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A Dynamic Model of Intermediated Consumer Credit and Liquidity

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Abstract

We construct a tractable model of consumer credit with settlement frictions (i.e., a consumer credit market that relies on a secondary market for privately issued debt claims to operate) to study the role of monetary policy in the efficient functioning of the payments system. In our framework, intermediaries hold reserves across periods to take advantage of rediscounting opportunities, and monetary policy influences the equilibrium allocation through the interest rate on reserves. We characterize the conditions for the existence of an allocation in which privately issued debt claims are not discounted in equilibrium. We also discuss the role of monetary policy in the payments system across different market structures in the intermediary sector and characterize the minimum size of the intermediary sector required to attain efficiency.

Keywords: Intermediation, liquidity, payments system, rediscounting

JEL classifications: E42, E58, G21

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1. INTRODUCTION

A prominent financial institution of modern economies is the payments system, a formal arrangement among market participants designed to facilitate the repayment of private debt claims. An important feature of these systems is the presence of financial intermediaries buying and selling debt claims originated in retail transactions and finally clearing all debts. These intermediaries usually operate by holding reserves that serve as an essential liquid instrument to help them achieve a desired portfolio of private claims. Because of intermediaries' active role in clearing private claims and their demand for reserves, many economists have highlighted the important role of monetary policy in helping the adequate functioning of the economy's payments system.

A widely held view among monetary economists is that the payments system works efficiently when it is satiated with liquidity (i.e., when reserves are plentiful for intermediaries involved in the clearing of payment instruments). This view is based on the Friedman rule, which provides a rationale for the optimum quantity of money in the economy.[1] The Friedman rule asserts that optimal monetary policy should aim at eliminating the opportunity cost of holding money balances, such as reserves held at the central bank, for transaction purposes. This monetary policy prescription has been shown to be optimal in a variety of economic environments. Woodford (1990) and Williamson and Wright (2010a,b) provide comprehensive surveys of optimal central bank policy across different monetary environments.

Absent in the majority of those papers is the explicit modeling of settlement imperfections, such as spatial separation, unsynchronized trading patterns, and imperfect information. A notable exception is Freeman (1996a), who first proposed a dynamic general equilibrium framework with explicit settlement frictions to study how monetary policy affects the clearing of private debt claims. Some subsequent important contributions to the payments literature include Freeman (1996b), Kahn and Roberds (1998), Williamson (1998), Temzelides

[1] For more on this principle, see Friedman (1959, 1969).

and Williamson (2001), Martin (2004), and Mills (2006).[2] Papers in this literature explicitly consider the underlying settlement frictions agents face when trading in the marketplace and study the role of government policies to achieve efficiency in the payments system. The main drawback of this literature is that the underlying framework makes it difficult to relate to standard monetary models with infinitely lived agents.

The goal of this paper is to provide a tractable model of the payments system within a unified monetary framework. This allows us to study the role of monetary policy in the efficient functioning of a consumer credit market with settlement frictions (i.e., a consumer credit market that relies on a secondary market for privately issued debt claims to operate). In particular, we seek to answer the following research questions. What are the properties of an efficient payments system? What is the relation between the franchise value of the rediscounting business and the efficiency of the payments system? Is a competitive rediscounting market always consistent with an efficient allocation? What is the role of intertemporal exchange among intermediaries in the operation of the payments system? Can the existence of credit arrangements among intermediaries lead to an efficient payments system? In this paper, we provide answers to these questions within a unified framework in monetary economics.

In our environment, consumers buy goods from merchants by issuing personal debt claims. Unsynchronized trading patterns in the environment imply that consumers and merchants do not meet again in the future to settle debt claims. A subset of agents, referred to as intermediaries, has the ability to sequentially interact with merchants and consumers, respectively. Consequently, they can buy debt claims from merchants and subsequently retire them by directly contacting the issuers. Our analysis, thus, considers a credit system in which financial intermediaries play a crucial role in the clearing of debt claims originated in retail transactions, giving rise to a rediscounting market.

The timing of events within the period is such that intermediaries must hold a portfolio of liquid assets to transfer their wealth across periods to take advantage of rediscounting

[2] Kahn and Roberts (2009) provide a comprehensive survey of the payments literature. Rocheteau and Nosal (2017) provide a useful textbook analysis of the economics of payments.

opportunities. The government supplies reserves that serve as a store of value for intermediaries, which influences the functioning of the payments system.[3] The interest rate paid on reserve balances affects the amount of funds flowing into the rediscounting market, which determines the prevailing discount rate. By setting the interest rate on reserves, the central bank affects the functioning of the intermediated credit system through the discount rate on private debt claims. Given this channel of transmission, we study the links between monetary policy and the efficient functioning of the payments system.

To better understand the role of government policy in the intermediated credit system, it makes sense to start the analysis by considering a laissez-faire economy. Precisely, we initially characterize the properties of an economy with a fixed supply of an outside asset that serves as reserves for intermediaries. Then, we consider two distinct market structures for the intermediary sector. First, we characterize equilibrium allocations in an economy with no intertemporal exchange across intermediaries (i.e., only spot trades are allowed among them). We refer to this scenario as an economy with *incomplete interbank markets*. Second, we study equilibrium allocations in an economy with perfect intertemporal exchange across intermediaries, referred to as an economy with *complete interbank markets*. In reality, interbank markets are somewhere in between these two extremes (i.e., neither inexistent nor completely frictionless). Studying the extreme scenarios is certainly analytically convenient and still provides useful insights on how actual interbank markets influence the functioning of the payments system.

The laissez-faire economy with incomplete interbank markets implies that intermediaries rely on a fixed supply of reserves as the *sole* store of value for taking advantage of rediscounting opportunities across periods. Surprisingly, we find that, depending on the fundamentals

[3] In our analysis, money and credit are complements, as opposed to many papers in the literature in which the underlying equilibrium implies that money and credit are substitutes in payments, such as Monnet and Roberds (2008), Sanches and Williamson (2010), Lotz and Zhang (2016), Araujo and Hu (2018), among others. However, notable exceptions are Gomis-Porqueras and Sanches (2013), who consider a model in which money and credit are complements. In that framework it is shown that the existence of a credit system increases the set of feasible government policies. So the complementarity between money and credit is due to the implementation of policies, not a market arrangement to improve payments.

of the economy, an equilibrium without discounting of private IOUs exists. In particular, an efficient payments system can arise endogenously if the initial wealth in the intermediary sector is sufficiently large relative to the size of the retail sector, even though intermediaries do not engage in any form of credit arrangement among themselves. In the economy with complete interbank markets, intertemporal exchange and credit arrangements among intermediaries are now feasible. We then show that an equilibrium without discounting always exists, regardless of the fundamentals. Thus, allowing intermediaries to trade in a frictionless credit market with perfect enforcement of their debt claims is sufficient to overcome the settlement frictions in the model (i.e., spatial separation and unsynchronized trading patterns).

Although an efficient payments system can arise endogenously in the laissez-faire economy, we show that the overall equilibrium allocation is never efficient because the allocation for intermediaries is suboptimal. This result is observed regardless of the market structure in the intermediary sector. We demonstrate that efficiency in the payments system requires that intermediaries earn zero profits in rediscounting. In other words, the franchise value of their rediscounting business must be zero to be consistent with efficiency in the payments system. As a result, the real return on reserves solely determines the value of future income for an intermediary when setting up a rediscounting business. In the absence of central bank intervention, we find that the real return on reserves is inefficiently low so that the consumption allocation for intermediaries is suboptimal.

Given these inefficiencies of the laissez-faire economy, our next step is to consider the role of active monetary policy by controlling the amount of reserves issued at each date and paying interest on reserve balances. We find that, regardless of the market structure in the intermediary sector, a version of the Friedman rule is optimal. In the economy with complete interbank markets, the Friedman rule leads to an efficient allocation, with perfect consumption smoothing for intermediaries and no discounting of private IOUs in the secondary market for debt claims. In the economy with incomplete interbank markets, the Friedman rule does not always lead to an efficient allocation, even though it eliminates the opportunity cost of holding reserves across periods. Finally, our analysis pins down the minimum size of the intermediary sector relative to the retail sector required to attain an efficient allocation.

The rest of the paper is organized as follows. Section 2 describes the basic framework. Section 3 characterizes efficient allocations by solving the planner's problem. Section 4 describes a laissez-faire economy with incomplete interbank markets. Section 5 characterizes equilibrium allocations for the laissez-faire economy with complete interbank markets. Section 6 discusses the role of optimal monetary policy, and Section 7 concludes.

2. MODEL

Our framework builds on Lagos and Wright (2005) and Rocheteau and Wright (2005). Time is discrete and continues forever. Each period is divided into two subperiods in which economic activity will differ. There is a frictionless centralized market (CM) in the first subperiod, while trade is decentralized (DM) in the second subperiod. A perishable consumption good is produced and consumed in each subperiod. We refer to the consumption good produced in the first subperiod as the CM good and to the consumption good produced in the second subperiod as the DM good.

