Can We Take the "Stress" Out of Stress Testing? Applications of Generalized Structural Equation Modeling to Consumer Finance

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Abstract

Financial firms, and banks in particular, rely heavily on complex suites of interrelated statistical models in their risk management and business reporting infrastructures. Statistical model infrastructures are often developed using a piecemeal approach to model building, in which different components are developed and validated separately. This type of modeling framework has significant limitations at each stage of the model management life cycle, from development and documentation to validation, production, and redevelopment. We propose an empirical framework, spurred by recent developments in the implementation of Generalized Structural Equation Modeling (GSEM), which brings to bear a modular and all-inclusive approach to statistical model building. We illustrate the "game changing" potential of this framework with an application to the stress testing of credit risk for a representative portfolio of mortgages; we also extend it to the analysis of the allowance for credit loss under the novel Current Expected Credit Loss (CECL) accounting regulation. We illustrate how GSEM techniques can significantly enhance every step of the modeling framework life cycle. We also illustrate how GSEM can be used to combine various risk management projects and tasks into a single framework; we specifically illustrate how to seamlessly integrate stress testing and CECL (or IFRS9) frameworks and champion, and challenger, modeling frameworks. Finally, we identify other areas of model risk management that can benefit from the GSEM framework and highlight other potentially fruitful applications of the methodology.

Keywords: GSEM, stress test, CCAR, CECL, credit risk, regulatory capital, allowances, mortgages

JEL Classification: C30, C50, G20, G21, G32

Introduction

Statistical models are at the core of risk — and business — management decisions at financial institutions. Empirical projections, or forecasts, are key components of financial accounting and regulatory reporting disclosures. Empirical projections are often a combination of outcomes from multiple models, with the different model components regularly developed and validated separately. This piecemeal approach to model building introduces limitations at every stage of the model management life cycle, from development to validation, production, and redevelopment, and it also constitutes a potential source of model risk.

Industry practitioners have highlighted complexity in models and documentation as a growing area of concern (Hughes, 2019). Complexity in models and model documentation impacts model risk, the development and evolution of models, model validation and production, and all aspects associated with maintaining a sound and robust model infrastructure. Costs and risks associated with model development, initial and ongoing validation, audit, production, redevelopment, infrastructure, and supervision increase with complexity. A robust and nimble modeling framework is particularly critical during periods of stress when model projections are often updated with increased frequency, and weaknesses in forecasts need to be assessed, analyzed, minimized or mitigated, and explained to various constituencies regularly.

In this paper, we illustrate, with practical examples, how state-of-the-art Generalized Structural Equation Modeling (GSEM) techniques can contribute to a modular approach to model building and help overcome many of the limitations of a piecemeal model development framework. At a high level, GSEM can be interpreted as a flexible modeling syntax that can accommodate a large variety of model specifications and can facilitate the design — and implementation — of complex modeling projects. The GSEM framework has been extensively applied in social, behavioral, educational, ecological, psychological, managerial, or health sciences research, to name a few (e.g., Fan, Chen, Shirkeyet al., 2016; Di and Karasoy, 2020; and Wolf and Brown, 2013). The GSEM framework has been implemented in popular statistical packages like Stata, R, Mplus, and SAS.[2] The analysis in this paper will be conducted using the Stata GSEM implementation.

Specifically, the empirical examples developed in this paper illustrate how GSEM can be used to develop a transparent framework for the analysis of portfolio credit loss, in which different modeling components are jointly developed and integrated into an overall loss projection framework. Our examples focus on two important financial and regulatory applications: stress testing of credit risk under the current Comprehensive Capital Analysis and Review (CCAR) framework, and the analysis of the allowance for credit loss under the Current Expected Credit

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[2] *GSEM* is described in the Stata reference documentation as the following: "Structural equation modeling is a way of thinking, a way of writing, and a way of estimating." The Stata 16 GSEM reference documentation represents a very useful reference.

Loss (CECL) framework. Our empirical examples focus on the analysis of a mortgage portfolio using a publicly available data set of U.S. mortgage loans, originated over the past 20 years, but the approach is generalizable to the analysis of credit risk across retail or wholesale portfolios and can be dutifully expanded beyond these areas.

Banking regulators in the U.S. have been conducting public annual stress tests of the largest bank holding companies since the onset of the great recession, starting with the 2009–2010 Supervisory Capital Assessment Program (SCAP). Typical stress test models are trained on the historical performance of macroeconomic factors and the historical loan loss experience of banks' portfolios. An important component of stress tests is the projection of credit losses postulated on prespecified macroeconomic conditions over a prescribed time frame, usually nine quarters. For the CCAR stress test exercise released in June 2020, the Federal Reserve projected total loan losses of $433 billion for the 33 participant firms under the severely adverse scenario. Credit cards portfolios contributed $144 billion, or 33 percent, to overall losses, and first mortgage portfolios contributed $19 billion, or 4.4 percent, while junior liens and home equity lines of credit (HELOCs) contributed $7 billion, or 1.6 percent, and other consumer loans, including auto loans, contributed $48 billion, or 11 percent, to overall losses.[3]

The COVID-19 crisis led to more frequent stress testing under a broader set of alternative macroeconomic scenarios. While potential projected losses from mortgage portfolios were at the heart of concerns over the soundness of banks' balance sheets during the last Great Recession, projected losses from mortgage portfolios under CCAR have been decreasing steadily in recent years as a result of improvements in underlying portfolio risk factors and economic factors, home prices in particular (Calem et al., 2019).

Another important focus of accounting and banking regulators is the analysis of the Allowance for Loan and Lease Losses (ALLL). The ALLL represents an estimate of credit losses within a portfolio of loans and leases. This estimated measure of credit loss reduces the amount of the loan portfolio reported on the bank's balance sheet. During the 2008 global financial crisis, the existing allowance methodology was found to delay the recognition of credit losses resulting in loan loss reserves that were not adequate. As a result, the Financial Accounting Standards Board (FASB) conducted a review of the ALLL methodology. This review culminated with the issuance of an accounting standards update (ASU 2016-13), which introduced CECL as a new standard for the analysis of ALLL. The new standard represents a significant departure from the incurred-loss standard that it replaces. The incurred-loss standard considers losses that are probable and estimable as of the reserve calculation date.

CECL differs from the incurred loss standard in many respects. CECL is built on the notion of forward-looking estimated "expected losses." This measurement of expected credit losses is based

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[3] See https://www.federalreserve.gov/publications/files/2020-dfast-results-20200625.pdf.

on relevant information about past events, including historical experience, current conditions, and reasonable and supportable forecasts that affect the collectability of loans. Institutions are expected to reserve for lifetime losses on loans at the time the loans are originated.[4] The American Bankers Association (ABA) described CECL as "the most sweeping change to bank accounting ever."[5]

The CECL framework is not prescriptive about models and methodologies and allows for a variety of quantification strategies. A recent ABA document covering CECL background and FAQs from bankers highlights the integration of CECL and CCAR as a key area of operational concern.[6] As the ABA document highlights, challenges for the integration of CECL and CCAR include the divergence of assumptions and objectives. At a very high level, CECL considers the analysis of lifetime losses on a static portfolio at a specific date, while CCAR adopts a dynamic view on a bank's balance sheet and considers the projection of portfolio losses over a prespecified time frame of nine quarters. CCAR and CECL also differ on their respective assumptions about macroeconomic scenarios, with CCAR postulating macroeconomic scenarios with different levels of severity over a nine-quarter time frame and CECL adopting a more flexible framework, based on reasonable and supportable forecasts.[7] Furthermore, the ALLL represents an integral component of the analysis of a bank's equity position within the CCAR framework.[8]

The conventional framework for the analysis of credit risk involves the projection of credit loss in terms of three components: probability of default (PD), loss rate given default (LGD), and exposure at default (EAD). The academic and industry/regulatory communities have embraced a piecemeal approach to model building in which each of the different components of credit loss are analyzed separately. Specifically, there is a vast literature dedicated to the analysis of loan default across retail asset classes, with a significant amount of research concentrated on the problem of mortgage default. For example, Deng, Quigley, and Van Order (2000), a widely cited paper in this literature, analyzes the risk of mortgage default as a competing risk process that simultaneously considers the probability of default and the probability of prepayment, a modeling approach that has been the workhorse of analysis in the specialized mortgage default literature. More recently, Gerardi, Lehnert, Sherlud, and Willen (2008) conducted an analysis of model performance preceding the financial crisis with a focus on predicting mortgage default. Other authors have

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[4] Banking regulators have issued Implementation and transition guidance. See the Board of Governors of the Federal Reserve System (BOG), https://www.federalreserve.gov/supervisionreg/topics/accounting.htm or https://www.federalreserve.gov/newsevents/pressreleases/files/bcreg20200826a2.pdf for recent guidance.

[5] See https://www.aba.com/advocacy/our-issues/cecl-implementation-challenges.

[6] See https://www.calbankers.com/sites/main/files/file-attachments/cecl-backgrounder.pdf?1497388167.

[7] See https://www.federalreserve.gov/supervisionreg/topics/faq-new-accounting-standards-on-financial-instruments-credit-losses.htm.

