Bargaining Shocks and Aggregate Fluctuations: Online Appendix

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Online appendix

This appendix includes details about the data, the empirical exercises, the model, and the quantitative results.

A Capital share data

A.1 U.S. data

While the definition of the capital share of income is conceptually straightforward, its measurement is challenging. For instance, we need to allocate ambiguous sources of income such as copyright royalties, deferred compensation, or proprietors' income between labor and capital. Also, we must decide how to impute indirect taxes. Finally, to go from the gross to the net share, we need to pick depreciation rates.

We now overview different measurements of the capital income share in the U.S. economy. These alternative calculations agree among themselves regarding the behavior of capital income share over middle and business cycle frequencies (see Figure A.2). Thus, for our purposes, picking one measure or another in the U.S. case is inconsequential (Muck et al. (2015) make a similar point). On the other hand, across countries, our empirical statements depend on available data.

We construct the net capital share in the corporate business sector from BEA Table 1.14, "Gross Value Added of Domestic Corporate Business in Current Dollars and Gross Value Added of Nonfinancial Domestic Corporate Business in Current and Chained Dollars," and focus on the data on non-financial corporate businesses. We compute the net capital share as compensation of employees (mnemonic A460RC1) relative to the sum of compensation and the net operating surplus (mnemonic W326RC1). Figure A.1 plots the resulting series.

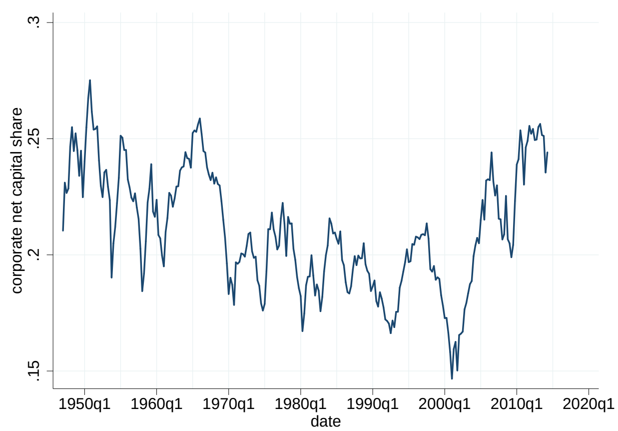


Figure A.1: Net capital share levels: Quarterly U.S. data.

We also consider some alternative measures of the U.S. capital share for comparison:

1. We compute the capital share as the reciprocal of wages over net value added (mnemonic A457RC1), effectively treating taxes as coming out of the capital share only.

2. BLS data on the (reciprocal of the) labor share in the overall business sector (mnemonic PRS84006173), the non-farm business sector (mnemonic PRS85006173), and in the corporate

non-financial sector (mnemonic PRS88003173). The BLS defines the labor share as the ratio of current labor compensation paid to current dollar output, imputing a cost for labor services by proprietors. See p. 7 of http://www.bls.gov/lpc/lpcmethods.pdf for the definition and http://www.bls.gov/data/#productivity for the data.

3. Data on the capital share as the reciprocal of the U.S. labor share in the Penn World Table (Feenstra et al., 2013).

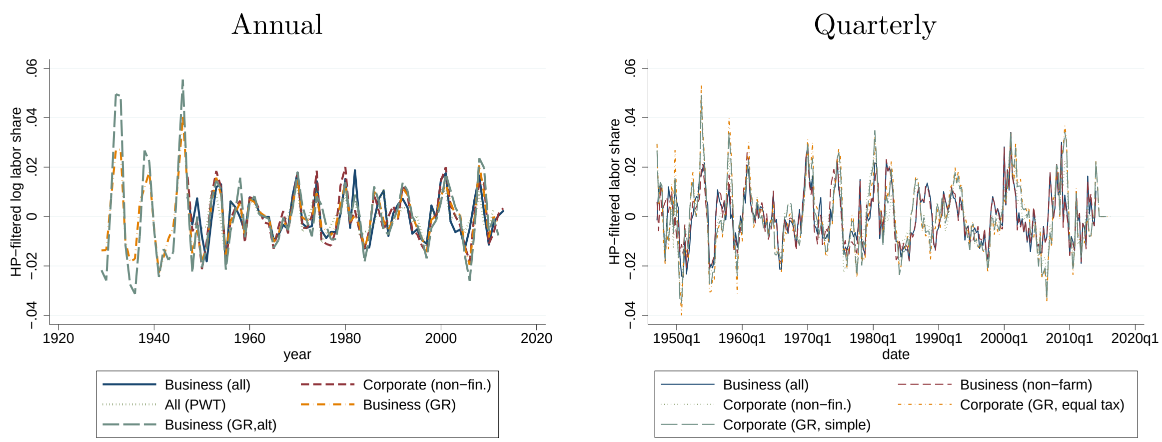


Figure A.2: Detrended labor shares in the U.S.

The different measures are reported in the two panels of Figure A.2. Figure A.3 compares the different measures of the labor share that are available in levels. The left panel shows the annual time series, and the right panel plots the shorter quarterly series. In both the annual and the quarterly data, there is no clear evidence of a trend in the labor share over the full sample period. However, most measures of the labor share are close to their minimum at the end of the sample period. In the quarterly data, adjusting for the share of taxes in corporate net value added only results in a roughly parallel shift of the labor share, whereas taking out net government production in the annual series changes the trend behavior. The different labor shares average between about 65% and 80%.[28]

Extending the comparison to include the BLS data comes at the cost of losing the level information. Figure A.4 shows that the raw data, indexed to 100 in 2009, correlates positively at higher frequencies, but may exhibit different time trends. Figure A.2, therefore, uses HP-filtered data on the log-labor share. Eyeballing both the annual and the quarterly filtered time series suggests a high agreement. Correlation tables (not shown here) confirm this impression: Raw time series sometimes exhibit low correlations, but filtered correlations are above 0.6 for annual data and above 0.7 for quarterly data except for correlations between manufacturing sectors and broader measures.

A.2 International and U.S. state-level data

- Long-run capital share data: We downloaded the data in Piketty and Zucman (2014) from http://gabriel-zucman.eu/capitalisback/ and use the net capital share ("net profit share")

[28] Giandrea and Sprague (2017) show that 2 pp. of the recent 7 pp. decline in the BLS measure of the labor share is due to the self-employed, for whom the BLS imputes capital income. We use only the corporate non-financial labor share to sidestep this issue. In the Piketty and Zucman data in the main text, we find an increase of 7 pp. from 2001 to 2010 in the net capital share and of 6 pp. in the gross capital share. In our calculations, we find an increase of 10 pp. in the net corporate labor share over this period (see Figure A.1).

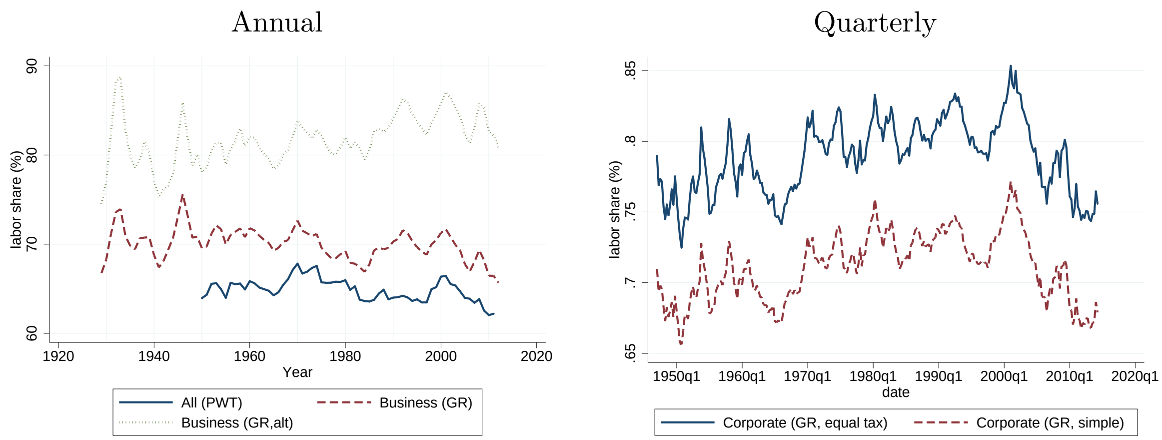


Figure A.3: Labor share levels data: Annual and quarterly.

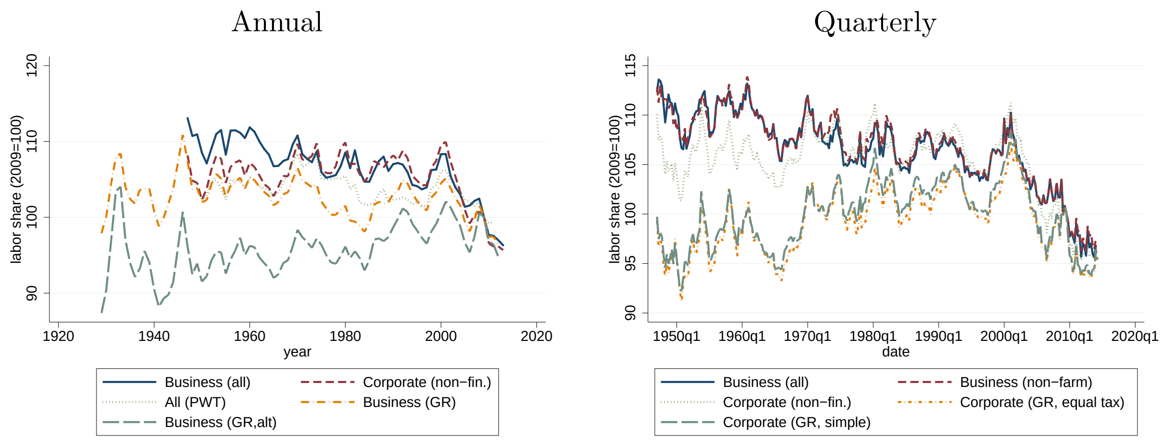


Figure A.4: Raw indexed labor shares: Annual and quarterly.

from the data sheets on "profits & wages in the corporate sector."

- OECD capital share data: We use the OECD business sector database cited in Blanchard (1997), downloaded from http://fmwww.bc.edu/ec-p/data/oecd/oecd.bsdb.html.

- ECLAC/CEPAL capital share data: We use the "CEPALSTAT Base de Datos," available at http://interwp.cepal.org/sisgen/ConsultaIntegrada.asp?idIndicador=2197&idioma=e to obtain the wage bill ("remuneración de los asalariados") and total profits ("excedente de explotación") on an annual basis in local currency. We compute the capital share as profits over the sum of profits to the wage bill, yielding the net capital share.

- U.S. state capital share and GDP data: We use the Bureau of Economic Analysis Regional Accounts section "Annual Gross Domestic Product By State" from http://www.bea.gov/regional/ to obtain data on "compensation of employees," "taxes on production and imports less subsidies" and "GDP in current dollars" to compute the gross capital share as one minus the compensation of employees over GDP minus taxes net of subsidies. All data are confined to (total) "private industry." Since the five-year periods in the states we are studying do not include 1997, when the BEA switched from SIC to NAICS, we pool the changes in GDP growth and the capital share based on either underlying classification.

- Annual GDP data: We use the data from the web appendix of Barro (2009) on real per capita GDP along with real GDP data from the Penn World Table (Feenstra et al., 2013). We detrend the data with a quadratic trend after taking logarithms.

- Stock market capitalization: We used the following (nominal) indices, downloaded from http://globalfinancialdata.com/ unless otherwise stated:
  - France: "France SBF Industrials," ticker symbol "_FISID."
  - Western Germany: "Germany CDAX Industrials Price Index," ticker symbol "_CXKNXD."
  - Spain: "Madrid SE Index (old)," ticker symbol, ESMADM."
  - Portugal: "Portugal Industrials," ticker symbol "PTINDUSM."
  - Argentina: "Buenos Aires SE General Index (IVBNG)," ticker symbol, "_IBGD."
  - Chile (financials): "Chile BEC Finance Index," ticker symbol "_FINANCD."
  - Chile (industrials): "Chile BEC Industrials Index (w/GFD extension)," ticker symbol "_INDUSTD."

- Price indices: We use consumer price indices to deflate the stock market indices. Except for Argentina and Chile, we downloaded the data from http://research.stlouisfed.org/fred2/:
  - France: Ticker symbol "FRACPIALLMINMEI."
  - Western Germany: Ticker symbol "DEUCPIALLMINMEI."
  - Spain: Ticker symbol "ESPCPIALLMINMEI."
  - Portugal: Ticker symbol "PRTCPIALLMINMEI."
  - Argentina: Global Financial Database "Argentina Consumer Price Index Inflation Rate," ticker symbol "CPARGM."
  - Chile: "Índice de Precios al Consumidor - Antecedentes históricos" from http://www.ine.cl/canales/chile_estadistico/estadisticas_precios/ipc/series_antecedentes_historicos/index.php

B Controlling for industry composition

To control for industry composition in the effect of capital income share movements in France, the U.K., and the U.S. as described in Section 2 of the main text, we use EU KLEMS data: http://www.euklems.net/. We compute the gross labor share as labor compensation relative to gross value added at basic prices. We drop the following industries from our calculations, as the division between labor and capital income is less straightforward than in other industries:

- Agriculture (code: "AtB").
- Mining (code: "C").
- Government (code: "L").
- Financial intermediation (code: "J").

We keep the most disaggregated industries available, leaving a total of 27 industries with data available for the three countries.

(a) Volatility in raw data (b) Volatility in HP-filtered data

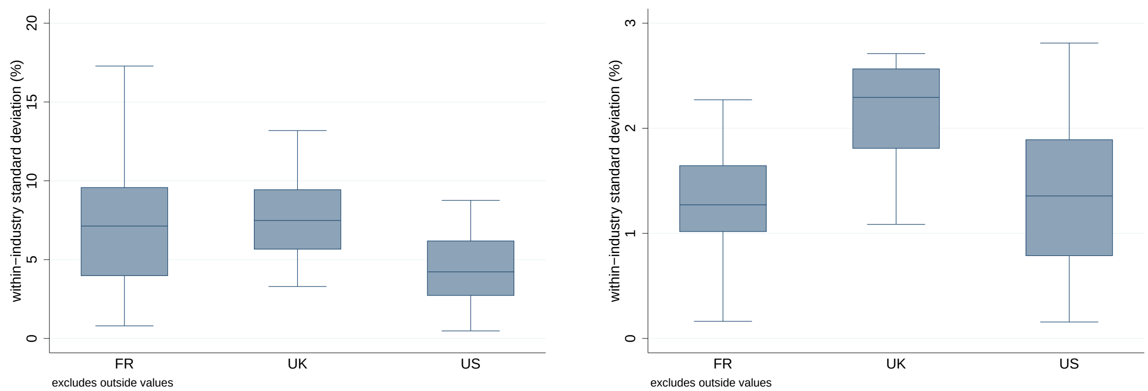


Figure B.5: Within-industry volatility of the gross labor share.

C Additional results on the international evidence

Here we present additional material on the international evidence: first, on the case studies and, second, on labor regulation and capital shares.

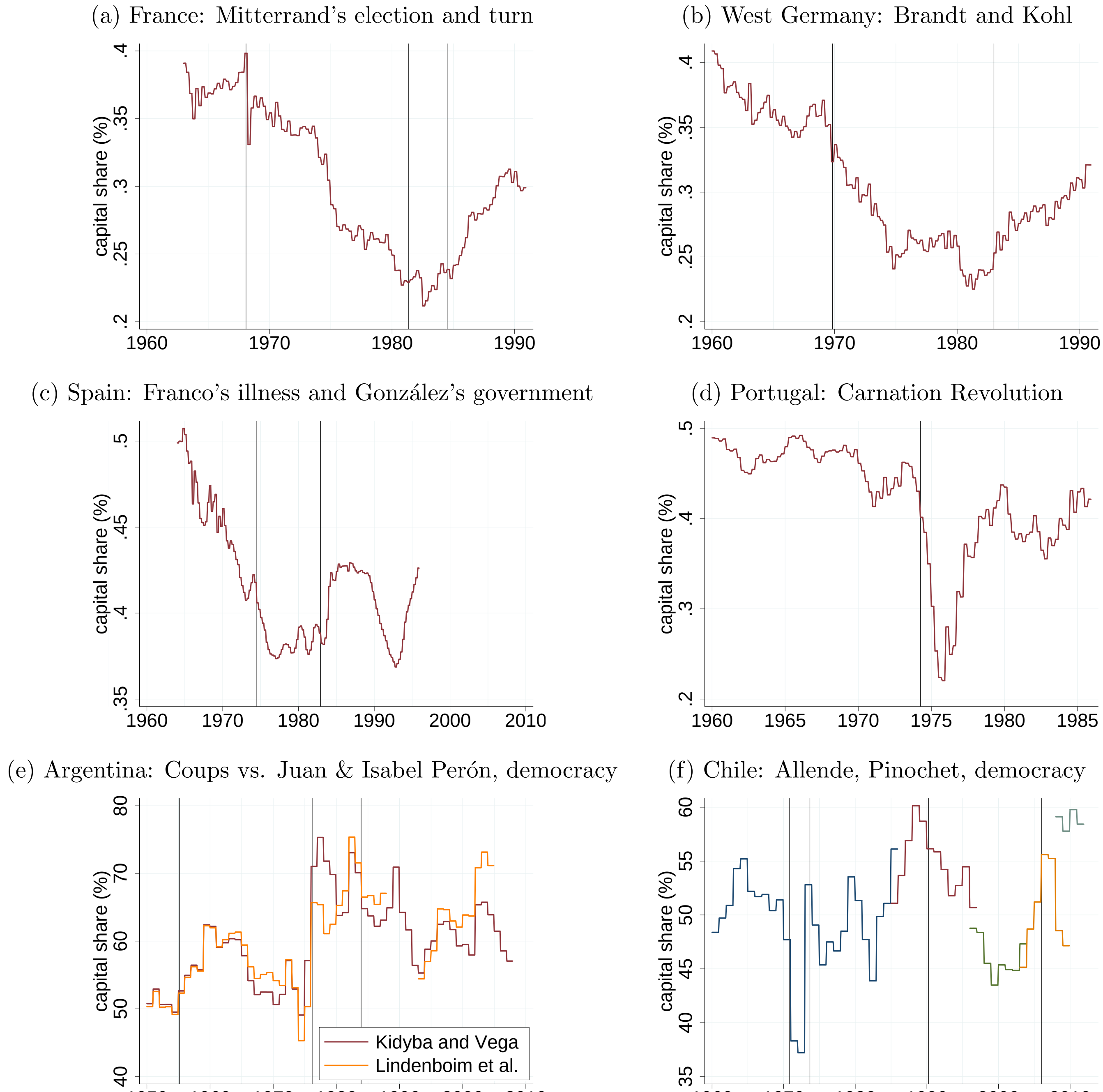
C.1 Three more case studies: West Germany, Spain, and Chile

Here we present three additional case studies –West Germany, Spain, and Chile– to complement the cases of France, Portugal, and Argentina described in the main text. Figure C.6 illustrates the three case studies from the main text and the three extra case studies here.

Panel (b) shows the case of West Germany. The appointment of Willy Brandt as the first Social Democratic chancellor since 1930 coincided with a sharp drop in the capital share. A prominent change introduced by the Social Democrat-led government was the strengthening of the role of workers' councils in its 1972 reform of the *Betriebsverfassungsgesetz* (the legal framework for worker co-determination within firms). Later, under Helmut Schmidt, co-determination was extended to all companies with at least 2,000 employees, rather than just the steel and coal industries. The increase in unions' power was halted and partly reversed when the Christian Democrats returned to power in October 1982 (second vertical line). For instance, in 1986, the Helmut Kohl-led government changed the *Arbeitsförderungsgesetz* (the Employment Promotion Act) to limit the strike tactics of unions. This policy reversal coincided with an increase in the capital share.

Our fifth case study is Spain (panel (c)). The last years of Franco's dictatorship were associated with increasing labor unrest, the breakdown of the system of government-controlled corporatist unions ("Organización Sindical Española") that had repressed wage growth, and a profound economic crisis. The capital share of income and the stock market plummeted. Only after 1982, with the election of the surprisingly pro-market Felipe González's government and the implementation of a wage restraint policy, did the capital share of income recover. In fact, González's economic team was keen on engineering a recovery of the profit rates of firms to help investment. See Solchaga (1997, pp. 198-199).

Our last case is Chile. Panel (f) plots the behavior of the capital income share and an index of the stock market in Chile throughout four periods: the *Unidad Popular* government of Allende (the area between the first two vertical lines), Pinochet's dictatorship (the area between the second and third vertical lines), the governments of Aylwin, Frei, and Lagos – the moderate left-wing first three democratic presidents after Pinochet (the area between the third and fourth vertical lines),



The graphs show the gross capital share for France, West Germany, Spain, Portugal, Argentina, and Chile. Overlain are black vertical lines that indicate major political events as described in the main text. The capital share data for Argentina come from Lindenboim et al. (2005) and Kidyba and Vega (2015). No continuous time series on the capital share is available for Chile, so we show the spells of available data.

Figure C.6: Capital share and major government changes.

and, finally, the more left-wing first presidency of Bachelet (the area to the right of the fourth vertical line). While we do not have a continuous time series, Panel (f) shows a sharp drop in the capital income share around the time of the election of Allende, a socialist candidate who supported a vigorous pro-labor agenda. The capital share recovers quickly around Pinochet's coup, with its violent policy against workers' unions, and falls after the transition to democracy and the return of a friendlier environment for workers' political action.

C.2 Additional results on the case studies

As a supplement to the time series discussed in the main text, we summarize the effect of the political events in the case studies in Table C.1. The table shows the change in the available capital share measure one year after the event year compared to one year before the event, i.e., two-year changes. Also, we show the corresponding changes in the employment-to-population ratio, computed using the Feenstra et al. (2013) data, and the change in the real stock index.

Table C.1: Political events, 2-year changes in capital shares, and employment

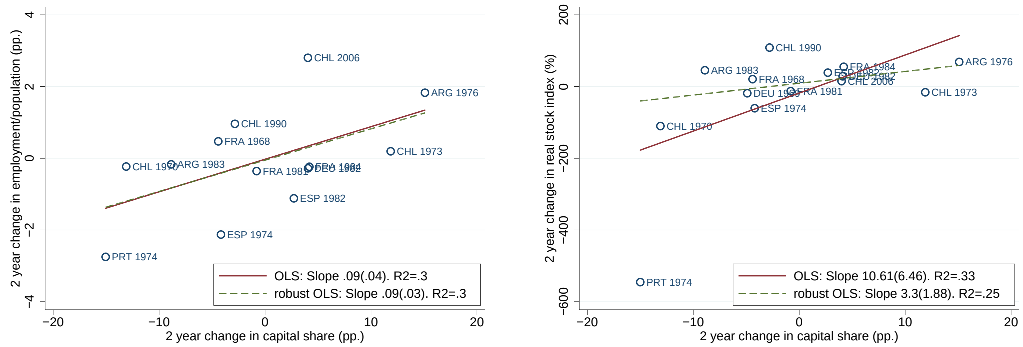
	Date		Δ capital share	Δ employment / population	Δ stock index
OECD data	Year	Month	pp.	pp.	%
Spain: Franco's illness	1974	7	-4.2	-2.1	-60.4
Spain: Gonzalez administration	1982	12	2.7	-1.1	38.8
Argentina: Coup against J. Peron	1955	9	5.5		
Argentina: Coup against I. Peron	1976	3	15.1	1.8	68.7
Argentina: Democratic transition	1983	12	-8.9	-.2	45.5
Chile: Allende election	1970	11	-13.1	-.2	-110.1
Chile: Pinochet coup	1973	9	11.9	.2	-15.9
Chile: Democratic election	1990	3	-2.8	1	108.7
Chile: Democratic transition	2006	3	4	2.8	15.1
France: 1968 strikes	1968	5	-4.4	.5	20.7
France: Mitterrand's election	1981	5	-.8	-.4	-12.8
France: Mitterrand's policy change	1984	7	4.2	-.2	55.2
Portugal: Carnation Revolution	1974	4	-15	-2.7	-545.3
W Germany: Brandt election	1969	10	-4.9		-18.6
W Germany: Kohl administration	1982	10	4.1	-.3	28.7

Figure C.7 shows the robust positive association between capital share changes and both employment and stock market changes. We compute both OLS regressions and weighted OLS regressions, where the weights are generated by Stata's robust regression `rreg` command that weighs down outliers and, in the case of the stock market change in Portugal, drops extreme observations. The relationship with capital share changes is always positive and particularly robust for employment changes, with implied t-statistics between 2.25 and 3.00. The t-statistics for the stock index slopes are somewhat lower, between 1.64 and 1.76. The R^2 statistics are 0.30 for the employment relationship and between 0.25 and 0.33 for the stock market regressions, indicating a moderate to strong correlation.

C.3 Additional results on labor regulation and capital shares

We report here some additional regression results on labor regulation and capital shares. Table C.2 documents the regression estimates for the three-year effects of changes in labor regulation on capital shares. Table C.3 does the same for the one-year effects of changes in labor regulation on capital shares. We try different specifications, such as examining only changes within the year of the political event or labor-regulation change, and using all countries for which we have labor

(a) Employment-population ratio (b) Real stock index



Heteroskedasticity-robust standard errors for slope coefficients in parentheses.

Figure C.7: Political events: 2-year changes in capital shares vs 2-year changes in employment and stock indices.

share and capital share data with OLS, following our algorithm blindly by assigning 1976 as the democratization date for Portugal, or using the Carnation Revolution date of 1974.

In both tables, "5y FE" refers to a fixed effect for the five-year period surrounding the event. "Initial conditions" are the level of capital share and real per capita GDP growth in the year before the estimation period. In the baseline IV regressions, we include only countries with political events, as defined in the main text. Alternatively, we include all observations for the countries with political events. For the OLS case, we also run the regression of the sample of all countries with capital share and labor regulation data.

Table C.2: Three-year effects of changes in labor regulation on capital shares: Regression estimates

Specification	5y FE	Initial conditions	Effect on capital share	t-stat	1-stage t-stat
All countries: OLS	–	–	-0.92	-5.20	.
	y	–	-0.86	-3.27	.
All countries: OLS	–	y	-0.90	-4.67	.
	y	y	-0.84	-2.94	.
Event countries: OLS	–	–	-1.49	-3.47	.
	y	–	-1.56	-3.72	.
	–	y	-1.40	-3.13	.
	y	y	-1.49	-3.41	.
Event episodes: OLS	–	–	-5.21	-3.16	.
	y	–	-6.21	-15.73	.
	–	y	-5.43	-2.48	.
	y	y	-5.22	-5.48	.
Carnation Revolution date					
Event episodes: IV	–	–	-6.94	-2.76	2.28
	y	–	-8.39	-4.04	2.45
	–	y	-7.94	-1.90	1.72
	y	y	-8.94	-2.99	1.76
Event countries: IV	–	–	-4.58	-3.06	2.09
	y	–	-4.06	-13.19	1.76
	–	y	-4.06	-2.42	2.01
Pure algorithm					
Event episodes: IV	–	–	-5.28	-2.14	2.27
	y	–	-5.35	-2.43	2.46
	–	y	-4.74	-1.59	1.89
	y	y	-4.96	-1.66	1.93
Event countries: IV	–	–	-3.99	-3.31	2.29
	y	–	-3.52	-7.72	1.81
	–	y	-3.66	-2.75	2.23
	y	y	-3.61	-9.76	1.79

Table C.3: One-year effects of changes in labor regulation on capital shares: Regression estimates

Specification	5y FE	Initial conditions	Effect on capital share	t-stat	1-stage t-stat
All countries: OLS	–	–	-0.45	-3.34	.
	y	–	-0.44	-3.84	.
All countries: OLS	–	y	-0.43	-3.29	.
	y	y	-0.43	-3.51	.
Event countries: OLS	–	–	-0.82	-2.71	.
	y	–	-0.84	-2.68	.
	–	y	-0.81	-2.73	.
	y	y	-0.84	-2.73	.
Event episodes: OLS	–	–	-3.03	-1.56	.
	y	–	-2.85	-1.24	.
	–	y	-2.89	-1.39	.
	y	y	-2.99	-1.12	.
Carnation Revolution date					
Event episodes: IV	–	–	-5.67	-1.86	2.34
	y	–	-5.35	-6.22	11.61
	–	y	-5.79	-1.75	2.24
	y	y	-5.61	-6.92	13.39
Event countries: IV	–	–	-2.35	-2.31	2.03
	y	–	-2.33	-6.04	2.79
	–	y	-2.25	-2.21	2.02
	y	y	-2.26	-4.95	2.70
Pure algorithm					
Event episodes: IV	–	–	-5.77	-1.85	2.37
	y	–	-5.86	-10.36	5.09
	–	y	-5.40	-1.81	2.37
	y	y	-6.13	-7.45	5.31
Event countries: IV	–	–	-2.10	-2.23	2.12
	y	–	-2.09	-7.98	2.79
	–	y	-2.06	-2.21	2.12
	y	y	-2.05	-7.73	2.76

D Additional evidence regarding right-to-work legislation

We repeat the same exercise as in Subsection 2.3, but now we look at real GDP growth instead of labor shares. Real GDP growth is computed using the change in state total private-sector GDP deflated by the national GDP deflator. Since the data start only in 1963, the year Wyoming adopted the new legislation, GDP growth in Wyoming is normalized to zero for the first year after adoption. Before 1997, we use private SIC industries. From 1997, we use private NAICS industries.

Figure D.8 reports the evolution of real state private industry GDP growth after the adoption of right-to-work legislation (in absolute levels and relative to the U.S.). Table D.4 presents a panel regression analysis of the data. Standard errors are clustered by state and industry, and two-sided p-values are in parentheses.

Change in real GDP growth and change relative to U.S.

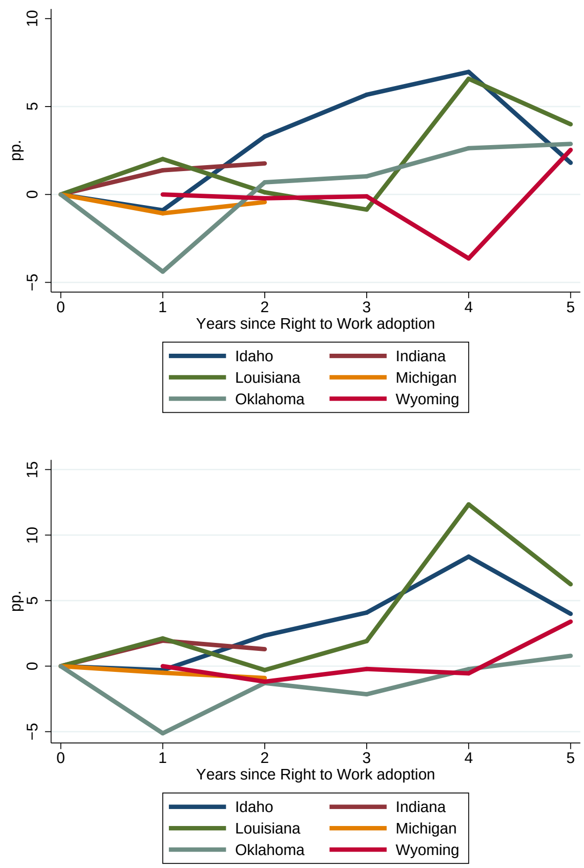


Figure D.8: Change in real state private industry GDP growth after right-to-work adoption.

Table D.4: State-industry panel regression: Right-to-work laws and real GDP growth

	Level	1y change	2y change	3y change	4y change	5y change
Controlling for state FE, and industry FE						
Right to Work	-0.01					
	(0.99)					
Change in RtW		0.82	0.16	0.87	1.03	0.87
		(0.14)	(0.88)	(0.45)	(0.26)	(0.31)
Controlling for state FE, quadratic trend, and industry FE						
Right to Work	0.64					
	(0.09)					
Change in RtW		0.94	0.14	0.85	1.03	0.88
		(0.10)	(0.90)	(0.45)	(0.25)	(0.28)

E Additional VAR results

Figure E.9 plots the IRFs from the small VAR with a quadratic trend.

