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Polarized Contributions but Convergent Agendas*

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Abstract

The political process in the United States appears to be highly polarized: Data show that the political positions of legislators have diverged substantially, while the largest campaign contributions come from the most extreme donor groups and are directed to the most extreme candidates. Is the rise in campaign contributions the cause of the growing political polarization? In this paper, we show that, in standard models of campaign contributions and electoral competition, a free-rider problem among potential contributors leads naturally to polarization of campaign contributors but without any polarization in candidates' policy positions. However, we go on to show that a modest departure from standard assumptions — allowing candidates to directly value campaign contributions (because of "ego rents" or because lax auditing allows them to misappropriate some of these funds) — delivers the ability of campaign contributions to cause policy divergence. Consistent with the model, we document that a candidate's share of contributions in U.S. House of Representatives races is higher when her opponent's agenda is more extreme.

Keywords: Polarization; Campaign Contributions; Agendas

JEL Codes: D72, H41

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1 Introduction

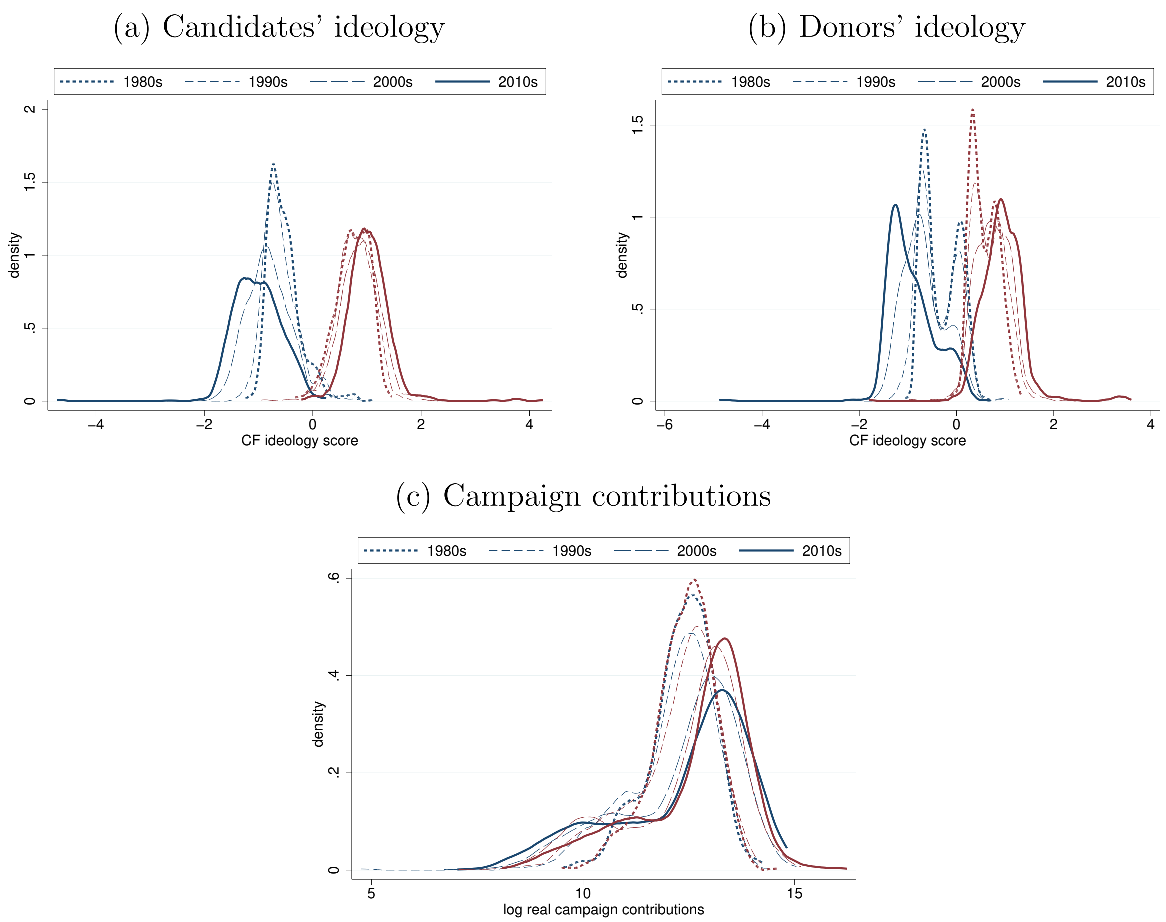
The polarization of politicians in the U.S. has risen significantly over the last decades (e.g., McCarty et al. (2006), Bonica (2014), Gentzkow et al. (2019)). The increased polarization of candidates has coincided with the increased polarization of donors and with increased campaign contributions, as panels (a) through (c) in Figure 1 show.[1] An active literature analyzes how ideologically extreme campaign donors may influence the polarization of elected politicians (e.g., La Raja and Wiltse (2012), Bonica et al. (2013), Barber (2016), La Raja and Schaffner (2015), Broockman and Malhotra (2020)). The possible connection between campaign donations and polarization has implications for campaign finance reform (Mann and Corrado (2014), Pildes (2020), Vandewalker (2020)). Yet, the relationship between contributions, donors, and candidates' agendas is not straightforward: We show that, under standard preferences, campaign donations can be a centripetal force even if only extreme donors ever contribute.

To analyze the connection between contributions and polarization, we present a theoretical model of policy formation, campaign contributions, and electoral success. Our general model nests the informed- and uninformed-voters framework of Baron (1994), applied to what he calls collectivist policies. It also accommodates the model of imperfectly targeted campaign spending of Herrera et al. (2008). In the model, the polarization of campaign contributions arises out of a natural incentive for moderate interest groups to "free ride" on the campaign contributions of more extreme interest groups. This polarization of contributors implies that candidates with more extreme positions receive larger contributions. However, despite the complete polarization of campaign contributors under the standard model specification (of preferences of lobbies and candidates), the model yields *convergence* of the politicians' policy positions.[2] In the absence of "informed voters" the candidates' agendas converge to the midpoint between the extreme lobbies. Adding the informed voters, as in Baron (1994), yields the equilibrium agendas between the midpoint of the extreme lobbies and the median informed voter.

These results all derive from the same force that leads to the polarization of campaign contributions in the first place. The intuition for this is quite straightforward. More extreme donor groups care more intensely about policies than do moderate interest groups. The more moderate interest groups therefore free-ride on the extreme lobbies, which have a greater incentive to make contributions. Furthermore, the extreme lobbies' incentives to contribute (and the amounts contributed) are increasing in the degree of polarization of policy platforms. Thus, moving towards an extreme

[1] Figure 1 plots densities of the campaign finance scores from Bonica (2016) for the four decades since 1980, separately for candidates (panel (a)) and donors (panel (b)), and separately for Republicans and Democrats. Both have shifted outward over time. Panel (c) plots the corresponding distribution of real campaign contributions, the mass of which has shifted to the right over time.

[2] Analyzing alternative model specifications reinforces the distinction between polarization of candidates and polarization of donors (see Section 3.2.2).



Data sources: Bonica (2016); FRED. We use data on all district-regular general election pairs with both a Democratic and a Republican candidate with at least eight donors in the data set. Campaign contributions are deflated by the core CPI index. See Appendix A for details.

Figure 1: The distribution of donors' and candidates' ideology scores has shifted away from the center over time, as campaign contributions increased: U.S. House of Representatives general election candidates, 1980–2014.

position increases the size of a candidate's campaign chest. However, such a move affects the opponent's contributions as well. And, under the (standard) assumption that donors' loss functions are convex, the increase in the opponent's contributions is in fact greater than that in the candidate's own campaign chest. Thus, if politicians care only about their probability of being elected, and if this probability depends on *relative* campaign expenditures, the result is a *convergence* of policy agendas.

The main mechanism in our model – that contributions are a centripetal force – is consistent with the data: Using the data for U.S. House races from Bonica (2016), which also underlies Figure 1, we show that a candidate's share of contributions is falling in the extremity of her own agenda, but rising in the extremity of her opponent's agenda. These correlations hold unconditionally, but also within districts over time and across districts for a given election. In our models, this behavior of relative contributions gives candidates the incentive to choose a centrist position.

A more basic model prediction that free-riding leads to a polarization of political contributors is also consistent with the data. While the data in Bonica (2016) do not allow us to compare the ideology of donors to non-donors, several papers support this notion: La Raja and Schaffner (2015, ch. 2, ch. 3) and Bafumi and Herron (2010) show that individual donors tend to come from ideological extremes. Broockman and Malhotra (2020) show not only that party donors hold more extreme views than party voters, but also that these differences are more pronounced among the top 1% of donors.[3] Many authors have found that contributors support candidates with similar ideological views (e.g., Barber et al. (2017)), with this result being strongest for groups with strong ideological positions (e.g., Langbein (1993)). Similarly, in a series of papers, Snyder (1990, 1992, 1993) has argued that ideological political action committees (PACs) do not fit a quid pro quo model of contributions, while Welch (1979) cites evidence that ideological PACs focus on close races as evidence in favor of models of contributions in support of a candidate with a given position. Finally, our emphasis on free-riding by interest groups is consistent with a substantial empirical literature that has found free-riding by donor groups to be important, albeit typically in the context of specific policies.[4] Hager et al. (2019) document evidence of free-riding among political activists.

In its simplest form, our model cannot speak to the data on candidates' polarization. As Figure 1 shows, measured agendas have diverged while our baseline model features complete convergence of agendas. Moreover, donors only contribute in equilibrium when candidates' agendas are distinct. Generalizing candidates' preferences allows our model to address this basic point. In our parsimonious model extension, candidates choose how much of their contributions they spend on campaigning (increasing their odds of winning office) and how much to consume.[5] Because the absolute level of contributions affects their level of consumption, candidates then balance the incentive to set a centrist agenda that maximizes their *share* of contributions with the incentive to set an extreme agenda that increases the *level* of contributions. We then obtain (partial) divergence of candidates' agendas, along with positive contribu-

[3]It is unclear how the ideology of institutional donors compares to the underlying population. However, La Raja and Schaffner (2015, ch. 3) show that also issue groups and, taken together, unions and business groups tend to donate funds to ideologically polarized candidates, while party committees tend to support centrist candidates. Poole and Romer (1985) provide evidence that political action committees provide the largest quantity of support for like-minded candidates.

[4]For example, Bloch (1993) finds that the degree of unionization is positively related to support for minimum wage legislation, while Kirchgässner and Pommerehne (1988) find that it is positively related to measures of social expenditure. Other authors have found a positive relationship between producer concentration in an industry and political influence, such as Esty and Caves (1983), Gardner (1987), Guttman (1980), Kalt and Zupan (1984) and Trefler (1993). Becker (1986) and Pincus (1975), hold the opposing view, however.

