

Working Papers

RESEARCH DEPARTMENT

WP 21-37

November 2021

<https://doi.org/10.21799/frbp.wp.2021.37>

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ISSN: 1962-5361

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Should Central Banks Issue Digital Currency?*

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November 3, 2021

Abstract

We study how the introduction of central bank digital currency affects interest rates, the level of economic activity, and welfare in an environment where both central bank money and private bank deposits are used in exchange. We highlight an important policy tradeoff: While a digital currency tends to promote efficiency in exchange, it may also crowd out bank deposits, raise banks' funding costs, and decrease investment. We derive conditions under which *targeted* digital currencies, which compete only with physical currency or only with bank deposits, raise welfare. If such targeted currencies are infeasible, we illustrate the policy tradeoffs that arise when issuing a single, universal digital currency.

Keywords: Monetary policy; public vs. private money; electronic payments; liquidity premium; disintermediation

JEL Classification: E42, E58, G28

*A previous version of this paper circulated under the title "Managing Aggregate Liquidity: The Role of a Central Bank Digital Currency." For helpful comments, we thank seminar and conference participants at the Bank of Canada, Bank for International Settlements, University of Bern, European Central Bank, Goethe University Frankfurt, the University of Mississippi, Queens University, the Federal Reserve Banks of New York, Richmond and St. Louis, the Southwestern University of Finance and Economics, the Economics of Payments IX Conference, the 2019 IMF Macro-Finance Research Conference, the Spring 2019 Midwest Macro Meetings, the 2019 ITAM-PIER Conference on Macroeconomics, the 2019 Cryptocurrencies and Blockchains Conference at the University of Chicago, the Bruce Smith Memorial Conference at the University of Texas, the 2020 NBER Summer Institute, the 2020 Virtual Search and Matching Congress, the 2021 ASSA Annual Meeting, and the 2021 WEAI Annual Conference. The views expressed in this paper are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.

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1 Introduction

The money used by households and firms is a combination of physical currency issued by the central bank and liabilities of private financial institutions, most notably bank deposits. In recent decades, it has become increasingly easy for transactions with bank deposits to be made electronically using payment cards, online account access, and mobile apps. Money issued by the central bank, in contrast, can generally be used only for purchases made with physical currency or for transactions directly between commercial banks that hold reserves on deposit at the central bank. The increased use of electronic means of payment has, therefore, represented a decline in the use of central bank money by households and firms, and this process has accelerated in many countries during the Covid-19 pandemic. Policy makers have expressed concern that this decline could have negative consequences for financial inclusion, contestability in payments services, and potentially for monetary policy as new types of electronic money and payments services are developed.

In response to these concerns, policy makers around the world are discussing the possibility of issuing central bank digital currency (CBDC). A central bank could, for example, issue cryptographic tokens that share some of the technological features of Bitcoin or other cryptocurrencies. Alternatively, a digital currency could be created simply by allowing households and firms to open deposit accounts at the central bank and use these accounts to make payments in much the same way they currently use private bank deposits. Depending on the design, a CBDC may allow central bank money to be used in a much wider range of situations, including online and large-value transactions where the use of physical currency is impractical. It could also allow a widely held form of central bank money to bear interest. Academics and policy makers have begun discussing a range of issues, from technical design features to political economy considerations, in an attempt to evaluate the potential benefits and costs of issuing digital currency.¹

One concern often raised in these discussions is that a central bank digital currency may crowd out private bank deposits and thereby lead to disintermediation of the banking system. If households and firms find this new option attractive, they may shift a substantial amount of funds out of private bank deposits and into the central bank digital currency. Such a shift could potentially raise bank funding costs and lead to a decline in bank lending and investment. A Bank of International Settlements report (BIS, 2018) expresses concern that “a flow of retail deposits into a CBDC could lead to a loss of low-cost and stable funding

¹ Early discussions of these issues were offered by Ali et al. (2014), Broadbent (2016), Fung and Halaburda (2016) and Skingsley (2016), among others. Auer et al. (2020) provides a recent overview of the policy discussion. Boar and Wehrli (2021) describe a survey of 65 central banks in which 86% reported currently studying central bank digital currency in some form.

for banks.” Mersch (2017) worries that “[a] consequence could be higher interest rates on bank loans.” Meaning et al. (2018) wonder if the benefits of a CBDC would be “outweighed by the negative consequences of the central bank disintermediating a large part of banks business models.”² Others are less concerned and believe that making central bank money a more attractive competitor to private bank deposits will necessarily benefit consumers and the broader economy.³ Several central banks have run or are planning pilot projects and at least one CBDC – the Bahamas’ Sand Dollar – is in full operation. However, as these debates indicate, the basic macroeconomic implications of introducing a central bank digital currency are not well understood.

We study how the introduction of a central bank digital currency affects interest rates, bank lending, output and welfare in an environment where both central bank money and private bank deposits are used in exchange. We build on the framework in Lagos and Wright (2005) and the subsequent New Monetarist literature, where money in some form is essential for exchange.⁴ Bankers in our model can issue deposits that serve as a means of payment. The ability of these deposits to facilitate exchange may give rise to a liquidity premium, which lowers banks’ funding costs and tends to increase investment. At the same time, however, bankers face credit constraints due to limited pledgeability of their returns, as in Kiyotaki and Moore (1997, 2005), Holmström and Tirole (1998) and others. These constraints tend to reduce bank lending and investment.

We show that introducing a central bank digital currency can often raise welfare in this environment, even if it leads to some disintermediation of banks. A key benefit of digital currency is that it increases production of those goods it can be used to purchase, which can potentially lead to higher total output. In addition, the central bank gains a new policy tool: the interest rate it pays on digital currency. This tool can be used to influence the efficiency of exchange and, in some cases, of aggregate investment. The optimal choice of this interest rate is sensitive to the design features of the digital currency, in particular, what existing form(s) of payment it competes with. We study two broad possibilities, one in which the central bank can issue *targeted* digital currencies, which only compete with a single existing form of payment, and the other in which a digital currency is *universal*, meaning that it necessarily competes with both physical currency and deposits.

The analysis of a targeted digital currency that competes only with physical currency

² This concern is also expressed in official reports from the Bank of England (2020, pp.35-36), Bank of Israel (2018, p.26), Danmark Nationalbank (2017, pp.13-14), European Central Bank (2020, pp.16-17), Norges Bank (2018, pp.35-36) and Sveriges Riksbank (2018, p.30).

³ See, for example, Bordo and Levin (2018) and Kumhof and Noone (2018).

⁴ For an overview of this literature, see the survey papers by Williamson and Wright (2010a, 2010b) and Lagos et al. (2017), as well as the many references therein.

is straightforward. Because it involves substituting one form of outside money for another, introducing a cash-like digital currency has no direct impact on bank funding or investment. The ability to pay interest on digital currency decouples the return on holding outside money from the rate of inflation, which allows the central bank to increase the real value of the stock of outside money, if desired. We show that a cash-like digital currency is desirable if and only if the social value of cash-based transactions is sufficiently large. In these cases, the optimal policy corresponds to a modified version of the Friedman rule.

Issuing a *deposit-like* currency, in contrast, will tend to crowd out bank deposits, raise the real interest rate on these deposits, and decrease bank-financed investment. At the same time, however, it will increase the aggregate stock of liquid assets in the economy, which promotes more efficient levels of production and exchange. The optimal interest rate on a deposit-like digital currency balances these competing effects. A deposit-like currency is desirable when productive projects are sufficiently scarce relative to the transactions demand for bank deposits and when credit market frictions are moderate.

Creating digital currencies that only compete with a single existing means of payment may not be technologically feasible, however. For example, it may not be possible to design a cash-like digital currency that cannot also be used in online or other transactions at a distance that currently use bank deposits. If a digital currency will necessarily compete with both cash and bank deposits, the central bank must take into account the potential interactions across these sectors. To illustrate these interactions, we study a *universal* digital currency that can be used in all transactions. The central bank is more restricted in this regime: it sets a single interest rate on a digital currency that is available for all uses. We provide conditions under which a universal digital currency can implement the same allocation as two targeted currencies. When it cannot, the welfare gain from a universal digital currency is smaller than from two targeted digital currencies but is often still positive. We show through examples that a universal digital currency may circulate either more or less widely than targeted digital currencies in equilibrium.

Our analysis demonstrates that the interest rate paid on a digital currency is a useful new policy tool. To further illustrate this point, we extend our model to allow the central bank to lend the proceeds it receives from a deposit-like digital currency back to private banks. Under this alternative policy, the digital currency represents *inside* rather than outside money, since the central bank holds a corresponding claim on the private sector.⁵ We show that, in this extended model, it is possible for the central bank to choose the interest rate on digital currency so that it is held in equilibrium but does not alter equilibrium allocations. This

⁵ See Lagos (2010) for a discussion of the distinction between inside and outside money, including situations where the public sector holds private claims.

result is in line with Brunnermeier and Niepelt (2019), who provide conditions under which the introduction of a digital currency has no effect on equilibrium allocations. However, the interest rate that achieves this outcome is often not the optimal policy. Instead, the central bank wants to use the new policy tool to alter total real money balances and the equilibrium liquidity premium, so that introducing a digital currency does affect allocations under the optimal policy and raises welfare.

Related literature. The literature on digital currencies is growing rapidly. A wave of recent papers discuss the possibility of a central bank digital currency and the many design choices it would bring. Bech and Garratt (2017) provide a useful starting point by laying out a taxonomy of types of money and comparing different types of possible digital currencies with existing payment options. Mancini-Griffoli et al. (2018) provide a comprehensive overview of the issues raised by a possible digital currency along with citations to many relevant papers. Among these, Kahn et al. (2019) and Kumhof and Noone (2018) provide interesting discussions of the design choices facing a central bank.

Our paper lies in the branch of this literature that uses dynamic general equilibrium models to analyze the macroeconomic effects of a central bank digital currency. The earliest paper in this branch is Barrdear and Kumhof (2021), which introduces a central bank digital currency into a quantitative DSGE model to assess its impact on GDP and to evaluate different monetary policy rules. The effects of issuing a digital currency in their framework come largely from the expansion of the assets held by the central bank, which directly lowers the real interest rate, rather than from having a new form of money *per se*. Our focus, in contrast, is on how the introduction of a new payment medium affects the liquidity premium on bank deposits and thereby alters equilibrium interest rates and investment.

In this respect, our paper is more closely related to recent work by Chiu et al. (2021) and Williamson (2021), both of which use New Monetarist models that share many features with ours.⁶ Chiu et al. (2021) follow Andolfatto (2021) in studying the effects of introducing a central bank digital currency when banks have market power. Williamson (2021) focuses on the efficiency gains that can arise when households hold direct claims on the central bank, in the form of a CBDC, rather than claims on financial intermediaries that are subject to incentive constraints. A CBDC may disintermediate banks in both of these papers, but only in situations where there is an overaccumulation of capital, which implies that disintermediation improves economic efficiency. In our model, in contrast, financial frictions may cause investment to be inefficiently low in equilibrium. A decline in bank deposits will

⁶ See also Davoodalhosseini (2021), who studies the monetary policy implications of a CBDC that competes with cash as a means of payment, and Dong and Xiao (2019), who study how the introduction of CBDC affects firms' financing decisions.

tend to worsen this inefficiency, which captures policy makers' concern that disintermediating banks could be costly from a social point of view. The optimal policy involves balancing the benefits of a CBDC against these costs.

Piazzesi and Schneider (2020) demonstrate that disintermediating banks can be costly in other ways as well. In their model, banks provide liquidity through both deposits and credit lines, and these two activities are complementary. A CBDC that crowds out deposits will also decrease the provision of credit lines, bringing additional losses. We view our analysis as a benchmark that captures the fundamental interaction between CBDC and bank deposits as (potentially) competing means of payment. While future work will likely continue to identify additional costs and benefits of introducing a CBDC, the fundamental tradeoffs we identify here are likely to be present in any setting where disintermediating banks is a concern.

The remainder of the paper is organized as follows. We present the model environment and derive the equilibrium conditions for a general formulation of the type(s) of currency available to agents in Section 2. We analyze equilibrium in a benchmark case without a digital currency in Section 3. We study the effects of introducing targeted of digital currencies in Section 4, of introducing a universal digital currency in Section 5, and of central bank lending in Section 6. Finally, we offer some concluding remarks in Section 7.

2 The Model

In this section, we describe the physical environment, which builds on Lagos and Rocheteau (2008) and Williamson (2012), among others. We also derive the conditions characterizing equilibrium for a general formulation of the type(s) of currency available to agents. Subsequent sections then specialize the analysis to study different digital currency designs.

2.1 The Environment

Time is discrete and continues forever. Each period is divided into two subperiods, the first with a frictionless centralized market and the second with decentralized trade. A perishable commodity is produced and consumed in each subperiod; we refer to these commodities as the centralized market (CM) good and the decentralized market (DM) good, respectively.

Agents. The economy is populated by three types of agents: buyers, sellers, and bankers. Buyers and sellers are infinitely lived and participate in both markets in each period. They can produce the CM good in the first subperiod using a linear technology that requires labor as input, and they also have linear utility over CM consumption. In the second subperiod, buyers want to consume but cannot produce, whereas sellers can produce but do not want

to consume. Each buyer is randomly matched with a seller with probability $\alpha \in [0, 1]$, so trade is bilateral. Each buyer has the period utility function

$$U^b(x_t^b, q_t) = x_t^b + u(q_t),$$

where $x_t^b \in \mathbb{R}$ denotes net consumption of the CM good and $q_t \in \mathbb{R}_+$ denotes consumption of the DM good. The function $u : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is strictly increasing, strictly concave, and continuously differentiable, with $u(0) = 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. Each seller has the period utility function

$$U^s(x_t^s, q_t) = x_t^s - w(q_t),$$

where $x_t^s \in \mathbb{R}$ denotes net consumption of the CM good and $q_t \in \mathbb{R}_+$ denotes production of the DM good. The function $w : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is strictly increasing, convex, and continuously differentiable, with $w(0) = 0$. There is a unit mass each of buyers and sellers, all of whom discount future periods at a common rate $\beta \in (0, 1)$.

