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Dynamic Pricing of Credit Cards and the Effects of Regulation

Suting Hong
Shanghai Tech University

Robert M. Hunt
Federal Reserve Bank of Philadelphia Consumer Finance Institute

Konstantinos Serfes
Drexel University

Consumer Finance Institute

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Dynamic Pricing of Credit Cards and the Effects of Regulation*

Suting Hong† Robert M. Hunt‡ Konstantinos Serfes§

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Abstract

We construct a two-period model of revolving credit with asymmetric information and adverse selection. In the second period, lenders exploit an informational advantage with respect to their own customers. Those rents stimulate competition for customers in the first period. The informational advantage the current lender enjoys relative to its competitors determines interest rates, credit supply, and switching behavior. We evaluate the consequences of limiting the repricing of existing balances as implemented by recent legislation. Such restrictions increase deadweight losses and reduce ex ante consumer surplus. The model suggests novel approaches to identify empirically the effects of this law.

Keywords: Financial contracts; Credit Card Accountability Responsibility and Disclosure Act; holdup; risk-based pricing; credit supply

JEL Classifications: D14, D18, D86, G28, K12

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†School of Entrepreneurship and Management, Shanghai Tech University. e-mail: hongst@shanghaitech.edu.cn.

‡Consumer Finance Institute, Federal Reserve Bank of Philadelphia, Philadelphia, PA 19106. e-mail: bob.hunt@phil.frb.org.

§School of Economics, LeBow College of Business, Drexel University, Philadelphia, PA 19104. e-mail: ks346@drexel.edu.

1 Introduction

We develop a parsimonious two-period model of financial contracts and competition to explore the consequences of flexibility or inflexibility in pricing over the life of a credit card relationship. We use the model to explore the relationship between competition among lenders, the role of public and private information about borrowers, the degree to which credit risk is incorporated into pricing, the effects of shocks to credit quality, and the ability of a lender to extract additional surplus from borrowers at a time when their outside alternatives become less attractive. We compare the level of consumer surplus for the cases of more and less flexible contracts and the factors that suggest why one or the other may be superior for borrowers. We use the model to illustrate the effects of regulatory changes that reduce the flexibility of contracts.

While the theoretical literature on credit contracts is voluminous, there are few papers that examine contracts with many of the empirical characteristics of a credit card contract. A number of models incorporate one or more important elements but too few to be useful for evaluating competition, consumer welfare, or recent policy interventions in the credit card market.[1] An important contribution of this paper is to evaluate the properties of an equilibrium in which consumers borrow using contracts that look very much like the credit card contracts we observe in the real world. In this paper, lenders offer long-term, unsecured lines of credit, and consumers decide whether to revolve their balances depending on shocks to their income. In the baseline model, lenders may decide to change the price (interest rate) charged on those revolving balances and on any new borrowing. Those prices need not be the same, and often will not be. Lenders' repricing decisions depend on their acquisition of new information as well as current competitive conditions. Consumers can choose to sever their existing credit relationship by switching to a new lender. Some consumers default in equilibrium, because of either income shocks or moral hazard. There is asymmetric information about the riskiness of consumers' income stream, so equilibrium prices, quantities, and switching behavior depend on the extent of adverse selection and, in some instances, advantageous retention.

In our model, there are two types of borrowers (high and low risk) and many lenders that compete over two periods for new customers and, in the second period, to retain their existing ones. Borrowers may draw on their credit lines in each period. In the second period, each lender learns the type of its existing customers before setting the interest rates it will charge on existing and new borrowing by those customers. Lenders also receive a noisy signal (e.g., a credit score) about the type of customers currently served by the other lenders. In the equilibrium with flexible contracts, each lender provides credit to both

[1] We describe the most closely related models and related empirical research in Section 2.2.

types of its existing customers in the second period. The informational advantage a lender enjoys with respect to its own customers in the second period enables it to earn rents in that period.[2] However, these rents are competed away in the first period, when all lenders share the same information. Thus, although lenders have market power in period 2, their discounted expected profits over the two periods are zero.

While simple, the model captures many aspects of the U.S. credit card market. For example, differential pricing of existing and new balances follows naturally from the competitive trade-offs lenders face. When we extend the model to allow for heterogeneity in the consumer's cost of default (Section 6), the lenders offer low rates for transferring balances and higher rates on any new balances, a common feature of the U.S. credit card market. In addition to setting prices, lenders also establish credit limits.

We apply this model to evaluate the effects of new regulations in the U.S. that limit the ability of lenders to increase interest rates on the existing balances of consumers who are not seriously delinquent.[3] In the regulated equilibrium, the inside lender is precluded from raising interest rates on the existing balances of its nondefaulting customers in the second period. Subject to this constraint, the inside lender will increase the rate on new borrowing by an amount that exceeds the incremental credit risk. This creates inefficiency in the second period because these prices distort the consumption decisions of some borrowers. The surplus of the borrowers in the second period is pinned down by the offers of the outside lenders, who are not constrained by the regulation. High-risk borrowers who carry debt from the first period will switch lenders in the second period, while low-risk borrowers generally do not. Regulation reduces the second-period rents earned by inside lenders, which, in turn, softens competition in the first period. The result is higher first-period interest rates, which makes all borrowers worse off. Finally, the regulation increases the deadweight loss, relative to the unregulated equilibrium (borrowers are worse off, while lenders break even across the two periods).[4]

Another important contribution of the model is that it suggests novel and potentially better approaches to identification in empirical studies of pricing restrictions on credit cards embodied in the Credit Card Accountability Responsibility and Disclosure Act (hereafter the CARD Act). Congress passed the CARD Act when the country was entering a financial crisis, resulting in the worst unemployment and wealth destruction since the Great Depression. Distinguishing between the effects of macroeconomic shocks and changes resulting from the CARD Act is a difficult exercise. With few exceptions,

[2]Throughout the paper, we refer to a lender interacting with its existing customers as an *inside* lender. Lenders seeking to attract new customers are referred to as *outside* lenders.

[3]This is one of several restrictions established by the Credit Card Accountability, Responsibility and Disclosure Act of 2009.

[4]We emphasize that these welfare results are based on a model in which the primary barrier to price competition is adverse selection resulting from asymmetric information in which consumers respond in a manner consistent with the usual neoclassical assumptions. Welfare results will differ under different assumptions. Nevertheless, our model is a useful benchmark for evaluating policy options.

comparing endogenous indicators (e.g., interest rates or credit lines) before and after the law came into effect cannot distinguish between these potential sources of causation. This is a major limitation of most existing studies of the CARD Act (see Section 2 for a discussion of the literature). Our model suggests that, if the CARD Act restrictions are indeed binding, we should observe distinct changes in the pattern of consumers switching lenders.[5] In particular, adverse selection in the acquisition of new accounts should increase.

The remainder of this paper is organized as follows. Section 2 reviews the institutional details of the credit card market and the policy debate surrounding the flexibility of a credit card contract. It also describes the existing literature. Section 3 introduces the model, and Section 4 establishes the existence of an equilibrium in which prices are both dynamic and which distinguish between old and new borrowing. Section 5 presents our results on the effects of limiting the repricing of existing balances. Section 6 extends the model by incorporating heterogeneity in the borrower's cost of default and by allowing the borrower to choose whether to default (moral hazard). Section 7 concludes. All proofs are in Appendix A, and a numerical example is presented in Appendix B.

2 Policy Debate and Existing Literature

2.1 The U.S. Credit Card Market in the 2000s

Today, there are roughly \$1 trillion in outstanding revolving balances in the U.S., accounting for 27 percent of consumers' nonmortgage debts.[6] About 71 percent of families have an open general-purpose credit card and there are more than 600 million of these cards in circulation.[7]

Credit cards embody a number of unique complementarities. On the one hand, they are a convenient form of payment, especially at the point-of-sale and on the Internet.[8] The consumer has the option to pay off the entire value of accumulated purchases at the end of the billing cycle. There is typically no cost to the consumer for using the card in this way because most credit cards do not charge interest on new purchases if the cardholder does not carry over a balance from the previous billing cycle. This is a popular feature for many consumers. In 2016, 58 percent of families with at least one credit card did not carry a balance on their cards (Bricker et al. 2017).

[5] For suggestive evidence that the CARD Act's pricing restrictions are binding, see Figures 1, 4, and 5 of Nelson (2018).

[6] These values are for year-end 2017, as reported in the Federal Reserve Board's monthly Consumer Credit - G19 report.

[7] The first statistic is from Bricker et al. 2017 and the second is from the Nilson Report.

[8] In 2016, general-purpose credit cards accounted for 34 billion transactions worth \$3 trillion. See Federal Reserve System Payments Study (2017).

Credit cards also embody a flexible credit feature, permitting the borrower to spread the payments on purchases over time. Most credit cards have an established credit limit that determines the maximum balance the consumer can borrow. After deciding to offer a credit card to a consumer, lenders incur little or no additional underwriting cost for each draw the consumer makes against the credit limit. In addition, subject to certain minimum payment requirements, the borrower is free to accelerate or decelerate repayments on the outstanding balance.

