Consumption and Time Use over the Life Cycle
Online Appendix

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Abstract

This online appendix defines stationary distribution, details computation steps, and describes calibration procedures. It also includes five charts that cannot be included in the main paper due to space limitation.

1 Appendix A. Definition of the Stationary Equilibrium

We focus on the stationary equilibrium of the economy where factor prices and agent distribution over state space are constant over time. Each agent's state is denoted by x. Let S denote the aggregate housing stock available for renting, D the aggregate consumption of home input, C_m the aggregate consumption of the market good, I_h the aggregate investment on housing, I_k the aggregate investment on physical capital, T^c the total transactions costs for trading housing.

Definition 1. A stationary equilibrium is given by government policies including tax rate τ, and pension $pen(t)$; an interest rate r, a rental rate η, and a wage rate w; value functions $V(x)$; allocations $c_m(x)$, $a'(x)$, $h'(x)$, $d(x)$, $s(x)$, $n_m(x)$, $n_h(x)$; bequest b; and a constant distribution of people over the state variables x, $v(x)$, such that the following conditions hold:

(i) Given the government policies, the interest rate, the wage, and the expected bequest, the value functions and allocations solve the described maximization problem for a household with state variables x.

(ii) $\upsilon(.)$ is the invariant distribution of households over the state variables.

(iii) The price of each factor is equal to its marginal product.

$$r = F_1^m(K, L) - \delta^k,$$
$$w = F_2^m(K, L).$$

(iv) The expected bequest is consistent with the actual bequest left

$$\int b\upsilon(dx) + \int_{t=0} (a(1+r) + (1-\delta^h)h)\upsilon(dx) = \int (1-\lambda_t)[(1+r)a' + (1-\delta^h)h']\upsilon(dx).$$

(v) The non-arbitrage condition holds for the rental agency

$$\eta = r + \delta^h.$$

(vi) Government budget is balanced at each period

$$\tau \int \varepsilon e_t w n_m \upsilon(dx) = \int_{t \geq T^r} pen(t)\upsilon(dx).$$

(vii) All markets clear.

$$S = \int_{h=0} s\upsilon(dx),$$
$$D = \int d\upsilon(dx),$$
$$K = \int a\upsilon(dx) - S,$$
$$C_m = \int c_m \upsilon(dx),$$
$$T^c = \int \varphi(h, h')\upsilon(dx),$$
$$L = \int \varepsilon e_t n_m \upsilon(dx),$$
$$I_h = \int [h' - (1-\delta^h)h]\upsilon(dx),$$
$$I_k = K' - (1-\delta^k)K,$$
$$F^m(K, L) = C_m + I_k + I_h + D + T^c.$$

2 Appendix B: Computation of the Model

Due to nonconvex transactions costs on housing and the collateralized borrowing constraint, we cannot use either an Euler equation approximation or the policy function iteration. Therefore, we solve the model using value function approximation.

We first discretize the income process into 5 points. The state space for owner-occupied housing and asset holdings are discretized into unevenly spaced grids. We chose 20 grid points for each of the asset variables. The choice variables are searched over 100 grid points for housing and assets and continuous for other variables. We use linear approximation to approximate valuation functions for the points not on the state grids.

We solve for the steady-state equilibrium as follows:

1. Make an initial guess of interest rate r, the wage rate w, pension, and the size of accidental bequests.

2. Set the value function after the last period to be 0 and solve the value function and policy functions for the last period of life for each of the points of the grid.

3. By backward induction, repeat step 2 until the first period in life.

4. Starting from the known distribution at age 25, compute the associated stationary distribution of households by forward induction using the policy functions.

5. Check whether market clearing conditions hold, whether the government budget is balanced, and whether the associated accidental bequests are consistent with the initial guess. If so, an equilibrium is found. If not, go to step 1 and update the initial guess.

3 Appendix C: Calibration

We use data from the National Income and Product Accounts and the Fixed Assets Tables for the years 1957-2007. Since our model has two assets and abstracts from government taxes and expenditures, we make the following adjustment.

In order to measure capital income share, we first remove income from the housing sector and the government sector from the national income accounts. Then we define private labor income, Y_{pl}, as compensation of employees. We define unambiguous capital income (UCI) as rental income, corporate profits and net interest. We define ambiguous capital income (ACI) as other income excluding employee compensation, UCI, and depreciation. Thus total private nonhousing income Y_p is the sum of Y_{pl}, ACI, UCI, and depreciation. Next, we allocate the ambiguous components of capital income and its depreciation into private capital income according to the share of capital income in measured total output. In other words, private capital income Y_{pk} is defined as

$UCI + dep_{UCI} + \alpha \cdot (ACI + dep_{ACI}) = \alpha \cdot Y_p$. Therefore ,the share of capital is calculated as

$$\tag{1} \alpha = (UCI + dep_{UCI})/(Y_p - ACI - dep_{ACI}).$$

We compute an average share of capital $\alpha = 0.240$.

