The Margins of Export: An Integrated approach*

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Abstract

Recent empirical work has highlighted two very important extensive margins that greatly impact bilateral trade flows: An exporting firm decides both which foreign markets it will serve, and how many of its products to sell in each destination. We show how trade costs affect both of these decisions, and aggregate up to the extensive margin of aggregate bi-lateral trade flows. Naturally, trade costs also affect the intensive margin of bilateral trade: how much of each product is sold. We also show how changes in trade costs have different repercussions on the extensive margins of trade over time. Our model captures how the different products developed by a firm compete within a common product market on the demand side. We then show how our multi-country model can replicate many of the stylized facts concerning the co-movements of the two extensive margins across both countries and firms. Our model remains highly tractable, even when considering the empirically relevant case of multiple asymmetric countries and trade costs.

Keywords: market structure, multiproduct firms, productivity heterogeneity, endogenous markups, trade liberalization

J.E.L. Classification: F12, R13.

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1 Introduction

Recent empirical evidence has highlighted how exporting firms use a very important adjustment margin across export destinations and over time: the product margin. International trade flows are dominated by the export patterns of multi-product firms that use this margin to adjust to different export market conditions. Differences in the geography of export market destinations induce significant adjustments in the number of products exported. This product margin response goes in the same direction as the aggregate bilateral trade responses to the same geographical variations: Firms export relatively more products to bigger, closer destinations, and to destinations that share other bi-lateral ties (such as a common language or colonial ties). However, the firm's intensive margin response at the product level (exports per firm per product) do not exhibit those same patterns.

In this paper, we develop a model of multi-product firms that captures the effects of geography (market size and bilateral trade barriers/enhancers) on this new export margin, as well as the firm export margin. We show how geography affects the decomposition of bilateral trade flows into different numbers of exporting firms, different exported product ranges per firm, and differences in the value of export shipments per product. For expositional purposes, we initially develop a two country version of our model, but then show how it can easily be extended to multiple asymmetric countries and asymmetric bilateral trade costs.

2 Literature Review

To be completed...

3 Closed Economy

We introduce multi-product firms in the model of Melitz and Ottaviano (2008). Consider an economy with L consumers, each supplying one unit of labor.

3.1 Preferences and Demand

Preferences are defined over a continuum of differentiated varieties indexed by $i \in \Omega$, and a homogenous good chosen as numeraire. All consumers share the same utility function given by

$$U = q_0^c + \alpha \int_{i \in \Omega} q_i^c di - \frac{1}{2}\gamma \int_{i \in \Omega} (q_i^c)^2 \, di - \frac{1}{2}\eta \left(\int_{i \in \Omega} q_i^c di \right)^2, \tag{1}$$

where q_0^c and q_i^c represent the individual consumption levels of the numeraire good and each variety i. The demand parameters α, η, and γ are all positive. The parameters α and η index the substitution pattern between the differentiated varieties and the numeraire: increases in α and decreases in η both shift out the demand for the differentiated varieties relative to the numeraire. The parameter γ indexes the degree of product differentiation between the varieties. In the limit when $\gamma = 0$, consumers only care about their consumption level over all varieties, $Q^c = \int_{i \in \Omega} q_i^c di$. The varieties are then perfect substitutes. The degree of product differentiation increases with γ as consumers give increasing weight to the distribution of consumption levels across varieties.

The marginal utilities for all goods are bounded, and a consumer may thus not have positive demand for any particular good. We assume that consumers have positive demands for the numeraire good ($q_0^c > 0$). The inverse demand for each variety i is then given by

$$p_i = \alpha - \gamma q_i^c - \eta Q^c, \tag{2}$$

whenever $q_i^c > 0$. Let $\Omega^* \subset \Omega$ be the subset of varieties that are consumed ($q_i^c > 0$). (2) can then be inverted to yield the linear market demand system for these varieties:

$$q_i \equiv L q_i^c = \frac{\alpha L}{\eta M + \gamma} - \frac{L}{\gamma} p_i + \frac{\eta M}{\eta M + \gamma} \frac{L}{\gamma} \bar{p}, \ \forall i \in \Omega^*, \tag{3}$$

where M is the measure of consumed varieties in Ω^* and $\bar{p} = (1/M) \int_{i \in \Omega^*} p_i di$ is their average price. The set Ω^* is the largest subset of Ω that satisfies

$$p_i \leq \frac{1}{\eta M + \gamma} (\gamma \alpha + \eta M \bar{p}) \equiv p_{\max}, \tag{4}$$

where the right hand side price bound $p_{\max}$ represents the price at which demand for a variety is driven to zero. Note that (2) implies $p_{\max} \leq \alpha$. In contrast to the case of C.E.S. demand, the price elasticity of demand, $\varepsilon_i \equiv |(\partial q_i / \partial p_i) (p_i / q_i)| = [(p_{\max}/p_i) - 1]^{-1}$, is not uniquely determined by the level of product differentiation γ. Given the latter, lower average prices $\bar{p}$ or a larger number of competing varieties M induce a decrease in the price bound $p_{\max}$ and an increase in the price elasticity of demand ε_i at any given p_i. We characterize this as a 'tougher' competitive environment.[1]

