1/\beta \hat{\rho} = 1 + i$ ; (ii) insert the arrival rate  $\hat{\alpha}_h = \mathcal{M}(1, 1 - \hat{u})$ ; and (iii) impose steady state. The result is

$$\frac{i}{\mathcal{M}(1, 1 - u)} = \frac{v'(q)}{g_1(q, \theta)} - 1. \quad (15)$$

This is the LW curve, determining  $q$  exactly as in Lagos and Wright (2005), except now  $u$  enters the equation. An increase in  $u$  affects  $q$  because it makes it less attractive to be a buyer, as discussed above, which reduces the demand for  $\hat{z}$ , which reduces  $q$  via the bargaining solution. Properties of the LW curve follow from well-known results in the literature. For example, simple conditions guarantee a unique  $q > 0$  solves (15) for any  $u \in (0, 1)$ , with  $\partial q / \partial u < 0$ .<sup>15</sup> Also,

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<sup>15</sup>Conditions in Lagos-Wright to make the RHS of (15) monotone in  $q$  are: (i)  $u'$  log-concave; or (ii)  $\theta \approx 1$ . A more general argument in Wright (2008) dispenses with these kinds of side conditions entirely, and proves there is (generically) a unique steady state  $q$  even if the RHS of (15) is not monotone.

letting  $q^*$  solve  $v'(q^*) = 1$ , we know  $q < q^*$  for all  $i > 0$ . Also,  $u = 1$  implies  $q = 0$ .

Summarizing these and some other properties, we have the following Proposition.

**Proposition 1** *For all  $i > 0$  the LW curve slopes downward in  $(u, q)$  space, with  $u = 0$  implying  $q \in (0, q^*)$  and  $u = 1$  implying  $q = 0$ . It shifts down with  $i$  and up with  $\theta$ . In the limit as  $i \rightarrow 0$ ,  $q \rightarrow q_0$  for all  $u < 1$ , where  $q_0$  is independent of  $u$ , and  $q_0 \leq q^*$  with  $q_0 = q^*$  iff  $\theta = 1$ .*

### 3.2 The Labor Market

Suppose that when  $f$  and  $h$  meet in MP they bargain over  $w$ , with threat points equal to continuation values, and  $\eta$  the bargaining power of  $f$ . It is routine to solve this for<sup>16</sup>

$$w = \frac{\eta[1 - \beta(1 - \delta)](b + \ell) + (1 - \eta)[1 - \beta(1 - \delta - \lambda_h)]R}{1 - \beta(1 - \delta) + (1 - \eta)\beta\lambda_h}. \quad (16)$$

If we substitute this and  $R = y + \alpha_f(\rho d - q)$  into (13), the free entry condition becomes

$$k = \frac{\lambda_f \eta [y - b - \ell + \alpha_f(\rho d - q)]}{r + \delta + (1 - \eta)\lambda_h}.$$

To reduce this to one equation in  $(u, q)$  we do three things: (i) as is standard, use the steady state condition  $(1 - u)\delta = \mathcal{N}(u, v)$  to solve for  $v = v(u)$  and market tightness  $\tau = \tau(u) = v(u)/u$ ; (ii) insert the arrival rates  $\lambda_f(u) = \mathcal{N}[1, \tau(u)]/\tau(u)$ ,  $\lambda_h(u) = \mathcal{N}[1, \tau(u)]$ , and  $\alpha_f(u) = \mathcal{M}(1, 1 - u)/(1 - u)$ ; and (iii) use the bargaining solution to eliminate

$$\rho d - q = g(q, \theta) - q = \frac{(1 - \theta)[u(q) - q]}{\theta u'(q) + 1 - \theta}.$$

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<sup>16</sup>To give details, notice the surplus for  $h$  is  $S^h = V_1^h(m) - V_0^h(m)$ , which after simplification yields  $S^h = w - b - \ell + \beta(1 - \delta - \lambda_h)\hat{S}^h$ . Similarly,  $S^f = R - w + \beta(1 - \delta)\hat{S}^f$ . As usual, one first maximizes the Nash product taking as given future surpluses  $\hat{S}^i$ , then imposes steady state, and rearranges to get the expression in (16).

The result is

$$k = \frac{\lambda_f(u)\eta \left\{ y - b - \ell + \alpha_f(u) \frac{(1-\theta)[u(q)-q]}{\theta u'(q) + 1 - \theta} \right\}}{r + \delta + (1 - \eta)\lambda_h(u)}. \quad (17)$$

This is the MP curve, determining  $u$  as in Mortensen-Pissarides (1996), except the total surplus here is not just  $y - b - \ell$ , but includes as an extra term the expected surplus from retail trade. Routine calculations show this curve is downward sloping. Intuitively, there are three effects from an increase in  $u$ , two from the textbook model plus a new one, all of which encourage entry: (i)  $\lambda_f(u)$  goes up (it is easier for  $f$  to hire); (ii)  $\lambda_h(u)$  goes down (it is harder for  $h$  to get hired, which lowers  $w$ ); and (iii)  $\alpha_f(u)$  goes up (it is easier for  $f$  to compete in the goods market).

Summarizing this and some other easily verified properties, we have:

**Proposition 2** *The MP curve slopes downward in  $(u, q)$  space. It shifts in with  $y$  or  $\eta$ , and out with  $k$ ,  $r$ ,  $\delta$ ,  $\theta$ ,  $b$  or  $\ell$ .*

### 3.3 Steady State Equilibrium

Propositions 1 and 2 imply LW and MP both slope downward in a box  $\mathcal{B} = [0, 1] \times [0, q^*]$  in  $(u, q)$  space, shown in Figure 3. Notice LW enters  $\mathcal{B}$  from the left at  $u = 0$  and  $q_0 \leq q^*$  and exits at  $(1, 0)$ , while MP enters where  $q = q^*$  at some  $\underline{u} > 0$ , with  $\underline{u} < 1$  iff  $k$  is not too big, and exits by either hitting the horizontal axis at  $u_0 \in (0, 1)$  or hitting the vertical axis at  $q_1 \in (0, q^*)$ . It is easy to check the latter case, shown by the curve labeled 1, occurs iff  $\eta(y - b - \ell) > k(r + \delta)$ , which is the usual condition to get  $u < 1$  in the MP model. In this case, there exists a nonmonetary steady state at  $(u_0, 0)$ , which is the standard MP equilibrium, plus at least one monetary steady state with  $q > 0$  and  $u < u_0$ . The Figure also shows cases labeled 2 and 3, where the MP curve intersects the vertical axis, and there either exist multiple or no monetary steady states, plus a steady state at  $(u, q) = (1, 0)$  where the KW and MP markets shut down. In

case 3  $(u, q) = (1, 0)$  is the only possibility; in case 2 there coexist equilibria with both markets open.

To understand which case is more likely, simply look at Propositions 1 and 2 concerning how changes in parameters shift the MP and LW curves. In any case, the discussion in the previous paragraph establishes existence of steady state equilibrium. Clearly we do not have uniqueness, in general, and in particular monetary and nonmonetary equilibria may coexist; it is possible however for the monetary steady state to be unique, as turns out to be the case in the calibrations below. If there exists any steady state with  $u < 1$ , which again is true iff  $\eta(y - b - \ell) > k(r + \delta)$ , then there will exist a monetary steady state. Once we have  $(u, q)$ , we easily recover all other endogenous variables, including vacancies  $v$ , arrival rates  $\alpha_j$  and  $\lambda_j$ , real balances  $z = g(q, \theta)$ , and so on.<sup>17</sup>

A convenient result from Propositions 1 and 2 is that changes in  $i$  shift only the LW curve, while changes in  $y, \eta, r, k, \delta, b$  or  $\ell$  shift only the MP curve, which makes it easy to analyze changes in parameters.<sup>18</sup> An increase in  $i$  e.g. shifts the LW curve in toward the origin, reducing  $q$  and  $u$  if equilibrium is unique (or in the ‘natural’ equilibria if we do not have uniqueness). The result  $\partial q / \partial i < 0$  holds in the standard LW model with a fixed  $\alpha_h$ , but now there is a general equilibrium multiplier effect via  $u$  that reduces  $\alpha_h$  and further reduces  $q$ . Similarly, an increase in  $b$  e.g. shifts the MP curve out, increasing  $u$  and reducing  $q$  if equilibrium is unique (or in the ‘natural’ equilibria). The result in

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<sup>17</sup>In particular, the nominal price level is  $p = M/g(q, \theta)$ , and the AD budget equation yields  $x$  for every  $h$  as a function of  $z$  and  $I_e$ . In the general case where AD utility is  $x + \Upsilon_e(\mathbf{x})$ , utility maximization determines individual demand as a function of  $e$  and  $\mathbf{p}$  (plus  $p$  which we already know), say  $\mathbf{x} = \mathbf{D}_e(\mathbf{p})$ . Market demand is  $\mathbf{D}(\mathbf{p}) = u\mathbf{D}_0(\mathbf{p}) + (1 - u)\mathbf{D}_1(\mathbf{p})$ , and equating this to the endowment  $\bar{\mathbf{x}}$  yields a standard system of GE equations that solve for  $\mathbf{p}$ . We get classical neutrality: if  $M$  changes, we can change  $p$  and  $\mathbf{p}$  proportionally without affecting the AD equilibrium conditions or  $(u, q)$ . We do not generally get superneutrality: any change in  $i$  shifts the LW curve, which affects  $(q, u)$  and the rest of the system. When  $\Upsilon_e$  does not depend on  $e$ , however, neither does  $\mathbf{D}_e(\mathbf{p})$ , in which case  $\mathbf{D}(\bar{\mathbf{p}})$  is independent of  $u$  and hence  $\mathbf{x}$  is independent of monetary factors – a version of the neoclassical dichotomy.

<sup>18</sup>The only parameter shifts both curves is bargaining power in the goods market,  $\theta$ . In the competitive search model in the next section,  $\theta$  does not appear.

$\partial u / \partial z > 0$  holds in the standard MP model with a fixed  $R$ , but now there is a multiplier effect via  $q$  that reduces  $R$  and further increases  $u$ .

Other experiments can be analyzed similarly. Summarizing what we know, we have:

**Proposition 3** *Steady state equilibrium always exist. One steady state is the nonmonetary equilibrium, which entails  $u < 1$  iff  $\eta(y - b - \ell) > k(r + \delta)$ . If this inequality holds, there also exists a monetary steady state. Assuming the monetary steady state is unique, a rise in  $i$  decreases  $q$  and increases  $u$ , while a rise in  $y$  or  $\eta$ , or a fall in  $k$ ,  $r$ ,  $\delta$ ,  $b$  or  $\ell$ , increases  $q$  and decreases  $u$ .*

## 4 Alternative Pricing Mechanisms

As discussed, there are reasons to consider alternatives to bargaining in the goods market. Here we consider competitive search equilibrium, with price posting and directed search. We also consider competitive equilibrium, with price-taking, which may be of interest because it can be reduced as a special case to something that looks like a common cash-in-advance or money-in-the-utility-function specification. We maintain bargaining in the labor market in this section, although as mentioned, one reinterpret the same equations as coming from competitive search simply by setting bargaining power in MP according to the Hosios (1990) condition (this is not the case in KW).

### 4.1 Price Posting

There are several ways to motivate the notion of competitive search. One is to have sellers first post the terms of trade, then have buyers direct their search to their preferred sellers, taking into account that they may not get served if too many other buyers show up. Or we can have buyers post to attract sellers. Or we can have market makers set up submarkets to which they try to attract

buyers and sellers by posting the terms of trade (so they can charge them an entrance fee, although this fee is 0 in equilibrium by free entry). These different stories all lead to the same set of equilibrium conditions.<sup>19</sup> We assume sellers post. Note that meetings are still probabilistic: if a group of  $B$  buyers direct their search towards a group of  $S$  sellers, the number of meetings is  $\mathcal{M}(B, S)$ , so  $Q = B/S$  determines  $\alpha_f = \mathcal{M}(Q, 1)$  and  $\alpha_h = \mathcal{M}(Q, 1)/Q$ .

We imagine  $f$  posting in the AD market the following message: “Conditional on  $e = 1$  in the next MP market, I commit to sell  $q$  units for  $d$  dollars in the KW market, but I can serve at most one customer, and you should expect queue length  $Q$ .” The equilibrium surplus  $h$  gets from participating in the KW market, from the perspective of the AD market, where he has to acquire the cash, is given by

$$\tilde{\Sigma} = -\tilde{d} + \beta\alpha_h(\tilde{Q})v(\tilde{q}) + \beta [1 - \alpha_h(\tilde{Q})] \rho\tilde{d},$$

where  $(\tilde{q}, \tilde{d})$  and  $\tilde{Q}$  are the best available terms and queue length (of course  $h$  always has the option of not participating, which yields  $\tilde{\Sigma} = 0$ ). Thus,  $f$  posts  $(q, d)$  to maximize  $V_1^f$ , which from (12) is simply  $\alpha_f(Q)(\rho d - q)$  plus a constant, subject to the constraint that in order to get  $Q > 0$  buyers must receive a surplus from him  $\Sigma$  equal to the market surplus  $\tilde{\Sigma}$ .

Formally, assuming  $f$  wants  $Q > 0$ , we can think of him choosing  $Q$  as well as  $(q, d)$  to solve

$$\begin{aligned} & \max_{q, d, Q} \mathcal{M}(Q, 1) (\rho d - q) & (18) \\ \text{st } \tilde{\Sigma} & = -d(1 - \beta\rho) + \beta \frac{\mathcal{M}(Q, 1)}{Q} [v(q) - \rho d] \end{aligned}$$

Using  $\beta\rho = 1/(1+i)$  and the equilibrium condition  $\Sigma = \tilde{\Sigma}$ , it is straightforward

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<sup>19</sup>See e.g. Moen (1997), Shimer (1996), Acemoglu and Shimer (1999), Julien et al. (2000), Burdett et al. (2001), Mortensen and Wright (2002) and Menzio (2007). A complication in monetary competitive search models is discussed by Faig and Huangfu (2005), but this can be finessed as in Rochteau and Wright (2005).

if tedious to derive the following conditions characterizing the solution

$$v'(q) = 1 + \frac{i}{\alpha_h(Q)} \quad (19)$$

$$\rho d = g[q, \epsilon(Q)] \quad (20)$$

$$\Sigma = \beta \alpha_h(Q) \{v(q) - v'(q)g[q, \epsilon(Q)]\}, \quad (21)$$

where  $g(\cdot)$  is defined in (14), and  $\epsilon(Q)$  is the elasticity of  $\mathcal{M}$  wrt  $B$  evaluated at  $Q$ . Notice (19) looks like the equilibrium condition in a standard LW model when buyers make take-it-or-leave-it offers, while (20) looks like the usual bargaining solution with  $\theta$  replaced by  $\epsilon(Q)$ . This means the Hosios (1990) conditions hold. As in the related literature, competitive search eliminates holdup problems on both the trade and entry (intensive and extensive) margins.

Let  $q(i, Q)$  be the  $q$  that solves (19), and notice it is strictly decreasing in  $i$  and  $Q$ . Substituting  $q(i, Q)$  into (21) give us an equation in  $Q$  and  $\Sigma$ . Denote the LHS of (21) by  $\Phi(q, Q)$ , and for the sake of tractability assume  $\Phi_1(q, Q) > 0$ ,  $\Phi_2(q, Q) < 0$ .<sup>20</sup> This implies there is a unique solution  $Q = Q(\Sigma) \geq 0$  to (21). Moreover, it is strictly decreasing, equals  $\bar{Q}(i) > 0$  when  $\Sigma = 0$ , and equals 0 when  $\Sigma \geq v(q^*) - q^* - iq^*$ . Often  $Q(\Sigma)$  is interpreted as the ‘demand’ for  $Q$ , determining the queue length a seller wants as a function of the market ‘price’  $\Sigma$ . The ‘supply’ of  $Q$  is simple: if  $\Sigma > 0$  then every  $h$  participates in KW, so  $Q = (1-u)^{-1}$ ; and if  $\Sigma = 0$  then  $h$  is indifferent to participating, so the ‘supply’ curve is flat at  $B \in [0, 1]$ .

Equilibrium equates ‘supply’ and ‘demand’ for  $Q$ , as in Figure 4.1. Letting  $\bar{u}(i) \equiv 1 - 1/\bar{Q}(i)$ , we have that  $q$  then depends on  $u$  as follows:

$$u \leq \bar{u}(i) \implies Q = (1-u)^{-1} \text{ and } v'(q) - 1 = i/\alpha_h(Q) \quad (22)$$

$$u > \bar{u}(i) \implies Q = \bar{Q}(i) \text{ and } v'(q) - 1 = i/\alpha_h[\bar{Q}(i)]. \quad (23)$$

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<sup>20</sup>Note that  $\Phi_2 < 0$  holds for the usual matching functions, while a sufficient condition for  $\Phi_1 > 0$  is that  $\epsilon(Q)$  is not too small. In the working paper (Berentsen, Menzio and Wright 2007) we discuss what happens more generally.

This is the LW curve with competitive search. It is downward sloping in  $(u, q)$  space and shifts in with  $i$ , as under bargaining. The only complication is that once we increase  $u$  beyond  $\bar{u}(i)$ , there is no  $\Sigma > 0$  that clears the market for  $Q$ , so we get  $\Sigma = 0$ ,  $Q = \bar{Q}(i)$ , and  $q = \bar{q}(i)$  where  $\bar{q}(i)$  solves (23). That is, the LW curve kinks and becomes horizontal at  $\bar{u}(i)$ . To find the point where it kinks, solve (21) with  $\Sigma = 0$  for  $\bar{q}(i) = \psi[\bar{u}(i)]$ , which implies  $\psi' \geq 0$ ; if  $\mathcal{M}(B, S)$  is Cobb-Douglas e.g. then  $\bar{q}(i)$  is independent of  $\bar{u}(i)$  and  $\psi(u)$  is horizontal.

The MP curve also needs to be modified. First, for  $u < \bar{u}(i)$ , we have

$$k = \frac{\lambda_f(u)\eta \left\{ y - b - \ell + \alpha_f(u) \frac{(1-\epsilon)[u(q)-q]}{\epsilon u'(q) + 1 - \epsilon} \right\}}{r + \delta + (1 - \eta)\lambda_h(u)}, \quad (24)$$

which is identical to (17) except we replace bargaining power  $\theta$  by the elasticity  $\epsilon = \epsilon(Q)$ , with  $Q = 1/(1 - u)$ . Second, for  $u > \bar{u}(i)$ , the result is the same except  $\alpha_f(u) = \mathcal{M}[\bar{Q}(i), 1]$  and  $\epsilon = \epsilon[\bar{Q}(i)]$  no longer depend on  $u$ . As Figure 4.2 shows, the MP curve is downward sloping, but now has a kink at  $\bar{u}(i)$ . Note that when  $u > \bar{u}(i)$ , the MP curve now depends on  $i$  directly.<sup>21</sup> But apart from technical modifications, posting is qualitatively similar to bargaining.

## 4.2 Price Taking

We now consider Walrasian price taking in the KW market. Search models with price taking go back to the Lucas and Prescott (1974) model of unemployment, where it takes time to get from one local labor market to another, but each one contains large numbers of workers and firms who behave competitively. We can tell the same story about our goods market, and have agents take the price of KW goods in terms of AD goods parametrically (although perhaps we should emphasize that money remains essential, because of anonymity, even though the market is Walrasian). We also generalize Lucas-Prescott by allowing agents to get into the goods market only probabilistically. Additionally, we now allow a

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<sup>21</sup>In the calibrated model, with high money demand elasticity, this is the case (Figure 7).

nonlinear opportunity cost  $c(q)$ , so that revenue is  $R = \zeta(y) + \alpha^f [q^f \rho - c(q^f)]$ .

Every  $f$  with  $e = 1$  wants to get into the KW market. Those that do choose  $q^f$  to maximize  $q^f \rho - c(q^f)$ , which implies  $c'(q^f) = \rho$ . Then in AD, with the usual manipulations, free entry implies

$$k = \frac{\lambda_f \eta \{ \zeta(y) - b - \ell + \alpha_f [q^f c'(q^f) - c(q^f)] \}}{r + \delta + (1 - \eta) \lambda_h}, \quad (25)$$

where  $\alpha_f$  is the probability  $f$  gets into KW.<sup>22</sup> Every  $h$  wants to get into this market, and those that do choose  $q^h$  to maximize  $v(q^h) + W_e^h [\rho(z - q^h)]$  st  $q^h \leq z$ . The constraint binds in equilibrium. In AD,  $h$  chooses  $\hat{z}$  to  $\max_{\hat{z}} \{-\hat{z} + \beta \hat{\alpha}_h v(\hat{q}) + \beta(1 - \hat{\alpha}_h) \hat{\rho} \hat{z}\}$ , where  $\hat{\alpha}_h$  is his probability of getting into KW. Taking the FOC, then using  $\partial q^h / \partial z = 1$  and  $\rho = c'(q^f)$ , we get

$$\frac{i}{\alpha_h} = \frac{v'(q^h)}{c'(q^f)} - 1. \quad (26)$$

Search-type frictions are captured by letting the measures of agents that get into a market depend on the measures that try to get in, which means  $\alpha^h(u) = \mathcal{M}^h(1, 1 - u)$  and  $\alpha^f(u) = \mathcal{M}^f(1, 1 - u)/(1 - u)$ . Goods market clearing implies  $\mathcal{M}^h q^h = \mathcal{M}^f q^f$ . Inserting  $q^h = q$  and  $q^f = q \mathcal{M}^h / \mathcal{M}^f = q/(1 - u)$ , as well as  $\lambda_h$  and  $\lambda_f$ , into (26) and (25), we get the LW and MP curves with Walrasian pricing in KW. A special case is the frictionless version, where everyone who wants gets in,  $\mathcal{M}^h = 1$  and  $\mathcal{M}^f = 1 - u$ . In this special case, with no frictions, the LW and MP curves are

$$i = \frac{v'(q)}{c'\left(\frac{q}{1-u}\right)} - 1$$

$$k = \frac{\lambda_f(u) \eta \left\{ \zeta(y) - b - \ell + \left[ \frac{q}{1-u} c'\left(\frac{q}{1-u}\right) - c\left(\frac{q}{1-u}\right) \right] \right\}}{r + \delta + (1 - \eta) \lambda_h(u)}$$

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<sup>22</sup>We emphasize that there are two distinct notions of entry here: first  $f$  pays  $k$  to get into the MP market (post a vacancy); then, once  $f$  produces, there is a probability  $\alpha_f$  that he gets into the KW market (if he does not he transits directly to AD). Similarly,  $h$  only gets into KW with probability  $\alpha_h$ . As a special case, of course,  $\alpha_f$  or  $\alpha_h$  or both can be 1.

