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An Analytical Framework

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The Conundrum of Zero APR: An Analytical Framework*

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Abstract

We document the prevalence of promotional pricing of credit card debt in the U.S. and develop an analytic framework to study how interest rates on multiperiod credit line contracts should be set when debt is unsecured and defaultable. We show that according to the basic theory of unsecured credit—suitably extended to allow for promotions—interest rates should price in the expected default risk on a period-by-period basis. The inspection of our model's mechanism implies that time-consistent consumption behavior is crucial for this result; accordingly, modeling time-inconsistent consumption behavior can be one means of rationalizing promotions—as we also discuss.

Keywords: zero APR, credit cards, promotional introductory offers, unsecured debt

JEL codes: E21, D91, G20

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1 Introduction

Zero APR credit cards are the hallmark of the U.S. credit card industry. Yet, to date, very little is known about the economic mechanisms that lead to such pricing behavior. The standard theory of credit card lending in consumer finance abstracts from the long-run nature of credit line contracts, imposing a zero profit condition on period-by-period basis, while models that do allow for multiperiod credit arrangements typically restrict interest rates to either be constant throughout the contract's duration or feature no commitment to future rates.[1] These exogenous restrictions imply that we do not know the predictions of the basic theory in this regard. While we do know that zero APR solicitations are a major marketing tool for credit card companies, we do not know how consumers use these offers and what they imply for debt pricing.

The goal of this paper is to fill these gaps by, first, documenting the impact of promotions on debt pricing and, second, providing a normative analysis of how interest rates should be set on multiperiod credit-line contracts. Our results show that, while promotions are widely used by credit card borrowers, their prevalence is inconsistent with the predictions of the canonical theory of unsecured credit suitably extended to allow for the possibility of promotions. The inspection of our model's mechanism implies that time-consistent consumption behavior is crucial for this result, and hence modeling time-inconsistent consumption behavior can be one means of rationalizing promotions—as we also show. Our analysis focuses on mechanisms as opposed to quantitative predictions and our findings have qualitative character.

The empirical patterns that motivate our analysis are derived from the supervisory collection by the Federal Reserve System for the purposes of the Dodd-Frank Act Stress Test (DFAST). This dataset comprises a panel of all general purpose credit card accounts reported by all bank

[1] When discussing the standard model of unsecured lending we mean a model similar to those in Athreya (2002), Chatterjee et al. (2007), Livshits et al. (2007), or Livshits et al. (2010).

holding companies subject to DFAST. Since the regulatory requirement underlying this dataset covers approximately 70 percent of all credit card accounts in the U.S., our analysis provides a unique look at the aggregate implications of promotional offerings. We identify 4 key stylized facts characterizing promotional lending for the theory to confront:

1. Approximately a quarter of credit card debt has an introductory promotional status, in most cases featuring a zero annual percentage rate (APR) for an introductory period of more than a year.[2]

2. The expiration of promotions is associated with a sizable rate hike, of 16 percentage points on average.

3. There is no systematic change in the borrower's observed default risk (credit score) between the origination of the promotion and its expiration, and both credit scores and delinquency rates on promotional debt are about average as for credit card debt overall.

4. Promotions are associated with a large movement of debt across credit cards, with nearly half of promotional debt coming from promotional transfers of balances accumulated on other credit cards.

Our analytical framework builds on the canonical models of unsecured credit used in consumer finance but extends them by introducing competitively priced long-lived credit lines with the option of setting an introductory promotional rate immediately after contract origination.[3] Our goal is to examine whether promotions could arise in that framework. Consistent with the Credit Card Accountability Responsibility and Disclosure (CARD) Act of 2009, we assume that

[2] APR refers to the yearly interest generated by a sum that credit card borrowers pay. APR is similar to the finance charge but it additionally includes any fees or additional costs associated with the transaction and does not take compounding into account.

[3] As referenced in footnote 1.

lenders partially commit to terms. Specifically, while being able to set promotions, lenders are prevented from hiking rates or slashing credit limits to force (early) debt repayment; however, they can raise credit limits, lower interest rates, or cancel the unutilized portion of outstanding credit lines. In addition, consistent with the data (fact 4), borrowers do not commit to lenders and—subject to an exogenous refinancing friction that sustains promotions in equilibrium—they can refinance expiring promotional debt. Importantly, promotions are different from setting a shorter maturity of credit lines, since the expiration of promotions does not eliminate the risk of default for lenders.

The key insight from our model is that equilibrium credit lines price in the expected default risk on a period-by-period basis into the current interest rates. In particular, if the probability of default is expected to be p in a given period after contract origination, the interest rate that applies to borrowing against that period should price in p as the default premium for that period. Considering the stylized facts listed above, this result implies that promotions are suboptimal unless the default risk is expected to sharply rise after contract origination—which we show is not the case in the data (see stylized fact 3 above). While our model is stylized, a version of this result extends to a broad class of models, as we discuss in Section (4), and it generally implies that having large asymmetries between interest rates that apply to ex ante similar periods is suboptimal.

Intuitively, such pricing arises in our model because rational and forward looking borrowers understand that competitive lenders must break even in expectation (by the zero profit condition). As a result, they understand that equilibrium contracts must be such that the expected present value of interest payments covers the cost of defaulting. Since the cost of defaulting depends on how much debt lenders expect the borrowers to draw ex post, forward looking borrowers prefer contracts that ex post make them internalize losses their borrowing behavior implies for lenders. This implies that borrowers prefer contracts with interest rates that most closely reflect the default risk that a marginal dollar of borrowing is associated with. As we show, the underlying allocation is constrained optimal in the sense that it solves the planning problem of choosing consumption under an analogous set of physical constraints, which here includes the option of the borrowers to revert to autarky (default).

It is important to stress that the above result does not involve the logic of the tax smoothing theorem from public finance (Atkinson and Stiglitz, 1976), which independently implies that lenders should strive to "smooth" interest rate burden across periods away from the equilibrium contract we identify. Instead, the result stems from the fact that interest rates are "nondistortionary" to the extent that they reflect default risk that borrowers want to internalize for their own sake. It is the excess level of interest rates above or below that level that would be distortionary in our setup, and only in that case the tax smoothing theorem would apply. Our result also does depend on the fact that borrowers may face some risk of refinancing promotional contracts. Our model intentionally assumes away that risk.

While our result relies on basic economics, it involves the assumption that consumers are rational and time consistent—as is the case in the standard model of consumption behavior. It has been argued based on empirical evidence that this may not be the case for all borrowers (Ausubel and Shui, 2005; Agarwal et al., 2015). To illustrate this point, the second part of the paper explores the hyperbolic discounting model of consumption behavior (Laibson, 1997) as a possible behavioral explanation of the prevalence of zero APR pricing in the data. We find that, in fact, the hyperbolic model offers a potent explanation of promotions. In particular, under the so-called naivete formulation of this model, consumers underestimate the importance of the reset rates because they erroneously expect to borrow less in the future. As a result, consumers prefer contracts that feature promotional rates and high reset rates. While the logic of our baseline model still applies within this model, lenders have an additional incentive to deviate from the fixed interest rate contract to exploit the irrational behavior of borrowers. This leads to promotional pricing. Interestingly, under the sophisticated formulation of hyperbolic discounting, while this mechanism is no longer operational, promotions can still arise. In that case borrowers favor promotions because they allow them to commit to less borrowing—since ex ante they prefer such an outcome for themselves.

The two mechanisms have ambiguous regulatory implications. In the naivete case, lenders exploit consumers' bounded rationality, and that could warrant consumer protection to limit such offerings in the marketplace (to the extent that the approach of maximizing the ex ante utility of consumers that is at odds with their ex post utility is of interest to regulators).[4] In contrast, in the sophisticated case, regulators could enhance lender commitment to credit limits. But while this would eliminate promotions, it would not affect welfare. Our analysis of the hyperbolic discounting model implies that distinguishing between these two mechanisms is crucial in case it is time inconsistency that drives promotions in the data.

The literature on promotional lending is scarce. The most closely related contributions are those that document the actual behavior exhibited by credit card borrowers when they select offers from a fixed menu. In this regard, Ausubel and Shui (2005) document the results from an experiment based on mailing multiple offers to consumers and show that consumers ex post choose contracts that are time inconsistent.[5] In a complementary work, Drozd and Kowalik (2022) study the potential implications of promotions for the collapse of credit card lending during the Great Recessions and explore the quantitative consequences of promotional lending. Their work is complementary to this work because it underscores the relevance of the analysis

[4]Regulators have to take a stand that the consumer's ex-ante preferences should be maximized as opposed to her ex-post preferences.

[5]Agarwal et al. (2015) present related evidence regarding the trade-off between interest rates and fees.

herein from the applied perspective.

The remainder of the paper is organized as follows. Section 2 discusses the data. Section 3 presents the baseline model and derives our main result. Section 4 extends the model and generalizes these results. Section 5 discusses the hyperbolic model. Finally, Section 6 concludes.

2 Data

Our data come from the supervisory collection of the Federal Reserve System for the purposes of the Dodd-Frank Act Stress Test (DFAST).[6] To the best of our knowledge, we are the first ones to document the patterns in this dataset as far as promotions go. The data comprise a panel of all general purpose credit card accounts reported by bank holding companies subject to DFAST in 2018 and 2019—which, according to estimates, covers approximately 70 percent of all credit lines.[7] We focus on these two years because they precede the COVID-19 crisis and should approximately characterize the credit card market under "normal" or "steady state" economic conditions. The variables include the typical information available on credit card statements short of an itemized list of purchases. There is no information about the borrowers aside from the information pertinent to the account (e.g., credit scores, zip code etc.). The reported statistics are based on a representative sample of all accounts drawn from the full dataset by the data provider and it is fixed for research purposes using this dataset. We define credit card debt as credit card balances carried over for at least one billing cycle, which corresponds to one month. We calculate debt for each month t by taking the difference between the balances on an

[6]The dataset is confidential but it is available for all researchers within the Federal Reserve System. Replication codes are available from the authors upon request.

[7]The Bureau of Consumer Financial Protection estimates that the Y14M dataset that we are using covers approximately 70 percent of all outstanding card balances in the U.S. (see CFPB (2019), page 18). The remainder of the market are cards issued by banks with assets of less than $100 billion, or cards issued by nonbanks, such as credit unions, which are institutions that do not fall under DFAST.

account during month $t-1$ and subtracting any payments made by the borrower during month t. Whenever we report an interest rate on an account it pertains to the APR rate posted on that account.[8] A promotional account is an account flagged as promotional by the lender, with the expiration of the promotion being inferred as when that flag is removed from the account.

2.1 Stylized facts

We summarize our key findings into 4 stylized facts for the theory to confront. Our first fact shows that promotions are relevant for the pricing of a large portion of credit card debt in the U.S.

Fact 1: *Approximately a quarter of credit card debt has an introductory promotional status; in most cases, promotions involve a zero APR for an introductory period of more than one year.*[9]

As shown in Table 1, promotional debt accounts for approximately 22.5 percent of the total credit card debt, and for prime borrowers, this ratio is as high as 27 percent. Approximately 80 percent of promotional accounts involve zero APR. The average time to expiration of the promotional period is 9 months, however, the length of the promotional period nears 20 months when weighted by debt and 16 months when it is not weighted by debt.[10] The credit score of credit account borrowers is approximately average.

Our second fact shows that promotions are sizable relative to reset rates and go to rates on nonpromotional credit card accounts.

Fact 2: *The expiration of a promotion involves a sizable rate hike by about 16 percentage points*

[8] The data that we use are proprietary, but our results can be replicated within the Federal Reserve System and the codes are available upon request after appropriate clearances are obtained by the requesting party.

[9] As defined in footnote 2.

[10] Here, duration pertains to the effective duration; that is, it measures how long the promotional flag remains on accounts in a continuous fashion. This measure may involve an extension of the initial promotion and hence does not necessarily imply that this is the duration of the promotional offerings per se.

Table 1: Promotional debt: prevalence and duration.

