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Abstract

This paper studies wage setting in a directed search model of multi-worker firms facing within-firm equity constraints on wages. The constraints reduce wages, as firms exploit their monopsony power over their existing workers, rendering wages less responsive to productivity in doing so. They also give rise to a time-inconsistency in the dynamic firm problem, as firms face a less elastic labor supply in the short run than the long run, making commitment to future wages valuable. Constrained firms find it profitable to fix wages, and doing so is good for worker welfare and resource allocation in equilibrium.
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1 Introduction

Large firms play an important role in the labor market: 70 percent of US private employment takes place in firms with 50 employees or more.[1] Worker compensation in large organizations is generally governed by formal salary structures—involving pay grades and salary ranges—with as many as 85 percent of firms reporting using such structures in practice.[2] The structures are administrative rules seeking to implement the firm's desired wage policies in the face of the various managers within the firm making decisions about individual worker compensation, by limiting managerial discretion conditional on worker performance. This paper studies wage setting subject to such within-firm rules, linking them to rigidity/stickiness in wages—features long held to characterize labor markets in practice.[3]

I study a labor market with search frictions and competitive search (Moen 1997), where firms employ a measure of workers and must pay all their equally productive workers the same. I refer to such constraints as firm wage constraints. I begin by showing, in the context of a static model, that introducing such constraints alters the tradeoffs firms face in choosing a wage to offer. In competitive search, firms set wages to resolve a tradeoff between the wage and vacancy costs of hiring: a higher wage increases hires per vacancy, but at the cost of having to pay those hires more. With firm wage constraints, this decision is influenced by the firm's incentive to profit from its existing workers via low wages, causing firms to set lower wages instead. With all firms affected, the equilibrium shifts toward lower wages in a way that hurts workers and benefits firms, increasing the profitability of vacancy creation and leading to overhiring. Moreover, in drawing wages down toward the worker's opportunity cost, the constraints also work to render wages less responsive to productivity.

I then show, in the context of a dynamic infinite horizon model, that the firm's wage-setting problem involves a time-inconsistency affecting allocations. In the initial period, the firm's incentive to profit from its existing workers leads the firm to set lower wages than an unconstrained firm would. Assuming commitment, the firm plans on higher wages in future periods, however. To understand these differing incentives in setting wages over time, note that the firm effectively faces a less elastic labor supply in the short run because it inherits a set of existing workers that are (to a degree) locked in due to the frictions and taken as given by the firm. In making plans for future periods the firm does not treat its future workforce

[1] See, e.g., Moscarini and Postel-Vinay (2012).

[2] See, e.g., WorldatWork Compensation Programs and Practices Survey (2016).

[3] While wage rigidities have long been held to play a role in the labor market (see, e.g., Bewley 1999), the sources of such rigidities are not fully understood. The question remains broadly relevant, however, as evidenced for example by recent research highlighting the role of wage rigidities for firm risk and borrowing (Favilukis and Lin 2016, Donangelo, Gourio, Kehrig, and Palacios 2019, Favilukis, Lin, and Zhao 2020).

as exogenous, however, making future labor supply more elastic. Of course, commitment to future wages is necessary to implement such a plan, as the firm would otherwise again choose lower wages ex post.[4]

To consider outcomes when firms cannot commit to future wages, I study Markov perfect equilibria. Analyzing Markov perfect equilibria in an environment with a time-inconsistency can be challenging because the decision-maker's objective does not coincide with maximizing their value function, which means that standard dynamic programming arguments cannot be directly applied.[5] Adopting a parsimonious approach that simplifies analyzing the model, I focus on equilibria that are consistent with the size-independence of the firm problem. With this, I provide an analytic characterization for the impact of firm wage constraints on wages, which drives the implications for hiring in the model.

While the baseline analysis is conducted in a setting with homogenous workers, to relate to the motivating evidence on salary structures more directly, it may be extended to feature explicit worker heterogeneity in a straightforward way. The analysis carries over directly. In this setting firms hire more productive workers at higher wages, with the constraints extending these wages to existing workers of similar productivity as well. Persistently higher productivity leads to persistently higher wages, and transitory increases/declines in productivity to transitory increases/declines in wages. The constraints work to ensure that wages reflect worker productivity throughout the firm, but also to compress wage differences across productivity types and realizations (in making wages less responsive to productivity).[6]

I then use the model in two applications related to the dynamics of wages. The first considers how such constraints influence the cyclical behavior of wages and labor market flows, and the second whether firms in this environment would find it profitable to fix wages—given the time-inconsistency—as well as the equilibrium implications of all firms doing so.

To study the impact of firm wage constraints on business cycle variation in wages and unemployment, I compare shock propagation in the firm wage model with the unconstrained

[4]The time-inconsistency of the dynamic firm problem is reminiscent of that in optimal capital taxation (Chamley 1986, Judd 1985) in that the firm prefers to tax labor (via low wages) more in the short run, where labor supply is less elastic.

[5]Time-inconsistencies appear in multiple contexts, due to either preferences directly or the economic environment, such as in problems of optimal fiscal or monetary policy. See Klein, Krusell, and Rios-Rull (2008) for a discussion on characterizing Markov perfect equilibria in problems with time-inconsistency, in the context of a study of optimal government spending.

[6]Broadly, one could interpret the different levels of the salary structures as corresponding to larger and more persistent differences in worker productivity, with smaller and more transitory productivity variation giving rise to wage variation within levels of the structure. If worker productivity increases over time on the job, the worker then rises in the structure as well. In this context the constraints compress wage differences both within and across levels of the structure.

case. The constraints render wages less responsive to shocks, leading to amplification in labor market flows. Parameterizing the constrained and unconstrained models to the same steady state, the amplification in labor market flows due to the constraints can be substantial, with a tenfold increase in the response of the vacancy-unemployment ratio to the shock relative to the unconstrained case. This allows the model mechanism to explain roughly a third of the observed variation in the vacancy-unemployment ratio. Alternative parametrization approaches yield more moderate differences between models, but qualitatively the constraints render wages less responsive to shocks.

To study the profitability and equilibrium implications of infrequent wage adjustment, I extend the model to allow firms to commit to a fixed wage for a probabilistic period of time. In an equilibrium where all firms fix wages for a probabilistic period of time, staggered across firms, longer wage durations have positive welfare implications. Longer wage durations make firms more forward looking in their wage setting, leading them to set higher wages. With all firms doing so, the equilibrium shifts toward higher wages in a way that improves the efficiency of resource allocation, reducing firm profits and the overhiring driven by too-low wages. Workers, both employed and unemployed, benefit in particular. These effects continue to hold also in the presence of substantial shocks, despite changes in labor market volatility associated with longer wage durations. In an environment characterized by firm wage constraints, fixed wages can thus be welfare-improving, despite the seeming "rigidities" in the labor market.

Longer wage durations are also privately optimal for firms. An individual firm deviating to a longer wage duration than equilibrium firms chooses a higher wage and grows faster than equilibrium firms. In particular, firm value increases as a result of the deviation—reflecting the value of commitment afforded by the wage—while the worker value of employment at the firm increases as well. I show that these effects continue to hold also when firms face substantial shocks, despite the fixed wage limiting the firm's ability to respond to them. This suggests the emergence of infrequent wage adjustment as an equilibrium outcome.

Finally, I demonstrate the emergence of a privately optimal equilibrium wage duration in the model. I show that, with a convex cost of longer wage durations, a firm's incentive to deviate toward longer wage durations diminishes as the equilibrium wage duration grows longer. Eventually, a privately optimal equilibrium wage duration emerges: an equilibrium duration where an individual firm would not prefer to deviate toward longer or shorter durations. This privately optimal equilibrium wage duration reflects the value of commitment, and is thus longer when agents are more patient, employment relationships longer term, and the value of output relative to the costs of hiring greater. It is also longer when the costs of longer durations are lower. Most characteristically, this theory predicts stable wages in stable long-term employment relationships.

Salary Structures The within-firm constraints relate to literature in personnel economics characterizing worker compensation within firms as governed by internal pay structures (see, e.g., Bewley (1999) for a description). Such structures involve a hierarchy of positions within the firm, with horizontal equity concerns typically viewed as limiting wage differences within levels of the hierarchy.[7]

Despite being standard practice, broader evidence documenting the prevalence and features of pay structures across firms remains limited. Industry surveys indicate, however, that 85 percent of (larger) firms used formal salary structures in administering worker compensation over the past ten year period. According to this evidence:[8] The structures are generally designed with the midpoints of the salary ranges targeting relevant market rates, and adjusted annually (with as many as 75 percent of firms reporting adjusting annually and 15 percent biennially). Salary increases are most commonly reported to be based on worker performance, but the worker's position in the range also plays a role, as does the market rate for the position.[9] Performance-based increases are typically based on formal performance evaluations, together with firm-level guidelines for corresponding increases in pay. The broad majority of firms also report conducting analyses of pay equity with respect to pay levels as well as increases. Overall, it is common to have multiple structures within a firm, applying to different geographic regions, job categories and regulatory categories, but following a common firm-level compensation philosophy.

Other evidence related to salary structures includes the survey work of Bewley (1999), who found that worker compensation in non-union firms with 50 employees or more is generally governed by formal salary structures, largely motivated by managers by internal equity concerns.[10] It has also been argued that such structures are important for explaining worker

[7] For example Lazear and Oyer (2012) note that a look inside firms shows that wage dynamics are driven largely by the jobs people hold, while individual jobs have a fairly narrow band of possible wages. Lazear and Shaw (2007) argue that managers and human resource professionals generally view compensation as more compressed than output, with the objective of making pay more equitable, while noting that measurement remains difficult for lack of data on individual output.

[8] This paragraph draws on surveys conducted by WorldatWork, a non-profit association for human resources professionals (with their permission). See their Compensation Programs and Practices Surveys 2015, 2016 and 2019, and Survey of Salary Structure Policies and Practices 2019.

[9] Years of service, education/certifications, and general increases were also mentioned. Outside job offers received by workers were not explicitly mentioned as a driver of salary increases.

[10] Of course, equity concerns may be at play in small organizations as well, despite the lack of a formal structure. A body of evidence emphasizes that workers are concerned with how their wages compare with peers (Card, Mas, Moretti, and Saez 2012, Bracha, Gneezy, and Loewenstein 2015, Breza, Kaur, and

compensation within firms: Baker, Gibbs, and Holmstrom (1994) offer an early case study, arguing that the hierarchy explains the bulk of wage differences within their firm. More recently, Lazear and Oyer (2004) and Bayer and Kuhn (2018) provide evidence that incorporating information on hierarchies allows explaining as much 80 percent of the cross-sectional variation in wages—a substantial increase over cross-sectional wage regressions that typically explain only about a third of this variation with observables.[11]

Salary structures and related guidelines are administrative tools implementing the firm's desired pay strategies in a setting where decisions about individual worker compensation are made by various managers within the organization. Explicit guidelines linking pay to performance serve to implement these strategies when individual managers' incentives need not align with those of the firm leadership. The rules may thus be viewed as a response to an agency problem where it is costly for the firm leadership to make informed decisions on an individual worker level. One might also motivate them based on workers caring about equity directly. The present paper does not formalize such agency problems, but simply takes the structures as a feature constraining wage setting in practice.

More than the specific form of the structure, what is relevant here is the notion of a policy that systematically connects the wages of different workers within the firm, influencing how the firm sets them. If new hires must be brought in on similar terms as comparable existing workers—in line with the structure—hiring wages will be influenced by the firm's incentive to profit from their existing workers.[12] The question then becomes: What are the implications for equilibrium outcomes in the labor market?

Related Literature The paper is related to a literature studying models of multi-worker firms in frictional labor markets. Some of these papers work with directed search, as in Rudanko (2011), Kaas and Kircher (2015) and Schaal (2017). Relative to these papers, the present work introduces the within-firm constraints, and considers the implications. I also relax an assumption commonly made in these papers that firms have full commitment to future wages, and show that doing so has allocative effects in the presence of constraints.[13]

Shamdasani 2018, Dube, Giuliano, and Leonard 2019). According to these authors, workers appear to prefer equal treatment with peers: Wage differentials appear to reduce effort and output, as well as lead to quits and withholding participation.

[11] Recent work studying multinational firms also finds that wages at a firm's foreign establishments reflect those at home, consistent with a firm-wide policy (Hjort, Li, and Sarsons 2020).

[12] This view is consistent with evidence that the wages of new hires are equally cyclical as those of existing workers (Gertler and Trigari 2009, Hagedorn and Manovskii 2013, Gertler, Huckfeldt, and Trigari 2020), conditional on match quality. See also Grigsby, Hurst, and Yildirmaz (2021).

[13] The competitive search equilibrium requires some degree of commitment (for a tradeoff between posted wages and resulting hiring rate), and I assume throughout that the firm has commitment to the current period

Other papers in this area work with random search and large-firm bargaining, such as Acemoglu and Hawkins (2014) and Elsby and Michaels (2013). These studies have not emphasized a time-inconsistency in the firm problem, despite the models seemingly sharing the feature that similar workers within the firm are paid the same. Two differences in wage setting protocols are worth noting. First, the surplus splitting rules applied in those models are generally micro-founded by a notion of the firm bargaining with each of its individual workers sequentially. That work does not discuss how such a process relates to firms making decisions about overall wage policies, however. Second, commitment plays a more central role in directed search, where the firm's wage setting problem centers around the tradeoffs between its offered wages and resulting hiring rate. The firm has an incentive to promise high wages ex ante but not pay them ex post. By contrast, in random search a firm's wages need not influence its hiring (as long as they remain within acceptable bounds).[14]

The Burdett and Mortensen (1998) model offers an alternative framework where firms set firm-level wages and—even though the model features random search—commitment to future wages matters for outcomes because wages affect workers' decisions to accept arriving job offers. The original model focuses on steady states (implicitly assuming commitment to future wages), but subsequent work extending the model to an explicitly dynamic setting points out that firms have an incentive to cut wages ex post. Dynamic extensions with different assumptions about firm commitment include Coles (2001), Moscarini and Postel-Vinay (2013), and Coles and Mortensen (2016).[15] By contrast, the present paper considers the impact of firm wages on wage dynamics in the Mortensen-Pissarides framework, a workhorse model of macro labor that has been widely used for thinking about labor market dynamics. This model allows comparing the constrained and unconstrained cases, whereas the Burdett-Mortensen framework imposes the constraints without offering an unconstrained benchmark. It also allows relating equilibrium outcomes to efficient allocations in a direct way, with an explicit planner problem involving the same frictions as in the decentralized market.[16] Further, the present model considers outcomes when workers are able to direct their search, and firms set wages and create vacancies accordingly. With vacancy creation responding to the

wage. This still leaves the constrained firm with a commitment problem, as the constraints prevent fully front-loading pay to the hiring period. For reference, the evidence suggests signing bonuses represent a relatively small share of earnings for the broad majority of workers (Lemieux, Macleod, and Parent 2009, Grigsby, Hurst, and Yildirmaz 2021). Of course, general performance bonuses may be viewed as a natural part of the firm pay policy considered here (with pay responding to worker productivity as in Section 4).

[14]For more on how outcomes differ from a similar model with random search and bargaining, see Footnotes 26 and 33.

[15]Moscarini and Postel-Vinay (2013) develop a dynamic extension assuming commitment to future wages, while Coles (2001) and Coles and Mortensen (2016) provide extensions relaxing commitment to future wages.

[16]See Appendix A.

lower wages associated with firm wage constraints, the equilibrium features overhiring rather than the high unemployment of Coles and Mortensen (2016).[17]

The Burdett-Mortensen framework takes an extreme view of the commitment problem facing firms, in its focus on continuous time formulations, if firms cannot commit for even an instant forward as in Coles and Mortensen (2016). This likely overstates the effects of monopsony. By contrast, the present model may be expected to overstate the effects of monopsony due to abstracting from endogenous separations/on-the-job search. Appendix B discusses the robustness of the wage setting mechanism to incorporating endogenous separations/quits in equilibrium, showing that outcomes need not change significantly. Formally extending the model to allow on-the-job search to gauge its impact on wage setting remains desirable, however.

The first application is motivated by the long-standing puzzle facing macroeconomists of why wages vary so little while unemployment varies so much over the business cycle (see, e.g., Bewley 1999), and a question of whether within-firm constraints on wages could play a role in generating rigidity in wages over time. In the context of search models it is related to literature on the unemployment volatility puzzle discussed by Hall (2005) and Shimer (2005), seeking mechanisms generating amplification in the responses of unemployment and vacancy creation to aggregate shocks. In an early contribution in this vein, Menzio (2005) also sought to think about the implications of firm wage policies for labor market dynamics. His work considers a random search model with on-the-job search where firms have private information about their productivity and bargain wages with their workers. By contrast, the present study highlights the commitment problem involved with directed search.[18]

The second application is motivated by the observation that wages adjust infrequently relative to labor market flows, and a related modeling tradition imposing fixed wages with staggered adjustment (Taylor 1999, 2016).[19] In the spirit of work studying the tradeoffs between rules and discretion in settings with time-inconsistencies, I consider whether the time-inconsistency in the firm problem could be viewed as motivating the adoption of fixed wage rules.[20] In the labor market context the application relates to the work of Gertler

[17]See their companion paper, Coles and Mortensen (2015).

[18]Snell and Thomas (2010) also consider the implications of equity concerns for the cyclical behavior of wages, in a (non-search) framework where equity concerns combine with the motive of risk-neutral firms to smooth the wages of risk-averse workers, resulting in wage rigidity.

[19]Barattieri, Basu, and Gottschalk (2014) and Grigsby, Hurst, and Yildirmaz (2021) document average durations of wages between 4 and 8 quarters. For European countries, Lamo and Smets (2009) report an average duration of wages of 15 months. Wage adjustment is less frequent than the monthly, or even weekly, frequencies labor market flows vary at.

[20]See, e.g., Athey, Atkeson, and Kehoe (2005), Amador, Werning, and Angeletos (2006).

and Trigari (2009), who study the impact of staggered wage setting on business cycles in unemployment and vacancy creation in a random search model. The present study shows that directed search can provide a stronger motivation for the emergence of infrequent wage adjustment, in implying that longer wage durations can be profitable for firms, as well as desirable from a worker and planner perspective (in a second best sense).[21]

The paper is organized as follows. Section 2 begins with a one-period model to illustrate the static tradeoffs involved with firm wages, while Section 3 turns to a dynamic infinite horizon model to illustrate the time-inconsistency. Section 4 relates the analysis to settings with explicit worker heterogeneity, and Section 5 extends the baseline model to allow longer wage commitments/fixed wages. Section 6 considers the implications for business cycles in wages and unemployment, as well as the impact of infrequent wage adjustment, in a quantitative setting. Appendixes A-G provide the corresponding planner problem and unconstrained firm problem, a discussion of the role of endogenous separations/quits, proofs, details on the parametrization and solution methods, additional figures, as well as a version of the model with firm-level shocks.

2 Static Model

This section begins by considering the impact of firm wages in the context of a static, one-period model, before proceeding to the dynamic model in the next section.

