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Aging and the Real Interest Rate in Japan: A Labor Market Channel*

Shigeru Fujita[†] Ippei Fujiwara[‡]

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Abstract

This paper explores a causal link between aging of the labor force and declining trends in the real interest rate in Japan. We develop a search/matching model that features heterogeneous workers with respect to their ages and firm-specific skills. Using the model, we examine the long-run implications of the sharp drop in labor force entry in the 1970s. We show that the changes in the demographic structure induce significant low-frequency movements in per capita consumption growth and the real interest rate. The model suggests that aging of the labor force accounts for 40 percent or more of the declines in the real interest rate observed between the 1980s and 2000s in Japan. We also examine the impacts of other long-term developments such as a slowdown of TFP growth and higher shares of female and non-regular workers.

JEL Classification: E24, E43

Keywords: Aging, Real interest rate, Japan

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1 Introduction

Japan's labor force has been rapidly aging. Since the early 1980s, the average age of its workforce has risen roughly three years. The *level* of the labor force has been falling to date since the late 1990s. At the same time, the Japanese economy has experienced a prolonged slowdown in growth and persistent declines in the real interest rate. In this paper, we explore a novel causal link between the aging labor force and the low-frequency declining trend in the real interest rate since the 1970s by using a search/matching framework that incorporates heterogeneities in the worker's age and skill.

The main driver of Japan's aging workforce is the end of the baby boom. The birth rate fell precipitously in the 1950s, which translated into a similarly sharp drop in labor force entry in the 1970s. We argue that this sharp drop in labor force entry caused low-frequency movements in key macroeconomic variables.[1] In our quantitative experiment, we feed the model the exogenous labor force entry path that mimics the data. We find that the transition to the new steady state takes more than 50 years, and during the transition, the economy exhibits low-frequency fluctuations that are consistent with the behavior of the Japanese economy.

There are two key features in our model. First, the model incorporates the empirical regularity that older workers are more productive than younger workers. In the model, a worker enters the labor force as a young worker with no experience. The worker gains their experience as they become old. Experienced workers enjoy higher productivity. Second, we also incorporate into the model what we believe is the salient feature of the Japanese labor market, namely, the importance of firm-specific skills.[2] The (old) worker's skill associated with their experience in the labor market tends to be firm specific; therefore, when the worker loses their job, they could lose the productivity premium associated with being an experienced worker. When an old worker indeed loses their experience premium, they need to look for a job in the labor market where young (inexperienced) workers are also looking for their entry-level jobs.

In our baseline model, we abstract away from the consumption/saving heterogeneity of workers with different ages and their employment status by assuming that all sources of

[1] Although the baby boom ended long ago, the birth rate kept falling. For example, the total fertility rate reached the bottom in 2005 at around 1.3. This steady decline in the birth rate has received much attention because it exemplifies the aging of Japanese society. However, the baby boom is a much more quantitatively important factor for the gradual progress of the labor force aging observed in the data.

[2] The importance of firm-specific human capital in Japan was first pointed out by Hashimoto and Raisian (1985). Their main point is that firm tenure plays a much more important role in earnings profiles in Japan than in the US. The authors link this finding to the lifetime employment practice in Japan. Kambayashi and Kato (2017) study whether the lifetime employment practice remains common in Japan and find that prime-age male workers, in particular, still enjoy job stability much more so than in the US (see also Kato (2001)).

incomes are pooled and consumed equally across the existing household members. One may view this assumption as being restrictive, given the motivation of this paper.[3] However, we make this assumption deliberately to highlight the key mechanism of our paper, namely, skill (productivity) heterogeneity in the workforce. In our model, the evolution of the demographic structure leads to changes in the aggregate skill mix of the workforce, resulting in low-frequency movements in aggregate labor productivity, per capita consumption, and thus the real interest rate. The relationship between consumption/saving heterogeneity and aging has already been studied in the literature, as discussed below. However, this "productivity channel" has so far been largely overlooked. Moreover, adding consumption/saving heterogeneity to our model is likely to only strengthen our results.

We show that the equilibrium transition path of the economy replicates important low-frequency features of the Japanese economy observed between 1970 and 2010. According to the model, the first decade of this 40-year period is characterized by rising per capita consumption, labor productivity, and the real interest rate. This period, however, is followed by a prolonged period of a gradual slowdown of the economy. In the data, per capita consumption growth and the real interest rate fell 1.3 percentage points and 2.3 percentage points, respectively, between the 1980s and the 2000s. Our experiment indicates that workforce aging (driven by the decline in labor force entry in the 1970s) accounts for roughly 40 percent of the decline. The model economy exhibits the low-frequency swing for the following reasons. First, the initial phase is characterized by the environment, where more young workers are gaining experience and thus per capita consumption is growing faster. However, as more workers reach a high productivity state, there is less room to grow further. This together with fewer entry flows of young workers implies a slowdown of per capita growth. In addition, the firm-specific nature of the worker's skill plays a role of putting a further downward pressure on labor productivity and thus the real interest rate in the latter part of the transition path.

While the focus of our paper is the impacts of aging, the Japanese economy has experienced a few other notable long-term changes as well, such as declines in TFP growth (e.g., Hayashi and Prescott (2002) and the subsequent related literature), increases in female labor force participation, and increases in non-regular workers (e.g., Esteban-Pretel et al. (2011)). Adding exogenous TFP growth is straightforward, and thus the impacts of TFP growth slowdown can easily be studied. It is, however, difficult to assess the impacts of the other two phenomena within the baseline model. We thus extend our model by introducing another dimension of heterogeneity to the model, gender heterogeneity, and calibrate the model by using additional information relevant to these additional developments. We find that these other long-run changes only enhance our earlier result on the real interest rate.

[3]For example, Ramey (2009) and Aguiar and Hurst (2013) emphasize the differences in consumption patterns between different age groups in the US.

Relation to the literature. A paper by Carvalho et al. (2016) relates the declining real interest rate in Japan to the aging of the economy as in our paper. Their paper focuses on the role of consumption/saving heterogeneity, and the driving force in their model is a longer life expectancy. Ikeda and Saito (2014) develop a model in which the level of the real interest rate is influenced by population growth in the presence of financial frictions. Katagiri (2012) studies a model with labor market frictions with sectoral shifts and explores the effects of an unexpected shock to the population growth forecast. Such a shock results in changes in the demand structure, thus causing the real interest rate to change. None of these existing papers recognizes the "productivity channel" stressed in this paper.

The declining trend in the real interest rate in the US also has received considerable attention as well. For example, the secular stagnation hypothesis *a la* Eggertsson and Mehrotra (2015) and Eggertsson et al. (2019) posits that a decline in the birth rate eventually leads to the oversupply of saving and a decrease in aggregate demand, resulting in the lower real interest rate. Using an overlapping generations model, Gagnon et al. (2016) conclude that a decline of 1.25 percentage points in the equilibrium interest rate can be accounted for by the demographic factors in the US. Another related but different explanation is the one based on the "global savings glut" put forth by Bernanke (2005). It emphasizes the role of global forces in causing the decline in the real interest rate, namely larger international capital flows into the US economy. Ferrero (2010) associates larger capital flows into the US with the demographic structure of capital suppliers (i.e., China and Japan). [4]

There are earlier attempts to evaluate the effects of aging on the macroeconomic trend. In particular, Miles (1999) and Rios-Rull (2001) consider overlapping generations models with age-dependent consumption/saving decisions. Both of these papers incorporate the age-dependent productivity differences as well. They both conclude that aging can have significant impacts on the trend of the macroeconomy, confirming the point above that incorporating consumption/saving heterogeneity will only strengthen the effects on the real interest rate.[5] These papers calibrate their models to European countries, whereas we pay attention to the effects that result from interactions between aging and the labor market characteristics in Japan, such as the importance of firm-specific skills.

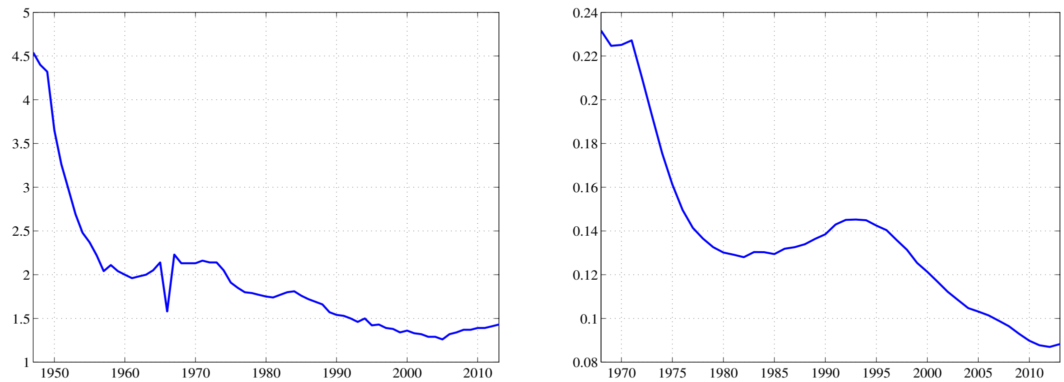
Our paper focuses on the labor market effects using the search/matching framework. We hypothesize that the stagnation of the Japanese economy, exemplified by lower labor productivity growth and the low real interest rate, is related to the structure of the Japanese labor market (in particular, the importance of the firm-specific human capital). The introduction of search frictions is also motivated by Bean (2004), who, using the language of the search/matching model, discusses the potential channels through which the natural rate of

[4] Ferrero (2010) concludes that "the international demographic transition is crucial for large U.S. external imbalances to be consistent with the persistent decline of world real interest rates observed in the data."

[5] The basic mechanism is that, given the usual life-cycle saving pattern, the aging of a society implies a higher capital-labor ratio and thus the lower marginal product of capital.

Figure 1: Labor Force Entry in Japan

(a) Birth Rate (b) Entry into the Labor Force



Notes: Panel (a) plots the total fertility rate. Panel (b) plots the share of workers between 15 and 24 in the total labor force between 15 and 64. *Sources*: Vital Statistics of Japan, Labour Force Survey.

unemployment is affected by the economy's demographic structure. A paper by Esteban-Pretel and Fujimoto (2012) develops a search/matching model with age heterogeneity and studies various structural changes facing the Japanese economy, including lower population growth. However, that paper is different from our paper along many dimensions.[6] Feyrer (2007) provides the strongest empirical evidence that supports the main idea of this paper. In his cross-country panel regression, he finds a strong correlation between the age structure of the workforce and aggregate productivity growth.[7]

The paper is organized as follows. The next section presents some basic facts about the aging of the Japanese labor force. Section 3 develops the model, which is calibrated in Section 4. Section 5 presents the main results of the paper. Section 6 extends the model by introducing gender heterogeneity in the workforce, which allows us to examine other secular changes. Section 7 concludes the paper.

2 Facts

Let us first present some basic macroeconomic facts pertaining to the main purpose of the paper.

[6]One of the differences is that Esteban-Pretel and Fujimoto (2012) consider the steady-state comparative static of lower population growth, while we trace out the dynamic effects induced by the baby boom.

[7]Feyrer (2007) notes, "(w)hile emphasizing the importance of demographics, this paper is agnostic as to the mechanisms through which demographic change and productivity are related." Our paper provides a mechanism that supports his evidence.

2.1 Aging of the Labor Force

Figure 1(a) plots the birth rate, more specifically, the total fertility rate, starting in 1948.[8] One can clearly see that the birth rate was very high in the late 1940s but fell dramatically in the following 10 years or so. This corresponds to the end of the (first) baby boom in Japan. The birth rate from then on was roughly flat at around 2 until the early 1970s. However, it started to fall in the early 1970s and kept dropping until 2005, when it hit the historical low at around 1.3. The effect of changes in the birth rate shows up 15-20 years later in the labor force. Figure 1(b) attempts to translate the behavior of the birth rate into entry flows into the labor force by plotting the share of workers between 15 and 24 years old in the total labor force.[9] This series behaves similarly to the birth rate with roughly a 15- to 20-year lag: It fell sharply in the 1970s, recovered temporarily around the early 1990s, and since then has been declining steadily. In our quantitative exercise below, the dramatic decline in labor force entry in the 1970s that is associated with the end of the baby boom is taken to be the exogenous force.[10]

In Figure 2(a), we present the average age of the labor force. The average age has increased from 36 years old to 42 years old, suggesting significant aging of the Japanese labor force over time. Another consequence of the slowdown in labor force entry is that it eventually leads to a decline in the labor force (in the absence of quantitatively meaningful immigration). Panel (b) shows that the labor force indeed started shrinking in the late 1990s. None of the large developed economies have experienced such a sustained decline in the labor force, and it exemplifies the serious nature of aging in Japan.[11] In Panel (c), we plot real hourly wage. The series starts in 1990 because of the data availability. But it is clear that after increasing through the mid-1990s, real wage has been falling.[12] Observe that the peak of real wage roughly coincides with that of labor force size in Panel (b).

2.2 Consumption Growth and the Real Interest Rate

Figure 3 presents the annual series of per capita consumption growth and the real interest rate between 1980 and 2013. To summarize the long-run trend of the two variables, Table 1 computes the average levels of the two variables for each of the three decades from 1980.

[8] The total fertility rate gives the number of children who are born to each woman in her childbearing years, which are usually defined as between 15 and 49 years old. We use the term the birth rate to mean the total fertility rate throughout the paper.

[9] We discard workers who are 65 or above from the total labor force so that the series is not influenced by the fact that a larger number of 65+ workers are in the labor force.

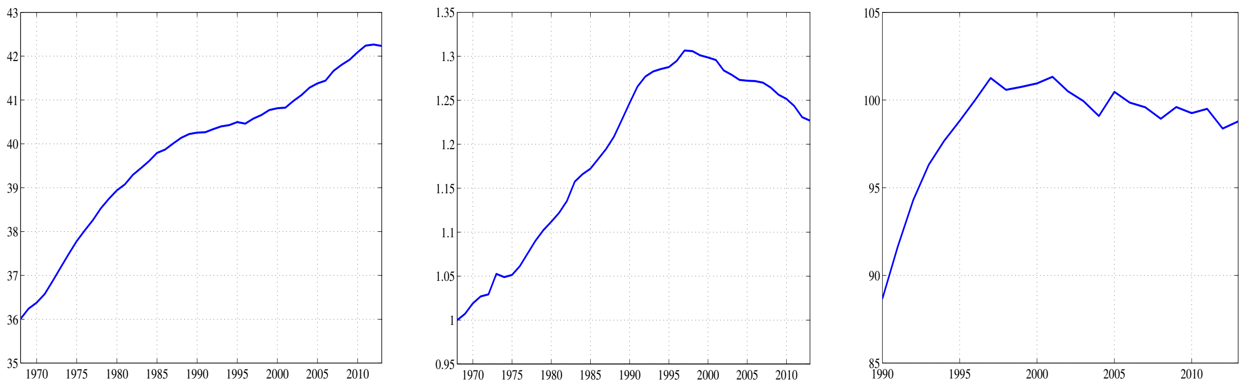
[10] Our exercise does not separately consider the smaller hump in labor force entry in the late 1980s and early 1990s.