[8] See "Regulatory Capital Rule: Revised Transition of the Current Expected Credit Losses Methodology for Allowances." *Federal Register* 85(190). Wednesday, September 30, 2020.

focused on the analysis of loss given default in mortgage portfolios (Goodman and Zhu, 2015; An and Cordell, 2020). While, Bellotti and Crook (2012) focus on the analysis of loss given default in credit cards. Furthermore, the analysis of exposure at default has been prevalent in the literature on credit risk in revolving credit, credit cards in particular (Qi, 2009; Leow and Crook, 2016).[9] Recent work by Banerjee and Canals-Cerdá (2013), Canals-Cerdá and Kerr (2015), and Breeden and Canals-Cerdá (2018) deploy a complete suite of models for the analysis of PD, LGD, and EAD for cards and mortgage portfolios, respectively, but analyze each model as a separate entity, and model projections are only combined at the final loss projection step.

The piecemeal approach to model building also represents the predominant framework of analysis in the growing literature on stress testing, as reflected in paper presentations at the annual Federal Reserve Stress Testing Research Conference organized by the Federal Reserve Bank of Boston. For example, Hale, Krainer, and McCarthy (2020) focus on comparing home equity loan default models with different levels of aggregation; Guo, Wu, and Zhao (2016) study the determinants of auto loan default; Haughwout, Tracy, and van der Klaauw (2017) focus on vintage effects in mortgage loan default models, while An and Cordell (2020) focus on loss severity on defaulted mortgages. Other papers in the stress testing literature include Frame, Gerardi, and Willen (2015) that analyze the supervisory stress testing framework of the government-sponsored enterprises (GSE) prior to the crisis, concentrating their attention on the analysis of default and prepay, while Bellotti and Crook (2013) focus on forecasting and stress testing credit card default. Financial institutions do not generally publish detailed descriptions of their model methodologies. However, the Federal Reserve in a recent publication documented best modeling practices across CCAR banks. This publication suggests that leading banks follow a piecemeal approach to modeling broadly consistent with the published empirical literature referenced in the previous paragraphs.[10]

The piecemeal approach to model building has also been broadly adopted in the growing literature that analyzes the implementation impact of CECL. Specifically, recent papers by Chae, Sarama, Vojtech, and Wang (2018), and Deritis and Zandi (2018) focus their attention on the analysis of CECL for mortgage portfolios, and their analysis of lifetime loan loss under CECL focus primarily on mortgage default and rely on simple assumptions about LGD to project losses. More precisely, the first paper assumes a fixed 45.5 percent LGD, while the second paper assumes an LGD of 35 percent across the board. Breeden (2018) compares CECL across modeling strategies and with the different components of loss estimated separately or aggregated into a single measure of loss. Canals-Cerdá (2020) conducts a descriptive forensic analysis of CECL using credit card data and

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[9] While stress tests of banks portfolios gained popularity during the financial crisis, they have a long history in banking. For example, the 2006 Basel II accord identifies stress tests as a key component of the banks risk management toolbox: "An IRB bank must have in place sound stress testing processes for use in the assessment of capital adequacy. (…) Examples of scenarios that could be used are (i) economic or industry downturns (…)." https://www.bis.org/publ/bcbs128.pdf.

[10] See https://www.federalreserve.gov/bankinforeg/bcreg20130819a1.pdf.

implements simple hurdle models of credit card loss to analyze the impact of macroeconomic forecasting error on CECL loss projections.

One of the objectives of this paper — perhaps the primary one — is to highlight the significant benefits that can be achieved by adopting a modular approach to model building, in which specific model components are analyzed and evaluated within the context of the overall framework at every step of the model project management life cycle. This is an interdisciplinary objective that requires coincident advancements in data, software, systems, and the risk management framework. We view GSEM as an incremental step forward toward that goal.

The paper proceeds as follows. In the next section, we provide a brief introduction to GSEM, CCAR, and CECL; a detailed description of each one of these topics is beyond the scope of this paper. In Section III, we briefly describe the data to be employed in the empirical examples and conduct a descriptive statistical analysis of the evolution over time of salient features of the data. In Section IV, we introduce the empirical application of the GSEM framework to the implementation of CCAR and CECL. In Section V, we present relevant empirical findings and discuss advantages of GSEM in this context. In Section VI, we present conclusions and highlight future areas of research. Tables and figures are presented in a separate section at the end of the paper.

A Brief Introduction to GSEM, Stress Testing, and CECL

The primary purpose of our paper is to highlight — with empirical examples — the game-changing potential of GSEM for the development and management of complex credit risk modeling projects. We illustrate this potential with an application that tackles the integration of CCAR and CECL. In this section, we provide a brief introduction to GSEM, stress testing, and CECL. An in-depth description of these topics is outside the scope of this paper.

A Hitchhiker's Guide to GSEM

The GSEM framework has been implemented in popular statistical packages like Stata, R, Mplus and SAS, and training courses, and introductory how-to guides are readily available. The analysis in this paper will be conducted using the Stata implementation of GSEM. The Stata manual provides a wealth of information about GSEM.

The next statement is a concise description of the power of the GSEM framework from the Stata manual:

> GSEM fits models to single-level or multilevel data. Latent variables can be included at any level. GSEM can fit models with mixed effects, including random effects such as unobserved effects (…), nested effects (…), and crossed effects (…). Structural equation modeling is not just an estimation method for a particular model (…).

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> Structural equation modeling is a way of thinking, a way of writing, and a way of estimating.[11]

GSEM incorporates a powerful and flexible syntax that facilitates the analysis of a rich set of alternative model specifications with minimum code changes. Table 1 presents some illustrative examples of popular models that can be easily implemented within the GSEM framework. As the table indicates, we can use GSEM to estimate simple regressions, Tobit regressions, seemingly unrelated regressions, instrumental variable regressions, survival and competing risk models, all kinds of discrete dependent variable models, and more, as well as combinations of models. Most of the examples in this table are described in detail in the GSEM Stata manual, along with helpful empirical illustrations.[12] The Stata Structural Equation Modeling reference manual includes nearly 700 pages of condensed technical information and about 60 relevant examples of GSEM applications.

In addition, GSEM integrates a powerful graphical interface that facilitates the process of model building and visualization of interactions across exogeneous, endogenous, and latent variables within complex models. For example, Figure 1 depicts the graphical representation of a seemingly unrelated regression model of the form

$$\text{gsem (Y1 <- X1 X2 X3) (Y2 <- X3 X4), cov(e.Y1 * e.Y2),}$$

with the first part of the model specification describing the structure of a two linear regression equations model and the second part of the model specification incorporating the ability to accommodate cross-equation contemporaneous correlations between residual components.

From the practical perspective considered in this paper, GSEM can be interpreted as a flexible modeling syntax for the design and development of complex modeling projects. The GSEM framework also represents a valuable tool that facilitates the implementation and integration of the different components of a credit risk modeling project within a single modeling framework. In addition, the postestimation commands that represent an integral part of GSEM facilitate the deployment into production of complex modeling projects. For example, in the particular case of the projection of loss in a consumer finance portfolio (credit cards, mortgages, or auto loans, for example), loss can be described in terms of standard PD, LGD, and EAD models within a unified GSEM framework, and model projections can easily be combined into an overall loss projection in a coherent and efficient way, as we will illustrate in the next sections.

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[11] See https://www.stata.com/manuals13/semgsem.pdf .

[12] The instrumental variables model is explained in https://statisticalhorizons.com/iv-in-sem.

We can expect the GSEM framework to continue to grow and improve by expanding the already extensive family of models and options, including postestimation options, the underlying optimization environment, its powerful graphical interface, and its simple syntax.

A Hitchhiker's Guide to Stress Testing

Banking regulators in the U.S. conducted the first official comprehensive stress test of systemically important financial institutions during the 2009–2010 period of financial distress. The annual CCAR stress test has been conducted annually ever since or on a more regular basis during the COVID-19 crisis.[13] The primary objective of the initial stress test was to ascertain capital needs across large bank holding companies (BHCs) such that the institutions would "still remain sufficiently capitalized at over the next two years and be able to lend to creditworthy borrowers should such losses materialize."[14] While our paper focuses primarily on a stress testing framework consistent with the analysis of credit risk under CCAR, similar stress tests have also been adopted by other regulators worldwide and for a variety of objectives.[15]

The conventional framework for the analysis of credit risk involves the analysis of projected credit loss in terms of three components: default, loss given default, and exposure at default. This framework has been broadly adopted by bank regulators, for example, in the calibration of regulatory capital[16] or as an important building block of the supervisory stress testing methodology, as described in the most recent Dodd–Frank Act Stress Test 2020 Disclosure:[17]

> For most loan types, losses in quarter t are estimated as the product of the projected PD, LGD, and EAD:
>
> $$\text{Loss}(t) = \text{PD}(t) \bullet \text{LGD}(t) \bullet \text{EAD}(t)$$
>
> (…). The Federal Reserve generally models PD as a function of loan characteristics and economic conditions. The Federal Reserve typically models LGD based on historical data, and modeling approaches vary for different types of loans. For certain loan types, the Federal Reserve models LGD as a function of borrower, collateral, or loan characteristics and the macroeconomic variables from the supervisory scenarios. For other loan types, the Federal Reserve assumes LGD is a fixed percentage of the loan balance for all loans in a category. Finally, the approach to modeling EAD varies by loan type and depends on whether the loan is a term loan or a line of credit.