Within a single period, consider a labor market with measure one workers, and a large number I firms. Each firm begins the period with n_i existing workers, for all $i \in I$. The total measure of matched workers in the beginning of the period is thus $N = \sum_{i \in I} n_i$, leaving $1 - N$ unmatched workers looking for jobs. All firms have access to a linear production technology with output z per worker, while workers who do not find jobs have access to a home production technology with output b $(< z)$ per worker.

In addition to their existing workers, firms can hire new workers in a frictional labor market. Firms seeking to hire must post vacancies, where posting v vacancies is subject to a convex cost $\kappa(v, n) = \hat{\kappa}(v/n)n$, where n is the firm's existing workforce and $\hat{\kappa}' > 0, \hat{\kappa}'' > 0$.[22] The search frictions in bringing these vacancies and unmatched workers together are

[21] Gertler and Trigari (2009) argue that firms (in their model) may find the added volatility associated with sticky wages profitable due to convexities in profits, but largely refrain from relating to efficient allocations, noting that in their model efficiency requires wages being driven to workers' opportunity cost.

[22] The convexity in the vacancy cost is introduced to help ensure that first order conditions characterize optimizing behavior, and the homothetic form (adopted from Kaas and Kircher (2015)) plays a role in allowing solving the dynamic model in a tractable way. Note that the derivatives $\kappa_v(v, n)$ and $\kappa_n(v, n)$ are

formalized with a matching function, with constant returns to scale. I denote the probability a worker finds a job in a market with tightness (vacancies per job seeker) θ by $\mu(\theta)$, with $\mu' > 0, \mu'' < 0$, and elasticity $\mu'(\theta)\theta/\mu(\theta)$ weakly decreasing. I denote the probability a vacancy is filled by $q(\theta)$, where $\mu(\theta) = \theta q(\theta)$.

In posting vacancies firms also specify the wage that will apply to those jobs and take into account that the offered wage will affect their ability to fill vacancies. Specifically, they expect the measure of job seekers they attract per vacancy to be such that job seekers are left indifferent between applying to this firm versus elsewhere, the latter yielding the equilibrium value of search U. Formally, given wage w_i, the market tightness θ_i they expect to face satisfies

$$U = \mu(\theta_i)w_i + (1 - \mu(\theta_i))b, \tag{1}$$

where the firm takes the value of search U as given (because the firm is small relative to the market). Here a worker applying to the firm finds a job with probability $\mu(\theta_i)$, attaining the wage w_i, and remains unmatched with probability $1 - \mu(\theta_i)$, attaining b. Per equation (1), the firm anticipates that offering a higher wage attracts more job seekers per vacancy, which increases the probability these vacancies are filled, $q(\theta_i)$. I denote these beliefs by $g(w; U)$.

Each firm chooses a wage and a measure of vacancies to maximize its profits:

$$\max_{w_i,v_i}(n_i + q(\theta_i)v_i)(z - w_i) - \kappa(v_i, n_i), \tag{2}$$

subject to constraint (1) characterizing firm beliefs regarding the market tightness to prevail in response to its hiring wage, and $w_i \geq b$ ensuring existing worker participation. The profits reflect the firm's n_i existing workers and $q(\theta_i)v_i$ new hires all producing z units of output at the firm wage w_i, with vacancies subject to the vacancy cost $\kappa(v_i, n_i)$.

Note that the firm problem is effectively independent of the firm's initial size. Defining the firm's rate of vacancy creation as $x_i := v_i/n_i$, one can scale and rewrite the problem as:

$$\max_{w_i,x_i}(1 + q(\theta_i)x_i)(z - w_i) - \hat{\kappa}(x_i), \tag{3}$$

subject to constraint (1) on the hiring wage and $w_i \geq b$ ensuring existing worker participation.

The size independence implies that heterogeneity in initial sizes across firms does not lead to systematic differences in wages or vacancy rates, as well as that firm growth is independent of size (Gibrat's law holds). I will focus on circumstances in which firms find it optimal to

functions of the ratio v/n only, and for expositional reasons I hence denote them as $\kappa_v(v/n)$ and $\kappa_n(v/n)$ in what follows.

hire, and constraint (1) thus binds (making the constraint $w_i \geq b$ superfluous).[23] With this, assuming that all firms are equally productive, I drop the firm indexes on w_i, θ_i, x_i in what follows.

The firm's first order condition for vacancy creation,

$$\kappa_v(x) = q(\theta)(z - w), \tag{4}$$

states that the firm creates vacancies to a point where the marginal cost of an additional vacancy, on the left, equals the expected profits from the additional workers hired, on the right.

The firm's first order condition for the wage,

$$1 + q(\theta)x = q'(\theta)g_w(w; U)x(z - w), \tag{5}$$

states that the firm raises the wage to a point where the marginal increase in wage costs, on the left, equals the marginal increase in profits from greater vacancy filling rates, as the higher wage increases job seekers per vacancy, on the right.

The firm wage policy is embodied in the single wage appearing in the firm problem above. The unconstrained firm problem, by contrast, may be written as:

$$\max_{w_i, v_i} n_i(z - w_i^e) + q(\theta_i)v_i(z - w_i) - \kappa(v_i, n_i),$$

subject to constraint (1) on the hiring wage w_i, and where the average wage of existing workers is denoted w_i^e.

The first order conditions for the unconstrained firm problem include the same condition for optimal vacancy creation as above (4), with a different condition for the hiring wage:

$$q(\theta)x = q'(\theta)g_w(w; U)x(z - w). \tag{6}$$

Here the firm again raises the wage to a point where the marginal increase in wage costs equals the increase in profits from greater vacancy filling rates—but with the difference that raising the wage is less costly for the unconstrained firm as the wage increase applies to new hires only.[24]

[23]Firms could, in some circumstances, prefer the corner solution of withdrawing from hiring altogether, paying their existing workers the minimum to keep them with $v_i = 0, w_i = b$. The present analysis focuses throughout on settings where interior solutions are optimal, checking in the quantitative exercises that the implied firm values dominate deviating to such a corner.

[24]For existing workers, outcomes depend on whether the firm has some pre-commitment to their wages or not. If not, the firm would optimally pay these workers as little as possible: their opportunity cost b, retaining them at minimum compensation. Note that doing so would mean paying new hires more than the firm's existing workers, something the latter might object to—bringing us to the original firm problem imposing equity.

Definition 1. *A competitive search equilibrium with firm wages is an allocation* $\{w, \theta, x\}$ *and value of search* U *such that the allocation solves the firm problem* (3) *with each job seeker applying to one firm:* $1 - N = xN/\theta$.

The effects of firm wage constraints on equilibrium outcomes may be summarized as follows:

Proposition 1. *The competitive search equilibrium with firm wages satisfying* (1), (4), (5), *and* $1 - N = xN/\theta$ *is unique, with a strictly lower hiring wage and greater market tightness, vacancy creation and employment than absent within-firm constraints. Firm profits from hiring are higher, and the worker values of searching for and accepting employment lower, than absent constraints.*

Intuitively, the constraints lead to downward pressure on wages, as the firms' incentive to profit from their existing workers via low wages reduces also those of new hires. With all firms affected, the equilibrium shifts toward lower wages in a way that hurts workers and benefits firms, encouraging vacancy creation and hiring in doing so.

Corollary 1. *The competitive search equilibrium with firm wages is inefficient.*

It follows immediately from the above that the firm wage equilibrium is inefficient, as allocations differ from the unconstrained case that is known to be efficient.[25] Firms exploit their monopsony power over their existing workers, leading to too-low wages and overhiring.

Note that in drawing wages down toward the workers' opportunity cost, the constraints also work to render wages less responsive to productivity on the job. The first order conditions for the wage, (5) and (6), imply that the wage may be written as a weighted average of the worker's opportunity cost and the value of output from the relationship:

$$w = (1 - \gamma)b + \gamma z, \tag{7}$$

where the weight on productivity γ differs between the two cases. In the unconstrained case the weight is determined by the matching function elasticity as $\gamma = 1 - \varepsilon$, whereas in the constrained case $\gamma = \frac{1-\varepsilon}{1-\varepsilon+\frac{1+q(\theta)x}{q(\theta)x}\varepsilon}$ instead. The constraints reduce γ and—to the extent that relevant hiring rates $q(\theta)x$ are well below one—significantly so. The constrained wage is

[25] For example Rogerson, Shimer, and Wright (2005) discuss the efficiency of the competitive search equilibrium. It is easy to see that the planner problem of maximizing total output $\sum_i[(n_i + q(\theta_i)v_i)z - \kappa(v_i, n_i)] + (1 - \sum_i(n_i + q(\theta_i)v_i))b$ subject to the adding up constraint $1 - \sum_i n_i = \sum_i x_i n_i/\theta_i$ yields the same optimality conditions as the unconstrained case.

thus less affected by productivity, with optimal vacancy creation (4) further implying that constrained firms' vacancy creation and hiring are more affected by productivity as a result.[26]

The next section extends the model to an explicitly dynamic setting.

3 Dynamic Model

Time is discrete and the horizon infinite. All agents are rational and discount the future at rate β. Each period a large number I firms inherit a measure of existing workers n_{it} from the previous period and hire new ones in a frictional labor market. Employment relationships are long term and end at the end of each period with probability δ. Labor productivity z_t is stochastic, and follows a Markovian process.

In posting vacancies, firms specify a fully state-contingent sequence of wages that will apply to those jobs. Given a sequence $\{w_{it+k}\}_{k=0}^{\infty}$ offered in period t, the market tightness θ_{it} firms expect to face is such that

$$U_t = \mu(\theta_{it}) E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (w_{it+k} + \beta\delta U_{t+1+k}) + (1 - \mu(\theta_{it}))(b + \beta E_t U_{t+1}), \tag{8}$$

where the firm takes the value(s) of search $\{U_{t+k}\}_{k=0}^{\infty}$ as given. Here a worker applying to the firm in period t finds a job with probability $\mu(\theta_{it})$, subsequently receiving the specified wages until a separation returns him to job search, and remains unmatched with probability $1 - \mu(\theta_{it})$, receiving b and continuing to search in the following period. Per equation (8), the firm anticipates that offering higher wages attracts more job seekers per vacancy, which increases the probability these vacancies are filled.

To ensure that existing workers are willing to stay with the firm, the value of employment $E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (w_{it+k} + \beta\delta U_{t+1+k})$ must exceed the value of quitting to look for a new job, $b + \beta E_t U_{t+1}$, at any point. This condition is guaranteed whenever the firm is hiring, because an existing worker's decision to stay is based on the same calculation as a new hire's decision to accept employment (per equation (8)). In what follows, I will focus on circumstances where firms always hire and (8) thus holds.[27]

[26] See Appendix C for more detail. Note that this formulation also allows comparing outcomes with those attained in a similar model with random search and bargaining, with γ corresponding to the workers' bargaining power parameter. In random search models the value of this parameter is typically chosen to yield the same outcome as in the unconstrained case above. Here the within-firm constraints have the effect of reducing the effective value of γ significantly, as firms apply their monopsony power in setting wages, also rendering wages less responsive to productivity in doing so. Relative to a similar model with random search and bargaining, the firm wage model studied here thus features both lower and more rigid wages.

[27] It remains necessary to verify that firms find it optimal to hire, rather than withdrawing from the market

For convenience, I adopt the following shorthand for equation (8):

$$X_t = \mu(\theta_{it})(E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k w_{it+k} - Y_t), \tag{9}$$

where I have defined the variables $X_t := U_t - b - \beta E_t U_{t+1}$ and $Y_t := b + \beta E_t U_{t+1} - E_t \beta\delta \sum_{k=0}^{\infty} \beta^k (1-\delta)^k U_{t+1+k}$. By way of interpretation, X_t represents the value of search, in satisfying $U_t = b + X_t + \beta E_t U_{t+1}$. Meanwhile, Y_t represents the value of forgone home production and search during employment, in satisfying $Y_t = b + E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k (b + X_{t+k})$. Firms take the values $\{X_t, Y_t\}_{t=0}^{\infty}$ as given, just as they do $\{U_t\}_{t=0}^{\infty}$.

The firm problem may then be written as:

$$\max_{\{w_{it}, v_{it}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t [(n_{it} + q(\theta_{it}) v_{it})(z_t - w_{it}) - \kappa(v_{it}, n_{it})] \tag{10}$$

$$\text{s.t. } n_{it+1} = (1-\delta)(n_{it} + q(\theta_{it}) v_{it}), \ \forall t \geq 0, \tag{11}$$

$$X_t = \mu(\theta_{it}) E_t (\sum_{k=0}^{\infty} \beta^k (1-\delta)^k w_{it+k} - Y_t), \ \forall t \geq 0, \tag{12}$$

and $E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k w_{it+k} - Y_t \geq 0$ for all $t \geq 0$ ensuring existing worker participation. Firms maximize the expected present value of profits, where in each period t the firm's existing and new workers produce z_t units of output at the firm wage w_{it}, with vacancies subject to the vacancy cost $\kappa(v_{it}, n_{it})$. In doing so, they take as given the law of motion for their workforce (11), the constraints (12) characterizing their beliefs regarding the market tightnesses prevailing in response to offered wages, as well as participation constraints.

The per-period wages w_{it} are generally not allocative in these models due to the long-term nature of the employment relationship, but the allocative wage variable is instead the present value of wages the worker expects to receive over the course of the employment relationship: $W_{it} = E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k w_{it+k}$. To consider allocative effects it would thus be desirable to write the problem in terms of these present values instead. (In choosing a sequence of per-period wages the firm of course chooses a sequence of present values as well, and vice versa.) Written in these terms, the firm problem reads:

$$\max_{\{W_{it}, v_{it}\}_{t=0}^{\infty}} n_{i0}(Z_0 - W_{i0}) + E_0 \sum_{t=0}^{\infty} \beta^t [q(\theta_{it}) v_{it} (Z_t - W_{it}) - \kappa(v_{it}, n_{it})], \tag{13}$$

$$\text{s.t. } n_{it+1} = (1-\delta)(n_{it} + q(\theta_{it}) v_{it}), \ \forall t \geq 0,$$

$$X_t = \mu(\theta_{it})(W_{it} - Y_t), \ \forall t \geq 0,$$

and paying their existing workers according to their outside option. The quantitative exercises check that hiring remains optimal relative to this alternative. In doing so I assume that existing workers are always free to quit, which disciplines the wages firms would pay if they chose not to hire. Section B discusses how the worker outside option of quitting disciplines the monopsony power firms have.

with $W_{it} - Y_t \geq 0$ for all $t \geq 0$ ensuring existing worker participation. Here the objective adds up the contributions of different cohorts of workers to firm value, as the firm hires over time (Z_t denotes the present value of z_t: $Z_t = E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k z_{t+k}$).

Writing the problem this way makes it clear that the firm's incentives in setting wages differ over time. In choosing the initial period present value wage W_{i0} the firm considers the impact on hiring within the period—the tradeoff between offering higher wages to attract more job seekers per vacancy versus the costs of paying the workers hired more—but the decision is also influenced by the firm's incentive to profit from its existing workers through low wages. In choosing the present value wages W_{it} for later periods the firm only considers the hiring margin, however, as it does not treat its future workforce as given when making plans. Intuitively, the firm faces a less elastic labor supply in the initial period than later on. The differing incentives in setting wages over time reflect a time-inconsistency in the firm problem, meaning the solution to this firm problem requires commitment to future wages on the part of the firm.

What happens if firms have full commitment to future wages? The optimality conditions for the firm problem (13) may be written as:

$$\kappa_v(x_t) = q(\theta_t)[Z_t - W_t - E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k \kappa_n(x_{t+k})], \ \forall t \geq 0, \tag{14}$$

$$1 + q(\theta_0)x_0 = q'(\theta_0) g_W^t(W_0) x_0 [Z_0 - W_0 - E_0 \sum_{k=1}^{\infty} \beta^k (1-\delta)^k \kappa_n(x_k)], \text{ and}$$

$$q(\theta_t)x_t = q'(\theta_t) g_W^t(W_t) x_t [Z_t - W_t - E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k \kappa_n(x_{t+k})], \ \forall t \geq 1. \tag{15}$$

The firm creates vacancies up to a point where marginal cost of an additional vacancy, on the left, equals the present value profits from additional workers hired, with vacancies filled with probability $q(\theta_t)$ leading to a present value of output Z_t net of the present value of wages W_t and the hires reducing the costs of vacancy creation going forward. The firm raises the present value wage to a point where the marginal increase in wage costs, on the left, equals the marginal increase in present value profits from the additional workers hired, as the higher present value wage increases job seekers per vacancy. In the initial period the firm considers the increase in wage costs to apply to the firm's existing and new workers alike, whereas an unconstrained firm would consider new hires only. In subsequent periods the firm considers the increase to apply to new hires only, however, as an unconstrained firm would. With full commitment to future wages, firms thus behave as unconstrained firms after the initial period.[28]

What happens if firms do not have full commitment to future wages? To think about firm behavior absent commitment to future wages, I consider Markov perfect equilibria. To this end, suppose the current aggregate state is denoted $S := (N, z)$. Based on (13), the firm problem may be written recursively as:

$$\max_{W,\theta,v} (n + q(\theta)v)(Z(S) - W) - \kappa(v, n) + \beta E_S V(n', S') \tag{16}$$
$$\text{s.t. } n' = (1-\delta)(n + q(\theta)v),$$
$$X(S) = \mu(\theta)(W - Y(S)), \tag{17}$$

together with the accounting equation

$$V(n, S) = q(\theta)v(Z(S) - W) - \kappa(v, n) + \beta E_S V(n', S'), \tag{18}$$

where W, θ, v solve (16). Here the objective takes into account the influence of the firm's existing workers on its wage setting decisions, while the accounting equation keeps track of the actual profits accruing from hiring, cohort by cohort. The fact that the two do not coincide reflects the time-inconsistency.

