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Price Setting with Customer Capital: Sales, Teasers, and Rigidity*

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Abstract

This paper studies price setting in an equilibrium search model of frictional product markets with long-term customer relationships. The theory gives rise to temporary sales when pricing is constrained to be anonymous across a firm's customer base. Equilibrium prices are inefficiently high, giving rise to overselling and excess trade, and the emergence of sale pricing can improve allocations by limiting this overselling. Pricing is also characterized by an asymmetry involving a stable regular price and variable sale price when firms face idiosyncratic shocks. Absent anonymous pricing, the theory gives rise to teaser pricing, which attains efficient allocations. Teaser pricing is also characterized by a stable regular price and variable teaser price, but in this case the seeming rigidity is not allocative.
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1 Introduction

This paper studies price setting in a model where customers are capital for firms: an equilibrium search model of frictional product markets with long-term customer relationships (Gourio and Rudanko 2014). The theory gives rise to temporary sales as an equilibrium outcome when pricing is constrained to be anonymous across a firm's customer base. Equilibrium prices are inefficiently high, leading to overselling and excess trade in the product market, and the emergence of sale pricing can improve allocations by limiting this overselling taking place. Equilibrium pricing is also characterized by an asymmetry involving a stable regular price and a variable sale price when firms face idiosyncratic shocks. Absent anonymous pricing, the theory gives rise to teaser pricing, with new customers facing a lower price than existing customers. Teaser pricing is also characterized by a stable regular price and variable teaser price, but in this case allocations are efficient and the seeming price rigidity not allocative.

A literature in macroeconomics has highlighted the prevalence of temporary sales in consumer pricing. Figure 1 illustrates this characteristic of consumer pricing, where a product's price undergoes repeated temporary downward shifts over time. The broadest evidence on consumer pricing draws on the micro data underlying the consumer price index, and research studying that data has documented the prevalence of temporary sales across a wide range of consumer product markets (see Table 1). This evidence indicates that there are also product markets where temporary sales do not appear to play a role, however, notably in markets for services. Temporary sales do not appear to be a characteristic of producer pricing either, as research studying the micro data underlying the producer price index has shown (Nakamura and Steinsson 2008).

How should one think about the emergence of temporary sales in consumer product markets and why they affect some markets and not others? This paper proposes a theory of temporary sales that builds on the premise that customer base concerns play a role in price setting in most markets. The theory distinguishes between markets where pricing is constrained to be anonymous—with the firm setting a common price across its customers at each point in time—and ones where pricing can reflect the long-term customer relationship in a more flexible way. The former is a natural feature of many retail markets with repeat customers, where the long-term customer relationship is implicit rather than explicit, as opposed to markets where long-term customer relationships are explicit and allow pricing accordingly.

The paper builds on the customer capital framework of Gourio and Rudanko (2014).

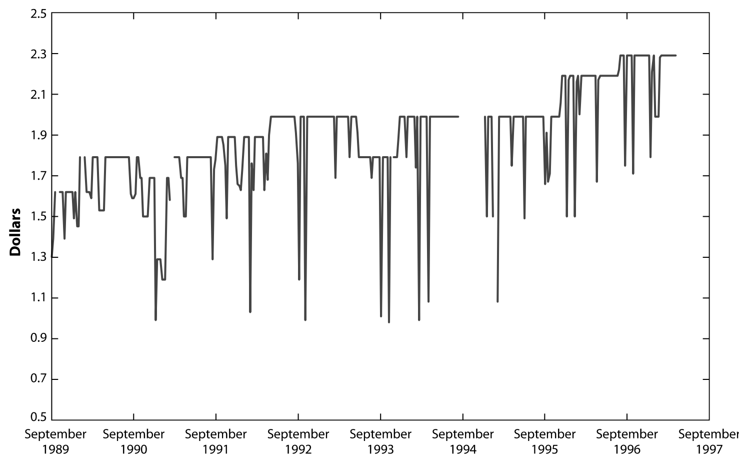

Figure 1: Illustrating Temporary Sales in Consumer Prices

Notes: The figure displays the evolution of the price of a package of Nabisco saltine crackers in a grocery store in Chicago 1989–1997. Source: Nakamura and Steinsson (2013).

In that model, firms produce heterogeneous products and buyers differ in their preferences over these products. Firms take on costly sales activities to inform potential buyers of their products and buyers search for products that fit their tastes. Bringing the two together, to allow buyers to assess whether a firm's product fits their tastes, involves coordination frictions. In addition to sales activities, firms use prices to influence customer acquisition: setting a low price attracts more searching buyers, leading to more new customers. And due to the search frictions, customers remain with the firm for a period of time.

Anonymous pricing creates a tension in the firm pricing problem between the firm's incentive to attract new customers via low prices versus its incentive to profit from existing customers via high prices. Attracting new customers requires a competitive price and costly sales activities to inform searching buyers of the firm's product, while existing customers that have already found the firm's product acceptable are willing to pay more and do not require the latter. A firm may find it optimal to set a competitive price and take on costly sales activities to attract new customers, or it may find it optimal to set a higher price without taking on sales activities, to focus on making profit on its existing customers instead.[1] In particular, equilibrium pricing can involve firms randomizing between a low "sale" price and a higher "regular" price across firms and over time.

Equilibrium outcomes depend on the prevalence of existing customer relationships in the

[1]Evidence from retail pricing supports the idea that firms spend resources on costly sales activities in conjunction with sale pricing (Hitsch, Hortacsu, and Lin 2021).

market. Sale pricing emerges as a unique equilibrium outcome when the share of buyers in existing customer relationships is sufficiently large. When few buyers are in existing customer relationships, and the pool of searching buyers is consequently large, firms find it profitable to focus on attracting new customers. As the share of buyers in existing customer relationships increases, the profitability of doing so falls, eventually leading to firms switching to randomizing between seeking to attract new customers versus focusing on profiting from existing customers instead. Firms dropping out of the market for new customers in turn serves to sustain that market, with a greater share of buyers in existing customer relationships implying a lower equilibrium probability of an individual firm seeking to attract new customers.

I show that equilibrium outcomes are inefficient, comparing to a corresponding planner problem. Absent sale pricing, firms price too high due to their incentive to profit from existing customers, with the high prices resulting in overselling and excess trade in the product market. Relative to this starting point, the emergence of sale pricing can be good for efficiency of resource allocation (in a second best sense) by limiting the excess selling taking place. Due to the firm focus on profiting from existing customers, equilibrium pricing also becomes too focused on buyer valuation for the product and too unresponsive to firm cost. The emergence of sale pricing only makes this rigidity with respect to cost more pronounced in that regular prices respond to cost even less, even if sale pricing does also introduce discontinuous jumps between sale price and regular price.

When firms face transitory idiosyncratic shocks, the theory gives rise to asymmetric pricing akin to that in Figure 1, with a relatively stable regular price that undergoes repeated, temporary, downward shifts of varying magnitude over time. In an equilibrium featuring sale pricing, a decline in firm cost leads to the firm setting a sale price that reflects the magnitude of the cost decline, and an increase in cost to the firm setting the regular price that is independent of cost. In a setting where firms face such shocks, sale pricing is thus triggered by sufficiently low cost realizations, with remaining firms setting the regular price. If firms also face similar shocks to buyer valuation for their product, sales can be triggered by sufficiently high valuation as well, with the remaining firms setting a regular price that reflects the valuation they face.[2]

When firms are able to keep track of individual customers and price accordingly—relaxing

[2] A literature in macroeconomics studies price setting with menu costs in settings where firms face idiosyncratic shocks, typically to cost or both cost and demand (e.g., Golosov and Lucas (2007), Gertler and Leahy (2008), Midrigan (2011), Klenow and Willis (2016)). Eichenbaum, Jaimovich, and Rebelo (2011) provide evidence that retail firms face frequent changes in cost, that prices do not always change with costs, but that price changes tend to be associated with cost changes.

Table 1: Frequency of Sales across Consumer Spending Categories

	Processed food	Unprocessed food	Household furnishings	Apparel	Recreation	Other	Vehicle fuel	Travel	Utilities	Services	All
Fraction observations	16.6	17.1	21.2	34.5	10.9	15.3	0.3	2.1	0.0	0.5	7.4
Fraction price changes	57.9	37.9	66.8	87.1	49.1	32.6	0.0	1.5	0.0	3.1	21.5
Expenditure weight	8.2	5.9	5.0	6.5	3.6	5.4	5.1	5.5	5.3	38.5	100

Notes: The table reports the share of price observations corresponding to a sale price and share of price changes associated with a sale within the data underlying the consumer price index during 1998-2005 by product category, in percent, and expenditure weighted. Note the higher prevalence of sales in the retail categories on the left than on the right. Source: Nakamura and Steinsson (2008), Table 2.

anonymous pricing—the theory predicts the emergence of teaser pricing instead. To profit from their existing customers, firms charge existing customers their full willingness to pay for the product. To attract new customers, firms simultaneously set a competitive price for them. Pricing thus involves an initial discount and a higher price in subsequent periods of the customer relationship. This added flexibility in pricing allows attaining efficient allocations in equilibrium. One could thus view the sale pricing emerging with anonymous pricing as a proxy for teaser pricing when the latter is not feasible. Despite efficiency of allocations, teaser pricing continues to feature high prices that are unresponsive to cost for existing customers, however. In this case the seemingly high and rigid pricing is thus not allocative.

I begin with a static model to illustrate ideas, before extending the analysis to a dynamic infinite horizon setting. The dynamic firm problem is characterized by a time-inconsistency, as firms have an incentive to promise low prices for the future to attract new customers, yet also an incentive to charge high prices today to profit from existing customers. The analysis focuses on Markov perfect equilibria where firms are free to reoptimize prices each period. Equilibrium outcomes remain transparent to analyze nevertheless, due to the structure of the model, allowing analytical characterizations of prices also in the dynamic setting.

Related literature Understanding the nature of price setting is a fundamental question in macroeconomics that has been considered by a significant body of research. Within this body of research, the present paper is related to work studying firm behavior and equilibrium outcomes when customer base concerns play a role in firm decision making. This literature goes back to Phelps and Winter (1970), with early contributions including Bils (1989) and Klemperer (1995). More recent studies formalizing the idea of customer base concerns in macroeconomic models with frictional product markets include Kleshchelski and Vincent (2009), Dinlersoz and Yorukoglu (2012), Drozd and Nosal (2012), Gourio and Rudanko (2014), Perla (2016), Paciello, Pozzi, and Trachter (2019), and Roldan and Gilbukh (2021). Most of these papers focus on the implications for firm price setting, but without connecting to sale or teaser pricing per se. The present paper develops the search-theoretic framework of Gourio and Rudanko (2014) to show that it gives rise to sale pricing when pricing is constrained to be anonymous across a firm's customer base, in addition to giving rise to teaser pricing in the absence of such constraints. The paper then proceeds to characterize the emergence of sale pricing, and implications for resource allocation and the dynamics of prices further.[3]

In a related vein, the paper also connects with Nakamura and Steinsson (2011), who study firm price setting when consumers have goods-level habit preferences (as in Ravn, Schmitt-Grohe and Uribe 2006) that also imply that a firm's current demand matters for future demand. They discuss the time-inconsistency of the firm pricing problem in this setting, and consider the implications for price rigidities under alternative assumptions about firm commitment. The present framework shares the time-inconsistency, but differs in considering an equilibrium search model where multi-customer firms set prices facing turnover in their customer base, with or without anonymous pricing, and accommodating also idiosyncratic shocks across firms.

In its focus on temporary sales and price rigidities, the present paper is also related to a strand of the macroeconomic literature on price setting that focuses on whether sale pricing may be viewed as restoring price flexibility when regular prices appear more rigid (Guimaraes and Sheedy 2011, Kehoe and Midrigan 2015, Kryvtsov and Vincent 2021). This literature models sale pricing in a relatively stylized way, in seeking to capture a richer set of elements, which leaves room for developing the theory further.[4] There also exists literature in theoretical industrial organization proposing models of sale pricing, such as Varian (1980), Salop and Stiglitz (1982) and Sobel (1984), but that research does not seek models that are easily incorporated into macroeconomic analysis.