Welfare can be evaluated using the indirect utility function associated with (1):

$$U = I^c + \frac{1}{2}\left(\eta + \frac{\gamma}{M}\right)^{-1}(\alpha - \bar{p})^2 + \frac{1}{2}\frac{M}{\gamma}\sigma_p^2, \tag{5}$$

where I^c is the consumer's income and $\sigma_p^2 = (1/M)\int_{i\in\Omega^*}(p_i - \bar{p})^2\,di$ represents the variance of prices. To ensure positive demand levels for the numeraire, we assume that $I^c > \int_{i\in\Omega^*} p_i q_i^c di = \bar{p}Q^c - M\sigma_p^2/\gamma$. Welfare naturally rises with decreases in average prices $\bar{p}$. It also rises with increases in the variance of prices σ_p^2 (holding the mean price $\bar{p}$ constant), as consumers then re-optimize their purchases by shifting expenditures towards lower priced varieties as well as the numeraire good. Finally, the demand system exhibits 'love of variety': holding the distribution of prices constant (namely holding the mean $\bar{p}$ and variance σ_p^2 of prices constant), welfare rises with increases in product variety M.

3.2 Production and Firm Behavior

Labor is the only factor of production and is inelastically supplied in a competitive market. The numeraire good is produced under constant returns to scale at unit cost; its market is also competitive. These assumptions imply a unit wage. Entry in the differentiated product sector is costly as each firm incurs product development and production startup costs. Subsequent production of each variety exhibits constant returns to scale. While it may decide to produce more than one variety, each firm has one key variety corresponding to its 'core competency'. This is associated with a core marginal cost c (equal to unit labor requirement).[2] Research and development yield uncertain outcomes for c, and firms learn about this cost level only after making the irreversible investment f_E required for entry. We model this as a draw from a common (and known) distribution $G(c)$ with support on $[0, c_M]$.

The introduction of an additional variety pulls a firm away from its core competency, which we model as higher marginal costs of production for those varieties. We think of these costs increases as

[1] We also note that, given this competitive environment (given N and $\bar{p}$), the price elasticity ε_i monotonically increases with the price p_i along the demand curve.

[2] For simplicity, we do not model any overhead production costs. This would significantly increase the complexity of our model without yielding much new insight.

also reflecting decreases in product quality/appeal as firms move away from their core competency. For simplicity, we maintain product symmetry on the demand side and capture any decrease in product appeal as an increased production cost. We label the additional production cost for a new variety a customization cost. A firm can introduce any number of new varieties, but each additional variety entails an additional customization cost (as firms move further away from their core competency). We index by m the varieties produced by the same firm in increasing order of distance from their core competency with $m = 0$ referring to the core variety. We then call $v(m, c)$ the marginal cost for variety m produced by a firm with core marginal cost c and assume $v(m, c) = \omega^{-m} c$ with $\omega \in (0, 1)$. This defines a firm-level 'competence ladder'. In the limit, as ω goes to zero, any firm will only produce at most its core variety and we are back to single product firms as in Melitz and Ottaviano (2008).

Since the entry cost is sunk, firms that can cover at least the marginal cost of their core variety survive and produce. All other firms exit the industry. Surviving firms maximize their profits using the residual demand function (3). In so doing, those firms take the average price level $\bar{p}$ and total number of varieties M as given. This monopolistic competition outcome is maintained with multi-product firms as any firm can only produce a countable number of products, which is a subset of measure zero of the total mass of varieties M.

The profit maximizing price $p(v)$ and output level $q(v)$ of a variety with cost v must then satisfy

$$q(v) = \frac{L}{\gamma} \left[p(v) - v \right]. \tag{6}$$

The profit maximizing price $p(v)$ may be above the price bound $p_{\max}$ from (4), in which case the variety is not supplied. Let v_D reference the cutoff cost for a variety to be profitably produced. This variety earns zero profit as its price is driven down to its marginal cost, $p(v_D) = v_D = p_{\max}$, and its demand level $q(v_D)$ is driven to zero. Firms with core competency $v > v_D$ cannot profitably produce their core variety and exit. $c_D = v_D$ is thus also the cutoff for firm survival. We assume that c_M is high enough that it is always above c_D, so exit rates are always positive. All firms with core cost $c < c_D$ earn positive profits (gross of the entry cost) on their core varieties and remain in the industry. Some firms will also earn positive profits from the introduction of additional varieties. In particular, firms with cost c such that $v(m, c) \leq v_D \iff c \leq \omega^m c_D$ earn positive profits on their m-th *additional* variety and thus produce at least $m + 1$ varieties. The total number of varieties produced by a firm with cost c is[3]

$$M(c) = \begin{cases} 0 & \text{if } c > c_D, \\ \max\{m \mid c \le \omega^m c_D\} + 1 & \text{if } c \le c_D. \end{cases} \tag{7}$$

The number of varieties produced is thus a step function of the firm's productivity $1/c$, as depicted in figure 1 below.