[5]If the candidates' preferences are log-linear in consumption and electoral success, the problem of the contributing lobbies remains largely unchanged and thus very tractable.

tions in equilibrium. Under mild functional-form assumptions, we derive a closed-form characterisation of the equilibrium. When candidates care little about the level of contributions, the divergence of agendas is small, and yet contributions are positive and the divergence of contributors is complete. The more candidates care about the level of contributions, the larger the policy divergence, as candidates strategically set their agendas to extract higher contribution levels.

We can use this augmented model to interpret the concurrent increase in the polarization of donors and candidates shown, for example, in Figure 1. The difference-in-difference estimates in Barber (2016) suggest that higher contribution limits for individual donors lead to more polarized candidates in U.S. states.[6] Since raising contribution limits shifts who the marginal contributor is, raising these limits is akin to making the (marginal) donors more extreme in our model. When candidates care directly about contributions, we show that the more extreme the marginal donors, the larger the equilibrium polarization of candidates – and the higher the contributions. A related comparative static of the model increases how much candidates care about their personal consumption and thus the level of contributions. When candidates are more "greedy," their agendas move towards the extremes and they extract larger contributions. The model is thus in line with the observation in La Raja and Wiltse (2012) that candidates are strategic in mobilizing contributions and can explain a rise in contributions even when the population of donors is stable.[7]

When we alter the preferences of donors, rather than those of candidates, we obtain another surprising result: Polarization of agendas without an inherent polarization of donors. Our benchmark model studied the standard case of donors having a convex loss function. When the donors' loss function is instead concave, the contributing donors in equilibrium are those "targeted" by the candidates, and not (generally) the extreme ones. But eliminating the inherent polarization of contributors for a given set of agendas does not eliminate polarization of the agendas. To the contrary, since the *relative* contributions to the two candidates are unaffected by their agenda choices, while the absolute contributions are increasing with agenda polarization, we obtain complete polarization of the candidates' positions in equilibrium whenever they put any utility weight on the contributions.

Literature. In an important early contribution, Austen-Smith (1987) considered competition between two rival lobbying groups and established a convergence result for candidate agendas. In this paper, we generalize the convergence result to a situation in which the identity and number of contributors is endogenous, and where free-riding leads to divergence of donor groups. We also establish the limits of this re-

[6] Barber (2016) finds the opposite for institutional donors. This could be rationalized if institutional donors are lobbies engaged in particularistic policies, as opposed to the collectivist policies we consider here.

[7] La Raja and Wiltse (2012) find that the extremism among small individual donors had not increased significantly until the early 2000s.

sult once candidates are allowed to value the absolute level of their contributions. The intuitive idea that donors and their campaign contributions may lead to polarization is present in Baron (1994) and Persson and Tabellini (2000) among others. Unlike Baron (1994), we endogenize which donors contribute, allowing for a public-good character of campaign contributions. While Baron (1994) attributes the convergence of agendas to the presence of informed voters, we show that the convergence arises even in the absence of informed voters. In the textbook treatment of Baron (1994) in Persson and Tabellini (2000), the probability of being elected is linear in campaign spending, so that campaign spending by one lobby does not diminish the marginal benefit of another lobby on the same side of the political spectrum. This rules out free-riding, which is central to our model.

An active literature analyzes models that can explain the measured polarization of politicians. Our divergence result grounded in greed as an incentive to please extreme donors is distinct from other explanations that rely on divergent candidate preferences (Roemer (1991), Lindbeck and Weibull (1993), or Krasa and Polborn (2014)). Apart from preferences, the literature has proposed uncertainty about candidate type (Bernhardt and Ingberman (1985)), or the threat of a third-party candidate's entry (Palfrey (1984) as explanations for divergent agendas; see also the survey by Osborne (1995)). More closely related are papers that analyze polarization and campaign finance. Similar to us, Le and Yalcin (2018) show that misappropriation of campaign donations can lead to polarization of agendas, when agendas would have converged in the absence of misappropriation. Our work differs from Le and Yalcin (2018) in highlighting that campaign contributions are a centripetal force. Unlike them, we analyze an environment with many potential donors in which not all donors contribute in equilibrium. Donors thus face a free-riding problem in our model and we endogenously obtain polarization of donors. In the context of particularistic policies in a two-dimensional policy space, Konishi and Pan (2020) show that campaign contributions by a single interest group can lead to polarization. In contrast, our model examines polarization in a one-dimensional policy space with many potential contributors. Herrera et al. (2008) explain polarization in a model that differs from ours in that it abstracts from the role of donors in campaign financing. Note that Dekel et al. (2008, 2009) study campaign spending with fixed agendas, while Jackson et al. (2007) establish the possibility of agenda divergence in a model where candidates spend their own resources in the absence of lobbyists.

The remainder of the paper is organized as follows. The basic model framework is presented in Section 2. Section 3 presents our key result on the convergence of policies with divergence of contributions in a simple version of our model. Section 4 analyzes the full model where candidates choose how much of their contributions to use for campaigning, which yields policy divergence in equilibrium. Section 5 analyzes comparative statics of the model, and derives implications for the effects of contribution limits and corruption. Section 6 concludes.

2 The General Model

In this section, we outline a relatively general model of agenda setting, campaign contributions, and electoral outcomes. In succeeding sections, we specialize this model in various ways in order to focus on specific forces that affect the decision-making of both candidates and contributors.

The model (game) has the following stages. First, candidates simultaneously set agendas. Second, donors decide on contributions. Third, candidates simultaneously choose campaign spending. Figure 2 summarizes this environment and anticipates the agents' decision problems at each stage.

1st stage	2nd stage	3rd stage
Candidates set agendas	Donors contribute	Candidates spend
$\max_{a_i} U(\Sigma_j c_j(i) - S_i) + p_i(S_i, S_{-i}, a_i, a_{-i})W$	$\max_{c_j(1), c_j(2)} -\mathbb{E}_{p(\cdot)}\lvert a - j\rvert^{\alpha} - \phi(c_j(1) + c_j(2))$	$\max_{S_i} U(\Sigma_j c_j(i) - S_i) + p_i(S_i, S_{-i}, a_i, a_{-i})W$

Figure 2: Summary of the model environment

Specifically, there are two political candidates, indexed by $i = 1, 2$, competing for election to one position. The game begins with each candidate selecting a policy platform or "agenda," denoted $a_i \in [0, 1]$, which they commit to implementing if elected.

There is a finite (and possibly large) number of potential donors. Each potential donor, or lobby, is identified with (and indexed by) its preferred agenda $j \in [0, 1]$. We will denote the left-most lobby by $\underline{j}$ and the right-most lobby by $\overline{j}$. A lobby's preferences over agendas, a, are represented by

$$V_j(a) = -|a - j|^{\alpha}, \tag{2.1}$$

with a common $\alpha > 0$. It is typical to assume that the loss function is convex, i.e., $\alpha \geq 1$, which implies that V is concave, so that the marginal distaste of a potential donor for an agenda is increasing as the agenda deviates further from the donor's preferred agenda. We also allow for the case of $\alpha \in (0, 1)$ in which the marginal distaste for alternative agendas is initially high and decreases as agendas become further removed from the donor's ideal point.[8]

[8] With apologies to Monty Python, the case $\alpha > 1$ corresponds to a world in which the People's Front of Judea and the Judean People's Front are both strongly preferred to the Romans, while the case $\alpha \in (0, 1)$ refers to the case where members of the People's Front of Judea despise the Judean People's Front almost as much as they do the Romans.

In the second stage of the game, after observing the agenda choices of each candidate, each donor j may elect to contribute a non-negative amount $c_j(i)$ towards candidate i's campaign at a cost to the donor of $\phi(c_j(1) + c_j(2))$. For now, we assume only that ϕ is (weakly) convex and strictly increasing in total contributions.

In the third stage of the game, each candidate i chooses how much to spend on the electoral campaign. This choice, S_i, is constrained both by the size of contributions and by the institutions governing the use of campaign funds. For example, in a country with relatively little corruption and accurate auditing of campaign donations, candidates may have to spend all of their contributions on campaigning, while in a country with a great deal of corruption and little auditing, candidates may be able to appropriate some or all of their campaign contributions for their own personal consumption. For now, we represent these constraints abstractly by choice set $B \subset \mathbb{R}^2_+$. That is, we require $(S_i, C_i) \in B$, where $C_i = \sum_j c_j(i)$ is the total of donors' contributions to candidate i.

The preferences of each candidate are likewise expressed somewhat abstractly as the sum of a term that captures the private benefit of campaign contributions net of campaign spending, and a term that reflects the expected benefit from winning the election:

$$U(C_i - S_i) + p_i\left(S_i, S_{-i}, a_i, a_{-i}\right) W, \tag{2.2}$$

where W represents the value the agent places on winning the election, $p_i(.)$ is the probability of i winning the election, and where the notation "$-i$" (for "not i ") has been used to denote the rival candidate. For simplicity, we assume that the candidates have no preference over agendas, except insofar as they affect the size of campaign contributions and the probability of electoral victory. In Section 4, we assume that the candidates' utility function U is continuously differentiable, strictly increasing and strictly concave with $\lim_{x \to 0} U'(x) = +\infty$.

The probability of winning the election has been conditioned on both the campaign spending of both candidates and their initial agenda choices, in order to encompass a wide array of voting mechanisms and political economy models. For now, we simply summarize the outcome of the fourth stage of the game — in which agents vote — simply in terms of the probability that a candidate wins the election as a function of both campaign spending and agenda choices, $p_i(S_i, S_{-i}, a_i, a_{-i})$. This reduced form specification allows us to capture a number of alternative, and not necessarily exclusive, possible assumptions about the way campaign expenditures and agendas affect election outcomes. For example, in Appendix C we show that this framework captures both the informed and uninformed voter model of Baron (1994) and the "get out the vote" model of Herrera et al. (2008). In each of these examples, the probability of winning the election is a strictly increasing continuously differentiable function of the candidate's share of total campaign spending $\frac{S_i}{S_i + S_{-i}}$ and a continuously differentiable function of policy agendas a_i and a_{-i}.

Assumption 1 *The probability of winning the election is represented by continuously* *differentiable function f*

$$p_i\left(S_i, S_{-i}, a_i, a_{-i}\right) = f\left(\frac{S_i}{S_i + S_{-i}}, a_i, a_{-i}\right),$$

with f strictly increasing in the first argument. Maintaining the symmetry assumption (that the probability of winning does not directly depend on the candidate's identity), we assume $p_i\left(0, 0, a_i, a_{-i}\right) = f\left(\frac{1}{2}, a_i, a_{-i}\right)$.

We maintain Assumption 1 throughout the paper.

3 Divergent Donors and Convergent Agendas

To begin, and to focus attention on the "public good" aspect of campaign contributions, we specialize the above model in two ways. First, we assume that the candidates do not value campaign contributions except insofar as these contributions increase the probability of electoral success, and that campaign contributions are the only source of funds for campaign expenditures. This assumption can be implemented either by setting U to 0 everywhere or by simply setting the constraint set $B = \{(S, C) \in \mathbb{R}_+^2 | S = C\}$. This is a relatively standard assumption in the literature, although, as we will see below, it has a significant effect on the results.

