Federal Reserve Bank of Philadelphia

Working Papers
Research Department

WP 19-25
May 2019
https://doi.org/10.21799/frbp.wp.2019.25

Financial Characteristics of Cost of Funds Indexed Loans

Patrick Greenfield
Federal Reserve Bank of San Francisco

Arden Hall
Federal Reserve Bank of Philadelphia Supervision, Regulation, and Credit Department

Consumer Finance Institute

ISSN: 1962-5361

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Patrick Greenfield† Arden Hall‡

Current version: May 22, 2019

Abstract

Two recent articles by Hancock and Passmore (2016) and Passmore and von Hafften (2017) make several suggestions for improving the home mortgage contract to make homeownership more achievable for creditworthy borrowers. Though the proposals in the two papers differ in some aspects, one common feature is an adjustable rate indexed to a cost of funds (COF) measure. Such indices are based on the interest expense as a fraction of liability balance for one or a group of depository institutions. One of these, the 11th District Cost of Funds (COF) Index, was in wide use in the 1980s and '90s, but use has fallen off since then. COF indices have the advantage that they are less volatile than market-based indices such as the 1-year U.S. Treasury rate, so that borrowers are not exposed to rapid increases in payments in a rising rate environment. We analyze COF-indexed ARMs from the point of view of the lender. First we develop a methodology for constructing a liability portfolio that closely tracks the specific COF index proposed by Hancock and Passmore (2016) and Passmore and von Hafften (2017). We then explore the financial characteristics of this liability portfolio. We show that the liability portfolio, and by implication, the mortgages it would fund, share a characteristic of fixed-rate mortgages: Values can vary significantly from par if rates change. This creates two problems for lenders: Pricing of COF-indexed ARMs is difficult because it depends not only on current interest rates but also on interest rates when principal is repaid, either through amortization or prepayment. Second, deviations from par make mortgage prepayment options valuable, so that lenders offering the product must manage option risk as well as interest rate risk. We conclude that while mortgages using a COF index have clear benefits for borrowers, they also are more difficult for lenders to price accurately. Further, once they are in lenders' portfolios, they increase the complexity of interest rate risk management. While these issues do not imply that COF indices cannot be part of innovative new mortgage designs, understanding their financial characteristics may contribute to the search for a better mortgage.

JEL Classifications: G12, G28
Keywords: Mortgages, cost of funds, interest rate risk, funds transfer pricing

*We would like to thank Wayne Passmore and Alexander H. von Hafften for sharing their COFI with us.

†Federal Reserve Bank of San Francisco, email: patrick.greenfield@sf.frb.org

‡Federal Reserve Bank of Philadelphia, email: arden.hall@phil.frb.org

Contents

1 Introduction

Recent papers by Hancock and Passmore (2016) and Passmore and von Hafften (2017) address issues of residential mortgage design, with the goal of devising a structure that is more affordable for potential homeowners while being feasible for banks and other investors to own. They have made several suggestions: Credit risk except for the extreme tail (catastrophe) should be borne by the private market rather than by the government, amortization should be accelerated through setting payments based on a fixed interest rate but with interest cost determined by a (usually lower) variable mortgage rate, the mortgage prepayment option should be eliminated, and a bank cost of funds index (COFI) should be used as the basis for setting mortgage interest rates. This note focuses exclusively on this last suggestion: The use of COFI as an adjustable-rate mortgage (ARM) index. Hancock and Passmore (2016) suggest that the COFI mortgage can both protect borrowers from much of the fluctuation in mortgage costs possible with ARMs based on market indices such as the 1-year U.S. Treasury rate, as well as provide significantly lower mortgage rates, while adequately compensating lenders. The purpose of this paper is to demonstrate some issues for lenders associated with the use of COFI as an ARM index. We believe the structure of COFIs creates complications for ARM lenders in pricing and managing interest rate risk. Hopefully, identifying and measuring the features of COFIs that are difficult for lenders can contribute in the search for better mortgage designs.

2 A financial model of the cost of funds

A COFI is the average rate on a portfolio of liabilities held by a depository institution or group of institutions. It is calculated as the ratio of interest expense to total liabilities. It is usually calculated on a monthly basis but then annualized. The COFI changes as the liability portfolio evolves over time with older liabilities maturing and new liabilities entering. As such, it represents a complex weighted average of current and historical interest rates. From this perspective, it is natural to consider modeling it with time series methods. A review of models of the 11th District Cost of Funds in Passmore (1993) confirms this, with all but one of the models up to that time based on the lagged value of the index and current and lagged values of a market interest rate.[1] Models proposed since then (Stanton and Wallace (1995) and Passmore and von Hafften (2017)) are also of this type. The most common form of these partial adjustment models is:

$$\text{COFI}_t = \alpha \text{COFI}_{t-1} + \beta r_t + \varepsilon_t. \tag{1}$$

These are clearly reduced form models, although Stanton and Wallace (1995) demonstrate that the model above can be derived from a structural model of the liability portfolio and the term structure of interest rates, under a restrictive set of assumptions. The one distinct model described in Passmore (1993) is a structural model based on an actual fixed-rate portfolio, which is estimated in that form. This model describes the liability portfolio as the sum of sub-portfolios each consisting of liabilities of the same original term. Each of these sub-portfolios contain all and only liabilities that have not yet matured. So, for example, the 3-month sub-portfolio can be

[1]The 11th District Cost of Funds is based on the liabilities of the members of the Federal Home Loan Bank of San Francisco. It was used extensively for ARMs in the 1970s and '80s

represented as

$$R_{3,t} = a_{1,t}r_{3,t-1} + a_{2,t}r_{3,t-2} + a_{3,t}r_{3,t-3}, \tag{2}$$

where $R_{3,t}$ is the overall rate on the 3-month sub-portfolio in period t, $a_{j,t}$ is the proportion of the liability issued i-months ago relative to period t ($\sum_j a_{j,t} = 1$), and $r_{3,t-i}$ is the rate on 3-month liabilities issued i months ago. We will refer to these sub-portfolios as "tenor sets." Then the COFI is just the weighted sum of the rates on the tenor sets for all original terms, or

$$\text{COFI}_t = b_{1,t}R_{1,t} + b_{2,t}R_{2,t} + b_{3,t}R_{3,t} + \cdots + b_{N,t}R_{N,t}, \tag{3}$$

where the potential original terms for liabilities are indexed from 1 to N, and b_i is the proportion of the i-th sub-portfolio in the total liability portfolio ($\sum_{j=1}^{N} b_{j,t} = 1$). With the appropriate a's and b's, this formulation can exactly match the COFI for a particular month, and so is a description of the index and not a model of it. However, the a's and b's can be expected to change in unpredictable ways over time as the liability portfolio evolves. This structure is approximated and made estimable by imposing two restrictions:

1. All $a_{i,t}$ are equal, so for liabilities with an original term of m-periods, $a_{i,t} = \frac{1}{m}$.
2. The $b_{i,t}$ are constant over time, so $b_{i,t} = b_i$ for all t.

