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Abstract

Larger firms (by sales or employment) have higher leverage. This pattern is explained using a model in which firms produce multiple varieties and borrow with the option to default against their future cash flow. A variety can die with a constant probability, implying that bigger firms (those with more varieties) have lower coefficient of variation of sales and higher leverage. A lower risk-free rate benefits bigger firms more as they are able to lever more and existing firms buy more of the new varieties arriving into the economy. This leads to lower startup rates and greater concentration of sales.

Keywords: Startup rates, leverage, firm dynamics

JEL Codes: E22 E43 E44 G32 G33 G34

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1 Introduction

In this paper, we document that among public firms leverage is increasing with firm size, consistent with similar recent findings regarding private firms (Dinlersoz, Kalemi-Ozcan, Hyatt, and Penciakova (2018)). We propose a model of firm dynamics with borrowing and default that is consistent with this fact. We then explore the model's potential to explain the decline in the business startup rate and the rise in business concentration since the late 1990s as a response to a fall in the return on safe assets.

In our model, the positive relationship between leverage and firm size is explained in the following way. First, business owners are assumed to be more impatient than lenders and therefore seek to borrow against the future cash flow of their companies. Second, firms manage different numbers of varieties (of products), and each variety is assumed to be subject to an independent extinction shock. This setup implies that larger firms (those with more varieties) have a lower coefficient of variation of sales and employment, which allows them to have higher leverage.[1] The lower dispersion of employment among larger firms that our model generates is consistent with the data reported in Davis, Haltiwanger, Jarmin, and Javier (2009).[2]

Firm entry and firm growth are driven by the arrival of ideas for new varieties. The ideas for new varieties occur to workers and, as such, the probability that an idea arises within a firm is proportional to the number of varieties owned by the firm. An idea has a fixed probability of success if it is implemented independently in a startup while its probability of success is drawn from a uniform distribution if implemented in the incumbent firm. A high probability of success can be thought of as a better match between the idea and the firm in which it arose. There will be a firm-size and firm-debt dependent threshold probability of success above which the firm will purchase the idea and implement it. The key benefit of implementing an idea in an existing firm is that, once the firm has successfully absorbed the new variety, it can borrow against a greater fraction of the future cash flow of the new variety compared to a startup. One implication of this is that larger firms will buy more ideas than smaller firms and, so, larger firms will spawn fewer

[1] The idea that leverage of firm is negatively related to volatility of the firm's cash flow is well known in finance; see for instance Leland (1998).

[2] For manufacturing firms in COMPUSTAT, Stanley, Amaral, Buldyrev, Havlin, Leschhorn, Maass, Salinger, and Stanley (1996) show that the standard deviation of sales declines with firm size. However, their findings are an underestimate of how rapidly volatility of sales growth shrinks with size because they limit attention to firms that survive from one year to the next. In contrast, Davis, Haltiwanger, Jarmin, and Javier (2009) take exit into account and show that the decline in the exit rate with size is a very important reason volatility of employment declines with firm size.

startups than smaller firms, which is consistent with evidence reported in Elfenbein, Hamilton, and Zenger (2010) and Gompers, Lerner, and Scharfstein (2005).

In our explanation of the decline in the startup rate and of the rise in business concentration since the late 1990s, the driving force is the decline in the risk-free interest rate over the same period. In the model, a decline in the risk-free interest rate benefits larger firms more than smaller firms, as larger firms are able to lever more against their future cash flow. This makes larger firms more willing to buy new ideas and implies both a lower startup rate and a higher fraction of total sales in bigger firms. More specifically, a decline in the risk-free rate decreases the threshold probability of success for which it becomes profitable to implement the idea in the incumbent firm. Given the observed decline in the risk-free rate between 1997 and 2015, we are able to match the observed decline in the startup rate over the same period by choosing the dispersion of the uniform probability distribution of success of an idea within the incumbent firm.

A crucial element of our model is the responsiveness of leverage to firm size. We obtain this information for the set of COMPUSTAT firms by regressing firm leverage (i.e., the ratio of net debt to total value of a firm) on the logarithm of firm sales (and other controls). Our panel regressions imply that firm leverage rises in a range between 2.4 to 2.6 percentage points for every doubling of firm size. Our cross-sectional regressions imply that the leverage rises in a range between 0.9 to 2.6 percentage points for every doubling of firm size. In the model, this responsiveness is not targeted but our model implied response lies within these range of estimates.

Regarding other implications, the model generates a rising profile of survival probabilities with age as in the data. In addition, a decline in interest rates causes incumbent firms to absorb more of the new varieties and, hence, lowers their probability of losing all varieties and exiting. This implies that survival probabilities conditional on age should rise over this period, and this is observed in the data as well.

Our paper is related to several strands of the recent literature on firm dynamics. First and foremost, it is related to the literature that has documented and studied the secular decline in the startup rate in the U.S. (Decker, Haltiwanger, Jarmin, and Miranda (2014), Hathaway and Litan (2014b), Decker, Haltiwanger, Jarmin, and Miranda (2016), among others). The reasons for the decline remain an active area of research with no settled answers. Hathaway and Litan (2014a) list several factors, including slowing population growth, increasing business consolidation, and rising burden of regulation and taxes. In terms of this general categorization of causes, our paper falls in the second category, namely, rising business consolidation. What we add to this perspective is the role of the decline in the risk-free rate in encouraging the growth of existing firms at the expense of startups. Karahan, Pugsley, and Sahin (2018) and Hopenhayn, Niera, and Singhania (2018) make the case for the role of declining population growth, and Neira and Singhania (2017) and Kaymak and Schott (2018) make the case for the role of changes in the corporate tax rate.

Second, Autor, Dorn, Katz, Patterson, and van Reenen (2017) show that in all six major industries (Manufacturing, Finance, Services, Utilities and Transportation, Retail, and Wholesale Trade) business concentration has risen since the mid-to-late 1990s, when measured by the share of sales accounted by the top 4 (or top 20) firms relative to total sales in their 4-digit industry.[3] Relatedly, there is work arguing that markups have risen in U.S. industries (Gutierrez and Philippon (2017) and De Loecker and Eeckhout (2017)). While our model generates a higher market share of large firms over time, we have not embedded a feature where a bigger market share leads to higher markups.

Third, many papers have highlighted the decline in real interest rates. Some have studied the causes of the decline and others have studies its consequences. In the first category are studies by Caballero, Farhi, and Gourinchas (2008), Mendoza, Quadrini, and Ríos-Rull (2009), Eichengreen (2015), Del Negro, Giannone, Giannoni, and Tambalotti (2017), Farhi and Gourio (2018), among others. The general view that emerges from these studies is that the decline in real interest rates is largely due to a rise in the convenience yield (i.e., a rise in the premium placed on safety and liquidity) that is unrelated to other determinants of real interest rate such as rate of growth of business sector productivity.[4] Consistent with this, we treat the model decline in the risk-free rate as stemming from a change in the preferences of lenders, specifically as a change in their degree of impatience. In the second category, Gopinath, Kalemli-Ozcan, Karabarbounis, and Villegas-Sanchez (2017) argue that, in a world with financial frictions, the decline in interest rates led to an increase in the misallocation of capital and lower productivity in Southern Europe. In our model as well, the decline in the risk-free rate generates an output loss because more new ideas get implemented in existing firms where the success probability is lower than in an independent startup.

[3]This rising concentration is most pronounced in sales but it is present for employment as well.

[4]Del Negro, Giannone, Giannoni, and Tambalotti (2017) attribute the majority of the 2 percentage point decline in the real yield on 10-yr U.S. Treasuries since the late 1990s to a rise in the convenience yield of safe assets.

On the theory side, our model relates to papers that focus on an inventor's decision to sell a new idea to an existing firm or commercially develop the idea via a startup (Anton and Yao (1994), Anton and Yao (2002), Chatterjee and Rossi-Hansberg (2012), Zábojník (2016)). In contrast to these studies we incorporate borrowing and financial frictions in the form of bankruptcy and examine the implications of this friction for the sell/startup decision.

There is an extensive macroeconomic literature on firm entry and firm dynamics (Jovanovic (1982), Hopenhayn (1992), Cooley and Quadrini (2001), Luttmer (2007), among others) to which our paper is connected. In all these studies, a firm is identified with a technology so that a new idea, if it goes into production, is automatically a new firm. In contrast, in our model new ideas occur to people and the inventor chooses the organizational form in which to implement his or her idea (through an incumbent firm or a startup).