If we additionally impose linear cost,  $c(q) = q$ , then  $u$  vanishes from LW and  $q$  vanishes from MP. In this special case, therefore, the LW curve is horizontal and the MP curve vertical.

In other words, the model dichotomizes in the case where: (i) there are no frictions; and (ii)  $c(q) = q$ . Actually, while (i) and (ii) are needed to solve LW for  $q$  independently of  $u$ , only the latter is needed to solve MP for  $u$  independently of  $q$ . Based on this, one can reinterpret the standard MP model as one where firms indeed sell their output in a market to households other than their own employees – for cash or credit, it is irrelevant in this case – since as long as the cost in this market is linear and pricing is Walrasian, firms get none of the gains from trade, and  $u$  is determined as in the standard model. There may or may not be monetary exchange lurking behind the scenes, but this does not affect vacancy creation or unemployment.<sup>23</sup>

One could say that when  $\alpha^h = \alpha^f = 1$  our model looks like a standard cash-in-advance economy, in the sense that there are no search or non-competitive pricing issues. One could also say it looks like a money-in-the-utility-function specification, since after all real balances do appear in the value (indirect utility) functions. This is all fine. It is because we like to go into more detail about the assumptions that make money useful, and to allow for search frictions and alternative pricing mechanisms, that we did not start with a cash-in-advance or money-in-the-utility-function specification. But for those wed to reduced-form approaches, we point out that a frictionless version of our model with Walrasian pricing leads to the same set of equations. To put it another way, one can derive the LW and MP curves *without* microfoundations for money. We prefer to be more explicit about the exchange process, not only for aesthetic reasons, but because this gives more general results, leads to additional insights, and is no more difficult.

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<sup>23</sup>One can get something similar in the bargaining model by setting  $\theta = 1$ .

## 5 Quantitative Analysis

Theory predicts an increase in inflation or interest rates increases unemployment, basically, because this raises the effective tax on cash-intensive goods markets, which reduces profit and employment (note that for the parameter values calibrated below, equilibrium is unique, so these results are unambiguous). We now ask how big the effects might be. As we said, the model is especially well suited to lower-frequency observations, as we abstract from complications that might matter in the short run like imperfect information, rigidities, etc. Although in principle the model could be used to address many questions, here we ask how Friedman’s “natural rate of unemployment” is affected by monetary factors. We focus on this: How well can the baseline model account quantitatively for low-frequency dynamics in unemployment when the driving force is counterfactually assumed to be exogenous changes in monetary policy?

We use the version with competitive search in the goods market and bargaining in the labor market. However, as is common, we set bargaining power  $\eta$  in the latter to the elasticity of the matching function à la Hosios (1990), which can be interpreted as imposing competitive search in MP (recall there is no analog in KW). We also tried other versions, including bargaining and price taking in KW, and the results were similar in terms of the big picture if not details.<sup>24</sup> Also, although so far we focused on steady states, the dynamic-stochastic generalization is presented in the Appendix. There we allow randomness in monetary policy, as described by an interest rate rule  $i_+ = \bar{i} + \rho_i(i - \bar{i}) + \epsilon_i$ ,  $\epsilon_i \sim N(0, \sigma_i)$ , and in productivity, as described by a similar stochastic process for  $y$ . For most of our experiments we take  $y$  to be constant in order to isolate the effects of money, and only use the model with real shocks as an aid in calibration.

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<sup>24</sup>This is not to say the mechanism does not matter: for given parameters it makes a difference e.g. if we have bargaining or posting. But if we change the mechanism and recalibrate parameters, we can get similar results (details to be posted on the website).

## 5.1 Parameters and Targets

We choose one quarter as the model period, and look at the years 1955-2005, the same sample discussed in the Introduction. We need to calibrate: (i) preferences as described by  $\beta$ ,  $\ell$  and  $v(q)$ ; (ii) technology as described by  $\delta$ ,  $k$ ,  $\mathcal{N}(u, v)$  and  $\mathcal{M}(B, S)$ ; (iii) policy as described by  $b$  and the process for  $i$ . In terms of functional forms, we assume KW preferences over  $q$  are given by  $v(q) = Aq^{1-a}/(1-a)$ . Following much of the literature, we take the MP matching function to be  $\mathcal{N}(u, v) = Zu^{1-\sigma}v^\sigma$ . We take the KW matching function to be  $\mathcal{M}(B, S) = S[1 - \exp(-B/S)]$ , the so-called urn-ball technology, which is a very parsimonious specification – it has no parameters – and one that can be derived endogenously using directed search theory (see e.g. Burdett et al. 2001 or Albrecht et al. 2006).

Calibration is now fairly standard. First, set  $\beta$  to match the average quarterly real interest rate, measured as the difference between the nominal rate and inflation. Then set the elasticity  $1 - \sigma$  of the MP matching function to the regression coefficient of the job-finding rate on labor market tightness, both expressed in logs. Then set UI so that in equilibrium the replacement rate is  $b/w = 0.5$ . Then set parameters of the  $i$  process  $(\bar{i}, \rho_i, \sigma_i)$  to match the average quarterly nominal rate, its autocorrelation and variance. Then set  $(\delta, k, Z)$  to match the average unemployment, vacancy and job-finding rates – although we can normalize the vacancy rate to 1 by choice of units, which affects the calibrated value of  $Z$  but nothing else. This leaves only the preference parameters  $(A, a, \ell)$ , which we now discuss.

We set  $(A, a)$  to match money demand in the data with that implied by theory. In the model,  $M/pY$  is given by  $M/p = g(q)$  over  $Y = \mathcal{M}(1, 1 - u)[g(q) - q] + (1 - u)y$ , where both  $q$  and  $u$  depend on  $i$ . In terms of data, we target average  $M/pY$  plus some measure of its responsiveness to  $i$ , using  $M1$

as our notion of money.<sup>25</sup> One method is to target directly the elasticity of  $M/pY$  wrt  $i$ , which we estimate to be around  $-0.7$  using several specifications and periods of different lengths, summarized in Figure 5.1. The implied money demand curve is shown in Figure 5.2, and fits well, at least until the mid 90s. A different tack is to simply match the slope of a regression line through the scatter in Figure 5.2, which also fits well, at least if we ignore the 80s. Although this method does not target the elasticity directly, the implied parameters do generate a money demand relation with an elasticity of around  $-1.4$ . We present results for both low and high elasticities, since both generate reasonable demand curves.<sup>26</sup>

TABLE 1: CALIBRATION TARGETS

Description	Value
average real rate $r$ (quarterly)	0.00816
average nominal rate $i$ (quarterly)	0.01803
autocorrelation of $i$	0.990
standard deviation of $i$	0.006
average money demand $M/pY$ (annual)	0.169
money demand elasticity (negative)	0.7 or 1.4
average unemployment $u$	0.058
average vacancies $v$ (normalization)	1
average UI replacement rate $b/w$	0.500
average job-finding rate $\lambda_h$ (monthly)	0.450
elasticity of $\lambda_h$ wrt $v/u$	0.280

The targets described above are summarized in Table 1 (we include the average vacancy rate even though it has been normalized to 1; again its only role is to determine the value of the constant in front of the MP matching function  $Z$ ). These targets are sufficient to pin down all parameters except

<sup>25</sup>We use  $M1$  mainly to facilitate comparison with the literature, but note that one can reformulate this kind of model so that demand deposits circulate in KW, either instead of or along with currency; see e.g. Berentsen et al. (2007), He et al. (2007), Chiu and Meh (2007), or Li (2007).

<sup>26</sup>These observations also pin down the share of the KW market: simply divide nominal spending in KW  $\mathcal{M}(1, 1 - u)M$  by total nominal spending  $pY$  to get  $\mathcal{M}(1, 1 - u)$  times money demand  $M/pY$ . Adjusting from an annual to a quarterly frequency,  $M1/pY$  is 0.676, and at the steady state  $u = 0.058$  our matching function yields  $\mathcal{M}(1, 1 - u) = 0.616$ , implying the KW market contributes just under 42% of total spending.

one: the value of leisure  $\ell$ . As is well known,  $\ell$  is difficult to calibrate and can matter a lot – this is at the heart of the difference between Shimer (2005) and Hagedorn-Manovskii (2007). Our approach is to be agnostic, and consider various strategies for  $\ell$ . In our UI (for ‘unemployment insurance’) calibration we impose  $\ell = 0$ , as in Shimer. In our BC (for ‘business cycle’) calibration we set  $\ell$  so that a real version of the model, with shocks to  $y$  calibrated to the data and constant  $i$ , generates cyclical fluctuations in  $u$  consistent with the evidence, which is close to Hagedorn-Manovskii.<sup>27</sup> Finally, in our BF (for ‘best fit’) calibration we choose  $\ell$  to minimize the deviations between HP-filtered  $u$  in the data and in the model, using first a low and then a high HP parameter of 1600 and 160000.

We spend little time in the paper on the UI calibration here (see the web site) since, as one should expect from the literature, it generates almost no response of  $u$  to shocks to either  $y$  or  $i$ . Our position is macro-labor economists have yet to settle on the best way to solve the ‘puzzle’ of getting  $u$  to move more in response to any shocks, so, for this exercise, we let  $\ell$  do the work, although other solutions to the ‘puzzle’ might work as well for our purposes. We therefore present versions of the model with low and high money demand elasticity, and for each we consider three  $\ell$  calibrations: the BC method, the BF method with a low HP filter, and the BF method with a high filter. Table 2.1 reports the calibrated values of the preference and technology parameters for the low money demand elasticity, and Table 2.2 for the high elasticity (to reduce clutter we omit policy parameters, which are the same in all cases:  $b = 0.494$ ,  $\bar{i} = 0.018$ ,  $\rho_i = 0.984$ , and  $\sigma_i = 8.9 \cdot 10^{-4}$ ).

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<sup>27</sup>For the record, Hagedorn-Maovskii do *not* pick  $\ell$  to match the volatility of  $u$ , but target some other observables. When we say we are close to them, we only mean that our ratio of  $b + \ell$  to  $y$ , which is what really matters, is close to theirs. We are aware of issues involved with high values of  $\ell$ , as in Hagedorn-Maovskii, including the critique by Costain and Ritter (2007), but we think the approach in Rogerson et al. (2008) can in principle address that problem.

TABLE 2.1: PARAMETERS WITH MD ELASTICITY 0.7

	Description	UI	BC	BF 1600	BF 160000
$\beta$	discount factor	0.992	0.992	0.992	0.992
$\ell$	value of non-market activity	0	0.477	0.491	0.491
$A$	KW utility weight	1.013	1.013	1.013	1.013
$a$	KW utility elasticity	0.037	0.037	0.037	0.037
$\delta$	job destruction rate	0.050	0.050	0.050	0.050
$k$	vacancy posting cost	$1.05 \cdot 10^{-2}$	$3.69 \cdot 10^{-4}$	$7.53 \cdot 10^{-5}$	$6.69 \cdot 10^{-5}$
$Z$	MP matching efficiency	0.364	0.364	0.364	0.364
$\sigma$	MP matching elasticity wrt $u$	0.720	0.720	0.720	0.720

TABLE 2.1: PARAMETERS WITH MD ELASTICITY 1.4

	Description	UI	BC	BF 1600	BF 160000
$\beta$	discount factor	0.992	0.992	0.992	0.992
$\ell$	value of non-market activity	0	0.474	0.486	0.485
$A$	KW utility weight	1.020	1.020	1.020	1.020
$a$	KW utility elasticity	0.018	0.018	0.018	0.018
$\delta$	job destruction rate	0.050	0.050	0.050	0.050
$k$	vacancy posting cost	$1.05 \cdot 10^{-2}$	$3.85 \cdot 10^{-4}$	$1.30 \cdot 10^{-4}$	$1.44 \cdot 10^{-4}$
$Z$	MP matching efficiency	0.364	0.364	0.364	0.364
$\sigma$	MP matching elasticity wrt $u$	0.720	0.720	0.720	0.720

## 5.2 Results

We first solve for recursive equilibrium in the general dynamic model presented in the Appendix, where we allow stochastic processes for both the interest rate  $i$  and productivity  $y$ . We then feed in the actual path of  $i$ , holding  $y$  constant, and calculate the implied path for  $u$ . This is our prediction for unemployment in the counterfactual case where the only impulses over the period were changes in monetary policy. We compare the predictions of the model and the data in terms of  $u$ , where  $u$  has been filtered to various degrees (in both the model and the data) to focus on long-run behavior. We then look at statistics, or more directly, at plots of the variables in question.

Consider first the case of a low money demand elasticity. Figure 6.1 summarizes the results of the BC calibration in two ways: scatter plots of trend (filtered)  $i$  vs.  $u$  and  $\pi$  vs.  $u$ ; and the time series of trend (filtered)  $u$  as well

as the raw (unfiltered) series. In Figure 1 we use a high HP filter parameter of 160000, while in Figure 6.2 we use a lower filter of 1600. As one can see from either figure, this version of the model implies that monetary policy alone can account for a little, but not that much, of the behavior of  $u$  over the sample. We get nothing like the big swings in  $u$  observed in the data, even after filtering, although qualitatively the model clearly does correctly predict the broad pattern of  $u$  rising in the first half and falling in the second half of the fifty-year sample.

To use one summary statistic, consider e.g. the runup in  $u$  over the worse part of the stagflation episode, between the first quarters of 1972 and 82.<sup>28</sup> As shown in Table 4.1, this version of the model accounts for only part of the increase in  $u$  during this episode. Depending on how much of the high frequency we filter out, one can say that  $u$  rose between roughly 20% and 40% during this ten-year period, while the model predicts an increase of less than 10%. Looking at unfiltered data, e.g., we only predict 8% as compared to an actual 41% increase – that is, only 20% of the observed increase in  $u$ . Similarly, the model can account for 12% to 13% of filtered unemployment during stagflation. So with the BC calibration method and a low money demand elasticity, one might conclude the model does not account for that much.

TABLE 4.1: LOW MD ELASTICITY, BC CALIBRATION

Observation	$u$ 1972Q1	$u$ 1982Q1	% change
unfiltered data	5.8	8.8	41
unfiltered model	5.8	6.3	08
low filtered data	5.3	8.2	43
low filtered model	5.8	6.1	05
high filtered data	5.7	7.1	22
high filtered model	5.8	6.0	03

<sup>28</sup>We did not chose this subsample to represent stagflation to ‘cook the results’ in any sense, but for the following three reasons. First, both  $i$  and  $u$  are close to their steady state values in 1972Q1. Second, we wanted to have exactly a decade of data. Third, 1982Q1 has the highest value of  $i = 15.1$  in the sample, as well as a a very high  $u = 0.088$ .

TABLE 4.2: LOW MD ELASTICITY, BF CALIBRATION

Observation	$u$ 1972Q1	$u$ 1982Q1	% change
unfiltered data	5.8	8.8	41
unfiltered model	5.4	14.5	91
low filtered data	5.3	8.2	43
low filtered model	5.4	8.5	44
high filtered data	5.7	7.1	22
high filtered model	5.8	7.1	20

TABLE 4.3: HIGH MD ELASTICITY, BC CALIBRATION

Observation	$u$ 1972Q1	$u$ 1982Q1	% change
unfiltered data	5.8	8.8	41
unfiltered model	5.8	6.4	10
low filtered data	5.3	8.2	43
low filtered model	5.8	6.3	08
High filtered data	5.7	7.1	22
High filtered model	5.8	6.1	05

TABLE 4.4: HIGH MD ELASTICITY, BF CALIBRATION

Observation	$u$ 1972Q1	$u$ 1982Q1	% change
Unfiltered data	5.8	8.8	41
Unfiltered model	5.8	8.7	40
Low filtered data	5.3	8.2	43
Low filtered model	5.9	8.3	34
high filtered data	5.7	7.1	22
high filtered model	6.0	6.9	14

Figures 6.3 and 6.4 report results of the same exercises using the BF calibration. Now the model accounts for much of the movement in trend  $u$  using a medium filter, and basically all of it using a high filter. And it is not as if we filtered out everything of interest: even with the high HP parameter  $u$  goes from below 5% to above 7% and back. Table 4.2 shows how we can account for basically the entire runup in filtered  $u$  during stagflation, although at a cost of overpredicting the increase in unfiltered  $u$ . One might conclude from this that the theory can account for most of the low-frequency behavior of  $u$ . But the BF calibration method is extreme, in the sense that its implied value of  $\ell$  generates excess volatility in  $u$  when we have shocks to  $y$  only – obviously, since the BC

calibration generates just the right volatility wrt  $y$  shocks. In fact,  $u$  is about twice as volatile wrt  $y$  shocks in the BF calibration. So one ought not claim too much. We find it nevertheless interesting that the theory in principle can account for so much when we get to choose  $\ell$ .

Figures 6.5-6.8 report results with a higher money demand elasticity. The BC version now generates a little more movement in  $u$  – e.g. Table 4.3 indicates that we can now account for 25, 19 or 23% of the runup in  $u$  during stagflation, depending on which filter we use (recall that with a less elastic money demand the numbers were 20, 12 and 13%). Also, the BF version matches quite well but now generates somewhat less excess volatility in  $u$  wrt real shocks. To describe the results another way, with a low HP filter the scatter plot between  $i$  and  $u$  generated by the model looks pretty similar to the data, and with a high filter the scatter plots look indistinguishable (this was pretty much true with a lower money demand elasticity, too). The general message is similar with a more elastic than with a less elastic money demand, although in the former case the model does a little better in accounting for the data.

To understand these results, consider the following intuitive argument. The initial impact of a change in  $i$  is to reduce  $M/p$ , which affects revenue  $R$  and hence employment. The size of the effect of  $i$  on  $M/p$  is determined by the money demand elasticity, as in any monetary model; the size of the effect of  $R$  on  $u$  is then determined by the value of leisure, as in the usual labor market model. A bigger money demand elasticity and smaller value of  $\ell$ , or vice-versa, generate similar net effects. To see this, consider the MP and LW curves drawn for the calibrated parameters in Figure 7. More elastic money demand implies the LW curve shifts more with  $i$ , while a higher value of  $\ell$  makes the MP curve flatter, and either makes  $u$  respond more to monetary policy.<sup>29</sup>

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<sup>29</sup>Of course, shifting these curves only describes comparisons across steady states, but this conveys the economic insights.

We conclude that combinations of parameters that are not unreasonable allow one to account for some of the behavior in trend  $u$ , although just how much depends on the exact calibration. We have no problem with the idea that much of trend  $u$  should be explained by productivity, demographics, taxes, etc. We still think it is interesting that money in principle has a role to play. One thing to do to make the results less dependent on  $\ell$  is to ask the following: how big would a shock to  $i$  have to be to make it equivalent to a given shock to  $y$ ? The answer is shown in Figure 8. For a low money demand elasticity, going from the Friedman rule to 10% annual inflation, i.e.  $i = 0$  to  $i = 0.13$ , is equivalent to around a reduction in  $y$  of around 3/4 of 1%. For a higher money demand elasticity, the answer is closed to 1.5%.

This suggests that money is important in the long run, even ignoring nominal rigidities, imperfect information, and other channels that may or may not be relevant in the short run. And these numbers are independent of the value of  $\ell$  or other aspects of the labor market. Monetary policy, like productivity, can have an impact on  $R$ , and these numbers simply give the equivalent effect on  $R$  from either  $i$  or  $y$ . The degree that changes in  $R$  translate into changes in  $u$  depend on how one calibrates the labor market, and especially  $\ell$ , but the comparison between changes in  $i$  and changes in  $y$  does not depend on these details. Also, to be clear, we emphasize that here we are referring to changes in  $y$  holding other things constant – including  $b$  and  $\ell$ . It is well known in the standard macro-labor model that the interesting equilibrium variables are independent of changing productivity in the market  $y$  and nonmarket activities  $b$  and  $\ell$  at the same rate.