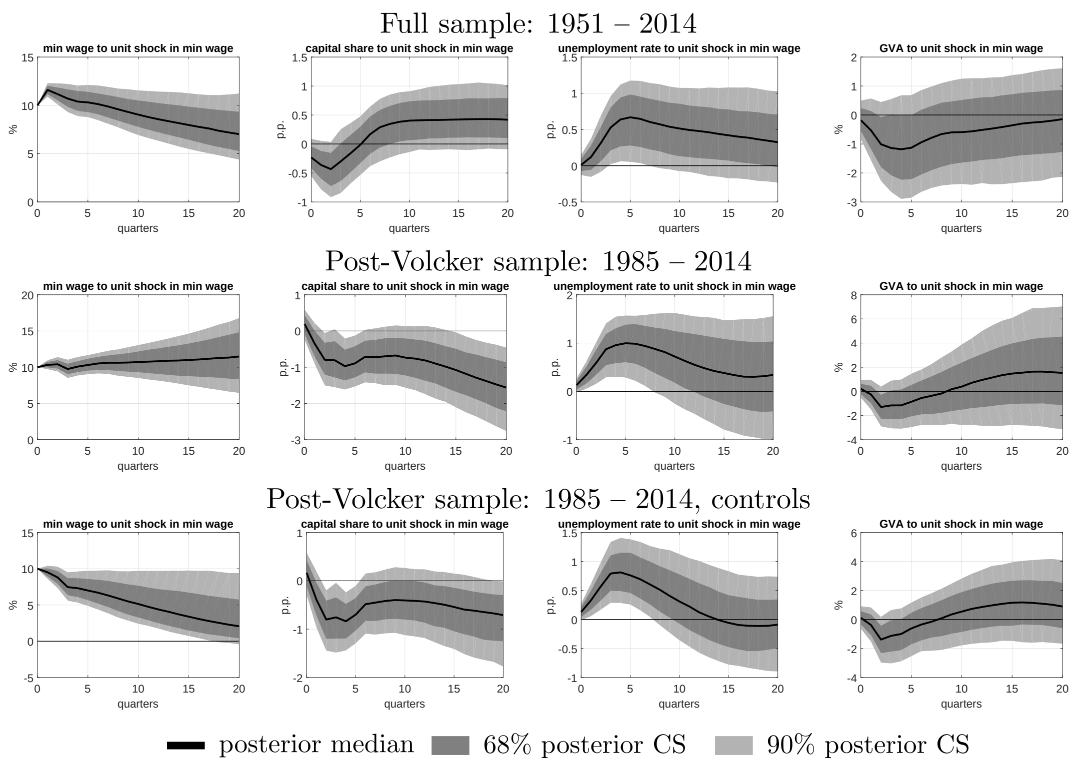


Figure E.9: Responses to a 10% real minimum wage shock in small VAR, quadratic trend.

F Large VAR

To check the robustness of the VAR exercise in the main text, we now present results using a larger VAR. In addition to the labor market and the non-corporate business sector, this VAR captures asset prices, consumption, and investment. As a result, we arrive at the following ten-variable VAR: (1) the (log) of the federal minimum wage relative to the PCE deflator, (2) the net capital share in the corporate non-financial sector, (3) the average of the total returns of consumer and manufacturing firms, (4) the unemployment rate, (5) non-farm labor productivity in the business sector, (6) labor market tightness, (7) capacity utilization, (8) real private investment, (9) real private consumption, and (10) the average corporate tax rate. Instead of using the cumulative total return in Greenwald et al. (2014), we use the (unweighted) average of the cumulative total return in the consumer and manufacturing sectors based on the five-sector Fama-French industry classification because we expect the minimum wage to be more important in these sectors.[29] Again, we use four lags in the estimation.

Minimum wage shocks are also clearly redistributive in this large VAR. Figure F.10 shows the IRFs to a typical minimum wage shock of 10%. Such a shock causes the capital share to drop by 0.25 to 0.5 pp. for two to five quarters with 68% posterior credibility. The labor market worsens, with unemployment rising by 0.5 to 1.5 pp. about a year after the initial shock. Labor productivity increases slightly with a delay, consistent with a selection effect. We also find that the stock market valuation drops significantly. Investment drops 5% at the peak and with it capacity utilization. Finally, there is a delayed decline in the average corporate tax rate. This decline may reflect the progressivity of the corporate tax code as corporate profits fall.

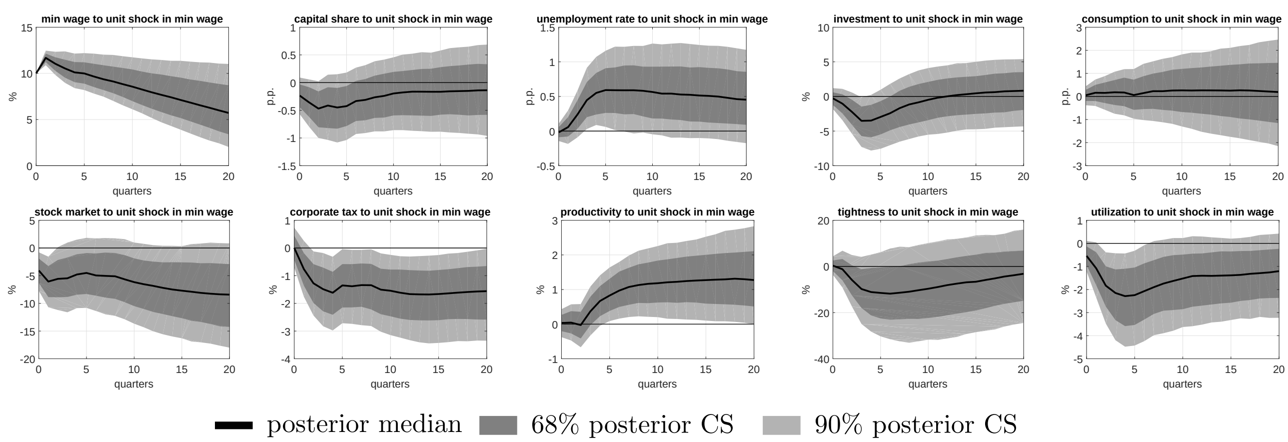


Figure F.10: Responses to a 10% real minimum wage shock in extended VAR: 1951–2014.

Many states set minimum wages above the federal level, particularly in the second half of our full sample. Hence, we incorporate state minimum wage changes in our analysis. More concretely, prior to estimation, we aggregate minimum wages across states by weighting them with the relative populations of each state. This weighting is imperfect given that the unemployment rate in our VAR is labor force weighted and stock returns are weighted by market capitalization.[30]

[29] We use these sectors because our empirical model does not speak much to the other three sectors. We focus on the non-financial corporate business sectors and thus drop the "other" sector that includes financial firms. See for the source data: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. Our results change little when we include only one of the sectors at a time.

[30] We use the data from Autor et al. (2016). Their coverage of Washington, D.C., has a gap, so we drop it. For the other states, we compute the change in the maximum of the state and the federal minimum wage, quarter by quarter.

Combining state and federal minimum wage strengthens the redistributive effects we estimate; see Figure F.11. After a minimum wage shock, there is a drop in the capital share that lasts for three to four years and peaks -1 to -1.5 pp. after six quarters. With a delay, unemployment rises significantly after five quarters, while stock values, consumption, and utilization fall. The differences in the size and shape of the IRFs of this exercise are not due to the different sample period compared to our large VAR baseline.

We also report several robustness exercises. First, in Figure F.12, we plot the IRFs from the large VAR in the post-1974 sample. Second, in Figure F.13, we plot the IRFs from the same VAR, but now with a quadratic trend and using shocks to the real effective state-level minimum wage. Third, in Figure F.14, we plot the IRFs of the same VAR with a quadratic trend for the full 1951-2014 sample.

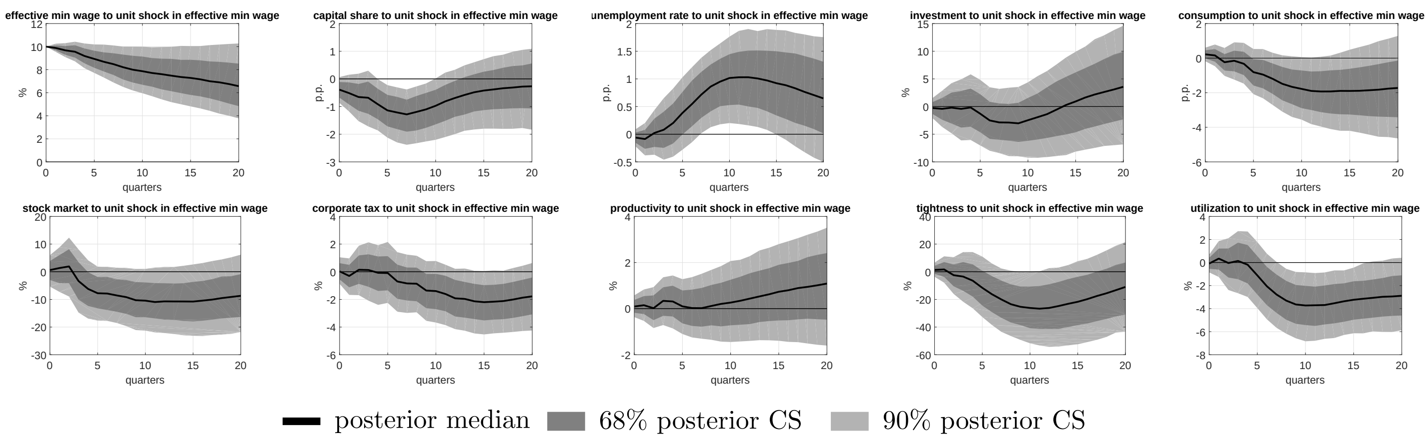


Figure F.11: Responses to a 10% real effective state-level minimum wage shock in extended VAR: 1974–2014.

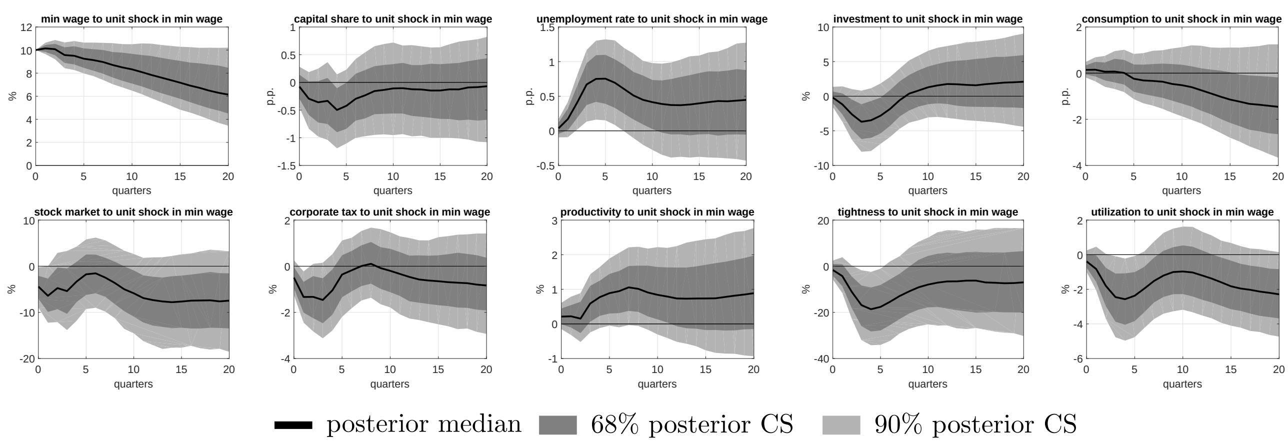


Figure F.12: Responses to a 10% real minimum wage shock in extended VAR: 1974–2014.

We deflate this nominal increase and average it across states using annual population weights.

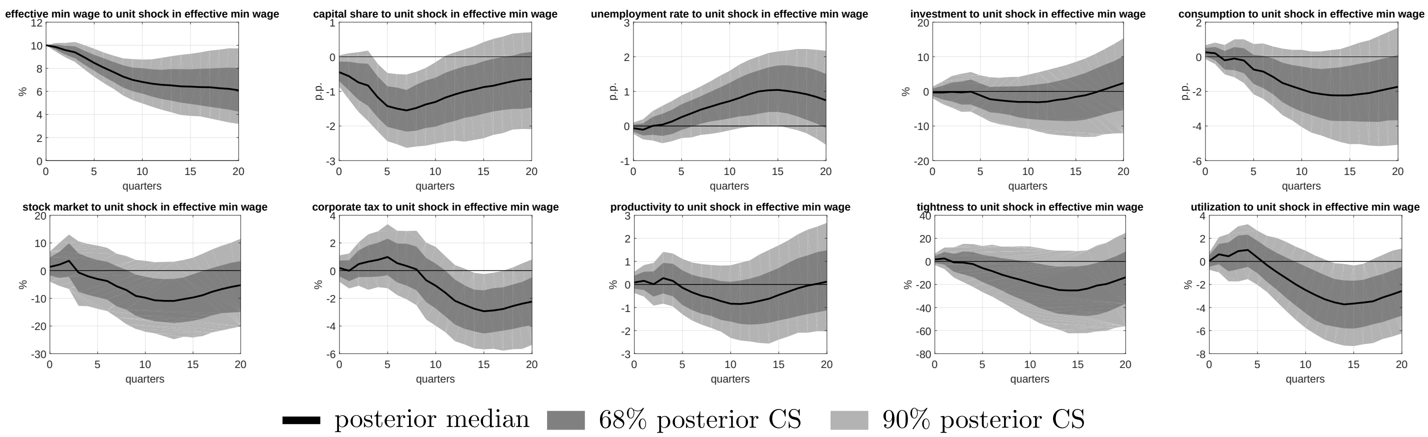


Figure F.13: Responses to a 10% real effective state-level minimum wage shock in extended VAR: 1974–2014, quadratic trend.

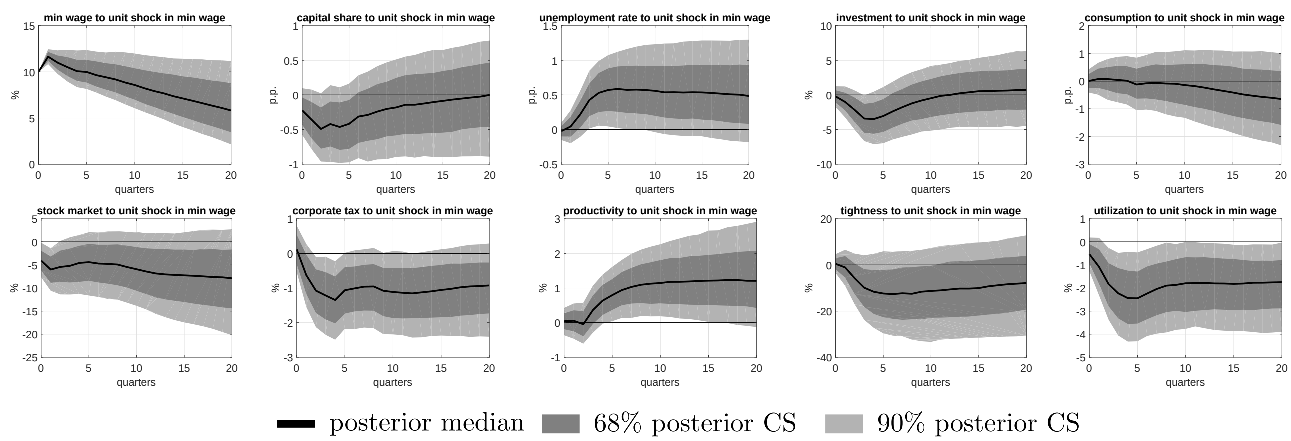


Figure F.14: Responses to a 10% real minimum wage shock in extended VAR: 1951–2014, quadratic trend.

G State minimum wage changes

Here we examine the relationship between changes in the maximum of statutory state minimum wage and federal minimum wages, deflated to constant 2010 real dollars, and three outcomes: (1) changes in the gross capital share, (2) changes in the unemployment rate, and (3) real GDP growth per capita. The state-level data are the same as in Section 2.3, except that we obtain the unemployment rate from the BLS via Federal Reserve Economic Data (FRED).[31] We regress the outcome variable on the current or lagged changes in the applicable nominal minimum wage, converted to 2010 dollars. In all specifications, we include state year fixed effects and cluster the standard errors by state. We also report variants that also include year fixed effects. For each specification, we report results for the full sample, and the sample of state-years with actual changes in the minimum wage.

Our results are the strongest for the capital share. Figure G.15 documents a significant negative relationship between changes in state minimum wages and the gross capital share within states in the specification that considers only state-years with changes in the minimum wage and includes year fixed effects. Table G.5 includes the detailed regression result that corresponds to the figure. Panel (a) reports regressions using the full sample, where columns (a1) through (a4) include state fixed effects, while columns (a5) through (a8) also include year fixed effects. Here, the point estimates point to a decline of the capital share in the year of the minimum wage increase and the year after, with a reversal after two years. The declines are, however, not significant with the year fixed effects. When we condition on changes in the minimum wage only and use only state fixed effects, we find no significant change in the capital share on impact (column (b1)), but a decline of 0.8 pp. with a one-year delay (column (b2)), and a partial reversal two years after (column (b3)). The results are similar when we estimate the impact and lagged effects simultaneously (column (b4)). With year fixed effects, we find an impact decline in the capital share of 0.42 (column (b5)) and a further decline of 0.46 in the year after (column (b6)), resulting in a cumulative decline of around 0.9 pp. for a one-dollar increase, similar to the estimate without year fixed effects. Two years after, this effect is partially reversed (column (b7)). Jointly estimating the effects yields similar signs, but smaller magnitudes and no statistical significance (column (b8)). Using all state-years for the estimation yields results similar to those with only state fixed effects, but insignificant results once we also include year fixed effects.

Tables G.6 and G.7 show the analogous results for economic activity, measured as changes in the unemployment rate or the real per capita GDP growth rate. For all state-years and with only state fixed effects, we find significant increases in the unemployment rate and decreases in real GDP growth on impact and two years after, as columns (a1) through (a4) show. A one-dollar increase in the minimum wage is associated with an increase in the unemployment rate of 0.7 pp. and a decrease in the GDP growth rate of 2.2 pp. in the year of the increase. These results weaken, however, once we introduce year fixed effects, in which case only the effect after two years remains significant for both the unemployment increase and the reduction in GDP growth (columns (a7) and (a8)). Conditioning on years with changes, our results for GDP growth are very similar: We find a significant drop on impact and after two years (columns (b1), (b3) and (b4)), but with year fixed effects the results become largely insignificant, except in column (b8), which also points to a decrease in GDP growth with a two-year lag. For the unemployment rate, the results are more subtle when we consider only years with changes in the minimum wage. With state fixed effects only, we estimate a 0.3 pp. drop in the unemployment rate on impact (column (b1)), followed by increases of 0.4 pp. and 0.8 pp. (columns (b2) and (b3)). Estimating the current and lagged effects jointly points to statistically and economically significant drops on impact and with lags of one and two years (column (b4)). With year fixed effects, however, the results are largely insignificant.

[31] The ticker symbols are AKURN, ALURN, . . . , downloadable from http://research.stlouisfed.org/fred2/.

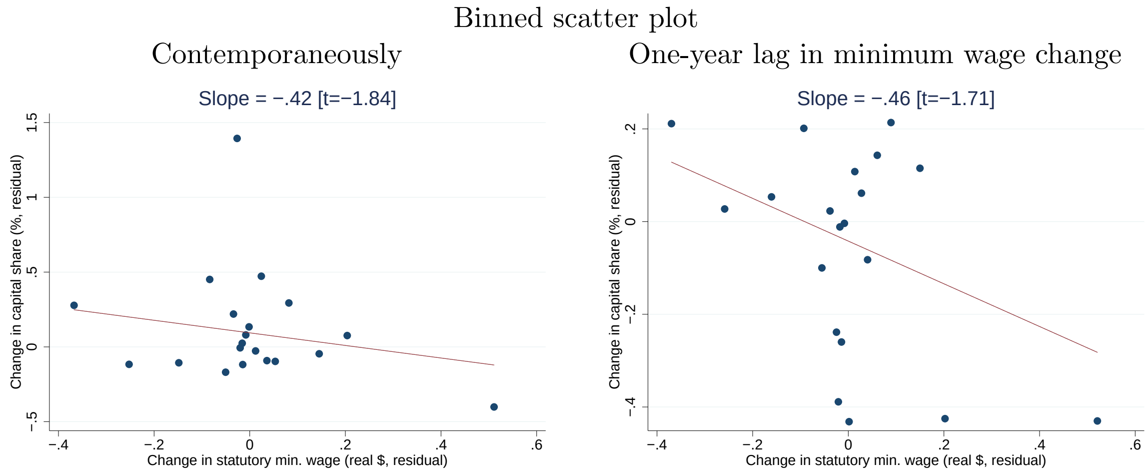


Figure G.15: Change in state statutory minimum wage and change in private-sector capital share: Contemporaneous and lagged relationship.

Table G.5: State panel regression: Statutory state minimum wage changes and gross capital share

(a) All state-years

Δ capital share (pp.)	(a1)	(a2)	(a3)	(a4)	(a5)	(a6)	(a7)	(a8)
Δ statutory min.wage (real USD)	-0.173*			-0.091	-0.223			-0.039
	(-1.77)			(-0.59)	(-1.61)			(-0.29)
L.Δ statutory min.wage (real USD)		-0.571***		-0.572***		-0.227		-0.220
		(-6.22)		(-5.79)		(-1.29)		(-1.26)
L2.Δ statutory min.wage (real USD)			0.139	0.316**			0.378*	0.406*
			(0.94)	(2.31)			(1.72)	(1.85)
R-squared	0.01	0.03	0.01	0.03	0.22	0.19	0.20	0.20
R-sq, within	0.00	0.02	0.00	0.01	0.00	0.00	0.00	0.00
Observations	1886	1838	1790	1686	1886	1838	1790	1686
States	51	51	51	51	51	51	51	51
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	No	No	No	No	Yes	Yes	Yes	Yes
Sample	All	All	All	All	All	All	All	All

(b) State-years with minimum wage changes only

Δ capital share (pp.)	(b1)	(b2)	(b3)	(b4)	(b5)	(b6)	(b7)	(b8)
Δ statutory min.wage (real USD)	-0.045			0.034	-0.420*			-0.114
	(-0.28)			(0.19)	(-1.84)			(-0.64)
L.Δ statutory min.wage (real USD)		-0.821***		-0.499***		-0.461*		-0.297
		(-4.49)		(-4.66)		(-1.71)		(-1.50)
L2.Δ statutory min.wage (real USD)			0.196	0.460**			0.524*	0.388
			(0.81)	(2.60)			(1.89)	(1.44)
R-squared	0.03	0.06	0.06	0.06	0.26	0.23	0.26	0.24
R-sq, within	0.00	0.02	0.00	0.02	0.00	0.00	0.01	0.01
Observations	841	797	796	1104	837	793	793	1103
States	51	51	51	51	51	51	51	51
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	No	No	No	No	Yes	Yes	Yes	Yes
Sample	Changes	Changes	Changes	Changes	Changes	Changes	Changes	Changes

Standard errors are clustered by state. t-statistics are in parentheses. $* \ p < 0.1$ $** \ p < 0.05$ $*** \ p < 0.01$

Table G.6: State panel regression: Statutory state minimum wage changes and unemployment

(a) All state-years

Δ unemployment rate (pp.)	(a1)	(a2)	(a3)	(a4)	(a5)	(a6)	(a7)	(a8)
Δ statutory min.wage (real USD)	0.694***			0.318***	-0.037			-0.018
	(7.79)			(4.24)	(-0.44)			(-0.20)
L.Δ statutory min.wage (real USD)		1.253***		0.992***		0.159*		0.126
		(14.44)		(12.29)		(1.86)		(1.55)
L2.Δ statutory min.wage (real USD)			0.873***	0.418***			0.287**	0.280**
			(10.80)	(4.31)			(2.51)	(2.44)
R-squared	0.04	0.12	0.06	0.14	0.69	0.69	0.69	0.69
R-sq, within	0.04	0.12	0.06	0.14	0.00	0.00	0.01	0.01
Observations	1770	1773	1776	1672	1770	1773	1776	1672
States	51	51	51	51	51	51	51	51
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	No	No	No	No	Yes	Yes	Yes	Yes
Sample	All	All	All	All	All	All	All	All

(b) State-years with minimum wage changes only

Δ unemployment rate (pp.)	(b1)	(b2)	(b3)	(b4)	(b5)	(b6)	(b7)	(b8)
Δ statutory min.wage (real USD)	-0.268*			0.304***	-0.188			-0.075
	(-1.88)			(3.24)	(-1.19)			(-0.61)
L.Δ statutory min.wage (real USD)		0.405***		0.983***		-0.142		0.118
		(3.80)		(11.51)		(-1.40)		(1.37)
L2.Δ statutory min.wage (real USD)			0.843***	0.403***			0.176	0.307**
			(3.72)	(3.46)			(0.92)	(2.17)
R-squared	0.04	0.05	0.04	0.12	0.73	0.74	0.75	0.71
R-sq, within	0.00	0.01	0.03	0.11	0.00	0.00	0.00	0.01
Observations	734	741	790	1096	730	737	787	1095
States	51	51	51	51	51	51	51	51
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	No	No	No	No	Yes	Yes	Yes	Yes
Sample	Changes	Changes	Changes	Changes	Changes	Changes	Changes	Changes

Standard errors are clustered by state. t-statistics are in parentheses. $* \; p < 0.1$ $** \; p < 0.05$ $*** \; p < 0.01$

Table G.7: State panel regression: Statutory state minimum wage changes and real per capita GDP growth

(a) All state-years

Growth (%)	(a1)	(a2)	(a3)	(a4)	(a5)	(a6)	(a7)	(a8)
Δ statutory min.wage (real USD)	-2.163***			-2.132***	0.089			-0.061
	(-6.61)			(-6.45)	(0.20)			(-0.13)
L.Δ statutory min.wage (real USD)		-1.835***		-0.875***		-0.246		-0.133
		(-7.38)		(-3.25)		(-0.81)		(-0.51)
L2.Δ statutory min.wage (real USD)			-1.405***	-1.452***			-0.533*	-0.544*
			(-5.41)	(-5.47)			(-1.86)	(-1.87)
R-squared	0.11	0.10	0.09	0.17	0.38	0.39	0.39	0.39
R-sq, within	0.05	0.04	0.02	0.11	0.00	0.00	0.00	0.00
Observations	1220	1223	1226	1122	1220	1223	1226	1122
States	51	51	51	51	51	51	51	51
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	No	No	No	No	Yes	Yes	Yes	Yes
Sample	All	All	All	All	All	All	All	All

(b) State-years with minimum wage changes only

Growth (%)	(b1)	(b2)	(b3)	(b4)	(b5)	(b6)	(b7)	(b8)
Δ statutory min.wage (real USD)	-1.958***			-2.760***	0.147			-0.317
	(-3.81)			(-7.20)	(0.29)			(-0.65)
L.Δ statutory min.wage (real USD)		-0.690		-1.026***		-0.122		-0.271
		(-1.49)		(-3.48)		(-0.28)		(-0.90)
L2.Δ statutory min.wage (real USD)			-2.331***	-2.093***			-0.648	-0.838***
			(-4.55)	(-7.46)			(-1.40)	(-2.70)
R-squared	0.18	0.13	0.15	0.23	0.53	0.54	0.54	0.48
R-sq, within	0.04	0.00	0.05	0.13	0.00	0.00	0.01	0.01
Observations	520	482	488	733	518	480	487	733
States	51	51	51	51	51	51	51	51
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	No	No	No	No	Yes	Yes	Yes	Yes
Sample	Changes	Changes	Changes	Changes	Changes	Changes	Changes	Changes

Standard errors are clustered by state. t-statistics are in parentheses. $* \ p < 0.1$ $** \ p < 0.05$ $*** \ p < 0.01$

H Model appendix

Our business cycle model with search frictions in the labor market is in the spirit of those in Andolfatto (1996) and Merz (1995) and builds on the formulation of Shimer (2010, ch. 3). Relative to the notation in Shimer (2010), we change the timing convention so that capital k_t and employment n_t are time t measurable, but not time $t-1$ measurable.

H.1 The household

There is a representative household that perfectly ensures its members against idiosyncratic risk. The following utility function represents its preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(c_{e,t} - hc^a_{e,t-1})^{1-\sigma}(1+(\sigma-1)\gamma)^{\sigma} - 1}{1-\sigma} n_{t-1} + \frac{(c_{u,t} - hc^a_{u,t-1})^{1-\sigma} - 1}{1-\sigma}(1-n_{t-1}) \right), \quad \text{(H.1)}$$

where $c_{e,t}$ and $c_{u,t}$ are the consumption of the employed and unemployed household members, respectively, and n_{t-1} denotes the fraction of employed households. The parameter $h \in [0,1)$ controls the strength of the external habit.

After matching with firms, employed household members draw an *iid* type. With probability $\zeta_{0,t}$, they have only $\zeta_1 \in (0,1)$ efficiency units of labor – otherwise they have one efficiency unit. For clarity, we denote variables referring to the high types with one efficiency unit by subscripts h and, for low types, by subscripts ℓ. We allow these employed members to receive a wage $w_{\ell,t}$, different from $w_{h,t}$ that productive workers receive. Minimum wages may imply that $w_{\ell,t} > \zeta_1 w_{h,t}$ in equilibrium. We denote variables with subscript ζ as weighted averages. For example, $w_{\zeta,t} \equiv (1-\zeta_{0,t})w_{h,t} + \zeta_{0,t}w_{\ell,t}$. Because we end up assuming that all employed household members with the low type receive the minimum wage w_ℓ, it may also have the interpretation of other welfare payments. For example, if $\zeta_1 = 0$, then w_ℓ might better be thought of as mandatory sick pay. The household perfectly insures the employed against variations in their type.

The household faces a lifetime budget constraint given the stochastic discount factor m_t:

$$\begin{aligned} a_{-1} = \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t} m_t \right) & (c_{e,t}n_{t-1} + c_{u,t}(1-n_{t-1}) \\ & - (1-\tau_n) \underbrace{((1-\zeta_0)w_{h,t} + \zeta_0 w_{\ell,t})}_{\equiv w_{\zeta,t}} n_{t-1} - (1-\tau_n)\omega_t(1-n_{t-1}) - T_t), \end{aligned}$$

where the present discounted value of consumption equals the beginning of the period financial wealth a_{-1} plus net labor income $(1-\tau_n)w_{h,t}$ for the fraction $1-\zeta_{0,t}$ of workers who are productive and $(1-\tau_n)w_{\ell,t}$ for the workers who are unproductive. We also define $w_{\zeta,t} = (1-\zeta_{0,t})w_{h,t} + \zeta_{0,t}w_{\ell,t}$. The household receives unemployment benefits ω_t and lump-sum transfers T_t.

Finally, when making its decisions, the household considers that workers lose their jobs at rate x and find new jobs at rate $f(\theta_t) = \xi\theta_t^{\eta}$, where θ_t is the recruiter-unemployment ratio that the household takes as given. Thus, the fraction of household members employed next period will be:

$$n_t = (1-x)n_{t-1} + f(\theta_t)(1-n_{t-1}). \quad \text{(H.2)}$$

H.1.1 Aggregation

Under perfect insurance within the household, a necessary condition for the household's optimality is that consumption of the employed and unemployed satisfies:

$$\beta^t(c_{e,t} - hc^a_{e,t-1})^{-\sigma}(1 + (\sigma-1)\gamma)^\sigma = \beta^t(c_{u,t} - hc^a_{u,t-1})^{-\sigma} = \lambda m_t,$$

where λ is the Lagrangian multiplier associated with the budget constraint. If $h = 0$ or given the initial condition that $c^a_{e,t-1} = c^a_{u,t-1}(1 + (\sigma-1)\gamma)$, it follows that:

$$c_{e,t} = c_{u,t}(1 + (\sigma-1)\gamma)$$

and

$$\begin{aligned}
c_t &\equiv c_{e,t}n_{t-1} + c_{u,t}(1 - n_{t-1}) \\
c_{u,t} &= \frac{c_t}{1 + (\sigma-1)\gamma n_{t-1}} \\
c_{e,t} &= \frac{c_t(1 + (\sigma-1)\gamma)}{1 + (\sigma-1)\gamma n_{t-1}}.
\end{aligned}$$

Hence, the utility function can be simplified as:

$$U_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t - hc^a_{t-1}\frac{1+(\sigma-1)\gamma n_{t-1}}{1+(\sigma-1)\gamma n^a_{t-2}}\right)^{1-\sigma}(1 + (\sigma-1)\gamma n_{t-1})^\sigma - 1}{1-\sigma}, \tag{H.3}$$

and the budget constraint becomes:

$$a_{-1} = \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t} m_t\right)(c_t - (1-\tau_n)w_{\zeta,t}n_{t-1} - (1-\tau_n)\omega_t(1-n_{t-1}) - T_t). \tag{H.4}$$

With $h > 0$, the household partially internalizes that increasing employment changes the size of habit one period ahead. Setting $h = 0$ recovers equation (4.1) in the main text.

H.1.2 Equilibrium conditions

We start the analysis of the labor market by writing the household problem using a recursive formulation:

$$V(a_{-1}, n_{-1}; S) = \max_{a(S'),c,n} \frac{(c - \hat{h}(n_{-1})c^a_{-1})^{1-\sigma}(1 + (\sigma-1)\gamma n_{-1})^\sigma - 1}{1-\sigma} + \beta\mathbb{E}[V(a(S'), n; S')|S] \tag{H.5}$$

subject to:

$$n = (1-x)n_{-1} + f(\theta)(1 - n_{-1}) \tag{H.6}$$

$$c = a_{-1} + (1-\tau_n)w_{\zeta,t}n_{-1} + (1-\tau_n)\omega_t(1-n_{t-1}) + T_t - \mathbb{E}[m(S')a(S')|S] \tag{H.7}$$

and where:

$$\hat{h}(n_{-1}) = h\frac{1 + (\sigma-1)\gamma n_{-1}}{1 + (\sigma-1)\gamma n^a_{-2}}.$$

Complete markets ensure that the household can pick next period's assets as a function of the future state S'.