Our paper contributes to the quantitative-theoretic literature on firm dynamics in the presence of equilibrium default risk (Cooley and Quadrini (2001), Arellano, Bai, and Zhang (2012), Arellano, Bai, and Kehoe (2016), Corbae and D'Erasmo (2017)) and borrowing constraints (Khan and Thomas (2013)). Khan and Thomas (2013) study how a large shock to the economy's financial sector can propagate through the economy; Arellano, Bai, and Zhang (2012) study how financial development affects firms' financing choices and economic growth; Arellano, Bai, and Kehoe (2016) examine how a positive shock to the volatility of sales can cause firms whose borrowings are subject to default risk to reduce production (so as to reduce default risk); and Corbae and D'Erasmo (2017) examine how proposed alterations to the U.S. corporate bankruptcy law (Chapters 7 and 13) might impact the long run frequency (and efficiency) of firm bankruptcies.

Finally, regarding the positive relationship between leverage and firm size in the U.S., our results reaffirm the findings reported in Rajan and Zingales (1995) for COMPUSTAT and more recently confirmed in Frank and Goyal (2009) and Dinlersoz, Kalemi-Ozcan, Hyatt, and Penciakova (2018) (which extends the analysis to private firms). It is worth pointing out that the most well-known model of firm leverage in macroeconomics – Cooley and Quadrini (2001) – predicts that firm size and leverage are negatively related and that in the commonly used models of firm leverage in finance, a firm's capital structure is independent of firm size (the so-called homogeneity property).

2 Firm Leverage and Firm Size

The goal of this section is to document that large U.S. firms tend also to be more leveraged. To so do, we use the COMPUSTAT database for the years 1978-2015. Our sample consists of all nonfinancial and nonutilities firms that report in U.S. dollars.

Table 1: Variable Name and Description

Variable	Description
SALES/TURNOVER (Net)	Sales
LT	Total Liabilities
DLC	Debt in Current Liabilities - Total
DLTT	Long-Term Debt - Total
CSHO	Common Shares Outstanding
PRCC_F	Price Close - Annual Fiscal
CHE	Cash and Short-Term Investments
PPENT	Property, Plant and Equipment - Total (Net)
EMP	Employees
CEQ	Common/Ordinary Equity - Total
OIBDP	Operating Income Before Depreciation
AT	Assets - Total
lnSale	ln(SALES/TURNOVER (Net))
Book_Val	AT
Mkt_Val	AT + CHSO*PRCC_F - CEQ
Mkt Leverage Ratio I	(DLC + DLTT - CHE)/Mkt_Val
Mkt Leverage Ratio II	(LT - CHE)/Mkt_Val
Book Leverage Ratio I	(DLC + DLTT - CHE)/Book_Val
Book Leverage Ratio II	(LT - CHE)/Book_Val
Cap_Ratio	PPENT/Mkt_Val or PPENT/Book_Val
Profit_Ratio	OIBDP/Mkt_Val or OIBDP/Book_Val

The top panel in Table 1 reports the observed variables taken directly from COMPUSTAT. The bottom panel describes the constructed variables that appear in our regressions. Our constructions follow the norms in the finance literature. The book value of the firm is what is reported as total assets in the COMPUSTAT dataset. The market value of a firm is the book assets excluding common equity plus the market value of its common equity outstanding. We experiment with two alternative measures of a firm's leverage ratio. In the first measure, we include only liabilities that arise as a result of the firm's active borrowing net of cash (DLC + DLTT - CHE), and in the second measure, we use total liabilities net of cash (LT - CHE) as our measure of net liabilities. For our measure of leverage, we divide these net liability measures by either market value of assets or book value.[5] We define the capital ratio of a firm to be the ratio of the value of its tangible capital to either the market value of assets or book value. We define the profit ratio as the ratio of operating income before depreciation to either market value or book value of assets. For each year, we include only those firms for which the book value of assets, the market value of assets, and the value of sales is at least $1 million in 2015 dollars[6] and the values of debt and cash are nonnegative. In addition, for a given leverage measure, only firms with leverage ratio between $[-1, 1]$ are included.

Table 2:
Relationship Between Leverage Ratio and Firm Size (Panel)

Dep Var	Lev I	Lev II	Lev I	Lev II
	Market		Book	
lnSale	0.034	0.037	0.036	0.038
	(21.83)	(20.39)	(20.85)	(20.13)
Cap_Ratio	0.287	0.440	0.634	0.592
	(22.40)	(27.14)	(49.01)	(43.65)
Profit_Ratio	-0.030	-0.137	-0.010	-0.010
	(-3.08)	(-9.43)	(-2.04)	(-1.89)
Firm FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
Num of Obs	182,984	183,496	211,096	203,009
Num of Groups	18,580	18,605	20,937	20,582
R^2	0.15	0.18	0.18	0.16

Table 2 reports our main results. All regressions are panel regressions with fixed effects for each firm and fixed effects for each calendar year. For the denominator in Cap_Ratio and Profit_Ratio, we use Mkt_Val (Book_Val) if the dependent variable is market (book) leverage. The coefficient of

[5]The resulting leverage measures are commonly used in the literature. For instance, book leverage based on debt is used in Rajan and Zingales (1995), Hennessy and Whited (2005), and Lemmon, Roberts, and Zender (2008), among many others; book leverage based on total liabilities is used in Baker and Wurgler (2002); market leverage based on debt is used in Whited (1992), Rajan and Zingales (1995), and Strebulaev and Yang (2013); and market leverage based on total liabilities is used in Baker and Wurgler (2002) and Michaels, Page, and Whited (forthcoming). See Frank and Goyal (2009) for a discussion of the pros and cons of these different leverage measures.

[6]The GDP deflator is used to determine the equivalent nominal cutoff for earlier years.

interest is the one for firm size measured as logarithm of sales. This coefficient is positive, highly statistically significant, and roughly the same value across the four regressions. The magnitude of the coefficient implies that every doubling of firm size increases leverage by 2.4 to 2.6 percentage points. Regarding the other coefficients, Cap_Ratio is highly significant across the regressions as well and positive as one would expect; the Profit_Ratio is generally not significant across the regressions.

Table 3:
Relationship Between Leverage Ratio and Firm Size (Cross-Section)

Dep Var	Lev I	Lev II	Lev I	Lev II
	Market		Book	
lnSale	0.013	0.025	0.021	0.038
	(49.22)	(79.20)	(62.77)	(109.49)
Cap_Ratio	0.293	0.400	0.486	0.378
	(107.72)	(127.90)	(126.06)	(91.98)
Profit_Ratio	0.004	-0.127	-.010	-0.012
	(0.97)	(-28.17)	(-16.90)	(-18.60)
Subindustry FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
Num of Obs	177,678	178,178	203,834	195,999
R^2	0.26	0.31	0.32	0.30

Table 3 reports the cross-sectional results. For these regressions, we include GICS subindustry fixed effects as well as time fixed effects. The coefficients on log sales are again highly statistically significant but generally somewhat lower than the coefficients reported in the previous table. As a point of comparison, our estimate of the effect of size on book leverage (Lev I, Book) of 0.021 is close to the value of 0.0178 in a similar regression reported in Dinlersoz, Kalemi-Ozcan, Hyatt, and Penciakova (2018, Table 4, first regression) for listed firms. For private firms, they find a higher coefficient of 0.0281 (Table 4, second regression).

In Appendix A we report results for regressions identical to the ones in Table 2 except that the logarithm of employment is used as a measure of firm size. The coefficients on size are very similar whether employment or sales are used as a measure of size. In Appendix A we also report the results of regressions where the dependent variable is transformed to be unbounded (as required by OLS). Specifically, letting ℓ denote either Lev I or Lev II, the transformed leverage measure is

$$\ln\left(\frac{\ell/2+1/2}{1-(\ell/2+1/2)}\right).$$

Running the panel regressions in Table 2, we find that the coefficients on lnSale range from 0.076 to 0.103 and remain highly significant.[7]

3 Model

To interpret these findings we now turn to a model. Time is discrete and a period should be thought of as a month. The state vector of a firm is the number of varieties owned by the firm at the start of a period, denoted $K \in \mathbb{K} = \{1, 2, 3, \ldots, K_{\max}\}$, and the firm's net debt position, denoted B. B can be positive (debt) or negative (assets). We will treat B as discrete and assume it takes values in the finite set $\mathbb{B}$, a sufficiently fine discrete approximation of an interval on the real line. The aggregate state of the economy is a distribution over $\mathbb{K} \times \mathbb{B}$. The mass at any point is the measure of firms on that point.