Bankers live for two periods, participate only in the centralized market, and consume only in old age. Each period, a new generation of bankers is born. Banker j is endowed at birth with an indivisible and nontradable project that requires one unit of the CM good as input and pays off $\gamma_j \in \mathbb{R}_+$ units of the CM good in the following period.⁷ Project returns are known in advance, publicly observable, and heterogeneous across bankers. The support of the distribution of project returns is $[0, \bar{\gamma}]$ with $\bar{\gamma} > \beta^{-1}$, which implies that some projects are socially efficient to operate but others are not. There is a measure $\eta > 0$ of bankers with each return γ in the support; the total measure of bankers is $\eta\bar{\gamma}$.

Bankers have no endowment; they must fund their project by issuing *deposits* in the centralized market when they are young. These deposits are risk-free claims on consumption in the following period's CM. The ability to issue deposits is limited by a pledgeability constraint: Only a fraction $\theta \leq 1$ of the project's return can be pledged to the bank's depositors. This friction prevents some banks whose projects are profitable at market interest rates from being able to borrow and invest. We assume $\theta > (\beta\bar{\gamma})^{-1}$, which ensures the most productive projects can be funded when the interest rate on deposits is β^{-1} .

Assets and exchange. Buyers and sellers are anonymous (i.e., their identities are unknown to each other and their trading histories are private information), which precludes credit in the decentralized market and makes a medium of exchange essential for decentralized trade. The possible media of exchange in our model are deposits issued by bankers and currency,

⁷ A banker in our model can be interpreted as the combination of an intermediary that issues a means of payment and a firm that operates a productive technology. It is straightforward to divide these two roles into separate institutions (a bank and a firm) in a way that leaves the results below unchanged.

both physical and digital.

The supply of bank deposits depends on the real interest rate, which determines how many bankers are able to satisfy the pledgeability constraint. The supply of currency is determined by the central bank according to a price-level targeting regime in which the gross inflation rate $\mu > \beta$ is assumed to be constant over time. In particular, the central bank stands ready to buy/sell CM goods each period at a predetermined price in either physical or digital currency. By enforcing the same price level target for physical and digital currency, the central bank is effectively offering to convert units of physical currency one-for-one into units of digital currency and vice versa. In this sense, the digital currency in our model is an electronic version of the physical currency and not a distinct item that might trade at a different price.⁸ The central bank uses lump-sum taxes/transfers to balance its budget each period.⁹

The extent to which each of these assets can be used in DM exchange depends on the verification technology available to the seller in a particular meeting. A fraction $\lambda_1 \in (0, 1)$ of sellers is endowed with the technology to recognize physical currency but not deposits. We interpret this assumption as capturing a variety of reasons why cash is used in practice, including concerns about the privacy of the transacting parties, fees, and/or a lack of access to the electronic payment network. The remaining fraction $\lambda_2 \equiv 1 - \lambda_1$ of sellers is endowed with the technology to recognize bank deposits but not physical currency. The meetings of these sellers correspond to transactions that in practice involve debit cards, checks, or other methods of directly transferring claims on a commercial bank from the buyer to the seller. These meetings represent transactions in which the value of the trade and/or the distance between parties make the use of physical currency impractical. We refer to a meeting in which the seller is able to verify physical currency as *type 1* and to a meeting in which the seller can verify bank deposits as *type 2*.¹⁰ A buyer finds out the type of seller she will potentially meet in the next DM before making her portfolio decision in the CM, which implies that she will choose to hold either currency or deposits for transactions purposes, but not both.

When we introduce digital currency into this environment, a key issue is the type(s) of

⁸ Private digital currencies like Bitcoin would be different in this regard, of course. See Bank for International Settlements (2015) for a discussion of the economic implications of private digital currencies and Fernández-Villaverde and Sanches (2019) for a model of private digital currency issue.

⁹ In other words, while we refer to the policy maker in the model as the “central bank,” it actually represents the consolidated public sector, as is common in dynamic general equilibrium models.

¹⁰ It is straightforward to add a third type of meeting in which both currency and deposits can be verified by the seller, as in Chiu et al. (2021). Doing so complicates the presentation without changing the basic insights of our model, as only one of the two forms of payment would typically be used in all such meetings. The important assumption for our purposes is that each form of payment can be used in some situations where the other cannot.

meeting in which it can be used. As the technological features of a potential CBDC are still largely undetermined, we study two broad possibilities. In Section 4, we assume the central bank is able to design *targeted* digital currencies that can only be used in a single type of meeting. A *cash-like* CBDC can only be verified by type 1 sellers, while a *deposit-like* CBDC can only be verified by type 2 sellers.¹¹ We derive conditions under which it is desirable to issue each type of targeted CBDC and properties of the optimal interest rates. In Section 5, we assume targeted CBDCs are technologically infeasible; instead, a digital currency can necessarily be verified by all sellers. In this case, the central bank has only a single policy instrument: the interest rate on the *universal* CBDC. We derive conditions under which a digital currency is desirable when it must be universal and characterize the optimal interest rate in this case.

Allocations and welfare. For discussions of optimal policy, we measure welfare using an equal-weighted sum of all agents' utilities. However, we allow for the possibility that some of the consumption that results from type 1 meetings, where physical currency is used, might have lower social value than private value. For example, a policy maker may put less weight on transactions involving illicit activities. Specifically, we follow Williamson (2012) in assuming that only a fraction $\nu \in [0, 1]$ of type 1 meetings generates social value. We can then write aggregate welfare as

$$\sum_{t=0}^{\infty} \beta^t \{x_t^b + x_t^s + x_t + \alpha [\lambda_1 (\nu u(q_{1t}) - w(q_{1t})) + \lambda_2 (u(q_{2t}) - w(q_{2t}))]\}, \quad (1)$$

where $x_t \in \mathbb{R}_+$ denotes the total CM consumption of old-age bankers. Feasibility of an allocation requires that the net consumption of all agents in the centralized market is no greater than the net output of bankers' investment projects. We focus on allocations characterized by a cutoff value $\hat{\gamma}_t$ above which a banker's project is operated and below which it is not. Feasibility in period t then requires

$$x_t^b + x_t^s + x_t \leq \eta \int_{\hat{\gamma}_{t-1}}^{\tilde{\gamma}} \gamma d\gamma - \eta (\tilde{\gamma} - \hat{\gamma}_t). \quad (2)$$

The right-hand side of this expression is the output from projects maturing at the current date minus total investment into new projects that will mature the following period. Net consumption of CM goods by all agents can be no larger than this difference.

¹¹ See Agur et al. (2021) for a model in which a single central bank digital currency is an imperfect substitute for both cash and deposits. While our approach of having digital currencies be a perfect substitutes for an existing payment method is perhaps somewhat extreme, it allow us to capture the key trade-offs faced by policy makers in a tractable macroeconomic framework. See Wang (2020) for an analysis of how the desirability of different designs is affected by concerns about tax avoidance.

Given quasi-linear preferences, the welfare properties of an allocation depend only on the sequences of DM consumption levels $\{q_{1t}, q_{2t}\}$ and of cutoff investment values $\{\hat{\gamma}_t\}$, which determine the total amount of CM consumption available in each period. As equations (1) and (2) make clear, the distribution of CM consumption across agents has no impact on welfare. In the analysis that follows, we summarize an allocation by these three quantities. In the remainder of this section, we derive buyers' demand for assets (deposits and currency), the supply of these assets, and the conditions that characterize an equilibrium of the model.

2.2 Asset Demand

Let $\phi_t \in \mathbb{R}_+$ denote the goods value of money in the centralized market in period t , so that the real value of M_t dollars can be written as $m_t \equiv \phi_t M_t$. Let i denote the net nominal interest rate paid on a digital currency by the central bank, which can be either positive or negative. The gross real rate of return on physical currency is then ϕ_{t+1}/ϕ_t and on digital currency is $(1+i)\phi_{t+1}/\phi_t$. Let $1+r_t$ denote the gross real interest rate on bank deposits. Finally, let $\mathbf{a} \equiv (m, d, e)$ denote an asset portfolio consisting of $m \in \mathbb{R}_+$ units of physical real money balances, $d \in \mathbb{R}_+$ units of bank deposits, and $e \in \mathbb{R}_+$ units of digital (or "electronic") real money balances, all measured in current CM consumption goods.

Bellman equations. Let $J_s(\mathbf{a}, t)$ denote the value function for a buyer entering the centralized market in period t holding portfolio \mathbf{a} . The index $s \in \{1, 2\}$ indicates what type of seller she will potentially meet in the next decentralized market. Let $V_s(\mathbf{a}', t)$ denote the value function of this same buyer when she arrives in the decentralized market with portfolio \mathbf{a}' . Using these two functions, we can write the Bellman equation for this buyer as

$$J_s(\mathbf{a}, t) = \max_{(x^b, \mathbf{a}') \in \mathbb{R} \times \mathbb{R}_+^3} [x^b + V_s(\mathbf{a}', t)],$$

where the maximization is subject to the budget constraint

$$x^b + \mathbf{p} \cdot \mathbf{a}' = \mathbf{R}_{t-1} \cdot \mathbf{a} + \tau_t.$$

The variable x^b is the buyer's net consumption of the CM good, which can be positive or negative. The price vector $\mathbf{p} \equiv (1, 1, 1)$ measures the cost of acquiring real money balances and deposits in terms of CM goods, while the vector

$$\mathbf{R}_{t-1} \equiv \left(\frac{\phi_t}{\phi_{t-1}}, 1 + r_{t-1}, (1+i) \frac{\phi_t}{\phi_{t-1}} \right)$$

measures the gross real returns on assets carried over from the previous period. Finally, τ_t

denotes the real value of any lump-sum transfer received by the agent.

The value function $V_s(\mathbf{a}', t)$ satisfies

$$V_s(\mathbf{a}', t) = \alpha [u(q_s(\mathbf{a}', t)) + \beta J(\mathbf{a}' - \mathbf{h}_s(\mathbf{a}', t), t + 1)] + (1 - \alpha) \beta J(\mathbf{a}', t + 1), \quad (3)$$

where $q_s(\mathbf{a}', t) \in \mathbb{R}_+$ denotes the buyer's consumption of the DM good and $\mathbf{h}_s(\mathbf{a}', t) \in \mathbb{R}_+^3$ denotes the payment she makes for this consumption out of her asset holdings \mathbf{a}' . The function $J(\mathbf{a}, t)$ in this expression represents the expected value of entering the centralized market before knowing the type of her potential meeting in the following period's decentralized market, that is,

$$J(\mathbf{a}, t) \equiv \lambda_1 J_1(\mathbf{a}, t) + \lambda_2 J_2(\mathbf{a}, t). \quad (4)$$

Bargaining. Throughout the analysis, we assume that the terms of decentralized trade are determined by Nash bargaining. For simplicity, we restrict attention to the case where the buyer has all the bargaining power. The bargaining problem can then be described as

$$\max_{(q_s, \mathbf{h}_s) \in \mathbb{R}_+^4} [u(q_s) - \beta \times \mathbf{R}_t \cdot \mathbf{h}_s]$$

subject to the seller's participation constraint

$$-w(q_s) + \beta \times \mathbf{R}_t \cdot \mathbf{h}_s \geq 0$$

and the liquidity constraint

$$\mathbf{h}_s \leq \mathbf{f}_s(\mathbf{a}). \quad (5)$$

The function \mathbf{f}_s enforces the fact that the buyer will only pay with assets her trading partner can verify. If, for example, type 1 sellers can only verify physical currency, we have $\mathbf{f}_1(\mathbf{a}) = (m, 0, 0)$. If these sellers can instead verify both physical and digital currency, we have $\mathbf{f}_1(\mathbf{a}) = (m, 0, e)$. In the sections that follow, we impose particular functions \mathbf{f}_s to capture different potential digital currency designs. For now, we only impose that type 1 sellers can verify physical currency but not bank deposits and that the reverse holds for type 2 sellers.

The solution to this bargaining problem implies the following schedule for DM output

$$q_s(\mathbf{a}, t) = \begin{cases} w^{-1}(\beta \mathbf{R}_t \cdot \mathbf{f}_s(\mathbf{a})) & \text{if } \mathbf{R}_t \cdot \mathbf{f}_s(\mathbf{a}) < \frac{w(q^*)}{\beta} \\ q^* & \text{otherwise} \end{cases} \quad (6)$$

and for payments

$$\mathbf{R}_t \cdot \mathbf{h}_s(\mathbf{a}, t) = \begin{cases} \mathbf{R}_t \cdot \mathbf{f}_s(\mathbf{a}) & \text{if } \mathbf{R}_t \cdot \mathbf{f}_s(\mathbf{a}) < \frac{w(q^*)}{\beta} \\ \frac{w(q^*)}{\beta} & \text{otherwise,} \end{cases}$$

where q^* is the surplus-maximizing quantity satisfying $u'(q^*) = w'(q^*)$. In other words, if the value of the buyer's spendable assets is large enough to induce the seller to produce q^* , the efficient level of trade occurs. If not, the buyer spends all that she can and the seller produces an amount smaller than q^* .

First-order conditions. Using this solution to the bargaining problem, a buyer's portfolio problem in the centralized market can be written as

$$\max_{\mathbf{a}' \in \mathbb{R}_+^3} \{-\mathbf{p} \cdot \mathbf{a}' + \alpha [u(q_s(\mathbf{a}', t)) - \beta \mathbf{R}_t \cdot \mathbf{h}_s(\mathbf{a}', t)] + \beta \mathbf{R}_t \cdot \mathbf{a}'\}. \quad (7)$$

Recall that the buyer knows the type of seller she will potentially meet in the next DM when making this portfolio choice in the CM. The slope of the objective function with respect to a given asset depends not only on whether the seller accepts the asset but also on whether the buyer is liquidity constrained. Define the function $L : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ by

$$L(A) = \begin{cases} \alpha \frac{u'(w^{-1}(\beta A))}{w'(w^{-1}(\beta A))} + 1 - \alpha & \text{if } \beta A \leq w(q^*) \\ 1 & \text{otherwise.} \end{cases} \quad (8)$$

This function measures the expected benefit of holding an extra unit of spendable assets. If the buyer's current spendable assets are insufficient to purchase the efficient quantity q^* , the increase will allow her to consume more if she is matched in the DM, which occurs with probability α . If she is not matched, or if she already has enough spendable assets to purchase q^* , she merely holds the extra unit of assets until the following CM.

