Labor-Market Matching with Precautionary Savings and Aggregate Fluctuations*

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May 2007

Abstract

We analyze a Bewley-Huggett-Aiyagari incomplete-markets model with labor-market frictions and aggregate shocks. Consumers are subject to idiosyncratic employment shocks against which they cannot insure directly. However, there is a complete set of assets for insuring against aggregate risk, and consumers use these as vehicles for precautionary saving. There are aggregate productivity shocks, and all firms are alike. The labor market has a Mortensen-Pissarides structure: firms enter by posting vacancies and match with workers bilaterally, with match probabilities given by an aggregate matching function. Wages are determined through Nash bargaining. The resulting model is shown to be remarkably similar to the Mortensen-Pissarides model, and yet it inherits the aggregate features of consumption and investment of typical real-business-cycle models. Aggregate fluctuations in employment and vacancies and in real wages depend crucially on the monetary value of being unemployed.

Keywords: incomplete market; matching; heterogenous agents
JEL Classifications: J63, J64, D52

*We thank the comments from the seminar participants at various places. The views expressed in this article are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

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1 Introduction

In this paper we build a model of the aggregate economy with two frictions that are becoming broadly viewed as central: labor markets involve search-matching costs, and consumers are risk-averse but cannot insure fully against idiosyncratic shocks. These features have each been studied in detail in previous work. Costs of search and matching have been argued to be central for labor markets and for business cycles: fluctuations in employment to a large extent involve the extensive margin and, arguably, an important fraction of these movements are of the "involuntary" kind, i.e., due to the frictions. There is also a widespread view that consumer risk aversion, and the implied desire to smooth consumption, is a key reason behind why investment fluctuates so much while consumption does not. On the individual level, moreover, studies of consumption strongly indicate that there is less than perfect individual consumption insurance, in particular against idiosyncratic shocks. The main purpose of the present paper is thus methodological: it is to demonstrate that it is possible to analyze models with labor-market frictions and risk-averse consumers facing partly uninsurable risk. The main analytical challenge in solving models with the aforementioned two frictions has been the interaction of two elements: first, there is a nontrivial and evolving distribution of wealth among consumers, which influences prices and aggregate quantities; and second, unlike in the one-firm neoclassical model, there is a distribution of wage outcomes influencing worker and firm decisions.

The need for a model of the macroeconomy which more adequately represents real-world savings, insurance, and work decisions on the microeconomic level is accompanied by quite promising initial indications of the resulting setting's aggregate implications. In a nutshell, at least in one of the calibrations that we entertain, we obtain plausible fluctuations in consumption and investment, as in the standard real-business-cycle (RBC) model, while also delivering reasonable labor-market outcomes. In particular, unemployment and vacancy levels are volatile (and roughly reproducing a realistic Beveridge curve), the real wage does not fluctuate nearly as much as in the standard RBC model, and the wage share of income is countercyclical. That is, we have a model of employment whose broad features are in line with, and arguably also an improvement upon, the standard RBC model, but employment movements here result from firm entry and exit only, i.e., they are due to changes in labor demand, and unemployment is always involuntary. The extent to which the latter—the involuntariness of unemployment—is important is still an open question, of course, which deserves much further research.

There is also an important normative motivation for the present work. Although we do not pursue normative analysis here, it is clear that the present framework allows the comparison of, and interaction between, aggregate (say, stabilization) policies and policies more directly aimed at insurance: social insurance policy, such as various forms of redistribution using taxes and transfers as well as labor-market regulation. As for aggregate policy, first, an example is the earlier work on the welfare costs of business cycles on a cross-section of consumers.[1] A theme of that work is that business cycles may influence different consumers differently; consumers with very low wealth may prefer a stable economic environment more than do consumers with high wealth, since the former are more vulnerable to risk. In this context, an open and quantitatively central question is how aggregate stabilization influences the nature of idiosyncratic shocks: in earlier analyses, idiosyncratic shocks were unmodeled, and assumptions had to be made as to whether and how these were linked to aggregate shocks.[2] The present setting, in contrast, has the advantage that because it models employment shocks—so that the nature of these shocks is endogenous—it provides an explicit answer to how stabilization policy will influence idiosyncratic shocks. Moreover, if, say, individual matches have heterogeneous productivity—an assumption not considered here but which is straightforward to incorporate—then aggregate policy would also influence the reservation wages; hence, also the support of individual wages would respond endogenously to this policy. Second, as for social insurance policy, it is usually studied in a context where its aggregate implications, or its impact on labor markets (via search behavior), is not in focus. Similarly, there is a significant body of theoretical research examining various labor-market policies, but these examinations typically ignore the implications for consumption insurance, as well as any related impact on aggregate capital formation.

[1] See, e.g., İmrohoroğlu (1989) and the review discussion in Lucas (2003).

[2] See Atkeson and Phelan (1994), Krusell and Smith (1999, 2002), Krebs (2006), and Mukoyama and Şahin (2006).

Our setting formally combines two well-known and much studied frameworks. One is the Bewley-Huggett-Aiyagari setting (Bewley (undated), Huggett (1993), and Aiyagari (1994)). There, the idiosyncratic risk is exogenous (of course, here, it is modeled) and consumers can only use a safe asset to insure against this risk. We consider the version of this model that includes aggregate shocks, as in Krusell and Smith (1998). Firms in that framework operate neoclassical production functions, which we assume firms use as well, but here they do so in the context of perfectly competitive and frictionless markets. In particular, we assume that they sell their output, and rent their capital, in perfectly competitive markets, though their participation in the labor market involves frictions. Thus, the other framework we use is the Diamond-Mortensen-Pissarides matching framework of the labor market (Diamond (1981), Pissarides (1985), and Mortensen and Pissarides (1994)). There, as in our model here, firms and workers match in pairs, and there is an aggregate matching function determining how many matches result for a given number of searching firms and workers. Wages are determined, period by period and without commitment, using Nash bargaining within each worker-firm pair: the wage maximizes a Cobb-Douglas function of the inside minus the outside indirect (present-value) utility of the worker and the equivalent measure for the firm. Firms enter at zero cost, but need to post vacancies at a cost in order to attract workers. We thus stay close to Pissarides (1985), or more specifically to a version of that model where firm-worker pairs, upon having decided to remain together, rent capital at competitive rental rates.[3]

We allow a complete set of assets for insurance against aggregate shocks; since we have a two-state process for aggregate productivity, this means that we can talk about a stock and a bond. As in the typical decentralization of the stochastic closed-economy growth model, we let consumers hold the capital (and rent it to firms), but here we have "equity" as well: equity is the claim to profits net of payments to capital and labor, and thus it is the return to the vacancy-posting costs. In our economy, because market incompleteness makes individuals evaluate the future differently, it may not be a priori clear how to price the firm, or what should guide firms' decisions; recall that firms' decisions are forward-looking since their wage bargaining involves the dynamic impact on

[3]See Pissarides (2000, Section 1.6).

future profits.[4] However, since there is no aggregate risk in our economy from period to period, aside from that contingent on aggregate productivity, our assumption of complete markets for this risk implies that consumers do agree on profit flows and that the price of equity and the firm decisions are well-defined.

There have not been many quantitative studies of the Diamond-Mortensen-Pissarides matching model with risk-averse workers and savings. The main and notable exceptions are Andolfatto (1996) and Merz (1995), who study quantitative implications of this type of model in a setting with complete consumption insurance for workers. Relative to these papers, thus, we provide a quantitative evaluation of with incomplete markets, with all its implications for consumer heterogeneity, wage setting, and so on. Further, recent quantitative studies of the labor-market model with frictions (see Hall (2005) and Shimer (2005)) conclude that calibration to labor-market flows and steady-state statistics implies insufficient movements in vacancies and unemployment relative to what we observe in the U.S. and in other economies, unless (real) wages are exogenously restricted to be rigid. Subsequently, Hagedorn and Manovskii (2006) pointed to an alternative calibration without these problems, and it is an open question how to judge these different views on how to choose parameters; we consider both here.

In Section 2, we first set up a Bewley-Huggett-Aiyagari dynamic general equilibrium model without aggregate fluctuations. We solve for and carefully discuss the steady state of this model, which is significantly different from the Aiyagari (1994) model. The reason is that workers' wages, due to the assumption of Nash bargaining, must depend on their asset holdings. With our calibration, wages are increasing in the asset holdings: rich workers have a higher outside option, since they can bear unemployment more easily with more saved resources to consume from. As a result, consumers see an additional return from saving: it will raise their future wage when employed. We show, however, that except for the very poorest workers, this channel is very weak quantitatively, which may be reassuring, since the effect of individual wealth on wages has not been argued to be large in the empirical literature.

[4] See, for example, Carceles-Poveda and Coen-Pirani (2005) for recent work on this issue.

In Section 3, we extend our model to incorporate aggregate uncertainty. Now, the distribution of asset and employment moves around endogenously, responding to aggregate shocks. When consumers optimize, they have to take the change in this distribution into account. This creates a computational complexity when solving the model's equilibrium. We extend the "approximate aggregation" result and associated computational algorithm, originally developed by Krusell and Smith (1998, 1997), for our model and its numerical characterization, with reassuring results: approximate aggregation obtains, despite the presence of several nontrivially determined aggregate equilibrium objects, such as labor-market tightness and asset prices. The results in this section, moreover, show that, as in the recent discussion of fluctuations in employment and vacancies in the context of the model with risk-neutral consumers, the calibration is key: fluctuations in the labor-market variables are large if and only if Hagedorn and Manovskii's calibration is adopted.

Section 4 concludes, and the Appendix contains a series of theoretical results as well as details on the numerical algorithms we used.

2 The model without aggregate shocks

In this section, we develop a model without aggregate shocks, and our focus is on a stationary equilibrium. The stationary version of the model is of independent interest, in part because it can be used for interesting comparative-statics exercises: it is possible to examine the effects on vacancies and unemployment of, say, a change in productivity. In addition, on a methodological level, it is also a challenging model to analyze due to the endogeneity of the wage function. As we show in this section, however, that challenge can be overcome to a large extent, and this is important for the computational feasibility of the model with aggregate shocks.

We first describe the matching technology and the asset structure. We then state the maximization problems of the different agents and describe the wage determination mechanism, which is based on bargaining within each worker-firm pair. After formally defining our stationary equilibrium we briefly discuss computation and finally present our results.

2.1 Matching

There is a continuum of consumers, with measure 1, in the economy. A consumer is either employed or unemployed. There are many firms, each of which operates with one "job" (position for a worker). The total value of the firms is represented by p.[5] Each job is filled by one worker and has a production function $zF(k)$. Here, z represents the aggregate productivity level (constant in this section), $F(\cdot)$ is an increasing and strictly concave production function, and k is the amount of capital stock employed in that job. Note that since z and $F(\cdot)$ are common across the firms (who act competitively in the market for capital stock), in equilibrium the same amount of k is employed at each filled job. Vacant jobs and unemployed workers are randomly matched each period according to an aggregate matching function. The aggregate matching function, $M(u, v)$, represents the number of matches in a period when there are u unemployed workers and v vacancies, and is specified as

$$M(u, v) = \chi u^{\eta} v^{1-\eta}.$$

In addition, $M(u, v) \leq v$ and $M(u, v) \leq u$ have to hold. Note that this aggregate matching function exhibits constant-returns-to-scale. The probability of a vacant job to be filled at the current period, λ_f, is

$$\lambda_f \equiv M(u, v)/v = M(u/v, 1) = \chi(v/u)^{-\eta} = \chi\theta^{-\eta}, \tag{1}$$

where $\theta \equiv v/u$ is the vacancy-unemployment ratio. The probability of an unemployed worker to be employed at the current period, λ_w, is

$$\lambda_w \equiv M(u, v)/u = (v/u)M(u/v, 1) = \theta\lambda_f. \tag{2}$$

Therefore, λ_f and λ_w are functions of θ. We assume that a match is separated with probability σ in each period. The assumption of a constant and exogenous separation rate is made for convenience. It is potentially an important source, or propagator, of fluctuations. However, it is arguably not likely to be the most important one, at least not for generating the negative correlation between

[5] Alternatively, we can interpret that there is one "representative firm", whose value is represented by p.

unemployment and vacancies.[6]

¿From the above assumptions, the transition of the unemployment rate u can be described by

$$u' = (1 - \lambda_w)u + \sigma(1 - u),$$

where a next period variable is denoted by a prime ($'$).

2.2 Asset structure

The consumers face idiosyncratic employment shocks, but we assume that there are no insurance markets for these idiosyncratic shocks. The consumers can hold only two kinds of asset—capital k, which is used as an input for production, and equity x, which is a claim for the firm's profit. Let r be the return to capital and d be the dividend paid to the holders of equity. We normalize the total amount of equities to one. Since we focus on the steady-state, there is only one aggregate state. The equity price p has to satisfy the following equation:

$$p = \frac{d + p}{1 + r - \delta}, \tag{3}$$

where r is the rental rate of capital and δ is the depreciation rate of capital. This comes from no-arbitrage: one unit of capital generates $1 + r - \delta$ unit of return next period, and one unit of equity generates $(d + p)/p$ unit of return (note that both assets are riskless).

Since the capital and the equity are essentially the same asset (both are riskless and provide same return) from the consumer's viewpoint, we do not have to keep track of the asset composition of the consumers. In the following, we define

$$a \equiv (1 + r - \delta)k + (p + d)x$$

and use a as the state variable for a consumer. Note that from (3),

$$a = (1 + r - \delta)(k + px)$$

holds.

[6] A temporary increase in the separation rate increases unemployment in a direct way but its impact on vacancies is less powerful than that resulting from increases in productivity or in "firm product demand".

2.3 Consumers

2.3.1 Employed consumers

The budget constraint for an employed consumer is

$$c + k' + px' = a + w, \tag{4}$$

where c is consumption and w is the wage rate. The wage is determined through Nash bargaining between the firm and the worker every period, and it turns out that the wage depends on the worker's asset level. The details of Nash bargaining are motivated and explained later.

For a given wage w, employed consumers choose their capital and equity holdings subject to (4) and the borrowing constraint $\underline{a}$:

$$\tilde{W}(w, a) = \max_{k', x'} u(c) + \beta \left[\sigma U(a') + (1 - \sigma) W(a')\right] \tag{5}$$

subject to

$$c + k' + px' = a + w,$$

$$a' = (1 + r - \delta)k' + (p + d)x',$$

and

$$a' \geq \underline{a}.$$

Here, $u(\cdot)$ is an increasing and concave utility function, β is the discount factor, $U(\cdot)$ is the value function of an unemployed worker, and $W(\cdot)$ is the value function of an employed worker, taking into account that the wage is renegotiated next period (defined later). Let the decision rule for a' for employed workers be $a' = \tilde{\psi}_e(w, a)$. Denote the wage resulting from the Nash bargaining (detailed later) as

$$w = \omega(a).$$

$W(a)$ is formally defined as

$$W(a) \equiv \tilde{W}(\omega(a), a). \tag{6}$$

Also define

$$\psi_e(a) \equiv \tilde{\psi}_e(\omega(a), a). \tag{7}$$

2.3.2 Unemployed consumers

The budget constraint for an unemployed consumer is

$$c + k' + px' = a + h \tag{8}$$

where h is the income for an unemployed worker. h can be thought as household production. This assumption is made so that an agent can earn some labor income even when she is unemployed. Alternatively, we can introduce an unemployment insurance system (such as the one in Hansen and İmrohoroğlu (1992)). In that case, a government and its budget constraint would need to be incorporated into our setup.