[3]A related paper that connects customer base concerns with sale pricing is Shi (2016), who considers the endogeneity of long-term customer relationships in frictional product markets. His theory does not distinguish between sale and teaser pricing, however, in focusing on single-customer firms.

[4]When it comes to broader market-wide shifts, the present theory tends to predict an increased frequency of sales during contractions driven by market-wide increases in costs or declines in buyer valuation, and vice versa, in the spirit of evidence in Kryvtsov and Vincent (2021). The theory also tends to predict an increased frequency of sales during expansions driven by greater numbers of buyers in the market, and vice versa, in the spirit of evidence in Chevalier, Kashyap, and Rossi (2003). These countervailing effects can leave the overall change in the frequency of sales across expansions and contractions ambiguous, in the spirit of evidence in Coibion, Gorodnichenko, and Hong (2015).

2 Static Model

Consider a competitive product market where buyers and sellers face search frictions in coming together to trade. The market has measure one buyers and a large number of firms. The firms produce heterogeneous products, at unit cost c. Buyers have unit demand, but heterogeneous tastes across these products, valuing a firm's product at either $u(>0)$ or zero.

Firms begin the period with some existing customers, of measure $n_i(>0)$, that have already found the firm's product acceptable. This leaves $1-\sum_i n_i(>0)$ buyers unmatched and searching in the market. Acquiring additional customers requires costly sales activities on the part of firms, to inform searching buyers of the firm's product. Selling effort s is subject to a cost $\kappa(s,n)=\hat{\kappa}(s/n)n$, where $\hat{\kappa}$ is increasing and convex. The convex cost reflects a firm's limited ability to increase selling effort locally, and the homothetic form that larger firms are less constrained by this due to operating in multiple locations.

The process of bringing firms and searching buyers together involves coordination frictions, captured with a matching function. If a firm exerts selling effort s and attracts b searching buyers, the measure of resulting new customer relationships is given by the constant returns to scale function $m(b,s)$. With this, the probability that a searching buyer becomes the firm's customer, $\mu(\theta)=m(b,s)/b$, becomes a decreasing and convex function of the queue of searching buyers per unit of selling effort at the firm, $\theta=b/s$. The probability that a unit of selling effort leads to a new customer, $\eta(\theta)=m(b,s)/s$, becomes an increasing and concave function of the same. The elasticity of the matching function, $\varepsilon(\theta)=\theta\eta'(\theta)/\eta(\theta)$, is assumed to be weakly decreasing, reflecting that customer acquisition becomes less responsive to queues when queues become longer.

In setting its price a firm faces a tradeoff between profit per customer and number of customers served. A higher price raises profit per customer, but also attracts fewer searching buyers and hence results in fewer new customers. A firm expects the queue of searching buyers it attracts to be such that searching buyers are left indifferent between choosing this firm versus any other in the market. Denoting the equilibrium buyer value of search by S, the queue of searching buyers θ_i a firm expects to attract with price p_i satisfies

$$S=\mu(\theta_i)(u-p_i), \tag{1}$$

where a searching buyer becomes a customer with probability $\mu(\theta_i)$, getting the good at valuation u and paying the price p_i. The relationship (1) determines the queue of buyers as a decreasing function of the firm's price, with firms taking the equilibrium value of search as given. I refer to this relationship by $\theta=g(p;S)$ in what follows.

Each firm chooses its price and selling effort to maximize its profits:

$$\begin{aligned}\max_{p_i,s_i}\ & (n_i+\eta(\theta_i)s_i)(p_i-c)-\kappa(s_i,n_i)\\ \text{s.t. } & S=\mu(\theta_i)(u-p_i) \text{ if } s_i>0,\\ & p_i\le u,\end{aligned}$$

taking its existing customer base and the equilibrium value of search as given. The profits reflect sales to existing and new customers at price p_i and production cost c, net of selling costs. If the firm is seeking to attract new customers, it faces constraint (1) characterizing the queue of searching buyers attracted by the firm's price. Either way, the price cannot exceed buyer valuation for the product, or buyers would not agree to trade.

Note that the problem is effectively independent of scale. Dividing by n_i and defining selling effort per existing customer as $\hat{s}_i=s_i/n_i$, the problem may be rewritten as

$$\begin{aligned}\max_{p_i,\hat{s}_i}\ & (1+\eta(\theta_i)\hat{s}_i)(p_i-c)-\hat{\kappa}(\hat{s}_i)\\ \text{s.t. } & S=\mu(\theta_i)(u-p_i) \text{ if } \hat{s}_i>0,\\ & p_i\le u,\end{aligned}\tag{2}$$

with the equilibrium value of search taken as given. Firm decisions are thus independent of the size of its existing customer base n_i.

The firm has two distinct options here. It can seek to attract new customers, which requires a price that is competitive in the market for searching buyers, or hold off on doing so to focus on making profit on its existing customers instead.

Option 1: Active selling to attract new customers If the firm seeks to attract new customers, it chooses a positive selling intensity $\hat{s}_i>0$ together with a price that is competitive in the market for searching buyers. This price will generally be strictly below buyer valuation for the product, as firms use prices to compete for buyers, implying that the second condition in the firm problem (2) becomes superfluous.

An interior optimum with active selling must satisfy first order conditions.[5] The first order condition for the selling intensity,

$$\kappa_s(\hat{s}_i)=\eta(\theta_i)(p_i^s-c),\tag{3}$$

states that the firm chooses a selling intensity where the marginal cost of selling, on the left, is equated to the profits from sales to the additional customers acquired, on the right.

[5] I restrict attention to circumstances where first order conditions are also sufficient for interior optimum.

The first order condition for the price,

$$1 + \eta(\theta_i)\hat{s}_i = -\eta'(\theta_i) g_p(p_i^s; S)\hat{s}_i(p_i^s - c), \tag{4}$$

states that the firm raises the price to a point where the increase in profits due to greater profit margins per customer, on the left, equals the decrease in profits due to reduced customer acquisition, on the right. Here the increase in price reduces the queue of searching buyers according to $g_p(p_i^s; S) = \mu(\theta_i)/(\mu'(\theta_i)(u - p_i^s)) < 0$.

The optimality condition (4) implies that the price may be written as a weighted average of firm cost and buyer valuation:

$$p_i^s = \gamma_i c + (1 - \gamma_i)u \quad \text{with} \quad \gamma_i = \varepsilon_i \Delta_i/(1 - \varepsilon_i + \Delta_i),$$

where $\varepsilon_i \in (0, 1)$ is the matching function elasticity and $\Delta_i = \eta(\theta_i)\hat{s}_i > 0$ the share of new versus existing customers in firm sales. In addition to firm cost and buyer valuation, this price reflects how effective pricing is at attracting new customers and how important new customers are for firm sales. If longer queues of searching buyers increase customer acquisition effectively (elasticity ε is large), the firm sets a lower price to attract more new customers. If new customers make up a large share of firm sales relative to existing ones (share Δ is large), the price is lower to attract more new customers.

Option 2: Profit from existing customers Alternatively, the firm can forgo attracting new customers to focus on making profits on its existing customers instead.

If the firm does not participate in the market for searching buyers, its selling intensity is zero and its pricing problem reduces to

$$\begin{aligned} &\max_{p_i}\ p_i - c \\ &\text{s.t.}\ \ p_i \leq u. \end{aligned} \tag{5}$$

The optimal price becomes $p_i^r = u$, with the firm taking the full gains from the relationship. This price is too high to attract searching buyers, but existing customers are willing to pay it.[6] Existing customers have already determined that they find the firm's product acceptable, whereas searching buyers must be enticed to find out, in the face of competition from other firms.

Which of the two options dominates for the firm depends on the equilibrium value of search S. The profits from seeking to attract new customers are decreasing in S, because

[6] In the dynamic model of Section 3 the price reflects also the fact that existing customers can return to search for a new firm if preferred.

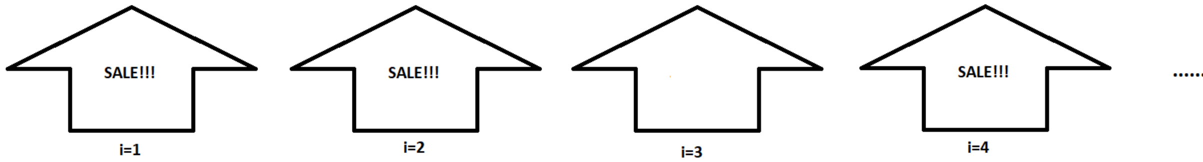

Figure 2: In the Sale Equilibrium Some Sellers Advertise a Reduced Price

a higher S means it is more costly for the firm to attract searching buyers. Meanwhile, the profits from focusing on profiting from existing customers are independent of S, at $p^r - c = u - c$. Thus, for sufficiently low values of S the firm prefers to seek to attract new customers and from some S upward the firm focuses on profiting from existing customers. Of course, in between there will generally be a value of search where firms are indifferent between the two alternatives, and might randomize between active selling at a lower price versus profiting from existing customers with a higher price instead.

Equilibrium firm behavior must maximize profits, as well as be consistent with a market clearing condition for searching buyers. From the optimality conditions, it is clear that if all firms are identical (aside from possible differences in size), their choices are also identical in that all actively selling firms choose the same selling intensity and price (resulting in the same queue length), while all remaining firms choose the same price. Denoting the total measure of existing customers across firms in the beginning of the period by $N = \sum_i n_i$ and the share of firms actively selling by α, the market clearing condition requires that the total measure of searching buyers across firms, $\theta\hat{s}\alpha N$, adds up to the total measure of searching buyers in the market $1 - N$.[7]

Definition 1. *A competitive search equilibrium with anonymous pricing specifies a share of firms actively selling α, corresponding sale price p^s, queue θ and selling intensity $\hat{s}$, as well as regular price p^r and value of search S such that: i) $\{p^s, \theta, \hat{s}\}$ solve the firm problem* (2) *with $\hat{s} > 0$, and p^r solves the firm problem* (5)*, ii) if $\alpha < 1$, firms are indifferent between active selling and not, and if $\alpha = 1$, firms weakly prefer active selling, and iii) the market for searching buyers clears: $1 - N = \theta\hat{s}\alpha N$.*

The equilibrium features sale pricing when $\alpha < 1$ in the sense that a share of firms charge a lower "sale" price and take on costly sales activities to inform searching buyers of their product, while the remaining firms charge a higher "regular" price without the latter.

[7]Each actively selling firm attracts $\theta\hat{s}n_i$ buyers. If share α of firms (of all sizes) sells, adding these buyers up across selling firms yields a total measure of searching buyers $\theta\hat{s}\alpha N$, which must equal $1 - N$.

If both strategies are pursued by some firms in equilibrium, then firms must be indifferent between them. The profits from active selling consist of the profits from existing customers $n[p^s - c]$ and the profits from new customers $n[\eta(\theta)\hat{s}(p^s - c) - \hat{\kappa}(\hat{s})] = -n\kappa_n(\hat{s})$, both positive.[8] Holding off on active selling yields greater profits on existing customers $n[u - c]$, but none on new customers. In such an equilibrium the profits from new customer acquisition must thus just make up for charging existing customers less: $p^s - c - \kappa_n(\hat{s}) = u - c$. By contrast, if all firms are actively selling, it must be that $p^s - c - \kappa_n(\hat{s}) \geq u - c$.

The following result characterizes equilibrium outcomes assuming a standard form for the selling cost and that the matching function elasticity declines from one toward zero as queues grow:

Proposition 1. *Let $\hat{\kappa}(\hat{s}) = \hat{s}^\varphi/\varphi$ such that $\varphi > 1$, $\lim_{\theta \to 0} \varepsilon(\theta) = 1$, and $\lim_{\theta \to \infty} \varepsilon(\theta) = 0$. There exists an $N^* \in (0, 1)$ such that when $N > N^*$, the competitive search equilibrium with anonymous pricing is unique and features sale pricing, and when $N \leq N^*$, the competitive search equilibrium with anonymous pricing is unique and does not feature sale pricing.*

The prevalence of existing customer relationships among buyers matters for outcomes in the market. When few buyers are in existing customer relationships to begin with, and the pool of searching buyers is consequently large, firms find it profitable to focus on attracting new customers. As the share of buyers in existing customer relationships increases, the profitability of doing so falls, eventually leading to firms switching to randomizing between seeking to attract new customers versus focusing on profiting from existing customers instead. Firms dropping out of the market for new customers in turn serves to sustain that market, with a greater share of buyers in existing customer relationships implying a lower equilibrium probability of an individual firm seeking to attract new customers.

