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The Cyclicality of Labor Force Participation Flows: The Role of Labor Supply Elasticities and Wage Rigidity*

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Abstract

Using a representative-household search and matching model with endogenous labor force participation, we study the cyclicality of labor market transition rates between employment, unemployment, and nonparticipation. When interpreted through the lens of the model, the behavior of transition rates implies that the participation margin is strongly countercyclical: the household's incentive to send more workers to the labor force *falls* in expansions. We identify two key channels through which the model delivers this result: (i) the procyclical values of non-market activities and (ii) wage rigidity. The smaller the value of the extensive-margin labor supply elasticity is, the stronger the first channel is. Wage rigidity helps because it mitigates increases in the return to market work during expansions. Our estimated model replicates remarkably well the behavior of transition rates between the three labor market states and thus the stocks, once these two features are in place.

Key Words: Labor force participation, unemployment, labor supply elasticity
JEL Codes: E24, J64

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1 Introduction

Search and matching models pioneered by Mortensen and Pissarides (1994) are now a standard tool to analyze labor market fluctuations. The focus of this literature, especially since Shimer (2005), has been mostly on the sources of cyclical variations in labor demand.[1] Recent experiences of the U.S. labor market, however, have spurred renewed interests among policymakers in the need for a better understanding of cyclical fluctuations in labor supply margins. This new trend is exemplified by numerous speeches by the Federal Reserve officials that engage in extensive discussions on underlying drivers of the movements in the labor force participation rate (LFPR).[2]

In this paper, we extend a canonical search and matching model by adding an extensive margin labor supply decision. Our aim is to develop a tractable yet quantitative framework that allows us to analyze labor market dynamics as a result of equilibrium responses of both the job-creation (labor demand) and labor force participation (labor supply) margins. In doing so, we also reevaluate the longstanding puzzle in macroeconomics that the value of the Frisch labor supply elasticity required to replicate aggregate labor market fluctuations is measurably larger than the values suggested by the micro-level evidence (see, for example, Chetty et al. (2011) for a review of this literature). We revisit this issue in a framework with search frictions, while it has traditionally been studied in models with a frictionless labor market. We show that our model replicates salient cyclical features of transition rates between three labor market states (employment, unemployment, and nonparticipation) and that small labor supply elasticities are in fact *necessary* to match their business-cycle movements. Throughout the paper, we emphasize the importance of the following two concepts for our results: (i) procyclicality of the values of non-market activities (as emphasized by Chodorow-Reich and Karabarbounis (2016)) and (ii) equilibrium wage rigidity (as in Hall (2005)). Note that our contribution is *not* to provide a novel channel for labor demand magnification, but to demonstrate that these two concepts are also important to understand the cyclical variations in the labor supply margin.

We first summarize cyclical properties of labor market transition rates and stocks by estimating sign-restriction vector autoregressions (VARs). In addition to the well-known cyclical pattern in transition rates between employment and unemployment, we show that the transition rate at which nonparticipants join the unemployment pool (NU rate) is countercyclical,

[1] This trend perhaps reflects "the consensus view point that shifts in labor demand account for most of the cyclical variation in labor input" (Hall, 2008).

[2] See, for example, Bernanke (2012), Bullard (2014), Plosser (2014), Yellen (2014), Williams (2017), and Kashkari (2017). These speeches are motivated by questions such as whether the unemployment rate is a sufficient measure of labor market slack or some information from the LFPR should also be considered.

while the exit rate from unemployment to nonparticipation (UN rate) is procyclical. Note that the behavior of these transition rates is particularly informative about the cyclicality of the labor supply margin, as they represent the pace of worker flows into and out of nonparticipation. Our VAR also reveals that transition rates between employment and nonparticipation are both procyclical in both directions. All of these patterns are consistent with the existing literature analyzing unconditional second moments of transition rates (e.g., Elsby et al. (2015), Krusell et al. (2017)), but we provide a more complete and nuanced description of the data.

Our model features the representative household that makes the participation decision. In the model, nonemployed household members differ with respect to their productivity at home, based on which the household optimally allocates them to, either active job search (unemployment) or nonparticipation (home production). The equilibrium determines the two key endogenous variables, labor market tightness and the participation margin. The latter is represented by the threshold value of home productivity, above (below) which a nonemployed household member stays out of the labor force (joins the unemployment pool). When looking at the empirical evidence through the lens of the model, we find that the participation margin must be strongly countercyclical, meaning that the household must be less willing to send an additional member to the unemployment pool in expansions than in recessions. The aforementioned countercyclicality of the NU rate and procyclicality of the UN rate directly reflect this mechanism. The conclusion that the participation margin must be strongly countercyclical underscores the importance of adopting a flow approach in modeling the labor market dynamics, as opposed to a stock approach. In the stock approach, the (weak) procyclicality of the LFPR is likely to be interpreted as indicating procyclicality of the participation margin. Our paper thus highlights how using labor market transition rates as the primitives of the analysis is important to understand the key underlying economic mechanisms behind the cyclicality of the LFPR.

To be more specific, consider the household's decision as to whether to send an additional member to the unemployment pool or to keep the member as a nonparticipant. In the model, there are two channels affecting this participation decision. The first channel is through the returns to market work: higher labor market tightness (and thus the higher job-finding rate) and higher wages during expansions motivate the household to send more workers to the unemployment pool. The second channel is through the cyclical fluctuations of the opportunity cost of market participation, i.e., the values of non-market activities (leisure and home production). Under standard preferences, a higher employment share in the household results in higher marginal values of leisure and home production (measured in market-goods consumption), keeping the household from sending more members to the labor force. The countercyclicality of the participation margin implies that the second channel must dominate the first one. The model can achieve this (i) if elasticities of labor supply are *small*, which implies a larger increase in the values of non-market activities and (ii) if wages do not rise as much in expansions. We adopt a simple form of equilibrium wage rigidity, proposed by Hall (2005), which not only enhances labor demand fluctuations but also plays a key role in replicating labor supply responses. Note that in models with a frictionless labor market, small labor supply elasticities and the lack of movements in wages imply the lack of employment variability. In models with search frictions, however, movements in the household's labor supply margin can be decoupled from labor demand (job-creation) fluctuations. Moreover, in such an environment, the lower the extensive-margin labor supply elasticity is and the larger the complementarity between home production and market-goods consumption is, the more countercyclical the participation margin becomes, without compromising labor market volatility. Our results indicate that, in models with search frictions, wage rigidity and the procyclical values of non-market activities together can provide a coherent mechanism that simultaneously explains labor demand and labor supply responses over business cycles.

Our estimated model matches the cyclicality of *all* transition rates across the three labor market states and the behavior of labor market stocks (the unemployment rate, the employment-to-population ratio, and the LFPR). Several notable results are as follows. First, our model reproduces the observation that the LFPR exhibits a small procyclical variation over the cycle, even though underlying transition rates all exhibit large volatilities. Second, our model matches the empirical pattern that separation rates from employment into unemployment (EU) and into nonparticipation (EN) move in opposite directions. Although our model assumes a constant separation rate out of employment, the share of separations flowing into the unemployment pool increases in downturns (and thus the share of the other flow falls). In replicating this pattern, the countercyclical participation margin again plays a key role. We show that the countercyclicality of the separation rate into unemployment contributes significantly to unemployment fluctuations. This is notable because, in two-state models, a constant separation rate implies no contributions of the EU rate to unemployment fluctuations, contrary to the data. Relatedly, our model maintains the strong negative correlation between unemployment and vacancies, known as the Beveridge curve.[3]

Relation to the literature. Earlier attempts that incorporate the extensive-margin labor supply decision into search models include Tripier (2003) and Veracierto (2008). They find

[3]It is known in the literature that two-state models with endogenous separations that deliver the countercyclical EU rate fail to replicate this robust empirical regularity. See Fujita and Ramey (2012) for details.

that unemployment tends to become procyclical, once the participation margin is endogenized. In their models, the participation margin is procyclical because of flexible wages and a large elasticity of labor supply. Their models are different from ours in many dimensions, but we show that a similar result arises in our model under the environment with flexible wages and large elasticities. More recently, Shimer (2012) studies the properties of a model similar to Tripier's. He pays close attention to the role of wage rigidity as we do in our paper, but without relating his findings to elasticities of labor supply. In Shimer's baseline model, the split between unemployment and nonparticipants is perfectly elastic and thus a rigid wage is not enough to mitigate the procyclical force in unemployment. Haefke and Reiter (2011) develop a search model with heterogeneous workers and endogenous participation decisions and evaluate its quantitative performance in light of micro evidence on labor supply elasticities. Based on a steady-state analysis, they also find that small elasticity values are consistent with fluctuations in the labor market stocks rate under empirically plausible degrees of wage rigidity. Galí (2010), Galí et al. (2012), and Campolmi and Gnocchi (2016) study a New Keynesian model with search frictions and endogenous participation.[4] Their models consider a richer environment with more real and pricing frictions and shocks. We study a simpler environment to emphasize key economic mechanisms. Importantly, none of the papers cited so far try to match the cyclicality of transition rates. We tackle this task of matching the cyclicality of transition rates as well as stocks, as it enables us to connect micro-level household decisions to aggregate labor market fluctuations. Ferraro and Fiori (2019) study asymmetric business cycles in a heterogeneous-agent model with endogenous participation in which labor supply is perfectly elastic. Their model matches the volatility and cyclicality (measured as the correlation with output) of labor market transition rates by exogenously making the opportunity cost of employment strongly procyclical. Last but not least, Krusell et al. (2017) develop a heterogeneous-agent search model with endogenous participation and look explicitly at transition rates, especially those between unemployment and nonparticipation. They emphasize the role of wealth heterogeneity and associated composition effects in explaining the cyclicality of these rates. Because we study a representative-agent environment, we necessarily abstract from such composition effects but provide complementary channels. Furthermore, we study the link between the cyclicality of labor force participation flows and extensive-margin labor supply elasticities.

As emphasized above, an important element of our model is the procyclicality of the values of non-market activities. In the context of a two-state model, Chodorow-Reich and Karabarbounis (2016) emphasize that this procyclicality reduces labor demand by making

[4] Erceg and Levin (2014) also develop a New Keynesian model in which three labor market states can be defined without introducing search frictions.

surplus less responsive to business cycle shocks, which poses more challenges to the resolution of the unemployment volatility puzzle (Shimer, 2005, Costain and Reiter, 2008, Hagedorn and Manovskii, 2008). We show that a procyclical value of non-market activities is necessary in models with endogenous labor force participation to match the cyclicality of labor market flows and stocks.[5]

This paper is organized as follows. Section 2 presents the empirical evidence. By estimating a sign-restriction VAR, we characterize the full dynamics of labor market transition rates, labor market stocks, vacancies, and real wage. Section 3 develops the model, which is then estimated in Section 4. Section 5 contains the paper's main quantitative results and discussions on the mechanism driving them. Section 6 concludes the paper.

2 Empirical Evidence

This section summarizes the cyclical behavior of transition rates across three labor market states and labor market stocks by estimating sign-restriction VARs. In contrast to the literature that focuses on the unconditional moments, we study full dynamic characteristics of the data. Our analysis extends a sign-restriction VAR study by Fujita (2011) to a three-state environment, incorporating transitions in and out of the labor force.

2.1 Data

We construct worker transition rates between employment (E), unemployment (U), and not-in-the-labor force (N), using the Current Population Survey (CPS) matched records. The literature has proposed various corrections to the data to deal with the margin errors, classification errors, and time aggregation bias.[6] Our main empirical results are based on flow series adjusted only for margin errors. We do not undertake any adjustments with respect to the other errors. Regarding classification errors, Elsby et al. (2015) propose an adjustment that they call "DeNUNifying." However, a paper by Kudlyak and Lange (2017) argues against the adjustment. We do not take a stand on this matter and instead confirm that our results are robust to this particular adjustment as well as the other data adjustments

[5]Our model features wage rigidity in the form of Hall (2005). This specification essentially decouples the movements in surplus from those in the opportunity cost of employment, allowing us to replicate observed labor market volatility, even with the endogenously procyclical opportunity cost of employment. But again, our paper's focus is not on labor demand fluctuations.

[6]Margin errors arise due to nonrandom attrition of survey participants, resulting in inconsistency between flow and stock data. See Abowd and Zellner (1985), Fujita and Ramey (2006), and Frazis et al. (2005) for earlier attempts to make the correction. However, the cyclicality of the data is not significantly affected by this correction (Fujita and Ramey, 2006).

(see Appendix A.1). Regarding the time aggregation bias, we measure transition rates in our model in a way that is consistent with the measurement practice used in the empirical analysis (see discussions in Section 3).

Our VAR analysis includes six transition rates: (i) EU, (ii) EN, (iii) UE, (iv) UN, (v) NE, and (vi) NU rates. The first letter represents the originating labor market state and the second letter the terminal state between two adjacent months. The VAR also includes real wage and job vacancies.[7] The sample period for our analysis is constrained by the availability of CPS microdata and spans between 1976 and 2016. We convert the monthly series into quarterly series by time averaging to smooth out high frequency variations of the data. All series are seasonally adjusted, logged, and then HP-filtered with a smoothing parameter equal to 10^5. We detrend all series because the data exhibit low-frequency variations that are difficult to endogenously analyze in our stationary models. We confirm the robustness of our results with respect to alternative detrending methods in Appendix A.1. We set the lag length at two quarters, suggested by the AIC.[8]

Note that the VAR does not explicitly include labor market stocks, because the impulse response functions (IRFs) of transition rates allow us to fully characterize the dynamics of the stock variables. That is, once we know the paths of transition rates, we can use the laws of motion for the stocks to trace their paths (and thus the paths of any functions of these stocks such as the unemployment rate), conditional on initial (steady-state) values of stocks and transition rates. Again, this approach is consistent with the view that transition rates are the primitives that drive labor market stocks.

2.2 Identifying Assumptions

Our sign restrictions are meant to identify the impulse that drives business cycle fluctuations in the U.S. labor market. We identify what we call an "aggregate profitability shock" (henceforth, an aggregate shock) by imposing restrictions on the signs of responses of transition rates between unemployment and employment. Specifically, we assume that in response to a positive (negative) aggregate shock, the EU rate falls (increases), while the UE rate increases (falls). We also assume that the shock leads to increases (declines) in vacancies and employment growth. These sign restrictions are assumed to hold for two quarters. The

[7] Real wage is measured by compensation per hour from the BLS's productivity and cost program, deflated by the total PCE price index. For the job vacancy series, we splice the help-wanted advertising index by the Conference Board and the job openings series reported by the Job Openings and Labor Turnover Survey (JOLTS), which has been available since December 2000. We multiplicatively adjust the level of the former series to the level that matches the level of the JOLTS series.

[8] The results are robust with respect to the lag length around two quarters.

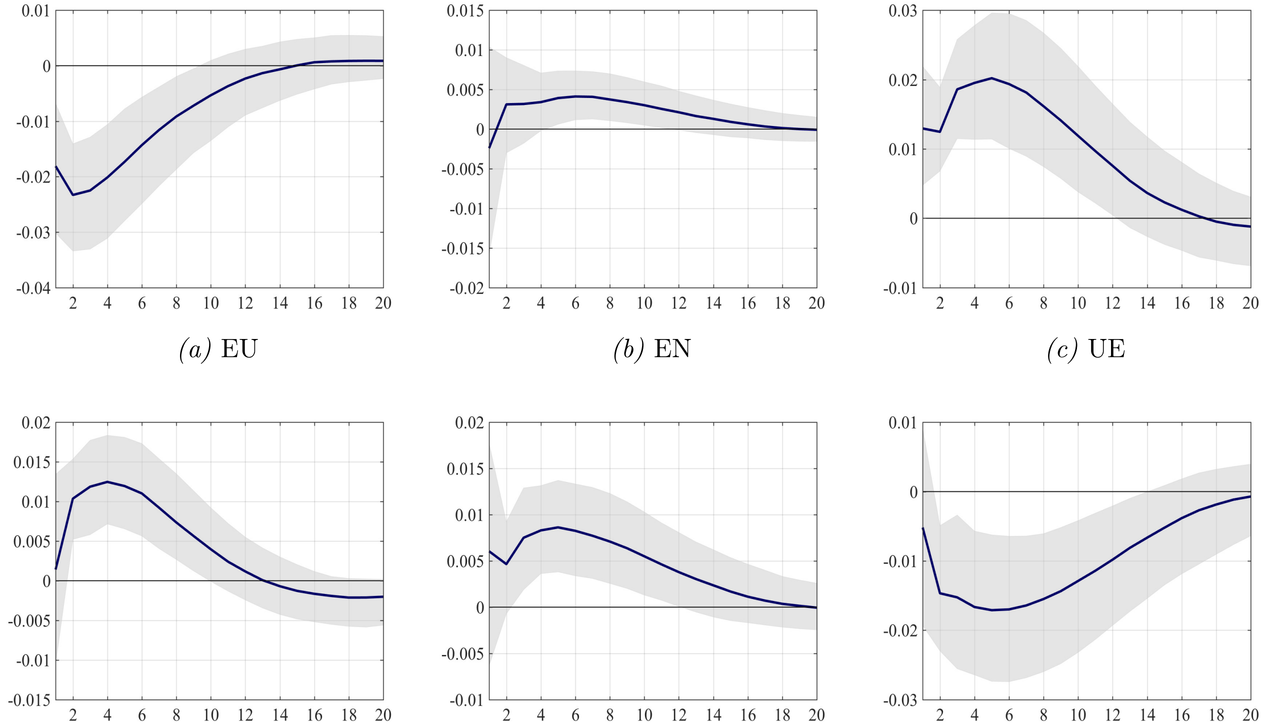
cyclical patterns of the EU and UE transition rates are well-established in the context of a two-state model of the labor market.[9] Since our main interest is to characterize the cyclicality of transition rates into and out of nonparticipation, imposing these restrictions on the directions of transition rates between employment and unemployment is sensible.

As is clear from our VAR setup, we do not attempt to identify various forms of more structural shocks. The spirit of our approach is similar to the one taken by highly influential papers in the literature such as Blanchard and Diamond (1990) and Haltiwanger and Davis (1999). These papers also use simple sign restrictions in a parsimonious VAR to identify a shock that has a similar interpretation. More recently, Fujita (2011) also follows a similar approach within the two-state model. Importantly, Fujita finds in his robustness checks that labor market variables respond very similarly to more fundamental shocks within his two-state framework. To further check the robustness of our results, we present the results from two alternative VARs in Appendix A.1. First, we present the results from a larger VAR that includes inflation data and distinguishes between demand- and supply-side shocks. Second, we also consider the case where the labor productivity series is directly used to identify a technology shock. In this VAR, the evolution of labor productivity is exogenous to the original eight variables, i.e., these variables do not enter the evolution for labor productivity, while the labor productivity series enters symmetrically in the remaining eight equations. We do not impose any sign restrictions in this VAR. The cyclical patterns of the labor market variables are found to be very similar across all cases.

2.3 Results

Figures 1 and 2 present the IRFs to a positive aggregate shock. Solid lines represent the median responses and shaded areas represent the 16th and 84th percentiles of the posterior distributions. Recall that we restrict the behavior of the EU and UE transition rates (Figures 1 (a) and (c)) for the first two quarters, and these restrictions indeed imply that the unemployment rate drops significantly and persistently (Figure 2 (c)). The persistent declines in the unemployment rate together with persistent increases in vacancies (Figure 2 (e)) form the Beveridge curve. Because we also restrict employment growth to be positive in the first two quarters, the employment-to-population ratio increases in a hump-shaped manner (Figures 2 (a) and (b)). Figure 2 (f) shows that real wage is only weakly procyclical, as has long been known in the business-cycle literature. Although the median response is positive throughout the five-year horizon, the 16-84 percentile error band tends to include

[9] Fujita (2011) does not restrict the signs of these transition rates because his main interest is on testing the cyclicality of these rates. His finding is indeed consistent with our sign restrictions.



