Federal Reserve Bank of Philadelphia

Working Papers
Research Department

WP 19-48
November 2019
https://doi.org/10.21799/frbp.wp.2019.48

Credit, Bankruptcy, and Aggregate Fluctuations

Makoto Nakajima
Federal Reserve Bank of Philadelphia Research Department

José-Víctor Ríos-Rull
University of Pennsylvania and
Visiting Scholar, Federal Reserve Bank of Philadelphia Research Department

Consumer Finance Institute

ISSN: 1962-5361

Credit, Bankruptcy, and Aggregate Fluctuations*

Makoto Nakajima
Federal Reserve Bank of Philadelphia

José-Víctor Ríos-Rull
University of Pennsylvania
University College, London
CAERP, CEPR, NBER

November 12, 2019

Abstract

We document the cyclical properties of unsecured consumer credit (procyclical and volatile) and of consumer bankruptcies (countercyclical and very volatile). Using a growth model with household heterogeneity in earnings and assets with access to unsecured credit (because of bankruptcy costs) and aggregate shocks, we show that the cyclical behavior of household earnings growth accounts for these properties, albeit not for the large volatility of credit. We find that tilting household consumption towards goods that can be purchased on credit and a slight countercyclicality in the terms of access to credit match the sizes of credit and bankruptcy volatilities. We also find that when the right to file for bankruptcy does not exist unsecured credit is countercyclical.

JEL Classifications: D91, E21, E32, E44, K35
Keywords: consumer credit, default, bankruptcy, debt, business cycle, heterogeneous agents, incomplete markets

*Nakajima: Research Department, Federal Reserve Bank of Philadelphia, Ten Independence Mall, Philadelphia, PA 19106. E-mail: makoto.nakajima@phil.frb.org. Ríos-Rull: Department of Economics, University of Pennsylvania. E-mail: vr0j@upenn.edu. Ríos-Rull thanks the National Science Foundation for Grants SES-0079504, SES-0351451, and SES-1156228. We have benefited from the comments of many people in the long and protracted story of this paper, especially those of Roberto Perotti, who discussed an earlier version of this paper. We also thank Serdar Ozkan who generously calculated for us many intermediate statistics needed to generate our estimates of the income process. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Philadelphia, or the Federal Reserve System.

1. Introduction

Gross total unsecured consumer credit has been consistently above 5% for the past 25 years. The number of personal bankruptcies in the U.S. has been often over 1 million per year (there are about 120 million households). Both variables are very volatile: unsecured consumer credit is three times more volatile than output and procyclical, while household bankruptcies are four times more volatile than output and countercyclical. Is this the result of the cyclical movements in household earnings over the cycle, or is it the result of independent movements in credit availability?

We extend models of household borrowing with bankruptcy protection to accommodate both purchases of credit goods aggregate fluctuations, and we take advantage of newly available data that allow researchers to decompose very precisely the cross-sectional volatility of earnings and its cyclical behavior to address this issue. We pose a model economy of the Bewley (1986)-Imrohoroğlu (1989)-Huggett (1993)-Aiyagari (1994) variety where there are aggregate shocks a la Krusell and Smith Jr. (1997, 1998) and household access to unsecured credit a la Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007)-Livshits, MacGee, and Tertilt (2007) where borrowing is done under a legal system that matches that in the United States, where households have easy access to filing for bankruptcy. In addition to those features we extend the model to include both gross (which we label "store") and net (which we label "bank") credit; this is to allow households to hold simultaneously assets and liabilities due to the existence of some goods that are purchased with credit. In our model aggregate shocks affect the total amount and distribution of households' efficiency units of labor in a way that is consistent with the movements in idiosyncratic earnings documented by Guvenen, Ozkan, and Song (2014) (with strong cyclical movements in skewness), as well as those features of total factor productivity (TFP). As is standard in equilibrium models of the real business cycle variety, the rate of return of the economy is determined by the interaction of the aggregate amount of efficient labor and net societal savings that take the form of real capital. Households respond to the shocks by varying their assets positions, which in some circumstances cause them to borrow and even to file for bankruptcy.

In particular, we ask four questions related to how access to credit affects the nature of business cycles:

1. Does the cyclical nature of earnings dynamics combined with the existence of unsecured credit to smooth consumption and credit for transaction motives account for the high volatility and procyclicality of credit and for the high volatility and countercyclicality of bankruptcy filings found in U.S. data?

2. What properties of earnings dynamics are behind the cyclical behavior of credit and bankruptcies?

3. To the extent that the mere cyclical movements in earnings and productivity are insufficient to replicate all the properties of the cyclical behavior of credit and bankruptcies, what other shocks could contribute?

4. What features of the U.S. consumer credit market are behind the behavior of credit, that is, is the existence of the possibility of having negative assets or access to bankruptcy filing or both crucial in shaping the nature of credit?

Answering these questions will shed some light on the process of how household consumption shapes aggregate credit. Our answers are:

1. Yes, the process for household earnings with explicit modeling of the countercyclicality of risk generates large fluctuations in credit and bankruptcies.

2. The key properties of earnings dynamics that matter for shaping the cyclical behavior of credit and bankruptcies are the differences over the cycle of earnings growth rates in the middle of the income growth distribution. A mere representation of earnings as replicating the process of the Solow residual does not result in credit and bankruptcy patterns like those in the data; in fact, a process for earnings where the higher moments of the earnings growth distribution are acyclical displays countercyclical bank credit. Also the relevant moment is not the variance (it is acyclical in the data) but the skewness. Moreover, it is not necessarily the cyclical behavior of the third central moment, but of the Kelly skewness (the skewness of the middle 80% of the distribution).

3. While the model with only shocks to earnings is capable of generating procyclical credit, the volatility that it generates is only about one quarter of that in the data. We find that an additional shock that shifts consumption from cash goods to credit goods (as the existence of procyclical durables would do) is capable of generating the additional volatility of credit that matches the data. Because of excessive volatility of bankruptcy filings, in this case, we find that an additional shock that makes access to credit *more* difficult in expansions will match the volatility of credit and bankruptcies and their comovements with output.[1]

4. Environments where households can have negative asset positions but do not have access to bankruptcy protection do not generate procyclical credit except to the extent to which it is linked to the purchase of credit goods and not to household borrowing. The overall cyclicality of credit is much smaller, since bank credit becomes countercyclical. Store credit is always procyclical as it depends on consumption, so economies where only this type of credit is available exhibit procyclical credit. Access to both credit and bankruptcy exacerbates the procyclicality of consumption as expansions allow more households access to cheaper credit given the persistence of earnings. We also find that the costs of business cycles are larger in economies with bankruptcy as recessions reduce the ability to borrow in the circumstances when it is more useful.

Some discussion of the findings is due. Perhaps the most important one is to account for why credit is procyclical. Partly and uninterestingly it is because the store credit is a function of aggregate consumption that moves up in expansions. The interesting part, however, is the unsecured or bank credit. Three forces contribute to its cyclical behavior: the cyclical behaviors of rates of return, of the borrowing premia and of the earnings process. Rates of return go up in expansions, which naturally deters the procyclicality of credit. So this force works against it. Business cycles are somewhat persistent so the outlook is better in expansions leading to a lower interest rate premia; this force pushes credit up in expansions. But this is not enough. When we compare processes for earnings with

[1] A shock that made access to credit *easier* in expansions could match the volatility of the premium, but because of the tight link between premia and bankruptcies, this is achieved at the expense of having a much higher volatility of bankruptcies.

different cyclical properties we see that the one calibrated to replicate the features in the data produces procyclical credit while another process where the higher moments of earnings growth are acyclical produces countercyclical credit. We conclude that both the persistence of aggregate conditions and the cyclical properties of the higher moments of earnings growth account for the procyclicality of credit.

We pose a statistic quarterly Markov model for earnings over the cycle that when aggregated into yearly data replicates the cyclical properties of earnings growth that Guvenen et al. (2014) document. The details of this process as stated are very important for shaping the cyclical behavior of credit and we need to have a fairly flexible functional form to be able to replicate those cyclical properties as well as the high earnings concentration of U.S. data.

Our findings require two qualifications in terms of the context in which we do the analysis: that lending institutions themselves do not contribute to economic instability (we model lenders as if they were banks with 100 percent reserves) and that the origin of the fluctuations in the business cycle is some form of TFP shock. Both of these feature are, to say the least, controversial and our questions should be explored in environments where lending institutions are heavily leveraged (or suffer some other vulnerabilities) and where the origin of aggregate fluctuation is related to either demand failures or credit troubles. We leave this analysis for the future.

Our work brings together various strands of literature. First, our model features aggregate fluctuations in growth models in which households are subject to uninsured idiosyncratic labor income shocks (Krusell and Smith Jr. (1997, 1998), Castañeda, Díaz-Giménez, and Ríos-Rull (1998) among many others). Second, we introduce unsecured credit and equilibrium bankruptcies (Chatterjee et al. (2007), Livshits et al. (2007) stemming from Eaton and Gersovitz (1981)). Third, we distinguish between goods by the timing of payments, with credit goods being paid a period later (Lucas and Stokey (1987), Erosa and Ventura (2002) among many others). Finally, the business cycles in our model pose earnings risk that varies over the cycle as argued by Storesletten, Telmer, and Yaron (2004) and Guvenen et al. (2014).

Regarding the effects of credit on consumption volatility, Athreya, Tam, and Young (2009) and to some extent the sovereign default literature (Arellano (2008), Chatterjee and Eyigungor (2012)) argue that unsecured credit does not facilitate consumption smoothing, since riskier types do not get access to cheaper credit. We find that a similar logic applies to consumption smoothing over the business cycles.

There are some papers that explore the business cycle properties of economies where households can file for bankruptcy. Nakajima and Ríos-Rull (2005) explore endowment economies where aggregate fluctuations are announced in advance. Unlike in this paper, they find that both bankruptcy and credit are extremely procyclical; the latter due to the wealth effect of good news, the former because credit loses its luster in good times and not having access to it is not that valuable. Gordon (2015) explores how the presence of aggregate risk affects the welfare assessment of different policies. In particular, the benefits of a more expensive bankruptcy filing cost are smaller in environments with aggregate risk, while aggregate risk barely changes the effects of the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) of 2005 that limited the ability of filing for bankruptcy of those with above median (state) earnings (newborns are slightly worse off under BAPCPA). Fieldhouse, Livshits, and MacGee (2016) explore the properties of endowment economies with aggregate and idiosyncratic shocks and unsecured credit and finds that aggregate fluctuations in earnings generate countercyclical bankruptcy filings, but credit is also countercyclical (it becomes procyclical with a countercyclical intermediation shock). Their model yields low volatilities of credit, interest rates, and debt well below the data. Luzzetti and Neumuller (2018) are concerned about the observed hump-shaped behavior of credit spreads after increases in default. In a small open economy with countercyclical income risk they argue that rational expectations imply an instantaneous response, while if lenders learn slowly about default risk the data can be rationalized. Herkenhoff (forthcoming) argues that the expansion of credit has contributed to make recessions larger and longer over the last 40 years by affecting how people search for jobs (indeed by allowing them to search longer). In the model, credit supply is assumed to expand in expansions (to match the fraction of the unemployed borrowing), which is crucial in generating the slow recovery of employment after a recession. The theory we offer in this paper provides a rationale for why credit expands at the end of a recession. There are other papers that focus on expansion of unsecured credit and the rise of consumer default (Livshits, MacGee, and Tertilt (2010, 2016), Drozd and Serrano-Padial (2017) and Athreya, Tam, and Young (2012) to name a few), while we look at the cyclical fluctuations of these variables.

Section 2 describes the business cycle properties of credit and bankruptcies. Section 3 poses the model of unsecured credit with aggregate fluctuations and credit and non-credit goods. Section 4 maps the model to data, with particular attention to the process for earnings. Section 5 puts the model to use, asking for its implied properties for the cyclical behavior of credit, interest rates and bankruptcies; for what are the important properties of the earnings process; for what other shocks may contribute to increase the variance of credit and tame that of bankruptcy filings; and for what is the exact role of unsecured credit in all of this. Section 6 concludes. Various appendices explore the robustness of some of our assumptions (Appendix A), provide additional details on our implementation of the earnings process (Appendix B), give the details of the computational strategy (Appendix C), provide more details of the U.S. data (Appendix D), and include some additional tables of interest (Appendix E).

2. Business Cycle Properties of the U.S. Economy

We construct time series for credit, bankruptcies, and the interest premium of credit card loans in order to compute the standard statistics (second moments of time series filtered with the Hodrick-Prescott (HP) filter) used for business cycle analysis.

Credit: We define credit as the balance of all credit card debt (Total Real Revolving Unsecured Consumer Credit Outstanding from the Flow of Funds, Federal Reserve Board (FRB)) divided by the corresponding GDP deflator at the end of the respective period. We use the data from 1980:I to 2018:II. Note that this is gross debt as we do not have access to the time series of the asset positions of households.

Bankruptcies: We use the number of bankruptcies (obtained from U.S. Courts) normalized by the number of households (from the Census). Annual data are available from 1980, but quarterly data

Table 1: Cyclical Properties of the U.S. Economy (Quarterly Data, $\lambda = 1600$)

Variable	SD%	Relative SD%	Auto-corr	Cross-Correlation of Output with x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}
Credit	3.81	**3.03**	0.91	0.09	0.24	**0.33**	0.41	0.46
All Bankruptcies	7.89	8.52	0.65	–0.27	–0.35	–0.47	–0.58	–0.52
Chap 7. Bankruptcies	9.55	**10.30**	0.57	–0.23	–0.31	**–0.43**	–0.54	–0.47
Borrowing premium	6.74	**6.32**	0.75	–0.57	–0.64	**–0.70**	–0.67	–0.62
Output	1.28	1.00	0.85	0.68	0.85	1.00	0.85	0.68
Consumption	0.98	0.78	0.83	0.69	0.81	0.86	0.72	0.59
Investment	6.15	4.88	0.84	0.62	0.78	0.90	0.77	0.56

Note: Data are from 1980:I to 2018:II, except for the default premia (from 1995:I to 2018:II) and bankruptcies (from 1995:IV to 2004:IV to exclude the effects of the BAPCPA in 2005).

are available only after 1995:IV. We use the data up to 2004 in order to avoid the effect from the bankruptcy law reform (BAPCPA) in 2005.

Premium: The interest rate premium for credit card loans is the difference between the interest rate of credit card accounts assessing interest (from the Flow of Funds, FRB) and the interest rate of 1-year Treasury Bills.[2] This series is only available since 1995.

Table 1 shows the business cycle properties of these variables as customarily reported (HP-filtered with the smoothing parameter of 1600) along with those of output, consumption and investment. Various properties deserve to be emphasized. Credit is very volatile and procyclical: its standard deviation is 3 times that of output and their correlation is 0.33.[3] It lags output slightly. The number of Chapter 7 bankruptcies is extremely volatile, but this time countercyclical. Its standard deviation is 10 times that of output while their contemporaneous correlation is minus 0.43. Bankruptcies also lag output slightly. Other types of personal bankruptcies follow very similar patterns. The average borrowing premium is also very volatile and strongly countercyclical: it has 6.3 times the standard deviation of output and

[2] There is another interest rate of credit card debt that is available that is associated with all credit card balances. It turns out, however, that the business cycle properties of both series are almost identical so we do not report them.

[3] While the large volatility is a statistic that is robust relative to the time period that we look at, this is not the case for the correlation. For instance, in the subperiod 1985:I-2014:IV the correlation is 0.05 as noted by Luzzetti and Neumuller (2018). Fieldhouse et al. (2016) point out that over the 1993-2006 period, such correlation was negative. They also point out that if the stock of revolving credit were to be adjusted using the credit card charge-off rate, the correlation with GDP could be even more negative.

maintains a contemporaneous correlation of minus 0.70. This premium has a pattern of simultaneity with output, neither leading nor lagging.

