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Financial Instability with Circulating Debt Claims and Endogenous Debt Limits

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Abstract

This paper develops a banking model in which intermediaries issue liabilities that circulate as a medium of exchange to finance loans to entrepreneurs, who use the proceeds to fund the accumulation of capital goods. The issuance of circulating liabilities, together with endogenous debt limits, gives rise to a franchise value for intermediaries. A competitive equilibrium with endogenous debt limits admits allocations that are characterized by a funding crisis and a self-fulfilling collapse of the banking system, with the intermediary's franchise value eroding over time. In view of these difficulties, I construct a sophisticated fiscal policy that provides a government guarantee for the franchise value, which results in the determinacy of equilibrium, with the constrained efficient allocation emerging as the unique outcome.

JEL classification: E42, E44, G21

Keywords: circulating bank liabilities, endogenous debt limits, franchise value, funding crisis, government guarantee

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1. INTRODUCTION

One of the earliest debates in monetary economics regarded the role of banks as money issuers. Because banks issue liabilities that are essentially transferable debt instruments, many economists have argued that these securities should unmistakably be included in any measure of monetary aggregates. The financial institutions that comprise our banking system have considerably evolved over time as a result of technological innovations and regulatory measures. Despite these transformations, their function as liquidity providers to households and firms has remained unchanged in the broader monetary system.

Although the literature on financial economics has developed sophisticated banking models, the role of banks as liquidity providers has proved difficult to incorporate into models of financial intermediation. Indeed, it has been a challenge to construct a framework in which intermediation and liquidity provision are essentially combined within the same firm or institution to implement a socially efficient allocation. A widely held view among monetary economists is that the banking firm's ability to issue circulating liabilities reduces its funding cost, which leads to the expansion of bank financed investment (i.e., an increase in the credit supply).

The modern literature on monetary economics, including the new monetarist school of thought, has made substantial progress in developing models of liquidity in which a wide array of assets can circulate as a medium of exchange. Despite all the progress made so far, the mechanisms linking the issuance of liquid liabilities by banks and the credit supply to firms involved in capital accumulation have not been fully exploited. This is a crucial aspect of banking that deserves a detailed analysis for a number of reasons.

First, fluctuations in the value of bank liabilities, including those of the so-called shadow banking system, can lead to substantial changes in the credit allocation to firms, consumers, and entrepreneurs. Second, economists would like to know under what conditions the banking system supplies the socially efficient amount of credit and how such an optimal amount is linked to its funding costs. Third, a funding crisis in which bank liability holders lose confidence in the banking system can lead to a substantial decline in investment, production, and consumption. In particular, it is necessary to understand the dynamics of a funding crisis to construct a mechanism to prevent it from happening in the first place.

In this paper, I develop a banking model in which intermediaries issue claims that circulate as a medium of exchange to finance loans to entrepreneurs, who use the proceeds to fund capital accumulation. Because consumption goods producers require capital goods as an input, changes in the flow of funds to the banking system affect production and consumption in the whole economy. The key friction in the environment is a lack of commitment: The members of the banking system cannot commit to their future promises when issuing circulating liabilities. As a result, we consider a competitive equilibrium with endogenous debt limits, as in Kehoe and Levine (1993) and Alvarez and Jermann (2000).

The banker's ability to issue liabilities that circulate as a medium of exchange, combined with endogenous debt limits, results in a *franchise value* for the members of the banking system. Those liabilities, which serve as a means of payment in goods markets, carry a liquid premium, lowering the banker's funding cost. To preserve their franchise value, bankers find it individually rational to make good on their promises to periodically retire old claims to support their exchange value (i.e., not to overissue those claims). Simultaneously, consumers who use bank liabilities as a medium of exchange believe that bankers will act in such a way to preserve their franchise value, which can result in a stable value for bank liabilities and credit supply to entrepreneurs.

In a competitive equilibrium with endogenous debt limits, the circulating liabilities of the banking system support a constrained efficient allocation, with the property that the value of those liabilities remains constant over time. Agents expect a constant franchise value for the members of the banking system, and those beliefs support a constrained efficient level of production, investment, and consumption in the economy.

The problem with the competitive banking system is that there exists a continuum of nonstationary, suboptimal equilibria characterized by a self-fulfilling collapse of the banking system with falling intermediation activity. In these equilibria, the agents believe that the franchise value erodes over time, which results in a declining flow of funds into the banking system and a persistent reduction in intermediation and capital accumulation. The economy converges to autarky as the balance sheet of the members of the banking system shrinks along the equilibrium trajectory. The existence of multiple equilibria driven by self-fulfilling beliefs is reminiscent of that in Diamond and Dybvig (1983), except that, in my analysis, the collapse of the banking system is characterized by a persistent depreciation of bank liabilities over time.

In view of these difficulties, I construct a fiscal intervention that provides a government guarantee for the liabilities of the banking system and show that such a policy results in the determinacy of equilibrium, with the constrained efficient allocation emerging as the unique outcome. The proposed fiscal rule constructs a compensation scheme for the members of the banking system that is conditional on the rate of return on debt claims, which is fully communicated to all agents. The government intervention provides a feasible (and credible) lower bound for the banker's franchise value, which makes the set of beliefs that imply a declining franchise value inconsistent with an equilibrium, allowing the government to uniquely implement the constrained efficient allocation and maintain financial stability.

2. RELATED LITERATURE

The analysis in this paper belongs to the new monetarist literature. Following the seminal contributions of Kiyotaki and Wright (1989, 1993), Trejos and Wright (1995), and Shi (1995), the literature has moved to a broader application of models of monetary exchange to include a wide array of assets that can serve the function of a medium of exchange. In particular, the generation of models with divisible money and divisible commodities, such as those of Shi (1997), Lagos and Wright (2003, 2005), and Rocheteau and Wright (2005), has opened the door to sophisticated analyses of the role of liquid assets in the efficiency and stability of dynamic economies.[1]

The literature has also produced some early models of banking and private money in a random search framework. These include the analyses in Cavalcanti, Erosa, and Temzelides

[1] These include Berentsen, Camera, and Waller (2007); Geromichalos, Licari, Suarez-Lledo (2007); Araujo and Camargo (2008); Lagos and Rocheteau (2008); Aruoba, Waller, and Wright (2011); Andolfatto, Berentsen, and Waller (2016); among others.