(a) EU *(b)* EN *(c)* UE

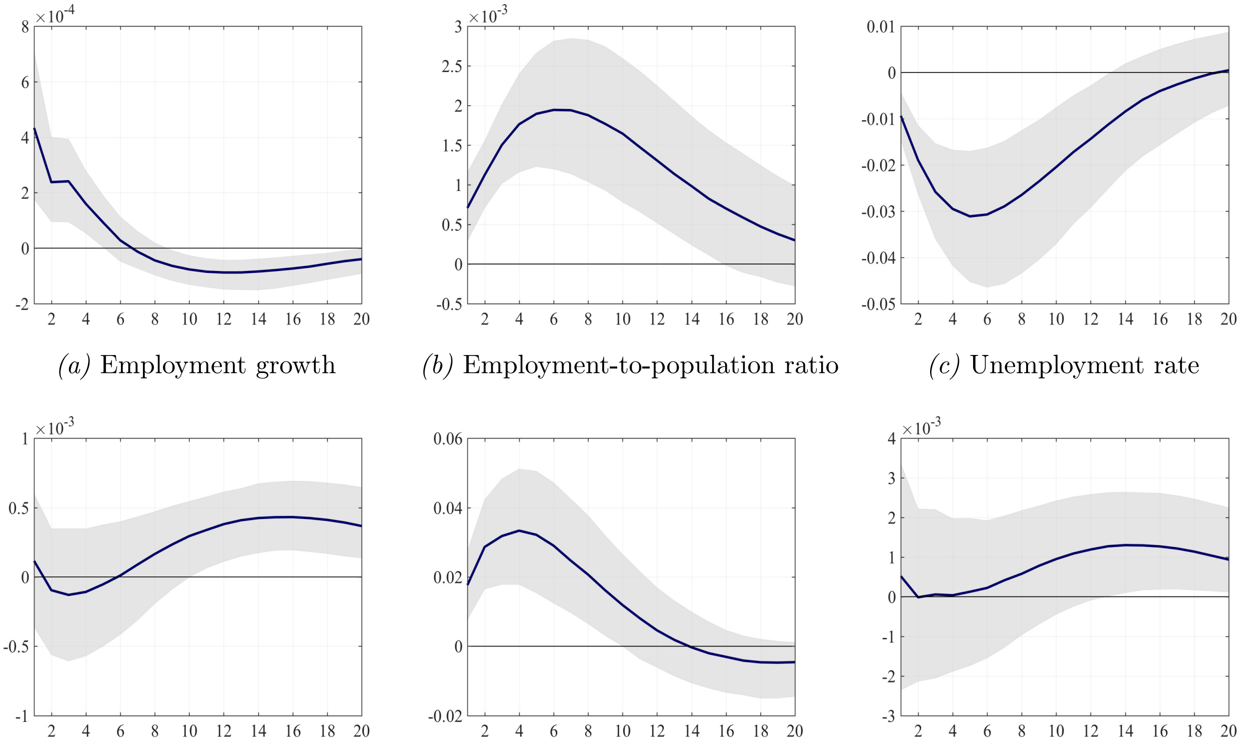
(d) UN *(e)* NE *(f)* NU

Figure 1: Empirical IRFs to a Positive Aggregate Shock: Transition Rates. *Notes*: Expressed as log deviations from steady-state levels. Shaded areas are error bands representing the 16th and 84th percentiles of the posterior distribution.

zeros.

Figure 2 (d) presents the response of the LFPR. It is known in the literature that the LFPR is only weakly procyclical.[10] But our VAR-based result provides more complete dynamic properties of the LFPR. Furthermore, by relating this response to the movements in transition rates, we provide a richer story that underlies the behavior of the LFPR. In Figure 2 (d), one can see that the weak procyclicality of the LFPR is due to the pattern that it takes several years before it starts to rise. Also, its volatility is minuscule: Based on the median response, the largest deviation from the steady-state level is about 0.0004 log point. In contrast, the employment-to-population ratio and the unemployment rate deviate from their steady-state levels as much as 0.002 log point and 0.03 log point, respectively. The small volatility of the LFPR is interesting, especially because volatilities of transition rates to and from nonparticipation are not particularly small compared with those of transition

[10] See, for example, Erceg and Levin (2014) and Van Zandweghe (2017).



(a) Employment growth *(b)* Employment-to-population ratio *(c)* Unemployment rate

(d) LFPR *(e)* Vacancies *(f)* Real wage

Figure 2: Empirical IRFs to a Positive Aggregate Shock: Stocks and Real Wage. *Notes*: Expressed as log deviations from steady-state levels (except for panel (a)). See notes to Figure 1.

rates between employment and unemployment.

Let us now discuss in greater detail the cyclical patterns of transition rates. First, consider the responses of EU and UE rates (Figure 1 (a) and (c)). Our VAR restricts the direction of the initial responses of these two variables, and we see that both of these responses are highly persistent. Second, compare the responses of separation rates into unemployment (EU) and into nonparticipation (EN) (Figure 1 (a) and (b)). The latter rate tends to be procyclical and partially offsets the countercyclicality of the former rate, thus making the overall separation rate out of employment less cyclical, although the sum remains countercyclical. Third, compare the responses of UE and NE rates, namely, job-finding rates from the two nonemployment pools (Figure 1 (c) and (e)). The literature has emphasized the strong procyclicality of the former, but a similar procyclicality applies to hiring from nonparticipation. The volatility of the latter is smaller because the pool of nonparticipants is much larger and includes a large number of individuals that are dormant in terms of labor force participation, such as retirees.

Next, compare two transition rates that constitute flows into the unemployment pool, namely, the EU and NU rates (Figure 1 (a) and (f)). Obviously, the countercyclical EU rate contributes to a higher unemployment rate in recessions. However, the entry rate from nonparticipation increases as well, thus also contributing to a higher unemployment rate.

Lastly, compare the behavior of UE and UN rates (Figure 1 (c) and (d)), both representing outflow rates from the unemployment pool. In contrast to the responses of inflow rates just discussed, they both move procyclically. Thus, in addition to the familiar procyclicality of the pace of hiring from the unemployment pool, the pace of exits to nonparticipation also increases in expansions. Again, the two-state analysis emphasizes the procyclicality of the UE rate on the countercyclical unemployment fluctuations, but our result here indicates that the procyclical UN rate also contributes to it. Note that in our model, movements in UN and NU rates are largely driven by the participation (labor supply) margin. In Section 5.3, we conduct a model-based exercise to quantify this "labor-supply channel" on unemployment fluctuations.

The cyclical movements in transition rates between unemployment and nonparticipation might sound counterintuitive, when one considers comments like "workers drop out of the labor force due to discouragement after failing to find a job" or "more workers join the labor force as the labor market conditions improve." These comments imply the movements of UN and NU rates that are opposite of the pattern we discussed above. However, they are not necessarily inconsistent with these comments in that our results are based on the transition *rates* rather than worker *flows*. Even though the UN rate falls in a downturn, the number of workers making UN transitions (thus dropping out of the labor force) might increase because the unemployment pool itself increases. We view transition rates as being primitive, and worker flows are determined as a result of movements in transition rates and their interactions with stocks.

The results regarding the cyclicality of UN and NU rates are not new. Elsby et al. (2015) point out that these patterns result from worker heterogeneity, particularly with respect to the labor force attachment. In a downturn, the composition of the unemployment pool shifts toward workers with strong attachment to the labor market. These attached workers are less likely to exit the labor force, thereby making the UN rate lower in a downturn. Similarly, the countercyclicality of the NU rate implies that the "need" for joining the labor force increases in a recession (for example, the spouse of a household head joining the labor force to compensate for the loss of hours or a job).[11] Krusell et al. (2017) study a model with wealth heterogeneity and a borrowing constraint and generate these cyclical patterns

[11] See Mueller (2017) for a general idea of countercyclicality of "attachment" of the unemployment pool.

in UN and NU rates. Our proposed model also sheds light on the underlying sources of these patterns in a representative-household setup.

Regarding the weak procyclicality of the LFPR, notice that, out of four transition rates that directly involve nonparticipation, three of them produce countercyclical forces for the LFPR: The procyclical responses of EN and UN rates increase flows into nonparticipation in expansions. The countercyclicality of the NU rate has the same effect. These forces are offset by the following procyclical forces. The first is the procyclicality of the NE rate. Second, even though the UN rate increases in a boom, the size of the unemployment pool falls more significantly, and thus the number of workers who move from U to N actually falls. We emphasized earlier that the LFPR varies very little over the business cycle, whereas underlying transition rates that drive its behavior exhibit larger variations. This empirical observation arises (in an accounting sense) due to the fact that movements in transition rates to and from nonparticipation tend to have offsetting effects on the participation rate.[12]

Let us summarize the empirical findings: (i) The EN separation rate is procyclical, which partially offsets the countercyclicality of the EU separation rate, therefore making the "total separation rate" less countercyclical; (ii) the UN rate is procyclical, while the NU rate is countercyclical; (iii) the job-finding rate from nonparticipation (NE rate) is as strongly procyclical and persistent as the one from unemployment; this procyclical NE rate is an important procyclical force of the LFPR; (iv) the LFPR is, however, only mildly procyclical and varies very little over the business cycle; and (v) the real wage is not very volatile and only mildly procyclical.

3 Model

This section presents our quantitative model that is intended to replicate the cyclical behavior of labor market transition rates and stocks. We maintain the representative-household structure throughout the paper. Details on the derivations are available in Appendix A.2.

3.1 Environment

There is a large number of identical households, each of which consists of a continuum of members on the unit interval. In each period t, the members can be either employed or

[12] Note that, as first-stage effects, the movements between E and U (EU and UE) are neutral to the LFPR. However, the procyclical movement in the former and the countercyclical movement in the latter also contribute to increasing the LFPR indirectly. That is, when an unemployed worker moves to employment, the probability that the worker exits the labor force is reduced relative to the case in which he or she remains unemployed (i.e., EN rate is much lower than the UN rate).

nonemployed (i.e., unemployed or out of the labor force). Employed members are paid a wage w_t and produce the consumption good with common productivity y_t, which follows an exogenous AR(1) process. The household decides the activity of the nonemployed members based on their productivity at home: either active job search (unemployed) or home production (out of the labor force). Home productivity h_i is assumed to be i.i.d. across members and time, and drawn from a distribution $\Phi(h_i)$.[13] We assume that workers out of the labor force can be "available to work" (N_t) or "permanently out of the labor force" ($\bar{N}$). Hence, the number of total nonparticipants is equal to $N_t + \bar{N}$. We assume that the latter group's productivity at home is permanently fixed at $\bar{h}$, where there is no incentive for them to take a job, even if they were to receive a job offer (as shown formally in Appendix A.2), essentially making $\bar{N}$ exogenous to the model. The introduction of $\bar{N}$ is needed to accommodate retirees or those who have no intention of participating in the labor market. Without $\bar{N}$, the steady-state levels of transition rates between unemployment and nonparticipation are too high relative to their observed levels.

Being unemployed means that the worker engages in active job search, receives unemployment insurance (UI) benefits (b), and finds market work at rate f_t. Unemployed workers also contribute to home production according to their own productivity but scaled down by a factor $\tau < 1$. We interpret τ as the fraction of time allocated to home production relative to what nonparticipants can allocate to home production. Nonparticipants contribute to home production according to their own productivity and can still receive job offers but at a reduced rate μf_t where $\mu < 1$. Krusell et al. (2017) adopt the same specification and this "passive job search" makes it easier to generate a large NE flow that exists in the data.[14]

Every household member that is out of the labor force at the beginning of t (N_{t-1}) draws new productivity at home h_i. On the other hand, we assume that among the unemployed at the beginning of t (U_{t-1}), only the fraction $1 - \lambda$ are allowed to draw the new home production value and make the same decision as nonparticipants. The fraction λ remain unemployed. The underlying reason for this persistence is that the unemployed are on average more "attached" to the labor force than those out of the labor force and thus tend to remain there, once they enter the pool (see, for example, Elsby et al. (2015) and Mueller (2017)). This parameter is useful mainly for matching the level of the UN transition rate. Without this parameter, workers switch between unemployment and nonparticipation too

[13] Bils et al. (2012) study the environment in which workers are permanently different in terms of relative efficiency between home production and market production. That formulation is obviously more realistic and thus appealing. For our purpose, however, we would like the model to be easy to aggregate.

[14] Note that the proportionality assumption in job-search efficiency (represented by a constant parameter μ), again adopted by Krusell et al. (2017), is empirically plausible as shown by Hornstein et al. (2014).

Table 1: Transition Rates in the Model

State in $t-1$	Transition Probability	State in t	Entry of Γ_t
E	$1-s+s\left[\Phi(h_t^*)+(\Phi(h_t^m)-\Phi(h_t^*))\mu\right]f_t$	E	$(1,1)$
E	$s\Phi(h_t^*)(1-f_t)$	U	$(1,2)$
E	$s\left[(\Phi(h_t^m)-\Phi(h_t^*))(1-\mu f_t)+(1-\Phi(h_t^m))\right]$	N	$(1,3)$
U	$\left[\lambda+(1-\lambda)(\Phi(h_t^*)+(\Phi(h_t^m)-\Phi(h_t^*))\mu)\right]f_t$	E	$(2,1)$
U	$\left[\lambda+(1-\lambda)\Phi(h_t^*)\right](1-f_t)$	U	$(2,2)$
U	$(1-\lambda)\left[(\Phi(h_t^m)-\Phi(h_t^*))(1-\mu f_t)+(1-\Phi(h_t^m))\right]$	N	$(2,3)$
N	$\left[\Phi(h_t^*)+(\Phi(h_t^m)-\Phi(h_t^*))\mu\right]f_t$	E	$(3,1)$
N	$\Phi(h_t^*)(1-f_t)$	U	$(3,2)$
N	$(\Phi(h_t^m)-\Phi(h_t^*))(1-\mu f_t)+(1-\Phi(h_t^m))$	N	$(3,3)$

Notes: First and third columns refer to end-of-the-period states. The last column refers to the entry of the transition matrix Γ_t. Last three rows are expressed as transition rates for available nonparticipants. For empirically relevant transition rates with respect to overall nonparticipants, multiply them by the ratio $\frac{N_t}{N_t+\bar{N}}$.

often relative to the observed frequency in the data.[15] Note that the earlier version of this paper (Cairó et al. (2019)) also presents a simpler model with $\lambda=0$ and $\bar{N}=0$ and shows that models with and without these features share the same economic intuitions.

Job separation occurs at the beginning of period t with a fixed probability s. After job separation, the participation decision is made and then the job-finding outcome is realized. Those who find a job can start working in t. The participation decision depends on the cutoff value h_t^*. Those who draw $h_i \leq h_t^*$ engage in active job search and otherwise they are out of the labor force. Recall that (available) nonparticipants also receive a job offer with probability μf_t. Among them, there is a job acceptance decision as well, characterized by another cutoff value h_t^m. Those who draw h_i higher than h_t^m reject the job offer. In our quantitative exercises, we assume that $\Phi(h_i)$ is log-normally distributed and thus it is possible that h_i is so high, albeit temporarily, that she chooses not to accept the job offer. However, the job rejection occurs only with small probability in our quantitative model and thus the variation in this margin is quantitatively unimportant.

Table 1 presents transition probabilities under our timing assumptions. The labor market

[15] There are various ways to structurally model this persistence. Krusell et al. (2017) do so by introducing heterogeneity in wealth; Bils et al. (2012) do so by introducing the "comparative advantage" of workers' productivity in market work and home production. However, our objective is to develop a tractable but quantitatively appealing framework that can be easily applied to broader analyses. We believe on this ground that our reduced-form specification is justified.

transitions can be summarized as follows:

$$\begin{bmatrix} E_t \\ U_t \\ N_t \end{bmatrix} = \Gamma_t' \begin{bmatrix} E_{t-1} \\ U_{t-1} \\ N_{t-1} \end{bmatrix}, \tag{1}$$

where Γ_t is the transition probability matrix (see Table 1 for each element and the location).

Labor market matching is governed by a Cobb-Douglas matching function: $m_t = \bar{m} S_t^{\alpha} v_t^{1-\alpha}$, where m_t is the number of matches, $\bar{m}$ the scale parameter, α the elasticity of the matching function, v_t the total number of job openings, and S_t the effective number of job seekers, given by:

$$S_t = \left[\Phi(h_t^*) + (\Phi(h_t^m) - \Phi(h_t^*))\mu\right] R_{t-1} + \lambda U_{t-1},$$

where $R_{t-1} \equiv sE_{t-1} + (1-\lambda)U_{t-1} + N_{t-1}$ represents the number of workers who draws h_i in the current period. Within this mass, the share $\Phi(h_t^*)$ engages in active job search while the share $\Phi(h_t^m) - \Phi(h_t^*)$ searches with reduced intensity μ. The mass λU_{t-1} remains in the unemployment pool. The job-finding rate per efficiency unit of search is $f_t \equiv m_t/S_t = \bar{m}\theta_t^{1-\alpha}$ and the job-filling rate per vacancy is $q_t \equiv m_t/v_t = \bar{m}\theta_t^{-\alpha}$, where $\theta_t \equiv v_t/S_t$ is labor market tightness.

3.2 Representative Household

The momentary household-level preferences are specified as:

$$U(C_t, L_t) = \frac{C_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \omega \frac{L_t^{1-\frac{1}{\nu}}}{1-\frac{1}{\nu}}, \tag{2}$$

where C_t is total consumption, L_t is total leisure hours, σ is the intertemporal elasticity of substitution, ν is the nonparticipation elasticity, and ω is a scale parameter. We assume that $\bar{l}$ leisure hours is available to each nonparticipant and is normalized to 1. Thus, total leisure hours L_t are given by $L_t = N_t + \bar{N}$.[16] Note that we write preferences in terms of utility of leisure (not in terms of disutility of participation). The parameter ν in Equation (2) represents the extensive-margin elasticity of nonparticipation.[17]

Total consumption consists of consumption of both market-produced goods C_{mt} and

[16] Note that we do not explicitly model hours of market work and job search. Thus, $\bar{l}$ can be interpreted as additional hours of leisure that each nonparticipant enjoys relative to participants.

[17] See footnote 25 for the mapping between ν and the extensive-margin labor supply elasticity.

home-produced goods C_{ht}, and is expressed as:

$$C_t = \left[\gamma C_{mt}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) C_{ht}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where γ is the weight parameter of the CES function and ε is the elasticity of substitution. This specification has been used in macro models with home production (e.g., Benhabib et al. (1991)).