To summarize, household credit is very procyclical and volatile; its volume, cheapness, and reliability all move together and much more (between 3 and 10 times) than output.[4] We now turn to develop a model suitable to explore the cyclical property of these variables.

3. The Model

The environment poses households subject to uninsurable earnings shocks that can save or borrow and can, and often do, unilaterally default on their debt. The defaulting protocol in the model captures the procedures defined in Chapter 7 of the United States Code (a.k.a. Bankruptcy Code). The environment aggregates into a neoclassical growth model with consumption and investment that is subject to aggregate risk.

The model poses two different types of credit: unsecured credit to move resources from the future into the present that we denote as *bank credit*, and credit that aids transactions that we denote as *store credit* and that is used by all households to purchase a fraction their consumption. Both types of credit are unsecured in the sense that the household can choose to default on them via a bankruptcy filing. However, the two types of credit are fundamentally different. Bank credit makes the household a borrower and (in general) it would not make sense for it to hold positive assets at the same time, while store credit coexists with either bank credit or positive assets as it is created in the process of consumption. The model is built so as to be mapped to a modern economy with a distribution of wealth capable of comparisons with data and with aggregate movements of consumption and investment. Finally, the model is subject to aggregate fluctuations (as in the work of Krusell and Smith Jr. (1997, 1998) and many others after them). In addition to posing aggregate risk in the household default model, our modeling poses a labor income process capable of accommodating the recent findings by Guvenen et al. (2014) with regard to the cyclical patterns of income risk.

4 These properties also held in annual terms. See Appendix D.

The environment involves the coexistence of various asset-type variables, which involves two fundamental difficulties. One is computational, because it is difficult to solve models with stock variables that are highly co-linear and not subject to severe transaction costs. The other is the delicate issue of how assets with non-identical payoffs in different states of the world open the possibility for households to devise sophisticated hedging strategies that we think are not present when people choose their credit levels and make their bankruptcy choices. We circumvent both problems by posing a particular market structure that allows for the use of the net asset position as the unique state variable. Such market structure is one where the lenders demand that borrowers have the same rate of return variability that savers have. In this case, the actual composition of the portfolio (whether the result say, of a lot of savings and a lot of store credit or of a little of each) is not relevant.

Lenders, on the other hand, will have different ex-post rates of return of asset depending on the state of nature the following period. The equilibrium requirement is that the different types of assets have an equal ex-ante rate of return.

We implement this market structure via competitive mutual funds that attract household savings and that pursue different economic activities: investment in physical capital, provision of different types of bank credit (because it is indexed by household type), and store credit. Sometimes we refer to those activities as if they were run by independent firms that borrow from the mutual funds, which amounts to the same outcome. An additional detail of the market structure is the existence of annuities, which are the natural implication of households that die exponentially and are replaced by newly born poor households.[5]

Production of goods is standard and is subject to aggregate productivity shocks that also shape the distribution of efficiency units of labor as recently documented by Guvenen et al. (2014). Bank credit is given to households and, because of individual specific bankruptcy risks, the terms of the loans are indexed by their characteristics, which are observable. Per the assumption of market incompleteness, the terms of these loans include the same component of aggregate risk that affect households' positive

[5] In Section 4 we discuss the role of this demographic assumption.

savings. Generally, high-risk borrowers are charged a high default premium, and, because of their persistence, recessions increase the default premium for all borrowers.

We assume that store credit is given to all households at the same rate, the result of not observing their individual characteristics. The rate has to be sufficient to cover the default risk and, like bank credit, it includes the same component of aggregate risk that affect households' positive savings.

We now turn to describe the model in detail, starting with the bankruptcy process (Section 3.1), the households (Section 3.2), the technology (Section 3.3) and the market arrangements (Section 3.4). We continue with an early discussion of the state of the economy (Section 3.5), which allows us to move into the problem faced by the household (Section 3.6) and the implied returns to the various types of credit providers (Section 3.7). At this stage we are able to specify in detail the components of the aggregate state of the economy (Section 3.8); we then move to determine the aggregate rate of return of the economy (Section 3.9) and we finish with the definition of equilibrium (Section 3.10).

3.1. Bankruptcy Filings in the Model

The procedure and the consequences of bankruptcy filings are modeled to capture those of the Chapter 7 consumer bankruptcy filings in the United States with the added consequence of being excluded from accessing the credit market for as long the Bankruptcy Code allows the public use of that information, which is 10 years. Let $h \in \{0, 1\}$ denote the credit history; $h = 1$ indicates a record of a bankruptcy filing in the household's recent credit history; and $h = 0$ indicates the absence of any such record. We will refer to h as simply the household's credit history, with the rating either good ($h = 0$) or bad ($h = 1$). A household with a good credit history can borrow and subsequently can file for bankruptcy. Upon filing (a process that we assume is always successful), the household experiences the following events:

- Creditors garnish a fraction ξ of the labor income and any savings it may have. This garnishment represents both partial repayment by the borrower before defaulting and the creditors' attempt to garnish income before the borrower files for bankruptcy. In case the borrower defaults both bank credit and store credit, the two institutions prorate the garnished amount based on the outstanding balance of each credit.

- The unsecured debt balance of the borrower that is not covered by the garnished labor income or savings is discharged. The creditors lose any future claims to the discharged debts.
- When filing for bankruptcy, the household is not permitted to save a positive amount in the current period, except for the exact amount necessary to use store credit.
- The household begins the next period with a bad credit history ($h = 1$). A household with a bad credit history ($h = 1$) cannot get any bank credit but can get store credit as long as it has enough savings to cover it. This assumption is broadly consistent with the experience of bankrupt individuals reported in Musto (1999). The household can save beyond its store credit requirements after the defaulting period.
- There is a positive probability λ that the bad credit history is cleared, so the household starts the next period with a good credit history ($h = 0$). This is a simple way of modeling the fact that a history of a bankruptcy filing remains on an individual's credit history for only a finite number of years.

3.2. Households

We now turn to the demographics, preferences and endowments of households.

Demographics Population is exponential, with a measure one of households each period. Households survive to the next with probability γ, and a measure $(1 - \gamma)$ of newly born households replaces the deceased households. Moreover, there are two types $i \in \{1, 2\}$ of households differing in patience, with υ_i being the fraction of type i. This concise life-cycle structure ensures that households are born with zero assets and therefore do not have time to accumulate as many precautionary assets as they would want, thus creating the conditions for an active loans market.[6]

Preferences Households care about two types of goods that we refer to as *cash goods* and *credit goods*. They have Leontief preferences over those two goods with η being the implied fraction of credit goods. As they are perfect substitutes in production and only differ by the means through which they can be acquired, we refer just to consumption unless there is an explicit need to distinguish them. Household preferences are described by the expected value of a discounted sum of period utilities:

$$E_0 \left\{ \sum_{t=0}^{\infty} \ (\beta_i \gamma)^t \ u(c_t) \right\}, \tag{1}$$

where $\beta_i \in (0,1)$ is the discount factor, $\gamma \in (0,1)$ is the survival probability, and c_t is consumption in period t. E_0 is the expectation operator taken with respect to information in period 0. The utility function u is continuous, strictly increasing, and strictly concave in consumption.

Endowments A household is born with zero assets and faces a stochastic endowment of efficient units of labor. In order to make the cyclicality of income risk consistent with the facts reported by Guvenen et al. (2014), we assume that individual shocks have a persistent and transitory component. The persistent shock p follows a finite-state Markov process with Markov transition matrix $\pi^p_{p'|pz'}$, (primes denote the value of variables in the next period) explicitly depending on z', tomorrow's realization of the aggregate shock. There is also a transitory (i.e. i.i.d) shock t with distribution π^t. With this process, aggregate labor L depends on the whole history of aggregate shocks.

3.3. Technology

Output is produced via $z\ F(K, L)$, where z is an aggregate productivity shock that follows a finite-state Markov process with Markov transition matrix $\Gamma^z_{z,z'}$, K and L are aggregate capital and aggregate labor measured in efficiency units, and F is a constant returns to scale aggregate production function, strictly

[6] See Krusell and Smith Jr. (1998) and Castañeda, Díaz-Giménez, and Ríos-Rull (2003) for justifications of these assumptions to help create both enough households with low assets and large wealth differences across them.

increasing, strictly concave, and satisfying Inada conditions. The good can become either a cash good or a credit good for consumption purposes and can also be invested in physical capital. The aggregate resource constraint is

$$C + K' = (1 - \delta)K + z\ F(K, L), \tag{2}$$

where C is aggregate consumption and δ is the depreciation rate of physical capital. As noted before ηC is the amount of credit good, and $(1 - \eta)C$ is the amount of the cash good.

3.4. Market Arrangements

We assume a complete annuities markets for survival risk: a household that saves $\gamma a'$ will receive a' only if it survives. This feature has no interesting consequences; it just avoids the cumbersome problem of dealing with the wealth of the dead. Households cannot trade securities that are contingent on either the aggregate shock or their own idiosyncratic shock but can smooth consumption by saving or borrowing, and by defaulting on their debt.

A household chooses three types of assets/debts, (y, b, d), where $y \in \mathbb{R}^+$ denotes savings, $b \in \mathbb{R}^+$ denotes bank credit, and $d \in \mathbb{R}^+$ denotes store credit. In order to have only one state variable that captures the asset position in the beginning of the following period we proceed by decomposing the rate of return of the three assets into two parts, one that is specific to the asset and only depends on variables that are known when the choice is made and one that is common to all three assets and that depends on the realization of next period's shock. We accomplish this by applying discounts to both bank and store credit so that the expected return *before* interest is zero. This makes the asset position before interest tomorrow the sum of assets and debts, and then we apply the rate of return of the economy to the net position. We will determine such rate below and we denote it $R^A(Z)$; the superindex denotes that it refers to the net position a at the beginning of a period, and the argument is the aggregate state vector.

Savings y then has a rate of return of $R^{A'}$. Bank credit b, as in the bankruptcy literature, has a discount rate, which depends on all aggregate and individual characteristics that help forecast the probability of default that we denote q^b. Its implied interest rate is $\left(1/q^b\right)\left(1+R^{A'}\right)-1$. Cash goods have to be paid immediately, while credit goods have to be paid the following period using store credit d. Like for bank credit, the discount rate of store credit, which we denote as q^d, has to restore in expected value what is lent. Unlike bank credit, where interest rates are household specific, store credit has interest rates that are the same for all households as we assume that stores do not distinguish the characteristics of shoppers, but it does depend on the aggregate state vector. The interest rate of store credit is then $(1/q^d)(1+R^{A'})-1$. With this convention, the asset position tomorrow is the sum of the three types of assets/debts $a' = y - b - d$.

3.5. State Variables

The aggregate state vector Z consists of the minimum set of variables that are both sufficient statistics for the relevant set of prices and for the forecasting of their own future values. We find it better to leave the discussion of what are exactly its components and why this is the case for later. In the meantime it suffices to refer to it as Z, and to point out that it consists of the aggregate shock z and whatever else that may be needed to determine and forecast prices.

We denote the individual state vector by $\{s, h, a\}$, where we use s to summarize the exogenous individual characteristics of a household $s = \{i, p, t\}$ (to save notation we include the permanent type attribute i that denotes patience here in addition to the persistent p and temporary t dimensions of the labor endowment shock), and we use $\pi^s_{s,s'|z'}$ to denote its transition. The household's credit history is $h \in \{0, 1\}$. The net asset position at the beginning of the period is denoted by a.

3.6. Household's Problem

To solve the their problem, households need to know wages, $w(Z)$, gross rates of return for their assets, $R^A(Z)$, discount prices for each type of bank credit, $q^b(Z, s, y, b, d)$, and discount prices for store credit, $q^d(Z)$. The variables that affect these discount prices are consistent with what is observed: all individual characteristics and choices for bank credit, and none for store credit.

Let us first consider the problem of a household with a good credit history ($h = 0$) that is considering whether to file for bankruptcy. Formally, $V(Z, s, 0, a)$ satisfies:

$$V(Z, s, 0, a) = \max\{V^{non}(Z, s, 0, a), V^{def}(Z, s, 0, a)\}. \tag{3}$$

If the household chooses not to default ($h' = 0$) it solves

$$V^{non}(Z, s, 0, a) = \max_{c,y,b,d,a'} \left\{ u(c) + \beta_i \gamma \sum_{z',s'} \Gamma^z_{z,z'} \pi^s_{s',s|z'} V(Z', s', 0, a') \right\}, \qquad \text{s.t.} \tag{4}$$

$$(1-\eta)c + \gamma[y - b\ q^b(Z, s, y, b, d)] = R^A(Z)\ a + p\ t\ w(Z), \tag{5}$$

$$\gamma\ q^d(Z)\ d = \eta c, \tag{6}$$

$$y, b \geq 0, \tag{7}$$

$$yb = 0, \tag{8}$$

$$a' = y - b - d, \tag{9}$$

$$Z' = \phi^Z(Z, z'). \tag{10}$$

The household chooses current consumption c, savings y, bank credit b and store credit d, all of which consolidate as net asset position a' the following period. Equation (5) is the budget constraint. Savings and credit balances are all multiplied by γ, because of the annuities. Bank credit b is also multiplied by its discount price $q^b(Z, s, y, b, d)$, where $1/q^b(.)$ is the individualized default risk premium, which is charged on top of the gross rate of return of savings of the following period. On the right-hand side, interest accrues to the consolidated asset position a. The last term in the budget constraint is labor income, which is the persistent (p) and transitory (t) labor endowment shocks that are part of s times the wage per efficiency unit $w(Z)$. Equation (6) is the amount of store credit implied by the household's consumption choice (a fraction η of consumption c) and is affected by an aggregate discount rate $q^d(Z)$ to be specified below but that makes cash producers and goods producers get the same expected return. Equation (7) writes as constraints what are really definitions: that savings and bank credit are non-negative. Equation (8) is a redundant condition that states that the household cannot save and borrow at the same time: in effect the household does not want to, given the rate of return differentials between both assets. Equation (9) is the law of motion of the net asset position. Equation (10) is the law of motion of the aggregate state, which will be consistent with agents' optimal decisions in equilibrium. Obviously, this way of writing the problem is a bit of an overkill and we could have eliminated equations (7) and (8) but we think that writing things this way is clearer.

The Bellman equation of a household with a good credit history ($h = 0$) conditional on defaulting ($h' = 1$) can be written compactly as

$$V^{def}(Z, s, 0, a) = \left\{ u(c) + \beta_i \gamma \sum_{z', s'} \Gamma^z_{z,z'} \pi^s_{s',s|z'} V(Z', s', 1, 0) \right\}, \qquad \text{s.t.} \tag{11}$$

$$\left[1 - \eta + \frac{\eta\ \gamma}{q^d(Z)}\right]\ c = (1 - \xi)\ p\ t\ w(Z), \tag{12}$$

$$Z' = \phi^Z(Z, z').$$

Equation (11) shows how saving is not possible when defaulting. Conditional on defaulting, asset position becomes zero next period discharging whatever debts the household may have had. The household has to prepay the store credit including the general premium for whatever defaulters may exist the following period as described in the budget constraint (equation (12)). The household can only consume out of its labor income using whatever is left after being garnished fraction ξ.