Assumption 2 *Candidates spend all contributions on campaigning, so that their objective becomes: $\bar{U} + p_i\left(\Sigma_j c_j(i), \Sigma_j c_j(-i), a_i, a_{-i}\right) W$.*

Second, we assume that the probability of electoral success does not directly depend on agendas, and is strictly increasing in a candidate's campaign spending. This has the effect of removing an obvious force for the convergence of agendas in equilibrium, and hence strengthens the nature of our convergence result.

Assumption 3 *The probability of electoral success does not directly depend on agendas: $p_i\left(\Sigma_j c_j(i), \Sigma_j c_j(-i), a_i, a_{-i}\right) = p_i\left(\Sigma_j c_j(i), \Sigma_j c_j(-i)\right) \forall a_i, a_{-i}$.*

This assumption rules out informed voters when we map our model to that of Baron (1994). While this assumption simplifies the analysis below, it also shows the contrast with Baron (1994), who claims that agendas converge because candidates try to capture informed voters. Instead, our analysis derives a convergence result in the absence of informed voters. The difference arises because we endogenize which lobbies contribute. We show in Appendix C.1 that our main results also hold in the specific setup of Baron (1994) with informed voters – and thus with a direct dependence of the probability of winning on agendas.

Under our assumptions, the third stage of the game described above is degenerate. We solve the game consisting of the first two stages by backward induction and focus on pure strategy subgame perfect equilibria.

3.1 Campaign Contributions

We first establish the identity of the contributing donors and the size of their contributions in an arbitrary subgame for given policy choices of the candidates, (a_1, a_2). For simplicity, and without loss of generality, we will adopt the convention that candidate 1 is to the left of candidate 2, or $a_1 \leqslant a_2$.

Consider the problem of a donor j, that is considering contributing to candidate 1. The donor solves the following problem, taking as given the opponent's campaign fund $C_2 = \sum_k c_k(2)$ and the total contributions $C_1(-j) = \sum_{k \neq j} c_k(1)$ of other donors to candidate 1's campaign:

$$\max_{c \geqslant 0} -p_1(c+C_1(-j), C_2)|a_1-j|^\alpha-(1-p_1(c+C_1(-j), C_2))|a_2-j|^\alpha-\phi(c+c_j(2)), \quad (3.1)$$

which is equivalent to

$$\max_{c \geqslant 0} p_1(c + C_1(-j), C_2)\Delta_j(a_1, a_2) - \phi(c + c_j(2)), \quad (3.2)$$

where we have defined the added benefit to donor j of policy a_1 over policy a_2 by

$$\Delta_j(a_1, a_2) = |a_2 - j|^\alpha - |a_1 - j|^\alpha. \quad (3.3)$$

This is a very well-behaved convex problem, with the first-order condition for an optimum given by

$$\frac{\partial p_1(C_1, C_2)}{\partial C_1}\Delta_j(a_1, a_2) \leq \phi'(c_j(1) + c_j(2)), \quad (3.4)$$

and symmetrically for contributions to candidate 2

$$\frac{\partial p_2(C_2, C_1)}{\partial C_2}\Delta_j(a_2, a_1) \leq \phi'(c_j(1) + c_j(2)), \quad (3.5)$$

with each of these conditions holding with equality if the contribution by donor j to candidate $i = 1, 2$ is positive.

Before we impose additional structure to characterize the contributions in more detail, it is useful to observe that donors contribute only because they care about the difference between candidates. This keeps them from contributing to both.

Lemma 1 *No donor ever makes positive contributions to both candidates.*

Proof. Let $c_k(i) > 0$. Then

$$\frac{\partial p_i(C_i, C_{-i})}{\partial C_i} \Delta_k(a_i, a_{-i}) = \phi'\left(c_k(i) + c_k(-i)\right).$$

Towards a contradiction, let $c_k(-i) > 0$. Then we must also have

$$\frac{\partial p_{-i}(C_{-i}, C_i)}{\partial C_{-i}} \Delta_k(a_{-i}, a_i) = \phi'\left(c_k(i) + c_k(-i)\right).$$

Since both derivatives (of p_i and of ϕ) are strictly positive, both $\Delta_k(a_{-i}, a_i)$ and $\Delta_k(a_i, a_{-i})$ must be strictly positive. But since $\Delta_k(a_{-i}, a_i) = -\Delta_k(a_i, a_{-i})$, that is a contradiction. ■

The additional properties of the solution depend on the curvature of the donor's preferences (given by the size of α), the curvature of the electoral success probability p_i, and the curvature of the cost of funds function ϕ. We get some of our starkest results when we assume that the cost of funds function ϕ is linear in contributions.

3.1.1 Donors with Deep Pockets

The starkest illustration of the key mechanism arises when we assume that donors have "deep pockets," i.e., that their cost of funds is linear (rather than strictly convex). The intuition derived here carries over to the more general case, as we illustrate in Appendix D.

Assumption 4 *The cost of contributions is linear:* $\phi(c) = \phi c$.

Under this specification, only the lobbies with the strongest policy preferences contribute in any equilibrium. More formally, in every subgame, $C_1 = \sum_{j \in J_1} c_j$ and $C_2 = \sum_{j \in J_2} c_j$, where $J_i = \arg\max_j \Delta_j(a_i, a_{-i}) = \arg\max_j \left(|a_{-i} - j|^\alpha - |a_i - j|^\alpha\right)$. Under the standard assumption about the preferences of the lobbies ($\alpha > 1$), this means that only the most extreme donors ever contribute, in contrast with Baron (1994), where all donors contribute.

Lemma 2 *Under Assumption 4, if $\alpha > 1$ then only the extreme donors possibly contribute in any subgame. That is, in every subgame, $C_1 = c_{\underline{j}}$ and $C_2 = c_{\overline{j}}$.*

Proof. Suppose not. Then there exists a j satisfying $\underline{j} < j < \overline{j}$ such that the first-order condition for contributing to one candidate holds with equality, or

$$\frac{\partial p_i(C_i, C_{-i})}{\partial C_i} \Delta_j(a_i, a_{-i}) = \phi. \tag{3.6}$$

But under the assumption of the lemma, for any $a_1 < a_2$, $\Delta_j(a_1, a_2)$ is strictly decreasing in j. But then the first-order condition (equation (3.4) or (3.5)) for at least one of the extreme donors is violated, a contradiction. Suppose then that $a_1 = a_2$. But then $\Delta_j(a_1, a_2) = 0$ for all j and no donor contributes. ■

This result follows from the fact that less extreme donors have an incentive to free-ride on the contributions of more extreme donors. In particular, the most extreme donors contribute up to the point where the marginal benefit from an extra contribution equals its marginal cost. However, since all non-extreme donors have a strictly lower marginal benefit, and yet face the same marginal cost, they find it optimal not to contribute.

Assumption 1 guarantees that the extreme donors make positive contributions whenever $a_1 \neq a_2$. Moreover, as we establish in in Appendix D, these positive contributions satisfy

$$\frac{C_1}{C_2} = \frac{\Delta_{\underline{j}}(a_1, a_2)}{\Delta_{\bar{j}}(a_2, a_1)}. \tag{3.7}$$

The relative contribution function defined in (3.7) incentivizes candidates to set centrist agendas. Figure 3 illustrates the shape of the relative contribution function from candidate 1's perspective in the case of $\alpha = 2$ and $\underline{j} = 0, \bar{j} = 1$. The horizontal axis shows the various agendas a_1 that candidate 1 could choose. The different lines correspond to different agenda choices for candidate 2. The vertical axis shows the relative contributions. Clearly, when a_1 and a_2 converge to $\frac{\underline{j}+\bar{j}}{2}$ (solid line), relative contributions are equal.[9] Moving away from the midpoint between the extreme lobbies, relative contributions decline smoothly along the solid line for $a_2 = \frac{1}{2}$. More generally, the candidate closer to the midpoint between the extreme lobbies has higher relative contributions. Candidate 1 maximizes her contributions by being marginally closer to the midpoint than candidate 2. Relative contributions change discontinuously as candidate 1 sets her agenda more towards the same extreme as candidate 2.

While altering the assumption regarding the preferences of the donors changes the identity of contributing donors in equilibrium (of a subgame), the logic of free-riding still applies: The donors who gain less utility free-ride on the donors who gain the most.

Lemma 3 *Under Assumption 4, if $\alpha < 1$ then the only lobbies contributing in any subgame equilibrium are either the ones most closely aligned with the candidate ($j_i \in \arg\min_j |a_i - j|$) or, if the closest lobby is more centrist than the candidate, possibly the second closest.*

Proof. The proof in Appendix B.1 amounts to determining which lobbies care the most, i.e., identifying $J_i = \arg\max_j (|a_{-i} - j|^\alpha - |a_i - j|^\alpha)$. ■

[9] Technically, contributions are both equal to zero when $a_1 = a_2$. By L'Hopital's rule, the relative contributions approach unity as a_1, a_2 converge to $\frac{\underline{j}+\bar{j}}{2}$.

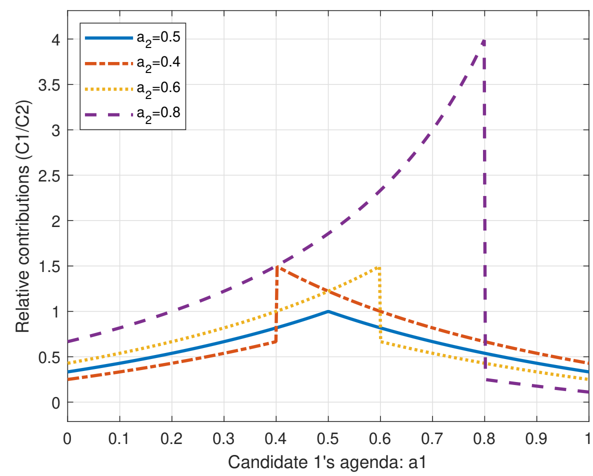


Figure 3: Relative contributions reward centrist agendas: relative contribution function (3.7) for candidate 1 given $\alpha = 2$ and $\underline{j} = 0, \bar{j} = 1$ and various values of a_2.

3.2 Political Agendas

So far, we have studied the outcome of the second stage of the game in which campaign contributions are determined given the agendas of the candidates. Having established the optimal behavior of the donors in the second stage of the game, we are now ready to consider the agenda-setting behavior of the candidates. Under Assumption 2 that candidates care only about electoral success, Assumption 3 that the winning probability does not directly depend on their agendas, and under Assumption 1 of the electoral-success probability function, the results above imply that the candidates care only about their relative contributions. Using this, we can establish our key results.

We proceed by analyzing the case with standard convex donor preferences ($\alpha > 1$) first, before turning to the case of the concave loss function ($\alpha < 1$).