To proceed, it is convenient to note that the firm problems satisfy a size-independence property where the firm's choices of wage W, implied tightness θ, and vacancy rate $x = v/n$ need not explicitly depend on firm size. Consistent with this, I consider equilibria where firm behavior is independent of size: $W(S), \theta(S), x(S)$ are independent of n and firm values correspondingly linear: $V(n, S) = \hat{V}(S)n$. Problem (16) implies that $W(S), \theta(S), x(S), \hat{V}(S)$ should satisfy:

$$\max_{W,x} (1 + q(\theta)x)(Z(S) - W) - \hat{\kappa}(x) + \beta(1-\delta)(1 + q(\theta)x)E_S\hat{V}(S') \tag{19}$$
$$\text{s.t. } X(S) = \mu(\theta)(W - Y(S))$$

together with the accounting equation

$$\hat{V}(S) = q(\theta)x(Z(S) - W) - \hat{\kappa}(x) + \beta(1-\delta)(1 + q(\theta)x)E_S\hat{V}(S'). \tag{20}$$

The first order conditions for vacancy creation and wages read, respectively:

$$\kappa_v(x) = q(\theta)(Z - W + \beta(1-\delta)E_S\hat{V}(S')), \text{ and} \tag{21}$$
$$1 + q(\theta)x = q'(\theta)g_W(W; S)x(Z - W + \beta(1-\delta)E_S\hat{V}(S')), \tag{22}$$

[28] For reference, Appendix A spells out the unconstrained firm problem explicitly.

where $g(W;S)$ again denotes the firm's beliefs regarding the tightness. Here the continuation values reflect the decrease in future vacancy costs from additional workers hired, with (21) and (22) thus implying the optimality conditions:[29]

$$\kappa_v(x_t) = q(\theta_t)[Z_t - W_t - E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k \kappa_n(x_{t+k})], \ \forall t \geq 0, \text{ and} \tag{23}$$

$$1 + q(\theta_t)x_t = q'(\theta_t) g_W^t(W_t) x_t [Z_t - W_t - E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k \kappa_n(x_{t+k})], \ \forall t \geq 0. \tag{24}$$

The firm creates vacancies up to a point where the marginal cost of an additional vacancy equals the present value profits from additional workers hired, with vacancies filled with probability $q(\theta_t)$ leading to a present value of output Z_t net of the present value of wages W_t and the hires reducing the costs of vacancy creation going forward. The firm raises the present value wage to a point where the marginal increase in wage costs equals the marginal increase in present value profits from the additional workers hired, as the higher present value wage increases job seekers per vacancy. The increase in wage costs in (24) again applies to the firm's existing and new workers alike, whereas an unconstrained firm would consider the increase to apply to new hires only. Absent commitment, this effect thus influences wage setting each period.

Finally, defining an equilibrium where firms reoptimize each period, we have:[30]

Definition 2. *A competitive search equilibrium with firm wages where firms reoptimize wages each period is an allocation $\{w_t, \theta_t, x_t, N_t\}_{t=0}^{\infty}$ and values of search $\{U_t\}_{t=0}^{\infty}$ such that the allocations solve the firm problem* (19) *with each job seeker applying to one firm,* $1 - N_t = x_t N_t/\theta_t$*, and law of motion* $N_{t+1} = (1-\delta)(N_t + \mu(\theta_t)(1-N_t))$*,* $\forall t \geq 0$.

The level effects of firm wages on equilibrium outcomes may be summarized as follows:

Proposition 2. *The competitive search equilibrium with firm wages satisfying* (23)*,* (24)*,* $1 - N_t = x_t N_t/\theta_t$ *and* $N_{t+1} = (1-\delta)(N_t + \mu(\theta_t)(1-N_t))$ *has a unique steady state, with a strictly lower present value hiring wage and greater market tightness, vacancy creation and*

[29]To see this, note that, given (21) and $\kappa_n(x) = \hat{\kappa}(x) - \kappa_v(x)x$, the accounting equation (20) implies $\hat{V}_t = -\kappa_n(x_t) + \beta(1-\delta)E_t\hat{V}_{t+1} = -E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k \kappa_n(x_{t+k})$. Note that these derivations make use of the size-independence property. In the more general case with $W(n,S), \theta(n,S), x(n,S)$, differentiating the accounting equation (18) yields $V_n(n,S) = -\kappa_n(x) + \beta(1-\delta)V_n(n',S') + nW_n(n,S)$ instead of the $V_n(n,S) = -\kappa_n(x) + \beta(1-\delta)V_n(n',S')$ applied in the text (assuming differentiability). The additional term $nW_n(n,S)$ appears because the firm's objective (16) does not coincide with the right hand side of (18). In the more general case the derivative term enters into the following equations (25) and (26), changing the nature of the problem from the dynamic system laid out in the text.

[30]In the case with full commitment to future wages, firms solve the sequence problem instead.

employment than absent within-firm constraints. Firm value from hiring is higher, and the worker values of searching for and accepting employment lower, than absent constraints.

The constraints lead to downward pressure on wages, due to the constrained firms' incentive to profit from their existing workers via low wages. With all firms affected, the equilibrium shifts toward lower wages in a way that hurts workers and benefits firms, encouraging vacancy creation and hiring in doing so.

Corollary 2. *The competitive search equilibrium with firm wages is inefficient.*

It follows immediately from the above that the firm wage equilibrium is inefficient, as allocations differ from the unconstrained case (see Appendix A). Firms exploit their monopsony power over their existing workers, leading to too-low wages and overhiring.

In drawing wages down toward the workers' opportunity cost, the constraints also work to render wages less responsive to productivity. The first order conditions imply that the present value wage may be written as a weighted average of the worker's opportunity cost and the value of output from the relationship:

$$W_t = (1-\gamma_t)Y_t + \gamma_t[Z_t - E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k \kappa_n(x_{t+k})], \tag{25}$$

where the weight γ_t differs between the constrained and unconstrained cases. In the unconstrained case, the weight is determined by the matching function elasticity as $\gamma_t = 1 - \varepsilon_t$, whereas in the constrained case $\gamma_t = \frac{1-\varepsilon_t}{1-\varepsilon_t + \frac{1+q(\theta_t)x_t}{q(\theta_t)x_t}\varepsilon_t}$ instead.[31] Here the worker's opportunity cost, $Y_t = b + E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k (b + X_{t+k})$, reflects the value of forgone home production and search during employment, while the value of output and reduction in subsequent hiring costs is given by $Z_t - E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k \kappa_n(x_{t+k})$. Per (25), the constraints work to render wages less responsive to productivity, with optimal vacancy creation implying that constrained firms' hiring becomes more responsive to productivity as a result.[32]

The first order conditions may also be used to derive a familiar dynamic equation char-

[31] In the case with full commitment to future wages, the initial weight takes the form of the (above) constrained weight, while subsequent weights take the form of the unconstrained weight. The equilibrium remains inefficient due to the departure in the initial period.

[32] This conclusion is more immediate with firm-level shocks than aggregate shocks, because the latter influence the worker's opportunity cost as well. The constraints reduce the direct dependence of wages on productivity with aggregate shocks as well, however.

acterizing the evolution of allocations based on the weights $\{\gamma_t\}_{t=0}^{\infty}$:[33]

$$\frac{\kappa_v(x_t)}{q(\theta_t)(1-\gamma_t)} = z_t - b + \beta(1-\delta)E_t[\frac{\kappa_v(x_{t+1})}{q(\theta_{t+1})(1-\gamma_{t+1})} - \frac{\gamma_{t+1}\theta_{t+1}\kappa_v(x_{t+1})}{1-\gamma_{t+1}} - \kappa_n(x_{t+1})]. \quad (26)$$

Equilibrium allocations $\{\theta_t, x_t, N_t\}$ thus follow the dynamic system given by: i) equation (26) with γ_t as defined above, ii) adding-up constraint $1 - N_t = x_t N_t/\theta_t$, and iii) law of motion $N_{t+1} = (1-\delta)(N_t + \mu(\theta_t)(1-N_t))$.

4 Heterogeneity and Hierarchies within the Firm

To relate to the motivating evidence on salary structures more directly, the analysis may be extended to feature explicit worker heterogeneity in a straightforward way. Doing so accommodates persistent differences in worker productivity within a firm, shocks to individual worker productivity over time, as well as explicit growth in worker productivity on the job.[34]

To that end, suppose individual worker productivity can take on a finite number of values $\{z^j(S)\}_{j=1}^J$ for each aggregate state S, with transitions between states characterized by probabilities π_{jk} where $\sum_{k=1}^J \pi_{jk} = 1$ for all $j = 1 \dots J$. Firms seek to hire all productivity types, with the labor market segmenting by type. The within-firm constraints, moreover, extend these hiring wages to existing workers of similar productivity as well.

The firm problem may then be written adding up across productivity types as:

$$\max_{\{W_j, v_j\}_{j=1}^J} \sum_{j=1}^J [(n_j + q(\theta_j)v_j)(Z_j(S) - W_j) - \kappa(v_j, n_j)] + \beta E_S V(\{n'_j\}_{j=1}^J, S') \quad (27)$$

$$\text{s.t. } n'_j = (1-\delta)\sum_{k=1}^J \pi_{kj}(n_k + q(\theta_k)v_k), \ \forall j = 1 \dots J,$$

$$X_j(S) = \mu(\theta_j)(W_j - Y_j(S)), \ \forall j = 1 \dots J,$$

together with the accounting equation

$$V(\{n_j\}_{j=1}^J, S) = \sum_{j=1}^J [q(\theta_j)v_j(Z_j(S) - W_j) - \kappa(v_j, n_j)] + \beta E_S V(\{n'_j\}_{j=1}^J, S').$$

[33] A similar characterization is provided by Shimer (2005) in a random search setting, with γ_t representing the worker bargaining power parameter. In random search models this bargaining power parameter is typically chosen such that allocations are efficient, meaning that outcomes in those models can be viewed as corresponding to the unconstrained case of the present analysis.

[34] One can consider productivity growth due to general or firm-specific human capital accumulation, differing in how separation shocks affect productivity.

Here the firm's existing and new workers of type j produce the present value of output Z_j at the type-specific wage W_j, with vacancies subject to the vacancy cost $\kappa(v_j, n_j)$.[35] The firm takes as given the type-specific laws of motion for its workforce, which depend on hiring across types, as well as the constraints characterizing its beliefs, where the searching workers' value of search X_j and opportunity cost Y_j depend on type.

Maintaining the focus on size-independent behavior, with $W_j(S), \theta_j(S), x_j(S)$ independent of the measures of workers within the firm and continuation values linear, $V(\{n_j\}_{j=1}^J, S) = \sum_{j=1}^J \hat{V}_j(S)n_j$, the firm problem (27) implies the type-specific problems:

$$\max_{W_j, x_j} \; (1 + q(\theta_j)x_j)(Z_j(S) - W_j) - \hat{\kappa}(x_j) + \beta(1-\delta)(1 + q(\theta_j)x_j)E_j E_S \hat{V}_{j'}(S')$$
$$\text{s.t. } X_j(S) = \mu(\theta_j)(W_j - Y_j(S)),$$

for all $j = 1, \ldots, J$, where

$$\hat{V}_j(S) = q(\theta_j)x_j(Z_j(S) - W_j) - \hat{\kappa}(x_j) + \beta(1-\delta)(1 + q(\theta_j)x_j)E_j E_S \hat{V}_{j'}(S').$$

The first order conditions for wages and vacancy creation remain essentially unchanged from the previous analysis.[36] In this setting, more productive workers are generally hired at higher wages, with the constraints extending these higher wages to existing workers of similar productivity as well. Persistently higher productivity leads to persistently higher wages, and transitory productivity increases/declines to transitory wage increases/declines. At the same time, the constraints continue to work to render wages less responsive to productivity—thus also compressing wage differences across productivity types within the firm.

Broadly, one could interpret the different levels of the hierarchies described in the motivation as corresponding to larger and more persistent differences in worker productivity within the firm, with smaller and more transitory productivity variation giving rise to wage variation within levels of the hierarchy. If worker productivity grows over time on the job, the worker then rises in the hierarchy as a result. The constraints work to ensure that wages reflect worker productivity throughout the organization, but also compress wage differences both within and across levels of the hierarchy.

[35] The vacancy cost takes a form where larger firms face a lower cost of creating the same measure of vacancies than smaller firms, which may be interpreted as the firms' existing workers contributing to recruiting activities. In writing the firm problem in this way I have assumed that more productive workers contribute to the recruiting of more productive workers and vice versa.

[36] The firm's first order conditions reflect (21) and (22):

$$\kappa_v(x_j) = q(\theta_j)(Z_j - W_j + \beta(1-\delta)E_j E_S \hat{V}_{j'}(S')), \text{ and}$$
$$1 + q(\theta_j)x_j = q'(\theta_j)g_W^j(W_j; S)x_j(Z_j - W_j + \beta(1-\delta)E_j E_S \hat{V}_{j'}(S')).$$

Having demonstrated how the model maps to a setting with heterogeneous workers, I return to the setting with homogenous workers in what follows.

5 Infrequent Wage Adjustment

In the presence of firm wage constraints, firms face a commitment problem leading to inefficient outcomes in the labor market. The commitment problem suggests firms may find it profitable to adopt rules regarding their wage setting, and that doing so may also be welfare improving. This section considers the parsimonious rule of firms simply fixing wages for a period of time, and reoptimizing only from time to time. I begin with the problem of an individual firm, and then turn to an equilibrium where all firms fix wages.

To that end, consider a firm setting a fixed wage w for a probabilistic period of time. The firm's beliefs regarding the market tightness continue to be determined by the constraint $X(S) = \mu(\theta)(h(w,S) - Y(S))$ each period, where $h(w,S)$ represents the present value of wages. For a firm setting a fixed wage w, expecting to reoptimize with probability α each period, this present value may be written as

$$h(w,S) = \frac{w}{1-\beta(1-\delta)(1-\alpha)} + E_S \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (1-\alpha)^k \beta(1-\delta)\alpha W^r(S^{k+1}),$$

where $W^r(S)$ denote the present value wages attained when reoptimizing.

The firm problem may be written as:

$$\begin{aligned} \max_{w,v}\ & (n+q(\theta)v)(Z(S)-h(w,S)) - \kappa(v,n) + \beta E_S(\alpha V^r(n',S') + (1-\alpha)V^f(n',w,S')) \\ \text{s.t. } & n' = (1-\delta)(n+q(\theta)v), \\ & X(S) = \mu(\theta)(h(w,S) - Y(S)). \end{aligned}$$

The objective corresponds to that in (16) except that the firm attains the continuation value of reoptimizing $V^r(n',S')$ only if the wage expires, and a corresponding value of holding the wage fixed $V^f(n',w,S')$ otherwise. The former satisfies the accounting equation

$$V^r(n,S) = \ q(\theta)v(Z(S)-h(w,S)) - \kappa(v,n) + \beta E_S(\alpha V^r(n',S') + (1-\alpha)V^f(n',w,S')),$$

where w, θ, v solve the above firm problem. The latter solves the problem

$$\begin{aligned} V^f(n,w,S) = \max_{v}\ & q(\theta)v(Z(S)-h(w,S)) - \kappa(v,n) + \beta E_S(\alpha V^r(n',S') + (1-\alpha)V^f(n',w,S')) \\ \text{s.t. } & n' = (1-\delta)(n+q(\theta)v), \\ & X(S) = \mu(\theta)(h(w,S) - Y(S)), \end{aligned}$$

with the firm continuing to optimize vacancy creation while the wage remains fixed. These problems may again be scaled, to arrive at size-independent problems.

Optimal wage setting takes into account that the chosen wage influences hiring for the duration of the wage rather than a single period. The first order condition for the wage reads

$$\begin{aligned}(1+q(\theta)x)h_w = q'(\theta)g_W(W;S)h_w x[Z-W+\beta(1-\delta)E_S[\alpha\hat{V}^r(S')+(1-\alpha)\hat{V}^f(w,S')]] \\ +\beta(1-\delta)(1+q(\theta)x)(1-\alpha)E_S\hat{V}^f_w(w,S'). \qquad (28)\end{aligned}$$

The firm raises the wage to a point where the increase in present value wage costs on the firm's existing and new workers today, on the left, equals the increase in present value profits from the additional workers hired *throughout the duration of the wage*, as the higher wage increases job seekers per vacancy, on the right. Recall that the firm faces a tradeoff between making profits on its existing workers via a low wage and optimizing on the hiring margin with a higher wage. Intuitively, the longer the duration of the wage—other things equal—the more weight the firm places on the hiring margin and the higher the wage.[37]

The first order condition for vacancy creation reads:

$$\kappa_v(x) = q(\theta)(Z-W+\beta(1-\delta)E_S(\alpha\hat{V}^r(S')+(1-\alpha)\hat{V}^f(w,S'))). \qquad (29)$$

The firm creates vacancies up to a point where the marginal cost of an additional vacancy equals the present value of profits from the additional workers hired. A wage that is closer to the optimal hiring wage should then, intuitively, also lead to increased vacancy creation to take advantage of the improved profitability of hiring.

Together with these changes in firm behavior, longer wage durations may also be expected to raise firm value due to the commitment involved, at least in the absence of shocks. Even though longer wage durations lead to reduced profits on the firm's existing workers, they increase the profitability of current and future hiring, due to the higher and thus more attractive wages sustained by the commitment.

Note that the above discussion holds the equilibrium value of search, and thus the behavior of other firms in the market, unchanged. If wage durations change across all firms, the equilibrium value of search will adjust in response to changes in firm behavior, altering the relationship firms face between their offered wages and hiring rate. Thus, the effects of

[37] The scaled firm problem may also be written as $\max_w Z(S) - h(w,S) + \hat{V}^f(w,S)$, with firm value comprised of the contributions of existing workers and new hires (today and in the future) separately. According to the implied first order condition $-h_w(w,S) + \hat{V}^f_w(w,S) = 0$, the firm raises the wage to a point where the decline in value from existing workers is equated to the increase in value from hiring (today and in the future). Thus, $\hat{V}^f_w(w,S) > 0$ at the equilibrium wage. The expression for h yields $h_w(w) = 1/(1-\beta(1-\delta)(1-\alpha)) > 0$.

longer wage durations on individual firms can differ from the equilibrium effects of longer wage durations affecting all firms.

To consider the equilibrium effects of longer wage durations, a competitive search equilibrium with firm wages and infrequent wage adjustment may be defined along the lines of Definition 2, but with each firm reoptimizing its wage with probability α each period, independently across firms, and otherwise holding it fixed. The distribution of these fixed wages across firms becomes part of the state S in this case, because the tightness at fixed-wage firms depends on the wage, and the equilibrium adding up constraint for job seekers thus requires keeping track of these wages.

The level effects of longer wage durations on equilibrium outcomes may be summarized as follows:

Proposition 3. *The competitive search equilibrium with firm wages and infrequent wage adjustment satisfying* (8)*,* (28)*,* (29)*,* $n_{it+1} = (1-\delta)(n_{it} + q(\theta_{it})x_{it}n_{it})$ *for all* $i \in I$ *and* $\sum_{i \in I} x_{it}n_{it}/\theta_{it} = 1 - \sum_{i \in I} n_{it}$ *has a unique steady state, with a present value hiring wage that is strictly increasing, and market tightness, vacancy creation and employment that are strictly decreasing in the duration of wages. Firm value is strictly decreasing, and the worker values of job search and employment strictly increasing in the duration of wages. Allocations and values do not reach their efficient levels for any* $\alpha \in [0, 1]$.

Longer wage durations continue to raise wages, but with all firms affected the equilibrium shifts as a result. On this level the market tightness and vacancy creation must move in the same direction, and both the tightness and vacancy creation fall as wages rise, reducing hiring. These changes bring the equilibrium closer to efficient allocations by causing the overhiring to subside. The equilibrium remains inefficient even as wage durations grow infinitely long, however: As long as firms discount the future, they place more weight on the profits they can make on their existing workers in the short run than on hiring later on.

6 Quantitative Illustration

This section considers the implications of firm wage constraints for labor market outcomes in a setting where firms face shocks to labor productivity. First, how firm wage constraints influence the responses of wages and hiring to aggregate shocks, and then the profitability and equilibrium implications of infrequent wage adjustment.

6.1 Parameterizing and Solving the Model

I begin with a parametrization and discussion of the solution approach.

