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Personal Bankruptcy as a Real Option

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Abstract

We provide a novel explanation to the longstanding puzzle of the “missing bankruptcy filings.” Even though a household with a negative net worth will receive contemporaneous benefit from bankruptcy, there may be greater insurance value from delaying the filing. Household bankruptcy is thus an American-style put option, which is not necessarily exercised even if the option is “in the money.” Based on the value functions in the household’s dynamic programming problem, we formulate the value of the bankruptcy option as well as the exercise price. We estimate a life-cycle model in which households choose the optimal time to exercise their bankruptcy option.

Keywords: personal bankruptcy, option value, life cycle, secured and unsecured debts

JEL Codes: R21, E21

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1 Introduction

One of the most interesting puzzles in household finance is why households do not default on their debt obligations more often. In the context of consumer bankruptcy, this issue is referred to by White (White, 1998, p. 206) as the puzzle of the “missing bankruptcy filings.”¹ In the context of mortgage default, previous studies have shown that consumers do not default on mortgages the moment their home equity turns negative (e.g., Elul et al., 2010), and even then, it is substantially negative (e.g., Bhutta, Dokko, and Shan, 2017). In this paper, we examine the question of why few borrowers with negative net worth file for personal bankruptcy.

We propose an explanation for the puzzle of missing bankruptcies using a purely rational expectations model. We show that bankruptcy is an American-style put option that is not necessarily exercised even if the option is in the money because of its insurance value.² As a result, it is not necessarily optimal for a household with a negative net worth to file for bankruptcy, even though the household would receive contemporaneous financial benefit from doing so, because there may be greater financial benefit from waiting.

We argue that personal bankruptcy is a real option rather than a financial option because it is not tradable, and the underlying asset is typically not replicated by existing financial assets. The option to file for bankruptcy is also irreversible in the sense that, under bankruptcy law in most countries, households can file for bankruptcy for only a limited number of times over the life cycle, and each additional bankruptcy filing becomes progressively costlier to the filer.³

In this paper, we highlight a variety of differences between the real option of bankruptcy filing and standard financial options. First, the household as an option holder changes the underlying asset price constantly through its consumption-saving choices. For example, the household can strategically choose to consume more, thus lower the price of the underlying assets, which increases the value of the put option. Second, the household can increase the volatility of the underlying asset price by allocating more savings to risky assets or taking a more levered position, which also increases the option value as predicted by standard option

¹Based on the 1992 Survey of Consumer Finances, White (1998) calculates that 15% of U.S. households would benefit financially from filing for bankruptcy, while in the U.S. data, only 0.66% of households actually filed per year between 1984 and 1994.

²Mahoney (2015) examines how bankruptcy protection serves as a form of insurance, in particular, health insurance.

³In the U.S., there is an eight-year waiting period between two Chapter 7 bankruptcy filings by the same household. In Canada, the Bankruptcy and Insolvency Act (BIA) entitles Canadian households to two cases of automatic discharge if they file for bankruptcy. While, in theory, it is possible to file for the third or additional times, individuals attempting this do not receive an automatic discharge of their eligible debts, but go through a court approval. The BIA makes it clear that the court shall refuse, suspend, or impose a conditional discharge upon such bankruptcy filers.

pricing theory. Third, the underlying asset price (i.e., the household value) depends not only on the risk and return of the household portfolio but also on the household's preferences, especially the risk attitude and the elasticity of intertemporal substitution (**EIS** hereafter). Fourth, since the household value depends on both the current consumption and the future value of its balance sheet, the option value from delaying the exercise depends on the discounted future value as well as the current consumption, as opposed to the standard American option in which the option value from delaying the exercise is simply the discounted future value from exercising the option.

We develop a life-cycle model in which a household has Epstein-Zin preferences, which separate risk aversion from the EIS. The household chooses its total savings and portfolio composition. The model includes both the risk-free liquid bond and the risky housing asset, featuring entry barriers and transaction costs. In addition, households can borrow from the unsecured debt market and the mortgage market. The model enables us to define the exercise price and the option value in the context of a dynamic programming problem.

We estimate the parameters of the full-blown life-cycle model via a simulated method of moments approach, using data on the universe of bankruptcy filings in Canada. A salient feature in the data is that the bankruptcy filing rate is hump shaped over the life cycle for both homeowners and renters. Relative to renters, the filing rate is much lower, and the peak occurs much earlier for homeowners. The shape and scale of the life-cycle profiles of filing rates for owners and renters serve as the key source of identification in our structural estimation, as they are informative about the preferences and the borrowing capacity of households. For example, consider the extreme case in which: (i) the household has a fixed borrowing capacity in the unsecured debt market; and (ii) the household is risk neutral and perfectly willing to substitute consumption intertemporally. Under such conditions, the household should borrow to the limit in period one, and then file for bankruptcy in period two. The hump-shaped filing rate indicates a low EIS and the increasing of borrowing capacity with age, at least in the early stage of the life cycle. The data also have information on whether a filer is a repeat filer, and the fraction of repeat filers among all filers is about 19%. This statistical moment is used to estimate the extent to which a filing history affects the borrowing capacity.

The simulated model matches the data moments fairly well, especially the profiles of bankruptcy filing rates of both homeowners and renters. Consistent with Canadian Bankruptcy law, we assume a household can file for bankruptcy twice over its lifetime. The model is well matched with the fraction of repeat filers among all filers. Using the estimated model, we compute the option value of bankruptcy, for both the first-time and second-time filings, as a function of the household's age, homeownership status, portfolio composition, and income states.

Our explicit modeling of homeownership and mortgages is an important contribution to

the literature. First of all, the rich setting allows us to study the role of leverage and illiquidity in determining the option value of bankruptcy and the patterns of bankruptcy filings. Our new finding in this setting is that the option value of bankruptcy filing is on average larger for homeowners than for renters because owners are exposed to more risks and face less liquidity.⁴ In our model, some homeowners are clearly the wealthy hand-to-mouth consumers as discussed in Kaplan, Violante, and Weidner (2014). As we show, the option value of bankruptcy is particularly high for these households.

Second, although most of the bankrupts in the data are renters, housing and mortgage affect the bankruptcy filings of renters through two channels. First, homeowners are relatively rich households; renters are relatively poor and more inclined to file for bankruptcy. Second, housing investment offers a higher return than the other assets that renters have access to; thus, it motivates renters to save for the down payment on a home, which lowers the bankruptcy filing rate of renters.⁵ As we show, renters have substantially different bankruptcy filing patterns if the housing market is shut down.

The remainder of the paper is organized as follows. Section 2 reviews the related literature and outlines our contribution. Section 3 presents the model and formulates the valuation of the bankruptcy option. Section 4 estimates the model via Simulated Method of Moments and presents simulation results. Section 5 shows how the valuation of bankruptcy option depends on household characteristics. Section 6 concludes and discusses future research.

2 Literature

To the best of our knowledge, this is the first paper to explicitly study the option value of household bankruptcy in a dynamic life-cycle context. The hypothesis that bankruptcy is an option and that some households may benefit from waiting has been examined by White (1998), who used data from the Panel Study of Income Dynamics (PSID) to show that the option value of bankruptcy can vary widely across wealth deciles. However, while White (1998) does examine bankruptcy as a real option, she does not explicitly model bankruptcy in a dynamic context. As we show, the bankruptcy decisions are very different in our dynamic life-cycle model compared with the static models. In static bankruptcy models (e.g., Fay, Hurst, and White, 2002), it is optimal for households to default when the net worth is negative. However, this is generally

⁴We assume house rent is riskless in the model. Although house rent is time varying in reality, it is not cited as reasons for bankruptcy filing, according to our data.

⁵In reality, renters can also invest in the stock market, which offers a higher return than housing, on average. Still, the access to housing market substantially improves the mean-variance efficiency of a household's portfolio, as shown in Flavin and Yamashita (2002).

not true in a dynamic model because the value of keeping the option open is higher. As a result, our model shows that only a small fraction of households with negative net worth file for bankruptcy, as documented in White (1998).

We contribute to the literature that attempts to explain the puzzle of missing bankruptcies. One strand of this literature argues that households should indeed maximize their economic benefits from bankruptcy or default, by acting strategically or ruthlessly (Bhutta et al., 2017). However, the lack of success of this strategic-default explanation in fully explaining the puzzle of missing bankruptcies is indicated by the large number of papers arguing that various behavioral factors and institutional frictions reduce individual’s propensity to default on debt or file for bankruptcy. Examples of such behavioral explanations include Gross and Souleles (2002), who ascribe suppressed defaults to social stigma and consequences of a bad reputation (p. 320); Guiso, Sapienza, and Zingales (2013), who ascribe avoiding default to views about fairness and morality (p. 1473); and Bhutta et al. (2017), who ascribe too few defaults to emotional and behavioral factors (p. 2433). We contribute to this literature by explaining the puzzle of missing bankruptcies, using a purely rational expectations model and modeling the value of bankruptcy as a real option over the life cycle.

Our valuation of the household bankruptcy option builds on the literature on dynamic programming (e.g., Bellman, 1957; Sargent, 1987; Stokey and Lucas, 1989). We show that a natural link exists between the option value of a bankruptcy filing and the value functions derived from the household’s dynamic programming problem. In particular, the underlying asset of the bankruptcy option is the household’s balance sheet, and the “price” of this underlying asset is well represented by the value function in the absence of bankruptcy filing choices. The exercise price of the option is the value that a household obtains from filing for bankruptcy. Similar to the valuation of an American option using a binomial tree model, the option value of bankruptcy filing can be computed by a backward induction approach.

Our household bankruptcy option’s valuation is related to the corporate finance literature which studies the valuation of firm debt and equity by considering equity holding as a call option written on the firm value (e.g., Black and Scholes, 1973; Merton, 1974; Leland, 1994; Leland and Toft, 1996). Several differences exist between the household bankruptcy option and the corporate default option. First, the household bankruptcy option is a put option, while the corporate default option is a call option. Second, the value of the underlying asset differs. In the corporate default option, the underlying asset price is the firm value, which is typically assumed to follow an exogenous stochastic process. By contrast, in our household finance model, the underlying asset is captured by the value function of the household’s dynamic programming problem, which is endogenously determined by the utility generated from the consumption streams. Last, the corporate finance literature typically assumes that the market is complete,

which facilitates option pricing through the construction of a risk-neutral probability measure. In our household finance model, the market is incomplete because the income risks and housing return risks are uninsurable. As a result, the risk preferences of households matter for the option value of bankruptcy.⁶

In our model, bankruptcy filing is triggered by income shocks and/or asset returns shocks. This feature relates our paper to the literature on income statement causes of bankruptcy, which shows how various exogenous shocks can impact household bankruptcy decisions. The standard hypothesis in this literature is that a negative income shock should increase bankruptcy filings. This hypothesis is supported by Gross and Notowidigdo (2011), who examine exogenous increases in U.S. state-level Medicaid coverage. However, the hypothesis is inconsistent with the empirical findings in Hankins, Hoekstra, and Skiba (2011) and Gross, Notowidigdo, and Wang (2014), with income shocks proxied by lottery winnings in the former paper and tax rebate payments in the latter paper. Gross, Notowidigdo, and Wang (2014) show that positive income shocks increase bankruptcy filings, which they explain by the relaxation of filing fee constraints after the shock. In addition, Hankins, Hoekstra, and Skiba (2011) find that the size of a positive income shock only delays, but does not prevent, bankruptcy filings.⁷

A central element of our analysis is that the characteristics of household balance sheets (e.g., secured liabilities, unsecured liabilities, assets serving as collateral) are key determinants in the bankruptcy decision because these characteristics determine the option value of bankruptcy. This feature relates our paper to the literature on the balance sheet causes of bankruptcy. An important early contribution is Fay, Hurst, and White (2002), who argue that the larger the net benefits of bankruptcy minus the costs of bankruptcy for a household, the greater the probability of a bankruptcy filing. Our focus on the household balance sheet also fits our paper into a larger literature, which emphasizes the importance of balance sheet characteristics in households' responses to exogenous shocks (e.g., Mishkin, 1978; Olney, 1999; Koo, 2003; Agarwal, Liu, and Souleles, 2007; Mian, Rao, and Sufi, 2013).