Unemployed consumers' optimization problem is:

$$U(a) = \max_{k',x'} u(c) + \beta \left[(1-\lambda_w)U(a') + \lambda_w W(a')\right] \tag{9}$$

subject to

$$c + k' + px' = a + h,$$

$$a' = (1 + r - \delta)k' + (p + d)x',$$

and

$$a' \geq \underline{a}.$$

Let the decision rule for a' for unemployed workers be $a' = \psi_u(a)$.

2.4 Firms

To create a job, a firm first posts a vacancy. We assume that there is a flow cost of posting a vacancy and denote it by ξ. The value of posting a vacancy, V, is

$$V = -\xi + \frac{1}{1+r-\delta}\left[(1-\lambda_f)V + \lambda_f \int J(\psi_u(a))\frac{f_u(a)}{u}da\right]. \tag{10}$$

Since the equity price is discounted at the rate $1 + r - \delta$, the firm discounts the future at the rate $1 + r - \delta$. With probability $1 - \lambda_f$, the vacancy remains unfilled. With probability λ_f, the vacancy is filled by a worker. $J(a)$ is the value of a job filled by a worker whose asset level is a. Since the matching process is random, the firm can be matched with any worker in the current period unemployment pool. $f_u(a)$ is the population of unemployed workers with the current asset level a. Thus $f_u(a)/u$ is the density function of the unemployed workers over a, and an unemployed worker with the current asset level a will have the next-period asset level $\psi_u(a)$. The integral calculates the expected value. In equilibrium, the firm will post the vacancy until $V = 0$.

The value of a filled job, given the wage w, is

$$\tilde{J}(w, a) = \max_k zF(k) - rk - w + \frac{1}{1+r-\delta}\left[\sigma V + (1-\sigma)J(\tilde{\psi}_e(w, a))\right], \tag{11}$$

Note that $\tilde{J}$ depends on a since with probability $(1-\sigma)$ the firm continues to be matched with the same worker, whose next-period asset level depends on a. $\tilde{J}$ and J are related by

$$J(a) \equiv \tilde{J}(\omega(a), a). \tag{12}$$

The firm's first-order condition implies that

$$r = zF'(k)$$

holds.

In equilibrium, the period profit is equal to

$$\pi(a) = zF(\tilde{k}) - r\tilde{k} - \omega(a), \tag{13}$$

with

$$\tilde{k} = \frac{\bar{k}}{1-u},$$

where $\bar{k}$ is aggregate capital stock. $\tilde{k}$ is the amount of capital stock for each job: from symmetry, capital is distributed evenly across jobs.

The dividend is paid out as the sum of profit minus the total vacancy cost:

$$d = \int \pi(a) f_e(a) da - \xi v, \tag{14}$$

where $f_e(a)$ is the population of the matched workers with wealth level a. Appendix A shows that with this dividend and the asset pricing formula (3), $p + d$ is equal to the sum of $J(a)$.

2.5 Wage determination

The wage is determined by the (generalized) Nash bargaining. The Nash bargaining solution solves the problem

$$\max_{w} \left(\tilde{W}(w,a) - U(a)\right)^{\gamma} \left(\tilde{J}(w,a) - V\right)^{1-\gamma}. \tag{15}$$

$\gamma \in (0,1)$ is a parameter that represents the bargaining power of the worker. The solution of this is $w = \omega(a)$. It is clear that the dependence of w on a stems from $\tilde{W}(w,a) - U(a)$ being a function of a.

The assumption of Nash bargaining requires brief discussion. First, does the idea that a worker's wealth can influence his wage "ring true"? Though the evidence is inconclusive, it is a logical implication of the assumption that there is no commitment to wages in advance (by workers or firms) and of worker rationality: if one's outside option is strong (for whatever reason, but high wealth is one such reason), why not bargain for a higher wage? Second, we could have considered a setting where firms can precommit to a wage before searching for workers (and before knowing whether they find a rich or a poor worker). Though this is certainly an interesting alternative to the assumption used here, we are not sure that commitment (especially many periods ahead) is a realistic assumption.[7] Moreover, if commitment is introduced without allowing firms to post different wages, and workers to direct search to different wages, then it gives firms more bargaining power than one might consider realistic: it amounts to letting firms make take-it-or-leave-it offers (and would lead wages to equal the exogenous unemployment value). If instead firms can post wages, and consumers can choose which job to apply for (so that they would be indifferent between a high-probability/low-wage outcome and a low-probability/high-wage outcome), firms would indeed compete and workers would obtain some surplus, but here the analysis is likely not simpler than for the present setting (without as well as with aggregate shocks), especially since we show below that the Nash bargaining case is much easier to solve than expected. Third and finally, for comparison with the earlier literature, the results under Nash bargaining ought to be known, since

[7] In the presence of aggregate shocks, precommitment introduces an additional issue: firms would need to consider changes in wages due to these shocks. See Rudanko (2006a,b) for different approaches.

Nash bargaining is the most commonly used assumption under risk neutrality.

2.6 Recursive stationary equilibrium

We define our object of study as follows.

Definition 1 (Recursive stationary equilibrium) *The recursive stationary equilibrium consists of a set of value functions* $\{\tilde{W}(w,a),\ \tilde{J}(w,a),\ W(a),\ J(a),\ U(a),\ V\}$*, a set of decision rules for asset holdings* $\{\tilde{\psi}_e(w,a),\ \psi_e(a),\ \psi_u(a)\}$*, prices* $\{r,\ p,\ \omega(a)\}$*, vacancy* v*, matching probabilities* λ_f *and* λ_w*, dividend* d*, and the distribution of employment and asset* μ *(which contains the information of* $f_e(a)$*,* $f_u(a)$*, and the unemployment rate* u*) which satisfy*

1. *Consumer optimization:*

   Given the job-finding probability λ_w*, prices* $\{r,\ p\}$*, and wage* w*, the individual decision rules* $\tilde{\psi}_e(w,a)$ *and* $\psi_u(a)$ *solve the optimization problems (5) and (9), with the value functions* $\tilde{W}(w,a)$ *and* $U(a)$*. Given the wage function* $\omega(a)$*,* $W(a)$ *and* $\psi_e(a)$ *satisfy (6) and (7).*

2. *Firm optimization:*

   Given prices r *and* w*, distribution* μ*, and the employed consumer's decision rule* $\tilde{\psi}_e(w,a)$*, the firm solves the optimization problem (11), with the value function* $\tilde{J}(w,a)$*. Given the wage function* $\omega(a)$*,* $J(a)$ *satisfies (12). Given the worker-finding probability* λ_f*,* r*, the unemployed consumer's decision rule* $\psi_u(a)$*, and* μ*,* V *satisfies (10).*

3. *Free entry:*

   The number of vacancy posted, v*, is consistent with the firm free-entry:* $V = 0$*.*

4. *Asset market:*

   The asset-market equilibrium condition

$$\int \psi_e(a) f_e(a) da + \int \psi_u(a) f_u(a) da = (1 - \delta + r)\bar{k} + p + d$$

holds: the left hand side is the total asset supply and the right hand side is the total asset demand. The no-arbitrage condition (3) holds. Dividend d satisfies (14). $\bar{k}$ satisfies $\tilde{k} = \bar{k}/(1-u)$, where $\tilde{k}$ satisfies the firm's first-order condition: $r = zF'(\tilde{k})$ for given r.

5. *Matching:*

   λ_f and λ_w are functions of v and u as in (1) and (2).

6. *Nash bargaining:*

   The wage function $\omega(a)$ is determined through Nash bargaining between the firms and the consumers by solving (15).

7. *Consistency:*

   μ is the invariant distribution generated by λ_w, σ, and the consumer's decision rules.

2.7 Computation

In the standard Bewley-Huggett-Aiyagari setting, computation of a steady state reduces to a one-dimensional fixed point problem in the value of the capital stock. An efficient algorithm for that model is to (i) guess on a value for the capital stock; (ii) obtain the prices (the wage and the rental rate) from the firm's first-order conditions for inputs (price equals marginal product); (iii) solve the consumer's dynamic-programming problem globally using nonlinear methods; (iv) simulate an agent's capital accumulation path for a large number of periods; and finally (v) compare the average capital stock held by the agent with the initial guess, and update. A key feature of this simple algorithm is that the distribution of capital across agents in a steady state need not be used in the computation, though upon convergence it is straightforward to compute it using, say, the method described in Huggett (1993).[8] The reason why the distribution is not needed in the computation of the steady-state prices is that consumers do not need to know it: they only need to know the two prices (which can be summarized by one number, the capital stock) to perform their utility

[8]The reason why only one agent's capital accumulation needs to be simulated is that the consumer's process for capital accumulation is ergodic under rather general conditions: the cross-section of capital holdings is in one-to-one correspondence with the time-series process for capital of any given agent.

maximization. The functional fixed-point problem that does need to be confronted in step (iii), finally, is computationally straightforward since it is a contraction mapping (and fast to solve, since it has only one endogenous state variable).

The present model cannot be computed with an algorithm that reduces to a one-dimensional fixed-point problem; instead one needs to solve a functional fixed-point problem. The reason is that the consumer needs to know not just two prices but the entire wage function, $\omega(a)$, to maximize utility. Thus, the algorithm we use works as follows: (i) guess on $\omega(a)$, along with r and θ; (ii) using the matching function and the given θ, compute the probability of finding a job; (iii)–(iv) as above, which also involves a straightforward contraction mapping and can be used to obtain the distributions of employed and unemployed consumers across capital holdings; (v) given the latter, obtain firm values by iterating on the firm's value function (this involves no maximization and is a contraction mapping); and (vi) given all value functions, perform the Nash bargaining. Now note that (vi) delivers an update for the wage function; (iv) an average total capital stock, which since that unemployment can be computed from θ allows us to find the per-firm capital stock, and hence we can obtain an update for r from the firm's capital rental decision; and (v) gives a value of entry, which should be zero, and hence leads us to adjust the guess for θ. The fixed-point problem in (ω, r, θ) could in principle be a very difficult one, but fortunately it turns out that it is easy to solve for this model, and arguably so for a large range of parameter values. The reason for this is that the ω function is mostly flat and that few agents have capital holdings over the range where it is not flat; see the results in Section 2.9 below. The detailed computational procedure can be found in Appendix B.

2.8 Calibration

One period is considered to be six weeks. Following the standard real business cycles literature, we choose $\delta = 0.0125$ and the value of β as 0.995. The household production parameter h is assumed to be 1.4. This figure turns out to be about 40% of the average equilibrium wage, similarly to Shimer's (2005) calibration. The borrowing constraint $\underline{a}$ is set at 0. The production function is $zF(k) = zk^{\alpha}$ with $\alpha = 0.36$. We use the utility function $u(c) = \log(c)$ and also consider the $u(c) = c^{1-\zeta}/(1-\zeta)$

with $\zeta = 5$. Matching-related parameters are calibrated following Shimer (2005). The separation rate (σ) is set to 0.05 based on the observation that jobs last about two and a half years on average. We set $\chi = 0.675$ to match the monthly job finding rate of 0.45. Shimer estimates the elasticity of the matching function as 0.72, and we set the corresponding parameter $\eta = 0.72$. We target the value of θ to be 1 following Shimer's procedure. The cost of posting a vacancy is set to $\xi = 0.7368$ so that V becomes zero with this target value of θ. Shimer sets the bargaining parameter $\gamma = 0.72$ by following the Hosios efficiency condition. We set the bargaining parameter to $\gamma = 0.72$ as well. In the benchmark, z is set to 1.00.

2.9 Results

Comparative statics in the model with risk neutrality can be computed analytically and are useful also for understanding short-run effects of productivity shocks, because θ (labor-market tightness) jumps immediately to its new long-run value in response to a permanent (unexpected) shock; unemployment then follows a constant law of motion to its new long-run value, and vacancies follow. This result comes from the assumption that θ is a "jump" variable: vacancy flows are not associated with any adjustment costs, and they react immediately so as to maintain the profit from entering at zero. The result also involves an immediate jump in the capital stock (in the version of the model with capital) to its new long-run value, and this result requires risk neutrality. In the present model, comparative statics do involve a jump in θ, but not to its new long-run value, since capital must adjust slowly in order to allow consumption smoothing. Thus, the long-run effect on θ is different—it is larger—than the short-run effect.

Various comparative-statics exercises are relevant. We examine preferences in order to see how important consumption smoothing is, and we look at the effects of productivity; these are contained in separate sections. We also examine the effects of those parameters over which the literature has not arrived at a consensus, such as the (monetary) value of unemployment and the Nash bargaining share.

2.9.1 Different preferences

Table 1 presents the summary statistics for different utility functions. One is log utility, and the other is $u(c) = c^{1-\zeta}/(1-\zeta)$ with $\zeta = 5$ (we kept the other parameters constant). Larger ζ is associated with higher precautionary savings and thus with higher $\bar{k}$. Higher $\bar{k}$ leads to larger profitability of each vacancy: v increases, θ increases, and u decreases. Naturally, p and d increase.

	ξ	θ	u	v	$\bar{k}$	p	d	w
log utility	0.7368	1.00	6.90%	0.069	104.74	1.02	0.0051	3.44
$\zeta = 5$	0.7447	1.00	6.90%	0.069	104.94	1.04	0.0052	3.45

Table 1: Summary statistics for the model without aggregate shocks. w is the average wage in the economy.

Figure 1 shows the wage as a function of asset holdings.

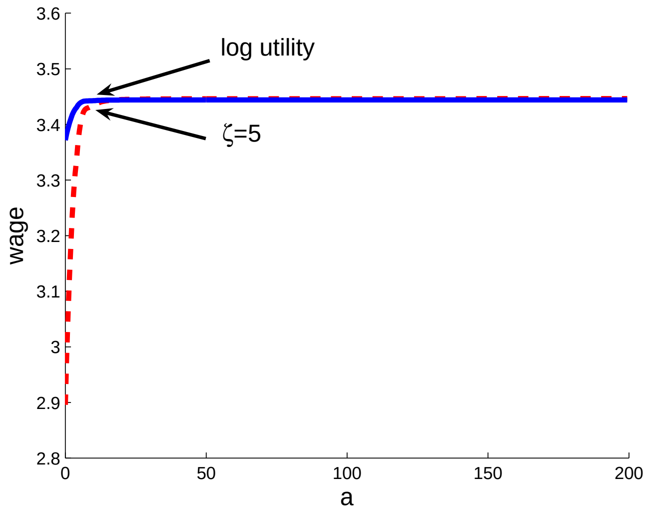

Figure 1: Wages for log utility and $u(c) = c^{1-\zeta}/(1-\zeta)$ with $\zeta = 5$.

The observed concavity of $\omega(a)$ follows, intuitively, from two features: (i) the function being increasing, which is due to the outside option being worse for consumers with a low stock of assets (since their buffer against unemployment shocks is lower), and (ii) a natural upper bound, which is given by that value which a risk-neutral agent would obtain in the bargaining (as in the Mortensen-Pissarides model), and this value is approached here for consumers with infinite asset holdings since they are "perfectly insured". It turns out that with our parametrization, the wages are only increasing significantly for the as that are very close to the borrowing constraint. Figure 1 compares the wage functions across different utility functions (log and $\zeta = 5$). The change in wages close to the borrowing constraint is larger with $\zeta = 5$. This is because the outside option $U(a)$ is very low for a nearly constrained worker, when the utility function has a large curvature.