Finally, the problem of a delinquent ($h = 1$) household is:

$$V(Z, s, 1, a) = \max_{c,y,d,a'} \Big\{ u(c) + \beta_i \gamma \sum_{z',s'} \Gamma^z_{z,z'} \pi^s_{s',s|z'} \Big\{ \lambda V(Z', s', 0, a') + (1 - \lambda) V(Z', s', 1, a') \Big\} \Big\}, \quad \text{s.t.} \tag{13}$$

$$(1 - \eta)\ c + \gamma\ y = R^A(Z)\ a + p\ t\ w(Z), \tag{14}$$

$$\gamma\ q^d(Z)\ d = \eta c, \tag{15}$$

$$y \geq d, \tag{16}$$

$$a' = y - d, \tag{17}$$

$$Z' = \phi^Z(Z, z').$$

The Bellman equation shows how, with probability λ, the credit history turns good $h' = 0$ and, with probability $1 - \lambda$, the credit history remains bad ($h' = 1$). Equations (16) and (17) determine that the household cannot borrow (its savings have to be higher than its store credit).

We denote decision rules by $c = g^c(Z, s, h, a)$ for consumption, $y = g^y(Z, s, h, a)$ for gross saving, $b = g^b(Z, s, h, a)$ for bank credit, $d = g^d(Z, s, h, a)$ for store credit, $a' = g^a(Z, s, h, a)$ for net assets and $h' = g^h(Z, s, h, a) \in \{0, 1\}$ for the default decision. Note that the standard model without store credit arises when $\eta = 0$, which naturally implies $d = 0$.

3.7. Credit Providers

As discussed before we impose that all assets pay ex-post the same rate of return to households. To achieve this goal it is easiest to decompose the rate of return of bank credit into the discount q^b on a promise to return $bR^A(Z')$ the next period, and the return next period itself $R^A(Z')$. The discount depends not only on today's aggregate state Z, but also on the individual characteristics of the household (s, y, b, d), which we assume are observable to the provider of bank credit. Formally, we have decomposed the interest rate to be paid for bank credit into a default risk part, $q^b(Z, s, y, b, d)$, and the aggregate return for households of their net assets $R^A(Z')$. Defined this way, lending is a linear technology, as lenders can increase the number of loans to households with characteristics (s, y, b, d). A lender that gives $b\ q^b(Z, s, y, b, d)$ to a borrower gets returns tomorrow in state z' that are

$$R^B(z', Z, s, y, b, d) = \frac{\sum_{s'} \pi^s_{s'|s,z'} \left\{ b\ R^A(Z') - g^h(Z', s', 0, y - b - d) \left[b\ R^A(Z') - \xi ptw(Z') \frac{b}{b+d} \right] \right\}}{b\ q^b(Z, s, y, b, d)}. \quad (18)$$

Note that we have written the state tomorrow Z' on the right-hand side. So in order to evaluate the rate of return of their lending, banks have to know the law of motion of the economy, ϕ^Z, which is implicitly used in this formula. We can also get the expected rate of return to $\{s, y, b, d\}$ borrowers:

$$E\{R^B(Z, s, y, b, d)\} = \sum_{z'} \Gamma^z_{zz'} \mathbb{R}^B(z', Z, s, y, b, d). \quad (19)$$

A similar decomposition of the rate of return can be applied to store credit. The difference with bank credit is that the discount term $q^d(Z)$ cannot be made conditional on the individual characteristics of the household as they are not observable by the store. So the rate of return of all store credit is

$$R^D(z', Z) = \frac{\int \sum_{s'} \pi^s_{s'|s,z'} \left\{ d\ R^A(Z') - g^h(Z', s', 0, y - b - d) \left[d\ R^A(Z') - y - \xi ptw(Z') \frac{d}{b+d} \right] \right\} dx(s, h, a)}{\int d\ q^d(Z)\ dx(s, h, a)}. \quad (20)$$

Again, we can calculate the expected rate of return of the store credit industry:

$$E\{R^D(Z)\} = \sum_{z'} \Gamma^z_{zz'}\ R^D\left(Z, z'\right). \quad (21)$$

3.8. Aggregate variables

We now turn to the aggregate variables of the economy. They have to include the variables that are sufficient statistics for the relevant set of prices, which in this case are q^d, q^b, R^A and w. Moreover, they also have to include all variables that are relevant to forecast those sufficient statistics. For our economy these include aggregate capital K, aggregate bank credit given the previous period, B^-, and aggregate store credit given also in the previous period, D^-, as well as the distribution of households over shocks s, credit histories h and net asset holdings a, that we denote x. So we have $Z = \{z, K, B^-, D^-, x\}$. Note that in models without store credit, aggregate capital and aggregate credit can be readily obtained by integrating: aggregate capital is just aggregate wealth while aggregate credit is just the integral over negative assets. However, with store credit, this is not the case. Aggregate net asset position does not tell us how much capital there is, nor the aggregates that determine the rates of return of bank and store credit. Hence, we keep track explicitly of the aggregate quantities of capital and of promises to pay store credit issuers and bank credit issuers.

We next use today's state to determine aggregate labor, L, and next period state:

$$L = \int p \ t \ dx(s, h, a), \tag{22}$$

$$Y = \gamma \int g^y(z, K, B^-, D^-, x, s, h, a) \ dx(s, h, a), \tag{23}$$

$$B = \int \gamma \ q^b[z, K, B^-, D^-, x, s, g^y(Z, s, h, a), g^b(Z, s, h, a), g^d(Z, s, h, a)] \\ g^b(z, K, B^-, D^-, x, s, h, a) \ dx(s, h, a), \tag{24}$$

$$D = \gamma \int q^d(z, K, B^-, D^-, x) \int g^d(z, K, B^-, D^-, x, s, h, a) \ dx(s, h, a), \tag{25}$$

$$K' = Y - B - D, \tag{26}$$

$$x'(s', 0, \widetilde{A})(Z) = (1-\gamma) \sum_s \pi^s_{s'|s,z'} \left\{ \int 1_{g^y - g^b - g^d \in \widetilde{A}} \ 1_{g^h=0} \ x(s, 0, da) + \right. \\ \left. \lambda \int 1_{g^y - g^b - g^d \in \widetilde{A}} \ x(s, 1, da) \right\} + \gamma \ x(s'_0, 0, \widetilde{A}) \ 1_{0 \in A}, \tag{27}$$

$$x'(s', 1, \widetilde{A})(Z) = (1-\gamma)\sum_s \pi^s_{s'|s,z'} \left\{ \int 1_{g^h=1} x(s, 0, da) + (1-\lambda) \int 1_{g^y - g^b - g^d \in \widetilde{A}} x(s, 1, da) \right\},$$

$$\text{for any suitable (Borel) set } \widetilde{A}. \qquad (28)$$

Equation (22) is aggregate labor and Equation (23) is aggregate positive savings from all households. Equations (24) and (25) are the amounts of credit given by banks and stores. Note that it is not the promised amount to repay that enters households' future state but the actual amount disbursed by the banks and stores. Equation (26) is the amount of capital for the next period, and it follows from the aggregate feasibility constraint. The evolution of the distribution of agents is described in equations (27) and (28). We have use shorthand notation to make these equations more compact.

3.9. Aggregate Rate of Return of the Economy

The aggregate shock z, physical capital K, and labor L determine the rate of return of physical capital R^K, which under competition is marginal productivity:

$$R^K(Z) = z\ F_K(K, L) + 1 - \delta. \qquad (29)$$

Total capital income in this economy is the sum of the income coming from the returns on physical capital and on loans. The income coming from physical capital is $K\left[R^K(Z) - 1\right]$. The income from bank loans is $B\left[R^B(Z) - 1\right]$ while that coming from store loans is $D\left[R^D(Z) - 1\right]$. The rate of return of the economy is the appropriate weighted average:

$$R^A(Z) = \frac{K\ R^K(Z) + B\ R^B(Z) + D\ R^D(Z)}{K + B + D}. \qquad (30)$$

Note that while we defined above the rates of return of loans in terms of the following period state, here we are just using the rates of return of the current period, which can be easily calculated given the aggregate state of the economy.

3.10. Equilibrium

We are now ready to define equilibrium. Some of its elements have already been described: the problem of the households and the requirements that aggregate laws of motion have to be consistent with individual decisions. What is left is the determination of prices. Factor price $R^K(Z)$ is already determined and $w(Z)$ is also determined from marginal productivity:

$$w(Z) = z\ F_L(K, L). \tag{31}$$

The key additional conditions are that the rates of return between different economic activities (physical investment, bank credit of various types, and store credit) have to be equated ex-ante as the rates of return of assets are likely to be different ex-post. The equilibrium condition that we impose is that from the point of view of savers (which we can think is done within mutual funds), all economic activities have the same expected rate of return. For this, we write the rate of return of capital next period as a function of tomorrow's aggregate shock z' and today's aggregate state Z by

$$R^K(z', Z) = z'\ F_K \left[K', \left(\sum_{s'} \pi^s_{s'|sz'} \sum_s p(s')\ t(s')\ x(s,.,.) \right) \right] + 1 - \delta. \tag{32}$$

Recall that we have already obtained aggregate capital the following period as a function of today's state and households decisions. The expected rate of return of capital is then

$$E\{R^{K'}(Z)\} = \sum_{z'} \Gamma_{zz'}\ R^K(z', Z). \tag{33}$$

Now we are ready to define equilibrium.

Definition 1 *A recursive equilibrium for this economy are value functions and decision rules for the household as well as actual ($R^A(Z)$, $R^B(Z,s,y,b,d)$, $R^D(Z)$, $R^k(Z)$) and expected ($E\{R^{B'}(Z,s,y,b,d)\}$, $E\{R^{D'}(Z)\}$, $E\{R^{b'}(Z)\}$) rates of return, wages $w(Z)$, discount bank credit $q^b(Z,s,y,b,d)$, and store credit $q^d(Z)$ rates such that*

1. *Households solve their problem.*

2. *Factor prices are marginal productivities.*

3. *Given loan discount prices q^b and q^d and household rates of return R^A, rates of return of bank and store credit satisfy equations* (18) *and* (20).

4. *All economic activities have the same expected rate or return:*

$$E\{R^{B'}(Z, s, y, b, d)\} = E\{R^{D'}(Z)\} = E\{R^{K'}(Z)\}. \tag{34}$$

A careful look at this definition confirms that the list of state variables is both necessary and sufficient for this definition of equilibrium. The distribution of net assets over households, while sufficient for the calculation of the evolution of household choices, is not sufficient to compute the rates of return of the different assets. For them we need to know the amount of capital K, bank credit B^-, and store credit D^- that the mutual fund holds. Hence the need to have them as state variables as posed above in Section 3.8.

We solve the model with a suitable extension of the Krusell and Smith Jr. (1997) method, which we detail in Appendix C.

4. Matching the Model to Data

Recall that our strategy is to replicate some of what we think are the main features of the U.S. economy and then use the model to answer the four questions that we have posed. We parameterize the model economy in three stages. In the first stage, we pin down parameters using targets that are independent of the details of the model and, therefore, can be imposed without solving the model. We think of these targets as uncontroversial. In the second stage, we estimate the parameters that characterize the cyclical and distributional properties of individual earnings. Some of these properties come from the Survey of Consumer Finances (SCF), while the bulk of them are those reported by Guvenen et al.

(2014) in their study of the cyclical properties of earnings. The data come at an annual frequency and the model is quarterly (because we are interested in business cycle properties), which requires that we aggregate the quarterly data generated by the model into yearly data. In the third stage, we pose the remaining parameters so that the steady-state version of the model replicates several targets from the data that we consider important to have confidence in the answers to the questions that we pose. This third stage involves solving the model many times as it is akin to using an exactly identified method of moments.

Before diving into the details of each stage in the following subsections, we want to make the case for two modeling design choices that we have made: the population structure is exponential (agents face a constant probability of death after which they are replaced by others) and the existence of types that differ in preferences. The rationale for these choices is the desire to replicate the amount of unsecured borrowing in the U.S. economy as well as its wealth concentration. The two demographic assumptions that we make suffice to achieve these two targets. The exponential population assumption (together with no altruism) makes all new agents be born with zero assets, which together with patience heterogeneity allows for large wealth accumulation while at the same time having a group of agents engaging in borrowing even when facing large interest rates. Alternative ways to generate large borrowing, large wealth and large wealth concentration are possible but more cumbersome. They involve adding more life-cycle features (retirement and bequest motives) and more shocks, such as shocks to the marginal rate of substitution that induce agents to increase consumption borrowing more and expenditure shocks that place agents in negative asset positions without being voluntarily lent resources. We have found it simpler to use discount rate shocks.[7]

[7] Krusell and Smith Jr. (1998) used preference shocks, while Castañeda et al. (2003) showed how earnings shocks and life-cycle features generate the observed wealth concentration. To get the features of credit, Livshits et al. (2007) posed a life-cycle model while Chatterjee et al. (2007) posed shocks to the marginal rate of substitution and expenditure shocks to generate enough debt.

4.1. First Stage: Parameters Set Ex-Ante

At this stage we choose functional forms and 7 parameters. The period utility function has constant relative risk aversion (CRRA), $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. We set $\sigma = 2$, *i.e.* an elasticity of intertemporal substitution of one-half, the standard value in the literature. We set γ, the probability of dying, to 0.005 at quarterly frequency, so that the average duration of life is 50 years. We pose it as a relatively short length because we are abstracting from retirement and want to think only of active households. The probability of credit history recovery is $\lambda = 0.0250$, which implies that, on average, defaulting households are excluded from the unsecured credit market for 10 years. The production function is Cobb-Douglas, $z\ K^{\theta} L^{1-\theta}$, with $\theta = 0.30$, close to the historical average of the capital share of income in the U.S. economy. Capital depreciates at an annual rate of 0.06 ($\delta = 0.015$), aiming for an investment-to-output ratio of 18 percent with a wealth-to-output ratio of 3 (in yearly terms) and a rate of return of 4% (also in annual terms).

We also pose progressive taxation on labor income to generate an after-tax income process, which is the base for household decision making. Following Heathcote, Storesletten, and Violante (2017) and many others before them, the tax and transfer policies are modeled as: $T(income) = income - \tau_0 income^{\tau_1}$. Here τ_0 is linked to the level of taxation while τ_1 is more related to its progressivity. We set $\tau_1 = 0.181$, which is estimated by Heathcote et al. (2017) using the Panel Study of Income Dynamics (PSID) combined with NBER's TAXSIM program. τ_0 is calibrated so that the government satisfies a balanced budget with public expenditures (which are not valued by households) amounting to about 19% of GDP.

4.2. Second Stage: The Earnings Process

We want the earnings process to replicate the properties of earnings in the U.S. in as many dimensions as possible, which include inequality properties, persistence properties, life-cycle properties and, especially, cyclical properties. The best cross-sectional data source of earnings in the U.S. is the SCF, which includes all sources of earnings but it is a small sample and has no relevant panel dimension. The best description of the dynamic and cyclical properties of earnings is in Guvenen et al. (2014) who have an enormous amount of data (a random 10% of the U.S. male population) for the period 1978-2011. The data come from the Master Earnings File of the U.S. Social Security Administration, which uses data from W2 forms for wage and salary workers. These data are impressive in their scope but miss elements of earnings that are important: business income, partly but not only of the self-employed, and also that part of capital gains that is associated to previous labor efforts (such as when the owner of a business sells it or when stock options are realized above the amount calculated when they were issued). Business earnings and capital gains from business sales are the territory of the highest earners, which are then going to be misrepresented when using W-2 data alone.[8] For this reason, we have taken the sample in Guvenen et al. (2014) to be representative of the bottom 99% of earners and we match the properties of the top 1% of earners in the SCF; further we have chosen the average duration of being in the top 1% to be 5 years.

Guvenen et al. (2014), when analyzing the cyclical behavior of the earnings distribution, partition the period since 1978 into expansions and recessions. According to their partition, expansions last on average 26.7 quarters and recessions 11.0 quarters.

Guvenen et al. (2014) compute the properties of the cross-sectional distribution of individual earnings growth rates. The picture that emerges from Figure 7 of their paper and from the first column of Table 2 (computed using data generously made available for us by the authors) can be summarized by the following properties:

- The mean of the growth rate of earnings is higher in expansions than in recessions.
- The variance of earnings growth is much higher for low earners compared to those in the 70-90th percentile.[9]

[8] See Kuhn and Rios-Rull (2016) for instance.