The allocation of workers is characterized by a participation threshold value for h_i, denoted as h_t^*, above (below) which workers are allocated to nonparticipation (unemployment). Given h_t^*, aggregate home production that is equal to consumption of home-produced goods C_{ht} is written as:

$$C_{ht} = \tau \check{h}_t U_t + \hat{h}_t N_t + \bar{h}\bar{N}, \tag{3}$$

where $\check{h}_t$ and $\hat{h}_t$ are the conditional means of h_i at the end of the period for unemployed workers and nonparticipants, respectively (i.e., $\check{h}_t \equiv \mathbb{E}(h_i|h_i < h_t^*)$ and $\hat{h}_t \equiv \mathbb{E}(h_i|h_i > h_t^*$, where $\mathbb{E}$ is the conditional expectation operator). Note that unemployed workers and nonparticipants contribute to home production according to their productivity h_i. However, unemployed workers spend a fraction τ of their time on home production. As discussed before, we assume that unemployment is a persistent state (captured by the parameter λ) and we allow only $(1-\lambda)U_{t-1}$ to draw new h_i. We ensure the persistence of the unemployment state by assuming that home productivity of λU_{t-1} is equal to $\check{h}_t$. This assumption is embedded in the first term of Equation (3).[18]

The household problem is formally written as:

$$V(\mathbf{\Omega}_t) = \max_{\{C_{mt}, A_{t+1}, E_t, U_t, N_t, h_t^*, h_t^m\}} \frac{C_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \omega \frac{L_t^{1-\frac{1}{\nu}}}{1-\frac{1}{\nu}} + \beta \mathbb{E}_t V(\mathbf{\Omega}_{t+1}),$$

where the set of state variables is given by $\mathbf{\Omega}_t = \{E_{t-1}, U_{t-1}, N_{t-1}, A_t; y_t\}$. This problem is subject to the evolution of labor market stocks (1) and the following budget constraint:

$$A_{t+1} + C_{mt} = w_t E_t + bU_t + (1+r_t)A_t + \Pi_t - T_t,$$

where A_t represents (zero net supply) wealth yielding the real return r_t, Π_t the firm's flow profits, and T_t the lump sum tax used to finance UI benefits b. Market-goods consumption

[18] An alternative specification could be to assume that home productivities of λU_{t-1} remain the same as in the previous period. We do not believe that this specification changes the main takeaways of our paper. However, it leads to an asymmetric response in the UN transition rate, which significantly reduces the tractability and simplicity of our model.

C_{mt} follows the usual Euler equation:

$$\Lambda_t^{C_m} = \beta \mathbb{E}_t \Big[\Lambda_{t+1}^{C_m} (1 + r_{t+1}) \Big],$$

where $\Lambda_t^{C_m} \equiv \frac{\partial U(C_t, L_t)}{\partial C_{mt}}$. The marginal values of employment ($\mathcal{V}_t^E$), unemployment ($\mathcal{V}_t^U$), and nonparticipation ($\mathcal{V}_t^N$) are given by:

$$\begin{bmatrix} \mathcal{V}_t^E \\ \mathcal{V}_t^U \\ \mathcal{V}_t^N \end{bmatrix} = \begin{bmatrix} w_t \\ b + z_{ht} \tau \check{h}_t \\ z_{lt} + z_{ht} \hat{h}_t \end{bmatrix} + \mathbb{E}_t \hat{\beta}_{t+1} \Gamma'_{t+1} \begin{bmatrix} \mathcal{V}_{t+1}^E \\ \mathcal{V}_{t+1}^U \\ \mathcal{V}_{t+1}^N \end{bmatrix},$$

where future values are discounted by the stochastic discount factor $\hat{\beta}_{t,t+1} \equiv \beta \frac{\Lambda_{t+1}^{C_m}}{\Lambda_t^{C_m}}$. z_{ht} and z_{lt} represent the marginal rates of substitution between C_{ht} and C_{mt} and between L_t and C_{mt}, respectively, and are written as:

$$z_{ht} = \frac{\Lambda_t^{C_h}}{\Lambda_t^{C_m}} = \frac{1-\gamma}{\gamma} \left(\frac{C_{mt}}{C_{ht}} \right)^{\frac{1}{\varepsilon}} \text{ and } z_{lt} = \frac{\Lambda_t^L}{\Lambda_t^{C_m}} = \frac{\omega L_t^{-\frac{1}{\nu}}}{\gamma C_t^{-\frac{1}{\sigma}} \left(\frac{C_t}{C_{mt}} \right)^{\frac{1}{\varepsilon}}},$$

where $\Lambda_t^{C_h} \equiv \frac{\partial U(C_t, L_t)}{\partial C_{ht}}$ and $\Lambda_t^L \equiv \frac{\partial U(C_t, L_t)}{\partial L_t}$. The conditions that determine h_t^* and h_t^m are discussed in Section 3.6.

3.3 Representative Firm

The representative firm produces consumption goods via a linear technology with labor as the only input $Y_t = y_t E_t$, where y_t is exogenous stochastic labor productivity that follows:

$$\ln y_t = (1 - \rho) \ln \bar{y} + \rho \ln y_{t-1} + \epsilon_t \text{ with } \epsilon_t \sim \mathcal{N}(0, \sigma^2).$$

Hiring new workers is subject to search frictions. To generate a persistent and hump-shaped behavior of vacancies and thus to explain better the observed behavior of the transition rates, we introduce a strictly convex hiring cost of the form:

$$\kappa \frac{(q_t v_t)^{1+\epsilon_v}}{1+\epsilon_v},$$

where κ is a scaling parameter and ϵ_v represents the degree of convexity. Several papers in the literature also assume the same convex hiring cost.[19] The firm maximizes its value by choosing the number of vacancies posted every period and thus its size. The decision is characterized by the standard job-creation condition:

$$\kappa(q_t v_t)^{\epsilon_v} = \mathcal{V}_t^J,$$

where $\mathcal{V}_t^J$ is the value of the filled job, which evolves according to:

$$\mathcal{V}_t^J = y_t - w_t + (1-s)\mathbb{E}_t \hat{\beta}_{t,t+1} \mathcal{V}_{t+1}^J.$$

3.4 Wages

We introduce equilibrium wage rigidity as in Hall (2005) and assume the following wage evolution:

$$w_t = (1-\delta_w) w_t^* + \delta_w w_{t-1},$$

where δ_w is the degree of wage rigidity and can be interpreted as the fraction of wages not renegotiated each period. We consider period-by-period Nash-bargained wage (w_t^*) as the wage norm.[20] The Nash-bargained wage is implicitly defined by:

$$\eta \mathcal{V}_t^J = (1-\eta)(\mathcal{V}_t^E - \mathcal{V}_t^A),$$

where η is the worker's bargaining power and $\mathcal{V}_t^E - \mathcal{V}_t^A$ represents surplus for the worker, given by the difference between the value of employment and the value of the outside option $\mathcal{V}_t^A$. Note that the household has the option of either sending the worker to the unemployment pool or out of the labor force, with probability $\Phi(h_t^*)$ and $1-\Phi(h_t^*)$, respectively. Thus, the outside option value for the bargaining is written as:

$$\mathcal{V}_t^A = \Phi(h_t^*)\mathcal{V}_t^U + (1-\Phi(h_t^*))\mathcal{V}_t^N.$$

For the convenience of the analysis below, we rewrite this equation as follows:

$$\mathcal{V}_t^A = g_t + \Phi(h_t^*)\tilde{\mathcal{V}}^U + (1-\Phi(h_t^*))\tilde{\mathcal{V}}^N,$$

[19] The same specification is used, for example, by Merz and Yashiv (2007), Gertler et al. (2008), and Moscarini and Postel-Vinay (2016).

[20] Papers that make use of a similar wage-norm specification include Krause and Lubik (2007), Blanchard and Galí (2007, 2010), Thomas (2008), Sveen and Weinke (2008), and Shimer (2012).

where g_t represents the flow opportunity cost of employment (FOCE), written as

$$g_t \equiv \Phi(h_t^*)(b + z_{ht}\tau\check{h}_t) + (1 - \Phi(h_t^*))(\hat{h}_t z_{ht} + z_{lt}). \tag{4}$$

The "tilded" value functions are defined by:

$$\begin{bmatrix} \tilde{\mathcal{V}}_t^E \\ \tilde{\mathcal{V}}_t^U \\ \tilde{\mathcal{V}}_t^N \end{bmatrix} \equiv \begin{bmatrix} \mathcal{V}_t^E \\ \mathcal{V}_t^U \\ \mathcal{V}_t^N \end{bmatrix} - \begin{bmatrix} w_t \\ b + z_{ht}\tau\check{h}_t \\ z_{lt} + z_{ht}\hat{h}_t \end{bmatrix}. \tag{5}$$

The FOCE is endogenously cyclical in our model, while in canonical two-state models, it consists only of b, which is constant over the cycle.

3.5 Resource Constraint

Given that there is no physical capital in the model and that financial assets are in zero net supply, the aggregate resource constraint is given by:

$$y_t E_t = C_{mt} + \kappa \frac{(q_t v_t)^{1+\epsilon_v}}{1 + \epsilon_v}.$$

3.6 Participation Decision

Before discussing how the participation margin is determined in our model, let us first discuss what the empirical regularities of transition rates imply about the cyclicality of the participation margin h_t^*. One can see from Table 1 that transition rates in the model are functions of labor market tightness (θ_t), the participation margin (h_t^*), and the acceptance margin (h_t^m). To simplify our discussion, first assume the acceptance probability $\Phi(h_t^m) = 1$. We then ask how the model can replicate the cyclical behavior of the transition rates (and thereby the stocks) through the movements in the participation margin, given that tightness is likely to be procyclical.[21]

In the data, the EU rate is countercyclical and the EN rate is procyclical. With the assumption $\Phi(h_t^m) = 1$, these two rates are expressed as $s\Phi(h_t^*)(1-f_t)$ and $s(1-\Phi(h_t^*))(1-\mu f_t)$, respectively. The job-finding rate f_t enters these expressions as a result of the "contemporaneous hiring" we allow for in the model, and the procyclical f_t creates a countercyclical force

[21] We find in our quantitative model that $\Phi(h_t^m)$ is indeed close to one and that the variations in h_t^m have minimal impacts on our results.

in these two rates.[22] Thus, the only way to make both of these model-based EU and EN rates move consistently with the data is through countercyclical $\Phi(h_t^*)$ and thus h_t^*. Next, the entry rate from nonparticipation (NU rate) is countercyclical in the data, as the EU rate is, and, in the model, it differs from the EU rate only by the constant factor s. Thus, the countercyclical h_t^* is sufficient for this to be countercyclical. Next, consider the UN rate, defined as $(1-\lambda)(1-\Phi(h_t^*))(1-\mu f_t)$. In order for this to behave procyclically as in the data, it is necessary that $\Phi(h_t^*)$ is countercyclical, assuming that f_t behaves procyclically. Lastly for UE and NE rates, which represent job-finding probabilities from unemployment and nonparticipation, respectively, both of these rates are driven by the term $[(1-\mu)\Phi(h_t^*) + \mu]\, f_t$. In the data, they are clearly procyclical and the countercyclicality of $\Phi(h_t^*)$ works against this empirical regularity. However, as far as f_t dominates the movements of these rates (as is the case in the model with no participation margin), our model can still be consistent with the procyclicality of these rates.[23]

The discussion so far clearly points to the following conclusion: In order to explain the cyclical behavior of transition rates, the participation margin must be countercyclical. Note that this conclusion underscores the importance of the flow approach, as opposed to the stock approach. In the stock approach, the (weak) procyclicality of the LFPR is likely to be interpreted as indicating procyclicality of the participation margin. Moreover, as is discussed in detail below, our model generates the weak procyclicality of the LFPR even though the participation margin is highly countercyclical.

We show in Appendix A.2 that the participation margin is determined by:

$$\frac{(1-\mu)f_t}{1-\mu f_t}(w_t + \tilde{\mathcal{V}}_t^E) + \frac{1-f_t}{1-\mu f_t}\left(b + \tau z_{ht} h_t^* + \tau z_{ht}(h_t^* - \check{h}_t)\frac{\lambda U_{t-1}}{\Phi(h_t^*)R_{t-1}} + \tilde{\mathcal{V}}_t^U\right)$$
$$= z_{lt} + h_t^* z_{ht} + \tilde{\mathcal{V}}_t^N. \qquad (6)$$

Recall that R_{t-1} represents the mass of workers who draws new h_i at the beginning of period t, $E_{t-1} + (1-\lambda)U_{t-1} + N_{t-1}$. This condition determines the participation margin h_t^*, such that the marginal returns from market participation (left-hand side) and non-market activities (right-hand side) are equal to each other. Note that the third term in the second parenthesis on the left-hand side of (6) exists due to our assumption about the persistence

[22]One can view this effect as capturing the time aggregation effect that Shimer (2012) emphasizes with respect to the observed countercyclicality of the EU rate.

[23]One can see that log linearizing the above expression results in $d\hat{f}_t + \frac{\Phi(h^*)(1-\mu)}{(1-\mu)\Phi(h^*)+\mu} d\hat{\Phi}(h_t^*)$ where the "hat" variables and variables without t represent their log deviations and steady-state values, respectively. The multiplicative factor in front of $d\hat{\Phi}(h_t^*)$ dampens the countercyclicality of $\Phi(h_t^*)$ and the dampening effect is larger as the value of μ gets larger.

in the unemployment state λ. To simplify our explanation, let us first imagine that $\lambda = 0$. The condition then takes the following simpler form:

$$\frac{(1-\mu)f_t}{1-\mu f_t}(w_t + \tilde{\mathcal{V}}_t^E) + \frac{1-f_t}{1-\mu f_t}\left(b + \tau z_{ht} h_t^* + \tilde{\mathcal{V}}_t^U\right) = z_{lt} + h_t^* z_{ht} + \tilde{\mathcal{V}}_t^N. \tag{7}$$

The "weight" terms on the left-hand side $\frac{(1-\mu)f_t}{1-\mu f_t}$ and $\frac{1-f_t}{1-\mu f_t}$ sum to one, and correspond to $\frac{\partial E_t}{\partial N_t}$ and $\frac{\partial U_t}{\partial N_t}$, respectively, i.e., the share who move to employment or unemployment when the household decides to allocate one additional nonparticipant to market work. The expressions for the weights are consistent with our assumptions about labor market transitions that those who enter the unemployment pool may move to employment within the same period and that those in N_{t-1} can directly move to employment with probability μf_t.

How is the condition (7) affected by the variations in y_t? In expansions, the job-finding rate f_t and wage w_t are to increase. The weight on the value of employment is increasing in f_t, while the weight on unemployment is decreasing in f_t. Shifting the weights toward employment away from unemployment further raises the left-hand side. On the other hand, the drivers of the value of nonparticipation are the values of leisure (z_{lt}) and home-produced goods consumption (z_{ht}). As emphasized above, the empirical evidence requires that the participation margin h_t^* be countercyclical. In order for that to be the case, z_{lt} and z_{ht} need to be procyclical. Suppose to the contrary that these two terms were to be constant (as in the linear utility case); then the participation margin would be procyclical, given that the left-hand side is procyclical. Note also that the smaller the labor supply elasticity ν is, the stronger the procyclicality of z_{lt}. The same is true for the elasticity of substitution ϵ. We set these two elasticity parameters to the values consistent with micro-level evidence. One should also recognize the importance of wage rigidity as well in achieving the countercyclical participation margin. That is, it mitigates the strong procyclical response of the value of employment.[24] In the case with $\lambda > 0$ (as in Equation (6)), the additional term (the third term in the second parenthesis on the left-hand side) makes the discussion less clear-cut. However, a similar logic still applies, as this term is small in our estimated model. We show in Section 5 that both procyclical values of non-market activities and wage rigidity are *equally* important in generating the countercyclical participation margin.

The existing literature has highlighted the importance of the procyclical values of non-market activities and wage rigidity in the context of labor demand magnification, while in this paper we emphasize their importance in understanding the variations in the labor supply

[24]Note that wage rigidity also contributes to magnifying the fluctuations of f_t, making the left-hand side more responsive. The convex hiring costs play an important role in controlling this force.

margin. Specifically, Chodorow-Reich and Karabarbounis (2016) provide strong evidence in favor of procyclical values of non-market activities and they show how this cyclicality could reduce labor demand responses. Our analysis of the participation margin extends their work by showing why the procyclical values of non-market activities are necessary to achieve the countercyclical participation margin. Regarding wage rigidity, it is well-known that it makes the job-finding rate more volatile, but we include wage rigidity not for that purpose, but for eliminating an otherwise procyclical labor supply response. Note also that, while rigid wage directly reduces the return to market work, the magnification effect on the job-finding rate encourages labor force participation. In this regard, the strictly convex hiring cost in our model helps to slow down this effect.

Regarding the acceptance threshold h_t^m, we show in Appendix A.2 that it is determined by the following condition:

$$w_t + \tilde{\mathcal{V}}_t^E = z_{lt} + h_t^m z_{ht} + \tilde{\mathcal{V}}_t^N. \tag{8}$$

The acceptance margin corresponds to the home productivity value that makes a nonparticipant indifferent between accepting a job (left-hand side) and remaining out of the labor force (right-hand side). Does the time-varying acceptance margin (h_t^m) change our previous analysis? No. First, it is clear from Equation (8) that the acceptance threshold h_t^m inherits similar countercyclicality of the participation threshold h_t^*. Second, note that the acceptance margin affects EN, UE, UN, and NE transition rates by adding extra terms, but the countercyclical acceptance margin reinforces the procyclicality of EN and UN rates. Moreover, we continue to expect procyclical f_t to be a dominant driver of UE and NE rates for sufficiently large values of λ and μ.

Let us recapitulate the main takeaways of the current analysis. To replicate the dynamics of labor market transition rates and stocks, we need a strongly countercyclical participation margin. In order to achieve that condition, values of non-market activities need to be strongly procyclical, which in turn requires smaller values of the labor supply elasticity and elasticity of substitution. Wage rigidity also helps, as it mitigates the procyclical responses of the return to market work. We show later that both of the two channels are equally important for our quantitative results.

4 Calibration/Estimation

The model frequency is monthly. Table 2 summarizes the parameter values chosen. We first set values for some of the parameters exogenously, as described in Panel A. The values of β, α, and η, and σ are all standard in the literature. The elasticity of substitution between C_m and

Table 2: Parameter Values and Implied Steady-State Values

Parameter	Interpretation	Value
	Panel A: Externally calibrated	
β	Discount factor	0.995
α	Elasticity parameter of matching function	0.5
η	Worker's bargaining power	0.5
σ	Intertemporal elasticity of substitution	1
ϵ	Elasticity of substitution between C_m and C_h	2.5
ν	Elasticity of nonparticipation	0.47
τ	Fraction of time devoted to home production for unemployed	0.89
σ_h	Standard deviation of log home productivity	1
$\bar{h}$	Home productivity for inactive nonparticipants	1.9935
ρ	Autoregressive parameter for log aggregate productivity	0.99
σ	Standard deviation for log aggregate productivity	0.0053
	Panel B: External steady-state restrictions	
g	Steady-state flow opportunity cost of employment	0.71
q	Steady-state job-filling probability	0.9
z_h	Steady-state marginal rate of substitution between C_h and C_m	1
$\bar{y}$	Steady-state aggregate productivity level	1
E	Steady-state employment-to-population ratio	0.6194
	Panel C: Estimated	
s	Total separation rate	0.0491
δ_w	Wage stickiness	0.9844
ϵ_v	Curvature of hiring cost function	30.32
$\bar{\omega}$	Ratio between z_l and $\hat{h}z_h$ in the steady state	0.9895
λ	Fraction of unemployed workers who do not draw home productivity	0.4571
b	Unemployment benefits	0.2199
μ	Relative job-search efficiency of nonparticipants	0.3304
$\bar{N}$	Unavailable nonparticipants	0.2992
$\bar{m}$	Scale parameter of matching function	0.6103
κ	Scale parameter of hiring cost function	7.79e+45
γ	CES weight parameter	0.4992
ω	Scale parameter of leisure in the utility function	0.0433
μ_h	Mean of log home production	-1.6365

C_h is set to $\varepsilon = 2.5$, following Aguiar et al. (2013). As these authors explain, the values used in the literature range from somewhat less than 2 to 4. To set nonparticipation (or leisure) elasticity ν, we follow Chetty et al. (2011, 2013) who suggest that the micro estimate of the extensive-margin labor supply (participation) elasticity is around 0.25, which translates into the nonparticipation elasticity at 0.47.[25] As discussed above, our model needs a small value for ν to match the data. We set τ equal to 0.89, which corresponds to the ratio of home production time between individuals that are unemployed and individuals that are out of the labor force in the American Time Use Survey (ATUS).[26]

We assume that the distribution of home production $\Phi(.)$ is lognormal with mean μ_h and standard deviation σ_h. We fix $\bar{h}$ to the 99th percentile of the distribution. Following Boerma and Karabarbounis (2019), we set $\sigma_h = 1$ and estimate μ_h as explained below. For the exogenous productivity process, we first normalize its steady-state level $\bar{y}$ to 1 and select its persistence and volatility to match the cyclical properties of the quarterly U.S. labor productivity (seasonally adjusted real output per person in the nonfarm business sector) between 1976 and 2016. After taking logs and deviations from an HP trend with smoothing parameter 10^5, the standard deviation of quarterly labor productivity equals 0.0176 and its quarterly autocorrelation is 0.899 in the data. With our selected values, our exogenous productivity series matches these moments.