Parametrization To compare the constrained and unconstrained models in terms of shock propagation, I first seek to parameterize the two models to the same steady state in observables: the speed of labor market flows, level of unemployment/employment, and wage/profit shares.[38] Appendix D provides details.

I adopt a monthly frequency, set the discount rate to $\beta = 1.05^{-1/12}$, and normalize steady-state labor productivity to $z = 1$. To be consistent with an average duration of employment of 2.5 years, I set the separation rate to $\delta = 0.033$. To then be consistent with an average unemployment rate of 5 percent, when steady-state unemployment in the model is $\delta(1-\mu(\theta))/(\mu(\theta)+\delta(1-\mu(\theta)))$, requires a steady-state job-finding probability of $\mu(\theta) = 0.388$. I adopt the matching function $m(v,u) = vu/(v^\ell + u^\ell)^{1/\ell}$ for this discrete time model, as in den Haan, Ramey, and Watson (2000), and target a steady-state level of θ of 0.43, as in Kaas and Kircher (2015). With this, fitting the above job-finding probability requires $\ell = 1.85$. Finally, I follow Kaas and Kircher (2015) in adopting the vacancy cost $\kappa(v,n) = \frac{\kappa_0}{1+\varphi}(v/n)^\varphi v$ where $\varphi = 2$. This leaves two remaining free parameters, κ_0 and b.

For a benchmark parametrization for the unconstrained model, following Shimer (2005), I adopt the value $b = 0.4$ and set κ_0 such that equation (26) holds in steady state with the unconstrained value of γ. I then seek alternative values of κ_0, b for the constrained model that hold the share of wage versus profit income across the two models unchanged, while ensuring that equation (26) holds with the constrained value of γ. It turns out that doing so requires holding the value of κ_0 unchanged across models, while raising b to bring wages in the constrained model to their levels in the unconstrained model (see Appendix D for details). The implied value of b for the constrained model is 0.89. I consider alternative parametrization approaches also.

The discussion of infrequent wage adjustment maintains the above parametrization, comparing outcomes in the constrained model to the corresponding planner allocation.

Solution Approach The baseline model where firms reoptimize each period is relatively straightforward to solve, as the equilibrium conditions reduce to a set of nonlinear difference

[38] This approach allows comparing firm responses to shocks in a setting where the shock is similarly sized relative to the profitability of hiring across models. Note that a similar approach is used in, e.g., Hall and Milgrom (2008), Hagedorn and Manovskii (2008), and Elsby and Michaels (2013), who adopt an explicit target for the vacancy cost (and hence firm profit rate).

equations that can be solved with standard methods.[39] The complete system of equations is provided in Appendix E. In solving the model, I also check that the solution characterized by the first order conditions dominates the corner solution of withdrawing from hiring for a period: zero vacancies and a low wage making existing workers indifferent between remaining employed and quitting to search for a new job (see Appendix B for a discussion).[40]

In the equilibrium with infrequent wage adjustment the distribution of wages becomes a state variable, because individual firms' choices of θ_{it}, x_{it} depend on their wage and in equilibrium these must satisfy the adding up constraint across firms $\sum_i x_{it}n_{it}/\theta_{it} + \sum_i n_{it} = 1$ each period. With aggregate shocks, this extended model may be solved by first linearizing the model equations and then aggregating across firms, arriving at a system where the average wage across firms becomes a sufficient statistic for the distribution of wages.

I also consider an environment where firms face firm-specific idiosyncratic shocks (see Appendix G). The model with firm-level shocks is solved on a grid for productivity directly, as in this case the only state variable is the firm's current productivity. For the baseline model, doing so involves solving a nonlinear system of equations in the equilibrium firm choices of $\{W, \theta, x\}$ for each possible productivity realization, and simulating the model to find the value of X consistent with the equilibrium adding up constraint. For the equilibrium with fixed wages, the set of unknowns is larger but a similar approach may be used. I use continuation to aid in solving these systems.

The next sections describe the results.

6.2 Firm Wages over the Business Cycle

How do firm wage constraints affect the responses of wages and hiring to aggregate shocks?

A side-by-side comparison of the two models shows that the firm wage model features more rigid wages in response to aggregate shocks than the unconstrained model. To illustrate, Figure 1 plots impulse responses to a one percent positive productivity shock across the two models. Wages increase in response to the shock in both models, but the increase in the firm wage model is only about a quarter of that in the unconstrained model, where the wage increase is almost identical to that of productivity. This allows the profitability of hiring to rise more in the firm wage model, whereas in the unconstrained model the wage increase absorbs the bulk of the productivity increase, leaving limited room for the profitability of

[39] The tractability is due to the structure of the problem together with the focus on equilibria that are consistent with the size-independence of the firm problem.

[40] See Appendixes C and F for these checks.

Figure 1: Impulse Responses in Firm Wage vs Unconstrained Model

Notes: The figure plots the percentage responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model and the unconstrained model. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$. The two models have the same steady-state levels of wage, market tightness, unemployment, as described in the section on parametrization.

hiring to rise. The result is an increase in the vacancy-unemployment ratio that is an order of magnitude greater in the firm wage model than the unconstrained model, with equally significant amplification in the increase in vacancy creation and drop in unemployment in response to the shock.[41]

The above impact of firm wages on the volatility of labor market flows is significant relative to the gap between model and data emphasized in the literature seeking to understand the cyclical volatility of unemployment and vacancy creation: In the model the vacancy-unemployment ratio increases by 6.5 times the increase labor productivity, while the relative standard deviation of the vacancy-unemployment ratio to labor productivity in the data is $38/2 \approx 19$ (Shimer 2005). The model thus generates roughly a third of the variation in the data.

One can also consider parametrizations where the difference in b across models is smaller, letting κ_0 adjust as well. The corresponding model comparison is shown in Figure 2, which

[41] The statement that the constraints lead to rigidity in wages refers to allocative rigidity, i.e. rigidity in the present value of wages. Strictly speaking, the unconstrained model does not pin down per-period wages without additional assumptions, so to arrive at a series for per-period wages for that model that speaks to allocative rigidity I make the symmetric assumption that the unconstrained firm pays all its workers the same at each point in time.

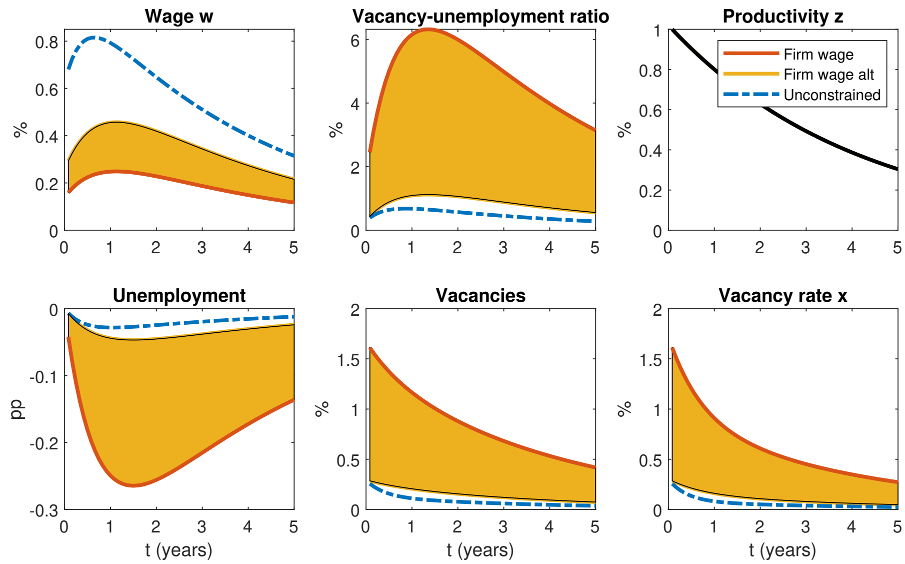

Figure 2: Impulse Responses in Firm Wage vs Unconstrained Model across Parametrizations
Notes: The figure plots the percentage responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model parameterized with alternative combinations of κ_0, b and the unconstrained model. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$.

plots a band of impulse responses under alternative combinations of κ_0, b, from only b adjusting across models (as in Figure 1) to only κ_0 adjusting. While the magnitude of the amplification in labor market flows ranges from substantial to moderate, the constrained model continues to feature more rigid wages and more volatile labor market flows throughout. Note, however, that this comparison generally involves a greater vacancy cost for the firm wage model—to counteract the increased tendency for hiring in that model—something that also works to dampen the responses of labor market flows to shocks. In the limiting case where b is unchanged across models, the vacancy cost is six times greater in the firm wage model.[42]

In sum, firm wage constraints give rise to rigidity in wages, amplifying business cycle fluctuations in labor market flows.

[42] With all model parameters held unchanged across models in the comparison, the qualitative features continue to hold with more modest magnitudes, while steady-state levels differ noticeably across models. Figure F.2 in Appendix F provides impulse responses.

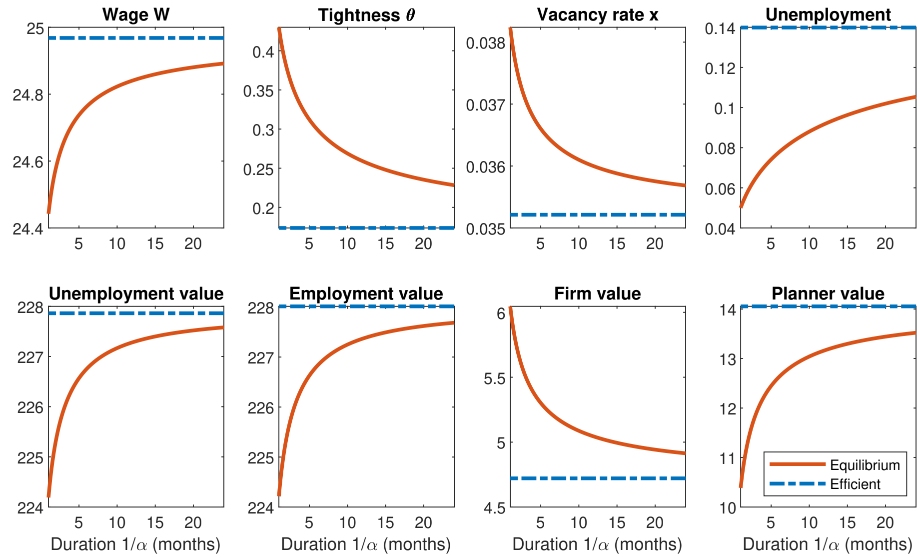

Figure 3: Equilibrium with Infrequent Wage Adjustment

Notes: The figure plots the non-stochastic steady state of the equilibrium with infrequent wage adjustment as a function of the equilibrium wage duration, together with efficient allocations. The firm and planner values plotted are the scaled values, but the unscaled values remain monotonic in wage duration.

6.3 Infrequent Wage Adjustment

This section turns to consider the equilibrium impact and profitability of fixed wages, and concludes by demonstrating the emergence of a privately optimal equilibrium wage duration.

Equilibrium Effects of Fixed Wages Equilibrium outcomes are inefficient due to too-low wages distorting hiring. Relative to this starting point, Figure 3 illustrates the equilibrium effects of all firms adopting simple fixed wage rules in wage setting, showing how outcomes change with the expected duration of wages. The figure focuses on level effects in a non-stochastic setting. As the figure shows, longer wage durations raise the level of wages, causing the overhiring taking place in equilibrium to subside, with market tightness and vacancy creation both falling. As this happens, unemployment rises toward its socially optimal level in this frictional market. The changes improve allocative efficiency, but fall short of achieving planner allocations as the wage remains below efficient throughout.

The bottom panel illustrates the impact on planner, worker and firm values. Planner value rises in the expected duration of wages as allocations approach efficient ones. Not everyone gains from this improvement in allocative efficiency, however. Workers, both employed and unemployed, benefit from longer wage durations due to the higher equilibrium wages. Firm profitability, on the other hand, falls. Longer wage durations reduce firm valuations, as firms become more focused on hiring with attractive wages than gouging their existing workers, and the equilibrium shifts as a result.

When firms face shocks, these level effects are combined with fixed wages limiting privately optimal responses to shocks. Figure F.4 provides an example illustrating how a fixed wage prevents a firm from taking advantage of a positive shock to labor productivity. The presence of shocks needs not overturn the above results, however. Figure F.5 demonstrates the impact of longer wage durations on equilibrium outcomes in a setting where firms face aggregate shocks to labor productivity. Longer wage durations raise the cyclical volatility of vacancy creation and unemployment, but the level effects remain important. Figure F.6 considers the effects of longer wage durations in a setting where firms face firm-level shocks, of an order of magnitude larger than aggregate shocks. Such shocks clearly matter for firm behavior, but longer wage durations continue to work to reduce the inefficiencies resulting from firms exploiting their monopsony power, and make workers better off. I return to describe the setting with firm-level shocks in the following section.

Single Firm Deviations from Equilibrium Longer wage durations are also privately optimal for firms. To illustrate, Figure 4 begins with the equilibrium where firms reoptimize wages each period (indicated by the circles), and considers an individual firm setting its wage for a longer duration, in a non-stochastic setting. The longer wage duration makes the firm more forward looking in setting its wage, placing more weight on the profitability of hiring over this longer duration. As a result, the firm sets a higher wage than equilibrium firms, attracting more job seekers per vacancy. As the higher wage raises the profitability of vacancy creation, the firm also creates more vacancies. For both reasons, the firm's hiring rate exceeds that of equilibrium firms.[43] In particular, the longer wage duration raises firm value, as well as making the firm's workers better off. The firm would thus prefer to extend its wage duration above the equilibrium duration.

The effects of longer wage durations remain qualitatively similar when considering an equilibrium where all firms reoptimize wages less frequently. To illustrate, Figure 5 begins with an equilibrium where firms reoptimize wages annually (indicated by the circles), and considers an individual firm setting its wage for a longer/shorter duration, in a non-stochastic setting. A longer wage duration raises the firm's wage, vacancy rate and hiring rate, and in particular firm value. A shorter wage duration has the opposite effect. The firm would thus

[43] The deviating firm begins to grow relative to equilibrium firms, which is why the figure considers small deviations only. If desired, the model is straightforward to extend to feature exit shocks, with exiting firms replaced by new entrants. In this case the deviations to longer wage durations would remain of finite expected duration.

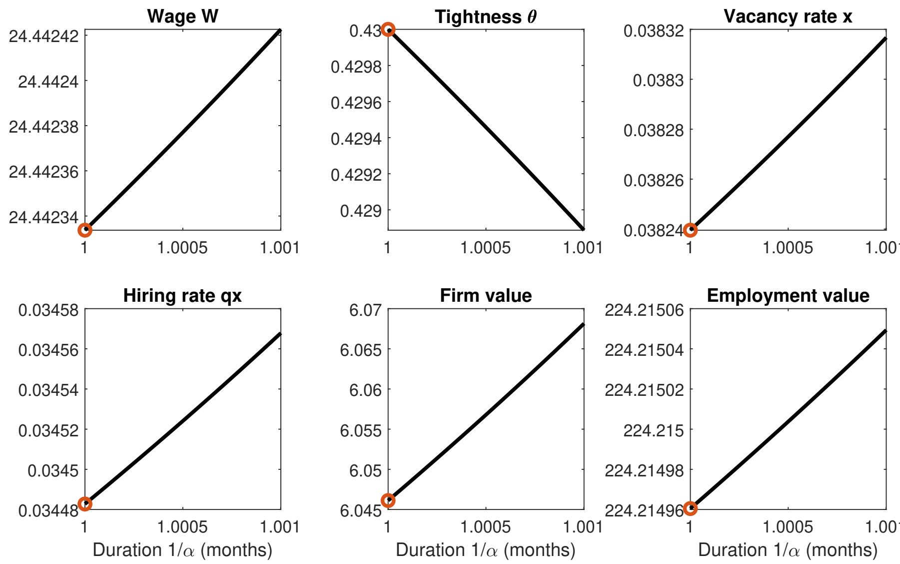

Figure 4: Single Firm Deviation to Longer Wage Duration

Notes: The circles denote the non-stochastic steady state of the equilibrium with firm wages where firms reoptimize monthly, and the figure plots corresponding values for a firm deviating to longer wage duration as a function of the expected duration of wages $1/\alpha$. The firm value plotted is the scaled value.

prefer to extend its wage duration above the equilibrium duration.

What happens when firms face shocks that make fixed wages costly by preventing privately optimal responses? The answer is relatively straightforward when it comes to aggregate shocks, because aggregate shocks are small and the first order effects of longer wage durations on levels thus likely to prevail. The answer becomes less clear when considering the notably larger firm-level shocks that firms face. To consider this, I turn to a version of the model with idiosyncratic firm-specific shocks to productivity, increasing the standard deviation of shocks by an order of magnitude in doing so.[44]

In a stationary equilibrium with idiosyncratic firm-specific shocks, firms respond to increased productivity by raising their wage, which attracts more job applicants per vacancy, as well as by creating more vacancies. This accelerates firm growth relative to less productive firms, and the firm expands. A decline in productivity has the opposite effect. The effects of single firm deviations to longer wage durations in this setting are illustrated in Figures F.7-F.9. These figures display the behavior of an individual firm setting its wage for a longer/shorter duration than equilibrium firms, both as a function of firm productivity and

[44] Appendix G lays out the version of the model with firm level shocks. Because the model has constant returns to scale on the firm level, it is not ideally suited for incorporating very persistent firm-specific differences in productivity, even if the curvature in vacancy costs does accommodate more transitory differences in equilibrium. As such, I reduce the persistence of shocks somewhat in the experiment with firm level shocks.

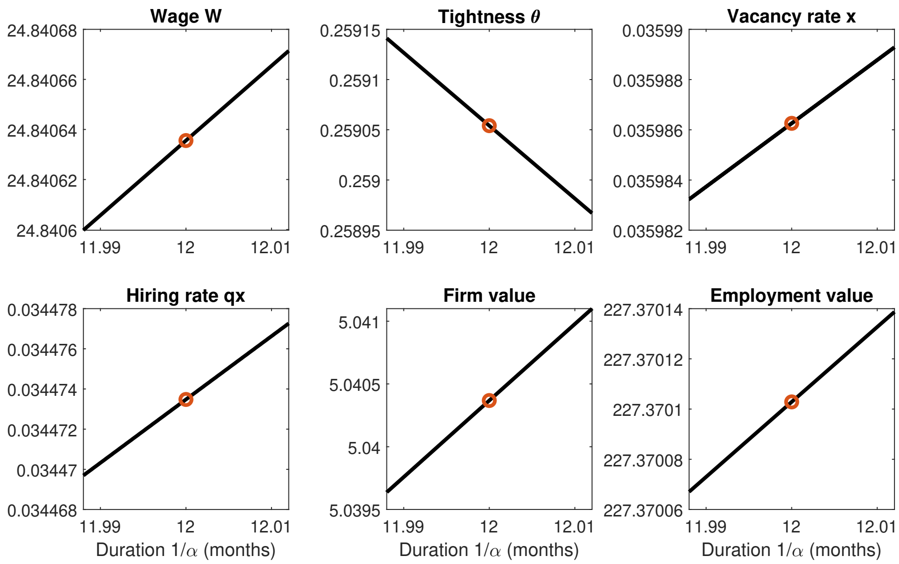

Figure 5: Single Firm Deviation to Longer/Shorter Wage Duration

Notes: The circles denote the non-stochastic steady state of the equilibrium with infrequent wage adjustment where firms reoptimize annually, and the figure plots corresponding values for a firm deviating to longer/shorter wage duration as a function of the expected duration of wages $1/\alpha$. The firm value plotted is the scaled value.

when equilibrium firms reoptimize wages monthly, annually, and biennially. Firm behavior remains consistent with the above discussion throughout. The impact of longer wage durations on firm value is summarized in Figure 6, which shows that an individual firm continues to prefer to extend its wage duration above the equilibrium duration throughout, despite the presence of such shocks.