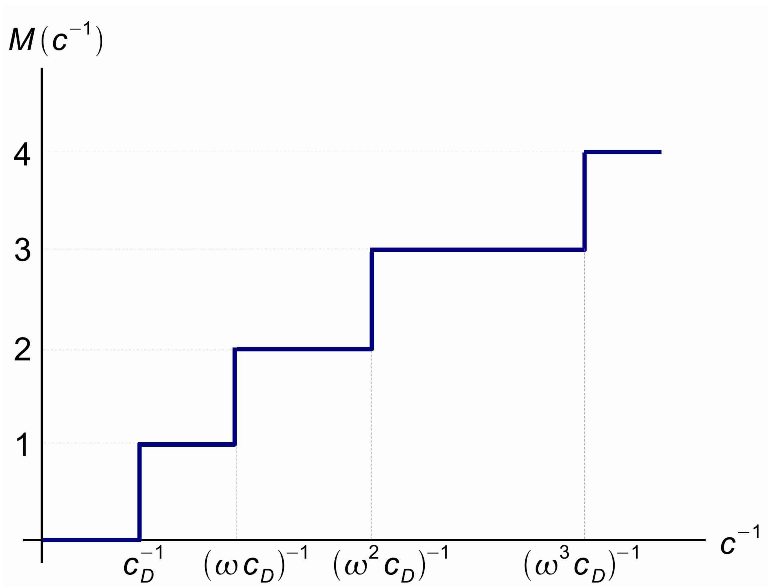

Figure 1: Number of Varieties Produced as a Function of Firm Productivity

The threshold cost v_D summarizes the competitive environment across all varieties produced by surviving firms. Let $r(v) = p(v)q(v)$, $\pi(v) = r(v) - q(v)v$, $\mu(v) = p(v) - v$ denote the revenue, profit, and (absolute) markup of a variety with cost v. All these performance measures can then

[3] Note that this is an integer number, and not a mass with positive measure.

be written as functions of v and v_D only:

$$p(v) = \frac{1}{2}\left(v_D + v\right), \tag{8}$$

$$\lambda(v) = \frac{1}{2}\left(v_D - v\right), \tag{9}$$

$$q(v) = \frac{L}{2\gamma}\left(v_D - v\right), \tag{10}$$

$$r(v) = \frac{L}{4\gamma}\left[\left(v_D\right)^2 - v^2\right], \tag{11}$$

$$\pi(v) = \frac{L}{4\gamma}\left(v_D - v\right)^2. \tag{12}$$

As expected, lower cost varieties have lower prices and earn higher revenues and profits than varieties with higher costs. However, lower cost varieties do not pass on all of the cost differential to consumers in the form of lower prices: they also have higher markups (in both absolute and relative terms) than varieties with higher costs.

All these performance measures can be aggregated to the firm level:

$$Q(c) = \sum_{m=0}^{M(c)-1} q\left(v\left(m,c\right)\right),$$

$$R(c) = \sum_{m=0}^{M(c)-1} r\left(v\left(m,c\right)\right),$$

$$\Pi(c) = \sum_{m=0}^{M(c)-1} \pi\left(v\left(m,c\right)\right), \tag{13}$$

where $Q(c)$, $R(c)$, $\Pi(c)$ denote total firm output, revenue, and profit. Firm-level measures for prices and markups are now best expressed as averages (weighted by relative output across varieties):

$$\bar{P}(c) = \frac{R(c)}{Q(c)} \qquad \text{and} \qquad \bar{\Lambda}(c) = \frac{\Pi(c)}{Q(c)}.$$

We also define an average cost measure at the firm-level in a similar way (average cost per unit produced):

$$\bar{C}(c) = \frac{C(c)}{Q(c)},$$

where $C(c) = R(c) - \Pi(c)$ is the firm's total production cost across all varieties. Given a competitive environment summarized by $v_D = c_D$, we show in the appendix that this firm's average production cost $\bar{C}(c)$ is a monotonic function of its core competency c. However, this key (inverse) measure of a firm's productivity now responds to the competitive environment (unlike the core competency measure c). We discuss this in further detail below. We note that one could also measure a firm's productivity directly as value added per worker $\Phi(c) = R(c)/C(c)$. This measure of firm productivity combines the effects of physical productivity $1/\bar{C}(c)$ as well as markups: $\Phi(c) = \left[\bar{\Lambda}(c)/\bar{C}(c)\right] + 1$. We show in the appendix that this alternate measure of productivity also varies monotonically with a firm's core competency c (again, holding the competitive environment constant).