There are three types of agents, referred to as consumers, merchants, and bankers. Consumers and merchants are both infinitely lived, with a $[0, 1]$-continuum of each type. A banker lives for two consecutive periods only. In each period, a measure $\alpha \in \mathbb{R}_+$ of new bankers is born. In the initial period, there is a measure α of old bankers. All agents discount future periods using the same subjective discount factor $\beta \in (0, 1)$.

The production possibilities in the economy are as follows. Each consumer has access to a divisible technology that allows him to produce one unit of the CM good with one unit of effort in the first subperiod. Each merchant has access to a divisible technology that allows him to produce one unit of the DM good with one unit of effort in the second subperiod. A banker does not have access to any production technology but receives an endowment of $e \in \mathbb{R}_+$ units of the CM good in the first period of his life.

The consumer's preferences are represented by

$$U^c\left(x_t^c, q_t^c\right) = x_t^c + u\left(q_t^c\right),$$

where $x_t^c \in \mathbb{R}$ is consumption of the CM good and $q_t^c \in \mathbb{R}_+$ is consumption of the DM good. The function $u : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is twice continuously differentiable, increasing, and strictly concave, with $u\left(0\right) = 0$ and $u'\left(0\right) = \infty$.

The merchant's preferences are represented by

$$U^m\left(x_t^m, q_t^m\right) = x_t^m - w\left(q_t^m\right),$$

where $x_t^m \in \mathbb{R}_+$ is consumption of the CM good and $q_t^m \in \mathbb{R}_+$ is production of the DM good. Assume that $w : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is continuously differentiable, increasing, and convex, with $w\left(0\right) = 0$. Assume that $u'\left(q\right)/w'\left(q\right)$ is strictly decreasing and that $\lim_{q \rightarrow 0} u'\left(q\right)/w'\left(q\right) = \infty$.

The banker's preferences are represented by

$$U^b\left(x_t^y, x_{t+1}^o\right) = v\left(x_t^y\right) + \beta v\left(x_{t+1}^o\right),$$

where $x_t^y \in \mathbb{R}_+$ denotes consumption of the CM good in the first period and $x_{t+1}^o \in \mathbb{R}_+$ denotes consumption of the CM good in the second period. The function $v : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is continuously differentiable, increasing, and strictly concave. Finally, assume $w\left(q^*\right) \leq \alpha\beta e$.

The physical environment where the agents interact is as follows. There exist two distinct locations in the economy, referred to as the bankers' location and the central location. Consumers and merchants visit the central location in the first subperiod, and trade is bilateral (decentralized) in the second subperiod. Specifically, a consumer is randomly and bilaterally matched with a merchant with probability one in the decentralized market. In the absence of other trading frictions (to be described next), an obvious trading arrangement would involve a merchant producing the DM good for a consumer in a bilateral match with repayment occurring in the following period in the central location, when the consumer is a producer of the CM good.

However, an important characteristic of the environment is that consumers and merchants do not overlap in the central location. Specifically, merchants arrive first and depart before all consumers arrive. We refer to this feature of the environment as the settlement friction, as in Freeman (1996b). As we shall see, this friction will prevent the *direct* settlement of private debt claims issued in the decentralized market.

A banker is born in the bankers' location in the first period of his life. In the second period, he visits the central location, returning to the bankers' location before the end of the period. Bankers can transport goods at no cost from the bankers' location to the central location and vice versa. See Figure 1 for a description of events within a period.

3. EFFICIENT ALLOCATIONS

We start by solving the planner's problem. In what follows, we consider equal weights across all generations. The planner chooses an allocation

$$(x_t^c, x_t^m, x_t^y, x_t^o, q_t) \in \mathbb{R} \times \mathbb{R}_+^4$$

in every period t to maximize the lifetime utility of consumers

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^c + u\left(q_t\right)\right]$$

subject to the feasibility conditions

$$\alpha x_t^y + \alpha x_t^o + x_t^m + x_t^c = \alpha e,$$

$$x_t^m \leq \alpha e, \tag{1}$$

the merchant's lifetime participation constraint

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^m - w\left(q_t\right)\right] \geq U^m, \tag{2}$$

and the banker's lifetime participation constraint

$$v\left(x_t^y\right) + \beta v\left(x_{t+1}^o\right) \geq U^b, \tag{3}$$

with $U^m \in \mathbb{R}_+$ and $U^b \in \mathbb{R}_+$ exogenously given. Constraint (1) arises because of the spatial separation in the environment and reminds us that the per capita amount of CM output that can be allocated to merchants cannot exceed the per capita endowment of bankers.

Because x_t^c can take on any real value in $\mathbb{R}$, we can rewrite the objective function as follows:

$$\sum_{t=0}^{\infty} \beta^t \left[\alpha e - \alpha x_t^y - \alpha x_t^o - x_t^m + u\left(q_t\right)\right].$$

Let $\beta^t \times \delta_t \in \mathbb{R}_+$ denote the multiplier on (1), $\eta \in \mathbb{R}_+$ the multiplier on (2), and $\alpha \times \beta^t \times \lambda_t \in \mathbb{R}_+$ the multiplier on (3). We can summarize the first-order conditions for the planner's problem as

$$-1 - \delta_t + \eta = 0, \tag{4}$$

$$-1 + \lambda_t v'\left(x_t^y\right) = 0, \tag{5}$$

$$-1 + \lambda_{t-1} v'\left(x_t^o\right) = 0, \tag{6}$$

$$u'\left(q_t\right) = \eta w'\left(q_t\right), \tag{7}$$

together with the complementary slackness conditions

$$\delta_t \times \left(x_t^m - \alpha e\right) = 0, \tag{8}$$

$$\eta \times \left\{ \sum_{t=0}^{\infty} \beta^t \left[x_t^m - w\left(q_t\right)\right] - U^m \right\} = 0, \tag{9}$$

$$\lambda_t \times \left[v\left(x_t^y\right) + \beta v\left(x_{t+1}^o\right) - U^b\right] = 0. \tag{10}$$

Note that condition (6) includes the term λ_{t-1}, which refers to the participation constraint of a banker born in period $t-1$. Increasing the consumption of an old banker at date t reduces the value of the objective function but relaxes the banker's participation constraint at time $t-1$, holding the other variables constant. The multiplier for that constraint is $\alpha\beta^{t-1}\lambda_{t-1}$. Because the banker discounts the future at the rate β, the marginal change in the Lagrangian with respect to an infinitesimal increase in x_t^o is given by $-\alpha\beta^t + \alpha\beta^{t-1}\lambda_{t-1} \times \beta v'\left(x_t^o\right) = -\alpha\beta^t\left[1 + \lambda_{t-1} v'\left(x_t^o\right)\right]$. As a result, when we write down the first-order conditions, the discount factor drops out of the calculations, leaving only the term λ_{t-1} multiplying the marginal utility of consumption.

Note that (5) and (6) imply the following:

$$v'\left(x_t^y\right) = v'\left(x_{t+1}^o\right) = \frac{1}{\lambda_t}.$$

Because $v : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing and concave, we have $x_t^y = x_{t+1}^o$. Note also that $\lambda_t > 0$ implies $v(x_t^y) + \beta v\left(x_{t+1}^o\right) = U^b$ at the optimum. Because $x_t^y = x_{t+1}^o$, we conclude that

$$x_t^y = x_{t+1}^o = x^* \equiv v^{-1}\left(\frac{U^b}{1+\beta}\right)$$

at all dates. In other words, the planner chooses an allocation with perfect consumption smoothing for each generation of bankers.

Condition (4) implies that the multiplier δ_t is constant at

$$\delta = -1 + \eta.$$

There are two possibilities: either $\delta = 0$ or $\delta > 0$. Consider the case $\delta = 0$. Then, we have

$$q_t = q^*$$

at all dates. Here q^* denotes the surplus-maximizing quantity (i.e., q^* is the unique solution to $u'(q) = w'(q)$).

The merchant's participation constraint implies

$$x^m = (1-\beta) U^m + w(q^*).$$

Because $x^m \leq \alpha e$, we must have

$$w(q^*) \leq \alpha e - (1-\beta) U^m.$$

As previously mentioned, we assume, throughout the analysis, that $w(q^*) \leq \alpha\beta e$. Then, for any lifetime utility level $U^m \leq \alpha e$, we can guarantee that the solution to the planner's problem is consistent with $\delta = 0$.

Finally, the consumption plan for a consumer satisfies

$$x^c = \alpha(e - 2x^*) - (1-\beta) U^m - w(q^*).$$

This concludes the description of an efficient allocation in our environment, given an exogenously set level of utility U^b for the bankers and an exogenously set level of utility $U^m \leq \alpha e$ for the merchant.