[9] This difference is high enough to have inspired Guvenen et al. (2014) to say that relative to earnings growth risk the 70-90th percentile of earnings is a "safer zone".

- The variance of earnings growth is acyclical.

- Skewness is strongly cyclical: it is negative and much larger in absolute value in recessions. But skewness is still too vague a description depending on how much weight we put on the extreme tails of the distribution. We have three measures of skewness that we can use:

  1. Pearson Skewness or just skewness (the third moment with respect to the mean) is the skewness measure that puts the most weight on the tails of the distribution. In our sample it is strongly negative and its absolute value is more than twice as large in recessions than it expansions.

  2. Kelly Skewness $[(P90 + P10 - 2\ P50)/(P90 - P10)]$ disregards the two deciles on the tails of the distribution. Its value is mildly positive in expansions and negative with twice its absolute value in recessions.

  3. Median Skewness, also known as Pearson's second measure of skewness [3 · (mean – median) /standard deviation], is somewhere in between the previous two: it takes into account the tails but does not put enormous weight on it. This one is also mildly positive in expansions and negative and twice as large in absolute value in recessions.

We want the earnings process that delivers these properties of earnings growth rates along with the main properties of earnings inequality in the U.S. (share of the top 1%, Gini index in annual terms) and a modicum of life cycle that we take to be the earnings ratio between those in their 50s and those in their 20s. We also want the cyclical movements in inequality to be consistent with output volatility.[10]

With all these in mind, we pose a two-stage Markov process where agents switch between a state designed to include the top 1%, the *high state*, and *the normal state* for the bottom 99%. This requires two parameters (the transition probabilities) that can be specified ex-ante.[11] Some features

[10] We discuss below how we make sure that we get output volatility even though it is jointly dependent on the exogenous movements of earnings and the endogenous movements of capital.

[11] We follow in this regard Castañeda et al. (2003) and Kindermann and Krueger (2014) who also pose a high earnings state to give a rationale to high wealth accumulation. We however do not aim at replicating the right tail of the

Table 2: Estimation Targets of Individual Earnings Process

	Data	Weight	Baseline	No-PS1	No-PS2
Annual Earnings Growth Rates Whole Sample					
Skewness (expansion)	-22.82	10.0	-21.59	-30.37	-30.48
Skewness (recession)	-46.45	10.0	-49.44	-30.55	-30.49
Kelly's skewness (expansion)	4.67	10.0	4.60	0.38	2.62
Kelly's skewness (recession)	-9.52	10.0	-9.44	0.35	-6.24
Mean (expansion)	1.36	1.0	1.44	0.20	0.66
Mean (recession)	-2.72	1.0	-3.84	-1.09	-2.68
Standard deviation (expansion)	53.00	1.0	51.17	51.02	51.70
Standard deviation (recession)	53.64	1.0	51.13	51.04	51.93
P10 (expansion)	-42.68	0.1	-54.34	-57.07	-56.86
P10 (recession)	-50.76	0.1	-64.33	-58.40	-63.75
P50 (expansion)	0.50	0.1	0.77	0.44	0.37
P50 (recession)	-0.70	0.1	-1.21	-0.82	-0.83
P90 (expansion)	47.94	0.1	61.17	58.39	60.66
P90 (recession)	40.65	0.1	51.04	57.16	54.68
Median Skewness (expansion)	0.049	-	0.039	-0.014	0.017
Median Skewness (recession)	-0.113	-	-0.154	-0.016	-0.107
Annual Earnings Growth Rates Earners in 70%-90% percentiles					
P10 of (expansion)	-25.71	0.1	-29.46	-30.19	-29.95
P10 of (recession)	-31.15	0.1	-34.59	-31.56	-35.22
P50 of (expansion)	-0.38	0.1	-0.60	-0.71	-0.49
P50 of (recession)	-1.06	0.1	-1.67	-2.02	-1.55
P90 of (expansion)	21.31	0.1	21.22	21.29	21.51
P90 of (recession)	18.97	0.1	20.41	20.05	20.66
Moments from Other Sources					
Top 1% share of earnings	12.23	10.0	12.24	12.24	12.44
Gini index of annual earnings	51.14	10.0	50.39	50.40	51.24
Earnings ratio between age 55 and 25	1.788	10.0	1.787	1.792	1.777
Standard deviation of HP-filtered output	1.284	10.0	1.287	1.284	1.284
Average weighted % deviation between model and data			3.05	18.53	8.18
Average unweighted % deviation between model and data			18.93	31.79	20.30

Note: All are in percentage. Data in the top panel are from Guvenen et al. (2014). Gini index and the top 1% share are computed from the Survey of Consumer Finances, 2004, using households of age 20-65. Earnings ratio between age 55 and age 25 is calculated using the American Community Survey (ACS), averaging between 2001 and 2005. Output is real GDP, from the Bureau of Economic Analysis. Output (real GDP) is detrended with the HP filter with the smoothing parameter of 1600.

of the process governing the earnings while in the normal state will depend on the aggregate state of the economy. Accordingly we pose two states for the aggregate shock z with the same duration of expansions and recessions that were calculated by Guvenen et al. (2014), which determine the persistence of the aggregate state, $\Gamma^z_{zz'}$. This gives two additional parameters that can be determined before proceeding to the actual estimation. All the predetermined parameters are reported in the first two panels of Table 4.

We assume that log earnings are the sum of a persistent, p, and a temporary, t, component. The temporary component follows a standard normal with zero mean and σ_e standard deviation and affects both the high and the normal state. The persistent component of the high state is constant. An agent can enter the normal state either from birth, and in this case it draws its initial persistent log earnings from a normal distribution with mean μ_0 and standard deviation σ_0, or from being in the high state, in which case it draws its persistent component of earnings from the ergodic distribution of the persistent earnings of the normal state that we denote $\overline{F}_p$, yet to be described. An agent that stays in the normal state has a persistent component of its earnings tomorrow given by

$$p' = \rho_p p + \epsilon, \qquad \text{where} \qquad \epsilon \sim SN(\mu, \sigma^2(p), \alpha^z). \tag{35}$$

That is, the persistent component of earnings follows an AR(1) process with persistence parameter, ρ_p. In order for the growth rate of earnings to exhibit skewness we pose a non-standard innovation, ϵ, amenable to display skewness. We assume that its variance is linked to the level of the persistent component of earnings and we also let its skewness depend on the aggregate state. Specifically, we pose an innovation, ϵ, that has three non-standard features:

1. The innovation is distributed according to a *skew normal* (SN) distribution which is characterized by three parameters, $(\mu^z, \sigma^2(p), \alpha^z)$. While its mean is $\mu^z + \sigma(p)\phi^z\sqrt{2/\pi}$ with

wealth distribution by means of choosing the size and duration of this state. That would require getting the right earnings not by the top 1% but by the top 0.1% or even the top 0.01%. For an alternative view of what accounts for the right tail of the wealth distribution see Benhabib, Bisin, and Luo (2017) and Hubmer, Krusell, and Smith Jr. (2018).

$\phi^z = \alpha^z/\sqrt{1+\alpha^{z2}}$, its variance is $\sigma^2(p)(1-2\phi^{z2}\pi)$, and its skewness is $(4-\pi)(\phi^z\sqrt{2/\pi})^3/(2(1-2\phi^{z2}\pi)^{3/2})$; when $\alpha^z = 0$ this distribution collapses to a standard normal with mean μ and standard deviation $\sigma(p)$, and we use heuristic language to loosely associate parameters $\mu^z, \sigma^2(p)$ and α^z with its mean, variance and skewness.[12] Therefore, to economize on the number of parameters we set $\mu^z = 0$, for both expansions (EXP) and recessions (REC).

2. Parameter $\sigma^2(p)$ is not constant. Instead, we assume that it changes linearly with the current level of persistent log earnings p:

$$\sigma(p) = \sigma_1 + \sigma_2\,\widetilde{p}, \tag{36}$$

where $\widetilde{p}$ is the difference between p and the ergodic mean of $\overline{F}_p$, normalized by the standard deviation of $\overline{F}_p$.

3. Parameter α^z depends on the aggregate state of the economy z and therefore takes two values that switch with transition $\Pi^z_{z,z'}$.

We then have to estimate 10 parameters: mean and standard deviation of the initial earnings draw (μ_0, σ_0); four parameters of the skewed normal innovation to the persistent shock (σ_1, σ_2, and the values for α^z for expansions and recessions); the persistence of the persistent process ρ_p; the standard deviation of the temporary shock σ_e; the value of income for the top 1% $\overline{p}$;[13] and, finally, the difference of the value of the productivity shock z between expansions and recessions because of its contributing to the variance of earnings (it multiplies the marginal productivity of labor).

4.2.1. Estimation

We use a minimum distance estimator to obtain the parameter values, targeting 24 moments. Two of those moments are from all earnings as reported in the 2004 SCF (the Gini Index and the share of the

[12] See for example Wikipedia contributors (2018) and references therein for details.

[13] Since the original stochastic process for normal p has unbounded support, there are agents with income higher than $\overline{p}$. However, when the stochastic process for normal p is discretized, the highest grid point of p is lower than $\overline{p}$.

top 1%). A third moment is the ratio of earnings of full-time males between those in their 50s and those in their 20s (from the American Community Survey (ACS), averaging between 2001 and 2015). The volatility of HP-filtered GDP between 1980 and 2018, which is closely (but not exactly) related to the differential value of z across expansions and recessions, is another targeted moment.

The other 20 moments that we target have to do with the cyclical properties of the earnings distribution studied by Guvenen et al. (2014). Some of them are reported in Tables A.8 and A.13 in their paper while others were generously provided to us by the authors. We report all of them in Table 2 along with the weights that we give to each one of them and the value of the moment generated by the estimated earnings model. The moments are means, standard deviations, the Pearson and Kelly skewness coefficients, the 10th, 50th and 90th percentiles of the growth rates of the whole earnings distributions, and the 10th, 50th and 90th percentiles of the growth rates of earners between the 70th and the 90th earnings percentiles, all of these calculated separately for expansions and recessions.[14]

We do not weigh all moments equally. We have given the highest weight to the 4 moments that deal with the cross-sectional distribution of earnings and to output volatility. We have also given the highest weight to both the Pearson and Kelly skewness statistics (both in expansions and recessions). We have given less weight to the means and standard deviations of the growth rates of earnings (again both in expansions and recessions). The smallest weights have gone to the selected percentiles of the distribution of growth rates of earnings. Overall, we think that the goodness of fit is quite good.

A relevant issue that we have to deal with is that in the model observed earnings are not determined uniquely by the individual shock; instead they are the product of the shock and the price of efficiency units of labor, which in turn is the joint outcome of the TFP shock and the aggregate quantities of capital and labor. While the quantity of labor can be calculated without solving the model, the quantity of capital cannot as it is the result of previous savings decisions. Fortunately, aggregate movements in capital are small at a cyclical frequency and they do not change the process for earnings very much

[14] The Median (or second Pearson measure) Skewness Coefficients are computed directly using the estimates of the selected percentiles of the whole earnings sample and, therefore, do not have independent informational content. They are also reported for the sake of completeness in Table 2.

(see Appendix A), so we have estimated the earnings process assuming constant capital.

Table 2 also provides two measures of goodness of fit. The first one is the average percentage deviation weighted by the estimation weights. Here we obtain a 3.05% deviation. The second one reports the unweighted average deviation, which is 18.9%. Recall that we have 24 targets and 10 moments in the estimation.

One last issue to raise is that we do approximate the stochastic process that we have described with a a finite-state Markov chain with 20 states (including that of the top 1%) for the persistent shock and 5 possible values for the transitory shock. Our method for discretizing the AR(1) process with a skewed normal distribution is a straightforward extension of the method developed by Tauchen (1986), and is explained in detail in Appendix B.

4.3. Third Stage: Parameters Internally Calibrated

Given our choices in stage 1, and our estimates in stage 2, we obtain the rest of the parameters by imposing that various aggregate statistics of the model economy have the same values as their data counterparts (akin to exactly identified method of moments). The 5 parameters involved in this stage are the patience parameters of both types, β_1, and β_2, the fraction of the patient agents, υ_1, the income garnishment parameter, ζ, and the proportion of credit goods, η. The 5 aggregate statistics that we target are tightly related to these parameters and they consist of the aggregate wealth-to-output ratio (3.0 annually), the total amount of bank credit (2.40% of GDP, calculated using households of age 20-60 in 1998-2016 waves of the SCF), the percentage of households with a net negative asset position (11.70%, again using the 1998-2016 SCF), the amount of store credit (3.47%, calculated by taking the difference between total credit reported by the Board of Governors Consumer Credit (G.19), and bank credit from the 1989-2016 SCF). The total number of bankruptcies that we target is 1.20% per year (Chapter 7 bankruptcies from the U.S. Bankruptcy courts).[15]

[15] The average annual bankruptcy rate for the overall population during 1998-2016 is 0.943 according to the U.S. Bankruptcy Court, but we adjust the ration upward by 30% because the working-age population has a higher fraction in debt than the overall population by about 30%.

Table 3: Calibration Targets and Model Performance

Target statistics in Percentages (annualized)	Data	Model
Capital / GDP	300.0	300.0
Bank Credit / GDP	2.400	2.410
Store Credit / GDP	3.470	3.470
Proportion in Net Debt	11.70	11.84
Proportion of Defaulters	1.220	1.215

Table 3 shows the targets in the data and how the Baseline economy replicates them (the estimated parameter values are reported in the third panel of Table 4). Our estimates imply a sharp distinction between the patient and the impatient ($\beta_1 = 0.9858$, versus $\beta_2 = 0.8440$), with the latter being responsible for most of the net debt while being about a fourth of the population. The estimate of the fraction of quarterly earnings recovered by lenders during default events, or income garnishment, $\zeta = 0.3963$. Livshits et al. (2007) estimate the parameter to be 0.355, which is close to ours, but one period in their model is 3 years, so the implicit assumption is that lenders can garnish earnings for 3 years upon default. The estimate of credit goods is that they are 22% of total consumption expenditures. We do not target explicitly the average interest rate of unsecured credit, but the model generates 11.9% per year, while it is 11.8% in the data (Board of Governors Consumer Credit (G.19), average interest rate of all credit card accounts assessing interest, minus 1-year Treasury rate).

5. The Contribution of Earnings Shocks to the Business Cycles Properties of Credit

We now turn to use the model to answer the questions that we posed, whether the cyclical behavior of earnings and productivity in a world with unsecured credit is capable of generating the cyclical properties of credit and bankruptcy that we see (Section 5.1); what are the relevant properties of earnings dynamics responsible for this behavior (Section 5.2); what other shocks may fill the gap between the model and data (Section 5.3); and what role does unsecured credit play in shaping this cyclical behavior (Section 5.4). We finish with a measurement of how the welfare costs of business

Table 4: Parameterization of Baseline Economy

Parameter	Value	Description
First Stage: Parameters Determined Ex-Ante		
σ	2.0000	EIS = 0.5
γ	0.0050	Average life = 50 years
λ	0.0250	Credit Exclusion upon Bankruptcy is 10 years on average
θ	0.3000	Labor share = 0.70
δ	0.0150	Annual depreciation rate = 0.06
τ_0	0.8004	Government balances budget with G/Y=0.19
τ_1	0.1810	Progressivity of income tax, estimated by Heathcote et al. (2017)
Second Stage: Parameters Determined by the Earnings Process		
Second Stage: Parameters Determined Ex-Ante		
ρ_1	0.9500	Persistence of staying top 1% earner
ρ_0	.999495	Probability of staying bottom 99% earner
$\Pi^z_{1,2}$	0.0375	Average duration of expansions = 26.7 quarters
$\Pi^z_{2,1}$	0.0909	Average duration of recessions = 11 quarters
Second Stage: Estimated Parameters		
z^{EXP}	0.0110	Differential productivity in expansions
μ_0	-1.8916	Mean earnings of newborns
σ_0	0.3041	Standard deviation of earnings of newborns
ρ	0.9945	Autocorrelation of persistent shocks to earnings
σ_p	0.6426	"Standard deviation" parameter of persistent innovations
$\widetilde{\sigma}_p$	-0.0965	Slope of "standard deviation" of persistent innovations
$\alpha^{\overline{z}}$	-1.9314	"Skewness" parameter of persistent innovations in expansions
$\alpha^{\underline{z}}$	-2.3252	"Skewness" parameter of persistent innovations in recessions
$\overline{p}$	2.5628	Earnings level of top 1%
σ_e	0.2295	Standard deviation of transitory shocks to earnings
Third Stage: Estimated Parameters		
β_1	0.9858	Discount rate of patient households
β_2	0.8440	Discount rate of impatient households
υ_1	0.7350	Fraction of patient households
ζ	0.3963	Fraction of income that can be garnished
η	0.2204	Fraction of goods that are purchased by credit

Note: All parameters are at quarterly frequency.