Statistic[a] **[in % unless otherwise noted]**	2019	2018
Fraction of debt with a promotional rate[b]	22.3	22.4
Fraction of prime debt with a promotional rate[c]	27.3	27.0
Average time to promotion expiration[d] [in months]	9.6 (8.3)	8.3 (7.5)
Average duration of promotional periods[d] [in months]	19.8 (15.7)	20.4 (16.5)
Fraction of zero APR promotional accounts[a]	80.4	83.3
Fraction of promotional accounts with APR $\leq$ 3%	84.1	85.7
Fraction of promotional accounts with APR $\leq$ 6%	88.1	89.6
Average credit score on all promotional accounts	727	728
Average credit score on zero APR promotional accounts[e]	731	726
Average credit score on nonpromotional accounts	696	698

[a]We calculate each respective statistic for each month in 2018 and 2019 and then average them over each respective year. [b]We define debt as credit card balances that are carried over for at least one cycle. We calculate it on the account level in each month t by taking the difference between the balances in month $t-1$ net of payments made by the borrower in month t. [c]Prime debt includes accounts with prime credit score (e.g., minimum 670 credit scores on the account). [d]Debt-weighted; unweighted values are in the parentheses. [e]Most aggressively discounted promotional cards: 0 APR with 3 percent or less balance transfer fee. Source: Federal Reserve System, Y14M.

on average.

As shown in Table 2, the expiration of promotions is associated with a 16 percentage point rate hike on average, which is substantial. The distribution is highly skewed, with even the 10th percentile hike being sizable, at approximately 5 percentage points. The last rows of the same table show that the average APR on promotional accounts is similar to the average reset rate on nonpromotional accounts.[11]

Our next fact shows that there is no a systematic change in the borrower's observed default risk (credit score) between the origination of the promotion and its expiration, with both credit scores and delinquency rates on promotional debt being approximately average as for the credit card market as a whole. The credit score of borrowers who accept promotional terms is also

[11]Nonpromotional accounts are accounts that could have been promotional in the past. The sample includes accounts that are nonpromotional at the time of measurement.

Table 2: Cost of promotional debt.

Statistic [in %]	2019	2018
Average APR discount vis-à-vis the later reset rate[a]	16.8	17.1
- 10th percentile of APR discounts on promotional accounts	4.83	6.0
- 50th percentile of APR discounts on promotional accounts	17.3	18.0
- 90th percentile of APR discounts on promotional accounts	25.0	25.2
Average APR on nonpromotional accounts[c]	18.9	18.2
Median APR on nonpromotional accounts	18.1	17.6
Average APR on accounts that were never promotional[d]	18.7	18.0
Median APR on accounts that were never promotional	18.0	17.3

Notes from the previous tables apply. [a]Debt-weighted statistics (reported percentiles are thus effectively for dollars of outstanding debt as a unit of observation). [b]APR discount is the difference between the promotional APR on the account and the nonpromotional reset rate on the same account. [c]Nonpromotional account as of the billing cycle at the time of measurement. [d]Nonpromotional account since origination of the account as of the billing cycle at the time of measurement. To obtain this statistic we average monthly statistics throughout our sample period (years 2018 and 2019).

about average.

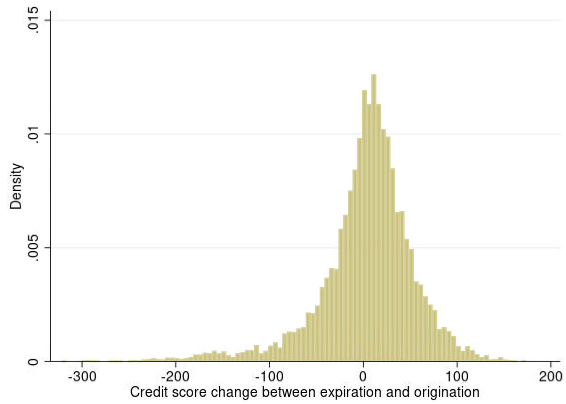
Fact 3: *There is no systematic change in the borrower's observed default risk (credit score) between the origination of the promotion and its expiration; both credit scores and delinquency rates on promotional debt are similar to the average values for credit card market as a whole.*

The average credit score on promotional accounts is 732 in the first 3 months after the start of the promotional period and 736 in the first three months after the end of the promotion.[12] Similarly, the account level difference in credit scores between the expiration (3 months average after expiration) and the origination (3 months average after origination) of a promotional account is only 3 points. The median is also positive, at 8 points. These changes are economically insignificant and the average or median borrowers' credit score goes up during the promotional period. Figure 1 shows a plot of the histogram of score changes in our data, which are widely dispersed but show no systematic pattern. In conclusion, these data show that there is no basis for lenders to expect a systematic change of default risk between the originations and expirations

[12]Since credit scores are sensitive to monthly changes in the credit card utilization we look at the 3-month averages around the events rather than a 1-month average.

of promotional accounts. As we later show, our basic theory could rationalize promotions had this been the case in the data, and it is thus an important property to note.

Figure 1: Histogram of credit score changes between the origination and expiration of promotional accounts.

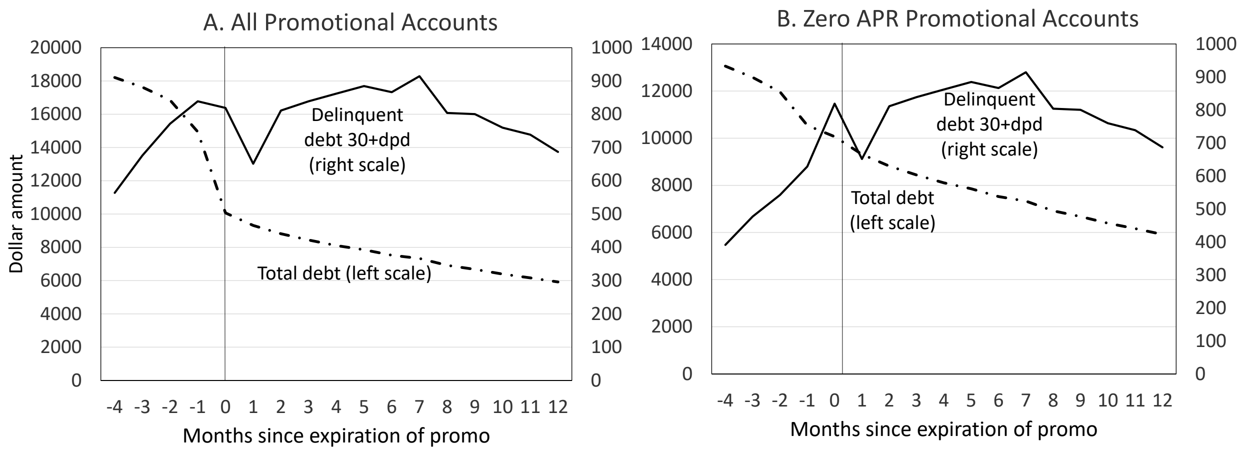


Notes: This figure shows a plot of the histogram of the changes in credit scores across promotional accounts during the promotional period. To calculate this change, we take the average credit score on the account over the first 3 months after the expiration of the promotion and subtract the average score on the same account over the first 3 months after the origination of the promotion. The credit score change is an unweighted statistic calculated across all promotional accounts throughout the sample period. Source: Federal Reserve System, Y14M.

Table 3 shows delinquency rates on promotional accounts in comparison to nonpromotional accounts. The reported delinquency rates correspond to the fraction of outstanding debt that is 30+ days past due and 120+ days past due and has not yet been written off by the lender—which occurs after approximately 180 days (or after a successful bankruptcy ruling).[13] Compared to nonpromotional accounts, delinquency rates on promotional accounts are lower before the expiration of the promotional period and substantially higher after the expiration. However, as Figure 2 shows, debt repayments rather than additional delinquent debt is the main reason why delinquency rates spike after the expiration. From the perspective of the entire life cycle of promotional debt, the delinquency rate on promotional debt appears to be about average.

[13]The reported statistics generally do not include accounts that are 180+ days past due and without any payment, nor accounts discharged via bankruptcy, since this leads to a statutory write off and the account is removed from our dataset.

Figure 2: Delinquent debt and total debt around the expiration of the promotions.



Notes: This figure shows a plot of the debt and delinquent debt that is 30+ days past due and has not (yet) been written off (typically after 180 days past due or after bankruptcy discharge). The left panel includes all promotional cards and the right panel reports the same for the most aggressively discounted promotional cards (0 APR cards with 3 percent or less balance transfer fee). The pool of accounts is fixed and they come from different time periods in 2018 and 2019, all centered around the expiration of the promotional period ("0" on the horizontal axis). Source: Federal Reserve System, Y14M.

While it is possible that some delinquent debt is recovered later on, as long as recoveries on promotional accounts are no higher than that on all accounts, the 3 percent balance transfer fee appears grossly insufficient to cover the default losses suffered by lenders on the zero APR promotional accounts (those accounts with zero APR, a 3 percent balance transfer fee, and a promotional period of at least 12 months).[14] Transaction fees are too small to make up the difference and there are also other costs associated with maintaining credit card accounts.[15] It thus appears that credit card lenders are losing money during the promotional period on the majority of these accounts.

Finally, our last (complementary) fact shows that much of the promotional debt is associated with active repricing of existing card debt and prevalent card flipping that underlies promotional activity.

Fact 4: *Promotions are associated with a large movement of debt across credit cards, with*

[14]The charge-off rate on credit card debt reported by the Federal Reserve Board of Governors was above 3 percent during this time period.

[15]The data reported by Evans and Schmalensee (2005) suggest that the cost of funds and charge-offs account for less than two-thirds of the total costs of the credit card industry.

Table 3: Average delinquency rates on credit card debt.

Statistic [in %]	30+ dpd[a]	120+ dpd
All promotional accounts:		
- 2 months before promotion expiration	4.6	2.7
- 2 months after promotion expiration	9.2	5.6
- 5 months after promotion expiration	11.3	7.0
0 APR promotional account (3% or less BT fee):[b]		
- 2 months before promotion expiration	4.5	2.8
- 2 months after promotion expiration	9.2	5.6
- 5 months after promotion expiration	11.3	7.0
All accounts	6.7	3.5
Nonpromotional accounts[c]	7.9	4.2

Notes from the previous tables apply. [a]30 or more days past due credit card debt that has not been written off by the lender. Delinquent credit card debt is generally written off after 180 days past due and after debt is discharged in bankruptcy court.[b]This category includes the most aggressively priced promotional account; that is, those with zero APR and a 3 percent or less balance transfer fee. [c]Accounts that are nonpromotional at measurement; together the two categories cover all accounts.

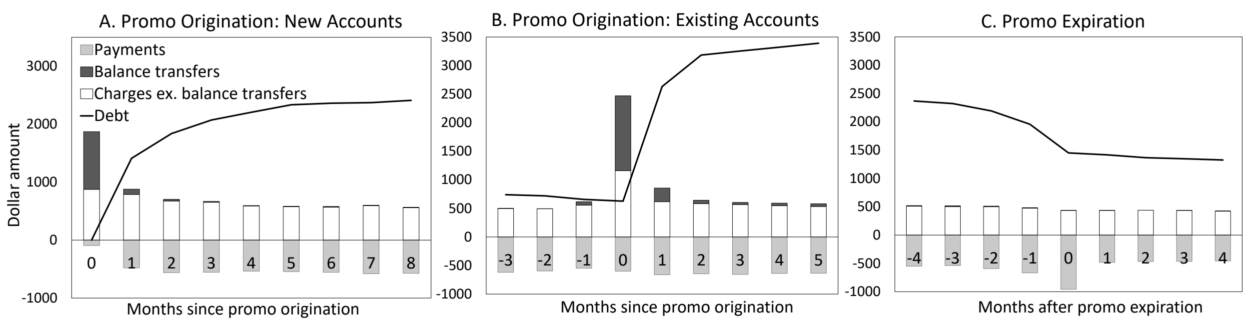
nearly half of promotional debt originating from promotional transfers of balances accumulated on other credit cards.[16]

The first two panels of Figure 3 show plots of charges and balance payments for a cohort of accounts originated in early 2019 and tracked until early 2020—for both newly originated promotional accounts (Panel A) and existing nonpromotional accounts that become promotional (Panel B). The solid line represents the stock of debt on these accounts, which corresponds to the cumulated net value of charges and (re)payments on the account since its origination. As shown, the bulk of debt accumulation on promotional accounts occurs in the first few months. Importantly, balance transfers are the key driver of charges early on, especially among the existing accounts (Panel B). Interestingly, while the expiration of promotions is definitely associated with accelerated debt repayment, a significant fraction of debt remains unpaid for months after the expiration (Panel C). This aspect of the data suggests how lenders may break even on even the most aggressively priced promotional accounts.