As with the partial adjustment models, this model would be estimated with Treasury rates, rather than actual bank liability rates so a term representing the average difference between Treasury and liability rates is included. With an error term to represent the impact of the restrictions, the model becomes

$$\text{COFI}_t = c + b_1R_{1,t} + b_2R_{2,t} + b_3R_{3,t} + \cdots + b_NR_{N,t} + \delta_t, \tag{4}$$

where c is the regression's intercept term. A unique feature of this model, relative to lagged adjusted models of the COFI, is that it defines a portfolio that could potentially be constructed and maintained in financial markets. Each sub-portfolio tenor set would consist of equal face value amounts of all outstanding issues of that original term. So the 3-month tenor set would be constructed by combining equal amounts of 3-month Treasuries issued 3, 2, and 1 month ago. Combining tenor sets for all maturities in face value amounts proportional to the appropriate b_i's would produce a portfolio whose average rate tracks the COFI. Note also that the tracking portfolio, once constructed, could be updated over time with no adjustment to the size of the portfolio. As each security matures, it is replaced with the same face value amount of a new liability of the same original term, issued at par. The fact that a tracking portfolio for this COFI model can be created and maintained has implications for leveraged investors in COFI-based assets. Such investors could, if they chose, construct a liability portfolio that tracked the COFI in order to have interest expense closely track interest income. This feature is clearly not available in the partial adjustment models of COFI described previously, since to construct a COFI liability for the current period, they would require a COFI liability issued a period earlier.

While it is not clear that COFI investors would want to create liability portfolios that track the COFI, the ability to do so provides two substantial advantages. First, since it can actually be constructed, the COFI tracking portfolio can be valued at any time based on the current yield curve. And based on the current value of a COFI liability, the value of a COFI-based asset can be calculated consistent with the values of other debt securities. In contrast, no spot value can be associated with the lagged adjustment models of the COFI. (Some of the papers cited previously attempt to value COFI assets but can only estimate average values over long intervals.) It will be shown next that the value of COFI liabilities can vary significantly from par, so this feature would be important to fixed income portfolio managers.

The second advantage of the portfolio-based model of COFI relates specifically to the management of the COFI assets specifically in banks. Banks base their pricing of loans on the interest rate that it would be necessary for them to pay were they to issue liabilities with principal cash-flow characteristics identical to the new loan. The interest rate on this notional liability is called the "matched-maturity marginal cost of funding," and this approach to pricing is usually referred to as funds transfer pricing.[2] This interest rate is then marked up for other costs such as credit risk or loan servicing, and a profit margin is added to produce the rate to be offered to the borrower.[3] The use of the matched-maturity marginal cost of funding methodology is essential to enable banks to compare the expected profitability of potential loans with differing cash-flow characteristics on a consistent basis.[4] It provides pricing discipline across banks' typically decentralized lending businesses. So if a bank wanted to originate a loan product indexed to the COFI, it would need to construct a notional COFI liability based on its current borrowing costs.

2.1 Methodology for constructing the COFI-tracking portfolio

For this paper we use the particular COFI presented in Passmore and von Hafften (2017) as the COFI. Their index equals the total interest expense of the domestic commercial banks divided by their total interest-bearing liabilities. As this is a quarterly index, we imply a monthly time series from it via linear interpolation, obtaining 393 consecutive months from March 1985 to November 2017, where the day of each month is the 1st of that month (e.g., 3/1/1985). This index is shown below in Figure 1 versus the 11th District COFI that is produced monthly by the Federal Home Loan Bank of San Francisco (FHLBSF).[5] As can be seen, the Passmore and von Hafften (2017) COF index is consistently lower than the 11th district COFI, especially in the late 1980s and after the financial crisis.

[2]For a definition see Board of Governors of the Federal Reserve System (2016) p. 6. Note that while the cash flow characteristics of the liabilities will be identical for simpler loans, they may not be for more complex products such as adjustable-rate loans or prepayable mortgages.

[3]Where the loan offered is prepayable, banks include an additional charge to compensate for their financial risk for the option given to the borrower.

[4]The fact that banks calculate the cost of funds for a matching liability does not mean that they create the matching liability when they fund the loan. Instead, decisions about acquiring new assets and new liabilities are made separately, and any resulting overall interest rate risk is managed centrally.

[5]See http://www.fhlbsf.com/resource-center/cofi/.

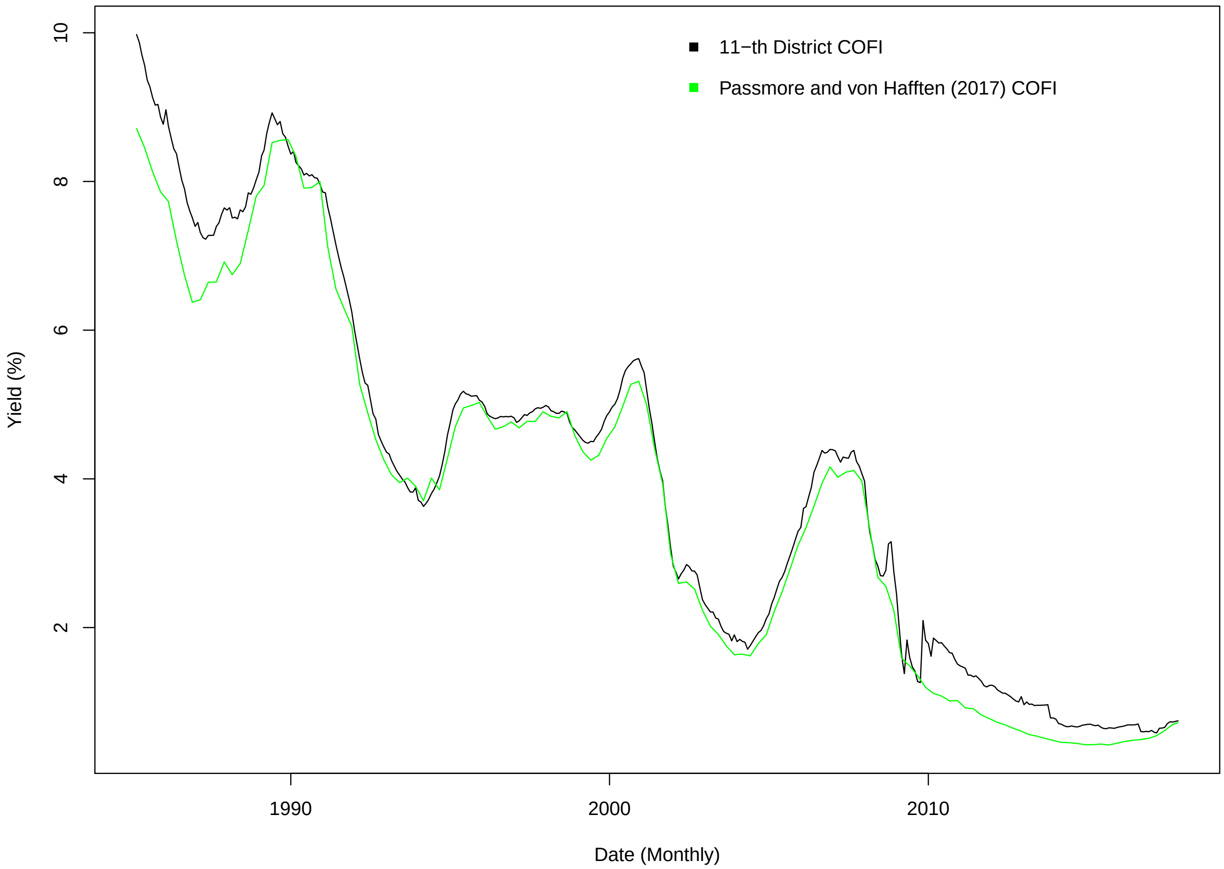

Figure 1: The 11th District COF and the Passmore and von Hafften (2017) COF indices from 3/1/1985 to 11/1/2017.