Historically, an important aspect of credit cards is the flexibility of the underlying contract. Most credit card lines do not have an expiration date, or a cost to close the account, so the length of the cardholder relationship is indeterminate.[9] Lenders typically reserve the right to revise contract terms at any time. Change-of-terms provisions in credit card contracts have existed for at least 40 years (Watkins 2009, FN 120). At least during the 2000s, creditors frequently exercised their rights to change terms.[10]

In recent years, the flexibility of the credit card contract has received increasing scrutiny, especially from policymakers. A particular concern has been the potential risk that repricing poses for consumers. For cardholders with large balances, raising interest rates on existing balances will increase the required minimum payment amount. A consumer might respond by switching those balances to another card, refinancing into another form of debt or by deciding to pay down the balance more rapidly. However, those options may not be available to all consumers who experience the repricing of their existing balances. Moreover, for some liquidity constrained consumers, the repricing of existing balances might affect their ability to service all their debts. More generally, raising interest rates at a time consumers may be experiencing a negative shock reduces the effectiveness of using revolving credit lines to smooth consumption (Athreya, Tam, and Young 2009).

An important question in the policy debate was why a credit card lender would choose to reprice outstanding balances in this way. Lenders argued that these interest rate increases reflected a revised assessment of the credit risk of an account, even when the borrower was still current on the account. Lenders have ready access to credit bureau information on their existing customers and can track changes in their customers' credit scores, new accounts, and repayment status on other accounts. Lenders argued that repricing accounts to reflect new information reduces the cross-subsidy from lower risk to higher risk customers and consumers would have an increased incentive to accelerate the pay down of balances now perceived to be more risky. The additional interest rates charged would also help to offset losses on

[9]For this reason, some legal scholars describe these as "at-will" contracts.

[10]Bar-Gill and Davis (2010) describe a June 2009 telephone survey in which one-third of 1,000 cardholders reported experiencing adverse changes in the terms of their accounts. Data for 2007-2008 from Argus Information & Advisory Services suggest that 17 percent of credit card accounts were repriced. See CFPB (2013), p. 28. Nelson (2018) estimates that prior to the CARD Act about 50 percent of accounts carrying a balance were repriced in ways subsequently prohibited by the act.

the accounts that are eventually charged off. Lenders also argued that the flexibility to reprice accounts over the lifespan of an account made it possible to offer credit on better terms even to those consumers with a less than perfect credit history.

Others have argued that lenders were engaged in a form of opportunistic holdup of their customers. According to this argument, credit card issuers might raise interest rates charged to those existing customers who would find it hard to shop for a competitive alternative. A stronger form of the argument suggested that the opportunity to hold up customers in the future might spur competition to acquire new customers by initially offering lower rates and other benefits (Levitin 2011). In that case, myopic or simply unlucky customers might find themselves worse off than if they obtained a product with fewer repricing opportunities.[11]

In 2008, the Federal Reserve Board and other federal regulators proposed a new rule containing substantive restrictions on credit card pricing, fees, and other account management practices. After taking into account comments on the proposal, the regulators published a revised rule in January 2009.[12] Congress incorporated many elements of these rules (but also made significant changes) in the legislation that became the CARD Act.[13] Title X of the 2010 Dodd-Frank Wall Street Reform and Consumer Protection Act transferred responsibility for rulemaking and enforcement of most provisions contained in the CARD Act to the Consumer Financial Protection Bureau, effective in July 2011.[14]

Most of the requirements relevant to this paper came into force in February 2010.[15] Under those rules, lenders cannot raise the interest rate on the existing balances of a customer unless the borrower is already more than 60 days late on the account. The lender is permitted to change interest rates on new balances at any time, provided that the consumer is given at least 45 days prior notice of the change.

As noted in the introduction, measuring the effects of the CARD Act is difficult because the timing of the proposed and final changes overlaps with the recent collapse in housing markets, which contributed to a financial crisis, a deep recession, and very slow recovery in the labor market. The credit card market experienced tumultuous changes during this period. Between 2008 and 2011, banks charged off well more than $160 billion in credit card balances.[16] The number and aggregate value of credit card lines fell by more than 20 percent in less than two years. There was a comparable drop in outstanding balances.

[11] See Gabaix and Laibson (2006) for a formalization of this intuition. As an empirical question, Nelson (2018) finds evidence supporting both arguments: Opportunistic repricing of prime borrowers and repricing for increased credit risk among subprime borrowers. See Figure 8 of that paper.

[12] The original proposed rules are at 73 *Federal Register* 28904 (May 19, 2008). The final pre-CARD Act rules are at 74 *Federal Register* 5498 (January 29, 2009) ("UDAP") and 74 *Federal Register* 5244 (Jan. 29, 2009).

[13] Public Law 111-24, 123 Stat. 1734 (2009).

[14] Public Law 111-203, 124 Stat. 1376 (2010).

[15] 75 *Federal Register* 7658 (February 2010).

[16] This is the value of gross charge-offs reported in the spreadsheets compiled for the FDIC's Quarterly Banking Profiles. Santucci (2016) estimates that 28 percent of the reduction in balances that existed in March 2009 occurred via charge-offs.

of mid-2018, inflation adjusted revolving credit balances remain about 15 percent below the peak level observed in 2007–2008. Mailings of prescreened offers for new credit card accounts fell approximately 80 percent between 2007 and 2009 before beginning to recover.[17] An important challenge both for theoretical and empirical research, then, is to develop approaches that allow for separately identifying the effects of the CARD Act and the financial crisis.

2.2 Existing Research

2.2.1 Theory

The theoretical literature on financial contracts is voluminous.[18] Much of the literature can be divided into models of one-period contracts and models of long-term debt, such as installment or balloon loans. Credit cards exhibit aspects found in each category. For example, they have the pricing flexibility of one-period contracts. Models with long-term debt allow for the emergence of additional information (either private or public). This permits one to study the role of insurance as well as potential hold-up problems, issues we also examine in this paper. Our model is one of asymmetric information and, potentially, relationship lending. These too are common features in the literature.

The theoretical contribution of this paper is not a general one, but rather it is a specific application to contracts that look and behave much more like actual credit cards than most models in the literature. For example, a model of one-period contracts is not particularly suitable for studying a product (and regulation) that may treat old and new debts differently. On the other hand, most models of long-term debt begin with a borrower who actually borrows entirely at the beginning of the relationship.[19] There are a few notable exceptions. In DeMarzo and Fishman (2007), the optimal financial structure of a firm is characterized by a combination of equity, long-term debt, and a revolving credit line. The latter is the closest analogue to a credit card, but in their model, the endogenous interest rate does not vary over time, nor does it vary across older and more recent draws.[20] Shockley and Thakor (1997) present a theoretical and empirical study of bank loan commitments to business. However, their model does not allow for ex-post repricing, nor does repricing appear to be a feature in loan data they study.

The model that is closest in spirit to ours is found in Sharpe (1990). This is a model with two period contracts and an information asymmetry between inside and outside lenders. We incorporate aspects of

[17]These are the authors' calculations using data from Mintel Comperemedia.

[18]See, for example, Bolton and Dewatripont (2005) and Freixas and Rochet (2008).

[19]As noted earlier, a significant share of credit cardholders pay off their balance every month. Our model, and the legislation we study, has different implications for cardholders who revolve and those who do not.

[20]One could argue, however, that the interest charged on long-term debt represents a separate rate paid on the earliest draw associated with establishing the firm.

this model into an environment that looks more like the credit card market and where prices on old and new debt can vary.

The number of papers that model dynamic pricing of the credit aspect of credit cards is relatively small. Park (2004) constructs a two-period model with adverse selection and shocks to credit quality to study the option value to consumers who wish to use their credit lines when their risk of default rises. While this has several implications for the ex-ante pricing of a credit card contract (e.g., teaser rates), Park does not consider ex-post repricing.[21] DellaVigna and Malmendier (2004) develop a model in which contracts are devised to exploit consumers with time inconsistent preferences and naive expectations. An extension of the model allows for dynamic competition, with the result that firms have an incentive to create switching costs, and fees tend to be back-loaded. Their extension explicitly allows for repricing, as we do in this paper. Neither of those papers model an asymmetry in the information created by lenders learning about their customers or consider differential repricing of old and new balances. In our model, lenders learn about the riskiness of their customers and use this information to set prices that influence the opportunity cost of switching to another lender; in effect this lowers switching costs for their most risky customers and induces advantageous retention.

Sanches (2011) examines long-term unsecured credit relationships in the Lagos-Wright model of search and matching. Sellers (lenders) are willing to finance the purchases of buyers (borrowers) who experience idiosyncratic liquidity shocks. The contract terms vary (become less generous) as the amount borrowed rises. There is no asymmetric information in the model. Borrowers do not switch lenders in equilibrium, but the potential to do so induces lenders to partially forgive the debt owed by a borrower who experiences many consecutive liquidity shocks. In other words, it is the lender that is vulnerable to hold-up.

The literature that explicitly studies the effects of the CARD Act is growing. Tam (2011) considers the consequences of the repricing restrictions studied in this paper by comparing calibration models of one-period versus multiperiod unsecured debt contracts. In Tam (2011), there is no asymmetric information, and the prices in the model do not distinguish between old and new borrowing. Tam evaluates the implications of equilibrium interest rates on consumption smoothing, but this is not a model suited to evaluating potential hold-up problems.

Ronen and da Silva Pinheiro (2016) develop a three-period model in which the consumer learns private information about his or her type (high-risk or low-risk) and has the option of incurring a cost to credibly signal this information to creditors.[22] In equilibrium, low-risk consumers will signal their

[21] In this dimension, Park's model is more closely related to the literature on bank loan commitments described previously. In Section 6, where we allow for endogenous default, we reproduce a number of the results found in Park (2004).