Using this function, the first-order condition for the real physical currency balances of a buyer who will potentially be in a type 1 match can be written as

$$L(\mathbf{R}_t \cdot \mathbf{f}_1(\mathbf{a}')) \leq \frac{\phi_t}{\beta \phi_{t+1}}, \quad (9)$$

with equality if $m' > 0$. The first-order condition for the deposits of a buyer who will potentially be in a type 2 match is

$$L(\mathbf{R}_t \cdot \mathbf{f}_2(\mathbf{a}')) \leq \frac{1}{\beta(1+r_t)}, \quad (10)$$

with equality if $d' > 0$. In addition, only buyers potentially entering type 1 meetings will hold physical currency and only buyers potentially entering type 2 meetings will hold bank deposits.¹² If type s sellers accept digital currency, the first-order condition for real digital currency balances of a buyer who will potentially be in a type s match is

$$L(\mathbf{R}_t \cdot \mathbf{f}_s(\mathbf{a}')) \leq \frac{\phi_t}{\beta(1+i)\phi_{t+1}}, \quad (11)$$

with equality if $e' > 0$. Equations (9) – (11) thus characterize the demand for each asset in the period- t CM.

2.3 Asset Supply

Deposits. To derive the supply of deposits, consider a banker born in period t with a project that returns $\gamma \in [0, \bar{\gamma}]$. Given a market interest rate r_t , this banker is willing to issue a deposit claim if

$$\gamma - (1 + r_t) \geq 0.$$

However, the promised repayment on this claim cannot exceed the value of the banker's pledgeable future income

$$1 + r_t \leq \theta\gamma. \quad (12)$$

Note that if $\theta < 1$ holds, this constraint is strictly tighter than the previous one, meaning that some bankers with projects that are profitable at the market interest rate will not be able to raise funds and invest.

Let $\hat{\gamma}_t \in \mathbb{R}_+$ denote the banker whose project's payoff satisfies the pledgeability restriction with equality in period t , that is,

$$\hat{\gamma}_t = \frac{1 + r_t}{\theta}. \quad (13)$$

The aggregate supply of deposits then equals the measure of bankers with project returns of at least $\hat{\gamma}$, which equals $\eta(\bar{\gamma} - \hat{\gamma})$, or

$$\eta \left(\bar{\gamma} - \frac{1 + r_t}{\theta} \right). \quad (14)$$

Note that, for any θ , a reduction in the interest rate leads to an increase in investment by allowing a larger number of bankers to issue debt claims. In other words, a lower interest

¹²These decisions reflect agents' strict preferences when the real return on an asset is less than β^{-1} . All buyers and sellers are indifferent about holding an asset whose real return equals β^{-1} . In this case, we simplify our notation by assuming, without any loss of generality, that only buyers potentially entering a meeting where an asset is accepted will hold that asset.

rate will lead to an expansion of the banking system and an increased supply of deposits.

Currency. The supply of both physical and digital currency is set by the central bank following a price-level target rule. We assume the target grows at a constant gross rate $\mu > \beta$, so that

$$\frac{\phi_t}{\phi_{t+1}} = \mu \quad \text{for all } t. \quad (15)$$

The central bank stands ready to exchange units of both physical and digital currency for CM goods at the desired price level each period.¹³ Letting $\bar{M}_t \in \mathbb{R}_+$ denote the supply of physical currency and \bar{E}_t the supply of digital currency, the central bank's budget constraint is

$$\phi_t (\bar{M}_t + \bar{E}_t) = \phi_t (\bar{M}_{t-1} + (1+i)\bar{E}_{t-1}) + \tau_t,$$

where the lump-sum tax/transfer τ_t is set to balance the budget each period.

2.4 Market Clearing

Because physical currency is only used in type 1 meetings, we can write its market-clearing equation as

$$\lambda_1 m_t = \phi_t \bar{M}_t. \quad (16)$$

Similarly, the fact that bank deposits are only exchanged in type 2 meetings allows us to write the market-clearing equation for the deposit market as

$$\lambda_2 d_t = \eta \left(\bar{\gamma} - \frac{1+r_t}{\theta} \right). \quad (17)$$

Market clearing for digital currency requires

$$\lambda_1 e_{1,t} + \lambda_2 e_{2,t} = \phi_t \bar{E}_t, \quad (18)$$

recognizing that $e_{s,t}$ will be zero whenever type s sellers do not accept digital currency. An equilibrium of the model consists of sequences of prices $\{r_t, \phi_t\}$, portfolio holdings $\{\mathbf{a}_{1t}, \mathbf{a}_{2t}\}$, and an allocation $\{q_{1t}, q_{2t}, \hat{\gamma}_t\}$ satisfying equations (9)-(13) and (15)-(18).

In the next section, we derive the properties of equilibrium in a benchmark model with no digital currency. We then introduce different types of digital currency in Sections 4 and 5, analyzing the resulting equilibrium allocations and welfare.

¹³ We could instead take the more standard approach of assuming that the total money supply grows at a constant rate μ . With both physical and digital currency, however, the relative supply of each type of currency is endogenous and the notation becomes more complex. Given that we focus on stationary allocations where money is valued, the simpler approach we take here is without any loss of generality.

3 Equilibrium with No Digital Currency

When there is no digital currency, the functions \mathbf{f}_s in the buyer's liquidity constraint in equation (5) are given by

$$\mathbf{f}_1(\mathbf{a}) = (m, 0, 0) \quad \text{and} \quad \mathbf{f}_2(\mathbf{a}) = (0, d, 0).$$

A buyer can only use her physical currency balances in a type 1 meeting and her bank deposits in a type 2 meeting. The Inada condition on buyers' utility function then implies that the first-order conditions (9) and (10) for buyers' portfolio choices will hold with equality. Combining these equations with the market-clearing conditions (16) and (17) yields

$$L\left(\frac{m_t}{\mu}\right) = \frac{\mu}{\beta} \tag{19}$$

and

$$L\left((1+r_t)\frac{\eta}{\lambda_2}\left(\bar{\gamma} - \frac{1+r_t}{\theta}\right)\right) = \frac{1}{\beta(1+r_t)}. \tag{20}$$

The fact that only period- t variables appear in each of these two equations shows that an equilibrium in our model is necessarily stationary. The equations also demonstrate a dichotomy between the money and deposit markets in our baseline model. Given the inflation rate $\mu > \beta$, equation (19) pins down real money balances independent of the interest rate on deposits. Meanwhile, equation (20) determines the equilibrium interest rate on deposits independent of the inflation rate. We think of our model as capturing long-run phenomena, in which case it is not unreasonable to think that standard monetary policy has a limited effect on real interest rates and the level of investment. Notice that inflation is not neutral, however. A higher inflation target leads to lower production and consumption in type 1 DM meetings and to lower welfare, as is standard in models of monetary exchange.

To guarantee the existence and uniqueness of equilibrium, we assume preferences are such that:

$$(i) \ AL(A) \text{ is strictly increasing} \quad \text{and} \quad (ii) \ \lim_{(1+r) \rightarrow 0} \frac{L^{-1}\left(\frac{1}{\beta(1+r)}\right)}{1+r} < \eta\bar{\gamma}.$$

The first assumption ensures that the demand for deposits is strictly increasing in the interest rate, while the second guarantees that the supply of deposits is large enough to meet the demand at some interest rate.¹⁴ The following proposition establishes the existence and uniqueness of equilibrium in the benchmark economy. Proofs of all propositions are contained

¹⁴If u is of the CRRA form and w is linear, for example, these assumptions are satisfied whenever the coefficient of relative risk aversion is less than one.

in the Appendix.

Proposition 1. *The economy with no digital currency has a unique equilibrium. There is a liquidity premium on deposits in this equilibrium if and only if $\eta < \frac{\lambda_2 w(q^*)}{\bar{\gamma} - \frac{1}{\beta\theta}}$. When this condition holds, the equilibrium interest rate is a strictly increasing function of η .*

When bankers' income is fully pledgeable ($\theta = 1$), the results for our benchmark model follow those in Lagos and Rocheteau (2008) closely. When η is sufficiently large, productive projects are plentiful and there is no liquidity premium on deposits, that is, $1 + r^N = \beta^{-1}$. In this case, an investment project is funded if and only if it returns at least β^{-1} , and production in type 2 DM meetings equals the surplus-maximizing quantity q^* . When η is smaller, productive projects are scarce and a liquidity premium emerges on deposits: $1 + r^N < \beta^{-1}$. This liquidity premium leads to overinvestment in the sense that some projects with a return lower than β^{-1} are funded, which decreases the total welfare derived from CM consumption. In addition, the quantity produced in type 2 DM meetings falls below q^* . An increase in η in this region leads to a larger supply of deposits at any interest rate, which decreases the equilibrium liquidity premium and moves the quantities of CM investment and of DM production in type 2 meetings toward their efficient levels.¹⁵

In the presence of credit market frictions ($\theta < 1$), the relationship between the liquidity premium on deposits and the efficiency of equilibrium investment changes, and our approach offers new insights. It remains true that when η is sufficiently large, there is no liquidity premium on deposits and production in type 2 DM meetings is at the efficient level. However, some bankers with socially productive projects no longer have enough pledgeable income to credibly repay their deposits. As a result, the equilibrium investment cutoff is *higher* than β^{-1} and the quantity of CM investment is inefficiently low. When η is smaller and high-return projects are scarce, a liquidity premium again emerges on deposits as $1 + r^N$ falls below β^{-1} . This lower interest rate now increases investment *toward* the efficient level. In other words, when $\theta < 1$, a liquidity premium on deposits can partially offset the effects of the credit market friction and thereby increase the total welfare derived from CM consumption. At the same time, however, the quantity of the DM good produced in type 2 meetings falls below the surplus-maximizing quantity q^* .

The result that CM investment can be inefficiently low depends on a combination of two assumptions: bankers face the pledgeability constraint (12) and have limited funds of their own. In Williamson (2021), bankers face a similar credit constraint but can work without limit when young and invest the proceeds in their bank. Investment will never be inefficiently

¹⁵ For studies of how liquidity premia affect the level of investment and macroeconomic outcomes in related environments, see Williamson (2012), Hu and Rocheteau (2013), Rocheteau and Rodriguez-Lopez (2014), Andolfatto et al. (2016), Hu (2021) and Cui et al. (2021), among others.

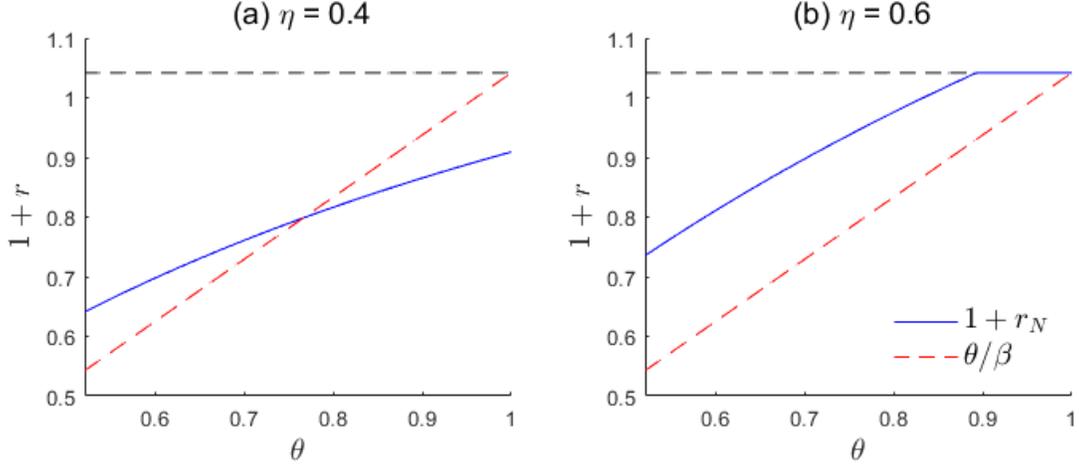


Figure 1: Equilibrium interest rate on deposits

low in that case, since any project that returns at least β^{-1} would be operated with internal funds if the banker cannot borrow more cheaply. Our assumption that bankers have zero income when young simplifies the notation, but similar results would obtain as long as this income is sufficiently limited. When we introduce a digital currency, the fact that investment may be inefficiently low creates the possibility that crowding out bank deposits is socially costly.

The following example illustrates the equilibrium outcome in our benchmark model and how this outcome varies with the pledgeability parameter θ .

Example 1. Suppose $u(q) = 2\sqrt{q}$, $w(q) = q$, and $\alpha = 1$. Then the unique solutions to equations (19) and (20) are

$$m^N = \frac{\beta}{\mu} \quad \text{and} \quad 1 + r^N = \min \left\{ \frac{\theta\eta\bar{\gamma}}{\theta\beta\lambda_2 + \eta}, \frac{1}{\beta} \right\}.$$

The equilibrium quantities of DM trade are

$$q_1^N = \frac{\beta^2}{\mu^2} \quad \text{and} \quad q_2^N = \min \left\{ \left(\frac{\beta\theta\eta\bar{\gamma}}{\theta\beta\lambda_2 + \eta} \right)^2, 1 \right\},$$

and the equilibrium investment cutoff is

$$\hat{\gamma}^N = \min \left\{ \frac{\eta\bar{\gamma}}{\theta\beta\lambda_2 + \eta}, \frac{1}{\beta\theta} \right\}.$$

Figure 1 depicts the equilibrium interest rate on deposits as a function of the pledgeability

parameter for two different values of η .¹⁶ Panel (a) corresponds to the case where η is small enough that there is a liquidity premium when $\theta = 1$, while panel (b) lies in the opposite case. The equilibrium interest rate is an increasing, concave function of θ in both cases, strictly so whenever $1 + r^N < \beta^{-1}$. The dashed line in each panel corresponds to $1 + r = \theta/\beta$, the interest rate at which the liquidity premium on deposits exactly offsets the effect of the pledgeability constraint, placing the equilibrium investment cutoff $\hat{\gamma}$ at the efficient value β^{-1} . When the liquidity premium is large enough that $1 + r^N$ lies below the dashed line, the investment cutoff is below β^{-1} and the equilibrium is characterized by overinvestment. As the figure shows, overinvestment will occur whenever both (i) η is small enough that there is a liquidity premium when $\theta = 1$ and (ii) θ is sufficiently close to 1. When $1 + r^N$ lies above the dashed line, in contrast, the equilibrium investment cutoff is above β^{-1} and the level of CM investment is inefficiently low. The figure shows that underinvestment always occurs in our model when η is sufficiently large (as in panel (b)) and $\theta < 1$, as well as when η is smaller (as in panel (a)) and θ is sufficiently small. For the example presented above, underinvestment occurs in equilibrium if

$$\theta < \min \left\{ \frac{\eta(\beta\bar{\gamma} - 1)}{\beta\lambda_2}, 1 \right\}.$$

It bears emphasizing that, while a liquidity premium may improve the efficiency of equilibrium investment in our model, it still reduces the quantity of the DM good produced in type 2 meetings below the surplus-maximizing value q^* . This tradeoff between the efficiency of DM exchange and the quantity of CM investment will be central to understanding the macroeconomic effects of introducing a digital currency in the sections that follow.