3.1 Creation of New Varieties

At the start of the period, with probability $p(K)$ the firm is confronted with the decision to purchase a new variety. We assume that

$$p(K) = \rho K, \tag{1}$$

where $\rho K_{\max} \leq 1$.[8] The form of the function captures the notion that ideas for new varieties occur to people working in the firm and employment is proportional to the number of varieties. If it chooses to purchase the new variety, with some probability s the new variety can be successfully produced. The value of s is drawn from a uniform distribution with support $[s_{\min}, 1]$. The closer s is to 1, the more likely it is that the idea will succeed and add to the portfolio of varieties owned by the firm. We can view s as index of quality of the match between the idea and the firm.

[7] For these regressions we exclude firm-year observations for which the leverage measure is exactly either 1 or -1.

[8] In the quantitative section we assume that $\rho K_{\max} \leq 2$ and allow firms for whom $\rho K > 1$ to get the option to purchase one variety for sure and the option to purchase a second variety with probability $\rho K - 1$ (see Appendix B for details).

If the firm chooses to not purchase the idea, or if the firm cannot purchase another idea because $K = K_{\max}$, we assume that the idea is implemented in a startup (equivalently, in a spinoff) and the probability of success is a constant σ.

There is a surplus from implementing the new variety in the incumbent firm if

$$s[W(K+1,B) - W(K,B)] \geq \sigma W(1,0). \tag{2}$$

Here $W(N,B)$ is the value of a firm that owns N varieties and has debt B. The l.h.s. is the expected gain to the incumbent firm if it absorbs the new variety and the r.h.s. is the gain to a startup if it absorbs the new variety (a startup has no existing debt). If the surplus is nonnegative, we assume that the new variety is absorbed by the incumbent and the inventor gets $\sigma W(0,1)$ through shares in the combined firm.[9] Therefore, post purchase, the value to the original owners of the firm rises by

$$sW(K+1,B) + (1-s)W(K,B) - \sigma W(1,0).$$

Let $\mathbb{M}(K,B,s)$ be an indicator function that takes the value 1 if the surplus from an acquisition is nonnegative and 0 otherwise. Then, $Z(K,B)$ is the value of the firm at the start of the period:

$$\begin{aligned} Z(K,B) &= [1-p(K)]W(K,B) \; + p(K)\times \\ &\int_{s_{\min}}^{1} [\max\{W(K,B), sW(K+1,B) + (1-s)W(K,B) - \sigma W(1,0)\}]\, \mathrm{U}(ds). \end{aligned}$$

Here $\mathrm{U}(ds)$ is short-hand for Uniform density over $[s_{\min}, 1]$.

For future reference, let $s^*(K,B)$ be the threshold value of s that solves (2) with an equality if the solution is in $[s_{\min}, 1]$. If the solution falls outside the support of s, we set $s^*(K,B) = 1$ if the solution exceeds 1 and set $s^*(K,B) = s_{\min}$ if the solution is less than $s_{\min}$.

3.2 Destruction of Existing Varieties

Let N denote the number of varieties owned by the firm after it has made its purchase decision and after it is known if a new purchase is successful or not (N is either K or $K+1$). At this juncture, each variety in existence receives an extinction shock with probability ϕ. We denote by

[9] Alternatively we could have required that inventors be compensated in cash. In this case, 1-variety firms would have to accumulate cash holdings to purchase ideas and such firms may choose not to do so. Then all firms would be 1-variety firms and there would be no firm growth.

$x(N, K')$ the probability that a firm that has N varieties at the end of the first subperiod ends up with $0 \leq K' \leq N$ varieties at the start of the second subperiod:

$$x(N, K') = \frac{N!}{K'!(N-K')!}(1-\phi)^{K'}\phi^{N-K'}.$$

3.3 Default and Debt Decision

On the financial side, we impose two important constraints. First, we require that dividend payments be nonnegative (i.e., no equity infusion). This constraint is expressed as

$$\pi K' - B + q(K', B')B' \geq 0. \tag{3}$$

In the above, π is the free cash flow from each variety and, hence, $\pi K'$ is the total free cash flow of the firm in the current period. For $B' > 0$, the function $q(K', B')$ gives the price of a unit of debt issued by the firm on which the firm might default next period and so $q(K', B')B'$ is the revenue raised from bond sales.[10] Second, we impose that the probability of default on debt issued by a firm cannot exceed θ. This constraint will serve to capture the fact that small firms have limited access to capital markets.[11]

A firm cannot repay its obligations fully if (3) cannot be satisfied for any level of new debt B' that respects the constraint on the probability of default. In this situation there is default on existing debt B and the firm enters bankruptcy. To formalize this, let $G(K')$ be the highest revenue from bond sales consistent with default probability constraint. That is,

$$\begin{aligned} & G(K') = \max_{B'} q(K', B')B' \\ & \text{s.t.} \\ & d(K', B') \leq \theta, \end{aligned}$$

[10] For $B' \leq 0$, it is understood that $q(B', K')$ is simply $1/(1+r)$ and the product is the funds saved by the firm.

[11] Alternatively, to limit leverage, we could have assumed that defaulting has some costs. One complication with this is that when a 1-variety firm loses its variety and defaults the cost would have to be a cost borne by the erstwhile owner of the firm because the recovery rate for creditors is already zero and cannot be negative. Given that we don't model owners explicitly, we chose not to follow this approach.

where $d(K', B')$ is the probability of default on debt next period. Let $\overline{B}(K')$ solve:

$$\pi K' - \overline{B}(K') + G(K') = 0.$$

Then bankruptcy occurs if the inherited debt $B > \overline{B}(K')$ since there cannot be any B' for which the firm will be able to pay back all its debts and still satisfy the default probability constraint. In the event of bankruptcy, the debt owed to creditors is reduced to $\overline{B}(K')$. The value of a firm in bankruptcy is then given by:

$$\begin{aligned}
&V^D(K', B) = \max_{B'} \pi K' - \overline{B}(K') + q(K', B')B' + \beta Z(K', B') \\
&\text{s.t.} \\
&q(K', B')B' = G(K') \\
&d(K', B') \leq \theta.
\end{aligned}$$

If there is a unique B' that attains $G(K')$, it is also the B' that (trivially) solves the firm's optimization problem under bankruptcy: It is the only choice that is available to the firm (any other choice will either violate the nonnegativity constraint or the default probability constraint). We assume that bankruptcy allows owners to retain rights over the firm's future cash flow but observe that the firm's dividend payment in bankruptcy is exactly zero. Thus bankruptcy in our model is a reorganization rather than a liquidation. If the firm does not default, that is $B \leq \overline{B}(K')$, the firm solves

$$\begin{aligned}
&V^R(K', B) = \max_{B'} \pi K' - B + q(K', B')B' + \beta Z(K', B') \\
&\text{s.t.} \\
&\pi K' - B + q(K', B')B' \geq 0 \\
&d(K', B') \leq \theta.
\end{aligned}$$

If a firm loses all its varieties then its continuation value becomes zero since it loses the ability to acquire new varieties ($p(0) = 0$) and therefore has no future cash flow it can borrow against. Hence $\overline{B}(0) = 0$. Then,

$$V^D(0, B) = 0 \text{ if } B > 0 \text{ and } V^R(0, B) = -B \text{ if } B \leq 0.$$

Since a firm with 0 varieties stays permanently in that state, we treat $K' = 0$ as exit of the firm.

Let $\mathbb{D}(K', B)$ be the indicator function for default. It is 1 if $B > \overline{B}(K')$ and 0 otherwise. We can now give the expression for default on bonds issued in the current period:

$$
\begin{aligned}
d(K', B') = {} & \left[p(K') \textstyle\int_{s_{\min}}^{1} \mathbb{M}(K', B', s)\mathrm{U}(ds)\right] \textstyle\sum_{K''=0}^{K'+1} x(K'', K'+1)\mathbb{D}(K'', B') + \\
& \left[1 - p(K') \textstyle\int_{s_{\min}}^{1} \mathbb{M}(K', B', s)\mathrm{U}(ds)\right] \textstyle\sum_{K''=0}^{K'} x(K'', K')\mathbb{D}(K'', B').
\end{aligned}
$$

The bracketed term multiplying the first summation term is the probability that the firm successfully adds to its product portfolio next period. The summation term is the probability of default conditional on having acquired a new variety. Correspondingly, the bracketed term multiplying the second summation term is the probability that it fails to acquire a new variety next period, and the summation term is the probability of default conditional on not having acquired a new variety.