[22] Hereafter, we refer to this paper as RdSP.

type and obtain better terms, while high-risk consumers will not signal and borrow under the original terms of the contract. The combination of costly signaling and the ability to reprice enable a separating equilibrium that partially mitigates an adverse selection problem. Equilibrium contracts exhibit fixed up-front fees, an interest rate, and a credit limit. The authors use their model to examine the consequences of limiting the repricing of balances, which reduces the ability of the market to implement a separating equilibrium. They find such restrictions lower up-front fees, increase interest rates for high-risk borrowers and lower credit limits for low-risk borrowers.

There are a number of important differences between the model constructed here and the one in RdSP. First, the source of asymmetric information is different. In this paper, the borrower and the current lender have private information, while in RdSP, all lenders have the same information about borrowers. In RdSP, borrowers can credibly signal their type; in this paper they cannot. In RdSP, there is only one episode of borrowing so there is no distinction between the pricing of existing and new balances. Finally, the changes in contract terms resulting from repricing restrictions identified in the model by RdSP are observationally equivalent to the predictions for a shock to credit quality. Thus, the model cannot easily be used to develop an identification strategy for empirical tests of the consequences of the CARD Act. The model developed in this paper suggests that changes in switching behavior are a promising approach to addressing the identification problem.

2.2.2 Empirics

A number of careful empirical studies document important properties of the credit card market. Ausubel (1999) and Agarwal et al. (2010) employ randomized credit card offers to identify the presence of substantial adverse selection. Calem and Mester (1995) provide additional evidence of adverse selection as well as switching costs during the 1980s. Calem, Gordy, and Mester (2006) show that this phenomenon continued to persist after the widespread adoption of credit scores. Nelson (2018) exploits the repricing of entire credit card portfolios to document a negative correlation between credit risk and price elasticity, which is to be expected in a market with adverse selection.

Trench et al. (2003) provides concrete examples of the types of private information lenders use to make pricing and credit limit decisions for their existing customers. These include purchases, actual payments, and account profitability. The fact that a specific consumer responded to a credit offer with specific terms reveals private information about the consumer to the lender making the offer.[23] Nelson

[23]Lenders do monitor detailed information about the offers made by their competitors. For example, such information is found in data collected by Mintel Comperemedia. While lenders do not observce specifically which consumers receive credit card offers, they are able to monitor the credit applications of consumers (hard inquiries).

(2018) shows that, controlling for observable credit risk, the interest rates charged by banks on mature accounts is correlated with the ex-post likelihood of default.

There are a number of empirical studies of the CARD Act. Data contained in CFPB (2013) document the practical elimination of over-the-limit fees and a significant reduction in the repricing of existing balances. That study describes an increase of about 200 basis points in the account-weighted average retail annual percentage rate (APR) of open credit card accounts over the period 2009–2010, primarily among consumers with prime credit scores.[24] At the same time, CFPB found a nearly comparable decline in effective cardholder costs (actual interest charges and fees divided by balances) over the period 2009–2012. The CFPB reports are careful to acknowledge the difficulty in making cause-and-effect arguments, especially in light of the recession that occurred over the period in which new regulations were contemplated and then implemented.

Agarwal et al. (2015) report the results of a difference-in-difference analysis of credit card accounts at eight large banks over the period 2008–2012.[25] This is accomplished by comparing variables for 150 million consumer and 7 million small business credit card accounts before and after the CARD Act restrictions came into effect (small business cards were not subject to those restrictions). The authors find consumer cardholder costs fell by 1.7 percent relative to average cardholder balances and a larger decline (5.5 percent) among subprime borrowers. They also did not find evidence of anticipatory increases in interest rates, at least relative to behavior on business accounts, before implementation of the CARD Act. An important caveat, however, is that their data do not account for repricing behavior that occurred before the beginning of their data (CFPB 2013).

A limitation of the data used by Agarwal et al. (2015) is that the unit of analysis is an account and not a consumer. While the authors can observe accounts being opened and closed, they cannot observe consumers switching balances from specific accounts to others or the pricing that would explain why a consumer decided to switch accounts.

Elliehausen and Hannon (2017) find that the CARD Act's restrictions on risk management practices adversely affected higher risk consumers. In particular, their credit card accounts declined after the CARD Act became effective.

Han, Keys and Li (2018) examine the credit card offers made to low-risk and high-risk consumers (subprime and prime borrowers, respectively) before and after enactment of the CARD Act. Using a difference-in-differences methodology, the authors find that credit offers fell disproportionately among

[24] Note that this was a period in which the reference rate for most credit cards (either the LIBOR or Prime rate) fell significantly. CFPB also found that approximately 150 million card accounts were repriced (upward) in the year before implementation of the CARD Act's limitations on repricing. See also Nelson (2018).

[25] This data set is similar to one of the data sets used in CFPB (2013).

subprime consumers and recovered less for this group as the economy recovered from the financial crisis. They also find that offered credit limits fell and interest rate spreads increased disproportionately among subprime consumers. This appears to be reflected in banks' credit card line management strategies before and after the CARD Act and the Great Recession (Santucci, 2015).

Nelson (2018) estimates a structural model using data before enactment of the CARD Act equilibrium and then imposes the CARD Act's price restrictions to study how the market responds. This helps to remove the effects of the financial crisis from his policy simulation.[26] In his model, lenders learn information about two characteristics of each borrower: i) their demand elasticity for credit and ii) their default risk. The ability of lenders to reprice because of information about the former characteristic increases markups and hence is welfare reducing, while the ability to reprice due to the latter characteristic mitigates adverse selection and so is welfare-enhancing. The CARD Act restricts repricing and as a result poses a trade-off between lower markups and mispriced risk (adverse selection). For borrowers and welfare, the net effect of the CARD Act depends on which of these two forces dominates in equilibrium. Nelson estimates that, for borrowers who remain in the market, lower markups dominate, making them better off and resulting in lower deadweight loss. At the same time, the inability to reprice in response to new information about credit risk leads some consumers (with more elastic demand) to exit the mar-ket because they face higher interest rates than before. In his simulations, a substantial proportion of consumers with very low credit scores drop out of the market.

In contrast, our model predicts that the CARD Act increases deadweight losses and hurts borrowers. The difference in results is attributable to important differences between the model presented here and the one found in Nelson (2018). In our model, consumers are liquidity constrained so they must use a credit card to make purchases. In addition, unlike Nelson (2018), our model allows for an intensive margin in borrowing. Liquidity shocks and consumers' impatience and, eventually, their type determine how much they borrow. Thus, in our model, regulation will change lenders' strategies, but there is no unraveling (partial or otherwise) at the level of the market. Further, by incorporating an intensive margin, our model can accommodate moral hazard which, in turn, motivates the adoption of credit limits by lenders and induces the commonly observed phenomenon of teaser rates. Thus, our contracts look and act more like credit card contracts.[27]

In Nelson (2018), there are two prices: interest rates charged on existing accounts and teaser rates offered on new accounts. In our model, there are three prices: an interest rate offered for balance transfers

[26] It should be noted, however, that during the time period in which he estimates his model (July 2008–June 2009), lenders were actively repricing accounts in anticipation of new regulations. See footnote 11.

[27] Switching costs are an important facet of the model and empirical results in Nelson (2018). Our model can accommodate heterogeneous switching costs, but we omit them from the exposition as they do not affect our qualitative results.

to new customers and (potentially) distinct interest rates charged for existing balance and new balances on existing accounts. This approach allows us to follow the actual re-pricing restrictions imposed by the CARD Act more closely. Allowing lenders to compete on these margins enables them to generate advantageous retention of existing customers. Second-period borrower utility is pinned down by the offers of outside lenders, taking into account the adverse selection problem induced by the offers of inside lenders. In the absence of regulation, inside lenders extract rents, where possible, by repricing existing balances. This is a transfer from some low-risk borrowers to lenders but does not distort second-period borrowing. The existence of rents in the second period induces competition in teaser rates in the first period, benefiting all consumers. Regulation that limits the repricing of existing, but not new, balances leads to higher prices on new balances, distorting borrowing in the second period. Relative to the unregulated equilibrium, the offers of outside lenders are more competitive, reducing potential rents earned by inside lenders, which, in turn, leads to higher-interest rates charged in the first period.

Several other empirical studies of the CARD Act are not as closely related to the questions examined in this paper. Jambulapati and Stavins (2014) study the closing of open revolving credit lines before and after the CARD Act regulations came into force. Han, Keys, and Li (2018) analyze data on mailings of prescreened offers of revolving credit during the financial crisis and subsequent implementation of the CARD Act. Debbaut et al. (2016) study the effects of the CARD Act's restrictions on marketing to consumers under the age of 21 and use it as source of exogenous variation in a study of the credit risk among young borrowers. Several papers study the effects of changes to the content of monthly billing statements required by the CARD Act. These include papers by Campbell, Gartenberg, and Tufano (2011), Jones, Loibl, and Tennyson (2015), Agarwal et al. (2015), and Keys and Wang (2016).