We construct a liability portfolio in the general manner described previously in §2 that mimics the monthly effective rate of the Passmore and von Hafften (2017) COF index (i.e., a given month's annual index value is divided by 12) out of sets of notional U.S. Treasury monthly pay par bonds with 3-month and 2- and 10-year tenors. We detail the par bonds used to construct the portfolio, the choice of tenors, the determination of portfolio weights, and the portfolio valuation below in §2.1.1, §2.1.2, and §2.1.3, respectively.

2.1.1 Constructing monthly pay Treasury par bond rates

The ideal bonds from which to construct the tracking portfolio would be regularly issued monthly pay bonds. Then the portfolio monthly average interest rate would just be the sum of all coupon payments divided by the face value of the portfolio. And the portfolio would be maintained at a constant face value by replacing each maturing bond with a newly issued par bond of the same original maturity. Unfortunately, no such set of regularly issued monthly pay bonds exists. However, by employing a yield curve model, we can construct hypothetical yield curves for monthly pay Treasury bonds consistent with market prices for Treasury bonds.

Consider a U.S. Treasury-based par bond that is issued with a maturity of T-years and a monthly coupon rate of $c_{12}^{\text{par}}(T)$. A liability portfolio can be constructed by issuing such par bonds across various tenors, T_i, each month. In a given month, j, a new T_i bond would be issued at par to pay off the principal of the now maturing T_i bond that was issued $12 \cdot T_i$-months prior, and coupon payments would be made on every T_i bond issued from months $\{j - 12 \cdot T_i, \ldots, j - 1\}$, resulting in a total of $12 \cdot T_i$ bonds of a given tenor T_i outstanding in the j-th month. For example, for bonds with an original maturity of 3 months, or 0.25 years, on the j-th month a new 3-month bond would be issued to pay the principal payment of the 3-month bond issued on month $j - 3$, and coupon payments would also be made on the 3-month bonds issued on months $j - 3$, $j - 2$, and $j - 1$, leaving the bonds issued on months $j - 2$, $j - 1$ and j outstanding.

To determine market-consistent rates for monthly pay Treasury bonds, we first determine a yield curve of U.S. Treasury zero rates in each month with a Svensson (1994) model such that the continuously compounded zero coupon yield or short rate, $r(t)$, has the following parametric form

$$\begin{aligned} r(t) = \beta_0 + \beta_1 \left(\frac{1 - \exp\left(-\frac{t}{\tau_1}\right)}{\frac{t}{\tau_1}} \right) + \beta_2 \left(\frac{1 - \exp\left(-\frac{t}{\tau_1}\right)}{\frac{t}{\tau_1}} - \exp\left(-\frac{t}{\tau_1}\right) \right) \\ + \beta_3 \left(\frac{1 - \exp\left(-\frac{t}{\tau_2}\right)}{\frac{t}{\tau_2}} - \exp\left(-\frac{t}{\tau_2}\right) \right), \end{aligned} \tag{5}$$

where $\{\beta_0, \beta_1, \beta_2, \tau_1, \tau_2\}$ are parameters estimated from bond prices. Specifically, we use the daily Svensson parameters that were estimated by Gürkaynak, Sack, and Wright (2006) using the prices of off-the-run U.S. Treasury notes and bonds outstanding on a given day from 1961 to the present (for the purposes of this paper we consider the end date to be November 1, 2017). As the spot rates are specified to be continuously compounded in Eq. (5), the discount factors or the prices of zero coupon bonds, $P(0, t)$, can be readily obtained by

$$P(0, t) = \exp\left(-r(t) \cdot t\right). \tag{6}$$

The par coupon rate, $c_p^n(T)$, for a given n-times a year coupon frequency and T-year maturity can be obtained from discount factors by

$$c_n^{par}(T) = \frac{n\left(1 - P(0, T)\right)}{\sum_{i=1}^{n \cdot T} P\left(0, \frac{i}{n}\right)}. \tag{7}$$

Since the COFI time series is monthly and uses the first day of a given month (e.g., 7/1/2017), we construct a monthly time series of Svensson parameters from the daily parameter estimations of Gürkaynak et al. (2006) by defining a given COFI time series entry's parameters as the parameters estimated for the first date available for that month in the Gürkaynak et al. (2006) time series, e.g., since July 3, 1961 is the first data entry for July 1961, we use that date's estimated parameters as that month's estimated parameters. Monthly zero rates and par coupon rates are then obtained from this monthly time series of parameters using Eqs. (5), (6), and (7), where $n = 12$ in Eq. (7) because we are constructing monthly paying par coupon bonds.

2.1.2 COFI-tracking portfolio construction

As discussed previously in §2.1.1, on a given month, j, the liability portfolio consists of sets of $12 \cdot T_i$ outstanding par bonds for each tenor set T_i. The overall rate of a given tenor set on month j is the set's average par coupon rate, or

$$C_j^{T_i} = \frac{\sum_{k=j-12 \cdot T_i}^{j-1} c_k^{T_i}}{12 \cdot T_i}. \tag{8}$$

Taking again the 3-month originally issued bond set, its average rate for month j would be the average of the 3-month bonds issued on months $j-3$, $j-2$, and $j-1$. To be clear, the current month's par coupons are not included in these averages as the COFI represents a lookback on the previous month's cost of funds. Rolling monthly average coupon rates are derived in this fashion for different tenor sets, and after some experimentation, we chose the 3-month, and 2- and 10-year par bonds to construct a COFI-tracking portfolio. The rates for those are shown in Figure 2 versus the monthly effective Passmore and von Hafften (2017) COFI.