Finally, we can now give the expression for $W(N, B)$ (i.e., the value of the firm following the merger decision but before the realization of the extinction shocks):

$$
W(N, B) = \sum_{K'=0}^{N} x(N, K') \left[[1 - \mathbb{D}(K', B)]V^R(K', B) + \mathbb{D}(K', B)V^D(K', B)\right].
$$

3.4 Closing the Model

We now turn to the two equilibrium conditions of the model. The first pertains to the pricing of bonds. We assume that lenders are risk neutral and lending is a competitive business. This implies that the price of a bond, conditional on the amount borrowed and the number of varieties in possession of the borrowing firm, must be such as to make the rate of return on the bond equal to the risk-free rate in expectation. To give the resulting expression for bond prices, define

$$
Q(K', B) = \begin{cases} 1 & \text{if } \mathbb{D}(K', B) = 0 \\ \frac{\overline{B}(K')}{B} & \text{if } \mathbb{D}(K', B) = 1. \end{cases}
$$

Q is the "recovery rate" on each bond next period: In the event there is no default, the rate is 1; in the event of bankruptcy, the rate is the ratio of $\overline{B}(K')$ to B (all bonds are treated equally). With this definition of the recovery rate, the requirement that investors break even on their loans implies that bond price is given by:

$$q(K', B') = (1+r)^{-1} \Big[p(K') \int_{s_{\min}}^{1} \mathbb{M}(K', B', s)\mathrm{U}(ds) \sum_{K''=0}^{K'+1} x(K'', K'+1)Q(K'', B') + \\ [1 - p(K') \int_{s_{\min}}^{1} \mathbb{M}(K', B', s)\mathrm{U}(ds)] \sum_{K''=0}^{K'} x(K'', K')Q(K'', B') \Big].$$

The second equilibrium condition pertains to the determination of ρ. While ρ is taken as parametrically given by firms, its value is determined endogenously to satisfy the constraint that the total measure of ideas for new varieties arriving into each period is M. To express this condition, let $\mu_\rho(K)$ $K \in \mathbb{K}$ denote the steady-state measure of firms with varieties K, given ρ. Then, ρ satisfies

$$M = \rho \sum_{K \in \mathbb{K}} K \mu_\rho(K).$$

4 Calibration

We now turn to our quantitative analysis. In this section we calibrate the model to data moments from 1997. In Section 5 we discuss that steady properties of the model, in particular, explain the mechansism via which a positive relationship between size and leverage arises in the model. In Section 6 we focus on the effects of a lower real interest rate. The motivation for this exercise, as well as the choice of 1997 as the year to which the model is calibrated, is discussed in more detail in Section 6.

The model has two market parameters, r and θ, one preference parameter, β, and four technological parameters, M, ϕ, σ and $s_{\min}$. In the model a period is a month, but all parameter values and model results reported in this section are annualized. Of these seven parameters, the three reported in Table 4 are set independently. The risk-free interest rate r is set to 2.16 percent, which is the trend value of the annual average real return on 3-month Treasury bills in 1997.[12] The exact value of β is not very important for our results but it is important that firms be more impatient than lenders. For the baseline calibration, we set β to 0.95. The value of M is a normalization and is set to a numerically convenient value of 120.

The top panel of Table 5 reports the remaining parameters that are set jointly to match data moments. The second and third columns give the observed and model values of these moments.

[12]The trend value in 1997 is the predicted value of a linear time trend regression over the period 1997-2015.

Table 4:
Parameters Set Independently

Parameter, Annualized	Value
r	0.0216
β	0.95
M	120

The fourth column lists the parameters that most affect the corresponding moment, and the final column reports the parameter values that achieve the targeted moments.

The first moment listed is the default probability on debt. For the target for this moment we use the bankruptcy rate for firms reported in Corbae and D'Erasmo (2017, Table 1).[13] The implied value of θ gives a maximum allowable annual default probability of 5 percent. The remaining two moments are the annual rate of entry of new firms and the survival rate of 1-year-old firms in 1997. As in the case of the real interest rate target, both rates are set to their respective trend values, as implied by linear trend regressions over the period $1997 - 2015$. The model parameter that most affects the entry rate is σ (the success probability of spinoffs) with the entry rate increasing in σ. The model parameter that most affects the survival rate is ϕ (the product extinction probability) with the survival rate declining in ϕ. The implied values of these two parameters are 0.91 and 0.195, respectively. The final parameter is $s_{\min}$ and its value is set to 0.87. There is no data moment to pin this parameter down, and there is range of values that will work: If $s_{\min}$ is lowered (raised), the value of σ can be lowered (raised) to match the same entry rate (no moments other than the entry rate are affected by a different choice of $s_{\min}$). However, the value of $s_{\min}$ will affect how the economy responds to a decline in the real interest rate, as discussed in Section 6.

The bottom panel of Table 5 reports some relevant nontargeted data moments along with their model counterparts. The model's response of leverage to logarithm of sales is in line with the data. The survival probabilities in the model rise with age, as they do in the data although the model slope is less steep. The probability of a variety getting a new idea is slightly higher than the probability of a variety becoming extinct.

[13]They report an overall bankruptcy rate of 0.96 percent for the period 1980-2014. Once we take into account that only around 90 percent of firms carry debt, bankruptcy rate conditional on debt is 1.1 percent. The historical default rate on corporate bonds for the period 1983-2017 reported in Moody's is 1.6 percent.

Table 5

Parameters Set Jointly				
Description of Target	Data	Model	Parameter	Value
Probability of default	0.011	0.012	θ	0.050
Annual entry rate of new firms	0.111	0.111	σ	0.910
Survival rate of 1-yr-old firms	0.843	0.844	ϕ	0.195
Normalization	-	-	$s_{\min}$	0.87

Nontargeted Moments: Data and Model		
Moments	Data	Model
Response of leverage to lnSales	$0.013-0.038$	0.026
Survival rate of 0-yr-old firms	0.77	0.83
Survival rate of 2-yr-old firms	0.87	0.86
Survival rate of 3-yr-old firms	0.88	0.87
Survival rate of 4-yr-old firms	0.90	0.88
Employment growth of 0-yr-old firms	0.99	0.94
Employment growth of 1-yr-old firms	0.92	0.94
Employment growth of 2-yr-old firms	0.93	0.94
Employment growth of 3-yr-old firms	0.94	0.94
Employment growth of 4-yr-old firms	0.96	0.94
Prob. a variety generates new idea (ρ)	-	0.209

The data for entry rates, survival rates, and employment growth are from the U.S. Census Bureau's Business Dynamics Statistics database (https://www.census.gov/ces/dataproducts/bds/data.html). Each data point reported is the predicted trend value from a linear time trend regression between 1997-2015 of the corresponding series. Model results are annualized via simulation. For instance, 0-yr-old firms are defined as all the firms that entered in the previous 12 model periods and survived. Another 12 model periods later, the surviving firms become 1-yr-old firms.

5 Model Properties

In this section we examine the properties of the model.

Why Do Firms Borrow and How Much?

Firms borrow because owners discount the future more than lenders. Furthermore, since both owners and lenders are risk-neutral, owners will typically attempt to get as much revenue from bond sales subject to the default probability constraint. The intuition for this is as follows. If the firm issues B' in the current period, it obtains $q(K', B')B'$ in the current period. To compensate the risk-neutral lenders, the firm's expected payment next period must be $q(K', B')B'(1 + r)$ and the discounted utility of this payment to the firm is $-\beta q(K', B')B'(1 + r)$. Then, the utility gain in the current period of issuing B' is $q(K', B')B'[1 - \beta(1 + r)]$. Since the term in square brackets is strictly positive, utility of the owners rises with revenue from bond sales. This net gain expression ignores the impact that a higher or lower debt level might have on the decision to acquire a new variety next period (see the subsection on Debt Overhang below). In the quantitative model, we find that default probability constraint binds for all firms and all firms borrow as much as they can subject to this constraint.