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[13] See https://www.federalreserve.gov/bankinforeg/ccar-and-stress-testing-as-complementary-supervisory-tools.htm

[14] See https://www.federalreserve.gov/newsevents/pressreleases/files/bcreg20090424a1.pdf

[15] See Schuermann (2014) for a broader overview of banks' stress testing.

[16] See https://www.bis.org/publ/bcbs128.pdf.

[17] Extract from page 25 of https://www.federalreserve.gov/supervisionreg/dfa-stress-tests.htm.

Financial institutions do not generally publish detailed descriptions of their model methodologies. However, the Federal Reserve in a recent publication documented best modeling practices across CCAR banks:[18] "BHCs with leading practices were able to break down losses into PD, LGD and EAD components, separately identifying key risk drivers for each of these components." This suggests that leading banks follow modeling practices broadly consistent with the framework adopted by bank regulators as described in the previous paragraph.

This paper also adopts the conventional framework to project credit loss in terms of its different components, while addressing the specific challenges associated with the modeling of credit risk in mortgage portfolios. However, in contrast with most of the prior research in the field, which has adopted a piecemeal approach to model building, we illustrate how the GSEM framework can contribute to a modular model-building approach. Furthermore, this paper illustrates how GSEM can leverage the potential synergies across modeling efforts within an organization by implementing a framework that combines CCAR and CECL modeling within a single GSEM framework as an illustrative and relevant application.

A Hitchhiker's Guide to CECL

During the 2008 global financial crisis, it was determined that the prescribed reserving methodology delayed the recognition of credit losses on loans and resulted in loan loss reserves that were not adequate. Postcrisis, the Financial Accounting Standards Board (FASB) considered enhancing standards on valuation and loan loss provisioning. In June 2016, the accounting standard setters issued an accounting standards update (ASU 2016-13) — and the Current Expected Credit Loss Framework (CECL) was born.[19]

The allowance for loan and lease losses (ALLL) under U.S. generally accepted accounting principles, prior to CECL implementation, is an incurred loss accounting methodology. Under the incurred loss methodology, the allowance is a valuation reserve established and maintained to cover losses that are probable and estimable as of the reserve calculation date. The methodology has been in place for about 40 years.

CECL represents an alternative framework for calculating the allowance for credit losses and a significant departure from the incurred loss standard. CECL is built on the notion of forward-looking estimated "expected losses." The measurement of expected credit losses is based on relevant information about past events, including historical experience, current conditions, and reasonable and supportable forecasts that affect the collectability of loans. Institutions are

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[18] See https://www.federalreserve.gov/bankinforeg/bcreg20130819a1.pdf.

[19] Banking regulators have issued implementation and transition guidance. See the Board of Governors of the Federal Reserve System (BOG), May 2018, BOG, June 2016, and BOG "Frequently Asked Questions on the New Accounting Standard on Financial Instruments – Credit Losses."

expected to reserve for lifetime losses on loans at the time the loans are originated. CECL will also require enhanced disclosures.[20]

Both CCAR and CECL are nonprescriptive about the models and loss projection methodology that banks should employ. CCAR loss projections rely on macroeconomic forecast scenarios published by bank regulators as well as banks' own macroeconomic forecasts under different assumptions about the severity of a potential economic downturn over a prespecified timeframe of nine quarters. In contrast, CECL considers lifetime loss and is nonprescriptive about economic projections but prescribes reasonable and supportable forecasts over reasonable timeframe and convergence to long-run economic conditions after that.

Data and Descriptive Analysis

The remaining sections illustrate with empirical examples how the GSEM framework can contribute to simplify and strengthen essential elements of a financial institution's risk management framework. Our examples focus on the analysis of a mortgage portfolio, but the approach is generalizable across retail or wholesale exposures. We begin this section with a description of the data and follow up with a high-level descriptive analysis highlighting relevant trends across mortgage origination cohorts.

Because the purpose of our analysis is illustrative, for each year in our data, we focus our attention on the problem of projecting stress loss and lifetime loss for the segment of new loans originated in each particular year.[21] The recently introduced CECL framework for the analysis of allowances for loan losses makes the analysis of loss projections for new loans particularly relevant. Specifically, CECL requires an institution to record lifetime expected credit losses at the time of loan origination. Because of this, institutions are likely to pay particular attention to the analysis of allowances for newly originated loans during periods of economic stress, especially capital constrained institutions. Some experts argue that CECL will depress new loan originations during periods of economic stress, and this effect may slow economic recovery. This is not a universally held view; other experts argue that CECL adds flexibility to the allowance framework, and financial institutions taking advantage of this flexibility can increase their allowances earlier, which may incentivize lending during economic downturns.[22] An analysis of the allowance requirements for

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[20] CECL applies to every organization required to issue financial statements in compliance with U.S. GAAP. Following U.S. GAAP is required by the Federal Deposit Insurance Act, which says that all insured depository institutions are required to be uniform and consistent with GAAP. FDI Act – SEC 37(a)(2)(A). Banks are likely to experience the largest implementation burden.

[21] A cohort analysis of all loans in a portfolio for the analysis of stress test or allowances for loan loss will require a consideration of risk differences across vintages.

[22] See Scott Blackley, Bill Nelson, Joseph A. Stieven, and Mark Zandi Hearing: "Assessing the Impact of FASB's Current Expected Credit Loss (CECL) Accounting Standard on Financial Institutions and the Economy" Subcommittee on Financial Institutions and Consumer Credit (Committee on Financial Services).

newly originated loans over the economic cycle of the type conducted in this paper will contribute to shed light on this discussion, although many factors contribute to the risk appetite and loan origination strategy of financial institutions.

The data

Our empirical analysis uses a publicly available mortgage panel data set of loans originated between 1999 and 2017, including their historical performance information. Freddie Mac makes this data set available for download in its website as part of a larger effort to increase transparency.[23] The data include loan-level credit performance data on a portion of fully amortizing fixed-rate mortgages that the company purchased or guaranteed. The data set covers approximately 24 million fixed-rate mortgages originated between January 1999 and September2017. Our empirical analysis is conducted on a 25 percent sample of the overall data set. Monthly loan performance data, including credit performance information up to and including property disposition, are being disclosed. Specific credit performance information in the data set includes voluntary prepayments and loans that were foreclosure alternatives and real estate owned. Specific actual loss data in the data set include net sales proceeds, mortgage insurance (MI) recoveries, non-MI recoveries, expenses, current deferred unpaid principal balance, and due date of last paid installment. We complement this data with macroeconomic information at the state level. Table 2 provides a brief description of risk drivers employed in our analysis.

Descriptive analysis

Figure 2 provides a graphical representation of the distribution of relevant variables, across origination cohorts, at the time of origination. The figure shows stable underwriting standards until 2008 and a tightening of underwriting standards after that, especially during the 2008–2013 period. In particular, the average origination credit score remained stable at around 720 until 2008 and increased to the mid-760s during the period from 2008 to 2013. Similarly, combined loan to value (LTV) remained relatively stable close to 0.8 until 2008 and decreased to the low 0.70s during the period from 2009 to 2013. The same tendency toward more conservative underwriting is also observed for the origination debt-to-income ratio after 2008. Origination loan balance has continued to increase over the years.

Figure 3 displays the historical performance of the unemployment rate and home price index across years as well as historical measures of prepay, default, and loss rate for defaulted loans by origination cohorts.[24] Our definition of *default* and *loss rate given default* is consistent with Frame, Gerardi, and Willen (2015), which employs the same data source in their analysis of GSE stress

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[23] Comprehensive information about the data set described in this section, including access to the overall data set, is available from http://www.freddiemac.com/news/finance/sf_loanlevel_dataset.html . Much of the data description in this section is extracted directly from the information provided at this website.

[24] Historical economic variables complementing the Freddie Mac data set are from Haver Analytics.

tests. We observe the highest nine-quarters ahead default rates for loans originated during the 2006–2008 origination cohorts. We also observe the highest nine-quarter ahead prepayment rates for loans originated during the 2000–2002 cohorts. The figure also reports loss rates for defaulted loans across origination cohorts and highlights a significant increase in realized loss rates starting with the 2005 cohort, reaching a peak with the 2007 cohort and staying elevated until the 2011 cohort. Figure 3 also depicts relevant macroeconomic variables; the figure highlights a significant increase in the unemployment rate as well as a downward shift in the house price index (HPI) during the 2006–2009 period.

Empirical Applications of GSEM to Consumer Finance: From CCAR to CECL

CCAR and CECL differ in their objectives and underlying assumptions, but both regulations share the common problem of projecting portfolio credit loss over a time horizon conditional on defined macroeconomic scenarios. Both regulations are nonprescriptive about empirical methodology, although best modeling practices across CCAR banks have emerged over the years.[25] As discussed in the introduction, the integration of CCAR and CECL is a key area of operational concern among industry professionals.[26] In the next paragraphs, we develop a common loss projection framework that can accommodate the requirements of both regulations, with a focus on a nine-quarter forward loan loss forecast under CCAR and the analysis of lifetime loan losses under CECL.

The loss projection framework

The loss projection workhorse for the stress testing of credit risk postulates the decomposition of projected loss as the product of the projected PD, LGD, and EAD. Different types of consumer finance portfolios experience distinctive modeling challenges. For example, the analysis of prepayment and loss, or recovery, given default is particularly challenging for mortgage and auto portfolios. While the analysis of exposure at default may have its own challenges for portfolios of revolving accounts, credit cards in particular (e.g., Canals-Cerdá and Kerr, 2015). The GSEM framework is flexible enough to accommodate these challenges. Furthermore, the GSEM framework can expand the universe of model specifications beyond these typically considered, as we discuss later in this paper.