(1999); Cavalcanti and Wallace (1999); Williamson (1999); Berentsen (2006); and Martin and Schreft (2006). Some authors have also exploited the role of banks as liquidity providers in the context of overlapping generations models, such as Champ, Smith, and Williamson (1996); Antinolfi, Huybens, and Keister (2001); and Azariadis, Bullard, and Smith (2001). In financial economics, some notable studies of the monetary aspects of banking include Andolfatto and Nosal (2009); Allen and Gale (2014); Donaldson, Piacentino, and Thakor (2018); Donaldson and Piacentino (2019); and Gorton and Ordonez (2020).

In recent years, some papers have incorporated banking arrangements into dynamic models of monetary exchange. Gu, Mattesini, Monnet, and Wright (2013) have constructed a model in which banks take deposits and make investments, with their liabilities circulating as a means of payment. Monnet and Sanches (2015) have developed a banking model to study banking regulation in a competitive and monopolistic environment. Sanches (2016) has constructed a model of circulating private claims in the context of the Lagos-Wright model to study the properties of private money.

More recently, Matsuoka and Watanabe (2019) have also used the Lagos-Wright framework to study banking arrangements in an environment with aggregate uncertainty. Finally, Andolfatto, Berentsen, and Martin (forthcoming) have integrated the Diamond (1997) model of banking and financial markets with the Lagos-Wright framework to study a number of issues in financial economics and monetary policy. My paper contributes to this literature by identifying the sources of financial instability in banking and by constructing a government intervention designed to rule out suboptimal outcomes.

3. MODEL

Time is discrete and the horizon is infinite. Each period contains four subperiods or stages. There are three commodities, referred to as the centralized market (CM) good, the decentralized market (DM) good, and the capital good. There are four types of agents, referred to as consumers, merchants, bankers, and entrepreneurs, with a $[0, 1]$-continuum of each type. Consumers, merchants, and bankers are infinitely lived. Entrepreneurs live for two periods only. A new generation of entrepreneurs is born each period, and there is a $[0, 1]$-continuum of old entrepreneurs in the initial period.

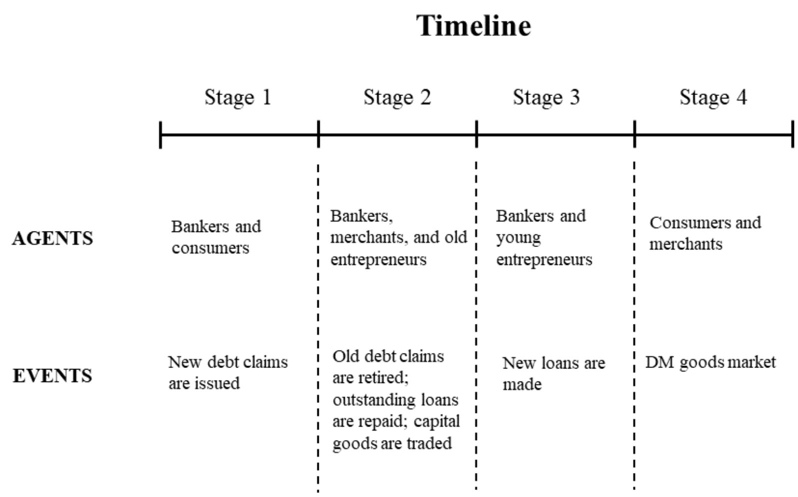
In the first stage, consumers and bankers interact in a centralized location. The consumer has access to a linear technology to produce the CM good, which requires effort as an input. The banker is endowed with the technology to issue a durable object, referred to as a debt claim, that perfectly identifies him as the issuer.

In the second stage, bankers, merchants, and old entrepreneurs get together in a centralized location. An entrepreneur is endowed with the technology to produce the capital good, which requires $e \in \mathbb{R}_+$ units of the CM good in period t to produce γ units of capital in period $t + 1$. Entrepreneurs are heterogeneous with respect to their productivity level $\gamma \in [0, \bar{\gamma}]$, which follows a distribution $G(\gamma)$. Merchants require the capital good as an input to produce the DM good in the fourth stage. A merchant who has previously produced and sold DM goods holds the debt claims issued by the bankers, which can be retired on demand to finance the purchase of capital goods.

In the third stage, young entrepreneurs and bankers trade in a centralized location. A typical banker issues debt claims in the first stage, retires old debt claims (i.e., those issued in the previous period) in the second stage, and makes new loans to young entrepreneurs in the third stage. Old entrepreneurs repay their loans in the second stage, after selling capital goods to the merchants.

In the fourth stage, the merchant can produce the DM good by supplying labor $l \in \mathbb{R}_+$ and capital $k \in \mathbb{R}_+$ to obtain $F(k, l)$ units. Consumers want to consume the DM good and use their debt claims (acquired from bankers in the first stage) to make a payment to the merchants. Following the literature, we refer to this market as the decentralized market (DM) and the good produced and consumed in the fourth stage as the DM good.

The banker can perfectly store the CM good across the first, second, and third stages. At the end of the third stage, there are two exclusive options for the banker: he can consume the CM good or used it as an input to produce β^{-1} units of the CM good in the following period (i.e., an intertemporal productive technology for the CM good), with $\beta \in (0, 1)$.



Let $x_t \in \mathbb{R}$ denote the consumer's net consumption of the CM good (i.e., a negative value means he is a net producer), and let $q_t \in \mathbb{R}_+$ denote consumption of the DM good. The consumer's preferences are represented by

$$\sum_{t=0}^{\infty} \beta^t \left[x_t + u\left(q_t\right)\right],$$

where $u : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable, increasing, and strictly concave, with $u(0) = 0$ and $u'(0) = \infty$. Let $x_t \in \mathbb{R}$ denote the merchant's net consumption of the CM good, and let $l_t \in \mathbb{R}_+$ denote his effort level to produce the DM good. The merchant's preferences are represented by

$$\sum_{t=0}^{\infty} \beta^t \left[x_t - w\left(l_t\right)\right],$$

where $w : \mathbb{R}_+ \to \mathbb{R}_+$ is continuously differentiable, increasing, and convex. The banker's preferences are represented by

$$\sum_{t=0}^{\infty} \beta^t x_t,$$

where $x_t \in \mathbb{R}_+$ is CM good consumption. Finally, we assume that entrepreneurs have linear utility and value old-age, CM good consumption only.