Our incomplete market model with uninsurable income shocks is related to the strand of literature that uses structural models to quantitatively study bankruptcy-related topics, although none of these earlier studies examines the option value of bankruptcy. Earlier examples in this literature include Athreya (2002, 2006) and Li and Sarte (2006); Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007); Livshits, MacGee, and Tertilt (2007); Livshits, MacGee, and

⁶Miao and Wang (2007) study the value of an entrepreneur's real option to invest in a project. Under the assumption of incomplete markets and exponential utility, they show analytically how the option value depends on the degree of absolute risk aversion.

⁷In a related study, Agarwal, Mikhed, and Scholnick (2019) examine the effect of lottery winnings on bankruptcy filings of winners' neighbors.

Tertilt (2016); and Pavan (2008). Pavan (2008) builds a life-cycle model in which agents have access to two assets: a risk-free bond and durable goods. Our model has the same choice over assets, but it explicitly includes secured debt (mortgages), which Pavan (2008) abstracts from. Campbell and Cocco (2015) use a dynamic model with endogenous mortgage decisions to study the high default rates on adjustable-rate mortgages during the recent U.S. housing downturn, but their paper does not consider personal bankruptcy. More recent contributions are Luzzetti and Neumuller (2016) who study the time series evolution of bankruptcy filings, and Mitman (2016) who studies how bankruptcy filings in the U.S. are affected by two important policies: the Bankruptcy Abuse Prevention and Consumer Protection Act and the Home Affordable Refinance Program.

3 The Model

This section introduces the household lifetime optimization model in which households are forward looking with rational expectations.⁸ Households have access to two types of assets – risk-free bonds and housing, and two types of liabilities – unsecured debt and mortgages. When a household files for bankruptcy, its debts are discharged and its assets are seized.⁹ We lay out the optimization problem of households using the language of dynamic programming, then we formulate the exercise price and valuation of the bankruptcy option based on the household’s value functions.

3.1 Environment

In this section, we introduce the exogenous processes of household income, the arrangement of the financial market, and the legal environment regarding bankruptcy filing.

⁸The decision-making unit in our model is a household. Bankruptcy filing can be done at the individual level, but it is still a household decision. In our bankruptcy filing data, 4.7% of the filings are explicitly joint filings of family members.

⁹In reality, the bankruptcy process is more complicated, with some assets being exempt from seizure in bankruptcy (e.g., a certain value of home equity), while some debts being not dischargeable (e.g., student loans of recent graduates). While these institutional details are interesting, we abstract from them in this paper to keep the model tractable. It is worth noting that most provinces in Canada have low levels of bankruptcy exemptions, making them less relevant in economic terms. The situation may be different in the U.S., where certain states offer unlimited homestead exemptions. These features and the incentives they generate are outside the scope of this study.

3.1.1 Income Process

A household makes consumption decisions and financial choices over its lifetime. At age $t = 21, 22, 23, \dots, T^r$ the household earns a stochastic income. At age $t = T^r$, the household retires and starts to receive retirement income. At each age, the household is faced with a exogenous death probability, which equals one at age $T + 1$, so that the maximum life span is T . In the quantitative analysis, we have $T^r = 65$ and $T = 80$.

In the beginning of age $t < T^r$, a household receives the exogenous labor income y_t . The stochastic process of the income stream is the following:

$$\log(y_t) = \log(\bar{y}_t) + \log(\tilde{y}_{i,t}), \quad (1)$$

where \bar{y}_t is the deterministic component of income that is typically hump shaped with respect to age in the data. The stochastic component of income, $\tilde{y}_{i,t}$, is specific to the i^{th} household, and it is further decomposed into

$$\begin{aligned} \log(\tilde{y}_{i,t}) &= z_{i,t} + \epsilon_{i,t}, \\ z_{i,t} &= \rho_y z_{i,t-1} + \eta_{i,t}, \end{aligned}$$

where $\epsilon_{i,t}$ is the purely transitory income shock, and $\eta_{i,t}$ is the persistent income shock. The parameter ρ_y determine how persistent $\eta_{i,t}$ is. $z_{i,t}$ is the accumulation of persistent shocks over ages.

After retirement, a household is no longer subject to persistent income shocks, but it is still subject to transitory income shocks. In other words, postretirement income is still represented by equation (1), except that the stochastic $\tilde{y}_{i,t}$ only has a transitory component after retirement.

Income shocks play two roles in the model. First, they give rise to precautionary saving motives; thus, households save even if they are extremely impatient. Second, they cause households to file for bankruptcy if they receive negative shocks and run low on assets. These two effects are both related to the borrowing capacity of households. A large borrowing capacity weakens the precautionary saving motives, resulting in low asset accumulation. When negative income shocks occur persistently, households sink deeper into debt and finally find it optimal to file for bankruptcy.

3.1.2 Financial Markets

Households in the model have access to two types of assets – risk-free bond and housing, and two types of liabilities – uncollateralized debt and mortgages. The risk-free bond offers a fixed rate of return of r_b , and the borrowing rates are r_d and r_m in the uncollateralized

debt market and the mortgage market, respectively. Here, we assume that r_d is a constant, but we allow the capacity of unsecured borrowing to vary with a household's age and income. Thus, we highlight the rational behavior that a bankruptcy filer should strategically max out the unsecured borrowing limit regardless of the borrowing rate. By contrast, Mitman (2016) assumes that households have no borrowing limit, but they face heterogeneous borrowing rates, which increase endogenously with the unsecured debt outstanding.¹⁰

Return on the housing asset consists of the rental return and price appreciation. Rental return is denoted r_h and is expressed as a percent of the current value of the housing asset.¹¹ The price appreciation rate has a mean value of μ and a stochastic term \tilde{p}_t that follows an AR(1) process. Let p_t be the house price at time t , the stochastic process is modelled as follows:

$$\begin{aligned} \log(p_t) &= \mu + \log(p_{t-1}) + \log(\tilde{p}_t), \\ \log(\tilde{p}_t) &= \rho_h \log(\tilde{p}_{t-1}) + \zeta_t, \end{aligned} \tag{2}$$

where ζ_t denotes the random shock and ρ_h governs the persistence of house price shocks. Thus, for homeowners (i.e., households with positive housing assets), the house price constitutes another source of uncertainty that can also cause bankruptcy.

We assume housing is illiquid – it is costly to buy and sell houses. The buying cost is ϕ_{buy} and is expressed as proportion of the total price of the house, and the selling cost is ϕ_{sell} and is shown as proportion of the total house price. In addition, we assume a minimum housing size of \underline{h} . Upon purchasing a house, a household needs to pay d proportion of the total house price as a down payment. Thus, a household needs to accumulate at least dph amount of wealth before becoming a homeowner.

Upon purchasing a house of size h at the price p , the mortgage debt outstanding is $m = (1 - d)ph$. Regarding the mortgage payment schedule, we assume that the homeowner pays a fixed amount each period, such that the mortgage balance in the end of the terminal period should be zero. Assume the household has mortgage debt m in the beginning of period t and x amount of mortgage payment. The following table shows the mortgage debt balance from period t until T .

To ensure the zero mortgage balance in the end of period T , the term $m(1 + r_m)^{T-t} - x \frac{1 - (1+r_m)^{T-t+1}}{1 - (1+r_m)}$ needs to be zero, which is equivalent to $x = \left[\frac{(1+r_m)^{T-t} - (1+r_m)^{T-t+1}}{1 - (1+r_m)^{T-t+1}} \right] m$. Thus, the mortgage payment in each period is an age-dependent fraction of mortgage debt outstanding.

¹⁰As illustrated in Figure 6 of Mitman (2016), the unsecured credit supply becomes inelastic to interest rates when the debt outstanding is sufficiently large, implying a de facto borrowing limit.

¹¹The rent changes as house price varies over time. As an alternative modeling strategy, Mitman (2016) assumes house rent is equal to the maintenance cost of housing.

	Before Payment	After Payment
t	m	$m - x$
t+1	$m(1 + r_m) - x(1 + r_m)$	$m(1 + r_m) - x[1 + (1 + r_m)]$
t+2	$m(1 + r_m)^2 - x[(1 + r_m) + (1 + r_m)^2]$	$m(1 + r_m)^2 - x[1 + (1 + r_m) + (1 + r_m)^2]$
...
T		$m(1 + r_m)^{T-t} - x \frac{1 - (1 + r_m)^{T-t+1}}{1 - (1 + r_m)}$

We use ρ_t to denote this fraction, i.e.,

$$\rho_t = \left[\frac{(1 + r_m)^{T-t} - (1 + r_m)^{T-t+1}}{1 - (1 + r_m)^{T-t+1}} \right]. \quad (3)$$

Given the down payment requirement d , the loan-to-value ratio (LTV) is $1 - d < 1$ when the mortgage is originated. However, the LTV may rise above one in our model, if the home owner receives a series of negative house price shocks. Households may face a greater incentive to file for bankruptcy when the LTV is larger than one, which implies that the household has negative equity in the housing market.¹²

3.1.3 Number of Filings

Households in the model are allowed to file for bankruptcy twice over the life cycle. This assumption is based on Canadian bankruptcy law and is consistent with the data. The 2009 amendments to the Bankruptcy and Insolvency Act (BIA) of Canada extended the right of an automatic discharge to second-time bankrupts. However, for bankrupts with debts that have been discharged more than twice, the BIA makes it clear that the court shall refuse, suspend, or impose a conditional discharge.¹³ According to Duggan et al. (2014), less than 1.4% of bankrupts in 2010-2012 are third-time filers, and about 0.1% are fourth-time filers. Thus, it is a reasonably good approximation to allow households in the model to file twice.

Given that we explicitly model the second chance of bankruptcy filing, we are able to show how the option value of the first-time filing differs from that of the second-time filing. In addition, we are able to study in the counterfactual experiments how the presence of a second chance to file for bankruptcy influences the option value of the first chance, and how it affects the likelihood and timing of the first-time filing.

¹²We do not model delinquency of homeowners outside of bankruptcy explicitly.

¹³BIA, at ss. 173(1)(j) and 172(2).

3.2 Dynamic Optimization Problem

In the beginning of each period, a household receives the death shock, the income shock, and the housing return shock. After these shocks are revealed, a household, if it survives, decides on whether to file for bankruptcy or not to maximize its lifetime utility. If the household chooses not to file for bankruptcy, it makes decisions on consumption, asset holdings, and debt holdings of the next period.

The value function of a household of age t is denoted by $V_t^{BF}(\Omega)$, where BF is the number of prior bankruptcy filings, with $BF = 0, 1, 2$. The state vector is $\Omega = (b, h, m, y, p)$, where b , h , and m are the holdings of bonds (or debts if $b < 0$), housing, and mortgage, respectively. Combined they are the balance sheet of a household.¹⁴ The state variable y predicts future income because of the AR(1) assumption for z_t , the persistent income shock. Similarly, p is included in the state space because it predicts future growth of house prices because of the serial correlation of house price shocks.

3.2.1 Bequest Motives

Bequest motives play a role in the option value of bankruptcy filing. Intuitively, without bequest motives, a household with the option to file for bankruptcy should always file in period T . However, the data do not show a rise of filing rates among very old households. Another relevant data observation is that many elderly households have a large net worth. Following DeNardi, French, and Jones (2010), we use a bequest function to capture the slow wealth decumulation and low bankruptcy filing rate of the elderly.

The bequest function is defined as:

$$B(w') = L(\kappa + w'), \quad (4)$$

which is interpreted as the value received by a household that bequeaths w' amount of net worth upon death.¹⁵ The bequest value increases with net wealth w' and the parameter L , which determines the strength of bequest motives. The parameter κ is nonnegative and measures the extent to which the bequest is necessary. If κ is large, then the bequest resembles luxury goods and it is optimal to leave bequests only when a household's wealth is above some threshold level.

¹⁴Since h is costly to adjust (buy/sell), we need to keep track of each one of the three state variables instead of just the net wealth.

¹⁵As we show later, a new entrant to the economy receives an initial endowment of wealth. We do not explicitly link the bequest to the initial endowment in our partial equilibrium framework, but we calibrate the initial endowment based on the Survey of Financial Security (SFS) data.

In the context of our model, we assume a household has chosen b' , h' , and m' , which denote amounts of the risk free bond (or uncollateralized debt if $b' < 0$), housing asset, and mortgage debt, respectively, at the beginning of period $t \leq T$. If the household dies at the end of period t , then the net worth left behind is:

$$w' = (1 + r)b' + (1 + r_h - \phi_{sell})p'h' - (1 + r_m)m', \quad (5)$$

with $r = r_b$ if $b' \geq 0$ and $r = r_d$ if $b' < 0$. The term $(1 + r_h - \phi_{sell})p'h'$ represents the net revenue from selling the house – the beneficiary needs to collect the rent, pay the selling cost, and pay off the mortgage balance.