We can also ask about the welfare cost of inflation. Some recent models where money is modeled with relatively explicit microfoundations generate bigger costs than traditional models – e.g. where the reduced-form literature finds that eliminating a 10% inflation is generally worth less than 1% of consump-

tion, and often much less, some more recent models imply this policy can be worth 3% to 4%, or even more.<sup>30</sup> However, these bigger effects usually occur only when there are holdup problems, as occur in bargaining models, and not in competitive search models like the one here. As Figure 9 shows, we can generate big welfare effects even with competitive search, where for each case, depending on the calibration, the effect measured as the percentage change in total consumption that is equivalent to reducing inflation from 10% to the Friedman rule.

We do not dwell too much on welfare, however, since the results depend on the assumption that the Hosios condition is satisfied in the labor market. In the working paper, we show the following. Given no constraints on policy, the optimum is to set  $i = 0$  and set fiscal policy (any combination of UI and taxes) to correct for discrepancies between bargaining power  $\eta$  and the elasticity of the matching function  $\sigma$  in the MP market. Given fiscal policy is set exogenously incorrectly, however, optimal unconstrained monetary policy is  $i \neq 0$ . We of course are constrained to  $i \geq 0$ , but if UI is too low e.g. the optimum is  $i > 0$ . Intuitively, labor market policy can lead to excessive firm entry, and in this case we improve efficiency by the inflation tax. The point is simply that the cost of inflation is sensitive, and can even be negative, depending on bargaining power and fiscal policy. Additional exploration of welfare and optimal policy is therefore left for future work.

## 6 Conclusion

This paper revisited a classic issue in macroeconomics: the relation between unemployment and monetary variables like inflation or nominal interest rates. We began by reviewing the data, and documenting that there is a clear positive long-

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<sup>30</sup>See e.g. Rocheteau and Wright (2007) or Craig and Rocheteau (2007) for summaries of recent findings, as well as a discussion of more traditional studies.

run relationship between these variables, after filtering out the higher-frequency movements. We then built a model, based on explicit microfoundations for both money and unemployment, consistent with these observations. The model takes seriously Friedman's (1977) suggestion that the natural rate of unemployment is determined by real factors, one of which is the cost of holding money balances. We think the framework provides a natural integration and an interesting extension of existing models of unemployment and monetary economics. We then considered some quantitative issues. While in principle the model can be used to address many questions, we focused on asking how well it could account for the low-frequency patterns in unemployment in the counterfactual case where the sole driving process is monetary policy.

After calibrating to standard observations, we found the answer depends mainly on two key parameters: the elasticity of money demand and value of leisure. The former influences the effect of monetary policy on real balances and hence on profit in the retail sector, where money is used as a medium of exchange, while the latter determines how profits translate into entry and employment. For conservative values of the money demand elasticity and value of leisure, depending on some details, we can account for about 20% of the increase in unemployment during the 1970s stagflation episode, which is not insignificant but does leave room for other factors. For less conservative but not unreasonable parameters, the model can account for the lion's share of movements in trend unemployment over the last half century. These results suggest that monetary factors may be important for labor market outcomes, not only theoretically but also quantitatively. Future research could attempt to hone these numerical results and explore other quantitative and theoretical questions.

## Appendix

We define equilibrium in the dynamic-stochastic model for case of wage bargaining in MP, price posting in KW, and price taking in AD (other combinations are similar). At the start of a period the state is  $s = (u, i, y)$ , where  $u$  is unemployment,  $y$  productivity, and  $i$  nominal interest on bonds purchased in the previous and redeemed in the current AD market. The state  $s$  was known in the previous AD market, including the return on the nominal bonds maturing this period. Although these bonds are not traded in equilibrium,  $i$  matters for the following reason. When  $s_+$  is revealed in the current AD, there is a response in the price  $p = p(s_+)$  and hence in the return  $\rho(s_+) = p(s)/p(s_+)$  on money brought in from the previous AD; this implies the no-arbitrage condition

$$1 = \beta(1 + i)\hat{\rho}(s),$$

where  $\hat{\rho}(s) = \mathbb{E}_{s_+}[\rho(s_+)|s]$ .

We assume  $i$  and  $y$  follow exogenous and independent processes,

$$\begin{aligned} y_+ &= \bar{y} + \rho_y(y - \bar{y}) + \epsilon_y, \epsilon_y \sim N(0, \sigma_y) \\ i_+ &= \bar{i} + \rho_i(i - \bar{i}) + \epsilon_i, \epsilon_i \sim N(0, \sigma_i). \end{aligned}$$

Unemployment changes endogenously as follows. The probability in MP an unemployed  $h$  finds a job and  $f$  fills a vacancy are  $\lambda_h[\tau(s)]$  and  $\lambda_f[\tau(s)]$ , where  $\tau(s)$  is the  $v/u$  ratio and  $v = v(s)$  was set in the previous AD market as a function of the current state, so that

$$u_+(s) = u - u\lambda_h[\tau(s)] + (1 - u)\delta.$$

Similarly, in KW the probability  $h$  meets a seller and  $f$  meets a buyer are  $\alpha_h[Q(s)]$  and  $\alpha_f[Q(s)]$ , where the  $B/S$  ratio  $Q(s)$  and terms of trade  $[d(s), q(s)]$  were posted in the previous AD market, and  $d$  is measured in units of  $x$  from that market.

After MP and KW, in the current AD market the realization of  $s_+$  becomes known. Firms then liquidate inventories, pay wages and dividends, create vacancies for the next MP, and post terms for the next KW. Also, households choose real balances for the next KW, while government collects taxes, pays UI and announces  $i_+$ . Once  $s_+$  is observed in AD, the real value of money brought in from KW is adjusted from  $z(s)$  to  $z(s)\rho(s_+)$ ; hence, in the KW market real balances are valued at  $z(s)\hat{\rho}(s)$ . Also, agents can commit within the period to any wage negotiated in MP, to be paid in units of  $x$  in the current AD market, but  $w(s)$  can be renegotiated when MP reconvenes next period.

We now present the value functions for  $h$ , keeping track of  $s$  as well as individual state variables, as appropriate. In MP, taking as given the equilibrium wage function  $w(s)$ ,

$$\begin{aligned} U_0^h(z; s) &= V_0^h(z; s) + \lambda_h [\tau(s)] \{V_1^h[z, w(s); s] - V_0^h(z; s)\} \\ U_1^h(z; s) &= V_1^h[z, w(s); s] - \delta \{V_1^h[z, w(s); s] - V_0^h(z; s)\}. \end{aligned}$$

Consider  $h$  in KW with arbitrary real balances  $z$  and, if  $e = 1$ , arbitrary wage  $w$ , taking as given  $[q(s), d(s), Q(s)]$ . In equilibrium it should be clear that  $h$  chooses either  $z = 0$  or  $z = d(s)$ . If  $z = 0$  then  $V_e^h(\cdot; s) = \mathbb{E}_{s_+} W_e^h(\cdot; s_+)$ ; if  $z = d(s)$  then

$$\begin{aligned} V_1^h(z, w; s) &= \alpha_h [Q(s)] \{v[q(s)] - d(s)\hat{\rho}(s)\} + d(s)\hat{\rho}(s) + \mathbb{E}_{s_+} W_1^h(0, w; s_+) \\ V_0^h(z; s) &= \alpha_h [Q(s)] \{v[q(s)] - d(s)\hat{\rho}(s)\} + d(s)\hat{\rho}(s) + \mathbb{E}_{s_+} W_0^h(0; s_+), \end{aligned}$$

using the linearity of  $W_e^h(\cdot; s_+)$ . And finally, in AD,

$$\begin{aligned} W_1^h(z, w; s_+) &= z + w + \Delta(s_+) - T(s_+) + \max_{z_+ \geq 0} \{-z_+ + \beta U_1^h(z_+; s_+)\} \\ W_0^h(z; s_+) &= z + b + \ell + \Delta(s_+) - T(s_+) + \max_{z_+ \geq 0} \{-z_+ + \beta U_0^h(z_+; s_+)\} \end{aligned}$$

We now present the value functions for  $f$ . In MP, given the equilibrium wage function  $w(s)$ ,

$$\begin{aligned} U_0^f(s) &= \lambda_f [\tau(s)] V_1^f[w(s); s] \\ U_1^f(s) &= (1 - \delta) V_1^f[w(s); s]. \end{aligned}$$

For  $f$  in KW with  $e = 1$  and wage obligation  $w$ , given  $[q(s), d(s)]$  and  $Q(s)$ ,

$$V_1^f(w; s) = y - w + \alpha_f [Q(s)] [d(s)\mathbb{E}_{s_+}\rho(s_+) - q(s)] + \beta \mathbb{E}_{s_+} U_1^f(s_+).$$

We do not actually need  $W_e^f$ , although it should be clear how to define it.

In MP, wage bargaining implies

$$w(s) = \max_w [V_1^f(w; s)]^\eta [V_1^h(z, w; s) - V_0^h(z; s)]^{1-\eta}$$

where we note that  $z$  vanishes on the RHS. In KW, let the surplus for  $h$  with wage  $w$  from either participating or not in the market be

$$\Sigma(s) = \max \{V_e^h[d(s), w; s] - V_e^h(0, w; s), 0\},$$

where we note that  $w$  vanishes from the RHS. Then  $[d(s), q(s), Q(s), \Sigma(s)]$  solve the generalized conditions for competitive search given in the text:

$$\begin{aligned} v'[q(s)] - 1 &= \frac{i}{\alpha_h [Q(s)]} \\ d(s)\mathbb{E}_{s_+}\rho(s_+) &= g[q(s), \epsilon(s)] \\ \Sigma(s) &= \beta\alpha_h [Q(s)] \{v[q(s)] - v'[q(s)]g[q(s), \epsilon(s)]\} \\ \Sigma(s) > 0 &\implies Q = [1 - u(s)]^{-1} \quad \text{and} \quad \Sigma(s) = 0 \implies Q = \bar{Q}(i). \end{aligned}$$

Finally, we construct a probability transition function  $\mathcal{P}(s_+, s)$  from the laws of motion for  $u$ ,  $i$  and  $y$  given above in the obvious way.

We can now define equilibrium as a list of value functions  $(U_e^j, V_e^j, W_e^j)$ , prices  $(w, d, q, \rho)$ , market tightness measures  $(\tau, Q)$ , and distribution  $\mathcal{P}$  satisfying the above conditions. More compactly, define the surplus from a match by

$$S(s) = V_1^h [z, w(s); s] + V_1^f [w(s); s] - V_0^h (z; s),$$

where  $w$  and  $z$  both vanish on the RHS. Then the list  $(S, d, q, \tau, Q, \mathcal{P})$  constitutes an equilibrium as long as:

(i) the surplus solves

$$S(s) = y - b - \ell + \alpha_f [Q(s)] \left[ \frac{d(s)}{\beta(1+i)} - q(s) \right] + \beta \mathbb{E}_{s_+} \{1 - \delta - (1 - \eta)\lambda_h [\tau(s_+)]\} S(s_+)$$

(ii) the KW terms of trade solve

$$\begin{aligned} v'[q(s)] &= 1 + i/\alpha_h [Q(s)] \\ d(s) &= \beta(1+i)g[q(s), \epsilon(Q(s))] \end{aligned}$$

(iii) KW tightness  $Q(s)$  solves

$$Q(s) = \begin{cases} [1 - u_+(s)]^{-1} & \text{if } u_+(s) \leq \bar{u}(i) \\ [1 - \bar{u}(i)]^{-1} & \text{if } u_+(s) > \bar{u}(i) \end{cases}$$

where  $\bar{u}(i)$  solves  $v[\phi(u, i)] - v'[\phi(u, i)]g[\phi(u, i), \epsilon\left(\frac{1}{1-u}\right)] = 0$ , and  $\phi(u, i)$  is defined by  $v'[\phi(u, i)] - 1 = i/\alpha_h\left(\frac{1}{1-u}\right)$ , as in the text.

(iv) MP tightness  $\tau(s)$  solves

$$k = \beta\lambda_f [\tau(s)]\eta S(s)$$

(v)  $\mathcal{P}(\cdot)$  is derived from the laws of motion.

It is a standard exercise to solve numerically for functions  $(S, d, q, \tau, Q, \mathcal{P})$ . See <http://www.wvz.unibas.ch/witheo/aleks/BMWII/BMWII.html> for details, including programs for calibration and simulation of the model.

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Figure 1.1: M1 growth and inflation