[16]A promotional balanced transfer features an introductory promotional rate.

The annualized flow of promotional balance transfers is about 15 percent relative to all card debt outstanding. This is reported in Table 4. Assuming the average expiration of an account of about 18 months, the volume of balance transfers is in line with the volume of expiring promotional debt. While we do not observe outbound balance transfers, the jump in payments around the expiration of the promotional status (Figure 3, Panel C) is consistent with the idea that balance transfers are accelerating debt repayments near the expiration date.

Figure 3: Charges and payments over the life cycle of promotional cards.



Notes: This figure shows the life cycle of new promotional accounts and newly promotional existing accounts. We plot monthly charges excluding balance transfers, such as fees, purchases, cash advances (white bar), inbound balance transfers (black bar), and balance (re)payments (gray bar). The accumulated stock of debt is the cumulation of monthly charges, balance transfers and payments (flows). Source: Federal Reserve System, Y14M.

A balance transfer carries an additional fee, even if it is promotional. This creates an additional source of revenue for lending even when the APR on the promotional account is zero. Table 4 reports the fees charged on balance transfers. As we can see, a typical balance transfer involves a fee of 3 percent of the transferred amount. Only in approximately 15 to 20 percent of cases balance transfers are free.

3 Basic theory of credit line pricing

In this section, we present our baseline theory and examine whether it can be consistent with the facts documented in the previous section. We relax some of the simplifying assumptions

Table 4: In-bound balance transfers to promotional cards.

Statistic [in %]	2019	2018
Annual flow of balance transfers to total credit card debt[a]	15	14
Annual flow of balance transfers to promotional card debt[a]	69	64
Promotional balance transfers as a fraction of total	94.2	94.4
- with zero balance transfer fee[b]	14.7	19.8
- with balance transfer fee $\leq 3\%$	65.0	68.6
- with balance transfer fee $\leq 6\%$	99.9	99.9

Notes from the previous tables apply. [a]Balance transfers represent inbound balance transfers (flow of balances coming in from some other account). [b]As a fraction of all promotional balance transfers.

made here in Section 4 and generalize our results to a broader class of models. The last sections consider hyperbolic discounting within this setup.

3.1 Environment

There are three periods denoted by $t = 1, 2, 3$.[17] The economy is populated by a large number of lenders and a large number of consumer families. Consumer families comprise a mass 1 of identical members (shoppers) who share consumption risk.[18] Lenders face zero cost of funds and must break even in expectation. There is no aggregate uncertainty and the law of large numbers is assumed throughout. The market is competitive and lenders compete in a Bertrand fashion. The presence of risk sharing within consumer families removes the risk of refinancing and gives the model the best chance of generating promotions. Information is symmetric.

[17]It is possible to generalize our main results to a model to multiple periods and instead assume that the contract's duration (maturity) is three periods. We choose a three period setup to simplify the exposition.

[18]This assumption removes refinancing risk that would otherwise be present and further discourage promotions.

3.1.1 Consumers

Each member of a family starts with income y and debt $b_0 > 0$ in period 1. With probability $0 < p < 1$, the income of all members of a given family switches to a low income level $y - \Delta$ in period 2, or period 3, where $0 < \Delta < y$ denotes the size of the negative income shock. The income shocks across families are independent but they are perfectly correlated across family members. We refer to income level y as the *high state* and income level $y - \Delta$ as the *low state*. As mentioned, this feature of our model removes the risk of refinancing, which could be another reason why promotions are suboptimal.

The family evaluates consumption streams according to the expected utility function given by $\mathbb{E}\left\{u(c_1) + \beta u(c_2) + \beta^2 u(c_3)\right\}$, where, because of risk sharing within the family, c_t pertains to the average consumption of its members in period t, $0 < \beta \leq 1$ is the discount factor, and u obeys the neoclassical assumptions.[19]

As a first pass, we assume that default on debt is nonstrategic and occurs in the low income state; that is, the family intends to repay its debt, and it is the catastrophic nature of the negative income shock that leads to default. We later generalize our results to allow for strategic (endogenous) default. Throughout, we restrict attention to type-identical allocations, which means that the decisions of each family member are the same whenever the state for that member is the same.

3.1.2 Lenders

Lenders compete in a Bertrand fashion and extend unsecured credit lines to individual family members to, in effect, maximize the utility of the consumer family subject to a zero profit condition that must hold in expectation. A credit line is a vector $\mathcal{C} = (r, l, R, L)$ and it comprises

[19]The flow utility u function is continuously differentiable, strictly increasing, and strictly concave and the Inada condition applies whenever consumption nears zero.

introductory terms specifying the introductory interest rate $r \geq 0$, the introductory credit limit $l \geq 0$ (balance transfer offer) and the reset terms that specify an interest rate $R \geq 0$ and a credit limit $L \geq 0$. The introductory terms apply in the first period after the contract is extended and they reset to R, L thereafter. There is no need to restrict how r relates to R, or how l relates to L, although we will be looking for promotional terms with $r \leq R$ and it is natural to also think that $l \leq L$. Each member of the family can hold one credit line at a time, which is a standard assumption in the literature.[20]

While lenders commit to terms as of contract origination, they are allowed to "sweeten" the terms later on; that is, we allow lenders to set new promotions by reducing interest rates or raising credit limits. Credit limits can be slashed when the line is not utilized but lenders cannot force early debt repayment by reducing credit limits or raising reset rates. These restrictions are consistent with the Credit Card Accountability Responsibility and Disclosure (CARD) Act of 2009.[21]

3.1.3 Timing and budget constraints

The timing of events in the model is as follows:

[20]While this assumption is at odds with the data, it is not clear to us what mechanism associated with nonexclusivity of contracts in the data could explain promotions.

[21]According to the Credit CARD Act of 2009, lenders must maintain interest rates on accounts up to five years. In particular, lenders cannot slash credit limits on the utilized portion of the credit line to force early debt repayment. They can also set promotional rates that expire after a specified period of time as long as it is longer than 6 months. Before the CARD Act, term changes were possible and took place occasionally. However, anecdotal evidence still suggests that banks recognized the value of reputation and avoided changing terms. For example, Capital One and Citi, which together account for approximately thirty percent of the market, before the CARD Act of 2009 were contractually committing themselves to offer opt out options from any rate changes other than the ones triggered by noncompliance (e.g. late payments, overdraft). In early 2008, Chase followed suit and also adopted an internal rule of not responding to any credit history changes unrelated to the account when reviewing terms. The OCC openly discouraged national banks from the practice of changing terms on credit cards (see OCC Advisory Letter, AL 2004-10). For this and other evidence, see the Appendix for H.R. 5244 "The Credit Cardholders' Bill of Rights: Providing New Protections for Consumers," Hearing before the Subcommittee on Financial Institution and Consumer Credit of the Committee on Finance Services, U.S. House of Representatives, One Hundred Tenth Congress Session, April 17, 2008, Serial no. 110-109. Pages 280, 327, 371, 373-379, and 410 are of particular interest.

1. At the onset of the first period, members of the family shop for credit lines. There is unimpeded access to credit and since we restrict attention to type-identical allocations all members accept the same terms. Let those terms be denoted by $\mathcal{C} = (r, l, R, L)$, where the notation is as explained above. After receiving the contracts, the consumer family chooses borrowing level $b_1 \leq l$ for each member, and the budget constraint determines consumption in the first period:

$$c_1 = y - b_0 + b_1 - rb_1^+, \tag{1}$$

where, throughout, we use the notation $b_1^+ := \max\{b_1, 0\}$, and rb_1^+ are interest payments made to the lender. If the income state switches to low in the second period, the consumer family defaults on their debt and each member's consumption is[22]

$$c_2^d = y - \Delta. \tag{2}$$

2. If the high income state y persists to the next period, the incumbent lender reprices the initial credit line to $\mathcal{C}' = (r', l', R, L)$ subject to the stylized CARD Act constraints, which boil down to $r' \leq R$ and $l' \geq b_1$. The consumer family then decides how much to consume in the second period, which amounts to borrowing or repaying $b_2 - b_1$, and results in debt $b_2 \leq l'$ placed on the repriced credit line.

3. Each member of the family then applies to refinance debt b_2 placed on the repriced incumbent's line. Access to credit is impeded at this point, and each individual member

[22] Implicitly, we assume that the initial lender retracts the unutilized portion of the credit line when default occurs and this way prevents the consumer from drawing more funds. The borrower cannot save borrowed funds from the first period and consume them in the default state. This is a standard assumption but we later consider a generalization of the model that relaxes these assumptions and we show that it has no bearing on our results.

manages to obtain a refinance offer $\mathcal{C}'' = (r'', l'')$ with probability $0 < \rho < 1$, with the $1 - \rho$ mass of members receiving no offer. A refinance offer allows the transfer of debt b_2 of an atomless member onto the new credit line and, as we show later, it is without a loss to assume that a refinance offer features $l'' \geq b_2^+$.[23] Refinance risk is shared among family members, which implies that the second-period consumption is

$$c_2 = y - b_1 + b_2 - (1 - \rho')r'b_2^+ - \rho' r'' b_2^+, \tag{3}$$

where $0 \leq \rho' \leq \rho$ is the mass of refinance offers that the family chooses to exercise (in equilibrium we will have $\rho' = \rho$ or $\rho' = 0$ by linearity). If the income switches to the low state in the third period, the consumer family defaults on their debt and consumption is

$$c_3^d = y - \Delta. \tag{4}$$

4. If income y persists in the third period, the family repays and third-period consumption of each member is

$$c_3 = y - b_2. \tag{5}$$

3.2 Consumer problem

It is convenient to set up the consumer problem by starting from the second period and working backwards. As of the second period, and starting in the high income state (repayment state), each consumer family member holds debt b_1 (savings if negative) and each member has a repriced incumbent's credit line $\mathcal{C}' = (r', l', R, L)$ on hand. The family chooses the second-period

[23] That is, the consumer family member requests refinancing of b_2^+. As we show below, it is optimal for the second period lender to refinance in full.

borrowing for each member, $b_2 \leq l'$, and exercises a fraction $0 \leq \rho' < \rho$ of mass ρ of refinance offers $\mathcal{C}''(\mathcal{C}', b_2) = (r''(\mathcal{C}', b_2), l'' = b_2)$. These choices solve

$$V(\mathcal{C}', b_1) = \max_{b_2 \leq l', 0 \leq \rho' \leq \rho} u(c_2) + \beta\left((1-p)\, u(c_3) + pu\left(c_3^d\right)\right), \tag{6}$$

where c_2, c_3, c_3^d are given by (3), (5) and (4), respectively. As of the first period, and given contract $\mathcal{C} = (r, l, R, L)$, the family chooses borrowing $b_1 \leq l$ to maximize

$$U(\mathcal{C}) := \max_{b_1 \leq l} u_1(c_1) + \beta\left((1-p)\, V(\mathcal{C}'(\mathcal{C}, b_1), b_1) + p\left(u_2\left(c_2^d\right) + \beta D\right)\right), \tag{7}$$

where c_1, c_2^d are given by (1) and (2), respectively. D stands for an exogenous continuation value function after default severs the relationship with the existing lender; $\mathcal{C}'(\mathcal{C}, b_1)$ denotes the equilibrium repricing policy of the incumbent lender.