The average wage in the economy is 3.44 and the minimum wage is 3.38. The mean-min wage ratio is 1.018. This means that the model generates only 1.8% wage differential between the average wage and the lowest wage paid in the labor market.[9] We find that wage differentials created by the heterogeneity of asset and Nash bargaining are small.

Figure 2 plots the asset holding densities $f_e(a)$ and $f_u(a)$ for the log utility case. (Asset distributions are similar for $\zeta = 5$ case.)

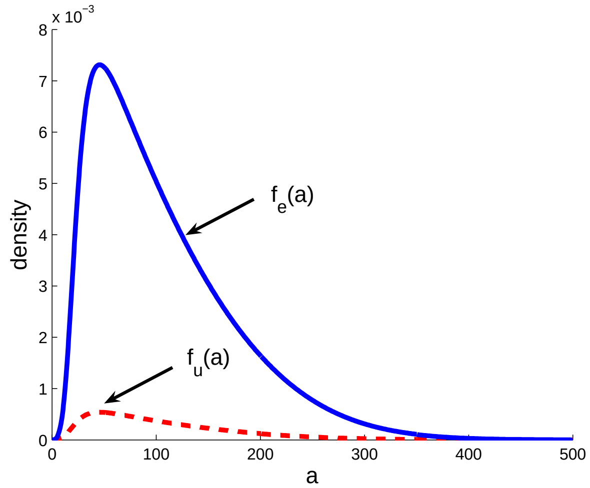

Figure 2: Asset distributions for log utility.

[9]See Hornstein, Krusell, and Violante (2006) for a study that utilizes this measure to analyze inequality in a matching model.

As can be seen from the asset distribution, consumers avoid to be at the very lower tail of the asset distribution. This is due to the additional savings incentive that arises in our model. The wealth distribution here, clearly, is not realistic, but this is probably not a major shortcoming; in the present setting, the only source of wealth inequality is unemployment shocks (there is no ex-ante consumer/worker heterogeneity, and there are no other shocks, such as wage shocks).

The additional incentive for saving can be seen easily from the Euler equations of the consumers. For employed consumers, the Euler equation is (when they are not borrowing-constrained)

$$u'(c_t) = \beta(1 + r - \delta) \left[\sigma u'(c^u_{t+1}) + (1 - \sigma)(1 + \omega'(a_{t+1}))u'(c^e_{t+1}) \right],$$

where c_t is the current consumption and c^u_{t+1} is the future consumption in the case of unemployment, c^e_{t+1} is the future consumption in the case of employment. For unemployed consumers, the Euler equation is

$$u'(c_t) = \beta(1 + r - \delta) \left[(1 - \lambda_w)u'(c^u_{t+1}) + \lambda_w(1 + \omega'(a_{t+1}))u'(c^e_{t+1}) \right].$$

These Euler equations are standard except for $\omega'(a_{t+1})$, which turns out to be positive in our calibration. Figure 3 plots $\omega'(a_{t+1})$ for the log utility case and $\zeta = 5$ case.

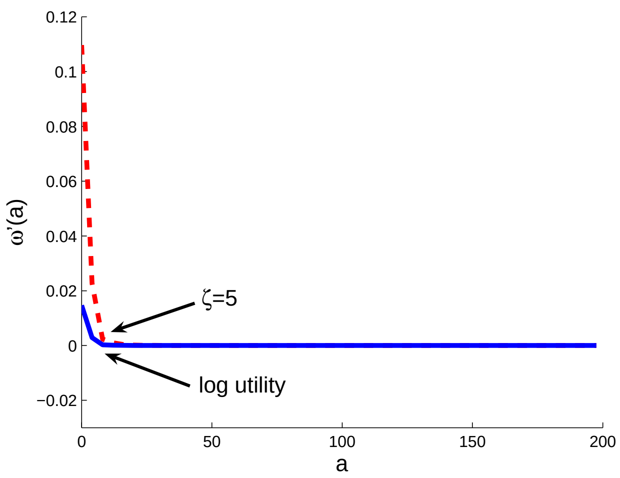

Figure 3: Derivative of the wage function for log utility and $u(c) = c^{1-\zeta}/(1-\zeta)$ with $\zeta = 5$.

$\omega'(a_{t+1})$ is large for the consumers with low asset holdings. Consumers with low asset holdings cannot insure themselves from being unemployed—since they have no other resources to consume (they are constrained by the borrowing limit), they suffer from low consumption when they are unemployed. This makes their bargaining position weaker. As the asset level increases, the consumers are better insured, and their value function becomes closer to linear. As a consequence, $\omega(a)$ becomes flatter. Note that when the utility function is linear (as in standard Diamond-Mortensen-Pissarides model), the wage doesn't depend on a. We will see later that the aggregate behavior of our model is very similar to the linear utility model.

In this model, a positive $\omega'(a_{t+1})$ provides an extra incentive to save. Figure 4 shows the benchmark asset distributions compared with the asset distributions when the wage is given exogenously and independently from the asset level (at the mean value of the benchmark). As can be seen in these figures, if we shut down the $\omega'(a_{t+1})$ channel by assuming that the wage is fixed exogenously, the stationary wealth distributions look different. In particular, there is a larger mass of consumers at the left tail of the wealth distribution. When the wage is Nash bargained, the consumers try to escape from the low asset holding level. As a result, most of the populations stay in the part where $\omega(a)$ is flat. Thus, the homogeneity of the wages across the population is generated by the endogenous choice of asset by the consumers.

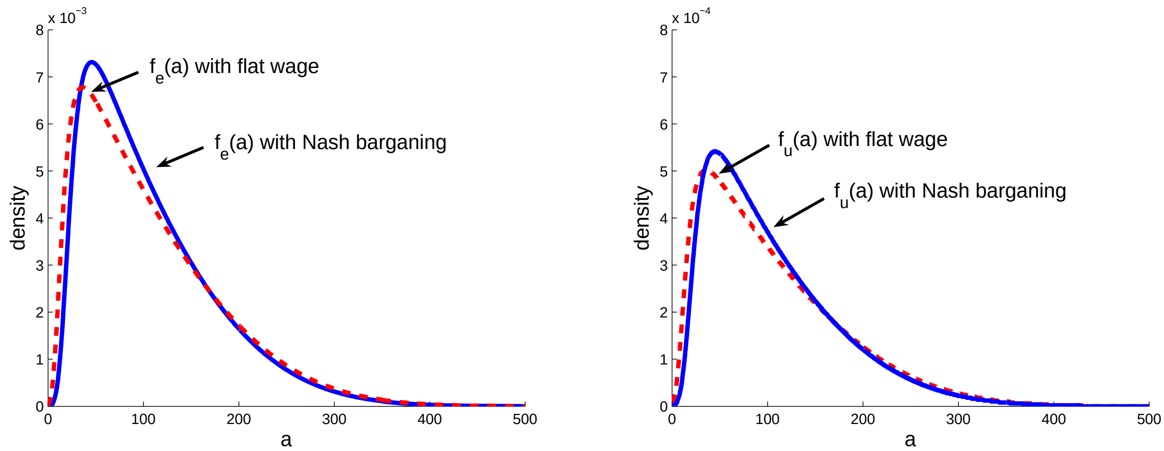

Figure 4: Asset distributions for the employed and the unemployed

2.9.2 Different aggregate productivity

As a prelude to the analysis with aggregate shocks, we computed the equilibrium with different values of z. Table 2 presents the summary statics for different z values.

	u	v	θ	$\bar{k}$	p	d	w
$z = 0.99$	6.95%	-2.0%	-2.7%	-1.6%	-2.2%	-2.2%	-1.6%
$z = 1.01$	6.85%	$+2.0\%$	$+2.7\%$	$+1.6\%$	$+1.9\%$	$+1.9\%$	$+1.6\%$

Table 2: Summary statistics for the model without aggregate shocks for different z values. All in % deviations from $z = 1.00$ case, except for u.

In the table (and all the following tables that involves the comparison across different z), all the results are shown as the % deviation from the $z = 1.00$ case, except for u. u is shown in the absolute level. Aggregate capital increases with z, $\bar{k} = 103.06$ for $z = 0.99$, $\bar{k} = 104.74$ for $z = 1.00$ and $\bar{k} = 106.44$ for $z = 1.01$. The vacancy-to-unemployment ratio (θ) increases with z as well. Similarly, unemployment rate is negatively correlated with the aggregate productivity shocks. Both equity prices and dividends are higher for higher z values. Note that quantitatively, the differences in u is very small compared to what we see in the business cycle data. This echoes with Shimer's (2005) finding that the Diamond-Mortensen-Pissarides model with linear utility generates very small unemployment and vacancy fluctuations. Figure 5 shows the wage functions for different values of z and assets. Wages move together with z.

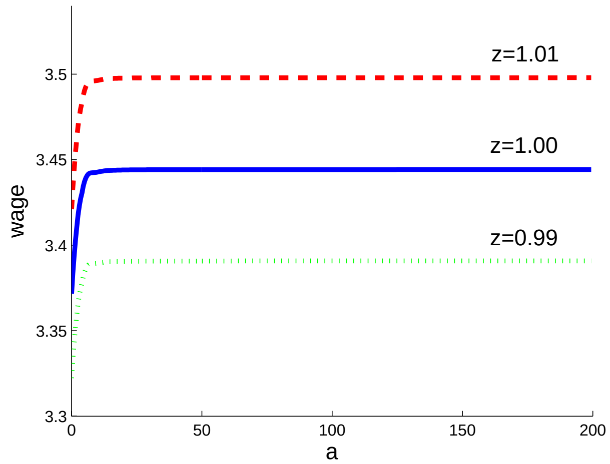

Figure 5: Wages for different z, Shimer calibration

We repeat the same exercise for the case of $\zeta = 5$. We re-calibrated ξ to make the free-entry condition hold with $\theta = 1$ when $z = 1.00$. This results in setting $\xi = 0.7447$. Table 3 summarizes the results. The response of the economy to the change in z is quite similar to that in the log utility case.

	u	v	θ	$\bar{k}$	p	d	w
$z = 0.99$	6.94%	-1.9%	-2.6%	-1.6%	-3.1%	-3.1%	-1.6%
$z = 1.01$	6.85%	$+2.1\%$	$+2.8\%$	$+1.6\%$	$+0.9\%$	$+0.9\%$	$+1.6\%$

Table 3: Different z: $\zeta = 5$ case. All in % deviations from $z = 1.00$ case, except for u.

2.9.3 An alternative calibration: Hagedorn-Manovskii (2006)

Hagedorn and Manovskii (2006) (HM henceforth) suggests that when the parameter values are calibrated differently from Shimer (2005), the Diamond-Mortensen-Pissarides model exhibits much larger labor market fluctuations in response to changes in z. HM demonstrated this in a model with linear utility. In this section, we re-calibrate our model similarly to HM and check whether their results hold in our model.

We change the values of three parameters. The home production (unemployment insurance) h and the Nash bargaining parameter γ are set as following. The home production parameter is raised to $h = 3.33$, which is about 98% of the equilibrium average wage. The Nash bargaining parameter is lowered to $\gamma = 0.05$. As a result of these changes, the worker takes about 97% of $zF(\tilde{k}) - r\tilde{k}$ in equilibrium. After these parameters are changed, we set ξ so that the free entry condition $V = 0$ holds with $\theta = 1$ when $z = 1.00$. This results in setting $\xi = 1.255$.

Table 4 shows the response of aggregate variables when z is changed.

	u	v	θ	$\bar{k}$	p	d	w
$z = 0.99$	7.77%	−28.8%	−36.7%	−2.5%	−28.9%	−28.9%	−0.8%
$z = 1.01$	6.32%	+27.7%	+39.3%	+2.2%	+27.3%	+27.3%	+0.8%

Table 4: Different z, HM Calibration. All in % deviations from $z = 1.00$ case, except for u.

As in the HM paper (where they used a linear utility function), the response of unemployment and vacancy is much larger with this calibration, and now comparable to the business cycle data. The profit of the firm changes more with z, and this leads to the larger change in θ and therefore v, resulting in a large response in u. When u changes substantially, the marginal product of capital changes and as a result, $\bar{k}$ also changes.

Figure 6 shows the change in wages.

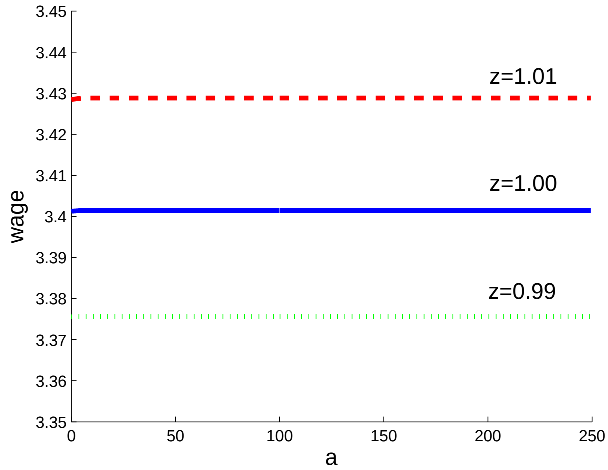

Figure 6: Wages, HM Calibration

First, note that the wage functions (as functions of a) are flatter than the benchmark wage functions. Second, here the wage responds somewhat less to the change in z, compared to the benchmark case. A smaller response in the wage implies a larger response in the firm's profit, contributing to a larger change in θ.

To analyze which element is important in generating large sensitivity of vacancy and unemployment in z, we repeat the same experiment for two additional cases.[10] First, h is kept at 3.33 and changed γ to the Shimer parameter value, 0.72. The result is in Table 5.

	u	v	θ	$\bar{k}$	p	d	w
$z = 0.99$	7.63%	−25.0%	−32.2%	−2.3%	−25.9%	−25.9%	−1.5%
$z = 1.01$	6.41%	+23.2%	+32.6%	+2.1%	+22.7%	+22.7%	+1.5%

Table 5: Different z: Effect of higher γ ($h = 3.33$ and $\gamma = 0.72$). All in % deviations from $z = 1.00$ case, except for u.

We can see that the response of u and v are a little smaller but quite similar to the HM calibration case. Thus, the calibration of γ is not crucial to the large response of u and v to the change in

[10]Again, ξ is re-calibrated in each experiment, so that $V = 0$ with $\theta = 1$ when $z = 1.00$. The value of ξ is 0.06077 in Table 5 and 15.207 in Table 6.

z. This is true despite a large change in the sensitivity of the wage to aggregate productivity: the wage responds much more now, quite like under the Shimer calibration.

Second, we kept $\gamma = 0.05$ and lowered h to 1.40. Table 6 shows the result.

	u	v	θ	$\bar{k}$	p	d	w
$z = 0.99$	6.95%	−2.3%	−3.1%	−1.6%	−2.4%	−2.4%	−1.2%
$z = 1.01$	6.84%	+2.3%	+3.2%	+1.6%	+2.4%	+2.4%	+1.2%

Table 6: Different z: Effect of lower h ($h = 1.40$ and $\gamma = 0.05$). All in % deviations from $z = 1.00$ case, except for u.

Even though the change in h is slight, the responses of u and v to z are muted substantially. This demonstrates that the value of h is crucial in determining the response of u and v to the change in z.