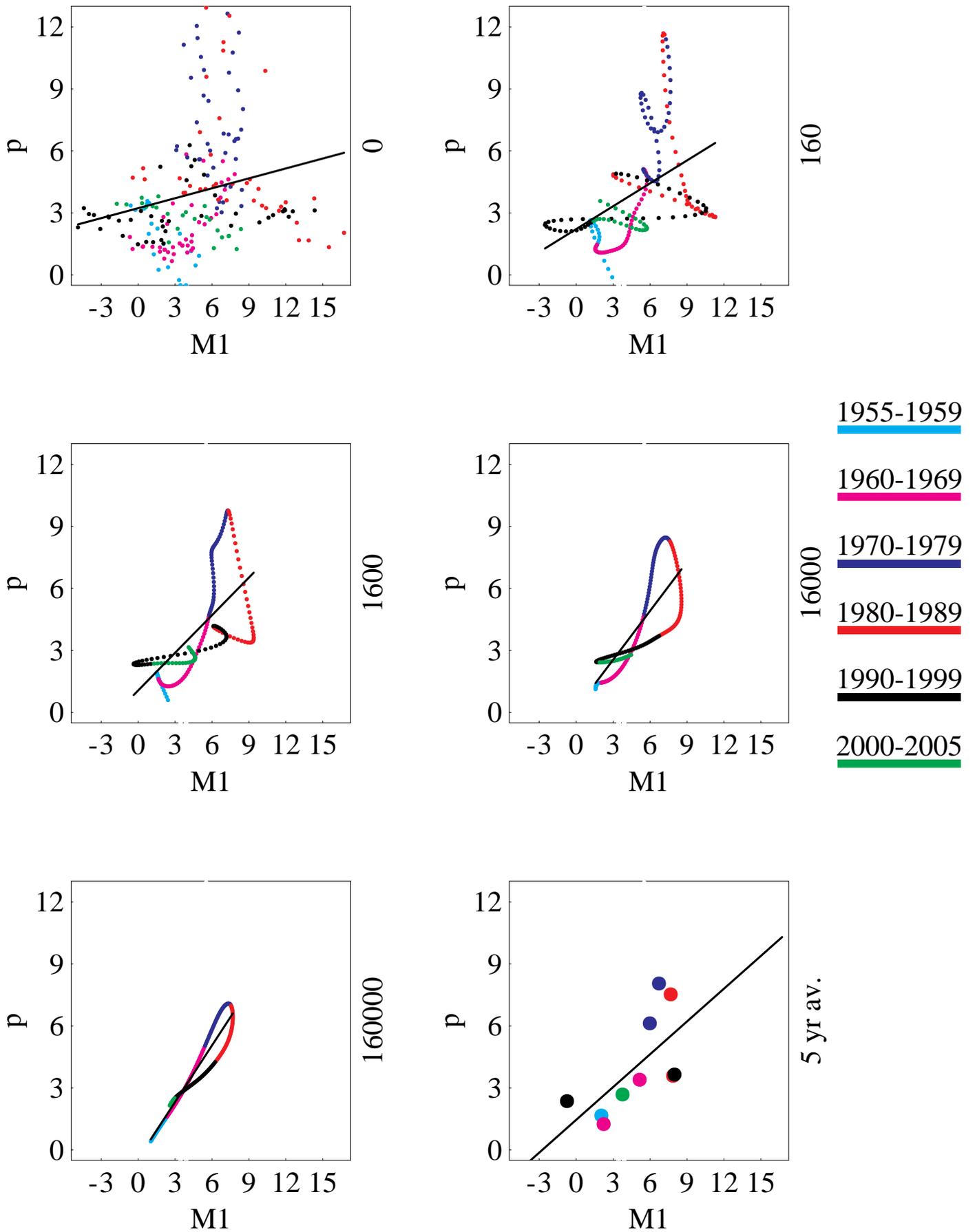


Figure 1.2: M2 growth and inflation

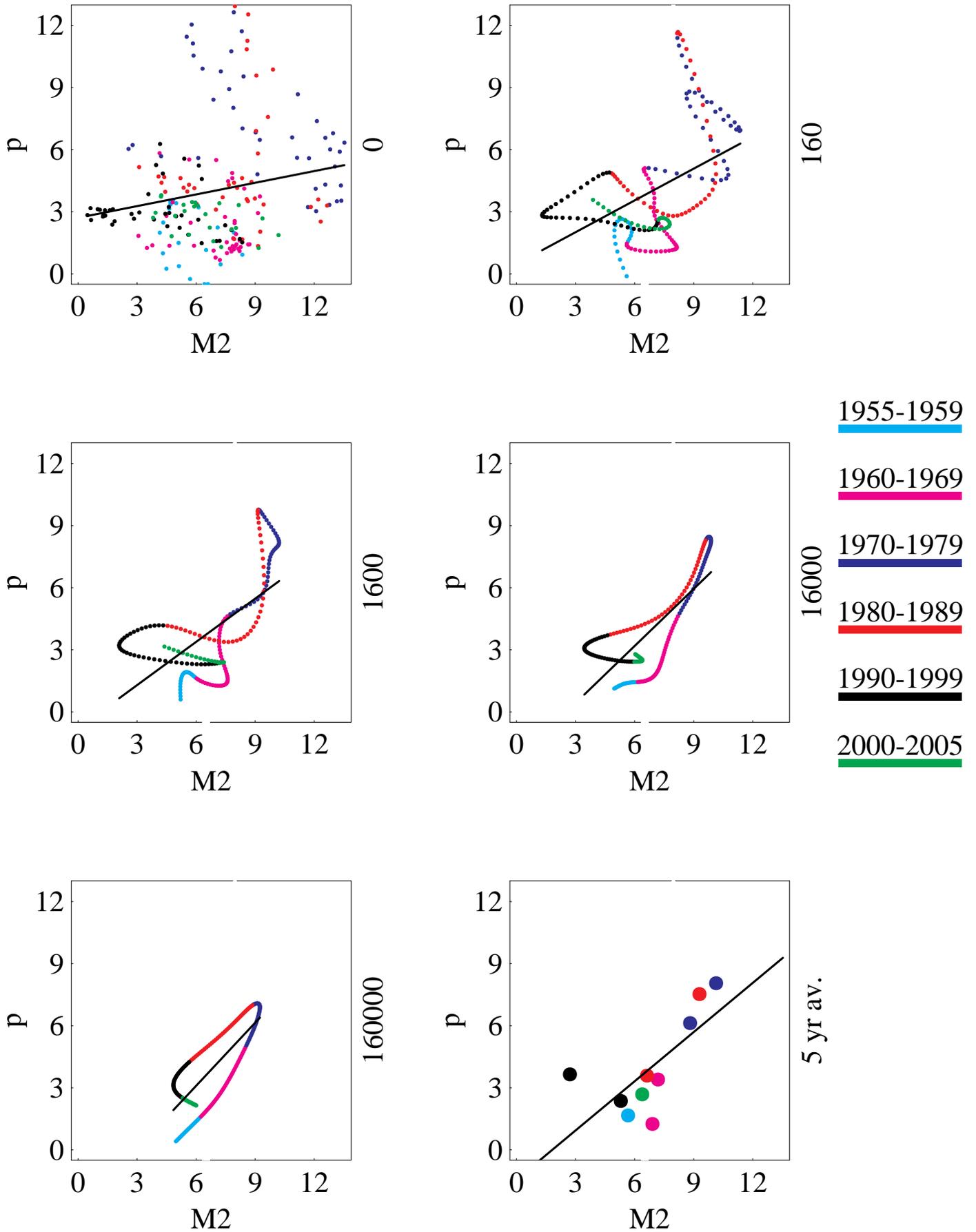


Figure 1.3: M0 growth and inflation

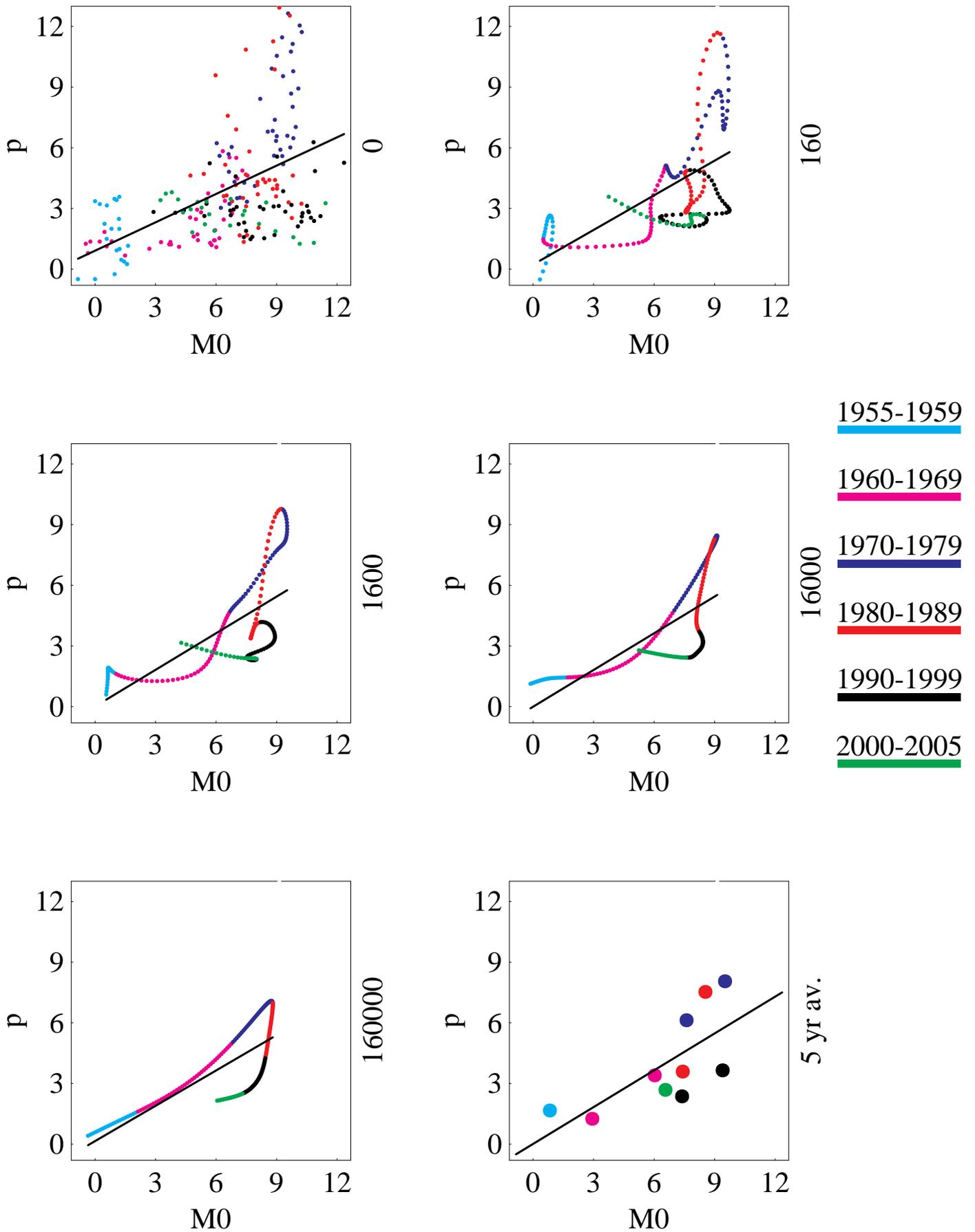


Figure 1.4: Inflation and interest rate

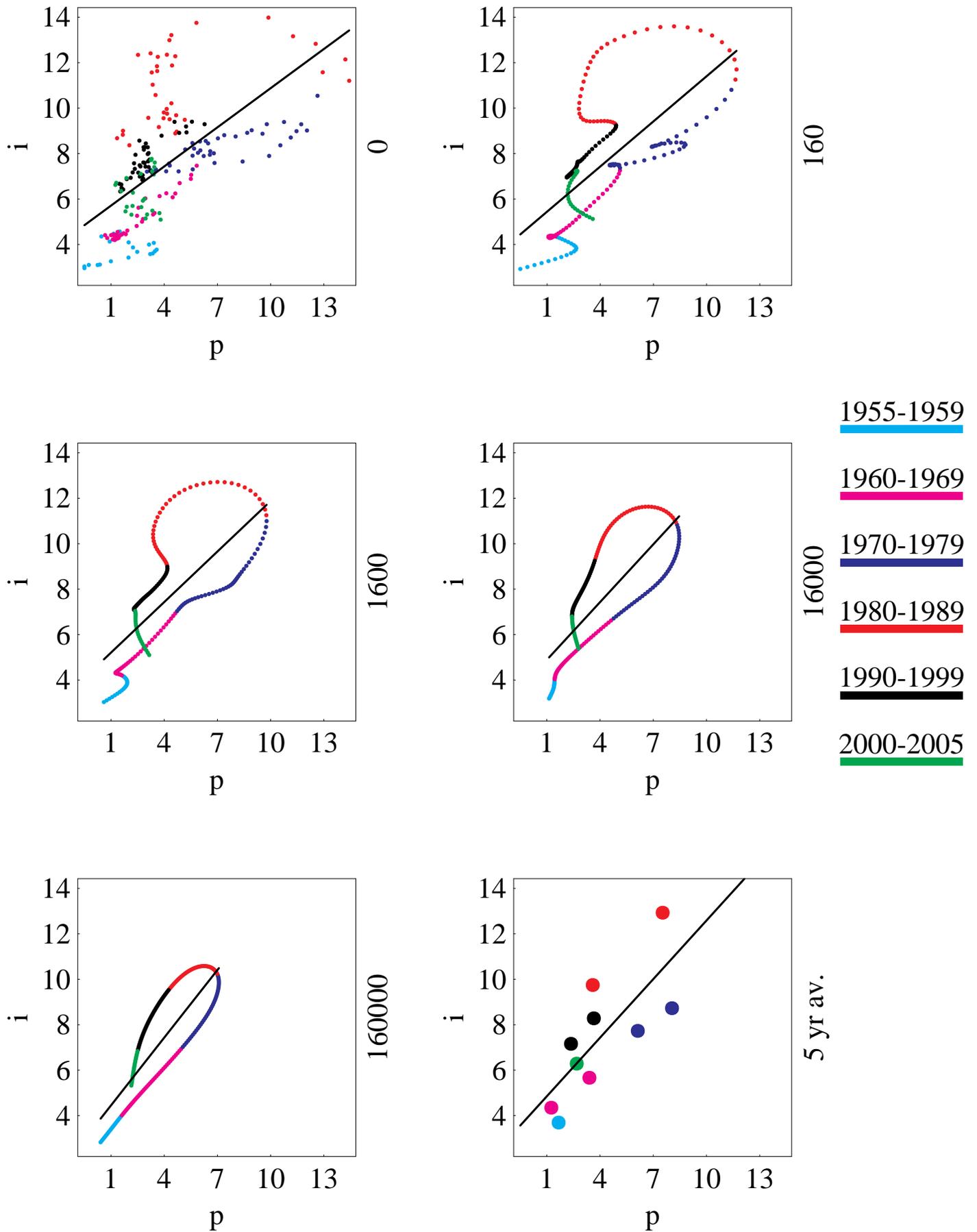


Figure 1.5: M1 growth and interest rate

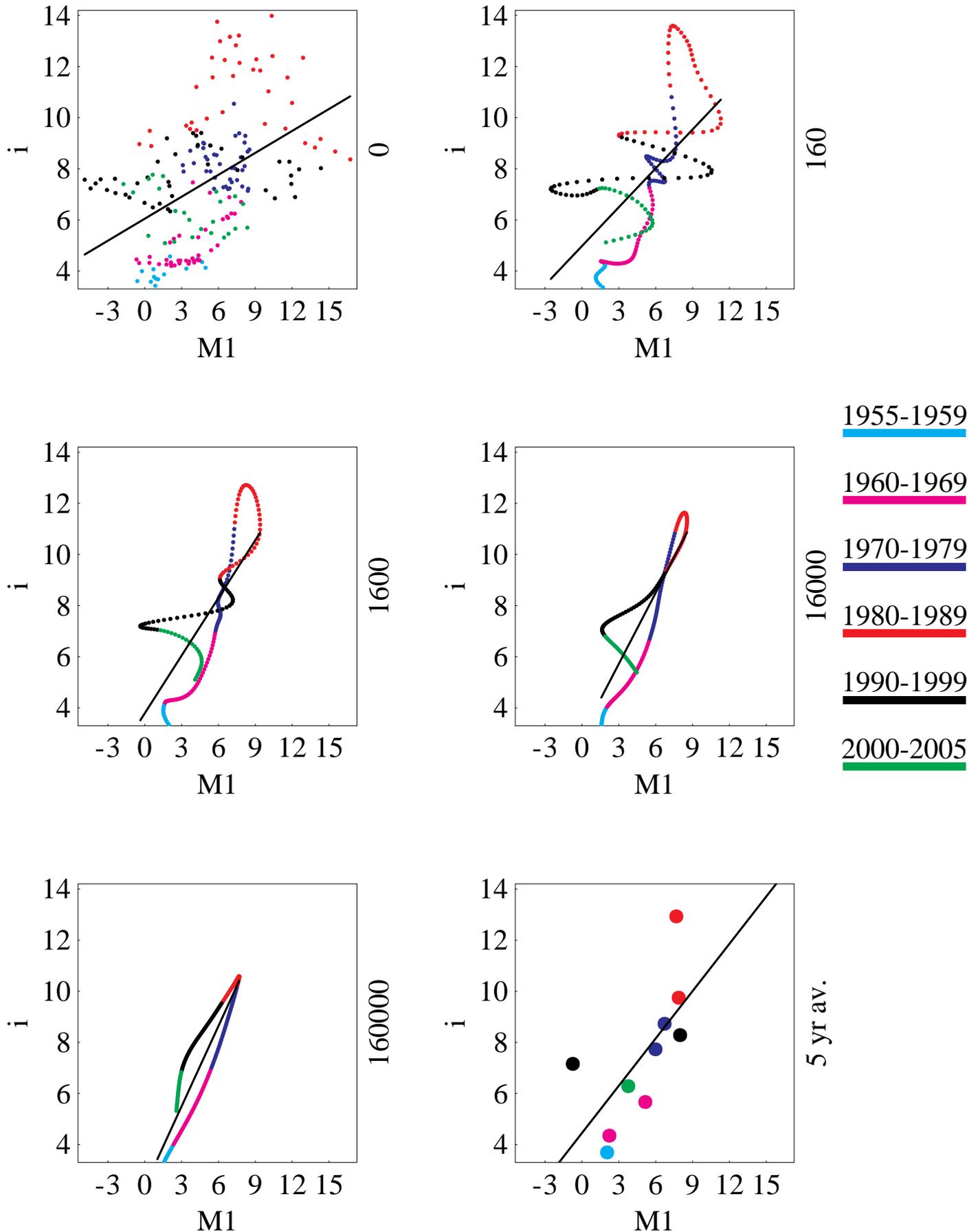


Figure 1.6: Inflation and unemployment

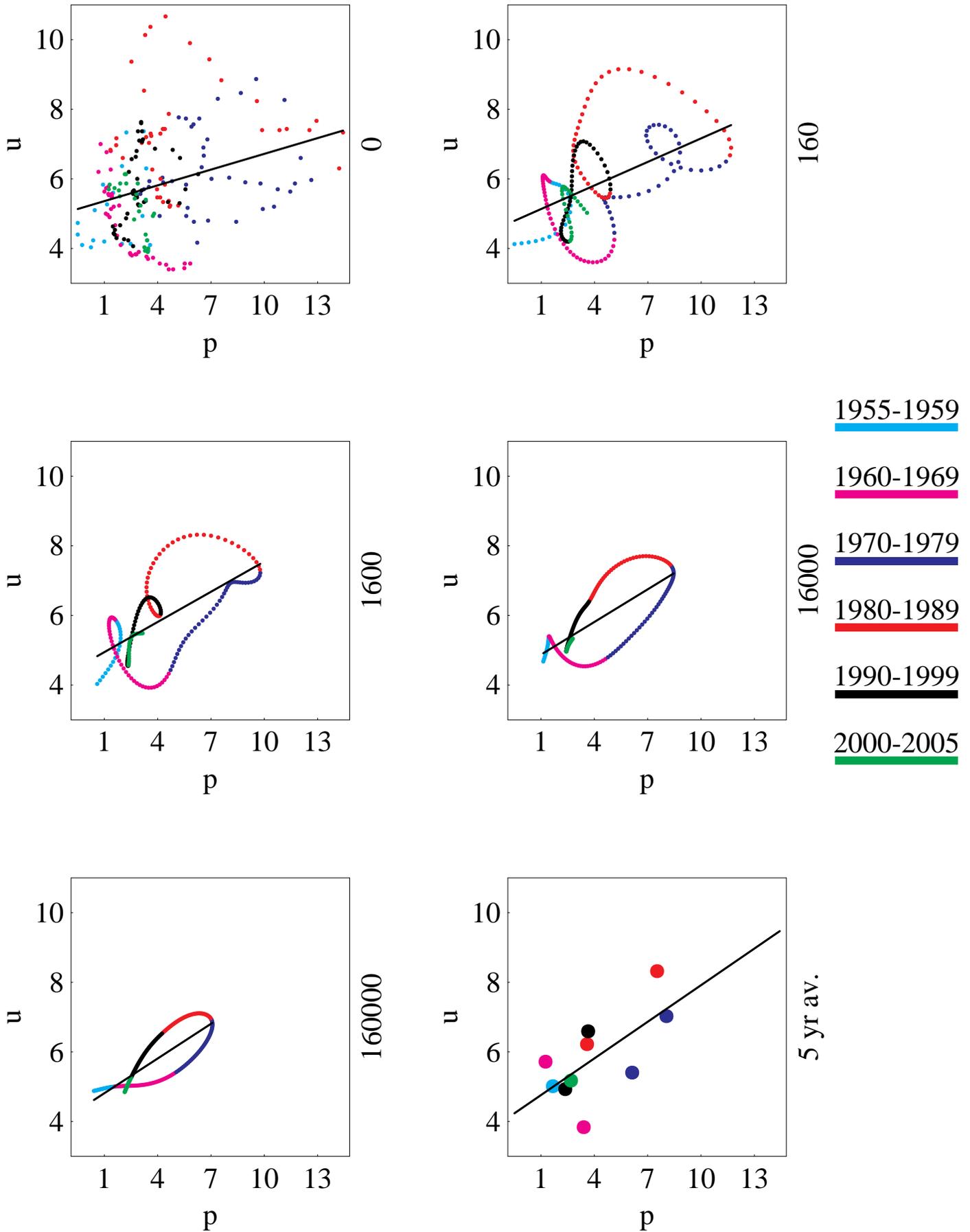


Figure 1.7: Interest rate and unemployment

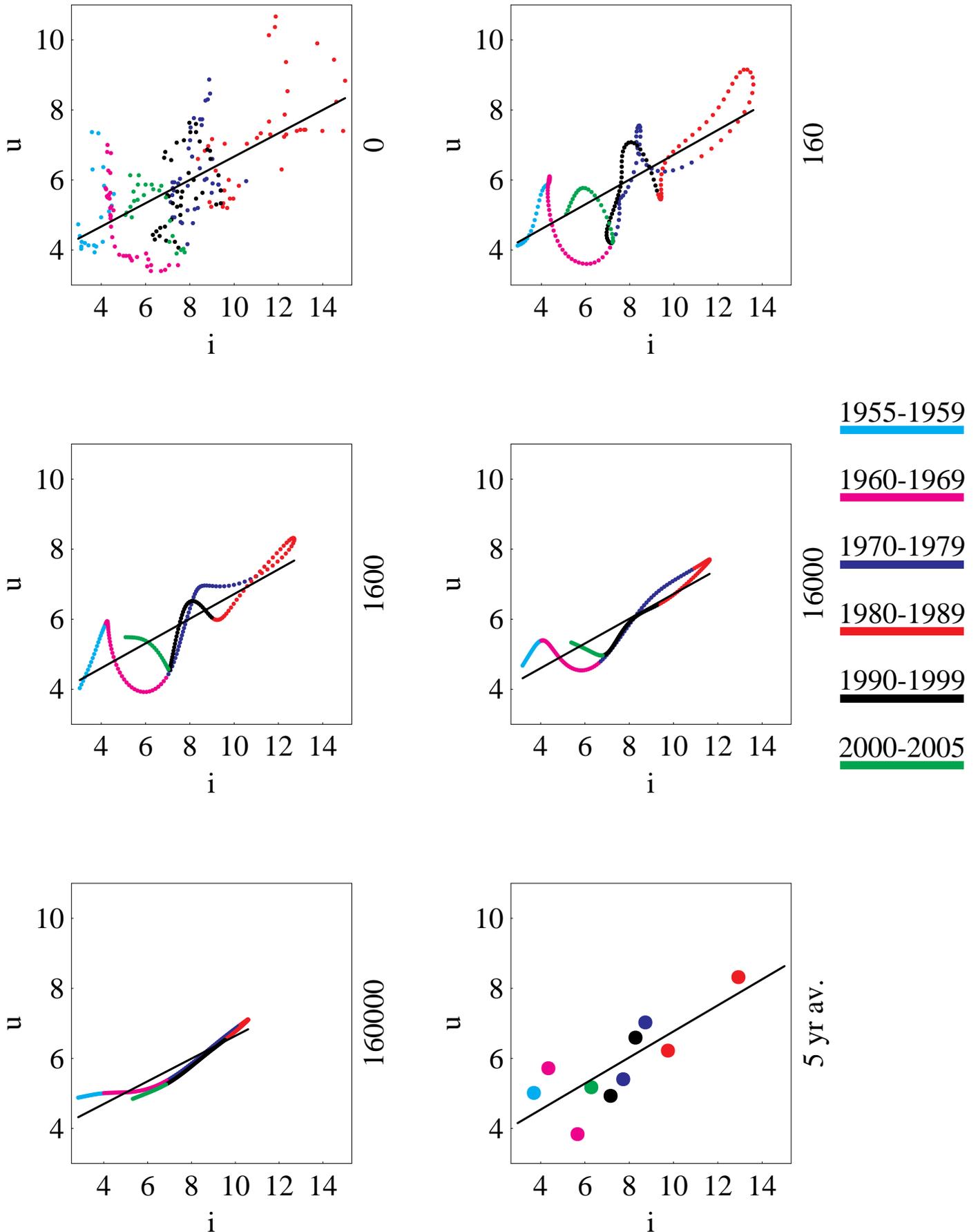
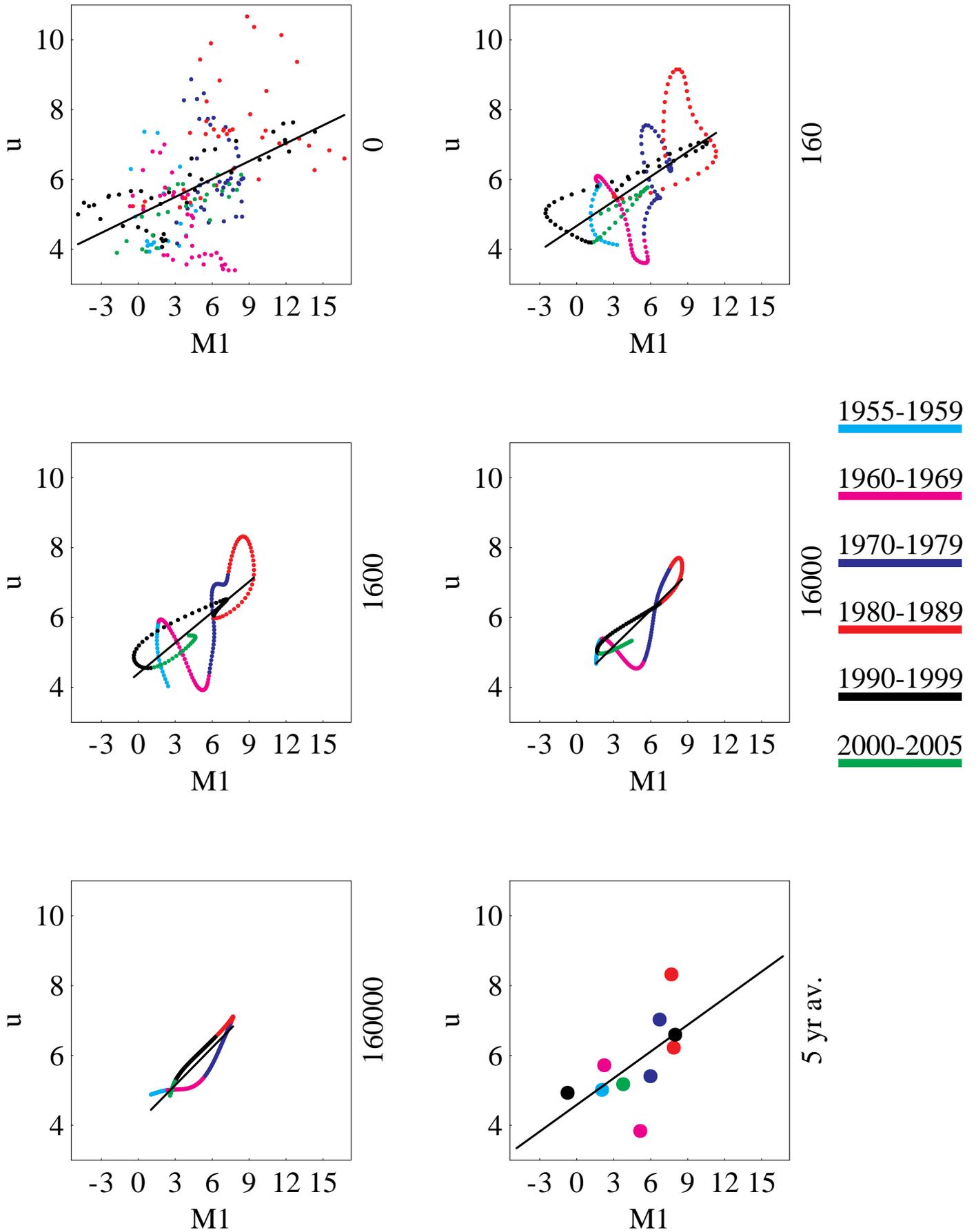