3.2.1 Political Agendas with Convex Donors' Loss Function: $\alpha > 1$

To simplify the main analysis, we begin with a simple observation:

Lemma 4 *Under Assumption 4 and if $\alpha > 1$, no candidate ever chooses a platform that is located further from the center (the other candidate) than the preferred point j of the donor that contributes to the candidate's campaign in equilibrium.*

Proof. Candidate 1 aims to maximize $C_1/C_2 = \Delta_1/\Delta_2$. Under our assumptions, at most two donors, which we denote $j_1 \leq j_2$ without loss of generality, contribute. But $\Delta_{j_1}(a_1, a_2)$ is increasing in a_1 for $a_1 < j_1$, while $\Delta_{j_2}(a_2, a_1)$ is decreasing in

a_1 in this range. That is, by locating further from the other candidate than the supporting donor's preferred point, a candidate would both lower her own campaign contributions and increase those of the opponent. This contradicts optimization by candidate 1. The same logic applies to candidate 2. ■

We now establish the first key result — the convergence of agendas when the candidates' sole objective is winning the election. In particular, we show that, for all specifications, there is convergence in agendas to some "central" agenda. We also show that, in general, the convergence will *not* be to a median agenda, and provide conditions under which agendas converge to the one preferred by the average donor.

Theorem 1 *If $\alpha > 1$ and the candidates' sole objective is winning the election, the unique equilibrium has both candidates locating (choosing platforms) in the midpoint between the two extreme potential donors, at $j_m = \frac{\bar{j}+\underline{j}}{2}$. Contributions are zero in equilibrium.*

Proof. This is an equilibrium because moving away from the midpoint increases one's opponent's contributions more than one's own. A candidate i, whose only objective is to win the election, will (strategically) maximize C_i/C_{-i}. That is, the candidate will take into account the effect of her choice of platform a_i on the contributions to her opponent. So, the problem of the (left) candidate 1 is:

$$\max_{a_1 \leqslant a_2} \frac{(a_2 - \underline{j})^\alpha - (a_1 - \underline{j})^\alpha}{(\bar{j} - a_1)^\alpha - (\bar{j} - a_2)^\alpha}. \tag{3.8}$$

(There is an analogous problem with $a_1 \geq a_2$.)

If $a_{-i} < \frac{\underline{j}+\bar{j}}{2}$, then candidate i will choose to locate to the right of a_{-i} ($a_i > a_{-i}$). To see this, simply observe that $\Delta_{\underline{j}}(a, a_{-i}) < \Delta_{\bar{j}}(a_{-i}, a)$ for $a < a_{-i}$ and $\Delta_{\bar{j}}(a, a_{-i}) > \Delta_{\underline{j}}(a_{-i}, a)$ for $a_{-i} < a \leqslant j_m$. Similarly, if $a_{-i} > \frac{\underline{j}+\bar{j}}{2}$, then candidate i will choose to locate to the left of a_{-i} ($a_i < a_{-i}$). Either way, candidate i can guarantee herself more than a 50% chance of winning the election. Thus, choosing any platform other than j_m cannot be part of a pure strategy equilibrium. In fact, since choosing j_m guarantees at least a 50% chance of winning the election (regardless of what the opponent's platform is), the only equilibrium has both candidates choosing j_m. ■

It is important to note that the above result is not a median voter result. The midpoint to which the platforms converge need not be the preferred point of a median voter (or a median donor for that matter).

This result contrasts with the analysis of collectivist policies in Baron (1994). Both Proposition 5 in Baron (1994) and our Theorem 1 predict convergence of politicians' agendas, but for different reasons: Baron (1994) attributes the convergence to the influence of informed voters, and the point of convergence is the median voter's preferred policy. When we consider the same parametric case, but recognize the free-rider problem among lobbies and endogenize which lobbies contribute, we show that the policy convergence arises even in the absence of informed voters, and that point of convergence is instead the midpoint between the extreme lobbies.

In Appendix C.1, we dispense with Assumption 3 by allowing agendas to influence the winning probability via informed voters as in Baron (1994). We then find that there is convergence to a point in between the midpoint between the most extreme donors and the median (informed) voter. This model extension thus yields two distinct centripetal forces, which may disagree regarding the center they pull towards.

3.2.2 Political Agendas with Concave Donors' Loss Function: $\alpha < 1$

While the analysis above has established our key result under the well-established (standard) specification of policy preferences of lobbies, we find it worthwhile to analyze the case of a concave donor loss function as well. This formulation of the model highlights the important distinction between polarization of the candidates and polarization of their contributors.

Again, we begin with a simplifying observation:

Lemma 5 *Under Assumption 4 and if $\alpha < 1$, no candidate ever chooses a platform that is not located at a preferred point j of some donor (which contributes to the candidate's campaign in equilibrium). That is, candidates do not locate (choose platforms) at points where there are no potential donors.*

Proof. Due to the concavity of the donor's loss function, moving away from the preferred point of a supporting coalition lowers the candidate's contribution more than the opponent's. The basic mechanism behind this argument is also behind the proof of Lemma 3 in Appendix B.1. ■

The last lemma implies that when $\alpha < 1$, only one donor contributes to each candidate.

Theorem 2 *If $\alpha < 1$ and the candidates' sole objective is winning the election and the number of potential donors is N, then there are N^2 distinct equilibria. The two candidates choose some donors' (not necessarily distinct) preferred points as their platforms. The contributions are $C_1 = C_2 = \frac{|\Delta|}{4\phi}$, where Δ is given by equation (3.3).*

Proof. The proof follows from Lemma 5. ■

Thus, there is no inherent polarization of contributors in this case arising from the free-rider problem, as was the case in the standard model. On the other hand, the candidates are not "punished" for picking an extreme agenda in this case — their *relative* contribution is unaffected by their choice of agenda.

Lastly, if $\alpha = 1$, we have a continuum of equilibria with platforms locating anywhere on $[\underline{j}, \overline{j}]$ and the identity (and number) of contributing donors being indeterminate in general.

The multiplicity of equilibria when $\alpha \leq 1$ is not robust to allowing the probability of an election victory to also depend directly on agendas. In particular, if there are any informed voters (who vote sincerely and are not affected by campaign spending), and there is a donor that has the same preferred point as the median voter, then platforms converge to the median voter's preferred point.

3.3 Campaign Finance and Agendas in the Data

The key economic mechanism behind the results above is that candidates are strategic, and internalize that by setting their agenda they influence not only their own donations, but also their opponent's donations. We now turn to the Database on Ideology, Money in Politics, and Elections (DIME) by Bonica (2016) to assess this mechanism. This dataset is well-suited for our purposes, because it provides information on campaign contributions along with a measure of ideology.

Specifically, we examine how the share of donations received by a candidates in an election relates to their own agenda and that of their opponents.[10] As in Figure 1, we focus on the general elections to the U.S. Congress when we can identify both a Republican and a Democratic candidate. Table 1 reports estimates of how the Republican candidate's share of campaign contributions relates to her own campaign finance score and that of her opponent.[11] We estimate the regressions using ordinary least squares and cluster standard errors at the state level. In the first column, we consider all observations, in the other columns we restrict the sample to those observations where the campaign finance (CF) score is estimated using at least eight donors, a number Bonica (2016) considers the threshold for reliable estimates of the ideological position. We then add year (election) fixed effects (column (3)), district by decade fixed effects that account for redistricting (column (4)), or both fixed effects (column (5)). To interpret the magnitudes, note that Republican candidates tend to have campaign finance scores around 0.9, whereas the average Democratic campaign finance score is about -0.8.[12] Standard errors are clustered by state to allow for correlation between districts and across elections.

All regressions imply that candidates receive a higher share of contributions when their opponent moves away from them. Similarly, candidates receive a higher share of contributions when they move to the center. For example, the coefficient of -0.272 in column (2) for the "opponent's CF score" implies that when the Democratic can-

[10] We construct our sample such that the Democratic and Republican share sum to one.

[11] Because we define the shares so that they add up to one and because the regressors are identical, using the Democratic share on the LHS yields numerically identical results.

[12] We report the regressions only for the Republican share, because the results for the Democratic share are identical up to flipping signs and labels.

	(1) All obs	(2) ≥ 8 givers	(3) +year FE	(4) +district FE	(5) +both FE
own CF score	-0.243***	-0.249***	-0.229***	-0.127***	-0.129***
	(0.03)	(0.04)	(0.05)	(0.04)	(0.04)
opponent's CF score	-0.278***	-0.272***	-0.297***	-0.119***	-0.123***
	(0.02)	(0.02)	(0.04)	(0.03)	(0.03)
R-squared	0.24	0.25	0.27	0.83	0.85
R-sq, within	0.24	0.25	0.25	0.05	0.06
Observations	3230	2367	2367	1716	1716
States	51	51	51	48	48
Years	18	18	18	17	17
Fixed effects	no	no	year	district	district, year
Rep CF score: average	0.89	0.90	0.90	0.90	0.90
Rep CF score: st.dev.	0.42	0.38	0.38	0.38	0.38
Dem CF score: average	-0.78	-0.82	-0.82	-0.84	-0.84
Dem CF score: st.dev.	0.47	0.43	0.43	0.42	0.42

Standard errors clustered at the state level in parentheses. *** p-value<0.01. See Appendix A for a detailed description of the data.

Table 1: Republican share of campaign contributions within district, own ideology, and opponent's ideology: U.S. House of Representatives, general election candidates, 1980 to 2014.

didate's CF score is one standard deviation (0.43) further to the left, then the Republican's share of contributions rises by 11.7 percentage points (-0.272×-0.43). If the Republican candidate herself moves one standard deviation to the center, she could expect a 9.4 percentage point higher contribution (-0.249×-0.38). This coefficient varies little across the first three specifications. Thus, the results are not driven by some underlying time trend. Instead, the results also hold within years across districts. Including district fixed effects, which change every ten years to allow for redistricting, the coefficient size is cut in half (columns (4) and (5)), but the coefficients remain significant. Thus, we observe the same patterns within (rezoned) districts over time and across districts for a given election. All estimates are highly significant across specifications. The regressions explain about 25% of the variation in the data without district fixed effects, and about 6% of the within-district, within-year variation in column (5).

These regressions are consistent with the basic mechanism we are emphasizing: Campaign contributions are a centripetal force. Candidates who move toward an extreme position increase the share of total contributions going to their opponent. Moving to the center increases a candidate's share of contributions.

However, such a regression result could not be generated by a literal interpretation of our model under standard (convex) donor preferences. The equilibrium of this benchmark model features complete convergence of agendas and zero contributions. We now enrich the model by introducing a motive for agents to care about the absolute level of their contributions. This generates separation of candidates' agendas and positive campaign contributions in equilibrium, while preserving the mechanism that the agendas of both a candidate and her opponent determine contributions.