Table 5: Data and Baseline Model Economy

	Data	Baseline Model
Standard Deviation Relative to Output		
Total credit	3.03	0.74
Bank Credit	NA	0.82
Bankruptcy filings	10.30	12.98
Total Credit Premium	6.32	3.68
Bank Credit Premium	NA	3.62
Consumption	0.78	0.77
Investment	4.88	1.82
Correlation with Output		
Total credit	0.33	0.96
Bank Credit	NA	0.76
Bankruptcy filings	–0.43	–0.44
Total Credit Premium	–0.70	–0.90
Bank Credit Premium	NA	–0.92
Consumption	0.86	1.00
Investment	0.90	0.98

Note: Quarterly frequency. HP filtered with the smoothing parameter of 1600.

cycles are affected by unsecured credit (Section 5.5).

5.1. Cyclical Earnings Movements and Credit Behavior

What are the implications that the cyclical behavior of earnings have for credit in the economy? We answer this question by comparing the business cycle statistics of the model economy described in the previous section with those in the data. The results are displayed in Table 5, which shows the standard deviations relative to and correlations with output of total credit, bankruptcy filings and the borrowing interest premium along with consumption and investment.[16]

The message of Table 5 is clear: credit is procyclical in the Baseline model economy as is seen in the data: all the indicators of a good state of credit (high volume, few bankruptcies, small credit premia) move together with output. The relative magnitudes, however, are somewhat off in the model economy:

[16] A more detailed version is in Table A5.

the volatility of credit is one fourth of that in the data, that of bankruptcies is one third more than in the data and that of the interest rate premia of unsecured debt is 60% of that in the data. In the model, but not in the data, we can look separately at the role played by bank credit that is made to households with negative asset positions, and store credit that is related to the consumption of the credit good, and we can see that bank credit behaves similarly to total credit. We take these properties to mean that the cyclical behavior of labor earnings over the cycle is the main ingredient in shaping the cyclical behavior of the part of unsecured credit that is used to smooth consumption.

5.1.1. The Role of Persistence of Aggregate Shocks

An important concern is the separation of expansions between their role as good times versus as forecasters of good times. To this end we pose an economy that differs from the Baseline only in that aggregate shocks are now i.i.d. (to have equal frequencies of expansions and recessions, the probability of expansions is set to 0.71). In this economy good times do not make future good times any more likely.

Table 6 compares the Baseline economy with the economy with i.i.d. aggregate shocks. They are quite different. The volatility of total credit is a fifth of the Baseline (and a twentieth of that in the data) and much less positively correlated with output. There are two reasons. First, consumption is less volatile and hence store credit is also much less volatile. Second, bank credit is now countercyclical as households want to borrow more in recessions. Bankruptcies are countercyclical but their volatility is about a third of the Baseline's because of the much lower accumulated debt of households and their smaller fear of being cut off from borrowing in the near future. The volatility of the premium is much smaller as recessions do not forecast future recessions and hence higher bankruptcies. These results confirm that the news effect (persistence) of aggregate shocks is crucial in shaping how credit responds to aggregate shocks, and thus for the Baseline model to replicate key cyclical properties of credit and default. Still news effects are not the only relevant component of credit; aggregate consumption is also relevant.

Table 6: Baseline Model and Model with i.i.d. Aggregate Shock

	Baseline Model	i.i.d.-Shock Model
Standard Deviation Relative to Output		
Total credit	0.74	0.15
Bank Credit	0.82	0.37
Bankruptcy filings	12.98	4.85
Total Credit premium	3.68	0.40
Bank Credit Premium	3.62	0.25
Consumption	0.77	0.33
Investment	1.82	3.53
Correlation with Output		
Total credit	0.96	0.42
Bank Credit	0.76	–0.77
Bankruptcy filings	–0.44	–0.99
Total Credit Premium	–0.90	–0.82
Bank Credit Premium	–0.92	–0.36
Consumption	1.00	0.93
Investment	0.98	0.99

Note: Quarterly frequency. HP filtered with the smoothing parameter of 1600.

5.2. Cyclical Behavior of Earnings and Credit

The cyclical movements of earnings that we have used here are estimated to replicate the properties found by Guvenen et al. (2014) of the cyclicality of the earnings distribution including the main ones that the cross-sectional variance of earnings is acyclical but the skewness is procyclical. To investigate how important this particular set of properties of the dynamics of earnings is, we explore two model economies that do not display procyclical skewness. In what we label "No-PS1", we set $\alpha^{EXP} = \alpha^{REC}$, and reestimate the value for z^{EXP} to ensure the same output volatility. The estimated process generates no cyclical differences in the properties of earnings: the standard deviation, the skewness and the Kelly skewness are all acyclical. In what we label "No-PS2", we pose a process where we allow for general cyclical asymmetries, by letting $\mu^{EXP} \neq 0$, as well as reestimating $\alpha^{EXP} \neq \alpha^{REC} \neq 0$ but where we change the target of cyclical skewness for a constant skewness while maintaining all other

Table 7: Parameterization of Model Economies without Procyclical Skewness

Parameter	Baseline	No-PS1	No-PS2
z^{EXP}	0.0110	0.0129	0.0109
$\alpha^{\bar{z}}$	-1.9314	-2.1283	-2.3744
$\alpha^{\underline{z}}$	-2.3252	-2.1283	-1.2513
μ^{EXP}	–	–	0.0418

Note: All parameters are at quarterly frequency.

targets as before.[17] This allows for cyclical differences in the properties of the cross-sectional earnings distributions other than skewness. Table 2 shows the implied target values while Table 7 shows the estimated parameters of these two economies and the Baseline.

Table 8 compares the properties of the model economies. The first panel displays the volatility properties of these economies and the second panel the correlations with output of the main credit variables. The picture is very sharp. Economy No-PS2 is very similar to the Baseline despite having different skewness (but similar Kelly skewness). However, economy No-PS1 is sharply different: the volatility of credit (0.18) is one-fourth of that in the Baseline (0.74), due both to bank and store credit. But what jumps out of Table 8 is that the correlation of bank credit with output is actually negative at -0.90 (see the second panel). Notice that bank credit is the one that matters most for default, and as a result both bankruptcy filings and the interest rate premium are much lower in No-PS1 (about 12% for bankruptcies and 9% for the premium) than in the Baseline. An interesting feature of Table 8 is that the credit premium is countercyclical while its components are procyclical. The reason for this is the countercyclical nature of bank credit and its important role in shaping the credit premium: in recessions there is more of this credit that has a much higher premium than store credit; hence the total effect on the premium is to make it countercyclical.

Why are economies No-PS1 and No-PS2 behaving so different while the Baseline and No-PS2 economies have very similar properties? To investigate this issue, Figure 1 looks in detail at the properties faced by

[17] Because of normalization of units, μ^{REC} and ^{EXP} are not separately identified. So we only add one parameter.

Table 8: Comparison of Earnings Processes: The Role of Procyclical Skewness

	Data	Baseline	No-PS1	No-PS2
Standard Deviation Relative to Output				
Total Credit	3.03	0.74	0.18	0.71
Bank Credit	Na	0.82	0.25	0.83
Store Credit	Na	0.77	0.43	0.72
Bankruptcy Filings	10.30	12.98	1.58	11.18
Total Credit Premium	6.32	3.68	0.33	3.27
Bank Credit Premium	Na	3.62	0.28	3.23
Store Credit Premium	Na	3.05	0.46	2.69
Consumption	0.78	0.77	0.43	0.72
Investment	4.88	1.82	3.07	1.98
Correlation with Output				
Total Credit	0.33	0.96	0.92	0.96
Bank Credit	Na	0.76	–0.90	0.76
Store Credit	Na	1.00	0.99	1.00
Bankruptcy Filings	–0.43	–0.44	–0.24	–0.46
Total Credit Premium	–0.70	–0.90	–0.63	–0.90
Bank Credit Premium	Na	–0.92	0.07	–0.92
Store Credit Premium	Na	–0.90	0.99	–0.90
Consumption	0.86	1.00	0.99	1.00
Investment	0.90	0.98	1.00	0.99

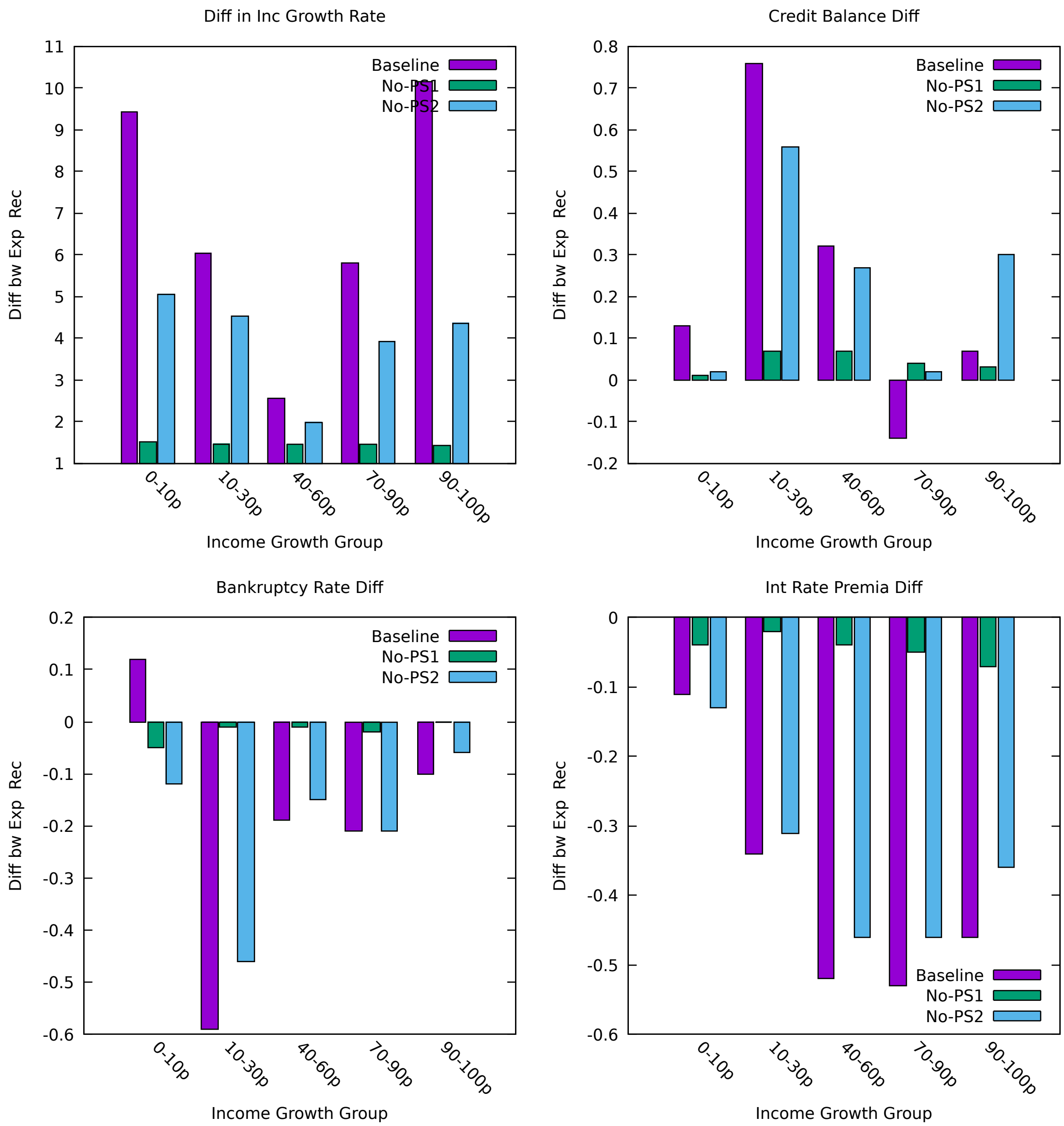
Note: In No-PS1 parameters of the earnings shock are calibrated such that all higher moments of earnings growth distribution are acyclical. In No-PS2 only skewness is acyclical; other moments like the Kelly skewness depend on the cycle.

various household groups that differ in their annual earnings growth rates.[18] We display the differences of the averages between expansions and recessions of the average annual earnings growth rate (top left), credit balances (top right), proportions filing for bankruptcy (bottom left), and interest risk premium (bottom right).

Let's start with the first panel. We see that, in the Baseline model with estimated earnings shocks, households in the bottom decile have their earnings growth shrink by about half while those in the top decile almost double earnings. More importantly, in the Baseline model, earnings growth rate is

[18] A detailed table with the information for this figure is available in Table A6.

Figure 1: Annual Income Growth and Borrowing/Defaulting



higher in expansions than in recessions, at 5.4 percentage points on average. The difference is small in the No-PS1 model (1.5pp), while it is in the middle in the No-PS2 economy (3.5pp). Comparing the Baseline and the No-PS2 economies, the differences of the averages in expansions and recessions are larger for the top and the bottom deciles (about 10pp in the Baseline and about 5pp in the No-PS2 economy), while the cyclical changes in the earnings growth rate of the two economies are smaller in the middle of the distribution. As expected, the differences between expansions and recessions are consistently small in the No-PS1 economy.

The second panel shows how the total credit balance changes between expansions and recessions in the three model economies. In the Baseline we see that for the bottom 10% as well as for the top 30% the size of credit balances are very similar in expansions and recessions. However, for the 10-30% the difference in credit balances in expansions and recessions is significant. A similar, albeit a bit smaller, pattern holds for the No-PS2 economy where the households in that 10-30% of earnings growth also experience a much larger level of credit in expansions than in recessions. This is in contrast to the same group in the No-PS1 economy where credit balances in expansions and recessions are quite similar to each other. The 40-60% group also has a sizable difference in the Baseline and in the No-PS2 economy while almost no difference in the No-PS1 economy. Looking at the other two variables, bankruptcy filings and the default premium for households in the various earnings growth rate groups, we see that in the Baseline and the No-PS2 economies there are large differences in the 10-30% groups and much smaller differences elsewhere, while they are all quite similar in the No-PS1 economy.

We conclude that the households that are mostly responsible for the procyclicality of credit and the countercyclicality of the bankruptcy filing rate are those in 10-30% (and to a smaller extent the 30-60%) earnings growth rate percentiles. The data as analyzed by Guvenen et al. (2014) display a very negative value of the earnings growth rate for those people in recessions relative to expansions and the Baseline economy replicates this feature. Turning off the cyclical patterns altogether as we do in economy No-PS1 eliminates these differences uniformly over all households, implying very small cyclical differences in credit volatility. In economy No-PS2 we eliminate total skewness, but not the cyclical differences in earnings growth rates for all groups. In particular, maintaining the differences in the Kelly skewness across expansions and recessions seems to be crucial property of the dynamics of the earnings distribution in generating a sizable aggregate credit and bankruptcy filings volatility.