3 The Model

We assume that the market comprises a continuum of consumers (borrowers) of unit mass and $N > 2$ lenders. Each consumer wishes to borrow to finance consumption for up to two market periods, $t = 1, 2$. Within each period there are two stages: a morning and an evening. The consumer is cash-constrained so he must borrow to finance consumption in the morning. The consumer's utility over the two stages is of a quasilinear form, in which the marginal utility from consumption in the morning is diminishing, while in the evening, when the loan should be repaid, marginal utility is constant. We denote the amount a consumer borrows in period t by b_t and the utility he derives from borrowing by $u(b_t)$, with $u' > 0$, and $u'' < 0$ (and $u(0) = 0$ and $u'(0)$ sufficiently high). In each period, a consumer either has sufficiently high income to repay the loan fully or the income is zero. The probability of high income is p.[28] Consumers are heterogeneous with respect to their quality as is captured by the probability p, with two types in the population: a high quality (H) and a low quality (L), with $0 < p_L \leq p_H \leq 1$ and $i \in \{H, L\}$. The fraction of high-quality consumers in the population is $\mu \in (0, 1)$. Hence, the expected (ex-ante) probability of high income is $\bar{p} \equiv \mu p_H + (1 - \mu) p_L$. The probability p_i is constant across the two lending periods.

Note that, in the evening, the borrower is either paying off or accruing interest on the amount borrowed in the morning of the same period. The model could be generalized to allow for transactors, who repay within the billing cycle and thus incur no interest. This can be done by dividing the day into three periods, with stochastic income realized at noon and in the evening. If the borrower realizes income at noon, he repays the balance without interest; otherwise, he either repays the balance with interest in the evening or defers repayment to the next period. We chose not to add this complication to the model because we are focusing on a regulation of the credit aspect of this product.

The timing of the game is as follows.

Period 1. The true quality of each consumer is unknown to all parties at the beginning of the first period (including the consumer himself). Lenders simultaneously make interest rate offers to each consumer for the first period but cannot commit to second-period rates. Borrowers have rational expectations, so that in period 1 they correctly anticipate the expected second-period interest rate offers they will receive. Each borrower borrows from one lender and decides how much to borrow.[29] At the end of the first period, each borrower either repays the loan fully (with probability p_i) or, with the complementary probability, carries a balance into the second period. We denote this balance (debt) by d.

Period 2. At the beginning of the second period, each borrower learns his true probability of repayment. In addition, the lender that lent to borrower i in the first period (we call him an inside lender, $j = I$) also learns the probability perfectly. All other lenders (outside lenders, $j = O$) receive noisy signals about the borrower's true p. This signal can be considered a credit score. We also assume that all lenders observe every borrower's debt d at the beginning of the second period and before they make

[28] In the main model, we do not allow the borrower to choose whether to default or not, but we do so in an extension in Section 6. Our main results do not change qualitatively.

[29] In reality, many consumers hold more than one credit card. However, Nelson (2018) argues that 80–90 percent of consumers concentrate their balances on a single card at a time. It is very common for lenders to incent consumers to single home by offering contracts with fixed annual fees and rewards based on purchase volume. Thus single homing of balances is not an implausible assumption for the model.

offers.[30] Moreover, we assume the inside lender also observes the signal the outsiders receive.[31] Borrower i receives interest rate offers from all lenders and chooses how much to borrow from one lender. More specifically, we assume that the inside lender moves first by committing to an interest rate policy, which is a function of the borrower's type, his or her indebtedness, and the signal received by the outside lender. Outside lenders are able to observe the interest rate policy of the insider lenders. All lenders then simultaneously make interest rate offers to individual borrowers. This timing is similar to the model found in Sharpe (1990) and ensures existence of an equilibrium in pure strategies.[32] At the end of period 2, either the borrower's income is sufficiently high to repay all loans or he defaults.

Each lender has access to unlimited capital at a cost of $\bar{r}$ per period. Let $s \in \{h, \ell\}$ be the noisy signal of consumer's quality the outside lenders receive at the beginning of period 2 for every borrower who borrowed from another lender in the first period. The signal's conditional distribution function is given by

$$\Pr(s = h|H) = \Pr(s = \ell|L) = \frac{1+\phi}{2} \text{ and } \Pr(s = h|L) = \Pr(s = \ell|H) = \frac{1-\phi}{2},$$

where $\phi \in [0, 1]$ measures the accuracy of the signal. If $\phi = 1$ the signal is perfect. On the other hand, if $\phi = 0$, the signal would be completely uninformative. The weaker this signal (lower ϕ) is, the greater will be the informational advantage enjoyed by the inside lender. Using Bayes' rule the updated fractions of high types upon simply observing the borrower's debt are

$$\Pr(H|d = 0) = \frac{p_H\mu}{\overline{p}} \equiv \mu(d = 0) \text{ and } \Pr(H|d \neq 0) = \frac{(1-p_H)\mu}{1-\overline{p}} \equiv \mu(d \neq 0). \tag{3.1}$$

In the rest of the paper, sometimes we suppress the dependence of μ on the level of debt, but it should be clear from the context whether in the Bayesian updating we should be using $\mu(d = 0) \geq \mu$ (when the borrower carries no debt) or $\mu(d \neq 0) \leq \mu$ (when the borrower carries debt). It is worth noting that observing the level of debt in our model is equivalent to observing whether the project was successful

[30] This matters only for the outside lenders because it affects the fraction of high types μ in the Bayesian updating. In the U.S., lenders can target prospective new customers by specifying criteria (including scores and debt levels) for lists of customers who will receive "prescreened" offers of credit.

[31] In practice, creditors do periodical reviews of their existing customers by obtaining "refreshed" credit scores from one of the bureaus.

[32] von Thadden (2004) shows that without the incumbent's commitment to an interest rate policy, the only equilibrium in Sharpe's model is in mixed strategies. In practice, credit card lenders do not engage in mixed strategies. Lenders use randomized experiments to test new pricing strategies, but those are costly. Yuan (2013) shows that randomized balance transfer offers yield $13 less profit per account (excluding mailing costs) than offers targeted on the basis of borrower characteristics. The authors' conversations with several market participants suggest that the vast majority of offers in a given credit card marketing campaign are based on a deterministic price schedule.

or not in Sharpe's model, except without the noise he assumes. But unlike that model, we also assume there exists additional information that can be used to assess the creditworthiness of the borrower. For example, all lenders know the credit score of the borrower. In addition, the inside lender enjoys additional information derived from its internal data on how the borrower uses his or her account.[33]

Let $r^n_{I2}(i,s)$ and $r^o_{I2}(i,s)$ denote the interest rate the inside lender offers to the borrower of type i for new (n) and old (o) debt, respectively, in period 2 when the signal the outsider lenders have received is s. In our notation, we will sometimes suppress the dependence of these rates on s. Since the inside lender knows the type of the borrower and the signal of the outside lenders (e.g., a credit score), he will condition his offers on all available information. Also, we denote by $r^n_{O2}(s)$ and $r^o_{O2}(s)$ the second-period offers of the outside lenders for new and old debt, respectively. In the first period, there are no insiders and outsiders, and lenders have no information about the quality of consumers. Thus, we denote the first period interest rate by r_1. We search for a perfect Bayesian Nash equilibrium in pure strategies.

4 Analysis

We begin our analysis with the borrower's problem, and then we solve the problem the lenders face. In both cases, we start from period 2, and then we proceed to period 1.

4.1 Borrower's problem

We solve the game backward, starting from the second period.

4.1.1 Period 2 borrower choice

Borrower i in period 2, who borrows from lender j, chooses b_2 to maximize his second-period expected utility

$$U_2 = \begin{cases} u(b_2) - \left(1 + r^n_{j2}(i)\right) b_2 p_i - \left(1 + r^o_{j2}(i)\right) b_1 (1 + r_1) p_i, & \text{if } d = b_1(1+r_1) \\ u(b_2) - \left(1 + r^n_{j2}(i)\right) b_2 p_i, & \text{if } d = 0. \end{cases} \tag{4.1}$$

[33] For example, in addition to obtaining credit scores generated from credit bureau data, many credit card lenders use behavioral scores, generated from data describing how their customers are using their accounts with the lender.

The first-order condition with respect to b_2, which does not depend on the level of debt d, is

$$u'(b_2) - \left(1 + r^n_{j2}(i)\right) p_i = 0. \tag{4.2}$$

Let $b^*_2(r^n_{j2}(i), p_i)$ denote the amount of new borrowing that satisfies this first-order condition. Note that this value does not depend on the size of the first period loan, but it does depend (negatively) on the interest rate and on the probability of repayment. In a number of cases in what follows, we suppress these dependencies in our notation to reduce the length of the expressions.

If the borrower's debt d at the end of the first period is zero, then the second period expected maximized utility, before the borrower learns his type, is

$$\begin{aligned} U^*_2(d=0) &= \mu\left[u\left(b^*_2(p_H)\right) - \left(1 + R^n_{j2}(H)\right) b^*_2(p_H) p_H\right] + \\ &\quad (1-\mu)\left[u\left(b^*_2(p_L)\right) - \left(1 + R^n_{j2}(L)\right) b^*_2(p_L) p_L\right], \end{aligned}$$

while if the borrower's debt is not zero, the expected utility becomes

$$\begin{aligned} U^*_2(d \neq 0) &= \mu\left[u\left(b^*_2(p_H)\right) - \left(1 + R^n_{j2}(H)\right) b^*_2(p_H) p_H\right] + \\ &\quad (1-\mu)\left[u\left(b^*_2(p_L)\right) - \left(1 + R^n_{j2}(L)\right) b^*_2(p_L) p_L\right] - \\ &\quad b_1(1+r_1)\left[\mu p_H\left(1 + R^o_{j2}(H)\right) + (1-\mu) p_L\left(1 + R^o_{j2}(L)\right)\right], \end{aligned}$$

where $R^n_{j2}(i)$ and $R^o_{j2}(i)$ denote the best interest rate offers borrower i *expects* to receive at the beginning of period 2.[34] In a rational expectations equilibrium, we should have $r^n_{j2}(i) = R^n_{j2}(i)$ and $r^o_{j2}(i) = R^o_{j2}(i)$.