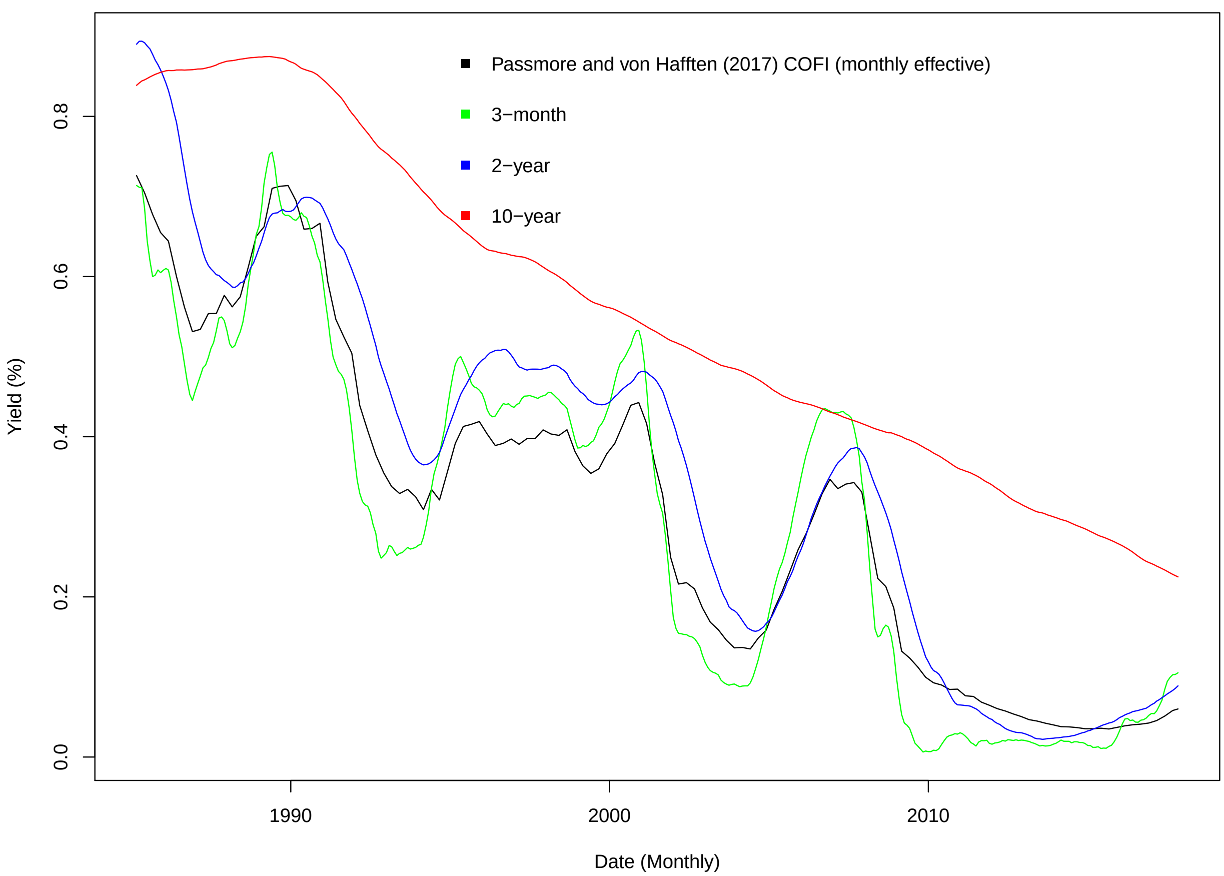

Figure 2: The 3-month and 2- and 10-year rolling average par bond coupons and the Passmore and von Hafften (2017) COFI (monthly effective), monthly from 3/1/1985 to 11/1/2017.

Portfolio weights of the tenor sets of par bonds are then obtained from a constrained linear least-squares regression of average par bond coupons of the tenor sets on the monthly effective Passmore and von Hafften (2017) COFI using the entire time series from 3/1/1985 to 11/1/2017 inclusive. The constrained regression is specified by

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \|Ax - b\|_2^2 \\ \text{subject to} \quad & \sum_{i=2}^{4} x_i = 1, \\ & 0 \le x_i \le 1, \quad i = 2, 3, 4, \end{aligned} \tag{9}$$

where

$$A_{393,4} = \begin{bmatrix} 1 & C_{393}^{3m} & C_{393}^{2y} & C_{393}^{10y} \\ 1 & C_{392}^{3m} & C_{392}^{2y} & C_{392}^{10y} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & C_{1}^{3m} & C_{1}^{2y} & C_{1}^{10y} \end{bmatrix}$$

and

$$b_{393,1} = \begin{bmatrix} COFI_{393} \\ COFI_{392} \\ \vdots \\ COFI_{1} \end{bmatrix},$$

where the subscripts $\{1, \ldots, 393\}$ within A and b represent the 393 months of data from 3/1/1981 to 11/1/2017 inclusive. Solving the constrained regression described in Eq. (9) results in an intercept coefficient and portfolio weights that are all significant, are shown in Table 1 with a R-squared value of 98.8%.[6] The regression's fit is shown in Figure 3.

[6]The minimization in Eq. (9) is a Quadratic Programming (QP) problem and can be solved by various standard convex optimization packages, where we use the R package `quadprog` by Weingessel (2013).

	Dependent Variable
	COFI (Monthly Effective)
3-month (b_2)	0.4814^{***}
	(0.0138)
2-year (b_3)	0.2184^{***}
	(0.0183)
10-year (b_4)	0.3002^{***}
	(0.0150)
Intercept (b_1)	-0.0008^{***}
	(0.00004)
Observations	393
R^2	0.9881
Residual Std. Error	1.99×10^{-5} (df = 387)
Note:	$^{\cdot}$p<0.1; *p<0.05; **p<0.01; ***p<0.001

Table 1: Summary table of the matching portfolio constrained least-squares regression on COFI (monthly effective).

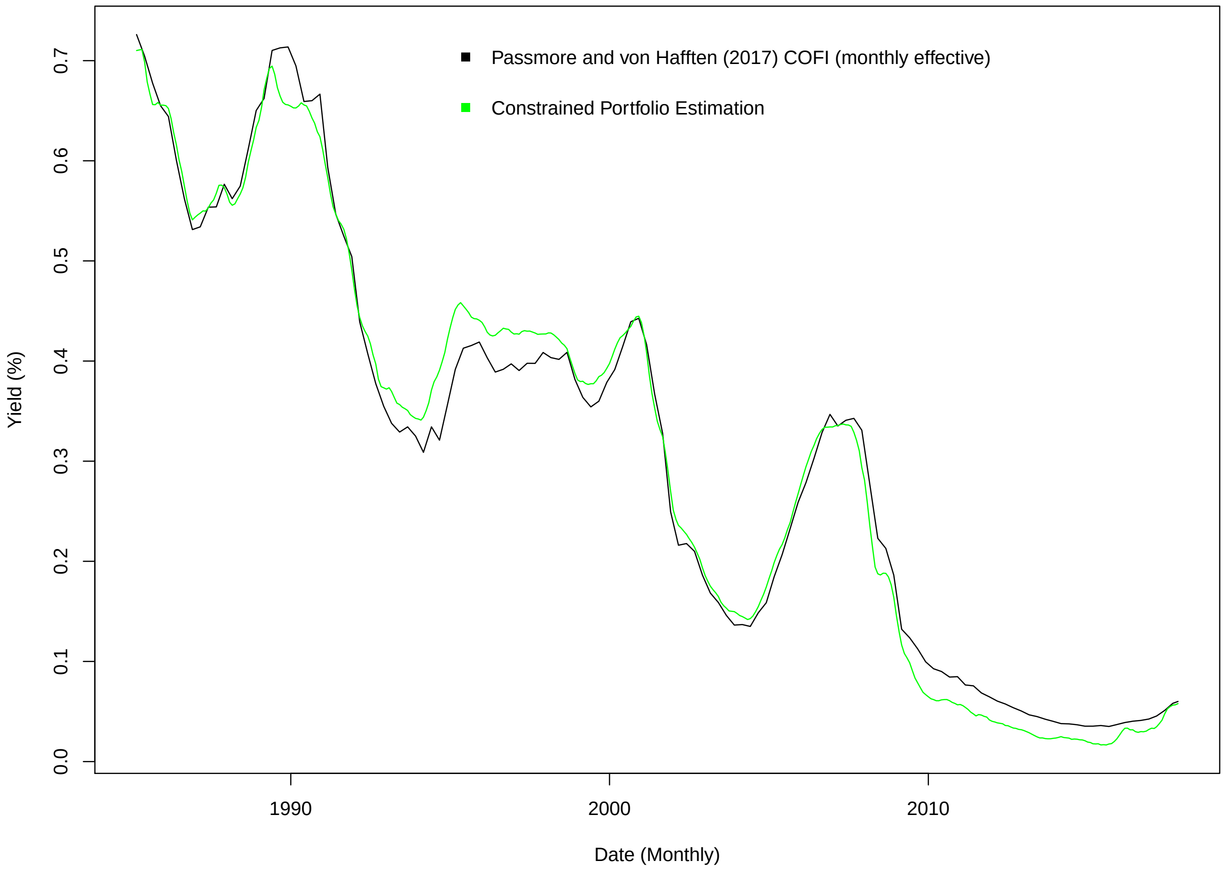

Figure 3: Matching portfolio constrained least-squares fit versus COFI (monthly effective), monthly from 8/1/1986 to 11/1/2017.