[11] Note that the labor force size has since increased mainly due to a higher female labor force participation rate, while population peaked around 2011 and has been on a downward trend.

[12] Note that nominal wage fell even more, given low inflation or deflation over the same period.

Figure 2: Average Age, Labor Force Size, and Real Wage

(a) Average Age | (b) Labor Force Size (1968=1) | (c) Real Wage



Notes: For panel (c), average nominal wage is deflated by the CPI. *Sources*: Labour Force Survey (Panels (a) and (b)); Monthly Labor Survey (Panel (c)).

The real interest rate averaged roughly 4 percent in the 1980s but fell considerably in the 1990s, averaging 2.3 percent. The gradual decline in the real interest rate has persisted into the 2000s, with an average level of 1.7 percent in that decade.[13] Per-capita consumption growth exhibits a similar declining low-frequency trend: It grew 2.3 percent in the 1980s and fell more than 1 percentage point over the past three decades.

In the model presented below, these two variables are linked by a single consumption Euler equation, given our large family assumption, even though the model features heterogeneity on the production side. Under that assumption, we examine how much of the variations in these two variables can be accounted for by the change in the demographic structure that is ultimately driven by the fall in labor force entry in the early 1970s.

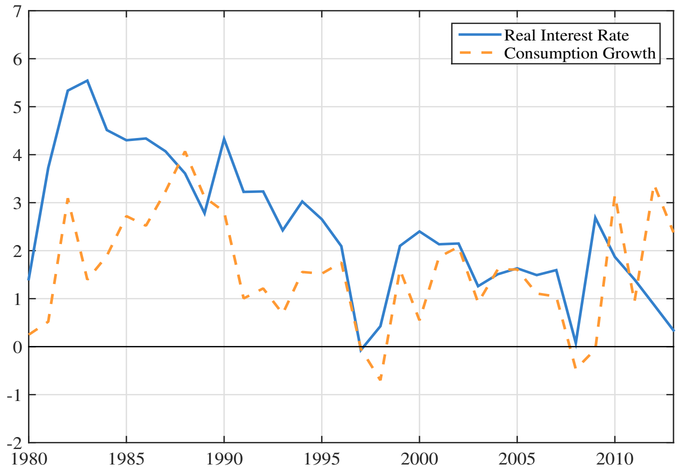
3 Model

This section lays out the model. The model incorporates what we believe are salient features of the Japanese labor market, aging and firm-specific human capital. Firms produce and sell goods to the household. Labor is the only input for production, and there are three types of workers, as specified below. Hiring is subject to search frictions. Population growth is governed by the exogenous but time-varying entry rate and the fixed death rate.

In the later section, we present the extended model with an additional heterogeneity, gender, and exogenous neutral technology growth. These additional features allow us to analyze other important developments, such as increases in female labor force participation

[13]Obviously, the level of the real interest rate changes depending on the nominal interest rate series we use. However, the size of the decline is largely insensitive to the alternative measures of nominal rates.

Figure 3: Real Interest Rate and Consumption Growth



Notes: Real interest rate equals the Treasury bill rate minus the realized inflation rate measured by the CPI. *Sources*: IMF International Financial Statistics; National Accounts of Japan; the Consumer Price Index of Japan.

Table 1: Long-Run Levels of Real Interest Rate and Consumption Growth

	Real Interest Rate (%)	Consumption Growth (%)
1980–1989	3.96	2.28
1990–1999	2.34	1.14
2000–2009	1.69	1.03
Difference	−2.27	−1.26

Notes: See notes to Figure 3. "Difference" refers to the difference between the levels in the 2000s and the 1980s, expressed as percentage points.

and lower TFP growth. Note also that in our baseline model, we also assume away capital accumulation. This is mainly because the causal link between aging and declining real interest rates that we emphasize in the paper can be understood more intuitively in the simpler model. The availability of saving through physical capital obviously influences the paths of consumption and the real interest rate, and the saving decision depends crucially on the expectation about the future demographic trend. Therefore, the results in the model with physical capital can be sensitive to the assumption on how agents form their expectation about the future path of the fertility rate. In Appendix B, we incorporate capital formation into our model (while maintaining the representative household structure) and conduct the quantitative analysis under a particular, yet plausible expectation formation on the demo-

Table 2: Summary of Worker Transitions

State at the End of $t-1$	Transition Probabilities	State at the End of t
$n_{y,t-1}$	$(1-\mu)s^i[f^i(\theta_t^i)+(1-s^i)]$	$n_{y,t}$
$n_{y,t-1}$	$\mu[s^e f^e(\theta_t^e)+(1-s^e)]$	$n_{o,t}^e$
$n_{y,t-1}$	$(1-\mu)s^i(1-f^i(\theta_t^i))$	$u_{y,t}$
$n_{y,t-1}$	$\mu s^e(1-f^e(\theta_t^e))$	$u_{o,t}^e$
$u_{y,t-1}$	$(1-\mu)f^i(\theta_t^i)$	$n_{y,t}$
$u_{y,t-1}$	$\mu f^e(\theta_t^e)$	$n_{o,t}^e$
$u_{y,t-1}$	$(1-\mu)(1-f^i(\theta_t^i))$	$u_{y,t}$
$u_{y,t-1}$	$\mu(1-f^e(\theta_t^e))$	$u_{o,t}^e$
$n_{o,t-1}^e$	d	–
$n_{o,t-1}^e$	$(1-d)s^e\delta f^i(\theta_t^i)$	$n_{o,t}^i$
$n_{o,t-1}^e$	$(1-d)s^e\delta(1-f^i(\theta_t^i))$	$u_{o,t}^i$
$n_{o,t-1}^e$	$(1-d)[s^e(1-\delta)f^e(\theta_t^e)+(1-s^e)]$	$n_{o,t}^e$
$n_{o,t-1}^e$	$(1-d)s^e(1-\delta)(1-f^e(\theta_t^e))$	$u_{o,t}^e$
$u_{o,t-1}^e$	d	–
$u_{o,t-1}^e$	$(1-d)f^e(\theta_t^e)$	$n_{o,t}^e$
$u_{o,t-1}^e$	$(1-d)(1-f^e(\theta_t^e))$	$u_{o,t}^e$
$n_{o,t-1}^i$	d	–
$n_{o,t-1}^i$	$(1-d)[s^i f^i(\theta_t^i)+(1-s^i)]$	$n_{o,t}^i$
$n_{o,t-1}^i$	$(1-d)s^i(1-f^i(\theta_t^i))$	$u_{o,t}^i$
$u_{o,t-1}^i$	d	–
$u_{o,t-1}^i$	$(1-d)f^i(\theta_t^i)$	$n_{o,t}^i$
$u_{o,t-1}^i$	$(1-d)(1-f^i(\theta_t^i))$	$u_{o,t}^i$

graphic trend that is based on the empirical evidence. We find that the results in that model remain similar to those in our baseline model. The channel through expectation is largely irrelevant in our baseline model, and thus an important virtue of our baseline model is that we can highlight our novel channel that works only through worker heterogeneity and search frictions.

3.1 Worker Transitions

In the economy, there are young and old workers. A mass of ϕh_{t-1}^y young workers is born every period and enters the labor market as jobless. Note that ϕ represents the "entry rate" expressed as a share of young workers in the previous period h^y_{t-1}. In our model, only young workers can reproduce, and this specification allows us to map the observed birth rate into ϕ. The key quantitative experiment below entails tracing the effects of a large decline in this variable.

Young workers become old with probability μ every period. Old workers die with probability d every period.[14] When young workers become old, they also become "experienced," having higher labor productivity by a factor of $1 + \gamma$. Jobs are subject to exogenous job destruction risks.[15] When jobs are destroyed, workers enter the matching market and look for a new job opportunity. We capture the specificity of human capital by assuming that experienced (i.e., old) workers lose their skills at the time of separation with probability δ. When they are hit by this shock, they become "inexperienced," losing their productivity premium γ.[16]

Note that if the skill is fully job specific, job separation results in a complete skill loss. With probability $1 - \delta$, the worker remains experienced and can be hired as an experienced worker, retaining the productivity premium. This structure implies that there are three types of workers in the model: (i) young and inexperienced workers, (ii) old and experienced workers, and (iii) old and inexperienced workers. Note that all young workers are inexperienced, and thus the terms "young" and "inexperienced" are equivalent in the model. Also, all experienced workers are old workers. However, there are old workers who used to be experienced but are currently inexperienced because of the skill loss that occurred at the time of job loss.[17]

We assume that the matching market is divided by skill level, meaning that experienced workers and inexperienced workers look for a job in two separate labor markets. In other words, firms hire workers separately for different types of jobs: (i) jobs that require previous experience and thus are suitable only for experienced (and hence old) workers, and (ii) entry-level jobs for which any workers can be employed. We call the matching market for the first type of jobs the "E-matching market" and for the latter type of jobs the "I-matching market." Each jobless worker finds a job with probability either $f^e(\theta^e_t)$ or $f^i(\theta^i_t)$, depending

[14] See Cheron et al. (2013) and Esteban-Pretel and Fujimoto (2012) for search/matching models with a more explicit demographic structure.

[15] In the model, job destruction and job separation are equivalent, and we use these terms interchangeably.

[16] Ljungqvist and Sargent (1998) first introduced the formulation to capture some specificity of human capital. But the model itself is silent about the nature of this specificity (for example, weather it is firm specific or occupation specific). We interpret the parameter δ as capturing the loss of firm-specific skills. Fujita (2018) uses a model that features a similar parameter and interprets it as capturing the loss of occupation-specific skill. His focus is on the US labor market, where the human capital formation tends to be associated with the worker's occupation tenure. See the discussion in Section 3.1.1 in Fujita (2018).

[17] We do not allow for old workers to regain their skills, once they are hit by the δ shock. This specification is intended to capture the key characteristic of the Japanese labor market, where workers tend to choose their "lifetime job" upon graduating from school, and if they lose the job after staying there for many years, it is difficult to rebuild their career.

on whether they undertake a job search in the E-matching market or I-matching market. Job finding probabilities are a function of labor market tightness in the respective matching market (θ_t^e and θ_t^i).[18]

Timing of events. We adopt the following timing of events in each period.

1. Demographic transitions occur.
2. Job destruction occurs. If the worker is old, the worker may lose their skill with probability δ at this point.
3. Job search takes place if the worker is jobless; whether or not the worker finds a job is then determined.
4. Production takes place.

Note that the worker who lost their job can possibly find a new job within the period. This assumption is often used in the literature. Table 2 lists all possibilities. The variables n and u, respectively, denote employment and unemployment. The subscripts y and o indicate the age group, young and old. The superscripts for the old worker e and i indicate the skill group, experienced and inexperienced. Job destruction probabilities are denoted by s with superscripts i and e, indicating that the two skill groups are subject to potentially different job destruction probabilities.

In addition to the transition probabilities listed in Table 2, the mass $\phi h_{y,t-1}$ enters the labor market and is added to $u_{y,t-1}$ and becomes $u_{y,t}$ in period t. Given worker transitions summarized in Table 2, the total number of young workers $h_{y,t}(= n_{y,t} + u_{y,t})$, the number of old experienced workers ($h_{o,t}^e = n_{o,t}^e + u_{o,t}^e$), the number of old inexperienced workers ($h_{o,t}^i = n_{o,t}^i + u_{o,t}^i$), the number of old workers ($h_{o,t} = h_{o,t}^e + h_{o,t}^i$), and finally the overall population (labor force, $h_{y,t} + h_{o,t}$), respectively, evolve according to:

$$h_{y,t} = (1-\mu)h_{y,t-1} + \phi h_{y,t-1}, \tag{1}$$

$$h_{o,t}^e = (1-d)(h_{o,t-1}^e - s^e \delta n_{o,t-1}^e) + \mu h_{y,t-1}, \tag{2}$$

$$h_{o,t}^i = (1-d)(h_{o,t-1}^i + s^e \delta n_{o,t-1}^e), \tag{3}$$

$$h_{o,t} = (1-d)h_{o,t-1} + \mu h_{y,t-1}, \tag{4}$$

$$h_t = h_{t-1} + \phi h_{y,t-1} - d h_{o,t-1}. \tag{5}$$

Note that the number of job seekers in each of the two matching markets is not equal to "u", because those who are separated at the beginning of each period can start looking for a

[18] We exclude the possibility that experienced workers look for a job in the I-matching market. Such an incentive does not exist under plausible calibrations. In our quantitative exercises below, we ensure that experienced workers are better off looking for a job in the E-matching market.

job within the period. Let $\bar{u}_t^i$ and $\bar{u}_t^e$ be the number of job seekers in the I- and E-matching markets, respectively. These variables are written as:

$$\bar{u}_t^i = (1-\mu)[u_{y,t-1} + s^i n_{y,t-1}] + (1-d)[\delta s^e n_{o,t-1}^e + s^i n_{o,t-1}^i + u_{o,t-1}^i], \tag{6}$$

$$\bar{u}_t^e = \mu[u_{y,t-1} + s^e n_{y,t-1}] + (1-d)[(1-\delta)s^e n_{o,t-1}^e + u_{o,t-1}^e]. \tag{7}$$

The terms inside the first square brackets in Equation (6) correspond to the number of young workers who were unemployed at the end of the previous period and who have just lost their job at the beginning of this period, respectively. The terms in the second square brackets are the number of old inexperienced job seekers. A similar interpretation applies to Equation (7). We define the share of old inexperienced workers within $\bar{u}_t^i$ as $\omega_{o,t}$:

$$\omega_{o,t} \equiv \frac{(1-d)[\delta s^e n_{o,t-1}^e + s^i n_{o,t-1}^i + u_{o,t-1}^i]}{\bar{u}_t^i}. \tag{8}$$

The matching function in each market, indicated by the superscript i or e, takes the Cobb-Douglas form $m_t^i = \overline{m}^i \left(\bar{u}_t^i\right)^{\alpha} \left(v_t^i\right)^{1-\alpha}$ and $m_t^e = \overline{m}^e \left(\bar{u}_t^e\right)^{\alpha} \left(v_t^e\right)^{1-\alpha}$ where v_t^i and v_t^e are the number job openings posted in the I- and E-matching markets, respectively, and $\bar{m}^i$ and $\bar{m}^e$ are scale parameters. The job finding probabilities in the two job search pools are written as $f^i(\theta_t^i) = \overline{m}^i \left(\theta_t^i\right)^{1-\alpha}$ and $f^e(\theta_t^e) = \overline{m}^e \left(\theta_t^e\right)^{1-\alpha}$, where θ_t^i and θ_t^e represent labor market tightness in the two matching markets and are defined as: $\theta_t^i = \frac{v_t^i}{\bar{u}_t^i}$ and $\theta_t^e = \frac{v_t^e}{\bar{u}_t^e}$. Last, we can also define the job filling probability for each job opening posted in the two markets as $q^i(\theta_t^i) = \overline{m}^i \left(\theta_t^i\right)^{-\alpha}$ and $q^e(\theta_t^e) = \overline{m}^e \left(\theta_t^e\right)^{-\alpha}$, respectively.