Figure 1.8: M1 growth and unemployment



# Figure 1.9: Inflation and employment

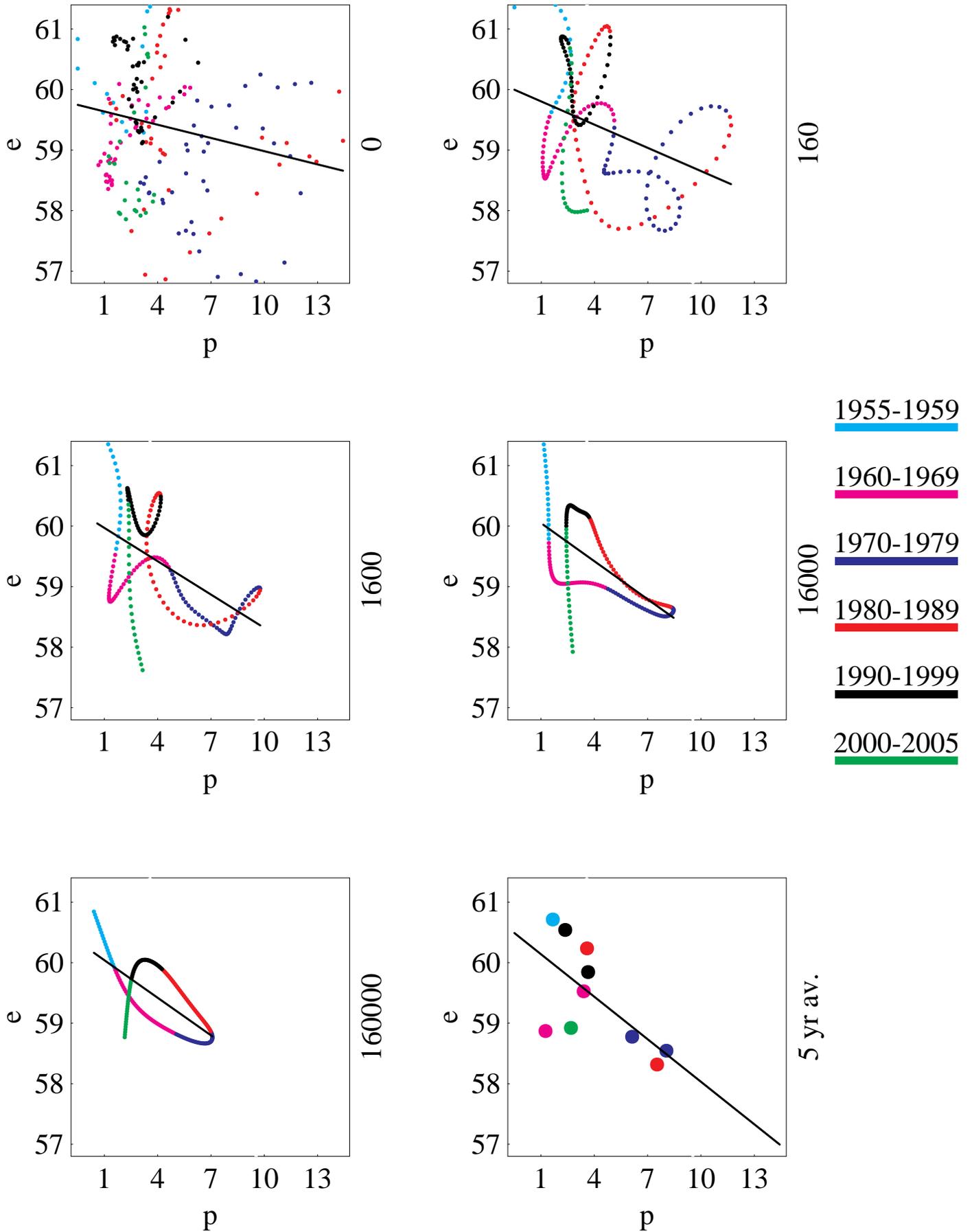


Figure 1.10: Interest rate and employment

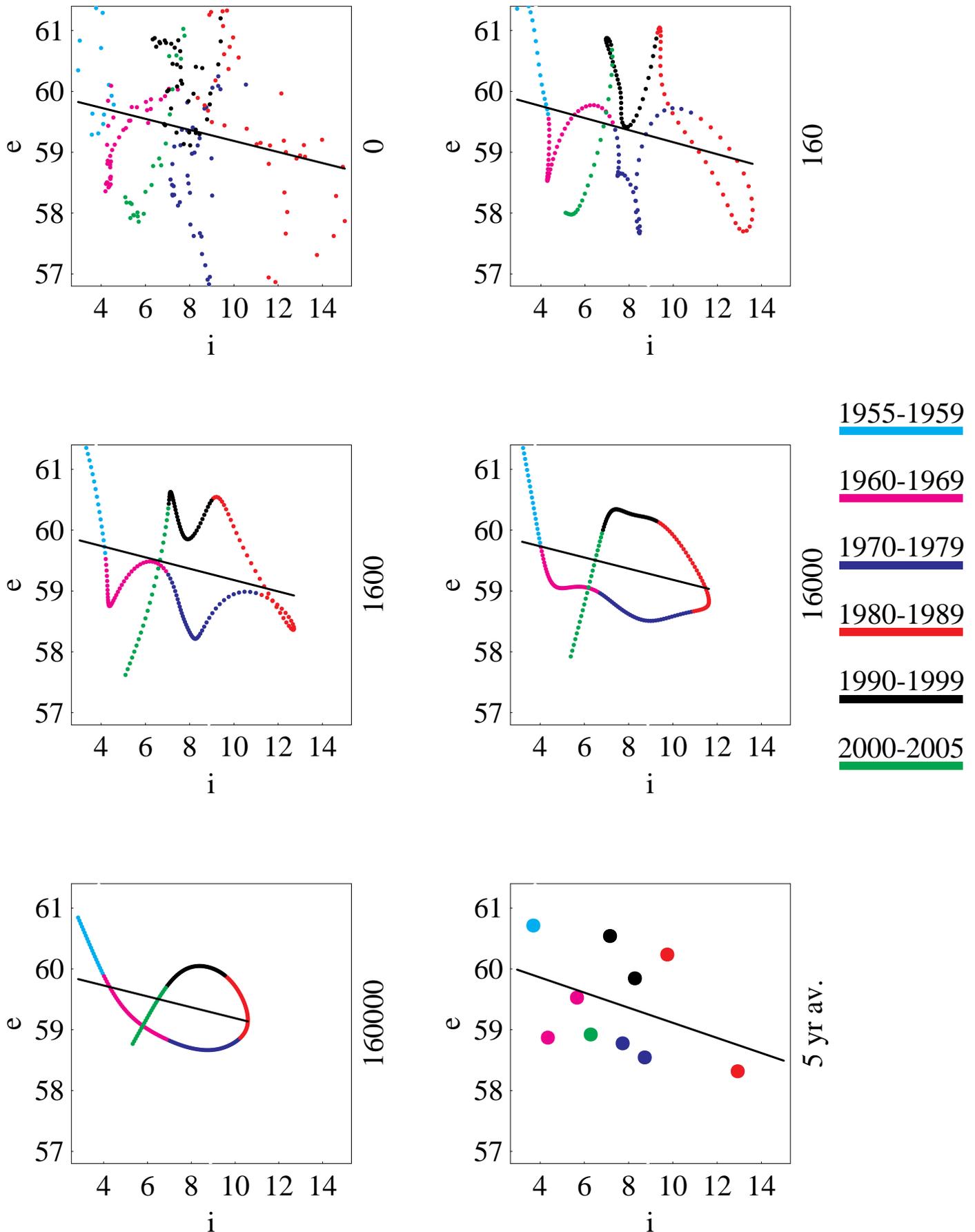


Figure 1.11: M1 growth and employment

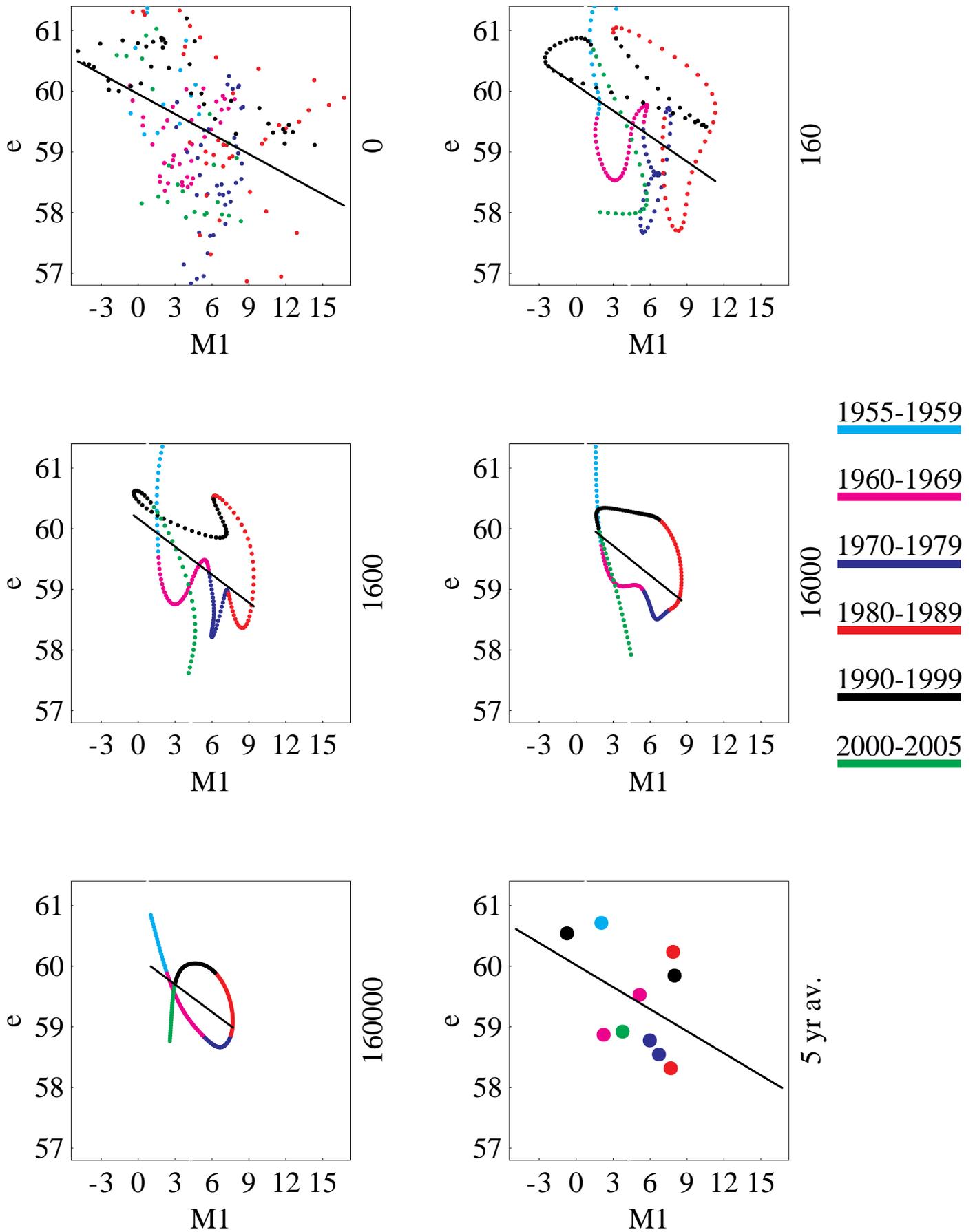


Figure 1.12: Interest rate and M1/PY

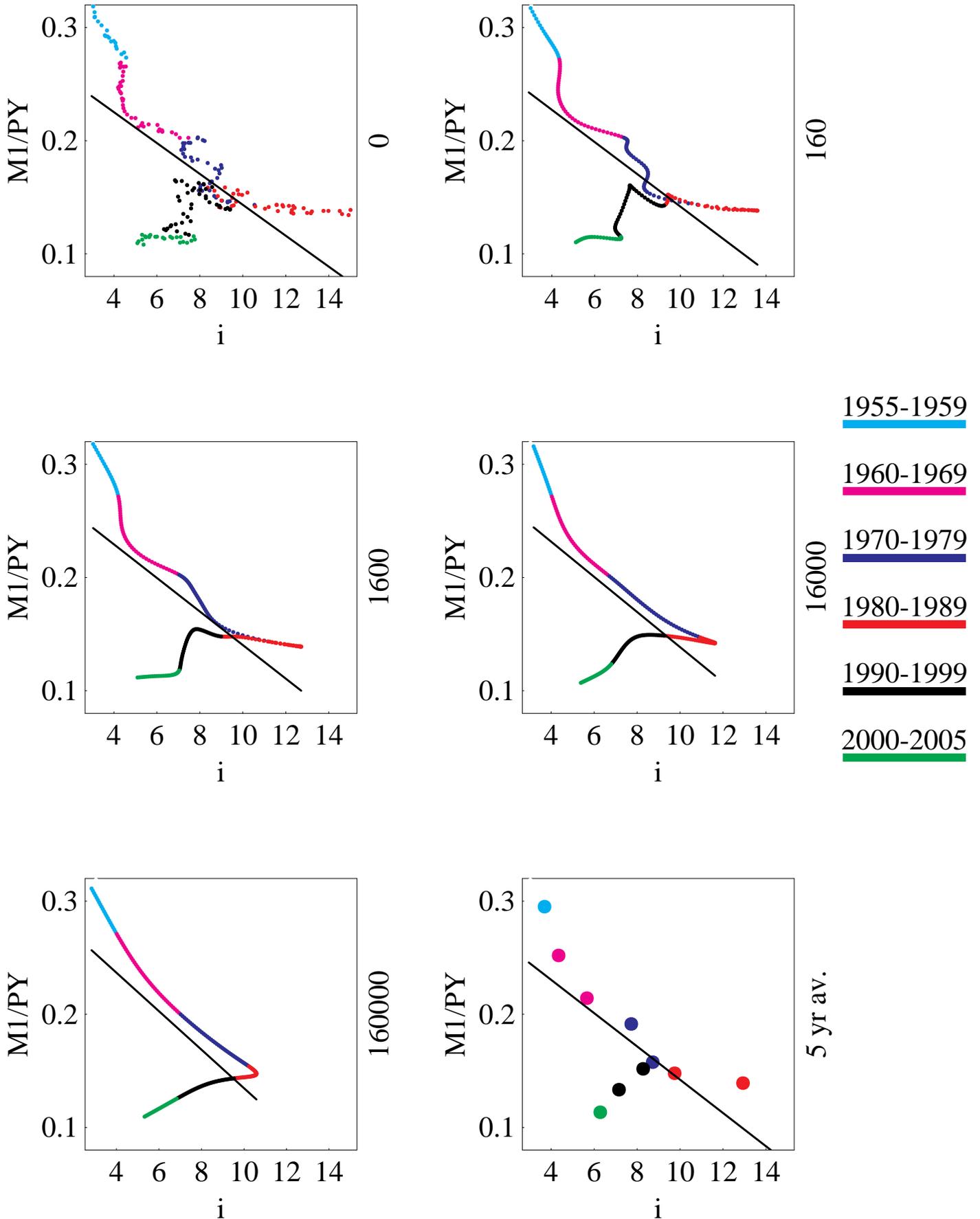


Figure 1.13: Interest rate and M2/PY

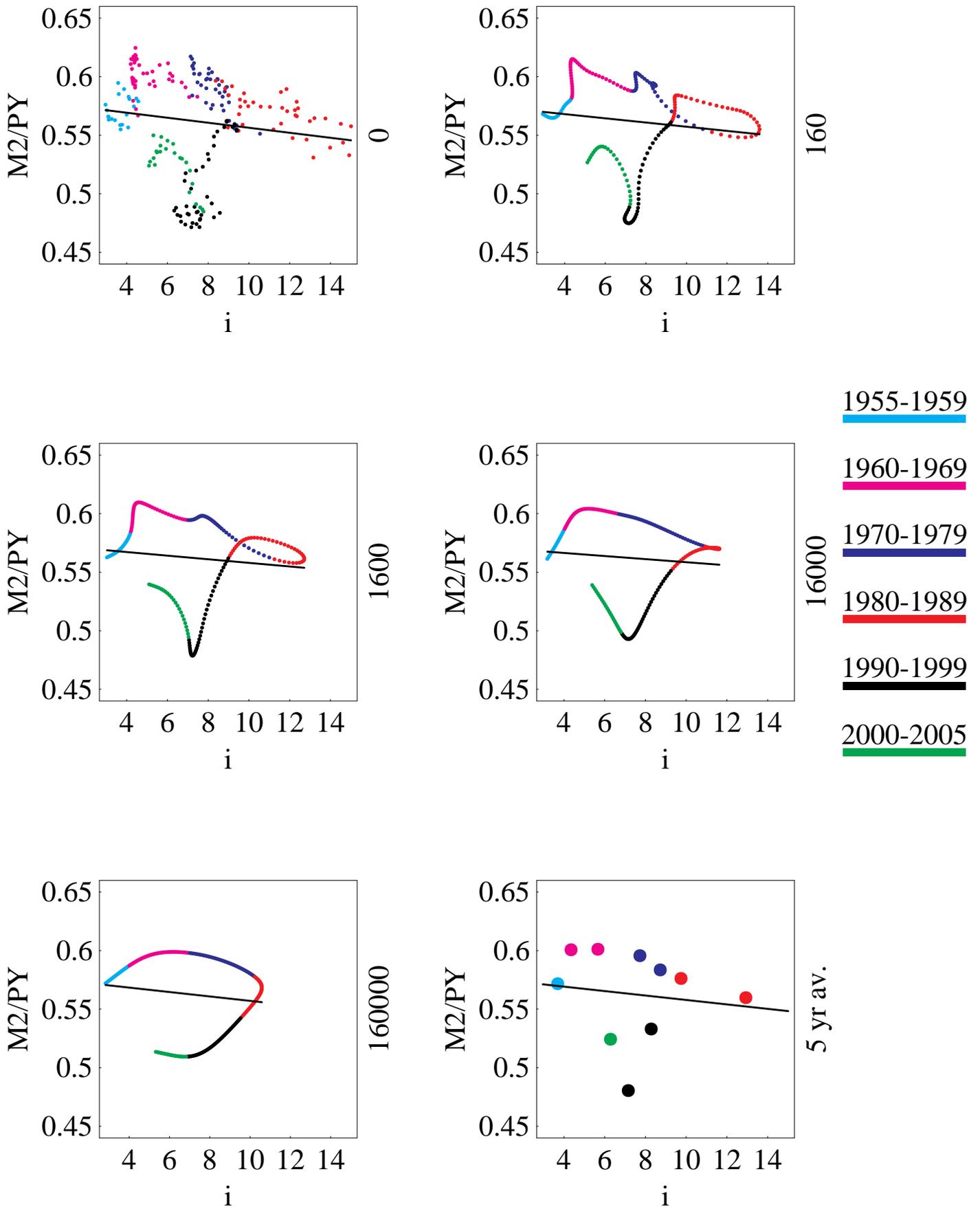


Figure 1.14: Interest rate and M0/PY

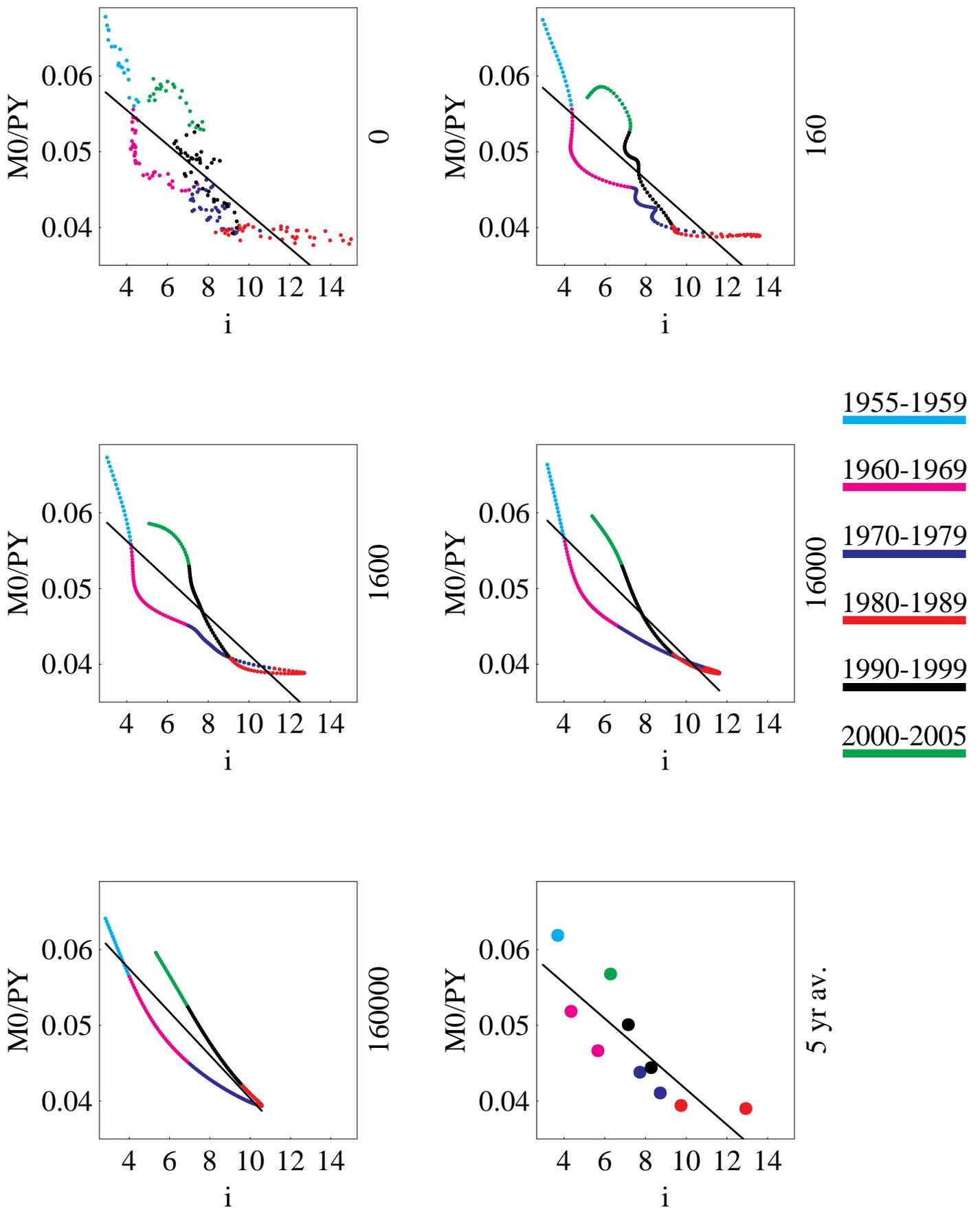


Figure 2: Timing

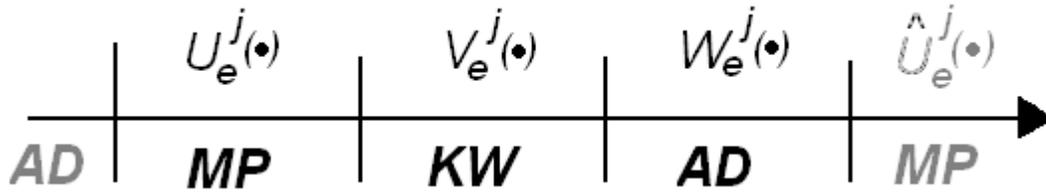