2.9.4 Comparison to the linear utility model

In this section, we compare our result to the linear Diamond-Mortensen-Pissarides model with a linear utility function. In Appendix C, we show that

$$\frac{y-h}{r+\sigma+\gamma\chi\theta^{1-\eta}} = \frac{\xi}{(1-\gamma)\chi\theta^{-\eta}}. \tag{16}$$

where y is defined as

$$y = \arg\max_{\tilde{k}} \; z\tilde{k}^{\alpha} - r\tilde{k}$$

that is,

$$y = z\left(\frac{r}{\alpha z}\right)^{\frac{\alpha}{\alpha-1}} - r\left(\frac{r}{\alpha z}\right)^{\frac{1}{\alpha-1}} \tag{17}$$

since

$$\tilde{k} = \left(\frac{r}{\alpha z}\right)^{\frac{1}{\alpha-1}}. \tag{18}$$

Our experiment is the following. Set all the parameters except for ξ the same as in our original models (with the benchmark Shimer calibration and the HM calibration). First, let $z = 1.00$ and set ξ so that (16) holds with $\theta = 1$.[11] Note that y is defined in (17) and $r = 1/\beta - 1 + \delta$. The

[11]This results in $\xi = 0.7183$ in the benchmark calibration and $\xi = 1.1005$ in the HM calibration.

steady-state condition for u

$$u = \frac{\sigma}{\sigma + \chi\theta^{1-\eta}} \tag{19}$$

can be solved for u, from $\theta = 1$. From (18) and $\tilde{k} = \bar{k}/(1-u)$, we can obtain the aggregate capital $\bar{k}$:

$$\frac{\bar{k}}{1-u} = \left(\frac{r}{\alpha z}\right)^{\frac{1}{\alpha-1}}. \tag{20}$$

Now, we change z. (16) can be solved for θ (note that the value of y is different for each z), then (19) can be solved for u, and finally (20) can be solved for $\bar{k}$. Table 7 summarizes the result.

		y	u	v	θ	$\bar{k}$
Shimer	$z = 0.99$-Linear	-1.6%	6.95%	-2.0%	-2.7%	-1.6%
	$z = 0.99$-Incomplete	-1.6%	6.95%	-2.0%	-2.7%	-1.6%
	$z = 1.01$-Linear	$+1.6\%$	6.85%	$+2.0\%$	$+2.7\%$	$+1.6\%$
	$z = 1.01$-Incomplete	$+1.6\%$	6.85%	$+2.0\%$	$+2.7\%$	$+1.6\%$
HM calibration	$z = 0.99$-Linear	-1.6%	7.78%	-29.2%	-37.2%	-2.5%
	$z = 0.99$-Incomplete	-2.2%	7.77%	-28.8%	-36.7%	-2.5%
	$z = 1.01$-Linear	$+1.6\%$	6.32%	$+28.2\%$	$+40.0\%$	$+2.2\%$
	$z = 1.01$-Incomplete	$+2.0\%$	6.32%	$+27.7\%$	$+39.3\%$	$+2.2\%$

Table 7: Summary statistics for the linear model and for the incomplete markets model. All in % deviations from $z = 1.00$ case, except for u.

The results are remarkably similar to those in our original incomplete market model with log utility (reproduced here from Tables 2 and 4).

3 The model with aggregate shocks

We now incorporate aggregate uncertainty: aggregate productivity shocks. There are two broad reasons for introducing aggregate shocks. One is substantive: the steady-state analysis above only gives an indication of aggregate dynamics, since a model with capital and consumption smoothing implies that labor-market tightness will have nontrivial short-run dynamics. Indeed, as indicated above, the model will generate stochastic processes for aggregates that, at least under some parameter configurations, are broadly in line with available U.S. data. The second reason for looking at the model with aggregate shocks is methodological: it provides a "how-to" for what we believe is a range of potentially very interesting applications. There are two methodological issues of importance: the evaluation of firm profits (how are they priced, and how are dynamic firm decisions made?) and the numerical implementation, which is significantly more involved than that in Krusell and Smith (1998, 1997).

Like in Krusell and Smith (1998), we assume that the aggregate productivity z is either good ($z = g$) or bad ($z = b$), with $g > b$, and follows a first-order Markov process, with the probability of moving from state z to state z' denoted $\pi_{zz'}$. Unlike in Krusell and Smith, however, we do not need to make additional assumptions about individual employment shocks (and their correlation with aggregates) since they are endogenous here.

3.1 Asset structure

Again, we assume that there are no insurance markets for the idiosyncratic shocks. Agents can hold only two kinds of asset—capital k and equity x. Now, the difference from the previous chapter is that capital and equity have different return structures, so the consumers face a portfolio choice problem.

Let S be the joint distribution of asset and employment across the consumers. Then the aggregate state at any given period can be described by (z, S).

The distribution of assets across the consumers in the next period is determined in the current period, since it depends on the consumers' asset accumulation and portfolio choice decisions, which are governed by the current state (z, S). The *distribution* of the employment states at the aggregate level in the next period is determined in the current period by the amount of vacancy (the firm's decision) and unemployment, which are also governed by (z, S). Thus, the next period state S' is determined by (z, S). Let's write this dependence as $S' = \Omega(z, S)$. It is important to note that even though S' is already determined by the current state, the employment state of each individual in the next period is still uncertain.

By above argument, there are only two uncertain *aggregate states* in the next period, that is, (g, S') and (b, S'). We can span these states by two "aggregate" assets that we have—capital and equity. Note that the asset markets are still incomplete, since there are no assets to insure the idiosyncratic risk.

Below, when we consider the consumer's decisions, we will work on the portfolio choice between two "Arrow securities"—securities that provides one unit of consumption good at each aggregate states.[12] This is without loss of generality, since we can create these securities by combining the capital and equity. For the ease of exposition, below we introduce an entity, called an "investment firm", who conducts this transformation.

Suppose that the current state is (z, S). Let $Q_{z'}(z, S)$ be the price of an Arrow-security that pays out one unit of consumption good when the next-period state is z'. Let the interest rate $r(z, S)$ and the equity price $p(z, S)$. Then, from no-arbitrage, the asset prices have to satisfy

$$Q_g(z, S)(1 - \delta + r(g, S')) + Q_b(z, S)(1 - \delta + r(b, S')) = 1 \tag{21}$$

and

$$p(z, S) = Q_g(z, S)[p(g, S') + d(g, S')] + Q_b(z, S)[p(b, S') + d(b, S')] \tag{22}$$

where $d(z, S)$ is the dividend. (21) and (22) show that there is a one-to-one mapping between $\{r(z, S), p(z, S)\}$ and $\{Q_g(z, S), Q_b(z, S)\}$ for a given $\{d(z, S)\}$.

3.2 Consumers

Consumers in the economy choose their demand for Arrow securities subject to a budget constraint. Again, we impose an exogenous borrowing constraint for each contingent claim at $\underline{a}$. Consumers in the economy differ in their employment status and asset holdings.

3.2.1 Employed consumers

Let $a'_{z'}$ be the demand of an Arrow security that pays out one unit of consumption good in the next period if the next period state is z'. The budget constraint for an employed consumer is

$$c + Q_g(z, S)a'_g + Q_b(z, S)a'_b = a + w$$

[12]Krusell and Smith (1997) also have two assets but do not use contingent claims in the implementation.

where c is consumption and w is the wage rate.

Let $\tilde{W}(w, a; z, S)$ be the value of being an employed consumer, given the wage w. An employed worker's optimization problem is

$$W(a; z, S) = \max_{a'_g \geq \underline{a}, a'_b \geq \underline{a}} \quad u(c) + \beta[\quad \pi_{zg}(\sigma U(a'_g; g, S') + (1-\sigma)W(a'_g; g, S')) \\ +\pi_{zb}(\sigma U(a'_b; b, S') + (1-\sigma)W(a'_b; b, S'))] \tag{23}$$

subject to

$$c + Q_g(z, S)a'_g + Q_b(z, S)a'_b = a + w$$

and

$$S' = \Omega(z, S).$$

Let the decision rule for $a'_{z'}$ be $\tilde{\psi}_e^{z'}(w, a; z, S)$. Here, $U(a; z, S)$ is the value of being an unemployed consumer with asset holding a and $W(a; z, S)$ is the value of being an employed consumer, taking into account that the wage depends on a and (z, S) through Nash bargaining. More formally, denoting the wage function as

$$w = \omega(a; z, S),$$

$W(a; z, S)$ is defined as

$$W(a; z, S) = \tilde{W}(\omega(a; z, S), a; z, S) \tag{24}$$

and we define the decision rule $\psi_e(a; z, S)$ as

$$\psi_e(a; z, S) = \tilde{\psi}_e(\omega(a; z, S), a; z, S). \tag{25}$$

3.2.2 Unemployed consumers

The budget constraint for the unemployed consumers is

$$c + Q_g(z, S)a'_g + Q_b(z, S)a'_b = a + h$$

where h is the income that the consumer receives when she is unemployed.

The unemployed worker's optimization problem is

$$U(a; z, S) = \max_{a'_g \geq \underline{a}, a'_b \geq \underline{a}} \quad u(c) + \beta[\quad \pi_{zg}((1-\lambda_w(z, S))U(a'_g; g, S') + \lambda_w(z, S)W(a'_g; g, S')) \\ +\pi_{zb}((1-\lambda_w(z, S))U(a'_b; b, S') + \lambda_w(z, S)W(a'_b; b, S'))] \tag{26}$$

subject to

$$c + Q_g(z,S)a'_g + Q_b(z,S)a'_b = a + h$$

and

$$S' = \Omega(z,S).$$

The job-finding probability $\lambda_w(z,S)$ is defined by (2). Let the decision rule for $a'_{z'}$ be $\psi_u^{z'}(a;z,S)$.

3.3 Firms

Again, we consider an representative firm which holds many jobs. In order to be able to get matched with a worker and produce, the firm post a vacancy. Let the vacancy cost be ξ. The value of a vacancy $V(z,S)$ is

$$\begin{aligned} V(z,S) = -\xi \quad & +Q_g(z,S)((1-\lambda_f(z,S'))V(g,S') + \lambda_f(z,S') \textstyle\int J(\psi_u^g(a;z,S);g,S')[f_u(a;S)/u]da) \\ & +Q_b(z,S)((1-\lambda_f(z,S))V(b,S') + \lambda_f(z,S) \textstyle\int J(\psi_u^b(a;z,S);b,S')[f_u(a;S)/u]da) \end{aligned} \tag{27}$$

where $J(a;z,S)$ is the value of a matched job (taking into account the wage bargaining) and $f_u(a;S)$ is the population of unemployed workers with asset a. The worker-finding probability $\lambda_f(z,S)$ is defined by (1). The firm will post vacancies $v(z,S)$ until $V(z,S)=0$.

Let us consider a job matched with a worker with asset holdings a. The firm rents capital from the consumers at a rental rate of $r(z,S)$ and pays the worker a wage of $\omega(a;z,S)$. The output is determined by the production function $zF(k)$. The value of a filled job given the wage w, $\tilde{J}(w,a;z,S)$, is

$$\begin{aligned} \tilde{J}(w,a;z,S) = \tilde{\pi}(w;z,S) \quad & +Q_g(z,S)(\sigma V(g,S') + (1-\sigma)J(\tilde{\psi}_e^g(w,a;z,S);g,S')) \\ & +Q_b(z,S)(\sigma V(b,S') + (1-\sigma)J(\tilde{\psi}_e^b(w,a;z,S);b,S')), \end{aligned} \tag{28}$$

where the flow profit $\tilde{\pi}(w;z,S)$ is defined as

$$\tilde{\pi}(w;z,S) = \max_k zF(k) - r(z,S)k - w. \tag{29}$$

From the first-order condition,

$$r(z,S) = zF'(k)$$

holds. In equilibrium, the capital stock per job is $\tilde{k} = \bar{k}/(1-u)$, where $\bar{k}$ is the aggregate capital stock. Thus, the equilibrium profit is

$$\pi(a; z, S) = zF(\tilde{k}) - r(z, S)\tilde{k} - \omega(a; z, S).$$

$\tilde{J}$ and J are related by

$$J(a; z, S) = \tilde{J}(\omega(a; z, S), a; z, S). \tag{30}$$

The dividend is determined by aggregating the profits of all matched firms in the economy as

$$d(z, S) = \int \pi(a; z, S) f_e(a; S) da - \xi v, \tag{31}$$

where $f_e(a; S)$ is the population of employed workers with asset a. Note that since wages paid by firms depend on the asset positions of the workers that they are matched with, dividends depend on the wealth distribution of the employed consumers in the economy.

3.4 Wage determination

Vacant jobs and unemployed workers are randomly matched each period according to the aggregate matching function $M(u, v)$, which is identical to the one we defined in the previous section. A realized match produces some pure economic rent that is shared by the firm and the worker through Nash bargaining. The wage that the firm pays a worker with asset holdings a is determined by

$$\max_{w} \left(\tilde{W}(w, a; z, S) - U(a; z, S)\right)^{\gamma} \left(\tilde{J}(w, a; z, S) - V(z, S)\right)^{1-\gamma}. \tag{32}$$

3.5 Investment firms

There are competitive investment firms who sell contingency claims to consumers by rearranging capital and equities. For the asset market equilibrium, the following has to hold for each z'.

$$\int \psi_e^{z'}(a; z, S) f_e(a; S) da + \int \psi_u^{z'}(a; z, S) f_u(a; S) da = (1 - \delta + r(z', S'))\bar{k}' + p(z', S') + d(z', S'). \tag{33}$$

Note that for the asset prices (21) and (22) have to hold.

3.6 Recursive equilibrium

The following defines the recursive equilibrium of the model. Appendix C shows that when the below conditions are satisfied, the resource balance condition (goods market clearing condition) is satisfied.