3.2 Firms

Each firm combines the three types of labor to produce the consumption good y_t as follows:

$$y_t = n_{y,t} + n_{o,t}^i + (1+\gamma) n_{o,t}^e. \tag{9}$$

The firm's profit maximization is subject to search frictions in hiring workers. Regarding the latter, the firm pays κ^i and κ^e per vacancy posted in I- and E-matching markets, respectively. The firm maximizes its value by choosing $n_{y,t}, n_{o,t}^i, n_{o,t}^e, v_t^i$, and v_t^e.

The firm's optimal decision is characterized by the following first-order conditions. The following two equations govern job creation (vacancy posting) in the two matching markets:

$$\omega_{o,t} J_{o,t}^i + (1-\omega_{o,t}) J_{y,t} = \frac{\kappa^i}{q^i(\theta_t^i)}, \tag{10}$$

$$J_{o,t}^e = \frac{\kappa^e}{q^e(\theta_t^e)}, \tag{11}$$

where $J_{y,t}$, $J_{o,t}^i$, and $J_{o,t}^e$ represent marginal gains to the firm from adding each of the three types of labor. The LHS of Equation (10) indicates that the gain from posting a vacancy in the I-matching market is influenced by the composition of the matching market $\omega_{o,t}$. In particular, if $J_{y,t} > J^i_{o,t}$ holds, then a higher $\omega_{o,t}$ would lower the value of the LHS and thus, in equilibrium, market tightness θ^i_t decreases. Last, the marginal values follow:

$$J_{y,t} = 1 - w_{y,t} + \hat{\beta}_{t,t+1}\Big[(1-\mu)(1-s^i)J_{y,t+1} + (1-s^e)\mu J^e_{o,t+1}\Big], \tag{12}$$

$$J^e_{o,t} = 1 + \gamma - w^e_{o,t} + (1-d)(1-s^e)\hat{\beta}_{t,t+1}J^e_{o,t+1}, \tag{13}$$

$$J^i_{o,t} = 1 - w^i_{o,t} + (1-d)(1-s^i)\hat{\beta}_{t,t+1}J^i_{o,t+1}. \tag{14}$$

The interpretation of these three equations is straightforward.

3.3 Household

The household sector consists of different types of workers in terms of their age, their skill level, and their labor market status. We assume that the representative household pools incomes of all members and allocates consumption across its members equally. The household is also assumed to be fully altruistic toward its future members as well. These assumptions imply that the member's age, skill, and labor market status do not matter for their consumption.[19]

The household maximizes the following value function $V(.)$:

$$V(\mathbf{s}^h_t) = \max h_t u\left(c_t\right) + \beta V(\mathbf{s}^h_{t+1}), \tag{15}$$

where c_t is per capita consumption. $\mathbf{s}^h_t$ is the vector of the state variables for the household. Recall that h_t represents the population in the economy, and thus Equation (15) implies that the household maximizes the total welfare of the dynasty into the indefinite future, with the future periods discounted by a common discount factor β per period. All employed workers supply labor inelastically with the same constant disutility level (regardless of their type). We also assume that jobless workers look for a new job with the same constant disutility level, again regardless of their type. Accordingly, hours of work and search do not enter the value function above. The budget constraint is given by:

$$h_t c_t + B_{t+1}h_t = \left(1 + r_t\right)B_t h_{t-1} + w_{y,t}n_{y,t} + w^e_{o,t}n^e_{o,t} + w^i_{o,t}n^i_{o,t} + b_y u_{y,t} + b^e_o u^e_{o,t} + b^i_o u^i_{o,t},$$

where B_t represents the per capita real bond holdings; r_t denotes the real interest rate; and b_y, b^e_o, and b^i_o represent unemployment insurance benefits that each type of jobless worker receives at the end of the period. The state variables of this problem $\mathbf{s}^h_t$ include $n_{y,t-1}$, $n^e_{o,t-1}$, $n^i_{o,t-1}$, $h^e_{o,t-1}$, and B_t.[20]

[19] We can extend our model by allowing for heterogeneity in the consumption side of the model, as in Fujiwara and Teranishi (2008) and Carvalho et al. (2016).

[20] Among the labor market stocks, only four are independent, and the remaining variables are substituted out when writing down the marginal value functions below.

The household's optimal decision is characterized by the standard consumption Euler equation:

$$c_t^{-\varepsilon} = \beta\left(1 + r_{t+1}\right) c_{t+1}^{-\varepsilon}, \tag{16}$$

where ε is the coefficient of relative risk aversion.

Marginal value functions. To simplify the notation, let us introduce the following four variables: $M_{y,t} \equiv \frac{\partial V(\mathbf{s}_t^h)}{\partial n_{y,t-1}}$, $M_{o,t}^e \equiv \frac{\partial V(\mathbf{s}_t^h)}{\partial n_{o,t-1}^e}$, $M_{o,t}^i \equiv \frac{\partial V(\mathbf{s}_t^h)}{\partial n_{o,t-1}^i}$, and $D_t \equiv \frac{\partial V(\mathbf{s}_t^h)}{\partial h_{o,t-1}^e}$. These variables follow:

$$\begin{aligned} M_{y,t} &= u'(c_t)(w_{y,t} - b_y) \\ &\quad + \beta\Big[(1-\mu)(1-s^i)(1-f^i(\theta_{t+1}^i))M_{y,t+1} + \mu(1-s^e)(1-f^e(\theta_{t+1}^e))M_{o,t+1}^e\Big], \end{aligned} \tag{17}$$

$$M_{o,t}^i = u'(c_t)(w_{o,t}^e - b_o^e) + (1-d)(1-s^i)(1-f^i(\theta_{t+1}^i))\beta M_{o,t+1}^i, \tag{18}$$

$$\begin{aligned} M_{o,t}^e = u'(c_t)(w_{o,t}^e - b_o^e) + (1-d)\beta\Big[&(1-s^e)(1-f^e(\theta_{t+1}^e))M_{o,t+1}^e \\ &- s^e\delta\big\{f^e(\theta_{t+1}^e)M_{o,t+1}^e - f^i(\theta_{t+1}^i)M_{o,t+1}^i + D_{t+1}\big\}\Big], \end{aligned} \tag{19}$$

$$D_t = u'(c_t)(b_o^e - b_o^i) + (1-d)\beta\Big[f^e(\theta_{t+1}^e)M_{o,t}^e - f^i(\theta_{t+1}^i)M_{o,t+1}^i + D_{t+1}\Big]. \tag{20}$$

Equations (17) and (18) give the marginal values of each type of employment to the household net of having one more unemployed worker of the corresponding type. In Equation (17), the first term represents the flow surplus of young worker employment, and the terms that follow capture the future possibilities that the worker stays young with probability $1-\mu$ or that the worker becomes old and experienced with probability μ. A similar interpretation applies to Equation (18). In Equation (19), the terms in the curly brackets capture the possibility of the skill loss that occurs with probability $s^e\delta$. The value D_t, given by (20), corresponds to the difference in the values of being unemployed as experienced and inexperienced old workers.[21]

3.4 Wages

We assume that each type of worker and the firm engage in Nash bargaining individually. Let $S_{y,t}$ be the joint surplus of the match between the young worker and the firm, i.e., $S_{y,t} = J_{y,t} + \frac{1}{u'(c_t)}M_{y,t}$. This surplus is split between the worker and the firm according to the worker bargaining power η and the firm bargaining power $1-\eta$. Using (12) and (17) in $\eta J_{y,t} = (1-\eta)\frac{1}{u'(c_t)}M_{y,t}$ and solving for wage, we obtain the following expression:

$$w_{y,t} = \eta + (1-\eta)b_y + \eta\hat{\beta}_{t,t+1}\Big[(1-\mu)(1-s^i)f^i(\theta_{t+1}^i)J_{y,t+1} + \mu(1-s^e)f^e(\theta_{t+1}^e)J_{o,t+1}^e\Big]. \tag{21}$$

[21] Note that $D_t = \frac{\partial V(\mathbf{s}_t^h)}{\partial h_{o,t-1}^e} = \frac{\partial V(\mathbf{s}_t^h)}{\partial u_{o,t-1}^e} - \frac{\partial V(\mathbf{s}_t^h)}{\partial u_{o,t-1}^i}$.

Applying similar algebra to the other two types of matches results in:

$$w^i_{o,t} = \eta + (1-\eta)b^i_o + (1-d)(1-s^i)\eta\hat{\beta}_{t,t+1}f^i(\theta^i_{t+1})J^i_{o,t+1} \tag{22}$$

$$\begin{aligned} w^e_{o,t} = \eta(1+\gamma) + (1-\eta)b^e_o + (1-d)\hat{\beta}_{t,t+1}\Bigg[&(1-s^e)f^e(\theta^e_{t+1})\eta J^e_{o,t+1}, \\ &+ \delta s^e\left\{f^e(\theta^e_{t+1})\eta J^e_{o,t+1} - f^i(\theta^i_{t+1})\eta J^i_{o,t+1} + \frac{D_{t+1}}{u'(c_{t+1})}\right\}\Bigg]. \end{aligned} \tag{23}$$

3.5 Resource Constraint

Assuming zero net supply of the bonds in the aggregate and $b_y u_{y,t} + b^e_o u^e_{o,t} + b^i_o u^i_{o,t} = T_t$ results in the following resource constraint:

$$Y_t = h_t c_t + \kappa^i v^i_t + \kappa^e v^e_t.$$

3.6 Detrending

In the model, population growth is nonzero in general. Therefore, nonstationary equations need to be detrended appropriately by h_t before the model is solved and the perfect foresight equilibrium path is computed. The detrended system of equations are presented in the Appendix A.[22]

4 Calibration

The calibration below intends to replicate the Japanese economy at the start of the aging process, assuming that the economy is in the steady state at that point. In the quantitative exercises below, we compute the transition path of the economy when labor force entry ϕ_t follows the path of its empirical counterpart. One period in the model is assumed to be one quarter in the actual economy. Note also that the first period in our experiment corresponds to the first quarter of 1970.

4.1 Demographic Transitions

We assume that each worker in the model spends on average 20 years as a "young" worker and 25 years as an "old" worker. What we have in mind as a career of a typical worker is that they enter the labor force when they are 20 years old, accumulate their experience over the next 20 years, become an experienced worker when they are 40 years old, spend the rest of

[22] Our extended model below features exogenously growing neutral technology as well and thus we also detrend the model with technology.

their career as an old worker, and then retire (or die) at 65 years old. This transition implies that $\mu = 1/80 = 0.0125$ and $d = 0.01$. Next, we translate the total fertility rate (TFR) into the labor force entry rate ϕ as follows. Note that the TFR represents the number of children who are born to a woman over her childbearing years, and it was above 4 at the peak in 1948. Suppose that childbearing years correspond to the 20-year period that each worker spends as a young worker.[23] The TFR of 4 implies that each young worker reproduces 2 young workers over this 20-year period, implying $\phi = 0.025$ (=2/80). But given that our young worker's definition is somewhat narrower than the actual childbearing years, we choose to use $\phi = 0.02$. Note that the steady state of the economy is characterized by a constant population growth. Specifically, Equation (1) implies that the growth rate is given by $g = \phi - \mu = 0.02 - 0.0125 = 0.0075$, where $g = \frac{h_t}{h_{t-1}} - 1$.

4.2 Labor Market Transitions

Next, let us discuss the calibration of parameter values related to labor market transitions. Transition rates are all expressed as quarterly values. First, to set separation rates, we refer to the several pieces of evidence presented by Lin and Miyamoto (2012) and Esteban-Pretel and Fujimoto (2012). Lin and Miyamoto (2012) show that the monthly aggregate job separation rate into unemployment averaged roughly around 0.4 percent over the period between 1980 and 2010. Based on this, we target the quarterly separation rate of 1.2 percent per quarter. The paper by Esteban-Pretel and Fujimoto (2012) shows that the separation rate declines as workers get older. A visual inspection of their result suggests that the calibration in which the young worker's separation rate is roughly twice as high as that of the old worker is plausible. These two considerations allow us to uniquely pin down s^i and s^e at 1.5 percent and 0.75 percent (per quarter). Next, we target the steady-state job finding rates $f^i(\theta^i)$ and $f^e(\theta^e)$ both at 35 percent. This value is roughly consistent with the monthly evidence presented by Lin and Miyamoto (2012). We also impose that, in the steady state $q^i(\theta^i) = f^i(\theta^i)$ and $q^e(\theta^e) = f^e(\theta^e)$. These assumptions allow us to determine the scale parameters of the matching functions $\overline{m}^i$ and $\overline{m}^e$ for a given value of α. By setting $\alpha = 0.5$, we can set both $\overline{m}^i$ and $\overline{m}^i$ to 0.35. The only remaining parameter within the steady-state stock-flow relationships is δ, which measures the risk of the skill loss. We simply set this parameter at 0.8 in our baseline calibration and show how the model's behavior changes as we lower its value and consider the sensitivity of our results with respect to a lower value.[24]

[23] This assumption is not exactly correct because childbearing years in the calculation of the TFR are between 15 and 49 years old. But we are considering only the middle 20-year period for simplicity.

[24] In calibrating our extended model that features two δ's, we bring in more information, which allows us to pin down the values of these two parameters. We find 0.8 in the current model being plausible. See Section 6.2 for more detail.

4.3 Remaining Parameters

The discount factor is set to 0.995, which implies that the steady-state annual real interest rate is 2 percent. Our choice is based on the evidence in Figure 3 where the real interest rate fluctuated around that level over the sample period. Note that, in our model, the steady-state real interest rate is invariant to the demographic structure. However, changes in the demographic structure induce slow-moving fluctuations in the real interest rate over a long period, which is how we address its effects. Note also that our results are largely insensitive to the steady-state level of the real interest rate. That is, whether we set the steady-state real interest rate at 4 percent or 1 percent instead of 2 percent has virtually no impact on the changes in the real interest rate along the transition path. The value of the coefficient of relative risk aversion (ε) is set to 2, a value that is within a plausible range in macro literature. It is also consistent with the time-series evidence on consumption growth and the real interest rate presented in Figure 3, where one can see that the volatility of the real interest rate is roughly twice as large as that of the growth rate of per capita consumption.