4. INTERMEDIATED CREDIT SYSTEM

To understand how the intermediated economy works, start with the bilateral meetings in the decentralized market. We consider a credit system in which a consumer purchases goods from a merchant by issuing a debt claim (i.e., a personal IOU) in a bilateral meeting. Because consumers and merchants do not overlap in the central location in the following period, the settlement of privately issued debt claims has to be intermediated by bankers.

Old bankers can purchase privately issued debt claims from merchants while visiting the central location, giving rise to a rediscounting market for privately issued debt. The claims can be subsequently redeemed in the central location upon the arrival of consumers. A banker who wants to rediscount claims in the central location must save in the first period of his life to purchase debt claims in the second. The difference between the face value of a debt claim and the discount at which the debt is purchased in the rediscounting market is the banker's profits per unit of debt.

Because the merchant knows that there is a market for rediscounting privately issued debt claims in the following period, he or she is willing to produce the DM good today in exchange for a consumer's debt claim. The price at which debt claims trade in the rediscounting market influences the amount of the DM good the merchant is willing to produce and sell to the consumer. Throughout the paper, we assume that all agents have access to a technology that permits them to perfectly identify the debt claims issued by a consumer so that counterfeiting will not be a problem. Additionally, we assume that consumers can fully commit to redeem previously issued debt claims so that default is not possible in the benchmark model.

A banker can rediscount private debt in the second period of his life only if he has access to a store of value when young. A starting point in our analysis is to endow the initial old bankers with $\bar{M} \in \mathbb{R}_+$ units of a durable, divisible, and perfectly recognizable object to serve as a record-keeping device across generations. As a result, a young banker can save part of his endowment by buying tokens from old bankers to sell to the next generation of bankers, raising funds to buy debt claims in the rediscounting market in the central location and to finance old-age consumption. We interpret the previously described arrangement as a laissez-faire economy with an exogenously given amount of outside assets (not necessarily supplied by a central bank). In what follows, we refer to the outside assets traded in the banker's location as *reserves*.

We can interpret the first and second periods of a banker's life cycle as follows. In the first period, each banker receives his endowment and uses part of it to set up a bank. The initial capital of the bank is invested in reserves, which will allow the bank to rediscount private debt at a profit in the following period.

In the bankers' location, there is a perfectly competitive market for reserves in which young and old bankers trade at the real price $\phi_t \in \mathbb{R}_+$ (in terms of the CM good). In the central location, there is a perfectly competitive rediscounting market in which old bankers and merchants trade privately issued debt claims at the real price $\rho_t \in \mathbb{R}_+$. Let $n_{t+1} \in \mathbb{R}_+$ denote the per-capita supply of private debt claims. Specifically, a unit of debt issued at date t is a promise to pay one unit of the CM good at date $t+1$.

4.1. Bankers

We start the analysis by formulating and solving the banker's problem in the first period of his life cycle. The banker chooses a consumption profile, reserve holdings, and the amount of rediscounting to maximize utility

$$\max_{\left(x_t^y, x_{t+1}^o, M_t, \hat{n}_{t+1}\right) \in \mathbb{R}_+^4} \left[v\left(x_t^y\right) + \beta v\left(x_{t+1}^o\right) \right]$$

subject to the first-period budget constraint

$$x_t^y + \phi_t M_t \leq e,$$

the second-period budget constraint

$$x_{t+1}^o + \rho_{t+1} \hat{n}_{t+1} \leq \phi_{t+1} M_t + \hat{n}_{t+1},$$

and the liquidity constraint

$$\rho_{t+1} \hat{n}_{t+1} \leq \phi_{t+1} M_t.$$

Here $M_t \in \mathbb{R}_+$ denotes reserve holdings in period t, and $\hat{n}_{t+1} \in \mathbb{R}_+$ is the amount of debt claims the banker decides to purchase in the rediscounting market at date $t+1$. The liquidity constraint is the key constraint in the banker's problem. To rediscount debt claims at a profit in the second period of his life cycle, the banker must accumulate reserves in the first period to sell to young bankers in the second period so that he has funds to purchase debt claims in the rediscounting market in the central location.

Let $\beta \times \mu_{t+1} \in \mathbb{R}_+$ denote the Lagrange multiplier on the liquidity constraint. The corresponding first-order conditions are given by

$$-\phi_t v'\left(x_t^y\right) + \beta\phi_{t+1} v'\left(x_{t+1}^o\right) + \beta\mu_{t+1}\phi_{t+1} = 0,$$

$$\left(1 - \rho_{t+1}\right) v'\left(x_{t+1}^o\right) - \mu_{t+1}\rho_{t+1} = 0,$$

$$x_t^y = e - \phi_t M_t,$$

$$x_{t+1}^o = \phi_{t+1} M_t + \left(1 - \rho_{t+1}\right)\hat{n}_{t+1}.$$

Additionally, we have the complementary slackness condition:

$$\mu_{t+1}\left(\rho_{t+1}\hat{n}_{t+1} - \phi_{t+1} M_t\right) = 0.$$

The previously derived first-order conditions can be combined to obtain the Euler equation:

$$\phi_t v'\left(x_t^y\right) = \beta \frac{\phi_{t+1}}{\rho_{t+1}} v'\left(x_{t+1}^o\right). \tag{11}$$

The left-hand side of (11) gives the marginal cost of an additional unit of reserves at date t. The banker gives up consumption at date t to increase his balances at the real price ϕ_t so that he can rediscount debt claims at date $t+1$. An additional unit of reserves at date $t+1$ allows him to purchase ϕ_{t+1}/ρ_{t+1} extra units of debt claims, increasing his old-age consumption.

Note that a banker obtains old-age income from two sources: the real return on reserves held across periods and the profits from rediscounting debt claims in the second period. Thus, the effective return on asset holdings in the Euler equation is $\frac{\phi_{t+1}}{\phi_t \rho_{t+1}}$.

We can also use (11) to derive the real price of debt claims in the rediscounting market at date $t+1$ as

$$\rho_{t+1} = \beta \frac{\phi_{t+1}}{\phi_t} \frac{v'\left(x^o_{t+1}\right)}{v'\left(x^y_t\right)}.$$

4.2. Consumers and Merchants

Consider the consumer's problem in the decentralized market. Let $n_{t+1} \in \mathbb{R}_+$ denote the amount of debt the consumer issues at date t in exchange for $q_t \in \mathbb{R}_+$ units of the DM good. Let $V^c_t \in \mathbb{R}_+$ denote the consumer's lifetime utility in period t *after* retiring debt claims issued in the previous period. The Bellman equation for the consumer is

$$V^c_t = u\left(q_t\right) + \beta\left(-n_{t+1} + V^c_{t+1}\right).$$

If we denote the merchant's lifetime utility at date t by $V^m_t \in \mathbb{R}$, then his Bellman equation can be written as

$$V^m_t = -w\left(q_t\right) + \beta\left(\rho_{t+1} n_{t+1} + V^m_{t+1}\right).$$

Note that V^m_t is the merchant's utility in period t after receiving payment for the debt claims he holds at the beginning of the period. As we have seen, bankers have to be willing to intermediate the settlement of private claims because consumers and merchants do not overlap in the central location to directly settle debt claims. For this reason, the value of a debt claim for the merchant is given by $\rho_{t+1} n_{t+1}$.

The consumer is willing to trade in the decentralized market if the terms of trade $(q_t, n_{t+1}) \in \mathbb{R}^2_+$ satisfy

$$u\left(q_t\right) - \beta n_{t+1} \geq 0.$$

The merchant is willing to trade if

$$-w\left(q_t\right) + \beta \rho_{t+1} n_{t+1} \geq 0.$$

We assume that the consumer makes a take-it-or-leave-it offer to the merchant in the decentralized market.[4] Given this bargaining protocol, the terms (q_t, n_{t+1}) are determined

[4] This assumption is without loss of generality for our analysis.

by solving the following problem:

$$\max_{(q_t, n_{t+1}) \in \mathbb{R}^2_+} \left[u(q_t) - \beta n_{t+1} \right]$$

subject to

$$-w(q_t) + \beta \rho_{t+1} n_{t+1} \geq 0.$$

The first-order conditions are

$$\frac{u'(q_t)}{c'(q_t)} = \frac{1}{\rho_{t+1}}$$

and

$$w(q_t) = \beta \rho_{t+1} n_{t+1}.$$

The amount of goods the consumer gets from the merchant depends on the discount rate that is expected to prevail in the following period. If there is no discounting (i.e., $\rho_{t+1} = 1$), the consumer gets the surplus-maximizing quantity q^* in exchange for debt claims with real value $\beta^{-1} w(q^*)$. If the discount rate is zero, the consumer credit market works smoothly, allowing consumers to purchase the surplus-maximizing quantity on credit. In the absence of discounting, we obtain efficient decentralized exchange even though the functioning of the credit system requires intermediation by profit-maximizing agents.