It should be noted that the net worth w' is not necessarily positive when a household dies for two reasons. First, although a household is required to pay off all the mortgage balance in period T , the household may die before period T ; thus, it may have negative housing equity if it has been hit by negative house price shocks. Second, even in period T , households can borrow from the uncollateralized debt market, which might cause the net worth w' to be negative after the household dies.

3.2.2 Households with the Filing Option

In the case of $BF < 2$, households have the bankruptcy filing option; thus, they face the following discrete choice problem regarding whether to file for bankruptcy or not:

$$V_t^{BF}(\Omega) = \max\{V_t^{BF,x}(\Omega), V_t^{BF,u}(\Omega)\}, \quad (6)$$

where $V_t^{BF,x}(\Omega)$ denotes the value if a household chooses to exercise the option (i.e., filing for bankruptcy) in the beginning of period t , and $V_t^{BF,u}(\Omega)$ is the value if the household chooses to hold the option unexercised.

Value of Filing With the recursive preferences taken from Epstein and Zin (1989) and Weil (1990), the value of choosing to exercise the option has the following recursive representation:

$$V_t^{BF,x}(\Omega) = \left\{ (1 - \beta)c^{1-1/\theta} + \beta \left[(1 - \nu_{t+1}) (E_t(V_{t+1}^{BF+1})^{1-\gamma})^{\frac{1}{1-\gamma}} + \nu_{t+1} (E_t B_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}} \right]^{1-1/\theta} \right\}^{\frac{1}{1-1/\theta}} \quad (7)$$

$$s.t. \quad c = y(1 - \phi); \quad b' = 0; \quad h' = 0; \quad m' = 0,$$

where ϕ is the cost of bankruptcy filing as a proportion of the household's income.¹⁶ By choosing to file for bankruptcy, the household sets the assets and liabilities to zero, and consumes an amount equal to the current income net of the bankruptcy filing cost. It should be noted that the survival value V_{t+1}^{BF+1} is a function of the state space $\Omega' = \{b', h', m', y', p'\}$, and the bequest value is a function of bequeathed net wealth w' , which is zero in this case.

The parameter θ is the EIS, which determines how substitutable consumption is intertemporally, and γ is the coefficient of relative risk aversion. When $\theta = 1/\gamma$, households have a more conventional constant relative risk-aversion preference. The parameter β determines the household's time preference because it specifies the weight on the future consumption as summarized in the value function relative to the current consumption.

Two aspects of the continuation value in equation (7) are noteworthy. First, the continuation value is a linear combination of the expected bequest value and the expected survival value, with the death probability ν_t as the weight. Second, the survival value V_{t+1}^{BF+1} has the superscript $BF + 1$, denoting that the household has filed for bankruptcy $BF + 1$ times.

Value of Non-Filing The value of choosing to hold the option unexercised is again the outcome of a discrete choice – whether to adjust housing investment or not. Let $V_t^{BF,a}$ and $V_t^{BF,na}$ be the value of adjusting and not adjusting housing asset, respectively. The discrete choice is

$$V_t^{BF,u}(\Omega) = \max\{V_t^{BF,a}(\Omega), V_t^{BF,na}(\Omega)\}. \quad (8)$$

The value of adjusting housing asset is the following:

$$V_t^{BF,a}(\Omega) = \max_{b' \geq -D^{BF}, h' \geq h} \left\{ (1 - \beta)c^{1-1/\theta} + \beta \left[(1 - \nu_{t+1}) (E_t(V_{t+1}^{BF})^{1-\gamma})^{\frac{1}{1-\gamma}} + \nu_{t+1} (E_t B_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}} \right]^{1-1/\theta} \right\}^{\frac{1}{1-1/\theta}}, \quad (9)$$

$$\begin{aligned} s.t. \quad c &= y + (1 + r)b - b' + (1 + r_h - \phi_{sell})p_t h - m - p_t h' [\phi_{buy} + d - (1 - d)\rho_t], \\ m' &= (1 + r_m)[p_t h' (1 - d)(1 - \rho_t)], \end{aligned} \quad (10)$$

where $r = r_b$ if $b' \geq 0$; and $r = r_d$ if $b' < 0$. The bequest value B_{t+1} is a function of bequeathed wealth w' , which is a function of the assets and liabilities the household chooses, as specified in equation (5).

As mentioned earlier, we assume a borrowing limit for each household; thus, we have the

¹⁶This cost may include both monetary and nonmonetary costs such as psychological cost.

constraint $b' \geq -D^{BF}$ in equation (9). The borrowing limit is a function of the household's age, income, as well as its bankruptcy filing history.

The term $(1 - \phi_{sell} + r_h)p_t h - m$ in the budget constraint represents the net revenue from selling the existing house. To purchase a new house, the household needs to pay ϕ_{buy} fraction of the house value as the buying cost. In addition, d fraction of the house value needs to be paid down. The new mortgage debt amounts to $(1 - d)$ fraction of the house value, for which the household needs to pay ρ_t fraction in the present period, with ρ_t given by equation (3). It is noteworthy that rental income and interest are collected at the beginning of the period. All the payments are also made at the beginning of the period, including the buying cost, down payment, and the first mortgage installment. The remaining mortgage debt grows into $(1 - d)(1 - \rho_t)(1 + r_m)$ fraction of the house value at the beginning of period $t + 1$.¹⁷

For the case of not adjusting the housing asset, we have

$$\begin{aligned}
V_t^{BF,na}(\Omega) = & \\
\max_{b' \geq -D^{BF}} & \left\{ (1 - \beta)c^{1-1/\theta} + \beta \left[(1 - \nu_{t+1}) (E_t(V_{t+1}^{BF})^{1-\gamma})^{\frac{1}{1-\gamma}} + \nu_{t+1} (E_t B_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}} \right]^{1-1/\theta} \right\}^{\frac{1}{1-1/\theta}}, \\
s.t. \quad & c = y + (1 + r)b - b' + r_h p_t h - \rho_t m, \\
& h' = h, \\
& m' = (1 + r_m - \rho_t)m,
\end{aligned} \tag{11}$$

where ρ_t is the mortgage payment in period t as a fraction of mortgage balance, which satisfies equation (3).

3.2.3 Households Without the Filing Option

If a household has filed for bankruptcy twice, then $BF = 2$, and it no longer has the option to file again. The household can either choose to adjust the housing asset or not to adjust. This is a discrete choice problem similar to the one facing those with $BF < 2$ and choose not to file. That is, for households with $BF = 2$, we also have

$$V_t^{BF}(\Omega) = \max\{V_t^{BF,a}(\Omega), V_t^{BF,na}(\Omega)\}, \tag{12}$$

where $V_t^{BF,a}(\Omega)$ and $V_t^{BF,na}(\Omega)$ are defined in exactly the same way as in the cases of nonfiling households with $BF < 2$, except that now these households have no access to uncollateralized

¹⁷If the household decides to become a renter, then $h' = 0$, and everything else in problem (9) is still valid.

borrowing. In other words, they face the following constraint in the optimization problems:

$$b' \geq 0.$$

3.3 Valuation of the Bankruptcy Filing Option

When the household files for bankruptcy, it essentially sells its balance sheet at a price of zero because it gives up all its assets and liabilities. Thus, the choice to go bankrupt is an American-style put option. The underlying asset of this option is the household balance sheet. This subsection discusses the valuation of this option based on the value functions of households introduced previously.

3.3.1 Exercise Price, Underlying Price, and Option Price

To understand the value of the bankruptcy filing option, first we need to find the exercise price of the option as well as the price of the underlying asset.

Since bankruptcy filing essentially sets the household's assets and liabilities to zero, it is tempting to think that the exercise price should be zero, and the price of the underlying asset should be the net worth of the household balance sheet, denoted w . However, this simple valuation implicitly assumes that households are risk neutral and perfectly willing to substitute consumption intertemporally. Given that households are risk averse, the option to file for bankruptcy is more valuable because the option functions as an insurance against future income shocks and asset return shocks. Further, the intertemporal substitution is not perfect; thus, a household with low income may find it optimal to smooth consumption by borrowing more from the debt market rather than to file for bankruptcy, even if $w < 0$.

In summary, the valuation of the bankruptcy filing option needs to take household preferences into account. The value functions introduced in the previous section summarize the utility the household receives from its balance sheet, given its risk preferences and intertemporal substitutability. Hence, the value functions are used to formulate the prices of underlying assets, the exercise price, and the valuation of bankruptcy filing options.

As for any American option, the price of the bankruptcy filing option at time t is the larger of two prices: (i) the price of exercising the option immediately, denoted p_t^x , and (ii) the price of holding the option unexercised, denoted p_t^u . That is,

$$p_t = \max\{p_t^x, p_t^u\}. \tag{13}$$

The price of exercising the option depends on how much the household gains from exercising the option relative to the underlying value of the household balance sheet. Specifically, p_t^x is

the difference between the exercise price of the household balance sheet and the price (or value) of the household balance sheet itself, which is the following in our model:

$$p_t^x = V_t^{BF,x}(\Omega) - V_t^{BF+1}(\Omega), \quad (14)$$

where $V_t^{BF,x}(\Omega)$, as defined in equation (7), is the total utility generated from the balance sheet and the income stream, if the option is exercised, while $V_t^{BF+1}(\Omega)$ is the total utility of the household without this bankruptcy filing option, which is equivalent to the total utility of a household that has exercised this bankruptcy filing option earlier. It should be noted that we follow standard economic theory to assume that a household evaluates its balance sheet through a utility function whose argument is the consumption stream.

To better understand the exercise price, we consider a household with $BF = 1$ that has one more chance to file for bankruptcy. Once this bankruptcy filing option is exercised, the household can no longer file for bankruptcy; thus, the term $V_t^{BF+1}(\Omega)$ in equation (14) represents the total utility a household can achieve without the bankruptcy filing option, and the difference $V_t^{BF,x}(\Omega) - V_t^{BF+1}(\Omega)$ represents the additional utility gained through exercising the option.

Similarly, the price of holding the option unexercised depends on how much the household gains from holding the option, relative to the underlying value of the household balance sheet, which is:

$$p_t^u = V_t^{BF,u}(\Omega) - V_t^{BF+1}(\Omega), \quad (15)$$

where $V_t^{BF,u}(\Omega)$, as defined in equation (8), is the value generated from the balance sheet and the income stream if the option is held unexercised in the present period. This value, as relative to the value of the underlying assets $V_t^{BF+1}(\Omega)$, is the payoff from holding the option unexercised.

Using equations (13), (14), and (15), the price of bankruptcy filing option is:

$$\begin{aligned} p_t &= \max\{p_t^x, p_t^u\} \\ &= \max\{V_t^{BF,x}(\Omega) - V_t^{BF+1}(\Omega), V_t^{BF,u}(\Omega) - V_t^{BF+1}(\Omega)\} \\ &= \max\{V_t^{BF,x}(\Omega), V_t^{BF,u}(\Omega)\} - V_t^{BF+1}(\Omega) \\ &= V_t^{BF}(\Omega) - V_t^{BF+1}(\Omega), \end{aligned} \quad (16)$$

where we have used the definition of $V_t^{BF}(\Omega)$ in equation (12).

Equation (16) expresses the option price as the difference between the value function with the option of bankruptcy filing and the value function without the option, if $BF = 1$, or the difference between the value function with two filings available and the value function with one filing, if $BF = 0$. In summary, the option price equals the additional value achievable to a

household because of the existence of the bankruptcy filing option.

3.3.2 Backward Induction

As with a standard American option, the price of the bankruptcy filing option can be computed via a backward induction approach. For the ease of exposition, we assume households have no bequest motives in this subsection. We also focus on households with $BF = 1$ (i.e., households with only one chance to file for bankruptcy).