While the interest rate on the constructed portfolio tracks COFI fairly closely, there are a few periods when it deviates. There are two potential reasons for the deviations. First, the two assumptions made to make the actual construction of the portfolio feasible cannot always be true. In the actual index, both the weights on individual issues in each tenor set and the weights on each tenor set in the portfolio must vary over time but are not allowed to vary in our matching portfolio. The second potential source of deviations is the unmodeled optionality in some of the bank liabilities included in the COFI. Because of switching costs and borrower inattention, banks have the opportunity to delay changing deposit rates in response to market rate changes. (See Driscoll and Judson (2013) and Hawkins and Arnold (2000).) As a result, deposit rates, and therefore the COFI, will tend to track market rates when they decline but lag them when rates increase.

To investigate whether this optionality affects the COFI, we added an additional variable, which we call the 3-month lookback variable, to the regression presented earlier, while maintaining the requirement that the coefficients for the three tenor sets sum to one:

$$\max\left(C_j^{3m} - C_{j-5}^{3m}, 0\right).$$

If bank deposit rate options affect the COFI, then the coefficient of this variable should have a negative and significant coefficient, and its addition should improve the fit of the model. The results for this model are presented in Table 2, and, as can be seen, both of these criteria are met. Further, we show in Figure 4 the fit of the estimation with and without this rate option, where the model including our proxy for the deposit rate option has a somewhat closer fit.

	Dependent Variable
	COFI (monthly effective)
3-month (b_3)	0.4116^{***}
	(0.0159)
2-year (b_4)	0.5430^{***}
	(0.0204)
10-year (b_5)	0.3111^{***}
	(0.0144)
$\max\left(C_j^{3m} - C_{j-5}^{3m}, 0\right)$ (x_2)	-0.4166^{***}
	(0.0508)
Intercept (b_1)	-0.0007^{***}
	(0.00004)
Observations	393
R^2	0.9901
Residual Std. Error	1.65×10^{-5} (df = 387)
Note:	$^{\cdot}p<0.1$; $^{*}p<0.05$; $^{**}p<0.01$; $^{***}p<0.001$

Table 2: Summary table of constrained least-squares regression of matching portfolio with 3-month lookback variable on COFI (monthly effective).

Although the second regression clearly demonstrates that the COFI is affected by the rate options on deposits held by banks, we do not incorporate this feature in our model of the index because there are no similar options traded in the market for which pricing data is available. Banks might be able to improve on our model by incorporating their short-term deposit rates in place of the 3-month rate we use. However, the major features of a match-funded liability portfolio for the COFI can be illustrated based on the model in Table 1.

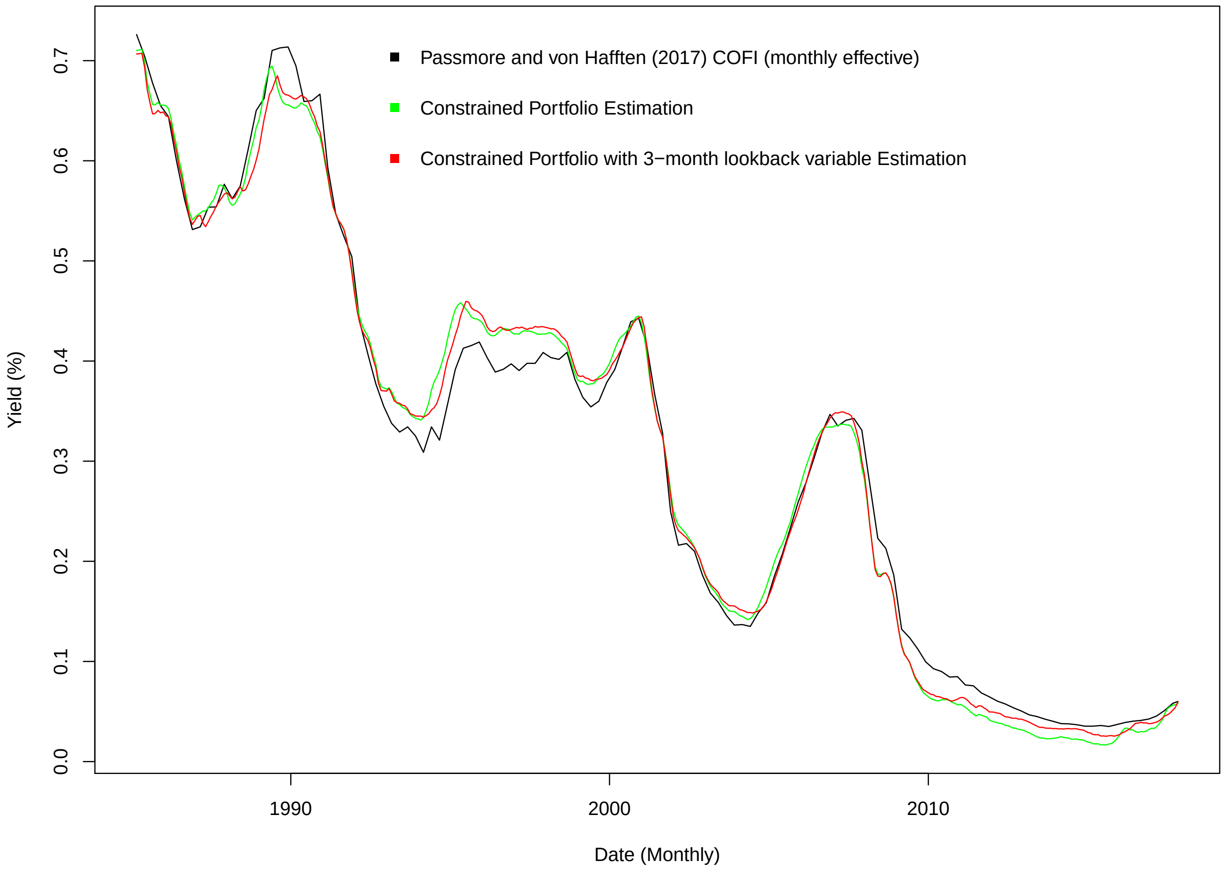

Figure 4: Matching portfolio with and without a 3-month lookback variable fits versus COFI (monthly effective), monthly from 8/1/1986 to 11/1/2017.