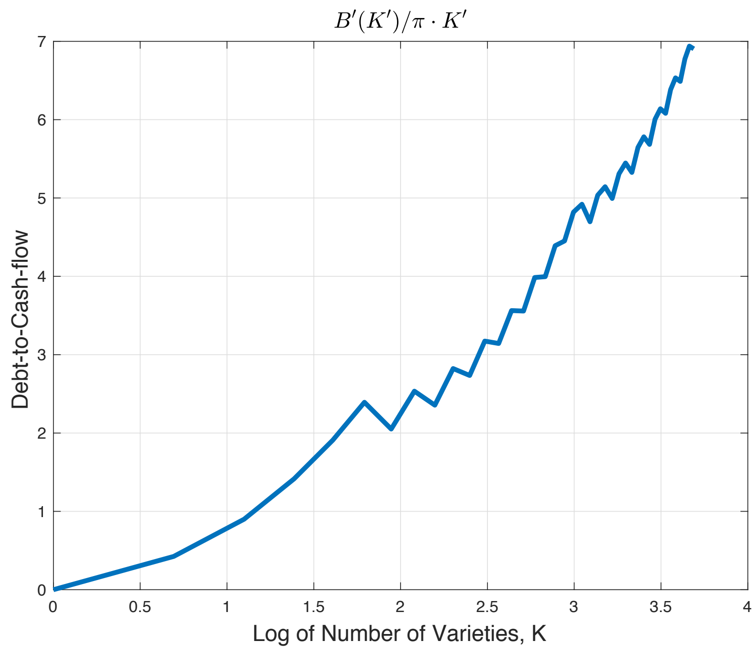
Why Does Leverage Rise with K?

For the purposes of understanding model mechanics, it is helpful to think of leverage as $B'(K')/\pi K'$. This ratio rises with K' because, on a per-variety basis, a firm with a larger number of varieties has less variable cash flow than a firm with fewer varieties and, therefore, is able to borrow more subject to the default probability constraint. We explain this below.

Consider a single variety firm, i.e., a firm with $K' = 1$. The probability that this firm will end up with $K'' = 0$ in the next period is at least $(1 - \rho)\phi$ (probability of the event that the firm does not get to make a purchase decision and then loses the one variety it has). Our calibration implies that this lower bound on the firm's exit probability is 0.15. Therefore, the probability of default on any amount of debt (no matter how little) is at least 0.15. Since this exceeds the calibrated value of $\theta = 0.05$, a single variety firm is shut out of the credit market and its equilibrium leverage is 0.

Firms with two or more varieties can borrow. For the lenders, the risk associated with leverage depends on the number of surviving varieties next period as a proportion of the number of varieties today. Ignoring for the moment the possibility of acquiring a new variety, a firm will, on average, have roughly $(1 - \phi)$ of its current varieties next period, and the variation around this proportion,

Figure 1:
Debt-to-Cash-Flow Ratio and Firm Size



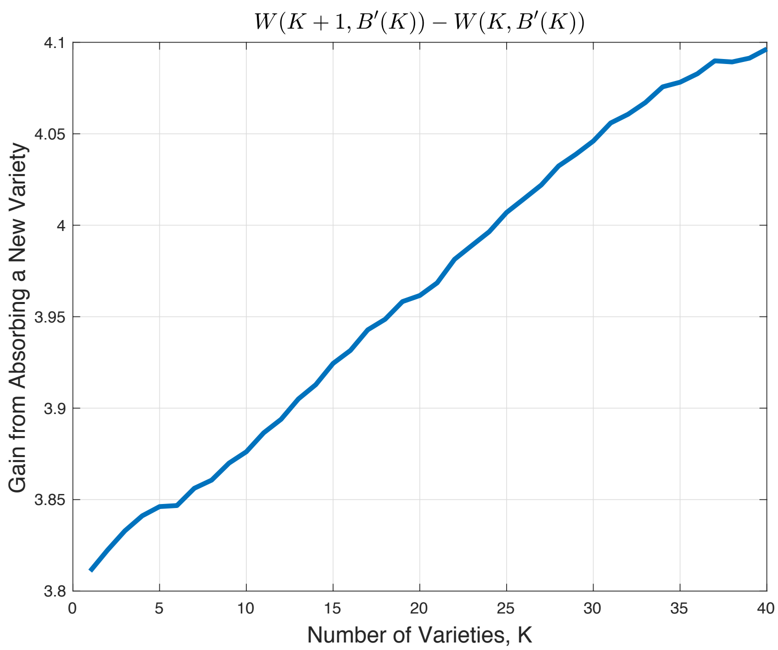
as measured by the variance, is $[(1-\phi)\phi/N]$.[14] Thus, the riskiness of the cash flow shrinks with N. Consequently, a bigger firm is able to borrow a higher proportion of its current-period cash flow before running into the default probability constraint. Figure 1 displays $[B'(K')/\pi K']$ against $\ln(K')$. As is evident, leverage is strongly increasing with (logarithm of) firm size: A firm that owns a larger number of varieties is able to borrow a greater multiple of its current-period cash flow.

While the maximum default probability on debt is 0.05, the average probability of default in our economy is around 0.01. This occurs because $d(K', B')$ is a step function in B' (here we are treating B' as a continuous variable). To see why, suppose that at B' the firm will default only if the number of surviving varieties next period is $\hat{K}$ or less. At some higher debt level, the firm will default if the number of surviving varieties next period is $\hat{K}+1$ or less. At the level of debt where the firm switches to defaulting with $\hat{K}+1$ varieties next period, there is a discrete jump up in the probability of default. Because of these discrete jumps, there is typically no debt level at which the

[14]Davis, Haltiwanger, Jarmin, and Javier (2009) document that dispersion of employment shrinks with firm size (measured as sales revenue). Taking into account firm exit is key for this fact.

default probability is exactly 0.05. Hence, the equilibrium default probability for a firm is always strictly less than 0.05.

Figure 2:
Firm Size and the Gain from Absorbing a New Variety



Why Are Larger Firms More Willing to Buy New Ideas?

New varieties are more valuable to larger firms because they are able to borrow a greater fraction of the present discounted value of the associated cash flow, which is the message of Figure 1. This point can be made more directly by plotting $[W(K+1, B'(K)) - W(K, B'(K))]$ against K, as done in Figure 2. The gain from absorbing a new variety is increasing in firm size over the range of K shown. The implication is that $s^*(K, B)$ (the s for which $s[W(K+1, B'(K)) - W(K, B'(K))] = \sigma W(0, 1)$) is declining in K: Larger firms are more likely to buy new varieties.[15]

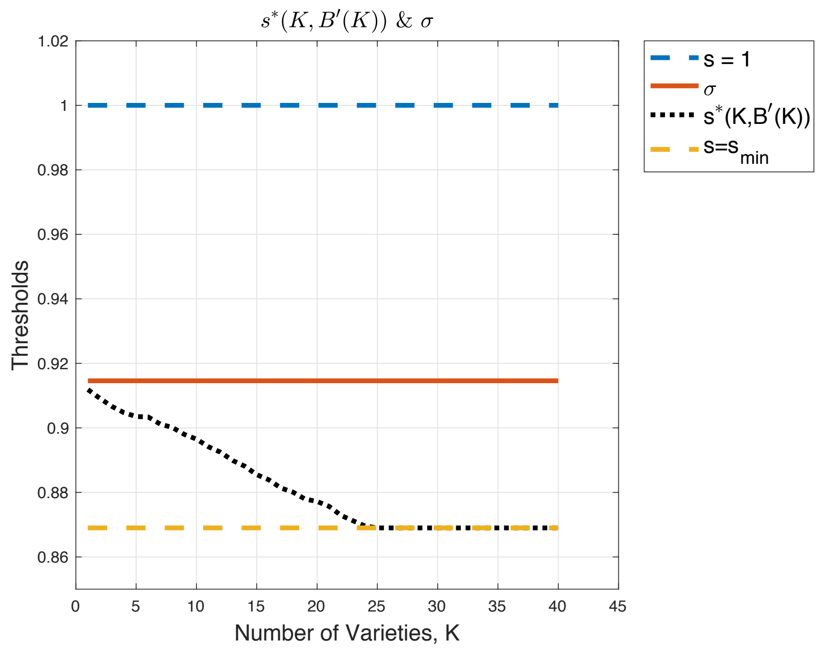
Misallocation

In our model, each new variety generates a positive cash flow, and, from an output maximization point of view, it is better if the new idea is implemented in the firm where the idea's success probability is highest. As we see in Figure 3, this does not always happen in equilibrium in our model. In the figure the blue and yellow dashed lines represent the boundaries of the support of s (1

[15] As K gets closer to $K_{\max}$ firms become pickier due to the approaching capacity constraint, and the threshold s^* rises.

and $s_{\min}$, respectively). The black dotted line plots the equilibrium threshold levels $s^*(K, B'(K))$: if the realized value of s falls above this line, the idea is implemented in the incumbent firm. The solid red horizontal line is drawn at the value of σ, the probability of success in a startup. Observe that if the realized value of s falls in the region between the solid red and the black dotted line, the idea will be implemented by the incumbent firm, while it would have a higher success probability as a startup. This happens because incumbents have better access to the credit market than startups. Note also that the likelihood of inefficient choice of organizational form rises with incumbent firm size.