Figure 3 illustrates these patterns in the context of a parameterized example. On the left side of each panel, few buyers are in existing customer relationships, and the equilibrium features all firms actively selling ($\alpha = 1$) at a price that attracts searching buyers. Moving right, the share of buyers in existing customer relationships increases, leading to the profitability of active selling declining, until firms become indifferent between the two pricing strategies at the vertical line. Moving further right, firms randomize between the two pricing strategies, remaining indifferent between them due to an increasing share of firms dropping out of active selling. When nearly all buyers are in existing customer relationships, the probability of active selling approaches zero.

[8] This uses the first order condition for the selling intensity (3) and that $\kappa_n(\hat{s}) = \hat{\kappa}(\hat{s}) - \hat{s}\kappa_s(\hat{s})$.

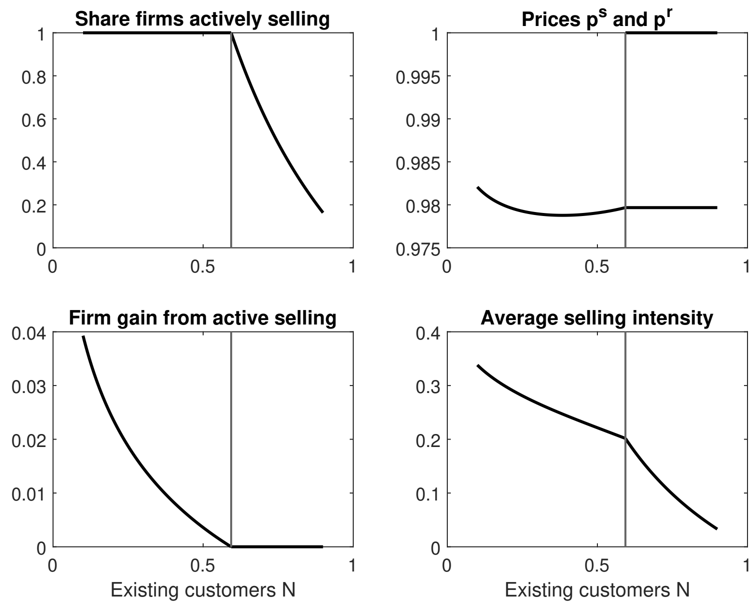

Figure 3: Equilibrium Outcomes as a Function of Existing Customer Relationships

Notes: The figure illustrates equilibrium outcomes as a function of the share of existing customer relationships in the market. The top left panel plots α, the top right the prices p^s, p^r, the bottom left a firm's net gain from active selling relative to forgoing doing so, and the bottom right the marketwide average selling intensity $\alpha\hat{s}$. When the share of existing customer relationships is sufficiently large, the equilibrium features sale pricing featuring two equilibrium prices. Here $u = 1$, $c = 1/2$, $\hat{\kappa}(\hat{s}) = \hat{s}^2/2$ and $\eta(\theta) = \theta/(1+\sqrt{\theta})^2$.

Efficient allocations How do equilibrium outcomes in this market compare to efficient allocations? To shed light on this question, this section turns to a planner problem.

A benevolent planner maximizes the value of output net of the costs of production and selling, facing the same frictions in creating customer relationships as market participants. The planner takes as given the existing customers at each firm, and decides how much selling effort each firm should take on as well as how to allocate searching buyers among firms:

$$\begin{aligned} \max_{\{s_i,\theta_i\}_{i\in I}} \; & \sum_{i\in I}(n_i + \eta(\theta_i)s_i)(u-c) - \sum_{i\in I}\kappa(s_i, n_i) \\ \text{s.t.} \; & \sum_{i\in I}\theta_i s_i = 1 - \sum_{i\in I} n_i. \end{aligned}$$

The planner does so subject to the constraint that the total measure of buyers allocated among firms equals the total measure of searching buyers.

The planner optimally allocates searching buyers such that the shadow value of additional buyers equals the gains from the additional customer relationships created: $\lambda = \eta'(\theta_i)(u-c)$, for each firm i. The optimality condition may equivalently be written as $\lambda = \mu(\theta_i)\varepsilon_i(u-c)$, which says that the efficient value of searching buyers is equated to the product of the buyer's probability of entering into a customer relationship and share ε of the gains from the relationship.

Correspondingly, the planner optimally allocates selling effort such that the costs of additional selling equal the gains from the additional customer relationships created: $\kappa_s(\hat{s}_i)+\lambda\theta_i = \eta(\theta_i)(u-c)$, for each firm i. The condition may also be written as $\kappa_s(\hat{s}_i) = \eta(\theta_i)(1-\varepsilon_i)(u-c)$, which says that the marginal selling cost is equated to the product of the probability of entering into a customer relationship and share $1-\varepsilon$ of the gains from the relationship.

To relate equilibrium outcomes to efficient allocations, it is convenient to note that the efficient allocation may be decentralized by firms setting price $p_i^p = \varepsilon_i c + (1-\varepsilon_i)u$, with the selling effort that satisfies the firm optimality condition (3) with this price.[9] Efficient pricing may thus also be expressed as a weighted average of firm cost and buyer valuation, but with different weighting than in equilibrium. Equilibrium prices, both p^s and p^r, are generally higher than efficient because firm pricing is influenced by the firms' incentive to profit from existing customers, whereas efficient pricing is not.

Proposition 2. *The competitive search equilibrium with anonymous pricing without sale pricing has a strictly higher price, selling intensity, volume of trade and firm profit than efficient.*

The equilibrium without sale pricing is inefficient, as too high prices lead to overselling and consequently excess trade in the product market. Even though the equilibrium with sale pricing is also inefficient, relative to this starting point the emergence of sale pricing may nevertheless be viewed as beneficial for resource allocation, in reducing the overselling taking place (as a share of firms withdraw from active selling).

Comparing the expressions for prices also shows that, due to the firm focus on profiting from existing customers, equilibrium pricing becomes too focused on buyer valuation for the product and consequently too unresponsive to firm cost. The emergence of sale pricing only makes this rigidity with respect to cost more pronounced in that regular prices respond to cost even less, even if sale pricing does also introduce discontinuous jumps between sale price and regular price.

Asymmetric pricing The price paths used to illustrate sale pricing (as in Figure 1) are typically characterized by an asymmetry, featuring a relatively stable regular price that

[9] The equilibrium value of search then satisfies $S = \mu(\theta_i)\varepsilon_i(u-c)$ and equilibrium selling intensity $\kappa_s(\hat{s}_i) = \eta(\theta_i)(1-\varepsilon_i)(u-c)$. These conditions coincide with the planner's optimality conditions, with the value of search coinciding with the planner's shadow value of searching buyers.

undergoes repeated temporary downward shifts of varying magnitude over time. A similar pattern emerges in the model when firms face idiosyncratic shocks.

To see this, consider the equilibrium with sale pricing, where firms are indifferent between charging a lower sale price and a higher regular price. Suppose then that a single firm faces a slightly lower production cost in this period than other firms. Instead of being indifferent between the two pricing strategies, the firm will strictly prefer to seek to attract new customers. Its profits from both strategies exceed other firms' due to its lower cost, but the profits from attracting new customers increase more because the firm benefits from lower costs on new customers as well.[10] The firm thus responds to the lower cost by setting a sale price and taking on sales activities, and with both price and selling intensity depending on the realized cost.[11]

On the other hand, if the firm faces a slightly higher production cost in this period than other firms, then the firm will strictly prefer to hold off on seeking to attract new customers, focusing on making profit on its existing customers instead. The firm thus responds to the higher cost by setting the regular price, which is independent of firm cost. Among the firms in the market, some charging the regular price and some the sale price, an individual firm's responses to increases and decreases in cost are thus asymmetric.

One can extend the logic to consider a setting where different firms face somewhat different costs this period, and anticipate the equilibrium to feature sufficiently low cost firms seeking to attract new customers—charging a sale price that reflects their cost while taking on sales activities—and sufficiently high cost firms focusing on making profit on existing customers—charging the regular price that is independent of their cost. In this setting the model generates sale pricing where sales are triggered by sufficiently low cost realizations, with sale prices reflecting cost and regular prices independent of cost.

One can also consider demand side factors such as buyer valuation for the firm product. In a setting where different firms face somewhat different buyer valuations this period, one can anticipate the equilibrium to feature sufficiently high valuation firms seeking to attract new customers—charging a sale price that reflects their buyer valuation while taking on sales activities—and sufficiently low valuation firms focusing on making profit on existing customers—charging a regular price that now also reflects their lower buyer valuation. In this setting the model generates sale pricing where sales are triggered by sufficiently high

[10] The difference in profits between seeking to attract new customers versus not, $[(1+\eta(\theta)\hat{s})(p^s - c) - \kappa(\hat{s}) - (u - c)]n$ with $p^s, \theta, \hat{s}$ satisfying the corresponding first order conditions, decreases in cost c.

[11] The firm's sale price, queue of buyers, and selling intensity satisfy the same expressions as equilibrium firms' but with a different cost: $p^s = \gamma c + (1-\gamma)u$, $S = \mu(\theta)(u - p^s)$, $\kappa_s(\hat{s}) = \eta(\theta)(p^s - c)$ with S unchanged.

buyer valuation, with both sale prices and regular prices reflecting buyer valuation—but where the switch to sale pricing limits increases in regular price in response to increased buyer valuation.

In a market where firms face both differing costs and differing buyer valuation in this period, sale pricing may be triggered by low costs or high buyer valuation for the firm's product. Sale prices reflect both cost and buyer valuation, but also regular prices can shift with buyer valuation.

Teaser pricing What happens if firms can keep track of individual customers and differentiate accordingly in pricing, instead of being constrained by anonymous pricing? In such settings the theory predicts the emergence of teaser pricing, as also often observed in consumer markets with more explicit long-term customer relationships.

In thinking about price setting when firms are able to differentiate among customers, the first thing to note is that firms optimally charge their existing customers their full valuation for the product, $p^e = u$. This is the most the firm can charge an existing customer while still retaining them, and is hence what a profit maximizing firm should charge its existing customers. With this, the firm's pricing problem reduces to a question of how to set the price p^n for new customers:

$$\begin{aligned} \max_{p_i^n, s_i} \; & \eta(\theta_i) s_i (p_i^n - c) - \kappa(s_i, n_i) \\ \text{s.t. } & S = \mu(\theta_i)(u - p_i^n) \text{ if } s_i > 0, \\ & p_i^n \leq u, \end{aligned}$$

taking its existing customer base and the equilibrium value of search as given. The profits reflect sales to new customers at price p_i^n and production cost c, net of selling costs. If the firm is seeking to attract new customers, it faces constraint (1) characterizing the queue of searching buyers attracted by the firm's price. Either way, the price cannot exceed buyer valuation for the product, or buyers would not agree to trade.

The firm problem is again effectively independent of scale. Dividing by n_i and defining selling effort per existing customer as $\hat{s}_i = s_i/n_i$, the problem may be written as

$$\begin{aligned} \max_{p_i^n, \hat{s}_i} \; & \eta(\theta_i) \hat{s}_i (p_i^n - c) - \hat{\kappa}(\hat{s}_i) \\ \text{s.t. } & S = \mu(\theta_i)(u - p_i^n) \text{ if } \hat{s}_i > 0, \\ & p_i^n \leq u, \end{aligned}$$

taking the equilibrium value of search as given.

The optimal selling intensity and price satisfy the corresponding first order conditions. The first order condition for selling,

$$\kappa_s(\hat{s}_i) = \eta(\theta_i)(p_i^n - c), \tag{6}$$

states that the firm chooses a selling intensity where the marginal cost of selling is equated to the profits from sales to the additional customers acquired.

The first order condition for the price,

$$\eta(\theta_i)\hat{s}_i = -\eta'(\theta_i)g_p(p_i^n; S)\hat{s}(p_i^n - c), \tag{7}$$

states that the firm raises the price to point where the increase in profits due to higher profit margins per customer equals the decrease in profits due to reduced customer acquisition. Here the increase in price again reduces the queue of searching buyers according to $g_p(p_i^n; S)$.