2.1.3 Portfolio valuation

The present value of the liability portfolio in the j-th month can be determined using the portfolio weights, b, and the rolling sets of T_i tenor par bonds, and the discount factors obtained in §2.1.2 and §2.1.1, respectively.[7] As discussed previously, in a given month, j, the liability portfolio is composed of $12 \cdot T_i$ outstanding par bonds for each tenor T_i. Consider a unit amount of total liability, such that b_i would be assigned to each set of T_i bonds, and each of the $12 \cdot T_i$ bonds within each i-th set would be allocated $\frac{1}{12 \cdot T_i}$-th of b_i. Since the present value of a given bond is the sum of its discounted future cash flows, the present value of the liability portfolio in the j-th month is the portfolio weighted sum of the discounted principal and coupon interest cash flows from all of the outstanding sets of T_i tenor bonds. Specifically, the portfolio value in month j is

[7]The regression in Table 2 has a small intercept term, suggesting that banks are able to create liabilities at slightly lower rates than the US Treasury. This difference would be incorporated into banks pricing decisions, but for the purpose of valuing the COFI portfolio, we do not make the intercept adjustment and use discount rates from Treasury rates as described earlier.

$$PV_j^{\text{Portfolio}} = b_1 + \sum_{i=2}^{4} \frac{b_i}{12 \cdot T_i} \cdot PV_j^{T_i}, \tag{10}$$

where

$$PV_j^{T_i} = c_{j-12 \cdot T_i}(T_i) + \sum_{k=j+1}^{j+12 \cdot T_i} \left[P(j,k) + c_{k-12 \cdot T_i}(T_i) \sum_{l=j}^{k} P(j,l) \right], \tag{11}$$

where $c_y(T_i)$ is the y-th month's T_i tenor par bond coupon with tenors $T_{2,\ldots,4} = \{0.25, 2, 10\}$-years. Using Eq. (10), the portfolio's present value (PV) over the estimation window was obtained and is shown in Figure 5. Unsurprisingly, the PV varies over time, indeed sometimes going below par. This is evidence that there is no way to guarantee that the liability matching a COF-indexed loan can be either originated or retired at par. This match funding problem can also be observed through the lens of portfolio duration (i.e., the weighted average of future payment times of a portfolio of securities). In particular, we show the Macaulay duration of the portfolio for each month in the estimation window in Figure 6. Like the PV, the portfolio duration varies over time, in this case it is between 0.925 and 1.522 years with an upward trend over the years. Analogous to the deviation from par issues exhibited in the PV, such variable duration would complicate asset/liability management duration matching.

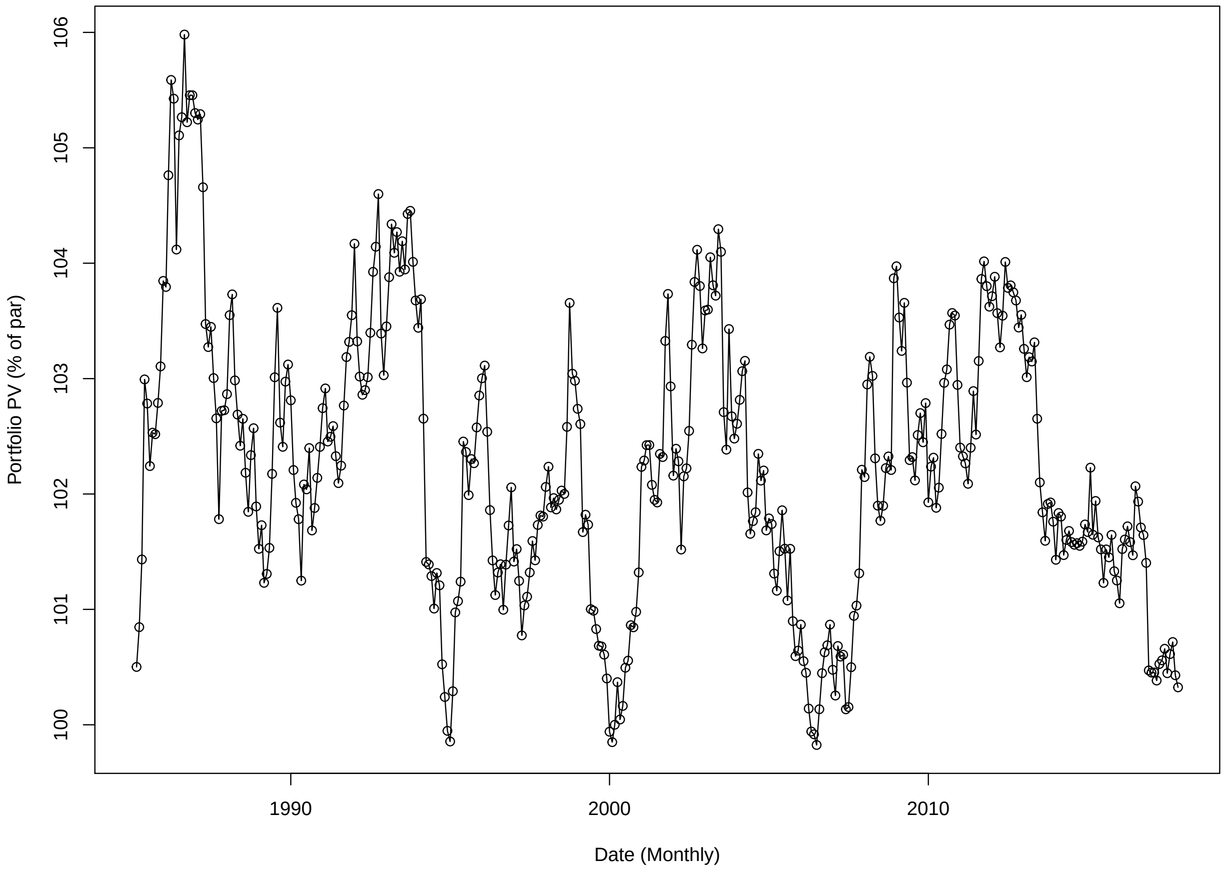

Figure 5: Matching portfolio present value, monthly from 3/1/1985 to 11/1/2017.