4 Targeted Digital Currencies

In this section, we assume it is technologically possible for the central bank to design digital currencies that can only be used in place of a single existing means of payment. We begin by studying a cash-like CBDC, which can be verified by type 1 sellers but not by type 2 sellers. We then turn to a deposit-like CBDC, which has the opposite features. In each case, we ask whether the digital currency is *desirable* in the sense that it increases welfare when the interest rate is chosen optimally. We show that a cash-like digital currency is desirable if and only if the welfare weight ν on type 1 DM consumption is sufficiently high and that the optimal interest rate corresponds to a modified Friedman rule. Optimal policy for a deposit-like CBDC is more complex because it must balance the desire to facilitate

¹⁶ The figure uses $\beta = 0.96$, $\lambda_1 = \lambda_2 = 0.5$, $\bar{\gamma} = 2$ and $\nu = 1$.

DM exchange with concerns about disintermediating banks and decreasing CM investment. Nevertheless, we show that a deposit-like CBDC tends to be desirable when productive projects are in scarce supply and when financial frictions are moderate. We then study the relationship between the two optimal interest rates in a dual-CBDC system, where both targeted CBDCs are issued.

4.1 A Cash-Like Digital Currency

A cash-like digital currency is one that can easily substitute for physical currency in transactions but not for bank deposits. These assumptions represent a design that aims to mimic both the features and the limitations of physical currency. Such a design may preserve users' privacy, for example, and allow transfer of funds without network connectivity. It may also minimize the fees and other costs associated with its use, particularly for small transactions. At the same time, the design may impose caps on balances and on transaction size that make digital currency impractical to use in large-value transactions. Balances may also be stored on a smart card or other device that must be physically present to transfer funds. In the context of our model, we capture this type of design by assuming that a cash-like CBDC can be verified by – and only by – type 1 sellers.

Equilibrium. With a cash-like digital currency, the functions \mathbf{f}_s in the buyer's liquidity constraint in equation (5) become

$$\mathbf{f}_1(\mathbf{a}) = (m, 0, e) \quad \text{and} \quad \mathbf{f}_2(\mathbf{a}) = (0, d, 0).$$

Buyers in a type 1 meeting can use their balances of physical and/or digital currency to make purchases, while buyers in a type 2 meeting can only use bank deposits. Comparing the first-order conditions for physical and digital currency in equations (9) and (11) shows that a buyer entering a type 1 meeting will only hold whichever currency offers the higher return. If the digital currency were to pay a negative interest rate, demand would be zero. If $i = 0$, type 1 buyers are indifferent between the two currencies and the equilibrium paths of $\{m_t\}$ and $\{e_{1t}\}$ are indeterminate, but total real money balances and all other equilibrium quantities are unchanged. Therefore, introducing a cash-like digital currency will only affect equilibrium consumption and welfare if it carries a positive interest rate.¹⁷

When $i > 0$, the first-order condition in equation (11) will hold with equality and the

¹⁷In practice, a cash-like digital currency may attract users without paying interest if it is more convenient, safer to use, or offers a loss-recovery mechanism. We interpret the interest rate i in our model as capturing these non-pecuniary benefits as well as explicit interest payments.

equilibrium quantity of real currency balances e_{1t} will satisfy

$$L\left(\frac{(1+i)}{\mu}e_{1t}\right) = \frac{\mu}{(1+i)\beta}.$$

Comparing this expression with equation (19) shows that $i > 0$ implies buyers entering a type 1 meeting will hold larger real currency balances when the digital currency is introduced. The solution to the bargaining problem in equation (6) then shows that the quantity of DM production in type 1 meeting also increases. Combining the different cases, we can express this quantity as

$$q_1^C(1+i) \equiv w^{-1}\left(\beta L^{-1}\left(\frac{\mu}{\beta \max\{1+i, 1\}}\right)\right). \quad (21)$$

Meanwhile, the dichotomy discussed above implies that the equilibrium quantities of deposits, CM investment, and DM production in type 2 meetings are unchanged. The following proposition summarizes these results, using a superscript C to denote equilibrium values with a cash-like digital currency.

Proposition 2. *Under a cash-like digital currency with $i > 0$, the unique equilibrium allocation satisfies $e_1^C > m^N$, $q_1^C > q_1^N$ and $(q_2^C, \hat{\gamma}^C) = (q_2^N, \hat{\gamma}^N)$.*

Optimal policy. The potential benefit of introducing a cash-like CBDC in our framework is that it allows the policy maker to increase the rate of return on currency while maintaining the same target for the price level and inflation.¹⁸ The policy maker's desired rate of return on currency depends critically on the parameter ν , which measures the fraction of type 1 DM meetings that result in socially valuable consumption. The following proposition shows that a cash-like digital currency raises welfare under the optimal policy if and only if ν is sufficiently high.

Proposition 3. *There exists $\bar{\nu} \in (0, 1)$ such that a cash-like digital currency is desirable if and only if $\nu > \bar{\nu}$. In this case, the optimal policy is given by*

$$1+i^C \equiv \frac{\mu}{\beta} \frac{\nu}{\alpha + (1-\alpha)\nu}. \quad (22)$$

When ν is large, the policy maker wants cash buyers to have access to a better means of payment, which we interpret as a desire to promote *financial inclusion*. When ν is small,

¹⁸ We think of this target as being determined by considerations outside the scope of our model, such as stabilization policy and the zero lower bound. See Andrade et al. (2019) for a recent discussion. If the policy maker could freely choose μ in our model, introducing a cash-like digital currency would have no effect, in line with the results in Williamson (2021). However, the ability to use digital currency in type 1 meetings would still have implications for the desirability of a universal CBDC in Section 5 below.

in contrast, concerns about facilitating illicit activities make a cash-like digital currency undesirable. The interest rate in equation (22) can be thought of as a modified version of the Friedman rule that optimally balances these two concerns. If a cash-like CBDC is not desirable, we normalize the optimal policy to $i^C = 0$.

4.2 A Deposit-Like Digital Currency

We next consider a deposit-like CBDC, which can be verified only by type 2 sellers. This assumption represents a CBDC design that shares the features and limitations of bank deposits as a medium of exchange. For example, individuals might hold accounts with the central bank, either directly or through an intermediary, and make payments using a debit card. However the balances are held, a design in which payments are processed over an existing bank-based network and have a similar fee structure would tend to be deposit-like.

Equilibrium. With a deposit-like digital currency, the functions \mathbf{f}_s in the buyer's liquidity constraint in equation (5) become

$$\mathbf{f}_1(\mathbf{a}) = (m, 0, 0) \quad \text{and} \quad \mathbf{f}_2(\mathbf{a}) = (0, d, e).$$

Buyers in a type 1 meeting can only use their physical currency balances to make purchases, while buyers in a type 2 meeting can use their bank deposits and/or digital currency balances. Comparing the first-order conditions for bank deposits and digital currency in equations (10) and (11) shows that a type 2 buyer would only choose to hold whichever asset offers the higher return. If the nominal interest rate on the digital currency, $1 + i$, is set below the equilibrium nominal interest rate on deposits in the benchmark model, $\mu(1 + r^N)$, the demand for the digital currency will be zero and it will have no effect on the equilibrium allocation. If these two interest rates are equal, type 2 buyers would be indifferent between deposits and digital currency. However, the first-order condition for deposits (10) together with the market-clearing condition (17) would imply that equilibrium digital currency holdings must again be zero. A deposit-like digital currency will only be held in equilibrium if the interest rate is set higher than the nominal interest rate on deposits in the benchmark model, that is,

$$1 + i > \mu(1 + r^N). \tag{23}$$

Note that this condition can be satisfied only if there is a liquidity premium on deposits in the benchmark model, that is, if $1 + r^N < \beta^{-1}$. In other words, the assets that back bank deposits must be in scarce supply for a deposit-like digital currency to play a role in our model.

Unlike in the previous section, a digital currency will not completely replace deposits as a medium of exchange when condition (23) holds. As some type 2 buyers switch to holding digital currency, the decline in deposit demand will cause the interest rate on deposits to increase. In equilibrium, the interest rate on deposits must rise until it equals that offered by the digital currency,

$$1 + i = \mu (1 + r^D), \quad (24)$$

where the superscript D denotes equilibrium values under a deposit-like regime. Because individual type 2 buyers are indifferent between the two assets in equilibrium, the first-order conditions (10) and (11) will both hold with equality and a type 2 buyer's total holding of spendable assets will satisfy

$$L \left((1 + r^D) \left[\frac{\eta}{\lambda_2} \left(\bar{\gamma} - \frac{1 + r^D}{\theta} \right) + e_2^D \right] \right) = \frac{1}{\beta(1 + r^D)}. \quad (25)$$

The higher interest rate on deposits, $r^D > r^N$, implies through equations (13) and (17) that the level of CM investment and the quantity of deposits issued by banks both decrease. In other words, a deposit-like digital currency disintermediates banks to some degree and crowds out bank-financed investment. At the same time, however, equation (25) shows that the higher rate of return leads type 2 buyers to hold larger total spendable assets, $e_2 + d$. These larger asset holdings translate, through the bargaining solution in equation (6), into larger production of the type 2 good in DM meetings. Finally, because the digital currency cannot be used in type 1 DM meetings, the demand for physical currency and the quantity produced in type 1 meetings are unchanged from the benchmark case. The following proposition summarizes these results.

Proposition 4. *With a deposit-like digital currency satisfying condition (23), the unique equilibrium allocation satisfies $e_2^D + d^D > d^N > d^D$, $(r^D, \hat{\gamma}^D, q_2^D) \gg (r^N, \hat{\gamma}^N, q_2^N)$, and $q_1^D = q_1^N$.*

Optimal policy. The results in Proposition 4 point to a tradeoff the central bank faces when setting the interest rate on a deposit-like digital currency. Raising this interest rate increases DM output in type 2 meetings and promotes efficient exchange, but decreases CM investment. The optimal policy balances these competing concerns. Using the stationarity of the equilibrium allocation and omitting the terms that are unaffected by a deposit-like digital currency, we can write the objective in equation (1) as

$$W^D (1 + i) \equiv \eta \int_{\frac{1+i}{\mu\theta}}^{\bar{\gamma}} (\beta\gamma - 1) d\gamma + \alpha\lambda_2 [u(q_2^D(1+i)) - w(q_2^D(1+i))] \quad (26)$$

where

$$q_2^D(1+i) \equiv w^{-1} \left(\beta L^{-1} \left(\frac{\mu}{\beta \max\{1+i, \mu(1+r^D)\}} \right) \right). \quad (27)$$

Without loss of generality, we can eliminate the max term by restricting the policy maker's choice set to $1+i \in [\mu(1+r^N), \mu/\beta]$. Using the definition of the cutoff productivity $\hat{\gamma}$ in equation (13), the slope of the objective function with respect to the policy choice can then be written as

$$\frac{dW^D}{d(1+i)} = - \left(\frac{\eta}{\mu\theta} \right) (\beta\hat{\gamma} - 1) + \alpha\lambda_2 [u'(q_2^D) - w'(q_2^D)] \frac{dq_2^D}{d(1+i)} \quad (28)$$

The first term in this expression measures the cost of a higher interest rate, which comes from disintermediating banks. A marginal increase in the interest rate on digital currency raises the equilibrium interest rate on deposits by μ^{-1} , as shown in equation (24). A marginal increase in the deposit rate, in turn, increases the productivity threshold for obtaining funding, $\hat{\gamma}$, by θ^{-1} , as shown in equation (13). Given that there is a measure η of bankers with each productivity level, the total measure of projects that lose funding due to a marginal increase in $1+i$ is thus $\eta/(\mu\theta)$. This quantity is multiplied by the social value of the marginal project, which produces $\hat{\gamma}$ units of CM output in the next period using one unit of CM input this period. The second term in equation (28) measures benefit of a higher interest rate from increased production and exchange of the DM good in type 2 meetings. The higher real return on deposits and digital currency leads buyers to hold larger real balances of spendable assets, which increases the quantity q_2^D produced and thereby increases the gains from trade when a buyer and a type 2 seller meet.

The following proposition shows that a deposit-like digital currency is desirable if productive projects are sufficiently scarce. It also shows that, in these cases, the solution to the optimal policy problem is interior when credit market frictions are present and is thus characterized by equality of the marginal cost and marginal benefit in equation (28). In the absence of credit market frictions, the optimal policy corresponds to the Friedman rule.