If $\rho_{t+1} < 1$, then the consumer gets less than the surplus-maximizing quantity, so DM output is suboptimal. Each unit of debt issued by the consumer is worth less than one unit of the CM good for the merchant when the discount rate is positive. Because the merchant anticipates a positive discount rate in the following period, he is willing to trade only if the disutility of production is less than that of the surplus-maximizing quantity.

4.3. Equilibrium

The market-clearing conditions in the market for reserves and the rediscounting market are

$$\bar{M}_t = M_t$$

and

$$\alpha \hat{n}_t = n_t,$$

respectively.

Let $m_t \in \mathbb{R}_+$ denote real reserve holdings per young banker. Using the previously derived first-order conditions, together with the market-clearing conditions, we obtain the following equilibrium relations in the laissez-faire economy:

$$x_t^y = e - m_t, \tag{12}$$

$$x_t^o = \frac{\phi_{t+1}}{\phi_t} m_t + (1 - \rho_t)\, \hat{n}_t, \tag{13}$$

$$v'\left(x_t^y\right) = \frac{\phi_{t+1}}{\phi_t \rho_{t+1}} \beta v'\left(x_{t+1}^o\right), \tag{14}$$

$$\left(\frac{1}{\rho_{t+1}} - 1\right)\left(\rho_{t+1}\hat{n}_{t+1} - \frac{\phi_{t+1}}{\phi_t} m_t\right) = 0, \tag{15}$$

$$\frac{u'\left(q_t\right)}{w'\left(q_t\right)} = \frac{1}{\rho_{t+1}}, \tag{16}$$

$$w\left(q_t\right) = \alpha\beta\rho_{t+1}\hat{n}_{t+1}. \tag{17}$$

Condition (15) indicates whether the liquidity constraint is binding. As we can see, a binding liquidity constraint implies that the debt claims sell below par value in the rediscounting market. Condition (16) gives DM output as a function of the discount rate, providing a link between real activity in decentralized markets and liquidity in the rediscounting market.

Combining (16) and (17), we can obtain per-capita debt issuance as a function of the discount rate by using the following relation:

$$\frac{u'\left(w^{-1}\left(\alpha\beta\rho_t\hat{n}_t\right)\right)}{w'\left(w^{-1}\left(\alpha\beta\rho_t\hat{n}_t\right)\right)} = \frac{1}{\rho_t}.$$

This condition implicitly defines purchases in the rediscounting market as a function of the discount rate, providing a demand relation for debt claims in the secondary market.

We can now formally define an equilibrium in the intermediated economy as follows.

Definition 1 *An equilibrium can be defined as a sequence* $\{x_t^y, x_t^o, \hat{n}_t, m_t, \rho_t, \phi_t, q_t\}_{t=0}^{\infty}$ *satisfying (12)-(17) with* $\rho_t\hat{n}_t \leq m_t \leq e$ *at all dates.*

For the remainder of this section, we restrict attention to stationary equilibria with the property that the value of reserves remains constant over time. Then, a stationary plan $(x^y, x^o, \hat{n}, m, \rho, q)$ will satisfy the following equilibrium conditions:

$$x^y = e - m, \tag{18}$$

$$x^o = m + (1-\rho)\,\hat{n}, \tag{19}$$

$$v'(x^y) = \frac{\beta}{\rho} v'(x^o), \tag{20}$$

$$\left(\frac{1}{\rho} - 1\right)(\rho\hat{n} - m) = 0, \tag{21}$$

$$\frac{u'(q)}{w'(q)} = \frac{1}{\rho}, \tag{22}$$

$$w(q) = \alpha\beta\rho\hat{n}. \tag{23}$$

Because consumption is nonnegative, real balances must satisfy the upper bound $m \leq e$ in equilibrium. Additionally, the liquidity constraint imposes the lower bound $m \geq \rho\hat{n}$. Thus, in addition to the previously described conditions, a stationary equilibrium must satisfy the following boundary conditions:

$$\rho\hat{n} \leq m \leq e. \tag{24}$$

Summing up, a stationary equilibrium can be defined as a stationary plan $(x^y, x^o, \hat{n}, m, \rho, q)$ satisfying (18)-(24).

4.4. Existence and Properties

We now provide conditions for the existence of a stationary equilibrium in the laissez-faire intermediated economy. Let $\hat{\Psi}(\cdot)$ denote the inverse of $u'(\cdot)/w'(\cdot)$.[5] Then, equilibrium conditions (22) and (23) imply the following relation:

$$w\left(\hat{\Psi}\left(\frac{1}{\rho}\right)\right) = \alpha\beta\rho\hat{n}.$$

[5] Recall that $u'(\cdot)/w'(\cdot)$ is strictly decreasing.

Define the function $\Psi(y) \equiv w\left(\hat{\Psi}(y)\right)$ for all $y > 0$. Then, per-capita rediscounting can be written as

$$\hat{n} = \frac{1}{\alpha\beta} \times \frac{1}{\rho}\Psi\left(\frac{1}{\rho}\right).$$

Given these equilibrium relations, we can rewrite the banker's Euler equation (20) as

$$v'(e-m) = \frac{\beta}{\rho}v'\left(m + \frac{1}{\alpha\beta}\left(\frac{1-\rho}{\rho}\right)\Psi\left(\frac{1}{\rho}\right)\right). \tag{25}$$

Also, we can rewrite the complementary slackness condition (21) as

$$\left(\frac{1}{\rho} - 1\right)\left[\frac{1}{\alpha\beta}\Psi\left(\frac{1}{\rho}\right) - m\right] = 0. \tag{26}$$

Then, a stationary equilibrium in the laissez-faire economy can be defined as a pair (m, ρ), satisfying equations (25) and (26) as well as the following inequalities:

$$\frac{1}{\alpha\beta}\Psi\left(\frac{1}{\rho}\right) \leq m \leq e. \tag{27}$$

Once we have determined reserve balances and the discount rate in a stationary equilibrium, we can derive the other equilibrium values by using (18)-(23).

4.4.1. Nonbinding Liquidity Constraint

Consider now the existence of a stationary equilibrium with a nonbinding liquidity constraint. As we have seen, a necessary condition for efficiency is to have a slack liquidity constraint so that debt claims trade at par value in the rediscounting market. Thus, it makes sense to start by characterizing equilibria without discounting of private IOUs. If the liquidity constraint is not binding, we have $\rho = 1$. Then, equation (25) implicitly defines equilibrium reserve balances m as

$$v'(e-m) = \beta v'(m). \tag{28}$$

The other equilibrium values are given by

$$q = q^*, \tag{29}$$

$$\hat{n} = \frac{w(q^*)}{\alpha\beta}, \tag{30}$$

$$x^{y} = e - m, \tag{31}$$

$$x^{o} = m. \tag{32}$$

Note that $x^y + x^o = e$ holds in equilibrium so that a banker earns zero profits when rediscounting debt claims. In that case, the banker's franchise value is zero, and the return on savings equals the return on assets (i.e., the banker earns no extra income from rediscounting). The amount of debt claims purchased by an old banker is given by (30), which is consistent with surplus maximization in a bilateral match.

To establish the existence of a steady state with a nonbinding liquidity constraint, we need to verify whether a slack liquidity constraint is consistent with the boundary equilibrium conditions, including the liquidity constraint for the banker. Condition (27) implies that a steady state with a nonbinding liquidity constraint exists provided reserve balances satisfy

$$\frac{w\left(q^{*}\right)}{\alpha\beta} \leq m. \tag{33}$$

Condition (33) states that the real value of reserve holdings must be at least the same as the socially efficient per-capita discounting amount. Thus, an equilibrium with a nonbinding liquidity constraint requires a minimum income level in the second period of the banker's life cycle. In other words, the real value of reserves in old age has to be sufficiently large to ensure that (33) holds in order to have an equilibrium with a nonbinding liquidity constraint.

As we have seen, the banker's old-age income can be broken down into two parts: (i) the gross returns from holding assets across periods and (ii) the earnings from rediscounting. Because no discounting is socially optimal, the banker's franchise value must be zero to obtain efficiency in the payments system. Then, it is necessary that reserves be an asset class that yields a sufficiently high return across periods to attain the required old-age income consistent with no discounting. As we will show, depending on the parameters, this is *not* a sufficient condition for overall efficiency.

To obtain a closed-form solution for equilibrium real balances, we make the following assumption.

Assumption 1 Assume that $v(x) = \ln x$.

