

Home Production and Social Security Reform

Online Appendix

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Abstract

This online appendix provides the definition of the stationary equilibrium and the detailed computation steps, and analyzes a one-good economy using NIPA data. It also includes three charts and one table that cannot be included in the main paper because of space constraints.

1 Appendix A. Definition of the Stationary Equilibrium

We focus on the stationary equilibrium of the economy where factor prices and agent distribution over the state space are constant over time. Each agent's state is denoted by x . Let S denote the aggregate housing stock available for renting, D the aggregate stock of home input, C_m the aggregate consumption of the market good, I_s the aggregate investment on housing, I_k the aggregate investment on physical capital.

Definition 1. A stationary equilibrium is given by government policies including tax rate τ , and pension $pen(t_r, y)$; an interest rate r and a wage rate w ; price of rental housing η ; value functions $V(x)$; allocations $c_m(x)$, $a'(x)$, $d(x)$, $s(x)$, $n_m(x)$, $n_h(x)$, $f'(x)$; bequest b ; and a constant distribution of people over the state variables x , $v(x)$, such that the following conditions hold:

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(i) Given the government policies, the interest rate, the wage, price of rental housing, and the expected bequest, the value functions and allocations solve the above-described maximization problem for a household with state variables x .

(ii) $v(\cdot)$ is the invariant distribution of households over the state variables.

(iii) The price of each factor is equal to its marginal product

$$\begin{aligned} r &= F_1^m(K, N_m) - \delta^k, \\ w &= F_2^m(K, N_m). \end{aligned}$$

(iv) The expected bequest is consistent with the actual bequest left

$$\int bv(dx) + \int_{t=0} (a(1+r))v(dx) = \int (1-\lambda_t)[(1+r)a']v(dx).$$

(v) No arbitrage condition holds

$$\eta = r + \delta^s.$$

(vi) Government budget is balanced at each period

$$\tau \int \min\{\varepsilon e_t w n_m, y_{\max}\}v(dx) = \int \text{pen}(t_r, y)v(dx).$$

(vii) All markets clear.

2 Appendix B: Computation of the Model

To compute the steady state of our model, we first discretize the income process into five points. The state space for average lifetime earnings and asset holdings is discretized into unevenly spaced grids. The upper bounds on the grids are chosen to be large enough so that they do not constitute a constraint on the optimization problem. We chose 15 grid points for the asset variables and 15 for the average lifetime earnings. The choice variables are searched over 150 grid points for assets, 100 points on market hours, and continuous for other variables. We use linear approximation to approximate valuation functions for the points not on the state grids.

We solve for the steady-state equilibrium as follows:

1. Make an initial guess of interest rate r , the wage rate w and tax rate τ .
2. Guess the size of accidental bequests.
3. Set the value function after the last period to be 0 and solve the value function

for the last period of life for each of the points of the grid. This yields policy functions and value functions in the last period.

4. By backward induction, repeat step 3 until the first period in life.
5. Compute the associated stationary distribution of households by forward induction using the policy functions starting from the known distribution over types of age.
6. Check whether the associated accidental bequests are consistent with the initial guess. If so, continue to step 7. If not, go back to step 2 and update accidental bequests.
7. Check market clearing conditions and government budget constraint. If both hold, an equilibrium is found. If not, go to step 1 and update the initial guess.

We solve for the equilibrium transition path as follows:

1. Make an initial guess of the size of accidental bequests, interest rates r_t , wage rates w_t and tax rate τ_1 , generate a linear function so that tax becomes zero once all claims have been satisfied.
2. Solve for value functions and policy functions for each cohort by backward induction.
3. Compute the distribution of households by forward induction using the policy functions starting from the known distribution before the reform.
4. Calculate government debt.
5. Check whether the associated accidental bequests are consistent with the initial guess, check market clearing conditions, and check if the government debt has been repaid in the last period. If all hold, an equilibrium transition path is found. If not, go to step 1 and update the initial guess.

3 Appendix C: Analyzing the One-Good Economy Using NIPA Data

We recalibrate the one-good economy using NIPA data in this appendix. As discussed in the main text, NIPA gives a lower capital K compared to CEX because of housing. As a result, capital output ratio is much smaller than that in the case when CEX housing data is used, 3.10 as opposed to 3.67; on the other hand, capital depreciation rate is larger, 6.5 percent versus 4.2 percent. The net effect is that capital income share α is 0.356, slightly larger than the 0.339 number calibrated in the one-good economy using CEX housing data. In terms of the other parameters as reported in Table A1, the weight on consumption is smaller and the elasticity between consumption and leisure is larger.

When we eliminate Social Security, with more capital going to market production, interest rate declines more than in the benchmark one-good economy and the wage

rate increases less. Households work much more and enjoy slightly more consumption. The total welfare gain, at 9.26 percent, is very similar to the 9.25 percent gain in the benchmark one-good economy.