Therefore, the second-period expected maximized utility of the representative borrower, prior to the realization of the first period income draw, is

$$EU^*_2 = \bar{p} U^*_2(d=0) + (1-\bar{p}) U^*_2(d \neq 0).$$

[34] Note here that R can be viewed as an expected interest rate. As it will become clear when we analyze the second period equilibrium, the interest rate a borrower will receive from the inside lender may also depend on the signal received by the outside lender.

4.1.2 Period 1 borrower choice

In the first period, the borrower chooses b_1 to maximize

$$u(b_1) - (1 + r_1)\, b_1 \bar{p} + \delta E U_2^*, \tag{4.3}$$

where $\delta \in (0, 1)$ is the common discount factor. The first-order condition with respect to b_1, assuming that second-period rates for new borrowing do not depend on b_1, is[35]

$$\begin{aligned} & u'(b_1) - (1 + r_1)\bar{p} - \delta(1 - \bar{p})(1 + r_1)\left[\left(\mu p_H\left(1 + R_{j2}^o(H)\right) + (1 - \mu) p_L\left(1 + R_{j2}^o(L)\right)\right)\right] \\ - \; & \delta(1 - \overline{p}) b_1 (1 + r_1)\left(\mu p_H \frac{\partial R_{j2}^o(H)}{\partial b_1} + (1 - \mu) p_L \frac{\partial R_{j2}^o(L)}{\partial b_1}\right) = 0. \end{aligned} \tag{4.4}$$

Let $b_1^*(r_1)$ denote the size of the first-period loan (to simplify notation we will sometimes suppress the dependence of b_1 on r_1). Note that the first-period loan does not depend on the type of the borrower, since the borrower does not know his type when he chooses b_1. At the beginning of the second period, the borrower learns his type. But his choice of b_2 occurs after the lenders have made their interest rate offers. These assumptions allow us to sidestep any signaling issues that would arise if the size of the loan could convey information about the type of the borrower.[36]

4.2 Lenders' problem

We solve the game backward, starting from the second period.

4.2.1 Period 2

The expected profit of the inside lender when it lends to borrower i at rates $r_{I2}^o(i)$ and $r_{I2}^n(i)$ is

$$\begin{aligned} \pi_{I2}(r_{I2}^o(i), r_{I2}^n(i); d \neq 0) \;\; = \;\; & \left((r_{I2}^n(i) - \overline{r})p_i - (1 - p_i)(1 + \overline{r})\right) b_2^* \\ + \;\; & \left((r_{I2}^o(i) - \overline{r})p_i - (1 - p_i)(1 + \overline{r})\right) b_1^*(1 + r_1), \end{aligned} \tag{4.5}$$

[35] This assumption is confirmed in equilibrium. Essentially, the lender cannot become better off by making second-period rates for new borrowing a function of existing debt. This is no longer true when we allow for endogenous default (see Section 6). However, second period rates for existing debt can depend on the level of debt.

[36] As noted previously, the fact that a borrower carries debt into the second period (but not the level of that debt) conveys probabilistic information about his type.

when the borrower has debt, $d = b_1(1 + r_1)$, from period 1 and

$$\pi_{I2}(r^n_{I2}(i); d = 0) = ((r^n_{I2}(i) - \overline{r})p_i - (1 - p_i)(1 + \overline{r}))\, b^*_2, \tag{4.6}$$

when the first-period debt is zero.

Given that lenders compete on a borrower-by-borrower basis, in equilibrium, it must be the case that the borrower's expected utility is maximized. Therefore, each lender j chooses its interest rates between old and new debt to maximize the borrower's expected utility subject to a constant profit constraint. The solution is presented in the Lemma below.

Lemma 1 *If the borrower carries debt into the second period, the second-period interest rates for old (o) and new (n) debt that maximize borrower expected utility subject to lender j making constant (equal to k) expected profit are:*

$$r^n_{j2}(i) = \frac{1 + \overline{r} - p_i}{p_i} \text{ and } r^o_{j2}(i) = \frac{1 + \overline{r} - p_i}{p_i} + \frac{k}{p_i b^*_1(1 + r_1)}.$$

At the beginning of period 2, and given that the amount borrowed – and hence the borrower's second-period utility – depends on the interest rate, the rate on new debt affects the size of the surplus in the lender-borrower relationship, while the rate on existing debt simply transfers surplus between the two parties. The lender offers a low, break-even, rate for new debt and a higher rate, if $k > 0$, for old debt. This trade-off provides the borrower with the highest possible utility (given a downward sloping demand for credit), while the lender operates on an iso-profit curve. The rate on new borrowing is efficient and incorporates optimally the cost of funds $\overline{r}$ and the probability of repayment p_i.

Using Lemma 1, when expected profits are zero, $k = 0$, the rates (break-even rates) for old and new debt coincide and are given by

$$\hat{r}_{I2}(i) \equiv \frac{1 + \overline{r} - p_i}{p_i},\ i = L, H. \tag{4.7}$$

If the borrower does not carry debt from the first period, $d = 0$, then the break-even interest rate, which applies to new debt only, is also given by (4.7). Because $p_H \geq p_L$, we have $\hat{r}_{I2}(H) \leq \hat{r}_{I2}(L)$.

Outside creditors update their prior beliefs about the probability of a high-quality borrower after they observe the signal s (and the level of debt d). This is given by Bayes' rule as follows, with the understanding that μ is a function of whether $d = 0$ or $d \neq 0$, as given by (3.1), but in the expressions below such a dependence is suppressed

$$\Pr(H|h) = \frac{(1+\phi)\mu}{(1+2\mu\phi-\phi)} \text{ and } \Pr(L|\ell) = \frac{(1+\phi)(1-\mu)}{(1-2\mu\phi+\phi)}. \tag{4.8}$$

The expected repayment probability, attached by an outsider who has received signal $s = h$, is

$$p(h) \equiv p_H \Pr(H|h) + p_L \Pr(L|h) = p_H \frac{(1+\phi)\mu}{(1+2\mu\phi-\phi)} + p_L \frac{(1-\phi)(1-\mu)}{(1+2\mu\phi-\phi)}, \tag{4.9}$$

while if the signal the outsider has received is ℓ the expected probability is

$$p(\ell) \equiv p_H \Pr(H|\ell) + p_L \Pr(L|\ell) = p_H \frac{(1-\phi)\mu}{(1-2\mu\phi+\phi)} + p_L \frac{(1+\phi)(1-\mu)}{(1-2\mu\phi+\phi)}. \tag{4.10}$$

.

The rate offer an outsider makes to borrower i depends on the signal $s \in \{h, \ell\}$ it receives. The expected profits of an outside lender assume similar expressions as (4.5) and (4.6) after replacing p_i with $p(s)$ and changing the notation for the interest rate offers. If a borrower with debt switches to an outsider, the outsider repays the borrower's debt, $b_1(1 + r_1)$, to the insider.[37]

Following the same logic, we developed for the insider (see Lemma 1), the break-even rates that maximize the surplus in the lender-borrower relationship, given the outside lender's available information, are given by

$$\hat{r}_{O2}(s) \equiv \frac{1+\overline{r}-p(s)}{p(s)},\ s = \ell, h. \tag{4.11}$$

It follows easily that (where now we explicitly account for the effect of debt on μ)

$$p_H > p(h, d = 0) > p(h, d \neq 0) > \overline{p} > p(\ell, d = 0) > p(\ell, d \neq 0) > p_L \tag{4.12}$$

and therefore

$$\hat{r}_{I2}(H) < \hat{r}_{O2}(h, d = 0) < \hat{r}_{O2}(h, d \neq 0) < \frac{1+\overline{r}-\overline{p}}{\overline{p}} < \hat{r}_{O2}(\ell, d = 0) < \hat{r}_{O2}(\ell, d \neq 0) < \hat{r}_{I2}(L). \tag{4.13}$$

[37]In the real world, many credit card solicitations include a balance transfer option.