Then, (28) implies that

$$m = \frac{e\beta}{1+\beta}, \tag{34}$$

so that condition (33) becomes

$$\frac{w\left(q^{*}\right)}{\alpha\beta} \leq \frac{e\beta}{1+\beta}. \tag{35}$$

Thus, a necessary and sufficient condition for the existence of a stationary equilibrium with a nonbinding liquidity constraint is given by

$$\frac{1+\beta}{\beta} w\left(q^{*}\right) \leq \alpha\beta e. \tag{36}$$

Recall that we have assumed, throughout the paper, that $w\left(q^{*}\right) \leq \alpha\beta e$. Because $\frac{1+\beta}{\beta} > 1$, the parametric assumption $w\left(q^{*}\right) \leq \alpha\beta e$ does not necessarily guarantee that (36) always holds. Thus, we can state our existence proposition as follows.

Proposition 2 *A stationary equilibrium with a nonbinding liquidity constraint exists if and only if (36) holds. The equilibrium quantities and prices satisfy (28)-(32).*

Note that bankers do not achieve perfect consumption smoothing in a stationary equilibrium with a nonbinding liquidity constraint. An individual banker consumes $\frac{e\beta}{1+\beta}$ in the first period and $\frac{e}{1+\beta}$ in the second. Although the payments system is efficient, the overall allocation of resources is suboptimal because bankers do not attain a perfectly smooth consumption profile in the laissez-faire economy.

4.4.2. Binding Liquidity Constraint

Consider now the case $w\left(q^{*}\right) \leq \alpha\beta e < \frac{1+\beta}{\beta} w\left(q^{*}\right)$. In this region of the parameter space, the liquidity constraint is necessarily binding. Under Assumption 1, and restricting attention to stationary equilibria, we obtain the following solution:

$$x^{y} = \frac{e}{1+\beta},$$

$$x^{o} = \hat{n} = \frac{\beta e}{1+\beta} \times \frac{u'\left(w^{-1}\left(\frac{\alpha\beta^{2}e}{1+\beta}\right)\right)}{w'\left(w^{-1}\left(\frac{\alpha\beta^{2}e}{1+\beta}\right)\right)},$$

$$\rho = \frac{w'\left(w^{-1}\left(\frac{\alpha\beta^2 e}{1+\beta}\right)\right)}{u'\left(w^{-1}\left(\frac{\alpha\beta^2 e}{1+\beta}\right)\right)} < 1,$$

$$q = w^{-1}\left(\frac{\alpha\beta^2 e}{1+\beta}\right) < q^*.$$

As we can see, DM output is below the surplus-maximizing quantity, and the banker's consumption profile is not perfectly smooth.

The socially efficient per-capita discounting volume is larger than the equilibrium old-age income for intermediaries when $w(q^*) \leq \alpha\beta e < \frac{1+\beta}{\beta} w(q^*)$. As a result, the flow of funds into the rediscounting market is insufficient to drive the discount rate to zero. The existence of discounting implies a strictly positive franchise value for the banker, which is socially inefficient. The following proposition summarizes these results.

Proposition 3 *If $w(q^*) \leq \alpha\beta e < \frac{1+\beta}{\beta} w(q^*)$, then the discount rate is positive in a stationary equilibrium. The ensuing allocation is inefficient.*

One reason for a relatively small amount of capital in the intermediary sector is the existence of barriers to entry into banking. In our analysis, this could be reflected in a small value for the parameter α, which makes it more likely that the economy will fall in the region $w(q^*) \leq \alpha\beta e < \frac{1+\beta}{\beta} w(q^*)$. If this situation arises, then there is insufficient capital in the intermediary sector to support the efficient volume of rediscounting in the secondary debt market. As a result, the payments system is inefficient, resulting in suboptimal production and consumption in decentralized markets.

So far, we have considered an economy without interbank credit markets in which intermediaries hold reserves as a store of value across periods to take advantage of rediscounting opportunities. We have found that an efficient payments system can arise in equilibrium when there is sufficient initial wealth in the intermediary sector relative to the size of the retail sector. Although the intermediated consumer credit system works smoothly in that case, the overall allocation of resources is inefficient because intermediaries do not attain perfect consumption smoothing. An efficient payments system requires a zero franchise value for the rediscounting business, so the real return on reserves is the unique factor determining intertemporal exchange for intermediaries of the same generation. We have shown that the return on reserves is not sufficiently large to attain perfect consumption smoothing for intermediaries. Our next step, then, is to investigate whether the existence of interbank credit markets in the bankers' location can be consistent with no discounting of private IOUs, regardless of the fundamentals of the economy, and can simultaneously improve consumption smoothing for intermediaries so that an efficient equilibrium allocation exists.

5. COMPLETE MARKETS

We now consider the introduction of an interbank market in the bankers' location in which an old banker can borrow from young bankers an amount $b_t \in \mathbb{R}_+$ at the beginning of the period with repayment at the end of the period. Let $r_t \in \mathbb{R}_+$ denote the intra-period real interest rate on a short-term loan. In what follows, we assume that the debt issued by bankers can be perfectly enforced in the bankers' location.

5.1. Bankers

Consider now the possibility of an interbank market in the bankers' location with repayment of debt claims at the end of the period. Then, we can write the banker's problem as

$$\max_{\left(x_t^y, x_{t+1}^o, M_t, l_t, \hat{n}_{t+1}, b_{t+1}\right) \in \mathbb{R}_+^6} \left[v\left(x_t^y\right) + \beta v\left(x_{t+1}^o\right) \right]$$

subject to the first-period budget constraint

$$x_t^y + \phi_t M_t \leq e + r_t l_t,$$

the first-period liquidity constraint

$$\phi_t M_t + l_t \leq e, \tag{37}$$

the second-period budget constraint

$$x_{t+1}^o + \rho_{t+1} \hat{n}_{t+1} + r_{t+1} b_{t+1} \leq \phi_{t+1} M_t + \hat{n}_{t+1},$$

and the second-period liquidity constraint

$$\rho_{t+1}\hat{n}_{t+1} \leq \phi_{t+1}M_t + b_{t+1}. \tag{38}$$

Here, $l_t \in \mathbb{R}_+$ denotes the loan amount in the interbank market in period t, and $b_{t+1} \in \mathbb{R}_+$ denotes the amount borrowed in period $t+1$. Note that a banker is a lender in the interbank market when young and a borrower when old. Note also that a young banker who decides to increase her lending in the interbank market necessarily reduces her reserve holdings because we have assumed that the market for reserves closes *before* the repayment of loans occurs at the end of the period. Thus, the decision to increase lending in the interbank market necessarily leads to higher consumption in the first period of the life cycle when $r_t > 0$.

Let $\mu_t \in \mathbb{R}_+$ denote the Lagrange multiplier on the first-period liquidity constraint (37), and let $\beta \times \varphi_{t+1} \in \mathbb{R}_+$ denote the Lagrange multiplier on the second-period liquidity constraint (38). The first-order conditions are given by

$$-\phi_t v'\left(x_t^y\right) - \phi_t\mu_t + \beta\phi_{t+1}v'\left(x_{t+1}^o\right) + \beta\varphi_{t+1}\phi_{t+1} = 0,$$

$$r_t v'\left(x_t^y\right) - \mu_t = 0,$$

$$\left(1-\rho_{t+1}\right)v'\left(x_{t+1}^o\right) - \varphi_{t+1}\rho_{t+1} = 0,$$

$$-v'\left(x_{t+1}^o\right)r_{t+1} + \varphi_{t+1} = 0,$$

$$x_t^y = e + r_t l_t - \phi_t M_t,$$

$$x_{t+1}^o = \phi_{t+1}M_t + \left(1-\rho_{t+1}\right)\hat{n}_{t+1} - b_{t+1}r_{t+1}.$$

In addition, we have the complementary slackness conditions:

$$\varphi_{t+1}\left(\rho_{t+1}\hat{n}_{t+1} - \phi_{t+1}M_t - b_{t+1}\right) = 0$$

and

$$\mu_t\left(\phi_t M_t + l_t - e\right) = 0.$$

Following the same steps as in the previous section, we can use the first-order conditions to derive the following equilibrium relations:

$$v'\left(x_t^y\right) = \beta \frac{\phi_{t+1}}{\phi_t \rho_{t+1}} \frac{1}{1+r_t} v'\left(x_{t+1}^o\right)$$

and

$$r_{t+1} = \frac{1}{\rho_{t+1}} - 1.$$

The first relation is the Euler equation for the allocation of consumption across periods, with the real intertemporal price given by $\frac{\phi_{t+1}}{\phi_t \rho_{t+1}} \frac{1}{1+r_t}$. A young banker can lend in the interbank market to increase his contemporaneous consumption, but he has to reduce his reserve holdings, leading to lower consumption in old age. The second relation is a no-arbitrage condition. In other words, there are no arbitrage opportunities in the interbank market provided that the real return on each unit of debt rediscounted in the central location equals the cost of borrowing in interbank markets.