The optimality condition (7) implies that the price may be written as a weighted average of firm cost and buyer valuation, $p_i^n = \varepsilon_i c + (1 - \varepsilon_i)u$, with the weight given by the matching function elasticity. Relaxing anonymous pricing thus leads to teaser pricing, where new customers pay a lower price than existing customers: $p^n < p^e$.

Proposition 3. *The competitive search equilibrium without anonymous pricing is unique and equilibrium allocations efficient.*

Relaxing anonymous pricing also leads to equilibrium allocations becoming efficient. To profit on existing customers, firms charge existing customers their full valuation for the product. Yet to attract new customers, firms simultaneously set a competitive price for them. This added flexibility in pricing is enough to attain efficient allocations. Anonymous pricing rules out teaser pricing, but in a sense the sale pricing that emerges with anonymous pricing could be viewed as a proxy for teaser pricing—that can also serve to improve on efficiency of resource allocation—when the latter is not feasible.

Finally, note that teaser pricing continues to feature high prices that respond little to cost for existing customers, but with initial discounts allowing attaining efficient allocations nevertheless. In this case the seemingly high and rigid pricing is thus not allocative.

Next, I proceed to extend this static analysis to an explicitly dynamic setting, which allows prices to fluctuate over time as well as across firms. The key ideas remain unchanged.

3 Dynamic Model

Consider a competitive product market where buyers and sellers face search frictions in coming together to trade, with the frictions giving rise to long-term customer relationships.

Time is discrete and the horizon infinite. The market has measure one buyers and a large number of firms. All agents are risk neutral and discount the future at rate β. The firms produce heterogeneous products, at unit cost $c(z)$, where z is a Markovian productivity shock. Buyers have unit demand per period, but heterogeneous tastes across products, valuing a firm's product at either $u(>0)$ or zero.

Firms begin each period with some existing customers, of measure $n_{it}(>0)$, that have already found the firm's product acceptable. This leaves $1-\sum_i n_{it}(>0)$ buyers unmatched and searching in the market. Acquiring additional customers requires costly sales activities on the part of firms, to inform searching buyers of the firm's product. Selling effort s_t is subject to a cost $\kappa(s_t,n_t)=\hat{\kappa}(s_t/n_t)n_t$, where $\hat{\kappa}$ is increasing and convex. The convex cost reflects a firm's limited ability to increase selling effort locally, and the homothetic form that larger firms are less constrained by this due to operating in multiple locations.

The process of bringing firms and searching buyers together involves coordination frictions, captured with a matching function. If a firm exerts selling effort s_t and attracts b_t searching buyers, the measure of resulting new customer relationships is given by the constant returns to scale function $m(b_t,s_t)$. With this, the probability a searching buyer becomes the firm's customer, $\mu(\theta_t)=m(b_t,s_t)/b_t$, becomes a decreasing and convex function of the queue of searching buyers per selling effort at the firm $\theta_t=b_t/s_t$. The probability that a unit of selling effort leads to a new customer, $\eta(\theta_t)=m(b_t,s_t)/s_t$, becomes an increasing and concave function of the same. The elasticity of the matching function, $\varepsilon(\theta)=\theta\eta'(\theta)/\eta(\theta)$, is assumed to be weakly decreasing, reflecting that customer acquisition becomes less responsive to queues when queues become longer.

Customer relationships last until severed for exogenous, idiosyncratic reasons with probability $\delta\in(0,1)$ each period, or until the customer or firm prefers to end the relationship. During the customer relationship, the firm supplies the customer with a unit of the product each period, with the customer paying the corresponding price each period.

In setting its prices a firm faces a tradeoff between profit per customer and number of customers served. Higher prices raise profit per customer, but also attract fewer searching buyers and hence result in fewer new customers. A firm expects the queue of searching buyers it attracts in any period to be such that searching buyers are left indifferent between choosing this firm versus any other in the market. Denoting the equilibrium buyer value of search by S_t, the queue of searching buyers θ_t a firm expects to attract with prices $\{p_{t+k}\}_{k=0}^{\infty}$ satisfies

$$S_t = \mu(\theta_t) E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (u - p_{t+k} + \beta\delta S_{t+1+k}) + (1-\mu(\theta_t))\beta E_t S_{t+1}. \tag{8}$$

Here a searching buyer becomes a customer with probability $\mu(\theta_{it})$, getting the good at valuation u and paying the price p_t until the relationship ends and the buyer returns to search. If the searching buyer does not become a customer, they continue to search next period. The relationship (8) determines the queue of buyers as a decreasing function of the firm's prices, with firms taking the equilibrium value of search as given.

To express equation (8) more compactly, I define the flow value of search as $x_t = S_t - \beta E_t S_{t+1}$ and the present value of these flows that a buyer forgoes during a customer relationship as $X_t = E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k x_{t+k}$. Further, I define the present values of products received and prices paid during the relationship as $U = E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k u$ and $P_t = E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k p_{t+k}$, respectively. With this, equation (8) may be written as $x_t = \mu(\theta_t)(U - P_t - \beta(1-\delta)E_t X_{t+1})$, expressing the equilibrium flow value of search as a product of the probability that a searching buyer becomes a customer and the resulting present value of products net of the present values of price and forgone search while a customer.

Each firm chooses its prices and selling effort $\{p_t, s_t\}_{t=0}^{\infty}$ to maximize its present value:

$$\begin{aligned} & E_0 \sum_{t=0}^{\infty} \beta^t [(n_t + \eta(\theta_t)s_t)(p_t - c_t) - \kappa(s_t, n_t)] \\ \text{s.t. } & n_{t+1} = (1-\delta)(n_t + \eta(\theta_t)s_t), \ \forall t \geq 0, \\ & x_t = \mu(\theta_t)(U - P_t - \beta(1-\delta)E_t X_{t+1}) \text{ if } s_t > 0, \ \forall t \geq 0, \\ & P_t \leq U - \beta(1-\delta)E_t X_{t+1}, \ \forall t \geq 0, \end{aligned} \tag{9}$$

taking its existing customer base and the equilibrium value of search as given. The per-period profits reflect sales to existing and new customers at price p_t and production cost c_t, net of selling costs. The firm is constrained by a law of motion for the customer base. If the firm is seeking to attract new customers in a given period, it also faces constraint (8) characterizing the queue of buyers attracted by the firm's prices. Either way, the present value of prices cannot exceed buyer valuation for the product adjusted for the buyer option to return to search for a new firm if preferred, or buyers would not agree to trade.

Firm value may equivalently be expressed as a sum of the contributions of different cohorts of customers as:

$$n_0[P_0 - C_0] + E_0 \sum_{t=0}^{\infty} \beta^t [\eta(\theta_t) s_t (P_t - C_t) - \kappa(s_t, n_t)],$$

denoting the present value of costs by $C_t = E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k c_t$. Firm value derives from the present-value profits on the initial cohort of customers of measure n_0 together with present-value profits on subsequent cohorts of customers of measure $\eta(\theta_t)s_t$. Prices, specifically the present value of prices P_t, determine present-value profits per customer, as well as customer acquisition (together with selling effort). Note that, with this, the firm problem (9) may equivalently be written with the firm choosing a sequence of present-value prices $\{P_t\}_{t=0}^{\infty}$ instead of corresponding per-period prices.

Writing the firm problem in these terms highlights a time inconsistency affecting firm price setting, as the tradeoffs the firm faces in the initial period differ from later periods. In setting P_0, the firm takes into account that raising this present-value price raises present-value profits on both existing and new customers, at the cost of reduced customer acquisition. But in setting P_t for subsequent periods, the firm only takes into account the impact on new customers. The firm's behavior in the initial period thus differs from subsequent periods. Implementing such a plan requires commitment on the part of the firm, as it would choose differently in subsequent periods if able to reoptimize then.

I proceed by considering price setting assuming that firms are free to reoptimize prices each period, in a Markov perfect equilibrium. To that end, I denote the market-wide state as $\Omega = (N, z)$. The firm problem may then be written recursively as:

$$\begin{aligned}
\max_{P,s} \; & (n + \eta(\theta)s)(P - C(z)) - \kappa(s, n) + \beta E_\Omega V(n', \Omega') \\
\text{s.t. } & n' = (1-\delta)(n + \eta(\theta)s), \\
& x(\Omega) = \mu(\theta)(U - P - \beta(1-\delta)E_\Omega X(\Omega')) \text{ if } s > 0, \\
& P \leq U - \beta(1-\delta)E_\Omega X(\Omega'),
\end{aligned}$$

where $V(n, \Omega) = \eta(\theta)s(P - C(z)) - \kappa(s, n) + \beta E_\Omega V(n', \Omega')$ and the firm takes its existing customer base and the equilibrium value of search as given.

A Markov perfect equilibrium generally prescribes a firm's choices as functions of all payoff-relevant state variables, here (n, Ω). Due to the scale-independence of the firm problem, it becomes natural and convenient to restrict attention to equilibria where firm choices are independent of scale as well. To see the scale-independence, I first define the scaled variables $\hat{s} = s/n$, $\hat{V}(\Omega) = V(n, \Omega)/n$. Scaling the recursive firm problem by n then yields a firm problem that is independent of n:[12]

$$\max_{P,\hat{s}} \; (1+\eta(\theta)\hat{s})(P-C(z)) - \hat{\kappa}(\hat{s}) + \beta(1-\delta)(1+\eta(\theta)\hat{s})E_\Omega \hat{V}(\Omega') \tag{10}$$
$$\text{s.t. } X(\Omega) = \mu(\theta)(U - P - \beta(1-\delta)E_\Omega X(\Omega')) \text{ if } s>0,$$
$$P \leq U - \beta(1-\delta)E_\Omega X(\Omega'),$$

where $\hat{V}(\Omega) = \eta(\theta)\hat{s}(P-C(z)) - \hat{\kappa}(\hat{s}) + \beta(1-\delta)(1+\eta(\theta)\hat{s})E_\Omega\hat{V}(\Omega')$ and the firm takes the equilibrium value of search as given. Due to the scale-independence of the firm problem, I restrict attention to firm choices that are also functions of the market-wide state Ω alone.

The firm has two distinct options here, as in the static problem. It can seek to attract new customers, which requires a present-value price that is competitive in the market for searching buyers, or hold off on doing so to focus on making profit on its existing customers instead.

Option 1: Active selling to attract new customers If the firm seeks to attract new customers, it chooses a positive selling intensity $\hat{s} > 0$ together with a present-value price that is competitive in the market for searching buyers. This price will generally be strictly below the buyers' willingness to pay, as firms use prices to compete for buyers, implying that the second condition in the firm problem (10) becomes superfluous.

An interior optimum with active selling must satisfy first order conditions.[13] The first order condition for the selling intensity,

$$\kappa_s(\hat{s}) = \eta(\theta)(P^s - C(z) + \beta(1-\delta)E_\Omega\hat{V}(\Omega')), \tag{11}$$

states that the firm chooses a selling intensity where the marginal cost of selling is equated to the present-value profits from sales to the additional customers acquired. Additional customers yield a present value of prices P^s net of costs C, as well as reducing the costs of sales activities in future periods, as reflected in the continuation value $\hat{V}$ (discussed below).

The first order condition for the present-value price,

$$1+\eta(\theta)\hat{s} = -\eta'(\theta)g_P(P^s;\Omega)\hat{s}(P^s - C(z) + \beta(1-\delta)E_\Omega\hat{V}(\Omega')), \tag{12}$$

states that the firm raises the present-value price to a point where the increase in firm value due to greater profit margins per customer equals the decrease in firm value due to reduced

[12]Note that in general one would expect that $P(n), s(n)$ depend on size. Differentiating value function gives $V'(n_t) = -\kappa_n(s_t,n_t) + \beta(1-\delta)E_tV'(n_{t+1}) + [\eta(\theta_t)(P_t - C_t + \beta(1-\delta)E_tV(n_{t+1})) - \kappa_s(s_t,n_t)]s'(n_t) + [\eta(\theta_t)s_t + \eta'(\theta_t)f'(P_t)s_t(P_t - C_t + \beta(1-\delta)E_tV'(n_{t+1}))]P'(n_t)$ and then using the FOC yields $V'(n_t) = -\kappa_n(s_t,n_t) + \beta(1-\delta)E_tV'(n_{t+1}) - n_tP'(n_t)$. In general the marginal value thus depends on the derivative of the decision rule. The final term vanishes if P is independent of size.