Because our empirical examples consider the problem of loss projections for a portfolio of 30-year fixed-rate mortgages, we parametrize projected loss at time t as a function of the loan balance at projection time t_0, the probability of default at time t, and the LGD over the loan balance at t_0, if the loan defaults.[27]

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[25] See https://www.federalreserve.gov/bankinforeg/bcreg20130819a1.pdf.

[26] See https://www.calbankers.com/sites/main/files/file-attachments/cecl-backgrounder.pdf?1497388167.

[27] Losses and recoveries from defaulted mortgage loans are realized over a long time interval. For the purpose of our analysis, we consider overall net loss as reported in the data.

$$Loss(t|t_0) = PD(t|t_0) \bullet LGD(t|t_0) \bullet Bal(t_0).$$

Our default model specification considers three possible outcomes over a sequence of time intervals (t-1,t]: mortgage default (D) or mortgage prepay (P) within the (t-1,t] timeframe, or mortgage still active (A) after t. Each one of these possible outcomes is relevant in its own right.

The empirical process described previously is depicted graphically in Table 3. For completeness, we characterize here the Maximum Likelihood (ML) function associated with that empirical process. The likelihood of observing a loan defaulting in period T > t_0 at a loss l is equal to

$$\left\{ \prod_{t=t_0+1}^{T-1} p_A(t) \right\} \cdot p_D(T) \cdot f(l|T),$$

where $p_D(.)$ represents the default probability, $p_D(.)$ represents the probability of the mortgage remaining active, and $f(l|.)$ represents the loss rate probability conditional on default at a specific future point in time. We can derive similar expressions for the probability of prepayment and the probability of a loan remaining active at the end of the observation period.

The empirical specification considered in our analysis for the probability of each possible event, {default, prepay, active}, is a multinomial logit conditional on a set of risk drivers $X_{it} = (Z_i, M_{it})$, where Z_i represents loan characteristics at observation time t_0 specific to outcome i, and M_{it} represents a set of macroeconomic risk drivers,

$$p_i(t|t_0) = \frac{exp\left(\varphi(X_{it}|\beta_{it})\right)}{\sum_{j=1}^2 exp\left(\varphi(X_{jt}|\beta_{jt})\right)} \quad i = D, P \quad p_A(t|t_0) = \frac{1}{\sum_{j=1}^2 exp\left(\varphi(X_{jt}|\beta_{jt})\right)}.$$

The loss given default is represented by a simple index specification:

$$l_i(t|t_0) = \gamma(X_{it}|\delta_{it}),$$

and where $\varphi(.|.)$ and $\gamma(.|.)$ represent parametric functions of observed characteristics X_{it}, which can include interactions and time variant coefficients that can take on different values across time intervals {(t-t_0) = 1, 2,...}. This parametrization is consistent with the typical model specification in the literature on discrete time survival models (Sueyoshi, 1995).

We consider two different implementations of this loss projection framework, with multiple model specifications in each case. First, we estimate a simpler implementation that projects aggregated loss over a nine quarter period as a function of aggregated default and aggregated loss given default over that period. Second, we consider a more elaborated model specification consistent with the standard stress testing framework, which requires quarterly projection of losses. For the purpose of computing CECL projected losses, we expand our analysis up to 20 quarters after origination and consider aggregated losses over the 10th to 20th quarter period as a function of

aggregated default and aggregated loss given default over that period. For the purpose of our analysis, we assume that the risk of loss after 20 quarters is insignificant, but this is not an intrinsic limitation of the framework.

The first model specification considered can be interpreted as a benchmark to the second more complex model specification that considers a more elaborated framework that allows for quarterly projection of losses. For the first model specification, we consider two performance time intervals: "from 0 to 9 quarters" for the purpose of nine quarter projections under CCAR and "from 9 to 20 quarters" to expand the projection to the lifetime loss scenario required under CECL. For the second model specification, we explicitly model quarterly loan performance up to the ninth quarter, consistent with the typical stress testing framework, and a performance time interval of "from 9 to 20 quarters" for loans still active after nine quarters. Thus, the two models differ in their model specifications over the first nine quarters.

For each time interval, we specify equations for the likelihood of "default" or "prepay," with "active" selected as the base state in the multinomial logit specification, as described previously. In addition, for each time interval, we also specify an equation to model LGD as a function of initial loan balance $Bal(t_0)$, as described previously. Thus, for each time interval, our model specification consists of three model equations. As a result, the simplest model considered requires estimation of six model equations, while the more complex model specification requires estimation of 30 different equations. From the perspective of the CECL regulation, we can interpret the initial nine quarter period as a timeframe encompassing a period of "reasonable and supportable forecast," for illustrative purposes.

In the next sections, we illustrate how GSEM can integrate two complex regulations. GSEM brings a modular approach to model building that can simplify and enhance not only the model development process but also all other steps of the model life cycle. GSEM can also bring clarity to the model development process by integrating different model components into a single framework and by focusing attention on the key modeling outputs at every stage of the model development process.

Table 4 illustrates how complex model structures can be encapsulated within a single GSEM command. Model 1.a considers the estimation of $Loss(t|t_0)$ over a nine quarter time frame in terms of its $PD(t|t_0)$ and $LGD(t|t_0)$ components. In Model 1.b, we expand the estimation of $Loss(t|t_0)$ beyond nine quarters to the lifetime projected loss scenario prescribed under CECL. In the empirical section of the paper, we address practical model specification challenges associated with an aggregated long projection timeframe. The GSEM specification in Model 2 considers the estimation of a lifetime loss scenario prescribed under CECL, but it also allows for quarterly loss projection over the initial nine quarters as usually required by the typical stress test exercise.

For Model 1, the target variable lgd_9q represents the loss rate on a mortgage that defaults within the first nine quarters after origination. Target variables 0.9q_out, 1.9q_out and 2b.9q_out, represent the different target outcomes in our multinomial logit specification: default (as 0), prepay (as 1) and active (as 2, the base outcome). Also, X_9q_lgd and X_9q_out represent the dependent model specification risk drivers, explanatory variables, or features. The GSEM framework offers a great level of flexibility. For example, different model equations can be associated with different risk drivers and include combinations of risk drivers, although these features are not explicitly depicted in the table for simplicity.

Discussion of empirical results

We estimate and analyze different parametrizations of aggregated and quarterly loss projection models over the whole sample with a five year, or 20 quarters, performance period from origination as our target performance time frame. The various model specifications considered differ primarily in their parameterization of the macroeconomic variables, which is in line with our focus on analyzing model performance under stress economic conditions. Risk drivers included in our models are directly derived from these in Table 2. We don't consider cohort effects or year effects, or regional dummies, or changes in underwriting standards beyond changes reflected in observed control variables. Model specifications for the performance period after the initial nine quarters do not include controls for macroeconomic conditions. Thus, model projections after the initial nine quarters implicitly consider a through-the-cycle estimation approach consistent with the CECL requirement of long-run average loss projections beyond a reasonable and supportable forecast period.

The complexity of the models estimated makes it difficult to report model parameter estimates in a traditional fashion. The simplest model specification considered, specification Model 1.a, contains three estimation equations; other model specifications contain six, 27, or 30 different equations. However, the management of estimated models is a relatively simple task with the available postestimation commands in the GSEM framework. For illustrative purposes, Table 5 presents model estimates for a simple specification associated with Model 1.a of Table 4. We will refrain from depicting estimation results for most estimated models.

The model estimates presented in Table 5 are broadly consistent with our intuition. A challenging aspect of empirical models that project aggregated outcomes over a relatively large nine-quarter performance time frame is the specification of macroeconomic risk drivers. In the model specification of Table 5, we control for macroeconomic conditions over the nine-quarter time interval by defining the largest increase, or decrease, in the macroeconomic risk drivers over the nine-quarter projection time frame considered at the state level. For the default and prepayment equations, results are presented as odd ratios. A casual examination of model parameters indicates that the probability of default decreases significantly with a higher FICO, a higher combined LTV, or a higher DTI ratio. Cash-out refinance loans, investment loans, and

manufactured houses loans also have higher default probability. Macroeconomic effects are also consistent with our intuition. Higher unemployment and lower HPI increase the probability of default, while a decrease in interest rates over the nine-quarter performance period, as well as a higher loan size, increase the probability of loan prepayment. Also consistent with our intuition are the results associated with the loss rate for defaulted accounts. A higher combined LTV at origination above 80 percent of home value increases LGD. Cash-out refinance loans and investment properties increase the LGD. Loss rates are also higher in judiciary states, and a decrease in HPI increases LGD as well.

Figure 4 depicts model projections and realized outcomes for different year cohorts across years for several outcome variables of interest using different model specifications associated with Model 1.a structure in Table 4; the solid black line represents realized outcomes. Most model specifications considered are highly sensitive to macroeconomic conditions and broadly match the performance of realized outcomes. One model specification in particular, which purposely does not include macroeconomic risk drivers, matches the performance of realized outcomes poorly, as it is clear from the figure. However, this simple model specification without macroeconomic controls serves as a useful reference that help us compare graphically the impact of macroeconomic risk drivers versus other measures of credit risk.