Definition 2 (Recursive equilibrium) *The recursive equilibrium consists of a set of value functions* $\{\tilde{W}(w,a;z,S),\ \tilde{J}(w,a;z,S),\ W(a;z,S),\ J(a;z,S),\ U(a;z,S),\ V(z,S)\}$*, a set of decision rules for asset holdings* $\{\tilde{\psi}_e^{z'}(w,a;z,S),\ \psi_e^{z'}(a;z,S),\ \psi_u^{z'}(a;z,S)\}$*, prices* $\{r(z,S),\ p(z,S),\ Q_g(z,S),\ Q_b(z,S),\ \omega(a;z,S)\}$*, vacancy* $v(z,S)$*, matching probabilities* $\lambda_f(z,S)$ *and* $\lambda_w(z,S)$*, dividend* $d(z,S)$*, and a law of motion for the distribution,* $S' = \Omega(z,S)$*, which satisfy*

1. *Consumer optimization:*

   Given the aggregate states, $\{z,S\}$*, job-finding probability* $\lambda_w(z,S)$*, prices* $\{r(z,S),\ p(z,S),\ Q_g(z,S),\ Q_b(z,S)\}$*, wage* w*, and the law of motion for the distribution,* $S' = \Omega(z,S)$*; the individual decision rules* $\tilde{\psi}_e^{z'}(w,a;z,S)$ *and* $\psi_u^{z'}(a;z,S)$ *solve the optimization problems (23) and (26), with the value functions* $\tilde{W}(w,a;z,S)$ *and* $U(a;z,S)$*. Given the wage function* $\omega(a;z,S)$*,* $W(a;z,S)$ *and* $\psi_e^{z'}(a;z,S)$ *satisfy (24) and (25).*

2. *Firm optimization:*

   Given the aggregate states, $\{z,S\}$*, prices* $\{r(z,S),\ Q_g(z,S),\ Q_b(z,S)\}$*, wage* w*, the law of motion for the distribution,* $S' = \Omega(z,S)$*, and the employed consumer's decision rule* $\tilde{\psi}_e^{z'}(w,a;z,S)$*, the firm solves the optimization problem (28) with (29), with the value functions* $\tilde{J}(w,a;z,S)$*. Given the wage function* $\omega(a;z,S)$*,* $J(a;z,S)$ *satisfies (30). Given the aggregate states,* $\{z,S\}$*, the worker-finding rate* $\lambda_f(z,S)$*, prices* $Q_g(z,S)$ *and* $Q_b(z,S)$*, and the unemployed consumer's decision rule* $\psi_u^{z'}(a;z,S)$*,* $V(z,S)$ *satisfies (27).*

3. *Free entry:*

   The number of vacancy posted, $v(z,S)$*, is consistent with the firm free-entry:* $V(z,S) = 0$*.*

4. *Asset markets clear:*

   The asset-market equilibrium condition (33) holds for each z' and the asset prices satisfy (21) and (22). Dividends $d(z,S)$ are given by (31). $\bar{k}$ satisfies $\tilde{k} = \bar{k}/(1-u)$, where $\tilde{k}$ satisfies the firm's first-order condition $r(z,S) = zF'(\tilde{k})$ for given $r(z,S)$.

5. *Matching:*

   $\lambda_f(z,S)$ and $\lambda_w(z,S)$ are functions of $v(z,S)$ and u as in (1) and (2).

6. *Nash bargaining:*

   The wage function $\omega(a;z,S)$ determined through Nash bargaining between the firms and the consumers by solving (32).

7. *Consistency:*

   The transition function $\Omega(z,S)$ is consistent with $\lambda_w(z,S)$, σ, and the consumer's decision rules.

3.7 Calibration

We follow the same calibration as in the previous section. Aggregate shocks take the values $z \in \{b,g\} = \{0.99, 1.01\}$. π_{ij} is the probability of the transition from state i to state j. Following Krusell and Smith (1999, 2002), we set the average business cycle duration to 2 years. Our model period is six weeks, therefore the average duration is 16 periods. From $1/(1-\pi_{bb}) = 1/(1-\pi_{gg}) = 16$, $\pi_{bb} = \pi_{gg} = 0.9375$.

3.8 Computation

The computation of our model is considerably more complex than standard incomplete markets models. In addition to the problems we faced in the model without aggregate shocks, now the distribution of asset and employment moves around endogenously over time, and the information of the distribution is necessary for computing the individual optimization problem.

This complexity is similar to that in Krusell and Smith (1998, 1997). In our setting, however, since unemployment is not summarized by an exogenously given stochastic process, the aggregate productivity shock z is no longer the sole determinant of the unemployment rate. As a consequence, we need more aggregate state variables than in previous settings.

Following Krusell and Smith (1998), we study a setting where consumer have boundedly rational perceptions of the evolution of the aggregate state, since we hope that "approximate aggregation" will hold. Thus, we assume that the consumer perceives next period's aggregate capital stock, $\bar{k}'$, to be a (log-)linear function of $(z, \bar{k}, u)$, where u is an additional state relative to the setting without labor-market frictions. Since there are other nontrivial market-clearing conditions, we also need to model the resulting outcomes—for θ, p, d, and Q_z—as simple (typically linear) functions of $(z, \bar{k}, u)$. After computing the equilibrium based on these assumptions and simulating the economy, we check whether these perceptions are accurate. This indeed turns out to be the case, as we will show below.

The main steps of our algorithm, using the $z = g$ case for illustration, are as follows.

1. Postulate the law of motion for $\bar{k}$ by assuming that $\bar{k}'$ is a function of $(z, \bar{k}, u)$.

2. Assume that θ, p, d, and Q_g are functions of $(z, \bar{k}, u)$ and guess on coefficients of prediction rules.

3. Calculate u' from $u' = (1 - \lambda_w(\theta))u + \sigma(1 - u)$.

4. Calculate Q_b by using Q_g from

$$Q_g(z, \bar{k}, u)(1 - \delta + r(g, \bar{k}', u')) + Q_b(z, \bar{k}, u)(1 - \delta + r(b, \bar{k}', u')) = 1.$$

5. Perform the individual optimization and Nash bargaining. We employed convergence criteria of 10^{-3} for decision rules for individual asset accumulation, 3×10^{-3} (about 0.1% of the average wage) for the Nash bargaining.

6. Simulate the economy using the result from the previous step. Generate data on $\bar{k}$, p, d, Q_g, θ. Check if it is consistent with the predictions in the first step. If not, revise the prediction

rules and continue until convergence.

The detailed algorithm that we use is described in Appendix D.

3.9 Results

The law of motion for capital stock is

$$\log \bar{k}' = 0.0788 + 0.9821 \log \bar{k} - 0.0016 \log u + 0.0428 \log z, \quad R^2 = 0.99999.$$

The "approximate aggregation" result seems to hold up very well in the present setting. The R^2 of this prediction rule is 0.99999, and we report additional accuracy checks in Appendix F (forecasting accuracy, as well as sensitivity to adding additional moments). The average aggregate capital is $\bar{k} = 104.80$ for $z = b$ and and $\bar{k} = 105.24$ for $z = g$.

The prediction rules for the other aggregate variables are

$$\log \theta = -2.8285 + 0.6122 \log \bar{k} + 0.0070 \log u + 1.4294 \log z, \quad R^2 = 0.99998,$$

$$\log(p + d) = -1.7439 + 0.3501 \log \bar{k} - 0.0484 \log u + 1.1504 \log z, \quad R^2 = 0.99984,$$

$$\log Q_g = 0.0122 + 1.0919 \log \tilde{Q}_g - 0.0013 \log \bar{k} - 0.000018 \log u, \quad R^2 = 0.99999,$$

$$\log Q_b = -0.0049 + 0.9687 \log \tilde{Q}_b + 0.0006 \log \bar{k} + 0.000006 \log u, \quad R^2 = 0.99999,$$

where $\tilde{Q}_g$ and $\tilde{Q}_b$ are functions of z, $\bar{k}$, and u (see Appendix D for details). Thus, almost all the variation in the left-hand-side variables $(\theta, p + d, Q_g, Q_b)$ can be explained by the predicted value in the right-hand side.[13]

Table 8 summarizes the statistics for each state from our simulations.

	u	v	θ	$\bar{k}$	p	d
$z = b$	6.92%	−1.3%	−1.7%	−0.2%	−1.2%	−30.2%
$z = g$	6.88%	+1.1%	+1.4%	+0.2%	+1.0%	+25.3%

Table 8: Summary statistics for the model with aggregate shocks.

[13]We notice a slightly lower R^2 for the asset price; the forecasting accuracy for this variable is as good as for θ.

All the values are shown as % deviation of the average value in each state from the total average, except for u. The average value for each state is shown for u. Average aggregate capital is $\bar{k} = 104.80$ for $z = b$ and and $\bar{k} = 105.24$ for $z = g$. The vacancy-to-unemployment ratio (θ) increases with z, but the fluctuations in θ are not realistic in magnitude; this is the fact discussed in Shimer (2006) and Hall (2006). Similarly, the unemployment rate is negatively correlated with the aggregate productivity shock, but the magnitude of the fluctuations is much below what we observe in the data. Equity prices are higher on average for $z = g$ states, although the magnitude of these fluctuations is small.

Figure 7 shows a sample path for aggregate capital obtained from simulating our model.

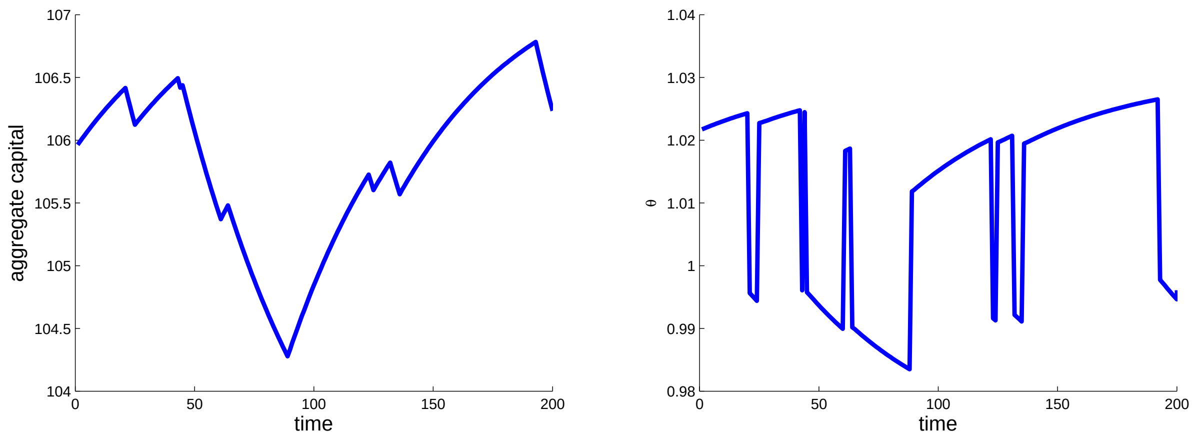

Figure 7: Sample paths of $\bar{k}$ and θ, Shimer calibration

$\bar{k}$ increases when $z = g$ and decreases when $z = b$, and θ is not constant over time conditional on an aggregate state: as the capital stock adjusts, the profitability of firm entry change. For example, if z moves from b to g and there is no immediate switch back, θ jumps, and the capital stock will start increasing, and as it increases, θ will keep increasing further.[14] The reason is that more capital availability will make capital cheaper to rent.

Figure 8 (left panel) shows a sample path for the unemployment rate and vacancy rate obtained

[14]The asset price follows a sample path very similar to that of θ, since it too is a jump variable.

from simulating our model. When the aggregate state switches from $z = b$ to $z = g$, θ jumps up. For a given u, this means a large increase in v. This will make u go down significantly in the following period. If the aggregate state is still g in the following period, θ will remain high (and even increase somewhat), but since u is now lower, v must fall as well (but remain higher than prior to the z switch). Subsequently, if z continues to be g, v and u will keep moving, and in opposite directions, since a rising θ reflects higher entry and a lower jobless rate.[15] When z switches from g to b, we see the opposite pattern. Note that the comparative steady states in the previous section describe the situation after the adjustment of k (and u) was completed, that is, after z has continued to be at the same state for a long time. The results in this section suggests that this adjustment is fairly slow. This explains why the unemployment fluctuations here are not as large as those we saw across steady states with different productivities.

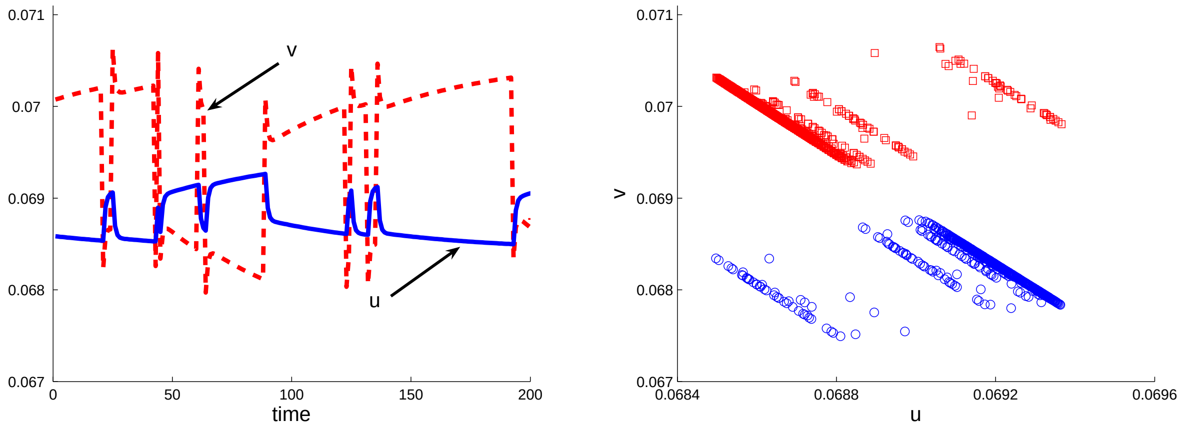

Figure 8: Sample paths of u and v (left panel) and unemployment rate (u), vacancy rate (v) plot (right panel), Shimer calibration. On the right panel, u and v values for $z = b$ are plotted as "circles" and u and v values for $z = g$ are plotted as "squares".

Figure 8 (right panel) shows combinations of the unemployment rates (u) and vacancy rates

[15] A decrease in u has the opposite effect, since the capital stock per job is $\tilde{k} = \bar{k}/(1-u)$, but this effect turns out to be smaller than the profitability effect.

(v) obtained from simulating our model for 2000 periods.[16] Most of the points in the figure are on either of the two "thick", circled (low-v, or $z = b$) or squared (high-v, or $z = g$), lines; they correspond to the paths u and v travel once the economy has been in one of the z states for more than one consecutive periods. On these thick lines, as time passes without a new z switch, u and v move in opposite directions, as θ adjusts. This can be compared with the θ movements in a model without capital and concave utility: there, θ can only take on as many values as there are values for z, and so will capital (if there is capital in the model); here, because of consumption smoothing, capital must move slowly, as must θ, within each aggregate state. The other lines in the figure are points the economy reaches in the period of a switch.[17]

3.10 Alternative calibration: Hagedorn and Manovskii (2006)

In this section, we examine the alternative calibration of the model by Hagedorn and Manovskii (HM). As in the model without aggregate shocks, we change three parameters: $h = 3.33$, $\gamma = 0.05$, and $\xi = 1.255$.

Approximate aggregation seems to obtain here as well for the main variables, though there are larger errors for θ and for the stock price (the additional robustness checks in Appendix F reveal similar findings). The law of motion for the capital stock is

$$\log \bar{k}' = 0.0901 + 0.9801 \log \bar{k} - 0.0010 \log u + 0.0383 \log z, \quad R^2 = 0.99999.$$

The prediction rules for the other aggregate variables are

$$\log \theta = -30.1104 + 6.4682 \log \bar{k} - 0.0077 \log u + 10.4304 \log z, \quad R^2 = 0.99912,$$

$$\log(p + d) = -26.5465 + 5.7540 \log \bar{k} - 0.0247 \log u + 10.9716 \log z, \quad R^2 = 0.99982,$$

$$\log Q_g = -0.01237 + 0.8831 \log \tilde{Q}_g + 0.0009 \log \bar{k} + 0.000021 \log u, \quad R^2 = 0.99999,$$

[16] The first 500 periods are discarded.

[17] The north-eastern-most and the south-western-most lines are lines the economy reaches after one switch, and the other lines (with a similar thickness to these lines) reflect the adjustment after the switch. The other scattered dots are the cases where a switch occurs during the adjustment.

and

$$\log Q_b = -0.0069 + 0.9486 \log \tilde{Q}_b + 0.0007 \log \bar{k} + 0.000006 \log u, \quad R^2 = 0.99999,$$

where $\tilde{Q}_g$ and $\tilde{Q}_b$ are functions of z, $\bar{k}$, and u (see Appendix D for details).