3.3 Lender problem

Lenders are Bertrand competitors and offer contracts that maximize consumers' utility subject to zero profits in expectation. Lenders have deep pockets and, as a matter of normalization, they face zero cost of funding. Let consumer policy functions be denoted by $b_1(\mathcal{C})$, $b_2(\mathcal{C}'(\mathcal{C}, b_1), b_1) \equiv b_2(., b_1)$ and $\rho'(\mathcal{C}'(\mathcal{C}, b_1), \mathcal{C}''(\mathcal{C}'(\mathcal{C}, b_1), b_2), b_2) \equiv \rho'(., b_2)$, where $\mathcal{C}''(\mathcal{C}'(\mathcal{C}, b_1), b_2)$ denotes the equilibrium refinance contract. The expected profit of the lender in the first period is

$$\Pi(\mathcal{C}) := (r-p)\, b_1^+(\mathcal{C}) + (1-p)\,(1-\rho'(., b_2))\,(r'-p)\, b_2^+(., b_1), \tag{8}$$

where the profit from the first period corresponds to the excess interest revenue over the default premium $r - p$ accrued on debt drawn in the first period b_1^+ and analogously in the second period assuming the borrower does not default—which takes place with probability $1-p$—with the interest rate accrued only on the nonrefinanced fraction $1-\rho'$ of the credit line. The second-period lender's zero profit implies that refinance interest rate corresponds to the default probability:

$$(r''-p)\, b_2^+(.,b_1) = 0 \Rightarrow r'' = p.$$

This expression implies that the equilibrium interest rate on the refinanced credit line is p regardless of the amount that the family may choose to borrow, which also validates our earlier assumption that the second-period lenders do not impose a stricter credit limit than b_2^+.[24]

3.4 Equilibrium

Bertrand competition requires that the initial lender's offer $\mathcal{C}$ maximizes the family's expected utility subject to that lender's zero profit condition in expectation:

$$\max_{\mathcal{C}} U(\mathcal{C}) \quad s.t. \quad \Pi(\mathcal{C}) = 0. \tag{9}$$

We refer to a contract that solves the above as the *equilibrium contract* and characterize it below. But, before we proceed, we restate the above contracting problem in a form that is amenable to analysis. To that end, we note the following key properties of the above contracting problem:

1) A rational consumer will anticipate any ex post repricing of the initial contract, so her decisions will depend on the anticipated repriced terms rather than the original terms. Conse-

[24] Without this assumption, the family could be forced to choose to partially refinance the line, with a residual debt $b_2 - l''$ still remaining on the incumbent's partially refinanced lines. Since this would never occur in equilibrium, we assumed that refinance offers are for an amount b_2^+ that borrowers request. The friction ρ is a technological constraint on the borrower's ability to access the market. An alternative approach would be to recast this friction as a search friction. However, this would require search costs being borne by lenders and priced into contracts. In such a case, the last condition would need to be modified to generate a strictly positive profit flow per accepted offer to cover the posting costs. It will later be clear that such an extension would make no difference in terms of results.

quently, without loss, we can assume that the initial lender offers anticipated terms right away; that is, without a loss, we assume $\mathcal{C}''(\mathcal{C}', b_2) = (R, L, .)$ and restrict attention to the subset of feasible contracts that the lender would not find profitable to reprice ex post.[25]

2) Equation (3) implies that the refinancing policy of the family exhibits a bang-bang property; that is, $\rho' = \rho$ if $R > p$, and $\rho' = 0$ otherwise.

3) Finally, the "kink" implied by refinancing both in consumer budget constraint and lender zero profit condition can be eliminated to obtain a more tractable problem featuring a single "stand-in" lender. This ensures a globally differentiable and well-behaved concave contracting problem. The intuition is straightforward: Since the risk of refinancing is removed, initial lenders can always raise interest revenue in the second period by setting the reset rate at a higher level due to the refinance friction $\rho > 0$, and what matters for the consumers is how much they pay on net for credit.[26] To show this formally, let us define

$$\mathcal{R}\left(\hat{R}\right) = \frac{1}{1-\rho} \max\left\{\hat{R} - p, 0\right\} + \min\left\{\hat{R}, p\right\}, \tag{10}$$

which, note, is a strictly increasing and nonnegative function for all $\hat{R} \geq 0$. It is easy to verify that $\mathcal{R}\left(\hat{R}\right) = \hat{R}$ for all $0 \leq \hat{R} \leq p$, and so the transformation has no bite unless $\hat{R} > p$. At $\hat{R} = p$, the function has a kink and steepens but it remains linear. It is easy to verify that for the optimal ρ', which is $\rho' = \rho$ if $R > r'' = p$ and $\rho' = 0$ otherwise, as noted in 2 above, the budget constraint in (1) boils down to $c_2 = y - b_1 + b_2 - \hat{R}b_2^+$, for all $\hat{R} \geq 0$. Finally, after making

[25] The proof of this fact is straightforward. Suppose the lender offers $\mathcal{C} = (r, l, R, L)$ and the borrower expects this contract to be repriced ex post to $\mathcal{C}' = (r', l', R, L)$, where by the CARD Act restrictions we must have $r' \leq R, l' \geq b_1$. By rational expectations, the borrowers' expectations are correct and this is what occurs ex post. The borrower will choose the exact same borrowing level b_1 and consumption c_1 if the initial contract is instead $\hat{\mathcal{C}} = (r, l, r', l')$, in which it is not repriced because the consumer's state is identical to that in the first case. Accordingly, we have $U(\mathcal{C}) = U(\hat{\mathcal{C}})$ and $\Pi(\mathcal{C}) = \Pi(\hat{\mathcal{C}})$, which proves the claim because the restricted set of contracts that are not repriced ex post attains the same value of the program.

[26] Note that refinance risk would only discourage promotions, which is another reason why they would be suboptimal in the models. Our analysis focuses on the most favorable case for promotions to arise by assuming away refinance risk.

the same substitution, the initial lender's zero profit condition is $\Pi = (r-p)\, b_1^+ + p\left(\hat{R}-p\right) b_2^+$, for all $\hat{R} \geq 0$. The function $R\left(\hat{R}\right)$ is a bijection and can be inverted to recover the original credit terms.

The lemma below combines these properties to restate the equilibrium contracting problem above in a form that is more amenable to further analysis. The transformed consumer problem is now a standard concave programming problem, since the budget constraint is linear and the objective function is strictly concave. Accordingly, constraints IC_1, IC_2, and CL of the maximization EQ in (11) correspond to, in that case, the necessary and sufficient Karush-Kuhn-Tucker (KKT) conditions for the consumer problem, and the rest is just a restatement of (9) after applying the above simplifications.[27] Importantly, the last restriction (condition 3 in the lemma) ensures that we restrict attention to contracts that would not be repriced ex post, as noted in point 1 above.

Lemma 1. *$\mathcal{C} = (r, l, R, L)$ is an equilibrium contract if and only if the following conditions are met:*

1) There exist $\hat{R} \geq 0$ such that $R = \mathcal{R}\left(\hat{R}\right)$ and $\hat{\mathcal{C}} = \left(r, \hat{R}, l, L\right)$ solves

$$EQ: \max_{r,\hat{R},l,L,b_1,b_2} u(c_1) + \beta(1-p)\, u(c_2) + \beta^2 (1-p)^2 u(c_3) + \beta U^d \tag{11}$$

where

$$BC \quad : \quad c_1 = y - b_1 + b_1 + r b_1^+, \; c_2 = y - b_1 + b_2 + \hat{R} b_2^+, \; c_3 = y - b_2, \tag{12}$$

[27]If borrowing is on the constraint (e.g. $b_1 = l$), that constraint can be either binding or nonbinding, and hence the inequality in IC_1, IC_2. If $b_1 < l$ we require an interior solution, and hence the constraint then must hold with equality.

subject to

$$
\begin{aligned}
IC_1 \quad &: \quad \left(u'\left(c_1\right)\left(1-r\right)-\beta\left(1-p\right)u'\left(c_2\right)\right)\mathbf{1}_{b_1=l}\geq 0, \\
&\quad \left(u'\left(c_1\right)\left(1-r\right)-\beta\left(1-p\right)u'\left(c_2\right)\right)\mathbf{1}_{b_1<l}=0, \\
IC_2 \quad &: \quad \left(u'\left(c_2\right)\left(1-\hat{R}\right)-\beta\left(1-p\right)u'\left(c_3\right)\right)\mathbf{1}_{b_2=L}\geq 0, \\
&\quad \left(u'\left(c_2\right)\left(1-\hat{R}\right)-\beta\left(1-p\right)u'\left(c_3\right)\right)\mathbf{1}_{b_2<L}=0 \\
ZP \quad &: \quad \left(r-p\right)b_1^{+}+p\left(\hat{R}-p\right)b_2^{+}=0, \\
CL \quad &: \quad b_1^{+}\leq l,\ b_2^{+}\leq L,
\end{aligned}
\tag{13}
$$

where $\mathbf{1}_{b_1<l}$ *is an indicator function that equals* 1 *if the subscripted constraint is true and 0 otherwise. (Note:* $U^d=p\left(u\left(c_2^d\right)+\beta D+\beta(1-p)u\left(c_3^d\right)\right)$ *stacks constant terms associated with default state.)*

3) The lender does not find it (strictly) profitable to reprice the contract $(r,l,\hat{R},L)$ *under the CARD Act restrictions.*

It is clear that the equilibrium interest rates r and $\hat{R}$ are determined if there is borrowing in equilibrium. For this reason, our analysis naturally focuses on such a case, although we do not need to explicitly assume it because it turns out to be without loss in the baseline setup. The lemma below establishes this result. (All proofs are in the appendix unless otherwise stated.)

Lemma 2. *In equilibrium,* $b_1>0$ *and* $b_2>0$.

3.5 Main result and the key intuition

Proposition (1) states the main result of our paper: the equilibrium contract that satisfies Lemma 1 is $r=p=R(=\hat{R})$, with nonbinding credit limits and no refinancing taking place in equilibrium ($\rho' = 0$). In particular, the equilibrium features no promotions, which is the key take away. The proof of this proposition and the intuition behind it is discussed in the text below and the more technical details are relegated to the appendix.

Proposition 1. *Equilibrium allocation satisfies*

$$u'(c_1) = \beta u'(c_2) = \beta^2 u'(c_3). \tag{14}$$

The supporting equilibrium contract is: $r = p = \hat{R}(= R)$, l, L is nonbinding, and it uniquely implements the constrained optimal allocation among all promotional contracts featuring a promotion: $r \leq R$ $\left(r \leq \hat{R}\right)$.[28]

The first part of the proposition states that the equilibrium consumption profile equalizes the marginal utility across the periods up to the discount factor β, that is, (14) holds. We begin by discussing the proof of how this condition leads to the state contract and after that we prove this condition applies.

Let us first consider contracts featuring nonbinding credit limits assuming the allocation is such that (14) holds. Given the consumer's Euler equations (constraints IC_1, IC_2 in 13 of EQ in (11)), satisfying the Euler equations with a nonbinding credit limit requires $r = p = \hat{R}$ and such a contract (trivially) satisfies the zero profit condition ZP in (13) for any b_1, b_2. This contract cannot be repriced to increase profits ex post, since $\hat{R}$ can only be lowered under the CARD Act restrictions and this will lower lender profits for any $b_1 > 0$ (and any credit limit/borrowing). Accordingly, by Lemma 1, the conditions for equilibrium are satisfied.

It turns out that this is a unique implementation; in particular, there is no contract with $r < \hat{R}$ and binding credit limits that also satisfies condition (14). To see this, note that, by the

[28] By "inactive" we mean that the Lagrange multipliers assigned to the IC_1, IC_2 constraints are all zero.

zero profit condition ZP in (13), $r < \hat{R}$ implies $\hat{R} > p$ (hence also $R > p$), but $\hat{R} > p$ (with binding L) violates condition 3 of Lemma 1 because, by (13), the lender earns $\hat{R} - p > 0$ on the marginal dollar borrowed in the second period and by relaxing the binding credit limit L in ii the lender can increase profits (the borrower is also better off).

We next prove that the allocation must satisfy (14), which we assumed is the case so far. To that end, we set up an auxiliary planning problem featuring the same objective function as in EQ but a looser set of constraints than those under EQ—which applies because of Lemma 2 (the resource constraint involving saving in any of the periods is different). In particular, let $c_1 = y - B$, $c_2 = y$, $c_3 = y$ be the autarkic consumption profile of the consumer, and consider a planning problem of choosing transfers T_1, T_2 and T_3 relative to autarkic allocation that solve

$$PL : \max_{T_1,T_2,T_3} u\left(c_1 + T_1\right) + \beta\left(1 - p\right) u\left(c_2 + T_2\right) + \beta^2 \left(1 - p\right)^2 u\left(c_3 + T_3\right) + \beta U^d \tag{15}$$

subject to

$$RC : T_1 + \left(1 - p\right) T_2 + \left(1 - p\right)^2 T_3 = 0, \tag{16}$$

where, as in Lemma 1, U^d captures the constants associated with default.