Looking more closely at Figure 4, we observe that our LGD models fit the realized average outcomes particularly well during the stress period 2005–2010, with the solid line representing realized outcomes. Some LGD model specifications perform particularly poorly during the period prior to the Great Recession. One possible reason for this level of model performance is that most defaults in our sample occur during the period of economic stress. Model default projections seem to track realized default rates better than prepayment rates. Finally, model projected loss rates derived from the estimated models as $PD(t|t_0) \bullet LGD(t|t_0)$ seem to track realized loss rates well, with the exception of the models that do not control for macroeconomic risk drivers as explained in the previous paragraph.

As illustrated in Table 4, Model 1.b provides a simple example of integration of a benchmark stress test model with a model of lifetime loss projection consistent with CECL. Model projections for this exercise are presented in Figure 5. This figure combines realized nine-quarter loss rates over an initial nine quarter period from origination, represented by a solid black line, with the long-run through-the-cycle CECL projected loss rate beyond nine quarters, represented by a gray dotted line, with CECL lifetime loss rate projections across several different model specifications. The long-run CECL projected loss rate is generated from a through-the-cycle model specification that does not include macroeconomic risk drivers. As the figure shows, CECL loss rate projections for newly originated accounts are highly sensitive to changes in economic conditions. The projections increased significantly for cohort years 2006 to 2008 and have remained relatively stable for years 2009 and beyond. The lower CECL loss rate projections in recent years are consistent with the

increase in loan origination standards, and historic low default rates, reported in Figure 2. We should point out that all model projections presented in this paper represent in-sample projections. As other authors have shown, a forecasting error in macroeconomic projections can have a significant impact on CECL projections; this is a well-known limitation of any forward-looking loss projection framework (Canals-Cerdá, 2020).

The second model specification considered, Model 2, provides quarterly projections up to the ninth quarter from origination, a requirement of the CCAR stress testing framework. The addition of quarterly projections in our framework increases model complexity significantly in Model 2, which includes 30 different equations, three equations per period. Alternatively, we could have considered a more traditional — and less complex — competing risk framework by specifying a single hazard function for each competing risk. However, Model 2 offers a good illustrative example of the ability of the GSEM framework to handle complex model structures. Because of the large number of equations, we do not report model parameter estimates in a traditional format similar to Table 5. Instead, we focus our attention on model projection performance.

Figures 6 and 7 provide quarterly projections over the initial nine quarter period after origination. Figure 6 compares model projections with realized outcomes (the solid black line) for the 2007 cohort at the beginning of the Great Recession. The model specification considered provides reasonable projections of default rate, prepay rate, and loss rate over nine quarters. Figure 7 compares quarterly loss projections and realized loss rates, the solid black line, across four different cohorts. We observe that nine quarter loss rates are significantly higher for the 2007 cohort, as expected, followed by 2000 cohort and 2003 cohort. The observed loss rate over a nine quarter period is close to zero for the 2012 cohort. Thus, perhaps it should not be surprising that our loss rate projects — while very low — are still significantly higher than the record low loss rates observed that cohort.

Conclusions

Financial firms, and banks in particular, rely heavily on complex suites of interrelated statistical models in their risk management and business reporting infrastructures, in particular. This dependence has continued to increase over the last decade in no small part as a result of the introduction of the regulatory stress test exercises at the onset of the financial crisis, and more recently with the introduction of the novel CECL framework for the calculation of allowances for credit loss. Alongside complex models, institutions have developed voluminous model documentation and equally intricate validation processes. The use of challenger and benchmark models has also become an integral part of a modern model risk management framework. In periods of significant uncertainty, benchmark models can contribute critical insights to the analysis

of model weaknesses and to the implementation of model overlays to mitigate model uncertainty.[28]

Research on processes and methods to reduce model complexity without a significant sacrifice in robustness and accuracy can have a significant practical impact across the board. This paper provides a practical introduction to the GSEM statistical framework in risk management. We illustrate the game-changing potential of this methodology with two empirical applications. The first model application considers a simple benchmark model for the analysis of stress testing and CECL in a consumer finance portfolios. The second model builds on the first application by introducing quarterly loss projection over the postulated stress test projection time frame, which is usually a requirement in stress testing. The first application illustrates how six different model equations can be combined within the GSEM framework to provide a complete projection of loss over a nine-quarter projection period postulated in the typical stress testing framework and CECL projections of expected lifetime loss. Our second application illustrates how 30 different model equations can be combined within the GSEM framework to produce standard stress testing and CECL lifetime loss projections. The first model can be interpreted as a benchmark or a challenger to the second one.

The examples considered illustrate how the GSEM framework can contribute to simplify the process of building and managing complex models and suites of models. For example, primary and challenger, or benchmark, models can be managed jointly or in parallel as part of a specific suite of models using the GSEM framework. Furthermore, models that share significant components and objectives, but serve different purposes, can also be integrated as part of a suite of models, as our combined stress testing and CECL examples illustrate. These valuable features of the GSEM framework have broad implications. Specifically, data management can be streamlined and strengthened because complete suites of models can more consistently rely on the same data sources that can fulfill multiple goals. Similarly, model documentation can be simplified because the definition of what constitutes a model can be streamlined and better aligned with regulatory and business requirements. More broadly, GSEM can contribute to develop or strengthen a coherent and robust modeling framework in which related models with dissimilar objectives can coexist: primary models, challenger models, benchmark models, models in production versus models in development, and other related models that may share some features but serve different objectives and depend on different assumptions, like stress test and CECL models. Consequently, other areas of model management can be streamlined accordingly: validation, audit, implementation/production, ongoing monitoring, and redevelopment. Improvements in all aspects of model management can also contribute to reduce the risk — and simplify the analysis

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[28] See https://www.federalreserve.gov/supervisionreg/srletters/sr1107.htm

— of model error at different states of the model life cycle. Furthermore, the use of GSEM opens access to a powerful set of hardcoded postestimation instructions part of the GSEM framework.

It is often the case that loan portfolios are divided into segments as part of the modeling process and then potentially different model specifications are considered across segments. In particular, portfolio segments can be specified based on risk criteria, like delinquency status, or business needs or criteria, like an origination channel. A very convenient feature of GSEM is its built-in flexibility to differentiate models and model specifications across segments with ease. This is clearly highlighted in the Stata documentation: "any model fit by GSEM can be simultaneously estimated for different groups with some parameters constrained to be equal across groups and others allowed to vary, and those estimates can be used to perform statistical tests for comparing the groups." Similarly, GSEM offers modelers the ability to expand the set of feasible models beyond these usually accessible in standard statistical packages by making use of latent variables to create mixtures or design interactions within and across modeling structures. These important features of GSEM hold great potential and should not be overlooked.

The focus of our analysis has been on the projection of losses. However, extensions of the GSEM framework to other important facets of the stress testing framework seem within reach as well, once the data on historical loss in our analysis is complemented with additional relevant information. For example, one can envision that projections of portfolio revenues could be computed along with loss projections for specific portfolios within the same GSEM framework and taking advantage of similar segmentation environment, when appropriate. This type of analysis could contribute additional consistency and coherence to the stress testing framework.[29]

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[29] See Duane, Schuermann, and Reynolds (2013) for additional discussion on this topic.

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Tables and Figures

Table 1: Examples of Standard Econometric Model Specifications in GSEM

Models for continuous dependent variables, time type variables	Models for discrete dependent variables and count type variables
Regression:	Logistic:
gsem (y <- x)	gsem (y <- x), logit
Seemingly unrelated regression:	Probit:
gsem (y1 <- x) (y2 <- z), cov(e.y1*e.y2)	gsem (y <- x), probit
Tobit regression:	Ordered Probit:
gsem (y <- x), family(gaussian, lcensored(0))	gsem (y <- x), oprobit
Instrumental variables:	Multinomial logit:
gsem (y1 <- x y2) (y2 <-z), cov(e.y2*e.y1)	gsem (y <- x), mlogit
Survival:	gsem (1b.y) (2.y 3.y <- x), mlogit
gsem (t <- x), exponential	gsem (1b.y) (2.y <- x) (3.y <- z), mlogit
gsem (t <- x, family(weibull, failure(f)))	Count and multiple counts models:
Multiple survivals and competing risk:	gsem (n1 <- x), poisson
gsem (t1 t2 <- x), exponential	gsem (n1 n2 <- x), poisson

Note: Y represents the dependent variable and/or endogenous risk driver, x and z represent the set of risk drivers. See Stata GSEM manual for additional examples.