Figure 3: MP and LW curves

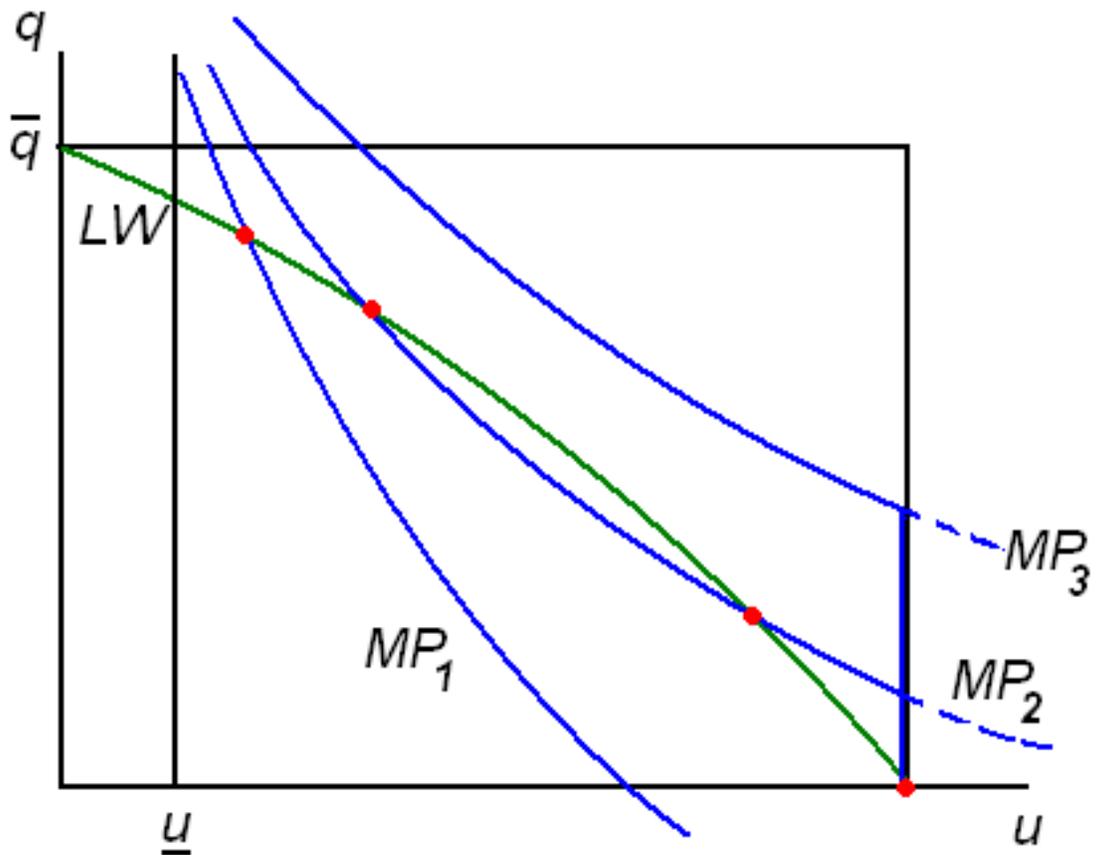


Figure 4.1: Supply and demand for  $Q$  in CSE

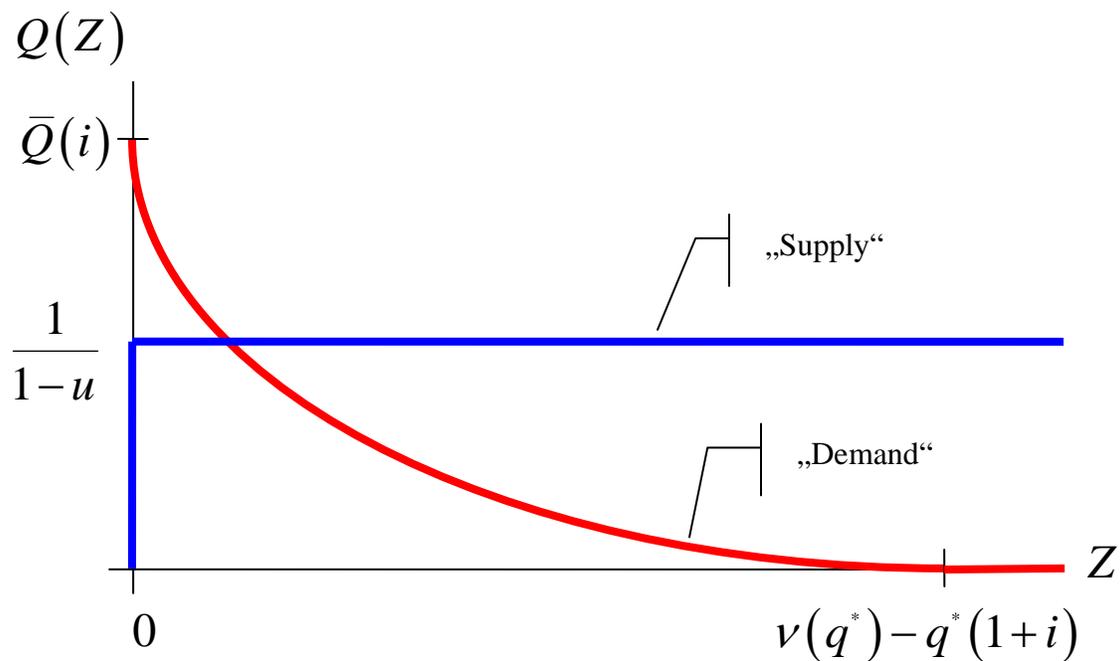
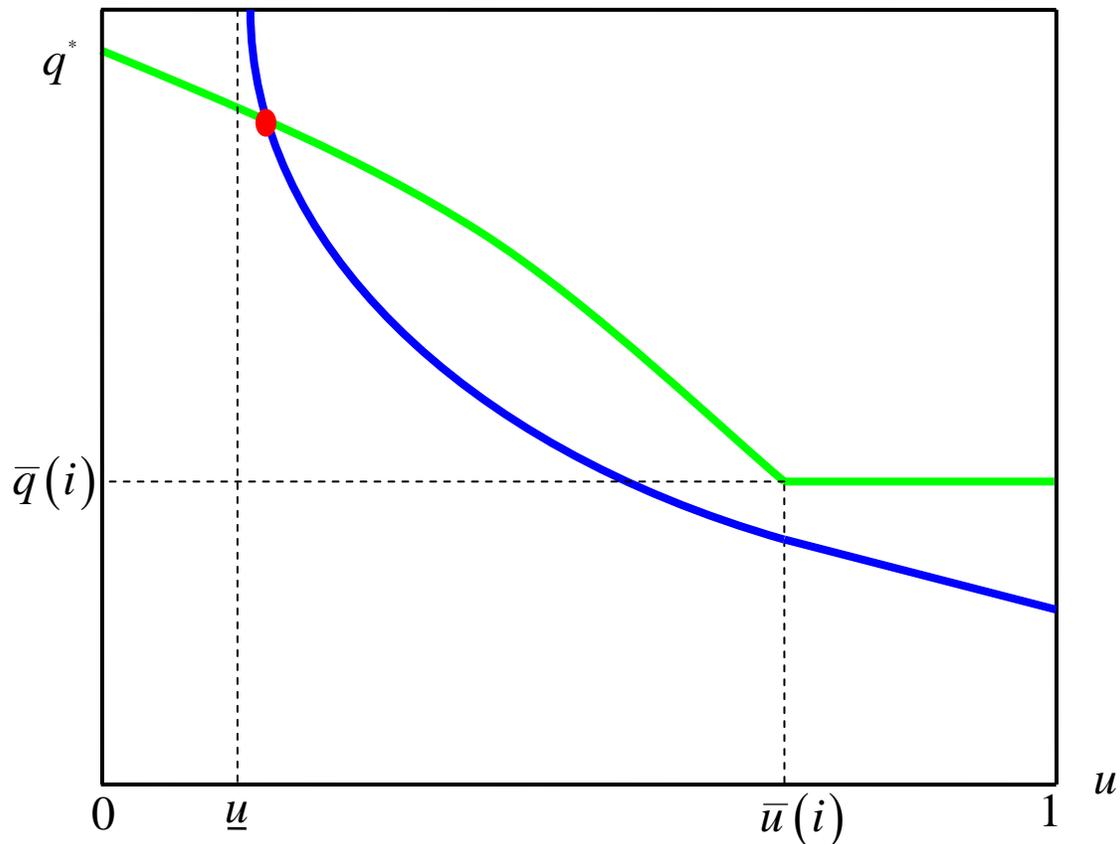
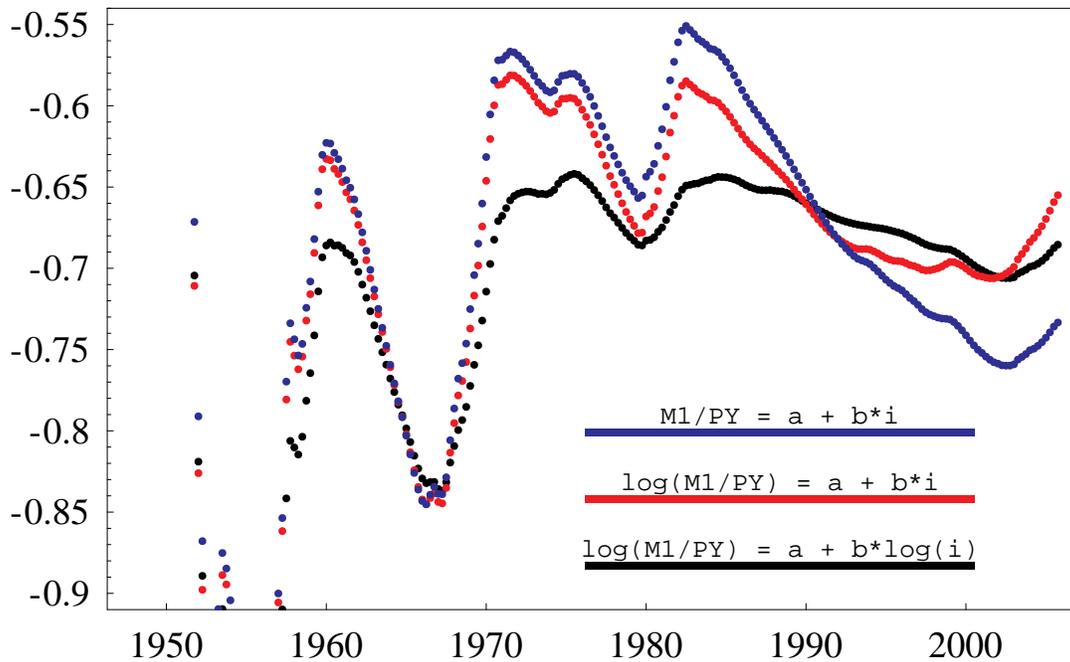


Figure 4.2: MP and LW curves in CSE



### Figure 5.1: M1/PY elasticities



Each point (x, y) displays the interest elasticity of money demand y calculated with data from 1948-x.

### Figure 5.2: Money demand fit

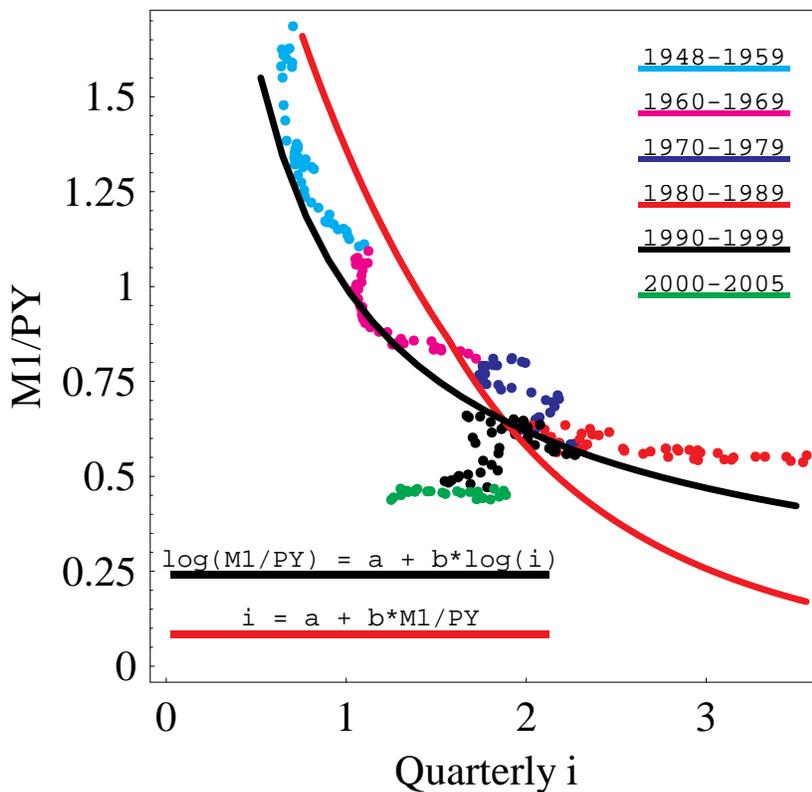


Figure 6.1: BC low elasticity 160000

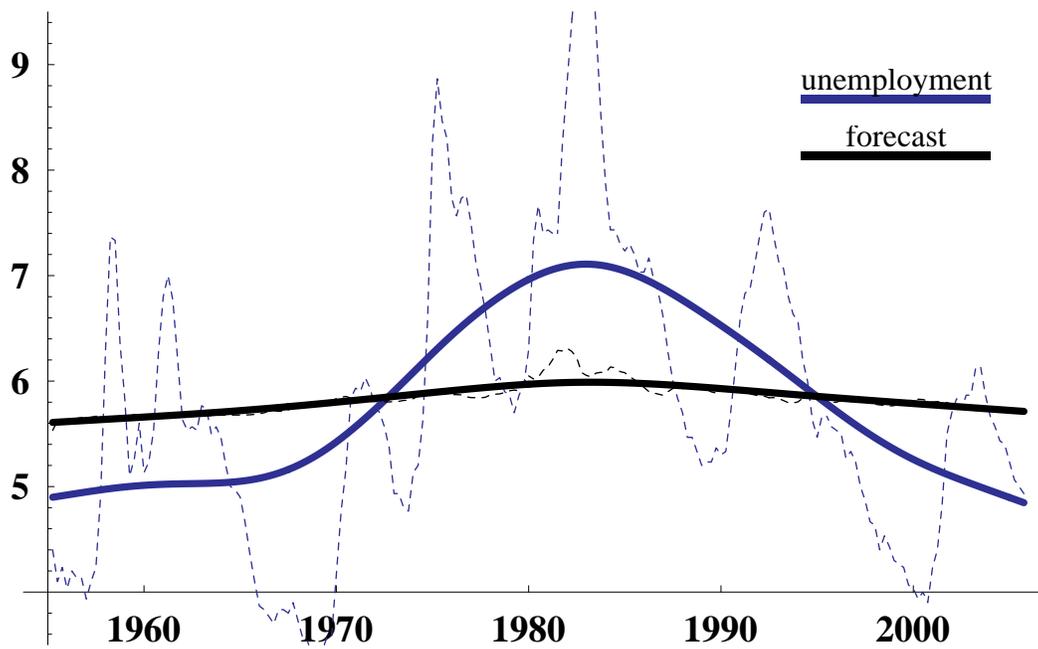
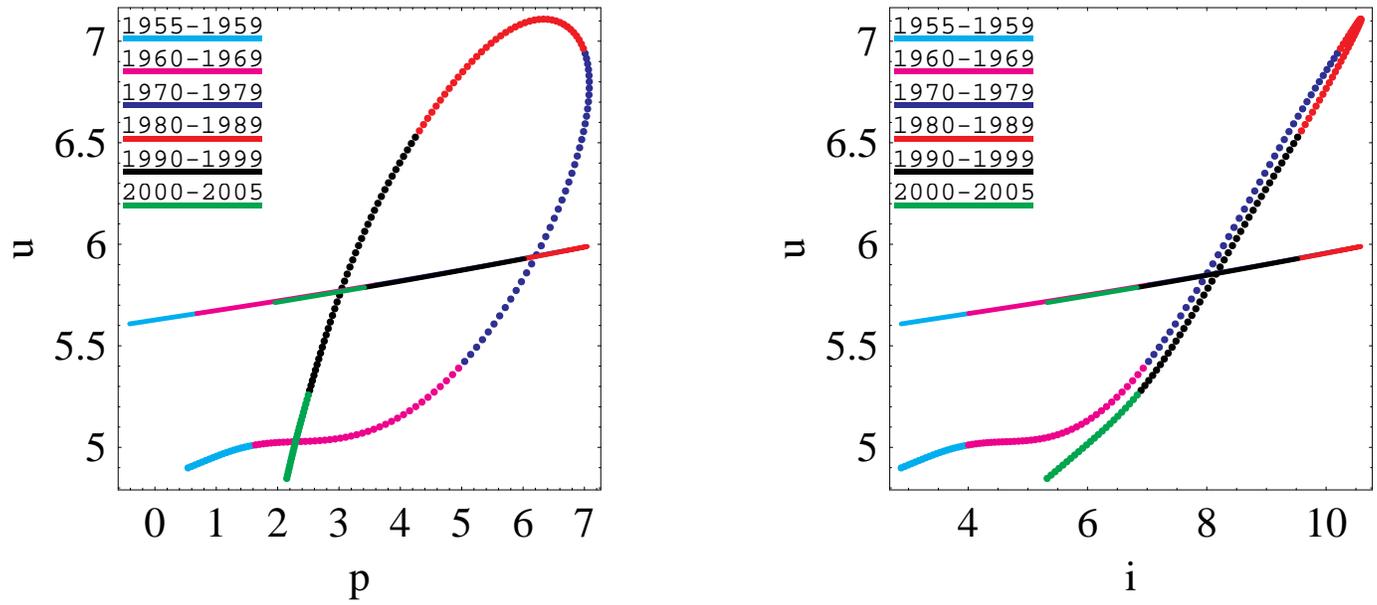