4. COMPETITIVE EQUILIBRIUM

I now describe the equilibrium allocations when all markets are perfectly competitive. In this economy, the bankers issue debt claims to finance capital accumulation and attain a positive consumption stream only if their debt claims carry a liquidity premium, which occurs when their rate of return is below the rate of time preference. Consequently, the bankers as a group operate a *banking system* that issues liabilities that circulate as a medium of exchange to finance capital investment in the form of loans to entrepreneurs and claims on a productive technology for the CM good.

The rate of return on debt claims determines the amount of funds that will flow into the banking system, which will influence the credit supply to entrepreneurs. Because the merchants require the capital goods supplied by the entrepreneurs to produce the DM good, it follows that DM good production and consumption are also influenced by the equilibrium rate of return on debt claims. Thus, the banking system plays a crucial role in the allocation of resources in the decentralized economy.

4.1. Bank loans

Young entrepreneurs are born without any wealth, so they need a loan to finance their investment project, and bankers can provide these funds in exchange for a repayment in the following period. Let $1 + r_t$ denote the interest rate in the market for bank loans. Because of perfect competition among bankers, the return on a bank loan must be the same as that on the productive technology for the CM good. Thus, we have

$$1 + r_t = \beta^{-1}$$

at all dates. Because they are indifferent, the bankers are willing to supply any amount satisfying their budget constraint (to be described below).

The demand for loans depends on the entrepreneur's productivity. Let $\rho_t \in \mathbb{R}_+$ denote the price of one unit of capital in terms of the CM good. A young entrepreneur has a profitable project if

$$\rho_{t+1}\gamma - e\beta^{-1} \geq 0.$$

Then, we can define the marginal entrepreneur γ_t^m as the type satisfying

$$\gamma_t^m = \frac{e}{\beta\rho_{t+1}}.$$

Any type $\gamma \in [\gamma_t^m, \bar{\gamma}]$ borrows in the market for bank loans to finance his project. As a result, the aggregate loan amount in period t is

$$L_t \equiv e\left[1 - G\left(\gamma_t^m\right)\right],$$

and the available capital in period $t+1$ is

$$k_{t+1} = \int_{\gamma_t^m}^{\bar{\gamma}} \gamma g\left(\gamma\right) d\gamma \equiv k\left(\gamma_t^m\right).$$

4.2. Consumers

Let $W\left(a,t\right)$ denote the value function of a consumer who enters the current period holding $a \in \mathbb{R}_+$ debt claims, and let $V\left(a,t\right)$ denote the value function in the decentralized market. The consumer's Bellman equation is then given by

$$W\left(a,t\right) = \max_{(x,a')\in\mathbb{R}\times\mathbb{R}_+} \left[x + V\left(a',t\right)\right]$$

subject to the budget constraint

$$x + \delta_t a' = a,$$

where $\delta_t \in \mathbb{R}_+$ denotes the price of a debt claim. In the decentralized market, we have

$$V\left(a,t\right) = \max_{q\in\mathbb{R}_+} \left[u\left(q\right) + \beta W\left(a - p_t q, t+1\right)\right]$$

subject to the liquidity constraint

$$p_t q \leq a,$$

where $p_t \in \mathbb{R}_+$ is the price of the date-t DM good in terms of the date-$(t+1)$ CM good.

Because $W(a,t)$ is an affine function, it follows that

$$V(a,t) = \max_{q \leq \frac{a}{p_t}} [u(q) - \beta p_t q] + \beta a + \beta W(0, t+1).$$

Then, the slope of the value function satisfies

$$V_1(a,t) = \begin{cases} \frac{1}{p_t} u'\left(\frac{a}{p_t}\right) \text{ if } a < p_t \hat{q}(p_t) \\ \beta \text{ if } a > p_t \hat{q}(p_t), \end{cases}$$

where $\hat{q}(p_t) \equiv (u')^{-1}(\beta p_t)$. If the liquidity constraint is slack, the marginal utility of an additional debt claim equals the discounted value of an extra unit of the CM good in the following period. If the liquidity constraint binds, then the marginal utility of an additional debt claim is greater than β because the debt claims now provide additional liquidity services in the decentralized market, giving rise to a liquidity premium.

Finally, the first-order condition for the optimal portfolio choice is

$$-\delta_t + V_1(a_t, t) \leq 0,$$

with equality if $a_t > 0$. If $\delta_t > \beta$, then we have

$$u'\left(\frac{a_t}{p_t}\right) = \delta_t p_t, \tag{1}$$

which implicitly defines the demand for debt claims as a function of its price and the relative price of the consumption goods.

4.3. Merchants

Let $H(a,t)$ denote the value function for a merchant who enters the current period holding $a \in \mathbb{R}_+$ debt claims, and let $Z(a,k,t)$ denote the value function in the decentralized market. The merchant's Bellman equation is given by

$$H(a,t) = \max_{(x,a',k') \in \mathbb{R} \times \mathbb{R}_+^2} \left[x + Z(a', k', t)\right]$$

subject to the budget constraint

$$x + \rho_t k' + \delta_t a' = a.$$

Here, $k' \in \mathbb{R}_+$ denotes the amount of capital the merchant takes into the decentralized market for DM good production. Note that the capital good depreciates completely at the end of the period.

In the decentralized market, we have

$$Z(a, k, t) = \max_{l \in \mathbb{R}_+} \left[-w(l) + \beta H(p_t F(k, l) + a, t + 1) \right].$$

In exchange for DM good production, the merchant receives a payment in terms of debt claims with a real value $p_t F(k, l)$. As we have seen, the merchant can retire these claims in the following period when meeting with the bankers in the centralized location.

The first-order conditions for capital accumulation and labor supply are

$$w'(l_t) = \beta p_t F_l(k_t, l_t) \tag{2}$$

and

$$\rho_t = \beta p_t F_k(k_t, l_t). \tag{3}$$

These conditions implicitly define the demand for capital and the labor supply for DM good production as a function of the relative price of goods p_t and the relative price of capital ρ_t.