Terminal Period For household of age T , after the realization of the income shock and asset return shocks, the price of exercising the option is:

$$\begin{aligned}
 p_T^x &= V_T^{BF=1,x} - V_T^{BF=2} \\
 &= \beta^{\frac{1}{1-\theta}} (1 - \phi) y_T - \beta^{\frac{1}{1-\theta}} [y_T + w_T(1 + \tilde{r})] \\
 &= \beta^{\frac{1}{1-\theta}} [-\phi y_T - w_T(1 + \tilde{r})],
 \end{aligned} \tag{17}$$

where we have set the continuation value to zero because of the no-bequest-motive simplification, so value functions depend on the current consumption only, as indicated by equation (7). For the case of exercising the option, consumption equals income net of filing cost; hence, $V_T^{BF=1,x} = \beta^{\frac{1}{1-\theta}} (1 - \phi) y_T$. For households without the option, the consumption equals the sum of income and net worth; hence, $V_T^{BF=2} = \beta^{\frac{1}{1-\theta}} [y_T + w_T(1 + \tilde{r})]$, where \tilde{r} denote the return on net worth.

From equation (17), it is clear that the exercise price falls with income and wealth. The exercise price is positive when the wealth is sufficiently negative and the income is sufficiently small.

The value of holding the option unexercised is zero in the terminal period because the option expires if it is not exercised; thus, $p_t^u = 0$. Therefore, the option price is

$$\begin{aligned}
 p_T &= \max\{p_T^x, p_t^u\} \\
 &= \beta^{\frac{1}{1-\theta}} \max\{0, -\phi y_T - w_T(1 + \tilde{r})\}.
 \end{aligned} \tag{18}$$

It is straightforward to see that the option price is weakly decreasing in wealth w_T . The smaller is w_T , the larger is p_T . This is consistent with the intuition in White (1998), which suggests that a household should file for bankruptcy when the net worth of the household portfolio is negative.

Second-to-Last Period In the beginning of age $T - 1$, after the realization of the income shock and return shock, the option price is

$$p_{T-1} = \max\{p_{T-1}^x, p_{T-1}^u\}.$$

Based on equation (14), the price of exercising the option is:

$$p_{T-1}^x = V_{T-1}^{BF=1,x}(\Omega) - V_{T-1}^{BF=2}(\Omega).$$

Here p_{T-1}^x does not depend on p_T — once the option is exercised, the continuation value in $V_{T-1}^{BF=1,x}(\Omega)$ and $V_{T-1}^{BF=2}(\Omega)$ does not depend on p_T , the option price in the next period.

On the other hand, the price of holding the option unexercised does depend on p_T because

$$\begin{aligned} p_{T-1}^u(p_T) &= V_{T-1}^{BF=1,h}(V_T^{BF=1}) - V_{T-1}^{BF=2} \\ &= V_{T-1}^{BF=1,h}(V_T^{BF=2} + p_T) - V_{T-1}^{BF=2}, \end{aligned} \quad (19)$$

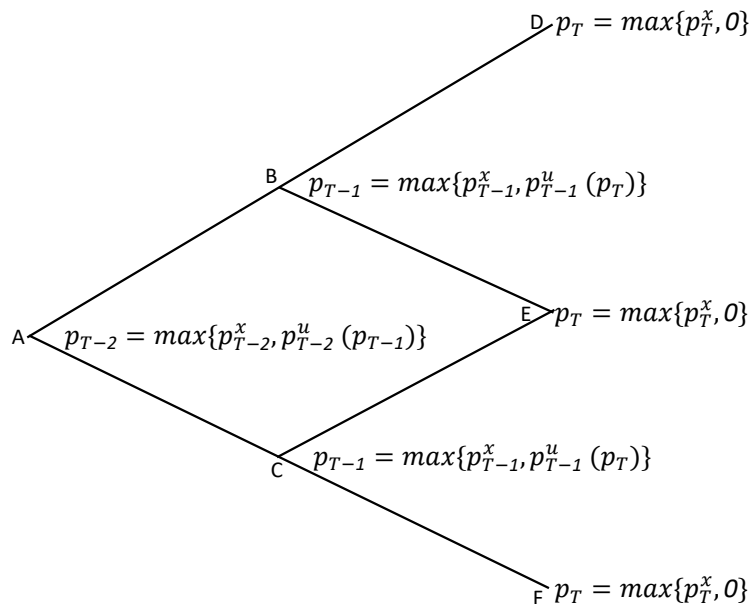
where $V_{T-1}^{BF=1,h}$ depends on the continuation value $V_T^{BF=1}$ as indicated by equations (8)-(11), while $V_T^{BF=1}$ itself is a function of p_T , as indicated by equation (12).

It is easy to see that $p_{T-1}^u(p_T)$ is an increasing function of p_T . Intuitively, if the option price is expected to be higher in the next period, the value of holding it unexercised is larger.

It should be noted that $p_{T-1}^u(p_T)$ also depends on consumption choices. By choosing to consume more, the household leaves less wealth to the next period, which leads to a tradeoff in the continuation value. First, the reduced wealth increases p_T because the option price in the last period is decreasing in wealth as shown in equation (18). Second, it decreases the household value $V_T^{BF=2}$, which is the value of not exercising the option in the last period. Therefore, the optimal consumption decision depends on this tradeoff.

Backward Induction The price of the bankruptcy option at age $t < T - 1$ is similarly calculated as the larger of two choices: to hold the option or to exercise the option. Figure 1 illustrates how the option of bankruptcy is priced using the backward induction technique. In each period, states of the world are represented by nodes. In each node, the option value is the maximum of values from two choices: to exercise (p^x) or to hold (p^u). The value from choosing to hold the option, p^u , is a function of the future option price except for the last period. For example, p_{T-1}^u depends on p_T as shown in equation (19).

Figure 1: Pricing of Bankruptcy Option



3.3.3 Discussion

We have shown the mapping between the valuation of bankruptcy filing and the value functions of a household's dynamic optimization problem. The valuation method illustrated in Figure 1 resembles the binomial tree model that is used to price an American option. However, difference exists between our bankruptcy filing model and the standard binomial tree model.

The most important difference is that, in a binomial tree model, the value of holding the option is simply the weighted sum of future values in different nodes, using a risk-neutral probability measure, discounted by a risk free rate. In Figure 1, p_t^u is an implicit function of the option price in the next period, discounted by the discount factor β .

Unlike the complete market model, risk aversion matters in our incomplete market model. From the dynamic optimization model laid out previously, how the future values affect p_t^u depends on the degree of risk aversion. Also, due to the insurance effect, the option is more valuable when the return on the portfolio is more risky because it insures the household against the downside risk. Similarly, the option is more valuable if the future income is riskier.

Finally, the option is more valuable when θ is large, which indicates a greater willingness to substitute consumption intertemporally. This intuition can be illustrated by considering a strategy in which the household borrows up to the limit to maximize current consumption, leaving a large negative net worth that increases the option value of filing for bankruptcy. The only utility cost of this strategy is the limited consumption in the next period when the household files for bankruptcy. This cost is high when θ is small, as households prefer

smoother consumption. However, if θ is infinitely large, households have no preference over smooth consumption, and the consumption in the current period perfectly substitutes the limited consumption in the next period. In this case, the option value can be easily boosted by the aforementioned strategy. In other words, the option value rises with the EIS.

4 Quantitative Analysis

In this section, we quantitatively study the model laid out previously. We use a Simulated Method of Moments approach to estimate a set of model parameters, exploiting the salient bankruptcy filing patterns of both homeowners and renters. Before the estimation, we briefly discuss the exogenous parameters that are either directly estimated from the data or taken from the existing literature.

4.1 Exogenous Parameters

The parameters of the exogenous process of income are estimated from the 1999-2017 waves of the Panel Study of Income Dynamics (PSID).¹⁸ The panel structure of the data allows us to estimate the income process using the variance and covariance structure of income shocks. To estimate the income process, we first decompose household labor income into a predictable component and an error term by regressing income on a set of demographic variables, including age, age-squared, year dummies, region dummies, education, occupation, and industry dummies. Next, we estimate the variance-covariance matrices of the error term. Finally, we find the variances of persistent and transitory shocks as well as the persistence parameter ρ_y to match the variance-covariance matrices. Additional details about this estimation procedure are available in Guvenen (2009).

The return to the risk-free bond is assumed to be $r_b = 0.02$, and the rate of uncollateralized borrowing is $r_d = 0.2$. The mortgage rate is $r_m = 0.03$, and the rental return on housing is $r_h = 0.02$.

The average real appreciation rate of house prices is 2% in the model. The literature generally finds that random shocks to house prices, as represented in equation (2), are moderately persistent; thus, we set $\rho_h = 0.15$. The variance of ζ is 0.015, which is close to the variance of persistent income shocks and much lower than the variance of shocks to stock returns, which is well over 0.1. Overall, the parameterization of house price appreciation indicates that the housing market is much less risky than the stock market, but it is also less efficient than the stock market as reflected in the serial correlation of house price growth rates.

¹⁸Comparable data for Canada are not accessible at this point.

In the baseline analysis, we assume a zero correlation between the persistent income shocks and house price shocks, i.e. $\text{corr}(\eta_{i,t}, \zeta_t) = 0$, where corr denotes the correlation coefficient. In Section 5, we explore the case in which $\text{corr}(\eta_{i,t}, \zeta_t) > 0$ and compare quantitative results under this assumption, especially the implied option values, with the baseline model.

The parameters determined outside the optimization model are presented in Table 1.

Table 1: Parameter Values

Parameter	Definition	Value
β	discount factor	0.95
r_b	bond rate	2%
μ	house price growth rate	2%
r_h	rent-price ratio of housing	2%
r_m	mortgage rate (nonfilers)	3%
r_d	unsecured debt rate	20%
ϕ_{buy}	house buying cost (% of house value)	0.02
ϕ_{sell}	house selling cost (% of house value)	0.06
d	down-payment requirement	0.2
\underline{h}	minimum housing size	1.8
ρ_y	persistence of income shocks	0.95
σ_y^2	variance of persistent income shock	0.015
σ_ϵ^2	variance of transitory income shock	0.05
ρ_h	persistence of housing return shock	0.15
σ_h^2	variance of housing return shock	0.08

This table reports the values of model parameters used in the quantitative analysis. Both house selling and house buying costs are proportions of house value. Minimum housing size is relative to the average annual income of households in the model, which is normalized to 1.

4.2 Debt Limit

The model allows households to have both secured debt (mortgage) and unsecured debt. For secured debt, we require household to pay 20% of the house value they purchase as a down payment, which effectively sets a LTV ratio at the level of 0.8 for homebuyers.¹⁹

Regarding the limit of unsecured debt, a higher debt limit (i.e., allowing more unsecured borrowing) affects bankruptcy in two ways. First, it raises the option value of bankruptcy – those who have not filed for bankruptcy have more flexibility in their financial planning; thus, it tends to lower the bankruptcy rate. Second, it increases the value of bankruptcy filing for households near the debt limit; hence, increases the bankruptcy filing rate. The relative

¹⁹Homebuyers in Canada who pay down less than 20% are required to purchase mortgage default insurance, so a down-payment requirement of 20% is natural benchmark in the model.

strength of these two effects depends on the nature of income and house price shocks, as well as the household’s preferences.

In reality, debt limits have an array of determinants, including the credit score, the income of the debtor, and the debt-to-income ratio.²⁰ To capture these various factors, we assume the following function for the debt limit for households with the bankruptcy filing options:

$$D^{BF=0} = \log(\alpha_0) + \alpha_{age} \times \log(age - 20) + \alpha_{inc} \times \log(income), \quad (20)$$

$$D^{BF=1} = \delta D^{BF=0} \quad (21)$$

$$D^{BF=2} = 0, \quad (22)$$

where *age* and *income* are the age and income of the household. The change of the credit limit with age is designed to mimic the growth of the credit score over a lifetime. Note that $BF = 1$ for households who have filed once. We assume these households can still borrow, but their borrowing capacity is only δ fraction of $D^{BF=0}$. This constraint is consistent with the empirical findings in Han and Li (2011) that households with a bankruptcy filing history have reduced access to credit.

We need to estimate the coefficients α_0 , α_{age} , and α_{inc} . It is difficult to estimate these coefficients directly from the data because the actual debt limit is not observable, as it includes all potential unsecured credit the household may get from lenders. What we can observe in our data is the amount of unsecured debt outstanding when a household files for bankruptcy, along with the income and wealth of the same household at the date of the filing. However, the amount of debt outstanding is generally different from the debt limit because the debtor might be able to borrow more than the debt outstanding when he or she files for bankruptcy.²¹ In addition, income and wealth prior to bankruptcy filing is not representative of these two variables of ordinary households that are not in financial distress.