Note that the shocks lead to significant variation in firm value, but remain small enough to keep firm value positive even in the lowest productivity state. The firm continues to hire throughout as well, preferring doing so over withdrawing from hiring and paying existing workers their outside option (Figure F.10 verifies this). Of course, in reality firms do also exit and let workers go when circumstances deteriorate sufficiently. Fully capturing such effects remains beyond the scope of the present model setup, limiting the magnitude of shocks that the exercise accommodates. The exercise confirms that the effects of longer wage durations survive incorporating substantial shocks, however.

The next section proceeds to demonstrate the emergence of a privately optimal equilibrium wage duration in the context of a convex cost of longer wage durations, instead of seeking to model such costs directly.

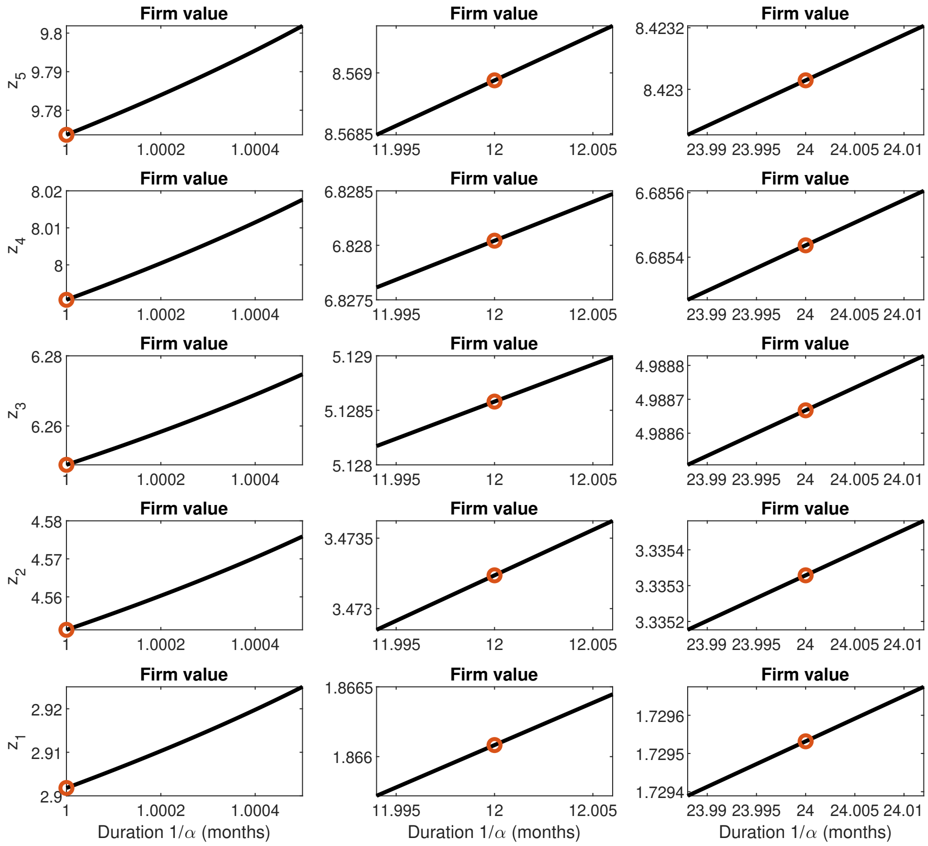

Figure 6: Single Firm Deviation to Longer/Shorter Wage Duration with Firm Shocks

Notes: The circles denote the equilibrium with infrequent wage adjustment where firms reoptimize monthly, annually and biennially, and the figure plots corresponding values for a firm deviating to longer/shorter wage duration as a function of the expected duration of wages $1/\alpha$. The model is solved on a 5-state grid for productivity, approximating an AR(1) with autocorrelation $\sigma_z = 0.88$ and standard deviation $\sigma_z = 0.2$ based on the Rouwenhorst method. The firm value plotted is the scaled value.

Privately Optimal Equilibrium Wage Duration If the costs of longer wage durations for firms are captured with a convex function $c(d)n$, firm value net of these costs may be written as a function of the duration d as $\hat{V}^0(d)n - c(d)n$, where $\hat{V}^0(d)$ refers to the scaled firm value discussed. The left panel of Figure 7 plots this firm value as a function of the equilibrium duration of wages, illustrating that longer wage durations make firms worse off (consistent with Figure 3 augmented with a growing cost of longer durations). By contrast, the right panel plots the change in firm value from an individual firm extending its wage duration above the equilibrium duration. The change is positive at low equilibrium durations, but falls in duration due to the growing costs involved. The figure demonstrates that there exists an equilibrium duration of wages that is also privately optimal for firms, in that individual firms do not have an incentive to deviate toward longer or shorter durations.

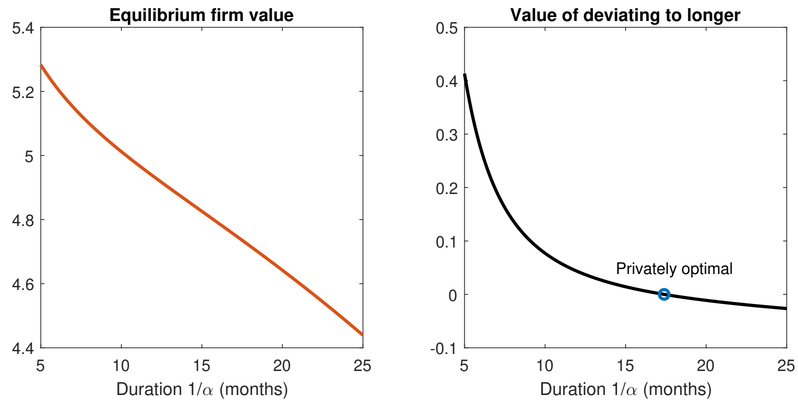

Figure 7: Privately Optimal Equilibrium Wage Duration

Notes: The figure demonstrates the emergence of a privately optimal wage duration in a setting with a convex cost of increasing wage duration. The left panel plots the scaled equilibrium firm value as a function of the duration of wages, and the right panel the change in firm value from an individual firm deviating to a longer wage duration than equilibrium firms. The cost function is quadratic and parameterized such that the privately optimal equilibrium wage duration is 17-18 months (Grigsby, Hurst, and Yildirmaz 2021), with $c(d)n = 0.00075d^2n$.

The privately optimal equilibrium wage duration is longer when firms value commitment to future wages more. Intuitively, the theory thus predicts longer wage durations when agents are more patient (value the future more), when employment relationships are more stable (separation rates and firm exit rates are lower), as well as when employment relationships are more valuable (the gains from market production relative to the costs of hiring are greater). Wage durations should also be longer when the costs of longer durations are lower, for example due to smaller shocks. These effects are indeed confirmed by the model.[45]

Most characteristically, this theory predicts stable wages in stable long-term employment relationships, as opposed to "spot" labor markets.

6.4 Firm Wages, Infrequent Adjustment and the Business Cycle

The above discussion raises the question of how firm wages affect business cycles in wages and labor market flows in the presence of infrequent wage adjustment. As discussed, firm wages directly dampen wage responses to shocks as well as motivate the emergence of infrequent wage adjustment. Because the two effects interact in the model, however, it is useful to

[45] A calculation using the above parametrization indicates that a one percent increase in the discount rate decreases wage duration by 0.5 percent and a one percent increase in the separation rate by 1.3 percent. A one percent increase in market productivity or decline in home productivity increases wage duration by six percent, while similar increases in the (parameters governing) the vacancy cost κ_0 and cost of longer wage durations c_0 decrease wage duration by 0.4 and 0.3 percent, respectively.

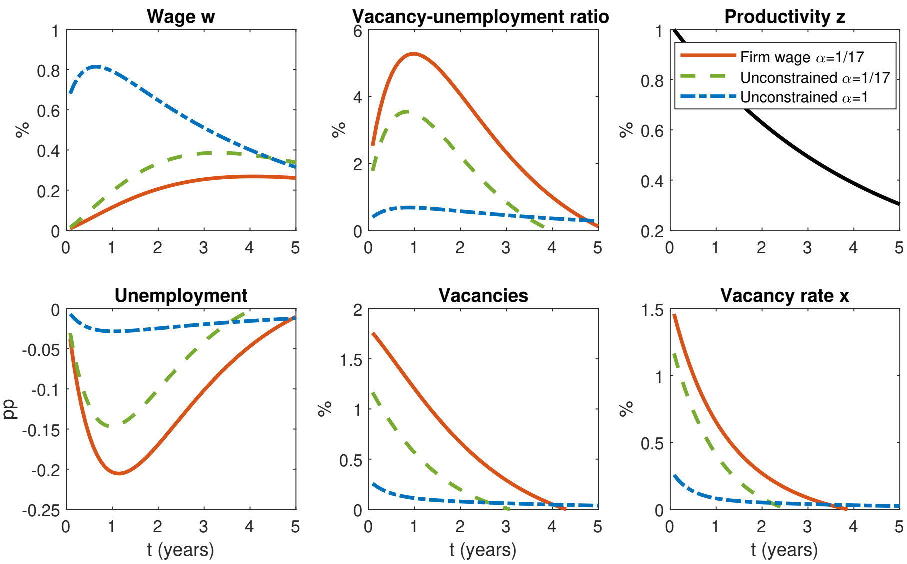

Figure 8: Impulse Responses with Infrequent Wage Adjustment

Notes: The figure plots the percentage responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model with infrequent wage adjustment and the unconstrained model with and without infrequent wage adjustment (duration of wages 17 months). Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$. The three models have the same steady-state levels of wage, market tightness, unemployment, as described in the section on parametrization.

revisit their combined effect on how wages and labor market flows respond to aggregate shocks.

To this end, Figure 8 provides a side-by-side comparison of the firm wage model with infrequent wage adjustment and the unconstrained benchmark model. Firm wages with infrequent adjustment continue to dampen wage responses to shocks and amplify those of labor market flows in a significant way. The response of the vacancy-unemployment ratio to the shock is roughly an order of magnitude greater than in the unconstrained model, confirming the earlier conclusion. The amplification is slightly weaker than absent infrequent wage adjustment, however, which can occur because infrequent wage adjustment weakens the direct effect of firm wages to dampen wage responses to shocks.

To isolate the roles of the two effects of firm wages, the figure also displays responses in the unconstrained model augmented with infrequent wage adjustment. Comparing the two versions of the unconstrained model shows that infrequent wage adjustment alone dampens wage responses and amplifies those of labor market flows significantly. Firm wages serve to motivate the emergence of such infrequent wage adjustment, as well as to dampen wage responses and amplify those of labor market flows further in a non-negligible way.

7 Conclusions

Large firms play an important role in the labor market, and worker compensation in large organizations is generally governed by firm-level policies. This paper developed a theory to shed light on the implications of such policies for labor market outcomes: a directed search model of multi-worker firms facing within-firm equity constraints on wages.

I showed that introducing such constraints reduces wages, as the firms' incentive to profit from their existing workers via low wages depresses also those of new hires. With all firms affected, the equilibrium shifts toward lower wages in a way that hurts workers and benefits firms, increasing the profitability of vacancy creation and leading to overhiring. In drawing wages toward the worker's opportunity cost, the constraints also work to render wages less responsive to productivity, leading to wage compression and rigidity.

I also showed that the constraints give rise to a time-inconsistency in the dynamic firm problem, as the firm effectively faces a less elastic labor supply in the short than the long run, making commitment to future wages valuable. A parsimonious approach nevertheless allowed providing an analytic characterization for the impact of such constraints on wages, driving the implications for hiring.

Finally, two applications demonstrated that such constraints give rise to rigidity in wages that amplifies business cycles in labor market flows, as well as implying that fixed wages can be both privately optimal for firms and good for worker welfare and resource allocation in equilibrium.

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A Unconstrained Firms and Efficient Allocations

An unconstrained firm's problem for hiring in period t may be written as

$$\max_{W_{it}, v_{it}} q(\theta_{it}) v_{it}(Z_t - W_{it}) - E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k \kappa(v_{it+k}, n_{it+k}) \text{ s.t. } X_t = \mu(\theta_{it})(W_{it} - Y_t),$$

with hired workers continuing to influence vacancy costs in subsequent periods according to the law of motion for the firm's workforce, $n_{it+1} = (1-\delta)(n_{it} + q(\theta_{it})v_{it})$ for all t. The first order conditions for this unconstrained problem coincide with (14) and (15) in the text.

By contrast, a benevolent planner maximizes the present value of producer and home output net of the costs of vacancy creation:

$$\begin{aligned}
&\max_{\{\theta_{it}, v_{it}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \sum_{i \in I} [(n_{it} + q(\theta_{it})v_{it}) z_t - \kappa(v_{it}, n_{it})] + [1 - \sum_{i \in I} (n_{it} + q(\theta_{it})v_{it})] b \right\} \\
&\quad \text{s.t. } n_{it+1} = (1-\delta)(n_{it} + q(\theta_{it})v_{it}), \ \forall i \in I, t \geq 0, \\
&\qquad \sum_{i \in I} v_{it}/\theta_{it} = 1 - \sum_{i \in I} n_{it}, \ \forall t \geq 0,
\end{aligned} \tag{30}$$

taking into account the law of motion for the workforce of each producer and that the planner's choices of θ_{it}, v_{it} must be consistent with the measure of unmatched workers each period, as the job seekers allocated to each producer, v_{it}/θ_{it}, must add up to the latter.

To connect these two problems, note that the planner objective may be rewritten, reorganizing terms and including the constraints (30) with Lagrange multipliers λ_t, as

$$E_0 \sum_{i \in I} \sum_{t=0}^{\infty} \beta^t [(n_{it} + q(\theta_{it})v_{it})(z_t - b) - \kappa(v_{it}, n_{it}) - \lambda_t (\frac{v_{it}}{\theta_{it}} + n_{it})],$$

subject to $n_{it+1} = (1-\delta)(n_{it} + q(\theta_{it})v_{it})$, $\forall i \in I, t \geq 0$.

Meanwhile, for the firm, substituting wages out and adding up across cohorts of workers hired over time yields the objective:

$$E_t \sum_{t=0}^{\infty} \beta^t [(n_{it} + q(\theta_{it})v_{it})(z_t - b) - \kappa(v_{it}, n_{it}) - X_t (\frac{v_{it}}{\theta_{it}} + n_{it})],$$

subject to the same laws of motion for the workforce n_{it} as above.

The unconstrained firm's objective thus coincides with the planner's, with the Lagrange multipliers replaced by the market value of search, and identical constraints on the two problems otherwise. Thus, the unconstrained equilibrium is efficient.

B Endogenous Separations/Quits

This section first highlights that workers are free to quit to look for a new job at any point, and that this assumption limits the monopsony power firms have in the model. It then extends the model to feature endogenous separations in equilibrium, showing that doing so need not change equilibrium wage setting in a significant way.

Endogenous separations/quits in the model Recall that in setting wages, firms face the constraint

$$U_t = \mu(\theta_t) E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (w_{t+k} + \beta\delta U_{t+1+k}) + (1-\mu(\theta_t))(b + \beta E_t U_{t+1}), \tag{31}$$

which tells them what tightness to expect in response to their offered present value wages. Here a worker's value of accepting a job, $E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (w_{t+k} + \beta\delta U_{t+1+k})$, includes the present value of wages together with the continuation value of returning to search upon separation. Meanwhile, the worker's value of remaining unmatched and continuing search is $b + \beta E_t U_{t+1}$. For workers to be willing to accept employment, the former must dominate the latter: $E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (w_{t+k} + \beta\delta U_{t+1+k}) \geq b + \beta E_t U_{t+1}$.

Due to the within-firm constraints, existing workers' wages satisfy the same constraint, meaning that an existing worker's value of remaining on the job, $E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (w_{t+k} + \beta\delta U_{t+1+k})$, exceeds the value of quitting to start search for a new job tomorrow, $b + \beta E_t U_{t+1}$. As long as the firm remains in the market for new hires each period, it thus must be paying wages that guarantee that none of its existing workers want to quit.

Whether remaining in the market for new hires is optimal for the firm hinges on what it can do to its existing workers if it does not hire (in which case it can ignore constraint (31)). The paper assumes that existing workers are always free to take the outside option of quitting to search for a new job next period (with value $b + \beta E_t U_{t+1}$), and this limits how low wages firms can pay them. In solving the model I check to make sure that firm value from remaining in the market (and setting wages according to the first order conditions as discussed) dominates the value of the firm withdrawing from the market for new hires and paying existing workers just enough to keep them from quitting.

The above checks confirm that the interior solutions considered are optimal when workers are free to quit. By contrast, if one assumed that firms can pay existing workers arbitrarily low wages with the workers forced to remain with the firm, then the firm would always prefer to do that (assuming wages could be unboundedly low) and the equilibrium look different from the paper. Thus, the assumption that workers are free to quit limits the monopsony power firms have in setting wages.

Endogenous separations/quits in equilibrium Does incorporating endogenous separations/quits in equilibrium alter wage setting in an important way? This section extends the model to consider this question explicitly.

To begin, there must be a reason giving rise to worker reallocation across firms. A natural reason driving such reallocation is that a worker may discover that a particular job is not a good fit for them for match-specific reasons, and seek a better fit.[46] I formalize such behavior in the extension laid out below.

Consider the version of the model with worker heterogeneity, assuming workers have permanent differences in productivity, with J types $\{z_j\}_{j=1}^{J}$. Suppose then that each worker faces a small probability p each period that their productivity with their current employer drops, with the worker starting the next period at a permanently lower productivity level. To be concrete, suppose productivity drops from z_j to z_{j-1} for $j = 2, ..., J$, and from z_1 to b for the lowest productivity type. The worker's inherent productivity type remains unaffected, however, leaving their outside options unchanged.

The firm pays workers based on their prevailing productivity, meaning that type z_{j+1} workers whose productivity declines to z_j following a p shock are paid the same per-period wage as workers with inherent productivity z_j. This wage may or may not be high enough to keep these workers with the firm, however, because their outside options are better than those of type j workers. If not, there will be quits in equilibrium.

In what follows, I first characterize equilibrium outcomes when p shocks cause workers to quit, and then consider conditions ensuring that doing so is optimal.