We assume that the worker bargaining power η is 0.5, which is often used in the literature, given the lack of direct evidence. We set the experience premium γ at 60 percent, which is based on the observed wage profile in Japan. The observed profile from the "Basic Survey on Wage Structure in Japan" shows that old workers (those who are between 40-64) on average make roughly 40 percent more than young workers (those who are between 20-39). Note that the slope of the observed profile does not directly measure $1 + \gamma$, because old workers in our model include those who have lost their firm-specific skill and thus have become inexperienced. By setting $\gamma = 0.6$, we can match the observed wage premium of 40 percent. We set the level of unemployment insurance benefits by assuming that each worker receives 60 percent of their productivity while unemployed. This procedure implies that $b_y = 0.6$ and $b_o^e = (1 + \gamma) \times 0.6 = 0.96$, given that productivity of the young worker is normalized to 1 and that the old experienced worker enjoys a 60 percent productivity premium. Regarding the old inexperienced worker, we assume that their benefit level is tied to their productivity level as an experienced worker, meaning that $b_o^i = b_o^e$.[25] The OECD reports that the benefits level as a share of previous income has historically been roughly constant at 65-70 percent in Japan, which is roughly consistent with our calibration.[26] Given the parameter values so far, we can compute all continuation values in the steady state of the economy. Using the steady-state values of J_y, J_o^e, and J_o^i in the free-entry conditions (10) and (11), we back out the vacancy posting costs in the two matching markets. This procedure yields $\kappa_i = 0.251$ and $\kappa_e = 0.337$.

[25]The assumption is reasonable, given the actual unemployment insurance benefit scheme in which the benefit level is tied to the worker's earnings prior to job loss.

[26]See https://data.oecd.org/benwage/benefits-in-unemployment-share-of-previous-income.htm.

Table 3: Initial Steady-State Values

	Average Wages		Employment Levels		Unemployment Rates	
Aggregate	$\frac{n_y w_y + n_o^e w_o^e + n_o^i w_o^i}{n_y + n_o^e + n_o^i}$	0.998	$\frac{n_o^i + n_o^e + n_y}{h}$	0.940	$\frac{u_o^i + u_o^e + u_y}{h}$	0.060
Young	w_y	0.818	$\frac{n_y}{h}$	0.537	$\frac{u_y}{h_y}$	0.079
Old	$\frac{n_o^e w_o^e + n_o^i w_o^i}{n_o^e + n_o^i}$	1.194	$\frac{n_o^e + n_o^i}{h}$	0.403	$\frac{u_o^e + u_o^i}{h_o^e + h_o^i}$	0.034
Old Exp.	w_o^e	1.310	$\frac{n_o^e}{h}$	0.305	$\frac{u_o^e}{h_o^e}$	0.026
Old Inexp.	w_o^i	0.831	$\frac{n_o^i}{h}$	0.098	$\frac{u_o^i}{h_o^i}$	0.056

4.4 Initial Steady State

Table 3 presents steady-state values of wages, employment levels, and unemployment rates at different levels of aggregation in the initial steady state. Note that employment levels are presented as a share of the total labor force (population), while unemployment rates are expressed as a share of the labor force of each type.

At the aggregate level, the unemployment rate is 6 percent, which is higher than the levels observed in the 1970s and 1980s. The main reason for this is that our model assumes that all young workers enter the labor force as jobless, and in the initial steady state, the entry rate is assumed to be very high. One can see that young workers' unemployment rate is 7.9 percent, while old workers' unemployment rate is much lower at 3.4 percent. Within old workers, experienced workers' unemployment rate is 2.6 percent, while inexperienced workers' unemployment rate is 5.6 percent. The reason for the higher unemployment rate for the latter group is that 80 percent (δ) of the job loss flow from experienced employment goes into this I-matching pool in our calibration.

Steady-state employment levels indicate that within all employed workers, roughly 43 percent are old workers and the rest are young workers. Roughly three-quarters of old workers are experienced (30 percent as a share of the labor force) and one-quarter are inexperienced. The average wage of all workers is about 1. Note that we normalize average productivity of the young worker and the old inexperienced worker at 1, and the experienced worker enjoys a 60 percent skill premium. The average wage of experienced workers is the highest, given the skill premium. There are two reasons why w_o^i and w_y differ from each other, even though their productivities are equal. First, the young worker's current wage incorporates the possibility that they become experienced in the future, which raises w_y relative to w_o^i, whereas the old worker's continuation value drops to zero, when hit by the d shock. However, the old worker's wage is pushed up because their flow outside option value b_o^i is higher (remember that we set b_o^i by linking it to their productivity as an experienced worker). The latter effect dominates the first effect and thus w_o^i is higher than w_y in our calibration. The average wage of all old workers is roughly 40 percent higher than the young worker's wage. As mentioned above, this measured age premium is consistent with the observed wage profile in Japan.

5 Quantitative Exercises

In this section, we analyze the perfect foresight equilibrium path of the economy, assuming that the labor force entry rate falls to a new steady-state value. The assumed path of the entry rate mimics the path of the series plotted in Figure 1. Demographic transitions take place only gradually, and thus it takes a long time for the economy to converge to a new steady state, even though ϕ_t itself reaches a new value relatively quickly.

5.1 Labor Force Entry Path

Recall that the labor force entry rate ϕ_t was set to 2 percent per quarter in the initial steady state. We assume that it falls to 1.1 percent roughly over the following 10-year period. This assumed path is plotted in Figure 4(a). The entry rate of 1.1 percent per quarter corresponds to the total fertility rate that is somewhat below 1.8, thus being roughly in line with the level of the birth rate after the baby boom. Remember that in our model, ϕ_t represents the labor force entry rate, and one can see in Figure 1(b) that labor force entry experienced a sharp drop throughout the 1970s. The assumed path of ϕ_t captures this decline over the 10-year period, and we associate period 1 in our model experiment with the first quarter of 1970.

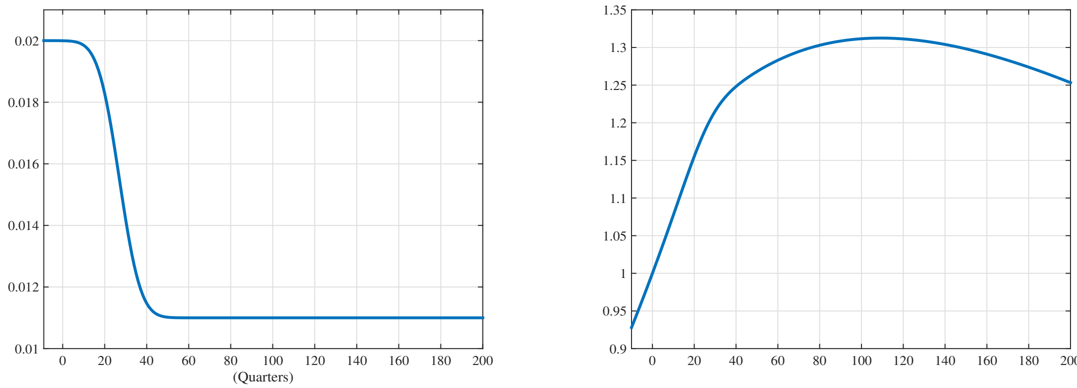
Importantly, with this assumed path of the entry rate, the labor force begins to fall at roughly the same timing as in the actual data. Figure 2(b) earlier showed that the Japanese labor force started shrinking in the late 1990s. The labor force in the model peaks around the 110th quarter, which corresponds to the late 1990s in our interpretation of the first period being the first quarter of 1970 (Figure 4(b)).

5.2 Labor Market Responses to Aging

Figure 5 presents the perfect foresight equilibrium paths of the key labor market variables. Labor productivity increases in the initial phase of the aging process because this phase corresponds to the situation in which more and more young workers are gaining experience, thus becoming more productive. However, after this initial phase is over, labor productivity growth slows down and eventually turns negative. The slowdown occurs simply because there are fewer young workers (whose future productivity is higher). The decline in the level of productivity in the later phase of aging is due to the presence of the firm-specific nature of the workers' skill in our model. Remember that old (experienced) workers are subject to the risk of job separation that is accompanied by the loss of their skills. As the

Figure 4: The Entry Rate and Labor Force

(a) Assumed Entry Rate Path (b) Implied Path of the Labor Force



Notes: The labor force is normalized to 1 in period 0.

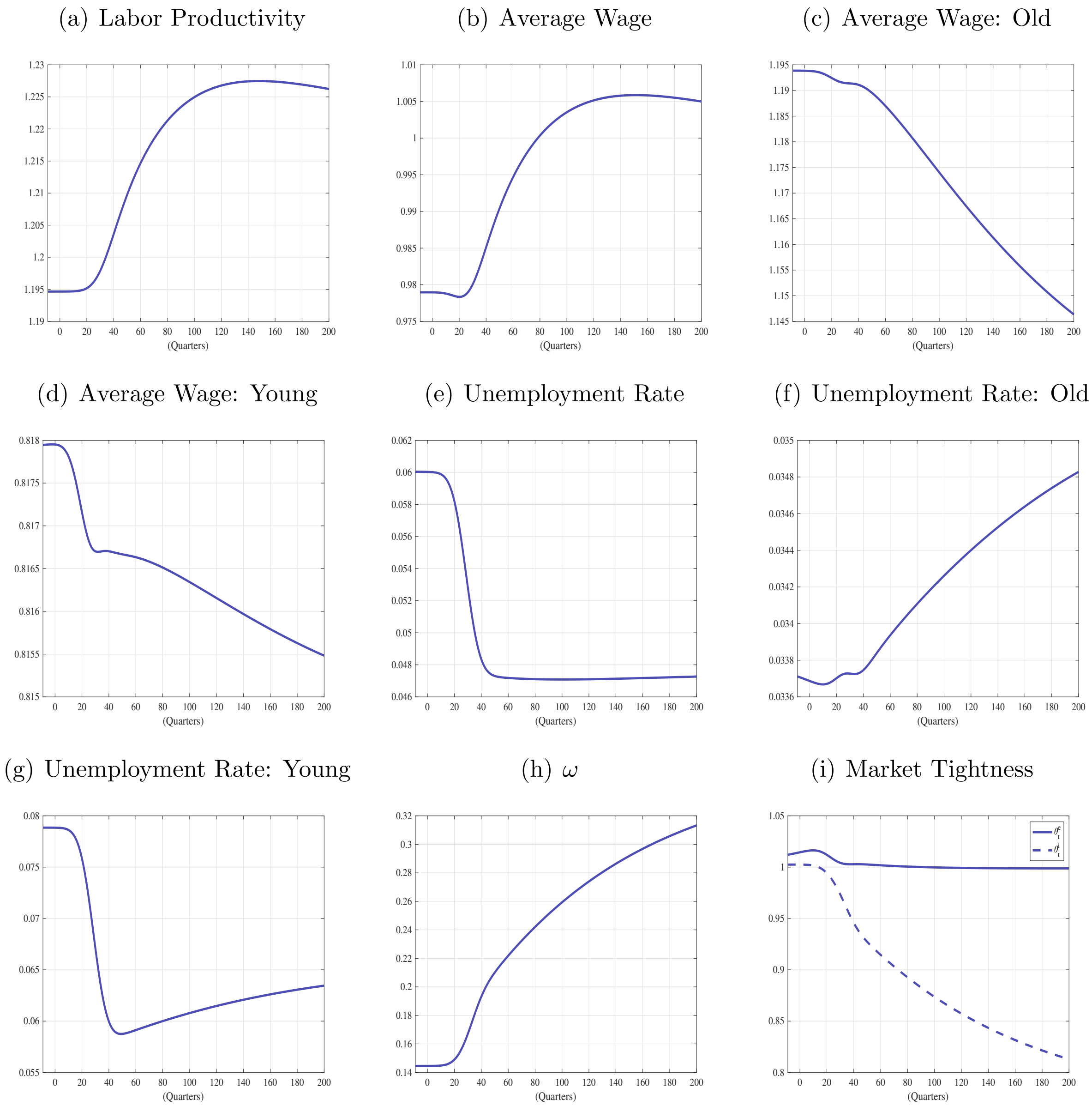
workforce gets older, there are more and more workers who are subject to this risk. When the experienced worker is hit by the δ shock, the worker is employable only at an entry-level job, and therefore, a higher share of old (experienced) workers eventually leads to a decline in labor productivity. The quantitative importance of this latter effect depends on the value of δ. Below, we will examine the impacts of the value of this parameter on our results.

The behavior of average wage is similar to that of labor productivity (Figure 5(b)). Overall, the hump-shaped pattern comes from the same mechanism behind the hump-shaped pattern in labor productivity. Note that in the data (Figure 2(c)), real wage stopped increasing in the late 1990s, around the same time as the labor force hit its peak. In our model, average wage hits the peak around 140th period, which corresponds to the mid 2000s, about 10 years after the peak of the labor force. Nevertheless, the overall comovement pattern between the labor force and average wage in our model is consistent with the empirical evidence.[27]

Average wage of old workers steadily fall over time (Figure 5(c)). The main reason for this result is the changing composition within these old workers. As the aging of the labor force progresses, the share of inexperienced workers (within old workers) increases, thereby putting downward pressure on the average wage of old workers. The average wage of young workers also falls over time, as shown in (d), which results from the increase in ω, the share of old inexperienced workers in their matching market. A higher ω lowers job creation in this market, because wages of old inexperienced workers are higher than those of young workers, and hence, a higher ω translates into a lower return to creating new jobs and lowering the outside option for all inexperienced workers. In Bean (2004), the author discusses several

[27]Note that our baseline model assumes no TFP growth. But with nonzero neutral technology growth (as in our extended model below), wages of all types also grow at the same rate. Lower TFP growth in the early 1990s would translate into a slowdown in wage growth in our model.

Figure 5: Perfect Foresight Equilibrium Paths (I)



(a) Labor Productivity (b) Average Wage (c) Average Wage: Old

(d) Average Wage: Young (e) Unemployment Rate (f) Unemployment Rate: Old

(g) Unemployment Rate: Young (h) ω (i) Market Tightness

potential labor market effects of aging, mentally applying the labor matching framework. The externality on young workers' wages we have just discussed is one of those effects, but, as can be seen in (d), the effect is quantitatively small. The largest effect on the aggregate wage behavior originates from the changing composition of workers itself rather than the equilibrium effects due to search frictions.