It is also difficult to infer these parameters based on the limits of credit cards. Theoretically, debt limit is the sum of the limits of all credit cards that a household has access to or can potentially access. However, it is impossible to observe the maximum number of credit cards available to a household or all limits on these cards.

We infer these parameter values by choosing them so that the model can match key data features, including bankruptcy filer profiles for both homeowners and renters, the average home-

²⁰In our model, we allow the debt limit to contract as a result of negative income shocks. This feature is designed to capture the fact that lenders may deny new credit cards to borrowers with reduced income (debt-to-income ratio requirements) or issue cards with lower initial credit limits.

²¹It is also possible at the date of bankruptcy filing that the filer has debt outstanding larger than the borrowing limit because the limit can be reduced before filing due to borrower’s increased credit risk.

ownership rate, the LTV ratio of homeowners and the share of housing wealth in total wealth. Thus, one of the innovations of this paper is to pin down these coefficients and to conduct experiments regarding how changes of these parameter values will impact bankruptcy filings.

4.3 Estimation

We use the Simulated Method of Moments to estimate the following vector of parameters: $\Theta = \{\gamma, \theta, \phi_{file}, L, \kappa, \alpha_0, \alpha_{age}, \alpha_{income}, \delta\}$. The estimation exploits the bankruptcy filing patterns over the life cycle for homeowners and renters, respectively. These patterns are summarized in the data moments, denoted by M^d . Given each set of parameter values for Θ , we compute the optimal decision rules of households, then simulate the model to calculate the model moments $M^s(\Theta)$ that are identically defined as the data moments.

The estimation searches over the plausible ranges of Θ to minimize the distance between the model moments and data moments, i.e.,

$$\mathcal{L} = \min_{\Theta} (M^s(\Theta) - M^d) \mathcal{W} (M^s(\Theta) - M^d)', \quad (23)$$

where \mathcal{W} is the weighting matrix. In our estimation, \mathcal{W} is the inverse of the diagonal matrix composed of the variances of the data moments.²² This weighting matrix improves the estimation efficiency because it puts more weights on moments with smaller variances.

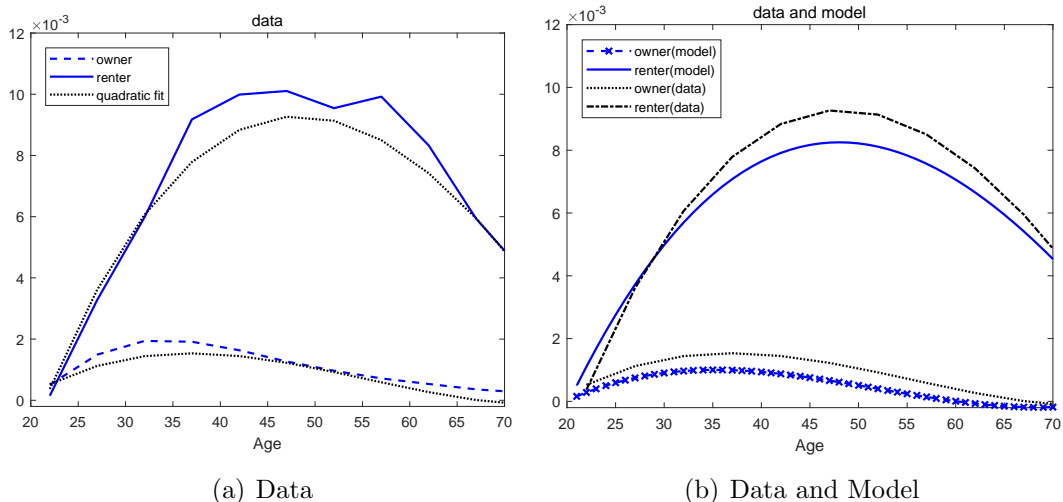
4.3.1 Data Moments

Our data are from the Office of the Superintendent of Bankruptcy, the single bankruptcy regulator to which every bankruptcy in Canada must be filed. Our data thus contain the universe of Canadian bankruptcy filings. Our database is extremely rich, including the full balance sheet, as well as the full income statement of every bankruptcy filer. This database allows us to examine data on the three main elements of our real option life-cycle model. First, we can observe the age of every bankruptcy filer; thus, we can examine the life-cycle characteristics of filers. Second, we are also able to observe whether any particular filer is a first-time filer or a repeat filer. Third, because we can examine the balance sheet of every filer, we can observe the current net worth of every bankruptcy filer and whether a filer is a homeowner or renter.

Figure 2 graphically illustrates how the bankruptcy filing rate varies with age for homeowners and renters, which is one of the main sources of identification for the model parameters. The filing rate of renters is defined as the number of renters who are bankruptcy filers, of a particular

²²These moments are from different sources, so it is impossible to estimate their covariances.

Figure 2: Age Profile of Bankruptcy Filing Rates



Note: Age profiles of bankruptcy rate for homeowners and renters. The left panel shows the data profiles along with the fit from a quadratic function of age. The right panel shows profiles from both the quadratic fit of the data and model simulation.

age, relative to the total number of renters of this age in Canada. The filing rate of homeowners is similarly defined. Figure 2 is based on the data on bankruptcy filers, homeowners, and renters for years 2007, 2008, 2009, and 2012.

Three patterns are evident. First, the age profiles of both homeowners and renters are hump shaped. Second, homeowners have a much lower filing rate than renters in each age. Third, the filing rate peaks between ages 30 and 35 for homeowners, but it peaks between ages 45 and 50 for renters. These profiles are well captured by a quadratic function of age, as shown by the dotted lines in the figure, which are the fitted profiles based on coefficients from regressing the proportion of filers on a quadratic function of age, controlling for year effects.

Coefficients of the quadratic function of age are reported in Table 2, for both homeowners and renters. The table also reports three additional data moments: housing share in total wealth (72.3%), homeownership rate (64%), and the fraction of repeat filers among all filers (19.2%). The second line of Table 2 reports the standard errors of these data moments.

4.3.2 Identification

The data moments in Table 2 are informative about the model parameters. We offer a brief discussion on the link between these moments and the model parameters to be estimated.

The life-cycle profiles of the bankruptcy filing rate are informative about how much a household can borrow. As we discussed earlier, the option value of a bankruptcy filing is larger if a household chooses to consume more currently, so that the net worth of its portfolio is

Table 2: Model and Data Moments

	Homeowner				Renter			
	<i>const.</i>	<i>age</i>	<i>age</i> ²	<i>age</i> ³	<i>const.</i>	<i>age</i>	<i>age</i> ²	<i>age</i> ³
Data	-0.0074	0.00059	-0.000012	7.22E-08	-0.0252	0.00161	-0.000022	7.33E-08
st. error	(0.0004)	(0.00003)	(0.000001)	(4.4E-09)	(0.0015)	(0.0001)	(0.000002)	(1.4E-08)
Model	-0.00585	0.00047	-0.000010	6.47E-08	-0.0192	0.00128	-0.000018	5.97E-08

	<i>House Share</i>	<i>Homeowner</i>	<i>Repeat Filers</i>
Data	0.723	0.640	0.192
st. error	(0.24)	(0.03)	(0.0005)
Model	0.904	0.672	0.250

Note: The table reports the model moments (i.e., moments calculated from the simulated data using parameter values reported in Table 3, along with data moments estimated from the actual data). The standard errors of the data moment estimates are reported in parentheses. Weighted by the inverse of the variances of data moments, the average distance between model moments and data moments is 452.

lower. Given a larger borrowing capacity, households have stronger incentives to accumulate debt over time, and then file for bankruptcy. Thus, the overall filing rate should increase with the borrowing capacity.

These life-cycle profiles are also indicative of how the borrowing capacity changes with age and income. The fact that the profiles are hump shaped suggests that the borrowing capacity is increasing with age or income, which provide incentives for households to postpone filing because they could make the option deeper in the money. As we will show in Figure 4, the borrowing limit indeed increases with age until the late stages of life cycle, both in the data and in our model.

As households move toward middle age, they accumulate wealth for both precautionary purposes and retirement, which lowers the option value of bankruptcy filing; hence, it lowers the filing rate. Therefore, the hump-shaped profiles illustrate the risk aversion of households because precautionary savings increase with risk aversion. The hump-shaped profiles are also indicative of how large the EIS parameter is because households save more for retirement if they are less willing to substitute consumption intertemporally.

Households save more, given a larger discount factor β or a stronger bequest motive, which effectively lowers bankruptcy filing rates, especially at the later stage of life. The larger amount of savings also leads to a higher homeownership rate. Hence, the profiles of the bankruptcy filing rate and the homeownership rate also contain information on these parameters.

Housing wealth accounts for about 72% of total wealth in the data. This moment helps identify the risk aversion parameter and the borrowing capacity. On the one hand, housing investment is reduced by the degree of risk aversion because the housing asset is risky, especially with leverage. On the other hand, housing investment increases with borrowing capacity

because the housing investment is illiquid with large adjustment costs, and a larger borrowing capacity provides liquidity.

The fraction of repeat filers among all filers is clearly indicative of how much the borrowing capacity falls after the first filing; thus, it helps identify the parameter δ . It is also possible to argue that the filing cost ϕ_{file} is closely associated with the overall filing rates – a larger cost leads to lower filing rates.

4.3.3 Initial Wealth Distribution

Our model assumes that a household enters the economy at age 21 with an initial endowment of wealth.²³ We need the initial distribution of wealth among the entering households in the estimation and simulation of the model. We obtain the initial distribution from the 1999-2012 waves of the SFS.²⁴ The data have 1,924 households aged below 23 with valid information on the household head’s age, housing asset, other financial assets, and mortgage balance. Among these households, 17% are homeowners. Among these homeowners, the average mortgage LTV ratio is 48%, and the average share of housing asset among total asset is 87.1%; 29.7% of homeowners have a zero mortgage balance.

Figure 3: Initial Wealth Distribution

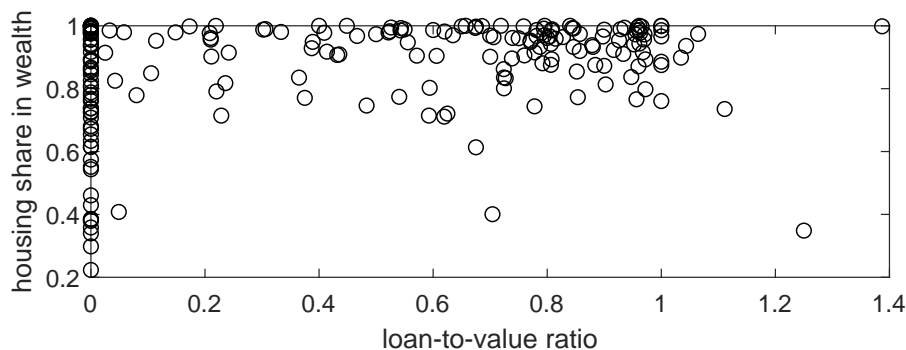


Figure 3 shows the joint distribution of the LTV ratio and the housing share in total wealth among the young homeowners in the data. The figure indicates that a large fraction of young homeowners have almost 100% of their wealth in housing while having a LTV ratio that is between 80% and 100%. These households are more likely to file for bankruptcy in our model in response to income shocks and house price shocks.

²³Part of this initial wealth may come from bequests.

²⁴The SFS is a survey conducted by Statistics Canada. It examines the net worth of a random sample of individuals, including the value of “all major financial and non-financial assets, and on the money owing on mortgages, vehicles, credit cards, student loans and other debts.” For more detail, see www23.statcan.gc.ca/imdb/p2SV.pl?Function=getSurvey&SDDS=2620.

4.4 Estimation Results

The row labeled “model” of Table 2 reports the moments generated from the model. The model matches the homeownership rate, housing share in total wealth, and the fraction of repeat filers reasonably well.

The eight moments related to bankruptcy filings are also matched reasonably well, as are the hump-shaped filing rates. Panel B of Figure 2 shows how the bankruptcy filing profiles generated from the model match the profiles from the data. Our model delivers a lower filing rate than in the data; however, the shapes and peaks of the profiles are well matched.

Consistent with the data, homeowners have a much lower filing rate in the model. The reason is that homeowners have much higher net wealth than renters, which is endogenous in our model. Specifically, to become homeowners, a household needs to accumulate sufficient wealth to meet the down-payment requirement. Therefore, only households with more positive income shocks are able to accumulate enough wealth to become owners. In the simulated data, the average net wealth of homeowners is 4.52 times larger than renters.