We impose several steady-state restrictions on the parameters listed in Panel B of Table 2. First, the steady-state value of the flow opportunity cost of employment is set to $g = 0.71$ (see equation (4)). The literature has used this value as plausible (e.g., Hall and Milgrom (2008)), lumping together UI benefits b with values of home production and leisure. Second, the steady-state job-filling rate is set to $q = 0.9$, following Fujita and Ramey (2007). Third, we normalize the steady-state value of z_{ht} at 1.[27] Finally, we set the employment-to-population ratio to 0.6194, which corresponds to the historical average over the period 1976-2016.

We estimate the remaining 13 parameters listed in Panel C of Table 2 by solving a constrained minimization problem of the weighted distance between the median IRFs from our VAR and the model IRFs. See Appendix A.2 for the estimation procedure in detail. This minimization problem is constrained in the sense that we impose that the steady-state levels of transition rates do not deviate from their historical averages by more than 30 percent.[28] We

[25]Let $\tilde{\nu}$ be the extensive-margin labor supply (participation) elasticity. We convert $\tilde{\nu}$ into ν by using $\nu = -\frac{1-N}{N}\tilde{\nu}$, where N is set to 0.347, which is the steady-state nonparticipation rate in our model.

[26]We identify home production in the ATUS as "unpaid household work," which includes (i) household activities such as food and drink preparation and cleaning, (ii) caring for and helping household members, (iii) purchasing goods and services, and (iv) travel related to unpaid household work. See, for example, Krantz-Kent (2009) for the detailed classification scheme. This ratio is calculated as an average of those between 16 and 64 years of age over the period 2003-2016. We exclude 65+ individuals from the calculation, because we view these individuals as being part of the unavailable pool in our model; however, including them in the calculation does not materially change the ratio.

[27]Within our calibration procedure, the model dynamics are invariant to the steady-state level of z_{ht}.

[28]The historical means are based on our margin-error adjusted series. We allow for some deviations, because different adjustments, proposed in the literature, lead to different average levels of transition rates, sometimes significantly. See Panel A of Table A.1 in Appendix A.1. Variations in historical means seem to

use six transition rates, vacancies, real wage, and the LFPR as our observables and weight the IRFs of these variables by their unconditional variances in evaluating the fit.[29] Note that, although we estimate 13 parameters, the model's steady-state equilibrium conditions put restrictions between those parameters. In practice, we set up this minimization problem such that the estimation routine searches for the best values of s, δ_w, ϵ_v, and $\bar{\omega}$, where $\bar{\omega} \equiv \frac{z_l}{\hat{h} z_h}$. Note also that the wage rigidity parameter δ_w does not appear in the steady state and thus is determined solely by the dynamics of the model.

Panel C of Table 2 presents the estimated parameter values and Table 3 presents the implied steady-state values of labor market stocks and transition rates. First, the relative efficiency of job search among available nonparticipants is estimated to be $\mu = 0.33$. Next, the size of unavailable nonparticipants is estimated to be $\bar{N} = 0.30$, which, together with the steady-state values of E and U, implies the steady-state value of available nonparticipants at $N = 0.05$. One possible empirical measure that roughly corresponds to this model concept is "persons who want a job" reported in the CPS. In the data, this group is classified as nonparticipants because they did not actively look for a job during the reference week, even though they expressed an interest in having a job. The average size of this pool amounts to 0.025 (as a share of 16+ population) over the period between 1994 and 2016. The empirical measure should, however, be considered a lower bound, given that there is still a large flow into E or U from nonparticipation even outside this group, such as new graduates from school. We thus view our estimate of available nonparticipants as plausible.

Next, the relative importance of z_l and $\hat{h} z_h$ in the steady state is estimated at $\bar{\omega} = 0.99$ and the level of UI benefits is estimated to be $b = 0.22$. The implied value of the scale parameter of the leisure function is $\omega = 0.0433$. The mean of $\log(h_i)$ is estimated to be -1.6365, which implies the mean of the distribution, $\exp(\mu + \sigma_h^2/2)$, at 0.32. The steady-state value of the FOCE (g) in the model can then be written as:

$$\begin{aligned} g &= \Phi(h^*)(b + z_h \tau \check{h}) + (1 - \Phi(h^*))(z_h \hat{h} + z_l) \\ &= 0.435 \times (0.220 + 1 \times 0.89 \times 0.090) + (1 - 0.435) \times (1 \times 0.516 + 0.510) = 0.71. \end{aligned}$$

Importantly, two elements of our estimation procedure are consistent with the empirical evidence by Chodorow-Reich and Karabarbounis (2016). First, the FOCE is strongly pro-

suggest that allowing for deviations of 30 percent is plausible.

[29] We do not include the employment-to-population ratio and the unemployment rate in the set of observables. These two variables are largely redundant, given that movements in transition rates imply clear cyclical patterns in those two variables. The behavior of the LFPR, on the other hand, is more subtle, thus including it in the set of observables helps identify some of the parameters more tightly. However, adding the two stock variables or dropping the LFPR does not materially change our results.

Table 3: Steady-State Performance

Empirical concept	Model concept	Target value	Steady-state value
UR	$\frac{U}{E+U}$	0.064	0.052
LFPR	$E+U$	0.630	0.653
EU rate	$s\Phi(h^*)(1-f)$	0.015	0.013
EN rate	$s\left[(\Phi(h_t^m)-\Phi(h_t^*))(1-\mu f_t)+(1-\Phi(h_t^m))\right]$	0.028	0.024
UE rate	$\left[\lambda+(1-\lambda)(\Phi(h_t^*)+(\Phi(h_t^m)-\Phi(h_t^*))\mu)\right] f_t$	0.251	0.325
UN rate	$(1-\lambda)\left[(\Phi(h_t^m)-\Phi(h_t^*))(1-\mu f_t)+(1-\Phi(h_t^m))\right]$	0.214	0.269
NE rate	$\left[\Phi(h_t^*)+(\Phi(h_t^m)-\Phi(h_t^*))\mu\right] f_t \frac{N}{N+\bar{N}}$	0.047	0.034
NU rate	$\Phi(h^*)(1-f)\frac{N}{N+\bar{N}}$	0.027	0.035

Notes: Steady-state values for $\Phi(h^*) = 0.435$, $f = 0.414$, $h^* = 0.165$, and $N = 0.048$.

cyclical in our model, given the procyclicality of the marginal rates of substitution z_{ht} and z_{lt}. In Table 4, we report that in our model, the elasticity of the FOCE with respect to labor productivity is 0.88, which is within the empirically estimated interval by Chodorow-Reich and Karabarbounis. Second, in our estimated model, the UI component ($\Phi(h^*)b = 0.096$) in the FOCE takes a relatively small share and represents less than 10 percent of the steady-state level of labor productivity. Chodorow-Reich and Karabarbounis show that UI benefits represent only 6 percent of labor productivity, once the take-up ratio and eligibility are taken into account. While we do not explicitly incorporate these elements that are present in reality, the participation margin $\Phi(h_t^*)$ can be compared to the decision to receive UI benefits. Moreover, these authors emphasize the countercyclicality of the (effective) amount of UI benefits, as the take-up rate is highly countercyclical. In our model, the UI component is highly countercyclical as the participation margin is highly countercyclical. Table 4 also shows that the simulated elasticity for UI benefits is close to the data.[30]

The total separation rate s is estimated to be 4.9 percent. The persistence parameter of the unemployment state is estimated to be $\lambda = 0.46$, which helps to bring the steady-state level of the UN rate closer to its empirical counterpart. The scale parameter of the matching function is estimated to be $\bar{m} = 0.61$. Together with the elasticity parameter exogenously set above, it implies the steady-state job-finding rate f_t at 0.414.

The wage rigidity parameter δ_w and the curvature parameter of the hiring cost function ϵ_v play important roles for the model dynamics. The estimated value of δ_w implies that the

[30] Chodorow-Reich and Karabarbounis (2016) do not directly report this elasticity. We compute this elasticity using their data and following their methodology: the data is logged and HP-filtered with a smoothing parameter equal to 1,600, and labor productivity is instrumented with the utilization adjusted TFP.

Table 4: Elasticities of FOCE and UI Income

	Flow opportunity cost of employment g_t	Unemployment insurance benefits $\Phi(h_t^*)b$
Model	0.88	-2.43
Data	(0.83, 1.11)	$(-1.99, -1.98)$

Note: This table reports elasticities for the flow opportunity cost of employment (g_t) and UI income ($\Phi(h_t^*)b$) in our estimated model. We simulate the model for 160 quarters (the length of our data) and regress the logged and HP-filtered series of g_t and $\Phi(h_t^*)b$ on labor productivity, which we instrumented using the utilization-adjusted TFP. We repeat this procedure 1,000 times and report the average elasticities. Results along "Data" are taken from Chodorow-Reich and Karabarbounis (2016).

data favors a high degree of wage rigidity. The curvature of the hiring cost is estimated to be large, given that job vacancies move gradually and persistently in response to the shock. Also, as explained in Section 3.6, for high degrees of wage rigidity, a high value of ϵ_v is needed to smooth the responses of the job-finding rate and to ensure that the participation margin is countercyclical. The large curvature value has also been previously used in the literature.[31]

One can see in Table 3 that, although the model is unable to match perfectly the steady-state values of these labor market transition rates and stocks, all values are near the empirical counterparts.[32]

5 Main Results

In this section, we demonstrate that the model replicates the key cyclical features of all labor market variables. We also assess the contribution of wage rigidity and the procyclical values of non-market activities for our results.

5.1 Model Dynamics

First, in Figure 3, we present responses of the two key endogenous variables in our model (solid blue lines): labor market tightness θ, and the participation margin h_t^*. The figure

[31] For example, Moscarini and Postel-Vinay (2016) use the same hiring cost function and set the curvature value at 50. The large curvature value implies a large value of the scale parameter κ, but it has no impact on our model dynamics, since it only shows up as a constant term in the log-linearized model.

[32] We can also compute the "job-to-job transition rate" in our model as those who separate at the beginning of t but return to work in the same period. The steady-state level of this rate is 1.2 percent per month in our model, which is lower than the average level of the empirical counterpart (at around 2 to 2.5 percent). The low level is not surprising given that the model does not allow for direct transitions between jobs. However, this series replicates strong procyclicality as observed in the data.

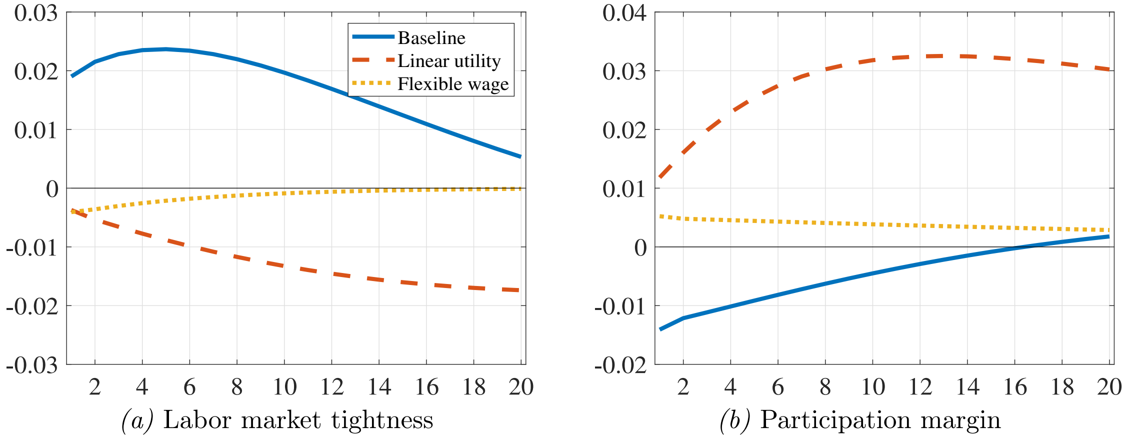


Figure 3: IRFs: Labor Market Tightness and Participation Margin. *Notes*: Responses to a one standard deviation positive productivity shock. Quarterly averages of monthly responses, expressed as log deviations from steady-state levels.

shows that our model successfully generates strongly procyclical and volatile labor market tightness, and a strongly countercyclical and volatile participation margin.

Figures 4 and 5 compare the model IRFs to their empirical counterparts. These figures demonstrate that our model performs well in replicating the cyclicality of transition rates and labor market stocks. In particular, the model successfully replicates the two key cyclical features of the LFPR: (i) it tends to fall initially before it increases above its steady-state level, and (ii) its variations over the business cycle are minuscule even though the underlying transition rates are much more volatile.

Let us now delve deeper into the cyclical patterns of transition rates (see Table 3 for their definition). First, consider separation rates into unemployment and nonparticipation (Figure 4 (a) and (d)). As emphasized before, the data show the former being countercyclical and the latter being procyclical, and our model is able to reproduce both features. The EU rate is defined by $s\Phi(h_t^*)(1-f_t)$, and the term $(1-f_t)$ captures the time aggregation effect, emphasized by Shimer (2012). That is, those who enter the unemployment pool (with probability $s\Phi(h_t^*)$) remain jobless only if they fail to find a job in that period (with probability $1-f_t$). In expansions, not only does the probability of entering into the unemployment pool fall, but also the probability of remaining there falls. Regarding the EN rate, given that $\Phi(h_t^m)$ remains close to 1, it is well approximated by $s(1-\Phi(h_t^*))(1-\mu f_t)$, for which, the procyclicality of the term $(1-\Phi(h_t^*))$ dominates its overall cyclicality, even though the time aggregation effect $(1-\mu f_t)$ weakens it.

Second, consider the UE and NE rates (Figure 4 (b) and (e)). In our model, the cyclical

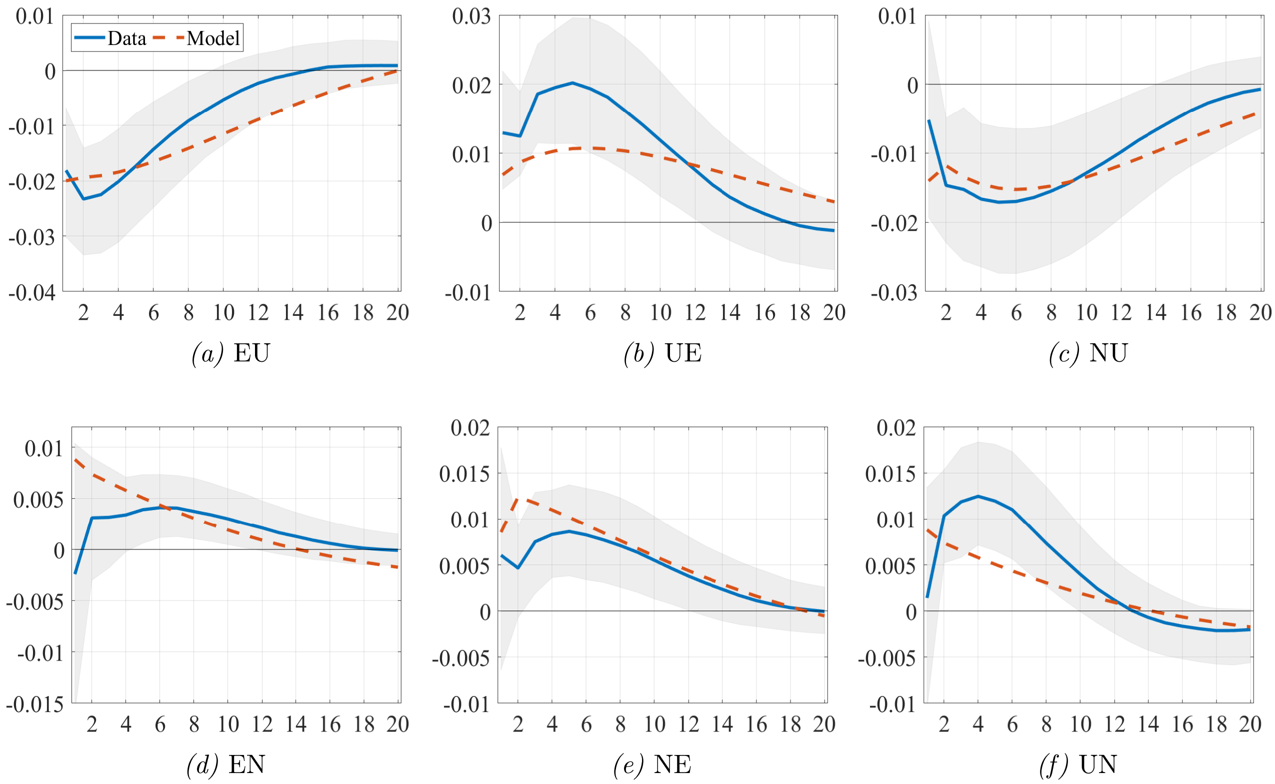


Figure 4: IRFs: Transition Rates. *Notes*: Responses to a one standard deviation positive productivity shock. Quarterly averages of monthly responses, expressed as log deviations from steady-state levels. Blue solid lines are the median empirical responses, and shaded areas are error bands representing the 16th and 84th percentiles of the posterior distribution.

movements in these two variables are dominated by the changes in f_t, although the procyclicality of these transition rates is somewhat mitigated by the presence of $\Phi(h_t^*) + (\Phi(h_t^m) - \Phi(h_t^*))\mu$. This whole term is countercyclical in our model, given the countercyclicality of $\Phi(h_t^*)$ and $\mu < 1$ (again, $\Phi(h_t^m)$ remains close to 1 and exhibits small variations).

Lastly, consider transition rates between unemployment and nonparticipation (Figure 4 (c) and (f)). Regarding the NU rate, it is straightforward to replicate the observed countercyclicality as far as f_t is procyclical and h_t^* is countercyclical. Cyclical fluctuations of $\frac{N_t}{N_t+N}$ are relatively small and thus are quantitatively unimportant. The UN rate behaves procyclically in our model as in the data. The same forces described for the cyclicality of the EN rate apply to this variable.