The key difference between the above planning problem and the lender problem underlying EQ in (11) is that the planner here can directly choose consumption in the high income state, while the lender chooses it indirectly by choosing credit terms. In particular, the planner does not need to obey the constraints IC_1, IC_2 and CL, and the resource constraint RC above replaces the zero profit condition ZP.

This auxiliary planning problem PL in (15) comes in handy because of how it relates to the equilibrium defined by EQ in (11). This is established in the lemma below, which shows that, if the consumer's budget constraint BC in (12) and the zero profit condition ZP in (13) are both satisfied for some consumption profile, the planner's resource constraint RC in 16 is automatically satisfied for the transfers T_1, T_2 and T_3 that sustain the same level of consumption. The converse also holds but in a narrower sense: any zero profit contract that sustains the planner's consumption profile in the sense of satisfying constraints IC_1, IC_2 and CL also satisfies the consumer's budget constraint BC and hence is a candidate solution to EQ. Consequently, by that lemma, if we find a unique solution to the planning problem PL in (15), and manage to identify a supporting zero profit contract that i) the lender would not find it profitable to reprice and ii) shows that the constraints IC_1, IC_2 and CL in EQ hold, we can be sure that we have found the equilibrium contract. Moreover, that contract is unique to the extent that there is no other contract that sustains the same consumption allocation because the solution to the above planning problem is unique by concavity.

Lemma 3. *1) Suppose c_1, c_2, c_3 is the consumption profile that satisfies the constraints of EQ in (11) and involves borrowing in both periods ($b_1 > 0, b_2 > 0$). Then, the transfers $T_1 = c_1 - (y - b_0)$, $T_2 = c_2 - y$, $T_3 = c_3 - y$ that sustain the same consumption profile under the maximization PL in (15) satisfy the resource constraint RC in (16). 2) Conversely, consider any nonnegative c_1, c_2, c_3, and associated transfers $T_1 = c_1 - (y - b_0)$, $T_2 = c_2 - y$, $T_3 = c_3 - y$ that satisfy the planner's resource constraint RC. Furthermore, suppose there exists a contract $\left(r, l, \hat{R}, L\right)$ satisfying $l \geq -T_2 - T_3\left(1 - \hat{R}\right)$, $L \geq -T_3$, as well as constraints IC_1, IC_2, ZP of EQ in (11) for $b_1 = -T_2 - T_3\left(1 - \hat{R}\right), b_2 = -T_3$. Then, the contract $\left(r, l, \hat{R}, L\right)$, consumption profile c_1, c_2, c_3, and b_1, b_2 satisfy all constraints of EQ.*

To derive (14), we use the lemma in conjunction with the necessary and sufficient conditions to the planning problem PL in (15).[29] Since this is a simple intertemporal consumption choice

[29]This is a standard concave programming problem featuring a linear constraints and a strictly concave objective function. There is a unique global maximum, and first order Lagrange conditions are both necessary and sufficient.

problem, the marginal conditions characterizing the solution involve the equalization of the marginal rate of substitution (MRS) implied by the objective function to the marginal rate of transformation (MRT) implied by the resource constraint RC in (16); in particular, we have

$$MRS_1 := -\frac{u_1'}{\beta(1-p)u_2'} = -(1-p)^{-1} =: MRT_1, \tag{17}$$

$$MRS_2 := -\frac{u_2'}{\beta(1-p)u_3'} = -(1-p)^{-1} =: MRT_2, \tag{18}$$

where u_t' denotes the marginal utility in period t. These conditions are intuitive and omit the discussion of what they entail. It is easy to combine these conditions to see that they boil down to (14), which we set out to establish. By Lemma (3), this completes the proof because we have shown that the contract $r = p = \hat{R}(= R)$ with a nonbinding credit limit yields zero profits, satisfies constraints IC_1, IC_2 under EQ, and the lender does not find it profitable to reprice that contract ex post (at the onset of the second period).

What is the economic intuition behind the above result? The fact that the equilibrium allocation must satisfy the planning problem in PL provides the key insight here. While interest rates are normally distortionary, and for this reason the solution to EQ should not coincide with that to PL, this is not the case because positive interest rates are used here to offset the distortion implied by the positive probability of default $p > 0$. To see this, note that the Euler equations in IC_1, IC_2 involve discounted future marginal utility by $1-p$, and it is that discount that the positive rate offsets vis-à-vis the planner's condition. This discount is brought about by the fact that, ex post, that is, after accepting the contract terms, the consumer will choose according to these terms, but ex ante the consumer understands that any additional dollar of debt borrowed in the first period is something she has to pay for so that lenders can break even. The consumer is thus optimizing ex ante via an appropriate selection of contract terms to ensure her own optimal choice ex post.

We should stress that this result does not involve the logic of the tax smoothing theorem from public finance (Atkinson and Stiglitz, 1976)—which independently applies to our model and implies that lenders should generally "smooth" interest rate burden across periods. Instead, our result stems from the fact that interest rates are "nondistortionary" to the extent that they reflect default risk that borrowers want to internalize for their own sake. It is the excess level of interest rates above or below that level that is distortionary in our setup and to which tax smoothing theorem applies.

4 Extensions and generalizations

This section generalizes our results by considering the possibility of hidden savings and strategic (endogenous) default. These features complicate the model and in general reinforce our results, and for this reason it is better to analyze them separately as a "robustness check." As mentioned, Lemma (1) and Lemma (3) both apply to the extensions considered here. The proof of Lemma (3) for the extended setup can be found in the Online Appendix. The proof of Lemma (1) is omitted because it would be analogous to the baseline case. We focus attention on characterizing the equilibria featuring positive borrowing $b_1 > 0, b_2 > 0$. Since Lemma 2 is specific to the baseline setup, hereafter we impose nonnegative borrowing as an assumption. The characterization of such equilibra is of interest because when there is no borrowing the interest rate is undetermined, and if there is borrowing in only one period one cannot speak about a promotional rate in a meaningful way.

4.1 Hidden savings

In our baseline model, consumption after default is equal to income. This makes sense but in practice debt collection and bankruptcy proceedings are time consuming, and borrowers may be able to divert part of the borrowed resources to raise their consumption even after they default and potentially are subject to debt collection and court-enforced screening. Here we extend our model to allow for such a possibility and show the results are robust to such a modification.

To that end, assume that consumer families have access to a constrained *hidden savings technology.* The use of this technology is potentially limited to a fraction of borrowed resources that are consumed in the first period, which is a parameter. Formally, given contract $\left(r, \hat{R}, l, L\right)$, the consumers in the modified setup choose $\left(b_t, b_t^d, c_t\right)_{t=1,2}$ to maximize

$$U\left(c_1, c_2, ...\right) = u\left(c_1\right) + \beta\left(1-p\right)u\left(c_2\right) + \beta^2\left(1-p\right)^2 u\left(c_3\right) + \beta U^d\left(b_1^d, b_2^d\right) \tag{19}$$

subject to the budget constraint given by

$$\begin{aligned} c_1 &= y - b_0 + b_1 - r\left(b_1 + b_1^d\right), \qquad (20) \\ c_2 &= y + b_1^d - \left(b_1 + b_1^d\right) + b_2 - \hat{R}\left(b_2 + b_2^d\right), \\ c_3 &= y + b_2^d - \left(b_2 + b_2^d\right), \\ b_1^+ &\leq l, b_2^+ \leq L \end{aligned}$$

and the additional *hidden savings constraint* is

$$b_1^d \leq \tau b_1\left(1-r\right), b_2^d \leq \tau b_2\left(1-\hat{R}\right), \tag{21}$$

and it modifies the utility flow from default state as follows:

$$U^d\left(b_1^d, b_2^d\right) = p\left(u\left(y-\Delta+b_1^d\right)+\beta D+\beta\left(1-p\right)u\left(y-\Delta+b_2^d\right)\right). \tag{22}$$

In the above budget constraint, the amount $b_1 + b_1^d$ represents total borrowing in the first period, where it is assumed that b_1 is the amount consumed right away and b_1^d is the amount saved for future period consumption and hidden from the lenders. This is clear from equation (22), where we can see that b_1^d increases consumption in the first period (similarly for b_2^d) and hence cannot be seized. When the consumer does not default, hidden savings b_1^d accrue to consumption, and since in that case the consumer repays $b_1 + b_1^d$, hidden savings have no effect on consumption in that period (as implied by the term "$b_1^d - \left(b_1 + b_1^d\right) = -b_1$" in the equation for c_2). Equation (21) states that only a fraction $\tau > 0$ of the borrowed funds b_1 can be hidden from lenders and consumed after default. The remaining part of the budget constraint is analogous to the baseline setup.

To characterize the constrained optimum, we set up a planning problem under which the planner chooses lump-sum transfers—here, T_1, T_1^d, T_2, T_2^d and T_3—to maximize the present discounted utility of the consumer:

$$\begin{aligned} U^{PL}\left(T_1, T_2, ...\right) = & u\left(c_1+T_1\right)+\left(1-p\right)\beta u\left(c_2+T_2\right)+\left(1-p\right)^2\beta^2 u\left(c_3+T_3\right) \\ & +\beta p\left(u\left(c_2^d+T_2^d\right)+\left(1-p\right)\beta u\left(c_3^d+T_3^d\right)\right), \end{aligned}$$

where, as before, $c_1 = y - b_0$, $c_2 = c_3 = y$, $c_2^d = c_3^d = y - \Delta$ correspond to the autarkic allocation. The maximization is subject to the planner's resource constraint (RC), which here is given by

$$T_1 + \left(1-p\right)T_2 + pT_2^d + \left(1-p\right)\left(\left(1-p\right)T_3 + pT_3^d\right) = 0, \tag{23}$$

and an additional constraint that must be added to the planning problem to "emulate" the distortion implied by the hidden savings constraint:

$$T_2^d \geq \tau T_1 + \kappa_1, T_3^d \geq \tau T_2 + \kappa_2, \tag{24}$$

where κ_1, κ_2 are arbitrary constants such that the level of savings is the same as in equilibrium. The marginal conditions derived from the above planning problem does not depend on constants κ_1, κ_2, and so introducing these constants is innocuous for the optimality conditions. Hidden savings constraint (24) captures the same trade off as equation (21) because, on the margin, the planner can similarly insure the agent against the low income realization by increasing consumption in that state at the expense of consumption in the high income state. The role of these constraints is to ensure that the planner is constrained in transferring resources from one state to the other when the agent is constrained.

Binding hidden savings constraint.— We begin by analyzing the above problem under the assumption that the hidden savings constraint binds. Using (23), we thus combine equations (23) and (23) to eliminate T_1 and plug in $T_2^d = -\tau (1-p) T_2 - \tau (1-p)^2 T_3 + \kappa$ whenever applicable. The relevant part of the Lagrangian for the planning problem pertaining to the first two periods is

$$L = u(c_1 + T_1) + (1-p)\beta u(c_2 + T_2) + \beta p u\left(c_2^d - \tau (1-p) T_2\right) + ... - \lambda (T_1 + (1-p) T_2 - ...),$$

where λ is the Lagrange multiplier on RC in (23) and it is the only constraint after plugging in the hidden savings constraint. The first order conditions with respect to T_1, T_2 are given by

$$T_1 : u_1' = \lambda$$

$$T_2 : \beta (1-p) u_2' + \beta p \tau u_{2d}' = \lambda (1 - p (1 + \tau (1-p))),$$

which gives the analog of (17) for the setup with hidden savings:

$$MRS_1 := \frac{u_1'}{\beta (1-p) u_2' + \beta p \tau u_{2d}' (1-p)} = (1 - p (1 + \tau (1-p)))^{-1} := MRT_1. \tag{25}$$

As for the consumer's Euler equation in the first period (applicable in equilibrium), we plug in $b_1^d = \tau b_1 (1-r)$. The relevant objective function for the choice of b_1 is

$$u (y - b_0 + b_1 - r (b_1 + \tau b_1 (1-r))) +$$
$$\beta (1-p) u \left(y - b_1 + b_2 - \hat{R} \left(b_2 + b_2^d\right)\right) + \beta p u (y - \Delta + \tau b_1 (1-r)) ...,$$

which yields the equilibrium Euler equation of the form:

$$\frac{u_1'}{\beta (1-p) u_2' + \beta \tau p u_{2d}' (1-r)} = (1 - (1 + \tau (1-r)) r)^{-1}. \tag{26}$$

It is clear from the comparison of the above Euler equation and the planner's condition in (25) that the unique zero profit contract that supports the planner's allocation is again $r = p$ and, by the analogous reasoning applied to the marginal conditions pertaining to last two periods, $\hat{R} = p$. We know this contract satisfies the zero profit condition. This can be directly verified by noting that lender profits in this case are

$$(r-p)\left(b_1+b_1^d\right)+(1-p)\left(\hat{R}-p\right)\left(b_2+b_2^d\right)=0.$$

Any other interest rate schedule would result in a different consumption level relative to the planning solution. As mentioned, Lemma 3 applies to the extended model, which we use here, and the proof of this fact can be found in the Online Appendix. The previous argument why binding credit limits cannot be used in conjuction with a promotion ($r<p$) to sustain the same allocation applies here without any modifications. Accordingly, our result applies to the extended model with a binding hidden savings constraint.