The first-order condition for the optimal amount of debt claims is $-\delta_t + \beta \leq 0$, with equality if $a_t > 0$. In the construction of a competitive equilibrium, we have assumed that $\delta_t > \beta$, which implies that merchants do not directly buy debt claims from the bankers.

4.4. Bankers

Let $J(n, s, t)$ denote the value function for a banker who has issued $n \in \mathbb{R}_+$ debt claims in the previous period and who enters the current period holding $s \in \mathbb{R}_+$ assets. These assets consist of loans made in the previous period and claims on the proceeds from the productive technology for the CM good. As we have seen, both assets yield the same rate of return in a competitive equilibrium. The banker's Bellman equation is given by

$$J(n, s, t) = \max_{(x,n',s')\in\mathbb{R}^3_+} \left[x + \beta J\left(n', s', t+1\right)\right]$$

subject to the budget constraint

$$x + s' + n = \beta^{-1}s + \delta_t n'$$

and the upper bound on debt claims

$$n' \leq \bar{b}_t.$$

When making his decisions at each date, the banker takes as given the price sequence $\{\delta_t\}_{t=0}^{\infty}$ and the sequence of individual debt limits $\left\{\bar{b}_t\right\}_{t=0}^{\infty}$. The latter will be set in such a way that bankers will always find it optimal to redeem their debt claims, as opposed to defaulting on their obligations and facing autarky.

If $\delta_t > \beta$, then the banker finds it optimal to issue as many debt claims as possible so that $n' = \bar{b}_t$. Because the return on assets equals the rate of time preference, the banker is indifferent between immediately consuming and reinvesting the proceeds from current earnings. Then, an optimal investment decision is to set $s' = \bar{b}_t$, which can be interpreted as the decision to voluntarily hold in reserve all proceeds from the sale of debt claims in the current period. In that case, the banker's consumption in period t is given by

$$x_t = \bar{b}_{t-1}\left(\beta^{-1}\delta_{t-1} - 1\right).$$

We define the banker's franchise value as the lifetime utility associated with a particular choice of asset returns, the sequence of debt limits, and the price sequence. The franchise value in period t is given by

$$\sum\nolimits_{\tau=t}^{\infty} \beta^{\tau-t}\bar{b}_{\tau-1}\left(\beta^{-1}\delta_{\tau-1} - 1\right).$$

4.5. Equilibrium

If $\delta_t > \beta$, then the value of all debt claims in circulation must be equal to the aggregate value of DM good production:

$$a_t = p_t F\left(k\left(\gamma_{t-1}^m\right), l_t\right). \tag{4}$$

Combining (1) and (4), we obtain

$$u'\left(F\left(k\left(\gamma_{t-1}^{m}\right),l_{t}\right)\right)=p_{t}\delta_{t}.$$

Using (2) to substitute for p_t, we find

$$u'\left(F\left(k\left(\gamma_{t-1}^{m}\right),l_{t}\right)\right)=\frac{\delta_{t}}{\beta}\frac{w'\left(l_{t}\right)}{F_{l}\left(k\left(\gamma_{t-1}^{m}\right),l_{t}\right)}. \tag{5}$$

This condition determines the merchant's effort level, given the predetermined capital stock. The price of debt claims influences effort level as follows: a lower price of claims increases the return on liquid assets and, consequently, the consumer's expenditure decision, raising the relative price p_t and inducing merchants to exert more effort in production.

Using (2) and (3) to substitute for ρ_t, we obtain

$$\beta u'\left(F\left(k\left(\gamma_{t}^{m}\right),l_{t+1}\right)\right)F_{k}\left(k\left(\gamma_{t}^{m}\right),l_{t+1}\right)=e\frac{\delta_{t+1}}{\beta}. \tag{6}$$

This condition determines the amount of capital accumulated between dates t and $t+1$, given the effort level decision at date $t+1$. Note that a lower anticipated value for the debt claims results in a larger amount of capital available for production at date $t+1$.

Our next step is to use (5) and (6) to implicitly define $\gamma_{t-1}^{m}=\gamma^{m}\left(\delta_{t}\right)$ and $l_{t}=l\left(\delta_{t}\right)$. Then, we can obtain DM production by defining $q\left(\delta_{t}\right)\equiv F\left(k\left(\gamma^{m}\left(\delta_{t}\right)\right),l\left(\delta_{t}\right)\right)$. Finally, the supply of debt claims as a function of their price can be defined as

$$a\left(\delta_{t}\right)\equiv\frac{u'\left(q\left(\delta_{t}\right)\right)q\left(\delta_{t}\right)}{\delta_{t}}.$$

It remains to specify the sequence of debt limits $\left\{\bar{b}_{t}\right\}_{t=0}^{\infty}$ in such a way that solvency is individually rational for the banker. First, we impose the market-clearing condition

$$\bar{b}_{t}=a\left(\delta_{t}\right)$$

at all dates. Second, we say that a particular price sequence $\{\delta_{t}\}_{t=0}^{\infty}$ is consistent with the full redemption of debt claims if and only if

$$\sum\nolimits_{\tau=t}^{\infty}\beta^{\tau-t}a\left(\delta_{\tau-1}\right)\left(\beta^{-1}\delta_{\tau-1}-1\right)\geq a\left(\delta_{t-1}\right)\left(\beta^{-1}\delta_{t-1}-1\right)+\delta_{t}a\left(\delta_{t}\right)$$

holds at all dates.

Following Kehoe and Levine (1993) and Alvarez and Jermann (2000), we set the sequence of debt limits to allow each banker to issue as many debt claims as possible without inducing him to opportunistically renege on his promises. The left-hand side gives the banker's lifetime utility, or the franchise value, in period t. The right-hand side gives the short-term payoff received if the banker decides not to hold in reserve the proceeds from the sale of debt claims in period t. In this case, he can increase current consumption by $\delta_t a(\delta_t)$, but he will inevitably default in period $t+1$, resulting in the autarkic payoff from date $t+1$ onward.

Rearranging the solvency constraints, we find that

$$-\delta_t a(\delta_t) + \sum\nolimits_{\tau=t+1}^{\infty} \beta^{\tau-t} a(\delta_{\tau-1}) \left(\beta^{-1}\delta_{\tau-1} - 1\right) \geq 0.$$

In equilibrium, the convertibility constraints hold with equality at all dates. Alvarez and Jermann (2000) have defined these limits as *not too tight* so that bankers can issue as many debt claims as it is consistent with individual rationality.