The equilibrium conditions for an interior equilibrium are:

$$\lambda = (c - \hat{h}(n_{-1})c^a_{-1})^{-\sigma}(1 + (\sigma - 1)\gamma n_{-1})^{\sigma} \tag{H.8}$$

$$\lambda m(S') = \beta V_a(a(S'), n; S') = \beta\lambda(S') = \beta(c(S') - \hat{h}(n)c^a)^{-\sigma}(1 + (\sigma - 1)\gamma n)^{\sigma}. \tag{H.9}$$

Thus, the stochastic discount factor of the economy is:

$$m(S') = \beta \frac{(c(S') - \hat{h}(n)c^a)^{-\sigma}(1 + (\sigma - 1)\gamma n)^{\sigma}}{(c - \hat{h}(n_{-1})c^a_{-1})^{-\sigma}(1 + (\sigma - 1)\gamma n_{-1})^{\sigma}}. \tag{H.10}$$

In equilibrium, $c^a = c$. In what follows, we use m_t as shorthand for $m(S_t)$ with $m_0 = 1$.

The marginal value of employment (after the type i is realized) is given by:

$$\begin{aligned} V_{i,n}(a_{-1}, n_{-1}; S) = {} & \left(\frac{c - \hat{h}(n_{-1})c^a_{-1}}{1 + (\sigma - 1)\gamma n_{-1}}\right)^{-\sigma} (1 - \tau_n)(w_i - \omega) \\ & - \left(\frac{c - \hat{h}(n_{-1})c^a_{-1}}{1 + (\sigma - 1)\gamma n_{-1}}\right)^{1-\sigma} \gamma \left(\sigma + (\sigma - 1)\frac{\hat{h}(n_{-1})c^a_{-1}}{c - \hat{h}(n_{-1})c^a_{-1}}\right) \\ & + \beta(1 - x - f(\theta))\mathbb{E}\left[(1 - \zeta_0')V_{h,n}(a(S'), n; S') + \zeta_0' V_{\ell,n}(a(S'), n; S')|S\right] \end{aligned} \tag{H.11}$$

and

$$V_{\ell,n}(a_{-1}, n_{-1}; S) = V_{h,n}(a_{-1}, n_{-1}; S) + \left(\frac{c - \hat{h}(n_{-1})c^a_{-1}}{1 + (\sigma - 1)\gamma n_{-1}}\right)^{-\sigma} (1 - \tau_n)(w_\ell - w_h). \tag{H.12}$$

Because all terms are independent of i except for the wage rate, the ex ante marginal value or the average marginal value is simply:

$$\begin{aligned} V_{\zeta,n}(a_{-1}, n_{-1}; S) = {} & \left(\frac{c - \hat{h}(n_{-1})c^a_{-1}}{1 + (\sigma - 1)\gamma n_{-1}}\right)^{-\sigma} (1 - \tau_n)(w_\zeta - \omega) \\ & - \left(\frac{c - \hat{h}(n_{-1})c^a_{-1}}{1 + (\sigma - 1)\gamma n_{-1}}\right)^{1-\sigma} \gamma \left(\sigma + (\sigma - 1)\frac{\hat{h}(n_{-1})c^a_{-1}}{c - \hat{h}(n_{-1})c^a_{-1}}\right) \\ & + \beta(1 - x - f(\theta))\mathbb{E}\left[V_{\zeta,n}(a(S'), n; S')|S\right]. \end{aligned} \tag{H.13}$$

For intuition, note that:

$$V_{\zeta,n}(a_{-1}, n_{-1}; S) = V_{h,n}(a_{-1}, n_{-1}; S) + \left(\frac{c - \hat{h}(n_{-1})c^a_{-1}}{1 + (\sigma - 1)\gamma n_{-1}}\right)^{-\sigma} (1 - \tau_n)\zeta_{0,t}(w_\ell - w_h). \tag{H.14}$$

The average marginal value of having an extra person employed is that of having the high type employed plus the (negative) expected value of the wage differential to the low type.

For future reference, it is useful to have a dynamic expression that uses (H.14) to write the $V_{h,h}$

analogue to (H.13):

$$
\begin{aligned}
V_{h,n}(a_{-1}, n_{-1}; S) = &\left(\frac{c - \hat{h}(n_{-1})c_{-1}^a}{1 + (\sigma - 1)\gamma n_{-1}}\right)^{-\sigma} (1 - \tau_n)(w_h - \omega) \\
&- \left(\frac{c - \hat{h}(n_{-1})c_{-1}^a}{1 + (\sigma - 1)\gamma n_{-1}}\right)^{1-\sigma} \gamma \left(\sigma + (\sigma - 1)\frac{\hat{h}(n_{-1})c_{-1}^a}{c - \hat{h}(n_{-1})c_{-1}^a}\right) \\
&+ \beta(1 - x - f(\theta))\mathbb{E}\left[V_{h,n}(a, n; S') + \left(\frac{c' - \hat{h}(n)c^a}{1 + (\sigma - 1)\gamma n}\right)^{-\sigma} (1 - \tau_n)\zeta_0'(w_\ell' - w_h')|S\right].
\end{aligned}
\tag{H.15}
$$

A useful equilibrium object is the value of having a worker employed at an arbitrary wage $\tilde{w}$ this period and at the equilibrium wage:

$$
\tilde{V}_{i,n}(a, n_{-1}; S) = \left(\frac{c - \hat{h}(n_{-1})c_{-1}^a}{1 + (\sigma - 1)\gamma n_{-1}}\right)^{-\sigma} (1 - \tau_n)(\tilde{w}_i - w_i) + V_{i,n}(a_{-1}, n_{-1}; S). \tag{H.16}
$$

$\tilde{V}_{in}$ differs from the marginal value of an extra worker employed at the equilibrium wage both this period and thereafter, i.e., V_{in}, by the marginal utility of income times the difference in the net wage income.

In what follows, we write $\mathbb{E}_t[\cdot]$ for the conditional expectation $\mathbb{E}[\cdot|S_t]$ and similarly index the value function instead of explicitly carrying the state vector and its other arguments.

H.2 The firm

There is a representative firm with n_{-1} workers and capital k_{-1}. It assigns a fraction $\nu_i, i = h, \ell$ of its $(1 - \zeta_0)n_{-1}$ type h and $\zeta_0 n_{-1}$ workers to recruiting and the remainder to production. Because the marginal product of type ℓ is ζ_1 both in production and in hiring, in equilibrium $\nu_h = \nu_\ell$ is optimal. Thus, we can drop the type i subscript and just write ν, so that $n_{-1}(1 - \nu)$ workers are producing. The firm produces the final good with the technology:

$$
\begin{aligned}
y_t &= \left(\alpha^{1/\varepsilon}(u_t k_{t-1})^{1-1/\varepsilon} + (1 - \alpha)^{1/\varepsilon}(e^{g_z t}(1 - \bar{\zeta}_t)z_t n_{t-1}(1 - \nu_t))^{1-1/\varepsilon}\right)^{\frac{\varepsilon}{\varepsilon - 1}} \\
&\equiv \omega(u_t k_{t-1}, z_t(1 - \bar{\zeta}_t)n_{t-1}(1 - \nu_t))
\end{aligned}
\tag{H.17}
$$

where $1 - \bar{\zeta}_t \equiv 1 - \zeta_{0,t} + \zeta_{0,t}\zeta_1 = 1 - \zeta_{0,t}(1 - \zeta_1)$ is the average number of available efficiency units. The constant elasticity of substitution between effective capital and labor in production is given by ε, labor-augmenting growth trend g_z, and the productivity process z_t that follows, for the moment, the AR(1) specified in the main text.

The law of motion for capital is:

$$
k_t = (1 - \delta(u_t))k_{t-1} + \chi i_t \left(1 - \frac{1}{2}\kappa\left(\frac{i_t}{k_{t-1}} - \tilde{\delta}\right)^2\right), \tag{H.18}
$$

where $\tilde{\delta} \equiv g_z - (1 - \delta(\bar{u}))$, χ is the marginal efficiency of investment, and

$$
\delta(u) = \delta_0 + \delta_1(u - 1) + \frac{1}{2}\delta_2(u - 1)^2. \tag{H.19}
$$

The firm's value is given by:

$$J(n_{-1},k_{-1}) = \mathbb{E}\sum_{t=0}^{\infty}\left(\prod_{s=1}^{t} m_t\right)\left((1-\tau_k)(y_t - w_t n_t) + \tau_k \delta(\bar{u})\bar{q}k_{t-1} - i_t\right),$$

where production and capital follow from equations (H.17) and (H.18) and employment growth satisfies:

$$n_t = (\nu_t \mu(\theta_t)(1-\bar{\zeta}_t) + 1 - x)n_{t-1},$$

where $\mu(\theta_t) \equiv f(\theta_t)/\theta_t$ is the hiring probability per efficiency units of recruiters. Given a LLN, this value function holds both before and after learning the current types of individual workers.

The firm's value can be expressed recursively as:

$$\begin{aligned} J(n_{-1},k_{-1}) = \max_{\nu,u,k,I}\Bigg(&(1-\tau_k)\left(\omega(uk_{-1}, z(1-\bar{\zeta}_t)n_{-1}(1-\nu), \Psi) - n_{-1}(w_\zeta)\right) + \tau_k \bar{\delta} k_{t-1} - I \\ &+ q\left(-k + (1-\delta(u))k_{-1} + \chi I\left(1 - \frac{1}{2}\kappa\left(\frac{I}{k_{-1}} - \tilde{\delta}\right)^2\right)\right) \\ &+ \mathbb{E}\left[mJ(n_{-1}(\nu\mu(\theta)(1-\bar{\zeta}_{t+1}) + 1 - x), k)\right]\Bigg). \end{aligned} \tag{H.20}$$

H.2.1 Firm optimality

At an interior solution for the share of recruiters, the following optimality condition holds:

$$(1-\tau_k)\underbrace{z_t(1-\bar{\zeta})\left((1-\alpha)\frac{Y_t}{z_t(1-\bar{\zeta}_{t+1})n_{t-1}(1-\nu_t)}\right)^{\frac{1}{\varepsilon}}}_{\equiv mpl_t} = \mu(\theta_t)(1-\bar{\zeta})\mathbb{E}[m_{t+1}J_{\zeta,n}(n_t,k_t,)]. \tag{H.21}$$

There is no analogous FOC with $\mathbb{E}[J'_{h,n}]$ or $\mathbb{E}[J'_{\ell,n}]$ on the RHS because one period ahead, the two types are identical.

Thus, the marginal value of overall employment is given by:

$$\begin{aligned} J_{\zeta,n}(n_{t-1},k_{t-1}) &= (1-\tau_k)\left(mpl_t \times (1-\nu_t) - w_{\zeta,t}\right) + \left(\nu_t\mu(\theta_t)(1-\bar{\zeta}_t) + 1 - x\right)\mathbb{E}\left[m_{t+1}J_{\zeta,n}(n_t,k_t)\right] \\ &= (1-\tau_k)\left(mpl_t\left(1 + \frac{1-x}{\mu(\theta_t)(1-\bar{\zeta}_{t+1})}\right) - w_{\zeta,t}\right), \end{aligned} \tag{H.22}$$

using equation (H.21) to substitute for $\mathbb{E}\left[m_{t+1}J_{\zeta,n}(n_t,k_t)\right]$. The constant taxes τ_k do not distort the recruiting decision because they affect costs and benefits proportionally.

Define as $n_{\ell,-} \equiv \zeta_0 n_-$ and $n_{h,-} \equiv (1-\zeta_0)n_-$ and $n_{\zeta,-} = \zeta_1 n_{\ell,-} + n_{h,-} = (1-\bar{\zeta})n_-$. y_t in (H.17) depends only on $n_{\zeta,-}$ and then:

$$\begin{aligned} mpl &\equiv \frac{\partial y}{\partial n_-} = \frac{\partial y}{\partial n_{\zeta,-}}\frac{\partial n_{\zeta,-}}{\partial n_-} = \frac{\partial y}{\partial n_{\zeta,-}}(1-\bar{\zeta}) \\ mpl_\ell &\equiv \frac{\partial y}{\partial n_{\ell,-}} = \frac{\partial y}{\partial n_{\zeta,-}}\frac{\partial n_{\zeta,-}}{\partial n_{\ell,-}} = \frac{\partial y}{\partial n_{\zeta,-}}\zeta_1 = \frac{mpl}{1-\bar{\zeta}}\zeta_1 \\ mpl_h &\equiv \frac{\partial y}{\partial n_{h,-}} = \frac{\partial y}{\partial n_{\zeta,-}}\frac{\partial n_{\zeta,-}}{\partial n_{h,-}} = \frac{\partial y}{\partial n_{\zeta,-}} = \frac{mpl}{1-\bar{\zeta}}. \end{aligned}$$

Using the type-specific marginal products, analogous equations hold for the marginal value of individual types. This reflects the linearity of the production and recruiting technologies. More concretely,

$$\begin{aligned} J_{\ell,n}(n_{t-1},k_{t-1}) &= (1-\tau_k)\left(mpl_t \times (1-\nu_t)\frac{\zeta_1}{1-\bar{\zeta}_t} - w_{\ell,t}\right) + (\nu_t\mu(\theta_t)\zeta_1 + 1 - x)\,\mathbb{E}\left[m_{t+1}J_{\zeta,n}(n_t,k_t)\right] \\ &= (1-\tau_k)\left(mpl_t \times (1-\nu_t)\frac{\zeta_1}{1-\bar{\zeta}} - w_{\ell,t}\right) + \frac{\zeta_1}{1-\bar{\zeta}_t}\left(\nu_t + \frac{1-x}{\mu(\theta_t)\zeta_1}\right)\mu(\theta_t)(1-\bar{\zeta})\mathbb{E}\left[m_{t+1}J_{\zeta,n}(n_t,k_t)\right] \\ &= (1-\tau_k)\left(mpl_t\frac{\zeta_1}{1-\bar{\zeta}_t}\left(1+\frac{1-x}{\mu(\theta_t)\zeta_1}\right) - w_{\ell,t}\right), \end{aligned} \tag{H.23}$$

and replacing ζ_i with 1 and ℓ with h:

$$J_{h,n}(n_{t-1},k_{t-1}) = (1-\tau_k)\left(mpl_t\frac{1}{1-\bar{\zeta}_t}\left(1+\frac{1-x}{\mu(\theta_t)}\right) - w_{h,t}\right), \tag{H.24}$$

As in the household case, $J_{\zeta,n} = (1-\zeta_{0,t})J_{h,n} + \zeta_{0,t}J_{\ell,n}$. We can also write

$$J_n(n_{t-1},k_{t-1}) = J_{h,n}(n_{t-1},k_{t-1}) - (1-\tau_k)mpl_t\frac{\bar{\zeta}_t}{1-\bar{\zeta}_t} + (1-\tau_k)\underbrace{\zeta_0(w_{h,t}-w_{\ell,t})}_{\equiv w_{h,t}-w_{\zeta,t}}.$$

$$\Leftrightarrow \quad J_{h,n}(n_{t-1},k_{t-1}) = J_n(n_{t-1},k_{t-1}) + (1-\tau_k)mpl_t\frac{\bar{\zeta}_t}{1-\bar{\zeta}_t} - (1-\tau_k)\zeta_0(w_{h,t}-w_{\ell,t}). \tag{H.25}$$

The difference between the values of having an average instead of a high type is, all else equal, negative because of the lower MPL of the average type, but lower if the average type receives a lower wage rate than the high type.

Now, define the marginal profit of employing a worker at an arbitrary (off-equilibrium) wage $\tilde{w}$ and at the equilibrium wage from then on, given employment and capital at the firm:

$$\tilde{J}_{i,n}(n,k) = (1-\tau_k)(w_{i,t}-\tilde{w}) + J_{i,n}(n,k). \tag{H.26}$$

The optimality condition for the utilization rate is:

$$\delta'(u_t)q_tk_{t-1} = (\delta_1 + \delta_2(u_t-1))\,q_tk_{t-1} = (1-\tau_k)\left(\alpha\frac{y_t}{u_tk_{t-1}}\right)^{1/\varepsilon}k_{t-1} \equiv (1-\tau_k)\frac{mpk_t}{u_t}, \tag{H.27}$$

and for investment:

$$1 = q_t\chi\left(\left(1-\frac{1}{2}\kappa\left(\frac{i_t}{k_{t-1}}-\tilde{\delta}\right)^2\right) - \kappa\left(\frac{i_t}{k_{t-1}}-\tilde{\delta}\right)\frac{i_t}{k_{t-1}}\right). \tag{H.28}$$

The optimality condition for capital k' is given by:

$$\begin{aligned} q_t &= \mathbb{E}[m_{t+1}J_k(n_t,k_t)] \\ &= \mathbb{E}\left[m_{t+1}\left(mpk_{t+1}(1-\tau_k) + \tau_k\bar{\delta} + \left((1-\delta(u_{t+1})) + \chi\kappa\left(\frac{i_{t+1}}{k_t}\right)^2\left(\frac{i_{t+1}}{k_t}-\tilde{\delta}\right)\right)q_{t+1}\right)\right], \end{aligned} \tag{H.29}$$

where the marginal product of physical capital is:

$$mpk_{t+1} \equiv u_{t+1} \left(\alpha \frac{Y_{t+1}}{u_{t+1} k_t} \right)^{\frac{1}{\varepsilon}}. \tag{H.30}$$

H.3 Wage determination

Under Nash bargaining, the equilibrium wage for type h solves, for a generic time-varying ϕ_t:

$$w_{h,t} = \arg\max_{\tilde{w}} \tilde{V}_{h,n,t}(\tilde{w})^{\phi_t} \tilde{J}_{h,n,t}(\tilde{w})^{1-\phi_t}.$$

The solution of this bargaining problem requires that, after plugging in from equations (H.26) and (H.16), the following condition holds:

$$(1-\phi_t)(1-\tau_k) \underbrace{V_{h,n}(a_t, n_{t-1})}_{\equiv V_{h,n,t}} \left(\frac{c_t - \hat{h}_{t-1} c^a_{t-1}}{1 + (\sigma - 1)\gamma n_{t-1}} \right)^{\sigma} = \phi_t (1-\tau_n) \underbrace{J_{h,n}(n_{t-1}, k_{t-1})}_{\equiv J_{h,n,t}}. \tag{H.31}$$

We use this expression to simplify equation (H.15), after multiplying (H.15) through by $(1-\tau_k)$ and dividing by the current marginal utility of consumption. We multiply and divide within the expectation operator:

$$\begin{aligned}
&(1-\phi_t)(1-\tau_k) V_{h,n,t} \left(\frac{c_t - \hat{h}_{t-1} c^a_{t-1}}{1 + (\sigma - 1)\gamma n_{t-1}} \right)^{\sigma} \\
=&(1-\phi_t)(1-\tau_k)(1-\tau_n)(w_{h,t} - \omega_t) - (1-\phi_t)(1-\tau_k) \left(\frac{c_t - \hat{h}_{t-1} c^a_{t-1}}{1 + (\sigma - 1)\gamma n_{t-1}} \right) \gamma \left(\sigma + (\sigma-1) \frac{\hat{h}_{t-1} c^a_{t-1}}{c_t - \hat{h}_{t-1} c^a_{t-1}} \right) \\
&+ (1 - x - f_t(\theta_t)) \mathbb{E}_t \left[\beta_t \left(\frac{(c_t - \hat{h}_{t-1} c^a_{t-1})/(1 + (\sigma-1)\gamma n_{t-1})}{(c_{t+1} - \hat{h}_t c^a_t)/(1 + (\sigma-1)\gamma n_t)} \right)^{\sigma} \frac{1-\phi_t}{1-\phi_{t+1}} \right. \\
&\left. \times (1-\phi_{t+1}) \left(\left(\frac{c_{t+1} - \hat{h}_t c^a_t}{1 + (\sigma-1)\gamma n_t} \right)^{\sigma} (1-\tau_k) V_{h,n,t+1} + (1-\tau_n)(1-\tau_k)\zeta_{0,t+1}(w_{\ell,t+1} - w_{h,t+1}) \right) \right].
\end{aligned}$$

Next, we substitute from equation (H.31), taking care to keep track of the future bargaining power terms:

$$\begin{aligned}
&\phi_t (1-\tau_n) J_{h,n,t} \\
=&(1-\phi_t)(1-\tau_k)(1-\tau_n)(w_{h,t} - \omega_t) - (1-\phi_t)(1-\tau_k) \left(\frac{c_t - \hat{h}_{t-1} c^a_{t-1}}{1 + (\sigma - 1)\gamma n_{t-1}} \right) \gamma \left(\sigma + (\sigma-1) \frac{\hat{h}_{t-1} c^a_{t-1}}{c_t - \hat{h}_{t-1} c^a_{t-1}} \right) \\
&+ (1 - x - f_t(\theta_t)) \mathbb{E}_t \left[\frac{1-\phi_t}{1-\phi_{t+1}} m_{t+1} (1-\tau_n) \left(\phi_{t+1} J_{h,n,t+1} + (1-\tau_k)(1-\phi_{t+1})\zeta_{0,t+1}(w_{\ell,t+1} - w_{h,t+1}) \right) \right].
\end{aligned}$$

Then, we substitute from equation (H.24) for current $J_{h,n}$ on the LHS and for future J_{hn} from (H.25). Also, divide by $(1-\tau_k)$:

$$\begin{aligned}
&\phi_t (1-\tau_n) \left(mpl_t \frac{1}{1-\zeta_t} \left(1 + \frac{1-x}{\mu(\theta_t)} \right) - w_{h,t} \right) \\
=&(1-\phi_t)(1-\tau_n)(w_{h,t} - \omega_t) - (1-\phi_t) \left(\frac{c_t - \hat{h}_{t-1} c^a_{t-1}}{1 + (\sigma - 1)\gamma n_{t-1}} \right) \gamma \left(\sigma + (\sigma-1) \frac{\hat{h}_{t-1} c^a_{t-1}}{c_t - \hat{h}_{t-1} c^a_{t-1}} \right)
\end{aligned}$$

$$+(1-x-f_t(\theta_t))\mathbb{E}_t\left[\frac{1-\phi_t}{1-\phi_{t+1}}m_{t+1}(1-\tau_n)\left(\phi_{t+1}\frac{J_{\zeta,n,t+1}}{1-\tau_k}+\phi_{t+1}\frac{\bar{\zeta}_{t+1}}{1-\bar{\zeta}}mpl_{t+1}+\zeta_{0,t+1}w_{\ell,t+1}-w_{h,t+1}\right)\right]. \tag{H.32}$$

If ϕ_t were constant, we could substitute out for future $J_{\zeta,n}$ conveniently from the recruiting optimality condition (H.21).

H.4 Market clearing

Market clearing involves, first, the resource constraint of the economy:

$$y_t \equiv \left(\alpha^{1/\varepsilon}(u_t k_{t-1})^{1-1/\varepsilon}+(1-\alpha)^{1/\varepsilon}(z_t(1-\bar{\zeta}_t)n_{t-1}(1-\nu_t))^{1-1/\varepsilon}\right)^{\frac{\varepsilon}{\varepsilon-1}}=c_t+i_t. \tag{H.33}$$

Second, the law of motion of capital:

$$k_t=(1-\delta(u_t))k_{t-1}+\chi i_t\left(1-\frac{\kappa}{2}\left(\frac{i_t}{k_{t-1}}-\tilde{\delta}\right)^2\right). \tag{H.34}$$

Third, the law of motion for employment:

$$n_t=(1-x)n_{t-1}+f_t(\theta_t)(1-n_{t-1}). \tag{H.35}$$

Finally, the recruiter-unemployment ratio (analogous to market tightness) is:

$$\theta_t=\frac{\nu_t n_{t-1}(1-\bar{\zeta}_t)}{1-n_{t-1}}. \tag{H.36}$$

H.5 Efficiency

Following Hosios (1990), we assess the allocative efficiency of the decentralized equilibrium. We consider a social planner's problem that is subject to the same set of distortionary taxes as the equilibrium allocation but recognizes the externalities embodied in the matching function. Because the external habit would introduce an additional externality, we set habit $h=0$ in this section to derive a cleaner result. We also eliminate type heterogeneity by setting $\zeta_0=0$. This implies that $w_\zeta=w_h$ and $J_{\zeta,n}=J_n$. For simplicity, we use this simpler notation in this section.

The planner solves:

$$W(n_{-1},k_{-1};S)=\max_{x,i,k,n,\nu,u}\frac{c^{1-\sigma}(1+(\sigma-1)\gamma n_{-1})^\sigma-1}{1-\sigma}+\beta\mathbb{E}[W(n,k;S')|S] \tag{H.37}$$

subject to:

$$c+i=(1-\tau_n)wn_{-1}+(1-\tau_n)\omega(1-n_{-1})+(1-\tau_k)(y-n_{-1}w)+\tau_k\bar{\delta}k_{-1}+T \tag{H.38a}$$

$$k=(1-\delta(u))k_{-1}+\chi i\left(1-\frac{\kappa}{2}\left(\frac{i}{k_{-1}}-\tilde{\delta}\right)^2\right) \tag{H.38b}$$

$$n=(1-x)n_{-1}+\xi(\nu n_{-1})^\eta(1-n_{-1}). \tag{H.38c}$$

Let λ_b be the multiplier on the budget constraint (H.38a), λ_k the multiplier on the law of motion for capital (H.38b), and λ_n the multiplier on the law of motion for employment n.

The optimality conditions for c, u, ν, i, n, and k are, respectively:

$$\lambda_b = \left(\frac{c}{1+(\sigma-1)\gamma n_{-1}}\right)^{-\sigma} \tag{H.39a}$$

$$\lambda_k k_{-1}\delta'(u) = \lambda_b \frac{mpk}{u} k_{-1}(1-\tau_k) \tag{H.39b}$$

$$\lambda_n \eta \underbrace{\xi\left(\frac{\nu n_{-1}}{1-n_{-1}}\right)^{\eta-1}}_{\equiv\mu(\theta)} n_{-1} = \lambda_b(1-\tau_k)mpl \times n_{-1} \tag{H.39c}$$

$$\lambda_b = \lambda_k \chi\left(1-\frac{\kappa}{2}\left(\frac{i}{k_{-1}}-\tilde{\delta}\right)^2 - \kappa\frac{i}{k_{-1}}\left(\frac{i}{k_{-1}}-\tilde{\delta}\right)\right) \tag{H.39d}$$

$$\lambda_n = \beta\mathbb{E}[W_n(S')|S] \tag{H.39e}$$

$$\lambda_k = \beta\mathbb{E}[W_k(S')|S]. \tag{H.39f}$$

We also have two envelope conditions with respect to n_{-1} and k_{-1}:

$$W_n = \lambda_n\left(1-x+\eta\nu\underbrace{\xi\left(\frac{\nu n_{-1}}{1-n_{-1}}\right)^{\eta-1}}_{\equiv\mu(\theta)} -(1-\eta)\underbrace{\left(\frac{\nu n_{-1}}{1-n_{-1}}\right)^{\eta}}_{\equiv f(\theta)}\right)$$

$$\lambda_b(1-\tau_n)(w-\omega) - \lambda_b(1-\tau_k)w + \lambda_b(1-\tau_k)mpl(1-\nu) - \lambda_b\frac{\gamma\sigma c}{1+(\sigma-1)\gamma n_{-1}} \tag{H.40a}$$

$$W_k = \lambda_k\left(1-\delta(u)+\tau_k\bar{\delta}+\left(\frac{i}{k_{-1}}\right)^2\kappa\chi\left(\frac{i}{k_{-1}}-\tilde{\delta}\right)\right)+\lambda_b mpk. \tag{H.40b}$$

We now guess and verify that, when we appropriately choose a constant bargaining power ϕ, the allocation of the planner's problem and the decentralized equilibrium coincide. Define:

$$q \equiv \frac{\lambda_k}{\lambda_b} \tag{H.41a}$$

$$m \equiv \beta\frac{\lambda_b'}{\lambda_b} \tag{H.41b}$$

$$J_n \equiv \eta\frac{W_n}{\lambda_b} \tag{H.41c}$$

$$\phi \equiv 1-\eta. \tag{H.41d}$$

Guessing that allocations are the same, we verify that we also obtain the private-sector optimality conditions for utilization, recruiting, investment, and capital. From equation (H.39) and the equilibrium for capital (H.40b) along with the optimality condition for employment (H.39e):

$$q\delta'(u) = \frac{mpk}{u}(1-\tau_k) \tag{H.39b'}$$

$$(1-\tau_k)mpl = \mathbb{E}\left[m'\frac{W_n'}{\lambda_b'}\Big|S\right]\eta\mu(\theta) = \mathbb{E}[m'J_n'|S]\mu(\theta) \tag{H.39c'}$$

$$q = \chi^{-1}\left(1 - \frac{\kappa}{2}\left(\frac{i}{k_{-1}} - \tilde{\delta}\right)^2 - \kappa\frac{i}{k_{-1}}\left(\frac{i}{k_{-1}} - \tilde{\delta}\right)\right)^{-1} \quad \text{(H.39d')}$$

$$q = \mathbb{E}\left[m'\left(q'(1-\delta(u)) + q'\left(\frac{i}{k_{-1}}\right)^2\kappa\chi\left(\frac{i}{k_{-1}} - \tilde{\delta}\right) + \tau_k\bar{\delta} + mpk'\right)\Big|S\right]. \quad \text{(H.39f')}$$

Therefore, we checked that the guess satisfies all the optimality conditions and the equilibrium condition for capital. We now check the remaining condition, the equilibrium condition for employment, using equation (H.39c'):

$$\eta^{-1}J_n = \left(\left(1 + \frac{1-x}{\mu(\theta)}\right)mpl - w\right)(1-\tau_k) + (1-x-f(\theta))\mathbb{E}[m'J_n'|S]\frac{1-\eta}{\eta}$$
$$(1-\tau_n)(w-\omega) + -\frac{\gamma\sigma c}{1+(\sigma-1)\gamma n_{-1}}. \quad \text{(H.40a')}$$

Plug in from equation (H.22) for $\left(\left(1 + \frac{1-x}{\mu(\theta)}\right)mpn - w\right)(1-\tau_k)$:

$$\frac{1-\eta}{\eta}J_n = (1-\tau_n)(w-\omega) + -\frac{\gamma\sigma c}{1+(\sigma-1)\gamma n_{-1}} + (1-x-f(\theta))\mathbb{E}[m'J_n'|S]\frac{1-\eta}{\eta}. \quad \text{(H.40a")}$$

Compare this to equation (H.32) with constant ϕ and dividing that equation through by $(1-\phi)$ and substituting from equation (H.22):

$$\frac{\phi}{1-\phi}J_n\frac{1-\tau_n}{1-\tau_k}$$
$$=(1-\tau_n)(w-\omega) - \left(\frac{c}{1+(\sigma-1)\gamma n_{t-1}}\right)\gamma\sigma + (1-x-f(\theta))\mathbb{E}\left[m'J_n'|S\right]\frac{\phi}{1-\phi}\frac{1-\tau_n}{1-\tau_k}. \quad \text{(H.32')}$$

Comparing this equation to equation (H.40a") shows that the two equations are equal with $\phi = 1-\eta$ if and only if $\tau_n = \tau_k$.

H.6 Detrended economy

In this subsection, we augment the model by allowing for a stochastic trend in z_t by allowing for the growth rate g_z to be stochastic:

$$\ln g_{z,t} = \ln(g_z) + \epsilon_{p,t}, \quad \text{(H.43)}$$

where $\epsilon_{p,t}$ is the permanent shock to productivity. Thus, we replace g_z^t by $\prod_{s=1}^t g_{z,s}$ in the production function. When $\epsilon_{p,t} = 0$ for all t, we recover the deterministic growth process from the main text.