The variable K is measured as private fixed nonresidential assets, and I_k is total private nonresidential investment. We calculate the average capital-output ratio $\frac{K}{Y} = 1.61$, and the investment-capital ratio $\frac{I_k}{K} = 0.10$. Given that the paper abstracts from population as well as technology growth, we set the depreciation rate for nonhousing capital δ^k at 0.10. The implied real interest rate is thus $r = \alpha\frac{Y}{K} - \delta^k = 0.05$. Note that this rate is somewhat higher than the 0.027 and 0.040 range typically used in the literature. Given that capital stock is measured with considerable error (Gomme and Rupert 2007), we decide to follow the literature and set our equilibrium real rate of interest at 0.04. Holding the nonhousing capital depreciation rate at 0.10, this implies a capital-output ratio of 1.71, slightly higher than the calculated 1.61. We set the depreciation rate on housing capital δ^h to 0.02, well within the range of those used in the literature.

References

[1] Gomme, Paul, and Peter Rupert, "Theory, Measurement and Calibration of Macroeconomic Models," *Journal of Monetary Economics* 54 (2) (2007), 460-497.
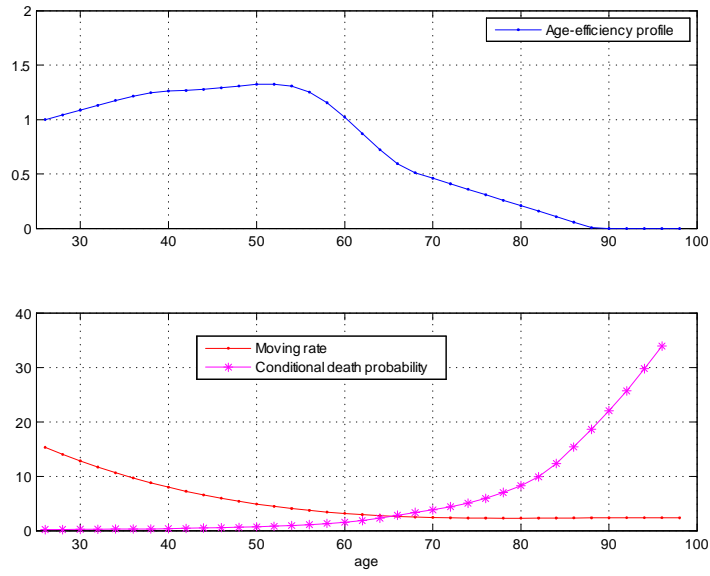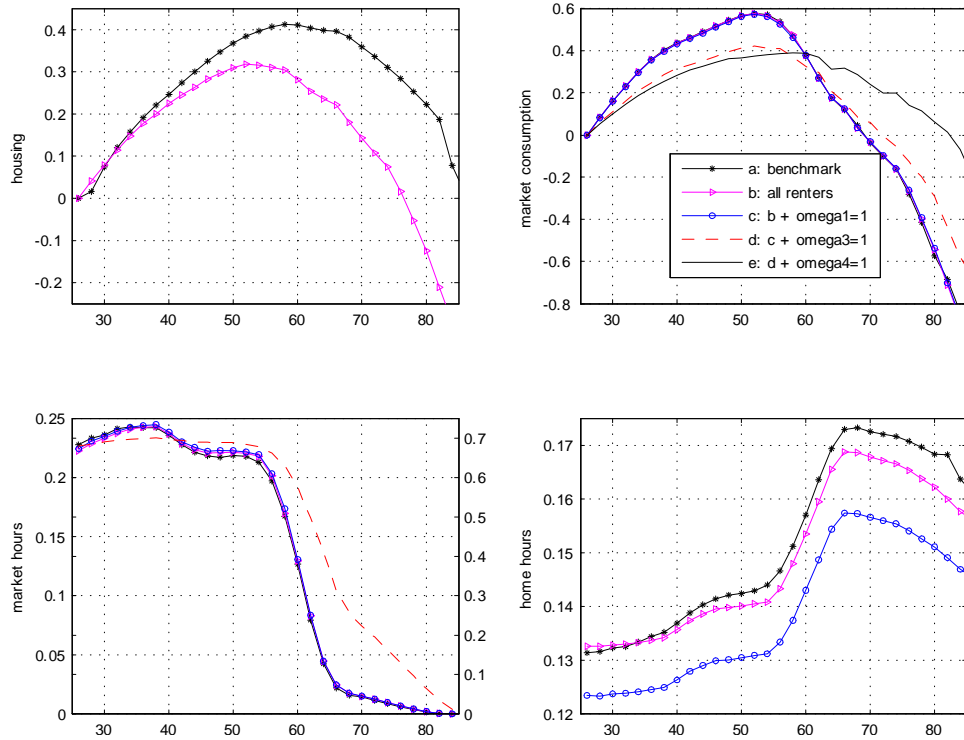
Figure A1. Exogenous Profiles – Benchmark