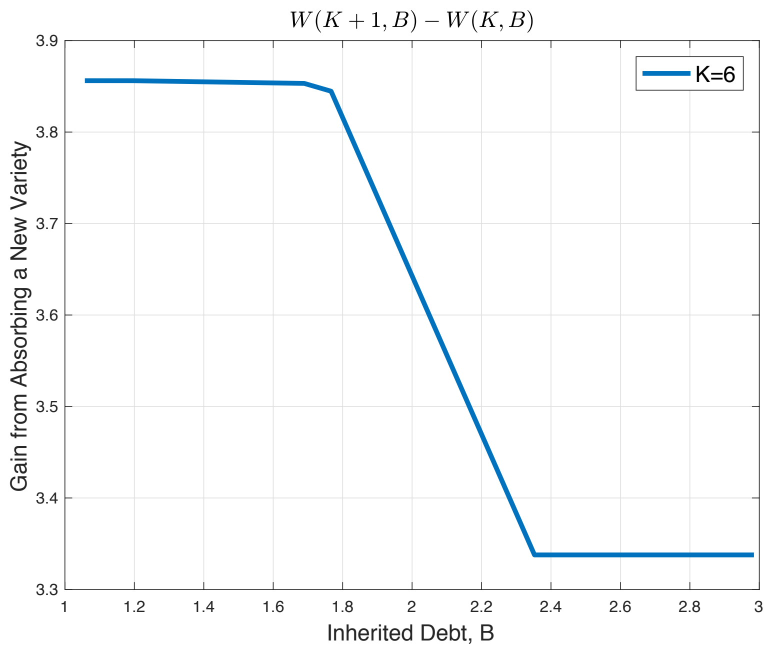
Figure 3:
Efficient and Inefficient Organizational Choice



Debt Overhang

As shown in Figure 4, given K, the gain from acquiring a new variety is decreasing in the level of inherited debt. Thus, greater indebtedness lowers the expected growth rate of a firm. This is a manifestation of the debt overhang effect. Given an inherited debt level B, there is a level of $\hat{K}$ at which default is triggered. If the firm acquires a new variety, value of $\hat{K}$ does not change but more destruction shocks would be needed for the surviving number of varieties to drop to or below $\hat{K}$, and, hence, the likelihood of default declines. In addition, conditional on default (that is, ending with a $K' \leq \hat{K}$), the expected number of varieties is higher if the firm acquires a new variety, and this will increase the recovery on the defaulted debt. On both counts, some portion of the cash flow of the new variety is captured by the firm's existing creditors, thereby blunting the firm's desire to acquire a new variety (the debt overhang effect on firm investment).

Figure 4:
Inherited Debt and the Gain from Absorbing a New Variety



In the figure, there are flat segments at low levels of debt and at high levels of debt. In the first case, the probability of default is zero and, so, the acquisition of a new variety does not affect the value of inherited debt (the default probability is still zero post acquisition). In the second case, the probability of default is 1 even with the acquisition of the new variety (and so it is also 1 preacquisition). Since the firm is defaulting for sure, the values of $W(K+1,B)$ and of $W(K,B)$ no longer depend on the value of B (under default, the total payment to existing creditors depends only on the number of surviving varieties) and so the net gain becomes independent of B.

What Is the Key Determinant of ρ?

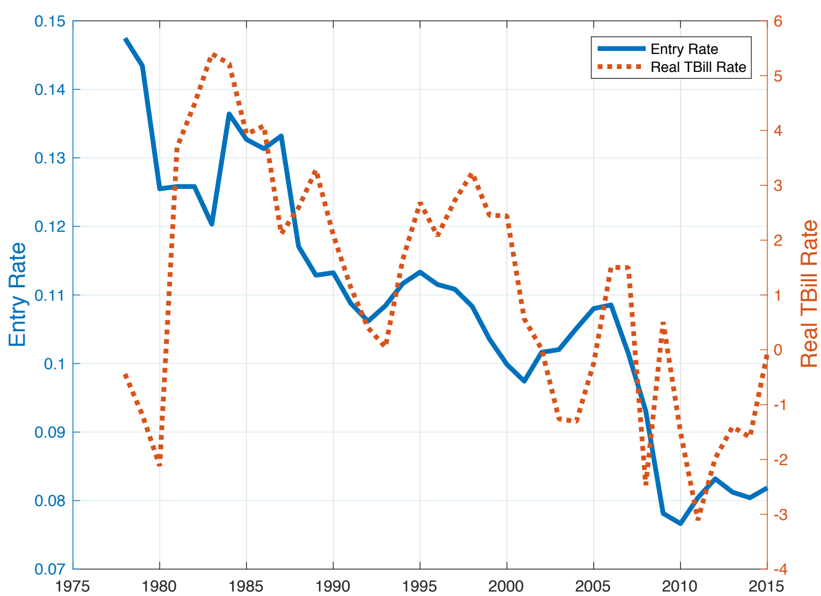
The key determinant of ρ is ϕ, the extinction probability. To see this, consider the case where all new ideas are successful. Then, the measure of new varieties in the economy will be M per period. Let E denote the measure of varieties in the economy. Then the measure of varieties exiting each period is ϕE. In the long run, the level of E will be determined by the condition that $M = \phi E$ and, hence, M/E must equal ϕ. But M/E is just ρ, which shows that in this case the key determinant of ρ is indeed ϕ.

The actual determination of ρ is somewhat more complicated because not all new varieties succeed and the success probability is partly endogenous: The varieties that are implemented in startups have a success probability of σ while those that are implemented in incumbent firms depend on the realized value of s. However, in our calibrated model these success probabilities are close to 1, so $M \approx \phi E$ and $\rho \approx \phi$.

6 Real Interest Rate and Firm Dynamics

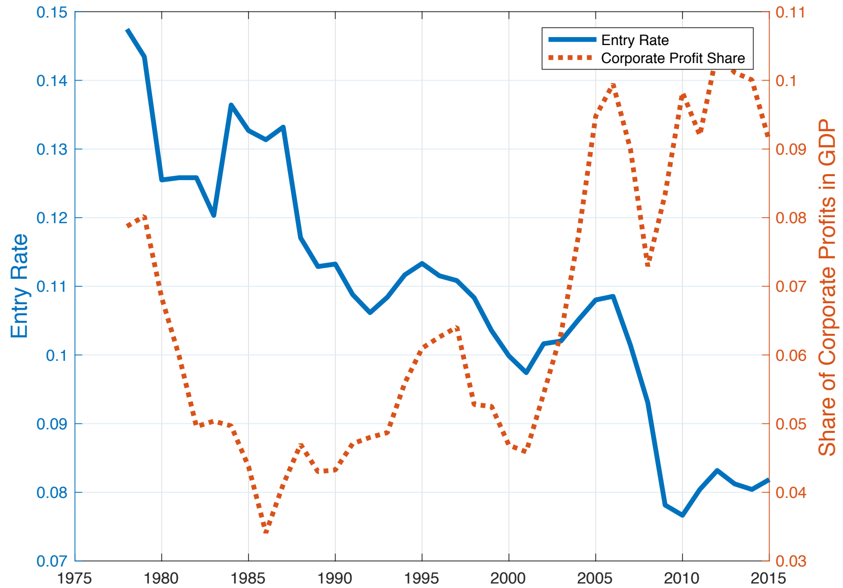
In this section, we examine the implications of a lower real interest rate on the steady state equilibrium of our economy. The motivation for this investigation is the well-known decline in the real interest rates and startup rates over the past few decades. Figure 5 shows the secular movement in both the startup rate and the real interest rate over the period 1978-2015.[16] As is evident, the entry rate has been falling since the late 1970s. The real interest has also been declining since the early 1980s, although, arguably, the downward trend is clearer since the late 1990s (see Del Negro, Giannone, Giannoni, and Tambalotti (2017)).

Figure 5:
Secular Trends in Real Interest Rates and Entry Rates



[16]The entry rate data are available since 1978.