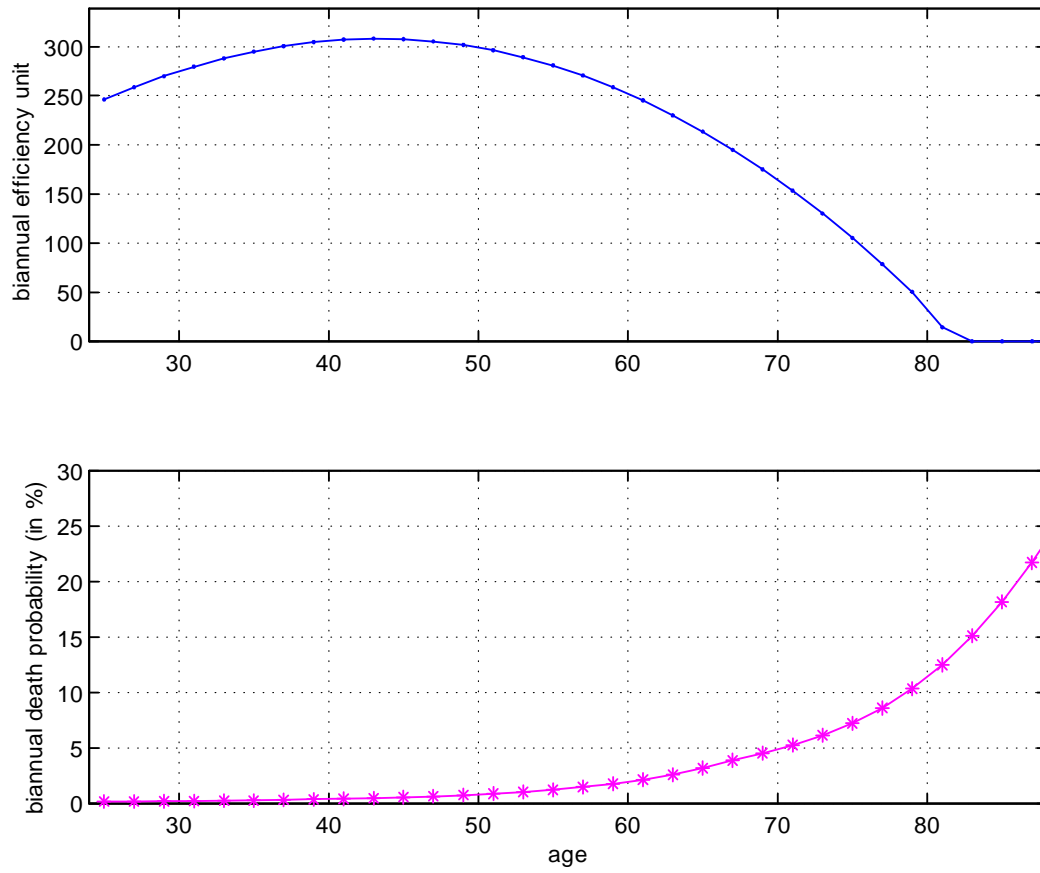


Figure A1. Life Cycle Profiles

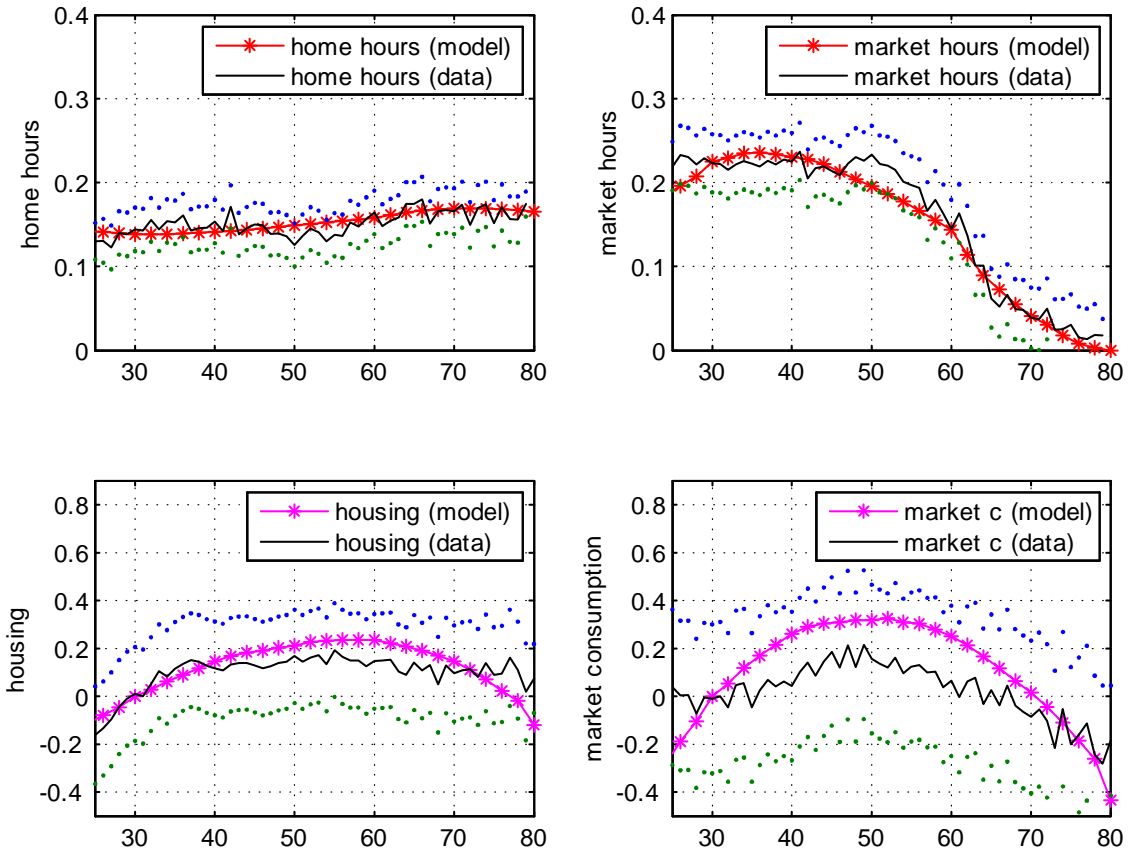


Figure A2. Life-cycle Labor Supply and Consumption Profiles — Benchmark (the dotted lines represent the two-standard-deviation error band)

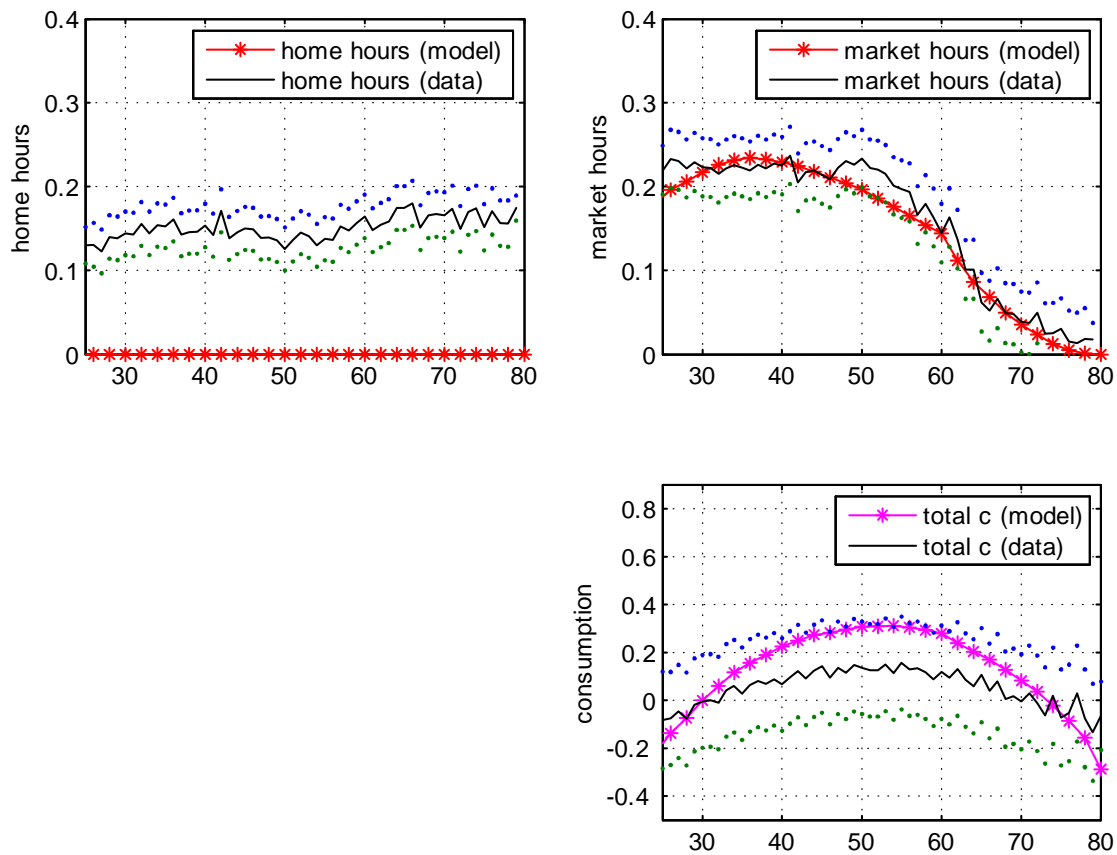


Figure A3. Life-cycle Profiles of Labor Supply and Consumption in the One-good Economy (the dotted lines represent the two-standard-deviation error band)

Table A1. Calibration to Match Data Moments – One-good Economy

Parameters	CEX Housing Data	NIPA Housing Data
β discount factor	0.955	0.952
τ Social Security tax	0.103	0.103
ζ_4 sub. betw. consumption and leisure	1.521	1.539
ω_4 weight on consumption	0.069	0.067