Given a mass of entrants N_E, the distribution of costs across all varieties is determined by the distribution of core competencies $G(c)$ as well as the optimal firm product range choice $M(c)$. Let $M_v(v)$ denote the measure function for varieties (the measure of varieties produced at cost v or lower, given N_E entrants). Further define $H(v) \equiv M_v(v)/N_E$ as the normalized measure of varieties per unit mass of entrants. Then $H(v) = \sum_{m=0}^{\infty} G(\omega^m v)$ and is exogenously determined from $G(.)$ and ω. Given a unit mass of entrants, there will be a mass $G(v)$ of varieties with cost v or less; a mass $G(\omega v)$ of first additional varieties (with cost v or less); a mass $G(\omega^2 v)$ of second additional varieties; and so and so forth. The measure $H(v)$ sums over all these varieties.

3.3 Free Entry and Flexible Product Mix

Prior to entry, the expected firm profit is $\int_0^{c_D} \Pi(c) dG(c) - f_E$. If this profit were negative, no firms would enter the industry. As long as some firms produce, the expected profit is driven to zero by the unrestricted entry of new firms. This yields the equilibrium free entry condition:

$$\begin{aligned} \int_0^{c_D} \Pi(c) dG(c) &= \int_0^{c_D} \left[\sum_{\{m | \omega^{-m} c \leq c_D\}} \pi\left(\omega^{-m} c\right) \right] dG(c) \\ &= \sum_{m=0}^{\infty} \left[\int_0^{\omega^m c_D} \pi\left(\omega^{-m} c\right) dG(c) \right] = f_E, \end{aligned} \tag{14}$$

which determines the cost cutoff $c_D = v_D$. This cutoff, in turn, determines the aggregate mass of varieties, since $v_D = p(v_D)$ must also be equal to the zero demand price threshold in (4):

$$v_D = \frac{1}{\eta M + \gamma} \left(\gamma \alpha + \eta M \bar{p} \right).$$

The aggregate varieties is then

$$M = \frac{2\gamma}{\eta} \frac{\alpha - v_D}{v_D - \overline{v}}, \tag{15}$$

where the average cost of all varieties

$$\overline{v} = \frac{1}{M}\int_0^{v_D} v dM_v(v) = \frac{1}{N_E H(v_D)}\int_0^{v_D} v N_E dH(v) = \frac{1}{H(v_D)}\int_0^{v_D} v dH(v)$$

depends only on v_D.[4] Finally, the mass of entrants is given by $N_E = M/H(v_D)$, which can in turn be used to obtain the mass of producing firms $N = N_E G(c_D)$.

3.4 Parametrization of Technology

All the results derived so far hold for any distribution of core cost draws $G(c)$. However, in order to simplify some of the ensuing analysis, we use a specific parametrization for this distribution. In particular, we assume that core productivity draws $1/c$ follow a Pareto distribution with lower productivity bound $1/c_M$ and shape parameter $k \geq 1$. This implies a distribution of cost draws c given by

$$G(c) = \left(\frac{c}{c_M}\right)^k, \ c \in [0, c_M]. \tag{16}$$

The shape parameter k indexes the dispersion of cost draws. When $k = 1$, the cost distribution is uniform on $[0, c_M]$. As k increases, the relative number of high cost firms increases, and the cost distribution is more concentrated at these higher cost levels. As k goes to infinity, the distribution becomes degenerate at c_M. Any truncation of the cost distribution from above will retain the same distribution function and shape parameter k. The productivity distribution of surviving firms will therefore also be Pareto with shape k, and the truncated cost distribution will be given by $G_D(c) = (c/c_D)^k, \ c \in [0, c_D]$.

When core competencies are distributed Pareto, then all produced varieties will share the same Pareto distribution:

$$H(c) = \sum_{m=0}^{\infty} G(\omega^m c) = \Omega G(c),$$

where $\Omega = \left(1 - \omega^k\right)^{-1} > 1$ is an index of multi-product flexibility (which varies monotonically with ω). In equilibrium, this index will also be equal to the average number of products produced across all surviving firms:

$$\frac{M}{N} = \frac{H(v_D)}{G(c_D)} = \Omega.$$

The Pareto parametrization also yields a simple solution for the cost cutoff c_D from the free

[4] We also use the relationship between average cost and price $\bar{v} = 2\bar{p} - v_D$, which is obtained from (8).