[13]I restrict attention to circumstances where first order conditions are also sufficient for interior optimum.

customer acquisition. Here the increase in present-value price reduces the queue of searching buyers according to $g_P(P^s;\Omega) = \mu(\theta)/(\mu'(\theta)(U - P^s - \beta(1-\delta)E_\Omega X(\Omega'))) < 0$.

The optimality condition (12) implies that the present-value price may be written as a weighted average of firm cost and buyer willingness to pay:

$$P^s = \gamma(C(z) - \beta(1-\delta)E_\Omega \hat{V}(\Omega')) + (1-\gamma)(U - \beta(1-\delta)E_\Omega X(\Omega')) \tag{13}$$

with $\gamma = \varepsilon\Delta/(1-\varepsilon+\Delta)$, where $\varepsilon \in (0,1)$ is the matching function elasticity and $\Delta = \eta(\theta)\hat{s} > 0$ the share of new versus existing customers in firm sales. In addition to firm cost and buyer willingness to pay, this price reflects how effective pricing is at attracting new customers and how important new customers are for firm sales. If longer queues of searching buyers increase customer acquisition effectively (elasticity ε is large), the firm sets a lower price to attract more new customers. If new customers make up a large share of firm sales relative to existing ones (share Δ is large), the price is lower to attract more new customers.

In terms of firm costs, additional customers imply both added costs of production as well as reducing the costs of customer acquisition going forward. Buyer willingness to pay, on the other hand, takes into account both the buyer valuation for the products and the forgone search during the relationship.

Option 2: Profit from existing customers Alternatively, the firm can forgo attracting new customers to focus on making profits on its existing customers instead.

If the firm does not participate in the market for searching buyers, its selling intensity is zero and its pricing problem reduces to

$$\max_P \; P - C(z) + \beta(1-\delta)E_\Omega \hat{V}(\Omega') \tag{14}$$
$$P \leq U - \beta(1-\delta)E_\Omega X(\Omega'),$$

taking the equilibrium value of search as given. The optimal present-value price becomes $P^r = U - \beta(1-\delta)E_\Omega X(\Omega')$, with the firm taking the full gains from the relationship. This price is too high to attract searching buyers, but existing customers are willing to pay it. Existing customers have already determined that they find the firm's product acceptable, whereas searching buyers must be enticed to find out, in the face of competition from other firms.

Which of the two options dominates for the firm in a given period becomes more subtle in the dynamic model than the static one. The firm problem retains essentially the same form as in the static model, however, with firm behavior continuing to depend on the prevailing value of search S_t in much the same way as before when holding other things equal.[14] The

[14] A higher current value of search S_t implies a higher x_t, whereas X_{t+1} depends on future values of search.

present-value profits from seeking to attract new customers are decreasing in S_t, because a higher value of S_t means it is more costly for the firm to attract searching buyers. Meanwhile, the present-value profits from focusing on profiting from existing customers are independent of S_t. Thus, for sufficiently low values of S_t the firm prefers to seek to attract new customers, and from some S_t up it focuses on profiting from existing customers instead. In between there will generally be a value of search where firms are indifferent between the two alternatives, and might randomize between active selling at a lower price versus profiting from existing customers with a higher price instead.

Equilibrium firm behavior must maximize firm value, as well as be consistent with a market clearing condition for searching buyers, each period. From the optimality conditions, it is clear that if all firms are identical (aside from possible differences in size), their choices are also identical in that all actively selling firms choose the same selling intensity and present-value price (with the same queue length), while all remaining firms choose the same present-value price. Denoting the total measure of existing customers across firms in the beginning of period t by $N_t = \sum_i n_{it}$, and the share of firms actively selling by α_t, the market clearing condition requires that the total measure of searching buyers across firms, $\theta_t \hat{s}_t \alpha_t N_t$, equals the total measure of searching buyers in the market, $1 - N_t$, for all t.[15]

Definition 2. *A competitive search equilibrium with anonymous pricing specifies shares of firms actively selling α_t, corresponding sale prices p_t^s, queues θ_t and selling intensities $\hat{s}_t$, as well as regular prices p_t^r and values of search S_t for $t \geq 0$ such that: i) $\{P_t^s, \theta_t, \hat{s}_t\}$ solves the firm problem* (10) *with $\hat{s}_t > 0$, and P_t^r solves the firm problem* (14)*, ii) if $\alpha_t < 1$, firms are indifferent between active selling and not, and if $\alpha_t = 1$, firms weakly prefer active selling, iii) the market for searching buyers clears: $1 - N_t = \theta_t \hat{s}_t \alpha_t N_t$, and iv) the law of motion $N_{t+1} = (1-\delta)(N_t + \mu(\theta_t)(1 - N_t))$ for all $t \geq 0$.*

The equilibrium features sale pricing in period t when $\alpha_t < 1$ in the sense that a share of firms charge a lower "sale" price and take on costly sales activities to inform searching buyers of their product, while the remaining firms charge a higher "regular" price without the latter.

If both strategies are pursued by some firms in equilibrium, then firms must be indifferent between them. The present-value profits from active selling consist of the profits from existing customers $n_t[P_t^s - C_t + \beta(1-\delta)E_t\hat{V}_{t+1}]$ and the profits from new customers $n_t[\eta(\theta_t)\hat{s}_t(P_t^s - C_t + \beta(1-\delta)E_t\hat{V}_{t+1}) - \hat{\kappa}(\hat{s}_t)] = -n_t\kappa_n(\hat{s}_t)$, both positive.[16] Holding off on active selling

[15] Each actively selling firm attracts $\theta_t \hat{s}_t n_{it}$ buyers. If share α of firms (of all sizes) sells, adding these buyers up across selling firms yields a total measure of buyers $\theta_t \hat{s}_t \alpha_t N_t$, which must equal $1 - N_t$.

[16] This uses the first order condition for selling intensity (11) and that $\kappa_n(\hat{s}) = \hat{\kappa}(\hat{s}) - \hat{s}\kappa_s(\hat{s})$.

yields higher profits on existing customers, $n_t[U - \beta(1-\delta)E_tX_{t+1} - C_t + \beta(1-\delta)E_t\hat{V}_{t+1}]$, but none on new customers. In such an equilibrium the present-value profits from new customer acquisition must thus just make up for charging existing customers less: $P_t^s - \kappa_n(\hat{s}_t) = U - \beta(1-\delta)E_tX_{t+1}$. By contrast, if all firms are actively selling, it must be that $P_t^s - \kappa_n(\hat{s}_t) \geq U - \beta(1-\delta)E_tX_{t+1}$.

Firms that are actively selling in period t have selling intensity and present-value price satisfying

$$\kappa_s(\hat{s}_t) = \eta(\theta_t)(P_t^s - C_t + E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\alpha_{t+k}\kappa_n(\hat{s}_{t+k})) \text{ and} \tag{15}$$

$$P_t^s = \gamma_t(C_t + E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\alpha_{t+k}\kappa_n(\hat{s}_{t+k})) + (1-\gamma_t)(U - \beta(1-\delta)E_tX_{t+1}). \tag{16}$$

These conditions correspond to (11) and (13), but with the effect of existing customers to reduce selling costs made explicit.[17] The remaining firms charge a higher present-value price that does not attract new customers: $P_t^r = U - \beta(1-\delta)E_tX_{t+1}$.

The corresponding per-period prices p_t^s and p_t^r satisfy

$$P_t^s = p_t^s + \beta(1-\delta)E_t(\alpha_{t+1}P_{t+1}^s + (1-\alpha_{t+1})P_{t+1}^r),$$
$$P_t^r = p_t^r + \beta(1-\delta)E_t(\alpha_{t+1}P_{t+1}^s + (1-\alpha_{t+1})P_{t+1}^r).$$

These relationships imply that the per-period regular price exceeds the per-period sale price, as firms randomize between the two pricing strategies and hence $p_t^r - p_t^s = P_t^r - P_t^s > 0$. The per-period sale price can also be lower than the per-period cost when the probability of a sale price is relatively low. If the probability of a sale price is low, the firm must implement any desired lower present-value sale price today with a low per-period sale price today, as prices are expected to be high in subsequent periods. (See Figure 4 for an illustration.)

As in the static model, the prevalence of existing customer relationships among buyers matters for outcomes in this market. By analogy, it would be natural to expect that when few buyers are in existing customer relationships to begin with, and the pool of searching buyers is consequently large, firms find it profitable to focus on attracting new customers. As the share of buyers in existing customer relationships increases, the profitability of doing

[17] The continuation values reflect the value of existing customers to reduce the costs of selling in periods in which the firm is actively selling. To see this, note that when a firm is actively selling today, its continuation value may be expressed as $\eta(\theta)\hat{s}(P^s - C(z)) - \hat{\kappa}(\hat{s}) + \beta(1-\delta)(1+\eta(\theta)\hat{s})E_\Omega\hat{V}(\Omega') = -\kappa_n(\hat{s}) + \beta(1-\delta)E_\Omega\hat{V}(\Omega')$, using the first order condition and $\kappa_n(\hat{s}) = \hat{\kappa}(\hat{s}) - \hat{s}\kappa_s(\hat{s})$. When a firm does not, this value reduces to $\beta(1-\delta)E_\Omega\hat{V}(\Omega')$ instead. If the firm randomizes with probability α, the expected continuation value becomes $-\alpha\kappa_n(\hat{s}) + \beta(1-\delta)E_\Omega\hat{V}(\Omega')$, reflecting the fact that actively selling firms benefit from existing customers via lower selling costs whereas the remainder of firms do not.

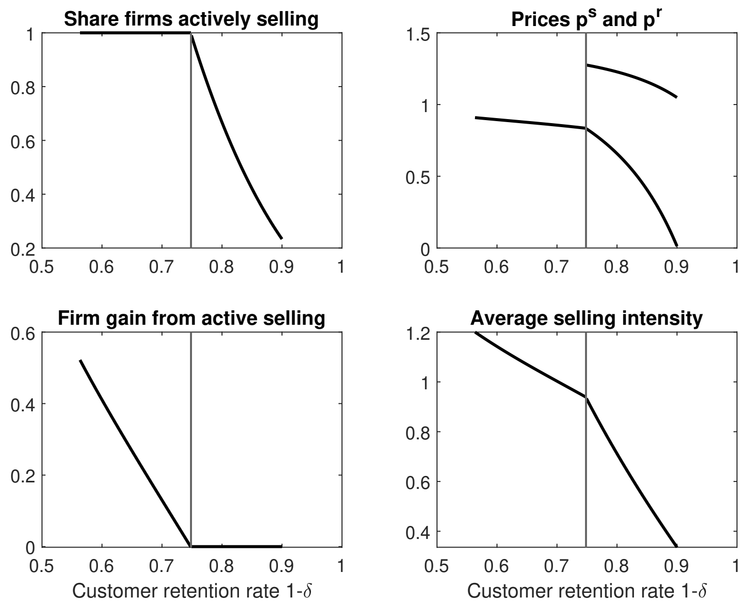

Figure 4: Steady State as a Function of the Customer Retention Rate $1-\delta$

Notes: The figure illustrates steady-state equilibrium outcomes as a function of the share of existing customer relationships in the market. The top left panel plots α, the top right the prices p^s, p^r, the bottom left a firm's net gain from active selling relative to forgoing doing so, and the bottom right the marketwide average selling intensity $\alpha\hat{s}$. When the share of existing customer relationships is sufficiently large, the equilibrium features sale pricing featuring two equilibrium prices. Here $u = 1$, $c = 1/2$, $\hat{\kappa}(\hat{s}) = \hat{s}^2/2$, $\eta(\theta) = \theta/(1+\sqrt{\theta})^2$ and $\beta = 1.05^{-1/24}$.

so falls, eventually leading to firms switching to randomizing between seeking to attract new customers versus focusing on profiting from existing customers instead. Firms dropping out of the market for new customers in turn serves to sustain that market, with a greater share of buyers in existing customer relationships implying a lower equilibrium probability of an individual firm seeking to attract new customers. In the dynamic model these current period outcomes further affect the prevalence of existing customer relationships, and hence market outcomes, in subsequent periods.