We next analyze what happens when the hidden savings constraint does not bind.

Nonbinding hidden savings constraint.— In this case the planner equalizes the marginal utility from consumption in the second period's high state and low state. That is, the allocation that solves the planning problem necessarily features $u_2'=u_{2d}'$, which means that the consumer is fully insured against the low income realization in the second period. This is easy to verify by working out the first order conditions implied by the corresponding Lagrangian with respect to T_2 and T_2^d, which we omit for brevity. Aside from this additional property, the planner's marginal conditions for the first two periods in the high income state are the same as in the baseline economy, with marginal rate of transformation given by $MRT_1=-(1-p)^{-1}$ and marginal rate of substitution given by $MRS_1=-u_1'/\left((1-p)\,\beta u_2'\right)$. Analogous conditions apply to the second period.

As for the consumer problem, the Euler equation implied by the marginal condition associated with b_1 is also identical to the baseline setup. Therefore, the contract $r=p=\hat{R}$ with nonbinding credit limits l,L is again a candidate supporting contract, and it obviously is a zero profit contract because it covers default risk on a period-by-period basis. What needs to be verified, however, is that the consumer's Euler equation that applies to hidden savings b_1^d also implies full insurance; that is, that the fact that the consumer under the proposed contract also chooses b_1 so that $u_2' = u_{2d}'$. If this is the case, the allocation is exactly the same.

To establish the latter property, let us rewrite the first period budget constraint as $c_1 = y - b_0 + (1-r)\, b_1 - rb_1^d$, which assuming $r = p$, implies $u_1' = \beta u_{2d}'$. Since the Euler equation in the high income state on this contract implies $u_1' = \beta u_2'$, the result now follows because the two equations together imply $u_2' = u_{2d}'$. Implementation via binding credit limits is not possible here for the same reasons. We summarize the above results in the proposition below.

Proposition 2. *With hidden savings, the equilibrium contract is* $r = p = \hat{R}(= R)$ *and features nonbinding credit limits.*

4.2 Strategic default

A model featuring strategic (endogenous) default is harder to analyze but the basic lesson from the baseline setup carries over. Recall that in the baseline setup lenders offered contracts so that borrowers could ex post see the probability of defaulting in the contracted terms to borrow the "right amount" from the ex ante perspective. This logic applies here but in a modified form: borrowers should seek contracts that reflect not only the probability of default but also the fact that this probability increases with borrowing, and hence credit should be further constrained. The conclusion from our analysis of strategic default is that, while strategic default leads to a more complex environment to analyze, there is no clear indication that it generates a robust mechanism resulting in frequent promotions.

To illustrate the additional forces introduced by strategic default, we simplify the baseline setup by assuming that income is constant at y ($\Delta = 0$), and instead consider a random stigma shock $s > 0$ that lowers the borrower's utility after default. The analysis of this model will show that there is no interaction between endogeneity of default decision and the kind of contracts that prevail in equilibrium.

Formally, in the modified setup the consumer in the second period solves

$$V(b_1) = \max_{b_2 \leq L} u\left(y - b_1 + b_2\left(1 - \hat{R}\right)\right) + \beta \mathbb{E} \max\left[u(y - b_2), u(y) - s_2\right], \tag{27}$$

where s_2 is an i.i.d. random variable distributed according to cdf F_2. In the first period, the consumer solves

$$U := \max_{b_1 \leq l} u_1(y - b_0 + b_1(1 - r)) + \beta \mathbb{E} \max\left[V(b_1), V^d - s_1\right], \tag{28}$$

where s_1 is an i.i.d. random variable distributed according to cdf F_1.

The probability of default is endogenous and it can be defined as follows. Let $\overline{s}_1(b_1)$ be the function of debt such that $V(b_1) = V^d - \overline{s}_1(b_1)$. Then, the probability of default in the second period is $p_2(b_1) = Pr(s_1 \leq \overline{s}_1(b_1)) = F_1(\overline{s}_1(b_1))$. Analogously, in the third period, let $\overline{s}_2(b_2)$ be the function that satisfies $u(y - b_2) = u(y) - \overline{s}_2(b_2)$, implying that the probability of default is $p_3(b_2) = Pr(s_2 \leq \overline{s}_2(b_2)) = F_2(\overline{s}_2(b_2))$.

The consumer's Euler equation now involves an additional term. This terms captures the fact that more debt increases the probability of default. Specifically, taking the derivative of that Euler equation with respect to b_1, we obtain

$$u_1'(1 - r) = (1 - p_2(b_1))\, u_2' - \beta \frac{d\overline{s}_1(b_1)}{db_1} \underbrace{\left(V(b_1) - V^d + \overline{s}_1(b_1)\right)}_{0} F_1', \tag{29}$$

where, note, the underbraced expression on the right-hand side is zero by the differentiation of the expectation of the maximized continuation value with respect to b_1, which we can rewrite as follows:

$$\mathbb{E}\max\left[V\left(b_1\right), V^d + s_1\right] = \int_{-\infty}^{\overline{s}_1(b_1)} V\left(b_1\right) dF_1\left(s\right) + \int_{\overline{s}_1(b_1)}^{\infty} \left(V^d - \overline{s}_1\left(b_1\right)\right) dF_1\left(s\right).$$

By definition of the cutoff $\overline{s}_1\left(b_1\right)$, the agent must be indifferent between defaulting or repaying at the cutoff point, implying $V\left(b_1\right) - V^d + \overline{s}_1\left(b_1\right)$, and the additional term in (29) drops out, implying that the Euler equation is identical as in the baseline model, that is, $u_1'\left(1-r\right) = \beta\left(1-p_2\left(b_1\right)\right)u_2'$. The derivation of the second period Euler equation follows analogously and implies $u_2'\left(1-\hat{R}\right) = \beta\left(1-p_2\right)\left(1-p_3\right)u_3'$.

Analogously, the planner maximizes

$$U^{PL}\left(T_1, T_2, T_3\right) = \max_{b_1 \leq l} u_1\left(y - b_0 + T_1\right) + \beta\mathbb{E}\max\left[V^{PL}\left(T_2, T_3\right), V^d - s_1\right],$$

where $V\left(T_2, T_3\right) = u\left(y + T_2\right) + \beta\mathbb{E}\max\left[u\left(y + T_3\right), u\left(y\right) + s_2\right]$. The resource constraint is as in the baseline setup but with endogenous default probabilities:

$$RC\left(T_1, T_2, T_3\right) := T_1 + \left(1 - p_2\left(T_2, T_3\right)\right)T_2 + \left(1 - p_2\left(T_2, T_3\right)\right)\left(1 - p_3\left(T_3\right)\right)T_3 = 0,$$

where default probabilities are defined analogously—and by that definition it should be clear that these probabilities are decreasing in transfers.

Given the Lagrangian $L = U^{PL}\left(T_1, T_2, T_3\right) - \lambda\left(RC\left(T_1, T_2, T_3\right)\right)$, the marginal rate of substitution involving the objective function is identical as in the baseline model and hence given by the expression on the left-hand side of (17) and (18). As for the marginal rate of transformation, implicit differentiation of $RC(T_1, T_2, T_3)$ gives

$$MRT_1 := -\frac{\partial RC/\partial T_1}{\partial RC/\partial T_2} = \left(1 - p_2 - \frac{\partial p_2}{\partial T_2} T_2 - (1-p_3)\frac{\partial p_2}{\partial T_2} T_3\right)^{-1},$$

and for the last two periods

$$MRT_2 := -\frac{\partial RC/\partial T_2}{\partial RC/\partial T_3} = \left((1-p_2)(1-p_3) - \frac{\partial p_2}{\partial T_3} T_2 - (1-p_3)\frac{\partial p_2}{\partial T_3} T_3 - (1-p_2)\frac{\partial p_3}{\partial T_3} T_3\right)^{-1}.$$

If the consumer borrows on net in each period, we have $T_2 < 0$ and $T_3 < 0$, in which case the additional terms on the right-hand side of the above expressions vis-à-vis the baseline model are all negative. Comparing the planner's conditions and the consumer's Euler equations, and assuming credit limits do not bind, it is clear that implementing this constrained optimum would require $r^{PL} > p_2$ and $\hat{R}^{PL} > p_3$, which in turn implies $MRT_2 = \left(1 - \hat{r}^{PL}\right)^{-1}$ and $MRT_2 = \left(1 - \hat{R}^{PL}\right)^{-1}$, respectively. But such a contract is not feasible because it violates the zero profit condition, here given by

$$ZP : (r - p_2) b_1 + (1 - p_2)\left(\hat{R} - p_3\right) b_2 = 0,$$

and hence this allocation cannot be implemented as an equilibrium. Accordingly, if the implementation of the constrained optimum is at all possible, it must involve binding credit limits. An additional assumption is needed to ensure that binding credit limits can constrain credit as needed to implement this allocation. This condition is a sufficient condition and it requires that consumer borrowing is decreasing in interest rates. Since there is an income and substitution effect, this places a restriction on consumer preferences. We assume preferences are such that this is the case below. The important result to note is that since we need L to be binding, it must be that $\hat{R} \leq p$, and hence a promotion is not possible. If this condition were violated the lender would have an incentive ex post to deviate and relax the credit limit L, which would violate the last condition of Lemma 1. We summarize these results in the statement below.

Assumption 1. *With nonbinding credit limit, we have $db_1/dr \leq 0$, $db_1/d\hat{R} \leq 0$, $db_2/dr \leq 0$,* and *$db_2/d\hat{R} \leq 0$, where by "d" we mean total derivatives.*

Proposition 3. *With endogenous default, and under Assumption 1, the equilibrium contract involves $r \geq p \geq \hat{R}$ and a binding credit limit l and L.*

5 Time-inconsistent preferences as source of promotions

One of the key features of our model is that the borrower makes time consistent decisions. Here we consider the hyperbolic model of consumption behavior to show that time inconsistency could indeed be a promising route to generate promotions in equilibrium. While there is some evidence in support of these preferences, more work is needed to determine the exact nature of the underlying mechanism because the naivete version and sophisticated version of the same preferences both work but have very different policy implications.

5.1 Setup

We modify our baseline model by assuming that consumers evaluate consumption streams according to

$$U(c_1, c_2, c_3) = u(c_1) + \beta\eta\mathbb{E}\left[u(c_2) + \beta u(c_3)\right], \tag{30}$$

and, as of the second period, they evaluate them according to

$$u(c_2) + \beta\eta\mathbb{E}\left[u(c_3)\right], \tag{31}$$

where $\eta < 1$ is an additional discount factor applied to the continuation value in every period. As we can see, the consumer's ex ante preferences assume that her "future self" is more patient, which is not the case ex post. There are two formulations of the hyperbolic discounting model in the literature that we consider next. The first one assumes that the consumer is not aware of the change in preferences and the second one assumes that the consumer is aware of it.