The banker's Bellman equation and the solvency constraints can be written as

$$J_t = a(\delta_{t-1}) \left(\beta^{-1}\delta_{t-1} - 1\right) + \beta J_{t+1}$$

and

$$\delta_t a(\delta_t) = \beta J_{t+1},$$

respectively. Combining these two conditions, we find that an equilibrium price sequence $\{\delta_t\}_{t=0}^{\infty}$ satisfies the difference equation

$$\delta_{t+1} a(\delta_{t+1}) = a(\delta_t) \tag{7}$$

at all dates. We can now formally define a competitive equilibrium with endogenous debt limits on private money creation.

Definition 1 *A competitive equilibrium with endogenous debt limits can be defined as a price sequence* $\{\delta_t\}_{t=0}^{\infty}$ *satisfying* $\delta_t \geq \beta$ *and (7).*

The previous definition allows us to characterize an equilibrium allocation in terms of the price of debt claims. My first step in the equilibrium analysis is to establish the existence of a stationary allocation.

Proposition 2 *Suppose that* $u(q) = (1-\sigma)^{-1} q^{1-\sigma}$, *with* $0 < \sigma < 1$, $w(l) = l$, *and* $F(k,l) = k^{\alpha} l^{1-\alpha}$, *with* $0 < \alpha < 1$. *Additionally, assume that* $g(\gamma) = 1$ *for all* $0 \leq \gamma \leq 1$ *and* $g(\gamma) = 0$ *otherwise. Then, there exists a unique stationary equilibrium with* $\delta_t = 1$ *for all* $t \geq 0$. *Such an allocation is constrained efficient.*

Proof. Conditions (5) and (6) can be written as

$$l = \chi(\delta)\left[\gamma^{-1}\left(1-\gamma^2\right)^{1-\alpha+\alpha\sigma}\right]^{\frac{1}{(1-\alpha)(1-\sigma)}} \equiv S(\gamma,\delta)$$

and

$$l = \lambda(\delta)\left(1-\gamma^2\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \equiv K(\gamma,\delta),$$

where $\chi(\delta)$ and $\lambda(\delta)$ are given by

$$\chi(\delta) \equiv \left(\frac{1}{2}\right)^{\frac{1-\alpha+\alpha\sigma}{(1-\alpha)(1-\sigma)}} \times \left(\frac{e\delta}{\alpha\beta^2}\right)^{\frac{1}{(1-\alpha)(1-\sigma)}}$$

and

$$\lambda(\delta) \equiv \left[(1-\alpha)\frac{\beta}{\delta}\left(\frac{1}{2}\right)^{\alpha(1-\sigma)}\right]^{\frac{1}{\alpha+\sigma(1-\alpha)}},$$

respectively.

Note that $d\chi/d\delta > 0$ and $d\lambda/d\delta < 0$. Also, we have

$$\chi(\beta) = \left(\frac{1}{2}\right)^{\frac{1-\alpha+\alpha\sigma}{(1-\alpha)(1-\sigma)}}\left(\frac{e}{\alpha\beta}\right)^{\frac{1}{(1-\alpha)(1-\sigma)}}$$

and

$$\lambda(\beta) = \left[(1-\alpha)\left(\frac{1}{2}\right)^{\alpha(1-\sigma)}\right]^{\frac{1}{\alpha+\sigma(1-\alpha)}}.$$

For any $\delta > \beta$, we have $S_1(\gamma,\delta) < 0$ for all $\gamma \in (0,1)$, $\lim_{\gamma\to 0} S(\gamma,\delta) = \infty$, and $\lim_{\gamma\to 1} S(\gamma,\delta) = 0$. For any $\delta > \beta$, we have $K_1(\gamma,\delta) < 0$ and $K_{11}(\gamma,\delta) < 0$ for all $\gamma \in (0,1)$, $\lim_{\gamma\to 0} K(\gamma,\delta) = \lambda(\delta)$, and $\lim_{\gamma\to 1} K(\gamma,\delta) = 0$. Thus, for any fixed $\delta > \beta$, a unique interior solution exists. The *Implicit Function Theorem* then implies $d\gamma^m/d\delta > 0$ and $dl/d\delta < 0$.

The solvency constraints imply

$$-\delta a(\delta) + \frac{\beta}{1-\beta} a(\delta) \left(\beta^{-1}\delta - 1\right) \geq 0.$$

This condition holds if and only if $\delta \geq 1$. In particular, it holds with equality if and only if $\delta = 1$. Thus, there exists a unique stationary equilibrium in which $\gamma^m = \gamma^m(1)$, $l = l(1)$, and $a = a(1)$, with $a(1)$ given by

$$a(1) = \left(\frac{1}{2}\right)^{\alpha(1-\sigma)} \left[1 - \gamma^m(1)^2\right]^{\alpha(1-\sigma)} l(1)^{(1-\alpha)(1-\sigma)}.$$

■

The competitive equilibrium admits a stationary allocation with the property that the *net* rate of return on debt claims is zero, which is the highest return for liquid assets consistent with the solvency constraints (i.e., the debt claims issued by the banking system have the property to maintain a stable value over time so that they serve as a "perfect" store of value for consumers and merchants in the decentralized market). This equilibrium return on bank liabilities supports a constrained efficient allocation. Thus, the competitive banking system is consistent with a constrained efficient production level, with consumers using bank liabilities as a medium of exchange and entrepreneurs borrowing from banks to finance capital accumulation.

5. FINANCIAL INSTABILITY

The problem with the competitive economy is that it admits a continuum of nonstationary, suboptimal equilibrium trajectories characterized by a self-fulfilling collapse of the banker's franchise value. In a suboptimal equilibrium, consumers anticipate a persistent decline in the franchise value, which results in a decrease in the supply of debt claims. Their beliefs become self-fulfilling, with intermediation activity diminishing along the equilibrium trajectory as the rate of return on debt claims converges to zero. The banking system balance sheet shrinks over time, and the supply of credit to entrepreneurs persistently contracts, leading to a declining amount of capital and production in the economy.