Capital, consumption, investment, the marginal value of employment, and wages grow with z_t, while all other variables are stationary. We denote detrended variables by $\sim$. To simplify notation, define the (detrended) marginal products of capital and labor as:

$$\widetilde{mpk}_t \equiv u_t\left(\alpha\frac{\tilde{y}_t}{u_t\tilde{k}_{t-1}}g_{z,t}\right)^{\frac{1}{\varepsilon}} = mpk_t \quad \text{(H.44)}$$

$$\widetilde{mpl}_t \equiv \tilde{z}_t(1-\bar{\zeta}_t)\left((1-\alpha)\frac{\tilde{y}_t}{\tilde{z}_t(1-\bar{\zeta}_t)n_{t-1}(1-\nu_t)}\right)^{\frac{1}{\varepsilon}}. \tag{H.45}$$

We substitute out for the number of recruiters by using the definition for market tightness:

$$n_{t-1}-\nu_{t-1}n_{t-1} = n_{t-1} - \frac{\theta_{t-1}}{1-\bar{\zeta}_t}(1-n_{t-1}). \tag{H.46}$$

Similarly, for the capital law of motion:

$$\tilde{k}_t = (1-\delta(u_t))g_{z,t}^{-1}\tilde{k}_{t-1} + \chi\tilde{i}_t\left(1-\frac{\kappa}{2}\left(\frac{\tilde{i}_t}{\tilde{k}_{t-1}}g_{z,t}-\tilde{\delta}\right)^2\right), \tag{H.47}$$

the resource constraint

$$\left(\alpha^{1/\varepsilon}(u_t\tilde{k}_{t-1}g_{z,t}^{-1})^{1/\varepsilon} + (1-\alpha)^{1/\varepsilon}(\tilde{z}_t(1-\bar{\zeta}_t)n_{t-1}(1-\nu_t))^{1-1/\varepsilon}\right)^{\frac{\varepsilon}{\varepsilon-1}} = \tilde{i}_t + \tilde{c}_t, \tag{H.48}$$

and the firm value with equilibrium choices for investment, capital, utilization, and recruiting:

$$\begin{aligned}\tilde{J}_t = \Bigg(&(1-\tau_k)(\tilde{y}_t - n_{t-1}\tilde{w}_{\zeta,t}) - \tilde{i}_t + \bar{\delta}\tilde{k}_{t-1}/g_{z,t} \\ &+ q_t\left(-\tilde{k}_t + (1-\delta(u))g_{z,t}^{-1}\tilde{k}_{t-1} + \chi\tilde{i}_t\left(1-\frac{1}{2}\kappa\left(\frac{\tilde{i}_t}{\tilde{k}_{t-1}}g_{z,t}-\tilde{\delta}\right)^2\right)\right) + \mathbb{E}_t\left[m_{t+1}g_z\tilde{J}_{t+1}\right]\Bigg).\end{aligned}$$

Since the constraint on capital accumulation binds, firm value is simply the present discounted value of the cash flow:

$$\tilde{J}_t = \left((1-\tau_k)(\tilde{y}_t - n_{t-1}\tilde{w}_{\zeta,t})) - \tilde{i}_t + \tau_k\bar{\delta}\tilde{k}_{t-1}/g_{z,t} + \mathbb{E}_t\left[m_{t+1}g_{z,t+1}\tilde{J}_{t+1}\right]\right). \tag{H.49}$$

We also have the marginal value of employment

$$\tilde{J}_{\zeta,n,t} = (1-\tau_k)\left(\widetilde{mpl}_t\left(1+\frac{1-x}{\mu(\theta_t)(1-\bar{\zeta}_t)}\right) - \tilde{w}_{\zeta,t}\right), \tag{H.50}$$

and the recruiting optimality condition:

$$(1-\tau_k)\widetilde{mpl}_t = \mu(\theta_t)(1-\bar{\zeta})\mathbb{E}_t[m_{t+1}g_{z,t+1}\tilde{J}_{\zeta,n,t+1}]. \tag{H.51}$$

Here, we use that $\mu(\theta) = \xi\theta^{\eta-1}$. It is useful to note that $f(\theta)(1-n_{-1}) = \xi(1-n_{-1})^{1-\eta}((1-\bar{\zeta})n_{-1}\nu)^{\eta} = (1-\bar{\zeta})n_{-1}\nu \times \xi\theta^{\eta-1}$, so that $f(\theta)(1-n_{-1}) = (1-\bar{\zeta})n_{-1}\nu\mu(\theta)$. This implies that the equilibrium laws of motion perceived by the household and the firm are, actually, identical.

Wage setting implies:

$$\begin{aligned}&\phi_t(1-\tau_n)\left(\widetilde{mpl}_t\frac{1}{1-\bar{\zeta}}\left(1+\frac{1-x}{\mu(\theta_t)}\right) - \tilde{w}_{h,t}\right) \\ =&(1-\phi_t)(1-\tau_n)(\tilde{w}_{h,t}-\tilde{\omega}_t) - (1-\phi_t)\left(\frac{c_t - \hat{h}_{t-1}c^a_{t-1}}{1+(\sigma-1)\gamma n_{t-1}}\right)\gamma\left(\sigma+(\sigma-1)\frac{\hat{h}_{t-1}\tilde{c}^a_{t-1}}{\tilde{c}_t - \tilde{h}_{t-1}\tilde{c}^a_{t-1}}\right)\end{aligned}$$

$$+(1-x-f_t(\theta_t))\mathbb{E}_t\left[\frac{1-\phi_t}{1-\phi_{t+1}}m_{t+1}g_{t+1}(1-\tau_n)\left(\phi_{t+1}\frac{\tilde{J}_{\zeta,n,t+1}}{1-\tau_k}+\phi_{t+1}\frac{\bar{\zeta}}{1-\bar{\zeta}}\widetilde{mpl}_{t+1}+\zeta_0(\tilde{w}_{\ell,t+1}-\tilde{w}_{h,t+1})\right)\right], \tag{H.52}$$

where $\tilde{h}_{t-1}=\hat{h}_{t-1}/g_{z,t}$ incorporates trend growth. Specifically, in equilibrium with $n^a_{t-2}=n_{t-2}$:

$$\tilde{h}_{t-1}=\frac{h}{g_{z,t}}\frac{1+(\sigma-1)\gamma n_{t-1}}{1+(\sigma-1)\gamma n_{t-2}}. \tag{H.53}$$

In our solution, we perturb $\ln(1-\tilde{h}_{t-1})$ and use the identity that $\tilde{h}_{t-1}=1-\exp(\ln(1-\tilde{h}_{t-1}))$.

Other equilibrium conditions are optimal utilization:

$$(\delta_1+\delta_2(u_t-1))\,q_t=(1-\tau_k)\frac{mpk_t}{u_t}, \tag{H.54}$$

optimal capital:

$$q_t=\mathbb{E}_t\left[m_{t+1}\left((1-\tau_k)\widetilde{mpk}_{t+1}+\bar{\delta}\frac{\tilde{k}_{t-1}}{g_{z,t}}+\left((1-\delta(u_{t+1}))+\kappa\chi\left(\frac{\tilde{i}_{t+1}}{\tilde{k}_t}g_{z,t+1}\right)^2\left(\frac{\tilde{i}_{t+1}}{\tilde{k}_t}g_{z,t+1}-\tilde{\delta}\right)\right)q_{t+1}\right)\right], \tag{H.55}$$

optimal investment:

$$q_t=\left(\left(1-\frac{1}{2}\kappa\left(\frac{\tilde{i}_t}{\tilde{k}_{t-1}}g_{z,t}-\tilde{\delta}\right)^2\right)-\kappa\left(\frac{\tilde{i}_t}{\tilde{k}_{t-1}}g_{z,t}-\tilde{\delta}\right)\frac{\tilde{i}_t}{\tilde{k}_{t-1}}g_{z,t}\right)^{-1}, \tag{H.56}$$

and the stochastic discount factor:

$$m_{t+1}=\beta_t g_{z,t+1}^{-\sigma}\left(\frac{\tilde{c}_t-\tilde{h}_{t-1}\tilde{c}^a_{t-1}}{\tilde{c}_{t+1}-\tilde{h}_t\tilde{c}^a_t}\frac{1+(\sigma-1)\gamma n_t}{1+(\sigma-1)\gamma n_{t-1}}\right)^{\sigma}. \tag{H.57}$$

Equations (H.47) to (H.57) determine:

1. Detrended capital $\tilde{k}_t$ from equation (H.47).
2. Detrended consumption $\tilde{c}_t$ from the resource constraint (H.48).
3. Detrended firm value $\tilde{J}$ from the Bellman equation (H.49).
4. Detrended marginal value of employment $\tilde{J}_{\zeta,n}$ from the envelope condition (H.50).
5. Recruiting intensity ν_t from equation (H.51).
6. Detrended wages $\tilde{w}_t$ from the Nash bargaining equation (H.52).
7. The utilization rate u_t from the utilization equation (H.54).
8. The shadow price of capital q_t from the capital equation (H.55).
9. Detrended investment $\tilde{i}_t$ from the investment equation (H.56).
10. The stochastic discount factor m_{t+1} from equation (H.57).

In addition, the following variables and equations matter:

11. Employment n_t is determined from equation (H.35).
12. Market tightness θ_t (or the number of recruiters) from equation (H.46).

And, for completeness, we add a few definitions:

13. The (detrended) marginal product of capital $\widetilde{mpk}_t$ from equation (H.44).
14. The (detrended) marginal product of labor $\widetilde{mpl}_t$ from equation (H.45).
15. Final goods production $\tilde{y}_t$

$$\tilde{y}_t \equiv \left(\alpha^{1/\varepsilon}(u_t\tilde{k}_{t-1}g_{z,t}^{-1})^{1/\varepsilon} + (1-\alpha)^{1/\varepsilon}(\tilde{z}_t(1-\bar{\zeta})n_{t-1}(1-\nu_t))^{1-1/\varepsilon}\right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{H.58}$$

16. The gross capital share cs_t from equation (H.59):

$$cs_t \equiv 1 - \frac{n_{t-1}w_{\zeta,t}}{\widetilde{y}_t}. \tag{H.59}$$

17. The net capital share ncs_t from equation (H.60):

$$ncs_t \equiv 1 - \frac{n_{t-1}w_{\zeta,t}}{\widetilde{y}_t} - \delta_t\frac{\tilde{k}_{t-1}}{\widetilde{y}_t g_{z,t}}. \tag{H.60}$$

18. To compute welfare in the presence of trend growth, we first shift the location of the value function V_t in (H.5) to $\hat{V}_t \equiv (1-\beta)V_t + \frac{1}{1-\sigma}$ for $\sigma \neq 1$:

$$\begin{aligned}\hat{V}_t &= (1-\beta)V_t + \frac{1}{1-\sigma}\\ &= (1-\beta)\frac{(c_t - \hat{h}(n_{t-1})c_{t-1}^a)^{1-\sigma}(1+(\sigma-1)\gamma n_{t-1})^\sigma}{1-\sigma} + \beta\mathbb{E}_t\left[(1-\beta)V_{t+1} + \frac{1}{1-\sigma}\right]\\ &\equiv (1-\beta)\frac{(c_t - \hat{h}(n_{t-1})c_{t-1}^a)^{1-\sigma}(1+(\sigma-1)\gamma n_{t-1})^\sigma}{1-\sigma} + \beta\mathbb{E}_t[\widehat{(1-\beta)V_{t+1}}].\end{aligned} \tag{H.61}$$

Let $\hat{V}_t = \prod_{s=0}^t g_s^{1-\sigma}\check{V}_t$, so that $\check{V}_t$ is the detrended version of the welfare measure:

$$\check{V}_t = (1-\beta)\frac{(\tilde{c}_t - \tilde{h}(n_{t-1})\tilde{c}_{t-1}^a)^{1-\sigma}(1+(\sigma-1)\gamma n_{t-1})^\sigma}{1-\sigma} + \beta\bar{g}^{1-\sigma}\mathbb{E}_t[\check{V}_{t+1}].$$

When $\sigma > 1$, we find it useful to actually compute

$$\begin{aligned}\tilde{V}_t &= \check{V}_t\frac{1-\beta\bar{g}^{1-\sigma}}{1-\beta} - \frac{1}{1-\sigma}\\ &= (1-\beta\bar{g}^{1-\sigma})\frac{(\tilde{c}_t - \tilde{h}(n_{t-1})\tilde{c}_{t-1}^a)^{1-\sigma}(1+(\sigma-1)\gamma n_{t-1})^\sigma - 1}{1-\sigma} + \beta\bar{g}^{1-\sigma}\mathbb{E}_t\left[\check{V}_{t+1}\frac{1-\beta\bar{g}^{1-\sigma}}{1-\beta} - \frac{1}{1-\sigma}\right]\\ &= (1-\beta\bar{g}^{1-\sigma})\frac{(\tilde{c}_t - \tilde{h}(n_{t-1})\tilde{c}_{t-1}^a)^{1-\sigma}(1+(\sigma-1)\gamma n_{t-1})^\sigma - 1}{1-\sigma} + \beta\bar{g}^{1-\sigma}\mathbb{E}_t\left[\tilde{V}_{t+1}\right].\end{aligned} \tag{H.62}$$

In this version of the model, there are the following exogenous processes:

19. Bargaining power

$$\log \phi_t = (1 - \rho_\phi) \log(\bar{\phi}) + \rho_\phi \log \phi_{t-1} + \epsilon_{\phi,t}. \tag{H.63}$$

20. Stationary labor productivity

$$\log z_t = (1 - \rho_z) \log(\bar{z}) + \rho_z \log z_{t-1} + \epsilon_{z,t}. \tag{H.64}$$

21. Permanent labor productivity

$$\log(g_{z,t}) = \log(g_z) + \epsilon_{p,t}. \tag{H.65}$$

22. Minimum wage

$$\log(\tilde{w}_{\ell,t}) = (1 - \rho_{w,\ell}) \log(\bar{w}_\ell) + \rho_{w,\ell} \log(\tilde{w}_{\ell,t-1}) + \epsilon_{w,\ell,t}. \tag{H.66}$$

23. Unemployment benefits

$$\log(\tilde{\omega}_t) = (1 - \rho_\omega) \log(\bar{\omega}) + \rho_\omega \log(\tilde{\omega}_{t-1}) + \epsilon_{\omega,\ell,t}. \tag{H.67}$$

H.7 Balanced growth path and data matching

Along the BGP of the economy, the discount factor becomes:

$$\bar{m} = \beta g_z^{-\sigma} \tag{H.68}$$

and the number of recruiters is given from (H.69):

$$(1 - \bar{\zeta})\bar{\nu}\bar{n} = \bar{\theta}(1 - \bar{n}) \quad \Leftrightarrow \quad \bar{n} - \bar{\nu}\bar{n} = \bar{n} - \frac{\bar{\theta}}{1 - \bar{\zeta}}(1 - \bar{n}). \tag{H.69}$$

In an initial calibration, we can normalize capacity utilization to be 1 along the BGP to get:

$$\bar{u} = 1 \tag{H.70}$$

$$\delta_1 = (1 - \tau_k)\overline{mpk}. \tag{H.71}$$

If δ_1 is given, rather than calibrated, utilization solves:

$$(\delta_1 + \delta_2(\bar{u}))\, \bar{q} = (1 - \tau_k)\frac{\overline{mpk}}{\bar{u}}. \tag{H.72}$$

Clearly, if $\bar{u} = \bar{u} = 1$, equation (H.71) holds.

The BGP optimality condition for capital can be written as:

$$1 = \bar{m}\left((1 - \tau_k)\overline{mpk} + (1 - (1 - \tau_k)\delta(\bar{u}))\bar{q} + \kappa\Big(\frac{\bar{i}}{\bar{k}/g_z}\Big)^2\Big(\frac{\bar{i}}{\bar{k}/g_z} - \tilde{\delta}\Big)\bar{q}\right)$$

$$\Leftrightarrow \quad \frac{\bar{q}/\bar{m} - (1 - (1 - \tau_k)\delta(\bar{u})) - \kappa\left(\frac{\bar{i}}{\bar{k}/g_z}\right)^2\left(\frac{\bar{i}}{\bar{k}} - \tilde{\delta}\right)}{1 - \tau_k} = \overline{mpk}. \tag{H.73}$$

If $\bar{u} = 1$ holds, the marginal product of capital does not depend on adjustment costs in the steady state.

Investment along the BGP is given by:

$$\bar{i} = \frac{(g_z - (1-\delta(\bar{u})))}{1 - \frac{1}{2}\kappa\left(\frac{\bar{i}}{\bar{k}/g_z} - \tilde{\delta}\right)^2} \frac{\bar{k}}{g_z}, \tag{H.74}$$

where $\tilde{\delta} \equiv 1 - \frac{1}{1-\delta_0} g_z$.

The steady-state price of capital is:

$$\bar{q} = \frac{1}{1 - \frac{1}{2}\left(\frac{\bar{i}}{\bar{k}/g_z} - \tilde{\delta}\right)^2 - \left(\frac{\bar{i}}{\bar{k}/g_z}\right)^2 \left(\frac{\bar{i}}{\bar{k}/g_z} - \tilde{\delta}\right)}. \tag{H.75}$$

If we cannot calibrate the adjustment costs in investment and utilization, then $\frac{\bar{i}}{\bar{k}/g_z}$, $\bar{u}$, $\bar{q}$, and $\overline{mpk}$ are jointly determined by equations (H.72), (H.73), (H.78), and (H.75). If $\bar{q} = \bar{u} = 1$, then $\frac{\bar{i}}{\bar{k}/g_z}$ and $\overline{mpk}$ are available in closed form.

Using the recruiting optimality condition (H.51), the wage equation (H.52) becomes:

$$\begin{aligned}
&\bar{\phi}(1-\tau_n)\left(\frac{\overline{mpl}}{1-\bar{\zeta}}\left(1 + \frac{1-x}{\mu(\bar{\theta})}\right) - \tilde{w}_{h,t}\right) \\
=&(1-\bar{\phi})(1-\tau_n)(\bar{w}_h - \bar{\omega}) - (1-\bar{\phi})\left(\frac{\bar{c} - \tilde{h}\bar{c}^a}{1+(\sigma-1)\gamma\bar{n}}\right)\gamma\left(\sigma + (\sigma-1)\frac{\tilde{h}\bar{c}^a}{\bar{c} - \tilde{h}\bar{c}^a_{t-1}}\right) \\
&+ (1 - x - \bar{f}(\bar{\theta}))(1-\tau_n)\left(\bar{\phi}\frac{\overline{mpl}}{1-\bar{\zeta}} + \bar{m}\bar{g}\bar{\phi}\frac{\bar{\zeta}}{1-\bar{\zeta}}\overline{mpl} + \bar{m}\bar{g}\zeta_0(\bar{w}_\ell - \bar{w}_h)\right).
\end{aligned}$$

Using $\bar{c}_a = \bar{c}$, we have that $(\bar{c} - \tilde{h}\bar{c}^a)\left(\sigma + (\sigma-1)\frac{\tilde{h}\bar{c}^a}{\bar{c}-\tilde{h}\bar{c}^a_{t-1}}\right) = \bar{c}(\sigma - \tilde{h})$. With this result, canceling $1-\tau_n$ and using that $f(\theta_t) \equiv \theta_t\mu(\theta_t) = \bar{\xi}\theta^\eta$ and that $1 = 1 - x - \bar{f} + \frac{\bar{f}}{\bar{n}}$ yields:

$$\begin{aligned}
&\bar{\phi}\frac{\overline{mpl}}{1-\bar{\zeta}}\left(1 + \bar{\theta} - \zeta_0(1-x-f)\bar{m}\bar{g}\right) \\
=&\bar{w}_h(1 - (1-x-f)\bar{m}\bar{g}\zeta_0) + (1-x-f)\bar{m}\bar{g}\zeta_0\bar{w}_\ell - (1-\bar{\phi})\bar{\omega} - \frac{1-\bar{\phi}}{1-\tau_n}\left(\frac{\bar{c}(\sigma-\tilde{h})}{1+(\sigma-1)\gamma\bar{n}}\right)\gamma.
\end{aligned} \tag{H.76}$$

We solve this equation for γ.

The marginal product of labor along the BGP is:

$$\overline{mpl} = (1-\bar{\zeta})\left((1-\alpha)\frac{\bar{y}}{\left(\bar{n} - \frac{\bar{\theta}}{(1-\bar{\zeta})}(1-\bar{n})\right)(1-\bar{\zeta})}\right)^{1/\varepsilon}.$$

Note that we can rewrite the definition of $\overline{mpk}$ as:

$$\frac{\bar{k}}{g_z} = \bar{n}(1-\bar{\nu})\left(\left(\frac{\alpha}{1-\alpha}\right)^{1/\varepsilon}\left(\frac{(\overline{mpk}/\bar{u})^{\varepsilon-1}}{\alpha} - 1\right)\right)^{-\frac{\varepsilon}{\varepsilon-1}} \overset{\varepsilon\to 1}{\to} \bar{n}(1-\bar{\nu})\frac{\alpha}{1-\alpha}(\overline{mpk}/\bar{u})^{-\frac{1}{1-\alpha}}.$$

This expression is useful to express output in terms of $\overline{mpk}$ and employment. Recall the expression for detrended production net of recruiting services:

$$
\begin{aligned}
&\bar{y} = \left(\alpha^{1/\varepsilon}(u_t \tilde{k}_{t-1} g_z^{-1})^{1/\varepsilon} + (1-\alpha)^{1/\varepsilon}(\tilde{z}_t(1-\bar{\zeta})n_{t-1}(1-\nu_t))^{1-1/\varepsilon}\right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \Leftrightarrow \\
&\bar{y} = \frac{(\overline{mpk})^{\varepsilon}}{\alpha}\frac{\bar{k}}{g_z}\bar{u}^{1-\epsilon} = \bar{n}(1-\bar{\nu})(1-\bar{\zeta})\frac{(\overline{mpk}/\bar{u})^{\varepsilon}}{\alpha}\left(\left(\frac{\alpha}{1-\alpha}\right)^{1/\varepsilon}\left(\frac{\overline{mpk}^{\varepsilon-1}}{\alpha}-1\right)\right)^{-\frac{\varepsilon}{\varepsilon-1}} \\
&\overset{\varepsilon\to 1}{\to} \bar{n}(1-\bar{\nu})(\overline{mpk}/\bar{u})^{-\frac{\alpha}{1-\alpha}}\frac{\alpha}{1-\alpha}.
\end{aligned}
\tag{H.77}
$$

The law of motion for capital gives us:

$$
\frac{\bar{c}}{\bar{y}} = 1 - \left(1 - \frac{1-\delta_0}{g_z}\right)\frac{\bar{k}}{\bar{y}g_z} = 1 - \left(1-\frac{1-\delta_0}{g_z}\right)\frac{\bar{k}g_z}{\bar{y}g_z}\frac{\alpha}{\overline{\overline{mpk}}^{\varepsilon}}\bar{u}^{\varepsilon-1}. \tag{H.78}
$$

The law of motion for employment implies:

$$
\bar{n} = \frac{f(\bar{\theta})}{x + f(\bar{\theta})}. \tag{H.79}
$$

If we combine equation (H.21) with (H.22):

$$
\bar{w}_{\zeta} = \overline{mpl}\left(1 - \frac{1-(1-x)\bar{m}g_z}{\bar{m}g_z\mu(\bar{\theta})(1-\bar{\zeta})}\right). \tag{H.80}
$$

We use this equation to set w_{ζ}.

For a given calibration target (e.g., $w_{\ell} = \frac{1}{3}w_{\zeta}$), we have:

$$
\bar{w}_h = \frac{\bar{w}_{\zeta} - \bar{w}_{\ell}}{1-\zeta_0}. \tag{H.81}
$$

Per definition:

$$
\mu(\bar{\theta}) = \frac{f(\bar{\theta})}{\bar{\theta}} = \xi\bar{\theta}^{\eta-1}.
$$

In general, we have the following unknowns and equations:

1. Employment $\bar{n}$ from the law of motion (H.79).
2. Capital $\bar{k}$ from the first-order condition (H.73).
3. Investment from the capital law of motion (H.78).
4. Capacity utilization, which follows from equation (H.70) when δ_1 is calibrated or, more generally, from (H.72).
5. The derivative of capacity utilization along the BGP δ_1 from equation (H.71).
6. The price of capital, which follows from equation (H.75).
7. Consumption $\bar{c}$ from the resource constraint (H.78).
8. Number of recruiters $\bar{n}\bar{\nu}$ from the definition of market tightness (H.69).

9. The stochastic discount factor $\bar{m}$ from no arbitrage (H.68).

10. Production $\bar{y}$ per definition (H.77).

and with these variables we can find market tightness $\bar{\theta}$ and the wages. In our calibration, we set the production function parameters as follows:

- Capital share: $\alpha = (\text{NIPA capital share})^{\varepsilon} \left(\frac{\bar{y}\bar{g}}{\bar{k}}\right)^{1-\varepsilon}$.
- Average depreciation rate: $\delta_0 = \frac{\text{NIPA depreciation}}{\bar{y}\bar{g}} \times \frac{\bar{y}\bar{g}}{\bar{k}}$.
- Rate of time preference: $\bar{\beta} = \bar{g}_z^{\sigma} \left(1 - \delta_0(1-\tau_k) + (1-\tau_k)\left(\alpha \frac{\bar{k}}{\bar{y}\bar{g}_z}\right)^{1/\varepsilon}\right)^{-1}$.

We can also fix $\bar{n}$ and choose γ:

1. Preference for leisure γ given n from wage setting.

2. Tightness $\bar{\theta}$ from the law of motion (H.79)

$$\bar{\theta} = \left(\frac{\bar{n}x}{\xi \times (1-\bar{n})}\right)^{1/\eta}. \tag{H.79'}$$

3. Capital-to-production ratio $\frac{\bar{k}}{\bar{y}\bar{g}}$ from the first-order condition (H.73).

4. Investment-to-production ratio from the law of motion of capital (H.78).

5. Capacity utilization, which follows from equation (H.70) when δ_1 is calibrated or, more generally, from (H.72).

6. The derivative of capacity utilization along the BGP δ_1 from equation (H.71).

7. The price of capital, which follows from equation (H.75).

8. Consumption-to-production ratio $\frac{\bar{c}}{\bar{y}}$ from the resource constraint (H.78).

9. Number of recruiters $\bar{n}\bar{\nu}$ from the definition of market tightness (H.69).

10. The stochastic discount factor $\bar{m}$ from equation (H.68).

11. Production $\bar{y}$ per definition (H.77).

The additional variables and the exogenous processes follow directly from the detrended economy.

H.8 U.S. business cycle data

To map observations into variables in the model we proceed as follows. First, we compute consumption as the sum of real services and non-durable consumption, divided by the civilian non-institutionalized population above 16. Specifically:

$$C_t = \frac{\frac{\text{DSERRA3Q086SBEA}_t}{\text{DSERRA3Q086SBEA in 2009}} \times \text{PCESVC96 in 2009} + \frac{\text{DGOERA3Q086SBEA}_t}{\text{DGOERA3Q086SBEA in 2009}} \times \text{PCNDGC96 in 2009}}{\text{CN16OV}_t}.$$

We multiply the base year (2009 average) value of the real consumption expenditure by the corresponding quantity index to obtain dollar amounts for longer horizons, i.e., before 1999.

We compute investment as the sum of consumer durables and gross private domestic investment, divided by the civilian non-institutionalized population above 16. Specifically:

$$I_t = \frac{\text{GDPIC96}_t + \frac{\text{DDURRA3Q086SBEA}_t}{\text{DDURRA3Q086SBEA in 2009}} \times \text{PCDGCC96 in 2009}}{\text{CN16OV}_t}.$$

Real GDP per capita is defined as the sum of real per capita investment and consumption:

$$Y_t = C_t + I_t.$$

H.9 Introducing product market power

An interesting extension of the model is to introduce market power for firms. To do so, we need to differentiate among firms. There is a representative final goods producing firm that produces aggregate output $\bar{y}_t$ as a CES aggregate of intermediate goods $y_t(i)$ with elasticity $\iota > 1$:

$$\bar{y}_t = \left(\int_0^1 y_t(i)^{1-1/\iota} di \right) \frac{\iota}{\iota - 1}. \tag{H.82}$$

Let $p_t(i)$ denote the price of each individual variety and $\bar{p}_t$ the optimal aggregate price index. Standard cost minimization for the representative final goods firm then implies that demand for variety i is given by:

$$y_t(i) = \bar{y}_t \left(\frac{p_t(i)}{\bar{p}_t} \right)^{-\iota}. \tag{H.83}$$

Each variety is produced according to the following production function:

$$\begin{aligned} y_t(i) &= \left(\alpha^{1/\varepsilon} (u_t(i) k_{t-1}(i))^{1-1/\varepsilon} + (1-\alpha)^{1/\varepsilon} (z_t n_{t-1}(i)(1-\nu_t(i)))^{1-1/\varepsilon} \right)^{\frac{\varepsilon}{\varepsilon-1}} - \Phi_t \\ &\equiv \psi(u_t(i) k_{t-1}(i), z_t n_{t-1}(i)(1-\nu_t(i)); \Phi_t), \end{aligned} \tag{H.84}$$

where $\Phi_t \geq 0$ is the fixed cost of operating. Along the BGP, it grows at the rate of labor productivity.

The intermediate goods producing firm takes its demand schedule (H.83) into account and has revenues of $p_t(i) \left(\frac{p_t(i)}{\bar{p}_t} \right)^{-\iota} \bar{y}_t$. Equivalently, revenue as a function of quantities becomes:

$$\bar{p}_t y_t(i)^{1-1/\iota} \bar{y}_t^{-1/\iota}.$$

In a symmetric equilibrium, each firm sets the same price so that $\bar{y}_t = y_t(i)$ and $\bar{p}_t = p_t(i)$ for all i. We choose the final good as the numeraire in the period.

With market power, as firms consider employing an extra worker or unit of capital, they take into account that the marginal revenue product is smaller than the marginal product. Importantly, the functional form for the match surplus $\tilde{J}_n(n, k)$ is unchanged but, as (H.22') shows, the marginal value of employment that enters into it reflects the lower marginal revenue product.

To see this, note that now the following optimality condition holds for recruiting:

$$(1-\tau_k)\underbrace{(1-1/\iota)z_t\left((1-\alpha)\frac{Y_t}{z_t(1-\bar{\zeta})n_{t-1}(1-\nu_t)}\right)^{\frac{1}{\varepsilon}}}_{\equiv mrpl_t} = \mu(\theta_t)\mathbb{E}[m_{t+1}J_n(n_t,k_t,)]. \tag{H.21'}$$

Thus, the marginal value of employment is given by:

$$\begin{aligned} J_{\zeta,n}(n_{t-1},k_{t-1}) &= (1-\tau_k)\left(mrpl_t \times (1-\nu_t) - w_t\right) + \left(\nu_t(1-\bar{\zeta}_t)\mu(\theta_t) + 1 - x\right)\mathbb{E}\left[m_{t+1}J_{\zeta,n}(n_t,k_t)\right] \\ &= (1-\tau_k)\left(mrpl_t\left(1+\frac{1-x}{\mu(\theta_t)(1-\bar{\zeta})}\right) - w_t\right), \end{aligned} \tag{H.22'}$$

using equation (H.21') to substitute for $\mathbb{E}\left[m_{t+1}J_{\zeta,n}(n_t,k_t)\right]$.

The optimality condition for the utilization rate becomes:

$$\begin{aligned} \delta'(u_t)q_tk_{t-1} = \left(\delta_1+\delta_2(u_t-1)\right)q_tk_{t-1} &= (1-\tau_k)(1-1/\iota)\left(\alpha\frac{y_t}{u_tk_{t-1}}\right)^{1/\varepsilon}k_{t-1} \\ &\equiv (1-\tau_k)\frac{mrpk_t}{u_t}. \end{aligned} \tag{H.27'}$$

The optimality condition for capital k' becomes:

$$q_t = \mathbb{E}\left[m_{t+1}\left(mrpk_{t+1}(1-\tau_k) + \tau_k\bar{\delta} + \left((1-\delta(u_{t+1})) + \chi\kappa\left(\frac{i_{t+1}}{k_t}\right)^2\left(\frac{i_{t+1}}{k_t}-\tilde{\delta}\right)\right)q_{t+1}\right)\right]. \tag{H.29'}$$

The marginal revenue product of physical capital is:

$$mrpk_{t+1} \equiv u_{t+1}(1-1/\iota)\left(\alpha\frac{y_{t+1}}{u_{t+1}k_t}\right)^{\frac{1}{\varepsilon}}. \tag{H.30'}$$

Market power also has an impact on the calibration. Monopolistic competition is an extra source of profits in the economy: In the detrended economy, the flow profit is $\bar{y}/\iota$ along the BGP. We consider two variants for calibrating the model with market power that keep the aggregate capital share in the economy unchanged:

1. No fixed cost, lower capital share in production. Here, we set the fixed cost of production Φ_t to zero. Then, we calibrate ι and adjust α so that the gross capital share in the economy is unchanged. Specifically, we target a capital share in production of $1-(1-0.31)(1-1/\iota)^{-1/\epsilon}$.

2. Fixed cost, same capital share in production. Here, we set the detrended fixed cost of production equal to the share of profits from monopolistic competition: $\tilde{\Phi}_t = \bar{y}/\iota$.

H.10 Identification: Additional relationships

Recall that we use three moments to pin down three parameters: ω_z, ω_ϕ, and κ/δ_0^2. In the main text, we show the three bivariate plots that show the important interaction terms among these three parameters. For completeness, we show here in Figure H.16 the additional bivariate plots. It is clear from this figure that the required standard deviations vary little with the adjustment cost and the adjustment cost depends little on ω_ϕ.

(a) Cyclicality of wages (b) Volatility of TFP (c) Volatility of investment

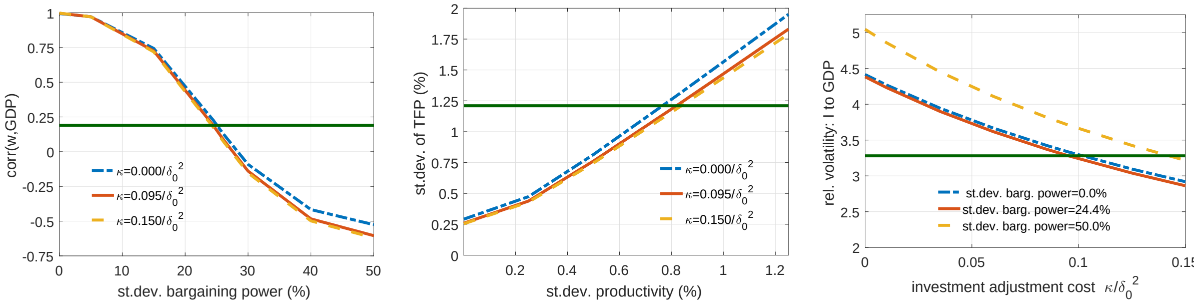
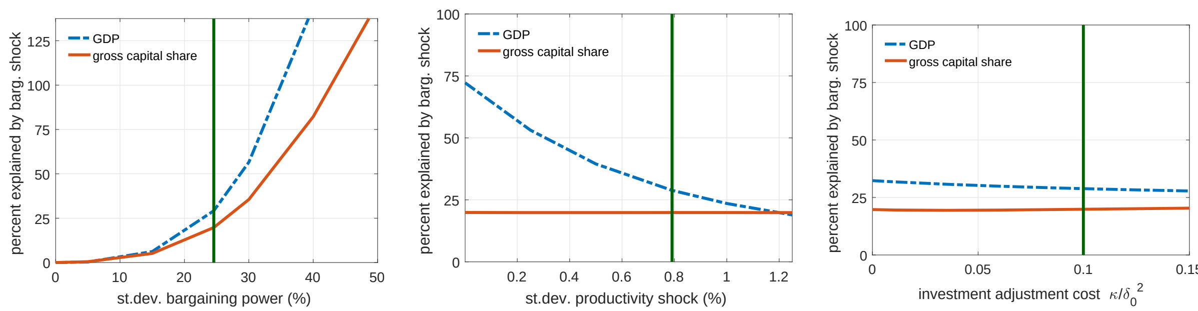


Figure H.16: Identifying ω_z, ω_ϕ, and κ/δ_0^2. Additional relationships.