entry condition (14):

$$c_D = \left[\frac{\gamma\phi}{L\Omega}\right]^{\frac{1}{k+2}}, \tag{17}$$

where $\phi \equiv 2(k+1)(k+2)\,(c_M)^k f_E$ is a technology index that combines the effects of better distribution of cost draws (lower c_M) and lower entry costs f_E. We assume that $c_M > \sqrt{[2(k+1)(k+2)\gamma f_E]\,/\,(L\Omega)}$ in order to ensure that $c_D < c_M$ as was previously anticipated. Note that, in the limit, when the marginal costs of non-core varieties becomes infinitely large ($\omega \to 0$), multi-product flexibility Ω goes to one (no multi product firms) and (17) boils down to the single-product case studied by Melitz and Ottaviano (2008). The average marginal cost across varieties is then

$$\overline{v} = \frac{k}{k+1} v_D$$

and the mass of available varieties (see (15) is

$$M = \frac{2(k+1)\gamma}{\eta}\frac{\alpha - c_D}{c_D}. \tag{18}$$

Since the cutoff level completely summarizes the distribution of prices as well as all the other performance measures, it also uniquely determines welfare from (5):

$$U = 1 + \frac{1}{2\eta}\left(\alpha - c_D\right)\left(\alpha - \frac{k+1}{k+2}c_D\right). \tag{19}$$

Welfare increases with decreases in the cutoff c_D, as the latter induces increases in product variety M as well as decreases in the average price $\bar{p}$ (these effects dominate the negative impact of the lower price variance).[5]

Increases in market size, technology improvements (a fall in c_M or f_E), or increases in product substitutability lead to decreases in the cutoff c_D and increases in both the mass of varieties produced, and the mass of surviving firms. Although the average number of varieties produced per firm remains constant at Ω, all firms respond to this tougher competition by decreasing the number of products produced: $M(c)$ is (weakly) decreasing for all $c \in [0, c_M]$. The average $M(c)$ remains constant due to the effects of selection (higher cost firms producing the fewest number of products exit). Thus, tougher competition induces firms to focus on the production of varieties that are

[5] This welfare measure reflects the reduced consumption of the numeraire to account for the labor resources used to cover the entry costs.

closer to its core competency. In addition, this tougher competitive environment induces firms to reallocate labor resources among the remaining products produced towards the production of the core varieties (lower m). Within-firm productivity $1/\bar{C}(c)$ thus increases due to the compounding effects of this reallocation and the product selection effect. Aggregate productivity (the inverse of the economy wide average cost of production) thus increases due to both a within-firm and across-firm selection effect. Output and sales per variety increases for all surviving products, and the distribution of markups across these products shifts down. Welfare increases due to higher productivity and product variety, and lower markups.

4 Open Economy

Consider a two economy world, H and F, with L^H and L^F consumers in each country. The markets are segmented, although any produced variety can be exported. This entails an additional customization cost (over and above the customization for the domestic market) with 'step cost' ladder $1/\theta^l$, $\theta^l \in (0,1]$, for exports to country $l = \{H, F\}$. There is also an iceberg trade cost $\tau^l > 1$ that is incurred once for each variety that is exported to l. For notational convenience, we subsume the first customization cost $1/\theta^l$ into this iceberg trade cost so that we can write the marginal cost of an exported variety from country $h = \{H, F\} \neq l$ to country l as $v_X^h(m,c) = \left(\theta^l \omega\right)^{-m} c$, with delivered cost $\tau^l v_X^h(m,c)$. ω^{-1} remains the step cost for varieties produced on each domestic market, leading to the same marginal cost function for variety m, $v_D(m,c) = \omega^{-m} c$.[6] Let $\omega^l \equiv \theta^l \omega \leq \omega$ denote the combined (inverse) step cost for exported varieties to country l. Throughout our analysis, we will allow for the possibility of $\theta^l = 1$ $\left(\omega^l = \omega\right)$, which is a natural benchmark of no step-differences between exported and domestic varieties.

Let $p_{\max}^l$ denote the price threshold for positive demand in market l. Then (4) implies

$$p_{\max}^l = \frac{1}{\eta M^l + \gamma}\left(\gamma\alpha + \eta M^l \bar{p}^l\right), \tag{20}$$

where M^l is the total number of products selling in country l (the total number of domestic and exported varieties) and $\bar{p}^l$ is their average price. Let $\pi_D^l(v)$ and $\pi_X^l(v)$ represent the maximized value of profits from domestic and export sales for a variety with cost v produced in country l.[7]

[6] Our model can easily accomodate differences in the step cost ω across countries. In this paper, we do not focus on those cross-country differences and assume the same ω for notational convenience.

[7] Recall that $v_X^h(m,c) \geq v_D(m,c)$ with a strict inequality whenever $\theta^l < 1$ and $m > 0$. In those cases, a firm that produces variety m at cost v for the domestic market cannot produce that same variety at cost v for the export market. Thus, in general, $\pi_D^l(v)$ and $\pi_X^l(v)$ do not refer to domestic and export profits for the *same* variety m.