What does an equilibrium where p shocks lead to quits look like? In such an equilibrium, worker search value satisfies:

$$\begin{aligned} U_{jt} =& \mu(\theta_{jt}) E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (1-p)^k [w_{jt+k} + \beta\delta U_{jt+k+1} + \beta(1-\delta)p(b + \beta U_{jt+k+2})] \\ &+ (1 - \mu(\theta_{jt}))(b + \beta E_t U_{jt+1}), \end{aligned} \tag{32}$$

where the value of getting hired differs from the baseline model due to the p shocks. When a worker is hired, they receive the wages paid until either the δ or p shock hits, valuing them

[46] Such shocks are the drivers of quits in the directed search model of Menzio and Shi (JPE 2011) (with single worker firms). Absent such shocks their model would not generate quits, with the equilibrium featuring a single wage across firms. (Recall that equilibrium allocations in that model are efficient, and without shocks leading to mismatch, there is no need for a planner to reallocate workers across firms.)

according to their present value. If the δ shock hits, the worker returns to search. If the δ shock does not hit but the p shock does, the worker quits, remaining at home for a period and returning to search the next.[47]

Reorganizing terms to express (32) as $X_{jt} = \mu(\theta_{jt})(W_{jt} - Y_{jt})$ implies $X_{jt} = U_{jt} - b - \beta E_t U_{jt+1}$ and $Y_{jt} = b + \beta E_t U_{jt+1} - E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (1-p)^k (\beta\delta U_{jt+k+1} + \beta(1-\delta)p(b + \beta U_{jt+k+2}))$.

Turning to the firm problem, suppose a firm has n_{jt} productivity z_j workers (not including the type $j+1$ workers hit with a p shock that quit). This measure of workers follows the law of motion $n_{jt+1} = (1-p)(1-\delta)(n_{jt} + q(\theta_{jt})v_{jt})$. With this, the firm problem involving productivity z_j workers becomes:

$$
\begin{aligned}
&\max_{W_j, v_j} (n_j + q(\theta_j)v_j)(Z_j(S) - W_j) - \kappa(v_j, n_j) + \beta E_S V_j(n_j', S') \\
&\text{s.t. } n_j' = (1-p)(1-\delta)(n_j + q(\theta_j)v_j), \\
&\qquad X_j(S) = \mu(\theta_j)(W_j - Y_j(S)),
\end{aligned}
$$

where $V_j(n_j, S) = q(\theta_j)v_j(Z_j(S) - W_j) - \kappa(v_j, n_j) + \beta E_S V_j(n_j', S')$. The present values now satisfy $W_{jt} = E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (1-p)^k w_{jt+k}$ and $Z_j = E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (1-p)^k z_{jt+k}$.

Scaling by n_j, the problem becomes:

$$
\begin{aligned}
&\max_{W_j, x_j} (1 + q(\theta_j)x_j)(Z_j(S) - W_j) - \hat{\kappa}(x_j) + \beta(1-\delta)(1-p)(1 + q(\theta_j)x_j) E_S \hat{V}_j(S') \\
&\text{s.t. } X_j(S) = \mu(\theta_j)(W_j - Y_j(S)),
\end{aligned}
$$

where $\hat{V}_j(S) = q(\theta_j)x_j(Z_j(S) - W_j) - \hat{\kappa}(x_j) + \beta(1-\delta)(1-p)(1 + q(\theta_j)x_j) E_S \hat{V}_j(S')$.

The first order conditions for this problem remain very similar to those in the paper:

$$
\begin{aligned}
\kappa_v(x_j) &= q(\theta_j)(Z_j(S) - W_j + \beta(1-\delta)(1-p) E_S \hat{V}_j(S')), \\
1 + q(\theta_j)x_j &= q'(\theta_j) g_W^j(W_j) x_j (Z_j - W_j + \beta(1-\delta)(1-p) E_S \hat{V}_j(S')).
\end{aligned}
$$

Turning to the adding up constraint for searching workers, suppose the sum of the n_{jt} workers across firms is denoted by N_{jt}. Searching type j workers include all type j workers not remaining with a firm (of measure $1 - N_{jt}$), except those hit with a p shock in the preceding period (of measure $\frac{p}{1-p} N_{jt}$) who will start search only in the subsequent period.[48] This brings

[47] The responses to the δ and p shocks have slightly asymmetric timing. This timing for p shocks is desirable for internal consistency, because a worker's decision to quit is based on the same calculation as a searching worker's decision to accept employment, in terms of comparing the value of the job to the value of search. Meanwhile, I have maintained the timing of the δ shocks unchanged from the body of paper, to minimize changes to the model.

[48] I assume the total measure of type j workers is one.

the total measure of searching type j workers to $1 - N_{jt}/(1-p)$. The adding up constraint for these workers may then be written as $x_{jt}N_{jt} = \theta_{jt}(1 - N_{jt}/(1-p))$, with the firm-level laws of motion for workers implying: $N_{jt+1} = (1-p)(1-\delta)(N_{jt} + \mu(\theta_{jt})(1 - N_{jt}/(1-p)))$.

For the above equations to characterize an equilibrium, workers must prefer to quit when a p shock hits, and firms prefer to let them go instead of raising wages to retain them.

What ensures that workers prefer to quit when a p shock hits? For a type $j+1$ worker, the value of quitting in period t following a p shock is $b + \beta E_t U_{j+1t+1}$. The value of waiting for a period and then quitting is $w_{jt} + \beta\delta E_t U_{j+1t+1} + \beta(1-\delta)E_t(b + \beta U_{j+1t+2}) = w_{jt} + \beta E_t U_{j+1t+1} - \beta(1-\delta)E_t X_{j+1t+1}$ (using that $U_{j+1t+1} = b + X_{j+1t+1} + \beta E_{t+1}U_{j+1t+2}$). The worker prefers to quit in period t if $w_{jt} < b + \beta(1-\delta)E_t X_{j+1t+1}$. This condition ensures that p shock workers prefer to quit immediately following a p shock.

What ensures that firms prefer the above wage setting behavior over retaining quitting workers? If the firm raised this period's per-period wage above the equilibrium wage, the firm value from type j workers would fall (because the equilibrium wage maximizes this value), but at some point the wage would become high enough to retain the type $j+1$ workers hit with a p shock, leading to a jump up in firm value (as long as the required wage is not too high). What would be the implied firm value from raising the wage to that point?

The calculation to determine the threshold wage retaining p shock workers is identical to the one above, determining the threshold wage as $\underline{w}_{j+1t} = b + \beta(1-\delta)E_t X_{j+1t+1}$. If the firm set this wage today, the present value of wages for the type j workers at the firm would be $\underline{W}_{j+1t} = \underline{w}_{j+1t} + \beta(1-\delta)(1-p)E_t W_{j+1t+1}$, assuming equilibrium wages prevail after the current period. This present value determines the tightness the firm faces in hiring this period via $X_{jt} = \mu(\theta_{jt})(\underline{W}_{j+1t} - Y_{jt})$. Firm value from productivity z_j workers would then be determined by the problem:

$$\max_{v_j} n_j^p(z_j(S) - \underline{w}_{j+1}(S)) + (n_j + q(\theta_j)v_j)(Z_j(S) - \underline{W}_{j+1}(S)) - \kappa(v_j, n_j + n_j^p) + \beta E_S V_j(n_j', S'),$$

where the firm begins with measure n_j^p type $j+1$ workers hit with a p shock in the preceding period. Here $n_j' = (1-p)(1-\delta)(n_j + q(\theta_j)v_j)$, as the p shock workers would quit after this period due to equilibrium wages.

Scaling by n_j (and denoting $s_j^p = n_j^p/n_j$, $x_j = v_j/n_j$), the problem becomes

$$\begin{aligned}\max_{x_j}\ & s_j^p(z_j - \underline{w}_{j+1}) + (1 + q(\theta_j)x_j)(Z_j(S) - \underline{W}_{j+1}) - (1 + s_j^p)\hat{\kappa}(x_j/(1+s_j^p)) \\ & + \beta(1-\delta)(1-p)(1 + q(\theta_j)x_j)E_S\hat{V}_j(S'),\end{aligned}$$

where θ_j is determined via $X_j(S) = \mu(\theta_j)(\underline{W}_{j+1}(S) - Y_j(S))$. For equilibrium wage setting to be optimal, equilibrium firm value must dominate the value attained here.

Numerical example: To consider the impact of endogenous separations on wages in concrete terms, I return to the parametrization in the paper. Suppose we augment the baseline parametrization by assuming that half of separations are exogenous and half endogenous. Maintaining the target that jobs last 2.5 years on average, this requires that $p = d = 0.0168$.[49] Suppose, further, that we assume worker productivity declines by 5% upon a p shock, with workers of inherent productivity $z_{j+1} = 1.05$ falling to $z_j = 1$.

According to the above conditions characterizing equilibrium outcomes with p shocks, the equilibrium specifies that workers of inherent productivity $z_{j+1} = 1.05$ begin jobs with wage 0.940 and workers of inherent productivity $z_j = 1$ with wage 0.911. When a type $j+1$ worker is hit with a p shock, they then face the lower equilibrium wage 0.911. Meanwhile, the threshold wage that would retain these workers with the firm is 0.937. The workers thus prefer to quit because of their better outside options.

Equilibrium firm value from workers of inherent productivity z_j is 6.038 (per existing type j worker). For every type j worker, each firm receives roughly 0.017 p-shock workers, consistent with the probability of the p shock hitting. In equilibrium firms let these workers quit. If a firm instead paid 0.937 today, retaining these workers, firm value from workers with productivity z_j would be 6.028 (per existing type j worker). Firms thus prefer to set the equilibrium wage, letting the p-shock workers quit.

Does having endogenous separations/quits occurring in equilibrium alter the level of wages significantly? The equilibrium wage of workers with inherent productivity $z_j = 1$ above is 0.911, while the steady-state wage of productivity $z = 1$ workers in the paper is 0.911. Wage setting is characterized by essentially the same optimality conditions across these two environments, with both shocks leading to separation, and the wage level little changed.

This example illustrates that wage setting need not be altered substantially by endogenous separations/quits taking place in equilibrium. Note, moreover, that it is also not clear that making the reallocation of mismatched workers easier (while maintaining the target for the duration of jobs) would alter this conclusion.

[49] An expected duration of employment of $1/(1-(1-p)(1-d)) = 30$ months implies a monthly total separation rate of 3.3%. According to the Job Openings and Labor Turnover Survey (JOLTS) of the U.S. Bureau of Labor Statistics, the overall quit rate is roughly half of the total separation rate, at 1.9% and 3.6% per month, respectively. The qualitative behavior of the example remains similar if one assumes all separations to be endogenous, as well as if one raises the total separation rate to 3.6% per month.

C Proofs and Details

The Static Model

Proof of Proposition 1 Equation (1) yields the derivative $g_w(w;U) = -\mu(\theta)/(\mu'(\theta)(w-b))$, equation (4) the wage $w = z - \kappa_v(x)/q(\theta)$ and the equilibrium condition the vacancy rate $x = \theta(1-N)/N$. Using these in (5) yields an equation determining equilibrium θ:

$$\frac{1}{\theta} + q(\theta)\frac{1-N}{N} = \frac{1-\varepsilon(\theta)}{\varepsilon(\theta)}\frac{1-N}{N}\frac{\kappa_v(\theta\frac{1-N}{N})}{z-b-\frac{\kappa_v(\theta\frac{1-N}{N})}{q(\theta)}},$$

where I denote the matching function elasticity by $\varepsilon(\theta) := \mu'(\theta)\theta/\mu(\theta)$. The left hand side is strictly decreasing and the right strictly increasing in θ, given the assumptions on the vacancy cost and matching function. The equation thus determines a unique equilibrium θ.

For the unconstrained model one simply leaves out the $1/\theta$ term on the left hand side, which implies that the tightness is strictly greater in the firm wage model, $\theta_{FW} > \theta_{SD}$, as is employment, $N + \mu(\theta)(1-N)$. From $x = \theta(1-N)/N$, the hiring rate in the firm wage model is strictly greater, $x_{FW} > x_{SD}$, as is total vacancy creation xN. The wage, from $w = z - \kappa_v(x)/q(\theta)$, is strictly lower in the firm wage model, $w_{FW} < w_{SD}$.

Firm profits from hiring may be written (using the first order condition for vacancies) as $q(\theta)x(z-w) - \hat{\kappa}(x) = -\kappa_n(x)$, which is increasing in x, implying these profits are greater in the firm wage model. Worker value from employment, w, is lower in the firm wage model.

Using the first order condition for vacancy creation to substitute out the wage, we have $U = b + \mu(\theta)(w-b) = b + \mu(\theta)(z-b) - \theta\kappa_v(\theta(1-N)/N)$. This expression is non-monotonic, with derivative $\mu'(\theta)(z-b) - \kappa_v(\theta(1-N)/N) - \kappa_{vv}(\theta(1-N)/N)\theta(1-N)/N$, which is strictly decreasing: positive at $\theta = 0$ but negative at the unconstrained θ (where $\mu'(\theta)(z-b) = \kappa_v(x)$) and beyond. It follows that the worker value of search U is strictly lower in the firm wage model.

Wages and Hiring Note that optimal wage setting implies

$$\frac{1+q(\theta)x}{q(\theta)x} = -\frac{q'(\theta)\mu(\theta)}{\mu'(\theta)q(\theta)}\frac{z-w}{w-b}$$

in the constrained model and

$$1 = -\frac{q'(\theta)\mu(\theta)}{\mu'(\theta)q(\theta)}\frac{z-w}{w-b}$$

in the unconstrained model. These equations follow from conditions (5) and (6), where (1) yields $g_w = -\mu(\theta)/(\mu'(\theta)(w-b))$. These equations imply that the wage can be written as the weighted average

$$w = (1-\gamma)b + \gamma z,$$

where $\gamma_c = [\frac{\frac{1}{qx}+1}{\frac{1}{\varepsilon}-1} + 1]^{-1}$ in the constrained case and $\gamma_u = [\frac{1}{\frac{1}{\varepsilon}-1} + 1]^{-1}$ in the unconstrained case. Here I denote the matching function elasticity as $\varepsilon := \mu'(\theta)\theta/\mu(\theta)$. For a simple illustration of wage outcomes, I treat ε as a constant.[50]

From the expressions above, it is easy to see that the wage is generally lower in the constrained case. To illustrate, note first that in a dynamic setting the steady-state hiring rate is related to the separation rate δ via $(1 + qx)(1 - \delta) = 1$. Adopting the values $\delta = 0.03$ and $\varepsilon = 0.5$ yields the weights $\gamma_c = 0.03$, $\gamma_u = 0.5$. The wage is clearly lower in the constrained case.

To consider how the wage responds to changes in productivity, hold γ fixed for a moment. From the expression for the wage it follows that the wage also responds less to changes in z in the constrained case, as $\Delta w = \gamma \Delta z$ with $\gamma_c < \gamma_u$.

In practice γ_c does respond to changes in market productivity, however (generally counteracting the above effect as an increase in z leads firms to place more weight on offering an attractive/high hiring wage instead of making profit on existing workers). Taking the change in γ_c into account, we have $\Delta w = \gamma_c \Delta z + (z - b)[\frac{\frac{1}{qx}+1}{\frac{1}{\varepsilon}-1} + 1]^{-2}[\frac{1}{\varepsilon} - 1]^{-1}(qx)^{-1}\frac{\Delta qx}{qx}$ in the constrained case, and $\Delta w = \gamma_u \Delta z$ in the unconstrained case.

Letting $z = 1$ and $\Delta z = 0.02$, we have $\Delta w = \gamma_u \Delta z = 0.01$ in the unconstrained case, implying a wage increase of half the increase in productivity. In the constrained case, we arrive at the approximate upper bound wage response $\Delta w \approx 0.0006 + 0.0027 = 0.003$, where I have used $\frac{\Delta qx}{qx} \approx 0.1$ and $z - b \approx 1$. Despite the increase in γ_c, the wage response in the constrained case thus remains a fraction of that in the unconstrained case.[51]

For hiring, note that the equilibrium condition $1 - N = xN/\theta$ implies that an increase in x is associated with an increase in $\theta = xN/(1 - N)$. With this, the left hand side of the optimality condition for vacancy creation $\kappa'(x)/q(\theta) = z - w$ is increasing in x. An increase in the right hand side thus implies an increase in x, as well as θ (and hence $\mu(\theta)$).

The Dynamic Model

Withdrawing from Hiring Consider the dynamic firm problem (13). In writing this problem I have implicitly assumed that the firm prefers to hire each period, with wages having to satisfy the job seeker constraint characterizing how the tightness responds to the offered present value wage each period. But, because the firm begins with a stock of existing workers, it could in some circumstances find it optimal to withdraw from hiring instead. If a firm did not hire in the initial period, it would optimally set the present value wage so low

[50] This would hold exactly with a Cobb-Douglas matching function, where the elasticity is constant.

[51] This conclusion continues to hold when comparing percent changes in the wage due to the large difference in wage responses.

as to make its existing workers indifferent between remaining with the firm and quitting into unemployment. Doing so would mean: $v_0 = 0$ and $W_0 - Y_0 = 0$.

These initial period choices would leave the rest of the firm problem as: $n_{i0}[Z_0 - Y_0] + E_t \sum_{t=1}^{\infty} \beta^t [q(\theta_{it}) v_{it} (Z_t - W_{it}) - \kappa(v_{it}, n_{it})]$. With commitment to future wages, subsequent choices would thus be consistent with those of unconstrained firms, and hence characterized by the first order conditions under standard conditions. In the initial period, it remains relevant to check that the value from the firm hiring throughout dominates the firm withdrawing from hiring in the initial period, however.

In the case of no commitment to future wages, problem (19), if a firm were to withdraw from hiring for a period, its (scaled) firm value would be $Z(S) - Y + \beta(1-\delta) E_S \hat{V}(S')$, where the continuation value $\hat{V}(S)$ follows (20). It remains relevant to verify that the firm values arising from solving the model based on the first order conditions described dominate this value from withdrawing from hiring for a period. In practice this requires keeping track of this alternative firm value throughout and checking that firm values dominate it. In practice this can restrict parameter values, as well as the range of N considered, as high measures of existing matches make hiring less profitable.

Proof of Proposition 2 From its definition, $Y = \frac{b + \beta(1-\delta)X}{1-\beta(1-\delta)}$. We thus have

$$X = \mu(\theta)(W - Y) = \mu(\theta)(W - \frac{b + \beta(1-\delta)X}{1-\beta(1-\delta)}) \Rightarrow \frac{X}{\mu(\theta)} = \frac{W - \frac{b}{1-\beta(1-\delta)}}{1 + \frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}}.$$

The first order condition for vacancy creation yields an expression for the present value wage

$$W = Z - \frac{\beta(1-\delta)\kappa_n(x)}{1-\beta(1-\delta)} - \frac{\kappa_v(x)}{q(\theta)}.$$

Using that $\kappa_n(x) = \hat{\kappa}(x) - x\kappa_v(x)$, we further arrive at

$$W = Z - \frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))}.$$

At the same time, combining the first order conditions for vacancy creation and wages yields

$$\frac{X}{\mu(\theta)} = \frac{\kappa_v(x)}{q(\theta)} \frac{1-\varepsilon}{\varepsilon} \frac{q(\theta)x}{1+q(\theta)x},$$

where the term $\frac{q(\theta)x}{1+q(\theta)x}$ equals $1/\delta$ in the constrained case, and reduces to 1 in the unconstrained case.