Table 9: Comparison of Model Economies: Additional Shocks

	Data	Baseline	Model 1	Model 2	Model 3	Model 4	Model 5
Additional Shock							
Credit good shock	–	–	+1.149	–	–	–	+1.333
Credit premium shock (%)	–	–	–	+0.065	–0.190	–	+0.137
Bankruptcy cost shock (%)	–	–	–	–	–	+0.51	–
Standard Deviation Relative to Output							
Total Credit	3.03	0.74	**3.03**	0.61	1.13	0.74	**3.03**
Bank Credit	NA	0.82	0.65	0.62	1.80	0.83	0.85
Store Credit	NA	0.77	5.30	0.75	0.80	0.77	6.05
Bankruptcy Filings	10.30	12.98	14.06	**10.31**	18.87	**10.32**	**10.30**
Total Credit Premium	6.32	3.68	5.68	2.48	**6.31**	2.44	2.96
Bank Credit Premium	NA	3.62	3.53	2.00	7.50	2.34	0.87
Store Credit Premium	NA	3.05	2.89	3.50	1.24	1.99	3.78
Consumption	0.78	0.77	0.77	0.75	0.80	0.77	0.75
Investment	4.88	1.82	1.81	1.85	1.84	1.82	1.87
Correlation with Output							
Total Credit	0.33	0.96	0.99	0.94	0.94	0.97	0.99
Bank Credit	NA	0.76	0.12	0.49	0.83	0.78	–0.62
Store Credit	NA	1.00	0.99	1.00	0.99	0.99	0.98
Bankruptcy Filings	–0.43	–0.44	–0.44	–0.52	–0.36	–0.56	–0.54
Total Credit Premium	–0.70	–0.90	–0.95	–0.89	–0.90	–0.85	–0.96
Bank Credit Premium	NA	–0.92	–0.91	–0.90	–0.92	–0.88	–0.41
Store Credit Premium	NA	–0.90	–0.88	–0.90	–0.58	–0.87	–0.90
Consumption	0.86	1.00	0.99	1.00	0.99	0.99	1.00
Investment	0.90	0.98	0.98	0.99	0.92	0.98	0.99

Note: Quarterly frequency. HP filtered with the smoothing parameter of 1600.

5.3. Other Shocks and Credit Volatility

What other shocks could be responsible for the additional volatility of credit and bankruptcy filings? We explore various possibilities: a *credit good shock* (a taste shock that increases the share of the credit goods out of total consumption), a *credit premium shock* (a shock to the borrowing markup), and a *bankruptcy cost shock* (a shock to the cost of filing for bankruptcy). The answers are in Table 9.

We start considering the shock that increases share of credit goods out of consumption goods in expansions. A rationale for this shock is the procyclical nature of durable goods that are both very procyclical and purchased overwhelmingly with credit (see for instance Gavazza and Lanteri (2018)). The size of the shock (its variance) replicates the standard deviation of total credit in the data. We see that a switch of 1.15% does the exact job.[19] This economy is labeled Model 1. In addition to replicating the volatility of credit in the U.S., it maintains its procyclicality and increases that of the total credit premium. It generates a volatility of bankruptcy filings that is slightly too large, 14 times the output volatility instead of 10 times.

We then turn to explore shocks to the ease of credit. To this end we pose shocks to the borrowing interest markup (both positive and negative) during expansions. These shocks are additive to whatever interest rates are determined by the market. In Model 2 we choose the value that replicates the volatility of bankruptcies while in Model 3 we choose the value that replicates the volatility of the credit premium (we have not been able to find values of this shock that are capable of matching the overall credit volatility by itself). Two things come out of this exercise: that a credit tightening shock is capable of reducing the bankruptcy filings volatility to the level in the U.S. data but only at the expense of further reducing the volatility of credit which in the Baseline is still too low, and that an expansionary shock to credit can replicate the volatility of the interest rate premium, but only at the expense of a volatility of bankruptcy filings that is almost twice that in the data. Moreover, the implied volatility of credit while going up a beat is still about a third of that in the data. We conclude that shocks to the markup by themselves do not seem to make the model more in line with the data.

The last type of shock that we consider hits the cost of bankruptcy filing, making it more costly in expansions, perhaps due to the fact that the courts are easier to satisfy in recessions in a similar vein than disability committees, or just because they are overwhelmed with filers. We choose such a shock on top of the Baseline economy to ensure that the volatility of bankruptcies is the same as in the data and we label this economy Model 4. While this shock reduces the volatility of bankruptcies, it also reduces that of the interest rate premia with which it is closely associated and it has literally no effect

[19] This value implies that on average $\eta = 0.236$ in expansions and $\eta = 0.183$ in recessions. The actual formula is $\eta = .22 + 1.15 + \widehat{c}$, where $\widehat{c}$ is the percentage deviation of consumption from its steady-state value.

on the volatility of credit. We conclude that is not a very effective shock in shaping credit.

Finally we pose two shocks simultaneously to generate in the model the same volatilities of credit and bankruptcy filings that we observe in U.S. data. These shocks are the credit good shock and the credit premium shock and we report the properties of this economy as Model 5. The shocks that are needed to achieve these two statistics are a large procyclical credit good shock and a sizable negative credit premium shock. That is, in expansions the share of credit goods increases and credit becomes more expensive. The first shock is responsible for the increase in credit volatility, while the second is needed to tame the bankruptcy volatility. This is achieved by tweaking even more consumption towards credit goods than in Model 1 where the credit shock was not accompanied by the credit premium shock.

Our findings are in contrast with those of other papers, for instance with Corbae and D'Erasmo (2014) who find that credit eases in expansions due to a more competitive lending sector or with Jermann and Quadrini (2012) who find that adverse financial shocks were a crucial ingredient for the Great Recession. While credit is strongly procyclical, we find that it is due to a combination of factors: in expansions the composition of expenditures is tilted towards goods that are credit intensive and there is also a better outlook of earnings for a large set of the population that takes the opportunity to increase consumption (a point made also by Athreya et al. (2009), but not in the context of business cycle fluctuations). These features by themselves would make bankruptcies too volatile so the credit shock puts limits to household borrowing.

We finish this section discussing the behavior of the borrowing premia vis a vis the bankruptcy rates. We have not been able to find a combination of the three shocks that replicate the volatilities of the three variables that we are looking at. This is because in the model there is a close connection between the borrowing premia and the subsequent bankruptcy rate amount, as the former is the expected value of the amount defaulted (weighted by the quantity). The two-state Markov structure that we have posed for the aggregate shock straitjackets the relation between bankruptcies and interest premia in a way that prevents the replication of the properties of the data. This can be noticed by comparing the cross-correlation of output with lagged variables in Table 1 and in Table A5. In the data for both bankruptcies the borrowing premia have a smooth pattern around output, with bankruptcies slightly (negatively) lagging output, while in the model this is the property of the premia but lagged bankruptcies become essentially uncorrelated with output, which we think is an artifact of the Markovian structure of the aggregate process. We also think that this is related to the credit spread puzzle in corporate bonds where default losses cannot explain the observed spreads, as explored in Chen, Collin-Dufresne, and Goldstein (2009). Longstaff, Pan, Pedersen, and Singleton (2011) in their research about sovereign bonds find large spreads and relate them to economy wide risk premia. Arellano, Mateos-Planas, and Ríos-Rull (2019) also find difficulties in matching the large spread between borrowing premia and bankruptcy rates.

5.4. Secure Credit and Unsecured Credit

We now turn to the question of whether the cyclical properties of credit are due to the fact that it is both available and unsecured. To this end in Table 10 we pose two additional economies that differ from the Baseline in their credit arrangements: one is an economy where households can borrow up to some limit but cannot default (*Model with no default*) and the other is an economy where all credit has to be backed by assets, and hence credit is just the amount of goods purchased on credit by households that are restricted to have a non-negative asset position (*Model with no bank credit*). To make these economies comparable we have recalibrated the economies with no default by posing a borrowing limit so that it has the same average amount of total credit as the Baseline economy (2.4% of GDP) and we have adjusted the value of the β of the patient agents so that aggregate wealth is identical across economies. The lack of bankruptcy protection induces commitment that eases access to credit, so its elimination shoots up the number of net borrowers from 11.8% in the Baseline to 24.1%, which reduces the average size of credit of each debtor. Without the credit limit that we impose the total amount of credit would be much larger.

The three economies have quite different properties. The economy with no default has a much lower total credit volatility, since bank credit is countercyclical, dampening total credit volatility. The commit-

Table 10: Comparison of Model Economies: Role of Credit

	Data	Baseline Economy	Model with No Default	Model with No Bank Credit
Standard Deviation Relative to Output				
Total Credit	3.03	0.74	0.39	0.74
Bank Credit	Na	0.82	0.20	–
Store Credit	Na	0.77	0.74	0.74
Consumption	0.78	0.77	0.74	0.74
Investment	4.88	1.82	1.90	1.91
Correlation with Output				
Total Credit	0.33	0.96	0.99	1.00
Bank Credit	NA	0.76	–0.65	–
Store Credit	NA	1.00	1.00	1.00
Consumption	0.86	1.00	1.00	1.00
Investment	0.90	0.98	0.97	0.99

Note: Quarterly frequency. HP filtered with the smoothing parameter of 1600.

ment guaranteed by the lack of bankruptcy protection allows households to borrow when they most need to, during recessions, without having to pay the premium that the Baseline displays. As a consequence of the countercyclical bank credit, consumption is slightly less volatile.

The economy with only store credit has similar properties to the Baseline despite having no bank credit. The volatility of total credit is the same. This is due to the fact that store credit follows consumption and in the Baseline economy both types of credit move similarly. In the economy with no bank credit, store credit and total consumption also move together, accounting for the similar behavior of total credit in both economies. Yet, consumption is less volatile than in the Baseline economy where the good news of the recovery allows the poorest households to increase their consumption via credit.

Unsecured credit is very important in shaping consumption and credit: it expands in expansions enough to make consumption more procyclical than it would otherwise be. Note that access to credit does not smooth aggregate consumption (a feature related to Athreya et al. (2009) in the context of idiosyncratic shocks in the steady-state economy). On the contrary, because expansions allow access to credit with better terms only in the economy with both credit and default, aggregate consumption is more volatile in

Table 11: Welfare Costs of Business Cycles under Different Credit Arrangements

Utility Loss in Consumption %	High-β	Low-β	Average
Baseline model	–0.860	–0.502	–0.765
Model with no default	–0.861	–0.297	–0.712
Model with no bank credit	–0.871	–0.320	–0.725
Expansion			
Baseline model	3.115	2.510	2.955
Model with no default	3.114	2.109	2.848
Model with no bank credit	3.103	2.427	2.924
Recession			
Baseline model	–9.334	–7.119	–8.747
Model with no default	–9.334	–5.685	–8.367
Model with no bank credit	–9.340	–6.406	–8.562
Fraction	73.5	26.5	100

Note: All economies have the same amount of store credit. Differences in welfare are expressed as consumption equivalence variations of expected lifetime utility of a newborn.

the Baseline economy. We further explore the welfare implications of such amplification of consumption in the next section.

5.5. Unsecured Credit and Welfare Costs of Business Cycles

In this section we investigate the normative properties of credit and bankruptcies. We compute the welfare costs of business cycles defined as the difference in ex-ante expected lifetime utility, converted into percentage changes in flow consumption (consumption equivalence variation), between the steady-state economy and the economy with aggregate shocks that exhibit countercyclical individual income risks.[20] We compute the welfare costs of business cycles independently for households with a high and low discount factor, for the three models in Section 5.4 differing in the market structure (Baseline model with unsecured credit and default, model without default, and model without bank credit).

[20] Specifically, we evaluate the utility of a newborn in steady state and compare it with the weighted average (using the unconditional distribution of expansions and recessions) of a newborn agent that faces the steady-state distribution of wealth and either an expansion or a recession. Obviously, we do this separately for high-β and low-β households and we average them.

The results are summarized in Table 11 and there are various things to note, many of them relatively unrelated to our concerns that we deal with first. The high-β, patient agents suffer the most disutility from fluctuations, but this is just because they care more about the future and their consumption grow over time. They barely borrow and this is why they do not care that much about which market structure they are in. The high-β households do not often borrow or default, yet the welfare costs of business cycles are around 0.86% of flow consumption.

On the other hand, there is an interesting difference between the Baseline model and the other two models for the low-β, impatient agents. In the Baseline model, the welfare costs of business cycles is 0.50% of flow consumption, significantly larger than in the models without default or credit. With the existence of unsecured credit, the ability of low-β agents to borrow is significantly restricted in recessions because of higher risk premia. The larger welfare cost of business cycles is consistent with a higher consumption volatility in the Baseline model.

This size of the cost of business cycles is larger than the value of the typical estimates of the welfare costs of business cycles, because of the cyclical movement of individual income risks, as in Storesletten, Telmer, and Yaron (2001).[21]

This result is related to the steady-state result by Athreya et al. (2009), who find that unsecured consumer credit under full information does not contribute to better consumption smoothing, since adjustments of credit cost based on risks of borrowers do not allow high risk borrowers to borrow to smooth consumption. The result that unsecured credit makes the welfare costs of business cycles larger corresponds to their result in the sense that unsecured credit with credit supply adjusting to the riskiness to borrowers does not facilitate consumption smoothing over the business cycles.

[21] Storesletten et al. (2001) find that welfare costs of business cycles for a newborn in their OLG economy is 1.50% with $\sigma = 2$, compared with Lucas' original calculation (with $\sigma = 2$) of 0.2%.

6. Conclusion

In this paper we have investigated the mechanisms that may be behind the procyclicality and high volatility of unsecured consumer credit and the countercyclicality and extremely high volatility of bankruptcy filings. In doing so we have extended the benchmark models of idiosyncratic shocks to earnings with unsecured credit to accommodate aggregate uncertainty and the existence of goods that are purchased with credit even by households with a positive asset position. We have found that the particulars of earnings in the U.S. that generate countercyclical risk of individual earnings are a central element in shaping the procyclicality of credit. Countercyclicality of individual earnings risk implies a countercyclical default risk premium, which raises borrowing in expansions and dampens borrowing in recessions. Moreover, we have found that the important property of earnings dynamics behind the procyclicality of credit is not skewness itself, but Kelly skewness (the skewness within the middle 80% of the distribution). Still, countercyclicality of earnings risk and persistence of fluctuation are not enough by themselves to generate the size of credit volatility that we observe, and we have found that shocks to the composition of consumption that make credit goods more attractive in expansions and shocks that make credit relatively more expensive in expansions (for a given interest rate and individual risk) are capable of filling the gap.

Many new questions arise to complement the questions that we have addressed and that pertain to the behavior of secured consumer credit (mostly mortgages) and firm credit, and to the explicit role played by lending institutions that borrow short and lend long. We leave these issues for future research.

References

Aiyagari, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109, 659–684.

Arellano, C. (2008): "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review*, 98, 690–712.

Arellano, C., X. Mateos-Planas, and J.-V. Ríos-Rull (2019): "Partial Default," Unpublished Manuscript, University of Minnesota.

Athreya, K., X. S. Tam, and E. R. Young (2009): "Unsecured Credit Markets Are Not Insurance Markets," *Journal of Monetary Economics*, 56, 83–103.

——— (2012): "A Quantitative Theory of Information and Unsecured Credit," *American Economic Journal: Macroeconomics*, 4, 153–83.

Benhabib, J., A. Bisin, and M. Luo (2017): "Earnings Inequality and Other Determinants of Wealth Inequality," *American Economic Review*, 107, 593–97.