5.2. Equilibrium

To construct an equilibrium allocation, we need to add the market-clearing condition in the interbank market:

$$l_t = b_t.$$

Then, we can combine the first-order conditions for the banker's and consumer's optimization problem with the market-clearing conditions to derive the following equilibrium relations:

$$x_t^y = e + r_t l_t - m_t, \tag{39}$$

$$x_t^o = \frac{\phi_{t+1}}{\phi_t} m_t + (1 - \rho_t)\, \hat{n}_t - l_t r_t, \tag{40}$$

$$v'\left(x_t^y\right) = \beta \frac{\phi_{t+1}}{\phi_t \rho_{t+1}} \frac{1}{1+r_t} v'\left(x_{t+1}^o\right), \tag{41}$$

$$\left(\frac{1}{\rho_t} - 1\right)\left(\rho_t \hat{n}_t - \frac{\phi_{t+1}}{\phi_t} m_t - l_t\right) = 0, \tag{42}$$

$$r_{t+1} = \frac{1}{\rho_{t+1}} - 1, \tag{43}$$

$$r_t\left(m_t + l_t - e\right) = 0, \tag{44}$$

$$\frac{u'\left(q_t\right)}{w'\left(q_t\right)} = \frac{1}{\rho_{t+1}}, \tag{45}$$

$$w\left(q_t\right) = \alpha\beta\rho_{t+1}\hat{n}_{t+1}. \tag{46}$$

A key change in the equilibrium relations is that the condition determining the slackness of the liquidity constraint in the rediscounting market now includes an extra term, given that old bankers can borrow in the interbank market to finance purchases in the rediscounting market. Also, condition (43) indicates that a positive interest rate in the interbank market necessarily implies a binding liquidity constraint in the rediscounting market.

In addition, an equilibrium allocation must satisfy the boundary conditions

$$\rho_t\hat{n}_t \leq \frac{\phi_{t+1}}{\phi_t}m_t + l_t \tag{47}$$

and

$$m_t + l_t \leq e \tag{48}$$

at all dates. We can now formally define an equilibrium for the intermediated economy with complete interbank markets.

Definition 4 *An equilibrium in the economy with complete markets can be defined as a sequence* $\{x_t^y, x_t^o, \hat{n}_t, m_t, l_t, \rho_t, \phi_t, q_t, r_t\}_{t=0}^{\infty}$ *satisfying (39)-(48) at all dates.*

As in the previous section, we restrict attention to equilibria in which the value of reserves is constant over time.

5.3. Existence and Properties

In what follows, we focus on stationary allocations in which the discount rate and the interest rate remain constant over time. A stationary equilibrium for the economy with complete interbank markets can be defined as a plan $(x^y, x^o, \hat{n}, m, l, \rho, q, r)$ satisfying

$$x^y = e + rl - m,$$

$$x^o = m + (1-\rho)\,\hat{n} - lr,$$

$$v'(x^y) = \beta v'(x^o),$$

$$\left(\frac{1}{\rho} - 1\right)(\rho\hat{n} - m - l) = 0,$$

$$r = \frac{1}{\rho} - 1,$$

$$r(m + l - e) = 0,$$

$$\frac{u'(q)}{w'(q)} = \frac{1}{\rho},$$

$$w(q) = \alpha\beta\rho\hat{n},$$

together with the boundary conditions $\rho\hat{n} \leq m + l \leq e$.

Consider now the existence of a stationary equilibrium with a nonbinding liquidity constraint. Because $\rho = 1$ implies $r = 0$, a stationary equilibrium with a slack liquidity constraint satisfies (28) and (30). Going back to the boundary conditions, we can see that a nonbinding liquidity constraint requires

$$\frac{w(q^*)}{\alpha\beta} \leq m + l,$$

with m defined as in (28). The only difference from the equilibrium conditions when interbank credit was not possible is the presence of the loan amount l on the right-hand side. Given these restrictions on the equilibrium allocation, we can define the equilibrium loan amount as

$$l = l^* \equiv \max\left\{\frac{w(q^*)}{\alpha\beta} - m, 0\right\}$$

so that the liquidity constraint holds as an equality at all dates. If we choose the loan amount in this way, then we have

$$l + m = \frac{w(q^*)}{\alpha\beta} \leq e.$$

Because the interest rate is zero in an equilibrium with a nonbinding liquidity constraint, the loan amount l^* is consistent with market clearing. Thus, we have shown that it is possible to construct an equilibrium with a nonbinding liquidity constraint in the economy with complete interbank markets regardless of the region of the parameter space. We summarize these findings in the following proposition.

Proposition 5 *In the economy with complete markets, an equilibrium with a nonbinding liquidity constraint always exists.*

We should emphasize that the previous result holds for a generic utility function $v(x)$. However, we can obtain a closed-form solution for the loan amount if, for instance, we consider the functional form in Assumption 1. In that case, we have

$$l^* = \max\left\{\frac{w(q^*)}{\alpha\beta} - \frac{e\beta}{1+\beta}, 0\right\}.$$

Note that the loan amount will be zero in the region of the parameter space where $\frac{1+\beta}{\beta}w(q^*) \leq \alpha\beta e$ and will be positive in the region where $\frac{1+\beta}{\beta}w(q^*) > \alpha\beta e \geq w(q^*)$. The intra-period interest rate is always zero in an equilibrium without discounting of private claims.

We have shown that, under complete interbank markets, an equilibrium without discounting always exists. In other words, allowing bankers to trade in a frictionless credit market with perfect enforcement of debt claims is sufficient to overcome the settlement frictions in the model (i.e., spatial separation and unsynchronized trading patterns). However, the allocation is not efficient because bankers do not attain a perfectly smooth consumption profile.

6. MONETARY POLICY

We now consider the existence of a central bank that manages the amount of reserves issued each period. As we have seen, an equilibrium without discounting of private IOUs is a necessary but not sufficient condition for efficiency. We will show that an active monetary policy that sets the interest rate on reserves equal to the rate of time preference can lead to efficient intertemporal exchange in the intermediary sector. However, overall efficiency is not always achieved when interbank credit markets are incomplete.

The government's budget constraint is given by

$$\phi_t \bar{M}_t = \phi_t (1 + i_{t-1}) \bar{M}_{t-1} + \tau_t. \tag{49}$$

Here, $\tau_t \in \mathbb{R}$ denotes the real value of transfers to old bankers, and $i_t \in \mathbb{R}_+$ is the interest rate on reserves. We assume that the government intervenes at the end of each period to ensure that the supply of reserves returns to the same level as that of the beginning of the period. In this case, the money supply follows the law of motion:

$$m_t = \frac{\phi_t}{\phi_{t-1}} m_{t-1}. \tag{50}$$

6.1. Incomplete Markets

We now consider the economy without interbank intertemporal exchange to evaluate the role of monetary policy. The solution to the banker's problem can be summarized by the first-order conditions:

$$\phi_t v'\left(x_t^y\right) = \beta\left(1+i_t\right)\frac{\phi_{t+1}}{\rho_{t+1}} v'\left(x_{t+1}^o\right),$$

$$x_t^y = e - \phi_t M_t,$$

$$x_{t+1}^o = \phi_{t+1}\left(1+i_t\right) M_t + \left(1-\rho_{t+1}\right)\hat{n}_{t+1} + \tau_{t+1}.$$

Additionally, we have the complementary slackness condition:

$$\left(\frac{1}{\rho_{t+1}} - 1\right)\left[\rho_{t+1}\hat{n}_{t+1} - \phi_{t+1}\left(1+i_t\right) M_t - \tau_{t+1}\right] = 0.$$

To obtain an equilibrium, we follow the same steps as in previous sections. In addition, we use the government budget constraint to obtain the value of the equilibrium transfers to old bankers. Then, we arrive at the following equilibrium conditions:

$$x_t^y = e - m_t, \tag{51}$$

$$x_t^o = \phi_t m_t + \left(1-\rho_t\right)\hat{n}_t, \tag{52}$$

$$v'\left(x_t^y\right) = \beta\left(1+i_t\right)\frac{\phi_{t+1}}{\phi_t\rho_{t+1}} v'\left(x_{t+1}^o\right), \tag{53}$$

$$\left(\frac{1}{\rho_t} - 1\right)\left(\rho_t\hat{n}_t - m_t\right) = 0, \tag{54}$$

together with (16), (17), (50), and the boundary conditions

$$\rho_t\hat{n}_t \leq m_t \leq e. \tag{55}$$

A formal definition of equilibrium is now provided.