4 Greedy Candidates and Divergent Agendas

In the previous section we established the key result that, at first glance, seems surprising: Even though the private provision of publicly valuable contributions leads to extreme donors being the largest (and in some cases, the only) contributors, and even though the contributions to a candidate are increasing with polarization, the agendas of competing candidates converge in equilibrium. Upon reflection, the result is quite intuitive: although polarization increases the absolute level of a candidate's contributions, it increases the level of the candidate's opponent's contributions even more, so that relative contributions decline. Under our (standard) assumption that it is relative contributions that matter for electoral success, and that politicians care only about electoral success, we obtain policy convergence.

This logic also suggests that in order for polarization to arise in equilibrium, the candidates in the model must value the absolute level of their contributions in addition to (or possibly instead of) their relative contributions. There are several compelling reasons why this might be the case. For example, candidates may derive some pure utility ("ego rents") from receiving a large quantity of contributions. Alternatively, to the extent that candidates may use their own funds to support their campaign, the larger the absolute level of contributions, the less of a candidate's own money will be spent on the campaign, leading candidates to value the absolute level of contributions. Finally, if contributions to a campaign need not be spent on the campaign, and may instead be used to finance the candidate's consumption, then candidates will also value a large absolute level of contributions.

We refer to this way of generating a preference for the absolute level of contributions as "greed." It has also been analyzed by Le and Yalcin (2018), albeit without many potential donors and the endogenous polarization of donors. In this section, we establish that greed can yield agenda polarization in our framework. We first illustrate with a simple modification how candidates do choose extreme agendas when they exogenously divert funds and care only about their consumption spending. We then show how the result generalizes in our full model with an endogenous choice of consumption and campaign spending.

4.1 Illustrative Variation of the Benchmark Model

Consider first an extremely simplified illustrative variation of the benchmark model: Allow the candidates to consume fraction $(1 - \gamma)$ of the contributions they collect, and make their preferences increasing in consumption and independent of winning the election.

Now, the candidates will choose their platforms with the sole goal of maximizing their own contributions. The candidates are no longer concerned with their opponents' contributions, since they do not care about winning the elections. We immediately obtain the desired result:

Theorem 3 *When candidates do not care about winning* ($W = 0$) *and spend a fixed fraction* γ *of contributions on campaigning* ($B = \{(S, C) \in \mathbb{R}^2_+ | S = \gamma C\}$)*, the unique equilibrium (for any* $\alpha > 0$*) has two candidates tailoring to the extreme donors. That is,* $a_1 = \underline{j}$ *and* $a_2 = \overline{j}$*.*

It is important to point out that allowing the candidates to consume a fixed portion of endowments does not affect the (subgame) equilibrium contributions as a function of the platforms. That is, equations (3.6) and (3.7) still hold and do not include γ. The basic intuition for this (somewhat surprising) result comes from the fact that the marginal productivity of the contributions (in affecting the probability of election victory) remains unchanged. While the left candidate's consumption lowers the productivity of the left donor's contributions, the right candidate's consumption of her (right donor's) contributions raises the productivity right back up. This observation will be important in allowing us to characterize the equilibria in the richer model of the next subsection.

4.2 Full Model

We now analyze the full model, which corresponds to endogenizing the choice of γ in the illustrative example. Again, the only aspect of the model we will alter is the preferences of the candidates.[13] They will now care about *both* personal consumption (out of the campaign contributions) *and* winning the elections. The candidates will get to decide how much to consume out of their campaign fund:

$$\max_{S_i \in [0,C]} U(C - S_i) + p_i(S_i, S_{-i})W, \tag{4.1}$$

where S_i is the amount the candidate i actually spends on the campaign, C is the amount contributed to the candidate by the donors, S_{-i} is campaign spending (net of consumption) of the opponent, and W is the value of winning the election. We assume the following functional form of the probability of winning the election, as in Baron (1994) without informed voters:

Assumption 5 *The probability of winning the election is* $p_i(S_i, S_{-i}) = \frac{S_i}{S_i + S_{-i}}$ *whenever* $S_i + S_{-i} > 0$*, and* $p_i(0, 0) = \frac{1}{2}$*.*

[13] We also have to slightly alter our equilibrium concept. Subgame perfect equilibrium does not exist in this full model, since subgames where only one of the candidates receives positive amount of contributions do not have a Nash equilibrium (the candidate with positive contributions wants to spend the smallest amount strictly greater than 0). We thus require that our equilibrium strategy profile is a Nash equilibrium in every subgame that has a Nash equilibrium.

In order to get a closed form solution, we will consider a particular functional form of the candidates' utility function as well

Assumption 6 *Candidates' utility of consumption is* $U(h) = \ln h$.

The candidates' inability to ex-ante commit to an allocation of donations between consumption and campaigning implies that the spending decision is simply pinned down by the first-order condition of the (ex-post) problem (4.1) in the third stage (if the candidate has any contributions to spend, of course):[14]

$$U'(h) = W\frac{\partial p_i(S_i, S_{-i})}{\partial S_i} \qquad \Leftrightarrow \qquad \frac{1}{C - S_i} = \frac{WS_{-i}}{(S_i + S_{-i})^2}. \tag{4.2}$$

As we will now establish, this expression dramatically simplifies the analysis, as it implies that, in every third-stage subgame with positive contributions to both candidates, the candidate spends the same fraction of her resources as her opponent. Thus, from the perspective of the contributors, the candidates' behavior resembles that in the illustrative example in Section 4.1, and the equilibrium in the second stage is still characterized by equations (3.6) and (3.7). To show that, we multiply the first-order conditions (4.2) by S_i, which yields:

$$\frac{S_1}{C_1 - S_1} = \frac{S_2}{C_2 - S_2} = \frac{WS_1S_2}{(S_1 + S_2)^2}. \tag{4.3}$$

That is, the candidates spend the same fraction γ of their funds on their campaigns (and consume the rest). The fraction γ spent on the campaigns solves

$$\frac{1}{\gamma} = \frac{(C_1 + C_2)^2}{WC_1C_2} + 1. \tag{4.4}$$

Now, the problem in stage 2 of the (extreme) contributor to candidate i is the familiar

$$\max_{c_i} p_i(S_i(c_i, C_{-i}), S_{-i}(c_i, C_{-i}))\Delta_i - \phi c_i \tag{4.5}$$

and the intuition developed in Section 4.1 applies. It is worth noting that the level of contributions does affect the fraction γ spent on the campaign. While the contributors do recognize this fact, their behavior is still captured by the familiar equation (3.7), since they are not concerned with the campaign spending per se, but only with the *relative* campaign spending of their preferred candidate (relative to the opponent's).[15]

[14] Equation (4.2) presumes that the probability of election win is differentiable, which is not the case if the opponent's spending is 0. Technically, the candidate's best response is not well-defined in that case, as optimal spending would be the smallest number greater than 0. Of course, this technical issues arises only off the equilibrium path.

[15] The key is the fact that the candidates spend the *same* fraction γ of their contributions on cam-

More formally, the marginal effect of the contributions on the probability of winning the election (taking the effect on candidates' behavior into account) is:[16]

$$\frac{dp_i(S_i(c_i, C_{-i}), S_{-i}(c_i, C_{-i}))}{dc_i} = \frac{C_{-i}}{(c_i + C_{-i})^2}. \tag{4.6}$$

This corresponds exactly to what we had in section 3.2. If we impose the functional form Assumption 5 in Section 3.2, we find the same result: $\frac{\partial p_i(c_i, C_{-i})}{\partial c_i} = \frac{C_{-i}}{(c_i + C_{-i})^2}$.

Turning finally to the candidates' choice of agendas in the first stage, the problem of candidate i is:

$$\max_{a_i} U(C_i(a_i, a_{-i}) - S_i) + p_i(S_i, S_{-i})W, \tag{4.7}$$

where S_i and S_{-i} are both understood to be affected by a_i via its effects on C_i and C_{-i}. In this choice of policy positions, the candidates trade off the gain in personal consumption from larger *absolute* level of contributions arising from a more radical position against the loss in the probability of winning the election that such radical position implies due to lower *relative* contributions. The first-order condition of this problem equates the corresponding marginal benefit and marginal cost.

$$U'((1-\gamma)C_i)\left(\frac{dC_i}{da_i} - \frac{dS_i}{da_i}\right) + W\frac{dp_i}{da_i} = 0.$$

Incorporating $\frac{dp_i}{da_i} = \frac{\partial p_i}{\partial S_i}\frac{dS_i}{da_i} + \frac{\partial p_i}{\partial S_{-i}}\frac{dS_{-i}}{da_i}$, we can simplify this condition using the first-order condition (4.2) from the third-stage, akin to an envelope condition, which yields

$$U'((1-\gamma)C_i)\frac{dC_i}{da_i} + W\frac{\partial p_i}{\partial S_{-i}}\frac{dS_{-i}}{da_i} = 0.$$

That is, in equilibrium, the marginal benefit of polarizing one's agenda, which is the marginal utility of consuming the additional contributions, is equated to the marginal cost, which comes from the lower election probability due to the opponent's higher donations (recall that $\frac{\partial p_i}{\partial S_{-i}}$ is negative).

Using these first-order conditions and exploiting the symmetric nature of our environment, we obtain a closed-form characterisation of the equilibrium:

Theorem 4 *Under Assumptions 4, 5, and 6, if* $\alpha > 1$ *the degree of polarization*

paigning, which was established by equation (4.3). We can thus use the expression $S_j = \gamma(c_i, C_{-i})C_j$.

[16]To derive (4.6), begin with $\frac{\partial p_i(S_i,S_{-i})}{\partial c_i} = \frac{\partial p_i(S_i,S_{-i})}{\partial S_i}\frac{\partial S_i}{\partial c_i} + \frac{\partial p_i(S_i,S_{-i})}{\partial S_{-i}}\frac{\partial S_{-i}}{\partial c_i}$. Since $S_i = \gamma c_i$, $\frac{\partial S_i}{\partial c_i} = \gamma + \frac{\partial \gamma}{\partial c_i}c_i$, and since C_{-i} is taken as given, $\frac{\partial S_{-i}}{\partial c_i} = \frac{\partial \gamma}{\partial c_i}C_{-i}$. Plugging these in yields: $\frac{\partial p_i(S_i,S_{-i})}{\partial c_i} = \frac{S_{-i}}{(S_i+S_{-i})^2}\left(\gamma + \frac{\partial \gamma}{\partial c_i}c_i\right) - \frac{S_i}{(S_i+S_{-i})^2}\frac{\partial \gamma}{\partial c_i}C_{-i}$. The terms containing $\frac{\partial \gamma}{\partial c_i}$ cancel out in equilibrium since $c_i = C_{-i}$ and $S_i = S_{-i}$, leaving us with $\frac{\partial p_i(S_i,S_{-i})}{\partial c_i} = \frac{\gamma S_{-i}}{(S_i+S_{-i})^2}$. Since $S_j = \gamma C_j$, this yields (4.6).