Remember that we assume that the insider observes the outsiders' signal. The following proposition characterizes the second-period equilibrium.[38]

Proposition 1 *The unique equilibrium in the second period is described as follows:*

1. *If the borrower does not carry debt into the second period:*

   a) *The outsiders offer $r_{O2}(s, d = 0) = \hat{r}_{I2}(L)$ for any signal s they receive. The insider also offers $r_{I2}(L, s, d = 0) = \hat{r}_{I2}(L)$ if $i = L$ regardless of the outsider's signal.*

   b) *The insider offers $r_{I2}(H, h, d = 0) = \hat{r}_{O2}(h, d = 0)$ if $i = H$ and $s = h$ and*

   c) *it offers $r_{I2}(H, \ell, d = 0) = \hat{r}_{O2}(\ell, d = 0)$ if $i = H$ and $s = \ell$.*

   d) *The borrower always obtains credit from the insider.*

2. *If the borrower carries debt into the second period:*

   a) *The outsiders offer $r^o_{O2}(s, d \neq 0) = r^n_{O2}(s, d \neq 0) = \hat{r}_{I2}(L)$ for old and new debt and any signal s they receive. The insider also offers $r^o_{I2}(L, s, d \neq 0) = r^n_{I2}(L, s, d \neq 0) = \hat{r}_{I2}(L)$ for old and new debt if $i = L$ regardless of the outsider's signal.*

   b) *The insider offers $r^n_{I2}(H, s, d \neq 0) = \hat{r}_{I2}(H)$ for new debt if $i = H$ and for any s and*

   c) *for old debt it offers*

$$r^o_{I2}(H, s, d \neq 0) = \hat{r}_{O2}(s, d \neq 0) + \frac{A(s)}{p(s, d \neq 0)p_H b^*_1(r_1)(1 + r_1)}, \ s = h, \ell \qquad (4.14)$$

[38]For the equilibrium described in Proposition 1 to be valid, we implicitly have assumed that the monopoly rate for the inside lender to the high type is higher than the break-even rate for the outsiders when the signal is ℓ. Otherwise, and given that demand for credit is downward sloping, the inside lender can earn higher profits by charging rates to the high types that are lower than the ones stated in the proposition. This applies to the case in which the borrower carries no debt from the first period.

where

$$
\begin{aligned}
A(s) \quad \equiv \quad & (u(b_2^*(r_{I2}^n(H,s,d\neq 0))) - u(b_2^*(\hat{r}_{O2}(s,d\neq 0))))p(s,d\neq 0) \\
- \quad & (1+\bar{r})(b_2^*(r_{I2}^n(H,s,d\neq 0))p(s,d\neq 0) - b_2^*(\hat{r}_{O2}(s,d\neq 0))p_H) \geq 0.
\end{aligned}
$$

d) The borrower always obtains credit from the insider.

At the end of the first period the uncertainty about the borrower's repayment probability is either resolved completely (for the inside lender) or partially (for the outside lenders). Outside lenders have no hope in attracting the high-type borrower, due to the inside lender's superior information. As a result, the outside lender offers an interest rate it knows can only attract the low type. In that case, the inside lender's informational advantage forces the outsider to assume the worst, and the equilibrium interest rate offered to the low type is pushed down to the break-even point for the low type.

Note that the second-period rate offers to the low-type borrower do not depend on whether he carries debt or not. In other words, the rates offered on old and new debt to this type are the same: the break-even rate associated with lending to this type.

In contrast, the inside lender enjoys rents when it lends to the high type. When the high type's debt is zero, the second-period equilibrium interest rate depends on the signal received by the outsider and the informational advantage of the insider.

When the high-type borrower's debt is positive, inside lenders have two instruments they employ to maximize their rents: the rate for old and the rate for new debt. The equilibrium rates the low type receives for new and old debt are equal and moreover are the same as in the no debt case. Head-to-head competition between the insider and the outsiders is what determines the equilibrium interest rate for the low type (as it is also true in the no debt case). For the high type, however, the equilibrium rate for new borrowing is lower and the rate for existing debt is higher, relative to the rate charged to the same borrower type when the level of debt is zero. The inside lender, given its informational advantage, has room to maneuver in this case. By offering a lower rate on new debt, the inside lender maximizes the borrower's expected utility. A higher rate on existing debt, as given by (4.14), allows the lender to extract the borrower's surplus, subject to the constraint that the borrower's utility must be at least as high as he could obtain by accepting an offer from one of the outside lenders, for existing debt and new borrowing, at their break-even interest rates $\hat{r}_{O2}(s,d\neq 0)$.[39]

[39] In the proof of Proposition 1, we show that the second term in the RHS of (4.14) is positive.

4.2.2 Period 1

Equation (4.14) reveals an instance in which second-period interest rates on old debt depend negatively on the accumulated principal and interest carried over from the first period, $b_1^*(1 + r_1)$. We need to rule out the possibility that this induces a positive relationship between interest rates and the amount borrowed in the first period. We do so in the following Lemma.

Lemma 2 *A higher first-period interest rate r_1 reduces first-period borrowing b_1 and vice versa, that is $db_1^*/dr_1 < 0$.*

Lenders are ex-ante homogeneous and compete in r_1 in the first period. The proposition below states the result.

Proposition 2 *There exists a unique first-period equilibrium interest rate r_1^*, which is increasing in the informativeness of the outsiders' signal. When the inside lender has no informational advantage, $\phi = 1$, the equilibrium rate is $r_1^* = (1 + \overline{r} - \overline{p})/\overline{p}$.*

Second-period expected profit for the inside lender is strictly positive. Our assumptions of Bertrand competition and the symmetry of lenders' information in the first period imply that the second-period profits accruing to the inside lenders will be entirely competed away. In expectation, lenders lose money in the first period and earn zero overall expected profit over both periods. In other words, lenders incur up front losses to acquire customer relationships that will, on average, generate profits over later stages of the relationships. Those profits are derived from the additional information about the borrower learned over the course of the relationship.

When the borrower carries debt, the rates for the low type do not change relative to those without debt. The inside lender offers two rates – one for existing and one for new debt – but they coincide. The rate offered to the high type for new debt is the efficient rate (which is lower than the offer when the debt is zero, $\hat{r}_{O2}(s, d = 0)$), while the rate for existing debt $r_{I2}^o(H, s, d \neq 0)$, see (4.14), is higher than $\hat{r}_{O2}(s, d \neq 0)$. Moreover, $r_{I2}^o(H, h, d \neq 0)$ is decreasing in ϕ, but for $r_{I2}^o(H, \ell, d \neq 0)$, there are opposing effects.[40] Figure 3, in Appendix B, depicts the first-period and second-period equilibrium

[40] Both terms of $r_{I2}^o(H, h)$ decrease with ϕ. The first term is clear why. The numerator of the second term is decreasing because it reflects the difference in borrower utility between the inside and the outside lender for new debt only. The inside lender's rate is fixed, but the outside lender's rate decreases with ϕ. The denominator is increasing because $p(s)$ and $b_1^*(r_1)(1 + r_1)$ are increasing. The last term is increasing since r_1 is increasing in ϕ and, as we show in footnote 52, we are on the increasing part of the total revenue curve. On the other hand, the effect of ϕ on $r_{I2}^o(H, \ell)$ is ambiguous since $p(\ell)$ is decreasing in ϕ.

rates derived from a constant relative risk aversion (CRRA) utility function and specific values for all the model parameters.

4.3 First best

A social planner would set the marginal utility of consumption equal to the marginal cost in each period. The marginal cost of \$1 consumption in period 2 is $1+\bar{r}$, while in period 1, is $(1+\bar{r})+\delta(1-\bar{p})(1+\bar{r})^2$. The period 1 marginal cost is calculated as follows. The \$1 borrowed in period 1 has cost $1+\bar{r}$ for that period and, if the borrower does not pay back, which happens with probability $1-\bar{p}$, the cost in period 2 is $(1+\bar{r})^2$, appropriately discounted using δ. Therefore, the first-best (fb) consumption levels in periods 1 and 2 must satisfy

$$u'(b_1^{fb}) = (1+\bar{r}) + \delta(1-\bar{p})(1+\bar{r})^2 \text{ and } u'(b_2^{fb}) = 1+\bar{r}. \tag{4.15}$$

Note that the social planner prefers perfect smoothing of consumption in period 2 across types and some smoothing across time depending on the probability of repayment and the discount rate. It is straightforward to verify that the rates that implement the first-best borrowing are: $\hat{r}_{I2}(L)$ and $\hat{r}_{I2}(H)$ for second-period borrowing as a function of the repayment probabilities, $(1+\bar{r}-\bar{p})/\bar{p}$ for first period borrowing and $\bar{r}$ for first-period borrowing that is rolled over in period 2.

Relative to the first best, the equilibrium in the market, described in Propositions 1 and 2, is inefficient. The inefficiency arises from two sources: i) the inside lender's market power due to the informational advantage it enjoys over the outside lenders and ii) the higher interest rate charged on debt that is rolled over in period 2 (expected lender profits in the second period cannot be negative), even when market power is zero.

It is important to note that for borrowers who carry debt into the second period, the unregulated equilibrium interest rates are efficient. The social planner wants to insure the borrower against the consumption variability due to the type uncertainty. This, however, is achieved only when the borrower carries debt into period 2, in which case the rate charged on predetermined balances is a transfer, while the rate on new borrowing is actuarially fair. This is reminiscent of the efficiency of monopoly in the presence of two-part-tariffs. Since the inside lender can use the rate on existing debt to earn profits it offers the efficient rates for new borrowing, which smooth consumption across types and induce the efficient amount of period 2 borrowing.