The aggregate unemployment rate falls rather sharply initially and then stays roughly flat thereafter. The initial fall in the aggregate unemployment rate comes directly from the decline in labor force entry. Remember that young workers enter the labor force as unemployed and thus a smaller volume of this flow directly reduces the unemployment rate. The effect can be seen more clearly in (g), where the unemployment rate of the young workers drops almost 2 percentage points. However, young workers' unemployment increases after the initial drop. The reason for the upturn is the increase in ω. As explained in the previous paragraph, the increase in the share of old workers in their matching market discourages job creation in that market (the dashed line in (h)), thus resulting in a higher unemployment rate for the young workers. The old workers' unemployment rate steadily increases over time, and the main reason for this is again a higher share of inexperienced workers within all old workers. The level of the inexperienced workers' unemployment rate is higher than that of experienced workers (mainly because $s_e < s_i$), and thus a higher share of inexperienced workers results in a higher unemployment rate among old workers.[28]

5.3 Consumption and the Real Interest Rate

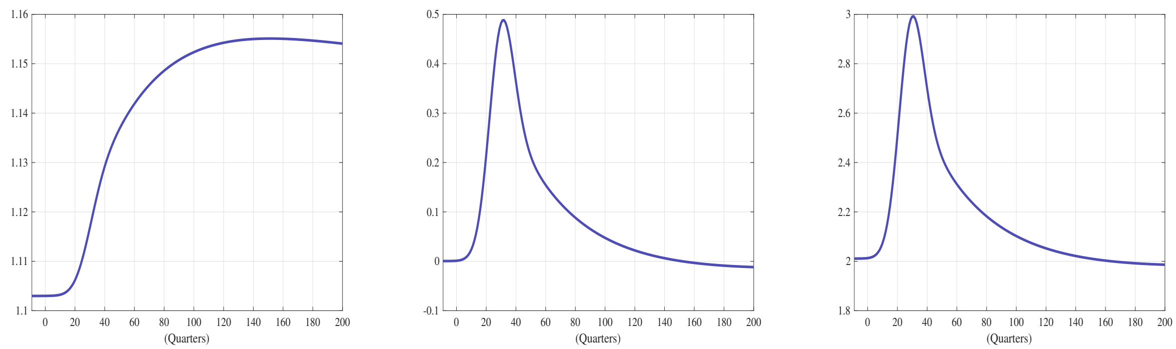
The per capita consumption level (Figure 6(a)) again takes a path similar to labor productivity. Per-capita consumption growth, expressed as the annualized growth rate in percent, increases roughly 0.5 percentage point at its peak and then gradually decreases. Given the value of the CRRA parameter of 2, the real interest rate rises about 1 percentage point and follows a similar gradual declining path. Note that the peak of these two variables occurs roughly 7 to 8 years after ϕ begins to fall. This timing corresponds to the late 1970s in the data. Recall that Table 1 shows that consumption growth and the real interest rate fell 1.3 percentage points and 2.3 percentage points, respectively, between the 1980s and 2000s. Our experiment implies that the aging of the labor force accounts for roughly 40 percent of these declines. Another subtle but important observation in Figure 6(b) and (c) is that, in the later phase of aging (from around the 150th quarter on), consumption growth turns negative (before coming back to its steady-state level again from below).[29] This negative growth occurs only in the economy with high δ as in our baseline calibration. As discussed with respect to the path of labor productivity, there are more and more old workers who are (re-)employed only in an entry-level job.

[28]Note that the unemployment rate in Japan increased substantially in the 1990s, and the model does not replicate the increase. This is not surprising, given that the model is not designed to capture the cyclical effects of lower aggregate demand or TFP. We will discuss this issue below, when we study the extended model with TFP growth.

[29]The negative consumption growth path corresponds to the path of the real interest rate that falls below its steady-state value of 2 percent, as can be seen in (c).

Figure 6: Perfect Foresight Equilibrium Paths (II)

(a) Consumption (b) Consumption Growth (c) Real Interest Rate



Notes: Expressed as annualized percent.

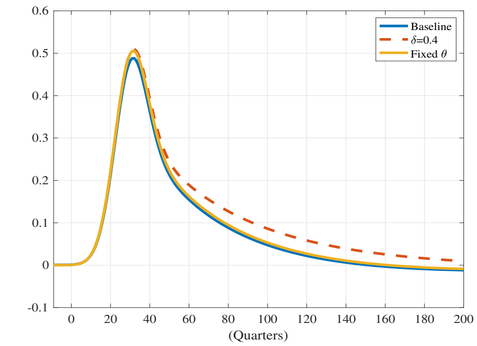
5.4 The Role of Skill Loss Probability and Search Frictions

One of the key features of our model is the possibility of the skill loss at the time of job loss among experienced workers. In our baseline calibration, we set this parameter to 0.8. We now present the results under an alternative value of this parameter, $\delta = 0.4$. This case is useful in highlighting the quantitative importance of skill specificity in our model. This section also studies the quantitative importance of the interaction between aging and labor market search frictions. We do so by solving for the equilibrium path, assuming that tightness measures in the two matching markets θ_t^e and θ_t^i are fixed at their respective initial steady-state levels. This assumption means that we drop the job creation conditions from the model and that the main channel that operates in the model is the changing workforce composition.

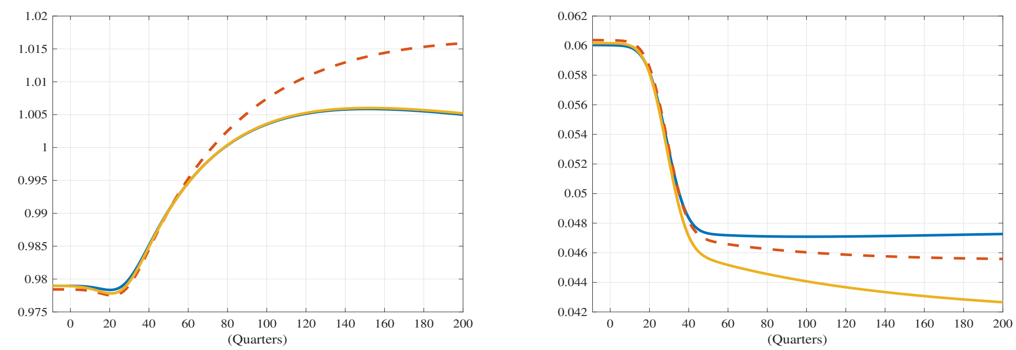
Figure 7 compares the paths of several variables under the baseline and two alternative calibrations. Under the lower δ value, consumption grows somewhat more in the initial stage of aging than under the baseline calibration, but more important, the deceleration of consumption growth occurs more gradually. The latter fact is a direct consequence of the smaller value of δ. As we discussed above, the source of the decline in the consumption level in the baseline case is a high value of this specificity parameter. With the smaller value of this parameter, consumption growth never turns negative, as shown in the first panel. The path of aggregate wage is consistent with that of consumption growth. It grows somewhat more rapidly initially than it does in the baseline case, and it keeps rising, albeit at a slower pace. Regarding the unemployment rate, it drops rapidly initially as before. But further gradual declines follow, in contrast to a roughly flat path in the baseline case. Remember that a high value of δ in the baseline calibration puts an upward pressure on the aggregate unemployment rate, because it implies a large number of experienced workers becoming inexperienced as the aging process progresses. Conversely, a smaller value of δ reduces this

Figure 7: Effects of a Smaller Value of δ and Fixed Labor Market Tightness

(a) Consumption Growth (b) Real Interest Rate



(c) Aggregate Wage (d) Unemployment Rate



effect. Overall, however, the model dynamics do not materially change, despite a rather big change in the value of δ.

The yellow lines in Figure 7 represent the paths under fixed labor market tightness. Panels (a) through (c) strongly suggest that impacts of aging on job creation are minimal on those variables: At the aggregate level, the paths of average wage, consumption growth, and thus the interest rate are hardly affected by the changing labor market tightness. As shown earlier (Figure 5 (i)), aging has an impact of lowering the labor market tightness of the I-matching market because of higher ω, leading to declines in job finding rates of inexperienced workers. As we can see from panel (d), this has a substantial impact on the aggregate unemployment rate, even though the impacts on the other variables are small.

6 Other Long-Term Developments

While our focus is on the effects of aging, there are a few other long-term developments in Japan. First, the female labor force participation rate (between 15-64 years of age) has increased substantially in Japan over the last several decades. According to the Labour Force

Survey, it stayed around 50 percent through the 1970s but increased to above 60 percent in the 2000s and rose further to above 70 percent in recent years.[30] Thanks to this development, the overall labor force size has increased somewhat since 2012, in contrast to the prediction of our baseline model.

Second, the share of so-called non-regular workers has also increased substantially over the past three decades.[31] The share of non-regular workers within all employed workers was about 15 percent in the mid-1980s, but steadily increased over the next three decades by about 15 percentage points. Furthermore, there is a large gender gap in the shares of non-regular workers. The share among female workers increased from 30 percent to 55 percent, while the share among male workers rose from 8 percent to 22-23 percent over the same period.

Third, it is well documented that TFP growth slowed down substantially after the burst of the bubble economy (see Hayashi and Prescott (2002) and the subsequent literature). According to the Japan Industry Productivity (JIP) database, it was growing at around 1 percent through the early 1990s, but then TFP growth appears to have since settled to a lower level.[32]

Fourth, in our model, the job destruction rates are exogenous and fixed, but the data show that it rose significantly in the late 1990s (Lin and Miyamoto (2012), Esteban-Pretel et al. (2010) and Esteban-Pretel et al. (2011)).

In this section, we extend our model to at least partially accommodate these long-run changes. Specifically, the major changes from our baseline model are (i) adding gender heterogeneity on top of existing skill heterogeneity and (ii) exogenously growing TFP. We do not explicitly model non-regular workers and keep job destruction rates exogenous in the extended model as well. However, as discussed below, we use the extended model to inform us of the potential impacts of the higher share of non-regular workers and higher job separation rates.

6.1 Extended Model

We present the full details of the extended model in the Appendix C, but, in essence, it features two blocks, each of which, respectively, corresponds to the labor market for female and male workers. The structure of each block remains largely the same as the baseline model. We assume that female and male inexperienced workers are identical in terms their

[30] Note that the overall female labor force participation rate (including 65+ population) has been mostly unchanged around 50 percent over the long period of time.

[31] The Labour Force Survey defines non-regular workers as the sum of part-time workers, temporary workers, dispatched workers from temporary labour agency, contract employees, and entrusted employees, and others.

[32] See Fukako et al. (2007) for data construction details of the TFP series. This series controls for the quality of labor input and capital utilization rates at the detailed industry level.

flow productivity, and accordingly, female and male workers compete for an entry-level job in the same matching market. But female and male experienced workers command different levels of skill premiums.

The job creation conditions are now modified as:

$$\sum_{\sigma}(\omega_{y,\sigma,t}J_{y,\sigma,t} + \omega_{o,\sigma,t}J^{i}_{o,\sigma,t}) = \frac{\kappa^{i}_{t}}{q^{i}(\theta^{i}_{t})}, \tag{24}$$

$$J^{e}_{o,\sigma,t} = \frac{\kappa^{e}_{\sigma,t}}{q^{e}_{\sigma}(\theta^{e}_{\sigma,t})}, \tag{25}$$

where σ refers to gender, taking either f for females or m for males. Equation (24) indicates all inexperienced workers (regardless of gender) look for a job in the same market, and (25) holds for each gender. As in the baseline model, ω refers to the share of a particular type of inexperienced workers in their matching market. The worker type is now distinguished by the two subscripts, age (young or old) and gender (female or male). The differences between female and male workers are reflected in the differences in some parameter values, as discussed below.

Production technology combines a total of six types of labor and features exogenous neutral technology A_t:

$$Y_t = A_t \sum_{\sigma}[n_{y,\sigma,t} + n^{i}_{o,\sigma,t} + (1+\gamma_{\sigma})n^{e}_{o,\sigma,t}],$$

where A_t grows at $z_t \equiv \frac{A_t}{A_{t-1}} - 1$. Given exogenous growth in A_t, unemployment benefits (b) and vacancy posting costs (κ), also exogenously grow at the same rate.

6.2 Calibration

We use the same values as in the baseline model for the following parameters: α (elasticity of the matching function), η (worker bargaining power), ϵ (coefficient of relative risk aversion), d (death probability), and μ (aging/skill-upgrading probability). Gender heterogeneity is reflected mainly in differences in separation rates for experienced workers (s^{e}_{σ}), experience premiums (γ_{σ}), and skill loss probabilities (δ_{σ}). In selecting these parameter values, we associate "inexperienced" workers in the model with non-regular workers in the data. This approach allows us to utilize several pieces of observable evidence in our calibration. In particular, the "Basic Survey on Wage Structure in Japan" reports wage profiles by gender and the regular/non-regular status.[33] The data reveal several important facts. First, wages-by-age profiles start at similar levels for all types (by gender and regular/non-regular status). Second, wage profiles are largely flat within the sample of non-regular workers for both

[33] See Figure 6 in the 2018 report of the survey. We use the observations from the 2018 survey for the calibration, but the basic pattern we discuss below applies to the data from the earlier surveys.

genders. Third, within regular workers, wages increase much more steeply with ages among males than among females. We can replicate these patterns by assuming that productivities of inexperienced workers within the model are the same for both genders (normalized at 1), and by setting γ_f and γ_m at 0.34 and 0.85, respectively. The job separation rate for the inexperienced workers remains the same as before (as both genders face the same separation rates when they are inexperienced). We set job separation rates for experienced workers s^e_σ and skill loss probabilities δ_σ by using the evidence on the relative shares of regular and non-regular workers within each gender. Specifically, the share of non-regular workers between 40 and 64 (as in our definition of "old" workers) was almost constant until the mid-1990s, at around 46 percent for females and around 8 percent for males. Since the share among the former group is large, we first set $\delta_f = 1$. We then choose s^e_f, s^e_m, and δ_m to replicate the shares of inexperienced old workers (i.e., 0.08 for males and 0.46 for females), while also satisfying the overall job separation rate at 0.75 percent. This procedure leads us to set $s^e_f = 0.016$, $s^e_m = 0.002$, and $\delta_m = 0.828$.