Figure 6.2: BC low elasticity 1600

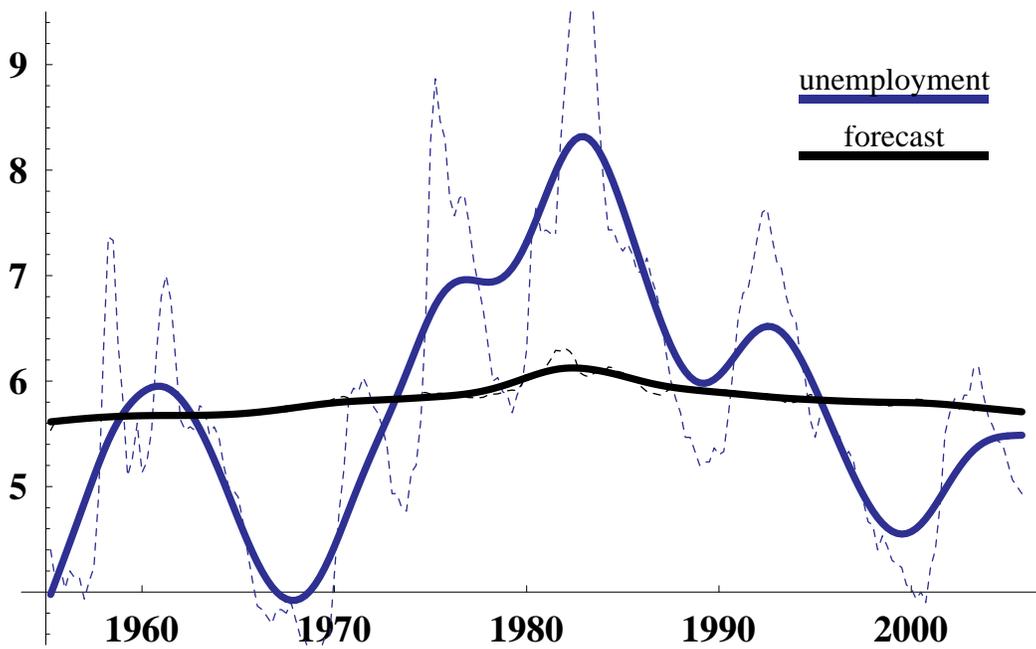
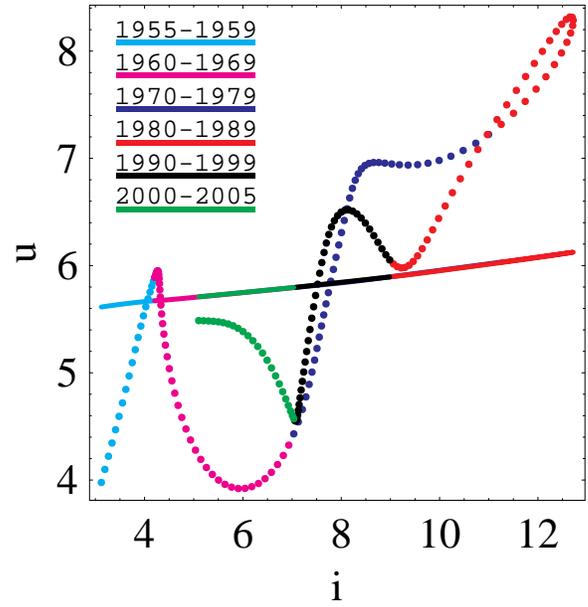
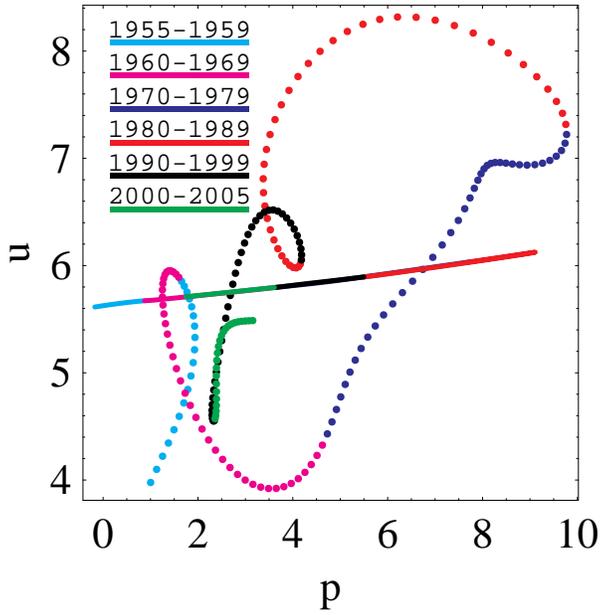


Figure 6.3: BF low elasticity 160000

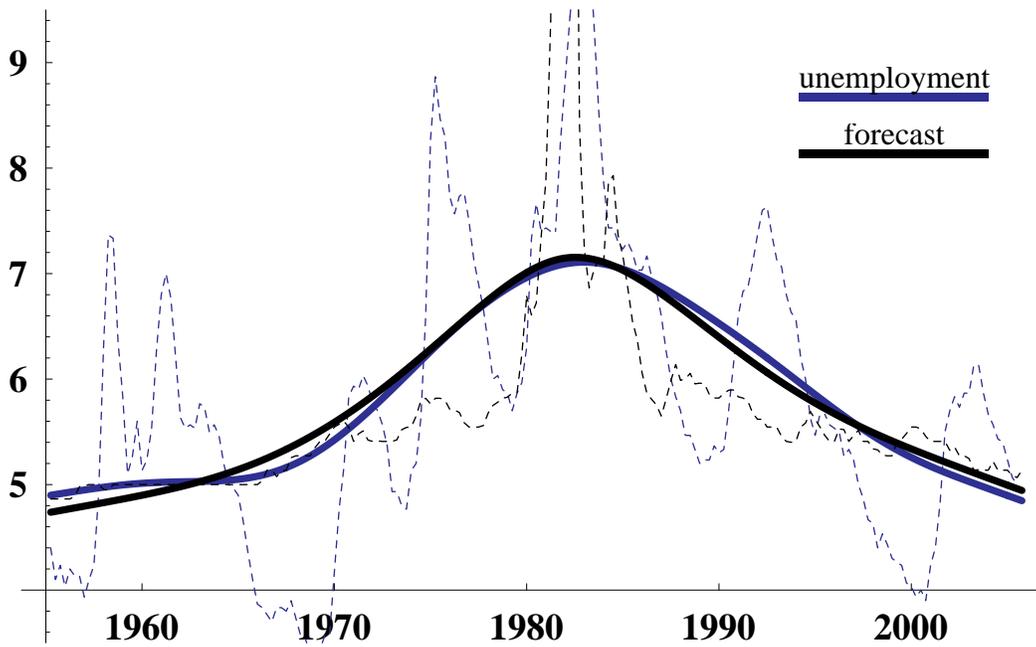
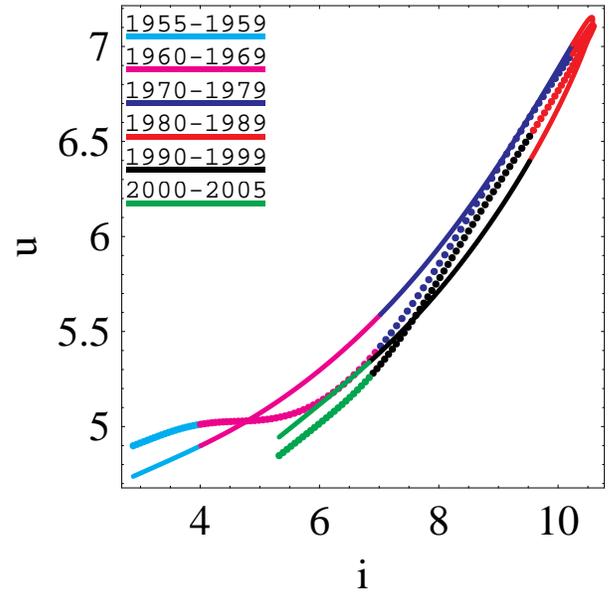
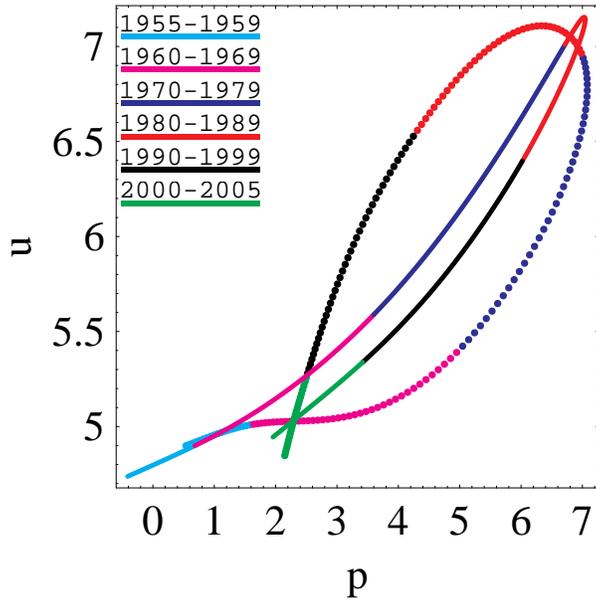


Figure 6.4: BF low elasticity 1600

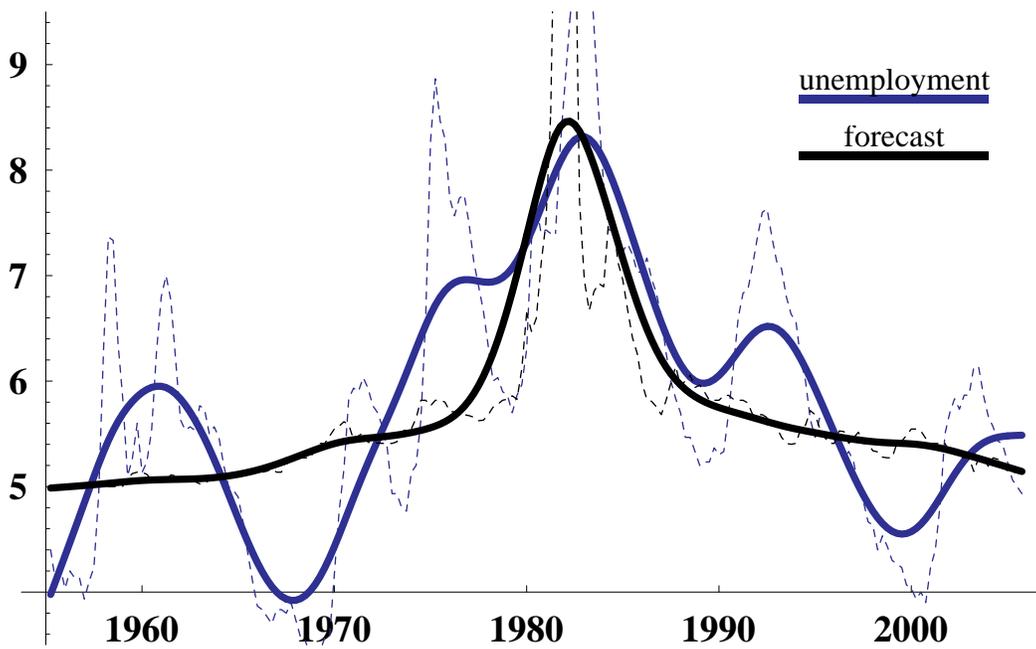
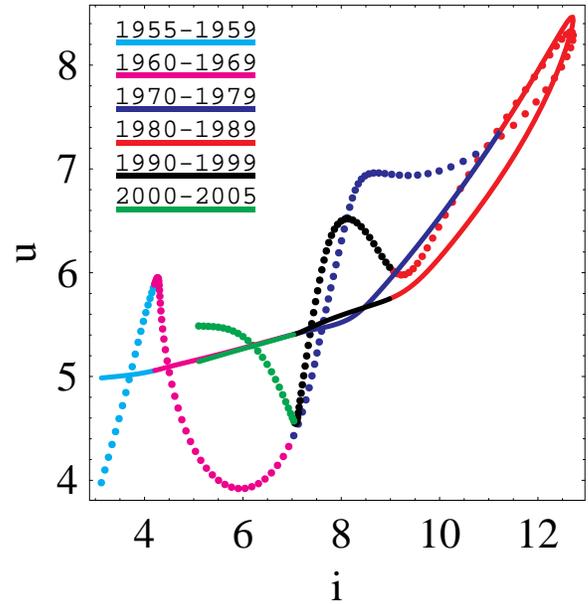
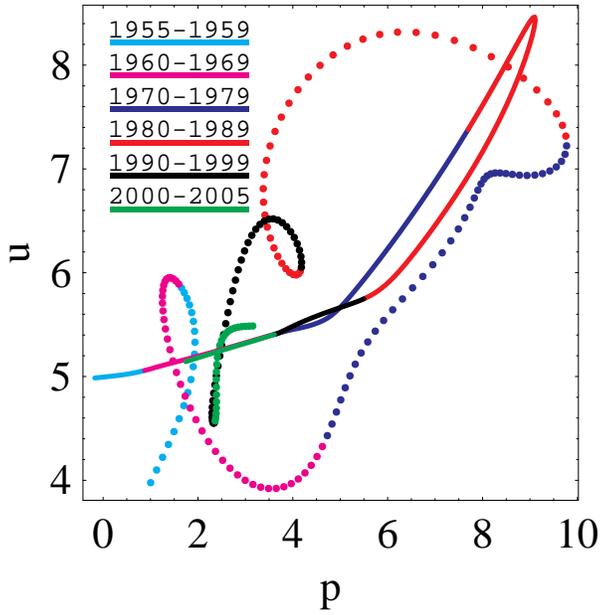


Figure 6.5: BC high elasticity 160000

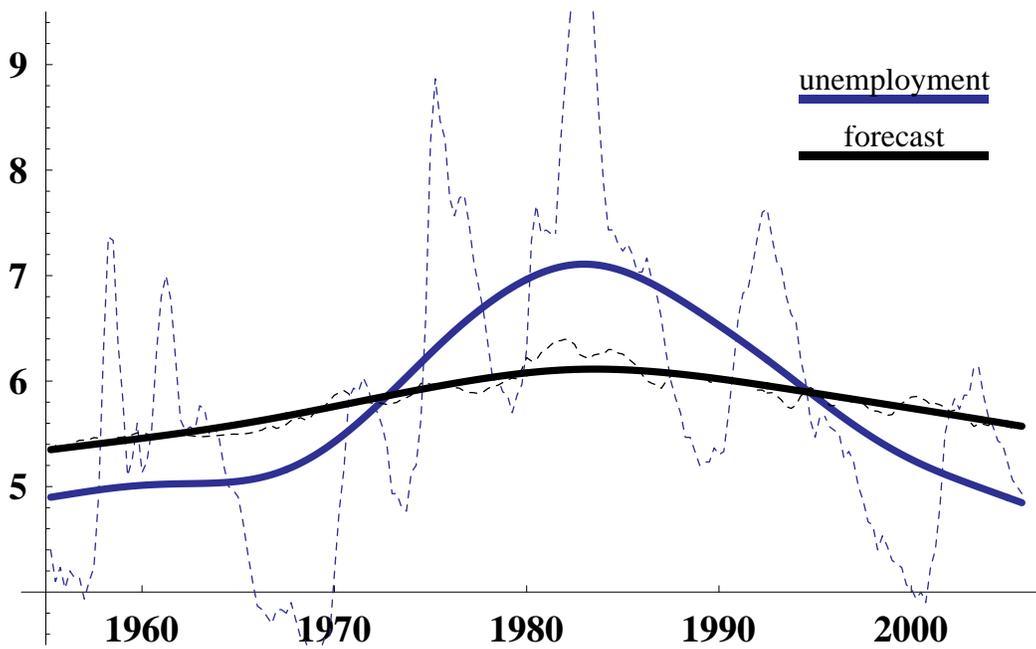
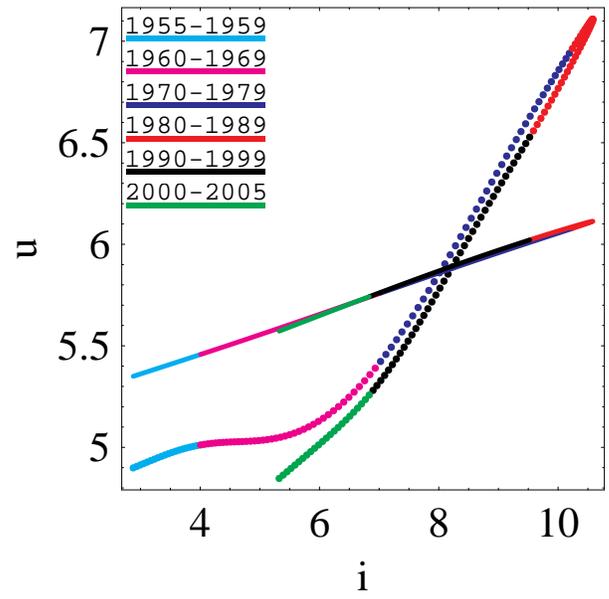
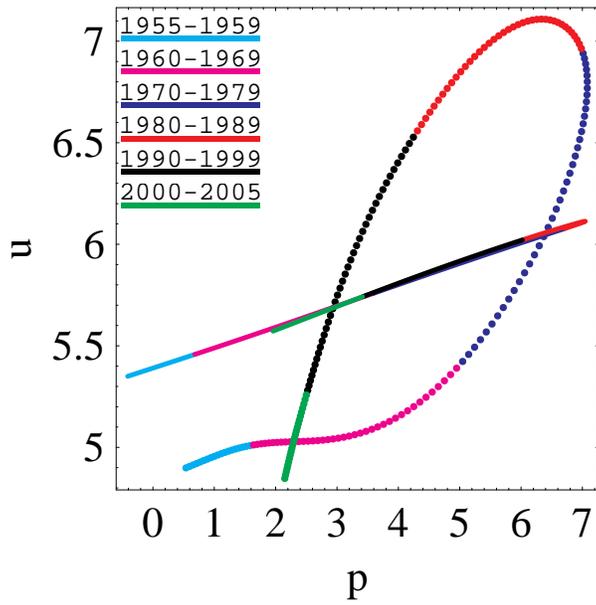


Figure 6.6: BC high elasticity 1600

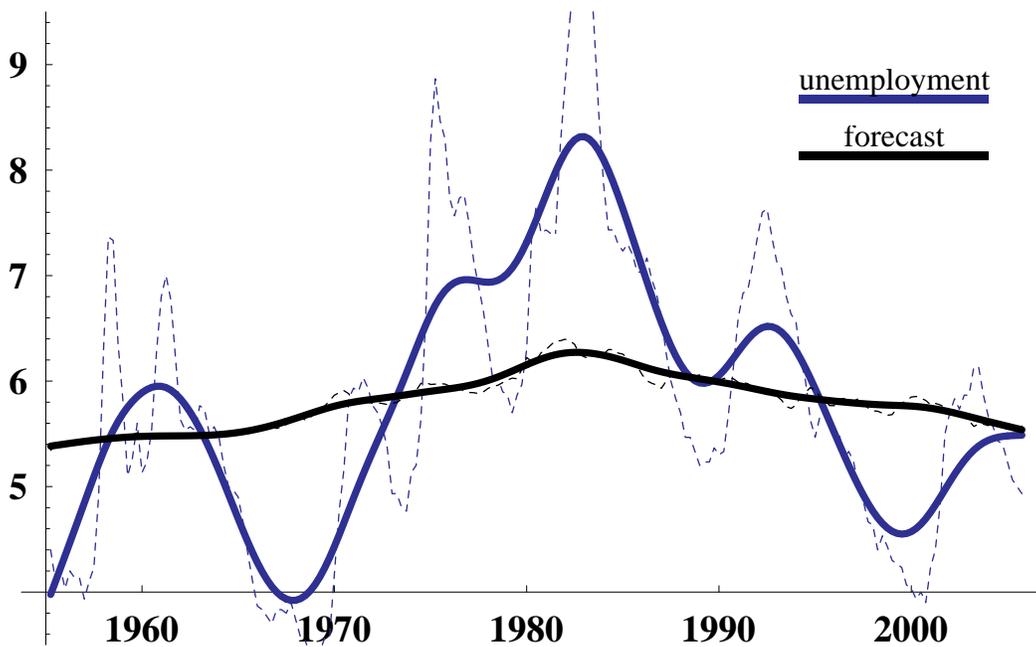
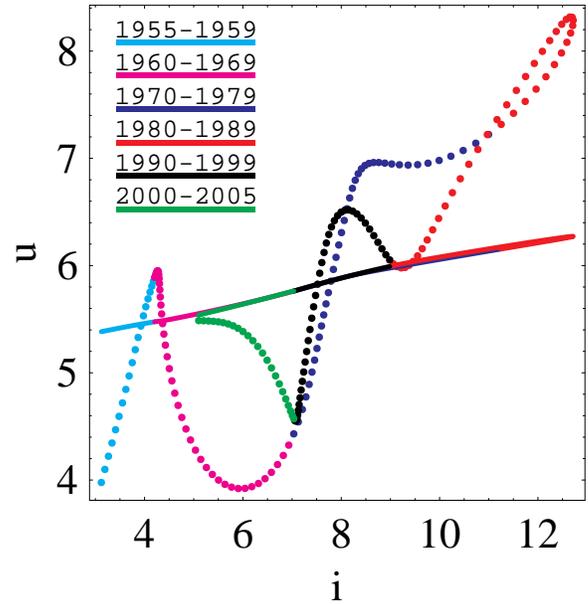
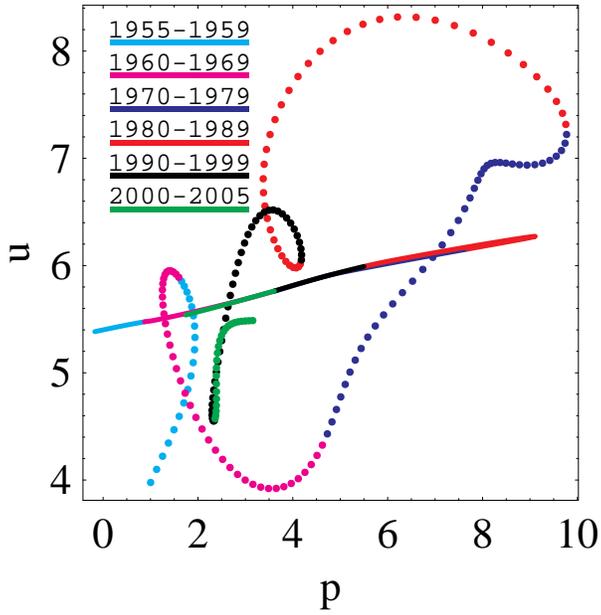