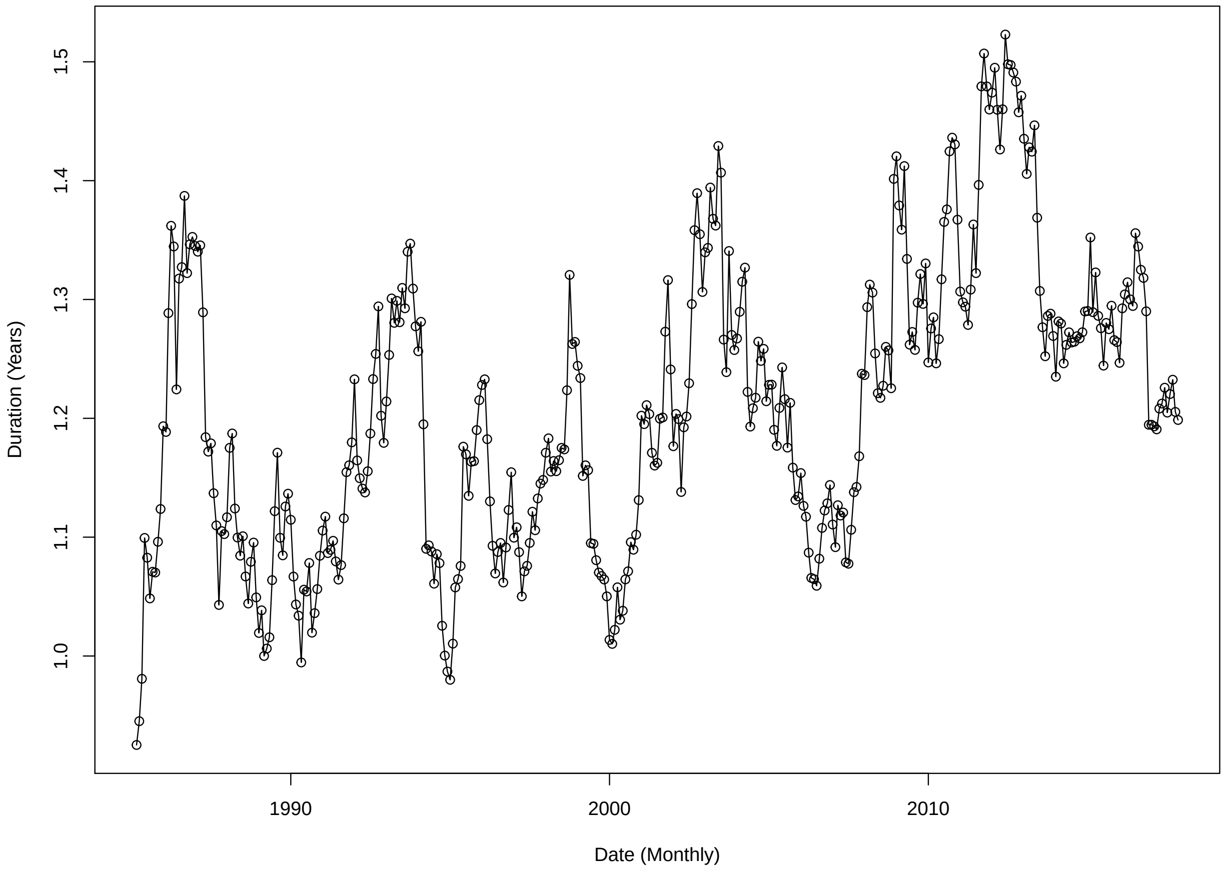

Figure 6: Matching portfolio Macaulay duration, monthly from 3/1/1985 to 11/1/2017.

3 Match funding fixed-term COFI assets

As described previously, the standard way that banks determine the rate they will offer on a loan starts with the construction of a notional liability whose principal cash flows will match those of the loan. While it is not necessary that the bank actually create this matched liability, its pricing for the loan should, for the reasons discussed earlier, be based on the interest rate for the matching liability. So while there are many other potential adjustments (for profit, risk, servicing costs, etc.), the pricing for a 3-year fixed-rate bank loan will be based on the interest rate the bank would need to pay to issue a new 3-year liability with the same principal repayment schedule at par. Applying this approach to COF-indexed loans immediately encounters a problem: The matching portfolio is not usually at par. From Figure 5, for brief periods, the COFI-matching portfolio was close to par, but generally its value has been above par from 1985 to 2017, reflecting the fact that interest rates have generally declined in that period. When the matching portfolio is at a premium, the bank would receive more than face value if it issued all the bonds in the portfolio. In other words, the bank could fund the loan with a matching portfolio whose face value was less than the loan amount. As a result, its actual interest expense would be less than the COFI rate. Of course, in those few periods when the matching portfolio was valued below par, the bank would need to issue bonds for more than par value to raise enough to fund the loan, and its interest expense would be higher than the COFI rate. Competition for borrowers when the COFI-matching portfolio is valued above par, and profitability requirements when the COFI-matching portfolio is below par, should force banks offering COF-indexed loans into incorporating the portfolio premium or discount into their pricing. This could be done by amortizing the premium or discount over the term of the loan and adding that adjustment to the price. So, for example, if the COFI-matching portfolio were at a 5% premium and a five-year loan was to be made, assuming the appropriate discount rate was 3%, then amortization of the premium would allow the bank to lower the rate charged the borrower by 1.08%. While there are additional problems using the COFI-matching portfolio as the matched maturity marginal cost of funding, this issue alone demonstrates that pricing loans at a constant spread to the COFI rate would not be feasible.

While amortization could resolve the problem of the market value of the COFI-matching liability portfolio differing from par at origination, it cannot resolve the problem when the loan is repaid. Notionally, each time principal is repaid on the COFI loan, a matching face amount of the liability portfolio must be repurchased. But while the mortgage repayments will be credited at par, the matching COFI liability will be repurchased at the then-current market price. So if the value of the COFI-matching liability portfolio is 102% of par in a particular month, and $100 of principal is repaid, then the bank would need to provide an additional $2 to repurchase the matching amount of the notional liability portfolio. And since the mortgage principal payments will be in the future, the deviations of the value of the COFI-matching liability portfolio from par will be unknown as of loan origination when the mortgage must be priced.

To measure the magnitude of the pricing problem created by variability of the value of the COFI portfolio, we performed the following experiment. Using historic data, we calculated the present value of COF-indexed loans to be issued in each month between March 1985 and November 2012. These loans were assumed to have an amortization period of 30 years but a term of five years, a structure chosen as an approximation of a prepayable mortgage. After calculating the present value for each month's loan, we derived the spread adjustment to the rate that would bring the present value to par. Figure 7 plots that spread adjustment (in basis points) over the period. Note that this is an ex-post calculation that would not be available to a bank making a loan and as such represents an unavoidable random element in the pricing of this COF-indexed loan. The ex-post spread varies between +5 and -20 basis points and is quite volatile.

The problem of the unknown future value of the COFI-matching portfolio is an intractable one with respect to loan pricing based on matched maturity marginal cost of funding. So, the fact that the COFI-matching portfolio need not be at par at either loan origination or maturity implies that profitability can vary considerably. And the fact that the value of the COFI-matching portfolio at maturity will not be known at origination means that the COFI-loan profitability cannot be determined at origination. This is a potentially serious problem for banks seeking to make rational pricing decisions for COFI ARMs vis-à-vis other types of loans they offer.

A bank might attempt to manage this uncertainty by assuming that, over a long enough period, the impact of variations in the value of the COFI-matching portfolio would cancel out so that long-term average profitability for a particular pricing strategy could be determined for COFI-based loans, although profitability for the next loan originated could not. The weakness of that approach is that both borrowers and bank loan officers whose incentive compensation is based on volume of loans originated would be motivated to make more COF-indexed loan when the product appeared cheap (when market rates had recently risen) than when it appeared expensive (when market rates had recently fallen). As a result, profitability realized over the long-term would be lower than the long term average profitability measured historically, even if history exactly repeated itself.