For setting unemployment benefit levels, we follow the same procedure as in the previous calibration, in which benefit levels are proportional to their productivity levels. The (de-trended) benefit levels are different from those used in the baseline model, because different experience premiums apply to female and male workers, as we have just discussed. We also set the replacement ratio (with respect to productivity) to 0.5 instead of 0.6 that is used in the baseline model for the reason explained below. This procedure implies that $b_y = 0.5$, $b^e_{o,f} = 0.5 \times (1 + \gamma_f) = 0.67$, and $b^e_{o,m} = 0.5 \times (1 + \gamma_m) = 0.925$. For $b^i_{o,\sigma}$, we continue to assume that it is tied to their productivities as an experienced worker and thus is equal to $b^e_{o,\sigma}$. We set the replacement ratio at 0.5 (instead of 0.6) in oder to maintain that $b^i_{o,m}$ is less than their market productivity of 1.[34]

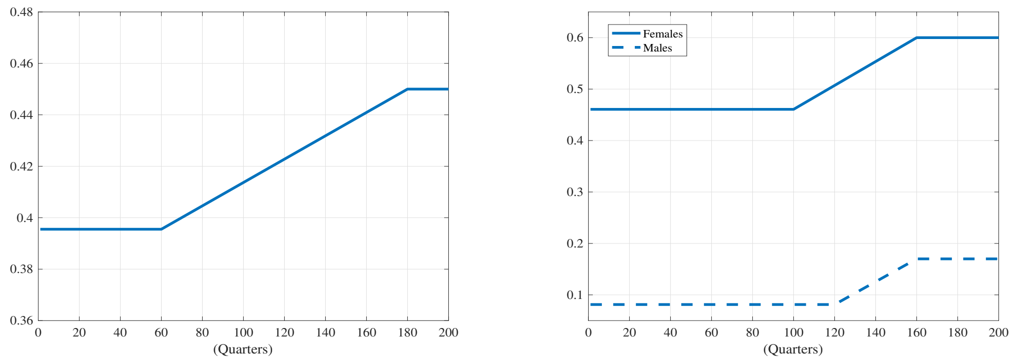
For job finding rates, we continue to assume the same steady-state job finding rates at 35 percent for all matching markets as in the baseline model. Esteban-Pretel et al. (2011) report job finding rates by gender and show that they are very similar to each other at a monthly rate of around 12 percent, which is roughly consistent with the quarterly value of 35 percent. As before, we set steady-state job filling rates at the same values. With $\alpha = 0.5$, the scale parameters $\overline{m}^i$ and $\overline{m}^e_\sigma$ are also equal to 0.35. Parameter values set so far allow us to compute steady-state values of jobs, $J_{y,\sigma}$, $J^e_{o,\sigma}$, and $J^i_{o,\sigma}$, which, in turn, can be used to back out (detrended) values of vacancy posting costs $\kappa^i = 0.433$, $\kappa^e_f = 0.568$, and $\kappa^e_m = 0.786$.

Last, we set exogenous TFP growth z_t at 0.009 per year based on the evidence in the JIP database mentioned above that TFP had been steadily growing at slightly less than 1 percent through the end of the 1980s. Lastly, we set the discount factor at 0.9961. Recall that we set $\beta = 0.995$ in the baseline model. The revised value incorporates the presence of

[34]Given $\gamma_m = 0.87$ (which is much higher than 0.6 in the baseline model), using the ratio of 0.6 puts $b^i_{o,m}$ above 1.

Figure 8: Experiments in the Model with Gender Heterogeneity

(a) Share of Female Workers (b) Share of Inexperienced Old Workers



Notes: Panel (a) plots the targeted path of the share of female workers in overall employment; solid and dashed lines in Panel (b) plot the shares of inexperienced workers within old female workers and old male workers, respectively.

z_t that grows at rate of 1.0022 per quarter, while maintaining the annualized real interest rate at 2 percent in the initial steady state. The initial steady-state values under this new calibration are summarized in Table A.3 in Appendix C.

6.3 Experiments

We first verify that the changes in lower labor force entry rates in this extended model play out similarly to those in our baseline model. We then conduct the following three additional experiments. Note that we study the effects of each experiment separately, and the effects should be viewed as the changes from the initial steady state induced by each specific experiment only.[35]

1. Lower TFP growth: The first experiment considers a permanently lower TFP growth rate. According to the JIP database, Japan's TFP growth appears to have dropped to a permanently lower level at around the burst of the bubble economy at the end of the 1980s. Specifically, we change z_t from 0.009 to 0.0025 (per year) at the beginning of the 1990s.

2. Higher female labor force participation: Although the model does not feature the labor force participation margin, we address the effects by looking at how a higher female

[35]A caveat to this approach is that the effects of these separate experiments may not add up linearly to produce the total effects. For example, a higher (exogenous) separation rate (which we study below) may have larger impacts on the aggregate variables when it hits an economy with a larger share of old workers than in an economy with constant labor force entry.

labor force entry rate impacts the economy. Note that our main experiment entails lower entry rates of both genders, and these lower rates occur starting in 1970. The current experiment increases the entry rate of female workers, and the timing of the change is also different. Specifically, we calibrate the exogenous path of female labor force entry by targeting the share of female workers in overall employment, which had been steady at around 40 percent through the mid-1980s, but has since gradually increased, reaching 45 percent in the mid-2010s. Figure 8(a) presents the path of the female employment share in the model, replicating the pattern that the female share starts to increase in 1985 and reaches the new steady state at 2014Q4 (recall that period 1 in our our experiment corresponds to 1970Q1).

3. Higher share of non-regular workers and job separation rates: We mentioned above that the shares of non-regular workers for both genders gradually have increased since the mid 1990s. The share is much higher among female workers and their increase is more drastic. In mapping these observed developments to the changes within the model, we target the share of inexperienced workers within old workers under the aforementioned interpretation that non-regular workers correspond inexperienced workers in the model. In the data, the share within female workers between 40 and 65 years old started increasing around 1995, while the share within male workers of the same age group started increasing somewhat later. We achieve the paths of these shares by increasing job separation rates among old workers. Job destruction rates for female and male old experienced workers start increasing in 1995 and 2000, respectively, and continue to increase until 2010. Figure 8(b) plots the paths of these shares in the model that replicate corresponding observed paths well.

6.4 Results

Table 4 summarizes the results on the paths of several key variables under the three experiments, as well as the same aging experiments as in the baseline model. The first two rows in each panel compare the effects of lower labor force entry rates in our baseline model and in the extended model with gender heterogeneity, respectively. Overall, the economy responds similarly to lower labor force entry rates and ensuing aging of the workforce, whether or not the model features gender heterogeneity. With the gender heterogeneity, the real interest rate increases a bit more initially to 2.87 percent in the first half of the 1980s, and over the next 25 years, it falls by 74 basis points, which is a similar amount as the decline in the baseline model. The behavior of real wage is consistent with the path of the real interest rate. Remember that our extended model features exogenous TFP that grows at 0.9 percent per year. For comparison, we present detrended wage. We see that aggregate wage increases over the initial two-and-half decades, before the path flattens in the subsequent years. The separation rate and the unemployment rate again behave similarly under our labor force entry experiment with and without gender heterogeneity. Both of these variables fall in the extended model for the same reasons as in the baseline model.

A permanent decline in TFP growth carries a straightforward implication for the real interest rate.[36] Note that the steady-state version of the Euler equation is written as $1+r = \frac{1}{\beta}(1+z)^{\varepsilon}$. The real interest rate responds proportionately to the change in z_t by a factor of $\frac{\varepsilon}{\beta}$, which is slightly larger than two. Thus, the decline in TFP growth from 0.9 percent to 0.25 percent implies roughly a 1.3 percentage point decline in the real interest rate. Also, the transition to a new steady state occurs quickly. Wages follow balanced growth paths and thus the TFP slowdown results in transitions to lower growth paths, which again occur quickly. As one can see from the bottom half of Table 4, the aggregate separation rate is hardly affected by the permanent change in the TFP growth rate. The aggregate unemployment rate falls slightly, reflecting small declines in group-level unemployment rates. The lower real rate (i.e., a higher stochastic discount factor) makes employment relationships more "durable" and thus more job creation follows, resulting in small increases in job finding rates and thus group-level unemployment rates.[37]

Higher shares of female workers and non-regular workers in the economy have similar impacts on key variables, at least qualitatively. Importantly, the real interest rate falls relative to the initial steady state. A higher share of female workers implies lower productivity growth, given that female workers have flatter wage profiles, and a higher share of inexperienced workers (= non-regular workers) also implies lower productivity growth. The predictions on aggregate wage under these two experiments are also consistent with this story. As in the first two rows, we present detrended wages for these two experiments as well. As shown on the third and fourth rows in Panel (b), aggregate wages decline slightly, as the shares of these workers increase as shown in Figure 8(b). In Figure 2(c), we saw that real wage started falling in the late 1990s, and between its peak and 2010, it fell a few percentage points. Even when all four effects are combined, the model does not quantitatively match this decline. However, the model does generate the overall stagnation of real wages since the mid-1990s, and all four changes contribute to this pattern. A higher share of female workers, which we replicate via higher entry of female workers, implies a slightly higher aggregate separation rate later in the simulation, owing to the changes in the composition of the work-

[36] Note that this experiment has the same implication in our baseline model.

[37] Esteban-Pretel et al. (2010) argue that the slowdown of TFP growth in the 1990s played a major role in the increases in the unemployment rate in that period. In their simulation, TFP growth was lower between 1990 and 2002 as observed in the data, before it starts growing again at a higher and constant rate from 2003 on (at the rate observed in 2002). Owning to the lower TFP growth rates between 1990 and 2002, their model generates a higher unemployment rate through lower job creation and higher job separation rate during that period. However, TFP growth overall remained low throughout the 2000s as well. In this sense, the observed pattern in TFP growth is consistent with a permanent downward shift at the beginning of the 1990s, as considered in our paper.

Table 4: Results of Experiments in the Extended Model

	70−74	75−79	80−84	85−89	90−94	95−99	00−04	05−09
	(a) Real interest rate (annualized %)							
Aging: Baseline	2.02	2.46	2.75	2.33	2.19	2.11	2.06	2.02
Aging: Extended	2.04	2.47	2.87	2.55	2.35	2.25	2.18	2.13
Lower TFP Growth	–	0.00	0.00	-0.06	-1.29	-1.30	-1.30	-1.30
Female Workers	–	0.00	-0.03	-0.15	-0.08	-0.08	-0.08	-0.08
Non-regular Workers	–	0.00	0.00	0.00	-0.02	-0.23	-0.60	-0.45
	(b) Aggregate wage (70−74 level = 100)							
Aging: Baseline	100.0	100.0	100.6	101.5	102.1	102.5	102.7	102.7
Aging: Extended	100.0	100.0	100.5	101.7	102.6	103.2	103.7	104.0
TFP Growth	100.0	104.6	109.4	114.4	117.7	119.2	120.7	122.2
Female Workers	100.0	100.0	100.0	99.9	99.7	99.5	99.3	99.1
Non-regular Workers	100.0	100.0	100.0	100.0	100.0	99.6	99.1	98.1
	(c) Aggregate separation rate (%)							
Aging: Baseline	1.26	1.26	1.25	1.23	1.22	1.22	1.22	1.22
Aging: Extended	1.19	1.19	1.18	1.16	1.15	1.13	1.13	1.12
TFP Growth	1.19	1.19	1.19	1.19	1.19	1.19	1.19	1.19
Female Workers	1.19	1.19	1.19	1.20	1.20	1.21	1.21	1.22
Non-regular Workers	1.19	1.19	1.19	1.19	1.19	1.25	1.38	1.45
	(d) Unemployment rate (%)							
Aging: Baseline	6.00	5.80	4.94	4.72	4.71	4.71	4.71	4.71
Aging: Extended	5.28	5.05	4.29	3.85	3.74	3.68	3.63	3.60
TFP Growth	5.29	5.29	5.29	5.29	5.27	5.27	5.27	5.27
Female Workers	5.29	5.29	5.29	5.46	5.48	5.48	5.49	5.50
Non-regular Workers	5.29	5.29	5.29	5.29	5.29	5.39	5.70	5.92

Notes: The last three rows in Panel (a) present the differences from the initial steady-state level. Panel (b) presents average detrended wages, except in the third row (TFP growth).

force. Note that we set s_f^e higher than s_m^e, while the separation rate among inexperienced workers is set at the same value for both genders. Thus, the higher entry rate of female workers results in a higher overall separation rate, although it is quantitatively small. For the last experiment, we achieve the the higher share of non-regular workers by exogenously raising separation rates, and thus the model generates a higher aggregate separation rate. In the data, the separation rate into unemployment increased steadily throughout the 1990s (see, for example, Esteban-Pretel et al. (2010) and Lin and Miyamoto (2012)). In our model, however, separation rates are used to match the increasing shares of non-regular workers, which start in the late 1990s, as plotted in Figure 8(b). Our model is thus unable to match the observed increase in the separation rate in the early 1990s. Between the mid-1990s and the mid-2000, the separation rate into unemployment increased about 40 percent, according to the aforementioned papers Esteban-Pretel et al. (2010) and Lin and Miyamoto (2012). In our model, the aggregate separation rate increases about 20 percent.[38] The paths of the unemployment rate under the last two experiments are largely accounted for by the changes in separation rates just discussed.

Overall, our additional experiments demonstrate these other developments have similar overall impacts on the real interest rate and other aggregate variables. In particular, higher shares of female workers and non-regular workers are consistent with slower labor productivity growth since the 1990s, and thus stagnation of real wage and the lower real interest rate. On the other hand, the unemployment rate nearly tripled in Japan during the 1990s and none of our experiments account for the increase, which clearly calls for explanations other than those considered in the current paper.[39]

7 Conclusion

This paper studied the implications of aging on the low-frequency behavior of the Japanese economy. We argued that the collapse in labor force entry in the 1970s associated with the end of the baby boom is an important driver of low-frequency fluctuations of the Japanese economy in the subsequent years. Our quantitative experiment suggests that the aging of the labor force accounts for roughly 40 percent of the declines in per capita consumption growth and the real interest rate over the period. We also studied the impacts of other secular developments in Japan, such as a decline in TFP growth and increasing shares of

[38]This calculation is based on the comparison between the 1995 and 2004 levels of the separation rate, while Panel (c) presents the average level in each 5-year window.

[39]Note that the main interest of this paper is in the implications of aging and a few other well-known long-term developments in Japan and is *not* in accounting for changes in the unemployment rate due to "cyclical" reasons. Importantly, the unemployment rate in Japan peaked in 2002 and has since fallen through 2019 (except in 2008 and 2009, when it increased temporarily because of the Great Recession) to a level similar to the level in the late 1980s.

female and non-regular workers. We confirm that these long-run changes only strengthen the basic mechanism in our baseline model.

In our paper, we abstract away from the potential important effect through consumption/saving heterogeneity, considered for example by Miles (1999), Rios-Rull (2001), Carvalho et al. (2016), and Gagnon et al. (2016). Incorporating consumption/saving heterogeneity is likely to strengthen our result, given a standard intuition in a OLG setup that aging results in increases in the supply of funds and thus a higher capital-labor ratio, lowering the real interest rate. However, gauging its quantitative impact requires a separate careful study. In the future work on the stagnation of the Japanese economy, it is particularly important to address misallocation of physical capital and the solvency of the public pension system on the consumer's saving behavior as well.

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Appendix

A Detrended System of Equations

In solving the model, we detrend nonstationary variables by total population h_t. We put together the detrended equations below. Note that $1 + g_t = \frac{h_t}{h_{t-1}}$. The tilde on each variable indicates a detrended variable.