Equating the above two expressions for $\frac{X}{\mu(\theta)}$ yields the equation:

$$\frac{\kappa_v(x)}{q(\theta)} \frac{1-\varepsilon}{\varepsilon} \frac{q(\theta)x}{1+q(\theta)x} = \frac{Z - \frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))} - \frac{b}{1-\beta(1-\delta)}}{1 + \frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}}.$$

Note that $q(\theta)x = \delta/(1-\delta)$ is a constant, which implies an increasing relationship between steady-state θ and x. With this, the left hand side of the above equation is strictly increasing in θ, rising from zero toward infinity as θ rises from zero to infinity, while the right hand side is strictly decreasing, falling from $Z - \frac{b}{1-\beta(1-\delta)}$ to negative values. Thus, there is a unique steady-state θ, and this value is strictly higher in the constrained case.

It follows that N and employment, $N/(1-\delta)$, are strictly greater in the constrained case. From $x = \delta/((1-\delta)q(\theta))$, x is higher in the constrained case. The expression for wages then implies that the present value hiring wage W is strictly lower in the constrained case.

Firm value from hiring may be written, using the first order condition for vacancies, as $q(\theta)x(Z-W) - \hat{\kappa}(x) + \beta(1-\delta)(1+q(\theta)x)\hat{V} = \hat{V} = -\beta(1-\delta)\kappa_n(x)/(1-\beta(1-\delta))$, which is increasing in x and thus greater in the constrained case where x is higher.

For worker values, the above expressions imply:

$$X = \mu(\theta)\frac{Z - \frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))} - \frac{b}{1-\beta(1-\delta)}}{1 + \frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}}.$$

The right hand side expresses X as a product $\mu(\theta)f(\theta)$ where $f(\theta)$ is strictly decreasing from a positive value to zero, with both equilibrium θ in the range where it remains positive. The derivative of the right hand side $\mu'(\theta)f(\theta) + \mu(\theta)f'(\theta)$ is strictly decreasing in this range. To see this, note that for values of θ starting at zero onward: μ' is positive and strictly decreasing, f is positive and strictly decreasing toward zero, μ is positive and increasing and f' is negative and decreasing. It follows that the derivative is strictly decreasing. At the unconstrained equilibrium, moreover, the derivative is strictly negative.

It follows that X is lower in the constrained case than the unconstrained case. Thus, the value of searching for employment $U = (b+X)/(1-\beta)$ and accepting employment $W + \beta\delta U/(1-\beta(1-\delta))$ are lower in the constrained case than the unconstrained case.

Wages and Hiring To arrive at equation (25), note that optimal wage setting implies

$$\frac{1+q(\theta_t)x_t}{q(\theta_t)x_t} = -\frac{q'(\theta_t)\mu(\theta_t)}{\mu'(\theta_t)q(\theta_t)}\frac{Z_t - W_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k})}{W_t - Y_t}$$

in the constrained case and

$$1 = -\frac{q'(\theta_t)\mu(\theta_t)}{\mu'(\theta_t)q(\theta_t)}\frac{Z_t - W_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k})}{W_t - Y_t}$$

in the unconstrained case. These equations follow from the first order conditions for the present value wage, where $X_t = \mu(\theta_t)(W_t - Y_t)$ yields $g^t_{W_t} = -\mu(\theta_t)/(\mu'(\theta_t)(W_t - Y_t))$. As in the static model, these equations imply that the wage can be written as the weighted average

$$W_t = (1-\gamma_t)Y_t + \gamma_t(Z_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k})),$$

with the same weights $\gamma_{ct} = [\frac{\frac{1}{q_t x_t}+1}{\frac{1}{\varepsilon_t}-1}+1]^{-1}$ in the constrained case and $\gamma_{ut} = [\frac{1}{\frac{1}{\varepsilon_t}-1}+1]^{-1}$ in the unconstrained case.

To shed light on the implications of changes in productivity for hiring, consider steady-state comparative statics. Note that the steady-state relationship $(1+q(\theta)x)(1-\delta) = 1$ implies that an increase in x is associated with an increase in θ. With this, the left hand side of the optimality condition for vacancy creation $\kappa'(x)/q(\theta) = Z - W - E\sum_k \beta^k(1-\delta)^k\kappa_n(x)$ is increasing in x. An increase in the right hand side thus implies an increase in x as well as θ (and hence $\mu(\theta)$). (Overall, functional forms and parameter values play a role in determining outcomes in the model, but in drawing the equilibrium wage toward the workers' opportunity cost, the constraints work to make wages less responsive to changes in productivity.)

Derivation of Equation (26) Note that the expression for the wage implies

$$W_t - Y_t = \gamma_t(Z_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k}) - Y_t) \text{ and}$$

$$Z_t - W_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k}) = (1-\gamma_t)(Z_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k}) - Y_t).$$

Note also that, from $Y_t = b + \sum_{k=1}\beta^k(1-\delta)^k(b + X_{t+k})$, $X_t = \mu(\theta_t)(W_t - Y_t)$ and the above, we have

$$\begin{aligned}
&Y_t - \beta(1-\delta)E_tY_{t+1} = b + \beta(1-\delta)E_tX_{t+1} = b + \beta(1-\delta)E_t\mu(\theta_{t+1})(W_{t+1} - Y_{t+1}) \\
&= b + \beta(1-\delta)E_t\mu(\theta_{t+1})\gamma_{t+1}(Z_{t+1} - \sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+1+k}) - Y_{t+1}) \\
&= b + \beta(1-\delta)E_t\mu(\theta_{t+1})\gamma_{t+1}\frac{\kappa_v(x_{t+1})}{q(\theta_{t+1})(1-\gamma_{t+1})} = b + \beta(1-\delta)E_t\frac{\gamma_{t+1}\theta_{t+1}\kappa_v(x_{t+1})}{(1-\gamma_{t+1})}.
\end{aligned}$$

Thus, we have

$$\begin{aligned}
&\frac{\kappa_v(x_t)}{q(\theta_t)(1-\gamma_t)} = Z_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k}) - Y_t \\
&= z_t - b + \beta(1-\delta)E_t[Z_{t+1} - \sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+1+k}) - Y_{t+1} - \kappa_n(x_{t+1}) - \frac{\gamma_{t+1}\theta_{t+1}\kappa_v(x_{t+1})}{(1-\gamma_{t+1})}] \\
&= z_t - b + \beta(1-\delta)E_t[\frac{\kappa_v(x_{t+1})}{q(\theta_{t+1})(1-\gamma_{t+1})} - \kappa_n(x_{t+1}) - \frac{\gamma_{t+1}\theta_{t+1}\kappa_v(x_{t+1})}{(1-\gamma_{t+1})}].
\end{aligned}$$

Infrequent Adjustment

Proof of Proposition 3 The equilibrium with infrequent adjustment is characterized by the first order conditions

$$\begin{aligned}\kappa_v(x) &= q(\theta)(Z - W + \beta(1-\delta)E_S(\alpha\hat{V}^r(S') + (1-\alpha)\hat{V}^f(w, S')))\\ (1 + q(\theta)x)h_w &= q'(\theta)g_W(W; S)h_w x[Z - W + \beta(1-\delta)E_S[\alpha\hat{V}^r(S') + (1-\alpha)\hat{V}^f(w, S')]],\\ &\quad + \beta(1-\delta)(1 + q(\theta)x)(1-\alpha)E_S\hat{V}^f_w(w, S').\end{aligned}$$

In steady state, $\hat{V}^f_w(w) = h_w(w)$ and $\hat{V}^r = \hat{V}^f(w) = -\beta(1-\delta)\kappa_n(x)/(1-\beta(1-\delta))$. The remaining equilibrium conditions are, as before, $X = \mu(\theta)(W - Y)$, $(1 + q(\theta)x)(1-\delta) = 1$ and $x = \theta(1-N)/N$. The latter two imply that $N = \mu(1-\delta)/(\delta + \mu(1-\delta))$.

As for the baseline dynamic model (see proof above), we have that:

$$\frac{X}{\mu(\theta)} = \frac{Z - \frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))} - \frac{b}{1-\beta(1-\delta)}}{1 + \frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}}.$$

Combining the first order conditions for vacancy creation and wages yields:

$$\frac{X}{\mu(\theta)} = \frac{\kappa_v}{q(\theta)}\frac{1-\varepsilon}{\varepsilon}\frac{qx}{1+qx}\frac{1}{1-\beta(1-\delta)(1-\alpha)}.$$

Combining the two yields the equation (using $\frac{1+qx}{qx} = \delta$):

$$\frac{\kappa_v(x)}{q(\theta)}\frac{1-\varepsilon}{\varepsilon}\frac{\delta}{1-\beta(1-\delta)(1-\alpha)} = \frac{Z - \frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))} - \frac{b}{1-\beta(1-\delta)}}{1 + \frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}}.$$

Note that $q(\theta)x = \delta/(1-\delta)$ is a constant, which implies an increasing relationship between steady-state θ and x. As before, the left hand side of the above equation is strictly increasing in θ, rising from zero toward infinity as θ rises from zero to infinity, while the right hand side is strictly decreasing, falling from $Z - \frac{b}{1-\beta(1-\delta)}$ to negative values. Thus, there is a unique steady-state θ, and this value is strictly higher than efficient and increasing in α. From $x = \delta/((1-\delta)q(\theta)$, x which is higher than efficient and increasing in α. It follows that N and employment, $N/(1-\delta)$, are strictly greater than efficient and increasing in α. The expression for wages

$$W = Z - \frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))},$$

then implies that the present value wage W is strictly below efficient and decreasing in α. Even with α approaching zero, the term $\delta/(1-\beta(1-\delta)(1-\alpha))$ remains strictly below one if $\beta < 1$, implying allocations fall short of efficient.

Firm value $Z - W - \beta(1-\delta)\kappa_n(x)/(1-\beta(1-\delta))$ is strictly greater than efficient and falls as α falls from one toward zero. For worker values, we have

$$X = \mu(\theta)\frac{Z - \frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))} - \frac{b}{1-\beta(1-\delta)}}{1 + \frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}}.$$

The right hand side is again decreasing from the efficient θ toward higher values, implying that X is below efficient and falls in α. Thus, the value of searching for and accepting employment are below efficient and fall in α. Firm and worker values also fall short of efficient even as α approaches zero.

D Calibration Details

The law of motion for matches implies that the steady-state level of unemployment satisfies:

$$u = 1 - N - \mu(\theta)(1-N) = \frac{\delta(1-\mu(\theta))}{\delta(1-\mu(\theta)) + \mu(\theta)}.$$

Given a value for δ, a target for steady-state u then determines $\mu(\theta)$.

Given a target for the tightness θ, the matching function parameter ℓ is then pinned down (uniquely) from $\mu(\theta) = \theta/(1+\theta^{\ell})^{1/\ell}$. This further pins down the steady-state value of $x = \theta(1-N)/N = \delta\theta/((1-\delta)\mu(\theta))$.

The above values must be consistent with the model equation (26) with the appropriate weight γ. Note that the text compares the constrained model where firms reoptimize each period—and the constrained γ thus applies to both sides of (26)—to the unconstrained model where the unconstrained γ applies to both sides of the equation. Given the above, equation (26) pins down a unique value of $(z-b)/\kappa_0$ for each of the two models. This still allows alternative combinations of b, κ_0 that are consistent with any such value.

The scaled steady-state firm value may be written, using the first order condition for vacancy creation, as

$$(1+q(\theta)x)(Z - W - \frac{\beta(1-\delta)\kappa_n(x)}{1-\beta(1-\delta)}) - \hat{\kappa}(x) = (1+q(\theta)x)\frac{\kappa_v(x)}{q(\theta)} - \hat{\kappa}(x) = \frac{\kappa_v(x)}{q(\theta)} - \kappa_n(x).$$

Flow profits per employed worker thus equal $(1-\beta)(\frac{\kappa_v(x)}{q(\theta)} - \kappa_n(x))/(1+q(\theta)x)$.

From the same first order condition, the wage may be written as:

$$w = W(1-\beta(1-\delta)) = (Z - \frac{\beta(1-\delta)\kappa_n(x)}{1-\beta(1-\delta)} - \frac{\kappa_v(x)}{q(\theta)})(1-\beta(1-\delta)).$$

For the share of profit and wage to remain unchanged across the two cases, given the above, κ_0 must remain unchanged across cases. Thus, only b adjusts across the two cases, essentially rising in the constrained model to keep the wage from falling.

If b is held fixed across cases, κ_0 must increase in the constrained case to keep hiring from rising while firm value rises and the wage falls.

E Solving: Firm Wages with Aggregate Shocks

The dynamic system for the firm wage equilibrium with aggregate shocks is given below. The last five equations define some variables of interest based on the solution (employment, unemployment, the vacancy-unemployment ratio, firm value, and firm value if the firm did not hire in the current period at all).

$$
\begin{aligned}
&\kappa_v(x_t) = q(\theta_t)(Z_t - W_t + \beta(1-\delta)\hat{V}_{t+1}) \\
&1 + q(\theta_t)x_t = q'(\theta_t)g_{Wt}x_t(Z_t - W_t + \beta(1-\delta)\hat{V}_{t+1}) \\
&g_{Wt} = -\mu(\theta_t)/(\mu'(\theta_t)(W_t - Y_t)) \\
&\hat{V}_t = -\kappa_n(x_t) + \beta(1-\delta)\hat{V}_{t+1} \\
&N_{t+1} = (1-\delta)(N_t + \mu(\theta_t)(1-N_t)) \\
&x_t = v_t/N_t \\
&\theta_t(1-N_t) = v_t \\
&X_t = \mu(\theta_t)(W_t - Y_t) \\
&W_t = w_t + \beta(1-\delta)W_{t+1} \\
&Y_t = b + \beta(1-\delta)(X_{t+1} + Y_{t+1}) \\
&z_{t+1} - 1 = \rho_z(z_t - 1) + \epsilon_{zt+1} \\
&Z_t = z_t + \beta(1-\delta)Z_{t+1} \\
&e_t = N_t + \mu(\theta_t)(1-N_t) \\
&u_t = 1 - e_t \\
&vuratio_t = v_t/u_t \\
&\hat{V}_{obj,t} = Z_t - W_t + \hat{V}_t \\
&\hat{V}_{objnh,t} = Z_t - Y_t + \beta(1-\delta)\hat{V}_{t+1}
\end{aligned}
$$

F Additional Figures

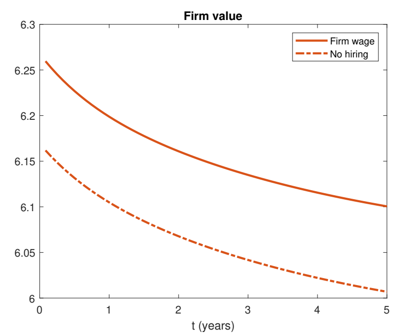

Figure F.1: Impulse Responses in Firm Wage Model: Optimality of Hiring

Notes: The figure refers to the impulse response in Figure 1, showing that the firm value attained by following the first order conditions dominates withdrawing from hiring for a period, throughout the response.

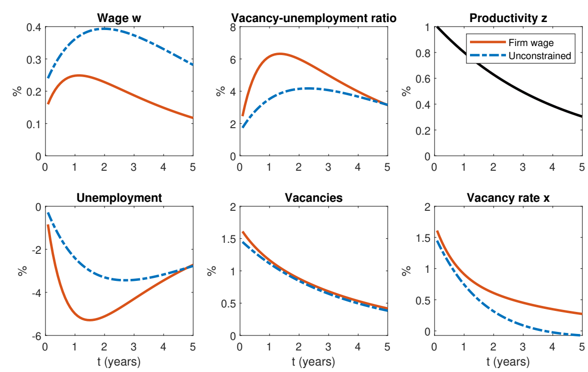

Figure F.2: Impulse Responses in Firm Wage Model: Identical Parameters

Notes: The figure plots the percentage responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model and the unconstrained model. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$. The two models have the same parameter values, with $b = 0.89$. Steady state unemployment is three times higher in the unconstrained model than in the firm wage model, with market tightness less than half of that in the constrained model.

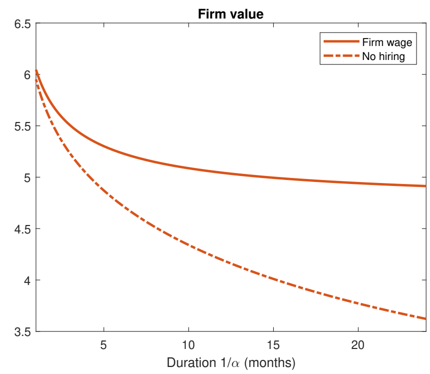

Figure F.3: Equilibrium with Infrequent Wage Adjustment: Optimality of Hiring

Notes: The figure refers to Figure 3, showing that the firm value attained by following the first order conditions dominates withdrawing from hiring for the duration of the wage, across equilibrium wage durations.

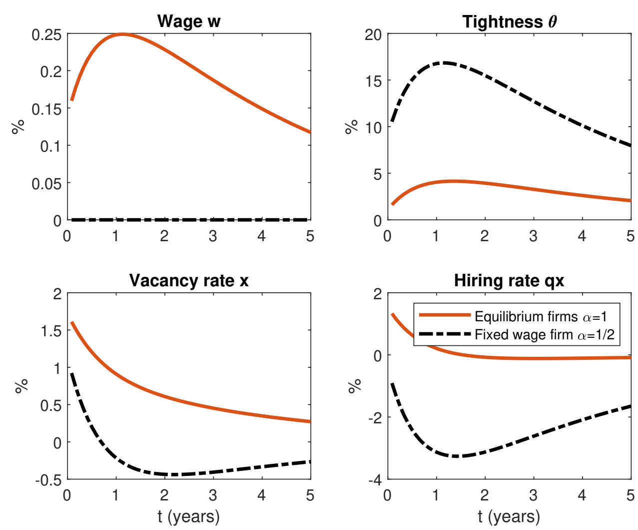

Figure F.4: Impulse Responses in Firm Wage Model: Impact of Fixed Wage

Notes: The figure plots the percentage responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model and for a single firm deviating to a longer wage commitment. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$.

Figure F.5: Equilibrium with Infrequent Wage Adjustment and Aggregate Shocks

Notes: The figure plots simulation means together with standard deviation bounds for the equilibrium with infrequent adjustment and aggregate shocks, as a function of the duration of wages. Labor productivity follows an AR(1) with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$. The firm and planner values plotted are the scaled values, but the unscaled values remain monotonic in wage duration.

Figure F.6: Equilibrium with Infrequent Wage Adjustment and Firm Shocks

Notes: The figure plots the equilibrium with infrequent wage adjustment with firm-level shocks, as a function of the equilibrium duration of wages. The model is solved on a 5-state grid for productivity, approximating an $AR(1)$ with autocorrelation $\rho_z = 0.88$ and standard deviation $\sigma_z = 0.2$ based on the Rouwenhorst method. The firm and planner values plotted are the scaled values, but the unscaled values remain monotonic in wage duration.