Finally, the three panels in Figure H.17 complete this discussion by showing the explained standard deviation of GDP and the gross capital share as a function of the standard deviation of the bargaining and productivity shocks and the investment adjustment. Only the size of the bargaining shocks has noticeable effects on the volatility of the capital share.



Note: The vertical lines indicate HP-filtered data moments.

Figure H.17: Explained standard deviation as a function of calibrated parameters.

H.11 Euler equation errors

Our model has two Euler equations: (1) The recruiting optimality condition (H.51) and (2) the capital optimality condition (H.55). We transform the Euler equation error to consumption units. To do so, take an Euler equation with a generic return R^i_{t+1}. Following Fernández-Villaverde and Rubio-Ramírez (2006), the Euler equation error in state s_t is:

$$EE(s_t) = \left| 1 - \frac{u_c^{-1}\left(\mathbb{E}_t\left[\beta_t g_z^{-\sigma} u_c(c(s_{t+1}); n(s_{t+1})) R^i(s_{t+1})\right]; n(s_t)\right)}{c(s_t)} \right|. \tag{H.85}$$

Here:

$$R^{\nu}_{t+1} = \left((1-\tau_k)\widetilde{mpl}_t\right)^{-1} \mu(\theta_t) g_{z,t+1}\tilde{J}_{n,t+1}$$

$$R^k_{t+1} = q_t^{-1}\left(\widetilde{mpk}_{t+1}(1-\tau_k) + \bar{\delta}\tau_k + \left((1-\delta(u_{t+1})) + \chi_{t+1}\left(\frac{\tilde{I}_{t+1}}{\tilde{k}_t}g_{z,t+1}\right)^2\left(\frac{\tilde{I}_{t+1}}{\tilde{k}_t}g_{z,t+1} - \tilde{\delta}\right)\right) q_{t+1}\right)$$

$$u_c^{-1}(\tilde{u}_c; n) = \tilde{u}_c^{-\frac{1}{\sigma}} \times (1 + (\sigma - 1)\gamma n).$$

The difficulty in our setup is that, because of the pruning, the state in terms of the endogenous observables is not uniquely defined: Any given level of capital can be reached by different combinations of the first-, second-, and third-order components of the solution. Thus, as in Andreasen et al. (2018), we resort to Monte Carlo integration (with a burn-in of 1,000 simulations). The pseudo-code below outlines the algorithm.

`Pseudo-code for Monte Carlo integration`

1. `Simulate the model for 6,000 periods.`
2. `Discard the first 1,000 periods and save the remaining 5,000 draws for the state` s_t `as` $\{s_t^{(\ell)}\}_\ell$.
3. `For` $\ell = 1, \ldots, 5,000$:
   (a) $s_t^{(\ell)}$, `compute the vector of current policies and stack it with the state vector:` $s_t^{+,(\ell)}$.
   (b) `For` $m = 1, \ldots, 1,000$:
      i. `Draw` $\epsilon_{t+1}^{(m)} \sim \mathcal{N}(0, I)$.
      ii. `Compute` $s_t^{+,(\ell,m)} = f(s_t^{(ell)}, \epsilon_{t+1}^{(m)})$,
   (c) `Average over` d:

$$EE(s_t^{(\ell)}) = \left| 1 - \frac{u_c^{-1}\left(1,000^{-1}\sum_{m=1}^{1,000}\left(\beta_t^{(\ell)} g_z^{-\sigma} u_c\left(c_{t+1}^{(\ell,m)}); n_{t+1}^{(\ell,m)}\right) R_{t+1}^{i}{}^{(\ell,m)}\right); n(s_t^{(\ell)})\right)}{c(s_t^{(\ell)})} \right|.$$

4. `Compute moments of` $EE(s_t)$.

We find that the implied Euler equation errors are reasonably small for both the capital and recruiting Euler equations. Table H.8(a) reports the mean of the Euler equation errors for both Euler equations along with their distribution. The average Euler equation error is below 10^{-2}, implying that agents would pay less than 1% of their period consumption to avoid the approximation error. The 99th percentile of approximation is below 2%. This is only a bit worse than the RBC analogue of our search model, as panel (c) shows. Errors in the search and matching model without bargaining shocks in panel (b) are smaller than in the RBC model.

Figure H.18 shows the mean, minimum, and maximum Euler equation errors also as a function of the endogenous state of the economy, i.e., capital and employment. The dependence is weak, though, except for some extreme values of employment and, even in this case, they still average below 1% of consumption.

H.12 Partial filter for bargaining power

H.12.1 Derivation

We derive the partial filter from (H.32) using the definition that:

$$w_{\zeta,t} \equiv (1 - \zeta_0) w_{h,t} + \zeta_0 w_{\ell,t} \quad \Leftrightarrow \quad w_{h,t} - w_{\ell_t} = \frac{w_{\zeta,t} - w_{\ell,t}}{1 - \zeta_0}$$

(a) Baseline search & matching model

Euler Equation	Mean	Min	p1	p5	Median	p95	p99	Max
Capital EE	-3.03	-7.87	-5.67	-4.99	-3.86	-2.42	-1.77	-1.26
Recruiting EE	-2.61	-7.43	-4.55	-3.81	-2.77	-2.20	-1.73	-1.16

(b) Search & matching model without bargaining shocks

Euler Equation	Mean	Min	p1	p5	Median	p95	p99	Max
Capital EE	-4.34	-7.80	-6.15	-5.47	-4.41	-3.95	-3.83	-3.62
Recruiting EE	-3.54	-7.46	-5.42	-4.70	-3.64	-3.11	-2.95	-2.52

(c) Hansen-Rogerson RBC model

Euler Equation	Mean	Min	p1	p5	Median	p95	p99	Max
Capital EE	-3.96	-8.04	-5.99	-5.30	-4.27	-3.45	-2.92	-2.29
Labor supply EE	-3.25	-7.18	-5.53	-4.83	-3.75	-2.66	-2.11	-1.56

Table H.8: Euler equation errors (expressed as $\log_{10}$): Mean and distribution.

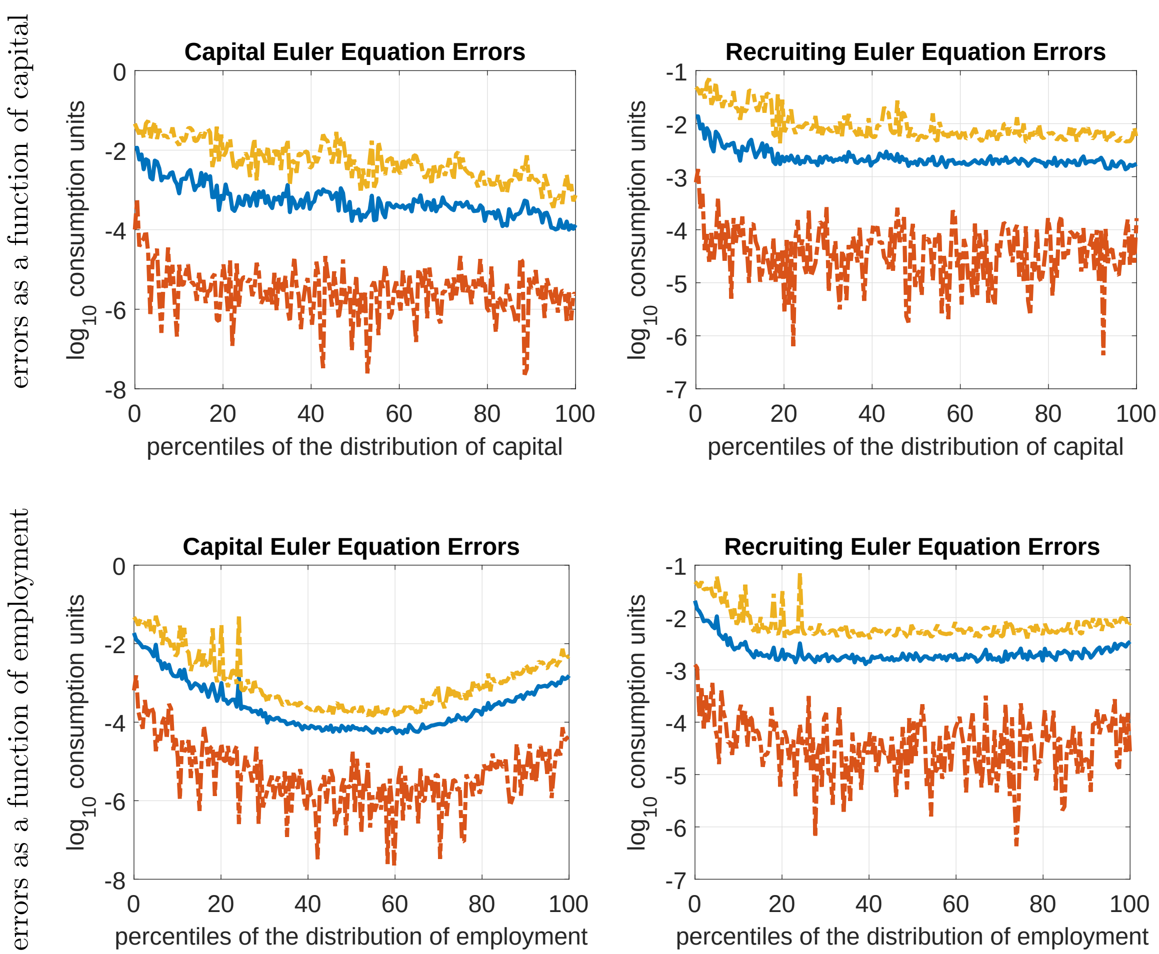


Figure H.18: Euler equation errors as a function of capital and employment: Mean, maximum, and minimum.

First, solve (H.32) for $w_{h,t}$. Here, we also set the habit parameter to zero:

$$
\begin{aligned}
w_{h,t} &= \phi_t mpl_t \frac{1}{1-\bar{\zeta}}\left(1+\frac{1-x}{\mu(\theta_t)}\right)+(1-\phi_t)\omega_t+\frac{1-\phi_t}{1-\tau_n}\left(\frac{c_t}{1+(\sigma-1)\gamma n_{t-1}}\right)\gamma\sigma \\
&-(1-x-f_t(\theta_t))\mathbb{E}_t\left[\frac{1-\phi_t}{1-\phi_{t+1}}m_{t+1}\left(\phi_{t+1}\frac{J_{\zeta,n,t+1}}{1-\tau_k}+\phi_{t+1}\frac{\bar{\zeta}}{1-\bar{\zeta}}mpl_{t+1}+\zeta_0(w_{\ell,t+1}-w_{h,t+1})\right)\right].
\end{aligned}
$$

Multiply by $(1-\zeta_0)$ and add $\zeta_0 w_{\ell,t}$ and plug in for $w_{h,t+1}-w_{\ell t+1}$:

$$
\begin{aligned}
w_{\zeta,t} &= \zeta_0 w_{\ell,t}+(1-\zeta_0)\phi_t mpl_t\frac{1}{1-\bar{\zeta}}\left(1+\frac{1-x}{\mu(\theta_t)}\right)+(1-\zeta_0)(1-\phi_t)\omega_t+(1-\zeta_0)\frac{1-\phi_t}{1-\tau_n}\left(\frac{c_t}{1+(\sigma-1)\gamma n_{t-1}}\right)\gamma\sigma \\
&-(1-\zeta_0)(1-x-f_t(\theta_t))\mathbb{E}_t\left[\frac{1-\phi_t}{1-\phi_{t+1}}m_{t+1}\left(\phi_{t+1}\frac{J_{\zeta,n,t+1}}{1-\tau_k}+\phi_{t+1}\frac{\bar{\zeta}}{1-\bar{\zeta}}mpl_{t+1}-\frac{\zeta_0}{1-\zeta_0}(w_{\zeta,t+1}-w_{\ell,t+1})\right)\right] \\
&= \zeta_0 w_{\ell,t}+(1-\zeta_0)\phi_t mpl_t\frac{1}{1-\bar{\zeta}}\left(1+\frac{1-x}{\mu(\theta_t)}\right)+(1-\zeta_0)(1-\phi_t)\omega_t+(1-\zeta_0)\frac{1-\phi_t}{1-\tau_n}\left(\frac{c_t}{1+(\sigma-1)\gamma n_{t-1}}\right)\gamma\sigma \\
&-(1-x-f_t(\theta_t))(1-\phi_t)\mathbb{E}_t\left[(1-\zeta_0)\frac{\phi_{t+1}}{1-\phi_{t+1}}m_{t+1}\left(\frac{J_{\zeta,n,t+1}}{1-\tau_k}+\frac{\bar{\zeta}}{1-\bar{\zeta}}mpl_{t+1}\right)\right]. \\
&+\zeta_0(1-x-f_t(\theta_t))(1-\phi_t)\mathbb{E}_t\left[\frac{1}{1-\phi_{t+1}}m_{t+1}(w_{\zeta,t+1}-w_{\ell,t+1})\right].
\end{aligned}
$$

Below, we model covariances and first moments using a VAR that includes $\tilde{\phi}_t, \ln m_t, \ln w_{\zeta,t}, \ln w_{\ell,t}, \ln mpl_t$, and $\ln\left(1+\frac{1-x}{\mu(\theta_{t+1})(1-\bar{\zeta})}\right)\equiv\tilde{\theta}_{t+1}$.

Write the VAR as:

$$
\begin{aligned}
X_{t+1} &= \mu_X + AX_t + B\epsilon_{t+1} \\
\epsilon_{t+1} &\overset{iid}{\sim} \mathcal{N}(0,I), \\
\Sigma &\equiv BB'.
\end{aligned}
$$

We use selection vectors (Kronecker deltas) e_m to select $m_{t+1}=\mathrm{e}_m X_{t+1}$ and analogously for other variables. We can, then, write:

$$
\begin{aligned}
\mathbb{E}_t[m_{t+1}] &= e^{\mathrm{e}_m(\mu_X+AX_t+\frac{1}{2}\mathrm{e}_m\Sigma\mathrm{e}_m')} \\
\mathrm{Cov}_t[m_{t+1},w_{\zeta,t+1}] &= \mathbb{E}_t[e^{\ln m_{t+1}}]\,\mathrm{Cov}_t[\ln m_{t+1},\ln w_{\zeta,t+1}]\mathbb{E}_t[e^{\ln w_{\zeta,t+1}}] \\
&= \exp(\mathrm{e}_m(\mu_X+AX_t)+\frac{1}{2}\mathrm{e}_m\Sigma\mathrm{e}_m')\mathrm{e}_m\Sigma\mathrm{e}_{w,\zeta}'\exp(\mathrm{e}_{w,\zeta}(\mu_X+AX_t)+\frac{1}{2}\mathrm{e}_{w,\zeta}\Sigma\mathrm{e}_{w,\zeta}') \\
&= \exp((\mathrm{e}_m+\mathrm{e}_{w,\zeta})(\mu_X+AX_t)+\frac{1}{2}(\mathrm{e}_m\Sigma\mathrm{e}_m'+\mathrm{e}_{w,\zeta}\Sigma\mathrm{e}_{w,\zeta}'))\mathrm{e}_m\Sigma\mathrm{e}_{w,\zeta}'. \qquad \text{(H.86)}
\end{aligned}
$$

Below, we use analogues to both expressions repeatedly and exploit log-normality to rewrite the expectational terms.

Last, we use our distributional assumption on the bargaining power process. To make sure that the bargaining power process remains bounded, we model the logistic transform of the bargaining power in the paper. This proves convenient here too:

$$
\tilde{\phi}_t \equiv \ln\frac{\phi_t}{1-\phi_t} = (1-\rho_\phi)\ln\frac{\bar{\phi}}{1-\bar{\phi}}+\rho_\phi\ln\frac{\phi_{t-1}}{1-\phi_{t-1}}+\omega_\phi\epsilon_{\phi,t}
$$

$$= \underbrace{(1-\rho_\phi)\ln\frac{\bar{\phi}}{1-\bar{\phi}}}_{\kappa_\phi} + \rho_\phi\tilde{\phi}_{t-1} + \omega_\phi\epsilon_{\phi,t}.$$

For future reference, note that $1+e^{\tilde{\phi}_t} = \frac{1}{1-\phi_t}$.

Now, start with the last term in the wage setting equation:

$$\begin{aligned}
T3_t \equiv & \mathbb{E}_t\left[\frac{1}{1-\phi_{t+1}} m_{t+1}(w_{\zeta,t+1}-w_{\ell,t+1})\right] \\
= & \mathbb{E}_t\left[(1+e^{\tilde{\phi}_t})e^{\ln m_{t+1}+\ln w_{\zeta,t+1}}\right] - \mathbb{E}_t\left[(1+e^{\tilde{\phi}_t})e^{\ln m_{t+1}+\ln w_{\ell,t+1}}\right] \\
= & e^{(\mathrm{e}_m+\mathrm{e}_{w,\zeta})(\mu_X+AX_t)+\frac{1}{2}(\mathrm{e}_m+\mathrm{e}_{w,\zeta})\Sigma(\mathrm{e}_m+\mathrm{e}_{w,\zeta})'} + e^{(\mathrm{e}_{\tilde{\phi}}+\mathrm{e}_m+\mathrm{e}_{w,\zeta})(\mu_X+AX_t)+\frac{1}{2}(\mathrm{e}_{\tilde{\phi}}\mathrm{e}_m+\mathrm{e}_{w,\zeta})\Sigma(\mathrm{e}_m+\mathrm{e}_{w,\zeta}+\mathrm{e}_{\tilde{\phi}})'} \\
& - e^{(\mathrm{e}_m+\mathrm{e}_{w,\ell})(\mu_X+AX_t)+\frac{1}{2}(\mathrm{e}_m+\mathrm{e}_{w,\ell})\Sigma(\mathrm{e}_m+\mathrm{e}_{w,\ell})'} - e^{(\mathrm{e}_{\tilde{\phi}}+\mathrm{e}_m+\mathrm{e}_{w,\ell})(\mu_X+AX_t)+\frac{1}{2}(\mathrm{e}_{\tilde{\phi}}\mathrm{e}_m+\mathrm{e}_{w,\ell})\Sigma(\mathrm{e}_m+\mathrm{e}_{w,\ell}+\mathrm{e}_{\tilde{\phi}})'}.
\end{aligned}$$

When $w_{\ell,t}$ is constant, $\mathrm{e}_{w,\ell}\mu_X \neq 0$, but $\mathrm{e}_{w,\ell}\Sigma = \mathbf{0}$.

Similarly:

$$\begin{aligned}
T2_t \equiv & \mathbb{E}_t\left[\frac{\phi_{t+1}}{1-\phi_{t+1}} m_{t+1} mpl_{t+1}\right] \\
= & \mathbb{E}_t\left[e^{\tilde{\phi}_{t+1}+\ln m_{t+1}+\ln mpl_{t+1}}\right] \\
= & e^{(\mathrm{e}_{\tilde{\phi}}+\mathrm{e}_m+\mathrm{e}_{mpl})(\mu_X+AX_t)+\frac{1}{2}(\mathrm{e}_{\tilde{\phi}}\mathrm{e}_m+\mathrm{e}_{mpl})\Sigma(\mathrm{e}_m+\mathrm{e}_{mpl}+\mathrm{e}_{\tilde{\phi}})'}.
\end{aligned}$$

For the term involving the future marginal value of employment to firms, we can use economic theory to plug in for the future value of employment to the firm:

$$\begin{aligned}
T1_t \equiv & \mathbb{E}_t\left[\frac{\phi_{t+1}}{1-\phi_{t+1}} m_{t+1} \frac{J_{\zeta,n,t+1}}{1-\tau_k}\right] \\
= & \mathbb{E}_t\left[\frac{\phi_{t+1}}{1-\phi_{t+1}} m_{t+1}\left(mpl_{t+1}\left(1+\frac{1-x}{\mu(\theta_{t+1})(1-\bar{\zeta})}\right) - w_{\zeta,t+1}\right)\right] \\
= & \mathbb{E}_t\left[e^{\tilde{\phi}_{t+1}+\ln m_{t+1}+\ln mpl_{t+1}+\ln\tilde{\theta}_{t+1}}\right] - \mathbb{E}_t\left[e^{\tilde{\phi}_{t+1}+\ln m_{t+1}+\ln w_{\zeta,t+1}}\right] \\
= & e^{(\mathrm{e}_{\tilde{\phi}}+\mathrm{e}_m+\mathrm{e}_{mpl}+\mathrm{e}_{\tilde{\theta}})(\mu_x+AX_t)+\frac{1}{2}(\mathrm{e}_{\tilde{\phi}}+\mathrm{e}_m+\mathrm{e}_{mpl}+\mathrm{e}_{\tilde{\theta}})\Sigma(\mathrm{e}_{\tilde{\phi}}+\mathrm{e}_m+\mathrm{e}_{mpl}+\mathrm{e}_{\tilde{\theta}})'} \\
& - e^{(\mathrm{e}_{\tilde{\phi}}+\mathrm{e}_m+\mathrm{e}_{w,\zeta})(\mu_x+AX_t)+\frac{1}{2}(\mathrm{e}_{\tilde{\phi}}+\mathrm{e}_m+\mathrm{e}_{w,\zeta})\Sigma(\mathrm{e}_{\tilde{\phi}}+\mathrm{e}_m+\mathrm{e}_{w,\zeta})'}.
\end{aligned}$$

Plugging ϕ_t for ϕ_t-terms and for the three terms, we have:

$$\begin{aligned}
w_{\zeta,t} = & \zeta_0 w_{\ell,t} + \frac{e^{\tilde{\phi}_t}}{1+e^{\tilde{\phi}_t}} mpl_t \frac{1-\zeta_0}{1-\bar{\zeta}}\left(1+\frac{1-x}{\mu(\theta_t)}\right) + \frac{1-\zeta_0}{1+e^{\tilde{\phi}_t}}\omega_t + \frac{1}{1+e^{\tilde{\phi}_t}}\frac{1-\zeta_0}{1-\tau_n}\left(\frac{c_t}{1+(\sigma-1)\gamma n_{t-1}}\right)\gamma\sigma \\
& - (1-x-f_t(\theta_t))\frac{1}{1+e^{\tilde{\phi}_t}}\mathbb{E}_t\left[(1-\zeta_0)\frac{\phi_{t+1}}{1-\phi_{t+1}} m_{t+1}\left(\frac{J_{\zeta,n,t+1}}{1-\tau_k} + \frac{\bar{\zeta}}{1-\bar{\zeta}} mpl_{t+1}\right)\right] \\
& - \zeta_0(1-x-f_t(\theta_t))\frac{1}{1+e^{\tilde{\phi}_t}}\mathbb{E}_t\left[\frac{1}{1-\phi_{t+1}} m_{t+1}(w_{\zeta,t+1}-w_{\ell,t+1})\right] \\
= & \zeta_0 w_{\ell,t} + \frac{e^{\tilde{\phi}_t}}{1+e^{\tilde{\phi}_t}} mpl_t \frac{1-\zeta_0}{1-\bar{\zeta}}\left(1+\frac{1-x}{\mu(\theta_t)}\right) + \frac{1-\zeta_0}{1+e^{\tilde{\phi}_t}}\omega_t + \frac{1}{1+e^{\tilde{\phi}_t}}\frac{1-\zeta_0}{1-\tau_n}\left(\frac{c_t}{1+(\sigma-1)\gamma n_{t-1}}\right)\gamma\sigma
\end{aligned}$$

$$- (1 - x - f_t(\theta_t))\frac{1}{1+e^{\tilde{\phi}_t}}\left(T1_t + \frac{\bar{\zeta}}{1-\bar{\zeta}}T2_t - \zeta_0 T3_t\right). \tag{H.87}$$

Given VAR estimates and noting that X_t contains $\tilde{\phi}_t$, we can solve (H.87) for $\tilde{\phi}_t$ using data on:

- The marginal product of capital mpl_t,
- Labor market tightness θ_t,
- The average (mean) wage rate $w_{\zeta,t}$
- The stochastic discount factor m_t,

taking the minimum wage rate $w_{\ell,t}$ and unemployment benefits ω_t as constant.

When $\zeta_0 = 0 \Rightarrow \bar{\zeta} = 0, T2_t = 0$ and $\omega_t = 0$, then equation (H.86) simplifies to:

$$w_{\zeta,t} = \frac{e^{\tilde{\phi}_t}}{1+e^{\tilde{\phi}_t}}mpl_t\frac{1}{1-\bar{\zeta}}\left(1+\frac{1-x}{\mu(\theta_t)}\right) + \frac{1}{1+e^{\tilde{\phi}_t}}\frac{1}{1-\tau_n}\left(\frac{c_t}{1+(\sigma-1)\gamma n_{t-1}}\right)\gamma\sigma$$
$$- (1 - x - f_t(\theta_t))\frac{1}{1+e^{\tilde{\phi}_t}}\left(\begin{array}{c} e^{(\mathrm{e}_{\tilde{\phi}}+\mathrm{e}_m+\mathrm{e}_{mpl}+\mathrm{e}_{\tilde{\theta}})(\mu_x+AX_t)+\frac{1}{2}(\mathrm{e}_{\tilde{\phi}}+\mathrm{e}_m+\mathrm{e}_{mpl}+\mathrm{e}_{\tilde{\theta}})\Sigma(\mathrm{e}_{\tilde{\phi}}+\mathrm{e}_m+\mathrm{e}_{mpl}+\mathrm{e}_{\tilde{\theta}})'} \\ e^{(\mathrm{e}_{\tilde{\phi}}+\mathrm{e}_m+\mathrm{e}_{w,\zeta})(\mu_x+AX_t)+\frac{1}{2}(\mathrm{e}_{\tilde{\phi}}+\mathrm{e}_m+\mathrm{e}_{w,\zeta})\Sigma(\mathrm{e}_{\tilde{\phi}}+\mathrm{e}_m+\mathrm{e}_{w,\zeta})'} \end{array}\right).$$

We estimate the VAR in the demeaned variables and add the model-implied mean.

H.12.2 Sampler

In our baseline model, we assume that the bargaining power is exogenous to the state of the economy and is driven by an AR(1) model. This implies an exclusion restriction on the estimated VAR. The exclusion restriction allows us to pull $\mathrm{e}_{\tilde{\phi}}(\mu_X + AX_t)$ out and write it simply as $\kappa_\phi + \rho_\phi\tilde{\phi}_t$ in (H.87).

Under the joint normality of the forecast errors, together with a flat prior this gives rise to a standard SUR algorithm for inference.[32] To begin, stack the model as follows:

$$Y_{SUR} = X_{SUR}\beta_{SUR} + v_{SUR}, \qquad v_{SUR} \sim \mathcal{N}(0, V \otimes I_T), \tag{H.88}$$

where $Y_{SUR} = [\mathrm{vec}(\tilde{\phi})', \mathrm{vec}(Y)']'$ and similarly for v_{SUR}. In addition, we have the definitions:

$$V = \begin{bmatrix} \Sigma_{\tilde{\phi},\tilde{\phi}} & \Sigma_{\tilde{\phi},Y'} \\ \Sigma_{Y,\tilde{\phi}} & \Sigma_{Y,Y} \end{bmatrix} \qquad \beta_{SUR} = \begin{bmatrix} \rho_\phi \\ A_Y \end{bmatrix},$$
$$X_{SUR} = \begin{bmatrix} \tilde{\phi}_{-1} & \mathbf{0}_{T\times n_Y(n_Y+3)} \\ \mathbf{0}_{T\times n_Y} & I_{m_z}\otimes X_Y \end{bmatrix} \qquad X_y = \begin{bmatrix} \tilde{\phi}_{-1} & Y_{-1} & [t]_{t=0}^{T-1} & \mathbf{1}_T \end{bmatrix}. \tag{H.89}$$

With these definitions, we transform the model to yield standard normal residuals: $\tilde{v} = \tilde{Y} - \tilde{X}\beta \sim \mathcal{N}(0, I)$. The transformed model gives rise to standard conditional Normal-Wishart posterior distributions. To implement the transformation, define U as the Cholesky decomposition of Σ such that $U'U = \Sigma$.

$$\begin{aligned} \tilde{X} &= ((U^{-1})'\otimes I_T)X_{SUR} & \tilde{Y} &= ((U^{-1})'\otimes I_T)\begin{bmatrix}\tilde{\phi} \\ Y\end{bmatrix} \\ N_{XX}(\Sigma) &= \tilde{X}'\tilde{X} & N_{XY}(V) &= \tilde{X}'\tilde{Y} \\ S_T(\beta) &= \tfrac{1}{\nu_0+T}\begin{bmatrix}(\tilde{\phi}-\rho_\phi\tilde{\phi}_{-1}) \\ (Y-XB)'\end{bmatrix}\begin{bmatrix}(\tilde{\phi}-\rho_\phi\tilde{\phi}_{-1}) \\ (Y-XB)'\end{bmatrix} + \tfrac{\nu_0}{\nu_0+T}S_0. \end{aligned}$$

[32] Without exclusion restrictions, the SUR model collapses to a standard scheme hierarchical Normal-Wishart posterior.

Here, S_0 is a prior over Σ, i.e., $\Sigma^{-1} \sim \mathcal{W}((\nu_0 S_0)^{-1}, \nu_0)$. Given a prior $\beta \sim \mathcal{N}(\bar{\beta}_0, N_0)$, the conditional posterior distributions are given by:

$$\beta|\Sigma, Y^T \sim \mathcal{N}(\bar{\beta}_T(V), (N_{XX}(\Sigma) + N_0)^{-1}), \tag{H.90a}$$

$$\Sigma^{-1}|\beta, Y^T \sim \mathcal{W}(S_T(\beta)^{-1}/(\nu_0 + T), \nu_0 + T), \tag{H.90b}$$

where $\bar{\beta}_T(V) = (N_{XX}(V) + N_0)^{-1}(N_{XY}(V) + N_0\bar{\beta}_0)$.

The algorithm, then, is:

1. Initialize $\Sigma^{(0)} = S_T(\bar{\beta}_T)$ and $\{\tilde{\phi}_t^{(0)}\}_t$.
2. Repeat for $i = 1, \dots n_G$:
   (a) Draw $\beta^{(i)}|\Sigma^{(i-1)}, \{\tilde{\phi}_t^{(0)}\}_t$ from (H.90a).
   (b) Draw $\Sigma^{(i)}|\beta^{(i)}, \{\tilde{\phi}_t^{(0)}\}_t$ from (H.90b).
   (c) Draw $\{\tilde{\phi}_t^{(i)}\}_t|\beta^{(i)}, \Sigma^{(i)}$ from (H.87).

H.12.3 Derivation with an alternative VAR

Running a VAR in a linear combination allows us to also use a model-implied FOC to factor the terms within the expectations. When $\zeta_0 = 0$, we would then only have to estimate the covariance term.

We begin our derivation as above (also with $h = 0$), but now use once that $\mathbb{E}_t[XY] = \text{Cov}_t[X, Y] + \mathbb{E}_t[X]\mathbb{E}_t[Y]$ to rewrite the term involving the future value of employment.

$$\begin{aligned}
w_{\zeta,t} &= \zeta_0 w_{\ell,t} + (1-\zeta_0)\phi_t mpl_t \frac{1}{1-\bar{\zeta}}\left(1 + \frac{1-x}{\mu(\theta_t)}\right) + (1-\zeta_0)(1-\phi_t)\omega_t + (1-\zeta_0)\frac{1-\phi_t}{1-\tau_n}\left(\frac{c_t}{1+(\sigma-1)\gamma n_{t-1}}\right)\gamma\sigma \\
&\quad - (1-x-f_t(\theta_t))(1-\phi_t)\mathbb{E}_t\left[(1-\zeta_0)\frac{\phi_{t+1}}{1-\phi_{t+1}} m_{t+1}\left(\frac{J_{\zeta,n,t+1}}{1-\tau_k} + \frac{\bar{\zeta}}{1-\bar{\zeta}} mpl_{t+1}\right)\right] \\
&\quad + \zeta_0(1-x-f_t(\theta_t))(1-\phi_t)\mathbb{E}_t\left[\frac{1}{1-\phi_{t+1}} m_{t+1}(w_{\zeta,t+1} - w_{\ell,t+1})\right] \\
&= \zeta_0 w_{\ell,t} + (1-\zeta_0)\phi_t mpl_t \frac{1}{1-\bar{\zeta}}\left(1 + \frac{1-x}{\mu(\theta_t)}\right) + (1-\zeta_0)(1-\phi_t)\omega_t + (1-\zeta_0)\frac{1-\phi_t}{1-\tau_n}\left(\frac{c_t}{1+(\sigma-1)\gamma n_{t-1}}\right)\gamma\sigma \\
&\quad - (1-x-f_t(\theta_t))(1-\phi_t)(1-\zeta_0)\left(\text{Cov}_t\left[\frac{\phi_{t+1}}{1-\phi_{t+1}}, m_{t+1}\frac{J_{\zeta,n,t+1}}{1-\tau_k}\right] + \mathbb{E}_t\left[\frac{\phi_{t+1}}{1-\phi_{t+1}}\right]\mathbb{E}_t\left[m_{t+1}\frac{J_{\zeta,n,t+1}}{1-\tau_k}\right]\right) \\
&\quad - (1-x-f_t(\theta_t))(1-\phi_t)\mathbb{E}_t\left[(1-\zeta_0)\frac{\phi_{t+1}}{1-\phi_{t+1}} m_{t+1}\frac{\bar{\zeta}}{1-\bar{\zeta}} mpl_{t+1}\right] \\
&\quad + \zeta_0(1-x-f_t(\theta_t))(1-\phi_t)\mathbb{E}_t\left[\frac{1}{1-\phi_{t+1}} m_{t+1}(w_{\zeta,t+1} - w_{\ell,t+1})\right].
\end{aligned}$$

Below, we model covariances and first moments using a VAR that includes $\tilde{\phi}_t$, $\ln(m_t \times mpl_t)$, $\ln\left(m_t \times \left((1+\theta_t)\frac{mpl_t}{1-\bar{\zeta}} - w_{\zeta,t}\right)\right)$), and $\ln(m_t \times (w_{\zeta,t} - \bar{w}_{\ell,t})))$.