Proposition 5. *There exists $\bar{\eta} > 0$ such that $\eta < \bar{\eta}$ implies a deposit-like digital currency is desirable. The optimal policy satisfies $1+i \in [\mu(1+r^N), \mu/\beta]$ if $\theta < 1$ and $1+i = \mu/\beta$ if $\theta = 1$.*

One case where it is easy to see that a deposit-like digital currency can raise welfare is when the equilibrium without a CBDC exhibits overinvestment. If $1+r^N < \theta/\beta$ holds, the marginal project being funded in the economy without CBDC returns less than β^{-1} . Introducing a CBDC that bears a slightly higher real return will crowd out these inefficient projects while at the same time increasing the quantity produced in type 2 DM meetings

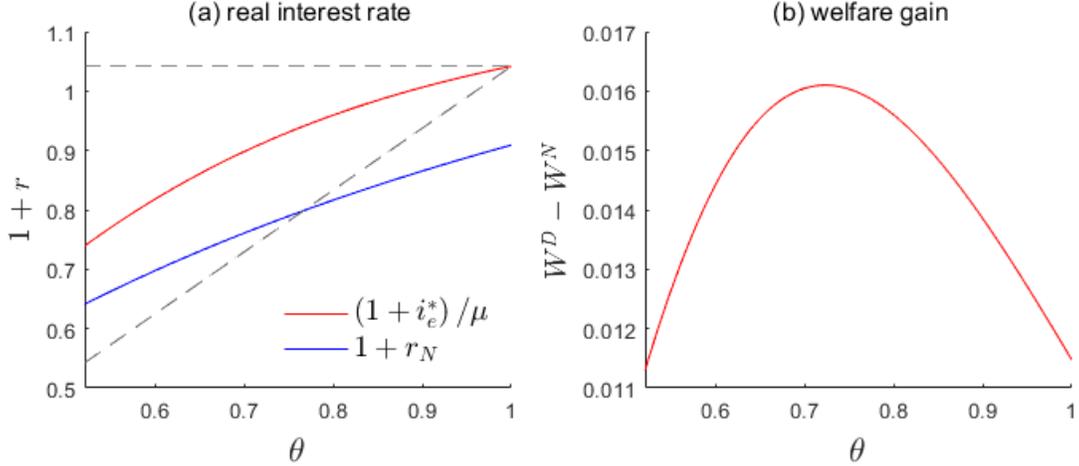


Figure 2: Optimal policy with a deposit-like CBDC

toward the efficient level. We record this result in the following corollary.

Corollary 1. *If $1 + r^N < \frac{\theta}{\beta}$, a deposit-like digital currency is desirable.*

Figure 2 depicts the optimal interest rate on a deposit-like CBDC and the associated welfare gain, using the same parameter values as panel (a) of Figure 1. Panel (a) in Figure 2 shows how, for this example, the optimal interest rate on a deposit-like digital currency is strictly higher than the nominal interest rate on deposits in the baseline economy for all values of θ . Moreover, the optimal interest rate converges to μ/β as θ approaches 1. Panel (b) shows that the welfare gain from introducing a digital currency is largest for intermediate values of the credit friction θ . Two competing forces are at work. On one hand, as θ increases, the costs associated with crowding out CM investment become smaller. In particular, the measure of projects that are crowded out by a marginal increase in the deposit rate and the productivity of the marginal project are both proportional to θ^{-1} . As θ increases, therefore, the disadvantages of introducing a CBDC become smaller and the policy maker also becomes willing to set a higher interest rate. On the other hand, however, an increase in θ decreases the liquidity premium in the baseline economy, which lowers the benefit of a CBDC in promoting DM production and exchange. In the example depicted here, the first effect dominates for lower values of θ , while the second effect dominates for higher values. As a result, the welfare gain of introducing a CBDC is largest for intermediate values of θ .

4.3 Discussion

Costly disintermediation. Whenever there is a liquidity premium on deposits, introducing a deposit-like CBDC can improve the efficiency of DM production and exchange by inducing type 2 buyers to hold larger total real money balances. If there were no interaction with CM investment, such a digital currency would always be desirable. The tradeoff in our model comes from the combination of CBDC crowding out bank deposits *and* inefficiently low investment due to the pledgeability constraint. This tradeoff distinguishes our approach from that in Williamson (2021), where a liquidity premium necessarily leads to overinvestment. Disintermediation in his model always raises welfare by moving the capital stock closer to the golden rule.¹⁹ Our model, in contrast, speaks directly to policy makers' concerns about disintermediation being socially costly. Our results outline the conditions under which a deposit-like CBDC is desirable despite these costs.

Market power. When banks have market power in the deposit market, the benefits of introducing a deposit-like CBDC may be larger than in our competitive framework. Andolfatto (2021) constructs a model with overlapping generations of households in which imperfectly competitive banks hold a portfolio of reserves and loans to firms. The introduction of a CBDC in his setting raises the interest rate on bank deposits in much the same way as in our model. However, this change has no effect on the loan rate or on bank lending in his model; instead, it simply decreases bank profits. Chiu et al. (2021) introduce Cournot competition for deposits into a modified version of the model we study here. They show that two distinct regimes arise. If the CBDC is only moderately attractive to households, there may be little or no use of it in equilibrium. Nevertheless, the availability of this outside option leads to both a higher interest rate on deposits and a larger quantity of deposits. In this region, their results are similar in spirit to those in Andolfatto (2021). If the interest rate on the digital currency is increased further, however, households begin to shift funds out of bank deposits and into the digital currency, which causes a decline in deposits and bank-funded investment, as in our model. To the extent that these effects are important, our competitive framework can be thought of as providing an upper bound on the costs of disintermediation and, therefore, a lower bound on the net benefits of introducing a deposit-like CBDC. Moreover, the policy tradeoff at the heart of our analysis is likely to appear in some form across different market structures whenever credit market frictions create the possibility that investment may be inefficiently low.

¹⁹In a related work, Hu (2021) constructs a model with a pledgeability constraint on banks and a role for currency in some anonymous meetings to study the implementation of optimal policy through the interest rate on excess reserves. He also finds that overinvestment necessarily occurs when a liquidity premium emerges on bank deposits.

Interest on reserves. An alternative way of increasing the stock of liquid assets in the economy would be for the central bank to provide reserves to private banks and rely on those banks to intermediate the reserves into bank deposits. The effectiveness of such a policy in our framework would depend critically on how bank reserves enter the pledgeability constraint in equation (12). If bankers can only pledge a fraction $\theta < 1$ of their matured reserve holdings, those banks with the highest productivities γ_j would be able to create some additional deposits. However, the ability of the banking system to create additional deposits would still be limited, making the policy less effective than issuing a deposit-like CBDC. This result is in line with Williamson (2021), who shows that allowing individuals to directly hold claims on the central bank is more efficient than having individuals hold indirect claims through commercial banks when collateral constraints bind.

If bankers could instead fully pledge their future income from reserve holdings to depositors, this alternative arrangement would be equivalent to a deposit-like CBDC in our framework. Adrian and Mancini-Griffoli (2021) call this approach a *synthetic CBDC*, since it generates the same allocation as a (deposit-like) CBDC but does not require the central bank to deal with retail clients. Proposals for a synthetic CBDC often aim to link depositors' claims as closely as possible to the underlying reserves. For example, creating narrow banks that hold only central bank reserves as assets can be interpreted as a way of increasing the pledgeability parameter θ for reserves. Our results here apply to synthetic CBDC as well as the case where depositors hold direct claims on the central bank.

Fiscal implications. Some observers have expressed concern about the fiscal implications of introducing an interest-bearing CBDC. Replacing physical currency with an interest-bearing digital currency would indeed tend to reduce seigniorage revenue. Such a change could potentially raise political-economy issues that are absent in our framework, including for central bank independence (see, for example, the discussion in Williamson, 2021). It is worth emphasizing, however, that paying interest on a *deposit-like* CBDC does not create a fiscal burden on the public sector. In fact, if the CBDC interest rate is set below μ/β , so that both deposits and CBDC carry a liquidity premium in equilibrium, introducing a deposit-like CBDC creates a net fiscal benefit. In the period a deposit-like CBDC is introduced, the public sector receives an inflow of goods in exchange for the newly issued currency. In our model, these funds are transferred lump-sum to households. In subsequent periods, the public sector taxes these same households to fund the interest payments on a CBDC. Given the quasi-linear specification of preferences, agents value any stream of transfers and taxes using discount factor β . As long as the real interest rate paid on a CBDC is less than β^{-1} , therefore, the present value of all future taxes will be smaller than the transfer received by households in the initial period. One interesting avenue for future research would be

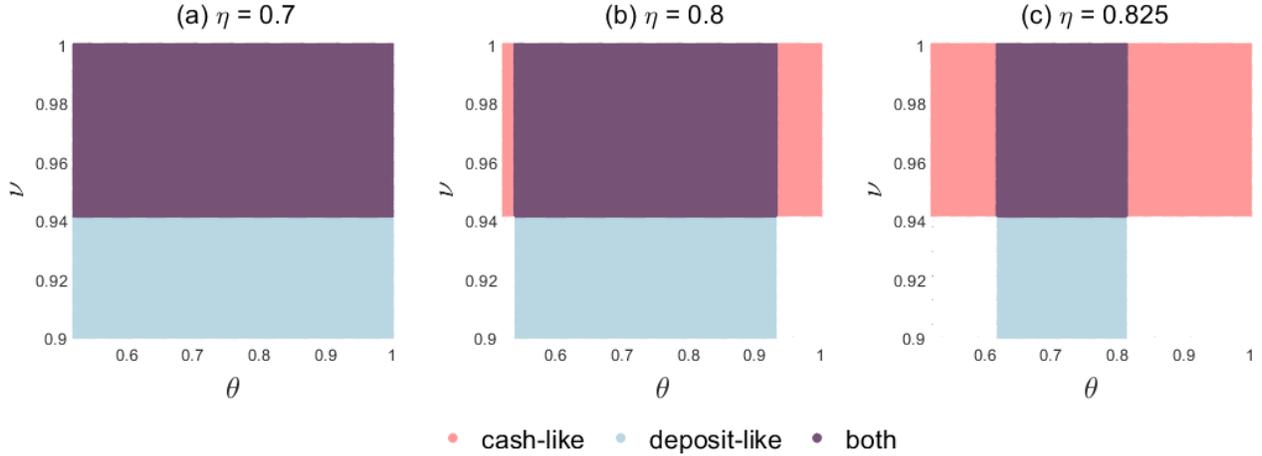


Figure 3: Desirability of targeted CBDCs

to explore possible implications of this benefit for fiscal policy, including the level of public debt, and for the balance of power between the fiscal authority and the central bank.

4.4 Dual CBDCs

Figure 3 shows the combinations of parameter values (θ, ν) under which each type of digital currency is desirable for three different values of the density η of the productivity distribution. In line with Proposition 3, a cash-like digital currency is desirable whenever the welfare weight ν given to the DM consumption of type 1 buyers is above a cutoff value $\bar{\nu}$. This cutoff is independent of both θ and η . A deposit-like CBDC is desirable for all values of the credit friction parameter θ when η is small enough, in line with Proposition 5, but only for intermediate values of θ when η is larger. The figure shows that, depending on parameter values, neither type of CBDC may be desirable, either type alone may be desirable, or both types may be desirable.

In this last case, the central bank does not need to choose between the two types of digital currency; it can issue both simultaneously. We call this approach a *dual-CBDC system*.²⁰ Using Propositions 2 and 4, it is straightforward to show that if we expand our model to allow the central bank to issue both targeted CBDCs at once, the equilibrium allocation will be $(q_1^C, q_2^D, \hat{\gamma}^D)$. In other words, the interest rate on the cash-like CBDC determines DM production in type 1 meetings, while the interest rate on the deposit-like CBDC determines both DM production in type 2 meetings and CM investment. Figure 3 indicates that a dual-CBDC system tends to be optimal when ν is large and θ is in an intermediate range.

²⁰ The possibility of simultaneously issuing two distinct types of CBDC is discussed in European Central Bank (2020, Section 5.2).

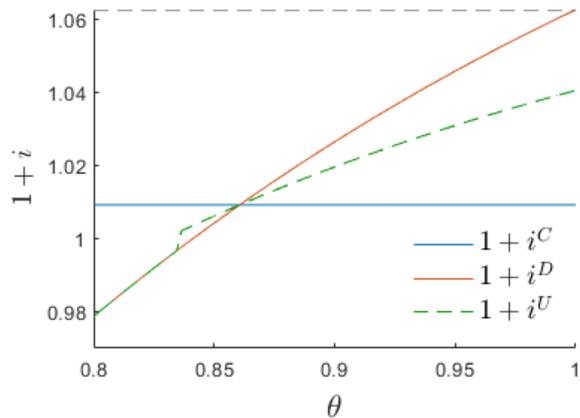


Figure 4: Optimal CBDC interest rates

When a dual-CBDC system is optimal, the interest rates the policy maker sets on the two types of CBDC will typically differ. To illustrate this point, Figure 4 plots the optimal interest rate on a cash-like CBDC (in blue) and on a deposit-like CBDC (in red) using the parameter values from panel (a) of Figure 3. The welfare weight ν on type 1 DM consumption is set to 0.95, which implies an optimal interest rate on a cash-like CBDC of about 1%. When θ is small, the optimal net interest rate on a deposit-like CBDC is negative. In this region, type 2 buyers would prefer to use the cash-like CBDC if possible. Implementing the desired allocation requires the design of the cash-like CBDC to be truly restrictive, meaning it cannot be used as a substitute for bank deposits. When θ is large, the opposite issue arises: the optimal interest rate on the deposit-like CBDC is higher than on the cash-like CBDC. Implementing the desired allocation in this region requires ensuring the cash-like CBDC cannot be used in place of deposits.

A similar issue can arise when the optimal policy involves only one type of CBDC. Consider, for example, parameter combinations in the northwest corner of panel (c) in Figure 3. Because ν is close to 1, the optimal interest rate on a cash-like CBDC is close to the Friedman rule, which is much higher than the equilibrium interest rate on deposits. The policy maker does not want a digital currency used in type 2 meetings in this region because disintermediating banks is too costly. Implementing the desired allocation in this region again requires having a design for the cash-like digital currency that prevents it from being used in type 2 meetings, despite its attractive interest rate.

This discussion raises an important question: Is it truly feasible to design such restricted-use digital currencies? Or would a CBDC necessarily be at least partially substitutable for both bank deposits and physical currency? For example, it may be difficult to prevent a cash-like CBDC from being used in some online or other transactions at a distance that

currently take place using bank deposits. The answer to this question will likely depend on design and technological features that are yet to be determined. If a single digital currency would end up competing with multiple existing payment methods, policy makers would need to take this fact into account when deciding whether to introduce the currency and when setting its interest rate. In the next section, we study the interactions that arise when a CBDC can necessarily be recognized by all sellers and study how these interactions change the optimal policy in our model.

5 A Universal Digital Currency

We now assume it is not technologically feasible to design targeted CBDCs that can be used in only a single type of DM meeting. Instead, a CBDC is necessarily *universal*, meaning that it can be recognized by all sellers. In this case, the central bank is more constrained. It has a single instrument – the interest rate on the universal CBDC – and must consider its effects on both types of DM meetings as well as on CM investment. In this section, we derive the conditions under which this new constraint does and does not bind at the optimal policy. We illustrate how the optimal policy changes when the constraint binds and show that a CBDC is often still desirable even when the central bank cannot restrict its use.