Figure 6.7: BF high elasticity 160000

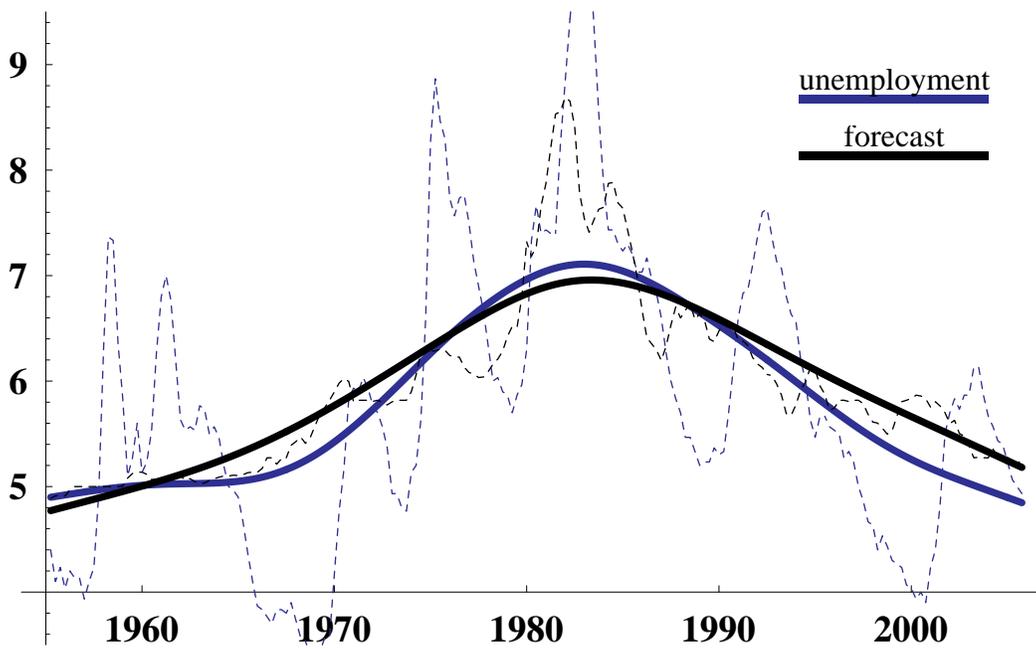
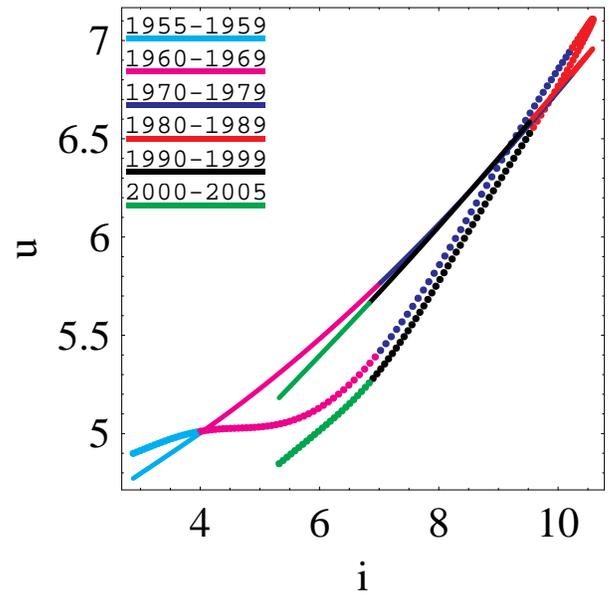
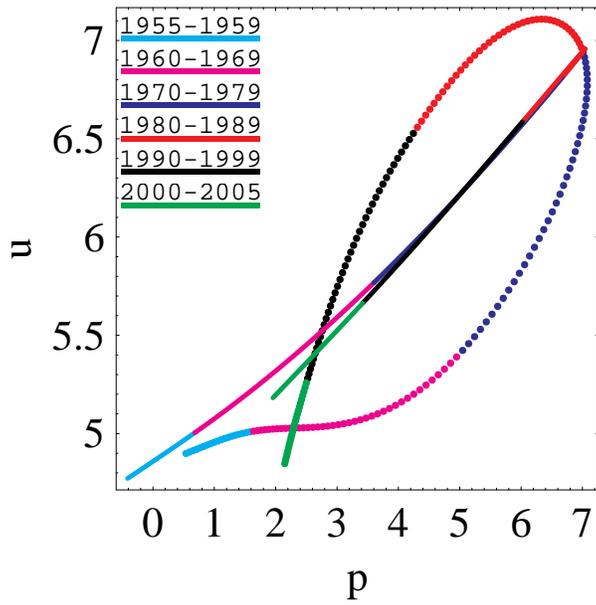
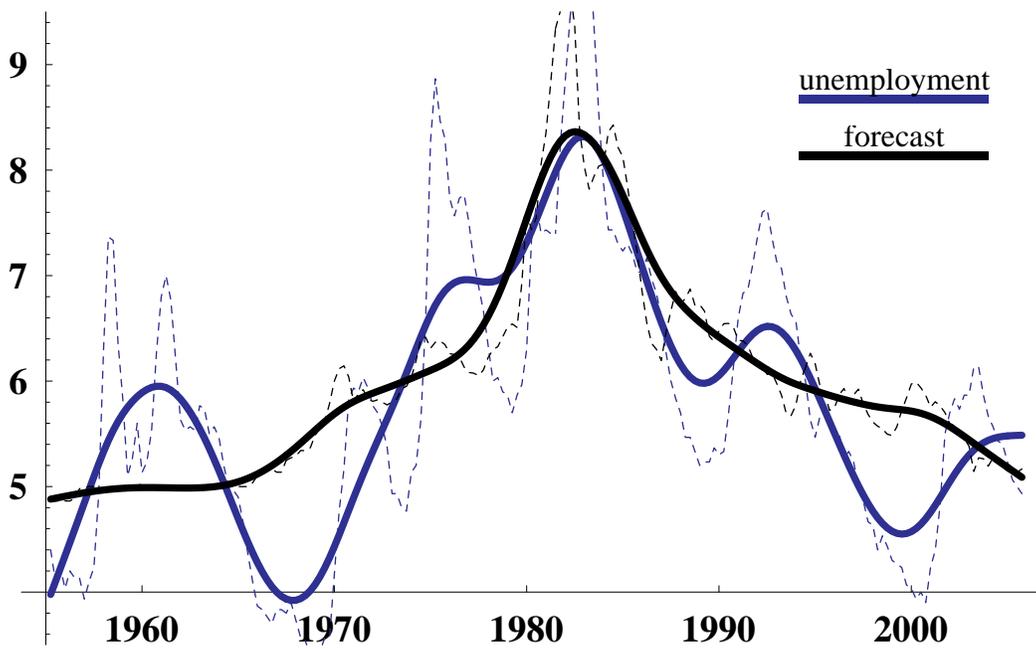
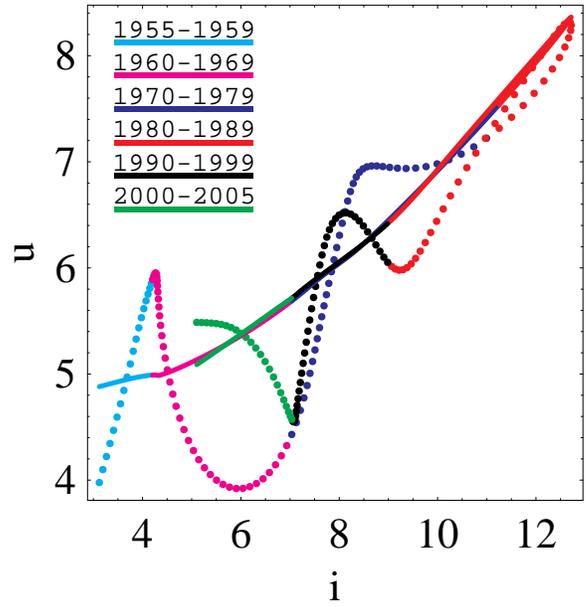
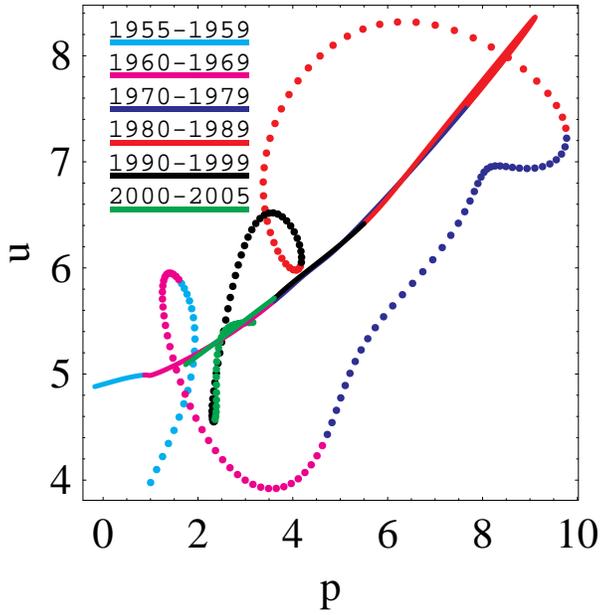
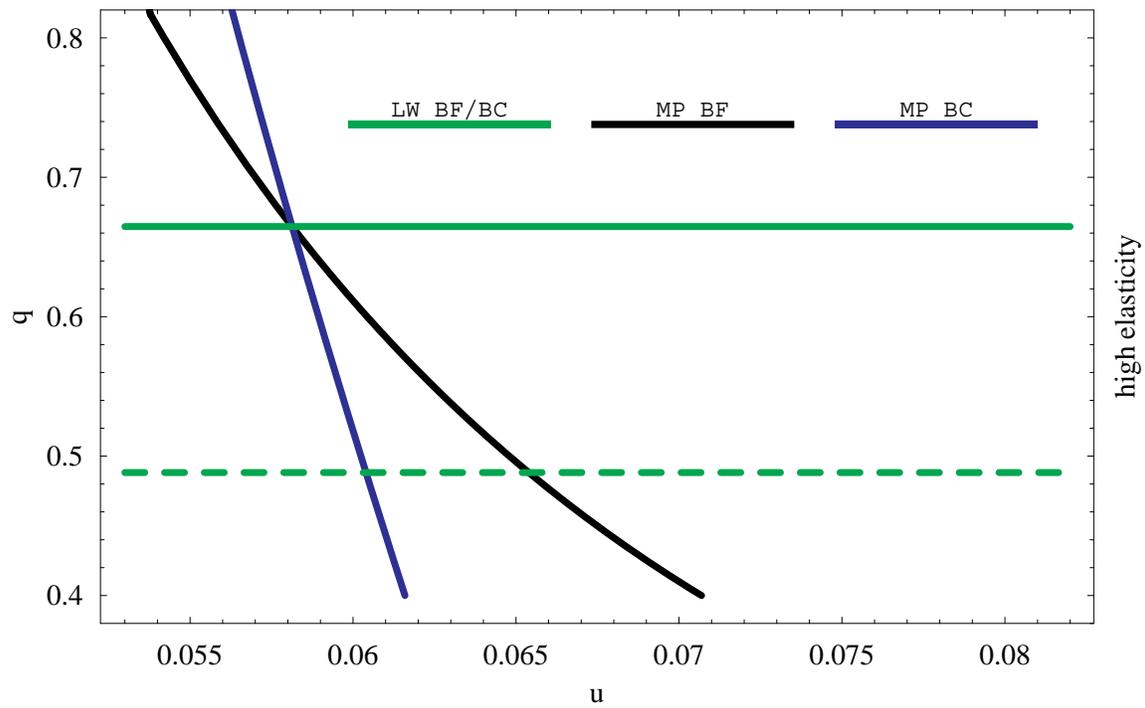
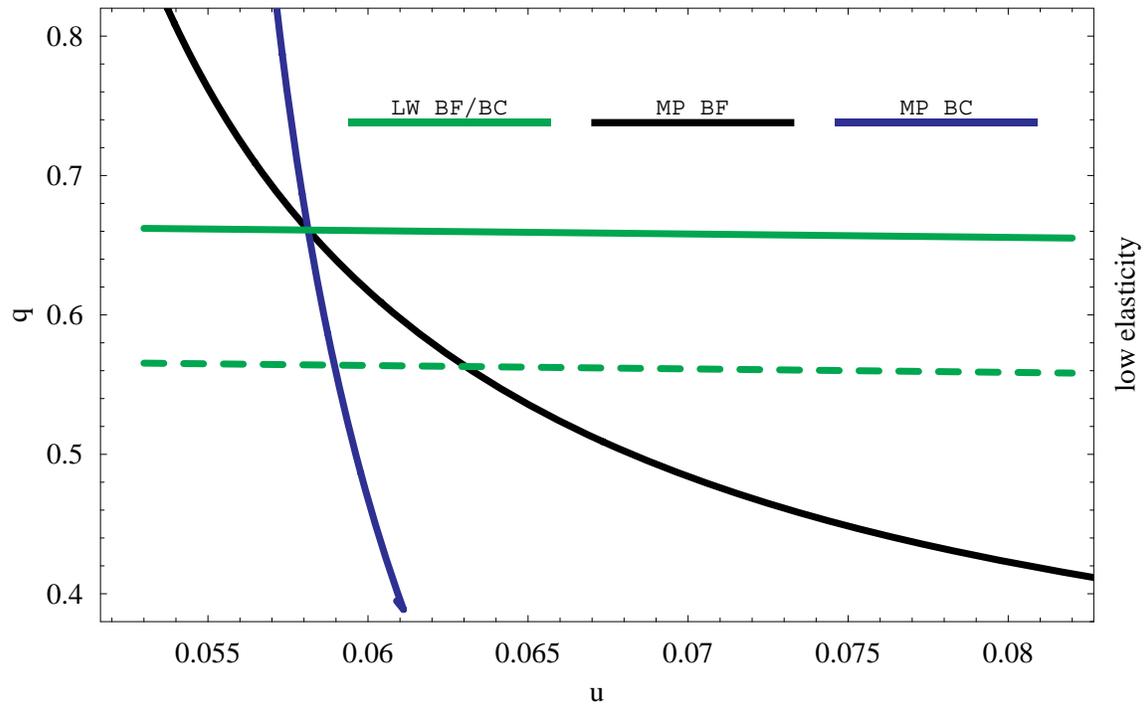


Figure 6.8: BF high elasticity 1600



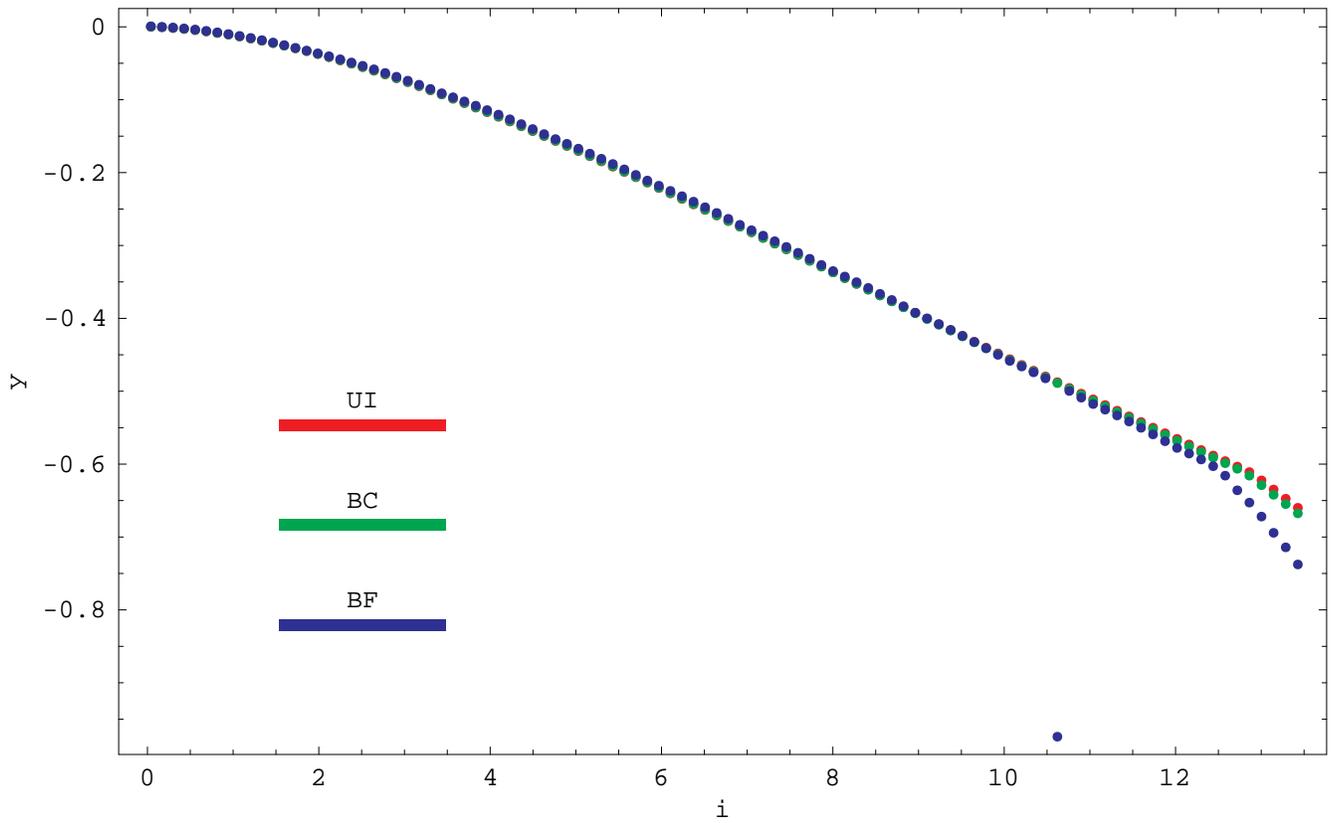
# Figure 7: LW and MP curves

Solid curves  $i = 7.4$ , Dashed curves  $i = 9$

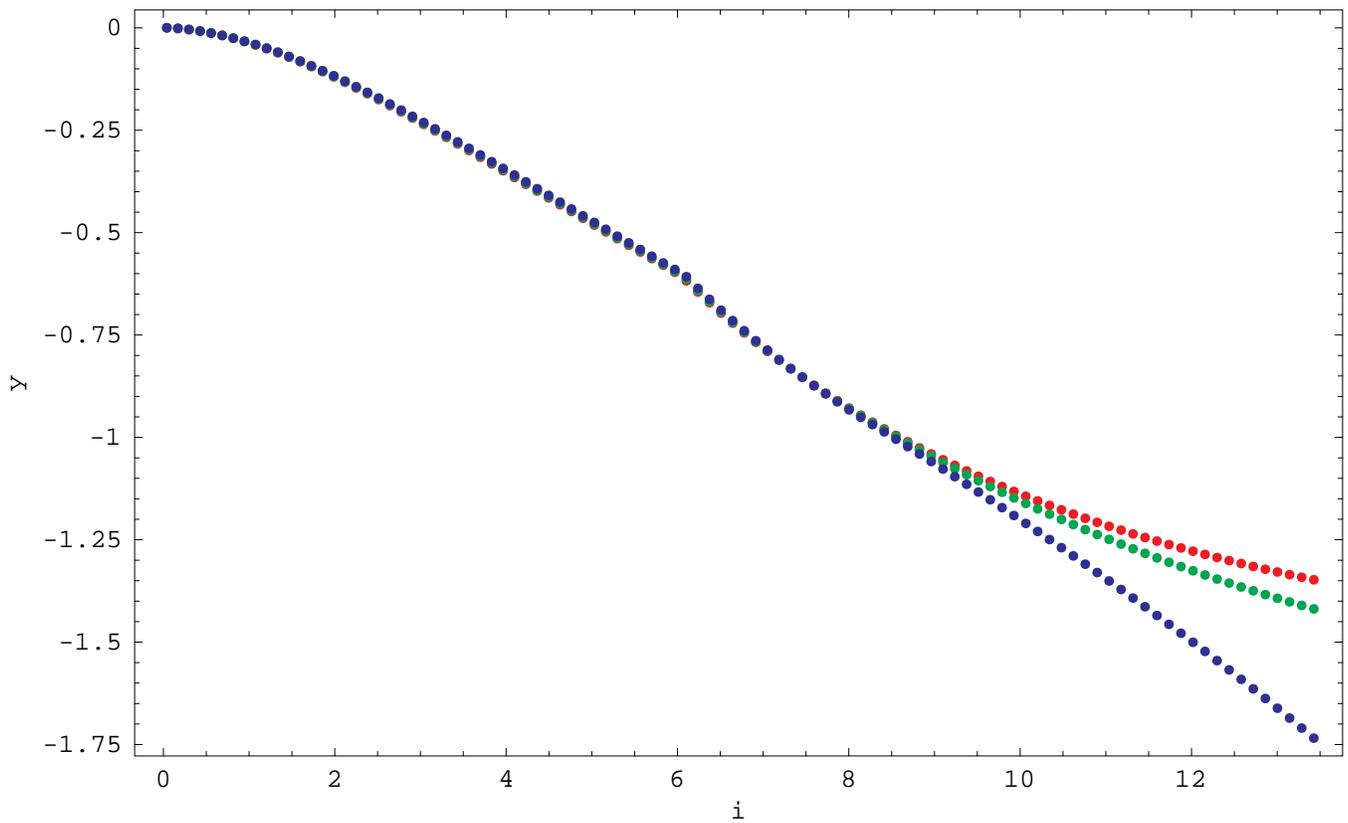


# Figure 8: Conversion

## Elasticity -0.7



## Elasticity -1.6



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