Plugging in from (H.21) and the law of motion for $\tilde{\phi}_t$:

$$
\begin{aligned}
w_{\zeta,t} = &\zeta_0 w_{\ell,t} + (1-\zeta_0)\phi_t mpl_t \frac{1}{1-\bar{\zeta}}\left(1+\frac{1-x}{\mu(\theta_t)}\right) + (1-\zeta_0)(1-\phi_t)\omega_t + (1-\zeta_0)\frac{1-\phi_t}{1-\tau_n}\left(\frac{c_t}{1+(\sigma-1)\gamma n_{t-1}}\right)\gamma\sigma \\
&-(1-x-f_t(\theta_t))(1-\phi_t)(1-\zeta_0)\left(\mathrm{Cov}_t\left[e^{\tilde{\phi}_{t+1}}, m_{t+1}\frac{J_{\zeta,n,t+1}}{1-\tau_k}\right] + e^{\kappa_\phi+\rho_\phi\tilde{\phi}_t+\frac{1}{2}\omega_\phi^2}\frac{1}{\mu(\theta_t)}\frac{mpl_t}{1-\bar{\zeta}}\right) \\
&-(1-x-f_t(\theta_t))(1-\phi_t)\mathbb{E}_t\left[(1-\zeta_0)e^{\tilde{\phi}_{t+1}}m_{t+1}\frac{\bar{\zeta}}{1-\bar{\zeta}}mpl_{t+1}\right] \\
&+\zeta_0(1-x-f_t(\theta_t))(1-\phi_t)\mathbb{E}_t\left[(1+e^{\tilde{\phi}_{t+1}})m_{t+1}(w_{\zeta,t+1}-w_{\ell,t+1})\right].
\end{aligned}
$$

Now, plugging in from (H.22) and using Stein's Lemma:

$$
\begin{aligned}
w_{\zeta,t} = &\zeta_0 w_{\ell,t} + (1-\zeta_0)\phi_t mpl_t \frac{1}{1-\bar{\zeta}}\left(1+\frac{1-x}{\mu(\theta_t)}\right) + (1-\zeta_0)(1-\phi_t)\omega_t + (1-\zeta_0)\frac{1-\phi_t}{1-\tau_n}\left(\frac{c_t}{1+(\sigma-1)\gamma n_{t-1}}\right)\gamma\sigma \\
&-(1-x-f_t(\theta_t))(1-\phi_t)(1-\zeta_0)e^{\kappa_\phi+\rho_\phi\tilde{\phi}_t+\frac{1}{2}\omega_\phi^2}\,\mathrm{Cov}_t\left[\tilde{\phi}_{t+1}, m_{t+1}\left(mpl_t\left(1+\frac{1-x}{\mu(\theta_t)(1-\bar{\zeta})}\right)-w_{\zeta,t}\right)\right] \\
&-(1-x-f_t(\theta_t))(1-\phi_t)(1-\zeta_0)e^{\kappa_\phi+\rho_\phi\tilde{\phi}_t+\frac{1}{2}\omega_\phi^2}\frac{1}{\mu(\theta_t)}\frac{mpl_t}{1-\bar{\zeta}} \\
&-(1-x-f_t(\theta_t))(1-\phi_t)\mathbb{E}_t\left[(1-\zeta_0)e^{\tilde{\phi}_{t+1}}m_{t+1}\frac{\bar{\zeta}}{1-\bar{\zeta}}mpl_{t+1}\right] \\
&+\zeta_0(1-x-f_t(\theta_t))(1-\phi_t)\mathbb{E}_t\left[(1+e^{\tilde{\phi}_{t+1}})m_{t+1}(w_{\zeta,t+1}-w_{\ell,t+1})\right].
\end{aligned}
$$

Define:

$$
\begin{aligned}
X_{1,t} &\equiv \tilde{\phi}_t \\
X_{2,t} &\equiv \ln\left(m_{t+1}\left(mpl_t\left(1+\frac{1-x}{\mu(\theta_t)(1-\bar{\zeta})}\right)-w_{\zeta,t}\right)\right) \\
X_{3,t} &\equiv \ln\left(m_{t+1}mpl_{t+1}\right) \\
X_{4,t} &\equiv \ln\left(m_{t+1}(w_{\zeta,t+1}-w_{\ell,t+1})\right).
\end{aligned}
$$

This gives:

$$
\begin{aligned}
w_{\zeta,t} = &\zeta_0 w_{\ell,t} + (1-\zeta_0)\phi_t mpl_t \frac{1}{1-\bar{\zeta}}\left(1+\frac{1-x}{\mu(\theta_t)}\right) + (1-\zeta_0)(1-\phi_t)\omega_t + (1-\zeta_0)\frac{1-\phi_t}{1-\tau_n}\left(\frac{c_t}{1+(\sigma-1)\gamma n_{t-1}}\right)\gamma\sigma \\
&-(1-x-f_t(\theta_t))(1-\phi_t)(1-\zeta_0)e^{\kappa_\phi+\rho_\phi\tilde{\phi}_t+\frac{1}{2}\omega_\phi^2+\mathrm{e}_2[\mu_x+AX_t]+\frac{1}{2}\mathrm{e}_2\Sigma\mathrm{e}_2'}\mathrm{e}_1\Sigma\mathrm{e}_2 \\
&-(1-x-f_t(\theta_t))(1-\phi_t)(1-\zeta_0)e^{\kappa_\phi+\rho_\phi\tilde{\phi}_t+\frac{1}{2}\omega_\phi^2}\frac{1}{\mu(\theta_t)}\frac{mpl_t}{1-\bar{\zeta}} \\
&-(1-x-f_t(\theta_t))(1-\phi_t)(1-\zeta_0)\frac{\bar{\zeta}}{1-\bar{\zeta}}e^{\kappa_\phi+\rho_\phi\tilde{\phi}_t+\mathrm{e}_3[\mu_x+AX_t]+\frac{1}{2}(\mathrm{e}_1+\mathrm{e}_3)\Sigma(\mathrm{e}_1+\mathrm{e}_3)'} \\
&+\zeta_0(1-x-f_t(\theta_t))(1-\phi_t)e^{\mathrm{e}_4[\mu_x+AX_t]+\frac{1}{2}\mathrm{e}_4\Sigma\mathrm{e}_4'}\left(1+e^{\kappa_\phi+\rho_\phi\tilde{\phi}_t+\frac{1}{2}(\mathrm{e}_1+\mathrm{e}_4)\Sigma(\mathrm{e}_1+\mathrm{e}_4)'}\right) \\
= &\zeta_0 w_{\ell,t} + (1-\zeta_0)\frac{e^{\tilde{\phi}_t}}{1+e^{\tilde{\phi}_t}}mpl_t\frac{1}{1-\bar{\zeta}}\left(1+\frac{1-x}{\mu(\theta_t)}\right) + \frac{1-\zeta_0}{1+e^{\tilde{\phi}_t}}\omega_t + \frac{1}{1+e^{\tilde{\phi}_t}}\frac{1-\zeta_0}{1-\tau_n}\left(\frac{c_t}{1+(\sigma-1)\gamma n_{t-1}}\right)\gamma\sigma \\
&-(1-x-f_t(\theta_t))\frac{1}{1+e^{\tilde{\phi}_t}}(1-\zeta_0)T1_t
\end{aligned}
$$

$$
\begin{aligned}
&- (1 - x - f_t(\theta_t))\frac{1}{1+e^{\tilde{\phi}_t}}(1-\zeta_0)\frac{1}{\mu(\theta_t)}\frac{mpl_t}{1-\bar{\zeta}}T2_t \\
&- (1 - x - f_t(\theta_t))\frac{1}{1+e^{\tilde{\phi}_t}}(1-\zeta_0)\frac{\bar{\zeta}}{1-\bar{\zeta}}T3_t \\
&+ \zeta_0(1 - x - f_t(\theta_t))\frac{1}{1+e^{\tilde{\phi}_t}}T4_t. \qquad \text{(H.91)}
\end{aligned}
$$

Finally, we proceed as above, with (H.91) taking the role of (H.86).

H.12.4 Measurement

To implement our filter, we need data on: (1) the real wage, (2) the marginal product of labor, (3) labor market tightness, (4) the unemployment rate, and (5) consumption. The data sources are the same as for our main model, so we just discuss the mapping into model variables.

1. We use raw consumption, real wages, and real GVA.
2. We set the counterpart to unemployment benefits to 40% of the initial wage rate and let this grow at the average wage growth rate over the entire sample to account for balanced growth. Similarly, we set the initial minimum wage according to the model steady state and then let it grow at the average wage growth rate over the entire sample.
3. We re-scale the average real wage (an index) to the steady-state wage in the model.
4. We implement our model for the Cobb-Douglas case of the production function. Thus, the average product of labor is proportional to the marginal product of labor.[33] We consider two different measures:
   - Real GVA in the business sector divided by (1-unemployment) and re-scaled.
   - Real GVA in the business sector population ratio instead of 1-unemployment.

   We first re-scale the average marginal product of labor to the steady-state marginal product of labor in the model. Second, we shift the marginal product of labor up so that it lies weakly above the real wage.
5. We compute the monthly job finding rate implied by labor market tightness as $f_m(\theta_t)$. We then adjust the job finding rate and the separation rate x for the quarterly data frequency in the following way: The quarterly separation rate is $x_q = (1 + x_m/100)^3 - 1$. The quarterly job finding rate is $f_q = f_m + (1 - f_m)f_m + (1 - f_m)^2 f_m$. This neglects within-quarter separations.
6. Given the real-wage rate, the static component of the household surplus turns negative in the 1990s. We shift the average disutility of working up until the implied average bargaining power in the data equals 0.5 for a first burn-in period, as in the model.
7. When we use the employment-to-population ratio to compute labor productivity, we also use the employment-to-population ratio to compute the disutility from working. However, our model is calibrated to an average employment-to-participation ratio of 0.95. To avoid

[33] Unfortunately, there is no easy way to differentiate in the data the marginal productivity of production workers (the object of interest in the model) from the marginal productivity of recruiters. Our prior is that the bias induced by considering the aggregate marginal productivity of both types of workers is negligible.

having the data counterpart to n_t in the model exceed unity, we re-scale the employment-to-participation ratio so that it averages 0.95 and has the same range (max − min) as the unemployment rate.

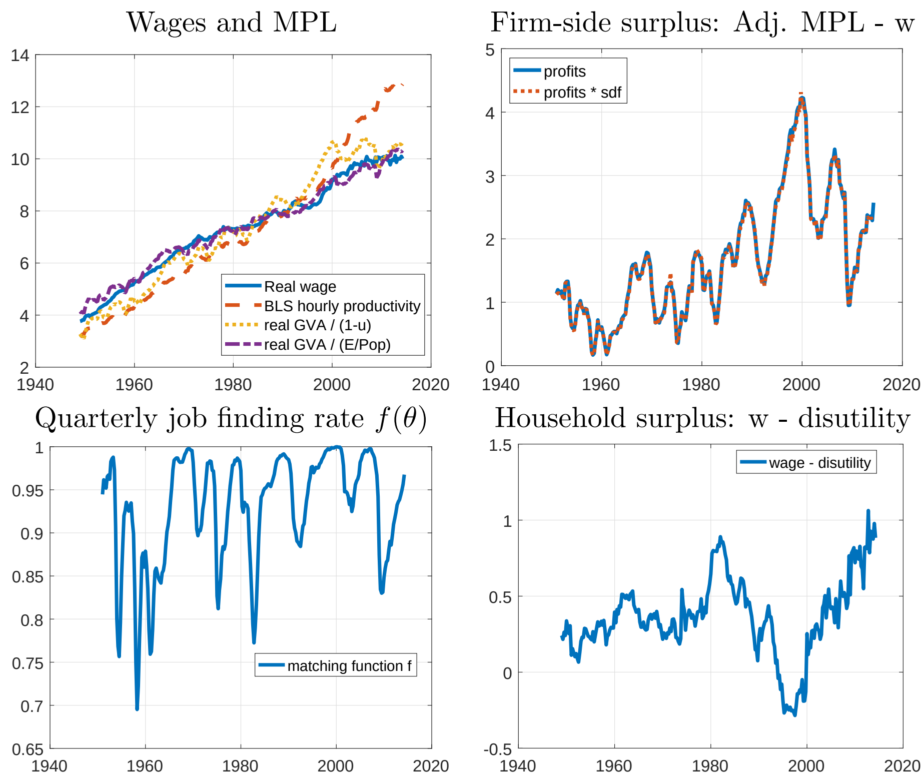


Figure H.19: Variables entering the filter

H.12.5 New hire wages

Since the wages of new hires are much more procyclical than the wages of continuing workers, Pissarides (2009) argues that wage rigidity à la Hall (2005) or Gertler and Trigari (2009) is unlikely to be a realistic way to make unemployment more volatile in search and matching models of the labor market. Given that all wages adjust every period in our model, wage stickiness is irrelevant in our setup. However, the cyclicality of real wages is a central calibration target and may affect our historical estimates.

As a robustness check, we consider a measure of the wage of new hires. Specifically, we use median hourly wages in the private sector from Haefke et al. (2013), extended by the Federal Reserve Bank of Philadelphia.[34] This series begins in 1979.Q1. The left panel of Figure H.20 shows the change in the real wages from Haefke et al. (2013) and the updated series for when they overlap. The correlation is very high at 0.85. The right panel of Figure H.20 plots the nominal series. We splice the nominal series together by adjusting the updated series down in the last quarter for which we have data from Haefke et al. (2013). We deflate the series with the PCE deflator.

Once we HP-filter this series, we find that the wage of new hires is as acyclical with respect to GDP (constructed, on a model-consistent basis, as the sum of real consumption and investment),

[34] We are grateful to Paul van Vliet for sharing his data.

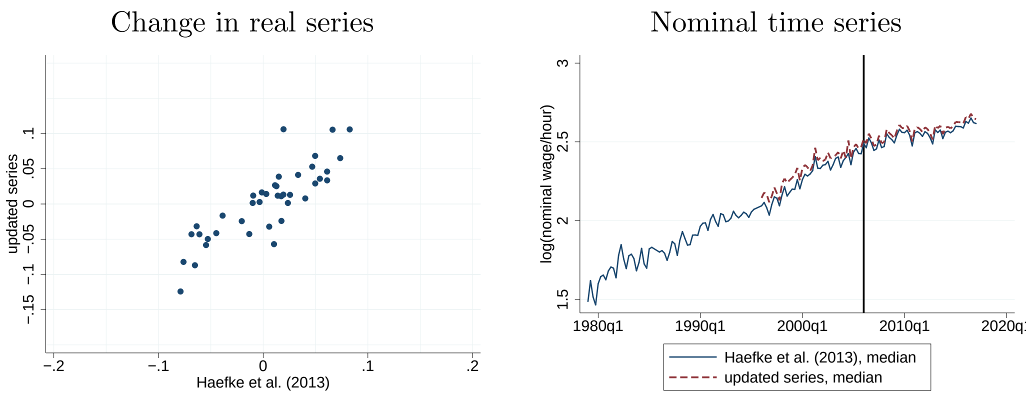


Figure H.20: Bargaining power process implied by the baseline calibration: Baseline wage measure and new hire wage

similar to the composition-adjusted employment cost index, which we also consider; see Table H.9.[35] Our finding is in line with Gertler et al. (2016). These authors report that wages of new hires from unemployment are no more procyclical than aggregate wages, based on SIPP data, once they control for match quality. Our finding differs somewhat from Haefke et al. (2013), who document that their measure of composition-adjusted real wage growth of new hires correlates much more strongly with labor productivity growth than the analogous hourly wage for all workers. While we replicate these authors' results (which are subject to large sample uncertainty) with their sample period, we find that the cyclicality of new wages and continuing wages is close in the extended sample. Haefke et al. (2013) exclude the early Volcker years (up to the end of 1983) from their baseline analysis and their sample ends in 2006.Q1. In that sample –but with new productivity data– we find a point estimate (t-statistics) of 0.56 (1.08) for the new hire wage, relative to 0.40 (3.79) for the overall wage rate and 0.41 (3.49) for the BLS wage rate. Adding the three observations raises the coefficient on the BLS wage to 0.46 (3.84) and lowers it to 0.46 (0.86) for the new hire wage. Adding 1983 and earlier years yields similar results.

Given that our series for new wages has acyclicality properties similar to those of the other measures of wages, it is not a surprise that our backed-out bargaining power is not materially affected. Figure H.21 compares the implied bargaining power. While the bargaining power implied by the new hire wage data is noisier (the new wage data, based on survey data, are themselves noisier), it closely mimics our baseline measure. Both backed-out bargaining power series have a high degree of comovement and identify similar peaks and troughs. The levels of the series are not comparable, because we calibrate the bargaining power to average 0.5 over the entire available sample, but not over the sub-sample shown for our baseline measure.

[35]Note that, in Table H.9, we change the length of the sample of aggregate variables in each row to make it consistent with the corresponding wage sample. That is why the statistics of these aggregate variables slightly vary across rows.

Table H.9: Business cycle statistics: Different wage measures and samples

	Volatility							
	Y [%]	$\frac{\text{std(I)}}{\text{std(Y)}}$	$\frac{\text{std(C)}}{\text{std(Y)}}$	std(ncs) [pp.]	std(cs) [pp.]	std(w) [%]	std(u) [%]	std(TFP) [%]
Baseline wage & sample	1.99	3.28	0.58	1.07	0.86	0.95	0.83	1.21
ECI wage & sample	1.77	3.06	0.47	0.94	0.72	0.48	0.77	0.96
New hire wage & sample	1.87	2.93	0.52	0.95	0.73	2.76	0.76	0.98
	Cyclicality							
	Y	I	C	ncs	cs	w	u	TFP
Baseline wage & sample	1.00	0.91	0.84	0.57	0.36	0.19	-0.76	0.78
ECI wage & sample	1.00	0.95	0.81	0.46	0.21	-0.25	-0.85	0.68
New hire wage & sample	1.00	0.95	0.83	0.50	0.28	-0.26	-0.81	0.71
	Persistence							
	Y	I	C	ncs	cs	w	u	TFP
Baseline wage & sample	0.87	0.82	0.78	0.76	0.74	0.67	0.90	0.78
ECI wage & sample	0.89	0.86	0.76	0.78	0.74	0.78	0.93	0.79
New hire wage & sample	0.90	0.87	0.82	0.79	0.74	0.16	0.93	0.80

Note: Quarterly data, HP-filtered with smoothing parameter $\lambda = 1,600$.

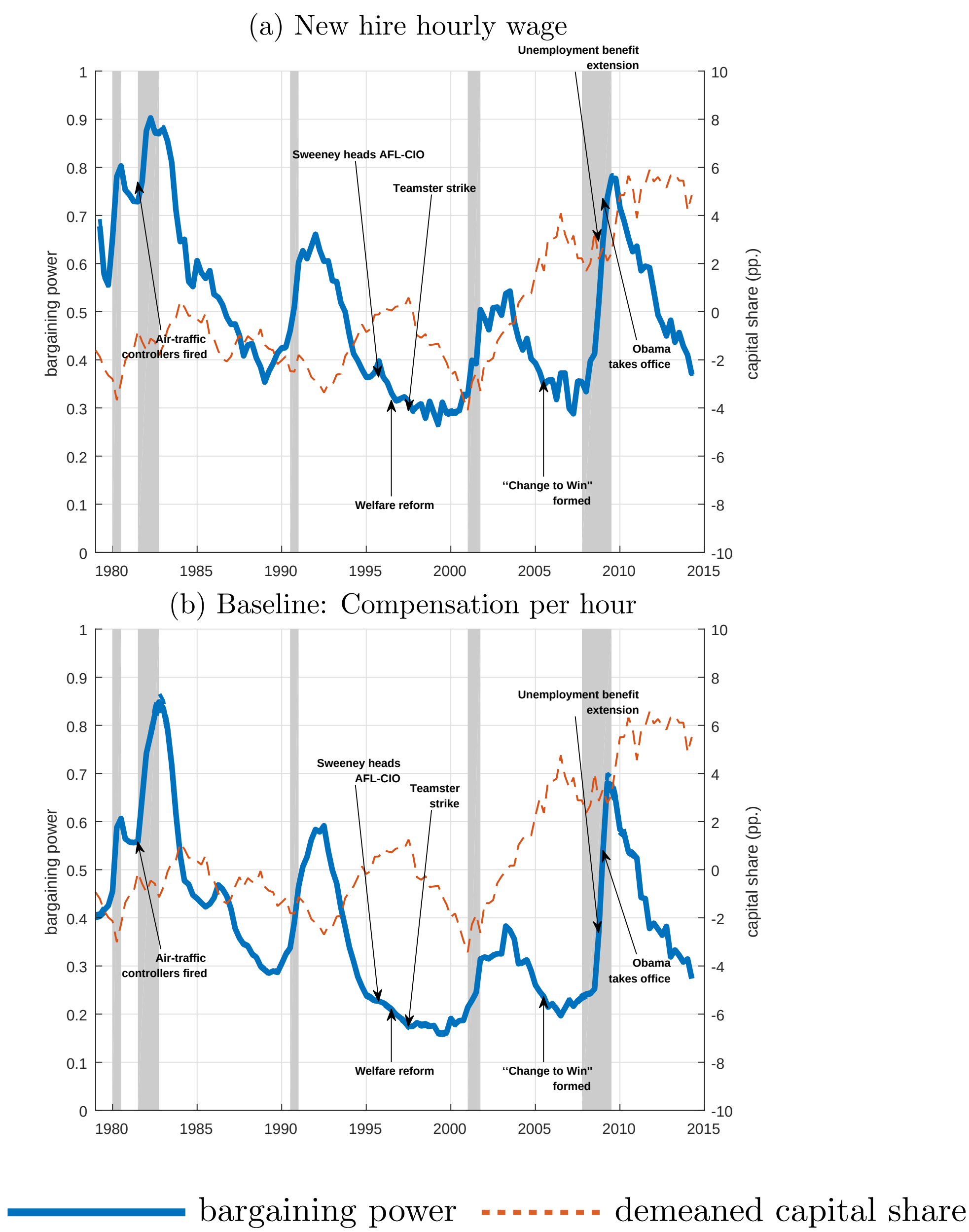


Figure H.21: Bargaining power process implied by the baseline calibration: Baseline wage measure and new hire wage

H.12.6 Additional results

When we use an alternative measure of labor productivity or detrend non-stationary variables prior to filtering, we find only small changes in the implied moments: See Table H.10(a) to (c).

Table H.10: Implied bargaining power process moments

(a) Productivity based on complement of the unemployment rate, alternative VAR and filter

	Median	5th percentile	95th percentile
In-sample autocorrelation	0.9615	0.9466	0.9678
Posterior autocorrelation	0.9483	0.9138	0.9814
In-sample AR(1) st.dev.	0.2795	0.2470	0.3541
Posterior AR(1) st.dev.	0.2861	0.2459	0.3718
In-sample average bargaining power	0.5004	0.4996	0.5014

(b) Productivity based on employment-to-population ratio

	Median	5th percentile	95th percentile
In-sample autocorrelation	0.9577	0.9530	0.9606
Posterior autocorrelation	0.9588	0.9264	0.9892
In-sample AR(1) st.dev.	0.2239	0.2109	0.2445
Posterior AR(1) st.dev.	0.2253	0.2051	0.2539
In-sample average bargaining power	0.4994	0.4964	0.5029

(c) Productivity based on BLS labor productivity

	Median	5th percentile	95th percentile
In-sample autocorrelation	0.9782	0.9712	0.9818
Posterior autocorrelation	0.9797	0.9570	0.9959
In-sample AR(1) st.dev.	0.2538	0.2232	0.3110
Posterior AR(1) st.dev.	0.2571	0.2207	0.3125
In-sample average bargaining power	0.5001	0.4969	0.5034

H.12.7 A comparison with an alternative bargaining power index

Levy and Temin (2007) propose to measure bargaining power as the inverse of the real unit labor cost. They call this measure the "bargaining power index." We compute their measure for our longer sample as the real hourly compensation divided by the real hourly output, both measured in the non-farm business sector.[36] This measure exhibits a pronounced downward trend, perhaps due to changes in the underlying industry or occupation mix. To compare our measures, we remove a quadratic trend from (the log of) their measure and do the same for our measure. Figure H.22 shows the resulting time series. Despite the very different methodological approaches, the two series move together. The overall correlation between the detrended series is 0.40. They track each other particularly well from the beginning of our sample period to the mid-1970s and again during the 2000s.

Quarterly changes in our filtered bargaining power tend to move with the changes in (log of) the Levy and Temin (2007) bargaining index, as Figure H.23 shows. The overall correlation is 0.28, and both measures pick up on the increased bargaining power due to the extension of unemployment

[36] Levy and Temin (2007) use the median, not the mean compensation. We prefer the mean because, due to demographic changes, who the median worker is has noticeably changed over the last few decades.

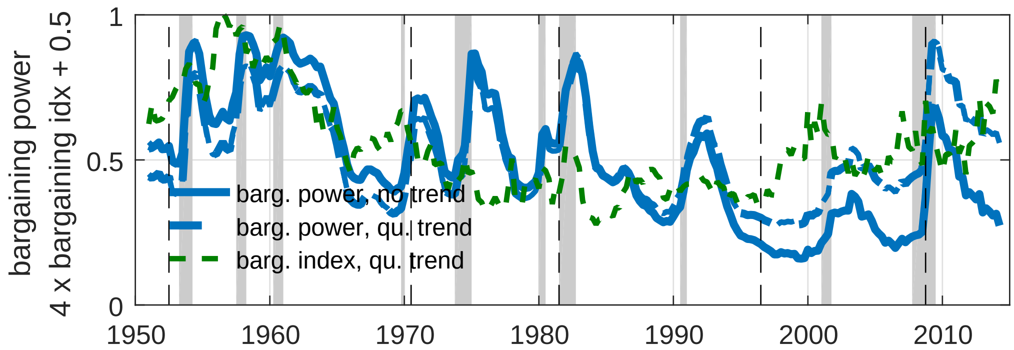


Figure H.22: Filtered bargaining power and Levy and Temin (2007) bargaining index

benefits in late 2008 and the reversal during several periods after the Great Recession. While both measures may contain measurement error, it is reassuring that the 10 lowest and highest changes in our bargaining power index also tend to be classified as such in the bargaining power index. The figure dates these data points, and their correlation is 0.38.

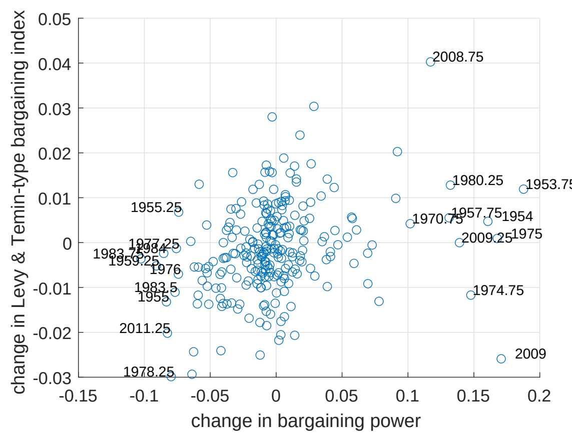


Figure H.23: Change in filtered bargaining power and Levy and Temin (2007) bargaining index

We have also argued in the main text that increases in the real minimum wage resemble increases in bargaining power, as pointed out by Flinn (2006). Figure H.25 plots the real minimum wage alongside our bargaining power – the real federal minimum wage (solid green line) and the real effective minimum wage, that is, the population-weighted average of the maximum of the state and federal minimum wage. Overall, the correlation between the real federal minimum wage and our bargaining power is a low 0.08. However, from 1974 on, when the federal minimum wage is unified and has the broadest coverage, the correlation is 0.53. For most of the period since 1974, we also have data on state minimum wages from Autor et al. (2016), and the correlation with the effective minimum wage is 0.41. Given that our filtered bargaining power was high before 1974 and the minimum wage was less broadly applicable, we view this evidence as consistent with the notion that our bargaining power index reflects redistributive measures such as the minimum wage when bargaining power is low.

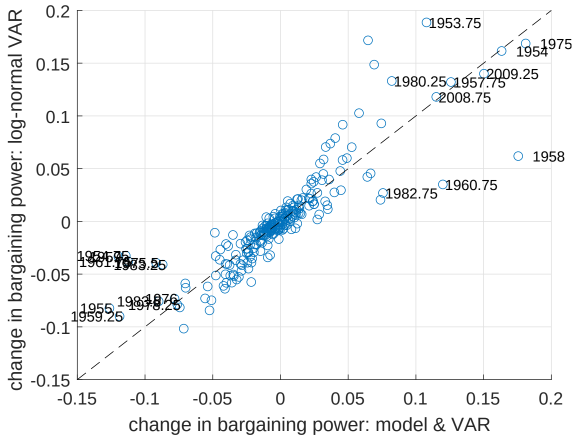
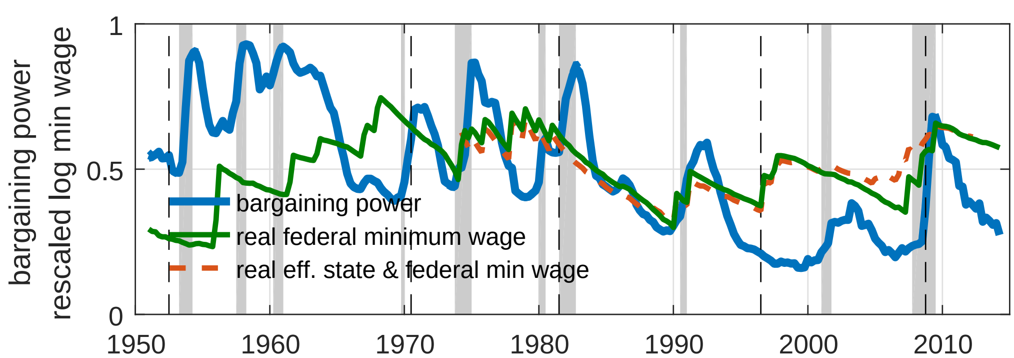


Figure H.24: Change in filtered bargaining power: Two alternative specifications



We demean the log real minimum wage measures, divide them by 100, and re-center them at 0.5 to ease the comparison.

Figure H.25: Filtered bargaining power and real minimum wage measures

H.12.8 Counterfactual unemployment

Our final exercise in this subsection is to filter the historical bargaining power with our calibrated stochastic process. Specifically, we set $\epsilon_{\phi,1} = \omega_{\phi}^{-1}\left(\tilde{\phi}_1 - (1-\rho_\phi)\log\frac{\bar{\phi}}{1-\bar{\phi}}\right)$ and for subsequent observations $\epsilon_{\phi,1} = \omega_{\phi}^{-1}\left(\tilde{\phi}_t - \rho\tilde{\phi}_{t-1} - (1-\rho_\phi)\log\frac{\bar{\phi}}{1-\bar{\phi}}\right)$.

With this series, we can compute the implied unemployment rate coming out of our model. More concretely, for each realization of the bargaining power shock, we take one draw of the labor productivity shock from its distribution and compute the difference to a world where the bargaining power shocks are set to zero, but the same productivity shocks. Figure H.26 displays the result for the unemployment rate. We also plot the simulated level of bargaining power to confirm that our procedure is consistent.