Figure 5 (a)–(c) present the responses of the three stock variables. Panel (e) shows a high degree of wage rigidity in the estimated model. This feature is of first-order importance for results on both volatilities of labor demand (i.e., f_t) and the cyclical patterns of labor

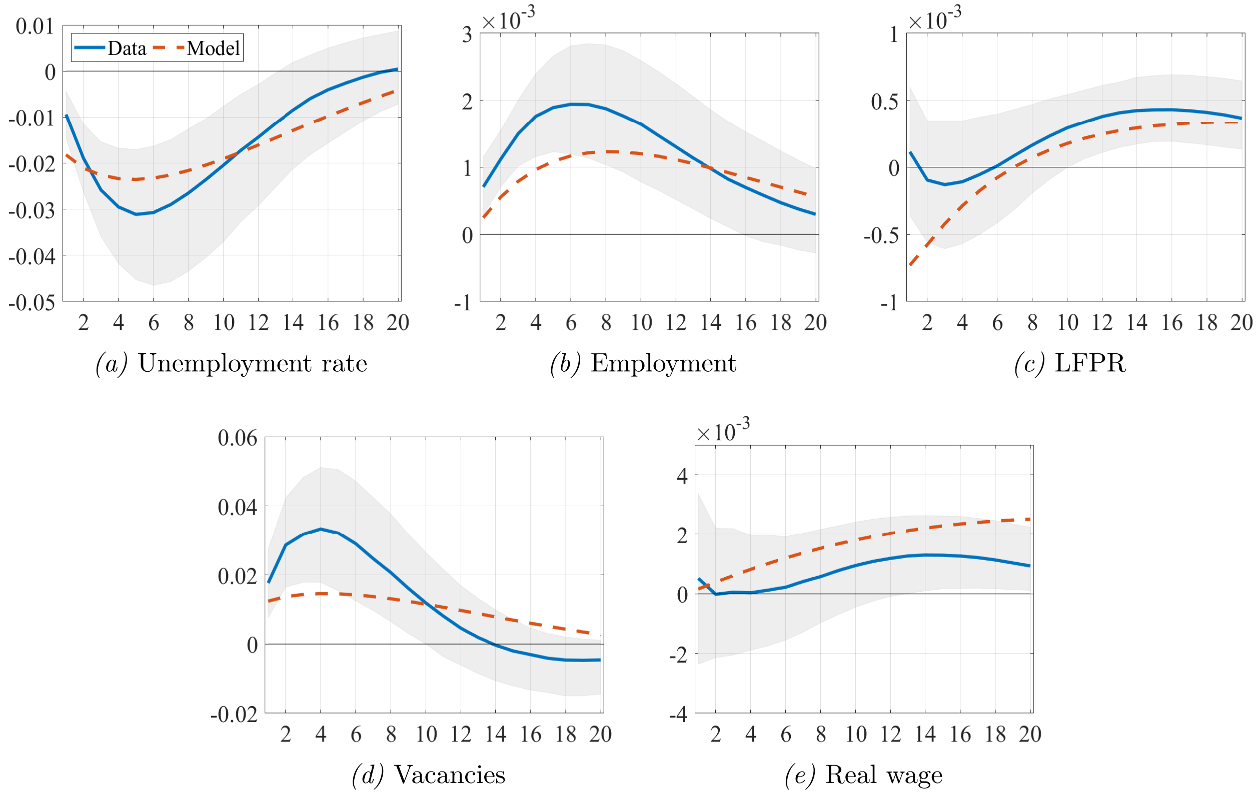
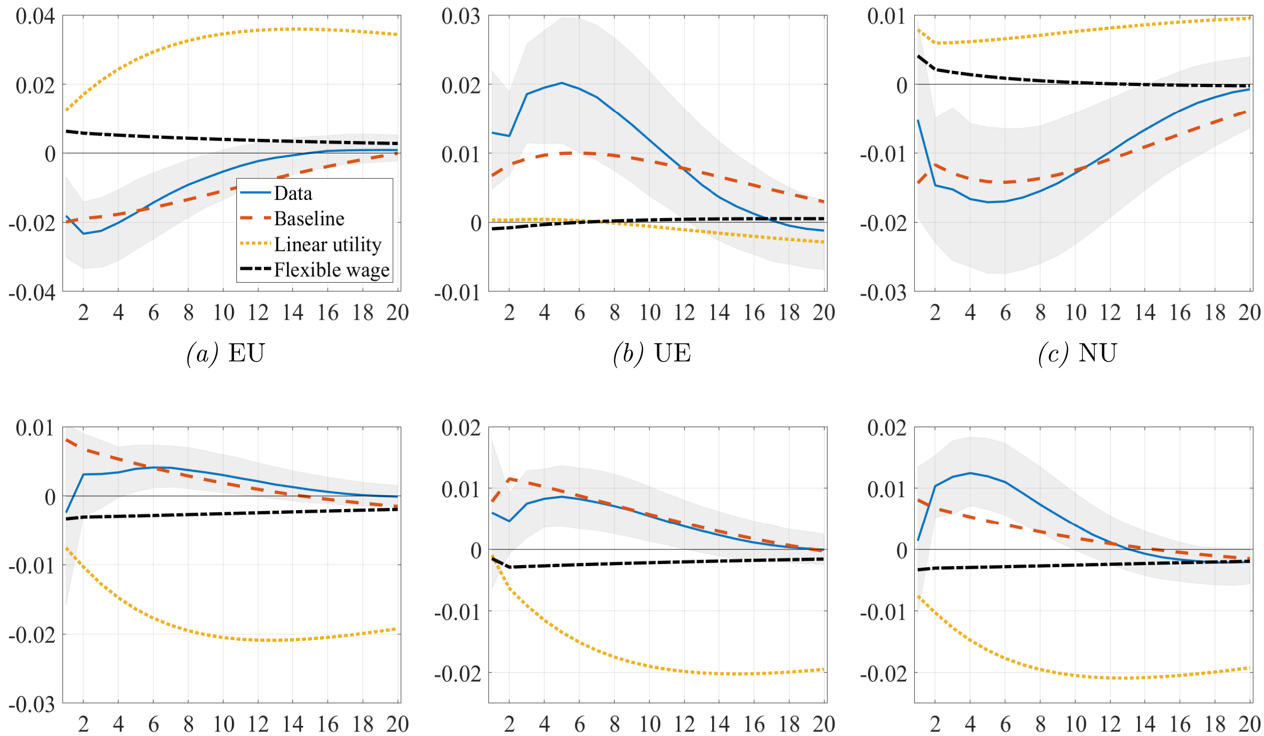


Figure 5: IRFs: Stocks and Real Wages. *Note*: See notes to Figure 4.

supply (i.e., transition rates into and out of nonparticipation). The model matches the countercyclical unemployment rate and the procyclical employment-to-population ratio, and their volatilities are roughly of the same magnitude as in the data. Vacancies increase in a hump-shaped manner. The sluggishness of the vacancy response is a direct result of our convex hiring cost and its large curvature.

The responses of the unemployment rate and vacancies show that the model replicates the so-called Beveridge curve. It is well known that, in two-state labor matching models, there is a tension between the model's capability of replicating the negative relationship between the two and how the separation decision is modeled (see, for example, Fujita and Ramey (2012)). In our setup, the underlying separation rate s is an exogenous parameter. Thus it is perhaps not surprising that our model is capable of replicating the strong negative correlation between the two. It is nevertheless worth noting that our model replicates both the strongly countercyclical EU rate and the Beveridge curve simultaneously. In addition, our model matches the fact that the overall separation rate (sum of EU and EN rates) is much less cyclical than either of the two but remains countercyclical. In sum, the quantitative



(a) EU *(b)* UE *(c)* NU *(d)* EN *(e)* NE *(f)* UN

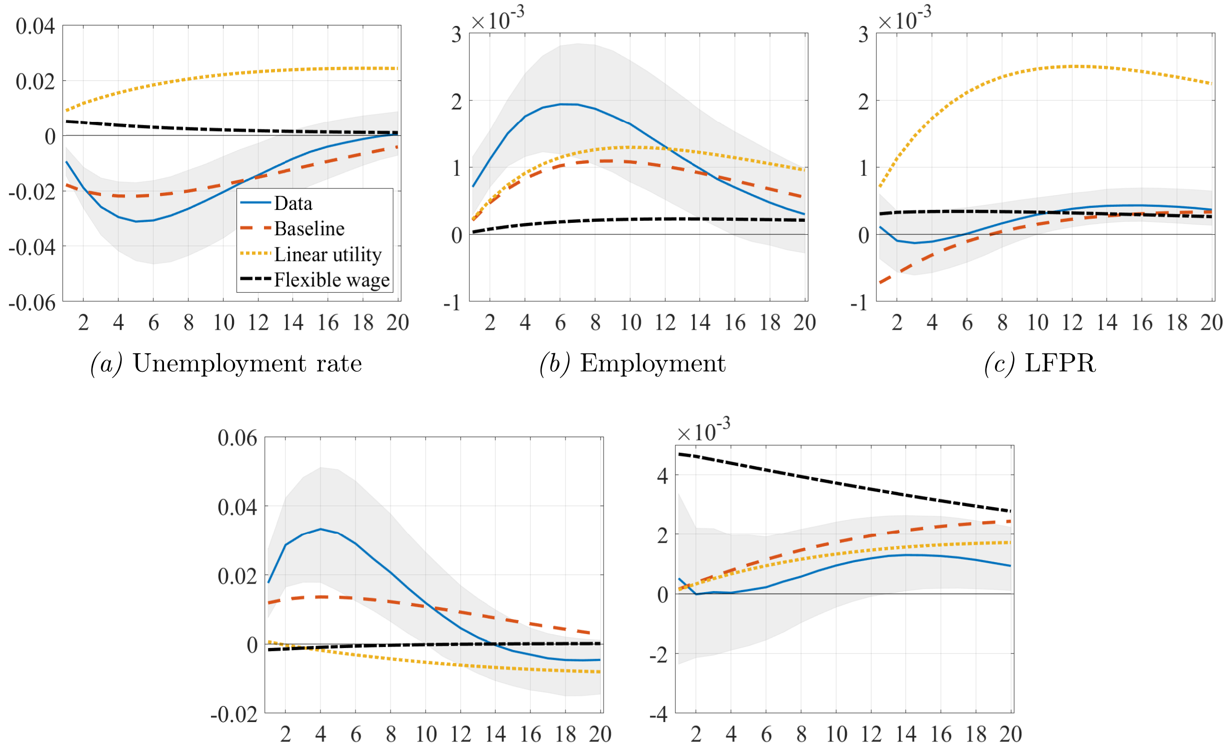
Figure 6: IRFs: Transition Rates. *Notes*: See notes to Figure 3.

performance of our model presents significant improvements over the existing literature.

5.2 Contributions of Wage Rigidity and Procyclical Values of Non-Market Activities

We earlier emphasized the importance of wage rigidity and procyclical values of non-market activities in delivering our quantitative results. In this section, we disentangle the contribution of each element separately and show that both elements are essential for our results.

Figures 6 and 7 compare our baseline results (red dashed lines) with those under the two counterfactual parameterizations. First, under the "linear utility" case (yellow dotted lines), we eliminate curvatures of the utility function by setting σ and ν to infinity, and make the two types of consumption goods perfectly substitutable by setting ϵ to infinity. We keep the values of the remaining parameters the same, including the wage rigidity parameter. Second, under the "flexible wage" case (black dash-dotted lines), we set the wage rigidity parameter δ_w to zero. Again, the values of the remaining parameters are kept the same, and thus the



(a) Unemployment rate *(b)* Employment *(c)* LFPR

(d) Vacancies *(e)* Real wage

Figure 7: IRFs: Stocks and Real Wage. *Note*: See notes to Figure 3.

values of non-market activities remain procyclical.

Figures 6 and 7 clearly show that shutting down either of the two features makes the model fail miserably. In either case, the model tends to generate three main counterfactual results: (i) a procyclical unemployment rate, (ii) wrong cyclical patterns of transition rates, and (iii) stronger procyclical responses of the LFPR than observed in the data. In both counterfactual cases, the model fails to explain the data for the same reason that the participation margin becomes procyclical (see Figure 3 (b)). More workers join the labor force in expansions, and as a result, labor market tightness becomes countercyclical (see Figure 3 (a)) due to the large increase in unemployment.

With flexible wages, returns to market participation increase too much in booms (Equation (6)). Similarly, the constant values of z_{ht} and z_{lt} imply that the cost of market participation does not increase in expansions, and thus even when wages do not rise, the incentive to join the labor force still rises, because the job-finding rate increases. Under both counterfactual cases, inflow rates to the unemployment pool (EU and NU rates) become procyclical, and outflow rates from the labor force (EN and UN rates) become countercyclical. It is well

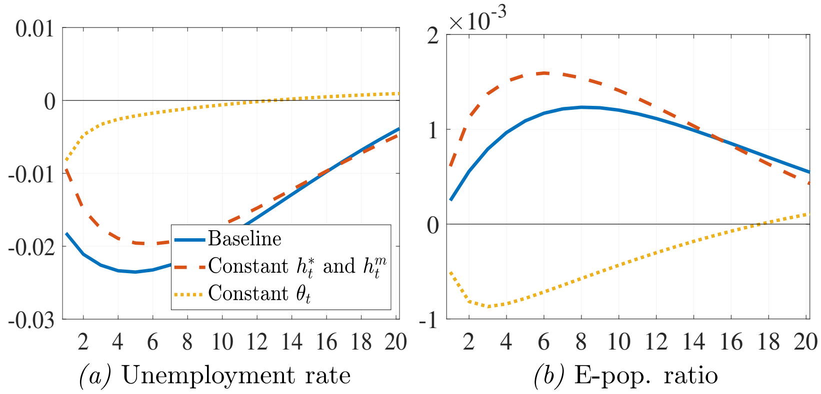


Figure 8: IRFs: Contributions of θ_t and h_t^*. *Notes*: See notes to Figure 3.

understood in the literature that wage rigidity helps magnify labor demand in the two-state models. However, when the participation margin is explicitly considered, not only does the effect pretty much disappear, but also the sign of the unemployment response flips.

In summary, the alternative parameterizations in this section demonstrate one of our key contributions to the literature: we clearly identify the countercyclical participation margin as a necessary condition to match the dynamics of the labor market and emphasize the importance of the procyclical values of non-market activities and wage rigidity in achieving that condition.[33]

5.3 Contributions of Labor Demand and Supply Responses

At the beginning of the paper, we noted the consensus view in the literature that shifts in labor demand account for most of the cyclical variation in labor input. Our model allows us to quantitatively evaluate this view. Specifically, we assess the contribution of labor demand by letting θ_t vary, while keeping h_t^* and h_t^m constant at their steady-state values. Similarly, we assess the contribution of labor supply by allowing h_t^* and h_t^m to vary, while keeping θ_t constant at its steady-state level.

Figure 8 presents the responses of three labor market stocks under these two counterfactual scenarios as well as under our baseline model. The same intuitions we discussed for our earlier experiments apply here. For the unemployment rate, both labor demand and labor supply contribute to lowering the unemployment rate in response to the positive shock, while for the other two variables, the two margins move them in opposite directions. We

[33] Krusell et al. (2017) also succeed in replicating the cyclical pattern of transition rates between unemployment and nonparticipation. In their model, the value of non-market activities is also procyclical, and this is achieved through procyclical movements in the wealth distribution.

can see that the participation margin makes nontrivial contributions to the variation in the unemployment rate especially in the short run. It is also interesting to note that for the movements in the LFPR, the contribution of one variable is largely offset by that of the other over the entire horizon, leaving the net movements in the LFPR small. With respect to the timing for the LFPR, the initial declines are largely due to the immediate declines in the participation margin, before the labor demand response (θ) starts pulling the LFPR up. This exercise suggests that both margins are quantitatively important for labor market dynamics. In particular, small variations in the LFPR do not immediately suggest that only the shifts in labor demand are operating in driving the aggregate labor market. Again, we reach this conclusion only with our flow-based analysis.

6 Conclusion

This paper studies quantitative properties of a labor search and matching model with endogenous labor force participation. Our model generates realistic cyclical movements in *all* labor market transition rates and stocks. In particular, the model replicates substantial cyclical variations in transition rates into and out of the labor force, along with small and weak procyclical variations in the LFPR. We spell out the key economic channels at work through various exercises and show that the procyclical values of non-market activities and wage rigidity are both critical in achieving the successful quantitative performance. Throughout the paper, we emphasize the importance of the careful modeling of transition rates, instead of focusing on stocks, to understand the cyclicality of the labor force participation margin.

Search frictions open the door to decouple shifts in labor demand from shifts in labor force participation: Changes in the participation margin influence the composition of nonemployed individuals (between nonparticipants and unemployed) and firms pull workers from the pool of individuals available (or waiting) to work. In such an environment, small values of labor supply elasticities are consistent with the observed cyclical behavior of transition rates. The unemployment pool expands in downturns not only because the pace of job loss increases and the pace of hiring slows down, but also because the entry rate into the pool from nonparticipation increases and the exit rate to nonparticipation slows down. Therefore, transitions from and to nonparticipation make important contributions to the countercyclicality of the unemployment rate.

This paper considers a simple environment in which only a neutral labor productivity shock hits a linear technology with labor as the only input. Thus, our model lacks necessary features to address policy relevant questions discussed, for example, by Bernanke (2012) and

Yellen (2014). However, our representative household framework can easily be extended to a full-fledged DSGE model, which enables us to address those questions. This paper lays out the foundation for this next step, and we believe that the key economic channels highlighted in this paper remain crucial in the extended environment as well.

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Online Appendices - Not Intended for Publication

A.1 Additional Empirical Evidence

This section shows that our empirical results are robust with respect to various alternative data series and specifications of the VAR.

A.1.1 Data

Figure A.1 presents transition rate series used in our analysis, adjusted by margin error, between 1976 and 2016.

A.1.2 Corrections to CPS Transition Rates

The literature has proposed various adjustments to the CPS gross flow data. We show here that our main results are robust with respect to these alternative datasets. We consider a total of nine different datasets following Krusell et al. (2017), and Table A.1 presents unconditional first and second moment statistics of these datasets. Although Krusell et al.