Figure 1: Graphical GSEM Example of a Seemingly Unrelated Regression Model
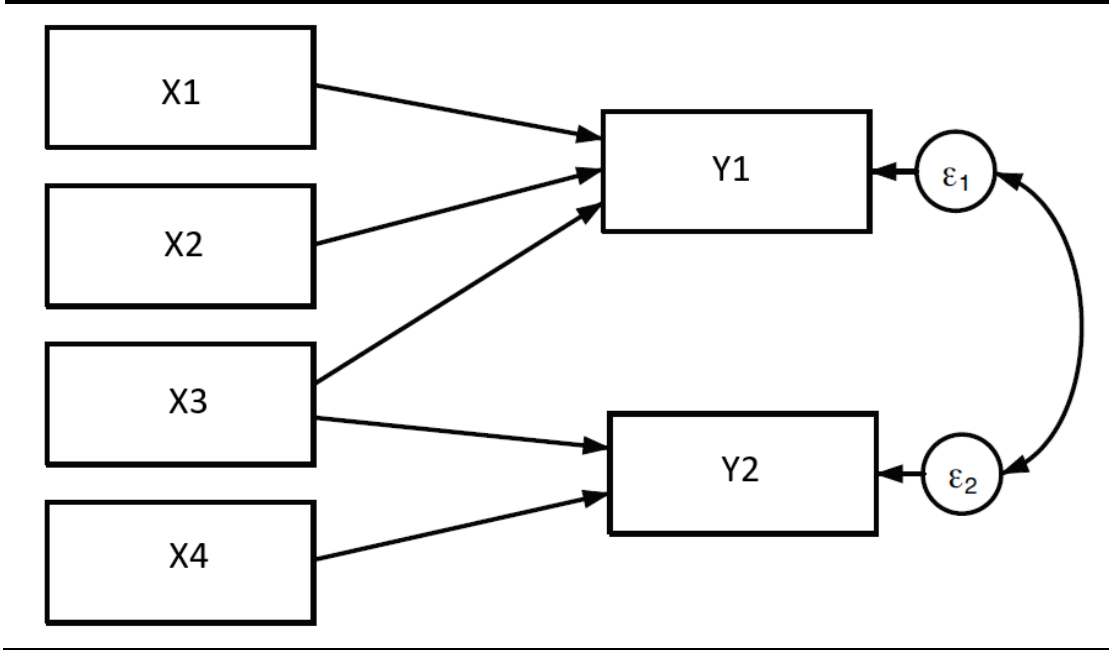
Table 2: Relevant Variable Definitions

MORTGAGE CHARACTERISTICS

Origination date	Mortgage origination month
Riskscore	Origination FICO credit score
Loan purpose	Loan Purpose (Purchase)
CLTV	Combined loan to value at origination
Occupancy type	Occupancy type (primary residence, investment property, second home, na)
Property type	Property type (condo, manufactured housing, single-family co-op, PUD, na).
Borrowers	Categorical control for number of borrowers
Channel	Channel (retail, broker, correspondent, na)
Debt-to-income	Debt-to-income at origination.
Loan size	Loan balance at origination
First-time buyer	First-time buyer dummy.
Statutory	Statutory foreclosure states (as in Crews and Merrill, 2008).

MACROECONOMIC VARIABLES

|	
Home price index (HPI)	Home price index change
Unemployment (UR)	Updated unemployment rate
Interest rate spread (IRR)	Measured as the difference of 10-year Treasury Note at a point in time with respect to its value on the date of loan origination

Note: Additional information about the data can be found at http://www.freddiemac.com/fmac-resources/research/pdf/user_guide.pdf
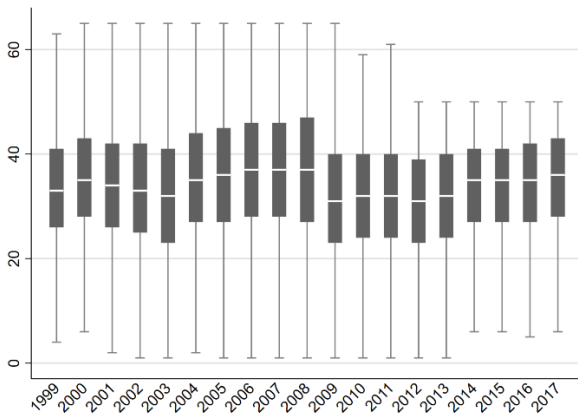
Figure 2: Distribution of Risk Drivers Across Origination Cohort Years
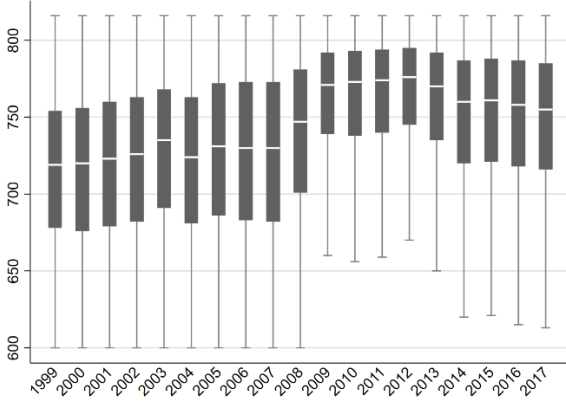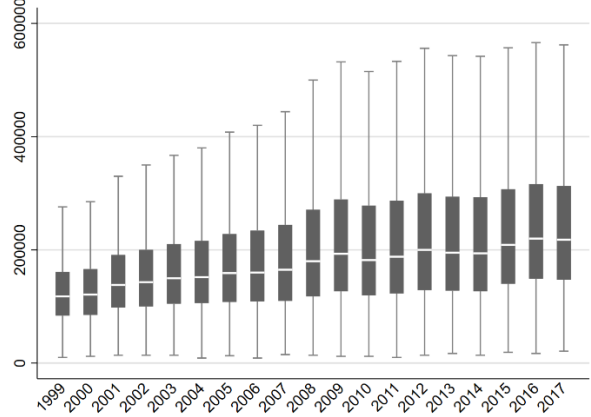

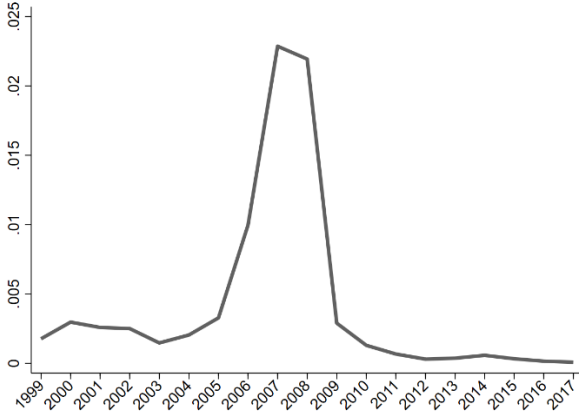
A. Origination Credit Score
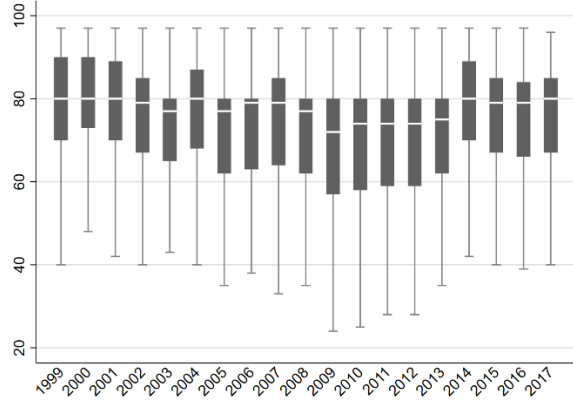
B. Origination Combined LTV


C. Origination Debt-to-Income

D. Origination Loan Balance

Note: Each cohort represents the sample of active loans in that particular year.

Figure 3: Time Series Performance of Realized Default and Prepayment and Loss Given Default Rate in Our Data, Along with Unemployment Rate and Home Price Index
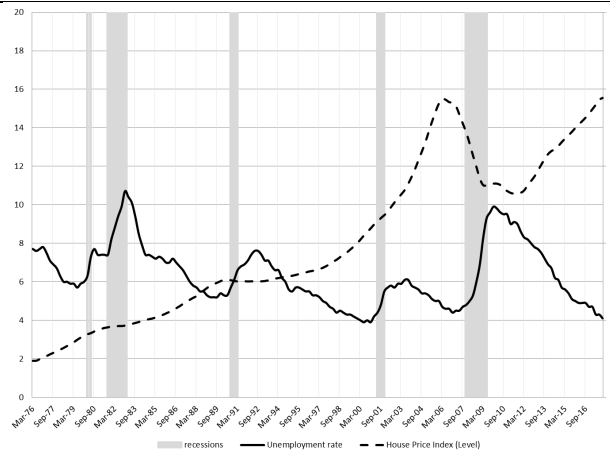

