ONLINE APPENDIX*

(not for publication)

The Trade-Comovement Puzzle

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All calculations for Section 1 have been performed using Mathematica and are documented in the following Mathematica notebooks available online:

1. File *Baseline-model-results.nb* contains calculations for the baseline model and baseline model under financial autarky. The file generates figure 1, and figures 4-5. Formulas for the figures come in part from this notebook and in part from other notebooks listed below.

2. File *GHH-baseline.nb* contains calculations for the GHH baseline model. The formulas from this notebook are used to generate figures listed in item 1 above.

3. File *Extended-baseline-results.nb* contains result collected from all variants of the extended baseline model. The file generates figures 2-3, and 6-8. The formulas for each individual model are derived in the following notebooks:

   (a) File *Extended-baseline.nb* contains derivation of the equilibrium system for the extended baseline model. The resulting system is used in *Extended-baseline-results.nb*.

   (b) File *Extended-baseline-FA.nb* contains derivation of the equilibrium system for the financial autarky (FA) extended baseline model. The resulting system is used and further processed in *Extended-baseline-results.nb*.

   (c) File *Extended-baseline-GHH.nb* contains derivation of the equilibrium system for the GHH extended baseline model. The resulting system is used and further processed in *Extended-baseline-results.nb*.

   (d) File *Extended-baseline-DTE.nb* contains derivation of the equilibrium system for the dynamic trade elasticity (DTE) extended baseline model. The resulting system is used and further processed in *Extended-baseline-results.nb*.

Dynare codes with calibrated models are in zipped file allDynare.zip. Refer to read-me.txt file for more detailed instructions how to run the codes.

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I. Relation of measure of comovement used in Section 1 to correlation coefficient

Note that the system in text can be written as:

$$\begin{aligned} \hat{y} &= (1-\nu)\hat{A} + \nu\hat{A}^* \\ \hat{y}^* &= (1-\nu)\hat{A}^* + \nu\hat{A}, \end{aligned}$$

where

$$0 < \nu < 1/2.$$

(To obtain the above $\hat{y}, \hat{y}^*$ need to be re-scaled to ensure weights sum to one. Since rescaling variables by a positive constant does not change the correlation coefficient this is without loss of generality.) For simplicity, we can normalize the variance of symmetric shock,

$$var(\hat{A}^*) = var(\hat{A}) = 1,$$

which implies

$$0 \leq cov(\hat{A}, \hat{A}^*) = corr(\hat{A}, \hat{A}^*) \leq 1.$$

By definition,

$$corr(y, \hat{y}) = \frac{cov\,(y, \hat{y}^*)}{var\,(\hat{y})},$$

hence

$$\begin{aligned} cov\,(\hat{y}, \hat{y}^*) &= cov((1-\nu)\hat{A} + \nu\hat{A}^*, (1-\nu)\hat{A}^* + \nu\hat{A}) = \\ &= \left((1-\nu)^2 \ + \nu^2\right) cov(\hat{A}, \hat{A}^*) \ + 2(1-\nu)\nu, \end{aligned}$$

and

$$var\,(\hat{y}^*) = var\,(\hat{y}) = var((1-\nu)\hat{A} + \nu\hat{A}^*) = ((1-\nu)^2 + \nu^2 + 2(1-\nu)\nu corr(\hat{A}, \hat{A}^*)),$$

from which we obtain

$$corr\,(\hat{y}, \hat{y}^*) = \frac{\left((1-\nu)^2 \ + \nu^2\right) corr(\hat{A}, \hat{A}^*) \ + 2(1-\nu)\nu}{(1-\nu)^2 + \nu^2 + 2(1-\nu)\nu corr(\hat{A}, \hat{A}^*)}.$$

The above expression is strictly decreasing in $\nu < 1/2$

$$\frac{\partial corr\,(\hat{y}, \hat{y}^*)}{\partial \nu} = -\frac{2(1 - corr(\hat{A}, \hat{A}^*)^2)(2\nu - 1)}{(1 - 2(1 - corr(\hat{A}, \hat{A}^*))(1-\nu)\nu))^2} < 0,$$

which proves the claim made in text (if $corr(A, A^*)$ is independent of trade).