$|a_1 - a_2|$ *is given by:*

$$|a_1 - a_2| = (\bar{j} - \underline{j})\frac{(4+W)^{\frac{1}{\alpha-1}} - W^{\frac{1}{\alpha-1}}}{(4+W)^{\frac{1}{\alpha-1}} + W^{\frac{1}{\alpha-1}}}. \tag{4.8}$$

Proof. See Appendix B.2. ∎

This equilibrium for $\alpha > 1$ has empirically plausible implications, as we discuss below.

For $\alpha < 1$, we obtain the even starker result of complete polarization.

Theorem 5 *Under the functional forms assumptions of Section 4.2, if $\alpha \leqslant 1$, the equilibrium features complete polarization, regardless of the value of W.*

Proof. This follows directly from Theorem 2. ∎

5 Model Implications

5.1 Greed, Polarization, and Contributions

The more the candidates value winning the election, the more of their campaign funds they spend on campaigning rather than consumption, i.e., the less greedy they behave. To see this, note that in a symmetric equilibrium, (4.4) implies that the fraction spent on campaigning is:

$$\frac{1}{\gamma} = \frac{4}{W} + 1 \quad \Leftrightarrow \quad \gamma = \frac{W}{4+W}, \tag{5.1}$$

If candidates do not value holding office, $\gamma = 0$. When candidates value holding office, γ increases towards unity. We thus use higher W interchangeably with lower greed.

To analyze the comparative statics of the equilibrium of the model with respect to greed, we use the following closed form solution for the equilibrium agendas, which we derived as part of the proof of Theorem 4:

$$a_l = \frac{(4+W)^{\frac{1}{\alpha-1}}\underline{j} + W^{\frac{1}{\alpha-1}}\bar{j}}{(4+W)^{\frac{1}{\alpha-1}} + W^{\frac{1}{\alpha-1}}}, \qquad a_r = \frac{(4+W)^{\frac{1}{\alpha-1}}\bar{j} + W^{\frac{1}{\alpha-1}}\underline{j}}{(4+W)^{\frac{1}{\alpha-1}} + W^{\frac{1}{\alpha-1}}}. \tag{5.2}$$

Note that when candidates care only about winning the election, i.e., as $W \to \infty$, agendas characterized in equation (5.2) converge to the midpoint between lobbies, $\frac{\underline{j}+\bar{j}}{2}$, as in Theorem 1 in Section 3.2.1. In contrast, when candidates have no office motivation, $W = 0$, the equilibrium agendas are those of the most extreme lobbies: $a_l = \underline{j}$ and $a_r = \overline{j}$.

This yields the first part of the following corollary. One can also show that campaign contributions move in the same direction as the polarization of agendas, leading to the second part of the corollary. Less greedy candidates lead to less polarized agendas and less campaign contributions.

Corollary 1 *(a) If $\alpha > 1$, polarization of agendas $|a_r - a_\ell|$ is decreasing in the value of winning the election, W.*
(b) If $\alpha > 1$, the total amount of campaign contributions in equilibrium is decreasing in the value of winning the election, W.

Proof. See Appendix B.3. ■

In contrast,

Theorem 6 *If $\alpha \leqslant 1$ and candidates put any weight on their private consumption, we obtain complete polarization in equilibrium.*

Proof. Recall from the analysis in Section 3 that a candidate's choice of platform affects the willingness to contribute of her own and her opponent's donors symmetrically. Thus, there is no cost to polarization, while there is still the benefit of raising the amount contributed (both to oneself and to the opponent). ■

We have thus shown that in the simple case when candidates have no greed and thus do not care about contribution levels, contributions are a centripetal force. With greed, we have shown that candidates strategically set their agendas to extract more donations, as suggested by La Raja and Wiltse (2012) in their empirical study of donors in U.S. elections. Our key departure from Baron (1994) is to allow lobbies to recognize the free-rider problem, and this allows us to overturn the convergence to the median voter in Baron (1994), replacing it with a different centripetal force aimed at the midpoint between the most extreme lobbies.

5.2 Contribution Limits and Polarization

Our model framework offers a tractable way of analyzing the effects of contribution limits on political outcomes. Barber (2016) estimates that when U.S. states had relatively higher individual contribution limits in state elections, they had relatively more ideologically polarized legislators. Seen through the lenses of our model, limiting how much any given lobby is permitted to contribute changes the identity of the marginal lobby. The free-rider problem still renders most of the potential donors inactive, but as the most extreme lobbies max out their contributions, the next most extreme contributors become the deciding agents in the contribution subgame. As a result, imposing contribution limits in our model is akin to compressing the distribution of contributors, i.e., increasing $\underline{j}$ and decreasing $\overline{j}$.

Unsurprisingly, restricting campaign contributions limits the extent of polarization obtained in the equilibrium of our general model.

Corollary 2 *Under the assumptions of Theorem 4, equilibrium polarization is increasing in the polarization of extreme lobbies* $(\overline{j} - \underline{j})$*, and thus weakly increasing in limits on campaign contributions.*

Proof. This is immediate from (4.8). ■

5.3 Corruption and Polarization

The effect of corruption on polarization in our model depends critically on what is meant by "corruption." On the one hand, corruption can be thought of as the ability of candidates to divert campaign contributions to private consumption. Mechanically, it can then be modeled as an additional constraint on diverting funds in the richer model.[17] From this perspective, corruption unequivocally associated with a greater degree of polarization.

But on the other hand, corruption can be thought of as the ability of a *successful* candidate to extract large office rents following the election. This then corresponds to a large value of the parameter W in our model. And this form of "corruption" unequivocally implies a *lower* degree of polarization (see Theorem 4).

Any empirical investigation of the relation between corruption and polarization has to take great care in defining the concept of corruption.

6 Conclusion

Most basic models of electoral competition predict that candidate policies should converge. Yet in practice, we observe a great deal of polarization both in candidate policies and in the identity of the donors that support them. In this paper, we have shown that polarization in the form of campaign contributions from extreme donors arises naturally in models of policy formation, lobbying and electoral success, as a result of the public-good characteristic of campaign contributions. In contrast to

[17] The general model requires $(S_i, C_i) \in B$. Here, we assume that the budget set B requires that candidates must spend at least a fraction $\underline{\gamma}$ of their campaign funds. For $\underline{\gamma} > \frac{W}{4+W}$, this constraint would be binding. More lenient constraints on the use campaign of campaign funds would then lower the constrained equilibrium γ.

a widely held intuition, we also show that under standard assumptions divergence among donors is consistent with complete policy convergence, albeit to a midpoint or average donor rather than a median voter, as candidates seek to maximize the *relative*, and not *absolute*, level of their campaign contributions. We document patterns in the data on campaign donations for U.S. House elections consistent with this model mechanism.

However, we go on to show that a small modification of our standard model that allows candidates to value the absolute level of their contributions in addition to their probability of election, either because of "ego rents" or because they are able to divert some contributions for private consumption, yields policy divergence in equilibrium. The extent of policy polarization in equilibrium depends on the relative strength of the motive for maximizing *relative* contributions (to maximize electoral success) versus the strength of the motive for maximizing *absolute* contributions, as well as on the dispersion of the distribution of potential donors.

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A Campaign finance data

We use the recipient level data from Bonica (2016). To convert contributions to real dollars, we use the core consumer price index (CPI), retrieved from `https://fred.stlouisfed.org/series/CPILFESL`.

Of all recipients, we keep only Republican and Democratic candidates for the House of Representatives, and also keep only those who are explicitly flagged as having run in the general election, or those who are incumbents, or those with general election information (either flagged as general election winners or losers, or with a positive number of votes in the general election). We also drop those without a dynamic recipient campaign finance score or with missing information on the number of givers, or with negative individual contributions. Next, we reshape the dataset into an election level dataset, keeping only observations with exactly two candidates – as some elections are uncontested, and on some occasions more than one candidate from one party may run in a general election, for example in several 2006 Texas House races. We use total receipts as our measure of campaign contributions, divided by the core CPI. For the regressions we use the logarithm of the resulting real variable. We define districts for the purpose of computing fixed effects to become a new district in 1982, 1992, 2002, and 2012 – thus allowing for redistricting.

B Proofs

B.1 Proof of Lemma 3

The argument that only the lobbies with the strongest preference contribute in any subgame equilibrium is identical to that in Lemma 2 and simply follows from the first-order conditions (3.4) and (3.5) of the donors. This proof thus amounts to identifying the set $J_i = \arg\max_j \left(|a_{-i} - j|^\alpha - |a_i - j|^\alpha\right)$ of lobbies with the strongest preference for candidate i.

To identify such a set of equilibrium contributors for the left candidate, for arbitrary $a_l < a_r$, consider the derivative of the j-lobby's preference for the left candidate, $\Delta_j(a_l, a_r) = -|a_l - j|^\alpha + |a_r - j|^\alpha$, with respect to the lobby's "location" (i.e., its policy bliss-point j):

$$\frac{\partial \Delta_j(a_l, a_r)}{\partial j} = \begin{cases} \alpha \left(\frac{1}{(a_l - j)^{1-\alpha}} - \frac{1}{(a_r - j)^{1-\alpha}} \right) > 0 & \text{for } j < a_l \\ -\alpha \left(\frac{1}{(j - a_l)^{1-\alpha}} + \frac{1}{(a_r - j)^{1-\alpha}} \right) < 0 & \text{for } j \in (a_l, a_r) \end{cases}$$

The value of this derivative to the right of a_r is irrelevant, as those lobbies would never contribute to the left candidate.

The first key observation is $\Delta_j(a_l, a_r)$ as a function of j peaks at a_l and monotonically falls of on either side of a_l. Thus, on either side of a_l, the closer a lobby is to a_l, the stronger is its preference for a_l over a_r. Consequently, the contributing lobby is either the lobby immediately to the left or the one immediately to the right of a_l.

Thus, the only point left to establish is that, if the closest lobby is more centrist, then the second closest lobby may contribute (but not when the closest lobby is more extreme than the candidate).[18] This follows from the observation that $\Delta_j(a_l, a_r)$ falls off "more quickly" to the right of a_l than to the left of a_l, implying that $\Delta_{j_1}(a_l, a_r)$ may be larger than $\Delta_{j_2}(a_l, a_r)$ even if j_1 is further away from a_l than j_2, as long as $j_1 < a_l < j_2$. This observation can be established by simply comparing the absolute values of the $\frac{\partial \Delta_j(a_l,a_r)}{\partial j}$ on either side of a_l. Recalling that the slope of Δ_j is positive to the left and negative to the right of a_l, simply observe that $\left.\frac{\partial \Delta_j(a_l,a_r)}{\partial j}\right|_{j=a_l-\delta} + \left.\frac{\partial \Delta_j(a_l,a_r)}{\partial j}\right|_{j=a_l+\delta} = -\alpha \left(\frac{1}{(a_r-a_l+\delta)^{1-\alpha}} + \frac{1}{(a_r-a_l-\delta)^{1-\alpha}} \right) < 0 \quad \forall \delta \in (0, \min\{a_l, a_r - a_l\})$. This implies that $\Delta_{j-\delta}(a_l, a_r) > \Delta_{j+\delta}(a_l, a_r) \;\; \forall \delta \in (0, \min\{a_l, a_r - a_l\})$.