5.1 Equilibrium and Optimal Policy

When a CBDC can be used universally, the buyer's liquidity constraints in equation (5) become

$$\mathbf{f}_1(\mathbf{a}) = (m, 0, e) \quad \text{and} \quad \mathbf{f}_2(\mathbf{a}) = (0, d, e).$$

A type 1 buyer can pay with any combination of physical and/or digital currency, while a type 2 buyer in a type 2 meeting can pay with any combination of digital currency and/or deposits. The equilibrium conditions for this case can be written as

$$\frac{\mu}{\beta \max\{1+i, 1\}} = L \left(\frac{\max\{1+i, 1\}}{\mu} e_{1t} \right), \quad (29)$$

$$\frac{1}{\beta(1+r_t)} = L \left((1+r_t) \left[\frac{\eta}{\lambda_2} \left(\bar{\gamma} - \frac{1+r_t}{\theta} \right) + e_{2t} \right] \right), \quad (30)$$

and

$$1+r_t \geq \frac{(1+i)}{\mu} \quad (31)$$

with equality if $e_{2t} > 0$. As before, a solution to these equations is necessarily stationary.

Given a choice of interest rate $1+i$, the analysis of equilibrium with a universal CBDC is a straightforward extension of the analyses with targeted CBDCs above. If the policy maker sets a negative nominal interest rate on the digital currency, $i < 0$, it will not be held by type 1 buyers. In this region, a universal digital currency will generate the same equilibrium allocation as a deposit-like currency and the results in Proposition 4 apply. If the policy maker sets a positive nominal interest rate, $i > 0$, the digital currency will replace physical currency for type 1 buyers. These are two possible cases in this region. If $1+i \leq \mu(1+r^N)$, the digital currency will not be held by type 2 buyers. In this case, a universal digital currency generates the same equilibrium allocation as a cash-like digital currency and the results in Proposition 2 apply. If, instead, $1+i > \mu(1+r^N)$, the digital currency will also be held by some type 2 buyers. In this case, the outcomes of type 1 meetings are determined by Proposition 2 and both the outcomes of type 2 meetings and CM investment are determined by Proposition 4.

The analysis of optimal policy with a universal digital currency, in contrast, is considerably more complex. The policy maker will choose the nominal interest rate $1+i$ in the interval $[\mu(1+r^N), \mu/\beta]$ to maximize

$$W^U(1+i) \equiv \eta \int_{\frac{1+i}{\theta\mu}}^{\bar{\gamma}} (\beta\gamma - 1) d\gamma + \alpha\lambda_1 [\nu u(q_1^U(1+i)) - w(q_1^U(1+i))] \quad (32)$$

$$+ \alpha\lambda_2 [u(q_2^U(1+i)) - w(q_2^U(1+i))],$$

where $q_1^U(1+i) = q_1^C(1+i)$ from equation (21) and $q_2^U(1+i) = q_2^D(1+i)$ from equation (27). This problem is equivalent to the optimal policy problem with two targeted CBDCs studied in Section 4.3 above, with the additional constraint that the two CBDC interest rates must be equal. In the next subsection, we study when and how this constraint changes the equilibrium allocation under the optimal policy. We then provide conditions in Section 5.3 that guarantee a CBDC is desirable when it must be universal.

5.2 Comparing Universal and Targeted CBDCs

Our next result provides a precise characterization of the conditions under which the constraint that the CBDC interest rate must be the same in both types of meetings does and does not bind at the solution to the optimal policy problem.

Proposition 6. *The optimal policy under a universal CBDC implements the same allocation as under two targeted CBDCs if and only if at least one of the following conditions holds:*

- (i) $1 + i^C = 1$ and $1 + i^D = \mu(1 + r^N)$,
- (ii) $1 + i^C = 1 + i^D$,
- (iii) $1 + i^C \leq 1 + i^D = \mu(1 + r^N)$, or
- (iv) $1 + i^C = 1 \geq 1 + i^D$.

It is straightforward to see that each of these four conditions is sufficient to ensure that a universal CBDC can implement the same allocation as two targeted CBDCs under the optimal policy. Condition (i) corresponds to a situation where neither of the targeted CBDCs are desirable. The same allocation can trivially be implemented with a universal CBDC by setting the interest rate low enough that no one chooses to hold it. In condition (ii), the two targeted CBDCs have exactly the same optimal interest rate. Setting the interest rate on a universal CBDC to this common value clearly leads to the same equilibrium allocation. Under condition (iii), a deposit-like digital currency is not desirable and the optimal interest rate on a cash-like digital currency is low enough that no type 2 buyers would choose to hold it. In this case, the optimal policy sets $i^U = i^C$ and the digital currency is only held by type 1 buyers. Finally, under condition (iv), a cash-like digital currency is not desirable and the optimal nominal interest rate on the deposit-like digital currency is non-positive. The optimal policy then sets $i^U = i^D < 0$, which ensures the digital currency is only held by type 2 buyers.

The less obvious part of Proposition 6 is that these four conditions are the *only* situations in a which a universal CBDC can implement the optimal allocation with two targeted CBDCs. In all other cases, the restriction that the CBDC interest rate must be the same in both types of meetings binds at the optimal policy and alters the resulting equilibrium allocation. In these cases, the policy maker sets the single CBDC interest rate considering both of the tradeoffs discussed above: between financial inclusion and facilitating illicit activity in type 1 DM meetings and between efficient exchange in type 2 DM meetings and CM investment.

The resulting optimal policy can impact CBDC usage along both the intensive and extensive margins. On the intensive margin, the optimal universal CBDC interest rate may be either higher or lower than the optimal rate on a targeted CBDC, which implies that buyers of a given type may hold either larger or smaller digital currency balances. On the extensive margin, a universal CBDC may be used in either fewer or more types of meetings than targeted CBDCs. We illustrate these possibilities by extending the examples discussed above to the case of a universal CBDC.

Intensive margin effects. Consider first the optimal interest rates for the two targeted CBDCs presented in Figure 4. The dashed green line depicts the optimal interest rate on a universal CBDC. When θ is large (above about 0.86), the optimal universal rate lies between the (higher) deposit-like rate and the (lower) cash-like rate. In this region, a universal CBDC will lead to higher real balances for type 1 buyers but to lower real balances for type 2 buyers compared with a dual-CBDC system. For one particular value of θ (about 0.86), the two restricted rates are equal, which implies condition (ii) of Proposition 6 is satisfied and a universal CBDC leads to the same outcome as the targeted CBDCs. For slightly lower values of θ , the optimal universal rate again lies again between the two optimal targeted rates. In this region, a universal CBDC leads to lower real balances for type 1 buyers and higher balances for type 2 buyers. This same pattern applies whenever a universal CBDC is used in both types of meetings under the optimal policy: compared to the allocation with two targeted CBDCs, production and exchange increase for one type of meeting and decrease for the other.

Extensive margin effects. Once θ falls below about 0.835 in the example in Figure 4, the universal interest rate that would optimally balance the policy maker's competing concerns becomes negative, which implies that type 1 buyers would prefer physical over digital currency. In this region, the optimal policy is to instead set the universal rate equal to the optimal deposit-like rate, $1 + i^D$, and have the digital currency only used in type 2 meetings. In other words, the constraints imposed by a universal CBDC lead in this case to a change in CBDC usage on the extensive margin, as type 1 buyers no longer hold a CBDC under the optimal policy.

This change in the extensive margin can also be seen in Figure 5, which depicts the type(s) of meeting in which a universal CBDC is used under the optimal policy for the same parameter values as Figure 3. In panel (a), a universal CBDC is used in both types of meeting under the optimal policy when θ is large, but only in type 2 meetings when θ is sufficiently small. Comparing this graph with panel (a) of Figure 3 verifies that, when ν is large and θ is small, a universal CBDC is used in fewer types of meetings than are the targeted CBDCs. At the same time, however, panel (a) also shows that a universal CBDC is used in more types of meetings when ν is small and θ is large. In this latter region, only a deposit-like CBDC is desirable and the optimal interest rate on this CBDC is positive. When the CBDC is universal, the policy maker can only prevent its use in type 1 meetings by setting an interest rate of zero or lower, which would substantially distort the allocation in type 2 meetings as well as CM investment. Instead, the optimal policy involves an interest rate lower than that on a deposit-like CBDC, but still positive, which implies the digital currency will be used in both types of meeting. Comparing the other panels of Figures 3 and

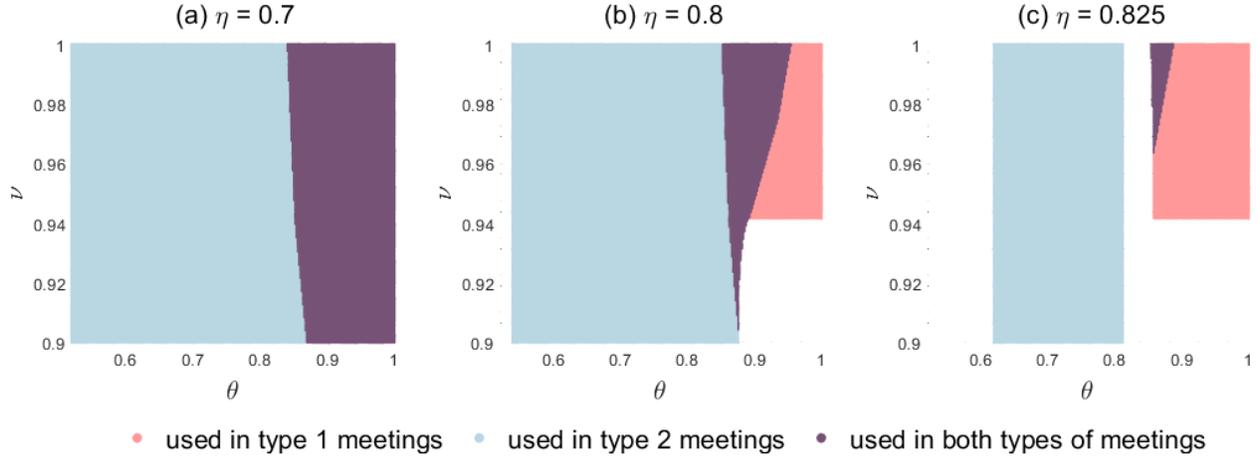


Figure 5: Desirability of a universal CBDC

5 yields additional examples where a universal CBDC circulates either more or less widely than two targeted CBDCs.

The two figures also highlight situations in which a targeted CBDC is desirable but a universal CBDC is not. In panel (b) of Figure 3, for example, a deposit-like CBDC is desirable when θ is around 0.9 and ν is small. For a universal CBDC to be attractive to type 2 buyers in this region, however, it would need to carry a positive interest rate and would, therefore, also attract type 1 buyers. Because ν is low, such a policy is unattractive and Figure 5 shows it is instead optimal not to issue CBDC. This region of parameter space corresponds to a situation in which the concern about facilitating illicit activity is strong enough to make an otherwise-useful CBDC undesirable. Another interesting case is the vertical white “stripe” in the middle of panel (c) in Figure 5. A cash-like CBDC is desirable in this region when $\nu > \bar{\nu}$. If the CBDC is universal, however, it would also be attractive to type 2 buyers. The resulting disintermediation of banks and decrease in CM investment would lower overall welfare and, therefore, the optimal policy is not to issue a CBDC in this region.

5.3 Desirability of a Universal CBDC

While the examples above illustrate how a universal CBDC often generates lower welfare than a pair of targeted CBDCs, Figure 5 also emphasizes that a universal CBDC is nevertheless often desirable. The next result provides three sets of sufficient conditions for a universal digital currency to raise welfare under the optimal policy.

Proposition 7. *A universal digital currency is desirable if any of the following sets of conditions holds:*

$$\begin{aligned}
(i) \quad & \nu > \frac{\alpha\beta}{\mu - (1-\alpha)\beta} \quad \text{and} \quad 1 + r^N > \frac{1}{\mu}; \\
(ii) \quad & \nu > \frac{\alpha\beta}{\mu - (1-\alpha)\beta} \quad \text{and} \quad 1 + r^N < \frac{\theta}{\beta}; \quad \text{or} \\
(iii) \quad & 1 + r^N < \frac{1}{\mu} \quad \text{and} \quad 1 + r^N < \frac{\theta}{\beta}.
\end{aligned}$$

In the first two cases, the policy maker would like to increase production and exchange in type 1 meetings and, therefore, the optimal interest rate on a cash-like digital currency would be positive. With a universal CBDC, however, the policy maker needs to also take into account its effects on type 2 meetings and CM investment. In case (i), the nominal interest rate on deposits in the baseline economy with no CBDC is strictly positive. The policy maker can, therefore, introduce a digital currency with a small positive interest rate that improves efficiency in type 1 meetings without affecting type 2 meetings and CM investment. In case (ii), the baseline economy exhibits overinvestment. A universal CBDC with a small positive interest rate will again improve efficiency in type 1 meetings and, if it causes the interest rate on deposits to rise, will also improve efficiency in type 2 meetings and CM investment. Finally, in case (iii), the baseline economy exhibits overinvestment and a negative nominal interest rate on deposits. In this case, a universal CBDC that offers a slightly higher interest rate can improve efficiency in type 2 meetings and in CM investment without affecting production and exchange in type 1 meetings. The logic of this third case can alternatively be stated in terms of the density η of productivities. As demonstrated in the proof of Proposition 5, the gross interest rate on deposits in the baseline economy will satisfy both conditions of case (iii) if η is small enough. We record this result as a corollary.

Corollary 2. *There exists $\bar{\eta}^U > 0$ such that $\eta < \bar{\eta}^U$ implies a universal digital currency raises welfare under the optimal policy.*

Turning to the question of the optimal interest rate on a universal digital currency, the logic of the first two cases in Proposition 7 can be extended to identify a lower bound. If the policy maker would like to increase production and exchange in type 1 meetings, then the CBDC interest rate should be large enough to at least ensure there is not overinvestment in the CM.