Table 9 summarizes the statistics for each state from the simulations.

	u	v	θ	$\bar{k}$	p	d
$z = b$	7.06%	-9.9%	-12.5%	-0.2%	-11.8%	-169.4%
$z = g$	6.71%	$+8.3\%$	$+10.5\%$	$+0.2\%$	$+9.9\%$	$+141.6\%$

Table 9: Summary statistics for the model with aggregate shocks, HM calibration

We can see that the cyclical properties of the variables are qualitatively the same as the benchmark calibration, but quantitatively much larger. The response of each variables to z are smaller than the comparative statics of the model without aggregate shocks (except for d). d tends to be very volatile since the vacancy posting behavior does not change much across states. In particular, the level of d can become negative when $z = b$.

Figure 9 illustrates the behavior of unemployment and vacancies.

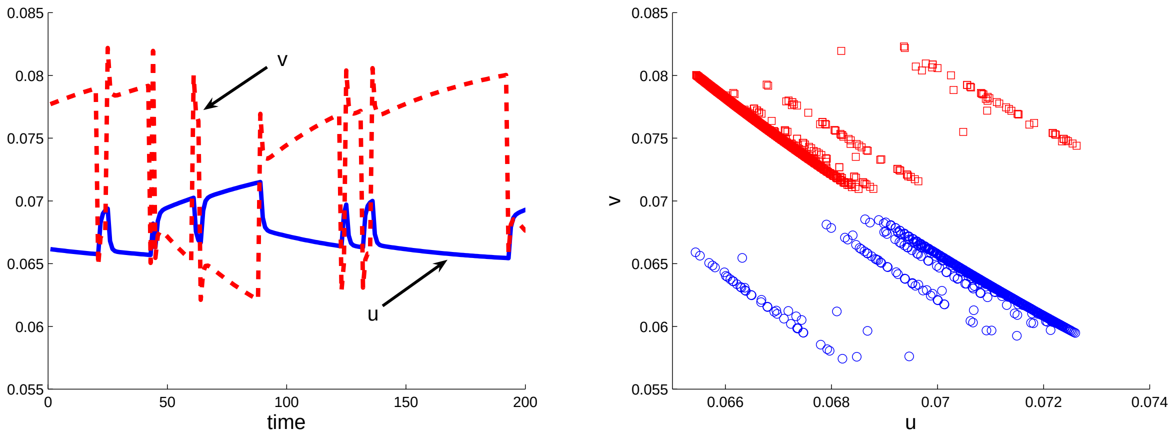

Figure 9: Sample paths of u and v (left panel) and unemployment rate (u), vacancy rate (v) plot (right panel), HM calibration. On the right panel, u and v values for $z = b$ are plotted as "circles" and u and v values for $z = g$ are plotted as "squares".

The qualitative properties of these paths are very similar to those for the benchmark (Shimer) calibration, but the magnitude of the fluctuations is much larger.

3.11 Cyclical fluctuations and the model

In this subsection, we evaluate the model's performance to match the fluctuations in aggregate variables. The standard deviation of output is 0.0069 for the Shimer calibration and 0.0075 for the HM calibration. In the U.S. economy, the unemployment rate and vacancies are negatively correlated. Shimer (2005) reports that the correlation of HP-filtered unemployment rate and vacancies is -0.894 for U.S. data.[18] In our model with Shimer calibration, the correlation coefficient of the cyclical components of unemployment rate and vacancies is -0.862 in the HM calibration this statistic is -0.863. Figure 10 shows the HP-filtered Beveridge curves for both calibration exercises.

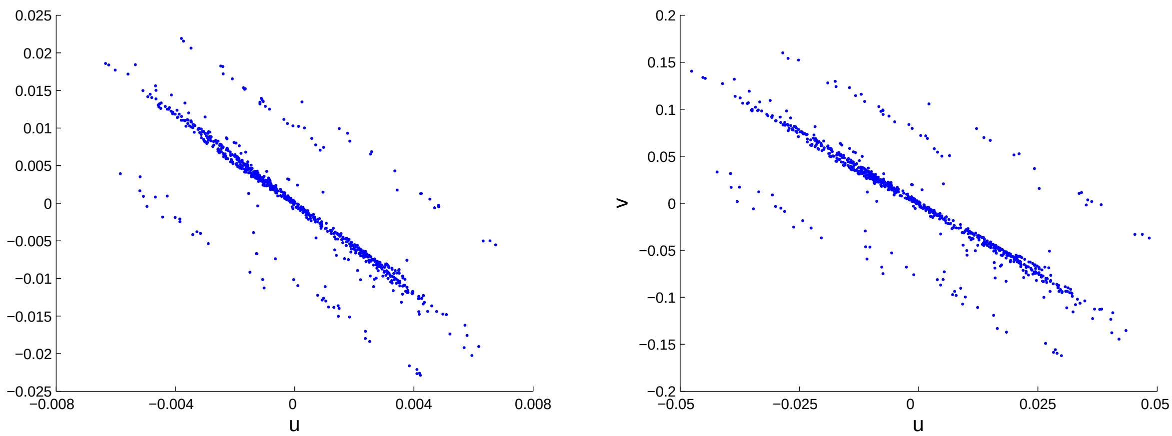

Figure 10: Beveridge curves for the Shimer calibration (left panel) and the HM calibration (right panel).

In the U.S. economy, the labor share is countercyclical. Ríos-Rull and Santaeulàlia-Llopis (2007) reports a correlation coefficient of -0.24 between the labor share and the output and Andolfatto (1996) reports this number as -0.38. Our model generates countercyclical labor share with a

[18]Shimer (2005) uses 10^5 as the smoothing parameter.

correlation of -0.80 with the Shimer calibration and -0.94 with the HM calibration.

Table 10 shows the standard deviation of investment, consumption, labor share, wage, stock prices, dividend, and vacancy-unemployment ratio relative to the standard deviation of output for the data and our model (both Shimer calibration and HM calibration). (The details of the calculation of the reported statistics can be found in Appendix G.)

	U.S. economy	Model: Shimer	Model: HM
Investment	3.14	3.51	2.93
Consumption	0.56	0.18	0.13
Labor share	0.43	0.03	0.51
Wage	0.44	0.96	0.38
Stock price	6.41	1.06	9.47
Dividend	1.81	28.62	161.05
Vacancy-unemployment ratio	16.27	1.46	9.91

Table 10: Standard deviation of detrended series divided by the standard deviation of output. All variables are logged and HP-filtered. Note that standard deviation of output is 0.0158 for the U.S. data, 0.0069 for the Shimer calibration and 0.0075 for the HM calibration.

Our benchmark (Shimer calibration) result produces less output fluctuations and much lower fluctuations in the vacancy-unemployment ratio. This type of finding has been documented by Andolfatto (1996), Hall (2005), and Shimer (2005), among others. They find that the standard Diamond-Mortensen-Pissarides model cannot generate the observed fluctuations in unemployment and vacancies. Our finding is similar—our results suggest that that market incompleteness does not improve the performance of the search-matching model in matching the magnitude of unemployment fluctuations. Since the benchmark result falls short in generating fluctuations in unemployment, it also cannot match the fluctuations in output.[19] The HM calibration generates much larger fluctuations in unemployment and vacancies, and the result comes closer to the actual data.

Table 10 also reports that our benchmark model underpredicts the fluctuations in stock prices.

[19]We calibrate the aggregate productivity shocks following Krusell and Smith (1998). In their calibration, unemployment is determined by an exogenously given Markov process which is calibrated to match the fluctuations in the data. Their calibration can produce realistic fluctuations in output, since there are significant fluctuations coming from both productivity and employment.

That is, it is also subject to the stock-market volatility puzzle. The HM result generates much larger fluctuations in stock prices. Dividends, however, fluctuate much more than in the data, and the puzzle of the relative volatility between stock prices and dividends remains unsolved.[20]

4 Conclusion

We constructed and solved a Bewley-Huggett-Aiyagari incomplete-markets model with labor-market frictions and aggregate shocks. We find that approximate aggregation holds, making model solution feasible despite a number of new elements relative to earlier models with aggregate shocks and nontrivially varying wealth distributions.

Our positive findings are that for all the parameter configurations considered, labor-market aggregates behave almost as in the linear utility model counterparts, and that the consumption and investment fluctuations behave as in the typical representative-agent real-business-cycle model. If we adopt Hagedorn and Manovskii's (2006) view, that is, we set the monetary value of unemployment to be very high, then the resulting setting has significant fluctuations in unemployment and vacancies, and the other aggregate variables behave in realistic ways as well (e.g., the labor share is countercyclical and real wages fluctuate relatively little). If, in contrast, we adopt the calibrations in Hall (2005) or Shimer (2005), the fluctuations in unemployment and vacancies are significantly lower than in the data.

One dimension in which the present model is not empirically satisfactory is in the dispersion of wealth and wages. Wages differ only because workers' bargaining power depend on their asset holdings; moreover, the effect of wealth on wages is very slight for most asset levels. Wealth, therefore, differs among workers only due to past employment luck. Since many elements of heterogeneity are left out—worker ability, match quality, preferences, etc.—it is an open question as to what our

[20] In our framework, the two volatility puzzles (those for the stock-market volatility and for labor market fluctuations) seem closely related. As can be seen from the results based on the HM calibration, if we can generate fluctuations in stock prices of a realistic magnitude, we will be able to generate more realistic labor-market fluctuations. This is because the stock price reflects future profits ($p + d$ is the sum of J in the economy), and future profits drive the firm's vacancy-posting decision (v increases as J increases). The HM calibration achieves this, while it overpredicts the volatility of the dividends.

target wealth dispersion should be. Future versions of the present setting ought to incorporate these elements, and it is an open question whether approximate aggregation will continue to hold then, and how the positive findings for aggregates that we obtain here will change.

Our model can serve as a framework for analyzing various stabilization and social-insurance policies, but we do not consider such studies here. It is, we think, another very promising avenue for future research.

Appendix

A Consistency in the valuation of the firm

This appendix establishes that the valuation of the firm is consistent between the individual job level and the aggregate level (equity price).

- Aggregate level (equity price):

$$\tilde{p} = d + q\tilde{p} \tag{34}$$

holds, where

$$d = \int \pi(a) f_e(a) da - \xi v$$

is dividend (same as the text) and q is the discount factor for the firm.

Note that $\tilde{p}$ is different from p in the text. $\tilde{p}$ is the value of the firm *before* the dividend is paid, and p is the value of the firm *after* the dividend is paid. They are related by $\tilde{p} = p + d$. In fact, (34) implies

$$p + d = d + q(p + d),$$

therefore $p = q(p + d)$. From (3), the discount factor q is equal to $1/(1 + r - \delta)$.

- Individual job level:

Define $J(a)$ and V with

$$J(a) = \pi(a) + q(\sigma V + (1 - \sigma) J(\psi_e(a))) \tag{35}$$

and

$$V = -\xi + q\left[(1 - \lambda_f)V + \lambda_f \int J(\psi_u(a)) \frac{f_u(a)}{u} da\right] \tag{36}$$

and

$$\tilde{p} = \int J(a) f_e(a) da. \tag{37}$$

Note that $V = 0$ in equilibrium.

Note also that in steady-state, assuming that $\psi_u(a)$ and $\psi_e(a)$ are increasing,

$$\int_{\underline{a}}^{a} f_e(a')da' = \lambda_w \int_{\underline{a}}^{\psi_u^{-1}(a)} f_u(a')da' + (1-\sigma)\int_{\underline{a}}^{\psi_e^{-1}(a)} f_e(a')da'$$

holds. Differentiating with respect to a, using Leibniz's rule,

$$f_e(a) = \lambda_w f_u(\psi_u^{-1}(a))\gamma_u(a) + (1-\sigma)f_e(\psi_u^{-1}(a))\gamma_e(a) \tag{38}$$

where

$$\gamma_u(a) = \frac{d\psi_u^{-1}(a)}{da} = \frac{1}{\psi_u'(\psi_u^{-1}(a))} \tag{39}$$

and

$$\gamma_e(a) = \frac{d\psi_e^{-1}(a)}{da} = \frac{1}{\psi_e'(\psi_e^{-1}(a))} \tag{40}$$

Plugging (35) into (37), we obtain

$$\tilde{p} = \int \pi(a)da + q\int (1-\sigma)J(\psi_e(a))f_e(a)da.$$

(Note that we used $V = 0$.) Adding vV (from (10)), which is equal to zero, to the right hand side,

$$\tilde{p} = \int \pi(a)da - \xi v + q\left[\lambda_f v\int J(\psi_u(a))\frac{f_u(a)}{u}da + \int (1-\sigma)J(\psi_e(a))f_e(a)da\right].$$

The first two terms equal d, thus if we show that

$$\lambda_f v\int J(\psi_u(a))\frac{f_u(a)}{u}da + \int (1-\sigma)J(\psi_e(a))f_e(a)da = \tilde{p}, \tag{41}$$

we are done.

To show this, plugging (38) into (37) we obtain

$$\tilde{p} = \int J(a)\lambda_w f_u(\psi_u^{-1}(a))\gamma_u(a)da + \int J(a)(1-\sigma)f_e(\psi_u^{-1}(a))\gamma_e(a)da.$$

For the first term, change the variable a by $a = \psi_u(a')$. Then $a' = \psi_u^{-1}(a)$. The first term will become

$$\lambda_w \int J(\psi_u(a'))f_u(a')\gamma_u(\psi_u(a'))\psi_u'(a')da',$$

where the final $\psi_u'(a')$ is Jacobian. Using $\lambda_w u = \lambda_f v$ and (39), this is equal to the first term in (41). The second term is similar, using (40).

B Computation of the model without aggregate shocks

1. Put discrete grids on a. We use a fine grid for value function and coarser grid for the wage function. For value function, we put 1000 grids with equal distance over $[0, 500]$. For the wage function, we put 125 grids.[21] In between the grids, the values of the functions are interpolated using the cubic spline interpolation.[22]

2. Guess $\omega(a)$.

3. Guess θ. Note that this will give us λ_w and λ_f. From the steady-state condition

$$u\lambda_w = (1-u)\sigma,$$

we know u. Since $\nu/u = \theta$, we also know ν.

4. Guess $\bar{k}$. Since $r = zf'(\tilde{k})$ where $\tilde{k} = \bar{k}/(1-u)$, we know r.

5. Now, let's work on worker's problem. Note that since

$$p = \frac{p+d}{1+r-\delta}, \tag{42}$$

we can define

$$a = (1+r-\delta)(k+px) \tag{43}$$

and write the employed worker's budget constraint as

$$a' = (1+r-\delta)(a+w-c).$$

We also know w as a function of a. So let's solve the worker's problem

$$W(a) = \max_{a'} u(c) + \beta\left[\sigma U(a') + (1-\sigma)W(a')\right]$$

subject to

$$a' = (1+r-\delta)(a+\omega(a)-c)$$

[21] In some experiments, we use 50 grids to gain stability.

[22] When the cubic spline interpolation does not perform well, we used linear interpolation.

and

$$U(a) = \max_{a'} u(c) + \beta \left[(1 - \lambda_w) U(a') + \lambda_w W(a') \right]$$

subject to

$$a' = (1 + r - \delta)(a + h - c).$$

If necessary, we can interpolate on $\omega(a)$.