The bargaining power shock explains 38.3% of the historical fluctuations in the unemployment rate. Its correlation with the historical unemployment rate is 0.25, and its relative standard deviation is 1.53. Multiplying these two numbers gives a share of the explained variance of 0.383. HP-filtered, the correlation is 0.86, and the relative standard deviation 1.29.[37]

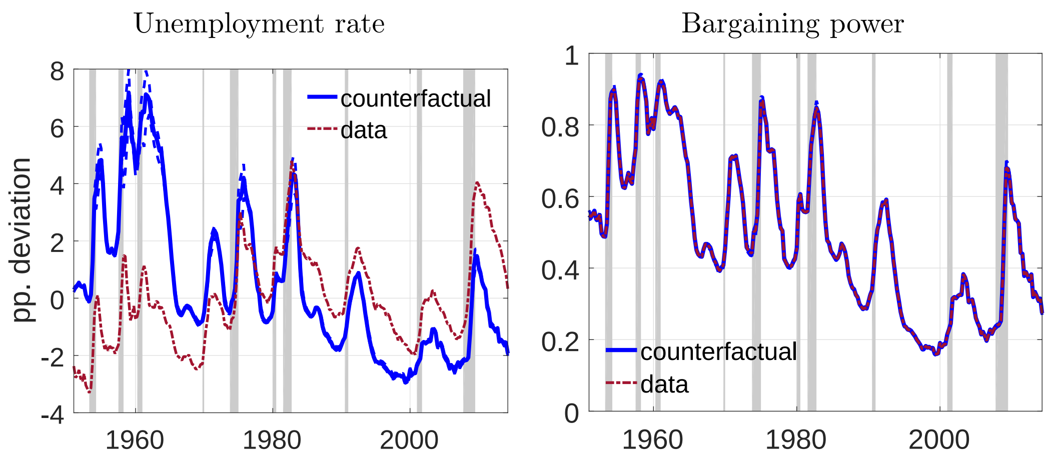


Figure H.26: Historical counterfactuals implied by bargaining power process

[37]The correlation is $\frac{\mathrm{Cov}[x,y]}{\mathrm{Var}[x]^{1/2}\,\mathrm{Var}[y]^{1/2}}$. Multiplying this correlation by the ratio of standard deviations gives the covariance relative to the variance. Because the variance is, up to first order, the sum of the variance due to bargaining shocks and the remainder due to all other shocks, the reported measure is the historical variance accounted for by the bargaining power shock.

H.13 Search and matching model: All GIRFs

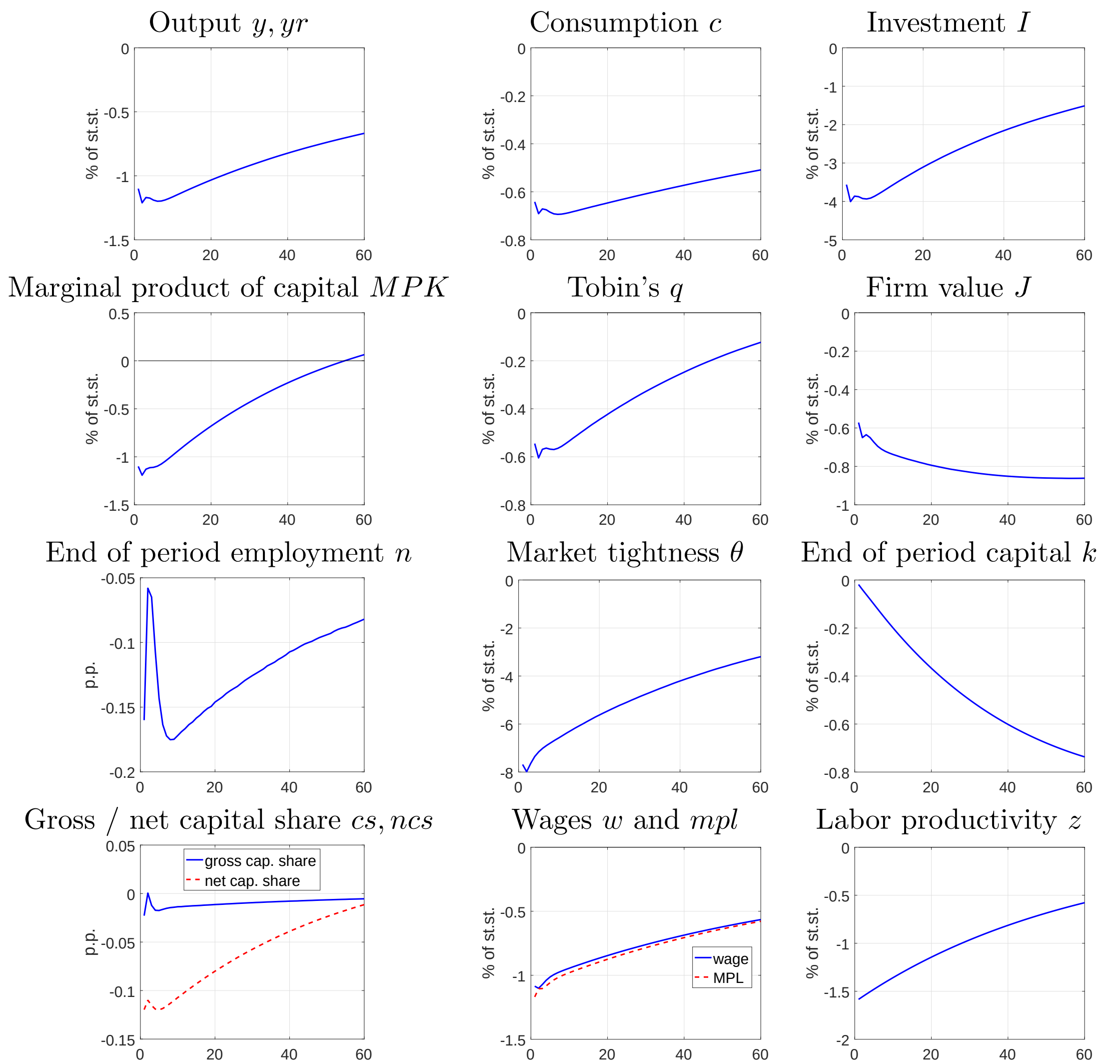


Figure H.27: GIRFs to a negative two standard deviation shock to labor productivity with Cobb-Douglas technology.

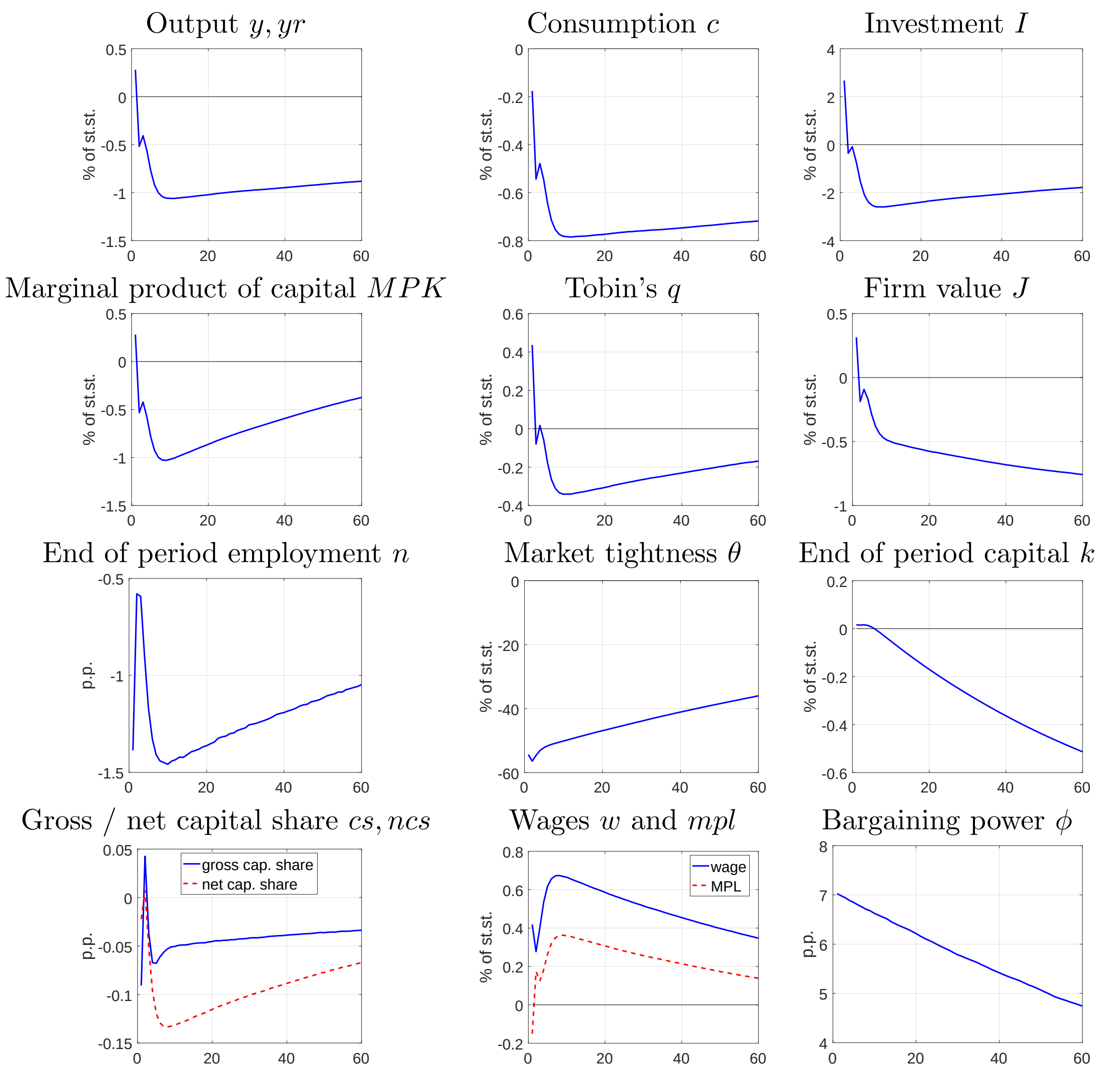


Figure H.28: GIRFs to a two standard deviation shock to workers' bargaining power with Cobb-Douglas technology.

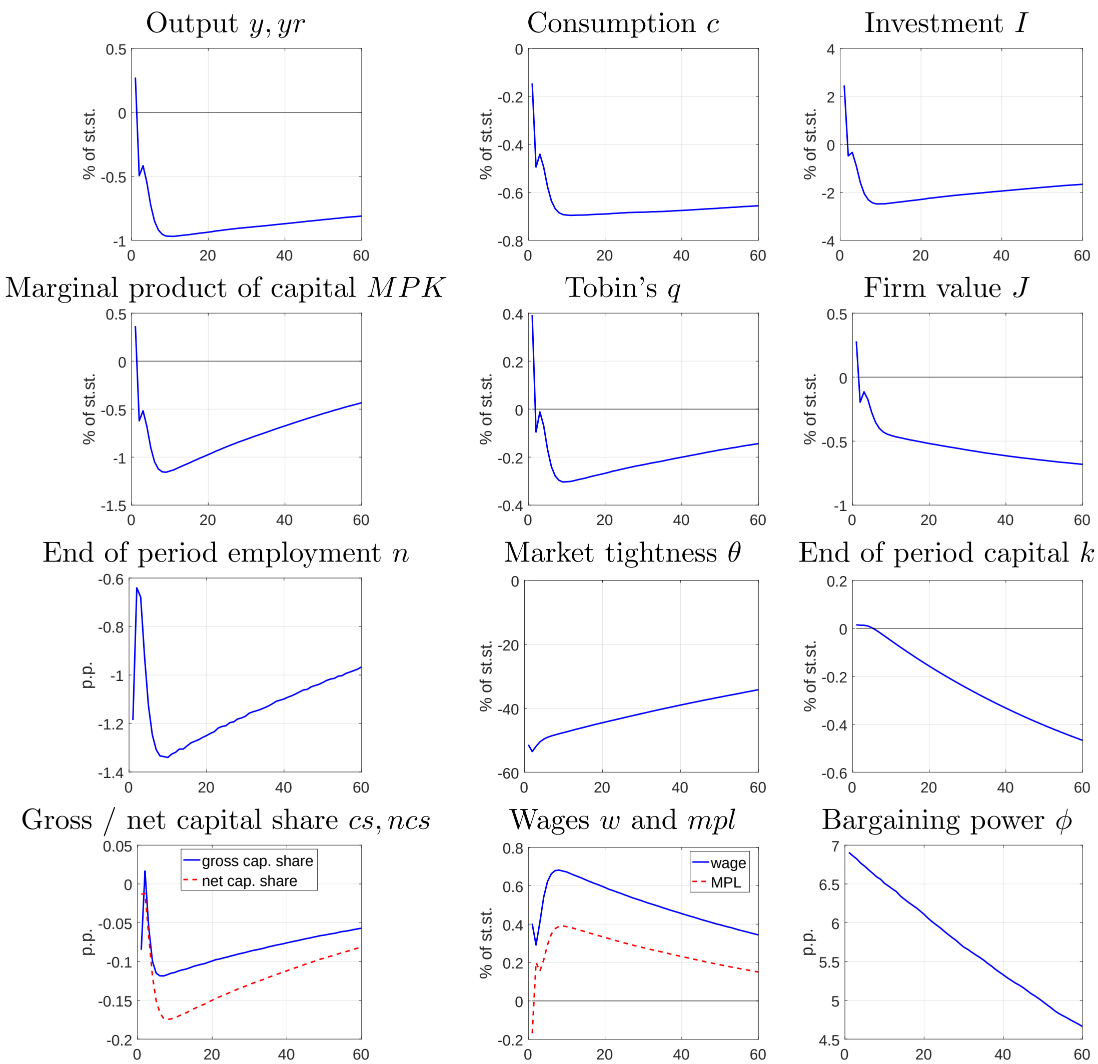


Figure H.29: GIRFs to a two standard deviation shock to workers' bargaining power with CES $\varepsilon = 0.75$.

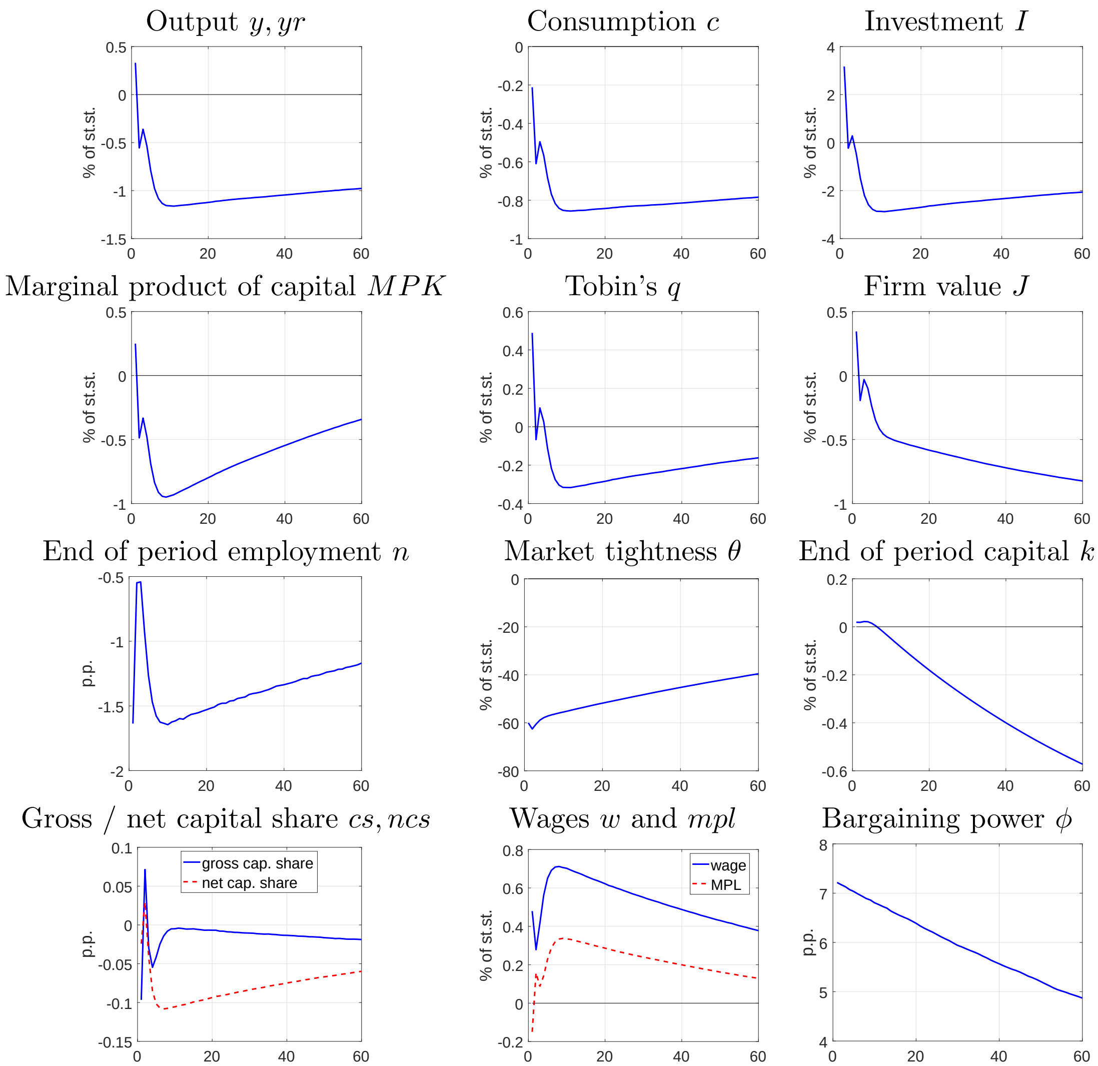


Figure H.30: GIRFs to a two standard deviation shock to workers' bargaining power with CES $\varepsilon = 1.25$.

H.14 GIRF comparison: Search and matching vs. RBC model

We benchmark our model against an RBC analogue to our economy. Since our baseline model features indivisible labor, its RBC analogue is closest to Hansen (1985) and Rogerson (1988). In keeping with our timing convention, however, labor is also hired and paid one period in advance. Also, employed and unemployed agents have the same consumption and hence the period utility function is simply:

$$U_t = \frac{(c_t - hc^a_{t-1})^{1-\sigma} - 1}{1-\sigma} - \gamma n_{t-1}.$$

Compared to the solution of the search model, this implies the following changes:

- The detrended habit function $\tilde{h}(\cdot)$ in (H.53) is constant at $\tilde{h} = hg_z^{-\frac{1}{1-\alpha}}$.
- The law of motion for employment (H.6) drops out as well as the recruiting optimality condition (H.21) – the fraction of recruiters ν_t and labor market tightness θ_t are not defined.
- There are alternative ways of setting wages that allow us to retain the assumption that labor is set one period in advance. We pick a structure where labor supply is predetermined:
  - The equation (H.50) for the marginal value of employment J_n is replaced by

    $$\widetilde{mpl}_t - w_t = 0.$$

    In words, the wage rate equals the marginal product of labor state by state – keeping the labor share of income constant with a Cobb-Douglas production function.
  - Wage-setting is replaced by an indifference condition for the household:

    $$\mathbb{E}_t \left[m_{t+1} g^{\frac{1}{1-\alpha}} \left((1-\tau_n) w_t - \sigma\gamma \frac{c_{t+1} - h\bar{c}_t}{1 + (\sigma-1)\gamma n_t} \right) \right] = 0.$$

    Households choose labor supply one period in advance so that, on expectation, they are indifferent between leisure and work.

We can then compare the responses to the common productivity shock, using the same deep parameters that we calibrated for our baseline model – except that we also recalibrate γ to make sure the employment levels in both models are the same.

We show the comparison of GIRFs from this model and our baseline model in Figures H.31 (for unitary elasticity of substitution), H.32 (for $\epsilon = 0.75$), and H.33 (for $\epsilon = 1.25$). Finally, in Figure H.34, we show the comparison of GIRFs with the RBC model with factor share shocks.

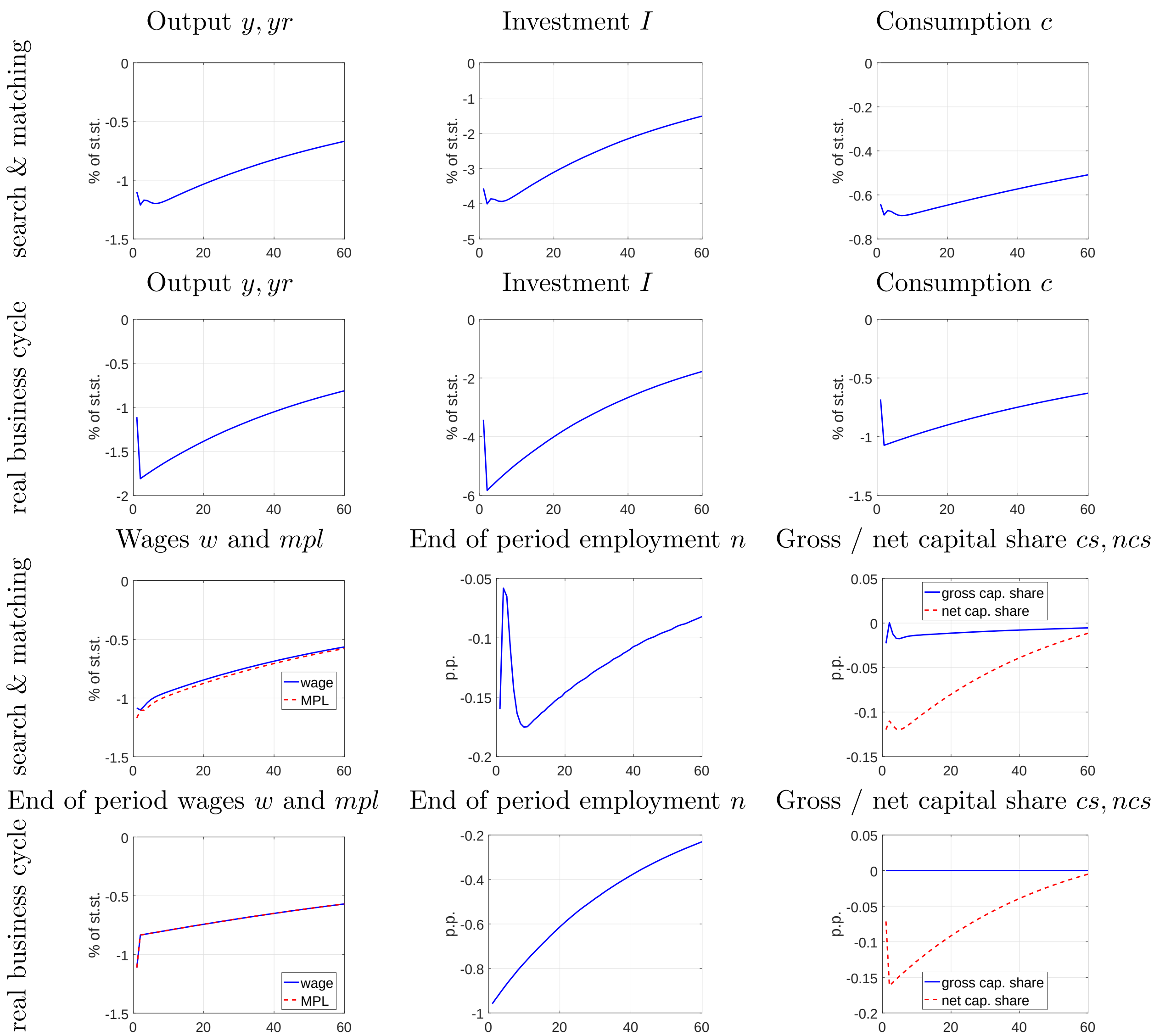


Figure H.31: GIRFs to a negative two standard deviation labor productivity shock: Search and matching vs. RBC model with Cobb-Douglas production function.

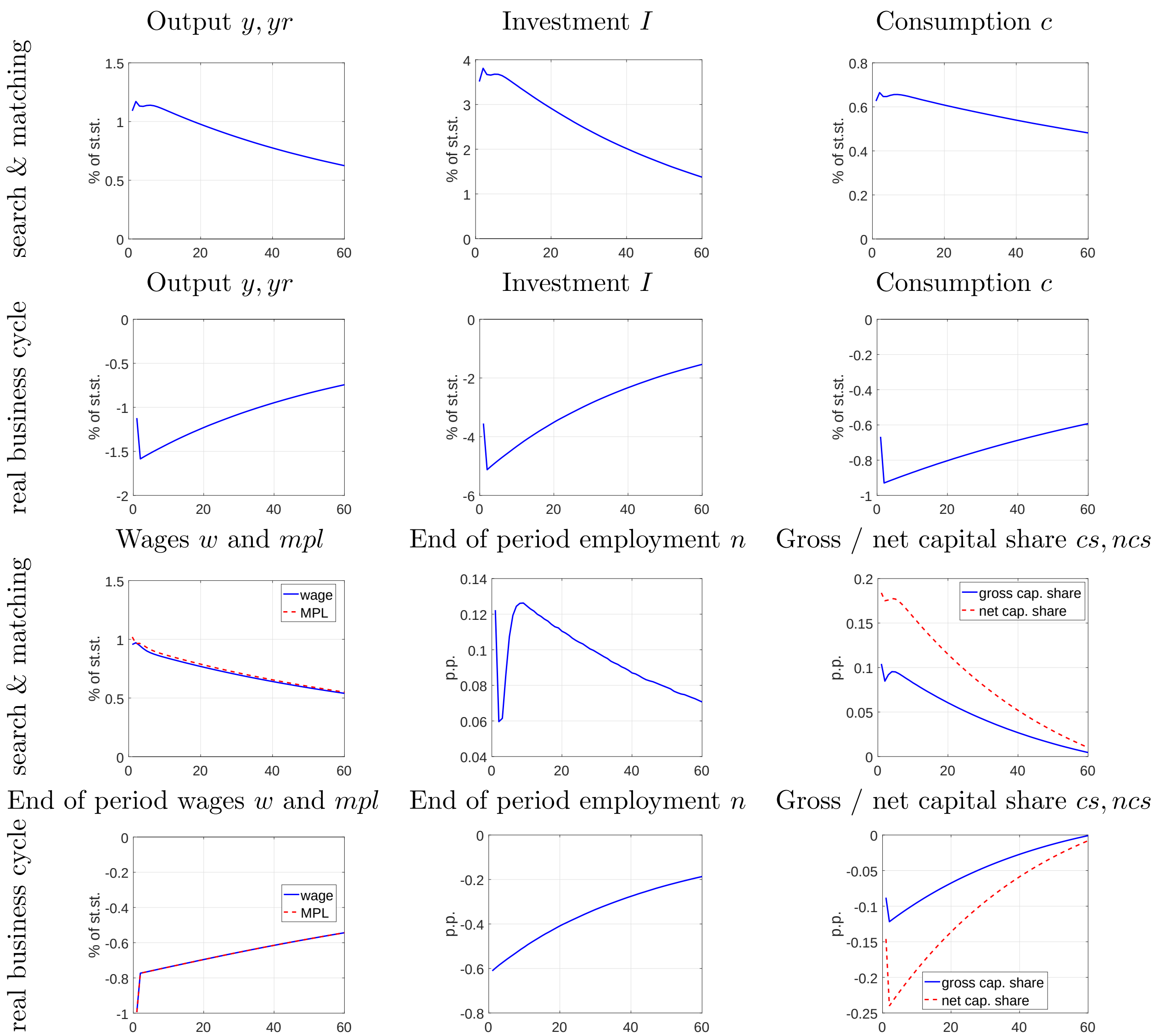


Figure H.32: GIRFs to a negative two standard deviation labor productivity shock: Search and matching vs. RBC model with CES $\varepsilon = 0.75$.

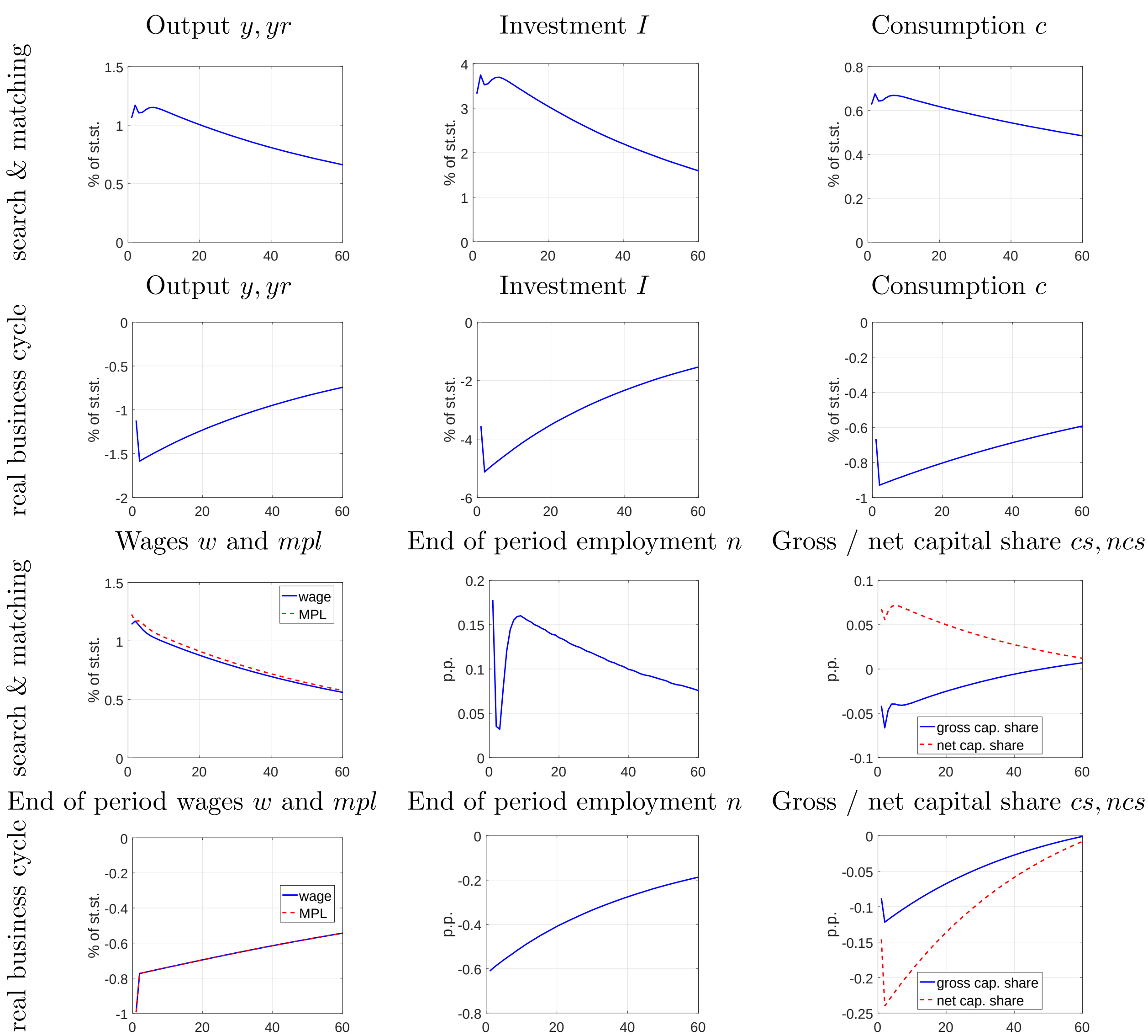


Figure H.33: GIRFs to a negative two standard deviation labor productivity shock: Search and matching vs. RBC model with CES $\varepsilon = 1.25$.

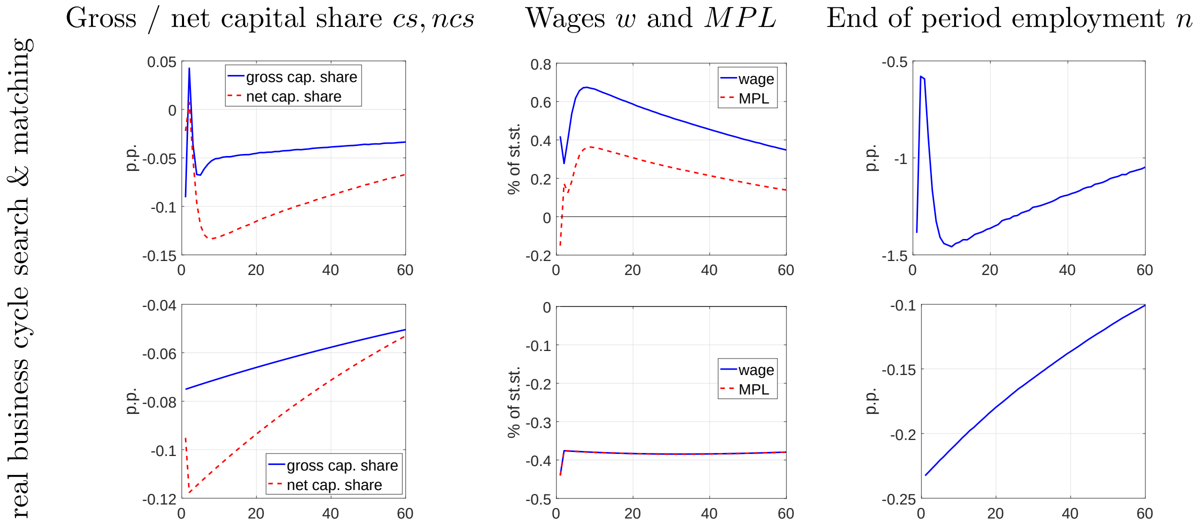


Figure H.34: Bargaining shock in search and matching model compared to factor share shock in RBC.

H.15 The Hosios rule and the welfare cost of political risk

The Hosios condition holds in our model without recipients of the minimum wage, i.e., when $\zeta_0 = 0$ and when $\tau_n = \tau_k$ and $\phi = 1 - \eta$. Below we compare our baseline calibration to one without minimum wage recipients but with unequal tax rates, and a calibration that also has equal tax rates so that the Hosios condition holds in the steady state.