To derive our next result, it is convenient to define the dynamic system in terms of the return on debt claims. Define the rate of return on a debt claim by $\pi_t \equiv \frac{1}{\delta_t}$. Then, we can define the demand for debt claims as a function of its rate of return by $\hat{a}(\pi_t)$, which satisfies $\hat{a}'(\pi) > 0$ for all $\pi > 0$. Given these changes, a competitive equilibrium with endogenous debt limits can be alternatively defined as a sequence $\{\pi_t\}_{t=0}^{\infty}$ satisfying $0 \leq \pi_t \leq \beta^{-1}$ and

$$\hat{a}(\pi_t) = \pi_{t-1}\hat{a}(\pi_{t-1}). \tag{8}$$

We can now formally state the main result in this section.

Proposition 3 *Suppose that $u(q) = (1-\sigma)^{-1} q^{1-\sigma}$, with $0 < \sigma < 1$, $w(l) = l$, and $F(k,l) = k^{\alpha}l^{1-\alpha}$, with $0 < \alpha < 1$. Additionally, assume that $g(\gamma) = 1$ for all $0 \leq \gamma \leq 1$ and $g(\gamma) = 0$ otherwise. Then, there exists a continuum of nonstationary equilibria with the property that the rate of return on debt claims converges monotonically to zero and the economy converges to autarky.*

Proof. Note that $\pi_{t-1} = 0$ when $\pi_t = 0$ because the demand function $\hat{a}(\pi)$ goes to zero as π converges to zero from above. If $\pi_t = \beta^{-1}$, then we have $\pi_{t-1} < \beta^{-1}$. As we have seen, $\pi = 1$ is a fixed point.

Using the *Implicit Function Theorem*, we have

$$\frac{d\pi_{t-1}}{d\pi_t} = \frac{\hat{a}'(\pi_t)}{\pi_{t-1}\hat{a}'(\pi_{t-1}) + \hat{a}(\pi_{t-1})} > 0$$

for any $0 \leq \pi_t \leq \beta^{-1}$. Thus, the law of motion for π_t is monotonically increasing so that (8) implicitly defines a strictly increasing mapping $\pi_{t-1} = f(\pi_t)$. From the *Inverse Function Theorem*, we find

$$\frac{d\pi_t}{d\pi_{t-1}} = \frac{\pi_{t-1}\hat{a}'(\pi_{t-1}) + \hat{a}(\pi_{t-1})}{\hat{a}'(\pi_t)}$$

for any $\pi_{t-1} \in \left[0, f^{-1}\left(\beta^{-1}\right)\right]$. In particular, we have

$$\left.\frac{d\pi_t}{d\pi_{t-1}}\right|_{\pi_{t-1}=\pi_t=1} = \frac{\hat{a}'(1) + \hat{a}(1)}{\hat{a}'(1)} > 1,$$

which implies that the mapping $\pi_t = f^{-1}(\pi_{t-1})$ crosses the 45-degree line from below at the point $(1, 1)$. As a result, for any initial condition $\pi_0 \in (0, 1)$, the dynamic system monotonically converges to zero. ■

According to the previous proposition, the competitive equilibrium admits allocations that are characterized by a collapse of the banking system driven by self-fulfilling beliefs that lead to the erosion of the banker's franchise value. To verify the persistent decline in the banker's franchise value along a nonstationary equilibrium trajectory, note that the dynamic system (8) can be written as

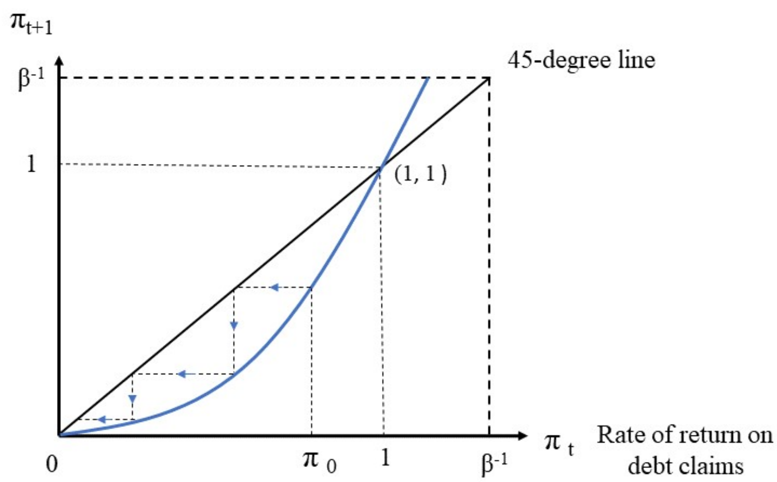
$$J_{t+1} = J_t h(J_t),$$

where $h(J) = \hat{a}^{-1}(\beta J)$. Because $h(J)$ is strictly increasing, a nonstationary equilibrium necessarily involves a monotonically decreasing franchise value J_t.[2] Because a declining franchise value is associated with a decreasing flow of intermediated credit in the economy, it follows that investment, production, and consumption persistently decline as the economy

[2] See appendix for the derivation of the equilibrium law of motion in terms of the franchise value J_t.

converges to autarky. Figure 1 provides an illustration of the dynamic system (8).

Figure 1: Self-fulfilling Funding Crisis



Another way to look at these nonstationary equilibria is by acknowledging that, along the equilibrium trajectory, consumers expect a depreciation of the value of bank liabilities. In a nonstationary equilibrium, the consumer acquires bank claims to subsequently trade in the decentralized market. Their expectation is such that for each unit of bank liabilities held in portfolio, the real return across periods is less than one. In other words, consumers expect an increasing depreciation of bank liabilities, which corresponds to a self-fulfilling inflationary process that accelerates indefinitely with the value of money going to zero. In other words, the competitive banking system can lead to a monetary system subject to hyperinflationary trajectories.

In a nonstationary equilibrium, consumers anticipate a declining path for the banker's franchise value, which leads to an expected depreciation of bank liabilities. In response to these movements, the consumer reduces the amount of bank liabilities in his portfolio. This process intensifies as his expectations become self-fulfilling, leading to a collapse of trading activity.