5.1.1 Naivete hyperbolic discounting

In this case the consumer erroneously believes that her future self will pay down debt faster than will be the case because the consumer is unaware that preferences will change. Formally, the consumer in the second period solves

$$b_2^{\eta} = \max_{b_2} \left[u\left(c\left(b_2; \mathcal{C}, b_1\right)\right) + \eta\beta u\left(c_3\left(b_2\right)\right)\right],$$

subject to (12), where $0 < \eta < 1$; however in the first period the consumer erroneously believes that her future self will make choices according to preferences featuring $\eta = 1$. Lenders are aware of this systematic error and their profit reflects b_2^{η} as opposed to $b_2 \equiv b_2^1$; that is, their zero profit condition is

$$ZP : (r - p)\, b_1^{+} + (1 - p)\left(\hat{R} - p\right) b_2^{\eta +} = 0.$$

For any b_1, which is a state variable as of the second period, it is easy to see that $b^{\eta} > b_2$. Therefore, as long as $\hat{R} > p$, the lender expects to make "excess" profits from the second period relative to what borrowers believe they will pay the lender, which is $(\hat{R} - p)b_2^{+}$. This wedge gives rise to promotions in equilibrium. The proposition below summarizes the key result for this setup.

Proposition 4. *Positive credit equilibrium features a promotional rate $r < p$.*

5.1.2 Sophisticated hyperbolic discounting

The sophisticated formulation is fundamentally different in that borrowers are rational but would like their future self to make choices according to preferences featuring $\eta = 1$. However, surprisingly, this setup can also result in promotional pricing under certain additional assumptions. We summarize these results in the proposition below.

Proposition 5. *The constrained optimal allocation satisfies $u_1' = \beta\eta u_2' = \beta^2 u_3'$. 1) If the consumer's unconstrained policy function $b_1(.)$ is decreasing in r, constrained optimal allocation can be implemented by the contract: i) $\hat{R} = 1 - \eta(1-p) > p$ (*implying $R = \frac{1}{1-\rho}(1 - \eta(1-p) - \rho p)$*), ii) L nonbinding, iii) $r < p$, and iv) l binding to ensure $u_1' = \beta\eta u_2'$. Alternatively, 2) if the consumer's unconstrained policy function $b_2(.)$ is decreasing in $\hat{R}$, the constrained optimum can also be implemented by the contract: i) $r = p = \hat{R}$, ii) nonbinding l, and iii) binding L to ensure $u_2' = \beta u_3'$.*

To explain the above proposition, let us first consider contracts with a nonbinding credit limit L in the second period. Observe that in that case the Euler equation of the consumer's future self is $u_2'\left(1 - \hat{R}\right) = \beta\eta(1-p))u$, while ex ante the consumer would like her future self to make decisions according to $u_2'\left(1 - \hat{R}\right) = \beta(1-p)u_3'$. If the credit limit does not bind, the consumer's ex-ante self would like the allocation to be such that $u_2' = \beta u_3'$, which is the same as in the baseline model. By that Euler equation that applies in equilibrium to the consumer problem, the reset rate that leads to this outcome must satisfy $1 - \hat{R} = \eta(1-p)$, which gives $\hat{R} = 1 - \eta(1-p) > p$. Plugging this into the transformation in (10), and given $1 - \eta(1-p) > p$, we obtain that $R = \frac{1}{1-\rho}(1 - \eta(1-p) - \rho p)$.

But, this is not all. Given $\hat{R} = 1 - \eta(1-p) > p$, note that the zero profit condition necessitates $r < p$. From the planning solution we know that the constrained optimal allocation must satisfy $u_1' = \beta\eta u_2'$, since there is an additional discount factor that applies in the first period. Given that the equilibrium Euler equation is $u_1'(1-r) = \beta\eta(1-p)u_2'$, implementing the same allocation with $r < p$ requires a binding credit limit l to constrain b_1.[30] This can only be the case when borrowing is decreasing in r so that a binding credit limit can implement the planner's consumption profile, as stated in the proposition.

Note that the contract discussed above is not the only implementation of the constrained optimal allocation, as also stated in the proposition. The alternative implementation is to set L binding so that the condition $u_2' = \beta u_3'$ holds for $r = p = \hat{R}$, since in that case relaxing the credit limit L ex post would not increase profits.[31] One way to obtain determinacy of the equilibrium with promotions is to restrict market incompleteness so that the latter implementation is not viable. For example, imposing $l \leq L$ could go a long way in eliminating that case. However, it should be noted that the mechanism that leads to promotional lending is less robust in this case.

The interesting implication of the hyperbolic model is that its regulatory implications crucially depend on which case we are dealing with. In particular, the naivete case may call for some degree of consumer protection, while in the sophisticated case there is no need for such an intervention.

6 Conclusions

We provided evidence that promotional pricing is prevalent in the U.S. credit market and showed that such pricing is at odds with the predictions of the canonical theory of unsecured credit

[30] Note that this is only possible as long as borrowing b_1 is decreasing in r, which under sensible parameterizations will be the case.

[31] Since this will increase the borrower's ex post utility the lack of commitment is likely to be an issue.

suitably extended to allow for promotions. Our work raises questions about the kind of features of the environment that might be missing from the standard theory of unsecured lending and which could explain the prevalence of promotions. Understanding pricing is not for its own sake, and it can be fruitful in guiding the development of better fitting theory and pay off in a better understanding of the regulatory needs of the U.S. credit card market. We hope that our work will inspire further research in this direction.

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Appendix

Omitted proofs

Proof of Lemma 2

Preliminaries: Let us compactly represent the consumer problem by the maximization: $U(\mathcal{C}) = \max_{b_1,b_2} \tilde{U}(\mathcal{C}; b_1, b_2)$, where $\tilde{U}$ is the same as the objective function of the maximization EQ in 9 but after plugging in from BC in 12 for c_1, c_2, c_3. Without a loss, note that the domain of b_1, b_2 can be restricted to a compact and connected set (low b_1 violated nonnegativity, and credit limits are always assumed finite on $\mathcal{C}$). The objective function is strictly concave.[32] By the basic results in convex programming (without differentiability), there exists a unique policy

[32] Let $\mathscr{C}\left(\mathcal{C}; b_1^i, b_2^i\right)$ be the budget set defined by BC constraints in 12 and nonnegativity of consumption. Consider $\left(b_1^i, b_2^i\right) \in \mathscr{C}\left(\mathcal{C}; b_1^i, b_2^i\right)$, $i = 1, 2$. Let $(b_{1\theta}, b_{2\theta}) = \theta\left(b_1^i, b_2^i\right) + (1-\theta)\left(b_1^i, b_2^i\right)$, and define period t consumption function $c_t(b_1, b_2)$ by the left-hand side of BC constraints in 12. As for the first period, note that $c_1(b_{1\theta}, b_{2\theta}) - \left(\theta c_1\left(b_1^1, b_2^1\right) + (1-\theta) c\left(b_1^2, b_2^2\right)\right) = -r\left(\theta b_1^{\theta+}\right) + r\theta b_1^{1+} + (1-\theta) r b_1^{2+} \geq 0$ by $\theta \max\left[b_1^1, 0\right] + (1-\theta)\max\left[b_1^2, 0\right] - \max\left[\theta b_1^1 + (1-\theta) b_1^1, 0\right] \geq 0$. Note that an analogous argument applies to the second period. Since u is strictly concave, the result now follows.

function $b_1(\mathcal{C}), b_2(\mathcal{C})$ that maximizes $\tilde{U}$. Accordingly, the consumer problem is a strictly concave programming problem despite the "kink" at $b_1 = 0$, $b_2 = 0$. Furthermore, directional partial derivatives can used to obtain the necessary and sufficient condition for the maximum at $b_1 = 0$, $b_2 = 0$. Recall that $\mathcal{C} = \left(r, l, \hat{R}, L\right)$ is an equilibrium contract if it maximizes $U(\mathcal{C})$ subject to ZP: $(r-p)\, b_1(\mathcal{C}) + (1-p)\left(\hat{R} - p\right) b_2(\mathcal{C}) = 0$. We now return to the proof of the lemma.

Step 1. (Particular case) Consider first the contract $\mathcal{C}^* = \left(r, \hat{R}, l, L\right)$ with $r = p = \hat{R}$, l, L nonbinding. Note that it is a zero profit contract for any b_1, b_2 by ZP in 13. We will show that the consumer borrows in both periods, i.e. $b_1(\mathcal{C}^*) > 0$, $b_2(\mathcal{C}^*) > 0$. To prove this, note that the consumer's Euler equations—given by IC_1, IC_2 in 13—imply $u_1'(c_1) = \beta u_2'(c_2) = \beta^2 u_3'(c_3)$, and hence

$$c_1 > c_2 > c_3. \tag{32}$$

Note that this applies regardless of whether the consumer borrows or saves (as noted above, at $b_1 = 0$ or $b_2 = 0$ we evaluate directional derivatives and obtain the same condition). By contradiction, i) suppose $b_1 > 0$, $b_2 \leq 0$. By (12), we know $c_1 = y - b_0 + b_1 - b_1^+ p$, $c_2 = y - b_1 + b_2 - b_2^+ p$, $c_3 = y - b_2$, and it is clear that in this case $c_3 > c_2$, which contradicts (32). ii) Suppose $b_1 \leq 0, b_2 > 0$. Note that the listed equations in part i now imply $c_1 < c_2$, which again contradicts (32). Finally, iii) suppose $b_1 < 0, b_2 < 0$, and note that it also contradicts (32) because the listed equations imply $c_3 > c_1$. This proves the claim for $\mathcal{C}^*$.

Step 2. (General case) Next, consider any equilibrium contract $\mathcal{C}^{**} = \left(r, \hat{R}, l, L\right) \neq \mathcal{C}^*$ that solves (9) (after transforming the contract space). By contradiction, suppose the consumer does not borrow in one of the periods. In particular, i) suppose the consumer chooses $b_1(\mathcal{C}^{**}) \leq 0$, $b_2(\mathcal{C}^{**}) > 0$. By the zero profit condition ZP, $\mathcal{C}^{**}$ must feature $\hat{R} = p$ (since the profit flow from the first period is zero). But, if so, $b_1(\mathcal{C}^{**}), b_2(\mathcal{C}^{**})$ is also a feasible choice under $\mathcal{C}^*$ and it results in the exact same level of consumption (budget constraint equations are identical in that case at that point). Yet, for $\mathcal{C}^*$, the consumer made a different choice, since we have shown in step 1 that $b_2(\mathcal{C}^*) > 0$. We have now obtained a contradiction because the above shows

$$U(\mathcal{C}^*) > \tilde{U}(\mathcal{C}^*, b_1(\mathcal{C}^{**}), b_2(\mathcal{C}^{**})) = \tilde{U}(\mathcal{C}^{**}, b_1(\mathcal{C}^{**}), b_2(\mathcal{C}^{**})) = U(\mathcal{C}^{**}),$$

where, from the left, the first part ($>$) follows from strict concavity of the consumer problem, the second part ($=$) follows by the fact that budget constraint in BC implies the same consumption at $b_1(\mathcal{C}^{**}), b_2(\mathcal{C}^{**})$ for $\mathcal{C}^*$ and $\mathcal{C}^{**}$, and the last part ($=$) follows by the hypothesis. Accordingly, $\mathcal{C}^{**}$ cannot be an equilibrium as hypothesized because it does not maximize the consumer's utility. Cases ii) and iii) follow by analogy. Q.E.D.