We re-calibrate the model for each parameter combination. Table H.11 shows the implied business cycle statistics. The violation of the Hosios condition in the steady state is immaterial for our results. Without minimum wages, we find a smaller role for bargaining power shocks for the capital share than in our baseline model. Equalizing the tax rates has no perceivable effect on the role of bargaining power shocks. The welfare implications, however, are unchanged up to the first digit; see Table H.12.

Intuitively, the actual bargaining power in our model is highly persistent, so that deviations from the Hosios condition are long-lasting independent of the steady-state calibration.[38] Thus, steady-state efficiency may have little effect on efficiency in the stochastic economy.

[38] We also re-calibrate the steady state of the model to attain the same calibration targets, including the employment level, independent of whether the Hosios condition holds.

Table H.11: Business cycle statistics: 1947Q1–2015Q2

	Y [%]	$\frac{std(I)}{std(Y)}$	$\frac{std(C)}{std(Y)}$	std(ncs) [pp.]	std(cs) [pp.]	std(w) [%]	std(u) [%]	std(TFP) [%]
Volatility								
U.S. data	1.99	3.28	0.58	1.07	0.86	0.95	0.83	1.21
Models								
Baseline	1.91	3.28	0.62	0.33	0.17	1.45	1.89	1.21
No bargaining shock	1.35	3.33	0.57	0.18	0.01	1.13	0.24	1.20
No minimum wage $\zeta_0 = 0$	1.96	3.28	0.62	0.31	0.11	1.37	1.82	1.21
Same, no bargaining shock	1.37	3.36	0.57	0.18	0.01	1.14	0.23	1.22
No minimum wage $\zeta_0 = 0$, $\tau_n = \tau_k = 0.3$	1.96	3.28	0.62	0.31	0.11	1.37	1.82	1.21
Same, no bargaining shock	1.37	3.39	0.56	0.18	0.01	1.14	0.23	1.22
Cyclicality	Y	$\frac{std(I)}{std(Y)}$	$\frac{std(C)}{std(Y)}$	std(ncs)	std(cs)	std(w)	std(u)	std(TFP)
U.S. data	1.00	0.91	0.84	0.57	0.36	0.19	-0.76	0.78
Models								
Baseline	1.00	0.96	0.97	0.80	0.13	0.19	-0.67	0.67
No bargaining shock	1.00	0.99	0.99	0.98	0.78	1.00	-0.95	1.00
No minimum wage $\zeta_0 = 0$	1.00	0.96	0.97	0.91	0.26	0.19	-0.69	0.68
Same, no bargaining shock	1.00	0.99	0.99	0.98	0.77	1.00	-0.95	1.00
No minimum wage $\zeta_0 = 0$, $\tau_n = \tau_k = 0.3$	1.00	0.96	0.97	0.91	0.26	0.19	-0.69	0.68
Same, no bargaining shock	1.00	0.99	0.99	0.98	0.78	1.00	-0.95	1.00
Persistence	Y	$\frac{std(I)}{std(Y)}$	$\frac{std(C)}{std(Y)}$	std(ncs)	std(cs)	std(w)	std(u)	std(TFP)
U.S. data	0.87	0.82	0.78	0.76	0.74	0.67	0.90	0.78
Models								
Baseline	0.81	0.77	0.83	0.80	0.71	0.80	0.70	0.79
No bargaining shock	0.81	0.81	0.80	0.80	0.48	0.79	0.83	0.79
No minimum wage $\zeta_0 = 0$	0.82	0.78	0.83	0.77	0.48	0.79	0.74	0.79
Same, no bargaining shock	0.80	0.81	0.80	0.80	0.45	0.79	0.83	0.79
No minimum wage $\zeta_0 = 0$, $\tau_n = \tau_k = 0.3$	0.82	0.78	0.84	0.78	0.51	0.79	0.74	0.79
Same, no bargaining shock	0.80	0.81	0.80	0.80	0.47	0.79	0.83	0.79

Note: Quarterly data, HP-filtered with smoothing parameter $\lambda = 1,600$. We average the monthly model-generated data first within quarters before HP-filtering.

Table H.12: Welfare effects of increased or reduced political distribution risk

Specification	std(Y) [%]	$\frac{\text{Std(C)}}{\text{StdY}}$	Std(cs) [%]	Std(n) [%]	Welfare change Consumption units
Baseline	1.91	0.62	0.17	1.89	0
No bargaining shock	1.35	0.57	0.01	0.24	+2.40%
No minimum wage recipients: Hosios fails					
No minimum wage $\zeta_0 = 0$	1.96	0.62	0.11	1.82	0
Same, no bargaining shock	1.37	0.57	0.01	0.23	+2.29%
No minimum wage recipients, equal taxes: Hosios condition holds					
No minimum wage $\zeta_0 = 0$, $\tau_n = \tau_k = 0.3$	1.96	0.62	0.11	1.82	
Same, no bargaining shock	1.37	0.56	0.01	0.23	+2.28%

H.16 Sensitivity analysis

In this final subsection, we include an extensive sensitivity analysis of the quantitative properties of the model.

H.16.1 The role of endogenous fluctuations in bargaining power

Table H.13: Business cycle statistics: 1947Q1–2015Q2

	Volatility							
	Y [%]	$\frac{\text{std(I)}}{\text{std(Y)}}$	$\frac{\text{std(C)}}{\text{std(Y)}}$	std(ncs) [pp.]	std(cs) [pp.]	std(w) [%]	std(u) [%]	std(TFP) [%]
U.S. data	1.99	3.28	0.58	1.07	0.86	0.95	0.83	1.21
Models								
S&M model: baseline	1.91	3.28	0.62	0.33	0.17	1.45	1.89	1.21
S&M model: no bargaining shock	1.35	3.33	0.57	0.18	0.01	1.13	0.24	1.20
S&M model w/ end. barg. power	1.71	3.28	0.60	0.48	0.35	1.59	1.64	1.21
S&M model w/ end. barg. power w/o shock	1.32	3.10	0.61	0.20	0.03	1.14	0.18	1.21
S&M model w/ varying ζ_0, ω	2.06	3.28	0.69	0.29	0.06	1.06	1.67	1.21
S&M model w/ varying ζ_0, ω w/o shock	1.22	3.71	0.50	0.16	0.01	0.96	0.24	1.07
	Cyclicality							
	Y	I	C	ncs	cs	w	u	TFP
U.S. data	1.00	0.91	0.84	0.57	0.36	0.19	-0.76	0.78
Models								
S&M model: baseline	1.00	0.96	0.97	0.80	0.13	0.19	-0.67	0.67
S&M model: no bargaining shock	1.00	0.99	0.99	0.98	0.78	1.00	-0.95	1.00
S&M model w/ end. barg. power	1.00	0.98	0.99	0.71	0.36	0.19	-0.66	0.71
S&M model w/ end. barg. power w/o shock	1.00	0.99	0.99	0.97	0.85	1.00	-0.89	1.00
S&M model w/ varying ζ_0, ω	1.00	0.90	0.93	0.94	0.19	0.19	-0.69	0.90
S&M model w/ varying ζ_0, ω w/o shock	1.00	0.99	0.99	0.97	0.80	0.99	-0.94	0.99
	Persistence							
	Y	I	C	ncs	cs	w	u	TFP
U.S. data	0.87	0.82	0.78	0.76	0.74	0.67	0.90	0.78
Models								
S&M model: baseline	0.81	0.77	0.83	0.80	0.71	0.80	0.70	0.79
S&M model: no bargaining shock	0.81	0.81	0.80	0.80	0.48	0.79	0.83	0.79
S&M model w/ end. barg. power	0.77	0.76	0.78	0.76	0.72	0.78	0.77	0.79
S&M model w/ varying ζ_0, ω	0.84	0.75	0.84	0.86	0.64	0.79	0.81	0.81
S&M model w/ varying ζ_0, ω w/o shock	0.81	0.81	0.81	0.81	0.49	0.79	0.83	0.80

Note: Quarterly data, HP-filtered with smoothing parameter $\lambda = 1,600$. We average the monthly model-generated data first within quarters before HP-filtering. "S&M model w/ end. barg. power" matches the regression coefficient of HP-filtered ϕ on HP-filtered u implied by the partial filter (which uses the baseline calibration) by making the bargaining power depend on the unemployment rate. "S&M model w/ varying ζ_0, ω" makes the share of minimum wage workers ζ_0 vary over time to replicate a regression coefficient of the share of hours worked below the minimum wage on the unemployment rate and another regression coefficient of the replacement rate on the unemployment rate.

H.16.2 The role of persistence

If redistribution shocks are short-lived, agents face little incentive to adjust their investment decisions to the realizations of the shock. Thus, when we calibrate shocks to be less persistent, there is a larger price effect (wages and the capital share become more volatile), but a smaller quantity effect. Recall that we consider two alternative calibrations. In one, the redistribution shock is a business cycle shock with a half-life of 3.5 years. In the other, it has a half-life of 20 years. The lower half-life is just below the posterior 5th percentile, while the higher half-life is below the 95th percentile implied by our filtering of U.S. data. When we re-calibrate the model, the differences with our baseline calibration are minor. With all persistences, bargaining shocks account for around 30% of the variation in GDP. The business cycle shock explains 26% of the gross capital share, compared with 19% in the baseline and 17% in the long-run calibration. See Table H.14 for details.

First, we consider different values of the persistence of the bargaining power shock in addition to the baseline value of $\rho_\phi = 0.98^{1/3}$. For the low persistence, we choose $\rho_\phi = 0.95^{1/3}$. For the high persistence, we choose $\rho_\phi = 0.9914^{1/3}$. For each value, we re-calibrate the model.

Table H.14 and Figure H.35 summarize the results. In short, the output effects of bargaining power shocks are roughly invariant to the persistence. In contrast, with shorter-lived shocks, the bargaining power shock explains more variation in the capital share. This is unsurprising, given that, as argued in the main text, steady-state changes in bargaining power have virtually no effects on capital shares.

Table H.14: Business cycle statistics with different persistence for the bargaining power shock and re-calibrated persistence and investment adjustment cost: 1947Q1–2015Q2.

	Volatility							
	Y [%]	$\frac{\text{std(I)}}{\text{std(Y)}}$	$\frac{\text{std(C)}}{\text{std(Y)}}$	std(ncs) [pp.]	std(cs) [pp.]	std(w) [%]	std(u) [%]	std(TFP) [%]
U.S. data	1.99	3.28	0.58	1.07	0.86	0.95	0.83	1.21
Models								
S&M, $\rho_\phi^3 = 0.95$	2.01	3.28	0.60	0.37	0.23	1.46	1.98	1.21
S&M, $\rho_\phi^3 = 0.95$: no barg. shock	1.32	3.17	0.60	0.18	0.01	1.13	0.20	1.18
S&M, $\rho_\phi^3 = 0.98$ (baseline)	1.91	3.28	0.62	0.33	0.17	1.45	1.89	1.21
S&M, $\rho_\phi^3 = 0.98$: no barg. shock	1.35	3.33	0.57	0.18	0.01	1.13	0.24	1.20
S&M, $\rho_\phi^3 = 0.9914$	1.92	3.28	0.64	0.33	0.16	1.44	1.88	1.21
S&M, $\rho_\phi^3 = 0.9914$: no barg. shock	1.34	3.41	0.56	0.18	0.01	1.12	0.24	1.18

	Cyclicality							
	Y	I	C	ncs	cs	w	u	TFP
U.S. data	1.00	0.91	0.84	0.57	0.36	0.19	-0.76	0.78
Models								
S&M, $\rho_\phi^3 = 0.95$	1.00	0.98	0.99	0.74	0.08	0.19	-0.77	0.69
S&M, $\rho_\phi^3 = 0.95$: no barg. shock	1.00	0.99	0.99	0.98	0.76	1.00	-0.95	1.00
S&M, $\rho_\phi^3 = 0.98$ (baseline)	1.00	0.96	0.97	0.80	0.13	0.19	-0.67	0.67
S&M, $\rho_\phi^3 = 0.98$: no barg. shock	1.00	0.99	0.99	0.98	0.78	1.00	-0.95	1.00
S&M, $\rho_\phi^3 = 0.9914$	1.00	0.94	0.95	0.83	0.18	0.19	-0.60	0.68
S&M, $\rho_\phi^3 = 0.9914$: no barg. shock	1.00	0.99	0.99	0.98	0.80	1.00	-0.95	0.99

	Persistence							
	Y	I	C	ncs	cs	w	u	TFP
U.S. data	0.87	0.82	0.78	0.76	0.74	0.67	0.90	0.78
Models								
S&M, $\rho_\phi^3 = 0.95$	0.83	0.81	0.83	0.82	0.79	0.80	0.82	0.79
S&M, $\rho_\phi^3 = 0.95$: no barg. shock	0.80	0.80	0.80	0.79	0.45	0.79	0.81	0.79
S&M, $\rho_\phi^3 = 0.98$ (baseline)	0.81	0.77	0.83	0.80	0.71	0.80	0.70	
S&M, $\rho_\phi^3 = 0.98$: no barg. shock	0.81	0.81	0.80	0.80	0.48	0.79	0.83	
S&M, $\rho_\phi^3 = 0.9914$	0.80	0.74	0.83	0.71	0.42	0.79	0.57	0.79
S&M, $\rho_\phi^3 = 0.9914$: no barg. shock	0.81	0.81	0.80	0.80	0.50	0.79	0.83	0.79

Note: Quarterly data, HP-filtered with smoothing parameter $\lambda = 1,600$. We average the monthly model-generated data first within quarters before HP-filtering.

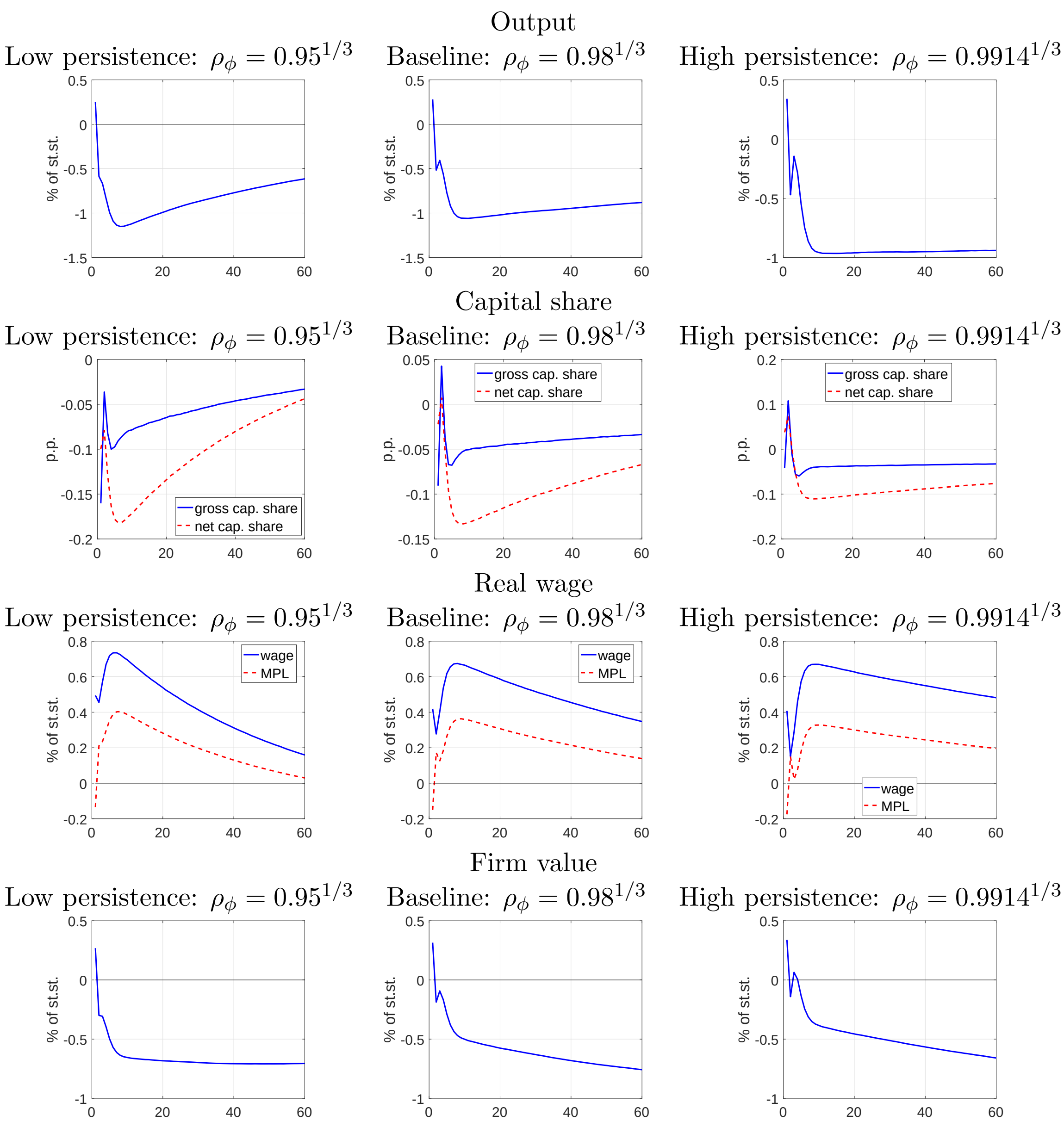


Figure H.35: GIRFs of output, capital share, real wage, and firm value with the model calibrated to different levels of persistence.

H.16.3 The role of the elasticity of substitution

We vary the elasticity of substitution between capital and labor to $\varepsilon = 1 \pm 0.25$. This range includes the estimates in both Oberfield and Raval (2014) and Karabarbounis and Neiman (2014) that we described in Section 5.

When we depart from $\varepsilon = 1$ (see Table H.15 for the complete results), the RBC model produces fluctuations in the capital share, but it cannot match the low but positive cyclicality of the gross capital share. With a low elasticity $\varepsilon = 0.75$, the RBC model generates a correlation of the gross capital share with output of 0.98 vs. 0.36 in the data. For a high elasticity $\varepsilon = 1.25$, the same correlation is -0.98. Similarly, the search and matching model with "no bargaining shocks" can produce sizable fluctuations in the gross capital share, but it fails to account for the cyclicality of capital shares, either getting the sign wrong ($\varepsilon = 1.25$) or considerably overstating it ($\varepsilon = 0.75$).

In comparison, our baseline model with $\varepsilon = 1.25$ matches well the cyclicality of the net capital share (0.65 vs. 0.57 in the data), but it misses the cyclicality of the gross share (-0.45 vs. 0.36). With $\varepsilon = 0.75$, the model can neither match the cyclicality of the gross capital share (0.73 vs. 0.36 in the data) nor the cyclicality of the net share (0.92 vs. 0.57 in the data). In both cases the bargaining shock accounts for between 30% and 42% of output fluctuations.

Table H.15 documents some properties of the model as we change the elasticity of substitution.

Table H.15: Elasticity of substitution and business cycle statistics: 1947Q1-2015Q2.

					Volatility			
	Y [%]	$\frac{\text{std(I)}}{\text{std(Y)}}$	$\frac{\text{std(C)}}{\text{std(Y)}}$	std(ncs) [pp.]	std(cs) [pp.]	std(w) [%]	std(u) [%]	std(TFP) [%]
U.S. data	1.99	3.28	0.58	1.07	0.86	0.95	0.83	1.21
				$\varepsilon = .75$				
S&M: with bargaining shock	1.88	3.28	0.61	0.33	0.21	1.28	1.72	1.21
S&M: no bargaining shock	1.29	3.34	0.56	0.21	0.11	1.00	0.17	1.18
RBC	1.66	3.28	0.59	0.25	0.13	0.84	0.65	1.21
				$\varepsilon = 1.25$				
S&M: with bargaining shock	2.15	3.28	0.62	0.28	0.21	1.51	2.26	1.21
S&M: no bargaining shock	1.31	3.30	0.57	0.11	0.06	1.20	0.23	1.16
RBC	1.66	3.28	0.59	0.34	0.13	0.84	0.65	1.21
					Cyclicality			
	Y	I	C	ncs	cs	w	u	TFP
U.S. data	1.00	0.91	0.84	0.57	0.36	0.19	-0.76	0.78
				$\varepsilon = 0.75$				
S&M: with bargaining shock	1.00	0.97	0.98	0.92	0.73	0.19	-0.72	0.74
S&M: no bargaining shock	1.00	0.99	0.99	0.98	0.98	1.00	-0.95	1.00
RBC	1.00	0.99	0.99	0.98	0.98	0.98	-0.96	0.99
				$\varepsilon = 1.25$				
S&M: with bargaining shock	1.00	0.96	0.97	0.65	-0.45	0.19	-0.74	0.65
S&M: no bargaining shock	1.00	0.99	0.99	0.97	-0.98	1.00	-0.95	1.00
RBC	1.00	0.99	0.99	0.98	0.98	0.98	-0.96	0.99
					Persistence			
	Y	I	C	ncs	cs	w	u	TFP
U.S. data	0.87	0.82	0.78	0.76	0.74	0.67	0.90	0.78
				$\varepsilon = 0.75$				
S&M: with bargaining shock	0.82	0.79	0.84	0.80	0.72	0.80	0.80	0.79
S&M: no bargaining shock	0.80	0.80	0.80	0.80	0.78	0.79	0.81	0.79
RBC	0.80	0.80	0.80	0.81	0.80	0.78	0.79	0.79
				$\varepsilon = 1.25$				
S&M: with bargaining shock	0.82	0.78	0.84	0.79	0.73	0.80	0.73	0.79
S&M: no bargaining shock	0.80	0.80	0.80	0.79	0.82	0.79	0.81	0.79
RBC	0.80	0.80	0.80	0.81	0.80	0.78	0.79	0.79

Note: Quarterly data, HP-filtered with smoothing parameter $\lambda = 1,600$. We average the monthly model-generated data first within quarters before HP-filtering.

H.16.4 The role of market power

In our baseline model, the profits largely go to physical capital. Market power introduces another source of profits: markups. Therefore, we augment our model to encompass market power; see Appendix H.9 for details. If these markups represent pure profits due to inelastic demand in a model of monopolistic competition, our analysis is virtually unchanged. If these markups compensate for the fixed cost of operating, we find a more volatile capital share. Re-calibrating the extended model, we set the elasticity of substitution between varieties to 10, corresponding to an 11% markup. We find (see Table H.16) that with market power these contributions of bargaining shocks to the capital share variation are slightly lower than in our baseline, while the contributions to the output variation are higher. Given that markups are constant, the RBC model cannot generate fluctuations in the capital share, unless there are fixed costs. With fixed costs, the RBC model can only account for 15% of the variation in the capital share. Table H.16 summarizes our findings on the role of market power.

Table H.16: Business cycle statistics: 1947Q1–2015Q2. The role of market power.

	Volatility							
	Y [%]	$\frac{\text{std(I)}}{\text{std(Y)}}$	$\frac{\text{std(C)}}{\text{std(Y)}}$	std(ncs) [pp.]	std(cs) [pp.]	std(w) [%]	std(u) [%]	std(TFP) [%]
U.S. data	1.99	3.28	0.58	1.07	0.86	0.95	0.83	1.21
				Fixed cost				
S&M: with bargaining shock	2.01	3.28	0.57	0.47	0.21	1.23	1.79	1.21
S&M: no bargaining shock	1.27	3.36	0.49	0.28	0.09	0.98	0.18	1.15
RBC	1.81	3.28	0.54	0.39	0.13	0.78	0.87	1.21
				No fixed cost				
S&M: with bargaining shock	2.19	3.28	0.62	0.35	0.16	1.36	2.13	1.21
S&M: no bargaining shock	1.27	3.30	0.57	0.17	0.01	1.09	0.20	1.14
RBC	1.89	3.28	0.60	0.25	0.00	0.92	1.06	1.21

	Cyclicality							
	Y	I	C	ncs	cs	w	u	TFP
U.S. data	1.00	0.91	0.84	0.57	0.36	0.19	-0.76	0.78
				Fixed cost				
S&M: with bargaining shock	1.00	0.97	0.96	0.93	0.75	0.19	-0.75	0.77
S&M: no bargaining shock	1.00	0.99	0.99	0.99	1.00	1.00	-0.96	1.00
RBC	1.00	0.99	0.99	0.99	1.00	0.97	-0.97	0.99
				No fixed cost				
S&M: with bargaining shock	1.00	0.96	0.97	0.85	0.11	0.19	-0.76	0.72
S&M: no bargaining shock	1.00	0.99	0.99	0.98	0.77	1.00	-0.95	1.00
RBC	1.00	0.98	0.99	0.98	NaN	0.96	-0.95	0.99

	Persistence							
	Y	I	C	ncs	cs	w	u	TFP
U.S. data	0.87	0.82	0.78	0.76	0.74	0.67	0.90	0.78
				Fixed cost				
S&M: with bargaining shock	0.82	0.78	0.84	0.84	0.80	0.80	0.78	0.79
S&M: no bargaining shock	0.80	0.80	0.80	0.81	0.79	0.79	0.81	0.79
RBC	0.80	0.80	0.80	0.81	0.80	0.77	0.79	0.79
				No fixed cost				
S&M: with bargaining shock	0.82	0.79	0.84	0.80	0.64	0.80	0.74	0.79
S&M: no bargaining shock	0.80	0.80	0.80	0.80	0.48	0.79	0.81	0.79
RBC	0.80	0.80	0.80	0.81	NaN	0.77	0.79	0.79

Note: Quarterly data, HP-filtered with smoothing parameter $\lambda = 1,600$. We average the monthly model-generated data first within quarters before HP-filtering.

H.16.5 The role of exogenous shocks

Table H.17 reports business cycle statistics with endogenous policy changes.

Table H.17: Business cycle statistics with endogenous policy changes: 1947Q1–2015Q2.

	Volatility							
	Y [%]	$\frac{\text{std(I)}}{\text{std(Y)}}$	$\frac{\text{std(C)}}{\text{std(Y)}}$	std(ncs) [pp.]	std(cs) [pp.]	std(w) [%]	std(u) [%]	std(TFP) [%]
U.S. data	1.99	3.28	0.58	1.07	0.86	0.95	0.83	1.21
Models								
S&M model: policy rule	2.02	3.28	0.68	0.33	0.13	1.39	1.90	1.21
S&M model: no bargaining shock	1.37	3.60	0.52	0.18	0.01	1.14	0.26	1.20
RBC model: baseline	1.90	3.28	0.60	0.24	0.00	0.91	1.03	1.21
	Cyclicality							
	Y	I	C	ncs	cs	w	u	TFP
U.S. data	1.00	0.91	0.84	0.57	0.36	0.19	-0.76	0.78
Models								
S&M model: policy rule	1.00	0.91	0.93	0.87	0.20	0.19	-0.59	0.70
S&M model: no bargaining shock	1.00	0.99	0.99	0.98	0.79	1.00	-0.94	0.99
RBC model: baseline	1.00	0.98	0.99	0.98	NaN	0.97	-0.96	0.99
	Persistence							
	Y	I	C	ncs	cs	w	u	TFP
U.S. data	0.87	0.82	0.78	0.76	0.74	0.67	0.90	0.78
Models								
S&M model: policy rule	0.83	0.74	0.85	0.81	0.62	0.79	0.71	0.79
S&M model: no bargaining shock	0.81	0.81	0.81	0.80	0.49	0.79	0.83	0.79
RBC model: baseline	0.80	0.80	0.80	0.82	NaN	0.77	0.79	0.79

Note: Quarterly data, HP-filtered with smoothing parameter $\lambda = 1,600$. We average the monthly model-generated data first within quarters before HP-filtering.

H.16.6 Alternative calibrations

We report two alternative calibrations: matching the industry- and occupation-adjusted wage rate (Table H.18) and matching the unemployment rate volatility and business cycle statistics (Table H.19).

Specifically, we calibrate our baseline model to the relative cyclicality of real compensation per hour in the non-farm business sector for the post-war sample. Alternatively, we can calibrate the model to the industry- and occupation-adjusted wage rate from the employment cost index, deflated by the PCE deflator. To form the longest possible sample, we splice together the SIC-based measure for wages and salaries and the NAICS-based measure for total compensation. The resultant sample covers 1980 to 2014, and we recompute the target moments: First, wages are now moderately countercyclical, with a correlation of -0.25 instead of +0.19. (For the baseline measure, the cyclicality is -0.02 for the subsample.) Second, investment is smoother, with a relative volatility of 3.06 instead of 3.28. Third, the average tax rate is lower (20% rather than 30%). Fourth, the share of depreciation is slightly higher (13.6% rather than 12.7%). Last, the volatility of TFP is 20% lower over the subsample. Hence, we recalibrate the model using the same strategy as before. Intuitively, we find that bargaining shocks are more important because wages are now more countercyclical. Bargaining shocks account for 73% of the volatility of GDP and 36% of the volatility of the gross capital share. See Table H.18 for more information.

Interestingly, our baseline model yields excess volatility of unemployment. The HP-filtered average quarterly unemployment rate has a standard deviation of 0.83% in the data, but of 1.89% in our calibration. This excess volatility is in stark contrast to the basic search model with only productivity shocks, which is well-known for failing to replicate the volatility of unemployment (Shimer, 2005). Cutting the volatility of the bargaining power shock by about 40%, however, matches the volatility of the unemployment rate. Therefore, bargaining power shocks can be a simple and empirically relevant way to reconcile search and matching dynamic macro models with the data. But as Table H.19 shows, in this case, our model explains only 6% of the volatility of the gross capital share, almost entirely because of bargaining power shocks. Only 5% of output volatility is due to bargaining shocks, but more than 70% of employment fluctuations are due to bargaining power shocks. Moreover, this calibration implies wages that are too pro-cyclical.

Table H.18: Matching the industry- and occupation-adjusted wage rate. Business cycle statistics: 1980Q1-2014Q4.

	Volatility							
	Y [%]	$\frac{\text{std(I)}}{\text{std(Y)}}$	$\frac{\text{std(C)}}{\text{std(Y)}}$	std(ncs) [pp.]	std(cs) [pp.]	std(w) [%]	std(u) [%]	std(TFP) [%]
U.S. data	1.77	3.06	0.47	0.94	0.72	0.48	0.77	0.96
Models								
S&M model: matching ECI wage	2.25	3.06	0.63	0.47	0.27	1.51	2.65	0.96
S&M model: no bargaining shock	0.95	3.06	0.58	0.14	0.01	0.82	0.14	0.86
RBC model: baseline	1.51	3.06	0.60	0.21	0.00	0.73	0.79	0.96

	Cyclicality							
	Y	I	C	ncs	cs	w	u	TFP
U.S. data	1.00	0.95	0.81	0.46	0.21	-0.25	-0.85	0.68
Models								
S&M model: matching ECI wage	1.00	0.96	0.97	0.73	0.13	-0.25	-0.78	0.50
S&M model: no bargaining shock	1.00	0.99	1.00	0.98	0.81	1.00	-0.97	1.00
RBC model: baseline	1.00	0.99	0.99	0.98	NaN	0.97	-0.97	0.99

	Persistence							
	Y	I	C	ncs	cs	w	u	TFP
U.S. data	0.89	0.86	0.76	0.78	0.74	0.78	0.93	0.79
Models								
S&M model: matching ECI wage	0.81	0.76	0.84	0.78	0.68	0.82	0.67	0.79
S&M model: no bargaining shock	0.80	0.80	0.80	0.80	0.48	0.79	0.81	0.79
RBC model: baseline	0.80	0.80	0.80	0.82	NaN	0.76	0.79	0.78

Note: Quarterly data, HP-filtered with smoothing parameter $\lambda = 1,600$. We average the monthly model-generated data first within quarters before HP-filtering.

Table H.19: Matching the unemployment rate volatility and business cycle statistics: 1947Q1-2015Q2.

	Volatility							
	Y [%]	$\frac{\text{std(I)}}{\text{std(Y)}}$	$\frac{\text{std(C)}}{\text{std(Y)}}$	std(ncs) [pp.]	std(cs) [pp.]	std(w) [%]	std(u) [%]	std(TFP) [%]
U.S. data	1.99	3.28	0.58	1.07	0.86	0.95	0.83	1.21
Models								
S&M model: matching std(u)	1.47	3.28	0.60	0.21	0.05	1.20	0.83	1.21
S&M model: no bargaining shock	1.37	3.32	0.58	0.19	0.01	1.15	0.24	1.21
RBC model: baseline	1.90	3.28	0.60	0.24	0.00	0.91	1.03	1.21

	Cyclicality							
	Y	I	C	ncs	cs	w	u	TFP
U.S. data	1.00	0.91	0.84	0.57	0.36	0.19	-0.76	0.78
Models								
S&M model: matching std(u)	1.00	0.98	0.99	0.95	0.27	0.77	-0.58	0.91
S&M model: no bargaining shock	1.00	0.99	0.99	0.98	0.78	1.00	-0.95	1.00
RBC model: baseline	1.00	0.98	0.99	0.98	NaN	0.97	-0.96	0.99

	Persistence							
	Y	I	C	ncs	cs	w	u	TFP
U.S. data	0.87	0.82	0.78	0.76	0.74	0.67	0.90	0.78
Models								
S&M model: matching std(u)	0.81	0.80	0.81	0.81	0.65	0.79	0.83	0.79
S&M model: no bargaining shock	0.81	0.81	0.80	0.80	0.48	0.79	0.83	0.79
RBC model: baseline	0.80	0.80	0.80	0.82	NaN	0.77	0.79	0.79

Note: Quarterly data, HP-filtered with smoothing parameter $\lambda = 1,600$. We average the monthly model-generated data first within quarters before HP-filtering.