Definition 6 *An equilibrium in the economy with active monetary policy can be described as a sequence* $\{x_t^y, x_t^o, \hat{n}_t, m_t, \rho_t, \phi_t, q_t\}_{t=0}^{\infty}$ *satisfying (16), (17), and (50)-(55), with* $\{i_t\}_{t=0}^{\infty}$ *taken as given.*

In what follows, we restrict attention to policies that maintain a constant interest rate on reserves over time. As a result, we can simplify our equilibrium definition as follows. Given $1+i$, a stationary equilibrium can be defined as a pair (ρ, m) satisfying

$$v'(e-m) = \frac{(1+i)}{\rho}\beta v'\left(m + \frac{1}{\alpha\beta}\left(\frac{1-\rho}{\rho}\right)\Psi\left(\frac{1}{\rho}\right)\right),$$

$$\left(\frac{1}{\rho}-1\right)\left[\frac{1}{\alpha\beta}\Psi\left(\frac{1}{\rho}\right) - m\right] = 0,$$

$$\frac{1}{\alpha\beta}\Psi\left(\frac{1}{\rho}\right) \leq m \leq e.$$

As in Section 4, stationary equilibria can have either a binding or a nonbinding liquidity constraint.

6.1.1. Nonbinding Liquidity Constraint

In an equilibrium with a nonbinding liquidity constraint, we can implicitly define real balances as a function of the interest rate on reserves as

$$v'(e-m(i)) = (1+i)\beta v'(m(i)). \tag{56}$$

Then, a stationary equilibrium with a nonbinding liquidity constraint exists provided that

$$\frac{w(q^*)}{\alpha\beta} \leq m(i). \tag{57}$$

Note that the interest rate on reserves influences old-age income so that the minimum required level of income consistent with no discounting can potentially be attained if the central bank sets the interest rate on reserves in such a way that it raises the real value of reserve balances in old age. If we consider the functional form in Assumption 1, we obtain the following closed-form solution for real balances:

$$m(i) = \frac{e\beta(1+i)}{1+\beta(1+i)}.$$

In this case, we have $m'(i) > 0$ so that a higher level of the interest rate on reserves raises real balances for intermediaries. Then, condition (57) becomes

$$1 + i \geq \frac{1}{\beta\left[\frac{\alpha\beta e}{w(q^*)} - 1\right]}.$$

To obtain a well-defined demand function for real balances, the interest rate on reserves must also satisfy the upper bound

$$1 + i \leq \frac{1}{\beta}.$$

Thus, a necessary and sufficient condition for the existence of a stationary equilibrium without discounting is

$$\frac{\alpha\beta e}{2} \geq w(q^*). \tag{58}$$

If (58) holds, then any level of the interest rate on reserves in the interval

$$\frac{1}{\beta\left[\frac{\alpha\beta e}{w(q^*)} - 1\right]} \leq 1 + i \leq \frac{1}{\beta} \tag{59}$$

leads to a stationary equilibrium with a nonbinding liquidity constraint. We summarize these results in the following proposition.

Proposition 7 *A stationary equilibrium with a nonbinding liquidity constraint exists if and only if* $\frac{\alpha\beta e}{2} \geq w(q^*)$ *and* $1 + i$ *lies in the interval (59).*

An equilibrium without discounting exists provided the policy rate is sufficiently large to attain the minimum required level of old-age income consistent with no discounting of private IOUs. As we have seen, the planner's solution also requires perfect consumption smoothing for bankers. We can attain perfect consumption smoothing if we set the interest rate on reserves equal to the subjective rate of time preference:

$$1 + i \to \frac{1}{\beta}.$$

In other words, the central bank should set the interest rate on reserves at the upper bound. Such a policy prescription is a version of the Friedman rule, which eliminates the opportunity cost of holding reserve balances across periods for the rediscounting of private IOUs. At the Friedman rule, we have

$$m\left(\beta^{-1}-1\right)=\frac{e}{2}$$

and

$$x^{o}=x^{y}=\frac{e}{2}.$$

The following proposition summarizes these findings.

Proposition 8 *If* $\frac{\alpha\beta e}{2}\geq w\left(q^{*}\right)$, *the Friedman rule* $1+i\rightarrow\frac{1}{\beta}$ *leads to an efficient allocation.*

To provide some intuition for the relation between the condition $\frac{\alpha\beta e}{2}\geq w\left(q^{*}\right)$ and efficiency in the allocation of resources, it is helpful to rewrite that condition as

$$\frac{e}{2}\geq\frac{1}{\alpha}\times\frac{w\left(q^{*}\right)}{\beta}.$$

The right-hand side gives the per-capita discounting volume consistent with an efficient allocation. The left-hand side gives the initial per-capita wealth in the banking system *at the Friedman rule* (i.e., when agents observe that the central bank has set the interest rate on reserves at the upper bound and form their expectations accordingly when forecasting prices in the future). If the initial per-capita wealth in the intermediary sector is at least as large as the socially efficient per-capita discounting volume, then the Friedman rule is consistent with an efficient allocation, and we can say that there is abundant liquidity flowing into the rediscounting market to drive the discount rate to zero.[6]

6.1.2. Binding Liquidity Constraint

We now consider the case $\frac{\alpha\beta e}{2}<w\left(q^{*}\right)\leq\alpha\beta e$. In this region of the parameter space, the liquidity constraint is binding for any value of the interest rate on reserves. We can then

[6] At the Friedman rule, the income of an old banker is $\frac{1}{\beta}\frac{e}{2}$. The government levies a lump-sum tax $-\left(1-\frac{1}{\beta}\right)\frac{e}{2}$ to finance the implementation of the Friedman rule. Thus, the banker's available wealth in old age is $\frac{e}{2}$, which is the same as his initial wealth.

solve for the equilibrium quantities and prices to arrive at the following allocation:

$$x^y = \frac{e}{1+(1+i)\,\beta},$$

$$x^o = \hat{n} = \frac{\beta e}{\frac{1}{1+i}+\beta} \times \frac{u'\left(w^{-1}\left(\frac{\alpha\beta^2 e}{\frac{1}{1+i}+\beta}\right)\right)}{w'\left(w^{-1}\left(\frac{\alpha\beta^2 e}{\frac{1}{1+i}+\beta}\right)\right)},$$

$$\rho = \frac{w'\left(w^{-1}\left(\frac{\alpha\beta^2 e}{\frac{1}{1+i}+\beta}\right)\right)}{u'\left(w^{-1}\left(\frac{\alpha\beta^2 e}{\frac{1}{1+i}+\beta}\right)\right)} < 1,$$

$$q = w^{-1}\left(\frac{\alpha\beta^2 e}{\frac{1}{1+i}+\beta}\right) < q^*.$$

Note that all variables now depend on the level of the interest rate on reserves set by the central bank. The equilibrium discount rate is strictly decreasing in the interest rate on reserves $1+i$ so that consumers are better off when the central bank raises the level of the interest rate on reserves. The utility of bankers can either increase or decrease. For instance, if we assume that $u(q) = (1-\sigma)^{-1} q^{1-\sigma}$ with $0 < \sigma < 1$ and $w(q) = q$, then it can be shown that the indirect utility of bankers is strictly decreasing in $1+i$.[7] For this economy, the welfare effects of an increase in the interest rate on reserves are ambiguous.

At the Friedman rule, the central bank sets the interest rate on reserves at the upper bound $1+i \to \beta^{-1}$. The associated equilibrium allocation is given by

$$x^y = \frac{e}{2},$$

$$x^o = \hat{n} = \frac{e}{2}\frac{u'\left(w^{-1}\left(\frac{\alpha\beta e}{2}\right)\right)}{w'\left(w^{-1}\left(\frac{\alpha\beta e}{2}\right)\right)} > \frac{e}{2},$$

$$\rho = \frac{w'\left(w^{-1}\left(\frac{\alpha\beta e}{2}\right)\right)}{u'\left(w^{-1}\left(\frac{\alpha\beta e}{2}\right)\right)} < 1,$$

$$q = w^{-1}\left(\frac{\alpha\beta e}{2}\right) < q^*.$$

[7] Precisely, the slope of the indirect utility of bankers with respect to $1+i$ is $\frac{\beta}{1+\beta(1+i)}\left(\frac{1-\sigma}{1+i}-1\right) < 0$.

The liquidity constraint is binding (and the discount rate is strictly positive) even though the implementation of the Friedman rule has eliminated the opportunity cost of holding reserves across periods. To provide some intuition for this result, note that $\frac{\alpha\beta e}{2} < w(q^*)$ can be rewritten as

$$\frac{e}{2} < \frac{1}{\alpha} \times \frac{w(q^*)}{\beta}.$$

As we can see, the socially efficient per-capita discounting volume is larger than the initial per-capita wealth in the intermediary sector at the Friedman rule. The existence of discounting implies a strictly positive franchise value for the banker, which is socially inefficient. The following proposition summarizes these results.