Bewley, T. (1986): "Stationary Monetary Equilibrium with a Continuum of Independently Fluctuating Consumers," in *Contributions to Mathematical Economics in Honor of Gérard Debreu*, ed. by W. Hildenbrand and A. Mas-Colell, Amsterdam: North Holland.

Castañeda, A., J. Díaz-Giménez, and J.-V. Ríos-Rull (1998): "Exploring the Income Distribution Business Cycle Dynamics," *Journal of Monetary Economics*, 42, 93–130.

——— (2003): "Accounting for the U.S. Earnings and Wealth Inequality," *Journal of Political Economy*, 111, 818–857.

Chatterjee, S., D. Corbae, M. Nakajima, and J.-V. Ríos-Rull (2007): "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default," *Econometrica*, 75, 1525–1589.

Chatterjee, S. and B. Eyigungor (2012): "Maturity, Indebtedness, and Default Risk," *American Economic Review*, 102, 2674–2699.

Chen, L., P. Collin-Dufresne, and R. Goldstein (2009): "On the relation between the credit spread puzzle and the equity premium puzzle," *Review of Financial Studies*, 22, 336–409.

Corbae, D. and P. D'Erasmo (2014): "Capital Requirements in a Quantitative Model of Banking Industry Dynamics," Working Paper 14-13, Federal Reserve Bank of Philadelphia.

Drozd, L. A. and R. Serrano-Padial (2017): "Modeling the Revolving Revolution: The Debt Collection Channel," *American Economic Review*, 107, 897–930.

Eaton, J. and M. Gersovitz (1981): "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies*, 48, 289–309.

Erosa, A. and G. Ventura (2002): "On Inflation as a Regressive Consumption Tax," *Journal of Monetary Economics*, 53, 410–40.

Fieldhouse, D., I. Livshits, and J. MacGee (2016): "Aggregate Fluctuations, Consumer Credit and Bankruptcy," Unpublished document, University of Western Ontario.

Gavazza, A. and A. Lanteri (2018): "Credit Shocks and Equilibrium Dynamics in Consumer Durable Goods Markets," 2018 Meeting Papers 384, Society for Economic Dynamics.

Gordon, G. (2015): "Evaluating default policy: The business cycle matters," *Quantitative Economics*, 6, 795–823.

Guvenen, F., S. Ozkan, and J. Song (2014): "The Nature of Countercyclical Income Risk," *Journal of Political Economy*, 112, 621–660.

Heathcote, J., K. Storesletten, and G. L. Violante (2017): "Optimal Tax Progressivity: An Analytical Framework," *The Quarterly Journal of Economics*, 132, 1693–1754.

Herkenhoff, K. F. (forthcoming): "The Impact of Consumer Credit Access on Unemployment," *Review of Economic Studies*.

Hubmer, J., P. Krusell, and A. A. Smith Jr. (2018): "A Comprehensive Quantitative Theory of the U.S. Wealth Distribution," Mimeo, Yale University.

Huggett, M. (1993): "The Risk-Free Rate in Heterogeneous-Agent, Incomplete-Insurance Economies," *Journal of Economic Dynamics and Control*, 17, 953–969.

İmrohoroğlu, A. (1989): "Cost of Business Cycles with Indivisibilities and Liquidity Constraints," *Journal of Political Economy*, 97, 1364–1383.

Jermann, U. and V. Quadrini (2012): "Macroeconomic Effects of Financial Shocks," *American Economic Review*, 102, 238–71.

Kindermann, F. and D. Krueger (2014): "High Marginal Tax Rates on the Top 1%? Lessons from a Life Cycle Model with Idiosyncratic Income Risk," NBER Working Papers 20601, National Bureau of Economic Research.

Krusell, P. and A. A. Smith Jr. (1997): "Income and Wealth Heterogeneity, Portfolio Choice, and Equilibrium Asset Returns," *Macroeconomic Dynamics*, 1, 387–422.

——— (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106, 867–896.

Kuhn, M. and J.-V. Rios-Rull (2016): "2013 Update on the U.S. Earnings, Income, and Wealth Distributional Facts: A View from Macroeconomics," *Quarterly Review*, 1–75.

Livshits, I., J. MacGee, and M. Tertilt (2007): "Consumer Bankruptcy: A Fresh Start," *American Economic Review*, 97, 402–418.

——— (2010): "Accounting for the Rise in Consumer Bankruptcies," *American Economic Journal: Macroeconomics*, 2, 165–93.

——— (2016): "The Democratization of Credit and the Rise in Consumer Bankruptcies," *The Review of Economic Studies*, 83, 1673–1710.

LONGSTAFF, F. A., J. PAN, L. H. PEDERSEN, AND K. J. SINGLETON (2011): "How sovereign is sovereign credit risk?" *American Economic Journal: Macroeconomics*, 3, 75–103.

LUCAS, R. E. AND N. L. STOKEY (1987): "Money and Interest in a Cash-in-Advance Economy," *Econometrica*, 55, 491–513.

LUZZETTI, M. N. AND S. NEUMULLER (2018): "The impact of learning on business cycle fluctuations in the consumer unsecured credit market," *Macroeconomic Dynamics*, 1–37.

MUSTO, D. K. (1999): "The Reacquisition of Credit Following Chapter 7 Personal Bankruptcy," Working Paper 99-22, Wharton Financial Institutions Center.

NAKAJIMA, M. AND J.-V. RÍOS-RULL (2005): "Defaults and Aggregate Fluctuations in Storage Economies," in *Frontiers in Applied General Equilibrium Modeling: In Honor of Herbert Scarf*, ed. by T. J. Kehoe, T. N. Srinivasan, and J. Whalley, Cambridge, UK: Cambridge University Press, chap. 7, 127–150.

STORESLETTEN, K., C. TELMER, AND A. YARON (2001): "The Welfare Cost of Business Cycles Revisited: Finite Lives and Cyclical Variation in Idiosyncratic Risk," *European Economic Review*, 45, 1311–1339.

STORESLETTEN, K., C. I. TELMER, AND A. YARON (2004): "Cyclical Dynamics in Idiosyncratic Labor Market Risk," *Journal of Political Economy*, 112, 695–717.

TAUCHEN, G. (1986): "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions," *Economics Letters*, 20, 177–181.

WIKIPEDIA CONTRIBUTORS (2018): "Skew normal distribution — Wikipedia, The Free Encyclopedia," https://en.wikipedia.org/w/index.php?title=Skew_normal_distribution&oldid=866693199, [Online; accessed 7-January-2019].

Appendix

A. A Comparison of Earnings with and without Movements in Aggregate Capital

In the second stage of matching the model to data (Section 4.2), when we estimate the parameters of earnings shock, we use wages per efficiency unit and output each period during simulations. However, it is virtually impossible to compute those for the model we eventually solve, since aggregate capital is endogenously determined. Therefore, in the second stage, we compute wage and output as if capital were constant over time, assuming that capital in the benchmark model is slow-moving anyway, and assuming fixed capital does not make a significant difference in terms of dynamics of wage and output. Table A1 justifies our assumption. In particular, Table A1 compares the cyclical properties of output, capital, labor supply, wage per efficiency unit, and the rate of return of capital, in the benchmark model (panel A) as well as the model that is solved as if capital is fixed (panel B). It is easy to see that, except for capital (which does not move in the fixed-capital model by assumption), the cyclical properties are quite similar between the two models.

Table A1: Comparison: Benchmark Model and Fixed-Capital Model

Variable	SD%	SD_x / SD_y	Auto-corr	Cross-Correlation of Output with x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}
A. Benchmark Model								
Output	1.28	1.00	0.73	0.52	0.73	1.00	0.73	0.52
Capital	0.14	0.11	0.97	–0.28	–0.09	0.16	0.36	0.50
Labor	0.64	0.50	0.95	0.10	0.33	0.60	0.75	0.80
Wage per efficiency unit	1.03	0.81	0.67	0.58	0.71	0.87	0.45	0.14
Rate of return of capital	0.03	0.03	0.74	0.55	0.75	1.00	0.71	0.47
B. Fixed-Capital Model								
Output	1.29	1.00	0.73	0.52	0.73	1.00	0.73	0.52
Capital	–	–	–	–	–	–	–	–
Labor	0.64	0.50	0.95	0.08	0.30	0.58	0.73	0.79
Wage per efficiency unit	1.05	0.82	0.68	0.58	0.71	0.87	0.45	0.15
Rate of return of capital	0.03	0.02	0.73	0.52	0.73	1.00	0.73	0.51

Note: Both models are simulated for 4100 periods and the data from the first 100 periods are discarded, before the generated time series are detrended using the HP filter with the smoothing parameter of 1600.

B. Discretization of AR(1) with Skewed Normal Innovations

This section describes how to discretize an AR(1) process $p' = \rho p + \epsilon$ where innovations are distributed according to skewed normal distribution $\epsilon \sim SN(\mu, \sigma^2, \alpha)$. The method is a straightforward extension of the method proposed by Tauchen (1986). Indeed, if the skewness parameter α is set to zero, the skewed normal distribution becomes a normal distribution $N(\mu, \sigma^2)$, and the method described here collapses to the method of Tauchen (1986). In the actual application, we assume σ^2 depends on the current p and μ depends on the current z, but it is straightforward to include these features:

1. We can compute the moments of the skewed normal innovations $SN(\mu, \sigma^2, \alpha)$. Let us call the mean μ_ϵ and the variance σ_ϵ^2.

2. We can also compute the mean and variance of the ergodic distribution of p, as $\mu_p = \mu_\epsilon/(1-\rho)$ and $\sigma_p^2 = \sigma_\epsilon^2/(1-\rho^2)$.

3. Set range for the discretized state space. In particular, following Tauchen (1986), set $\overline{p} = \mu_p + 2\sigma_p$ and $\underline{p} = \mu_p + 2\sigma_p$.

4. Set $N =$ the number of grids used to approximate shocks to p.

5. Set the grids $p_1 = \underline{p}, p_2, ..., p_{N-1}, p_N = \overline{p}$ with equal space between p's. In other words, the distance between adjacent grid points p_i and p_{i+1} is $4\sigma_p/(N-1)$ for all $i = 1, 2, ..., N-1$. Since p is a shock to log-earnings, this means that the distance between log of grids is constant.

6. Denote $F(\epsilon)$ as the cumulative density function (CDF) of the skewed normal innovation. Of course, it depends on the three parameters, but we omit them for ease of notation.

7. For a grid-i, the discretized transition probabilities from grid p_i to p_j, $j = 1, 2, ..., N$, $\pi_{i,j}$, can be computed as follows:

$$\pi_{i,1} = F\left(\frac{p_1 + p_2}{2} - \rho p_i\right), \tag{A1}$$

$$\pi_{i,j} = F\left(\frac{p_j + p_{j+1}}{2} - \rho p_i\right) - F\left(\frac{p_{j-1} + p_j}{2} - \rho p_i\right) \qquad j = 2, 3, ..., N-1, \tag{A2}$$

$$\pi_{i,N} = 1 - F\left(\frac{p_{N-1} + p_N}{2} - \rho p_i\right). \tag{A3}$$

The basic idea here, which is the same idea as in Tauchen (1986), is that the probability of transitioning from a grid i to a grid j is the probability that $p' = \rho p_i + \epsilon$ falls into the interval defined by $[\frac{p_{j-1}+p_j}{2}, \frac{p_j+p_{j+1}}{2}]$. The only difference here from Tauchen (1986) is that ϵ is distributed according to a skewed normal distribution instead of normal distribution.

C. Computational Strategy

We solve the model numerically, since there is no analytical solution. In particular, our solution method is based on the *approximate aggregation approach* developed by Krusell and Smith Jr. (1997) and Krusell and Smith Jr. (1998). Although, in the same spirit as Krusell and Smith Jr. (1998), we pose agents with bounded rationality in the size of the state space, taking advantage of the approximate aggregation feature of this type of environment.There are two differences that make our computation strategy more involved than Krusell and Smith Jr. (1998). First, while agents only need to forecast next period capital in Krusell and Smith Jr. (1998), agents in our model need to forecast the discount price of store credit q^d and the return of savings R^a, and capital and total labor supply in efficiency units in the next period. Moreover, the labor supply in the next period is conditional on the realization of the aggregate shock in the next period, because of the cyclicality of individual earnings shock. Agents need to forecast R^a because R^a is the weighted average of returns of capital, bank credit, and store credit. Second, agents need to forecast a state-dependent price of bank credit, but this can be computed using the optimal default decision rule.

Specifically, in the algorithm described in Appendix C.1, we assume that agents summarize the aggregate state of the economy by the aggregate productivity shock, capital stock, and aggregate labor supply in efficiency unit (z, K, L), instead of using the proper state space (z, K, B^-, D^-, L). After describing the algorithm, in Appendix C.2, we will show that (z, K, B^-, D^-, L) has sufficiently high predictive power over the important variables to be forecasted.

C.1. Computation Algorithm

We specify the forecasting functions in the following log-linear functional form:

$$\log K' = \log \phi_K(z, K, L) = \phi^0_{K,z} + \phi^1_{K,z} \log K + \phi^2_{K,z} \log L, \tag{A4}$$

$$\log q^d = \log \phi_q(z, K, L) = \phi^0_{q,z} + \phi^1_{q,z} \log K + \phi^2_{q,z} \log L, \tag{A5}$$

$$\log R^a = \log \phi_R(z, K, L) = \phi^0_{R,z} + \phi^1_{R,z} \log K + \phi^2_{R,z} \log L, \tag{A6}$$

$$\log L'(z') = \log \phi_L(z, z', K, L) = \phi^0_{L,z,z'} + \phi^1_{L,z,z'} \log K + \phi^2_{L,z,z'} \log L. \tag{A7}$$

Notice that labor supply in efficiency unit in the next period, L', depends on the current aggregate state (z, K, L) as well as on the realization of the aggregate shock in the next period z'. We are looking for parameters $\Phi = \{\phi^0_{K,z}, \phi^1_{K,z}, \phi^2_{K,z}, \phi^0_{q,z}, \phi^1_{q,z}, \phi^2_{q,z}, \phi^0_{R,z}, \phi^1_{R,z}, \phi^2_{R,z}, \phi^0_{L,z,z'}, \phi^1_{L,z,z'}, \phi^2_{L,z,z'}\}$ for $\forall z, z'$ that best forecasts the equilibrium values.

Algorithm 1 (Solution Algorithm for the Approximate Recursive Equilibrium)

1. *Guess parameters of forecasting functions Φ_0. The initial guess can be obtained by running the complete market version of the model and running regressions with simulated data. In particular, since L is exogenous, the coefficients of the forecasting function for L can be obtained without running the full model.*

2. *With Φ_0, $(K', q^d, R^a, L'(z'))$ can be forecasted. Furthermore, using the aggregate state variables z, K and L, wage w can be computed using the marginal condition.*

3. *Guess the bank credit price function $\widetilde{q}^b_0(z, K, L, s, a')$. Using the law of motion for $a' = y - b - d$, we can construct the guess for bank credit price function conditional on (y, b, d) instead of a': $q^b_0(z, K, L, s, y, b, d)$.*

4. *Guess the value function $V_0(z, K, L, s, h, a)$.*

5. *Using the Bellman equations described in Section 3.2, update the value function and obtain $V_1(z, K, L, s, h, a)$.*

6. *Check convergence of the value function. If the distance between* $V_0(z, K, L, s, h, a)$ *and* $V_1(z, K, L, s, h, a)$ *is smaller than a predetermined tolerance criterion, convergence is achieved. Go to the next step. Otherwise, update the value function using* $V_1(z, K, L, s, h, a) = V_0(z, K, L, s, h, a)$ *and go back to step 5.*

7. *Record the optimal decision rules* $c = g^c(z, K, L, s, h, a)$, $y = g^y(z, K, L, s, h, a)$, $b = g^b(z, K, L, s, h, a)$, $d = g^d(z, K, L, s, h, a)$, $a' = g^a(z, K, L, s, h, a)$, *and* $h' = g^h(z, K, L, s, h, a)$ *associated with the converged value function.*

8. *Using the optimal decision rules obtained in the previous step and equation (18), compute the updated bank credit price function* $\widetilde{q}^b_1(z, K, L, s, a')$.