Another potential way to mitigate the value uncertainty of the COFI-matching portfolio would be to only use the COFI for long-term loans with a fixed amortization schedule. On such a loan, the error described previously could still occur, but, over a long enough period, perhaps there would be deviations from par in both directions so that the aggregate error, expressed as a spread adjustment, would be less volatile. Of course, ensuring a long loan term would imply not allowing prepayment. Not allowing prepayment is, in fact, a feature of the mortgage design proposed by Passmore and von Hafften (2017), although they propose it for reasons unrelated to the use of the COFI. They propose using the index in a mortgage design whose required payment is structured to amortize the loan more quickly than implied by the COFI. Particularly in a declining rate environment, this considerably shortens the effective term of the mortgage. So their mortgage design could not be relied upon to deliver the long maturity that could mitigate the uncertainty of COF-indexed mortgage profitability.

While Passmore and von Hafften (2017) propose a nonprepayable mortgage, Hancock and Passmore (2016) do not, and historically, COFI ARMs, like almost all American mortgages, have been prepayable. This makes some consideration of the prepayment option relevant. Prepayment will be an issue in pricing to the extent that the borrower's use of it significantly reduces profitability to lenders. Since the likelihood borrowers will prepay and the loss to the lender when they prepay, are both related to the size of the mortgage's deviation from par, the cost of the option to the lender is determined by the volatility of the instrument underlying the prepayment option. The underlying for a COFI ARM is the COFI-matching portfolio. That, from Figure 5 seems sufficiently volatile to induce prepayments.

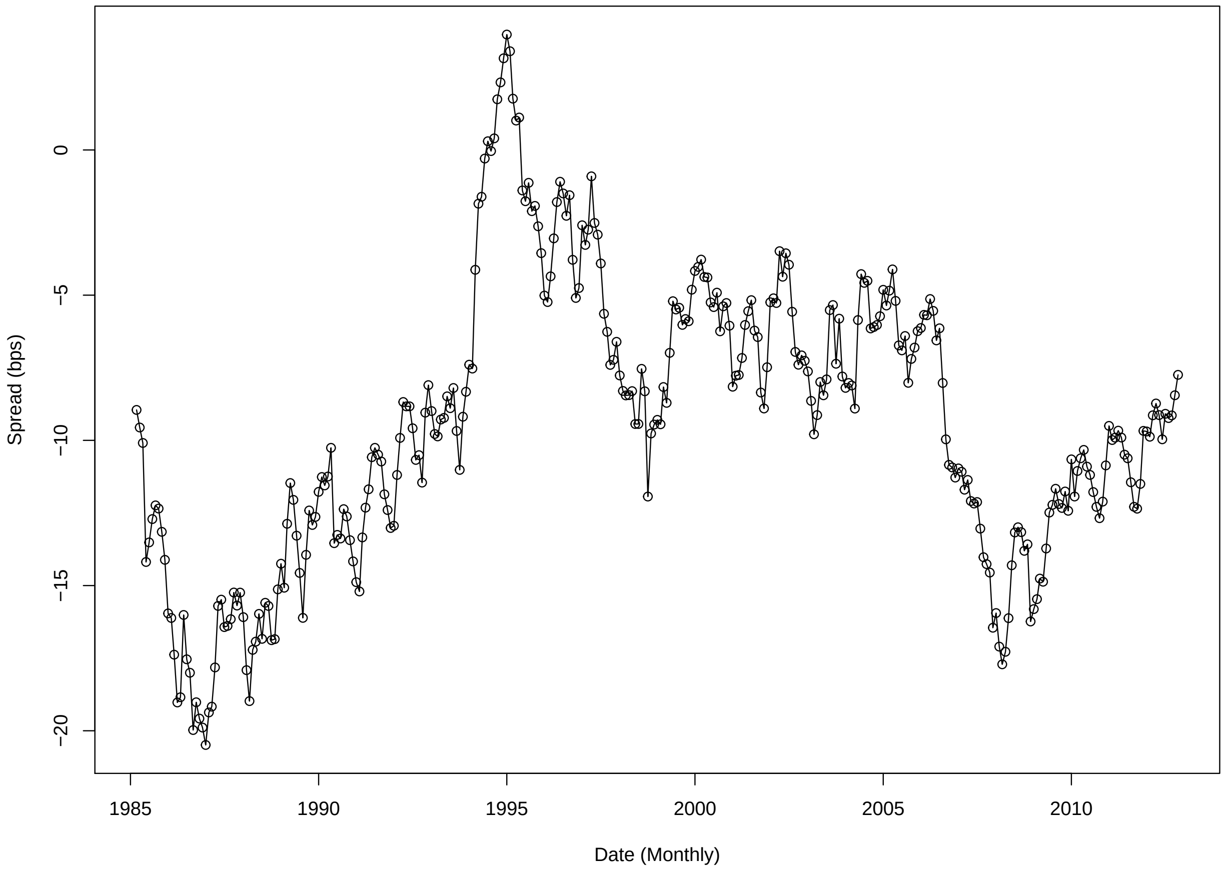

Figure 7: Spreads, monthly from 3/1/1985 to 11/1/2012.

On the other hand, it seems likely that the prepayment option on a COFI ARM would be less valuable than that on a 30-year fixed-rate mortgage. From Figure 6, the COFI-matching portfolio has a duration of 1 to 1.5 years. For comparison, Table 3 presents Macaulay durations for amortizing loans with various terms and various coupon interest rates. Comparing the table with the duration of the COFI-matching portfolio shows that, in duration terms, it matches a 2- to 3-year amortizing loan. Thirty-year amortizing mortgages have a much longer duration and so will have more volatility in value, and, as a result, a more valuable prepayment option [8] Therefore a COFI ARM should be seen as intermediate between fixed-rate mortgages and ARMs based on market indices in terms of prepayment risk to lenders.[9]

[8] It could be argued that the underlying instrument for a fixed-rate mortgage prepayment option is not a 30-year amortizing loan, but a 30-year amortizing loan whose term is adjusted for prepayments not motivated by interest rates. Such an instrument would have a shorter duration but is unlikely to have duration as short as 1.5 years.

[9] Current ARMs often include extended initial periods, interim rate caps and floors, and lifetime interest rates that have the potential to increase the value of the prepayment option in some circumstances.

Annual Rate (%)	30 Years	3 Years	2 Years
	Duration (Years)		
12	7.56	1.45	1.00
9	9.01	1.47	1.01
6	10.78	1.50	1.02
3	12.82	1.52	1.03

Table 3: Amortizing loan durations.

4 Conclusion

This paper has explored the financial characteristics of loans whose coupon rate is indexed to a COF measure. These derive from the fact that the liability portfolio from which a COFI is derived frequently has a nonpar value. We demonstrate the difficulties that the financial management of such loans presents. In essence, offering a COFI-based product introduces a new and unique pricing risk whose size is difficult to estimate. This will be a particular problem for depository institutions that use matched maturity marginal COF pricing for decentralized management of profitability and risk.

But do the issues raised previously mean that COFI-based mortgages are infeasible or a worse choice than fixed-rate mortgages or adjustable-rate mortgages based on a market index? Clearly COFI-based mortgages are feasible because they once had a large share of the American mortgage market.

Our analysis places COFI-based mortgages in a middle position, for both borrowers and lenders, between traditional fixed-rate mortgages on one side and standard adjustable-rate mortgages on the other. As such, they could very well represent the preferred alternative for some borrowers and some lenders. However, their viability will depend upon the amount banks will charge to bear the risks inherent in COF-indexed assets. Based on that characterization, the COFI does not represent a uniquely preferable mortgage feature.

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