The following result characterizes steady state outcomes assuming a standard form for the selling cost and that the matching function elasticity declines from one toward zero as queues grow:

Proposition 4. *Let $\hat{\kappa}(\hat{s}) = \hat{s}^{\varphi}/\varphi$ such that $\varphi > 1$, $\lim_{\theta\to 0}\varepsilon(\theta) = 1$, and $\lim_{\theta\to\infty}\varepsilon(\theta) = 0$. There exists a $\delta^* \in (0,1)$ such that when $\delta < \delta^*$, the competitive search equilibrium with anonymous pricing has a unique steady state and this steady state features sale pricing, and when $\delta \geq \delta^*$, the competitive search equilibrium with anonymous pricing has a unique steady state and this steady state does not feature sale pricing.*

Figure 4 illustrates these patterns in the context of a parameterized example. On the left side of each panel, customer turnover is high and few buyers consequently in existing customer relationships. The equilibrium features all firms actively selling ($\alpha = 1$) at a price that attracts searching buyers. Moving right, customer turnover declines and the share of buyers in existing customer relationships increases, leading to the profitability of active selling declining, until firms become indifferent between the two pricing strategies at the vertical line. Moving further right, firms randomize between the two pricing strategies, remaining indifferent between them due to an increasing share of firms dropping out of active selling. When nearly all buyers are in existing customer relationships, the probability of active selling approaches zero.

Efficient allocations How do equilibrium outcomes in this market compare to efficient allocations? To shed light on this question, this section turns to a planner problem.

A benevolent planner maximizes the value of output net of the costs of production and selling, facing the same frictions in creating customer relationships as market participants. The planner takes as given the existing customers at each firm and decides, for each period, how much selling effort each firm should take on as well as how to allocate searching buyers among firms:

$$\begin{aligned} \max_{\{\theta_{it}, s_{it}\}_{t=0, i\in I}^{\infty}} \; & E_0 \sum_{t=0}^{\infty} \beta^t \sum_{i\in I} [(n_{it} + \eta(\theta_{it})s_{it})(u - c_t) - \kappa(s_{it}, n_{it})] \\ \text{s.t. } \; & n_{it+1} = (1-\delta)(n_{it} + \eta(\theta_{it})s_{it}), \;\; \forall i \in I, t \geq 0, \\ & \sum_{i\in I} \theta_{it} s_{it} = 1 - \sum_{i \in I} n_{it}, \;\; \forall t \geq 0. \end{aligned}$$

The planner does so subject to the law of motion for the customer base of each firm and the constraint that the total measure of buyers allocated among firms equals the total measure of searching buyers.

The planner optimally allocates searching buyers such that the shadow value of additional buyers equals the gains from the additional customer relationships created:

$$\lambda_t = \eta'(\theta_{it})[u - c_t + E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k (u - c_{t+k} - \kappa_n(\hat{s}_{it+k}) - \lambda_{t+k})], \tag{17}$$

each period and for each firm. The gains accrue for as long as the relationship lasts, with existing customers forgoing search during that time. Correspondingly, the planner optimally allocates selling effort such that the costs of additional selling equal the gains from the additional customer relationships created:

$$\kappa_s(\hat{s}_{it}) + \lambda_t\theta_{it} = \eta(\theta_{it})[u - c_t + E_t \sum_{k=1}^{\infty} \beta^k(1-\delta)^k(u - c_{t+k} - \kappa_n(\hat{s}_{it+k}) - \lambda_{t+k})], \tag{18}$$

each period and for each firm.

To relate equilibrium outcomes to efficient allocations, it is convenient to note that the efficient allocation may be decentralized by each firm setting present-value price

$$P_t^p = \varepsilon_t(C_t + E_t \sum_{k=1}^{\infty} \beta^k(1-\delta)^k\kappa_n(\hat{s}_{t+k})) + (1-\varepsilon_t)(U - \beta(1-\delta)E_tX_{t+1}) \tag{19}$$

with the selling effort that satisfies the firm's optimality condition (15) with this present-value price, each period.[18] Efficient pricing may thus also be expressed as a weighted average of firm cost and buyer willingness to pay, but with different weighting than in equilibrium. Equilibrium present-value prices, both P^s and P^r, are generally higher than efficient because firm pricing is influenced by the firms' incentive to profit from existing customers, whereas efficient pricing is not.

Note that the planner problem does not pin down efficient per-period prices without additional assumptions, leaving the efficient dynamics of per-period prices undetermined. If efficient pricing must satisfy the anonymity property discussed—all customers facing a common per-period price each period—then efficient per-period prices are determined via the relationship $P_t^p = p_t^p + \beta(1-\delta)E_tP_{t+1}^p$, with efficient per-period prices reflecting efficient present-value prices.

Proposition 5. *The steady state of the competitive search equilibrium with anonymous pricing that does not feature sale pricing has strictly higher present-value price, selling intensity, volume of trade and firm profit than efficient.*

The equilibrium without sale pricing is inefficient, as too high prices lead to overselling and consequently excess trade in the product market. Even though the equilibrium with

[18]To see this, note that (17) may be rewritten as $\mu(\theta_{it})\varepsilon_{it}[u-c_t+E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k(u-c_{t+k}-\kappa_n(\hat{s}_{it+k})-\lambda_{t+k})] = \lambda_t$, which indicates that the efficient value of search is equated to the product of the probability of entering into a customer relationship and share ε_{it} of the gains from the relationship. Further, (18) may be rewritten as $\eta(\theta_{it})(1-\varepsilon_{it})[u - c_t + E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k(u - c_{t+k} - \kappa_n(\hat{s}_{it+k}) - \lambda_{t+k})] = \kappa_s(\hat{s}_{it})$, which indicates that the marginal selling cost is equated to the product of the probability of entering into a customer relationship and share $1-\varepsilon_{it}$ of the gains from the relationship. By contrast, (19) implies that the equilibrium value of search satisfies $x_t = \mu(\theta_{it})\varepsilon_{it}[U_t - C_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k(\kappa_n(\hat{s}_{it+k})+x_{t+k})]$ and equilibrium selling and equilibrium selling effort $\kappa_s(\hat{s}_{it}) = \eta(\theta_{it})(1-\varepsilon_{it})[U - C_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k(\kappa_n(\hat{s}_{it+k})+x_{t+k})]$. These conditions coincide with the planner's optimality conditions with the value of search coinciding with the planner's shadow value of searching buyers.

sale pricing is also inefficient, relative to this starting point the emergence of sale pricing may nevertheless be viewed as beneficial for resource allocation, in reducing the overselling taking place (as a share of firms withdraw from active selling).

Comparing the expressions for prices also shows that, due to the firm focus on profiting from existing customers, equilibrium pricing becomes too focused on buyer willingness to pay for the product and too unresponsive to firm cost. The emergence of sale pricing only makes this rigidity with respect to cost more pronounced in that regular prices respond to cost even less, even if sale pricing does also introduce discontinuous jumps between sale price and regular price.

Asymmetric pricing The price paths used to illustrate sale pricing (as in Figure 1) are typically characterized by an asymmetry, featuring a relatively stable regular price that undergoes repeated temporary downward shifts of varying magnitude over time. A similar pattern emerges in the model when firms face transitory idiosyncratic shocks.

To see this, consider a steady state with sale pricing, where firms are indifferent between charging a lower sale price and a higher regular price. Suppose then that a single firm faces a slightly lower production cost this period than the other firms, with costs in future periods unaffected. Instead of being indifferent between the two pricing strategies, the firm will strictly prefer to seek to attract new customers. Its present-value profits from both strategies exceed other firms' due to its lower cost, but the profits from attracting new customers increase more because the firm benefits from lower costs on new customers as well.[19] The firm thus responds to the lower cost by setting a sale price and taking on sales activities, and with both price and selling intensity depending on realized cost.

On the other hand, if the firm faces a slightly higher production cost this period than other firms, then the firm will strictly prefer to hold off on seeking to attract new customers, focusing on making profit on its existing customers instead. The firm thus responds to the higher cost by setting the regular price, which is independent of cost. Among the firms in the market, some charging the regular price and some the sale price, an individual firm's responses to increases and decreases in cost are thus asymmetric.

One can extend the logic to consider a setting where different firms face somewhat different costs this period, and anticipate the equilibrium to feature sufficiently low cost firms

[19]The difference in present-value profits between seeking to attract new customers versus not decreases in production cost. This difference reads $n_t[(1+\eta(\theta_t)\hat{s}_t)(P^s_t - C_t + \beta(1-\delta)E_t\hat{V}_{t+1}) - \kappa(\hat{s}_t) - (U - \beta(1-\delta)E_tX_{t+1} - C_t + \beta(1-\delta)E_t\hat{V}_{t+1})]$ where $P^s_t, \theta_t, \hat{s}_t$ satisfy the corresponding first order conditions, which decreases in C_t.

seeking to attract new customers—charging a sale price that reflects their cost while taking on sales activities—and sufficiently high cost firms focusing on making profit on existing customers—charging the regular price that is independent of their cost. In this setting the model generates sale pricing where sales are triggered by sufficiently low cost realizations, with sale prices reflecting cost and the regular price independent of cost.

One can also consider demand side factors such as buyer valuation for the firm product. In a setting where different firms face somewhat different buyer valuations this period, one can anticipate the equilibrium to feature sufficiently high valuation firms seeking to attract new customers—charging a sale price that reflects their buyer valuation while taking on sales activities—and sufficiently low buyer valuation firms focusing on making profit on existing customers—charging a regular price that also reflects their lower buyer valuation. In this setting the model generates sale pricing where sales are triggered by sufficiently high buyer valuation, with both sale prices and regular prices reflecting buyer valuation—but where the switch to sale pricing limits increases in regular price in response to increased buyer valuation.

In a market where firms face both differing firm costs and differing buyer valuation this period, sale pricing may be triggered by low costs or high buyer valuation for the firm's product. Sale prices reflect both cost and buyer valuation, but also regular prices can shift with buyer valuation.

Teaser pricing If firms can keep track of individual customers and differentiate accordingly in pricing, the theory predicts the emergence of teaser pricing, as also often observed in settings with more explicit long-term customer relationships.

In thinking about price setting when firms are able to differentiate among customers, the first thing to note is that firms optimally charge their existing customers their full willingness to pay for the product, in present value terms $P_t^e = U - \beta(1-\delta)E_t X_{t+1}$.[20] This is the most that the firm can charge an existing customer while still retaining them, and is hence what a profit maximizing firm should charge existing customers.

With this, the firm's pricing problem reduces to a question of how to set the price P_t^n for

[20]An existing customer is willing to remain with the firm as long as $E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (u - p_{t+k}^e + \beta\delta S_{t+k+1}) \geq \beta E_t S_{t+1}$, meaning if the present value of remaining weakly exceeds the present value of returning to search. In the notation introduced, this inequality reads $U - P_t^e - \beta(1-\delta)E_t X_{t+1} \geq 0$.

new customers:

$$\max_{\{P_t^n, s_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t [\eta(\theta_t) s_t (P_t^n - C(z_t)) - \kappa(s_t, n_t)]$$
$$\text{s.t. } n_{t+1} = (1-\delta)(n_t + \eta(\theta_t) s_t), \ \forall t \geq 0,$$
$$x_t = \mu(\theta_t)(U - P_t^n - \beta(1-\delta) E_t X_{t+1}) \text{ if } s_t > 0, \ \forall t \geq 0,$$
$$P_t^n \leq U - \beta(1-\delta) E_t X_{t+1}, \ \forall t \geq 0,$$

taking its existing customer base and the equilibrium value of search as given. Firm value from customer acquisition derives from present-value profits on current and future cohorts of new customers of measure $\eta(\theta_t)s_t$, reflecting sales at present-value price P_t^n and production cost C_t, net of selling costs. The firm is constrained by the law of motion for the customer base. If the firm is seeking to attract new customers, it faces constraint (8) characterizing the queue of searching buyers attracted by the firm's price. Either way, present-value prices cannot exceed buyer willingness to pay for the product, or buyers would not agree to trade.