Corollary 3. *If $\nu > \frac{\alpha\beta}{\mu - (1-\alpha)\beta}$, the optimal interest rate on a universal CBDC satisfies $1 + i \geq \frac{\theta\mu}{\beta}$.*

In summary, when a central bank cannot target a CBDC to compete only with a single existing type of payment, it must choose the CBDC interest rate to balance a variety of concerns. A universal digital currency should not be too attractive relative to cash, to avoid facilitating illicit activities. It should offer an efficient alternative to bank deposits, but not so much as to unduly disintermediate banks. Our analysis shows that, despite the constraints, a CBDC is often desirable. In these cases, the CBDC may only be used in one type of meeting under the optimal policy, even though it is universally accepted. It will tend to compete with bank deposits when productive projects are scarce and/or credit market frictions are strong. In contrast, a universal CBDC will tend to compete with physical currency only when productive projects are plentiful and credit market frictions are small.

6 Central Bank Lending

When the central bank issues \bar{E}_t units of digital currency, it receives $\phi_t \bar{E}_t$ units of CM good in exchange. Our analysis above assumes these goods are distributed to agents as lump-sum transfers. Might the crowding-out effect we identify be mitigated or eliminated if the central bank were instead to lend these goods back to banks in the deposit market? To answer this question, we now extend our model to include a central bank lending policy.

For concreteness, suppose the central bank introduces a deposit-like digital currency and sets the interest rate i so that the equilibrium quantity of digital currency held by type 2 buyers is positive.²¹ In addition, suppose the central bank lends an amount b of goods (measured per type 2 buyer) into the deposit market in each period. Letting d_t continue to denote the deposit of a typical type 2 buyer, the market-clearing condition for deposits in equation (17) becomes

$$\lambda_2 (d_t + b) = \eta \left(\bar{\gamma} - \frac{1 + r_t}{\theta} \right). \quad (33)$$

Note that this equation can also be interpreted as the balance-sheet identity for the banking system; on the left-hand side are the liabilities of banks to depositors and the government, while the funded projects on the right-hand side are the banking system's assets. Looking at buyers' portfolio-choice problem, it is still the case that type 2 buyers will hold both deposits and digital currency only if they offer the same rate of return. Combining the relationship in (33) with the first-order condition of a type 2 buyer in equation (10) yields

$$\frac{1}{\beta(1 + r_t)} = L \left((1 + r_t) \left(\frac{\eta}{\lambda_2} \left(\bar{\gamma} - \frac{1 + r_t}{\theta} \right) + e_{2t} - b \right) \right). \quad (34)$$

²¹ It is straightforward to show that the analysis with a universal digital currency is similar.

The equilibrium conditions for the model with a deposit-like digital currency and central bank lending are then equations (19), (24), and (34).

When $b = 0$, equation (34) reduces to (25) and the equilibrium allocation is the same as in the previous section. In particular, the real value of digital currency held by each type 2 buyer is e_2^D , which we assume here is positive. When the central bank instead sets $b > 0$, the equilibrium real interest rate $1 + r_t$ does not change, since equation (24) must still hold. It follows immediately that the level of bank lending does not change because the measure of projects that meet the funding constraint, given on the right-hand side of equation (33), is unchanged. The left-hand side of this equation shows that central bank lending crowds out buyers' deposits in banks one-for-one, so that banks' total liabilities $d_t + b$ remain unchanged. Equation (34) shows that central bank lending increases the digital currency holdings of type 2 buyers, so that the difference $e_{2t} - b$ is unchanged. In other words, for each dollar lent by the central bank to banks, private agents decrease their bank deposits and increase their digital currency holdings by exactly one dollar. Since bank deposits and digital currency holdings yield the same, unchanged return, this shuffling of funds has no effect on equilibrium allocations. In particular, the central bank cannot mitigate a digital currency's impact on banks by lending to them. Instead, central bank lending causes further disintermediation of private deposits. We summarize this result in the following proposition.

Proposition 8. *Suppose $e_2^D > 0$. If the central bank lends an amount $b \in [0, d^D]$ in the deposit market, (i) private bank deposits decrease by b , (ii) digital currency held by type 2 buyers increases by b , and (iii) equilibrium consumption allocations are unchanged.*

This result is related to Brunnermeier and Niepelt (2019), who establish an equivalence result between the use of public and private money. Their result can be seen in the context of our model as follows. Suppose the central bank were to introduce a digital currency and set the interest rate $(1 + i)$ equal to the equilibrium nominal interest rate on deposits when there is no digital currency, $\mu(1 + r^N)$. In the absence of central bank lending, our results from the previous sections show that the digital currency will not be held in equilibrium, that is, $e_2^D = 0$. Now suppose the government lends $b > 0$ in the deposit market. Proposition 8 shows that type 2 buyers will substitute b units of digital currency for bank deposits, so that e_2^D becomes positive. In this way, the central bank could introduce a digital currency that is held in equilibrium without changing the equilibrium allocation of resources, in line with the Brunnermeier-Niepelt equivalence result.²²

²² See also Niepelt (2020) and Fernández-Villaverde et al. (2021). In practice, additional operational issues such as haircuts and acceptable collateral arise when the central bank lends to the private sector in the process of creating CBDC; see Assenmacher et al. (2021).

Of course, our analysis in the previous sections shows that setting $(i + i) = \mu (1 + r^N)$ is often not the optimal policy. Instead, Proposition 5 and Figure 3 show that the policy maker can often raise welfare by setting the interest rate higher. Proposition 8 demonstrates that central bank lending cannot mitigate the resulting tradeoff between efficiency in exchange and CM investment because it substitutes one form of inside money (bank deposits) with another (a CBDC backed by loans), leaving real allocations unchanged. The fundamental tradeoff in our model arises when the policy maker sets $1 + i$ to increase the real stock of outside money, which promotes efficient exchange but tends to crowd out inside money and decrease investment.

7 Concluding Remarks

The introduction of a central bank digital currency would represent a potentially historic innovation in monetary policy. If households and firms choose to hold and use significant quantities of such a currency, it could lead to a substantial shift in aggregate liquidity, that is, in the types of assets that are used in exchange and that carry a liquidity premium. While the possibility of such a shift has been widely discussed in policy circles, its macroeconomic implications remain uncertain.

Our analysis shows how a fairly standard model in the New Monetarist tradition can generate insight into these issues. In particular, it highlights important policy tradeoffs that arise when digital currency competes with cash and with bank deposits as a medium of exchange. If a digital currency provides current cash users with a better means of payment, a tradeoff arises between promoting financial inclusion and facilitating illicit activities. If a digital currency competes with bank deposits, a tradeoff arises between promoting efficient exchange and efficient investment. If the central bank is able to design separate, targeted digital currencies for each of these uses, it can set the interest rate and other design features for each one to manage the relevant tradeoff. If a central bank digital currency is instead universal, the central bank must design the single digital currency with both tradeoffs in mind.

Our analysis shows that a cash-like digital currency is desirable if the financial inclusion motive is sufficiently strong. A deposit-like digital currency is desirable if the supply of productive projects is small relative to the transactions demand for deposit-like money and tends to be desirable when financial frictions are moderate. If digital currency is universal, these same patterns apply but interact in ways that may lead the currency to circulate either more or less widely than targeted digital currencies would. Taken together, our results show how a digital currency could potentially be an important tool for central banks in managing

aggregate liquidity and provide guidance for using this new tool.

The introduction of a central bank digital currency also raises issues that lie outside the scope of our analysis, of course. For example, policy makers have expressed concern that, by providing a safe alternative to bank deposits, a digital currency could facilitate runs on the banking system in periods of financial stress.²³ Digital currencies can also be held and used internationally much more easily than physical currency, which could potentially alter capital flows and interact with domestic monetary policy. Understanding these issues and how they relate to the fundamental effects of a CBDC identified in our analysis is a promising area of ongoing research.

²³ See, for example, Williamson (2021) and Schilling et al. (2021)

Appendix: Proofs

Proposition 1. *The economy with no digital currency has a unique equilibrium. There is a liquidity premium on deposits in this equilibrium if and only if $\eta < \frac{\lambda_2 w(q^*)}{\bar{\gamma} - \frac{1}{\beta\theta}}$. When this condition holds, the equilibrium interest rate is a strictly increasing function of η .*

Proof. Equations (19) and (20) immediately imply that any equilibrium is stationary, with m_t and r_t constant over time. We denote the equilibrium values by m^N and r^N , respectively. Because $\mu > \beta$, the properties of L imply that equation (19) has a unique solution for real money balances, $m^N \in (0, w(q^*)/\beta)$. Moreover, the equation implicitly defines m^N as a function of μ in this region, with

$$\frac{dm^N}{d\mu} = \frac{L\left(\frac{m^N}{\mu}\right) + \frac{m^N}{\mu} L'\left(\frac{m^N}{\mu}\right)}{L'\left(\frac{m^N}{\mu}\right)} < 0.$$

That is, equilibrium real money balances are strictly decreasing in the inflation rate μ .

We can write the market-clearing equation (20) for deposits as

$$\lambda_2 \frac{L^{-1}\left(\frac{1}{\beta(1+r)}\right)}{1+r} = \eta \left(\bar{\gamma} - \frac{1+r}{\theta} \right). \quad (35)$$

The left-hand side of (35) is the demand for deposits, and the right-hand side is the supply. Our assumptions imply that the left-hand side is a continuous, strictly increasing function of $1+r$ that starts below $\eta\bar{\gamma}$ and approaches $\lambda_2 w(q^*)$ as $1+r \rightarrow \beta^{-1}$. The demand for deposits becomes vertical (i.e., a correspondence) when $1+r = \beta^{-1}$, including all points greater than or equal to $\lambda_2 w(q^*)$. The right-hand side of (35) starts at $\eta\bar{\gamma}$ and is a decreasing, linear function of $1+r$. It follows that equation (35) has a unique solution, $1+r^N$, satisfying

$$1+r^N \begin{cases} < \\ = \end{cases} \frac{1}{\beta} \quad \text{as} \quad \eta \begin{cases} < \\ \geq \end{cases} \frac{\lambda_2 w(q^*)}{\bar{\gamma} - \frac{1}{\beta\theta}}.$$

In the first case, we can differentiate through equation (35) to obtain

$$\frac{\partial(1+r^N)}{\partial\eta} = -\frac{\frac{\beta}{\lambda_2} (1+r^N)^2 \left(\bar{\gamma} - \frac{1+r^N}{\theta} \right) L'(A^N)}{L(A^N) + A^N L'(A^N) - \frac{\eta}{\lambda_2} \frac{(1+r^N)^2}{\theta} L'(A^N)} > 0,$$

where

$$A^N \equiv \eta \left(\frac{1+r^N}{\lambda_2} \right) \left(\bar{\gamma} - \frac{1+r^N}{\theta} \right),$$

which shows that the equilibrium interest rate is strictly increasing in η in this region. \square

Proposition 2. *Under a cash-like digital currency with $i > 0$, the unique equilibrium allocation satisfies $e_1^C > m^N$, $q_1^C > q_1^N$ and $(q_2^C, \hat{\gamma}^C) = (q_2^N, \hat{\gamma}^N)$.*

Proof. Consider first an artificial economy with no physical currency, so that only digital currency is used in type 1 DM meetings. A type 1 buyer's real money balances, e_1 , would satisfy the first-order condition

$$L\left(\frac{(1+i)}{\mu}e_1\right) = \frac{\mu}{\beta(1+i)},$$

which implicitly defines e_1 as a function of the interest rate $1+i$. The buyer's holding of real physical currency balances in the economy with no CBDC, m^N , is equal to this value of e_1 when $i = 0$. Differentiating through this condition and solving yield

$$\frac{de_1}{d(1+i)} = -\frac{L\left(\frac{(1+i)}{\mu}e_1\right) + \frac{(1+i)}{\mu}e_1L'\left(\frac{(1+i)}{\mu}e_1\right)}{\frac{(1+i)^2}{\mu}L'\left(\frac{(1+i)}{\mu}e_1\right)} > 0,$$

where the fact that the numerator is positive follows from our assumption that $AL(A)$ is strictly increasing in A . It follows that $e_1^C > m^N$ holds whenever $i > 0$.

To see that the digital currency leads to an increase in DM production in type 1 meetings, note that the quantity produced satisfies

$$\alpha \frac{u'(q_1^C)}{w'(q_1^C)} + 1 - \alpha = \frac{\mu}{\beta(1+i)} < \frac{\mu}{\beta} = \alpha \frac{u'(q_1^N)}{w'(q_1^N)} + 1 - \alpha.$$

This inequality implies

$$\frac{u'(q_1^C)}{w'(q_1^C)} < \frac{u'(q_1^N)}{w'(q_1^N)},$$

which, in turn, implies $q_1^C > q_1^N$.

Finally, because a cash-like digital currency cannot be used in type 2 DM meetings, it will not be held by type 2 buyers. The quantities of deposits, investment, and type 2 DM production therefore remain unchanged. \square

Proposition 3. *There exists $\bar{\nu} \in (0, 1)$ such that a cash-like digital currency is desirable if and only if $\nu > \bar{\nu}$. In this case, the optimal policy is given by equation (22).*

Proof. The first-order condition for the policy maker's choice of interest rate can be written as

$$\alpha \lambda_2 [\nu u' (q_1 (1 + i)) - w' (q_1 (1 + i))] q_1' (1 + i) \leq 0,$$

with equality if $i > 0$. The proof of Proposition 1 establishes that q_1 is strictly increasing in $1 + i$. Therefore, the optimal choice of interest rate has $i > 0$ if and only if the expression in square brackets is positive when evaluated at $i = 0$, that is,

$$\nu u' (q_1^N) - w' (q_1^N) > 0 \quad \text{or} \quad \nu > \frac{w' (q_1^N)}{u' (q_1^N)} \equiv \bar{\nu}.$$

Because q_1^N satisfies

$$\frac{\mu}{\beta} = \alpha \frac{u' (q_1^N)}{w' (q_1^N)} + 1 - \alpha,$$

we have

$$\bar{\nu} = \frac{\alpha \beta}{\mu - (1 - \alpha) \beta}.$$

By varying $1 + i$, the policy maker can implement any quantity of DM trade between q_1^N and q^* in type 1 meetings. The optimal choice has the property that q_1 satisfies

$$\frac{w' (q_1)}{u' (q_1)} = \nu \leq 1$$

The equilibrium value of q_1 when $i > 0$ satisfies the first-order condition

$$\frac{\mu}{\beta (1 + i)} = \alpha \frac{u' (q_1)}{w' (q_1)} + 1 - \alpha.$$

Combining these equations yields the optimal policy in equation (22). □

Proposition 4. *With a deposit-like digital currency satisfying condition (23), the unique equilibrium allocation satisfies $e_2^D + d^D > d^N > d^D$, $(r^D, \hat{\gamma}^D, q_2^D) \gg (r^N, \hat{\gamma}^N, q_2^N)$, and $q_1^D = q_1^N$.*

Proof. When $1 + i$ is set so that equation (23) holds, the equilibrium interest rate and the investment cutoff rise to

$$1 + r^D = \frac{1 + i}{\mu} > 1 + r^N \quad \text{and} \quad \hat{\gamma}^D = \frac{1 + i}{\mu \theta} > \hat{\gamma}^N,$$

respectively. The new quantity of deposits is determined by the supply function at the

investment cutoff point:

$$d^D = \frac{\eta}{\lambda_2} \left(\bar{\gamma} - \frac{1+i}{\theta\mu} \right).$$

The quantity of digital currency held by buyers heading into a type 2 meeting is

$$e_2^D = \frac{\mu}{1+i} L^{-1} \left(\frac{\mu}{\beta(1+i)} \right) - \frac{\eta}{\lambda_2} \left(\bar{\gamma} - \frac{1+i}{\theta\mu} \right).$$

Note that the resulting value of e_2^D is positive if and only if $1+i > \mu(1+r^N)$, and e_2^D is a strictly increasing function for $1+i \in (\mu(1+r^N), \mu/\beta)$. It is also easy to show that d^D is strictly decreasing in $1+i$ in this region and that the sum $e_2^D + d^D$ is strictly increasing.