To summarize the main findings in this section, I have shown that the competitive economy is subject to a self-fulfilling collapse of intermediation because agents believe the banker's franchise value will erode over time, leading to a funding crisis in the banking system characterized by the persistent depreciation of bank liabilities. Given that the ensuing allocation is suboptimal, we now consider the existence of a government intervention that can ensure the determinacy of equilibrium, with the constrained efficient allocation potentially emerging as the unique equilibrium outcome.

6. GOVERNMENT GUARANTEE

In this section, I develop a sophisticated government policy that guarantees a minimum value for the franchise value regardless of the private sector demand for debt claims. As we shall see, such an intervention will prevent a self-fulfilling depreciation of bank liabilities to occur along the equilibrium trajectory, ultimately leading to the determinacy of equilibrium.

Suppose the government establishes a clearinghouse in the centralized location and requires that all debt claims be redeemed through the clearinghouse. Each banker is required to deposit funds with the clearinghouse to ensure the redemption of debt claims. The government will now set the individual debt limits $\left\{\bar{b}_t\right\}_{t=0}^{\infty}$ on each banker. We can then interpret the variable $n' \in \mathbb{R}_+$ in the banker's portfolio problem as his obligation with the clearinghouse. Additionally, assume that any issuance by an individual banker is publicly observable. By setting the individual debt limits and requiring member banks to fully cover their outstanding liabilities with clearinghouse deposits, the government limits the clearinghouse exposure to each member bank.

The government then sets the limit $\bar{b}_t = \hat{a}\left(\pi_t\right) - \mu\tau\left(\pi_t\right)$ at each date, where $\tau : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a continuously differentiable and increasing function and $\mu \in [0, 1]$. The choice of individual debt limits must be feasible so that

$$\hat{a}\left(\pi_t\right) \geq \mu\tau\left(\pi_t\right) \tag{9}$$

holds at all dates. We can interpret the term $\mu\tau\left(\pi_t\right)$ as a clearinghouse tax (or subsidy) on member banks, which varies according to the rate of return on debt claims. As we shall see, the key to a successful intervention is to construct a compensation scheme within the clearinghouse conditional on the expected rate of return on debt claims to prevent a collapse in the value of bank liabilities. As a result, the banker's franchise value is given by

$$\hat{J}_t = \left[\hat{a}\left(\pi_{t-1}\right) - \mu\tau\left(\pi_{t-1}\right)\right]\left(\beta^{-1} - \pi_{t-1}\right) + \beta \hat{J}_{t+1}.$$

The solvency constraints now take the form:

$$\begin{aligned} & \sum\nolimits_{\tau=t}^{\infty} \beta^{\tau-t}\left[\hat{a}\left(\delta_{\tau-1}\right) - \mu\tau\left(\pi_{\tau-1}\right)\right]\left(\beta^{-1}\delta_{\tau-1} - 1\right) \\ \geq \; & \left[\hat{a}\left(\pi_{t-1}\right) - \mu\tau\left(\pi_{t-1}\right)\right]\left(\beta^{-1}\delta_{t-1} - 1\right) + \hat{a}\left(\pi_t\right) - \mu\tau\left(\pi_t\right) \end{aligned}$$

at all dates.

The government budget constraint is given by

$$\upsilon_t + \mu\tau\left(\pi_t\right) + \left[\hat{a}\left(\pi_{t-1}\right) - \mu\tau\left(\pi_{t-1}\right)\right]\pi_{t-1} = \hat{a}\left(\pi_{t-1}\right)\pi_{t-1},$$

where $\upsilon_t \in \mathbb{R}$ denotes a lump-sum tax on consumers (and $\upsilon_t < 0$ implies a lump-sum transfer).

We can now formally define a competitive equilibrium in the economy with government intervention.

Definition 4 *A competitive equilibrium with endogenous debt limits and an active fiscal policy is a sequence* $\{\pi_t, \upsilon_t\}_{t=0}^{\infty}$ *satisfying (9),* $0 \leq \pi_t \leq \beta^{-1}$*,*

$$\upsilon_t = \mu\left[\pi_{t-1}\tau\left(\pi_{t-1}\right) - \tau\left(\pi_t\right)\right], \text{ and}$$

$$\left[\hat{a}\left(\pi_{t-1}\right) - \mu\tau\left(\pi_{t-1}\right)\right]\pi_{t-1} = \hat{a}\left(\pi_t\right) - \mu\tau\left(\pi_t\right) \tag{10}$$

at all dates.

Given the previous equilibrium definition, consider now the policy rule:

$$\tau\left(\pi\right) = \frac{\eta}{\hat{a}\left(1\right)}\hat{a}\left(\pi\right) - \left[\eta - \hat{a}\left(1\right)\right] \tag{11}$$

with a constant $\eta > \hat{a}(1)$. Then, set the parameter μ such that $\mu < \frac{\hat{a}(1)}{\eta}$. This policy implies that, if the rate of return on debt claims falls below a certain threshold value, each banker receives a subsidy from the government via a clearinghouse transfer. If the rate of return on debt claims goes above that threshold value, a banker is required to pay a tax to the government clearinghouse.

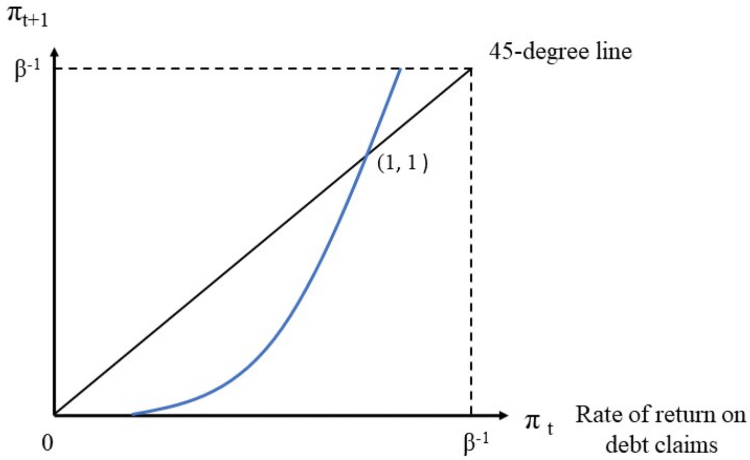
The policy rule (11) implies the after-tax revenue:

$$[\hat{a}(1) - \mu\eta] \frac{\hat{a}(\pi)}{\hat{a}(1)} + \mu [\eta - \hat{a}(1)] .$$

This revenue function shows that, even if the rate of return on debt claims converges to zero, the banker's revenues converge to $\mu [\eta - \hat{a}(1)] > 0$. Thus, the government intervention provides a revenue guarantee for the members of the banking system. If the private sector demand for debt claims declines to considerably low levels, the fiscal policy implies that the banker's franchise value reaches a strictly positive lower bound (or a floor).