Figure A2. Investigating the Model's Different Mechanisms (benchmark: -*;
all-renters: -square; no housing: -circle; no home production: - -; no home production
and no leisure: −) (note: in the right bottom panel, market hours of the no home
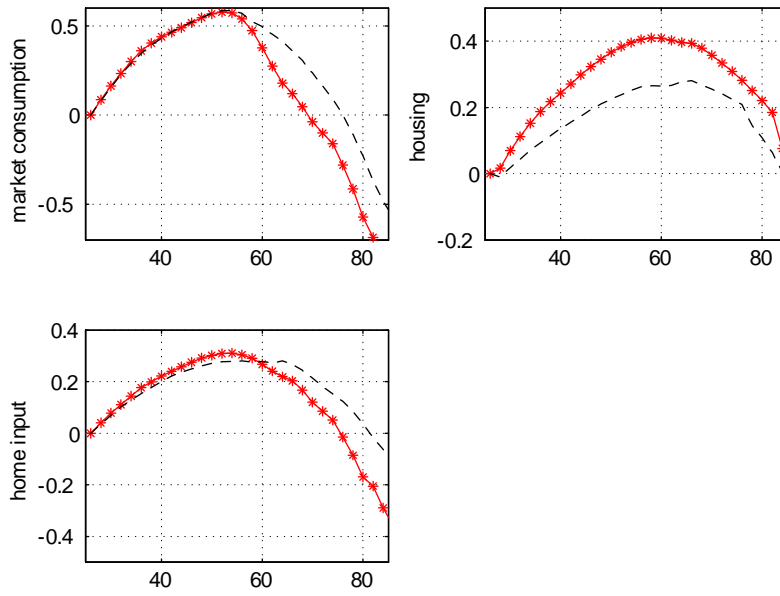production case uses the right vertical axis)

Figure A3. Consumption Profiles over the Life Cycle (-*: benchmark; - -: separable utilities)
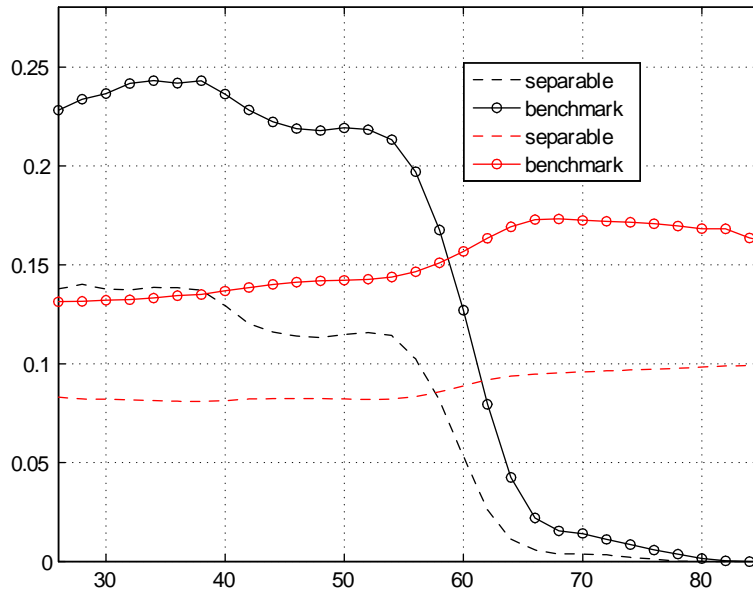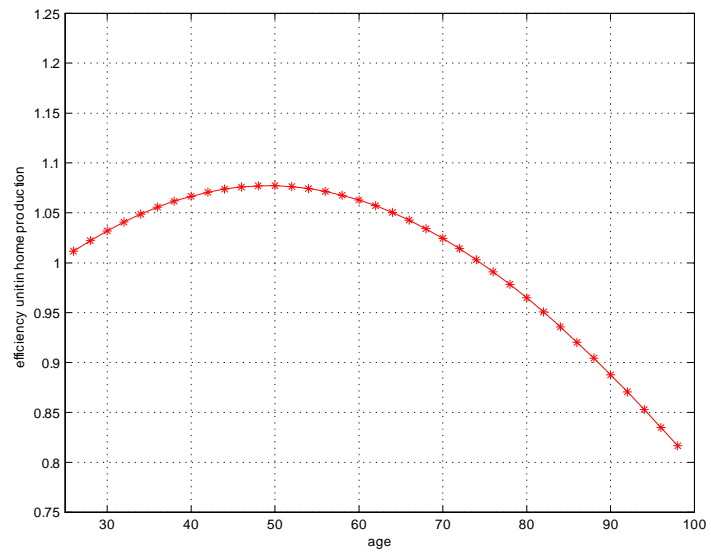

Figure A4. Hours over the Life Cycle

Figure A5. Alternative Home Production Efficiency Profile