Table 3: Parameter Estimates

γ	θ	ϕ	L	κ	α_0	α_{age}	α_{inc}	δ
2.219	0.558	0.728	4.464	3.400	0.135	0.420	1.300	0.475
(0.372)	(0.048)	(0.174)	(1.185)	(0.290)	(0.025)	(0.031)	(0.093)	(0.122)

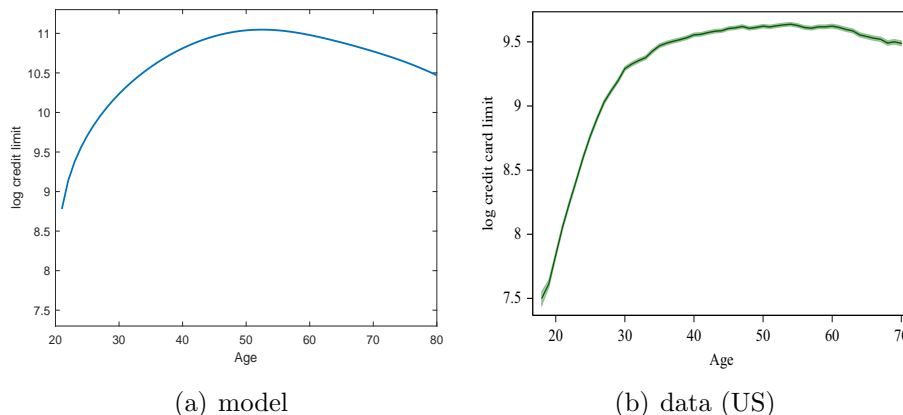
This table reports model parameter values estimated via the Method of Simulated Moments using the moments reported in Table 2. Standard errors are reported in the parentheses.

The model parameter values are reported in Table 3. The risk aversion parameter of $\gamma = 2.602$ and the EIS parameter of $\theta = 0.656$ are in line with conventional values.²⁵ The filing cost is about 70% of a household’s permanent income at the time of filing. This is interpreted as including both the monetary and nonmonetary costs, such as time costs or psychological costs (e.g., bankruptcy stigma).

The borrowing capacity increases significantly with age and income. With $\alpha_{inc} = 1.3$, the borrowing capacity increases by 1.3%, if the permanent income rises by 1%. This elasticity is much larger than the rise of borrowing capacity with age, which is $\alpha_{age} = 0.420$. The logarithm of average borrowing capacity in the simulated data is illustrated in panel A of Figure 4. This capacity rises with age until about 55 years old, then declines slightly. This profile has a similar shape to the borrowing capacity by age, based on U.S. data, reported by Fulford (2015), which is shown in panel B.

²⁵For example, the estimated γ is 1.625 in Alan (2006) and 4.409 in Cooper and Zhu (2016).

Figure 4: Limit of Unsecured Debt



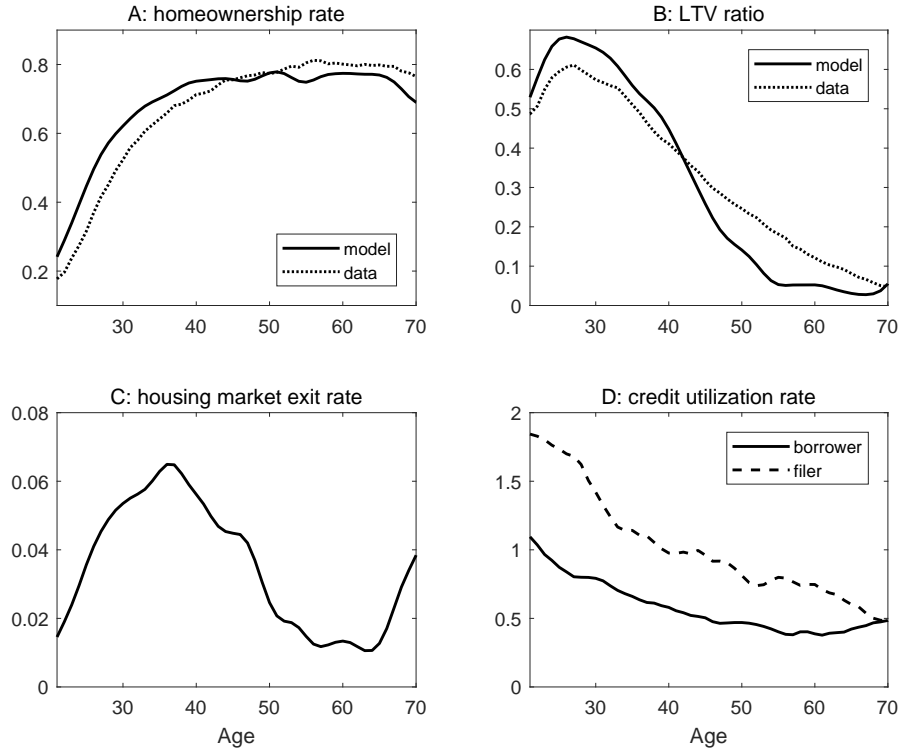
Note: Log of the limit of unsecured debt from the estimated model and the data. Panel B shows the credit card limit from the data, which is based on U.S. data, taken from Fulford (2015). It is noteworthy that the credit card limit will not capture all available and potential credit limit in our model. This feature may partly explain the difference in levels between panels A and B.

Figure 5 shows the life-cycle profiles of four endogenous variables from the model. Panels A and B report the homeownership rate and LTV ratio of homeowners by age from the model, as well as their data counterparts estimated from the 1999-2012 waves of the Survey of Financial Security. These profiles match those from the real data well, as shown in the figure, although they are not targeted in the model estimation. It is important for the model to match the homeownership profile because the bankruptcy filing pattern of homeowners is a major source of parameter identification. It is also important to match the LTV ratio because households with a high LTV ratio are more vulnerable to house price shocks, hence they are more likely to file for bankruptcy.

In Table A1 of Appendix B, we also report the elasticities of model moments with respect to parameter values, evaluated in the neighborhood of the parameter values reported in Table 3. The table shows how the borrowing limit changes with age and income (i.e., α_{age} and α_{inc}), which has strong implications on bankruptcy filing patterns. For example, with a larger α_{age} or α_{inc} , households tend to file later in the life cycle. Intuitively, this is because they can borrow more at a later stage of life; hence, benefit more from bankruptcy filing.

Panel C of Figure 5 shows the housing market exit rate, defined as the fraction of homeowners who choose to exit the housing market. Households in our model choose to exit the housing market for two reasons. First, they liquidate housing assets after the realization of a series of negative income or housing return shocks. This type of exit is likely to be associated with bankruptcy filing. As the figure shows, the exit rate peaks at around age 30-40, which is also the age with the highest proportion of bankruptcy filings for homeowners, indicating that some of the exiting homeowners are also bankruptcy filers. The second reason for exiting the housing

Figure 5: Other Profiles



market is wealth decumulation at an older age because of the lumpiness of the housing asset. This type of exit is reflected in the rise of the exit rate after retirement.

Panel D of Figure 5 reports the age profile of the utilization rate of unsecured borrowing capacity, defined as the balance of unsecured debt as a fraction of the borrowing limit.²⁶ The utilization rate is much higher among filers than average borrowers, indicating that indebtedness is positively correlated with bankruptcy filing. For filers, the credit utilization rate is greater than 100%. This is because these households tend to receive negative income shocks, which lower their borrowing capacity to such an extent that it is smaller than their credit balance accumulated prior to the negative income shocks.²⁷

²⁶Fulford and Schuh (2017) show that credit card utilization exhibits a similar age profile for all borrowers in the U.S.

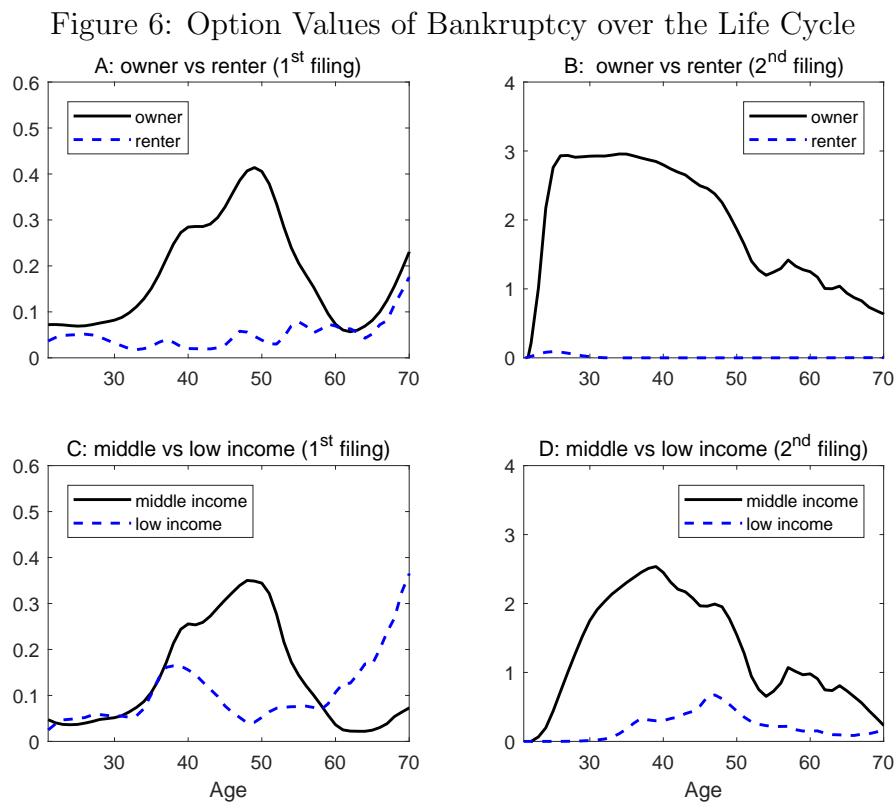
²⁷We compare these utilization rates from our model with utilization rates on all revolving credit computed from a Canadian credit bureau data set. The average revolving credit utilization rate for borrowers who never filed for bankruptcy and those who have filed are comparable to the values depicted in panel D of Figure 5.

5 Option Value

This section returns to the main point of our paper: the option value of bankruptcy filing. Based on the simulated data, we present the option values of different types of households. We further study the determinants of the option value in the simulated data.

5.1 Graphical Illustration

We consider two aspects of bankruptcy filing: the option value of bankruptcy filing and the filing decision. For the latter, we present the difference between the exercise price and the option price. Recall that a household files for bankruptcy only when the exercise price p^x equals the option value p , because by definition, the option value is the larger of the price to hold and the price to exercise, i.e., $p = \max\{p^x, p^u\}$.



Note: The average option value of bankruptcy filing by age and types of households.

Figure 6 presents the option values of bankruptcy filing over the life cycle, where the definition of option value, or option price, is given in Section 3.3. The value of a first filing is shown in panels A and C, and the value of a second filing is in panels B and D. Comparing these two sets of panels, it is clear that the option value is higher for the second filing than

for the first filing. This difference is partly driven by our plausible model assumption that the first-time filing does not affect households' access to the mortgage market, and access to the unsecured debt market is affected only through the reduced debt capacity, which is about 47.5% of the capacity prior to filing according to our estimation.²⁸ However, after the second filing, households are excluded from credit markets. As a result, the household exercises the first filing option at a lower value than the second-filing option.