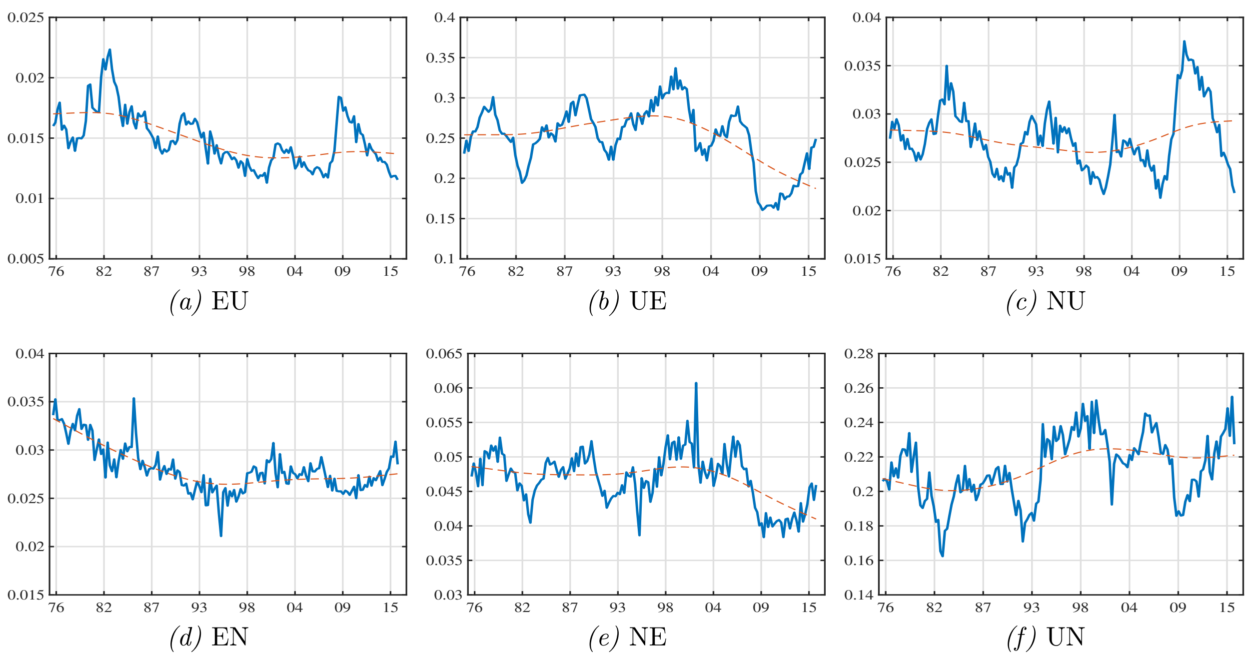


Figure A.1: Transition Rates. *Notes*: Solid blue lines are quarterly averages of monthly data. Red dashed lines are the HP-filtered trend.

Table A.1: U.S. Data, 1976-2016

Class. error adj.	Other adj.	EU	EN	UE	UN	NE	NU
		Panel A: Mean					
Unadjusted		0.015	0.029	0.253	0.224	0.045	0.026
Abowd-Zellner		0.014	0.015	0.228	0.150	0.021	0.019
DeNUNified		0.015	0.029	0.258	0.158	0.045	0.018
Unadjusted	ME	0.015	0.028	0.251	0.214	0.047	0.027
Abowd-Zellner	ME	0.014	0.014	0.226	0.138	0.022	0.020
DeNUNified	ME	0.015	0.028	0.254	0.149	0.047	0.019
Unadjusted	ME and TA	0.019	0.030	0.300	0.253	0.049	0.036
Abowd-Zellner	ME and TA	0.017	0.014	0.263	0.153	0.023	0.025
DeNUNified	ME and TA	0.019	0.029	0.304	0.168	0.049	0.024
		Panel B: Standard deviation					
Unadjusted		0.112	0.047	0.127	0.078	0.061	0.120
Abowd-Zellner		0.121	0.106	0.140	0.141	0.138	0.102
DeNUNified		0.112	0.047	0.131	0.096	0.061	0.118
Unadjusted	ME	0.107	0.051	0.127	0.074	0.066	0.117
Abowd-Zellner	ME	0.116	0.114	0.141	0.139	0.142	0.103
DeNUNified	ME	0.107	0.051	0.131	0.088	0.066	0.111
Unadjusted	ME and TA	0.087	0.052	0.146	0.084	0.068	0.098
Abowd-Zellner	ME and TA	0.098	0.116	0.159	0.150	0.144	0.096
DeNUNified	ME and TA	0.088	0.052	0.151	0.096	0.068	0.093
		Panel C: Correlation with GDP					
Unadjusted		-0.760	0.596	0.774	0.653	0.792	-0.755
Abowd-Zellner		-0.734	0.675	0.766	0.664	0.764	-0.537
DeNUNified		-0.760	0.596	0.772	0.595	0.791	-0.728
Unadjusted	ME	-0.758	0.545	0.769	0.650	0.762	-0.732
Abowd-Zellner	ME	-0.717	0.626	0.755	0.661	0.738	-0.487
DeNUNified	ME	-0.749	0.544	0.765	0.633	0.755	-0.718
Unadjusted	ME and TA	-0.706	0.542	0.768	0.650	0.758	-0.676
Abowd-Zellner	ME and TA	-0.655	0.628	0.756	0.662	0.735	-0.317
DeNUNified	ME and TA	-0.701	0.548	0.764	0.635	0.752	-0.651

Notes: The volatility and correlation is computed using all series logged and HP-filtered with a smoothing parameter equal to 10^5. ME stands for margin error correction, and TA for time-aggregation correction.

(2017) look at similar statistics, our filtering method and the sample period are different from theirs, and thus we present the results for completeness. As pointed out by Krusell et al. (2017), the adjusted flows using the Abowd and Zellner (1985) misclassification correction are systematically below their unadjusted counterparts. Also, the deNUNified correction

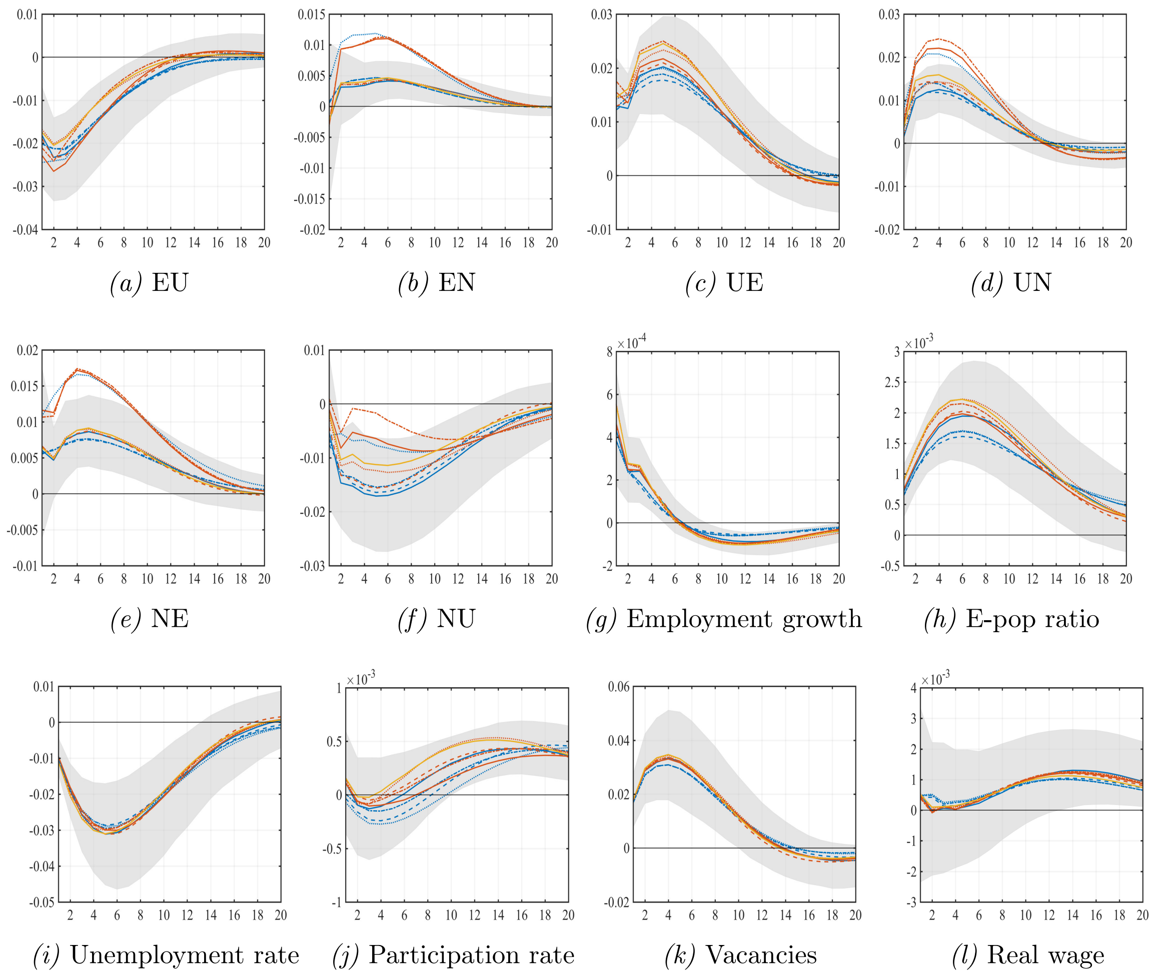


Figure A.2: Results with Alternative Datasets. *Notes*: Shaded areas are error bands for our baseline results (16th and 84th percentiles of the posterior distribution). Solid blue: baseline results; Blue dashed: unadjusted; Blue dotted: adjusted for MC (AZ flows); Blue dash-dotted: adjusted for MC (DeNUNified flows); Red solid: adjusted for MC+ME (AZ flows); Red dashed: adjusted for MC+ME (DeNUNified flows); Red dotted: adjusted for ME+TA; Red dash-dotted: adjusted for MC+ME+TA (AZ flows); Yellow solid: adjusted for MC+ME+TA (DeNUNified flows). MC stands for misclassification error correction, ME for margin error correction and TA for time-aggregation correction.

proposed by Elsby et al. (2015) has a similar effect for the average transition rates between unemployment and nonparticipation. Importantly, the cyclicality of flows are similar for the unadjusted and adjusted flows, except perhaps for the NU rate in that its cyclicality falls in the adjusted data.

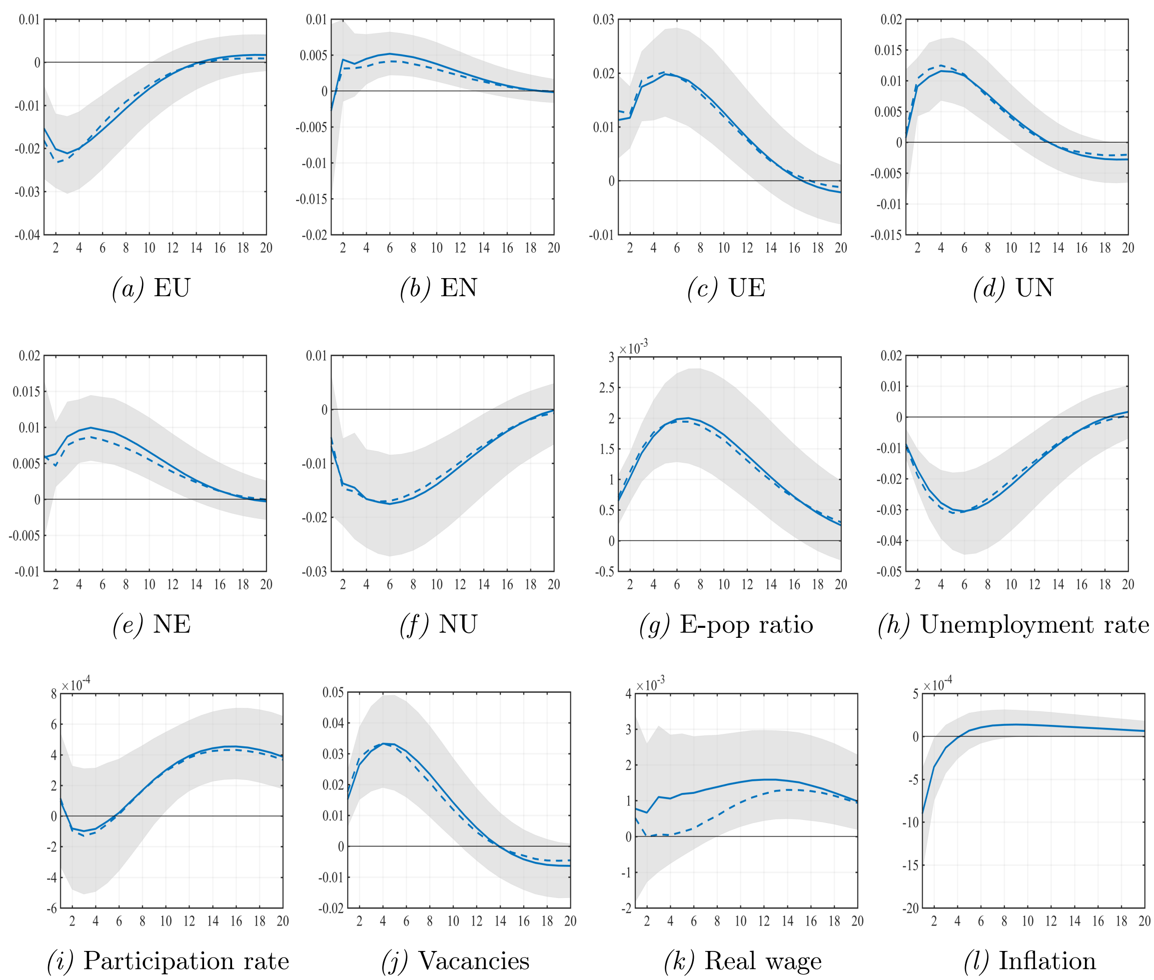


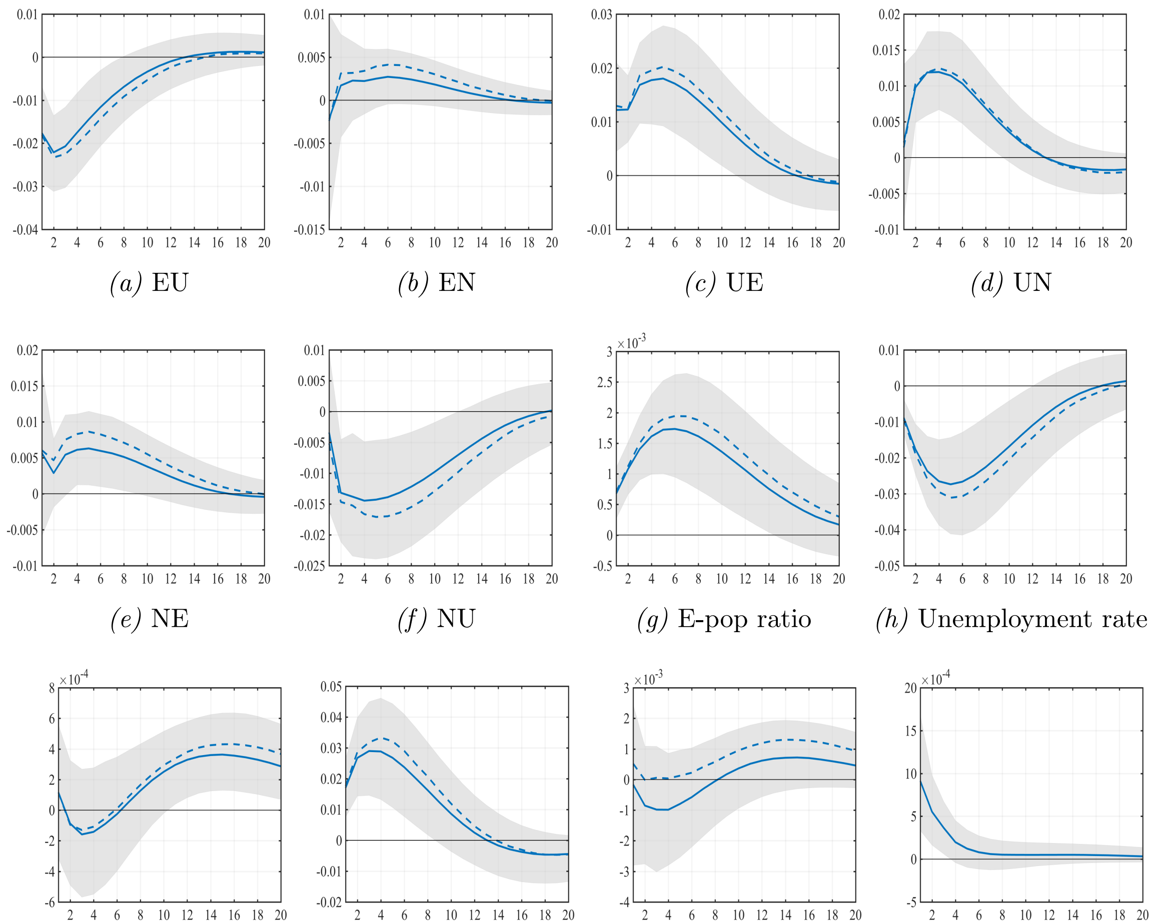
Figure A.3: Responses to a Technology Shock. *Notes*: Shaded areas are error bands for the baseline model. Except for inflation, impulse responses are expressed as log deviations. Inflation responses are expressed as level differences in the quarterly rates.

A.1.3 Robustness of the VAR

We show in this section that our results are robust with respect to (i) alternative datasets, (ii) structural shocks, and (iii) alternative detrending methods.

Figure A.2 presents the results of re-estimating the VAR using the various datasets. Overall, our results are fairly robust with respect to the different adjustment procedures.

In the main body of the paper, our VAR considers a generic shock, which we call "aggregate shock," by claiming that labor market responses with respect to more fundamental shocks are similar. We demonstrate that similarity in this section by estimating two alternative VAR models.



(a) EU *(b)* EN *(c)* UE *(d)* UN

(e) NE *(f)* NU *(g)* E-pop ratio *(h)* Unemployment rate

(i) Participation rate *(j)* Vacancies *(k)* Real wage *(l)* Inflation

Figure A.4: Responses to a Demand Shock. *Notes*: Shaded areas are error bands for the baseline model. Except for inflation, impulse responses are expressed as log deviations. Inflation responses are expressed as level differences in the quarterly rates.

First, we estimate a VAR that includes the same eight variables as before (six transition rates, vacancies, and wages) plus the headline PCE inflation rate. The PCE inflation rate is computed by taking log differences of price levels between the two adjacent quarters and thus should be understood as a quarterly rate. All series are seasonally adjusted, logged, and HP-filtered with a smoothing parameter of 10^5, and we set the lag length of the VAR at two quarters.

Based on this VAR, we identify two shocks that one can call "demand shock"' and "technology or supply shock." We impose exactly the same sign restrictions on the responses of EU and UE transition rates, vacancies, and employment growth for both the demand- and supply-side shocks. We distinguish between demand and supply shocks based on the pattern in the inflation response. In response to a positive demand shock, the price level increases for the first two quarters. Note that confining the responses of the price level rather than the inflation rate is less restrictive in the sense that the price-level restriction allows for the inflation rate to fall at any point after the first period, as long as the price level restriction is satisfied in the following periods. This restriction together with the previous restrictions on labor market variables imply a short-run "Phillips-curve-like" relationship, i.e., negative relationship between inflation and the unemployment rate. The unemployment rate tends to fall in response to a positive demand shock in our identification because of the higher UE transition rate and lower EU transition rate. Next, we assume that a positive supply shock leads to a fall in the price level for the two quarters after the shock. Note that this supply innovation can include both technology and cost-push shocks. Results are available upon request. The identification in this case follows the same procedure as in Fujita (2011).

Figures A.3 and A.4 present the responses to positive supply and demand shocks, respectively. Solid lines represent the median responses and shaded areas represent the 16th and 84th percentiles of the posterior distributions. Dashed lines are the median responses in our baseline specification presented in Figure 2. The overall pattern of responses of labor market variables is remarkably similar to our main results except for the fact that the demand shock implies the Phillips-curve relationship.

Next, we estimate a VAR model that includes the same variables as in our baseline specification and labor productivity as an exogenous variable. All series are seasonally adjusted, logged, and HP-filtered with a smoothing parameter of 10^5. We set the number of lags based on the AIC information criteria. Then, we shock this system with a one standard deviation labor productivity shock. Figure A.5 shows that the estimated IRFs in this case are similar to the baseline IRFs.