Figure 6:
Entry Rate and Corporate Profits

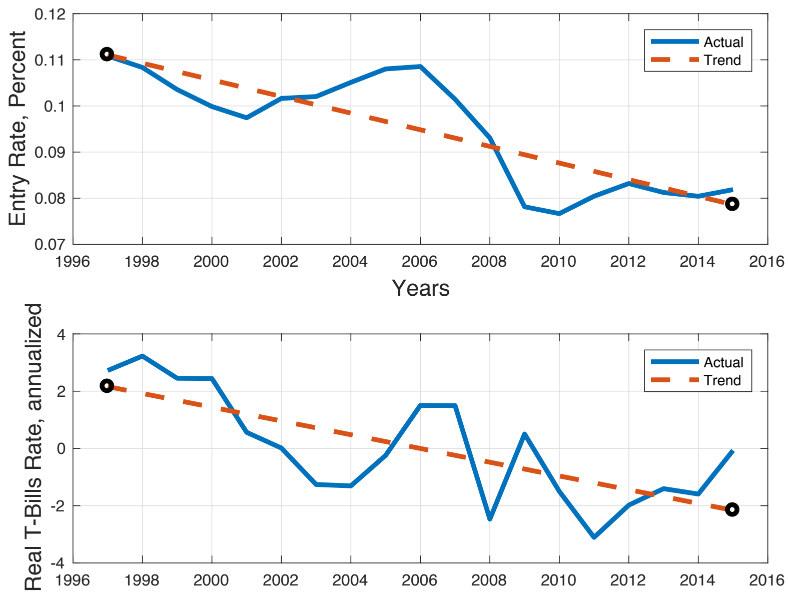


Importantly, as shown in Figure 6, the share of corporate profits in GDP has risen strongly since the late 1990s. This fact is a drawback for studies that explain the falling startup rate as a response to declining profits (Karahan, Pugsley, and Sahin (2018), Hopenhayn, Niera, and Singhania (2018)). In contrast, profits per variety do not play a crucial role in our model because M, the number of ideas arriving into the economy, is constant.[17] But we have another margin that effects the entry rate, namely, the choice of organization within which a new idea is implemented. In our economy, a decline in r leads to more ideas being implemented in existing firms instead of startups, leading to lower entry rates for firms.

It is to investigate this possibility that we chose to calibrate our economy to 1997 and will now examine how the steady state of the economy changes as the interest rate declines. The top panel of Figure 7 shows the trends in the annual short-term real interest rate and the entry rate between 1997 and 2015. The trend line for the real interest rate shows a decline from 2.16 percent (our calibration of r) to -2.16. To understand the effect of lower interest rates, we will lower interest in our model to 0 percent – keeping all other parameters unchanged — and analyze the new steady state. When we reduce r from 0.0216 to 0, the startup rate falls from around 0.11 to 0.08. This

[17]We could imagine a world where the higher profit share evident in the data induces more effort in the production of new ideas. In this case, our model would have a force that could elevate entry rates.

Figure 7:
Trends in Real Interest Rate and Entry Rate, 1997-2015



change is the same as the decline in the trend value of the entry rate between 1997 and 2015 as shown in the bottom panel of Figure 7. This is not a coincidence, as we can generate a bigger or smaller decline in the entry rate in response to the decline in r by changing the value of $s_{\min}$. Recall that $s_{\min}$ was not pinned down by the calibration of the model, but the chosen value generates enough sensitivity to explain the declining trend in startups since 1997 as a response to lower real interest rates (the mechanism is explained in more detail below).

The mechanism through which lower interest rates lead to lower startup rates in the model can be explained with a very simple example. Imagine a situation where a worker gets an idea for a new variety that, if it succeeds, will generate a payoff of \$ Y next period. If the worker were to implement the idea in a startup himself, the idea succeeds with probability of σ and he cannot borrow against the future cash flow of the idea even if it is successful. If the idea is sold to an incumbent, it succeeds with probability s and the incumbent gets to borrow $X < Y$ risk-free against the future cash flow if it is successful. Under these circumstances, the value of the idea to an incumbent is

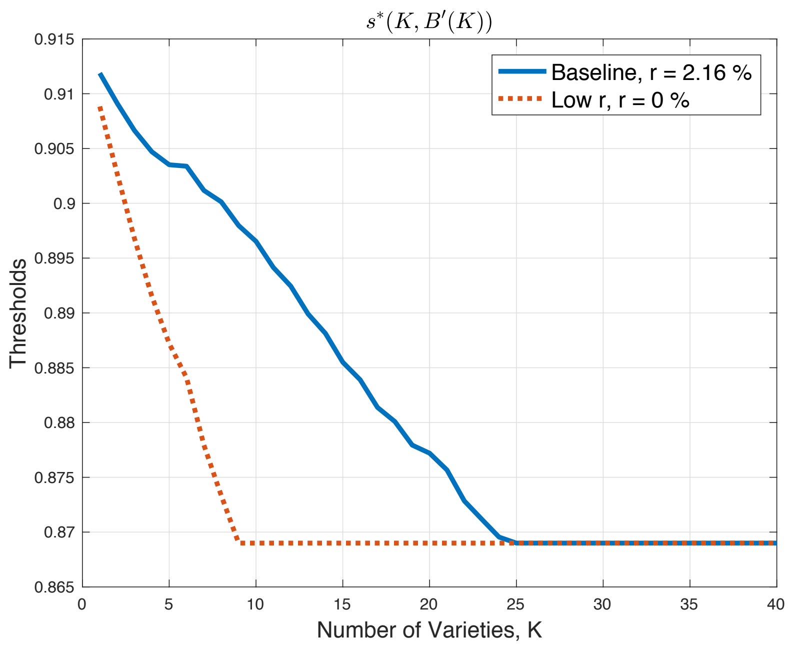
$$s\left[\left(\frac{X}{1+r}\right) + \beta(Y - X))\right]$$

and the value to a startup is

$$\sigma\beta Y.$$

Notice that as r falls, the value to the incumbent of a new idea increases but the value to the spinoff does not change. Consequently, the threshold value of s above which the incumbent can buy the idea shifts down and the range of s values for which an idea generates a startup shrinks. Although simple, the example captures the essence of the tradeoff between selling an idea and implementing it in a startup. In our model, a startup cannot immediately borrow against its future cash flow (as in the example) while incumbents can borrow (even though it is not at the risk-free rate as in this example). As the real interest declines, the price of the incumbent firm's bonds increases and it becomes more attractive for them to buy ideas.

Figure 8:
s Thresholds and the Real Interest Rate



To confirm this effect, Figure 8 plots the threshold value of s above where a firm of size K will purchase an idea. The solid blue line shows this threshold for the baseline model, and the orange dotted line shows it for the equilibrium with $r = 0$ (and no other changes in any parameters). Observe that for each K the threshold s is either unchanged or lower in the low interest rate equilibrium.

We now turn to the other equilibrium effects of a drop in the real interest rate, which are reported in Table 6. Turning first to the top panel, we see that there is a modest increase in the responsiveness of leverage to sales. One reason underlying this effect is the change in the distribution of firms, which shifts toward larger firms. From Figure 1 presented earlier, it should be reasonably clear that a linear regression of leverage on log of firm size would predict a negative value of leverage for small firms. Indeed, in COMPUSTAT many small firms have positive net assets because they hold substantial amounts of cash (see, for instance, Opler, Pinkowitz, Stulz, and Williamson (1999) and Duchin (2010)). Our model does not have a reason for savings by firms, and when there are fewer small firms, as in the low interest rate steady state, the relationship between leverage and log sales becomes stronger.

We see a modest increase in the bankruptcy rate in the low interest rate economy, which is also the result of the shift from small to larger firms. Generally speaking, a small firm's default probability is more sensitive to leverage (i.e., there are bigger upward jumps in probability of default as leverage increases) and, hence, smaller firms are typically further away from θ (the maximum allowed probability of default) in terms of their equilibrium default probability.

The total measure of varieties declines in the low r equilibrium. This is because incumbent firms become less choosy about the new ideas they purchase (the s threshold falls) and so the fraction of new ideas that are actually successful declines, leading to a lower measure of varieties. Since we keep the number of new varieties arriving into the economy constant at M, the decline in the measure of varieties requires that the probability that a variety generates a new idea, ρ, increase slightly.

Turning now to the bottom panel of Table 6, the first line reports the change in entry rates in the model and in the data. Recall that the value of $s_{\min}$ was picked to generate the drop in trend entry rates between 1997 and 2015. This parameter determines the support of the uniform distribution from which the success probability of an idea for an incumbent is drawn. If $s_{\min}$ is lowered, the range expands, and, therefore, any given change in threshold $s^*(\cdot)$ has less of an effect on entry rate when r falls.