An analogous argument applies to the identification of the equilibrium contributors of the right candidate. ■

B.2 Proof of Theorem 4

To establish that polarization increases with greed (decreases with W) when $\alpha > 1$, start with an (interior) equilibrium (a_1^*, a_2^*) which occurs when $W = W^*$.

The contributions of the extreme lobby c_i are given by:

$$c_i = \frac{\Delta_i^2 \Delta_{-i}}{\phi(\Delta_i + \Delta_{-i})^2}. \tag{B.1}$$

In equilibrium, $c_i = C_i$.

The fraction γ of contribution spent on campaigning (by both candidates) is an outcome of the third-stage game (see problem (4.1)) and given by:

$$\frac{1}{\gamma} = \frac{(\Delta_i + \Delta_{-i})^2}{W \Delta_i \Delta_{-i}} + 1. \quad \Leftrightarrow \quad \gamma = \frac{W \Delta_i \Delta_{-i}}{W \Delta_i \Delta_{-i} + (\Delta_i + \Delta_{-i})^2}. \tag{B.2}$$

In a symmetric equilibrium, $\Delta_i = \Delta_{-i} = \Delta$, so that:

$$\gamma = \frac{W \Delta_i \Delta_{-i}}{W \Delta_i \Delta_{-i} + (\Delta_i + \Delta_{-i})^2}. = \frac{W \Delta^2}{W \Delta^2 + 4\Delta^2} = \frac{W}{W + 4}, \tag{B.3}$$

[18] This can be true trivially because the more centrist lobby may be right in between the two candidates, in which case it would contribute to neither.

which is independent of agendas. γ increases towards 1 as W increases.

Note, however, that the endogeneity of γ could still influence agenda choices. We now show that it does not:

$$
\begin{aligned}
\frac{\partial \gamma}{\partial a_i} &= \frac{W\left(\Delta_{-i}\frac{\partial \Delta_i}{\partial a_i} + \Delta_i \frac{\partial \Delta_{-i}}{\partial a_i}\right)(W\Delta_i\Delta_{-i} + (\Delta_i + \Delta_{-i})^2)}{\left(W\Delta_i\Delta_{-i} + (\Delta_i + \Delta_{-i})^2\right)^2} \\
&\quad - W\Delta_i\Delta_{-i}\frac{W\left(\Delta_{-i}\frac{\partial \Delta_i}{\partial a_i} + \Delta_i \frac{\partial \Delta_{-i}}{\partial a_i}\right) + 2\left(\frac{\partial \Delta_i}{\partial a_i} + \frac{\partial \Delta_{-i}}{\partial a_i}\right)(\Delta_i + \Delta_{-i})}{\left(W\Delta_i\Delta_{-i} + (\Delta_i + \Delta_{-i})^2\right)^2} \\
&= \frac{W\left(\Delta_{-i}\frac{\partial \Delta_i}{\partial a_i} + \Delta_i \frac{\partial \Delta_{-i}}{\partial a_i}\right)(\Delta_i + \Delta_{-i})^2}{\left(W\Delta_i\Delta_{-i} + (\Delta_i + \Delta_{-i})^2\right)^2} - W\Delta_i\Delta_{-i}\frac{2\left(\frac{\partial \Delta_i}{\partial a_i} + \frac{\partial \Delta_{-i}}{\partial a_i}\right)(\Delta_i + \Delta_{-i})}{\left(W\Delta_i\Delta_{-i} + (\Delta_i + \Delta_{-i})^2\right)^2} \\
&\propto \left(\Delta_{-i}\frac{\partial \Delta_i}{\partial a_i} + \Delta_i \frac{\partial \Delta_{-i}}{\partial a_i}\right)(\Delta_i + \Delta_{-i})^2 - \Delta_i\Delta_{-i}2\left(\frac{\partial \Delta_i}{\partial a_i} + \frac{\partial \Delta_{-i}}{\partial a_i}\right)(\Delta_i + \Delta_{-i}) \\
&= \frac{\partial \Delta_i}{\partial a_i}(\Delta_i + \Delta_{-i})(\Delta_{-i}(\Delta_i + \Delta_{-i}) - 2\Delta_i\Delta_{-i}) \\
&\qquad + \frac{\partial \Delta_{-i}}{\partial a_i}(\Delta_i + \Delta_{-i})(\Delta_i(\Delta_i + \Delta_{-i}) - 2\Delta_i\Delta_{-i}).
\end{aligned}
$$

Note that, $\Delta_i = \Delta_{-i} = \Delta$ in equilibrium. Thus $(\Delta_i(\Delta_i + \Delta_{-i}) - 2\Delta_i\Delta_{-i}) = 2\Delta^2 - 2\Delta^2 = 0$ and similarly for the symmetric term, so that $\frac{\partial \gamma}{\partial a_i} = 0$. Thus, the endogeneity of γ does not influence the agenda choice in equilibrium.

Now consider the candidate i's ex-ante maximization problem of choosing the agenda:

$$\max_{a_i} U((1-\gamma)C_i(a_i, a_{-i})) + p_i(\gamma C_i(a_i, a_{-i}), \gamma C_{-i}(a_i, a_{-i}))W, \tag{B.4}$$

Using that $\frac{\partial \gamma_i}{\partial a_i} = \frac{\partial \gamma_{-i}}{\partial a_i} = 0$ and that $\gamma_i = \gamma_{-i} = \gamma$ in equilibrium, the first-order condition of the problem (B.4) can be expressed as:

$$U'((1-\gamma)C_i)\frac{1-\gamma}{\gamma}\frac{\partial C_i}{\partial a_i} = \left(-\frac{\partial p_i}{\partial S_{-i}}\frac{\partial C_{-i}}{\partial a_i} - \frac{\partial p_i}{\partial S_i}\frac{\partial C_i}{\partial a_i}\right)W, \tag{B.5}$$

which, for $U(c) = \ln c$, further simplifies to

$$\frac{1}{\gamma C_i}\frac{\partial C_i}{\partial a_i} = \left(-\frac{\partial p_i}{\partial S_{-i}}\frac{\partial C_{-i}}{\partial a_i} - \frac{\partial p_i}{\partial S_i}\frac{\partial C_i}{\partial a_i}\right)W. \tag{B.6}$$

We now use the properties of the contribution function and winning probability function to simplify further. Using the first-order conditions of the contributing lobbies, i.e., equation (3.4), we have that $\frac{\partial p_i(S_i(C_i),S_{-i})}{\partial C_i} = \frac{\phi}{\Delta_i}$.

$$\frac{\phi}{\Delta} = \frac{\partial p_i(S_i(C_i), S_{-i})}{\partial C_i} = \frac{\partial p_i(S_i(C_i), S_{-i})}{\partial S_i}\frac{\partial S_i}{\partial C_i} = \frac{\partial p_i(S_i(C_i), S_{-i})}{\partial S_i}\gamma \tag{B.7}$$

Differentiating with respect to the other candidate's campaign spending:

$$\frac{\partial p_i(S_i, S_{-i})}{\partial S_{-i}} = -\frac{\partial p_i(S_i, S_{-i})}{\partial S_i}\frac{S_{-i}}{S_i} = -\frac{\partial p_i(S_i, S_{-i})}{\partial S_i}, \tag{B.8}$$

where the last equality uses that the equilibrium is symmetric.

Contributions change as follows:

$$\frac{\partial C_i}{\partial a_i} = \frac{\left(2\Delta_i\Delta_{-i}\frac{\partial\Delta_i}{\partial a_i} + \Delta_i^2\frac{\partial\Delta_{-i}}{\partial a_i}\right)\phi(\Delta_i + \Delta_{-i})^2 - \Delta_i\Delta_{-i}^2\phi 2(\Delta_i + \Delta_{-i})\left(\frac{\partial\Delta_i}{\partial a_i} + \frac{\partial\Delta_{-i}}{\partial a_i}\right)}{\phi^2(\Delta_i + \Delta_{-i})^4}.$$

Using that $\Delta_i = \Delta_{-i} = \Delta$ in equilibrium, we can simplify to:

$$\frac{\partial C_i}{\partial a_i} = \frac{1}{4\phi}\left(\Delta^2\left(2\frac{\partial\Delta_i}{\partial a_i} + \frac{\partial\Delta_{-i}}{\partial a_i}\right) - \Delta^2\left(\frac{\partial\Delta_i}{\partial a_i} + \frac{\partial\Delta_{-i}}{\partial a_i}\right)\right) = \frac{1}{4\phi}\frac{\partial\Delta_i}{\partial a_i}.$$

Note that contributions in a symmetric equilibrium are $C_i = C = \frac{\Delta}{4\phi}$ so that:

$$\frac{1}{C_i}\frac{\partial C_i}{\partial a_i} = \frac{1}{\Delta}\frac{\partial\Delta_i}{\partial a_i}. \tag{B.9}$$

One can similarly show that:

$$\frac{1}{C_{-i}}\frac{\partial C_{-i}}{\partial a_i} = \frac{1}{\Delta}\frac{\partial\Delta_{-i}}{\partial a_i}. \tag{B.10}$$

Plugging into (B.6):

$$\frac{1}{\gamma}\frac{1}{\Delta}\frac{\partial\Delta_i}{\partial a_i} = \frac{\phi}{\gamma\Delta}\left(\frac{1}{4\phi}\frac{\partial\Delta_{-i}}{\partial a_i} - \frac{1}{4\phi}\frac{\partial\Delta_i}{\partial a_i}\right)W. \tag{B.11}$$

Canceling

$$\frac{\partial\Delta_i}{\partial a_i} = \frac{1}{4}\left(\frac{\partial\Delta_{-i}}{\partial a_i} - \frac{\partial\Delta_i}{\partial a_i}\right)W. \quad \Leftrightarrow \quad (4 + W)\frac{\partial\Delta_i}{\partial a_i} = W\frac{\partial\Delta_{-i}}{\partial a_i}. \tag{B.12}$$

Note that $\Delta_r = (\bar{j} - a_\ell)^\alpha - (\bar{j} - a_r)^\alpha$, so that:

$$\Delta_r = (\bar{j} - a_\ell)^\alpha - (\bar{j} - a_r)^\alpha \Rightarrow \qquad \frac{\partial \Delta_r}{\partial a_r} = \alpha(\bar{j} - a_\ell)^{\alpha-1}$$
$$\Delta_\ell = (a_r - \underline{j})^\alpha - (a_\ell - \underline{j})^\alpha \Rightarrow \qquad \frac{\partial \Delta_\ell}{\partial a_r} = \alpha(a_r - \underline{j})^{\alpha-1}.$$