- Labor market stock-flow balance equations:

$$
\begin{aligned}
(1+g_t)\tilde{n}_{y,t} =& (1-\mu)[1-s^i+s^i f^i(\theta^i_t)]\tilde{n}_{y,t-1} + f^i(\theta^i_t)(1-\mu)\tilde{u}_{y,t-1}\\
(1+g_t)\tilde{u}_{y,t} =& (1-f^i(\theta^i_t))(1-\mu)\tilde{u}_{y,t-1} + s^i(1-f^i(\theta^i_t))(1-\mu)\tilde{n}_{y,t-1} + \phi\tilde{h}_{y,t-1}\\
(1+g_t)\tilde{n}^e_{o,t} =& (1-d)[1-s^e+(1-\delta)s^e f^e(\theta^e_t)]\tilde{n}^e_{o,t-1} + [1-s^e+s^e f^e(\theta^e_t)]\mu\tilde{n}_{y,t-1}\\
&+ f^e(\theta^e_t)(\mu\tilde{u}_{y,t-1} + (1-d)\tilde{u}^e_{o,t-1})\\
(1+g_t)\tilde{u}^e_{o,t} =& (1-d)(1-f^e(\theta^e_t))\tilde{u}^e_{o,t-1} + (1-f^e(\theta^e_t))\mu[s^e\tilde{n}_{y,t-1} + \tilde{u}_{y,t-1}]\\
&+ (1-d)s^e(1-\delta)(1-f^e(\theta^e_t))\tilde{n}^e_{o,t-1}\\
(1+g_t)\tilde{n}^i_{o,t} =& (1-d)[1-s^i+s^i f^i(\theta^i_t)]\tilde{n}^i_{o,t-1} + (1-d)f^i(\theta^i_t)(s^e\delta\tilde{n}^e_{o,t-1} + \tilde{u}^i_{o,t-1})\\
(1+g_t)\tilde{u}^i_{o,t} =& (1-d)(1-f^i(\theta^i_t))(\tilde{u}^i_{o,t-1} + s^i\tilde{n}^i_{o,t-1} + s^e\delta\tilde{n}^e_{o,t-1})\\
(1+g_t)\tilde{h}_{y,t} =& (1-\mu)\tilde{h}_{y,t-1} + \phi\tilde{h}_{y,t-1}\\
(1+g_t)\tilde{h}^e_{o,t} =& (1-d)(\tilde{h}^e_{o,t-1} - s^e\delta\tilde{n}^e_{o,t-1}) + \mu\tilde{h}_{y,t-1}\\
(1+g_t)\tilde{h}^i_{o,t} =& (1-d)(\tilde{h}^i_{o,t-1} + s^e\delta\tilde{n}^e_{o,t-1})\\
(1+g_t)\tilde{h}_{o,t} =& (1-d)\tilde{h}_{o,t-1} + \mu\tilde{h}_{y,t-1}\\
g_t =& \phi\tilde{h}_{y,t-1} - d\tilde{h}_{o,t-1}\\
(1+g_t)\tilde{\bar{u}}^i_t =& (1-\mu)[s^i\tilde{n}_{y,t-1} + \tilde{u}_{y,t-1}] + (1-d)[\delta s^e\tilde{n}^e_{o,t-1} + s^i_{t-1}\tilde{n}^i_{o,t-1} + \tilde{u}^i_{o,t-1}]\\
(1+g_t)\tilde{\bar{u}}^e_t =& \mu[s^e\tilde{n}_{y,t-1} + \tilde{u}_{y,t-1}] + (1-d)[(1-\delta)s^e\tilde{n}^e_{o,t-1} + \tilde{u}^e_{o,t-1}]
\end{aligned}
$$

- Production and the resource constraint:

$$
\begin{aligned}
\tilde{y}_t =& \tilde{n}_{y,t} + \tilde{n}^i_{o,t} + (1+\gamma)\tilde{n}^e_{o,t}\\
\tilde{y}_t =& c_t + \kappa^i\tilde{v}^i_t + \kappa^e\tilde{v}^e_t
\end{aligned}
$$

- All marginal value functions, the free entry conditions ((10) and (11)), and the wage equations ((21), (22), and (23)) are stationary equations and thus remain the same. The consumption Euler equation (16) and the stochastic discount factor require no detrending, either.

B Endogenous Capital

Does capital accumulation alter our main conclusion that changes in the demographic structure induce significant low-frequency movements in the real interest rate? This section studies this question while maintaining the representative household structure.

B.1 Modifications to the Baseline Model

The production function in Equation (9) is now replaced by

$$y_t = [n_{y,t} + n^i_{o,t} + (1+\gamma)n^e_{o,t}]^{1-\bar{\alpha}} K_t^{\bar{\alpha}},$$

where $\bar{\alpha}$ denote the capital share. K_t is the capital stock rented by the firm and is not firm-specific. Capital accumulation is subject to the investment-growth adjustment cost *a la* Christiano et al. (2005):

$$K_{t+1} = (1-\delta)K_t + \left[1 - s\left(\frac{I_t}{(1+g_t)I_{t-1}}\right)\right] I_t,$$

where I_t and g_t denotes investment and the trend growth rate, respectively; the latter is pinned down solely by the population growth; and δ is the depreciation rate. The adjustment cost function $s(\cdot)$ has the property that $s(1) = 0$, $s'(1) = 0$ and $s'' > 0$. The other related equations such as the profit maximization problem and the resource constraint (the budget constraint) are also modified in a straightforward manner.

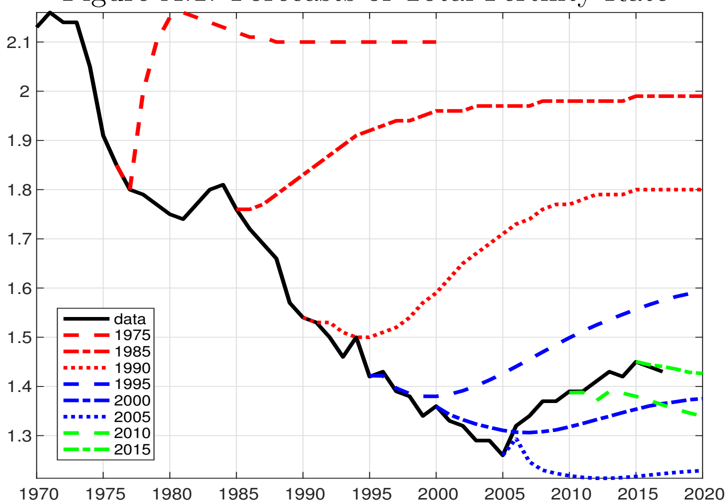
Newly added parameters are set at the conventional values. The capital share $\bar{\alpha}$ and the depreciation rate δ are set at 1/3 and 0.025, respectively. Regarding the investment growth adjustment cost, we set it to the estimated value in Fujiwara et al. (2011): $s'' = 4.82$. To see how sensitive our results can be to the level of adjustment cost, we also consider the case when the value of s'' is reduced to half of the estimated value.

B.2 Expectation about the Future Demographic Trend

Simulation results in the model with capital are more sensitive to the assumption about the agents' expectation about the future demographic trend. In our earlier quantitative exercises, we assume perfect foresight for the path of the labor force entry rate. In our baseline model, the results are robust with respect to whether we assume perfect foresight or a more realistic expectation formation. This is not the case in the current model, in which saving through physical capital is available. In particular, under perfect foresight, consumption jumps immediately, reflecting very large wealth effects in the first period, when the future path of the fertility rate is perfectly predicted.

We assume in the current model that whenever agents observe the change in the fertility rate, they perceive it as a permanent shift in the steady-state labor force entry rate that

Figure A.1: Forecasts of Total Fertility Rate



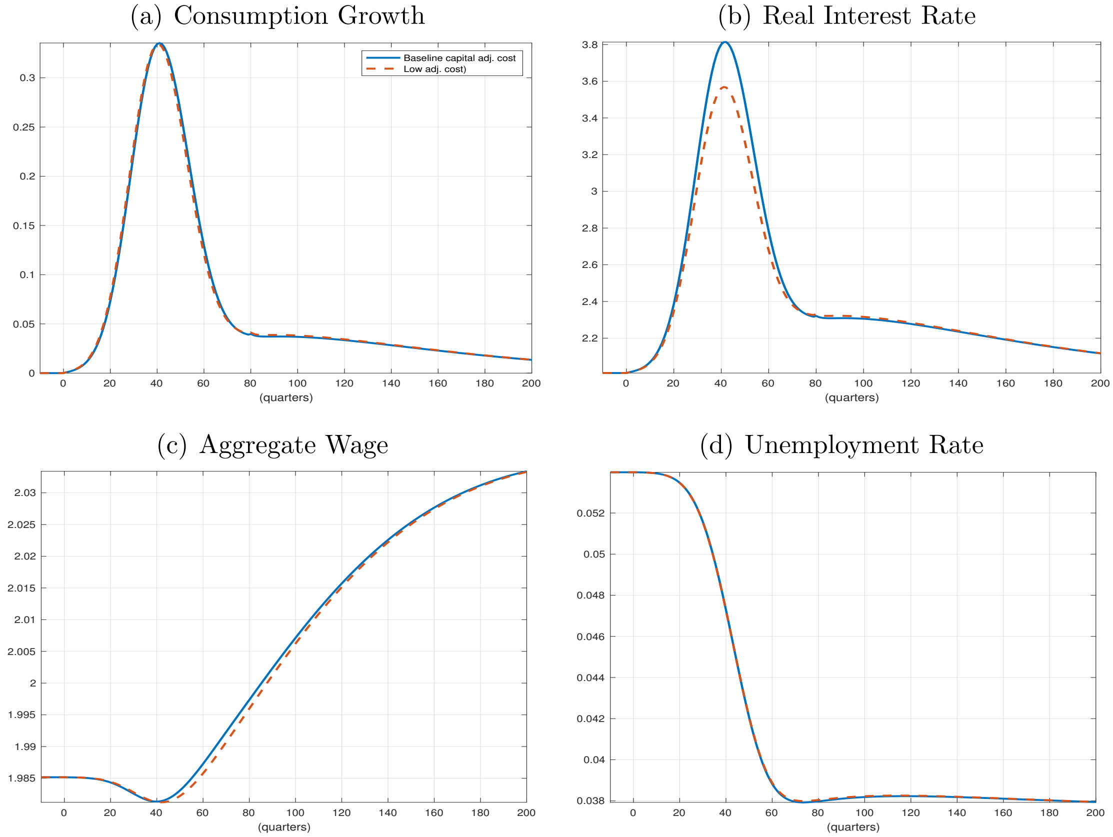
Source: National Institute of Population and Social Security Research.

stays constant at that level from there on. This expectation formation scenario appears more plausible than perfect foresight and indeed is consistent with the available empirical evidence. Figure A.1 plots the actual path of the fertility rate (black solid line) together with its real-time forecasts published by the National Institute of Population and Social Security Research. The actual path was on a long-term declining trend between the early 1970s through the early 2000s, but these declines were hardly expected ex-ante. The figure actually indicates that the decline at each point in time was viewed only as a short-run phenomenon. In the medium to long-run, the fertility rate was expected to recover. Given the difficulty of precisely mimicking the expectation formation such as the one shown in Figure A.1, we assume that each drop in the entry rate is perceived to be permanent and to stay there going forward.

B.3 Results

Figure A.2 presents simulation results from models with capital accumulation. The dashed lines give the results under a smaller capital adjustment cost, as discussed above. We can see that the results are not sensitive to the level of capital adjustment cost. The results in Figure A.2 are overall similar to those in our baseline model (without capital accumulation). The key feature of our baseline model remains intact: as the labor force entry rate falls, the share of experienced workers increases initially, which implies higher labor productivity growth, but as aging deepens, productivity growth slows down. Consumption growth and the real interest rate consequently also fall. One of the clear differences is that consumption growth and thus the real interest rate remain higher than the initial steady-state levels for a much longer period, after the steep declines throughout the 1980s (between quarters 41 and 80).

Figure A.2: Perfect Foresight Equilibrium Paths (Endogenous Capital)



Similarly, aggregate wage continues to increase at a higher rate later in the sample, whereas in our baseline model, the slowdown of wage growth is more apparent. The accumulation of physical capital keeps labor productivity higher than in our baseline model, thus offsetting some of the effects within our baseline model.

Table A.1 compares the observed declines in per capita consumption growth and the real interest rate to the simulated ones in models with capital. With capital accumulation, the decline in consumption growth becomes smaller than in our earlier model. The presence of capital as a saving device enhances consumption smoothing. However, reflecting the movements in the marginal product of capital, the decline in the real interest rate actually becomes larger. Specifically, 75 percent of the observed decline in the real interest rate can be attributed to the declining labor force entry rate. Even when the investment growth adjustment cost is set at a much smaller value than the estimated one, 60 percent of the decline can be explained solely by aging via skill (productivity) heterogeneity. As discussed above, the results depend on the assumption about the forecastability of the demographic trend, but the impact of aging on the real interest rate is likely to be larger (under a plausible expectation formation about the future demographic trend) when capital is endogenously

Table A.1: Real Interest Rate and Consumption Growth (Endogenous Capital)

	Real Interest Rate (%)	Consumption Growth (%)
Data	−2.27	−1.26
Baseline	−1.02	−0.51
Capital	−1.72	−0.32
Capital (smaller adj. cost)	−1.37	−0.32

Notes: Data refer to the difference between the average levels in 1980s and 2000s. See Table 1. The decline in the two variables in the model is computed as the difference between the peak level and the level at the 160th quarter (40 years later) after the start of the decline in the entry rate.

determined. The low-frequency dynamics of the workforce composition matter for the determination of the real interest rate, regardless of whether the model incorporates accumulation of physical capital or not.

C Extended Model

Here we present the details of our extended model that features gender heterogeneity. Subscript $\sigma \in \{f, m\}$ identifies females and males, respectively.

C.1 Worker Transitions

For completeness, all transition probabilities are summarized in Table A.2. There is a single matching market for all inexperienced workers regardless of their gender and age. The same job destruction rate applies to both female and male inexperienced workers. Female and male inexperienced workers face different future possibilities, represented by the different environment and parameter values when they become experienced. These differences create the gender wage gap, even among inexperienced workers. There are two matching markets for experienced workers, separately for female and male workers.