Proposition 9 *If* $\frac{\alpha\beta e}{2} < w(q^*) \leq \alpha\beta e$, *the Friedman rule does not result in an efficient allocation.*

Our analysis has shown that the Friedman rule is consistent with an efficient allocation only if the initial wealth in the intermediary sector is sufficiently large relative to the size of the retail sector. Otherwise, there is no efficient equilibrium. As previously mentioned, one reason for a relatively small amount of capital in the intermediary sector is the existence of barriers to entry into banking. For instance, holding other factors constant, an increase in the number of bankers operating in the rediscounting market leads to a larger value for α, which makes it more likely that the economy falls in the region $\frac{\alpha\beta e}{2} \geq w(q^*)$, in which case setting the interest rate on reserves equal to the rate of time preference leads to an efficient allocation. When α is relatively small, it is more likely that the economy falls in the region $\frac{\alpha\beta e}{2} < w(q^*) \leq \alpha\beta e$, in which case an equilibrium without discounting does not exist for any value of the interest rate on reserves. As a result, the payments system is inefficient, implying suboptimal production and consumption in decentralized markets.

6.2. Complete Markets

Consider now the role of monetary policy in the economy with complete interbank markets, as described in Section 5. The solution to the banker's problem can be summarized by the first-order conditions

$$v'\left(x_t^y\right) = \beta \frac{(1+i_t)\,\phi_{t+1}}{(1+r_t)\,\phi_t \rho_{t+1}} v'\left(x_{t+1}^o\right), \tag{60}$$

$$r_{t+1} = \frac{1}{\rho_{t+1}} - 1, \tag{61}$$

$$x_t^y = e + r_t l_t - \phi_t M_t,$$

$$x_{t+1}^o = \phi_{t+1}(1+i_t) M_t + \tau_{t+1} + \left(1-\rho_{t+1}\right)\hat{n}_{t+1} - b_{t+1} r_{t+1}.$$

In addition, we have the complementary slackness conditions:

$$\left(\frac{1}{\rho_{t+1}} - 1\right)\left[\rho_{t+1}\hat{n}_{t+1} - \phi_{t+1}(1+i_t) M_t - \tau_{t+1} - b_{t+1}\right] = 0$$

and

$$r_t\left(\phi_t M_t + l_t - e\right) = 0.$$

After using the government budget constraint to obtain the value of transfers to old bankers and applying the market-clearing conditions, we arrive at the equilibrium relations

$$x_t^y = e + r_t l_t - m_t, \tag{62}$$

$$x_t^o = m_t + (1-\rho_t)\,\hat{n}_t - l_t r_t, \tag{63}$$

$$\left(\frac{1}{\rho_t} - 1\right)\left(\rho_t \hat{n}_t - m_t - l_t\right) = 0, \tag{64}$$

$$r_t\left(m_t + l_t - e\right) = 0, \tag{65}$$

together with (16), (17), (50), (60), (61), and the boundary conditions

$$\rho_t \hat{n}_t \leq m_t + l_t \leq e. \tag{66}$$

A formal definition of equilibrium can now be provided.

Definition 10 *An equilibrium in the complete markets economy with active monetary policy can be described as a sequence* $\{x_t^y, x_t^o, \hat{n}_t, m_t, \rho_t, \phi_t, q_t, r_t, l_t\}_{t=0}^{\infty}$ *satisfying (16), (17), (50), and (60)-(66), with* $\{i_t\}_{t=0}^{\infty}$ *taken as given.*

We continue to restrict attention to policies that maintain a constant interest rate on reserves over time. Following the same steps as in Section 5, it is straightforward to show that a stationary equilibrium with a nonbinding liquidity constraint exists provided

$$\frac{w(q^*)}{\alpha\beta} \leq m(i) + l,$$

where real balances $m(i)$ are defined in (56). Then, we can define the equilibrium loan amount as

$$l(i) \equiv \max\left\{\frac{w(q^*)}{\alpha\beta} - m(i), 0\right\}.$$

As a result, we can restate Proposition 5 as follows.

Proposition 11 *In the economy with complete interbank markets and active monetary policy, an equilibrium with a nonbinding liquidity constraint always exists for any value of the interest rate on reserves.*

Note that condition (56) implies $m(i) \to \frac{e}{2}$ as $i \to \frac{1}{\beta} - 1$. From (62) and (63), we can see that the Friedman rule implies perfect consumption smoothing for the banker, as in the planner's solution. Thus, we can make the following statement.

Proposition 12 *The Friedman rule is consistent with an efficient allocation in the economy with complete markets.*

The role of monetary policy is to set the interest rate on reserves in such a way that the opportunity cost of holding reserves across periods for the rediscounting of private IOUs is completely eliminated in the payments system, which can be achieved by following the Friedman rule. With complete interbank markets, the Friedman rule is always consistent with an efficient allocation, even though unique implementation cannot be guaranteed.

7. CONCLUSIONS

We have shown that an efficient payments system can arise endogenously in an economy with a secondary market for debt claims even if intermediaries cannot engage in credit contracts when rediscounting privately issued claims. Additionally, government intervention is not necessary for the existence of an equilibrium without discounting of private IOUs. As we have seen, an equilibrium in which debt claims trade at par value arises when the initial wealth in the intermediary sector is sufficiently large relative to the size of the retail sector. Otherwise, private IOUs sell below par value in the secondary market, and the payments system is inefficient. If intermediaries are allowed to trade in a frictionless short-term credit market among themselves, then an equilibrium without discounting always exists, regardless of the fundamentals of the economy.

The overall allocation of resources in the economy is suboptimal in the absence of government intervention because intermediaries do not attain perfect consumption smoothing. We have shown that efficiency in the payment system requires a zero franchise value for the rediscounting business, so intervention in the market for reserves serves the role of raising the real return on asset holdings to promote efficient consumption smoothing across intermediaries. A version of the Friedman rule is always the optimal policy in our framework. It usually leads to an efficient allocation, except when the initial wealth in the intermediary sector is small relative to the size of the retail sector. In that case, the Friedman rule does not imply an efficient payments system nor an efficient intertemporal exchange among intermediaries.

REFERENCES

[1] L. Araujo, T-W. Hu. "Optimal monetary interventions in credit markets," *Journal of Economic Theory* 178 (2018), pp. 455-487.

[2] S. Freeman. "The payments system, liquidity, and rediscounting," *American Economic Review* 86 (1996a), pp. 1126-1138.

[3] S. Freeman. "Clearinghouse banks and banknote over-issue," *Journal of Monetary Economics* 38 (1996b), pp. 101-115.

[4] M. Friedman. *A Program for Monetary Stability*. New York: Fordham University Press

(1959).

[5] M. Friedman. "The optimum quantity of money" in *The Optimum Quantity of Money and Other Essays*. Chicago: Aldine Publishing Company (1969).

[6] P. Gomis-Porqueras, D. Sanches. "Optimal monetary policy in a model of money and credit," *Journal of Money, Credit and Banking* 45 (2013), pp. 701-730.

[7] C. Kahn, W. Roberds. "Payment system settlement and bank incentives," *Review of Financial Studies* 11 (1998), pp. 845-870.

[8] C. Kahn, W. Roberds. "Why pay? An introduction to payments economics," *Journal of Financial Intermediation* 18 (2009), pp. 1-23.

[9] R. Lagos, R. Wright. "A unified framework for monetary theory and policy analysis," *Journal of Political Economy* 113 (2005), pp. 463-484.

[10] S. Lotz, C. Zhang. "Money and Credit as Means of Payment: A New Monetarist Approach," *Journal of Economic Theory* 164 (2016), pp. 68-100.

[11] A. Martin. "Optimal pricing of intraday liquidity," *Journal of Monetary Economics* 51 (2004), pp. 401-424.

[12] D. Mills. "Alternative central bank credit policies for liquidity provision in a model of payments," *Journal of Monetary Economics* 53 (2006), pp. 1593-1611.

[13] C. Monnet, W. Roberds. "Optimal pricing of payment services," *Journal of Monetary Economics* 55 (2008), pp. 1428-1440.

[14] G. Rocheteau, E. Nosal. *Money, Payments, and Liquidity*, Second edition. Cambridge, MA: MIT Press (2017).

[15] G. Rocheteau, R. Wright. "Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium," *Econometrica* 73 (2005), pp. 175-202.

[16] D. Sanches, S. Williamson. "Money and credit with limited commitment and theft," *Journal of Economic Theory* 145 (2010), pp. 1525-1549.

[17] T. Temzelides, S. Williamson. "Payments systems design in deterministic and private information environments," *Journal of Economic Theory* 99 (2001), pp. 297-326.

[18] S. Williamson. "Payments systems with random matching and private information," *Journal of Money, Credit and Banking* 30 (1998), pp. 551-569.

[19] S. Williamson, R. Wright. "New monetarist economics: Methods," *Federal Reserve Bank of St. Louis Review* 92 (2010a), pp. 265-302.

[20] S. Williamson, R. Wright. "New monetarist economics: Models," *Handbook of Monetary Economics*, (eds.) B. Friedman and M. Woodford. Amsterdam: Elsevier (2010b).

[21] M. Woodford. "The optimum quantity of money," *Handbook of Monetary Economics*, vol. 2, (eds.) B. Friedman and F.H. Hahn. Amsterdam: North-Holland (1990).

Figure 1 – Events Within the Period