The cost cutoffs for profitable domestic production and for profitable exports must satisfy:

$$\begin{aligned} v_D^l &= \sup\left\{c : \pi_D^l(v) > 0\right\} = p_{\max}^l, \\ v_X^l &= \sup\left\{c : \pi_X^l(v) > 0\right\} = \frac{p_{\max}^h}{\tau^h}, \end{aligned} \tag{21}$$

and thus $v_X^l = v_D^h/\tau^h$. As was the case in the closed economy, the cutoff v_D^l, $l = \{H, F\}$, summarizes all the effects of market conditions in country l relevant for all firm performance measures. The profit functions can then be written as a function of these cutoffs:

$$\begin{aligned} \pi_D^l(v) &= \frac{L^l}{4\gamma}\left(v_D^l - v\right)^2, \\ \pi_X^l(v) &= \frac{L^h}{4\gamma}\left(\tau^h\right)^2\left(v_X^l - v\right)^2 = \frac{L^h}{4\gamma}\left(v_D^h - \tau^h v\right)^2. \end{aligned} \tag{22}$$

As in the closed economy, $c_D^l = v_D^l$ will be the cutoff for firm survival in country l. Similarly, $c_X^l = v_X^l$ will be the firm export cutoff (no firm with $c > c_X^l$ can profitably export any varieties). A firm with core competency c will produce all varieties m such that $\pi_D^l\left[v_D(m,c)\right] = \pi_D^l\left(\omega^{-m}c\right) \geq 0$, and will export the subset of varieties m such that $\pi_X^l\left[v_X(m,c)\right] = \pi_X^l\left[\left(\omega^l\right)^{-m}c\right] \geq 0$. The total number of varieties produced and exported by a firm with cost c in country l are thus

$$M_D^l(c) = \begin{cases} 0 & \text{if } c > c_D^l, \\ \max\left\{m \mid c \leq \omega^m c_D^l\right\} + 1 & \text{if } c \leq c_D^l, \end{cases}$$

$$M_X^l(c) = \begin{cases} 0 & \text{if } c > c_X^l, \\ \max\left\{m \mid c \leq \left(\omega^l\right)^m c_X^l\right\} + 1 & \text{if } c \leq c_X^l. \end{cases}$$

We can then define a firm's total domestic and export profits by aggregating over these varieties:

$$\Pi_D^l(c) = \sum_{m=0}^{M_D^l(c)-1} \pi_D^l\left[v\left(m,c\right)\right], \qquad \Pi_X^l(c) = \sum_{m=0}^{M_X^l(c)-1} \pi_X^l\left[v_X^l\left(m,c\right)\right].$$

Entry is unrestricted in both countries. Firms choose a production location prior to entry and paying the sunk entry cost. We assume that the entry cost f_E and cost distribution $G(c)$ are identical in both countries (although this can be relaxed). We also assume the same Pareto parametrization (16) for core competencies in both countries. A prospective entrant's expected profits will then be given by

$$
\begin{aligned}
&\int_0^{c_D^l} \Pi_D^l(c)dG(c) + \int_0^{c_X^l} \Pi_X^l(c)dG(c) \\
&= \sum_{m=0}^{\infty} \left[\int_0^{\omega^m c_D^l} \pi_D^l \left(\omega^{-m} c\right) dG(c) \right] + \sum_{m=0}^{\infty} \left[\int_0^{\left(\omega^l\right)^m c_X^l} \pi_X^l \left[\left(\omega^l\right)^{-m} c \right] dG(c) \right] \\
&= \frac{1}{2\gamma(k+1)(k+2)\left(c_M\right)^k} \left[L^l \Omega \left(c_D^l\right)^{k+2} + L^h \Omega^h \left(\tau^h\right)^2 \left(c_X^l\right)^{k+2} \right] \\
&= \frac{\Omega}{2\gamma(k+1)(k+2)\left(c_M\right)^k} \left[L^l \left(c_D^l\right)^{k+2} + L^h \frac{\Omega^h}{\Omega} \left(\tau^h\right)^{-k} \left(c_D^h\right)^{k+2} \right],
\end{aligned}
$$

where we define $\Omega^h \equiv \left[1 - \left(\omega^h\right)^k\right]^{-1}$ in an analogous way to Ω and use the relationship $c_D^h = \tau^h c_X^l$. Setting the expected profit equal to the entry cost yields the free entry condition

$$
L^l \left(c_D^l\right)^{k+2} + L^h \rho^h \left(c_D^h\right)^{k+2} = \frac{\gamma\phi}{\Omega}, \tag{23}
$$

where $\rho^h \equiv \left(\Omega^h/\Omega\right)\left(\tau^h\right)^{-k} < 1$ is a measure of 'freeness' of trade to country h that incorporates both the 'physical' trade cost τ^h as well as the step differences between domestic and export market customization. The technology index ϕ is the same as in the closed economy case. The two free entry conditions for $l = H, F$ can be solved to yield the cutoffs in both countries

$$
c_D^l = \left[\frac{\gamma\phi}{\Omega L^l} \frac{1-\rho^h}{1-\rho^l\rho^h} \right]^{\frac{1}{k+2}}.
$$