In addition to the evolutions of different types of workers summarized in Table A.2, the mass $\phi_t h_{y,\sigma,t-1}$ enters the labor market as jobless every period. Given our timing assumption, the number of job seekers in each matching market is, respectively, written as:

$$\bar{u}^i_t = (1-\mu)\sum_\sigma [s^i n_{y,\sigma,t-1} + u_{y,\sigma,t-1}] + (1-d)\sum_\sigma [\delta_\sigma s^e_\sigma n^e_{o,\sigma,t-1} + s^i n^i_{o,\sigma,t-1} + u^i_{o,\sigma,t-1}],$$

$$\bar{u}^e_{\sigma,t} = \mu[s^e_\sigma n_{y,\sigma,t-1} + u_{y,\sigma,t-1}] + (1-d)[(1-\delta_\sigma)s^e_\sigma n^e_{o,\sigma,t-1} + u^e_{o,\sigma,t-1}].$$

Table A.2: Summary of Worker Transitions in the Extended Model

State at End of $t-1$	Transition Probabilities	State at End of t
End of $t-1$		End of t
$n_{y,\sigma,t-1}$	$(1-\mu)(1-s^i+s^i f^i(\theta^i_t))$	$n_{y,\sigma,t}$
$n_{y,\sigma,t-1}$	$\mu(1-s^e_\sigma+s^e_\sigma f^e_\sigma(\theta^e_{\sigma,t}))$	$n^e_{o,\sigma,t}$
$n_{y,\sigma,t-1}$	$(1-\mu)s^i(1-f^i(\theta^i_t))$	$u_{y,\sigma,t}$
$n_{y,\sigma,t-1}$	$\mu s^e_\sigma(1-f^e_\sigma(\theta^e_{\sigma,t}))$	$u^e_{o,\sigma,t}$
$u_{y,\sigma,t-1}$	$(1-\mu)f^i(\theta^i_t)$	$n_{y,\sigma,t}$
$u_{y,\sigma,t-1}$	$\mu f^e_\sigma(\theta^e_{\sigma,t})$	$n^e_{o,\sigma,t}$
$u_{y,\sigma,t-1}$	$(1-\mu)(1-f^i(\theta^i_t))$	$u_{y,\sigma,t}$
$u_{y,\sigma,t-1}$	$\mu(1-f^e_\sigma(\theta^e_{\sigma,t}))$	$u^e_{o,\sigma,t}$
$n^e_{o,\sigma,t-1}$	d	$-$
$n^e_{o,\sigma,t-1}$	$(1-d)(1-s^e_\sigma+s^e_\sigma(1-\delta_\sigma)f^e_\sigma(\theta^e_{\sigma,t}))$	$n^e_{o,\sigma,t}$
$n^e_{o,\sigma,t-1}$	$(1-d)s^e_\sigma\delta_\sigma f^i(\theta^i_t)$	$n^i_{o,\sigma,t}$
$n^e_{o,\sigma,t-1}$	$(1-d)s^e_\sigma(1-\delta_\sigma)(1-f^e_\sigma(\theta^e_{\sigma,t}))$	$u^e_{o,\sigma,t}$
$n^e_{o,\sigma,t-1}$	$(1-d)s^e_\sigma\delta_\sigma(1-f^i(\theta^i_t))$	$u^i_{o,\sigma,t}$
$u^e_{o,\sigma,t-1}$	d	$-$
$u^e_{o,\sigma,t-1}$	$(1-d)f^e_\sigma(\theta^e_{\sigma,t})$	$n^e_{o,\sigma,t}$
$u^e_{o,\sigma,t-1}$	$(1-d)(1-f^e_\sigma(\theta^e_{\sigma,t}))$	$u^e_{o,\sigma,t}$
$n^i_{o,\sigma,t-1}$	d	$-$
$n^i_{o,\sigma,t-1}$	$(1-d)(1-s^i+s^i f^i(\theta^i_t))$	$n^i_{o,\sigma,t}$
$n^i_{o,\sigma,t-1}$	$(1-d)s^i(1-f^i(\theta^i_t))$	$u^i_{o,\sigma,t}$
$u^i_{o,\sigma,t}$	d	$-$
$u^i_{o,\sigma,t}$	$(1-d)f^i(\theta^i_t)$	$n^i_{o,\sigma,t}$
$u^i_{o,\sigma,t}$	$(1-d)(1-f^i(\theta^i_t))$	$u^i_{o,\sigma,t}$

Except for the summation over σ, the interpretations of these equations are the same as in the baseline model. The matching function in each market takes the Cobb-Douglas form $m^i_t = \overline{m}^i(\bar{u}^i_t)^\alpha(v^i_t)^{1-\alpha}$ and $m^e_{\sigma,t} = \overline{m}^e_\sigma(\bar{u}^e_{\sigma,t})^\alpha(v^e_{\sigma,t})^{1-\alpha}$. Job finding and filling rates are defined in a usual manner.

C.2 Equilibrium Conditions

The representative firm combines the six types of labor to produce the consumption goods.

$$Y_t = A_t \sum_\sigma [n_{y,\sigma,t} + n^i_{o,\sigma,t} + (1+\gamma_\sigma) n^e_{o,\sigma,t}],$$

where A_t represents neutral technology that grows at a constant rate z_t. The optimal decision is characterized by the following job creation conditions:

$$\sum_\sigma (\omega_{y,\sigma,t} J_{y,\sigma,t} + \omega_{o,\sigma,t} J^i_{o,\sigma,t}) = \frac{\kappa^i}{q^i(\theta^i_t)},\ \ J^e_{o,\sigma t} = \frac{\kappa^e_\sigma}{q^e_\sigma(\theta^e_{\sigma,t})},$$

where

$$\omega_{y,\sigma,t} = \frac{(1-\mu)[s^i n_{y,\sigma,t-1} + u_{y,\sigma,t-1}]}{\bar{u}^i_t},\ \ \omega_{o,\sigma,t} = \frac{(1-d)[\delta_\sigma s^e_\sigma n^e_{o,\sigma,t-1} + s^i n^i_{o,\sigma,t-1} + u^i_{o,\sigma,t-1}]}{\bar{u}^i_t}.$$

The values of different types of jobs are expressed as:

$$\begin{aligned}
J_{y,\sigma,t} &= A_t - w_{y,\sigma t} + \hat{\beta}_{t,t+1}[(1-\mu)(1-s^i)J_{y,\sigma,t+1} + \mu(1-s^e_\sigma)J^e_{o,\sigma,t+1}],\\
J^e_{o,\sigma,t} &= (1+\gamma_\sigma)A_t - w^e_{o,\sigma,t} + (1-d)(1-s^e_\sigma)\hat{\beta}_{t,t+1} J^e_{o,\sigma,t+1},\\
J^i_{o,\sigma,t} &= A_t - w^i_{o,\sigma,t} + (1-d)(1-s^i)\hat{\beta}_{t,t+1} J^i_{o,\sigma,t+1}.
\end{aligned}$$

The representative household consists of workers that differ in gender, age, skill level, and labor market status. But given the income pooling assumption, the household decision is characterized by the standard Euler equation. The marginal values of employment, denoted by M, to the household (net of having one more unemployed worker of the corresponding type) are written as:

$$\begin{aligned}
M_{y,\sigma t} &= u'(c_t)(w_{y,\sigma,t} - b_{y,\sigma})\\
&\quad + \beta\big[(1-\mu)(1-s^i)(1-f^i(\theta^i_{t+1}))M_{y,\sigma,t+1} + \mu(1-s^e_\sigma)(1-f^e_\sigma(\theta^e_{\sigma,t+1}))M^e_{o,\sigma,t+1}\big],\\
M^i_{o,\sigma,t} &= u'(c_t)(w^e_{o,\sigma,t} - b^e_{o,\sigma}) + (1-d)(1-s^i)(1-f^i(\theta^i_{t+1}))\beta M^i_{o,\sigma,t+1},\\
M^e_{o,\sigma,t} &= u'(c_t)(w^e_{o,\sigma,t} - b^e_{o,\sigma}) + (1-d)\beta\big[(1-s^e_\sigma)(1-f^e_\sigma(\theta^e_{\sigma,t+1}))M^e_{o,\sigma,t+1}\\
&\quad - s^e_\sigma\delta_\sigma\{f^e_\sigma(\theta^e_{\sigma,t+1})M^e_{o,\sigma,t+1} - f^i(\theta^i_{t+1})M^i_{o,\sigma,t+1} + D_{\sigma,t+1}\}\big],\\
D_{\sigma,t} &= u'(c_t)(b^e_{o,\sigma} - b^i_{o.\sigma}) + (1-d)\beta[f^e_\sigma(\theta^e_{\sigma,t+1})M^e_{\sigma,o,t} - f^i(\theta^i_{t+1})M^i_{o,\sigma,t+1} + D_{\sigma,t+1}].
\end{aligned}$$

Note that $D_{\sigma,t}$ represents the difference in the values of being unemployed as experienced and inexperienced old workers. The joint surplus $(M+J)$ is split between the firm and the worker based on the bargaining power η and $1-\eta$, and wages of different types of workers can be written as:

$$
\begin{aligned}
w_{y,\sigma,t} =& A_t[\eta + (1-\eta)b_{y,\sigma}] + \eta\hat{\beta}_{t,t+1}[(1-\mu)(1-s^i)f^i(\theta^i_{t+1})J_{y,\sigma,t+1} + \mu(1-s^e_\sigma)f^e_\sigma(\theta^e_{\sigma,t+1})J^e_{o,\sigma,t+1}],\\
w^i_{o,\sigma,t} =& A_t[\eta + (1-\eta)b^i_{o,\sigma}] + (1-d)(1-s^i)\eta\hat{\beta}_{t,t+1}f^i(\theta^i_{t+1})J^i_{o,\sigma,t+1},\\
w^e_{o,\sigma,t} =& A_t[\eta(1+\gamma_\sigma) + (1-\eta)b_{o,}{}^e] + (1-d)\hat{\beta}_{t,t+1}\Big[(1-s^e_\sigma)f^e_\sigma(\theta^e_{\sigma,t+1})\eta J^e_{o,\sigma,t+1}\\
&+ \delta s^e_\sigma\left\{f^e_\sigma(\theta^e_{\sigma,t+1})\eta J^e_{o,\sigma,t+1} - f^i(\theta^i_{t+1})\eta J^i_{o,\sigma,t+1} + \frac{D_{\sigma,t+1}}{u'(c_{t+1})}\right\}\Big].
\end{aligned}
$$

After imposing zero net supply of the bonds in the aggregate, the following resource constraint closes the model.

$$
Y_t = h_t c_t + \kappa^i v^i_t + \sum_\sigma \kappa^e_\sigma v^e_{\sigma,t}.
$$

C.3 Detrending

In the extended model, both population and technology growth rates are nonzero in general. Therefore, nonstationary equations need to be detrended appropriately by h_t, $h_{m,t}$, $h_{f,t}$ and A_t before the model is solved and the perfect foresight equilibrium path is computed. The detrended system of equations are available upon request.

C.4 Initial Steady-State Equilibrium

Table A.3 presents steady-state values of wages, employment levels, and unemployment rates at different levels of aggregation in the initial steady state. Note that employment levels are presented as a share of the total labor force (population), while unemployment rates are expressed as a share of the labor force of each type.

Table A.3: Initial Steady-State Values in the Extended Model

	Average Wages		Employment Levels		Unemployment Rates	
Female	$\frac{w_{y,f}n_{y,}+w^e_{o,f}n^e_{o,f}+w^i_{o,f}n^i_{o,f}}{n_{y,f}+n^e_{o,f}+n^i_{o,f}}$	1.053	$\frac{n_{y,f}+n^e_{o,f}+n^i_{o,f}}{h_f}$	0.942	$\frac{u_{y,f}+u^e_{o,f}+u^i_{o,f}}{h_f}$	0.058
Young	$w_{y,f}$	0.981	$\frac{n_{y,f}}{h_f}$	0.537	$\frac{u_{y,f}}{h_{y,f}}$	0.079
Old	$\frac{w^e_{o,f}n^e_{o,f}+w^i_{o,f}n^i_{o,f}}{n^e_{o,f}+n^i_{o,f}}$	1.148	$\frac{n^e_{o,f}+n^i_{o,f}}{h_f}$	0.404	$\frac{u^e_{o,f}+u^i_{o,f}}{h^e_{o,f}+h^i_{o,f}}$	0.030
Exp.	$w^e_{o,f}$	1.294	$\frac{n^e_{o,f}}{h_f}$	0.218	$\frac{u^e_{o,f}}{h^e_{o,f}}$	0.006
Inexp.	$w^i_{o,f}$	0.978	$\frac{n^i_{o,f}}{h_f}$	0.186	$\frac{u^i_{o,f}}{h^i_{o,f}}$	0.056
Male	$\frac{w_{y,m}n_{y,m}+w^e_{o,m}n^e_{o,m}+w^i_{o,m}n^i_{o,m}}{n_{y,m}+n^e_{o,m}+n^i_{o,m}}$	1.320	$\frac{n_{y,m}+n^e_{o,m}+n^i_{o,m}}{h_m}$	0.951	$\frac{u_{y,m}+u^e_{o,m}+u^i_{o,m}}{h_m}$	0.049
Young	$w_{y,m}$	0.988	$\frac{n_{y,m}}{h_m}$	0.537	$\frac{u_{y,m}}{h_{y,m}}$	0.079
Old	$\frac{w^e_{o,m}n^e_{o,m}+w^i_{o,m}n^i_{o,m}}{n^e_{o,m}+n^i_{o,m}}$	1.752	$\frac{n^e_{o,m}+n^i_{o,m}}{h_m}$	0.413	$\frac{u^e_{o,m}+u^i_{o,m}}{h^e_{o,m}+h^i_{o,m}}$	0.008
Exp.	$w^e_{o,m}$	1.819	$\frac{n^e_{o,m}}{h_m}$	0.380	$\frac{u^e_{o,m}}{h^e_{o,m}}$	0.003
Inexp.	$w^i_{o,m}$	0.995	$\frac{n^i_{o,m}}{h_m}$	0.034	$\frac{u^i_{o,m}}{h^i_{o,m}}$	0.056
Aggregate	$\frac{w_y n_y+w^e_o n^e_o+w^i_o n^i_o}{n_y+n^e_o+n^i_o}$	1.215	$\frac{n_y+n^e_o+n^i_o}{h}$	0.947	$\frac{u_y+u^e_o+u^i_o}{h}$	0.053
Young	w_y	0.985	$\frac{n_y}{h}$	0.537	$\frac{u_y}{h_y}$	0.079
Old	$\frac{w^e_o n^e_o+w^i_o n^i_o}{n^e_o+n^i_o}$	1.513	$\frac{n^e_o+n^i_o}{h}$	0.410	$\frac{u^e_o+u^i_o}{h^e_o+h^i_o}$	0.017
Exp.	w^e_o	1.612	$\frac{n^e_o}{h}$	0.316	$\frac{u^e_o}{h^e_o}$	0.004
Inexp.	w^i_o	0.988	$\frac{n^i_o}{h}$	0.094	$\frac{u^i_o}{h^i_o}$	0.056