II. A comment on the effect of trade on the volatility of capital

Consider a two-period storage endowment economy featuring dynamic trade elasticity DTE. The allocation is described by the following planning problem:

$$\max\{u(c) + u(c^*) + u(c_{+1}) + u(c^*_{+1})\}$$

subject to home country constraints:

$$
\begin{aligned}
c + \Delta k &= G(d, f) \\
c_{+1} - \Delta k &= d_{+1} + f_{+1} \\
d + d^{*} &= A \\
d_{+1} + d^{*}_{+1} &= 1,
\end{aligned}
$$

and analogous constraints for the foreign country.

In Mathematica notebook *Extended-baseline-storage-endowment-Appendix.nb*, we show the following result:

Proposition 1 $\hat{c} = \hat{c}_{+1} = \hat{c}^{*} = \hat{c}^{*}_{+1}$, *and*

$$\frac{d\sigma(\Delta k)}{d\bar{x}} \frac{\bar{x}}{\sigma(\Delta k)} \times 100 = \frac{4\bar{x}}{1 - 4\bar{x}^2} \times 100$$

The above result implies that storage technology perfectly smooths out the impact of shocks on consumption across countries and across time (to a first order approximation). However, the volatility of Δk increases with trade $\bar{x}$ to achieve that. The effect is large. For example, for $\bar{x} = 5\%$, the formula implies that a one percent increase in the volume of trade raises the volatility of Δk by 20%! In extended baseline DTE with $\psi = 0$, this number is from 24% to 26% depending on $\rho \in [1, 2]$ (see calculations in Mathematica notebook Extended-baseline-DTE-variance-k.nb).

To develop the intuition for the above result, note from the market clearing condition implies that any a surge in the supply of good f by some $\Delta > 0$ can be smoothed out without changing d/f and d^{*}/d^{*} ratio in the first period by using storage as follows:

$$\begin{bmatrix} 1 - \bar{x} & \bar{x} \\ x & 1 - \bar{x} \end{bmatrix} \begin{bmatrix} \Delta k \\ \Delta k^{*} \end{bmatrix} = \begin{bmatrix} 0 \\ \Delta \end{bmatrix}$$

and hence

$$\begin{bmatrix} \Delta k \\ \Delta k^{*} \end{bmatrix} = \begin{bmatrix} -\frac{\bar{x}}{1-2\bar{x}} \\ \frac{1-\bar{x}}{1-2\bar{x}} \end{bmatrix} \Delta.$$

It is clear that the size of the adjustment is increasing in trade. As discussed in the paper, the extended baseline DTE model penalizes the volatility of capital because of the convex adjustment cost on capital accumulation and decreasing marginal product of capital. This results in the discussed in the paper countervailing effect of trade on the size of transfers.

III. Volatility ratio across countries in the sample

Table 1 reports the estimated volatility ratio for each country in our sample.[1]

IV. Data sources for Section 2

Bilateral trade statistics were taken from International Monetary Fund, Direction of Trade Statistics, 2005. From SourceOECD.org, Quarterly National Accounts: Gross Fixed Capital Formation ("P51:

[1] The table omits Ireland, as the series required for volatility ratio were too short to apply the HP filter.

Table 1: Volatility ratio in a cross-section of major industrialized countries.

	Detrending method	
Country	Hodrick-Prescott filter (1600)	Linearly detrended
Australia	0.88	0.78
Austria	2.76	2.31
Belgium	1.21	1.27
Canada	1.27	1.24
Denmark	1.17	1.52
Finland	1.67	1.31
France	0.77	0.86
Germany	1.38	1.36
Italy	1.07	1.12
Japan	0.68	0.63
Korea	0.59	0.65
Netherlands	0.99	0.77
Norway	1.18	1.21
Portugal	1.07	1.04
Spain	1.89	1.21
Sweden	1.59	2.14
Switzerland	1.05	0.87
United Kingdom	0.90	0.67
United States	1.20	0.88
Median	1.17	1.12