As in the closed economy, the threshold price condition in country l (20), along with the resulting Pareto distribution of all prices for varieties sold in l (domestic prices and export prices have an identical distribution in country l) yield a zero-cutoff profit condition linking the variety cutoff $v_D^l = c_D^l$ to the mass of varieties sold in country l :

$$
M^l = \frac{2\left(k+1\right)\gamma}{\eta} \frac{\alpha - c_D^l}{c_D^l}. \tag{24}
$$

Given a positive mass of entrants N_E^l in both countries, the total mass of varieties sold in country l will also be given by $M^l = \Omega G(c_D^l) N_E^l + \Omega^l G(c_X^h) N_E^h$. The first term represents the number of varieties produced for the domestic market by the N_E^l entrants in l; and the second term represents the number of exported varieties by the N_E^h entrants in country h. This condition (holding for each country) can be solved for the number of entrants in each country:

$$
\begin{aligned}
N_E^l &= \frac{(c_M)^k}{\Omega\left(1-\rho^l\rho^h\right)}\left[\frac{M^l}{\left(c_D^l\right)^k} - \rho^l\frac{M^h}{\left(c_D^h\right)^k}\right] \\
&= \frac{2\left(c_M\right)^k(k+1)\gamma}{\Omega\eta\left(1-\rho^l\rho^h\right)}\left[\frac{\alpha-c_D^l}{\left(c_D^l\right)^{k+1}} - \rho^l\frac{\alpha-c_D^h}{\left(c_D^h\right)^{k+1}}\right].
\end{aligned}
\tag{25}
$$

4.1 Trade Liberalization

When trade costs are symmetric ($\rho^l = \rho^h = \rho$), then the cost cutoffs in both countries decrease monotonically as trade costs are reduced (ρ increases) – including the transition from autarky ($\rho = 0$). This increase in the toughness of competition induces the same firm and product reallocations that were previously described for the closed economy: firms drop their marginal products and focus on products closer to their core competency; they also re-allocate their labor resources towards the production of those 'core' varieties (lower m). Thus, firm productivity increases due to these compounding effects. The inter-firm reallocations (the lowest productivity firms exit) generate an additional aggregate productivity increase.

5 The Margins of Export

In order to examine how the margins of export vary across destinations, we now extend our model to an arbitrary number of countries and asymmetric trade costs. Let J denote the number of countries, indexed by $l = 1, ..., J$. We assume that firms everywhere face the same step cost ω^{-1} for varieties produced for their domestic market, but now allow the additional customization cost for exports from l to h, $\left(\theta^{lh}\right)^{-1} \geq 1$, to vary across country-pairs. This leads to differences in the combined (inverse) step-cost $\left(\omega^{lh}\right)^{-1} \equiv \left(\theta^{lh}\omega\right)^{-1} \geq 1$ across country-pairs. We also allow the iceberg trade cost $\tau^{lh} > 1$ to vary across country-pairs. As with our two-country version, we define the overall 'freeness' of trade for exports from country l to h as $\rho^{lh} \equiv \left(\Omega^{lh}/\Omega\right)\left(\tau^{lh}\right)^{-k} < 1$, where $\Omega^{lh} \equiv \left[1-\left(\omega^{lh}\right)^k\right]^{-1}$. We also allow for the possibility of internal trade cost so that τ^{ll} may also be above 1. If not, then $\rho^{ll} = 1$, since $\Omega^{ll} = \Omega$ by definition. We continue to assume that firm productivity $1/c$ is distributed Pareto with shape k and support $[0, c_M]$ in all countries.[8]

[8] Differences in the support for this distribution could also be introduced as in Melitz and Ottaviano (2008).

In this extended model, the free entry condition (23) in country l becomes:

$$\sum_{h=1}^{J} \rho^{lh} L^h \left(c_D^h\right)^{k+2} = \frac{\gamma\phi}{\Omega} \qquad l = 1, ..., J.$$

This yields a system of J equations that can be solved for the J equilibrium domestic cutoffs using Cramer's rule:

$$c_D^l = \left(\frac{\gamma\phi}{\Omega}\frac{\sum_{h=1}^{J}|C_{hl}|}{|P|}\frac{1}{L^l}\right)^{\frac{1}{k+2}}, \tag{26}$$

where $|P|$ is the determinant of the trade freeness matrix

$$P \equiv \begin{pmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1M} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{M1} & \rho_{M2} & \cdots & \rho_{MM} \end{pmatrix},$$

and $|C_{hl}|$ is the cofactor of its ρ_{hl} element. Cross-country differences in cutoffs now arise from two sources: own country size (L^l) and geographical remoteness, captured by $\sum_{h=1}^{J}|C_{hl}| / |P|$ (an inverse measure of market access). Countries benefiting from a larger local market or better market access have lower cutoffs, and exhibit tougher competition.