Plugging in (B.12) for $i = r, -i = \ell$:

$$(4 + W)\alpha(\bar{j} - a_r)^{\alpha-1} = W\alpha(a_r - \underline{j})^{\alpha-1}. \tag{B.13}$$

Equivalently:

$$(4 + W)^{\frac{1}{\alpha-1}}(\bar{j} - a_r) = W^{\frac{1}{\alpha-1}}(a_r - \underline{j}) \tag{B.14}$$

and

$$a_r = \frac{(4 + W)^{\frac{1}{\alpha-1}}\bar{j} + W^{\frac{1}{\alpha-1}}\underline{j}}{(4 + W)^{\frac{1}{\alpha-1}} + W^{\frac{1}{\alpha-1}}}. \tag{B.15}$$

We can also explicitly solve for the left agenda and polarization:

$$a_\ell = \frac{(4 + W)^{\frac{1}{\alpha-1}}\underline{j} + W^{\frac{1}{\alpha-1}}\bar{j}}{(4 + W)^{\frac{1}{\alpha-1}} + W^{\frac{1}{\alpha-1}}} \tag{B.16}$$

$$a_r - a_\ell = (\bar{j} - \underline{j})\frac{(4 + W)^{\frac{1}{\alpha-1}} - W^{\frac{1}{\alpha-1}}}{(4 + W)^{\frac{1}{\alpha-1}} + W^{\frac{1}{\alpha-1}}}. \tag{B.17}$$

■

B.3 Proof of Corollary 1

Part (a):
From (B.15), when candidates do not value holding the office ($W = 0$), then $a_r = \bar{j}$: The candidates set their agendas to the bliss point of the lobby they are targeting. In the other extreme, as $W \to \infty$ and candidates are only office-motivated, there is convergence to $\frac{\bar{j}+\underline{j}}{2}$. In between, a_r is decreasing in W (increasing in greed), and since $a_r + a_\ell = \bar{j} + \underline{j}$, the opposite is true for a_ℓ. Polarization, defined as $|a_1 - a_2| = a_r - a_\ell$, increases in greed.

Part (b):
Note that in the symmetric equilibrium, campaign contributions satisfy $C \propto \Delta$.

Define $\delta \equiv \frac{1}{2}(a_r - a_\ell)$ and the average agenda $a_m \equiv \frac{a_r + a_\ell}{2}$. We can then re-write Δ as:

$$\Delta = |a_r - \underline{j}|^\alpha - |a_\ell - \underline{j}|^\alpha = (a_m + \delta - \underline{j})^\alpha - (a_m - \delta - \underline{j})^\alpha. \tag{B.18}$$

From before, we know that $a_m = \frac{1}{2}(\bar{j} + \underline{j})$, which is invariant to W. We thus have that:

$$\frac{dC}{dW} \propto \frac{d\Delta}{dW} = \left(\alpha(a_m + \delta - \underline{j})^{\alpha-1} + (a_m - \delta - \underline{j})^{\alpha-1}\right) \frac{d\delta}{dW}. \tag{B.19}$$

Since $\frac{d\delta}{dW} < 0$ by part (a), campaign contributions decrease in W. ■

C Mapping Existing Frameworks into Our Model

C.1 Informed and Uninformed Voters.

The framework closest to our model is that of Baron (1994). However, we are interested only in collective policies (those that affect everyone), and choose to drop the particularistic policy considerations. The mapping from Baron (1994) into our framework is then quite simple: Normalize the total number of voters to one and assume that they are divided into separate groups of informed and uninformed voters with the measure of uninformed voters given by θ. The probability that candidate 1 wins the election, given spending levels and agendas, is then given by

$$p_1^{UV}(S_1, S_2, a_1, a_2) = \theta \frac{S_1}{S_1 + S_2} + (1 - \theta) \begin{cases} \frac{a_1 + a_2}{2} & \text{if } a_1 < a_2, \\ \frac{1}{2} & \text{if } a_1 = a_2, \\ 1 - \frac{a_1 + a_2}{2} & \text{if } a_1 > a_2. \end{cases} \tag{C.1}$$

Theorem 7 *If $\alpha > 1$, informed voters are distributed uniformly on $[0, 1]$, and the candidates' sole objective is winning the election, then the unique equilibrium has both candidates locating (choosing platforms) at agenda a^* located between the median informed voter's preferred policy $\frac{1}{2}$ and the midpoint between the two extreme potential donors, at $j_m = \frac{\bar{j} + \underline{j}}{2}$. Specifically, a^* solves*

$$\theta \frac{|u'_{\bar{j}}(a^*)|}{|u'_{\underline{j}}(a^*)| + |u'_{\bar{j}}(a^*)|} + (1 - \theta) a^* = \frac{1}{2}. \tag{C.2}$$

Contributions are zero in equilibrium.

Proof. This is an equilibrium because moving marginally from a^* leads to exactly offsetting discontinuous changes in the two components of the probability of an election win. Moving further away from a^* then lowers a candidate's probability of winning: Moving towards the median voter yields constant marginal gain from informed voters but increasing marginal loss from relative contributions. On the other hand, moving towards the midpoint j_m yields a decreasing marginal benefit from the (relative) contributions, while generating constant marginal cost from the informed voters. ■

C.2 Spending to "Get Out the Vote."

Another model that fits neatly into our general framework is a slight modification of Herrera et al. (2008), in which campaign spending increases the proportion of potential voters who turn out to vote.[19] There are two office-motivated candidates, who first simultaneously choose their agendas and then their spending. The voters have both idiosyncratic and aggregate (unknown) candidate bias. The voters also care about the policy choices — they have Euclidean preferences with their ideal points distributed uniformly on $[0, 1]$. Campaign spending by the candidates is necessary to motivate the electorate to vote. However, the spending is not perfectly targeted and brings some of the opponent's supporters to the polling stations. The fraction of candidate i's supporters who turn out to vote is then $(tS_i + (1-t)S_{-i})$, where $t \in (\frac{1}{2}, 1]$ is the accuracy of campaign targeting. As Herrera et al. (2008) show, the probability of the (left) candidate 1 winning the election is

$$p_1^{GV}(S_1, S_2, a_1, a_2) = F\left(a_1 - a_1^2 - a_2 + a_2^2 + 2\beta\left(t - \frac{1}{2}\right)\frac{S_1 - S_2}{S_1 + S_2}\right), \tag{C.3}$$

where F is the c.d.f. of the distribution of the aggregate bias for candidate 1, and β is measure of the dispersion of the idiosyncratic bias (which is distributed uniformly on $[-\beta, \beta]$). Since

$$\begin{aligned}\frac{S_1 - S_2}{S_1 + S_2} &= \frac{S_1}{S_1 + S_2} - \frac{S_2}{S_1 + S_2} \\ &= \frac{S_1}{S_1 + S_2} - \left(1 - \frac{S_1}{S_1 + S_2}\right) \\ &= 2\frac{S_1}{S_1 + S_2} - 1,\end{aligned}$$

[19] We omit the policy preferences of the candidates (parties), which are present in Herrera et al. (2008).

we can map this model into ours with:

$$f(x, a_1, a_2) = F\left(a_1 - a_1^2 - a_2 + a_2^2 + 2\beta\left(t - \frac{1}{2}\right)(2x - 1)\right), \qquad x \equiv \frac{S_1}{S_1 + S_2}.$$

D Donors with Increasing Marginal Cost of Funds

This appendix highlights that the key findings of the paper do not rely on the stark assumptions we made for illustrative purposes.

The key to the extreme free-riding results obtained in Section 3.1.1 is that the marginal cost of contributing the first dollar is strictly positive, and that the extreme donors never tire of contributing (that their marginal cost of doing so does not increase). If the marginal cost of contributing the first dollar is small and/or the marginal cost of contributing rises with the level of contributions, we obtain less extreme results in which more than one donor may contribute to each candidate. For the purposes of generalization, let the preferences of a donor j now be represented by

$$u_j(a) = -|a - j|^\alpha - \phi c^\sigma, \tag{D.1}$$

where $\sigma > 1$, and note that

$$\lim_{c \to 0} \frac{d\phi(c)}{dc} = \lim_{c \to 0} \sigma\phi c^{\sigma-1} = 0.$$

In this case, the only donor that may *not* contribute to any candidate is the one that is indifferent between the candidates. For all other donors, the first-order condition with respect to contribution holds with equality for at least one candidate, or

$$\begin{aligned} \tfrac{\partial p_1(C_1,C_2)}{\partial C_1}\Delta_j(a_1, a_2) &= \phi\sigma c_j(1)^{\sigma-1} \quad \text{whenever} \;\; \Delta_j(a_1, a_2) > 0, \\ \tfrac{\partial p_2(C_2,C_1)}{\partial C_2}\Delta_j(a_2, a_1) &= \phi\sigma c_j(2)^{\sigma-1} \quad \text{whenever} \; \Delta_j(a_2, a_1) > 0. \end{aligned} \tag{D.2}$$

Denoting the marginal productivity of contributions to candidate 1 by $\theta_1 = \partial p(C_1, C_2)/\partial C_1$, and defining θ_2 analogously, we obtain

$$c_j(i) = \left(\tfrac{\theta_i}{\phi\sigma}\Delta_j(a_i, a_{-i})\right)^{\frac{1}{\sigma-1}} \quad \text{whenever} \;\; \Delta_j(a_i, a_{-i}) > 0, \tag{D.3}$$

so that the most extreme donors are still the largest contributors. To complete the characterization, we simply need to note that

$$C_i = \sum_{\Delta_j(a_i, a_{-i}) > 0} c_j(i). \tag{D.4}$$

Given our assumptions on p_i, it is relative contributions that matter for the probability of election. Noting that by homogeneity of degree zero, $p_1(C_1, C_2) = p_1(C_1/C_2, 1) \equiv \tilde{p}(C_1/C_2)$, and hence using symmetry we obtain

$$\begin{aligned} \frac{\partial p_1(C_1, C_2)}{\partial C_1} &= \tilde{p}'\left(\frac{C_1}{C_2}\right)\frac{1}{C_2}, \\ \frac{\partial p_2(C_2, C_1)}{\partial C_2} &= \tilde{p}'\left(\frac{C_1}{C_2}\right)\frac{C_1}{C_2^2}, \end{aligned}$$

so that we can solve for relative contributions:

$$\frac{C_1}{C_2} = \left(\frac{\sum_j \left(\max\left\{\Delta_j(a_1, a_2), 0\right\}\right)^{1/(\sigma-1)}}{\sum_j \left(\max\left\{\Delta_j(a_2, a_1), 0\right\}\right)^{1/(\sigma-1)}}\right)^{\frac{\sigma-1}{\sigma}}, \tag{D.5}$$

which reduces to

$$\frac{C_1}{C_2} = \frac{\Delta_{\underline{j}}(a_1, a_2)}{\Delta_{\bar{j}}(a_2, a_1)}, \tag{D.6}$$

when $\phi(c) = \phi c$ (Assumption 4) and $\alpha > 1$.