Gross fixed capital formation," "VOBARSA: Millions of national currency, volume estimates, OECD reference year, annual levels, seasonally adjusted"), GDP in constant prices ("B1_GE: Gross domestic product - expenditure approach," "VOBARSA: Millions of national currency, volume estimates, OECD reference year, annual levels, seasonally adjusted"). Our measure of labor is civilian employment or employment from the Quarterly National Accounts or the International Labor Organization (based on data availability). GDP is available from 1980Q1 to 2011Q4 for all countries in our sample. Employment data are missing for some countries for some years. Since labor data are often not seasonally adjusted, we apply the X-12-ARIMA Seasonal Adjustment Program from census.gov. Nominal GDP series come from World Development Indicators. Gross Fixed Capital Formation, GDP in constant prices and Civil Employment series come from SourceOECD.org, Quarterly National Accounts. The series for physical capital have been constructed using the perpetual inventory method with a constant depreciation of 2.5%. Aggregate GDP for blocks of countries has been computed from growth rates of GDP in constant prices (recent years, varies by country) weighted by the nominal GDP of each country in 2004 (we applied the growth rates backward).

V. Additional model versions and parameterization

In this section, we present two additional quantitative experiments:

1. GHH model with trade elasticity equal to 2 instead of 1.17 as in the main text of the paper.

The other parameters are chosen to match the same targets as in the text.

2. The baseline model with separable utility function of the form: $\frac{c^{1-\sigma}}{1-\sigma} - \theta\frac{l^{1+\eta}}{1+\eta}$, where $1/\eta$ is the labor elasticity. We present results for the value of $1/\eta$ equal to 1 and 2. For each of these two cases, we choose the remaining parameters to match the same targets as in the baseline.

Table 2 below presents the parameter choice for each of the models. Table 3 presents the results for business cycle statistics. Table 4 presents performance for the trade-comovement puzzle.

Table 2: Parameter values.

Parameter		Value
	Separable utility with $1/\eta = 1$	
ρ	elasticity of substitution	1.17
$\omega_1^D, \omega_1^F, \omega_1^W$	preference weights country 1	0.81, 0.0.168, 0.1732
$\omega_2^D, \omega_2^F, \omega_2^W$	preference weights country 2	symmetric to country 1
$\omega_3^D, \omega_3^F, \omega_3^W$	preference weights country 3	0.1732, 0.1732, 0.6535
θ	disutility from labor	0.013
ψ	capital adjustment cost	4.2
$\zeta_1, \zeta_2, \zeta_3$	persistence of the productivity shock	$0.84, 0.84, 0.91$
$\mu_{12}, \mu_{13}, \mu_{23}$	Cross-correlation of productivity shocks	0.52, 0.66,0.66
$\sigma_1, \sigma_2, \sigma_3$	Standard deviation of productivity shocks	0.0096,0.0096,0.00694
	Separable utility with $1/\eta = 2$ (if different from case above)	
θ	disutility from labor	0.022
ψ	capital adjustment cost	4.3
$\mu_{12}, \mu_{13}, \mu_{23}$	Cross-correlation of productivity shocks	0.52, 0.65,0.65
$\sigma_1, \sigma_2, \sigma_3$	Standard deviation of productivity shocks	0.0091,0.0091,0.0066
	GHH with $\rho = 2$ (if different from GHH in text)	
$\omega_1^D, \omega_1^F, \omega_1^W$	preference weights country 1	0.698, 0.072, 0.23
$\omega_2^D, \omega_2^F, \omega_2^W$	preference weights country 2	symmetric to country 1
$\omega_3^D, \omega_3^F, \omega_3^W$	preference weights country 3	0.23, 0.23, 0.54
η	inverse of Frisch elasticity	1
θ	multiplier on disutility from labor	0.363
ψ	capital adjustment cost	5.9
$\zeta_1, \zeta_2, \zeta_3$	persistence of the productivity shock	$0.805, 0.805, 0.875$
$\mu_{12}, \mu_{13}, \mu_{23}$	Cross-correlation of productivity shocks	0.49, 0.62,0.62
$\sigma_1, \sigma_2, \sigma_3$	Standard deviation of productivity shocks	0.0079,0.0079,0.00545