The mass of varieties M^l sold in each country l (including domestic producers in l and exporters to l) is still given by (24). Given a positive mass of entrants N_E^h in country h, there will be $G(c_X^{hl})N_E^h$ firms exporting $\Omega^{hl}G(c_X^{hl})N_E^h$ varieties to country l. Summing over all these varieties (including those produced and sold in l) yields[9]

$$\sum_{h=1}^{J} \rho^{hl} N_E^h = \frac{M^l}{\Omega\left(c_D^l\right)^k}.$$

The latter provides a system of J linear equations that can be solved for the number of entrants in the J countries using Cramer's rule:[10]

$$N_E^l = \frac{\phi\gamma}{\Omega\eta\,(k+2)\,f_E}\sum_{h=1}^{J}\frac{\left(\alpha - c_D^h\right)}{\left(c_D^h\right)^{k+1}}\frac{|C_{lh}|}{|P|}. \tag{27}$$

[9] Note that $c_D^l = \tau^{hl} c_X^{hl}$.

[10] We use the properties that relate the freeness matrix P and its transpose in terms of determinants and cofactors.

5.1 Bi-Lateral Trade Patterns

We now investigate the predictions of this multilateral trade model for the composition of bi-lateral trade flows. A variety produced in country l at cost v for the export market to h generates export sales

$$r_X^{lh}(v) = \frac{L^h}{4\gamma}\left[\left(v_D^h\right)^2 - \left(\tau^{lh}v\right)^2\right].$$

Then $EXP^{lh} = N_E^l \Omega^{lh} \int_0^{c_X^{lh}} r_X^{lh}(v)dG(v)$ represents the aggregate bi-lateral trade from l to h across the $N_E^l \Omega^{lh} G(c_X^{lh})$ exported varieties. This aggregate trade flow can be decomposed into the product of the number of exporting firms, $N_X^{lh} \equiv N_E^l G(c_X^{lh})$, the average number of exported varieties per firm, Ω^{lh}, and the average export flow per variety, $\bar{r}_X^{lh} \equiv \left[\int_0^{c_X^{lh}} r_X^{lh}(v)dG(v)\right]/G(c_X^{lh})$. This last term, capturing the product-intensive margin of trade only depends on the characteristics of the import market h:

$$\bar{r}_X^{lh} = \frac{L^h}{2\gamma\,(k+2)}\left(c_D^h\right)^2.$$

Lower trade barriers to from l to h will clearly increase the export flow $r_X^{lh}(v)$ for any exported variety. However, the lower trade barriers will also induce new varieties to be exported to h. Since these new exported varieties will have the lowest trade volumes, these two effects will generate opposite forces on the average export flow $\bar{r}_X^{lh}$. Given our parametrization, these opposing forces exactly cancel out. We do not emphasize this exact result, but rather the presence of opposing forces generating the relationship between trade costs and average exports per variety. On the other hand, increases in importer country size generate unambiguous predictions for this intensive margin of trade: Increases in country size toughen the selection effect for exported varieties (skewing the distribution towards varieties with higher trade volumes), and also generates increases in export flows $r_X^{lh}(v)$ for the varieties with the largest trade volumes (lower v).

Trade costs τ^{lh} as well as differences in importer characteristics generate ambiguous effects on the average number of exported varieties per firm: Higher trade costs or tougher competition in h will both reduce the number of exported varieties by any given exporting firm. However, they will also generate a selection effect among firms: lower productivity firms exporting the smallest number of varieties exit the export market. Given our parametrization, these opposing forces cancel out, leaving the average number of exported varieties Ω^{lh} unchanged. Again, we emphasize the presence of competing forces for this margin of trade. However, changes in the additional step cost associated with customization for the export market in h do generate unambiguous predictions for the average number of exported varieties per firm: decreases in this additional cost will increase the average number of exported varieties, as all firms export more varieties.

Lastly, exporter and importer country characteristics, as well as trade barriers will have a predictable effect on the number of exporting firms:

$$N_X^{lh} = N_E^l G(c_D^h) \left(\tau^{lh}\right)^{-k}.$$

There are no countervailing forces at this final extensive margin: anything that makes it harder for firms from country l to break into the export market in h (higher trade barriers or tougher competition in h) will decrease the number of exporting firms. Holding those forces constant, an increase in the number of entrants (into production) in l will proportionally increase in the number of exporting firms to any given destination.

6 Conclusion

To be completed...

7 References

To be completed...