In our baseline specification, all variables were detrended using the HP filter with a smoothing parameter equal to 10^5. We show here the results based on different detrending methods. Figure A.6 plots the median responses that would result after detrending the data using (i) the HP filter with a smoothing parameter equal to 1,600 (dashed lines), (ii) a linear trend (dotted lines), and (iii) a cubic trend (dash-dotted lines). Solid lines give the median responses, and shaded areas are error bands from our baseline specification.

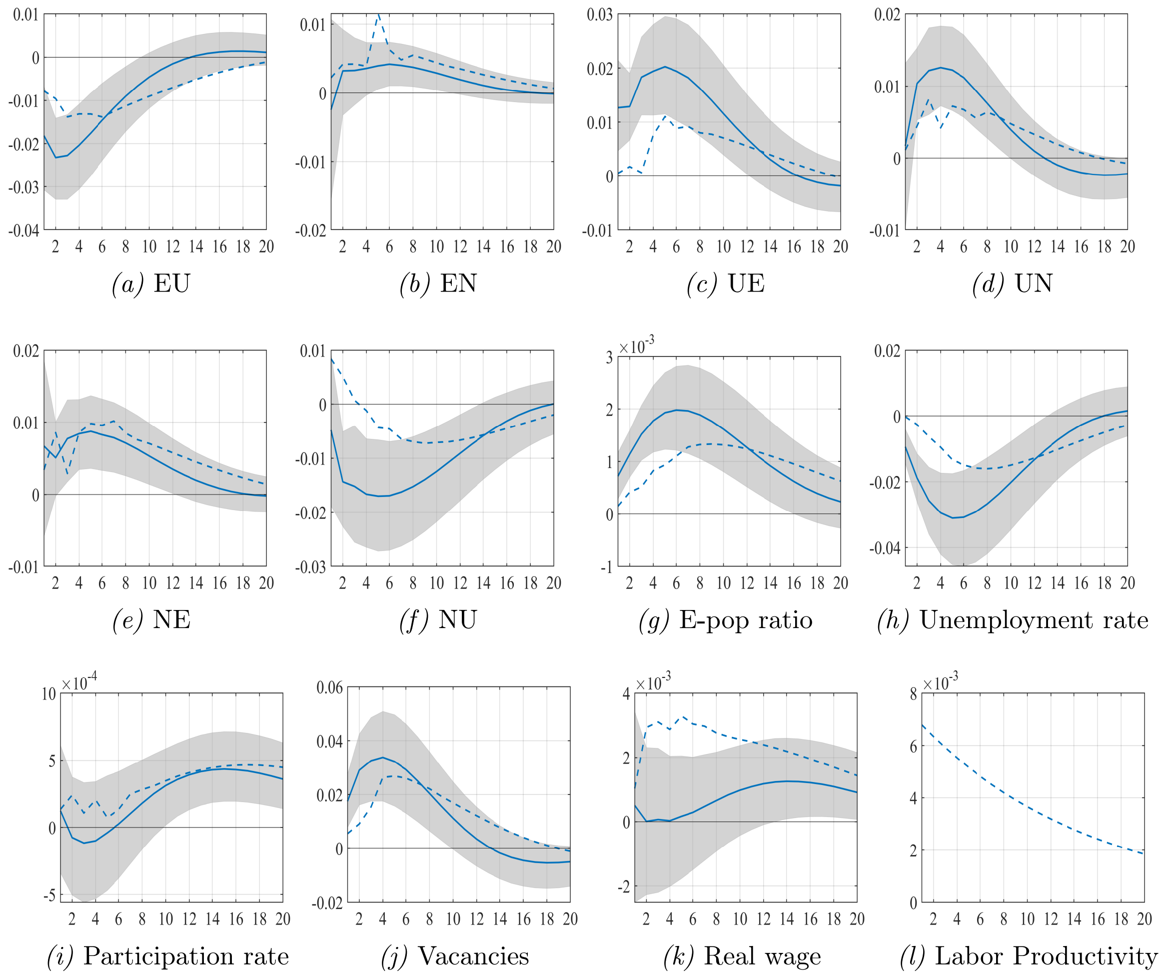
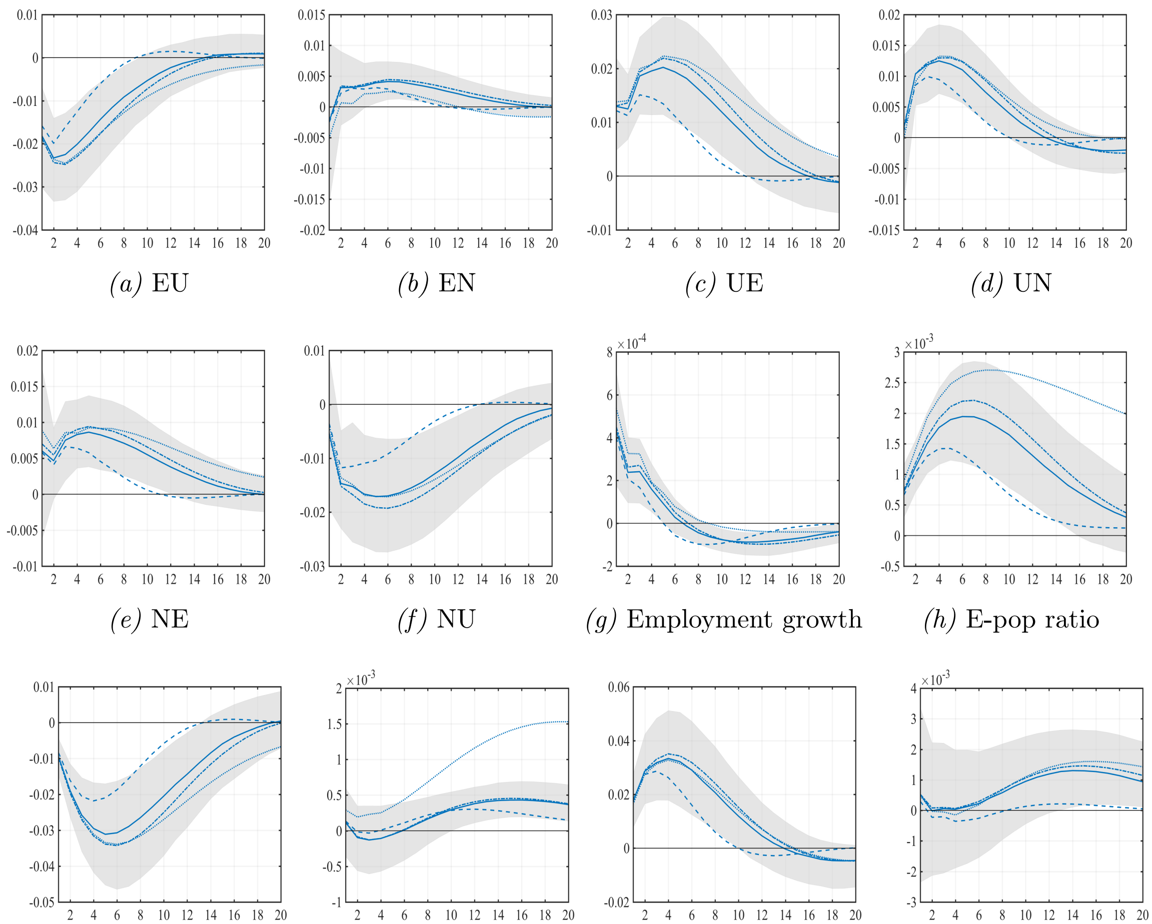


Figure A.5: Responses to a Labor Productivity Shock. *Notes*: Shaded areas are error bands for the baseline model.



(a) EU *(b)* EN *(c)* UE *(d)* UN

(e) NE *(f)* NU *(g)* Employment growth *(h)* E-pop ratio

(i) Unemployment rate *(j)* Participation rate *(k)* Vacancies *(l)* Real wage

Figure A.6: Results with Alternative Detrending Methods. *Notes*: Solid lines and shaded areas represent the median responses and error bands of our baseline VAR. Dashed lines: HP-filtered with smoothing parameter of 1,600; Dotted lines: linear trend; Dash-dotted lines: cubic trend.

A.2 Model Derivations

The household problem can be formulated as follows:

$$V(\mathbf{\Omega}_t) = \max_{\{C_{mt}, A_{t+1}, E_t, U_t, N_t, h_t^*, h_t^m\}} \frac{C_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \omega \frac{L_t^{1-\frac{1}{\nu}}}{1-\frac{1}{\nu}} + \beta \mathbb{E}_t V(\mathbf{\Omega}_{t+1}).$$

This problem is subject to the laws of motion for labor market stocks and the budget constraint:

$$\begin{bmatrix} E_t \\ U_t \\ N_t \end{bmatrix} = \Gamma_t' \begin{bmatrix} E_{t-1} \\ U_{t-1} \\ N_{t-1} \end{bmatrix}, \tag{A.1}$$

$$A_{t+1} + C_{mt} = w_t E_t + bU_t + (1 + r_t)A_t + \Pi_t - T_t. \tag{A.2}$$

The transition matrix Γ_t' is summarized in Table 1. The optimization problem also makes use of $L_t = N_t + \bar{N}$, $C_t = \left(\gamma C_{mt}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) C_{ht}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$, and $C_{ht} = \tau \check{h}_t U_t + \hat{h}_t N_t + \bar{h}\bar{N}$ together with the following conditional means of h_i:

$$\check{h}_t = \frac{\int_0^{h_t^*} h_i d\Phi(h_i)}{\Phi(h_t^*)}, \tag{A.3}$$

$$\begin{aligned} \hat{h}_t =& \frac{(1-\mu f_t)(\Phi(h_t^m) - \Phi(h_t^*))}{(1-\mu f_t)(\Phi(h_t^m) - \Phi(h_t^*)) + 1 - \Phi(h_t^m)} \left[\frac{\int_{h_t^*}^{h_t^m} h_i d\Phi(h_i)}{\Phi(h_t^m) - \Phi(h_t^*)}\right] \\ &+ \frac{1 - \Phi(h_t^m)}{(1-\mu f_t)(\Phi(h_t^m) - \Phi(h_t^*)) + 1 - \Phi(h_t^m)} \left[\frac{\int_{h_t^m}^{\infty} h_i d\Phi(h_i)}{1 - \Phi(h_t^m)}\right]. \end{aligned} \tag{A.4}$$

In writing Equations (A.3) and (A.4), we incorporate the threshold rule that those with $h_i \leq h_t^*$ enter U_t, those with $h_t^* < h_i \leq h_t^m$ enter N_t but are willing to accept job offers, and those with $h_i > h_t^m$ enter N_t but reject job offers. $\check{h}_t$ represents the conditional mean of h_i below the participation margin h_t^*, while $\hat{h}_t$ represents the conditional mean of h_i above the participation margin. The latter incorporates the possibility of job rejection for $h_i > h_t^m$, and thus the conditional means for the two intervals are weighted appropriately.

The FONCs for market-goods consumption and wealth yield the usual Euler equation:

$$\Lambda_t^{C_m} = \beta \mathbb{E}_t \Big[\Lambda_{t+1}^{C_m} (1 + r_{t+1})\Big],$$

where $\Lambda_t^{C_m} \equiv \frac{\partial U(C_t, L_t)}{\partial C_{mt}}$ is the marginal utility of market-goods consumption. The FONCs for the remaining choice variables are given by:

$$E_t : 0 = \Lambda_t^{C_m} w_t + \beta \mathbb{E}_t \frac{\partial V(\mathbf{\Omega}_{t+1})}{\partial E_t} - \Lambda_t^E, \tag{A.5}$$

$$U_t : 0 = \Lambda_t^{C_m} b + C_t^{-1/\sigma} \frac{\partial C_t}{\partial C_{ht}} \tau \check{h}_t + \beta \mathbb{E}_t \frac{\partial V(\mathbf{\Omega}_{t+1})}{\partial U_t} - \Lambda_t^U, \tag{A.6}$$

$$N_t : 0 = C_t^{-1/\sigma} \frac{\partial C_t}{\partial C_{ht}} \hat{h}_t + \omega L^{-\frac{1}{\nu}} + \beta \mathbb{E}_t \frac{\partial V(\mathbf{\Omega}_{t+1})}{\partial N_t} - \Lambda_t^N, \tag{A.7}$$

$$h_t^* : 0 = C_t^{-1/\sigma} \frac{\partial C_t}{\partial C_{ht}} \frac{\partial C_{ht}}{\partial h_t^*} + \Lambda_t^E \frac{\partial E_t}{\partial h_t^*} + \Lambda_t^U \frac{\partial U_t}{\partial h_t^*} + \Lambda_t^N \frac{\partial N_t}{\partial h_t^*}, \tag{A.8}$$

$$h_t^m : 0 = C_t^{-1/\sigma} \frac{\partial C_t}{\partial C_{ht}} \frac{\partial C_{ht}}{\partial h_t^m} + \Lambda_t^E \frac{\partial E_t}{\partial h_t^m} + \Lambda_t^N \frac{\partial N_t}{\partial h_t^m}, \tag{A.9}$$

where Λ_t^E, Λ_t^U, and Λ_t^N are the Lagrange multipliers associated with the constraints on E_t, U_t, and N_t, i.e., following the expression (A.1), respectively. Note that we can write the marginal value functions with respect to the predetermined labor market stocks as:

$$\begin{bmatrix} \frac{\partial V(\mathbf{\Omega}_t)}{\partial E_{t-1}} \\ \frac{\partial V(\mathbf{\Omega}_t)}{\partial U_{t-1}} \\ \frac{\partial V(\mathbf{\Omega}_t)}{\partial N_{t-1}} \end{bmatrix} = \Gamma_t' \begin{bmatrix} \Lambda_t^E \\ \Lambda_t^U \\ \Lambda_t^N \end{bmatrix}.$$

Using these three equations to substitute out $\frac{\partial V(\mathbf{\Omega}_{t+1})}{\partial E_t}$, $\frac{\partial V(\mathbf{\Omega}_{t+1})}{\partial U_t}$, and $\frac{\partial V(\mathbf{\Omega}_{t+1})}{\partial N_t}$ in Equations (A.5), (A.6), and (A.7), respectively, and dividing them by $\Lambda_t^{C_m}$, we obtain:

$$\begin{bmatrix} \mathcal{V}_t^E \\ \mathcal{V}_t^U \\ \mathcal{V}_t^N \end{bmatrix} = \begin{bmatrix} w_t \\ b + z_{ht} \tau \check{h}_t \\ z_{lt} + z_{ht} \hat{h}_t \end{bmatrix} + \mathbb{E}_t \tilde{\beta}_{t+1} \Gamma_{t+1}' \begin{bmatrix} \mathcal{V}_{t+1}^E \\ \mathcal{V}_{t+1}^U \\ \mathcal{V}_{t+1}^N \end{bmatrix}, \tag{A.10}$$

where $\mathcal{V}_t^i = \frac{\Lambda_t^i}{\Lambda_t^{C_m}}$ with $i \in \{E, U, N\}$.

Next, the participation condition (A.8) can be rewritten as:

$$z_{ht} \frac{\partial C_{ht}}{\partial h_t^*} + \mathcal{V}_t^E \frac{\partial E_t}{\partial h_t^*} + \mathcal{V}_t^U \frac{\partial U_t}{\partial h_t^*} + \mathcal{V}_t^N \frac{\partial N_t}{\partial h_t^*} = 0. \tag{A.11}$$

Note that given the laws of motion for labor market stocks, the partial derivatives of the three stocks with respect to h_t^* can be written as:

$$\frac{\partial E_t}{\partial h_t^*} = \Phi'(h_t^*)(1-\mu) f_t R_{t-1}, \quad \frac{\partial U_t}{\partial h_t^*} = \Phi'(h_t^*)(1-f_t) R_{t-1}, \quad \frac{\partial N_t}{\partial h_t^*} = -\Phi'(h_t^*)(1-\mu f_t) R_{t-1}. \tag{A.12}$$

Recall $R_{t-1} \equiv sE_{t-1} + (1-\lambda)U_{t-1} + N_{t-1}$, which represents the mass of workers who draws new h_i at the beginning of period t. Using (A.12), we can write (A.11) as:

$$-z_{ht}\frac{\frac{\partial C_{ht}}{\partial h_t^*}}{\frac{\partial N_t}{\partial h^*}} + \frac{(1-\mu)f_t}{1-\mu f_t}\mathcal{V}_t^E + \frac{1-f_t}{1-\mu f_t}\mathcal{V}_t^U = \mathcal{V}_t^N. \tag{A.13}$$

Note that the "weight" terms in front of $\mathcal{V}_t^E$ and $\mathcal{V}_t^U$ sum to one and reflect the fact that $\frac{\partial E_t}{\partial h_t^*} + \frac{\partial U_t}{\partial h_t^*} = -\frac{\partial N_t}{\partial h_t^*}$: a change in N_t has to be absorbed by either E_t or U_t with these proportions. Regarding the first term on the left-hand side, $\frac{\partial N_t}{\partial h^*} < 0$ as shown in (A.12) and $\frac{\partial C_{ht}}{\partial h_t^*} > 0$ as in:

$$\begin{aligned}\frac{\partial C_{ht}}{\partial h_t^*} &= \tau U_t \frac{\partial \check{h}_t}{\partial h_t^*} + \frac{\partial \hat{h}_t}{\partial h_t^*}N_t, \\ &= \frac{\Phi'(h_t^*)(h_t^* - \check{h}_t)}{\Phi(h_t^*)}\tau U_t + \frac{(1-\mu f_t)\Phi'(h_t^*)(\hat{h}_t - h_t^*)}{(1-\mu f_t)(\Phi(h_t^m) - \Phi(h_t^*)) + 1 - \Phi(h_t^m)}N_t. \end{aligned} \tag{A.14}$$

This partial derivative captures the effect that by raising h_t^*, the conditional means of home production productivities $\hat{h}_t$ and $\check{h}_t$ increase. Using Equations (A.14) in (A.13) results in the following participation condition:

$$z_{ht}(\hat{h}_t - h_t^*) + \tau z_{ht}(h_t^* - \check{h}_t)\frac{1-f_t}{1-\mu f_t}\left[1 + \frac{\lambda U_{t-1}}{\Phi(h_t^*)R_{t-1}}\right] + \frac{(1-\mu)f_t}{1-\mu f_t}\mathcal{V}_t^E + \frac{1-f_t}{1-\mu f_t}\mathcal{V}_t^U = \mathcal{V}_t^N. \tag{A.15}$$

This marginal condition for h_t^* equates returns (or costs) to participation and non-participation. When either $\lambda = 0$ (no persistence in the unemployment state) or $\tau = 0$ (no contribution of the unemployed to home production), the condition can be rewritten more intuitively. First, assuming $\lambda = 0$ (while keeping $\tau > 0$), and using the "tilde" notation for the marginal value functions (as in Equation (5)), the above condition reduces to:

$$\frac{(1-\mu)f_t}{1-\mu f_t}(w_t + \tilde{\mathcal{V}}_t^E) + \frac{1-f_t}{1-\mu f_t}(b + \tau z_{ht}h_t^* + \tilde{\mathcal{V}}_t^U) = z_{lt} + h_t^* z_{ht} + \tilde{\mathcal{V}}_t^N. \tag{A.16}$$

The flow values of nonparticipation at the margin consist of $z_{lt} + h^* z_{ht}$, while those of participation comes from w_t and $b + \tau z_{ht}h_t^*$, weighted appropriately based on labor market transition laws. When $\tau = 0$, regardless of the value of λ, $\tau z_{ht}h_t^*$ simply drops out from the flow value of unemployment. As discussed in Section 3.6, this condition demonstrates that we need procyclical values of non-market activities, z_{lt} and z_{ht}, in order to achieve a countercyclical participation margin h_t^* and that rigid wage is helpful in achieving it.