A. *Loan default rate within nine quarters from origination by origination cohort year*
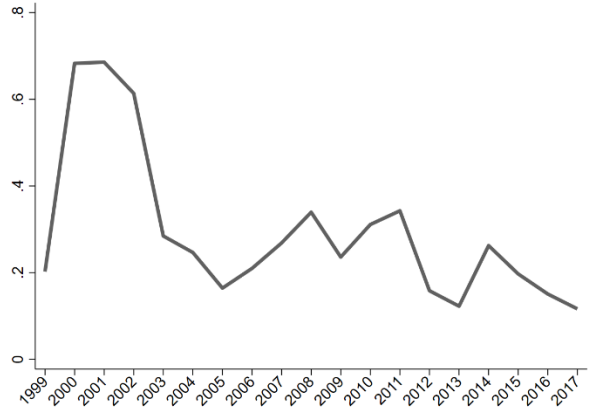
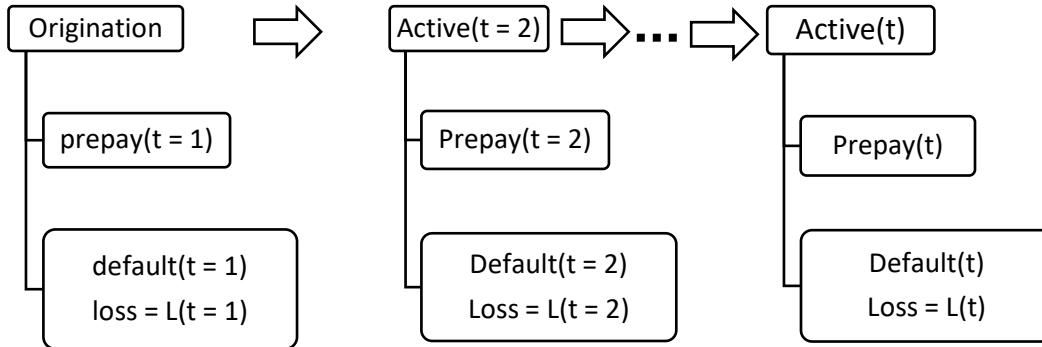B. *Loan prepay rate within nine quarters from origination by origination cohort year*
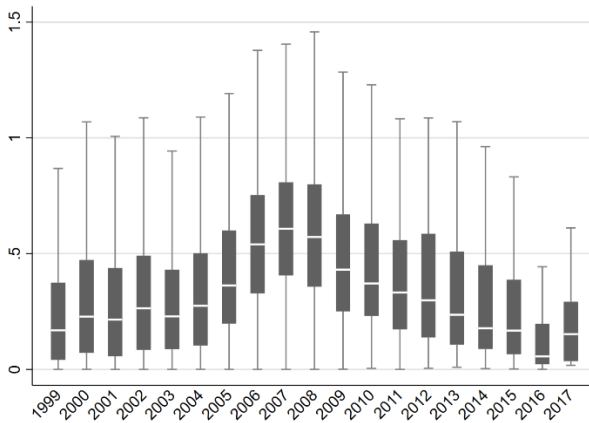
C. *Loss rate for loans defaulted nine quarters from origination (1 represents 100%).*

D. *State unemployment rate and House Price Index*

Table 3: Graphical Representation of the Econometric Model Structure

Table 4: Description of Different Modeling Frameworks Considered Under GSEM

Model 1.a: Stress testing with aggregated forecast outcomes, three estimation equations.

gsem (lgd_9q <- `X_9q_lgd') (0.9q_out 1.9q_out 2b.9q_out <- `X_9q_out ')

Model 1.b: Stress testing with aggregated forecast outcomes + CECL model extension, six estimation equations.

gsem (lgd_9q <- `X_9q_lgd') (0.9q_out 1.9q_out 2b.9q_out <- `X_9q_out ')   ///
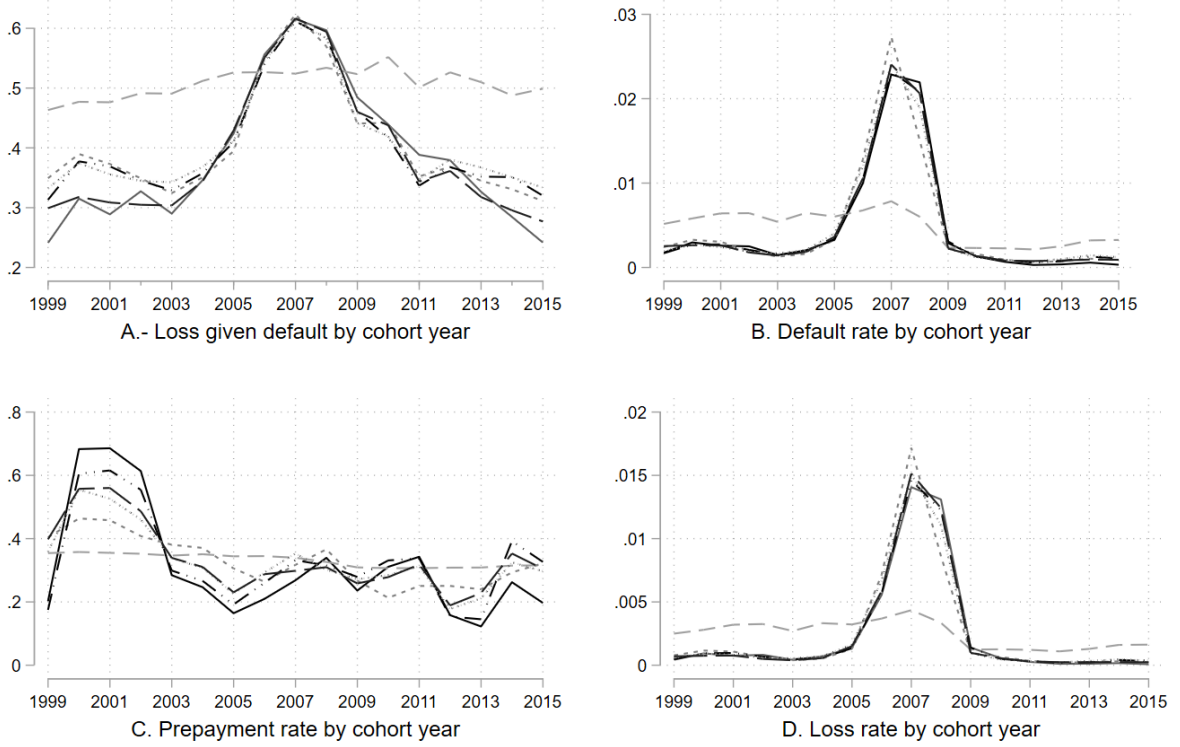   (lgd_9to20q <- `X_9to20q_lgd') (0.9to20q_out 1.9to20q_out 2b.9to20q_out <- `X_9to20q_out ')

Model 2: Stress testing with quarterly forecasts + CECL model extension, 30 estimation equations.

gsem (lgd_q1 <- `X_q1_lgd') (0.q1_out 1.q1_out 2b.q1_out <- `X_q1_out ')   ///
      (lgd_q2 <- `X_q2_lgd') (0.q2_out 1.q2_out 2b.q2_out <- `X_q2_out ')   ///
       …
      (lgd_q9 <- `X_q9_lgd') (0.q9_out 1.q9_out 2b.q9_out <- `X_q9_out ')   ///
      (lgd_9to20q <- `X_9to20q_lgd') (0.9to20q_out 1.9to20q_out 2b.9to20q_out <- `X_9to20q_out ')

Table 5: Parameter Estimates PD & LGD Model (Odds Ratio)

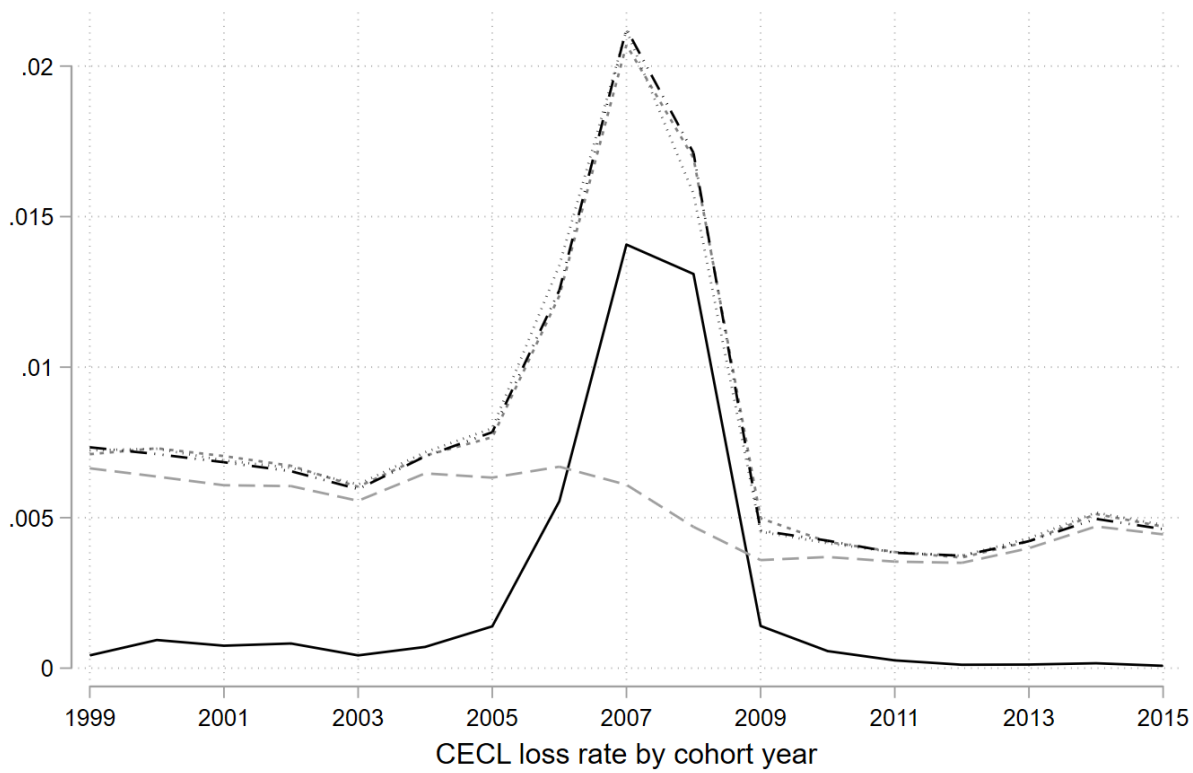|	Default		Prepay		LGD	
	coef.	t-val	coef.	t-val	coef.	t-val
RISKSCORE						
660 to 700	0.614	-26.44	1.025	5.75	-0.016	-3.48
700 to 740	0.419	-44.79	1.021	5.18	-0.021	-4.33
740+	0.200	-78.47	1.061	15.69	-0.047	-9.03
CLTV						
80%	2.469	45.85	0.873	-47.17	0.036	7.38
80% + to 90%	3.985	70.94	0.880	-37.73	-0.091	-18.64
90%+	6.032	75.24	0.848	-41.21	-0.142	-23.58
LOAN PURPOSE						
Cash out refi.	2.204	40.15	0.833	-61.94	0.068	13.71
NA	2.140	40.92	0.948	-19.73	0.043	9.1
OCCUPANCY						
investment	2.354	33.71	0.811	-37.39	0.125	20.19
PROPERTY TYPE						
Mult. Units	1.167	3.63	0.786	-29.04	0.195	18.58
Condo	1.004	0.17	1.023	5.15	0.022	3.29
Manuf. House	1.510	7.98	0.567	-36.05	0.058	4.52
BORROWERS # (multiple)	0.460	-53.38	1.116	48.64	-0.046	-12.46
CHANNEL (other than retail)	0.727	-22.07	0.962	-16.48	-0.009	-2.37
DTI						
25% to 40%	1.424	14.67	1.052	19.04	-0.002	-0.29
40% +	1.996	29.01	1.052	16.95	-0.005	-0.91
LOAN SIZE						
50K to 100K	0.881	-3.94	1.348	46.51	-0.188	-23.36
100K+	0.955	-1.46	2.243	129.15	-0.310	-38.56
Judiciary	1.075	4.83	1.007	2.87	0.126	33.53
NEG. CHG UR 27M	0.844	-9.93	0.739	-184.82	0.001	0.23
POS CHG UR 27M	1.270	49.94	1.031	29.81	0.028	22.42
NEG PCT CHG HPI 27M	1.011	32.84	0.976	-214.19	0.002	22.01
POS PCT CHG HPI 27M	0.948	-45.34	1.016	266.09	-0.006	-20.26
NEG PCT CHG IRR 27M	0.612	-28.85	1.747	188.06	-0.046	-10.85
POS PCT CHG IRR 27M	0.475	-24.97	0.781	-50.59	-0.045	-5.95
CONSTANT	0.012	-81.88	0.172	-179.23	0.692	50.56
OBS #			4434522		22378	
LLF			-2709344			