For low risk, type H, consumers who do not carry debt into the second period, the unregulated equilibrium is not efficient. Second-period rates offered induce a variability in consumption across the two types and underborrowing. Given the positive second period profits, first period-rates are below the break-even period 1 rate so the rates on existing debt are higher than the break-even level, so that profits across the two periods are zero. A social planner (who does not care whether lenders break even or not) would prefer a rate on existing debt in period 2 equal to $\bar{r}$, since the probability of repayment has already been accounted for once in period 1 when the borrower decides his b_1. Any rate on existing debt higher that $\bar{r}$ distorts first-period borrowing. This is most evident by inspecting the borrower's first-order condition as given by (A.2) and setting $\phi = 1$. This yields

$$u'\left(b_1^*\right) = (1+\bar{r}) + \delta \frac{(1-\bar{p})}{\bar{p}}(1+\bar{r})^2. \tag{4.16}$$

Comparing (4.16) with (4.15), we can observe that $b_1^{fb} > b_1^*$; a social planner would want the borrower to borrow more in period 1, when there are no information asymmetries in period 2. Therefore, when ϕ is high enough there is underborrowing in period 1 relative to the first best. However, when ϕ is low, r_1^* is also low and first-period borrowing may exceed the first best. We summarize in the proposition below.

Proposition 3 *The borrowing comparison between the first best and equilibrium across the two periods is as follows:*

- *Period 1: First-best borrowing, b_1^{fb}, is below the equilibrium, b_1^*, for low values of ϕ, while for high values of ϕ it can be higher.*
- *Period 2: First-best borrowing, b_2^{fb}, coincides with the equilibrium for the L types, $b_2^*(L)$, and for the H types who carry debt $b_2^*(H, d \neq 0)$. The H types who carry no debt borrow less than the first-best, $b_2^*(H, d = 0) < b_2^{fb}$.*

In Appendix B, we offer a numerical example, and in Figure 4, we depict the first-best and equilibrium borrowings across the two periods that are derived from that example. A question that arises next is whether the regulation mitigates or exacerbates the inefficiencies. Figure 7 depicts the first-best welfare along with the welfare under regulation and no regulation.

5 Regulation

We focus primarily on one aspect of the CARD Act: limitations on the ability of lenders to raise interest rates on existing balances in the absence of a significant delinquency (60 or more days under the CARD Act) on the part of the borrower. In the model, we assume that a regulation takes in effect before period 1 and prohibits the inside lender from raising the rate on existing debt in the second period relative to the first-period rate. The inside lender is free to choose the rate for new debt. Outside lenders face no constraints. We analyze the impact of regulation on both types of borrowers and on the two-period equilibrium interest rates and compare these with the model without this restriction. The proposition below summarizes the two-period regulated equilibrium.[41]

Proposition 4 *The two-period regulated equilibrium is described as follows.*

- ***First period.*** *There exists a unique equilibrium first-period rate, r_1^{reg}, which is higher than the first-period rate, r_1^*, in the unregulated equilibrium (but still less than $(1 + \bar{r} - \bar{p})/\bar{p}$).*
- ***Second period.*** *For the borrowers who carry no debt in the second period, the equilibrium is the same as in Proposition 1.*
- *For the borrowers who carry debt into the second period:*
  - *Low types: Pay the same rates as in the unregulated equilibrium for existing debt and new borrowing but switch lenders (borrow from an outside lender).*
  - *High types: Pay lower rates for existing debt and higher rates for new borrowing and do not switch lenders.*

The effect of the regulation is on the borrowers who carry debt from the first period. Absent regulation, outside lenders make equilibrium rate offers (regardless of the signal they receive) so that lenders break even by lending to the low type. Regulation induces switching, in equilibrium, on the part of the low types. This occurs because the inside lender is constrained by the regulation not to change the rate on

[41] We assume that the first-period interest rate is higher than $\hat{r}_{I2}(H)$. This is definitely true for sufficiently high ϕ. In the numerical example in Appendix B, it is true for all ϕ. We have not been able to produce an example in which the first-period rate falls below $\hat{r}_{I2}(H)$ but also we have been unable to prove the opposite in general. Under this assumption, H types never switch lenders in the unregulated equilibrium.

existing debt. For its existing borrowers revealed to be the low type, this rate generates expected losses in the second period. To break even, the inside lenders offer a higher rate on new debt than the efficient rate. This puts the inside lender at a disadvantage relative to the outside lenders, who can now beat the inside lender's offer.

In the unregulated equilibrium, the inside lender earns positive profits from the high-type borrowers. Thus, relative to when it deals with the low type, the incumbent enjoys some additional room to maneuver in the regulated equilibrium. As a result, it can retain the high-type borrower by raising the interest rate on new borrowing to the point that the borrower is just indifferent between switching lenders (while the rate on existing debt is kept low by the regulation).[42] The effect of the regulation is to lower the inside lender's rents from the H types.

Because regulation lowers profits for the inside lender in the second period, the first-period interest rate is higher than without regulation. Lender total discounted profits are zero with or without the regulation, while borrowers become worse off in period 1 since they pay a higher first-period rate.[43]

Figure 1 depicts the mechanism behind the main result in Proposition 4. The unregulated second-period equilibrium is determined by the tangency between the borrower's indifference curve and the lender's iso-profit curve. The borrower's indifference curve is pinned down by the competitive offers of the outside lenders. The first-period equilibrium interest rate, r_1^{reg}, is lower than the unregulated second-period rates for existing debt.[44] Hence, the effect of regulation is to force the inside lender to lower its rate for existing (old) debt. To retain some of its lost profits, it must increase the rate on new debt. Nevertheless, this trade-off generates some inefficiency (the regulated equilibrium is not at a tangency), which results in lower profits for the inside lender in the second period.

[42]This is reminiscent of the waterbed effect in the theory of regulation. For example, Genakos and Valletti (2011) show that although regulation in the mobile telephony reduced termination rates, it also led to an increase in mobile retail prices.

[43]This result is reminiscent of Gehrig and Stenbacka (2007), where information sharing in later periods between banks competing in repeated lending markets leads to reduction in expected future rents which, in turn, relaxes ex-ante competition.

[44]As we show in the proof of Proposition 4, this is always true for the low type and the high type when the signal the outsiders have received is low, while for the high type when the signal is high depends on the insider's informational advantage.

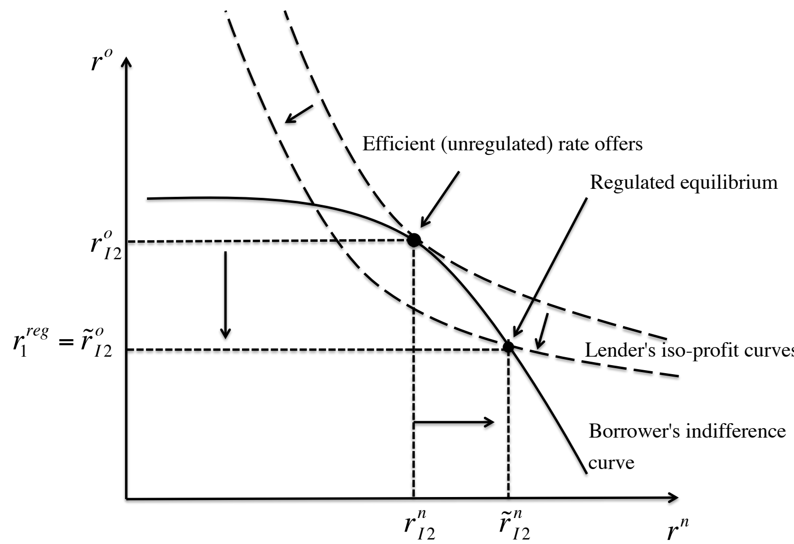

Figure 1: The effect of regulation on second-period interest rates for the high-type borrower with debt. The borrower's indifference curve is unchanged and determined by the offers of the outside lenders. The inside lender moves to a lower iso-profit curve.

The next corollary summarizes the welfare implications of the regulation (for a proof, see the proof of Proposition 4).

Corollary 1 *The effect of the regulation on consumer (borrower) surplus and total welfare is as follows (for those borrowers who carry debt):*

- ***Consumer Surplus:*** *For both types of borrowers, consumer surplus is the same in the second period as in the unregulated equilibrium. Given the higher first period interest rate, consumer surplus is lower in the first period than in the unregulated equilibrium.*

- ***Total surplus:*** *Given that expected profits do not change and consumer surplus decreases, regulation increases the deadweight loss (relative to the unregulated equilibrium).*

In the numerical example of Section B, we plot in Figure 7 the total surplus in the first best, regulated and unregulated cases for all values of ϕ.

6 Extension: Endogenous default / moral hazard

The purpose of this section is to demonstrate that our main results do not change qualitatively when we allow the borrower to decide whether to default or not, when his income is high. In the presence of potential moral hazard, lenders impose nonbinding credit limits and the equilibrium interest rate in the second period on new borrowing can be lower than the rate on existing debt. Given the complexity of the problem, we focus on the second period, taking first-period interest rate and amount borrowed as given.