Lastly, the acceptance condition (A.9) can be written as:

$$z_{ht}\frac{\partial C_{ht}}{\partial h_t^m} + \mathcal{V}_t^E \frac{\partial E_t}{\partial h_t^m} + \mathcal{V}_t^N \frac{\partial N_t}{\partial h_t^m} = 0. \tag{A.17}$$

The partial derivatives can be written as:

$$\frac{\partial C_{ht}}{\partial h_t^m} = (\hat{h}_t - h_t^m)\mu f_t \Phi'(h_t^m) R_{t-1}, \quad \frac{\partial E_t}{\partial h_t^*} = \Phi'(h_t^m)\mu f_t R_{t-1}, \quad \frac{\partial N_t}{\partial h_t^*} = -\Phi'(h_t^m)\mu f_t R_{t-1}.$$

Using these expressions, the acceptance condition (A.17) reduces to:

$$w_t + \tilde{\mathcal{V}}_t^E = z_{lt} + \hat{h}_t z_{ht} + \tilde{\mathcal{V}}_t^N. \tag{A.18}$$

This condition characterizes h_t^m as the indifference value of home production between accepting or rejecting the job offer.

To confirm that a worker that is permanently out of the labor force ($\bar{N}$) is not willing to join the labor force, note that we can write the value for such a worker $\mathcal{V}^{\bar{N}}$ as:

$$\mathcal{V}_t^{\bar{N}} = z_{lt} + z_{ht}\bar{h} + \mathbb{E}_t \tilde{\beta}_{t+1} \mathcal{V}_{t+1}^{\bar{N}}.$$

Notice that $\mathcal{V}_t^{\bar{N}} \geq \mathcal{V}_t^E$ as long as the following condition is satisfied:

$$\bar{h} \geq \frac{1}{z_{ht}} \left(\mathcal{V}_t^E - \mathbb{E}_t \tilde{\beta}_{t+1} \mathcal{V}_{t+1}^{\bar{N}} - z_{lt} \right).$$

The steady-state version of this is written as:

$$\bar{h} \geq \frac{1}{z_h} \left((1-\beta)\mathcal{V}^E - z_l \right). \tag{A.19}$$

Define also the minimum value for $\bar{h}$ such that, in the steady state, a worker is indifferent between being permanently out of the labor force and employed:

$$\bar{h}^{min} = \frac{1}{z_h} \left((1-\beta)\mathcal{V}^E - z_l \right).$$

In our calibration, condition (A.19) is always satisfied and, in the steady state, $\bar{h}^{min}$ corresponds to the 19th percentile of the h_i distribution, well below the 99th percentile to which it is fixed.

A.3 Estimation Procedure

We estimate the 13 parameters listed in panel C of Table 2 by solving a constrained minimization problem. This problem is constrained in that we impose that the steady-state levels of transition rates do not deviate from their historical averages by more than 30 percent. The objective function is the weighted distance between the median IRFs from our VAR and the model IRFs. We use six transition rates, vacancies, real wage, and the LFPR as our observables and weight their IRFs by their unconditional variances in evaluating the fit.

Of the 13 parameters, there are only eight parameters that can be freely estimated because of the steady-state equilibrium restrictions between them. For tractability purposes, we search for the best parameter values for $\{s,\ \delta_w,\ \epsilon_v,\ \bar{\omega}\}$ and the steady-state values for the unemployment rate (u), job-finding rate (f), UN transition rate (UNr), and N. We take this approach because it reduces the number of nonlinear conditions to be solved in steady state. The degree of wage rigidity (δ_w) does not appear in the steady state and thus is determined only from the dynamics of the model.

We compute the steady state of the model as follows. For given parameter vales for $\{\beta,\ s,\ \eta,\ \sigma_h,\ \tau\}$ and the steady-state moments for $\{z,\ f,\ g,\ \bar{\omega},\ E,\ U,\ N,\ UNr\}$, we can recover parameter values for $\{\lambda,\ \mu,\ \bar{h},\ b\}$ and steady-state values for $\{h^*,\ h^m,\ w,\ \mathcal{V}^E,\ \mathcal{V}^U,\ \mathcal{V}^N,\ \mathcal{V}^a,\ \mathcal{V}^J,\ \hat{h},\ \check{h}\}$ from the following nonlinear steady-state conditions:

$$
\begin{aligned}
UNr &= (1-\lambda)\left[(\Phi(h^m)-\Phi(h^*))(1-\mu f)+1-\Phi(h^m)\right], \\
EU+EN &= UE+NE, \\
UE+UN &= EU+NU, \\
\begin{bmatrix} \mathcal{V}^E \\ \mathcal{V}^U \\ \mathcal{V}^N \end{bmatrix} &= \begin{bmatrix} w \\ b+z_h\tau\check{h} \\ z_l+z_h\hat{h} \end{bmatrix} + \beta\Gamma' \begin{bmatrix} \mathcal{V}^E \\ \mathcal{V}^U \\ \mathcal{V}^N \end{bmatrix}, \\
\mathcal{V}^J &= y-w+(1-s)\beta\mathcal{V}^J, \\
\mathcal{V}^a &= \Phi(h^*)\mathcal{V}^U+(1-\Phi(h^*))\mathcal{V}^N, \\
g &= \Phi(h^*)\left(b+\tau z_h\check{h}\right)+(1-\Phi(h^*))(1-\bar{\omega})z_h\hat{h}, \\
\eta\mathcal{V}^J &= (1-\eta)\mathcal{V}^a, \\
h^m &= \hat{h}+\frac{\mathcal{V}^E-\mathcal{V}^N}{z_h}, \\
\frac{\partial C_h}{\partial h^*} &= \frac{\Phi'(h^*)\left(h^*-\check{h}\right)}{\Phi(h^*)}\tau U+\frac{(1-\mu f)\Phi'(h^*)(\hat{h}-h^*)}{(1-\mu f)(\Phi(h^m)-\Phi(h^*))+1-\Phi(h^m)}N, \\
R &= sE+(1-\lambda)U+N,
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial N}{\partial h^*} &= -\Phi'(h^*)(1-\mu f)R, \\
\mathcal{V}^N &= -z_h \frac{\frac{\partial C_h}{\partial h^*}}{\frac{\partial N}{\partial h^*}} + \frac{(1-\mu)f}{1-\mu f}\mathcal{V}^E + \frac{1-f}{1-\mu f}\mathcal{V}^U, \\
\hat{h} =& \frac{(1-\mu f)(\Phi(h^m)-\Phi(h^*))}{(1-\mu f)(\Phi(h^m)-\Phi(h^*))+1-\Phi(h^m)} \frac{\int_{h^*}^{h^m} h_i d\Phi(h_i)}{\Phi(h^m)-\Phi(h^*)} \\
&+ \frac{1-\Phi(h^m)}{(1-\mu f)(\Phi(h^m)-\Phi(h^*))+1-\Phi(h^m)} \frac{\int_{h^m}^{\infty} h_i d\Phi(h_i)}{1-\Phi(h^m)}.
\end{aligned}
$$

We can then recover the parameter values for $\{\kappa, \gamma, \bar{N}, \bar{m}, \omega\}$ and sequentially compute the steady-state values for the following variables:

$$
\begin{aligned}
R &= sE + (1-\lambda)U + N, \\
S &= \lambda U + \left[\Phi(h^*) + \mu(\Phi(h^m)-\Phi(h^*))\right] R, \\
\theta &= \frac{f}{q}, \\
\bar{m} &= \frac{f}{\theta^{\epsilon^v}}, \\
\bar{N} &= 1 - E - U - N, \\
L &= N + \bar{N}, \\
Y &= E, \\
\kappa &= \frac{\mathcal{V}^J}{(qv)^{\epsilon^v}}, \\
C_m &= Y - \kappa \frac{(qv)^{\epsilon^v}}{1+\epsilon^v}, \\
C_h &= \tau \check{h} U + \hat{h} N + \bar{h} \bar{N}, \\
\gamma &= \left[z_h \left(\frac{C_h}{C_m} \right)^{\frac{1}{\varepsilon}} + 1 \right]^{-1}, \\
C &= \left[\gamma C_m^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) C_h^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \\
\omega &= \bar{\omega} z_h \hat{h} L^{\frac{1}{\nu}} C^{\frac{1}{\sigma}} \gamma \left(\frac{C}{C_m} \right)^{\frac{1}{\varepsilon}}, \\
\Lambda^{C_m} &= C^{-\frac{1}{\sigma}}, \\
z_l &= \frac{\omega L^{\frac{1}{\nu}}}{\gamma C^{-\frac{1}{\sigma}} \left(\frac{C}{C_m} \right)^{\frac{1}{\varepsilon}}}.
\end{aligned}
$$

A.4 List of Model Equations

Table A.2: Steady-State Equations

$\begin{bmatrix} E \\ U \\ N \end{bmatrix} = \Gamma' \begin{bmatrix} E \\ U \\ N \end{bmatrix}$	(E, U)
$N = 1 - \bar{N} - E - U$	(N)
$\begin{bmatrix} \mathcal{V}^E \\ \mathcal{V}^U \\ \mathcal{V}^N \end{bmatrix} = (I - \beta\Gamma')^{-1} \begin{bmatrix} w \\ b + z_h \tau \check{h} \\ z_h \hat{h}(1+\bar{\omega}) \end{bmatrix}$	$(\mathcal{V}^E, \mathcal{V}^U, \mathcal{V}^N)$
$\mathcal{V}^J = \frac{1-w}{(1-s)\beta}$	$(\mathcal{V}^J)$
$\kappa(qv)^{\epsilon_v} = \mathcal{V}^J$	(κ)
$\frac{\partial C_h}{\partial h^*} = \frac{\Phi'(h^*)\left(h^* - \check{h}\right)}{\Phi(h^*)}\tau U + \frac{(1-\mu f)\Phi'(h^*)\left(\hat{h} - h^*\right)}{(1-\mu f)(\Phi(h^m) - \Phi(h^*)) + 1 - \Phi(h^m)} N$	$(\partial C_h / \partial h^*)$
$R = sE + (1-\lambda)U + N$	(R)
$\frac{\partial N}{\partial h^*} = -\Phi'(h^*)(1-\mu f)R$	$(\partial N / \partial h^*)$
$\mathcal{V}^N = -z_h \frac{\frac{\partial C_h}{\partial h^*}}{\frac{\partial N}{\partial h^*}} \times \frac{(1-\mu)f}{1-\mu f}\mathcal{V}^E + \frac{1-f}{1-\mu f}\mathcal{V}^U$	(h^*)
$h^m = \hat{h} + \frac{1}{z_h}\left(\mathcal{V}^E - \mathcal{V}^N\right)$	(h^m)
$\eta \mathcal{V}^J = (1-\eta)\left[\mathcal{V}^E - (\Phi(h^*)\mathcal{V}^U + (1-\Phi(h^*))\mathcal{V}^N)\right]$	(w)
$C_m = E - \frac{\kappa(qv)^{1+\epsilon_v}}{1+\epsilon_v}$	(C_m)
$C_h = \tau \check{h} U + \hat{h} N + \bar{h}\bar{N}$	(C_h)
$C = \left(\gamma C_m^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) C_h^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$	(C)
$z_h = \frac{1-\gamma}{\gamma}\left(\frac{C_m}{C_h}\right)^{1/\epsilon}$	(γ)
$\hat{h} z_h \bar{\omega} = \frac{\omega(N+\bar{N})^{-1/\nu}}{C^{-1/\sigma}\gamma\left(\frac{C}{C_m}\right)^{1/\epsilon}}$	(ω)
$h^* = \Phi^{-1}(h^*)$	(μ_h)
$\check{h} = \frac{\int_0^{h^*} h_i d\Phi(h_i)}{\Phi(h^*)}$	$(\check{h})$
$\hat{h} = \frac{(1-\mu f)(\Phi(h^m) - \Phi(h^*))}{(1-\mu f)(\Phi(h^m) - \Phi(h^*)) + 1 - \Phi(h^m)} \frac{\int_{h^*}^{h^m} h_i d\Phi(h_i)}{\Phi(h^m) - \Phi(h^*)} + \frac{1 - \Phi(h^m)}{(1-\mu f)(\Phi(h^m) - \Phi(h^*)) + 1 - \Phi(h^m)} \frac{\int_{h^m}^{\infty} h_i d\Phi(h_i)}{1 - \Phi(h^m)}$	$(\hat{h})$
$z_l = \bar{\omega}\hat{h} z_h$	(z_l)
$g = \Phi(h^*)(b + z_h \tau \check{h}) + (1 - \Phi(h^*)) z_h \hat{h}(1+\bar{\omega})$	(b)
$\theta = f/q$	(θ)
$S = (\Phi(h^*) + (\Phi(h^m) - \Phi(h^*))\mu)R + \lambda U$	(S)
$\theta = v/S$	(V)
$f = \bar{m}\theta^{1-\alpha}$	$(\bar{m})$

Table A.3: Model Equations

$\left[\mathcal{V}_t^E, \quad \mathcal{V}_t^U, \quad \mathcal{V}_t^N\right] = \left[w_t, \quad b + z_{ht}\tau\check{h}_t, \quad z_{lt} + z_{ht}\hat{h}_t\right] + \mathbb{E}_t\hat{\beta}_{t+1}\left[\mathcal{V}_{t+1}^E, \quad \mathcal{V}_{t+1}^U, \quad \mathcal{V}_{t+1}^N\right]\Gamma_{t+1}$	(Household values)
$\mathcal{V}_t^J = y_t - w_t + (1-s)\mathbb{E}_t\hat{\beta}_{t,t+1}\mathcal{V}_{t+1}^J$	(Job value)
$\kappa\left(q_t v_t\right)^{\epsilon_v} = \mathcal{V}_t^J$	(Job creation)
$\frac{\partial C_{ht}}{\partial h_t^*} = \frac{\Phi'(h_t^*)\left(h_t^* - \check{h}_t\right)}{\Phi(h_t^*)}\tau U_t + \frac{(1-\mu f_t)\Phi'(h_t^*)\left(\hat{h}_t - h_t^*\right)}{(1-\mu f_t)(\Phi(h_t^m) - \Phi(h_t^*)) + 1 - \Phi(h_t^m)}N_t$	($\partial C_{ht}/\partial h_t^*$)
$R_{t-1} = sE_{t-1} + (1-\lambda)U_{t-1} + N_{t-1}$	(Workers drawing h_i)
$\frac{\partial N_t}{\partial h_t^*} = -\Phi'(h_t^*)(1-\mu f_t)R_{t-1}$	($\partial N_t/\partial h_t^*$)
$\mathcal{V}_t^N = -z_{ht}\frac{\frac{\partial C_{ht}}{\partial h_t^*}}{\frac{\partial N_t}{\partial h_t^*}} + \frac{(1-\mu)f_t}{1-\mu f_t}\mathcal{V}_t^E + \frac{1-f_t}{1-\mu f_t}\mathcal{V}_t^U$	(Participation)
$z_{ht}\left(\hat{h}_t - h_t^m\right) + \left(\mathcal{V}_t^E - \mathcal{V}_t^N\right) = 0$	(Acceptance)
$w_t = (1-\delta_w)w_t^* + \delta_w w_{t-1}$	(Wage evolution)
$\eta\mathcal{V}_t^J = (1-\eta)\left[\mathcal{V}_t^E - (\Phi(h_t^*)\mathcal{V}_t^U + (1-\Phi(h_t^*))\mathcal{V}_t^N)\right]$	(Surplus sharing for w_t^*)
$C_{mt} = y_t E_t - \kappa\frac{(q_t v_t)^{1+\epsilon_v}}{1+\epsilon_v}$	(Resource constraint)
$C_{ht} = \tau\check{h}_t U_t + \hat{h}_t N_t + \bar{h}\bar{N}$	(Home production/consumption)
$C_t = \left(\gamma C_{mt}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)C_{ht}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$	(Aggregate consumption)
$g_t = \Phi(h_t^*)(b + z_{ht}\tau\check{h}_t) + (1-\Phi(h_t^*))(\hat{h}_t z_{ht} + z_{lt})$	(Opportunity cost of employment)
$z_{ht} = \frac{\Lambda_t^{C_h}}{\Lambda_t^{C_m}}$	(Marginal rate of substitution between C_{ht} and C_{mt})
$z_{lt} = \frac{\Lambda_t^L}{\Lambda_t^{C_m}}$	(Marginal rate of substitution between leisure and C_{mt})
$\Lambda_t^{C_h} = C_t^{-\frac{1}{\sigma}}(1-\gamma)\left(\frac{C_t}{C_{ht}}\right)^{\frac{1}{\varepsilon}}$	(Marginal utility of C_{ht})
$\Lambda_t^{C_m} = C_t^{-\frac{1}{\sigma}}\gamma\left(\frac{C_t}{C_{mt}}\right)^{\frac{1}{\varepsilon}}$	(Marginal utility of C_{mt})
$\Lambda_t^L = \omega L_t^{-\frac{1}{\nu}}$	(Marginal utility of leisure)
$\hat{h}_t = \frac{(1-\mu f_t)(\Phi(h_t^m) - \Phi(h_t^*))}{(1-\mu f_t)(\Phi(h_t^m) - \Phi(h_t^*)) + 1 - \Phi(h_t^m)}\frac{\int_{h_t^*}^{h_t^m} h_i d\Phi(h_i)}{\Phi(h_t^m) - \Phi(h_t^*)} + \frac{1 - \Phi(h_t^m)}{(1-\mu f_t)(\Phi(h_t^m) - \Phi(h_t^*)) + 1 - \Phi(h_t^m)}\frac{\int_{h_t^m}^{\infty} h_i d\Phi(h_i)}{1 - \Phi(h_t^m)}$	($\mathbb{E}(h_i \mid h_t^* < h_i < h_t^m)$)
$\check{h}_t = \frac{\int_0^{h_t^*} h_i d\Phi(h_i)}{\Phi(h_t^*)}$	($\mathbb{E}(h_i \mid h_i < h_t^*)$)
$L_t = N_t + \bar{N}$	(Aggregate leisure)
$f_t = \bar{m}\theta_t^{1-\alpha}$	(Job finding rate)
$q_t = \bar{m}\theta_t^{-\alpha}$	(Job filling rate)
$S_t = \left[\Phi(h_t^*) + (\Phi(h_t^m) - \Phi(h_t^*))\mu\right]R_{t-1} + \lambda U_{t-1}$	(Aggregate job search)
$\theta_t = v_t/S_t$	(Market tightness)
$\left[E_t, \quad U_t, \quad N_t\right] = \left[E_{t-1}, \quad U_{t-1}, \quad N_{t-1}\right]\Gamma_t$	(Labor market stock evolutions)
$E_t + U_t + N_t + \bar{N} = 1$	(Population normalization)
$\ln y_t = (1-\rho)\ln\bar{y} + \rho\ln y_{t-1} + \varepsilon_t$	(Aggregate productivity)