Figure 4: Realized vs. Projected Outcomes for Modeling Framework 1.a in Table 4

A.- Loss given default by cohort year

B. Default rate by cohort year

C. Prepayment rate by cohort year

D. Loss rate by cohort year

Note: The solid black line represents realized outcomes; any other lines represent model projections under alternative model specifications.

Figure 5: Realized vs. Projected Outcomes for Modeling Framework 1.b in Table 4

Note: The solid black line represents realized outcomes; any other lines represent model projections under alternative model specifications.

Figure 6: Cumulative Nine Quarters Default, Prepay, and Loss Rate Projection for the 2007 Cohort
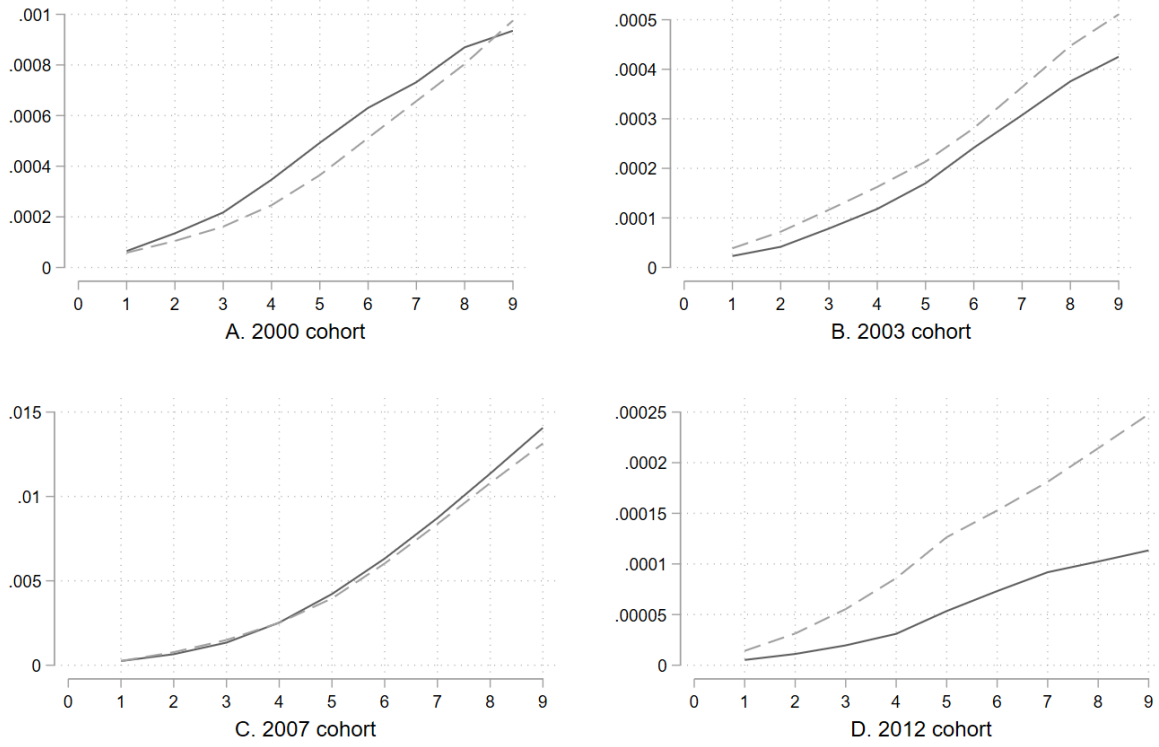

A. cumulative default rate

B. Cumulative prepay rate

C. Cumulative loss rate
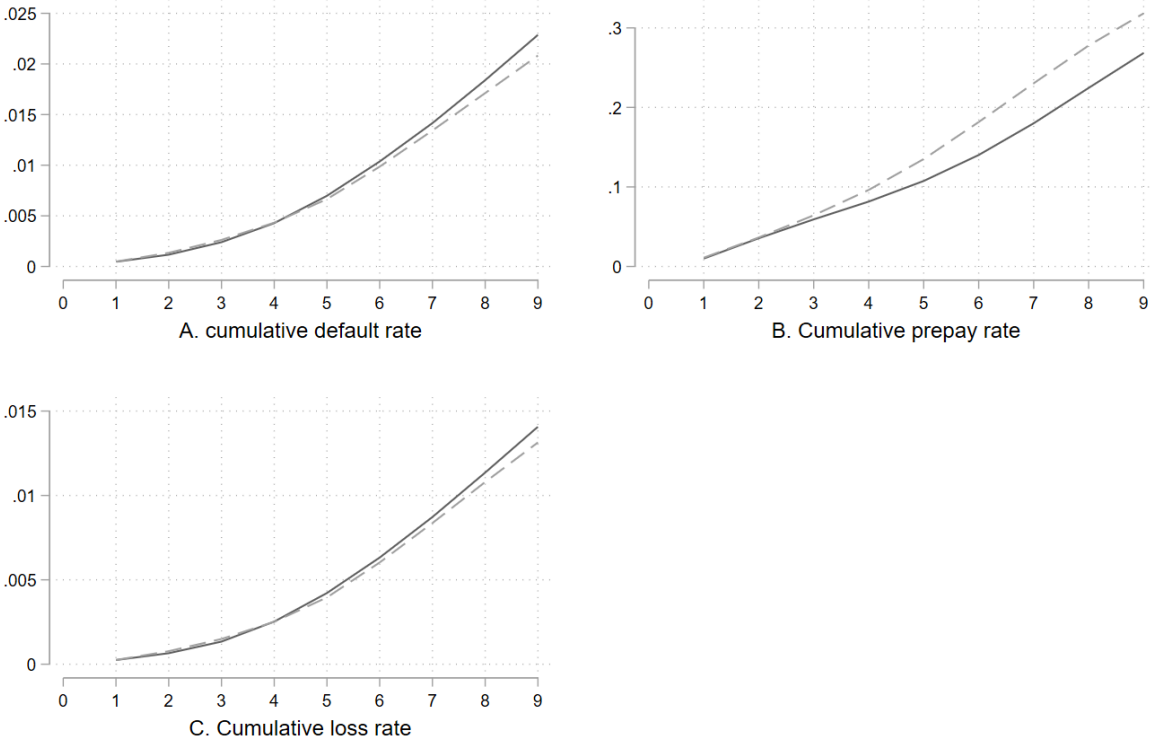
Note: Realized outcomes are represented by a solid black line; model projections are represented by a dotted line. The 2007 cohort represents loans originated in year 2007.

Figure 7: Cumulative Nine Quarters Loss Rate Projection Across Cohorts

Note: Realized loss rates are represented by a solid black line; model projections are represented by a dotted line. Each cohort represents loans originated in a specific year.