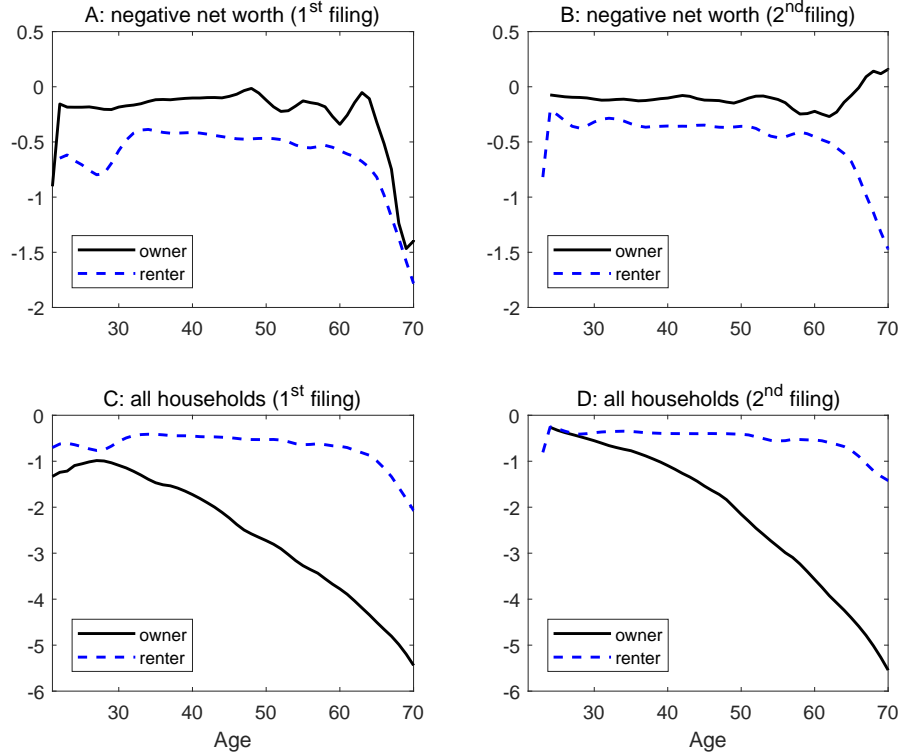
Intuitively, under our assumption of only two bankruptcy filings over the life cycle, the second-time filing is the last chance to file over the life cycle. For this reason, the option to file a second time is very highly valued by households; thus, households will be reluctant to exercise this option unless they are in significant financial distress. On the other hand, when making a decision regarding first-time filing, households have in mind that there is another chance for a second bankruptcy filing at a later date, so they are more willing to exercise the option of a first-time bankruptcy filing.

By comparing panels A and B of Figure 6, it is clear that the option value is higher, on average, for homeowners than for renters. This is driven by two mechanisms. First, because of self-selection, homeowners generally have higher incomes than renters; thus, their borrowing capacity is larger, which, in turn, results in the larger option value. Second, housing investment is both risky and illiquid, and the risk is further amplified by the leverage from mortgage borrowing. Thus, the option value is higher for homeowners because of its role as an insurance mechanism against house price shocks. Similarly, as shown in panels C and D of Figure 6, the option value is also higher for middle-income households (i.e., households with income between the 40th and 60th percentiles) than for lower-income households (i.e., households with income between the 10th and 20th percentiles). This is because middle-income households have a larger borrowing capacity, and because they are more likely to be homeowners. As we move from middle-income households to higher-income households, we find that the option value falls because their net worth is higher.

Figure 6 also shows that the average option value is hump shaped prior to retirement, especially for homeowners. This is driven by a number of factors. First, borrowing capacity, on average, increases with age until before retirement, which causes the option value to rise. Second, households accumulate wealth over time, which causes the option value to fall with age. Third, the major cost of bankruptcy filings is the (partial) exclusion from the credit market. This exclusion is less important for older households, which causes the option value to rise with age. This last factor also explains why the option value tends to rise again among older households.

²⁸These assumptions are consistent with the facts documented in Han and Li (2011), based on data from the Survey of Consumer Finances.

Figure 7: Difference Between Exercise Price and Option Price



Note: The difference between exercise price option price by age and types of households. For each type, the figure shows the average difference among households with negative net worth.

Figure 7 shows the difference between the exercise price and the option price (i.e., $p^x - p$) for both owners and renters of different ages. Panels A and B are for households with negative net worth. For such households, the difference between the exercise price and the option price is still mostly negative. In other words, even when net worth is below zero, households still find it not optimal to file for bankruptcy because the value of holding the option unexercised is larger than the exercise price. Among households with negative net worth, $p^x - p$ is closer to zero for homeowners than for renters. Thus, conditional on having negative net worth, homeowners are more likely to file for bankruptcy, which is intuitive because homeowners are subject to both income shocks and house price shocks. In addition, homeowners are more financially leveraged because they have access to both secured and unsecured debt markets.

Panels C and D of Figure 7 show the average values of $p^x - p$ by age and homeownership status for all households, rather than conditional on having negative net worth. These values are much farther from zero than in panels A and B, indicating that the value of holding the option unexercised is much larger than the exercise price (i.e., $p^u > p^x$) on average. For homeowners, $p^x - p$ is even farther from zero, consistent with the data fact that homeowners have much lower bankruptcy rates. It is also noteworthy that the homeowner's $p^x - p$ falls with age after

age 30-35, while renter’s $p^x - p$ does not show quick declines until after retirement. These observations are also consistent with the different life-cycle patterns of bankruptcy filings of owners and renters.

5.2 Regression Results

While Figures 6 and 7 illustrate how the option value and the exercise rule depend on age, income, homeownership status, and the number of bankruptcy filings, they do not show comprehensively the roles played by various other factors in our model, including wealth, leverage, and borrowing capacity. For a more complete picture, we estimate regression models of how the option value and bankruptcy filing probability are affected by all these various variables, for homeowners and renters, separately. It should be noted that the regressions are not for estimating causal effects but for illustrating how the option value and bankruptcy filing probability are correlated with the relevant variables.

As in the section on graphical illustration, we consider both the option value and the filing decision, with the filing decision captured by the probability of bankruptcy filing. To further understand why households with negative net worth do not always file for bankruptcy, we separately consider households that have negative net worth. The regression models are based on the simulated data. Table 4 reports the results from these regressions.

Table 4: Determinants of Option Price and Filing Probability

Owner 1st file	<i>Const.</i>	<i>Age</i>	<i>Age</i> ²	<i>Age</i> ³	<i>Income</i> <i>permanent</i>	<i>netW</i>	<i>Dlim</i>	<i>House</i> <i>share</i>	<i>LTV</i>
<i>p(option)</i> <i>All</i>	0.261	0.014	-0.0015	2.45E-05	0.072	-0.003	0.063	0.114	0.111
<i>netW</i> < 0	0.389	0.001	-0.0010	1.67E-05	0.087	-0.158	0.034	0.135	0.072
<i>Prob(file)</i> <i>All</i>	-0.731	0.012	-0.0006	4.39E-06	-0.082	-0.226	0.074	0.684	0.740
<i>netW</i> < 0	-0.523	0.012	-0.0012	1.50E-05	-0.083	-0.598	0.040	0.543	0.749
Renter 1st file	<i>Const.</i>	<i>Age</i>	<i>Age</i> ²	<i>Age</i> ³	<i>Income</i> <i>permanent</i>	<i>netW</i>	<i>Dlim</i>		
<i>p(option)</i> <i>All</i>	0.451	0.005	-0.0008	1.47E-05	0.121	-0.111	0.061		
<i>netW</i> < 0	0.853	-0.083	0.0024	-2.05E-05	0.117	-0.236	0.006		
<i>Prob(file)</i> <i>All</i>	-2.753	-0.119	0.0044	-4.71E-05	0.027	-4.997	-0.075		
<i>netW</i> < 0	-5.848	-0.040	0.0023	-2.73E-05	0.270	-6.853	-0.994		
Renter 2nd file	<i>Const.</i>	<i>Age</i>	<i>Age</i> ²	<i>Age</i> ³	<i>Income</i> <i>permanent</i>	<i>netW</i>	<i>Dlim</i>		
<i>p(option)</i> <i>All</i>	1.070	0.108	-0.0052	7.41E-05	0.050	-0.321	0.685		
<i>netW</i> < 0	4.206	-0.109	0.0007	2.41E-05	-0.106	-0.371	1.801		
<i>Prob(file)</i> <i>All</i>	-3.697	0.020	0.0008	-1.71E-05	-0.113	-5.220	-0.113		
<i>netW</i> < 0	-3.365	-0.053	0.0028	-3.33E-05	-0.050	-4.747	-0.316		

Note: Dependent variables include $p(option)$, which denotes the option value of bankruptcy filing, or probability of bankruptcy filing, denoted $Prob(file)$. We use an ordinary least squares model for option value and the Probit model for the probability of filing. Debt limit, net worth, and LTV ratio are denoted by $Dlim$, $netW$ and LTV , respectively.

As the table shows, net worth clearly has a negative effect on both option value and filing

probability, for both renters and owners. Everything else equal, wealthier households unequivocally value the option less, and they are less likely to exercise the option.

For both homeowners and renters, the borrowing limit is positively associated with the option value. Intuitively, the more a household can borrow, the more important the bankruptcy filing option is. However, a larger borrowing capacity also means the household can borrow more to increase the exercise price of the bankruptcy filing option. As a result, the effect of borrowing limit on filing probability can be either positive or negative, depending on whether the borrowing limit increases the option value more than the exercise price. As the table shows, for homeowners, the borrowing limit is positively associated with the probability of bankruptcy filing; thus, homeowners with more borrowing capacity (after controlling for their higher income and more wealth) are more likely to file for bankruptcy than owners with less borrowing capacity. However, the borrowing limit is negatively associated with the filing probability for renters, indicating that renters with more borrowing capacity are less likely to file for bankruptcy than renters with less borrowing capacity because the larger borrowing capacity increases option value more than its exercise price.

The top panel of Table 4 focuses on the option value and the probability of bankruptcy filing of the first filing for owners.²⁹ Housing share in total wealth is positively associated with option value and the probability of filing in all cases considered. For example, among all homeowners, a 1% increase in housing share is associated with a 0.68% increase in the probability of bankruptcy filing. This positive correlation is because housing investment is subject to risks; thus, the insurance value of the bankruptcy filing option is more important. The LTV ratio also has large positive regression coefficients, indicating that households with more leverage value the option more and exercise the option more.

The effect of permanent income on option value works through two channels. On the one hand, households with a higher permanent income are more likely to climb the housing ladder; hence, they value the option more. On the other hand, a higher permanent income means households have a larger income stream both in the present and in the future; hence, they value the option less. The regression coefficients show that the first channel is more important except for renters who have one filing history, as indicated by the negative effect of income on option value. The regression results also indicate that a higher permanent income implies a lower probability of bankruptcy filing. The only exception is that renters without any filing history are more likely to file when the permanent component of income is high, which is likely to be caused by the larger borrowing limit associated with higher income.

The regression results also shed light on what prevents households with negative net worth

²⁹Very few homeowners file for bankruptcy a second time.

from filing for bankruptcy. For this subset of households, the regression coefficients on net worth are still negative, yet they are larger than the corresponding coefficient in the regression for all households, indicating that households with smaller amounts of debt tend not to file for bankruptcy. Income is also a contributing factor, as indicated by the mostly negative coefficients on income. Finally, for renters, a larger borrowing limit discourages bankruptcy filings, indicating that some renters with negative net worth revolve debt rather than file for bankruptcy.

5.3 Correlation Between Housing Returns and Income

Thus far, we have assumed that income shocks are uncorrelated with house price shocks, i.e., $corr(\eta_{i,t}, \zeta_t) = 0$. In this subsection, we explore the implication of a positive correlation coefficient. Cocco (2005) decomposes the household’s income shocks into an aggregate component and an idiosyncratic component. He also shows that aggregate income shocks are positively correlated with house price shocks, with a coefficient of correlation as high as 0.553. However, the correlation between house price shocks and idiosyncratic income shocks is zero by construction in his model. In our model, we do not separate income shocks into an aggregate component and an idiosyncratic component. To construct a measure of correlation of income shocks and house price shocks similar to Cocco (2005), we assume that house price shocks are correlated with persistent income shocks in our model, $\eta_{i,t}$. To gain insights into how this correlation affects our quantitative results, we assume that $corr(\eta_{i,t}, \zeta_t) = 0.2$, which is in between Cocco’s two correlation coefficients. We resimulate the model using the parameters estimated in Table 3 and this correlation between shocks to house prices and income.

Table 5: Model Moments and Option Values

	House Share	Home-ownership	2nd filing	Option Value			
				1 st filing		2 nd filing	
				Renter	Owner	Renter	Owner
$corr(\eta_t, \zeta_t) = 0$	0.904	0.672	0.250	1	1	1	1
$corr(\eta_t, \zeta_t) = 0.2$	0.918	0.556	0.310	0.99	1.60	0.93	1.06

Note: Moments from the baseline model and the model in which there is a positive correlation between persistent income shocks and house price shocks. *corr* denotes correlation coefficient.