The optimal selling intensity and price satisfy the corresponding first order conditions. The first order condition for selling,

$$\kappa_s(\hat{s}_t) = \eta(\theta_t)(P_t^n - C_t - E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k \kappa_n(\hat{s}_{t+k})),$$

states that the firm chooses a selling intensity where the marginal cost of selling is equated to the profits from sales to the additional customers acquired. The first order condition for the price,

$$\eta(\theta_t)\hat{s}_t = -\eta'(\theta_t) g_P^t (P_t^n) \hat{s}_t (P_t^n - C_t - E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k \kappa_n(\hat{s}_{t+k})), \tag{20}$$

states that the firm raises the price to a point where the increase in profits due to higher profit margins per customer equals the decrease in profits due to reduced customer acquisition. The increase in price again reduces the queue of searching buyers according to $g_P(P_t^n; \Omega_t)$.

The optimality condition (20) implies that the present-value price may be written as a weighted average of firm cost and buyer willingness to pay,

$$P_t^n = \varepsilon_t (C_t + E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k \kappa_n(\hat{s}_{t+k})) + (1-\varepsilon_t)(U - \beta(1-\delta) E_t X_{t+1})$$

with the weight given by the matching function elasticity. Relaxing anonymous pricing thus leads to teaser pricing, where new customers pay a lower price than existing customers, as $p_t^e - p_t^n = P_t^e - P_t^n > 0$ for all t.

Proposition 6. *The competitive search equilibrium without anonymous pricing is unique and equilibrium allocations efficient.*

Relaxing anonymous pricing also leads to equilibrium allocations becoming efficient. To profit on existing customers, firms charge existing customers their full willingness to pay for the product. Yet to attract new customers, firms simultaneously set a competitive price for them. This added flexibility in pricing is enough to attain efficient allocations. Anonymous pricing rules out teaser pricing, but in a sense the sale pricing that emerges with anonymous pricing could be viewed as a proxy for teaser pricing—that can also serve to improve on efficiency of resource allocation—when the latter is not feasible.

Finally, note that teaser pricing continues to feature high prices that respond little to cost for existing customers, but with initial discounts allowing attaining efficient allocations nevertheless.[21] In this case the seemingly high and rigid pricing is thus not allocative.

4 Conclusions

This paper has studied price setting in an equilibrium search model of frictional product markets with long-term customer relationships (Gourio and Rudanko 2014). The theory gives rise to temporary sales as an equilibrium outcome when pricing is constrained to be anonymous across a firm's customer base. Equilibrium prices are inefficiently high, leading to overselling and excess trade in the product market, and the emergence of sale pricing can improve allocations by limiting this overselling. Equilibrium pricing is also characterized by an asymmetry involving a rigid regular price and variable sale price when firms face idiosyncratic shocks. Absent anonymous pricing the theory gives rise to teaser pricing, which attains efficient allocations. Teaser pricing is also characterized by a stable regular price and a variable teaser price, but in this case the seeming rigidity is thus not allocative.

From the perspective of this theory, sale pricing may thus be viewed as a welfare-improving feature of product markets with repeat customers where firms set a common price for their customers. Where feasible, teaser pricing does better still, however, in avoiding distortions in customer acquisition due to the firms' price-setting power over existing customers. That both pricing schemes feature a relatively stable regular price and a variable sale/teaser price highlights the fact that per-period prices are not the relevant allocative prices in settings with long-term customer relationships, calling for considering broader mea-

[21] The per-period price of existing customers p_t^e satisfies $P_t^e = p_t^e + \beta(1-\delta)E_t P_{t+1}^e$, where $P_t^e = U - \beta(1-\delta)E_t X_{t+1}$. Transitory idiosyncratic shocks to firm cost do not affect these prices.

sures of expected prices instead.

References

Bils, M. (1989): "Pricing in a Customer Market," *Quarterly Journal of Economics*, 104(4), 699–718, doi: 10.2307/2937863.

Chevalier, J. A., A. K. Kashyap, and P. E. Rossi (2003): "Why Don't Prices Rise during Periods of Peak Demand? Evidence from Scanner Data," *American Economic Review*, 93, 15–37, doi: 10.1257/000282803321455142.

Coibion, O., Y. Gorodnichenko, and G. H. Hong (2015): "The Cyclicality of Sales, Regular and Effective Prices: Business Cycle and Policy Implications," *American Economic Review*, 105, 993–1029, doi: 10.1257/aer.20121546.

Dinlersoz, E. M., and M. Yorukoglu (2012): "Information and Industry Dynamics," *American Economic Review*, 102, 884–913, doi: 10.1257/aer.102.2.884.

Drozd, L., and J. Nosal (2012): "Understanding International Prices: Customers as Capital," *American Economic Review*, 102, 364–95, doi: 10.1257/aer.102.1.364.

Eichenbaum, M., N. Jaimovich, and S. Rebelo (2011): "Reference Prices, Costs, and Nominal Rigidities," *American Economic Review*, 101(1), 234–262, doi: 10.1257/aer.101.1.234.

Gertler, M., and J. Leahy (2008): "A Phillips Curve with an Ss Foundation," *Journal of Political Economy*, 116(3), 533–572, doi: 10.1086/589522.

Golosov, M., and R. E. Lucas, Jr. (2007): "Menu Costs and Phillips Curves," *Journal of Political Economy*, 115(2), 171–199, doi: 10.1086/512625.

Gourio, F., and L. Rudanko (2014): "Customer Capital," *Review of Economic Studies*, 81(3), 1102–36, doi: 10.1093/restud/rdu007.

Guimaraes, B., and K. D. Sheedy (2011): "Sales and Monetary Policy," *American Economic Review*, 101, 844–76, doi: 10.1257/aer.101.2.844.

Hitsch, G. J., A. Hortacsu, and X. Lin (2021): "Prices and Promotions in U.S. Retail Markets," *Quantitative Marketing and Economics*, 19(3), 289–368, doi: 10.1007/s11129-021-09238-x.

Kehoe, P., and V. Midrigan (2015): "Prices Are Sticky After All," *Journal of Monetary Economics*, 75, 35–53, doi: 10.1016/j.jmoneco.2014.12.004.

Klemperer, P. (1995): "Competition When Consumers Have Switching Costs," *Review of Economic Studies*, 62, 515–539, doi: 10.2307/2298075.

Klenow, P., and J. Willis (2016): "Real Rigidities and Nominal Price Changes," *Economica*, 83, 443–472, doi: 10.1111/ecca.12191.

Kleshchelski, I., and N. Vincent (2009): "Market Share and Price Rigidity," *Journal of Monetary Economics*, 56(3), 344–352, doi: 10.1016/j.jmoneco.2009.02.006.

Kryvtsov, O., and N. Vincent (2021): "The Cyclicality of Sales and Aggregate Price Flexibility," *Review of Economic Studies*, 88, 334–377, doi: 10.1093/restud/rdaa050.

Midrigan, V. (2011): "Menu Costs, Multi-Product Firms and Aggregate Fluctuations," *Econometrica*, 79, 1139–1180, doi: 10.3982/ECTA6735.

Nakamura, E., and J. Steinsson (2008): "Five Facts about Prices: A Reevaluation of Menu Cost Models," *Quarterly Journal of Economics*, 123, 1415–1464, doi: 10.1162/qjec.2008.123.4.1415.

——— (2011): "Price Setting in Forward-Looking Customer Markets," *Journal of Monetary Economics*, 58, 220–233, doi: 10.1016/j.jmoneco.2011.06.004.

——— (2013): "Price Rigidity: Microeconomic Evidence and Macroeconomic Implications," *Annual Review of Economics*, 5, 133–63, doi: 10.3386/w18705.

Paciello, L., A. Pozzi, and N. Trachter (2019): "Price Dynamics with Customer Markets," *International Economic Review*, 60(1), 413–446, doi: 10.1111/iere.12358.

Perla, J. (2016): "Product Awareness, Industry Life Cycles, and Aggregate Profits," Unpublished manuscript, University of British Columbia.

Phelps, E., and S. Winter (1970): "Optimal Price Policy under Atomistic Competition," in *Microeconomic Foundations of Employment and Inflation Theory*, pp. 309–337. W. W. Norton & Company.

Ravn, M., S. Schmitt-Grohe, and M. Uribe (2006): "Deep Habits," *Review of Economic Studies*, 73(1), 195–218, doi: 10.1111/j.1467-937X.2006.00374.x.

Roldan, P., and S. Gilbukh (2021): "Firm Dynamics and Pricing under Customer Capital Accumulation," *Journal of Monetary Economics*, 118, 99–119, doi: 10.1016/j.jmoneco.2020.01.011.

Salop, S., and J. E. Stiglitz (1982): "The Theory of Sales: A Simple Theory of Equilibrium Price Dispersion with Identical Agents," *American Economic Review*, 72, 1121–1130.

Shi, S. (2016): "Customer Relationship and Sales," *Journal of Economic Theory*, 166, 483–516, doi: 10.1016/j.jet.2016.09.012.

Sobel, J. (1984): "The Timing of Sales," *Review of Economic Studies*, 51, 353–368, doi: 10.2307/2297428.

Varian, H. R. (1980): "A Model of Sales," *American Economic Review*, 70, 651–659.

A Appendix

Proof of Proposition 1 An equilibrium without sale pricing is characterized by $\{\theta, p^s, \hat{s}\}$ that satisfy first order conditions for optimal selling intensity (3) and price (4) together with the market clearing condition $1-N = \theta\hat{s}N$, such that active selling dominates, $p^s - \kappa_n(\hat{s}) \geq u$.

The first order condition for price yields, using the other two equations to substitute out price and selling intensity, the following equation for equilibrium θ:

$$\theta + \eta(\theta)\frac{1-N}{N} = \frac{\varepsilon(\theta)}{1-\varepsilon(\theta)}\frac{1-N}{N}\frac{\kappa_s(\frac{1-N}{N\theta})}{u-c-\frac{\kappa_s(\frac{1-N}{N\theta})}{\eta(\theta)}}. \tag{21}$$

The left hand side of (21) is positive and strictly increasing in θ, increasing from zero when $\theta = 0$ toward infinity when θ approaches infinity. The right hand side of (21) is positive when $\theta > \underline{\theta}$, where $\underline{\theta} > 0$ is such that $u - c - \kappa_s(\frac{1-N}{N\underline{\theta}}) = 0$. In this region, the right hand side is strictly decreasing, approaching infinity when θ approaches $\underline{\theta}$ (from above) and approaching zero when θ approaches infinity. Equation (21) thus determines a unique θ. The remaining equations then determine unique values of $p^s, \hat{s}, S$ based on this θ.

Both θ and $\hat{s}$ decrease in N. To see this, note that combining (3) and (4) implies $\frac{1}{\hat{s}} + \eta(\theta) = \frac{\varepsilon(\theta)}{1-\varepsilon(\theta)}\frac{\kappa_s(\hat{s})}{u-c-\kappa_s(\hat{s})/\eta(\theta)}$, which implicitly determines $\hat{s}$ as a strictly increasing function of θ when $\theta > \underline{\theta}$. With this, the market clearing condition implies that both θ and $\hat{s}$ are strictly decreasing in N.

The profitability condition may be written, using (3) and (4), as:

$$p^s - \kappa_n(\hat{s}) - u = -\frac{\eta(\theta)\hat{s}}{1+\eta(\theta)\hat{s}}(p^s - c)\frac{\varepsilon(\theta)}{1-\varepsilon(\theta)} - \kappa_n(\hat{s}) = -\frac{\kappa_s(\hat{s})\hat{s}}{1+\eta(\theta)\hat{s}}\frac{\varepsilon(\theta)}{1-\varepsilon(\theta)} - \kappa_n(\hat{s})$$
$$= -\kappa_n(\hat{s})[1 - \frac{\varphi}{\varphi-1}\frac{1}{1+\eta(\theta)\hat{s}}\frac{\varepsilon(\theta)}{1-\varepsilon(\theta)}] \geq 0.$$

The first term is positive but the term in brackets is strictly decreasing in N, as both θ and $\hat{s}$ are strictly decreasing in N. The term in brackets approaches a positive value when N approaches zero (from above) and turns negative at some threshold N^* (where the profitability condition thus holds as an equality). When $N \leq N^*$, the profitability condition thus holds and there exists a unique equilibrium without sale pricing. When $N > N^*$, such an equilibrium does not exist.