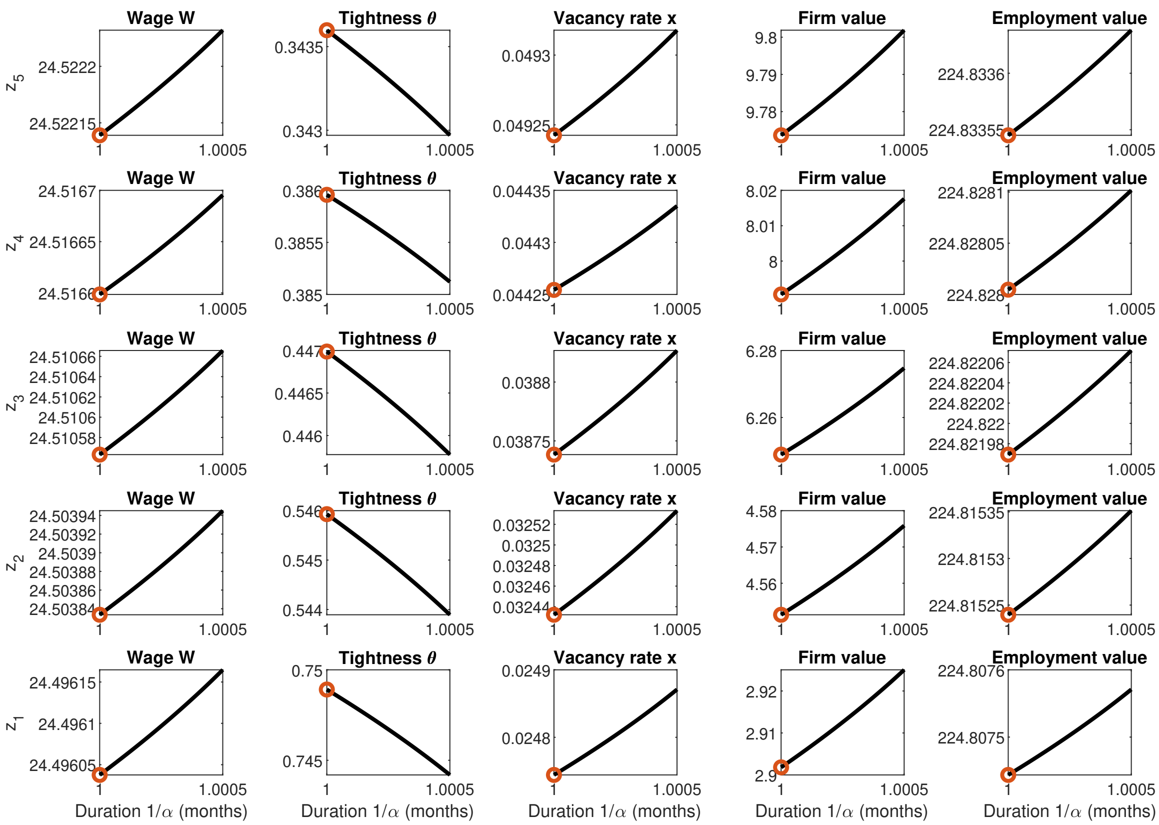

Figure F.7: Single Firm Deviation to Longer Wage Duration with Firm Shocks

Notes: The circles denote the equilibrium with infrequent wage adjustment where firms reoptimize monthly, and the figure plots corresponding values for a firm deviating to longer wage duration as a function of the expected duration of wages $1/\alpha$. The model is solved on a 5-state grid for productivity, approximating an AR(1) with autocorrelation $\sigma_z = 0.88$ and standard deviation $\sigma_z = 0.2$ based on the Rouwenhorst method. The firm value plotted is the scaled value.

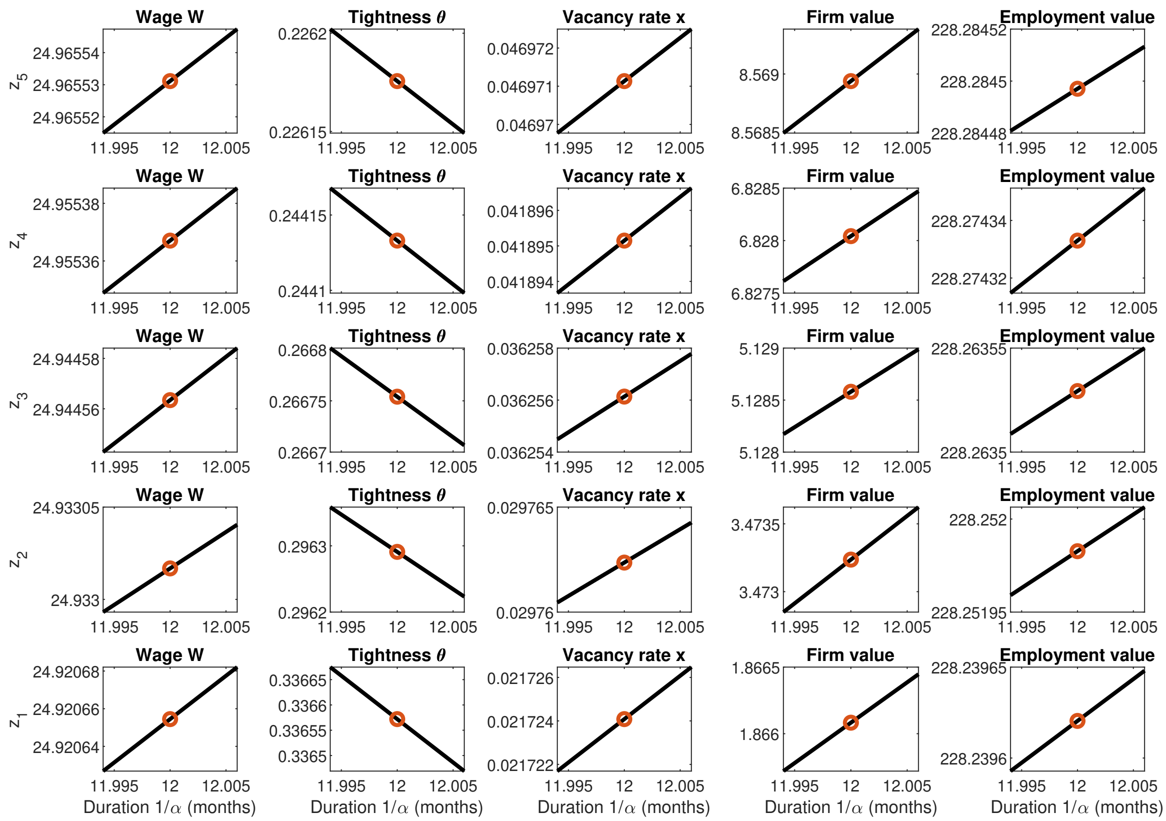

Figure F.8: Single Firm Deviation to Longer/Shorter Wage Duration with Firm Shocks

Notes: The circles denote the equilibrium with infrequent wage adjustment where firms reoptimize annually, and the figure plots corresponding values for a firm deviating to longer/shorter wage duration as a function of the expected duration of wages $1/\alpha$. The model is solved on a 5-state grid for productivity, approximating an AR(1) with autocorrelation $\sigma_z = 0.88$ and standard deviation $\sigma_z = 0.2$ based on the Rouwenhorst method. The firm value plotted is the scaled value.

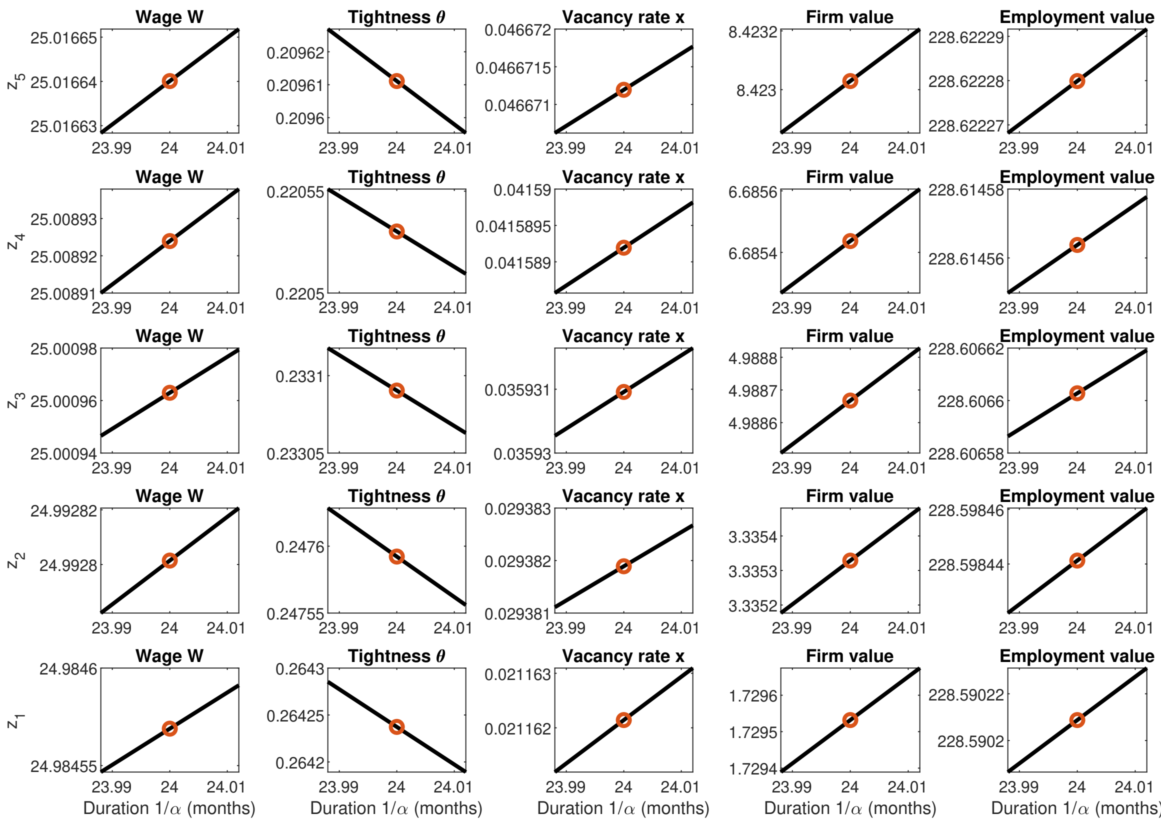

Figure F.9: Single Firm Deviation to Longer/Shorter Wage Duration with Firm Shocks

Notes: The circles denote the equilibrium with infrequent wage adjustment where firms reoptimize biennially, and the figure plots corresponding values for a firm deviating to longer/shorter wage duration as a function of the expected duration of wages $1/\alpha$. The model is solved on a 5-state grid for productivity, approximating an AR(1) with autocorrelation $\sigma_z = 0.88$ and standard deviation $\sigma_z = 0.2$ based on the Rouwenhorst method. The firm value plotted is the scaled value.

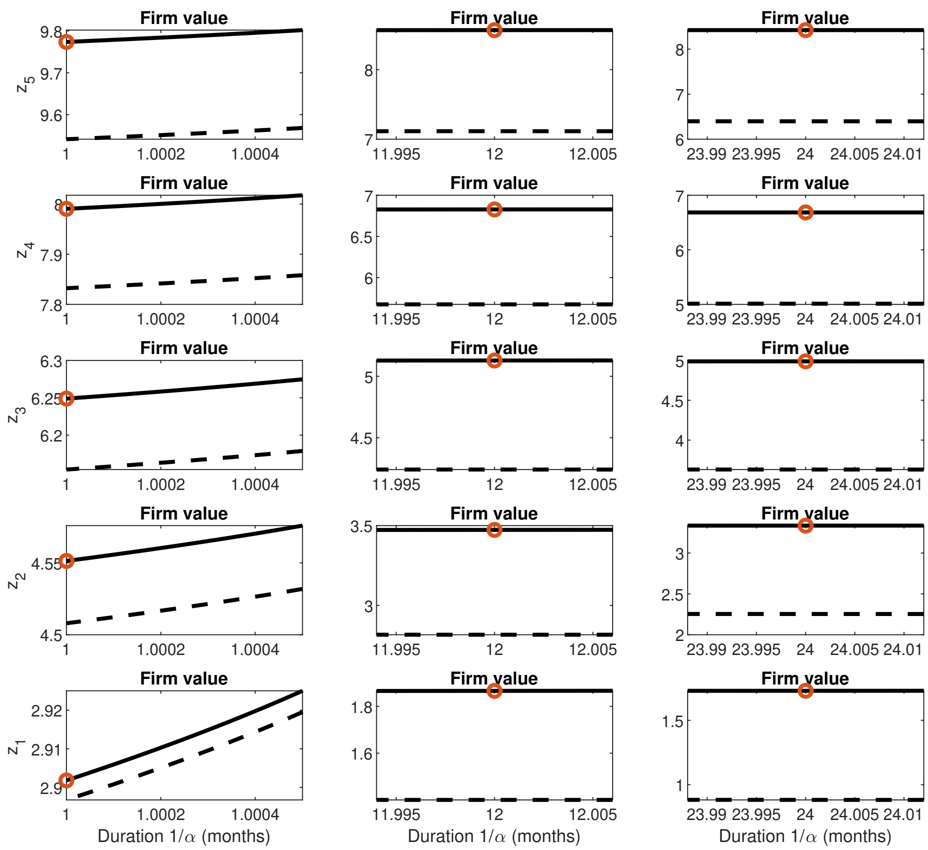

Figure F.10: Firm Deviation with Firm Shocks: Optimality of Hiring

Notes: The circles denote the equilibrium with infrequent wage adjustment where firms reoptimize monthly, annually and biennially, and the figure plots corresponding values for a firm deviating to longer/shorter wage duration as a function of the expected duration of wages $1/\alpha$. It shows that the firm value attained by following the first order conditions dominates withdrawing from hiring for the duration of the wage. The model is solved on a 5-state grid for productivity, approximating an AR(1) with autocorrelation $\sigma_z = 0.88$ and standard deviation $\sigma_z = 0.2$ based on the Rouwenhorst method. The firm value plotted is the scaled value.

G Model with Firm-Level Shocks

In a stationary equilibrium with idiosyncratic firm-specific shocks to productivity, the aggregate measure of matches N and value of search U (and hence also X, Y) remain constant, while shocks lead to reallocation of labor across firms over time.[52]

The firm problem may be written:

$$\max_{W,v}(n + q(\theta)v)(Z - W) - \kappa(v, n) + \beta E_z V(n', z')$$
$$\text{s.t. } n' = (1 - \delta)(n + q(\theta)v),$$
$$X = \mu(\theta)(W - Y),$$

where the continuation value satisfies $V(n, z) = q(\theta)v(Z - W) - \kappa(v, n) + \beta E_z V(n', z')$.

Scaling by size yields the size-independent problem:

$$\max_{W,x}(1 + q(\theta)x)(Z - W) - \hat{\kappa}(x) + \beta(1 - \delta)(1 + q(\theta)x)E_z\hat{V}(z')$$
$$\text{s.t. } X = \mu(\theta)(W - Y),$$

where $\hat{V}(z) = q(\theta)x(Z - W) - \hat{\kappa}(x) + \beta(1 - \delta)(1 + q(\theta)x)E_z\hat{V}(z')$.

Infrequent Wage Adjustment Consider a firm setting a fixed wage for a probabilistic period of time. The firm's beliefs regarding the market tightness continue to be determined by the constraint $X = \mu(\theta)(h(w, z) - Y)$ each period, where $h(w, z)$ represents the present value of wages.[53]

The firm problem may be written

$$\max_{w,v}(n + q(\theta)v)(Z - h(w, z)) - \kappa(v, n) + \beta E_z(\alpha V^r(n', z') + (1 - \alpha)V^f(n', w, z'))$$
$$\text{s.t. } n' = (1 - \delta)(n + q(\theta)v),$$
$$X = \mu(\theta)(h(w, z) - Y).$$

Here the value of reoptimizing satisfies

$$V^r(n, z) = q(\theta)v(Z - h(w, z)) - \kappa(v, n) + \beta E_z(\alpha V^r(n', z') + (1 - \alpha)V^f(n', w, z'))$$

[52] I abstract from entry and exit but one could easily incorporate exit shocks into the firm problem, with exiting firms replaced by new ones. The behavior of new and existing firms is identical if new firms enter with at least one worker.

[53] Suppose z lives on a grid and the transition probability matrix is denoted Π. Denote the vector of equilibrium present values of wages for a reoptimizing firm across z as $\mathbf{W}^r$ and the present value of wages of a firm holding its wage w fixed as $\mathbf{W}^f(w)$. We have that $\mathbf{W}^f(w) = w\mathbf{i} + \beta(1-\delta)[\alpha\Pi\mathbf{W}^r + (1-\alpha)\Pi\mathbf{W}^f(w)]$, where $\mathbf{i}$ a vector of ones. This gives the present value wage for a firm with wage w as $\mathbf{W}^f(w) = (I - \beta(1 - \delta)(1 - \alpha)\Pi)^{-1}(w\mathbf{i} + \beta(1 - \delta)\alpha\Pi\mathbf{W}^r)$. I denote the components of this vector in the text by $h(w, z)$.

with the above firm choices. The value of holding the wage fixed satisfies

$$
\begin{aligned}
V^f(n', w, z') = \max_v & q(\theta)v(Z - h(w,z)) - \kappa(v,n) + \beta E_z(\alpha V^r(n', z') + (1-\alpha)V^f(n', w, z')) \\
\text{s.t. } & n' = (1-\delta)(n + q(\theta)v), \\
& X = \mu(\theta)(h(w,z) - Y).
\end{aligned}
$$

Scaling these problems, firms reoptimizing wages solve

$$
\begin{aligned}
& \max_{w,x} (1 + q(\theta)x)(Z - h(w,z)) - \hat{\kappa}(x) + \beta(1-\delta)(1 + q(\theta)x)E_z(\alpha \hat{V}^r(z') + (1-\alpha)\hat{V}^f(w, z')) \\
& \text{s.t. } X = \mu(\theta)(h(w,z) - Y).
\end{aligned}
$$

Here the value of reoptimizing satisfies

$$
\hat{V}^r(z) = q(\theta)x(Z - h(w,z)) - \hat{\kappa}(x) + \beta(1-\delta)(1 + q(\theta)x)E_z(\alpha \hat{V}^r(z') + (1-\alpha)\hat{V}^f(w, z'))
$$

with the above choices. The value of holding the wage fixed satisfies

$$
\begin{aligned}
\hat{V}^f(w, z') = \max_x & q(\theta)x(Z - h(w,z)) - \hat{\kappa}(x) + \beta(1-\delta)(1 + q(\theta)x)E_z(\alpha \hat{V}^r(z') + (1-\alpha)\hat{V}^f(w, z')) \\
\text{s.t. } & X = \mu(\theta)(h(w,z) - Y).
\end{aligned}
$$