Proof of Lemma 3:

1) "$\Rightarrow$" Consider the given consumption profile c_1, c_2, c_3, and define the associated transfers under PL in (15) as follows $T_1 = c_1 - y - b_0$, $T_2 = c_2 - y$, $T_3 = c_3 - y$. By the hypothesis, c_1, c_2, c_3 satisfy BC in (12) and ZP in (13). Expanding the zero profit condition (the second line below) and plugging in from the budget constraint in (12) (the third line below) yields RC in (16) as claimed:

$$\begin{aligned}
\Pi = (r-p)\, b_1 + (1-p)\left(\hat{R} - p\right) b_2 = 0 \\
(-b_1 + rb_1) + (1-p)\left(b_1 - b_2 + \hat{R}b_2 + (1-p)\, b_2\right) = 0 \\
-(c_1 - (y - b_0)) - (1-p)(c_2 - y) - (1-p)^2 (c_3 - y) = 0 \\
\Rightarrow T_1 + (1-p)\, T_2 + (1-p)^2 T_3 = 0.
\end{aligned}$$

2) "$\Leftarrow$" Let c_1, c_2, c_3 be the consumption profile that solves PL in (15) (that is, $c_1 = y - b_0 + T_1,\ c_2 = y + T_2,\ c_3 = y + T_3$, and T_1, T_2, T_3 solve PL). Given the supporting rate schedule $\left(r, \hat{R}\right)$ from the statement of the lemma, we use the budget constraint BC in (12) to back out the unique values of b_1 and b_2 that ensure BC holds in periods 2 and 3 as follows:

$$\underbrace{y + T_3}_{c_3} = y - b_2 \Rightarrow b_2 = -T_3 \tag{33}$$

$$\underbrace{y + T_2}_{c_2} = y - b_1 + \underbrace{b_2}_{b_2 = -T_3}\left(1 - \hat{R}\right) \Rightarrow b_1 = -T_2 - T_3\left(1 - \hat{R}\right)$$

Next, we show that the first period budget constraint

$$c_1 = y - b_0 + b_1\,(1 - r) \tag{34}$$

is satisfied for $c_1 = y - b_1 + T_1$ iff ($\Leftrightarrow$) the zero profit condition ZP in (13) holds for $b_1 = -T_2 - T_3\left(1 - \hat{R}\right)$ and $b_2 = -T_3$, which, note, we have shown above satisfy (33). Plugging the planner's consumption $c_1 = y - b_0 + T_1$ into (34), we obtain

$$y - b_0 + T_1 = y - b_0 + b_1\,(1 - r)\,.$$

Using the fact that $T_1 = -T_2\,(1 - p) - T_3\,(1 - p)^2$ by RC in (16), the above gives

$$\left(-T_2\,(1 - p) - T_3\,(1 - p)^2\right) = b_1\,(1 - r)\,.$$

Plugging in for T_2, T_3 from equations for b_2, b_3 in (33), we obtain

$$-\left(\underbrace{-b_1 + b_2\left(1-\hat{R}\right)}_{-T_2}\right)(1-p) - \left(\underbrace{-b_2}_{T_3}\right)(1-p)^2 = b_1(1-r),$$

which after basic manipulations yields the zero profit condition $b_1(r-p) + \left(\hat{R}-p\right) b_2 (1-p) = 0$, which holds by the hypothesis. This finishes the proof because the last expression is ZP in (13) and the reasoning works in reverse "$\Leftarrow$" (and hence a stronger "iff" statement applies as stated).

Proof of Proposition 4

Assume, by the way of contradiction, that the optimal contract is $r = p = \hat{R}$ ($R = p$) with credit limits being slack or binding. We will show that, if $\eta < 1$, in either case, the lender has an incentive to deviate from this contract by lowering r and raising $\hat{R}$.

Consider now two economies: the first economy, referred to as the *hyperbolic economy*, is the economy with an additional discount $\eta < 1$ applied to the continuation value from the next period onward. As explained in text, preferences are time inconsistent because the discount factor in the second period as of the first period is β, and becomes $\eta\beta$ only after the first period ends. The second economy, referred to as the *baseline economy*, is an economy with exactly the same discounts as of the first period, with the only difference being that these discounts do not change after the first period ends; that is, there is no time inconsistency and the ex ante consumer problem is exactly identical. In the hyperbolic economy the profit function is

$$\Pi^{\eta} := (r-p)\, b_1\left(r,\hat{R},l,L\right) + (1-p)\left(\hat{R}-p\right) b_2^{\eta}\left(r,\hat{R},l,L\right),$$

where $b_2^{\eta}\left(r,\hat{R},l,L\right)$ denotes the borrower's ex post policy function. In contrast, in the baseline economy, it is

$$\Pi := (r-p)\, b_1\left(r,\hat{R},l,L\right) + (1-p)\left(\hat{R}-p\right) b_2\left(r,\hat{R},l,L\right),$$

where $b_2\left(r,\hat{R},l,L\right)$ is the borrower's ex ante policy function. It is clear that the proof of Proposition 1 applies to baseline economy. This can be verified by repeating each step. Consequently, the equilibrium contract is $r = p = \hat{R}$, l, L nonbinding.

Consider now a deviation from $\hat{R}$ by some $d\hat{R} > 0$, applied to both economies and evaluated at that contract. By the implicit function theorem, the required offsetting change in dr^{η} to keep profits constant in the hyperbolic economy is

$$dr^{\eta} := -\frac{(1-p)b_2^{\eta}(p,p,.)}{b_1(p,p,.)}d\hat{R}.$$

This can be calculated by implicitly differentiating the above profit function at $r = p = \hat{R}$. Assume the borrowing constraint in the first period is maintained slack if it was nonbinding initially and continues to be binding if it was binding initially ($\hat{R}$ is small enough not to affect the binding pattern of the constraint). We repeat the same calculation for the baseline economy, which removes superscript η from the above expression. To simplify, from now on we use shorthand notation and write b_1 instead of $b_1(p,p,.)$ and so on and so forth. We note the following

$$dr^\eta = \frac{(1-p)b_2^\eta}{b_1} d\hat{R} < \frac{(1-p)b_2}{\bar{b}_1} d\hat{R} = dr, \tag{35}$$

since, trivially, $b_2^\eta > b_2$, and $b_1 = b_1$, where dr is the required adjustment for the baseline economy. We also know that $U^\eta \equiv U$, since the ex ante consumer problem is identical across the two economies. The first order change in the consumer's utility implied by this deviation in the two economies can thus be written as follows:

$$\begin{aligned} dU^\eta &= \frac{\partial U^\eta}{\partial r} dr^\eta + \frac{\partial U^\eta}{\partial \hat{R}} d\hat{R} \\ dU &= \frac{\partial U}{\partial r} dr + \frac{\partial U}{\partial \hat{R}} d\hat{R}. \end{aligned}$$

Taking the difference side-by-side, and using the fact that $\frac{\partial U^\eta}{\partial r} = \frac{\partial U}{\partial r}$ (ex ante preferences are identical, $U^\eta \equiv U$), we obtain

$$dU^\eta - dU = \frac{\partial U^\eta}{\partial r} \left(dr^\eta - dr \right) > 0,$$

since we have shown in (35) that $dr^\eta < dr$ and we know $\frac{\partial U^\eta}{\partial r} < 0$ (r strictly contracts the budget constraint and the utility function is strictly increasing in consumption). We have now shown that there exists a deviation from the proposed contract that is profit feasible and raises the consumer's ex ante utility, a contradiction. We have also established the direction of this variation (lower r and higher $\hat{R}$).

Online Appendix (not intended for publication)

Proof of Lemma 3 for extended setup from Section 4

Here we combine both extensions of the baseline model from Section 4 and prove the analog of Lemma 3.

1) "⇒" Consider the given consumption profile $c_1, c_2, c_3, c_2^d, c_3^d$, and define the associated transfers under PL as follows $T_1 = c_1 - y - b_0$, $T_2 = c_2 - y$, $T_3 = c_3 - y$, $T_2^d = c_2^d - y + \Delta$, $T_3^d = c_3^d - y + \Delta$. By the hypothesis, $c_1, c_2, ...$ satisfy the budget constraint and the zero profit condition. Expanding the zero profit condition

$$\Pi = (r - p_2)\left(b_1 + b_1^d\right) + (1 - p_2)\left(\hat{R} - p_3\right)\left(b_2 + b_2^d\right) = 0$$

$$\left(-\left(b_1 + b_1^d\right) + r\left(b_1 + b_1^d\right)\right) + (1 - p_2)\left(\left(b_1 + b_1^d\right) - \left(b_2 + b_2^d\right) + \hat{R}\left(b_2 + b_2^d\right) + (1 - p_3)\left(b_2 + b_2^d\right)\right) = 0,$$

and plugging in from the budget constraint (12)

$$-\left(\underbrace{c_1 - (y - b_0)}_{b_1 - r\left(b_1 + b_1^d\right)} + \underbrace{c_2^d - y + \Delta}_{b_1^d}\right) +$$
$$-(1 - p_2)\left(\underbrace{c_2 - y}_{-b_1 + b_2 - \hat{R}\left(b_2 + b_2^d\right)} + \underbrace{\left(c_2^d - y + \Delta\right)}_{-b_1^d} + (1 - p_3)\left(\underbrace{c_3 - y}_{-b_2}\right) + p_3\left(\underbrace{c_3^d - y + \Delta}_{b_2^d}\right)\right) = 0,$$

we obtain RC

$$-\left(\underbrace{c_1 - (y - b_0)}_{T_1} + p_2\left(\underbrace{c_2^d - y + \Delta}_{T_2^d}\right)\right) - (1 - p_2)\left(\underbrace{c_2 - y}_{T_2} + (1 - p_3)\left(\underbrace{c_3 - y}_{T_3}\right) + p_3\underbrace{\left(c_3^d - y + \Delta\right)}_{T_3^d}\right) = 0$$

$$\Rightarrow T_1 + pT_2^d + (1 - p_2)\left(T_2 + (1 - p_3)T_3 + p_3 T_3^d\right) = 0.$$

2) "$\Leftarrow$" Let $c_1, c_2, c_3, c_{2d}, c_{3d}$ be the consumption profile that solves the planning problem. Given the supporting rate schedule $\left(r, \hat{R}\right)$ from the statement of the lemma, we use the budget constraint BC in (12) to back out the unique values of b_1 and b_2 that ensure BC holds in periods 2 and 3:

$$\underbrace{y - \Delta + T_2^d}_{c_2^d} = y - \Delta + b_{1d} \Rightarrow b_1^d = T_2^d$$

$$\underbrace{y - \Delta + T_3^d}_{c_3^d} = y - \Delta + b_2^d \Rightarrow b_2^d = T_3^d$$

$$\underbrace{y + T_3}_{c_3} = y - b_2 \Rightarrow b_2 = -T_3 \tag{36}$$

$$\underbrace{y + T_2}_{c_2} = y - b_1 + \underbrace{b_2}_{-T_3}\left(1 - \hat{R}\right) - \underbrace{b_2^d}_{T_3^d}\hat{R} \Rightarrow b_1 = -T_2 - T_3\left(1 - \hat{R}\right) - T_3^d\hat{R}$$

Next, we show that the first period budget constraint

$$c_1 = y - b_0 + b_1 - r\left(b_1 + b_1^d\right) \tag{37}$$

is satisfied for $c_1 = y - b_1 + T_1$ and $c_2^d = y - \Delta + T_2^d$ iff ($\Leftrightarrow$) the zero profit condition ZP in (13) holds for $b_1 = -T_2 - T_3\left(1 - \hat{R}\right) - T_3^d\hat{R}$ and $b_2 = -T_3$, $b_1^d = T_2^d$, $b_2^d = T_3^d$, as defined above. Plugging in the planner's consumption $c_1 = y - b_0 + T_1$ to (37), and using the fact that $T_1 = -T_2^d p_2 - (1 - p_2)\left(T_2 + T_3^d p_3 + (1 - p_3) T_3\right)$ by RC, as well as the formulas for b_1, b_2, we obtain

$$
\begin{aligned}
c_1 =& y - b_0 + b_1 - r\left(b_1 + b_1^d\right) \\
y - b_0 + T_1 = y - b_0 +& b_1 - r\left(b_1 + b_1^d\right) \\
\left(-T_2^d p_2 - (1-p_2)\left(T_2 + T_3^d p_3 + (1-p_3) T_3\right)\right) =& b_1 - r\left(b_1 + b_1^d\right) \\
-b_2^d p_2 - (1-p_2)\left(-b_1 + b_2\left(1-\hat{R}\right) - b_3^d \hat{R} + b_3^d p_3 - (1-p_3) b_2\right) =& b_1 - r\left(b_1 + b_1^d\right) \\
(r-p)\left(b_1 + b_1^d\right) - (1-p_2)\left(b_2\left(1-\hat{R}\right) - b_3^d \hat{R} + b_3^d p_3 - (1-p_3) b_2\right) =& 0 \\
(r-p)\left(b_1 + b_1^d\right) + (1-p_2)\left(\hat{R} - p_3\right)\left(b_2 + b_3^d\right) =& 0,
\end{aligned}
$$

which finishes the proof because the last expression is ZP and the reasoning works in reverse "$\Leftarrow$" (and hence a stronger "iff" statement applies as stated).