An equilibrium with sale pricing is characterized by $\{\theta, p^s, \hat{s}\}$ that satisfy first order conditions for optimal selling intensity (3) and price (4), together with the profitability condition $p^s - \kappa_n(\hat{s}) = u$. The probability of a sale, $\alpha = (1-N)/(N\theta\hat{s})$, must satisfy $\alpha < 1$.

The first order condition for price yields, using the other two equations, selling intensity $\hat{s}$ as a function of θ: $\hat{s} = (\frac{\varepsilon(\theta)}{1-\varepsilon(\theta)}\frac{\hat{s}\kappa_s(\hat{s})}{-\kappa_n(\hat{s})} - 1)/\eta(\theta) = (\frac{\varepsilon(\theta)}{1-\varepsilon(\theta)}\frac{\varphi}{\varphi-1} - 1)/\eta(\theta)$. This selling intensity is positive when $\theta \leq \overline{\theta}$, where $\overline{\theta} > 0$ is such that $\frac{\varepsilon(\overline{\theta})}{1-\varepsilon(\overline{\theta})}\frac{\varphi}{\varphi-1} = 1$. In this region, $\hat{s}$ is strictly decreasing in θ, approaching infinity as θ approaches zero (from above) and falling to zero when θ approaches $\overline{\theta}$ (from below).

At the same time, combining the first order condition for selling with the profitability condition also implies that $\theta = \eta^{-1}(\kappa_s(\hat{s})/(u - c + \kappa_n(\hat{s})))$, which implicitly determines $\hat{s}$ as a strictly increasing function of θ, increasing from zero when $\theta = 0$ toward a positive upper bound $\overline{\hat{s}}$ as θ approaches infinity.

The intersection of these two curves determines unique values for θ and $\hat{s}$, and the remaining equations then determine unique p^s, S based on these. This characterizes an equilibrium with sale pricing iff $\alpha = (1-N)/(N\theta\hat{s}) < 1$. An equilibrium with sale pricing thus only exists if $N > 1/(1+\theta\hat{s})$.

Note that the threshold for N is the same across the two cases, with the equilibrium conditions coinciding with $\alpha = 1$.

Proof of Proposition 2 For the equilibrium without sale pricing, equilibrium θ is characterized by equation (21). The corresponding equation for efficient allocations is derived similarly based on planner first order conditions. This efficient counterpart is otherwise identical but omits the θ on the left hand side, implying that the efficient θ is strictly greater than in equilibrium. It follows that the efficient volume of trade $N + \mu(\theta)(1-N)$, selling intensity $\hat{s} = (1-N)/(N\theta)$, and total selling $\hat{s}N$ are all lower than in equilibrium.

Firm profits from customer acquisition may be written (using the first order condition for selling) as $\eta(\theta)\hat{s}(p^s - c) - \hat{\kappa}(\hat{s}) = -\kappa_n(\hat{s})$, which is increasing in $\hat{s}$, implying these profits are greater in equilibrium than efficient.

Proof of Proposition 3 The first order conditions and market clearing condition reduce to the same equations for allocations as in the planner problem.

Proof of Proposition 4 For a steady state without sale pricing, combining the first order conditions for selling intensity (11) and price (12) implies the relationship:

$$\frac{x}{\mu(\theta)} = \frac{\kappa_s(\hat{s})}{\eta(\theta)} \frac{\varepsilon}{1-\varepsilon} \frac{\eta(\theta)\hat{s}}{1+\eta(\theta)\hat{s}}.$$

At the same time, the product market constraint $x = \mu(\theta)(U - P^s - \beta(1-\delta)X)$ together with the expression for the steady state present value $X = \frac{x}{1-\beta(1-\delta)}$ implies

$$\frac{x}{\mu(\theta)} = \frac{U - P^s}{1 + \frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}}.$$

Here the present-value price may be expressed, using the first order condition for selling intensity, as

$$P^s = C + \frac{\beta(1-\delta)\kappa_n(\hat{s})}{1-\beta(1-\delta)} + \frac{\kappa_s(\hat{s})}{\eta(\theta)},$$

or (using that $\kappa_n(\hat{s}) = \hat{\kappa}(\hat{s}) - \hat{s}\kappa_s(\hat{s})$) as

$$P^s = C + \frac{\beta(1-\delta)\hat{\kappa}(\hat{s})}{1-\beta(1-\delta)} + \frac{(1-\delta)(1-\beta)\hat{s}\kappa_s(\hat{s})}{\delta(1-\beta(1-\delta))}.$$

Equating the two expressions for $\frac{x}{\mu(\theta)}$ then yields the equation:

$$\frac{\kappa_s(\hat{s})}{\eta(\theta)} \frac{\varepsilon}{1-\varepsilon} \frac{\eta(\theta)\hat{s}}{1+\eta(\theta)\hat{s}} = \frac{U - C - \frac{\beta(1-\delta)\hat{\kappa}(\hat{s})}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)\hat{s}\kappa_s(\hat{s})}{\delta(1-\beta(1-\delta))}}{1 + \frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}},$$

which may also be written (using $\eta(\theta)\hat{s} = \delta/(1-\delta)$) as

$$1 = \frac{1-\varepsilon}{\varepsilon} \frac{u - c - \beta(1-\delta)\hat{\kappa}(\hat{s}) - \hat{s}\kappa_s(\hat{s})(1-\delta)(1-\beta)/\delta}{\hat{s}\kappa_s(\hat{s})(1-\delta)(1-\beta(1-\delta)(1-\mu(\theta)))}. \tag{22}$$

With $\hat{s} = \delta/((1-\delta)\eta(\theta))$, equation (22) determines a unique θ. To see this, note that the right hand side is positive for $\theta \geq \underline{\theta}$, where $\underline{\theta}$ is such that the second term in the numerator equals zero. In this region, the right hand side is strictly increasing in θ, increasing from zero at $\underline{\theta}$ toward infinity as θ approaches infinity. The equation thus has a unique solution.

This θ is strictly increasing in δ, as the right hand side of (22) is strictly decreasing in δ.

The profitability condition may be written as

$$P^s - U + \beta(1-\delta)X - \kappa_n(\hat{s}) = -\kappa_n(\hat{s})[1 - \frac{1-\varepsilon}{\varepsilon}(1-\delta)\frac{\varphi}{1-\varphi}] \geq 0.$$

The first term is positive but the term in brackets is strictly increasing in δ, with θ strictly increasing in δ. The term in brackets approaches a positive value when δ approaches one and turns negative at some threshold δ^* (where the profitability condition holds as equality). When $\delta \geq \delta^*$, the profitability condition thus holds and there exists a unique steady state without sale pricing. When $\delta < \delta^*$, such a steady state does not exist.

For a steady state with sale pricing, the product market constraint $x = \mu(\theta)(U - P^s - \beta(1-\delta)X)$ together with the profitability condition implies that

$$\frac{x}{\mu(\theta)} = -\kappa_n(\hat{s}).$$

At the same time, the product market constraint $x = \mu(\theta)(U - P^s - \beta(1-\delta)X)$ together with the expression for the steady state present value $X = \frac{x}{1-\beta(1-\delta)}$ again implies

$$\frac{x}{\mu(\theta)} = \frac{U - P^s}{1 + \frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}}.$$

The first order condition for selling yields a slightly different expression for the present-value price here (as the firm is not always actively selling):

$$P^s = C + \frac{\beta(1-\delta)\alpha\kappa_n(\hat{s})}{1-\beta(1-\delta)} + \frac{\kappa_s(\hat{s})}{\eta(\theta)} = C + \frac{\beta\delta\kappa_n(\hat{s})}{\hat{s}\eta(\theta)(1-\beta(1-\delta))} + \frac{\kappa_s(\hat{s})}{\eta(\theta)},$$

or (using $\kappa_n(\hat{s}) = \hat{\kappa}(\hat{s}) - \hat{s}\kappa_s(\hat{s})$),

$$P^s = C + \frac{\beta\delta\hat{\kappa}(\hat{s})}{\eta(\theta)\hat{s}(1-\beta(1-\delta))} + \frac{(1-\beta)\kappa_s(\hat{s})}{\eta(\theta)(1-\beta(1-\delta))}.$$

Equating the two expressions for $\frac{x}{\mu(\theta)}$ yields the equation:

$$-\kappa_n(\hat{s}) = \frac{U - C - \frac{\beta\delta\hat{\kappa}(\hat{s})}{\eta(\theta)\hat{s}(1-\beta(1-\delta))} - \frac{(1-\beta)\kappa_s(\hat{s})}{\eta(\theta)(1-\beta(1-\delta))}}{1 + \frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}},$$

or

$$1 = \frac{u - c - \frac{\beta\delta\hat{\kappa}(\hat{s})}{\eta(\theta)\hat{s}} - \frac{(1-\beta)\kappa_s(\hat{s})}{\eta(\theta)}}{-\kappa_n(\hat{s})(1-\beta(1-\delta)(1-\mu(\theta)))}. \tag{23}$$

The first order condition for price combined with the profitability condition, $1 + \eta(\theta)\hat{s} = \frac{\varepsilon}{1-\varepsilon}\frac{\hat{s}\kappa_s(\hat{s})}{-\kappa_n(\hat{s})}$, further determine $\hat{s}$ as a function of θ: $\hat{s} = (\frac{\varepsilon}{1-\varepsilon}\frac{\varphi}{\varphi-1} - 1)/\eta(\theta)$. This selling intensity is positive when $\theta \leq \overline{\theta}$, where $\overline{\theta} > 0$ is such that $\frac{\varepsilon(\overline{\theta})}{1-\varepsilon(\overline{\theta})}\frac{\varphi}{\varphi-1} = 1$. In this region, $\hat{s}$ is strictly decreasing in θ, approaching infinity as θ approaches zero (from above) and falling to zero when θ approaches $\overline{\theta}$ (from below).

With this selling intensity, equation (23) determines a unique θ. To see this, note that in this region the numerator is positive when $\theta > \underline{\theta}$ for some $0 < \underline{\theta} < \overline{\theta}$. From $\underline{\theta}$ to $\overline{\theta}$, the right hand side of (23) is strictly increasing from zero at $\underline{\theta}$ toward infinity as θ approaches $\overline{\theta}$. The equation thus determines a unique θ, implying also a unique $\hat{s}$.

These values characterize a unique steady state with sale pricing iff $\alpha = \frac{\delta}{(1-\delta)\eta(\theta)\hat{s}} = \frac{\delta}{(1-\delta)(\frac{\varepsilon}{1-\varepsilon}\frac{\varphi}{1-\varphi}-1)} < 1$. The θ increases and $\hat{s}$ decreases in δ, as the right hand side of (23) is a strictly decreasing function of δ. The probability of sale pricing thus increases in δ, and in order for this probability to not exceed one, δ must thus be below some threshold. When δ is strictly below this threshold, there exists a unique steady state with sale pricing. When δ is above the threshold, such a steady state does not exist.

Note that the threshold for δ, is the same across the two cases, with the equilibrium conditions coinciding with $\alpha = 1$.

Proof of Proposition 5 A steady state without sale pricing is characterized by (see proof of Proposition 4)

$$\frac{\kappa_s(\hat{s})}{\eta(\theta)}\frac{\varepsilon}{1-\varepsilon}\frac{\eta(\theta)\hat{s}}{1+\eta(\theta)\hat{s}} = \frac{U - C - \frac{\beta(1-\delta)\hat{\kappa}(\hat{s})}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)\hat{s}\kappa_s(\hat{s})}{\delta(1-\beta(1-\delta))}}{1 + \frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}}. \tag{24}$$

The efficient steady state is characterized by the same equation but with the term $\eta(\theta)\hat{s}/(1+\eta(\theta)\hat{s}$ replaced by one. With $\hat{s} = \delta/((1-\delta)\eta(\theta))$, equation (24) and its efficient counterpart determine a unique θ for both equilibrium and efficient case. (The left hand side is positive and strictly decreasing in θ, approaching infinity as θ approaches zero from above, and approaching zero as θ approaches infinity. The right hand side is positive and strictly increasing when $\theta \geq \overline{\theta}$, where $\overline{\theta}$ is such that the numerator is zero.) From the difference between the cases, it further follows that θ is strictly higher in equilibrium than efficient.

Proof of Proposition 6 This follows from the first order conditions and other equilibrium conditions reducing to the same equations for allocations as in the planner problem.