In the model, with lower r, survival rates and employment growth for all age groups increase slightly. The reason for this is because s^* declines and more ideas are implemented within existing firms, increasing their employment growth. Survival rates also go up because existing firms are less likely to lose all their varieties and exit. But overall, the changes are quite small. In the data, we see that survival rate and employment growth do increase for firms that are one year old or older, and the change is more pronounced than what the model generates. For 0-year-old firms, both survival rate and employment growth shrink. This might be because, in a world where there is competition between existing firms for new ideas, young firms may become more disadvantaged relative to large firms as interest rates decline. Our model does not take such effects into account.

Finally, we turn to the prediction of our model regarding business concentration. In the low interest rate environment, existing firms absorb more of the new ideas and grow faster, so we would expect business concentration to rise. Recall that Autor, Dorn, Katz, Patterson, and van Reenen (2017) found that the share of sales in the top 4 or top 20 firms in the six major industries has risen since the mid-to-late 1990s. Since our model has a distribution of firms, we will examine the share of sales accounted for by the top *measure* (as opposed to number) of firms.

Table 7 quantifies this prediction of the model. For the baseline model, we use $H(K, B)$ to first determine the measure of firms for each K. Then, starting with the firms with the largest number of varieties, we include firms with progressively fewer varieties until 0.1, 0.5, and 1 percent of the total measure of firms is included. The first column of numbers reports the resulting measures. Then, we compute the fraction of aggregate cash flow (our measure of output) accounted for each of the three measures of firms. These fractions are reported in the second column of numbers. Thus, in the baseline model, the top 0.1 percent of firms by size accounts for 2 percent of total output, the top 0.5 for 9 percent of output, and the top 1 percent for 13 percent of output. For the final column of numbers, using the new distribution of firms for the low interest rate equilibrium, we determine the share of output accounted for by the largest firms for the *measures reported in column 2*. Thus, the comparison between the last two columns holds fixed the number (more precisely, the measure) of top firms. The comparison reveals that the low interest rate economy is substantially more concentrated: For the top (by size) 0.22, 1.12, and 2.24 measures of firms, the share of output rises by 1, 4, and 11 percentage points, respectively.

Table 6:
Equilibrium Effects of Low Interest Rates

Moments		Baseline	Low r Eqbm	
Response of leverage to sales		0.025	0.037	
Fraction of firms that declare bankruptcy		0.012	0.016	
Steady state measure of varieties		516.0	513.0	
Prob. of a variety generating a new idea (ρ), ann.		0.209	0.210	
	Data 1997	Baseline	Low r Eqbm	Data 2015
Entry rate of new firms	0.11	0.11	0.08	0.08
Survival rate of 0-yr-old firms	0.77	0.83	0.83	0.76
Survival rate of 1-yr-old firms	0.84	0.84	0.85	0.87
Survival rate of 2-yr-old firms	0.87	0.86	0.86	0.89
Survival rate of 3-yr-old firms	0.88	0.87	0.87	0.91
Survival rate of 4-yr-old firms	0.90	0.88	0.88	0.91
Employment growth of 0-yr-old firms	0.99	0.94	0.94	0.90
Employment growth of 1-yr-old firms	0.92	0.94	0.94	0.95
Employment growth of 2-yr-old firms	0.93	0.94	0.95	0.97
Employment growth of 3-yr-old firms	0.94	0.94	0.95	0.98
Employment growth of 4-yr-old firms	0.96	0.94	0.95	0.96

The data for entry rates, survival rates, and employment growth are from the U.S. Census Bureau's Business Dynamics Statistics database (https://www.census.gov/ces/dataproducts/bds/data.html). Each data point reported is the predicted trend from a linear time trend regression between 1997-2015 of the corresponding series.

Table 7:
Effect on Business Concentration of Low Interest Rates

		Share of Output	
	Measure of firms	Baseline	Low r Eqbm
Top 0.1 percent by Size (K) in Baseline	0.22	0.02	0.03
Top 0.5 percent by Size (K) in Baseline	1.12	0.09	0.13
Top 1.0 percent by Size (K) in Baseline	2.24	0.13	0.24

7 Conclusion

We presented a model in which firms manage collections of product varieties. The arrival into the economy of new varieties and the extinction of existing varieties are random events. Since firms manage collections of varieties, the random process of *product variety* entry and exit induces a stochastic process for the entry, growth, and exit of *firms*. A firm's access to capital markets plays a key role in our theory of firm dynamics. Our model generates a positive relationship between firm size and firm leverage that is consistent with the evidence for U.S. firms. Our theory implies that a decline in the risk-free rate will result in larger firms purchasing more of the new varieties entering the economy in any period, resulting in fewer startups and greater concentration of sales among top firms. Thus our paper connects the decline in the startup rate and the rise in business concentration since the late 1990s to the decline in the risk-free rate over this same period.

Appendix A

The two tables below report on the results of the robustness checks of our main regressions with respect to alternative measures of size and leverage.

Table 8:
Relationship Between Leverage Ratio and Firm Size (Panel)

Dep Var	Lev I	Lev II	Lev I	Lev II
	Market		Book	
lnEMP	0.031	0.028	0.039	0.040
	(18.39)	(14.86)	(18.82)	(18.43)
Cap_Ratio	0.292	0.456	0.610	0.558
	(22.92)	(27.76)	(44.85)	(39.26)
Profit_Ratio	-0.004	-0.110	-0.100	-0.161
	(-0.44)	(-7.91)	(-13.69)	(-21.92)
Firm FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
Num of Obs	174,457	174,936	193,161	186,246
Num of Groups	18,053	18,075	19,808	19,477
R^2	0.15	0.17	0.17	0.14

Table 9:
Relationship Between Leverage Ratio and Firm Size (Panel with Logit)

Dep Var	Lev I	Lev II	Lev I	Lev II
	Market		Book	
lnSale	0.076	0.078	0.103	0.082
	(20.89)	(14.92)	(17.38)	(15.07)
Cap_Ratio	0.627	1.096	1.523	1.448
	(21.33)	(25.92)	(42.70)	(37.42)
Profit_Ratio	-0.019	-0.375	-0.226	-0.027
	(-0.83)	(-8.77)	(-4.02)	(-1.90)
Firm FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
Num of Obs	182,984	183,496	211,051	203,001
Num of Groups	18,580	18,605	20,937	20,582
R^2	0.13	0.14	0.15	0.11

Appendix B

This Appendix describes the choice problem of a firm with $\rho K > 1$. We assume that such a firm gets the opportunity to buy one idea for sure and gets an opportunity to buy a second idea with probability $\rho K - 1$ knowing whether the first purchase (if made) was successful. Given that the firm gets two opportunities (potentially) to buy ideas and makes decisions sequentially, we will divide the first subperiod into two parts.

Let $\tilde{K}$ denote the number of varieties the firm owns at the start of the second part of the first subperiod. Then, $\tilde{K}$ is either K or $K+1$, depending on whether the idea encountered in the first part of the subperiod was accepted and, if accepted, whether it was successfully implemented. At this juncture, the value of the firm that can absorb a new variety (i.e., with $\tilde{K} < k_{\max}$) is

$$Z(K, \tilde{K}, B) = [2 - \rho K] W(\tilde{K}, B) + [\rho K - 1] \times$$
$$\int_{s_{\min}}^{1} \left[\max\{W(\tilde{K}, B), sW(\tilde{K}+1, B) + (1-s)W(\tilde{K}, B) - \sigma W(1,0)\} \right] \mathrm{U}(ds).$$

Here $W(\tilde{K}, B)$ has the same interpretation as in the main text: It is the value of the firm after the merger decisions have been made but before the product extinction shocks are realized. The firm's decision problem after the extinction shocks are realized is exactly the same as any other firm and as described in the main text.

Moving to the first part of the first subperiod, the firm has the opportunity to buy one idea for sure. If it can absorb a new variety, it will purchase this idea if

$$s[Z(K, K+1, B) - Z(K, K, B)] \geq \sigma W(1,0).$$

The l.h.s. is the expected gain to the firm if it absorbs the new variety and the r.h.s. is the gain to spinning off. If the surplus is nonnegative, we assume that the new variety is absorbed by the incumbent and the inventor gets $\sigma W(0,1)$.

When this firm enters the period, its value is given by

$$Z(K, B) = \int_{s_{\min}}^{1} [\max\{Z(K, K, B), sZ(K, K+1, B) + (1-s)Z(K, K, B) - \sigma W(1,0)\}] \mathrm{U}(ds).$$

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