The quantity traded in type 2 meetings satisfies

$$\frac{\mu}{\beta(1+i)} = \alpha \frac{u'(q_2^D)}{w'(q_2^D)} + 1 - \alpha.$$

Given our assumptions on preferences, we have $q_2^D > q^N$ when $1+i > \mu(1+r^N)$. \square

Proposition 5. *There exists $\bar{\eta} > 0$ such that $\eta < \bar{\eta}$ implies a deposit-like digital currency is desirable. The optimal policy satisfies $1+i \in [\mu(1+r^N), \mu/\beta]$ if $\theta < 1$ and $1+i = \mu/\beta$ if $\theta = 1$.*

Proof. The objective function in equation (26) need not be concave in $1+i$, but it is continuous on the closed interval $1+i \in [\mu(1+r^N), \mu/\beta]$ and, therefore, an optimal policy exists. To characterize this policy, it is useful to set up an *auxiliary problem* in which the policy maker directly chooses a real interest rate $1+r \in (0, \frac{1}{\beta}]$ to maximize

$$\hat{W}(1+r) \equiv \eta \int_{\frac{1+r}{\theta}}^{\bar{\gamma}} (\beta\gamma - 1) d\gamma + \alpha\lambda_2 [u(\hat{q}_2(1+r)) - w(\hat{q}_2(1+r))],$$

where

$$\hat{q}_2(1+r) = w^{-1} \left(\beta L^{-1} \left(\frac{1}{\beta(1+r)} \right) \right).$$

Unlike with CBDC, which only allows the policy maker to increase the equilibrium deposit rate, this auxiliary problem allows the policy maker to either increase or decrease $1+r$. The Inada conditions on u imply that \hat{W} is strictly increasing when $1+r$ is sufficiently close to zero and, therefore, the auxiliary problem has a solution. Let $1+\hat{r}$ denote this solution. (If there are multiple solutions, let $1+\hat{r}$ denote the smallest one.)

If $1+\hat{r} > 1+r^N$, then, by definition, we must have $\hat{W}(1+r) > \hat{W}(1+r^N)$. Moreover, the nominal interest rate $1+i = \mu(1+\hat{r})$ is contained in the policy maker's choice set for

the original problem and, therefore, introducing a digital currency that bears this interest rate raises welfare. To establish the first part of the proposition, therefore, it suffices to show that $1 + \hat{r} > 1 + r^N$ holds when η is sufficiently small.

The slope of the auxiliary objective \hat{W} is given by

$$\frac{d\hat{W}}{d(1+r)} = -\frac{\eta}{\theta} \left[\frac{\beta(1+r)}{\theta} - 1 \right] + \frac{\lambda_2}{-L' \left(L^{-1} \left(\frac{1}{\beta(1+r)} \right) \right)} \frac{1 - \beta(1+r)}{\beta(1+r)^3}$$

The second term in this expression is positive for all values of $1 + r < \beta^{-1}$. The first term is also positive when $1 + r < \theta/\beta$ and, therefore, the solution to the auxiliary problem must satisfy

$$1 + \hat{r} > \frac{\theta}{\beta}$$

for all values of η . As established in Proposition 1, $1 + r^N$ is a strictly increasing function of η . Moreover, as η approaches the lower bound in Assumption 1, $1 + r^N$ approaches zero. It follows that there exists $\bar{\eta} > 0$ such that $1 + r^N < 1 + \hat{r}$ holds for all $\eta < \bar{\eta}$, which establishes the first part of the proposition.

For the second part of the proposition, first assume $\theta < 1$ and evaluate the derivative in equation (28) at $1 + i = \mu/\beta$. At this interest rate, there is no liquidity premium, which implies that type 2 buyers will be satiated in real balances and the quantity produced in type 2 DM meetings will be q^* . The second term in the derivative is thus zero. Because $\theta < 1$, the first term in the derivative is negative. It follows that the solution to the optimal policy problem must be lower, with $1 + i < \mu/\beta$.

Finally, when $\theta = 1$, the derivative in equation (28) is strictly positive for all $1 + i < \frac{\mu}{\beta}$ and, therefore, the optimal policy is $1 + i = \frac{\mu}{\beta}$. \square

Proposition 6. *The optimal policy under a universal CBDC implements the same allocation as under two restricted-use CBDCs if and only if at least one of the following conditions holds:*

- (i) $1 + i^C = 1$ and $1 + i^D = \mu(1 + r^N)$,
- (ii) $1 + i^C = 1 + i^D$,
- (iii) $1 + i^C \leq 1 + i^D = \mu(1 + r^N)$, or
- (iv) $1 + i^C = 1 \geq 1 + i^D$.

Proof. Define $1 + r^C \equiv (1 + i^C) / \mu$, $1 + r^D \equiv (1 + i^D) / \mu$, and $1 + r^U \equiv (1 + i^U) / \mu$ as the optimal real interest rate for the cash-like, deposit-like, and universal CBDCs, respectively.

Note that the aforementioned set of conditions can be written in terms of the real interest rate as

$$\begin{aligned}
(i) \quad 1 + r^C &= \frac{1}{\mu} \quad \text{and} \quad 1 + r^D = 1 + r^N, \\
(ii) \quad 1 + r^C &= 1 + r^D, \\
(iii) \quad 1 + r^C &\leq 1 + r^D = 1 + r^N, \text{ or} \\
(iv) \quad 1 + r^C &= \frac{1}{\mu} \geq 1 + r^D.
\end{aligned}$$

The equilibrium allocation $(q_1, q_2, \hat{\gamma})$ under a universal CBDC will be the same as with two restricted-use CBDCs if and only if each type of buyer faces the same rate of return on spendable assets under both regimes. For type 1 buyers, this requirement can be written as

$$\text{If } 1 + r^C > \frac{1}{\mu}, \text{ then } 1 + r^U = 1 + r^C; \text{ otherwise, } 1 + r^U \leq \frac{1}{\mu}. \quad (36)$$

In other words, if a cash-like CBDC is desirable, the interest rate on a universal CBDC must equal the optimal cash-like rate. If a cash-like CBDC is not desirable, the interest rate on the universal currency must be low enough that it does not change the quantity of real balances held by type 1 buyers. The requirement for type 2 buyers is

$$\text{If } 1 + r^D > 1 + r^N, \text{ then } 1 + r^U = 1 + r^D; \text{ otherwise, } 1 + r^U \leq 1 + r^N. \quad (37)$$

The logic here is similar. If a deposit-like CBDC is desirable, the interest rate on a universal CBDC must equal the optimal deposit-like rate. If it is not desirable, the interest rate on the universal currency must be low enough that no type 2 buyer chooses to hold it.

It is straightforward to show that each of the four conditions in the proposition is sufficient to guarantee that the requirements (36) and (37) are satisfied. Under condition (i), the optimal policy is $1 + r^U = \min \{1/\mu, 1 + r^N\}$. Under condition (ii), it is $1 + r^U = 1 + r^C = 1 + r^D$. Under condition (iii), the optimal policy sets $1 + r^U = 1 + r^C$ to satisfy requirement (36); the inequality in the condition then guarantees that (37) is satisfied as well. Finally, under condition (iv), the optimal policy sets $1 + r^U = 1 + r^D$ to satisfy (37) and the inequality in the condition guarantees that (36) is also satisfied.

The less obvious part of the proposition is that it is also necessary for at least one of these conditions to hold if a universal CBDC is to implement the same allocation as with two restricted-use CBDCs. We establish this part of the result by showing that if conditions (i) – (iii) are **not** satisfied, then requirements (36) and (37) imply that condition (iv) is necessarily satisfied.

We begin with conditions (i) and (ii). If condition (i) is not satisfied, then we either have $1 + r^C > 1/\mu$ or $1 + r^D > 1 + r^N$, or both. In other words, at least one type of restricted-use CBDC is desirable. If condition (ii) is not satisfied, the desired interest rates in the two types of meetings are different. Requirements (36) and (37) then imply that either $1 + r^C = 1/\mu$ or $1 + r^D = 1 + r^N$ must hold. In other words, if neither (i) nor (ii) is satisfied and a universal CBDC can implement the same allocation as two restricted-use CBDCs, it must be the case that one restricted-use CBDC is desirable and the other is not.

Now suppose that, in addition, condition (iii) is not satisfied, meaning either $1 + r^C > 1 + r^D$ or $1 + r^D > 1 + r^N$. Suppose first that $1 + r^C > 1 + r^D$ held. Requirements (36) and (37) imply

$$1 + r^U \leq \min \{1 + r^C, 1 + r^D\} \quad (38)$$

and, therefore, we would have $1 + r^U < 1 + r^C$. Requirement (36) would then imply $1 + r^C = 1/\mu$ must hold. Given that we have supposed $1 + r^C > 1 + r^D$, it follows that condition (iv) is satisfied. If we instead suppose $1 + r^D > 1 + r^N$ held, then (37) would require $1 + r^U = 1 + r^D$ and the fact that only one restricted-use CBDC is desirable would imply $1 + r^C = 1/\mu$. Combining these results with equation (38) would imply $1 + r^D \leq 1/\mu$ and condition (iv) is again satisfied, as desired. \square

Proposition 7. *A universal digital currency is desirable if any of the following sets of conditions holds:*

$$\begin{aligned} (i) \quad & \nu > \frac{\alpha\beta}{\mu - (1 - \alpha)\beta} \quad \text{and} \quad 1 + r^N > \frac{1}{\mu}; \\ (ii) \quad & \nu > \frac{\alpha\beta}{\mu - (1 - \alpha)\beta} \quad \text{and} \quad 1 + r^N < \frac{\theta}{\beta}; \quad \text{or} \\ (iii) \quad & 1 + r^N < \frac{1}{\mu} \quad \text{and} \quad 1 + r^N < \frac{\theta}{\beta}. \end{aligned}$$

Proof. To establish that a universal CBDC is desirable, we must show that there exists an interest rate $1 + i$ such that welfare $W^U(1 + i)$ from equation (32) is strictly higher than welfare with no digital currency, W^N . We address each of the three cases in turn.

(i) Because $\nu > \bar{\nu}$, a CBDC that pays a positive but sufficiently small interest rate will increase the middle term on the right-hand side of equation (32), which corresponds to the surplus from type 1 DM meetings. Moreover, $\mu(1 + r^N) > 1$ implies that if the net CBDC interest rate is set sufficiently close to zero, it will not be held by type 2 buyers and will not affect CM investment. It follows that there exists $\varepsilon > 0$ such that a universal CBDC with interest rate $1 + i \in (1, 1 + \varepsilon)$ raises welfare.

(ii) As in case (i), a CBDC with a sufficiently small positive interest rate will increase the surplus from type 1 DM meetings. Separately, $1 + r^N < \theta/\beta$ implies that there is overinvestment in the equilibrium with no digital currency and, therefore, setting the CBDC interest rate slightly above $\mu(1 + r^N)$ would increase both the first and third terms on the right-hand side of equation (32). By choosing the CBDC interest rate to be slightly above the *smaller* of these two values, the policy maker can ensure that some welfare terms increase while no others decrease. In other words, there exists $\varepsilon > 0$ such that a universal CBDC with interest rate

$$1 + i \in (\min\{1, \mu(1 + r^N)\}, \min\{1, \mu(1 + r^N)\} + \varepsilon)$$

raises welfare.

(iii) In this case, there is again overinvestment in the equilibrium with no digital currency, so setting $1 + i$ slightly larger than $\mu(1 + r^N)$ will increase the first and third terms on the right-hand side of equation (32). In addition, $\mu(1 + r^N) < 1$ implies that setting the CBDC interest rate sufficiently close to $\mu(1 + r^N)$ ensures that it will not be used in type 1 DM meetings. It follows that there exists $\varepsilon > 0$ such that a universal CBDC with interest rate $1 + i \in (1 + r^N, 1 + r^N\varepsilon)$ raises welfare. \square

Corollary 3. *If $\nu > \frac{\alpha\beta}{\mu - (1-\alpha)\beta}$, the optimal interest rate on a universal CBDC satisfies $1 + i \geq \frac{\theta\mu}{\beta}$.*

Proof. The proof follows similar reasoning to case (ii) in Proposition 7. Assume that $(1 + i)/\mu < \theta/\beta$ holds at the optimum. Because this condition implies overinvestment, we can find an $\varepsilon > 0$ with $(1 + i)/\mu < (1 + i + \varepsilon)/\mu < \theta/\beta$ such that the interest rate $1 + i + \varepsilon$ results in a higher value for the welfare function, W^U . But this contradicts the fact that $(1 + i)/\mu$ is a solution to the welfare maximization problem. \square

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