Table 5 reports some moments for the case of $corr(\eta_{i,t}, \zeta_t) = 0.2$, along with the corresponding moments from the baseline model. As the table shows, the homeownership rate is almost 12% lower when house price and income shocks are correlated relative to the baseline model, although the share of housing in total wealth for homeowners is little changed. As illustrated in Figure A1 in Appendix C, with the positive correlation, homeowners are more likely to file

for bankruptcy on average, relative to the case with zero correlation between income shocks and house price shocks, because housing investment is now riskier compared with the baseline model. On average, households become more likely to file for bankruptcy a second time, compared with the case with zero correlation of income shocks and house price shocks.

Table 5 also reports how the correlation between house price and income shocks affects option values. In this table, the option values from the baseline model are normalized to 1, so that the option values from the model with a positive correlation are reported as relative to the baseline model. The table shows that the option values of owners are much higher when house price and income shocks are correlated, especially for the first-time filers. The option values of renters become slightly smaller because renters on average have higher income or wealth in the model with $corr(\eta_{i,t}, \zeta_t) = 0.2$. This result is because homeownership is more selective as more middle-income households choose to rent than when there is no correlation between income and house price appreciation, which is reflected in the lower homeownership rate in this alternative model set up.

6 Conclusion

This paper contributes to the literature by studying the option value of bankruptcy filing, both theoretically and quantitatively. We theoretically derive the option value from the household’s dynamic programming problem, showing that the option is based on the household’s portfolio, whose price is represented by the value function of the household’s dynamic programming problem. The option value depends on both the household portfolio and on the household’s preferences. The value can be computed by backward induction, similar to the binomial tree model, used for the pricing of American options.

We offer two key insights to resolve the longstanding “missing bankruptcy filings” puzzle. First, as an American-style put option, the option to file for bankruptcy is not necessarily exercised when the option is in the money because of the insurance functionality of the option. This is a standard result in option pricing theory. Second, since the household portfolio is neither tradable nor replicated by tradable financial assets, the “price” of the household portfolio is not the net worth of the portfolio, but the utility derived from the portfolio. Consequently, the option is not in the money even when the portfolio’s net worth is negative.

Quantitatively, we estimate a structural model with two assets (i.e., housing and bond) and two liabilities (i.e., mortgage and unsecured debt). This rich setting allows us to explore how the option value of bankruptcy depends on different dimensions of the household portfolio, including leverage, liquidity, indebtedness, and others. The model is estimated based on a rich data set on personal bankruptcy filings in Canada. From the data, we present stylized facts

regarding bankruptcy filings of homeowners and renters and the percent of repeat filers. These data facts are exploited to identify the parameters of the structural model.

The model explicitly allows households to file for bankruptcy twice over the life cycle, which is roughly consistent with the institutional arrangements in Canada (and the U.S.). We calculate the option value of both the first filing and the second filing, and we show that the second filing generally has a higher option value than the first filing.

An important finding in our paper is that the capacity of unsecured debt borrowing has a strong effect on the option value and the optimal timing of bankruptcy filings. Exploiting observable patterns regarding bankruptcy filings, we estimate the borrowing capacity as a function of age and permanent income. We also study how the option value depends on households' risk attitude, elasticity of intertemporal substitution, and the discount rate.

Our results indicate that the option value of bankruptcy is higher for households with lower net worth, households with more leverage in their balance sheet, and/or households with higher income. The option value also changes with age: On average, it rises with age; thus, the majority of households do not exercise the option in their lifetime.

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Appendices

A Data

A.1 Data Description

The data used in this paper consist of the universe of household bankruptcy filings in Canada, provided by the Office of the Superintendent of Bankruptcy (OSB). There are both similarities and differences between the personal bankruptcy systems in Canada and the United States. Bankruptcy in Canada is federally regulated by a single regulator, the OSB, to which every bankruptcy filing must be made. This setting is very different from the U.S., where there are 94 separate bankruptcy court districts, to which bankruptcy filings are made, each of which have separate filing procedures. The single Canadian bankruptcy regulator is an important reason for our ability to access the large, Canada-wide database used in this paper.

A.1.1 Personal Bankruptcy in Canada

There are two types of personal insolvency in Canada: “bankruptcy,” in which the filer writes off unsecured debt in exchange for liquidating nonexempt assets that are used to repay debts to creditors; and “proposal,” which is a negotiated agreement with creditors to reduce or delay debt repayments without any liquidation of assets. These mechanisms are broadly similar to Chapter 7 and Chapter 13 bankruptcies in the U.S., respectively. Canadian bankruptcy (and U.S. Chapter 7) can be considered as “liquidation,” while Canadian proposal (and U.S. Chapter 13) can be considered as “restructuring.” This paper focuses only on Canadian bankruptcy, rather than proposals. This is because a restructuring process requires idiosyncratic negotiations with creditors, while under a liquidation process, various assets and liabilities are dealt with according to bankruptcy law.

A.1.2 Renters versus Homeowners in Bankruptcy Law

An important focus of this paper is on the distinction between homeowners and renters in bankruptcy. The distinction between owners and renters under bankruptcy flows directly from bankruptcy law, where the characteristics of balance sheets define the various costs of bankruptcy and benefits of bankruptcy. The critical distinction in bankruptcy law is between secured liabilities (e.g., mortgage debt) and unsecured liabilities (e.g., credit card debt). Secured liabilities (e.g., mortgages) are secured by collateral; for example, the house forms the collateral of the mortgage debt. Because the mortgage debt is secured by the house, if a mort-

gage holder defaults on the debt by filing for bankruptcy, then the individual will lose the ownership of the house to the creditor (i.e., the mortgage providing bank). The greater the positive equity in the home accumulated by the debtor at the date of the bankruptcy filing (i.e., current market value of the housing asset minus the mortgage debt outstanding), the greater the cost of the bankruptcy filing to the debtor because this will be the amount lost by the debtor to the creditor, when the ownership of the collateralized house transfers to the creditor, following default on that debt.

The implications of this is that homeowners who have secured debt such as mortgages have an incentive to avoid defaulting on the mortgage debt by filing for bankruptcy because they will lose ownership of the house and positive nonexempt equity in the house they have accumulated. Unsecured debt (e.g., credit card debt) is treated very differently than secured debt (e.g., mortgage debt) under bankruptcy law. Because unsecured (e.g., credit card) debt is not collateralized, if the debtor files for bankruptcy, then the unsecured creditor may receive the remainder of the bankruptcy estate after secured and preferred creditors' claims are paid. Usually, this implies that unsecured creditors receive little on their claims. Thus, the larger the amount of uncollateralized debt accumulated prior to the date of the bankruptcy filing, the greater the benefit to the defaulting debtor, because this debt is essentially written off, and thus the greater the losses to the unsecured creditor (i.e., the credit card provider).

In our bankruptcy balance sheet data (described next), we can observe the full balance sheet of every Canadian bankruptcy filing. While the vast majority of households that file for bankruptcy have access to credit card debt, only a proportion of households that file for bankruptcy are homeowners who have mortgage debt outstanding. Because we are able to observe the full balance sheet of every bankruptcy filer, we can distinguish between homeowners with a mortgage outstanding (who report a real estate asset and a mortgage liability), and individuals who do not own a home, who we designate “renters” (who do not report a real estate asset and a mortgage liability). (A very tiny fraction of bankruptcy filers report a housing asset with no mortgage debt outstanding – i.e., a house that has been paid in full.)

A.1.3 Data Sources

Every bankruptcy filing in Canada has to be made to the OSB by a bankruptcy trustee. The trustee is typically a professional accountant licensed by the OSB to act in bankruptcy filings. The trustee is considered an “officer of the court” and is designed to be impartial between creditors and debtors. The values of all balance sheet data used in this paper are determined by the trustee rather than by the individual bankruptcy filer, based on legal standards established by the OSB.

Balance Sheet Data The trustee of every bankruptcy filer is required to fill in OSB Form 79, which reports the full balance sheet of the filer as of the date of the filing. The current market value of various assets need to be reported, including house, car, land, securities, pension plans, cash, etc. In other words, a central advantage to the asset data used in this paper is that we can observe the current market value of the house of the bankruptcy filer, rather than the value as of the date of the real estate transaction. On the liabilities side of Form 79, the trustee needs to report the current amounts of all liabilities outstanding, including secured liabilities (e.g., mortgages) as well as unsecured liabilities (e.g., credit cards). When we examine housing assets and liabilities, our data allow us to observe both the current amount of the market value of the housing asset (as determined by the trustee), as well as the current value of the secured mortgage debt outstanding. We can thus observe the current LTV ratio of the housing asset. We can also observe the positive or negative equity in the house acquired by the owner (i.e., if the current market value is more or less than the current mortgage debt outstanding). Form 79 also reports on a variety of demographic information for each bankruptcy filer, including the age of the filer. Based on this age data, we can generate the life-cycle profiles of bankruptcy filers who are homeowners or renters, described next.

B Elasticities of Moments with Respect to Parameter Values

Table A1: Elasticities of Moments with Respect to Parameter Values

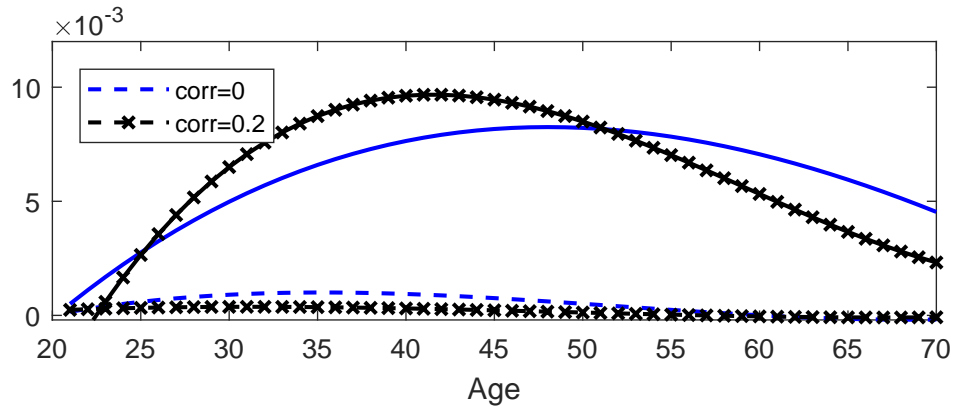
	Homeowner				Renter				<i>Hshare</i>	<i>HOR</i>	<i>2nd file</i>
	<i>const.</i>	<i>age</i>	<i>age</i> ²	<i>age</i> ³	<i>const.</i>	<i>age</i>	<i>age</i> ²	<i>age</i> ³			
γ	-1.42	-3.36	-4.55	-5.38	6.03	12.2	29.1	71.73	1.35	-0.33	-4.26
θ	-7.94	-4.98	-3.10	-1.66	-19.7	-23.8	-37.2	-71.5	1.27	0.13	8.96
ϕ	-15.0	-13.1	-12.0	-11.2	15.5	16.8	24.7	44.0	-1.06	-1.44	10.1
L	-3.67	-4.03	-4.00	-3.86	8.07	12.4	25.2	57.9	0.85	-1.56	0.62
κ	-24.4	-24.5	-24.9	-25.4	-40.2	-39.3	-48.6	-71.4	0.15	-1.30	7.58
α_0	9.80	10.2	11.0	11.9	-2.07	-2.55	-5.88	-16.0	2.48	0.71	-0.10
α_{age}	-2.57	0.22	2.08	3.49	38.2	35.2	37.4	41.7	2.65	-0.86	2.39
α_{inc}	-5.51	0.11	4.08	7.21	59.2	74.4	13.4	27.5	0.46	-0.57	-4.47
δ	1.56	1.38	1.30	1.26	-6.60	-9.17	-18.8	-43.9	0.08	-0.13	7.37

Note: This table reports the elasticities of model moments with respect to parameter values. *Hshare*, *HOR* and *2nd file* refer to share of housing, homeownership rate, and fraction of second-time filers, respectively.

C Bankruptcy Filing Patterns When $\text{corr}(\eta_{i,t}, \zeta_t) > 0$

Figure A1 compares the bankruptcy filing patterns with a positive correlation between income and house price shocks with these patterns in the baseline model with zero correlation between these shocks. More results on the case of $\text{corr}(\eta_{i,t}, \zeta_t) > 0$ is given in Section 5.3.

Figure A1: Filing Profiles with Correlated Income and House Price Shocks



Note: This figure shows the bankruptcy filing patterns when the stochastic components of income and house price growth rate are correlated, with a correlation coefficient of 0.2. The solid lines are for renters, and dashed lines are for homeowners.