As we shall see in a moment, this property of the equilibrium allocation implies that bankers will find it individually rational to maintain their promises and not default on their liabilities, even if the rate of return on debt claims goes to zero. This occurs because bankers want to preserve the government subsidy, which supports their franchise value if the return on debt claims enters a declining trajectory. As a result, the beliefs that the franchise value necessarily erodes as the rate of return on debt claims goes to zero is inconsistent with an equilibrium allocation. Figure 2 provides an illustration of the dynamic system in the presence of an active fiscal policy.

Figure 2: Government Guarantee and Global Determinacy



Note that, on the equilibrium path, the government does not levy any tax on agents. The anticipation of an active fiscal policy off the equilibrium path is sufficient to rule out self-fulfilling beliefs involving a persistent decline of the franchise value. The design of the fiscal policy involves either a subsidy or a tax on the members of the banking system, depending on the rate of return on debt claims. It might also be necessary to tax nonbank agents off the equilibrium path following some strongly declining trajectories for the rate of return on debt claims. But feasibility is always required in the equilibrium.

Agents internalize that these policies are indeed feasible on and off the equilibrium path, which influences their beliefs. Once the intervention is fully communicated, there is no *anticipated* tax or transfer associated with the implementation of the constrained efficient allocation. The following proposition formally states the uniqueness of equilibrium in the presence of the proposed fiscal intervention in (11).

Proposition 5 *By selecting the function* $\tau : \mathbb{R}_+ \to \mathbb{R}$ *and* $\mu \in [0, 1]$ *as in (11), it is possible to design a government intervention that results in a unique equilibrium in which* *the constrained efficient allocation is implemented.*

Proof. Suppose that $\tau : \mathbb{R}_+ \to \mathbb{R}$ is given by (11), and set $\mu < \frac{\hat{a}(1)}{\eta}$ with $\eta > \hat{a}(1)$. First, note that $\pi_t = \pi_{t-1} = 1$ satisfies (10) Second, π_{t-1} converges to a strictly positive value as π_t goes to zero. Third, $\pi_t = \beta^{-1}$ implies $\pi_{t-1} < \beta^{-1}$.

Using the *Implicit Function Theorem*, we find that

$$\frac{d\pi_{t-1}}{d\pi_t} = \frac{\left[1 - \frac{\mu\eta}{\hat{a}(1)}\right] \hat{a}'(\pi_t)}{\left[1 - \frac{\mu\eta}{\hat{a}(1)}\right] \left[\hat{a}'(\pi_{t-1})\,\pi_{t-1} + \hat{a}(\pi_{t-1})\right] + \mu\left[\eta - \hat{a}(1)\right]} > 0$$

for any $\pi_t \in \left[0, \beta^{-1}\right]$. Using the *Inverse Function Theorem*, we have

$$\left.\frac{d\pi_t}{d\pi_{t-1}}\right|_{\pi_t = \pi_{t-1} = 1} = \frac{\left[1 - \frac{\mu\eta}{\hat{a}(1)}\right] \hat{a}'(1) + (1-\mu)\,\hat{a}(1)}{\left[1 - \frac{\mu\eta}{\hat{a}(1)}\right] \hat{a}'(1)} > 1,$$

which implies that the law of motion for π_t crosses the 45-degree line from below at $(1, 1)$. Thus, we conclude that $\pi_t = 1$ for all $t \geq 0$ is the unique solution to the dynamic system (10). ■

The proposed fiscal rule establishes a state-contingent fiscal scheme tied to the expected rate of return on debt claims, which is fully communicated to all agents. The government guarantee for the franchise value makes the set of beliefs that imply a declining franchise value inconsistent with an equilibrium allocation, so it successfully attains the determinacy of equilibrium, with the constrained efficient allocation emerging as the unique outcome.

7. CONCLUSIONS

I have constructed a banking model with the following properties: 1. bankers issue liabilities that circulate as a medium of exchange; 2. entrepreneurs borrow from bankers to finance capital accumulation; 3. bankers are subject to endogenous debt limits. The main implication of these ingredients is that the price of the bankers' liabilities influences the amount of funds flowing to the banking system to finance capital accumulation and production in all sectors of the economy. As we have seen, the competitive equilibrium with endogenous debt limits attains a constrained efficient allocation, characterized by a stable value of bank liabilities. However, it also admits nonstationary allocations that are characterized by a self-fulfilling collapse of the banking system and persistent depreciation of bank liabilities.

In view of these difficulties, I have proposed a sophisticated fiscal intervention that provides a government guarantee for the banker's franchise value and have shown that such an intervention leads to the determinacy of equilibrium, with the constrained efficient allocation emerging as the unique outcome. Such an intervention has proved successful in the theoretical analysis because the policy is fully communicated to agents, who believe that the government agency responsible for its implementation will take the prescribed actions both *on* and *off* the equilibrium path. In reality, these assumptions are unlikely to hold, as governments usually act to contain a crisis when it is too late or do not fully specify their plan of action for every possible sequence of events.

APPENDIX

I now show how to rewrite the dynamic system for the competitive equilibrium in terms of the banker's franchise value J_t. As we have seen, the equilibrium conditions describing the evolution of the dynamic system reduce to

$$J_t = \hat{a}\left(\pi_{t-1}\right)\left(\beta^{-1} - \pi_{t-1}\right) + \beta J_{t+1} \tag{12}$$

and

$$\hat{a}\left(\pi_t\right) = \beta J_{t+1}. \tag{13}$$

Because $\hat{a}\left(\pi\right)$ is strictly increasing, its inverse function $h\left(J\right) = \hat{a}^{-1}\left(\beta J\right)$ is also strictly increasing. Then, we can use (12) to obtain

$$J_t = J_t - \beta h\left(J_t\right) J_t + \beta J_{t+1} \Leftrightarrow h\left(J_t\right) J_t = J_{t+1}$$

as claimed.

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