Table 3: Business Cycle Statistics: Data and Models[a]

Statistic	Data [b] (median)	Separable utility $1/\eta = 1$	Separable utility $1/\eta = 2$	GHH $\rho = 2$	GHH
A. Correlation					
domestic with foreign					
TFP (measured)	0.44	0.52	0.52	0.49	0.49
GDP	0.52	0.52	0.52	0.52	0.52
Consumption	0.41	0.64	0.63	0.97	0.84
Employment	0.42	0.54	0.53	0.59	0.62
Investment	0.50	0.53	0.54	0.61	0.58
GDP with					
Consumption	0.71	0.96	0.96	0.55	0.85
Employment	0.60	0.95	0.96	0.99	0.99
Investment	0.71	0.99	0.99	0.99	0.99
Net exports	-0.20	-0.64	-0.64	0.67	0.42
Terms of trade with					
Net exports	-0.31	-0.74	-0.75	0.99	0.71
B. Volatility relative to GDP					
Consumption	0.79	0.27	0.27	0.19	0.22
Investment	3.04	3.04	3.04	3.04	3.04
Employment	0.71	0.22	0.29	0.47	0.46
Net exports	0.59	0.05	0.05	0.14	0.02

Notes: The table reports business cycle statistics for each model variant described in the paper relative to the values in the data. [a]Statistics based on logged and Hodrick-Prescott filtered time series with a smoothing parameter $\lambda = 1600$.
[b]Unless otherwise noted, data column refers to the median in our sample of countries for the period 1980Q1-2011Q4.

Table 4: Implied Slope of Trade-Comovement: Fraction Relative to Data.

Model specification	Implied slope relative to data
GHH with trade elasticity 2	19%
Separable utility $1/\eta = 1$	9.5%
Separable utility $1/\eta = 2$	13%

Notes: The table reports the implied slope between trade and comovement (output correlation) by each model specification relative to the corresponding value for the data. Data value is derived from the OLS regression reported in the paper. The slope value for the models has been calculated by increasing bilateral trade intensity from the calibrated median value of bilateral trade to 90th percentile, and accordingly adjusting trade openness with rest of the world.

VI. Transition from the p50 to the p90 steady state of imports.

To get a sense of the dynamics of the adjustment cost friction in the DTE model, in Figure 1 below, we plot the dynamics of transition from the p50 to the p90 steady state of imports from the foreign country to the home country over steady state GDP in the home country, evaluated at steady state prices. Half of the transition to the new steady state happens within 30 quarters, while 90% of the transition happens within 93 quarters.

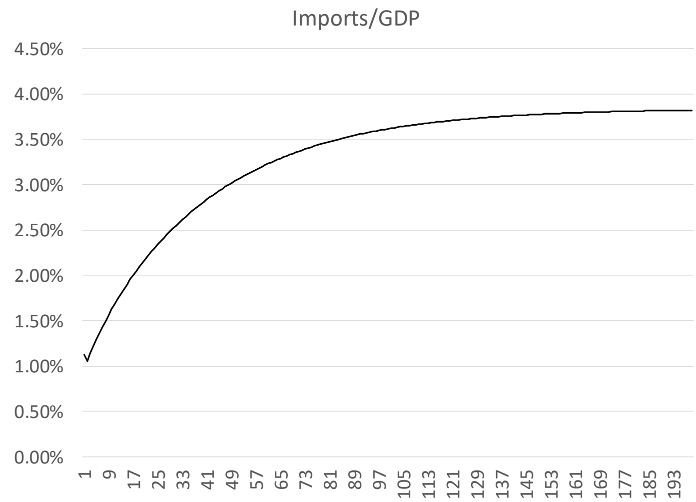

Figure 1: Import over steady state GDP along the transition path.