6. Now, we know the decision rules of asset (and employment) for workers, so we can easily calculate the invariant distribution for the worker's idiosyncratic states. These are calculated by iterating over the density functions, $f_u(a)$ and $f_e(a)$, until the densities converge. (The initial condition that we used for this iteration is that everyone holds $\bar{k}$ amount of asset.)

7. Let's move on to the firm's problem. First calculate

$$J(a) = zF(\tilde{k}) - r\tilde{k} - \omega(a) + \frac{1}{1 + r - \delta}(1 - \sigma) J(\psi_e(a)).$$

until convergence. (We already used the fact that $V = 0$.)

8. Calculate the right hand side of V as

$$-\xi + \frac{1}{1 + r - \delta} \lambda_f \int J(\psi_w(a)) \frac{f_u(a)}{u}.$$

This should be zero in equilibrium. If this is positive, our θ is too low, and if this is negative, our θ is too high.

9. We can calculate d from

$$d = \int \pi(\omega(a)) f_e(a) da - \xi \nu.$$

Here, $\pi(\omega(a)) = zf(\tilde{k}) - r\tilde{k} - \omega(a)$. From (42), we can calculate p. Summing up (43) for each individual i,

$$\int a_i di = (1 + r - \delta) \left[\int k_i di + p \int x_i di \right].$$

Since $\int k_i di = \bar{k}$ and $\int x_i di = 1$, $\bar{k}$ should satisfy

$$\bar{k} = \frac{1}{1 + r - \delta} \int a_i di - p.$$

If this $\bar{k}$ is different from the initial guess, we have to update.

10. Finally, we can calculate the new $\omega(a)$ for each a from Nash bargaining:

$$\max_{w} \left(\tilde{W}(w,a) - U(a)\right)^{\gamma} \left(\tilde{J}(w,a) - V\right)^{1-\gamma}.$$

Here, we can use $V = 0$. $\tilde{W}(w,a)$ is the solution of

$$\tilde{W}(w,a) = \max_{a'} u(c) + \beta\left[\sigma U(a') + (1-\sigma)W(a')\right]$$

subject to

$$a' = (1 + r - \delta)(a + w - c).$$

$J(w,a)$ is the solution of

$$\tilde{J}(w,a) = zF(\tilde{k}) - r\tilde{k} - w + \frac{1}{1+r-\delta}(1-\sigma)J(\tilde{\psi}_e(w,a)).$$

11. Repeat until convergence.

C Comparison to the linear model

In this section, we compare our result to the linear Diamond-Mortensen-Pissarides model. The linear model we consider is the following. The consumer maximizes

$$\sum_{t=0}^{\infty} \beta^t c_t$$

subject to

$$c + \frac{a'}{1+r-\delta} = a + w$$

when working, and

$$c + \frac{a'}{1+r-\delta} = a + h$$

when unemployed. Here, $a = k + px$ holds, as in the original model. Because of the linear utility, $1 + r - \delta = 1/\beta$ holds. Now, since the workers are indifferent in terms of the timing of the consumption, without loss of generality, we can let $a' = a$ for everyone, and set

$$c_t = \frac{r-\delta}{1+r-\delta}a + w$$

for employed, and

$$c_t = \frac{r-\delta}{1+r-\delta}a + h$$

for unemployed. Since the first terms of the right hand sides are constant and common across the employment states, we can factor them out in the utility function and consider $c_t = w$ and $c_t = h$, without loss of generality.

The value functions become

$$W = w + \frac{1}{1+r-\delta}[\sigma U + (1-\sigma)W]$$

and

$$U = h + \frac{1}{1+r-\delta}[(1-\lambda_w)U + \lambda_w W].$$

On the firm side, the value of a filled job is

$$J = y - w + \frac{1}{1+r-\delta}[\sigma V + (1-\sigma)J],$$

where y is defined as

$$y = \arg\max_{\tilde{k}} \; z\tilde{k}^{\alpha} - r\tilde{k}$$

that is,

$$y = z\left(\frac{r}{\alpha z}\right)^{\frac{\alpha}{\alpha-1}} - r\left(\frac{r}{\alpha z}\right)^{\frac{1}{\alpha-1}}$$

since

$$\tilde{k} = \left(\frac{r}{\alpha z}\right)^{\frac{1}{\alpha-1}}.$$

The value of vacancy is

$$V = -\xi + \frac{1}{1+r-\delta}[\lambda_f J + (1-\lambda_f)V]. \tag{44}$$

From the free entry, $V = 0$.

Now, since $W-U$ and $J-V$ is linear in w, the Nash bargaining solution results in the simple surplus-sharing rule:

$$W - U = \gamma S$$

and

$$J - V = (1-\gamma)S, \tag{45}$$

where

$$S = (W - U) + (J - V) \tag{46}$$

is the total surplus.

¿From (44), (45), and the free-entry condition,

$$S = \frac{(1+r-\delta)\xi}{(1-\gamma)\lambda_f}$$

holds. From (46) and the value functions,

$$S = \frac{(1+r-\delta)(y+\xi-h)}{r+\delta+(1-\gamma)\lambda_f+\gamma\lambda_w}$$

holds. Combining these two, simplifying, and using the definitions of λ_f and λ_w, the following holds:

$$\frac{y-h}{r+\sigma+\gamma\chi\theta^{1-\eta}} = \frac{\xi}{(1-\gamma)\chi\theta^{-\eta}}.$$

This is the same as the equation (21) in Hornstein, Krusell, and Violante (2006). This equation is shown as equation (16) in the main text.

D Resource balance/goods market equilibrium

In this subsection, we show that the resource balance condition (goods market equilibrium condition)

$$\bar{c} + \bar{k}' = (1-\delta)\bar{k} + zF(\tilde{k})(1-u) - \xi v + hu$$

holds, where

$$\tilde{k} = \frac{\bar{k}}{1-u},$$

$$\bar{c} \equiv \int c_e(a;z,S)f_e(a;S)da + \int c_u(a;z,S)f_u(a;S)da,$$

$$c_e(a;z,S) \equiv a + \omega(a;z,S) - Q_g(z,S)\psi_e^g(a;z,S) - Q_b(z,S)\psi_e^b(a;z,S), \tag{47}$$

and

$$c_u(a;z,S) \equiv a + h - Q_g(z,S)\psi_u^g(a;z,S) - Q_b(z,S)\psi_u^b(a;z,S). \tag{48}$$

Note that the asset market equilibrium condition (33) holds.

The first step is to show that

$$\int af_e(a;S)da + \int af_u(a;S)da = (1-\delta + r(z,S))\bar{k} + p(z,S) + d(z,S) \tag{49}$$

is implied by the asset market equilibrium in the last period and the law of motion for the individual states.

Assuming that the decision rules for a' are increasing in a, the law of motion for asset distribution is as follows:

$$\int_{\underline{a}}^{a'} f_e(\tilde{a};S')d\tilde{a} = \lambda_w \int_{\underline{a}}^{(\psi_u^{z'})^{-1}(a';z,S)} f_u(a;S)da + (1-\sigma)\int_{\underline{a}}^{(\psi_e^{z'})^{-1}(a';z,S)} f_e(a;S)da \tag{50}$$

$$\int_{\underline{a}}^{a'} f_u(\tilde{a};S')d\tilde{a} = (1-\lambda_w)\int_{\underline{a}}^{(\psi_u^{z'})^{-1}(a';z,S)} f_u(a;S)da + \sigma\int_{\underline{a}}^{(\psi_e^{z'})^{-1}(a';z,S)} f_e(a;S)da. \tag{51}$$

Here, $(\psi_u^{z'})^{-1}(a';z,S)$ denotes the value of a that satisfies $a' = \psi_u^{z'}(a;z,S)$.

To derive (49), we use a one-period forwarded version,

$$\int a'f_e(a';S')da' + \int a'f_u(a';S')da' = (1-\delta + r(z',S'))\bar{k}' + p(z',S') + d(z',S').$$

From (33), what we need to show is

$$\int a'f_e(a';S')da' + \int a'f_u(a';S')da' = \int \psi_e^{z'}(a;z,S)f_e(a;S)da + \int \psi_u^{z'}(a;z,S)f_u(a;S)da. \tag{52}$$

Differentiating (50) with respect to a',

$$f_e(a';S') = \lambda_w f_u((\psi_u^{z'})^{-1}(a';z,S);S)\rho_e(a';z,S) + (1-\sigma)f_e((\psi_e^{z'})^{-1}(a';z,S);S)\rho_u(a';z,S) \tag{53}$$

where

$$\rho_u(a';z,S) = \frac{d(\psi_u^{z'})^{-1}(a';z,S)}{da'} = \frac{1}{(\psi_u^{z'})'((\psi_u^{z'})^{-1}(a';z,S);z,S)} \tag{54}$$

and

$$\rho_e(a';z,S) = \frac{d(\psi_e^{z'})^{-1}(a';z,S)}{da'} = \frac{1}{(\psi_e^{z'})'((\psi_e^{z'})^{-1}(a';z,S);z,S)} \tag{55}$$

Now, multiply a' to both sides of (53) and integrate:

$$\begin{aligned}&\int a' f_e(a';S')da' \\ &\quad = \lambda_w \int a' f_u((\psi_u^{z'})^{-1}(a';z,S);S)\rho_e(a';z,S)da' \\ &\qquad +(1-\sigma)\int a' f_e((\psi_e^{z'})^{-1}(a';z,S);S)\rho_u(a';z,S)da'\end{aligned} \tag{56}$$

Changing variables by $a' = \psi_u^{z'}(a;z,S)$ (therefore $a = (\psi_u^{z'})^{-1}(a';z,S)$), the first term in the right hand side becomes

$$\lambda_w \int \psi_u^{z'}(a;z,S) f_u(a;S)\rho_e(\psi_u^{z'}(a;z,S);z,S)(\psi_u^{z'})'(a;z,S)da,$$

where $(\psi_u^{z'})'(a;z,S)$ is a Jacobian. From (54), this is equal to

$$\lambda_w \int \psi_u^{z'}(a;z,S) f_u(a;S)da.$$

Similarly, the second term of the right hand side of (56) is equal to

$$(1-\sigma)\int \psi_e^{z'}(a;z,S) f_e(a;S)da.$$

Therefore, (56) becomes

$$\int a' f_e(a';S')da' = \lambda_w \int \psi_u^{z'}(a;z,S) f_u(a;S)da + (1-\sigma)\int \psi_e^{z'}(a;z,S) f_e(a;S)da.$$

Similarly, we can show

$$\int a' f_u(a';S')da' = (1-\lambda_w) \int \psi_u^{z'}(a;z,S) f_u(a;S)da + \sigma\int \psi_e^{z'}(a;z,S) f_e(a;S)da.$$

Summing up these two, we obtain (52).

Next, integrating (47) and (48) for everyone gives us

$$\begin{aligned}&-\bar{c} + \int a f_e(a;S)da + \int a f_u(a;S)da + \int \omega(a;z,S) f_e(a;S)da + hu \\ &\quad = \int Q_g(z,S)\psi_e^g(a;z,S) f_e(a;S)da + \int Q_b(z,S)\psi_e^b(a;z,S) f_e(a;S)da \\ &\qquad + \int Q_g(z,S)\psi_u^g(a;z,S) f_u(a;S)da + \int Q_b(z,S)\psi_u^b(a;z,S) f_u(a;S)da\end{aligned}$$

From (49), the left hand side of this is equal to

$$-\bar{c} + (1-\delta + r(z,S))\bar{k} + p(z,S) + d(z,S) + \int \omega(a;z,S) f_e(a;S)da + hu. \tag{57}$$

In equilibrium, there are $(1-u)$ jobs in the economy and each job employs $\tilde{k} = \bar{k}/(1-u)$ amount of capital. Thus, (29) becomes

$$\pi(a; z, S) = zF(\tilde{k}) - r(z, S)\tilde{k} - \omega(a; z, S).$$

From (31) and $\int f_e(a; S)da = (1-u)$, (57) is equal to

$$-\bar{c} + (1-\delta)\bar{k} + p(z, S) + zF(\tilde{k})(1-u) - \xi v + hu.$$

Therefore, what we have to show is

$$\begin{aligned} &\bar{k}' + p(z, S) \\ &\quad = \int Q_g(z, S)\psi_e^g(a; z, S)f_e(a; S)da + \int Q_b(z, S)\psi_e^b(a; z, S)f_e(a; S)da \\ &\qquad + \int Q_g(z, S)\psi_u^g(a; z, S)f_u(a; S)da + \int Q_b(z, S)\psi_u^b(a; z, S)f_u(a; S)da. \end{aligned}$$

From (33), the right-hand-side is equal to

$$Q_g(z, S)[(1-\delta + r(g, S'))\bar{k}' + p(g, S') + d(g, S')] + Q_b(z, S)[(1-\delta + r(b, S'))\bar{k}' + p(b, S') + d(b, S')].$$

From the asset pricing equations (21) and (22), this is equal to $\bar{k}' + p(z, S)$.

E Algorithm with aggregate shocks

Since we have many state variables, we use relatively small numbers of grids. There are 60 grids in a direction for the value functions, 15 grids in a direction for the wage function, 4 grids in $\bar{k}$ direction and 4 grids in u direction. For a direction, we put more grids for a close to 0 to accommodate more curvature. We use cubic spline interpolation in a direction and linear interpolation in other directions.

1. Assume the law of motion for aggregate capital.

$$\log \bar{k}' = a_0 + a_1 \log \bar{k} + a_2 \log u + a_3 \log z. \tag{58}$$

   Assume the prediction rules for the current aggregate variables as functions of aggregate state.

$$\log \theta = b_0 + b_1 \log \bar{k} + b_2 \log u + b_3 \log z. \tag{59}$$

Note that u' can be calculated once θ is given.

$$u' = (1 - \lambda_w(\theta))u + \sigma(1 - u). \tag{60}$$

Asset prices:

$$\log(p(z, \bar{k}, u) + d(z, \bar{k}, u)) = c_0 + c_1 \log \bar{k} + c_2 \log u + c_3 \log z. \tag{61}$$

$$\log Q_z(z, \bar{k}, u) = \begin{cases} d_0 + d_1 \log \tilde{Q}_z(z, \bar{k}, u) + d_2 \log \bar{k} + d_3 \log u & \text{if } z = g \\ e_0 + e_1 \log \tilde{Q}_z(z, \bar{k}, u) + e_2 \log \bar{k} + e_3 \log u & \text{if } z = b, \end{cases} \tag{62}$$

where $\tilde{Q}_z(z, \bar{k}, u) \equiv \pi_{zz}/(1 - \delta + r(z, \bar{k}', u'))$ (note that $\bar{k}'$ and u' are obtained as functions of z, $\bar{k}$, and u by the above equations). $\tilde{Q}_z$ is the exact value of Q_z when $g = b$. We expect that Q_z is not too different from $\tilde{Q}_z$ when shocks are not too large. When $z = g$, we can calculate Q_b by

$$Q_g(z, \bar{k}, u)(1 - \delta + r(g, \bar{k}', u')) + Q_b(z, \bar{k}, u)(1 - \delta + r(b, \bar{k}', u')) = 1. \tag{63}$$

When $z = b$, this can be used to calculate Q_g, given Q_b. In total, we have 20 coefficients to iterate on.