9. *Check convergence of the bank credit price function. If the distance between* $\widetilde{q}^b_0(z, K, L, s, a')$ *and* $\widetilde{q}^b_1(z, K, L, s, a')$ *is smaller than a predetermined tolerance criterion, convergence is achieved. Go to the next step. Otherwise, update the bank credit price function using the following formula (where* ψ_q *controls the speed of updating) and go back to step 3.*

$$\widetilde{q}^b_{new}(z, K, L, s, a') = (1 - \psi_q)\ \widetilde{q}^b_0(z, K, L, s, a') + \psi_q\ \widetilde{q}^b_1(z, K, L, s, a').$$

10. *Simulate the model for* $T_0 + T_1$ *periods. Simulating the model gives the sequence of aggregate variables* $\{z_t, K_t, L_t, q^d_t, R^a_t\}_{t=1}^{T_0+T_1}$.

    (a) *Use the type distribution in the steady state as the type distribution in period* 1, x_1. K_1 *and* L_1 *are also obtained from the steady-state equilibrium. Set* z_1.

    (b) *At the beginning of period* t, (z_t, K_t, L_t) *are known. The type distribution of households,* x_t, *is also available. Using the forecasting functions with the guess for coefficients* Φ_0, ($\widetilde{K}_{t+1}$, $\widetilde{R^a_t}$, $\widetilde{q^d_t}$, $\widetilde{L}_{t+1}(z')$ *can be forecasted using the state variables* (z_t, K_t, L_t).

    (c) *Wage* w_t *can be computed using* (z_t, K_t, L_t).

    (d) *Given prices in period* t, *solve the household's optimization problem to obtain optimal decision rules in period* t.

    (e) *Using the type distribution of households* x_t *and optimal decision rules, true* $(K_{t+1}, L_{t+1}(z'),)$ *can be computed. The updated distribution* x_{t+1} *can also be computed.*

    (f) *Draw* z_{t+1}. *Conditional on* z_{t+1}, $L_{t+1} = L'_{t+1}(z_{t+1})$ *is obtained. Now we have the next period aggregate state* $(z_{t+1}, K_{t+1}, L_{t+1})$. *Go back to step (b). Continue until period* $T_0 + T_1$.

11. *Using the sequence of aggregate variables* $\{z_t, K_t, L_t(z'), L_t, q^d_t, R^a_t\}_{t=T_0+1}^{T_0+T_1}$, *update the forecasting function. Notice that the first* T_0 *periods are dropped. Use ordinary least squares (OLS) regression. Regressions then give updated parameters of forecasting functions* Φ_1.

12. *Check convergence of the forecasting functions. If the distance between* Φ_0 *and* Φ_1 *is smaller than a predetermined tolerance criterion, an approximate recursive equilibrium is obtained. Otherwise, update the forecasting functions using the following formula (where* ψ_ϕ *controls the speed of updating) and go back to step 3.*

$$\Phi_0^{new} = (1 - \psi_\phi)\ \Phi_0^{old} + \psi_\phi\ \Phi_1.$$

Table A2: Predictive Power of Forecasting Functions

	Adj R^2 of Forecasting Functions		Avg Abs Error in %
	z_1 (EXP)	z_2 (REC)	
K'	1.00000	0.99999	0.0030
$L'(z' = z_1\ (EXP))$	1.00000	0.99999	0.0052
$L'(z' = z_2\ (REC))$	1.00000	1.00000	–
q^d	0.71345	0.28825	0.0019
R^A	0.99964	0.97825	0.1361

Note: Based on a simulation of 4,000 periods with the Baseline model economy. Forecast errors in L' are not contingent on the realization of aggregate shock in the next period.

C.2. Predictive Power of Forecasting Functions

Table A2 summarizes the predictive power of the forecasting functions in the Baseline model economy with the estimated earnings shock represented. We show the predictive power in two criteria. First is the adjusted R^2, as in Krusell and Smith Jr. (1998). The second is average absolute error. Since forecasting functions are conditional on the aggregate state, we show adjusted-R^2 for both expansions and recessions. We make five remarks. First, perhaps not surprisingly, the adjusted R^2 is extremely high for L'. Even though the distribution of individual productivity is exogenous, there is a tiny loss in predictive power by not keeping track of the exact type distribution. The average absolute prediction error of L' is also extremely small, at 0.005%. Second, the adjusted R^2 is extremely high for K' as well. Since this is a quarterly model, capital moves slowly, which makes the forecasting function very accurate. The average absolute prediction error for K' is 0.003%. Third, adjusted R^2 for R^A is slightly lower for recessions, at 0.978. This is because the realized return of capital significantly depends on the distribution of debtors, and more importantly, how long the recession is. The latter matters because more defaults occur in the first period when the economy falls into a recession. The average absolute prediction error is slightly higher at 0.14%, but R^A is on average 4% (0.04), so the errors are small in terms of the level. Fourth, adjusted R^2 is low for the discount price of store credit q^d. This is because the movement of q^d is tiny, as most borrowers of store credit repay. Indeed, the average absolute prediction error is only 0.002%. We also run an alternative version of the Baseline model economy in which q^d is fixed at its steady-state value, and find that the cyclical properties of the model are basically unchanged. This implies that the (small absolute) errors in predicting q^d does not matter much in terms of cyclical dynamics of the model. Finally, our assessment of the performance of the forecasting functions is more than satisfactory for our purposes.

D. Details of Business Cycle Properties of the U.S. Economy

Credit: We define credit as the balance of all credit card debt (from Flow of Funds, FRB) divided by GDP deflator in corresponding years. It is gross debt. We use the data from 1980 to 2017 for annual data and from the first quarter of 1980 to the second quarter of 2018 for quarterly data. Table A3 summarizes cyclical properties of the U.S. economy in annual frequency. In terms of timing, we use the balance at the end of the respective period; for example, credit in the second quarter of 2014 is the balance of consumer credit in the end of the second quarter of 2014.

Cyclical properties of credit in annual frequency are close to quarterly counterparts, regardless of the choice of the smoothing parameter. Relative volatility is 3.4 with the smoothing parameter of 625 and 3.6 with the smoothing parameter of 100. Credit is procyclical in annual data, with the contemporaneous correlation of 0.38 (smoothing parameter of 6.25) to 0.55 (100).

Bankruptcies: The number of bankruptcies is obtained from U.S. Courts. The numbers are normalized by the number of households (from the Census) in their respective years but the normalization does not matter much since the number of households has been steadily growing. The annual data are available from 1980, but quarterly data are available only after 1995:III. We use the data up to 2004 in order to avoid the effect from the bankruptcy law reform (BAPCPA) in 2005.

In the annual data, the number of bankruptcy filings is extremely volatile as well, with the relative standard deviation around 7. The correlation with output depends on the smoothing parameter of the HP filter, though. With the smoothing parameter of 6.25, the number of bankruptcy filings is mildly countercyclical. The contemporaneous correlation with output is -0.26 for total bankruptcy filings, and -0.19 for Chapter 7 bankruptcy. With the smoothing parameter of 100, the number of filings becomes almost acyclical. The contemporaneous correlation with output is 0.06 for the total number of consumer bankruptcy filings and 0.14 for Chapter 7 bankruptcy filings. With the smoothing parameter of 6.25, the number of filings is countercyclical, with the correlation of –0.26 for total bankruptcy filings and –0.19 for Chapter 7 bankruptcy filings.

Premium: The interest rate premium for credit card loans is computed taking the difference between the average interest rate of credit card loans (from the Flow of Funds, FRB) and the interest rate of 1-year Treasury Bills. The average interest rates of credit card loans are available only after 1995. There are two kinds of credit card interest rates, one associated with all credit card balance, and the other associated with credit card accounts with interest assessed. Which one to look at does not matter much at the end, since the differences in terms of cyclical properties are minor. So we show the interest rate of accounts assessing interest.

The premium is very volatile and strongly countercyclical in annual frequency as well, regardless of the choice of the HP smoothing parameter. With the smoothing parameter of 6.25, relative volatility is 6.0, and the contemporaneous correlation with output is –0.83. With the smoothing parameter of 100, relative volatility is 4.4 and the correlation is –0.78.

Table A3: Cyclical Properties of the U.S. Economy (Annual Data)

Variable	SD%	Relative SD%	Auto-corr	Cross-Correlation of Output with x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}
HP-parameter=6.25								
Credit	3.92	3.44	0.45	–0.09	0.08	0.38	0.47	0.07
All bankruptcy	7.95	6.77	0.43	0.21	–0.23	–0.26	0.24	0.53
Chap 7. bankruptcy	8.84	7.53	0.43	0.20	–0.16	–0.19	0.21	0.49
Default premium	5.83	6.01	0.28	0.36	–0.37	–0.83	–0.43	0.25
Output	1.14	1.00	0.31	–0.38	0.31	1.00	0.31	–0.38
Consumption	0.89	0.78	0.39	–0.40	0.42	0.89	0.29	–0.25
Investment	5.51	4.85	0.25	–0.14	0.35	0.90	0.13	–0.58
HP-parameter=100								
Credit	6.37	3.55	0.58	0.13	0.41	0.55	0.50	0.13
All bankruptcy	11.12	6.21	0.58	–0.17	–0.15	0.06	0.41	0.54
Chap 7. bankruptcy	12.56	7.02	0.58	–0.16	–0.08	0.14	0.42	0.52
Default premium	7.13	4.35	0.42	0.10	–0.48	–0.78	–0.49	0.07
Output	1.80	1.00	0.61	0.07	0.61	1.00	0.61	0.07
Consumption	1.75	0.98	0.75	0.16	0.67	0.91	0.63	0.24
Investment	8.12	4.52	0.57	0.39	0.66	0.82	0.25	–0.38

Note: Output (real GDP), consumption (real personal consumption expenditure), investment (real gross domestic investment) are from BEA. Aggregate and average hours are from BLS. Credit is revolving debt balance (from Flow of Funds, FRB) divided by GDP deflator (from BEA). Number of bankruptcies (from U.S. courts) is normalized by the number of households (from Census), but the normalization does not matter much since the number of households is growing steadily. Default premia are the differences between the average interest rate on credit card balances assessing interest (FRB) and 1-year Treasury rate. Except for the numbers of bankruptcies (up to 2004, to exclude the effect from the BAPCPA in 2005) and default premia (from 1995, due to data availability), data are from 1980 to 2017.

E. Additional Tables of Interest

Table A4: Cross-Sectional Variances of Earnings and Consumption

	Cross-sectional Variances			
	Earnings All Economies	Consumption Baseline	Consumption Ec. Wo Credit	Consumption Ec. Wo Default
Coef of Var (All)	1.0803	0.9085	0.9060	0.9082
Coef of Var (Low-β)	1.0803	1.0085	1.0031	1.0060

Table A5: Cyclical Properties of the Baseline Model Economy

		Relative	Auto-	Cross-Correlation of Output with				
Variable	SD%	SD%	corr	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}
Credit	1.00	0.78	0.74	0.46	0.70	0.98	0.74	0.55
Bank credit	1.40	1.09	0.89	0.07	0.28	0.53	0.61	0.63
Bankruptcies	18.17	14.18	0.13	–0.37	–0.44	–0.52	0.03	0.14
Borrowing Premium	5.61	4.37	0.73	–0.58	–0.74	–0.91	–0.56	–0.27
Premium (bank credit)	7.21	5.63	0.74	–0.58	–0.75	–0.94	–0.62	–0.33
Output	1.28	1.00	0.74	0.52	0.74	1.00	0.74	0.52
Consumption	1.02	0.79	0.71	0.51	0.73	0.99	0.71	0.49

Note: Quarterly frequency. HP filtered with the smoothing parameter of 1600.

Table A6: Annual Income Growth and Borrowing/Defaulting

(Percent)	Baseline Economy All	Exp	Rec	Diff	No-PS1 All	Exp	Rec	Diff	No-PS2 All	Exp	Rec	Diff
Inc growth	–0.3	1.5	–3.9	5.43	–0.2	0.2	–1.2	1.46	–0.4	0.7	–2.8	3.48
0-10p	–101	–97.8	–107	9.43	–101	–100	–102	1.52	–102	–101	–106	5.06
10-30p	–28.6	–26.7	–32.7	6.05	–28.4	–27.9	–29.4	1.47	–29.3	–27.8	–32.4	4.53
40-60p	0.3	1.1	–1.5	2.56	0.3	0.7	–0.7	1.45	0.2	0.9	–1.1	1.99
70-90p	29.5	31.4	25.6	5.81	29.4	29.9	28.4	1.45	29.9	31.2	27.2	3.93
90-100p	95.0	98.3	88.2	10.15	95.0	95.5	94.1	1.42	95.9	97.3	93.0	4.37
Credit	5.50	5.59	5.32	0.27	5.60	5.62	5.57	0.05	5.45	5.53	5.27	0.26
0-10p	1.99	2.03	1.90	0.13	2.01	2.01	2.00	0.01	1.96	1.96	1.94	0.02
10-30p	5.48	5.73	4.97	0.76	5.58	5.60	5.54	0.07	5.36	5.55	4.99	0.56
40-60p	6.92	7.02	6.71	0.32	7.05	7.07	7.01	0.07	6.89	6.97	6.70	0.27
70-90p	5.61	5.56	5.71	–0.14	5.71	5.73	5.69	0.04	5.55	5.55	5.53	0.02
90-100p	3.40	3.42	3.35	0.07	3.46	3.47	3.45	0.03	3.37	3.47	3.16	0.30
Bankruptcy	1.20	1.12	1.37	–0.24	1.20	1.19	1.21	–0.02	1.20	1.13	1.36	–0.22
0-10p	4.44	4.48	4.36	0.12	4.51	4.50	4.55	–0.05	4.31	4.27	4.40	–0.12
10-30p	2.32	2.13	2.73	–0.59	2.36	2.36	2.37	–0.01	2.38	2.23	2.69	–0.46
40-60p	0.43	0.37	0.56	–0.19	0.39	0.39	0.40	–0.01	0.43	0.38	0.53	–0.15
70-90p	0.39	0.32	0.53	–0.21	0.36	0.35	0.37	–0.02	0.41	0.34	0.55	–0.21
90-100p	0.26	0.23	0.33	–0.10	0.25	0.25	0.26	–0.00	0.26	0.24	0.30	–0.06
Def prem	3.70	3.55	4.02	–0.46	3.71	3.70	3.74	–0.04	3.72	3.59	4.00	–0.41
0-10p	2.28	2.24	2.35	–0.11	2.32	2.30	2.35	–0.04	2.25	2.21	2.34	–0.13
10-30p	3.77	3.67	4.00	–0.34	3.75	3.75	3.77	–0.02	3.78	3.68	3.99	–0.31
40-60p	3.85	3.68	4.20	–0.52	3.85	3.84	3.88	–0.04	3.88	3.73	4.19	–0.46
70-90p	3.67	3.49	4.02	–0.53	3.70	3.69	3.74	–0.05	3.69	3.54	4.00	–0.46
90-100p	3.27	3.12	3.58	–0.46	3.34	3.32	3.39	–0.07	3.28	3.16	3.52	–0.36

Note: Percentiles are sorted according to annual income growth rate (shown in the first panel). Credit balance is in percent relative to steady-state annual output. Bankruptcy is annual frequency in percent. Default premium in annualized in percent.