2. Start the loop on individual optimization and Nash bargaining.

   (a) Outside loop: give the initial value for the wage function $\omega(a; z, \bar{k}, u)$.

   (b) Give the initial values for the value functions, $W(a; z, \bar{k}, u)$ and $U(a; z, \bar{k}, u)$.

   (c) Inside loop: for each value (grid points) of a, $\bar{k}$, u, z, we will perform the worker's individual optimization.

   (d) Repeat until $W(a; z, \bar{k}, u)$ and $U(a; z, \bar{k}, u)$ converge.

   (e) Now we can calculate $J(a; z, \bar{k}, u)$ by using the worker's decision rule and noting that $V(a; z, \bar{k}, u) = 0$.

   (f) Based on the above functions, we can calculate $\tilde{W}(w, a; z, \bar{k}, u)$ and $\tilde{J}(w, a; z, \bar{k}, u)$, and perform Nash bargaining for each aggregate states. That is, the Nash bargaining delivers the wage function $\omega(a;z,\bar{k},u)$ based on $\tilde{W}(w,a;z,\bar{k},u)$, $U(a;z,\bar{k},u)$, $\tilde{J}(w,a;z,\bar{k},u)$ and $V(a;z,\bar{k},u)(=0)$.

(g) Revise the wage function $\omega(a;z,\bar{k},u)$. Repeat until convergence.

Note that with aggregate shocks, the wage depends not only on a but also on the aggregate state, $(z,\bar{k},u)$. The wage function, $\omega(a;z,\bar{k},u)$, is defined on $15 \times 2 \times 4 \times 4$ grid points. (The Nash bargaining is performed at each of these points.) For each $(z,\bar{k},u)$, the wage function (as a function of a) is interpolated using the cubic spline interpolation in between the grid points. In the simulation (next step), we need to compute the wages for $(\bar{k},u)$ that are not on the grids. In that case, we first compute a wage function $\omega_{z^*\bar{k}^*u^*}(a)$ for a specific $(z^*,\bar{k}^*,u^*)$, by linearly interpolating in $\bar{k}$ and u directions. Then $\omega_{z^*\bar{k}^*u^*}(a)$ is used to compute the wages at each a (interpolated by cubic spline in between a grids).

3. Simulation.

(a) Give initial values for a, employment status, $\bar{k}$, u, and z. (Note that the sum of a is equal to $[(1-\delta+r)\bar{k}+p+d]$, so once we give $\bar{k}$, u, and z, we know what should be the sum of a.)

(b) Using

$$\begin{aligned} V(z,S) = -\xi \quad & +Q_g(z,S)((1-\lambda_f(\theta))V(g,S') + \lambda_f(\theta)\int J(\psi_u^g(a;z,S);g,S')[f_u(a;S)/u]da) \\ & +Q_b(z,S)((1-\lambda_f(\theta))V(b,S') + \lambda_f(\theta)\int J(\psi_u^b(a;z,S);b,S')[f_u(a;S)/u]da) \end{aligned}$$

equal zero (using the prediction rules (58), (59), and (60) for $\bar{k}'$ and u' in evaluating the future value function), we can calculate the equilibrium value of θ. Then we know $v = u\theta$. Record this *data* of θ.

(c) Calculate u' from the information on this equilibrium θ and (60). Note that this u' may not be the same as the u' predicted using (60), if the prediction rules are incorrect. Record this *data* of u' for the later use.

(d) In this step, we will calculate the sum of a'_g and the sum of a'_b from the consumer's decisions. Call them A'_g and A'_b. From the asset market equilibrium,

$$A'_g = (1 - \delta + r(g, \bar{k}', u'))\bar{k}' + p(g, \bar{k}', u') + d(g, \bar{k}', u') \tag{64}$$

and

$$A'_b = (1 - \delta + r(b, \bar{k}', u'))\bar{k}' + p(b, \bar{k}', u') + d(b, \bar{k}', u') \tag{65}$$

has to hold. These may not hold if the consumers' prediction rules are incorrect. Here, we search for the values of Q_g, Q_b, and $\bar{k}'$ so that these two equations and (63) hold. Let us explain how we find these values in the case of $z = g$. (When $z = b$, g and b are reversed everywhere.) The main idea here follows Krusell and Smith (1997).

Note that (64) can be rewritten as

$$\bar{k}' = \frac{A'_g - p(g, \bar{k}', u') - d(g, \bar{k}', u')}{1 - \delta + r(g, \bar{k}', u')} \tag{66}$$

and (65) can be rewritten as

$$\bar{k}' = \frac{A'_b - p(b, \bar{k}', u') - d(b, \bar{k}', u')}{1 - \delta + r(b, \bar{k}', u')}.$$

Therefore,

$$\frac{A'_g - p(g, \bar{k}', u') - d(g, \bar{k}', u')}{1 - \delta + r(g, \bar{k}', u')} = \frac{A'_b - p(b, \bar{k}', u') - d(b, \bar{k}', u')}{1 - \delta + r(b, \bar{k}', u')}. \tag{67}$$

holds. We will search for Q_g that satisfy (67)—we can expect that A'_g is decreasing in Q_g and A'_b is decreasing in Q_b.[23] Note that Q_b can be calculated as a (decreasing) function of Q_g, from (63). To calculate A'_g and A'_b for each Q_g, we re-calculate the optimization problem for a given Q_g.

$$\begin{aligned}\hat{W}(Q_g, a; z, \bar{k}, u) = \max_{a'_g, a'_b} \quad u(c) + \beta[\quad & \pi_{zg}(\sigma U(a'_g; g, \bar{k}', u') + (1-\sigma)W(a'_g; g, \bar{k}', u')) \\ & + \pi_{zb}(\sigma U(a'_b; b, \bar{k}', u') + (1-\sigma)W(a'_b; b, \bar{k}', u'))]\end{aligned}$$

subject to

$$c + Q_g a'_g + Q_b(Q_g) a'_b = a + \omega(a; z, \bar{k}, u),$$

[23] We use (58), (59), (60), (61) and $r = zF'(\tilde{k})$ to calculate the $p(z', \bar{k}', u') + d(z', \bar{k}', u')$ and $r(z', \bar{k}', u')$. Thus they are not functions of our unknowns.

$$a'_g \geq \underline{a},$$

$$a'_b \geq \underline{a},$$

and $\bar{k}'$, u' given.

Unemployed consumers:

$$\begin{aligned}\hat{U}(Q_g, a; z, \bar{k}, u) = \max_{a'_g, a'_b} \quad u(c) + \beta[\quad & \pi_{zg}((1-\lambda_w)U(a'_g; g, \bar{k}', u') + \lambda_w W(a'_g; g, \bar{k}', u')) \\ & +\pi_{zb}((1-\lambda_w)U(a'_b; b, \bar{k}', u') + \lambda_w W(a'_b; b, \bar{k}', u'))]\end{aligned}$$

subject to

$$c + Q_g a'_g + Q_b(Q_g) a'_b = a + h,$$

$$a'_g \geq \underline{a},$$

$$a'_b \geq \underline{a},$$

and $\bar{k}'$, u' given.

Calculate A'_g and A'_b with this method for different values Q_g, until we find a Q_g that makes (67) hold with equality—if we are in a rational expectations equilibrium, this Q_g has to be equal to the Q_g from the prediction rule (62).

Then we calculate $\bar{k}'$ from (66) and Q_b from (63). Record these *data* of Q_g, Q_b, and $\bar{k}'$.

(e) From

$$d(z, \bar{k}, u) = \int \pi(a; z, \bar{k}, u) f_e(a; \bar{k}, u) da - \xi v,$$

where $\pi(a; z, \bar{k}, u) = \tilde{\pi}(\omega(a; z, \bar{k}, u); z, \bar{k}, u)$, we can obtain the *data* of $d(z, \bar{k}, u)$. (We already know all the values to calculate the first term, and v was obtained in an earlier step.)

(f) From

$$p(z, \bar{k}, u) = Q_g(z, \bar{k}, u)[p(g, \bar{k}', u') + d(g, \bar{k}', u')] + Q_b(z, \bar{k}, u)[p(b, \bar{k}', u') + d(b, \bar{k}', u')],$$

we can calculate the *data* of $p(z, \bar{k}, u)$. Here, Q_g and Q_b are obtained in an earlier step, and for $p(z', \bar{k}', u') + d(z', \bar{k}', u')$, we use the prediction rules (58), (59), (60), and (61).

(g) With a random number generator, obtain z' and move to the next period. Individual employment status and individual asset status are also forwarded to the next period. the individual asset holdings are represented by the density function.[24] Repeat from step (b) for N periods. Discard the first n periods from the sample. We set $N = 2000$ and $n = 500$ in our program.

(h) Using all above data ($\bar{k}$, z, u, θ, Q_z, p, d), we can revise the laws of motion, labor market tightness functions, and pricing functions by running ordinary least squares regressions.

(i) Repeat until the prediction rules (the laws of motion, labor market tightness functions, and pricing functions) predicts the simulated data with sufficient accuracy (high R^2). As is written in the main text, we can find the prediction rules that are very accurate. In Shimer calibration, all R^2 are larger than 0.9999. In HM calibration, all R^2 are larger than 0.999. R^2 here is defined as

$$R^2 \equiv 1 - \frac{\sum_{t=501}^{2000}(m_t - m_t^p)^2}{\sum_{t=501}^{2000}(m_t - \bar{m})^2},$$

where m_t is the simulated value of a variable, m_t^p is the predicted value of m_t using the prediction rule (law of motion) that the consumers used in the optimization step and the simulated value of the right-hand-side variables, and $\bar{m}$ is the average of m_t.

F Accuracy of the computational algorithm

F.1 Forecasting accuracy

Agents in our model use a linear law of motion to forecast next period's capital stock ($\bar{k}'$). Similarly, they use linear prediction rules to predict the other aggregate variables ($\theta, (p + d), Q_g$ and Q_b) for the current period. To evaluate the agents's forecasting and prediction abilities we compute the errors that agents make in 1 period ahead and 25 years (200 periods) ahead forecasts.

[24] The period-0 density is given exogenously, but we discard sufficient number of initial periods from the sample in the following regression to remove the influence of the initial distribution. For the Shimer calibration, we used the uniform distribution on $[0, 2\bar{k}]$. In HM calibration, since the distribution moves slowly, we used the stationary distribution from the no-aggregate-shocks model as the initial distribution. In both cases, we checked that the results are not sensitive to the choice of the initial distribution.

	1 period ahead		25 years ahead	
Variable	$corr(x,\hat{x})$	max % error	$corr(x,\hat{x})$	max % error
$\bar{k}'$	1.000000	0.0035	0.999905	0.0325
θ	0.999991	0.0238	0.999978	0.0305
$p+d$	0.999998	0.0187	0.999991	0.0226
Q_g	1.000000	0.0029	1.000000	0.0027
Q_b	1.000000	0.0043	1.000000	0.0050
r	1.000000	0.0021	0.999976	0.0205
u'	0.999994	0.0049	0.999977	0.0084

Table 11: Forecasting and prediction accuracy for the Shimer calibration where x is the value of the variable from the simulations and $\hat{x}$ is the predicted value.

We assume that the agents in the economy have the knowledge of the the current period's capital stock and the unemployment rate. By only using this information and next period's technology shock, agents can predict 1 period ahead aggregate variables by using the linear prediction rules. We start from period 501 of our simulation and compute 1 period ahead capital stock, interest rate, unemployment rate and and current period's $(\theta, (p+d), Q_g$ and $Q_b)$. Then we compare these predicted values with values that we observe from the simulations. We report two measures of accuracy: the correlation between the values implied by the linear rules and the simulations and the maximum percentage deviation from the value implied by the simulation.

We also report the forecasting and prediction errors for 25 years ahead forecasts.

F.2 Additional moments

To check the robustness, we checked whether we can improve R^2 significantly by including the variance of a in the regression. We found that the improvements are less than 10^{-5} for all the regressions in the HM case. [More to be added later.]

	1 period ahead		25 years ahead	
Variable	$corr(x, \hat{x})$	max % error	$corr(x, \hat{x})$	max % error
$\bar{k}'$	1.000000	0.0030	0.999959	0.0179
θ	0.999565	1.0225	0.999590	0.9512
$p + d$	0.999992	0.3000	0.999989	0.3904
Q_g	1.000000	0.0041	1.000000	0.0055
Q_b	1.000000	0.0053	1.000000	0.0128
r	0.999994	0.0100	0.999980	0.0200
u'	0.999755	0.1826	0.999590	0.2438

Table 12: Forecasting and prediction accuracy for the HM calibration where x is the value of the variable from the simulations and $\hat{x}$ is the predicted value.

G Data for the U.S. economy and calculation of the cyclical statistics of the model

Output (data): We compute the logarithm of quarterly real GDP per capita and detrend it by using HP filter. The smoothing parameter that we use is 1600. The standard deviation of the cyclical component is 0.0158.

Stock prices and dividends (data): We have computed the standard deviation of the cyclical component of the stock market prices and dividends for 1951-2004 period.[25] We have used monthly data for stock prices and dividends which are normalized by CPI. We have adjusted the monthly data to quarterly frequency. For stock prices we have picked the monthly value for the 3rd, 6th, 9th and 12th month of each year. For the dividends, we have summed up the monthly flows for three months. The standard deviation of cyclical component of natural logarithm of stock prices is 0.1012 and for dividends it is 0.0286. Dividends fluctuate far less than stock prices. The finding that a typical economic model produces much less fluctuation in the stock prices (compared to the dividend fluctuation) has been labeled as "stock market volatility puzzle" in the literature (Shiller (1981)).

Vacancy-unemployment ratio (data): The vacancy-unemployment ratio is constructed by

[25] This dataset is constructed by Robert Shiller. See `http://www.irrationalexuberance.com/index.htm`.

calculating the ratio of the Help Wanted Advertising Index to the unemployment, measured in index units per thousand workers.[26] The data are quarterly and span the period of 1951 to 2005. First we have detrended the natural logarithm of the vacancy-unemployment ratio and then computed the standard deviation of the cyclical component of the series.

Investment, consumption and wage (data): We report these statistics from Andolfatto (1996).

Labor share (data): Ríos-Rull and Santaeulàlia-Llopis (2007) find that the standard deviation of labor share is 43% of that of output. Andolfatto (1996) reports this number as 68% of that of output.

Model: We have simulated our model for 2000 periods, discarded the first 500 periods, and then adjusted the generated data to quarterly frequency. Then we have detrended the series by using HP filter with a smoothing parameter of 1600 for output, investment, consumption, average wage, labor share, stock prices, dividend, and vacancy-unemployment ratio. The values of these variables at time t are calculated by using the simulated data as follows: output is calculated as $z_t\bar{k}_t^{\alpha}(1-u_t)^{1-\alpha}$, investment is $\bar{k}_{t+1}-(1-\delta)\bar{k}_t$, consumption is $z_t\bar{k}_t^{\alpha}(1-u_t)^{1-\alpha}-\bar{k}_{t+1}+(1-\delta)\bar{k}_t-\xi v_t+hu_t$, average wage is the average wage of the employed agents in the economy at time t, labor share is average wage divided by $z_t(\bar{k}_t/(1-u_t))^{\alpha}$, stock price is p_t, dividend is d_t, and vacancy unemployment ratio is v_t/u_t.

[26]This dataset was constructed by Robert Shimer. See `http://home.uchicago.edu/ shimer/data/mmm/`.

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