6.1 Borrower's problem

Borrower i in period 2, who borrows from lender j, chooses b_2 to maximize his second-period expected utility. After he chooses b_2 the cost of bankruptcy c is a random draw from a distribution $F(c)$ on $[0,\Gamma]$ with mean $\bar{c}$. We assume that $F(\cdot)$ satisfies the monotone hazard rate (MHR) property.[45] The borrower, after observing the cost of default and given that his income is not zero, decides whether to default or not. The second-period utility of the borrower is given as follows

$$U_2 = \begin{cases} u\left(b_2\right) - \left(1 + r^{n/d}\right) b_2 - \left(1 + r^o\right) b_1 \left(1 + r_1\right), & \text{if } d = b_1(1+r_1) \text{ and no default} \\ u\left(b_2\right) - c, & \text{if } d = b_1(1+r_1) \text{ and default} \\ u\left(b_2\right) - \left(1 + r^{n/nd}\right) b_2, & \text{if } d = 0 \text{ and no default} \\ u\left(b_2\right) - c, & \text{if } d = 0 \text{ and default,} \end{cases}$$

where n/d in the superscript of r stands for new (borrowing)/debt, while n/nd stands for new (borrowing)/ no debt. We assume that $u(b_2)$ exhibits a (weakly) decreasing Arrow-Pratt degree of absolute risk aversion and has a degree of relative risk aversion that is less than one.[46]

If the borrower carries debt from the first period, and assuming that his income realization is high, he chooses to default if and only if the cost of default is low, that is, $c \leq (1 + r^{n/d})b_2 + (1 + r^o)b_1(1 + r_1)$, where r^o is the interest rate for old debt. Let $C^d \equiv (1+r^{n/d})b_2 + (1+r^o)b_1(1+r_1)$. So, the probability of default is $F\left(C^d\right)$. If the borrower does not carry debt from the first period, he chooses to default if and only if $c \leq (1 + r^{n/nd})b_2$. Let $C^{nd} \equiv (1 + r^{n/nd})b_2$. The probability of default is $F(C^{nd})$.

[45] The MHR property states that $(1 - F(\theta))/f(\theta)$ is strictly decreasing in θ and is satisfied by many commonly used distributions such as the normal and the uniform. See Bagnoli and Bergstrom (2005).

[46] Both of these assumptions are satisfied, for example, by $u = b^c/c$, with $c < 1$ and $u = \log(1 + b)$.

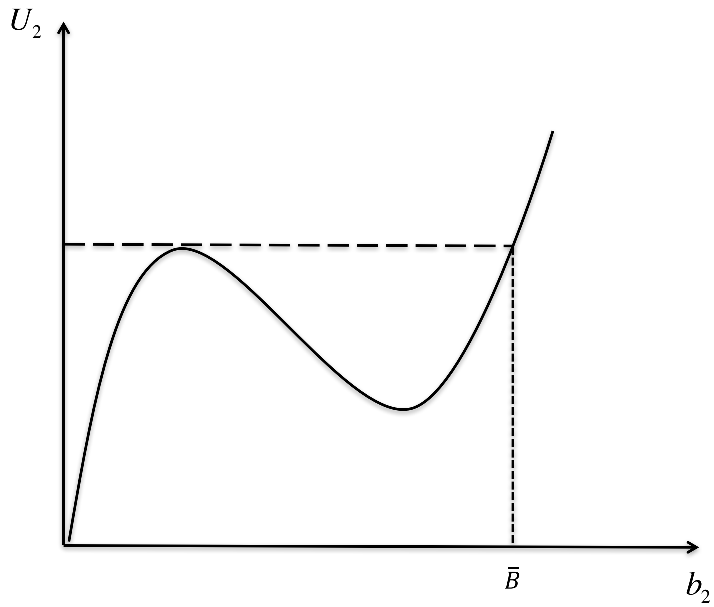

Figure 2: Second-period expected utility as a function of the amount borrowed. If the credit limit is set below $\overline{B}$, then it is not binding. The borrower in this case will opt for the interior maximum.

The borrower's expected utility if he carries no debt from the first period, where now we also account for the high income realization probability p_i, is[47]

$$U_2^{nd} = u(b_2) - p_i \int_0^{C^{nd}} cf(c)dc - p_i C^{nd} \int_{C^{nd}}^{\infty} f(c)dc - (1-p_i)\overline{c}, \tag{6.1}$$

while if the borrower carries debt from the first period his expected utility is

$$U_2^{d} = u(b_2) - p_i \int_0^{C^{d}} cf(c)dc - p_i C^{d} \int_{C^{d}}^{\infty} f(c)dc - (1-p_i)\overline{c}. \tag{6.2}$$

Finally, we assume that the upper bound of the support of the cost of default distribution Γ is below a threshold, that is defined in the proof of Proposition 5.[48]

The following proposition summarizes the results from the borrower's utility maximization problem.

[47]With probability $1 - p_i$ the borrower's income is low in which case he defaults with certainty and incurs the expected (i.e., the average) cost of bankruptcy $\overline{c}$.

[48]This assumption, which states that even the highest cost of bankruptcy is not too high, ensures that the lender does not find it profitable to induce the borrower to borrow up to the credit limit on the increasing portion of the borrower's expected utility to the right of the interior maximum (because default at that point is certain); see Figure 2. Alternatively, we could have assumed that the probability of default is always less than one but increases rapidly after borrowing exceeds a certain threshold and so a lender's profit function is also decreasing.

Proposition 5 *The borrower's expected utility exhibits a sideways "S" shape, see Figure 2. The lender will set a nonbinding credit limit to the left of $\overline{B}$ to prevent the borrower from borrowing a lot and then defaulting with certainty. There exists a unique interior level of borrowing that maximizes expected utility, and the demand for credit is downward sloping with respect to the interest rate.*

6.2 Lenders' problem

We now turn to the lenders' profit maximization problem. We begin by assuming that the borrower carries debt into the second period. The expected profit function of the lender in period 2 is

$$\begin{aligned}\pi_2^d &= p_i(1-F(C^d))\left((1+r^{n/d})b_2^d+(1+r^o)(1+r_1)b_1\right)-(1+\overline{r})(b_2^d+(1+r_1)b_1)\\ &= (p_i(1-F(C^d))(1+r^{n/d})-(1+\overline{r}))b_2+(p_i(1-F(C^d))(1+r^o)-(1+\overline{r}))(b_1(1+r_1)).\end{aligned}$$

The following Lemma, which is the extension of Lemma 1 to the endogenous default case, summarizes the results from the lender's second-period problem.

Lemma 3 *If the borrower carries debt into the second period, the second-period interest rates for old (o) and new (n) debt that maximize the borrower's expected utility subject to lender j making zero expected profit are:*

$$\tilde{r}^{n/d}=\frac{1+\overline{r}-p_i(1-F(C^d))}{p_i(1-F(C^d))}+\frac{f(C^d)C^d}{1-F(C^d)} \textit{ and } \tilde{r}^o<\frac{1+\overline{r}-p_i(1-F(C^d))}{p_i(1-F(C^d))}. \tag{6.3}$$

The rate on new borrowing is higher than the rate on existing debt.

The rate on new borrowing is the efficient rate and balances optimally the following effects (assuming the rate decreases): A higher b_2 increases the cost according to the cost of funds $\overline{r}$, it increases the profit to the lender in case of repayment (which happens with probability $p_i(1-F(\cdot))$), and increases the probability of default according to the hazard rate $f(\cdot)/(1-F(\cdot))$. From (6.3), it is clear that the rate on new debt must be higher than the rate on existing debt. The difference with the efficient rate in the case of exogenous default, see Lemma (1), is that when default is endogenous the efficient rate increases by an amount equal to the hazard rate to optimally induce the borrower to borrow less.

In sum, when we allow for moral hazard and endogenous default, we can obtain, in equilibrium, a reversal in the ranking of the interest rates between old and new debt, relative to when default is exogenous.[49] In addition, a nonbinding credit limit arises in equilibrium.[50] However, the main results and insights we derived with exogenous default will hold here qualitatively, at least for the second period, (i.e., holding r_1 and b_1 fixed). All one has to do is to redefine all the key break-even interest rates from the main analysis, see (4.13), by adding to them the term that accounts for the hazard rate, $f(\cdot)/(1 - F(\cdot))$, and define the probability of repayment as $p_i(1 - F(\cdot))$ instead of p_i. The regulation then will force the inside lender to increase the rate on new borrowing, as it would in the case of exogenous default. In other words, the intuition from Figure 1, and the switching behavior of borrowers in particular, can be readily extended to the case of endogenous default.

7 Conclusion

The main mechanism and intuition of our model can be described as follows. Lenders, in period 2, have two instruments at their disposal to attract new borrowers and to retain existing ones: the rate on existing debt (for the borrowers who carry debt) and the rate on new borrowing. The CARD Act regulation, which aims at protecting borrowers from a hold-up problem, restricts the inside lenders from raising the interest rate on existing debt on the basis of information learned during the lending relationship (i.e., between period 1 and 2 in our model). Since first-period competition among lenders to attract first-time borrowers is intense, resulting in low rate offers, second-period rate offers to own borrowers are likely to increase. However, outside lenders are unaffected by the regulation, and they are free to offer any rates they wish for new borrowing and existing debt to induce borrowers to switch lenders. As a result, the utility of any borrower in the second period is pinned down by the competitive outside offers and unaffected by the regulation. Since inside lenders must match the utility offered by their outside competitors, but are unable to raise the rate on existing debt, they raise the rate on new borrowing (above the levels justified by the new information about the creditworthiness of the borrowers). Hence, second-period profits for all lenders decrease, although the present value of expected profits is zero.

High-risk borrowers (low types) switch lenders in the regulated equilibrium and pay the same rates as in the unregulated equilibrium. Low-risk borrowers (high types) do not switch lenders, pay lower rates on

[49] Lemma 3 presents the break-even rates. But the inside lender will earn in equilibrium strictly positive rents from the high type borrower and therefore the rate on existing (old) debt will rise. Hence, we highlight the possibility that with moral hazard, and unlike the case where default was exogenous, we can observe an equilibrium in which rates on new borrowing are higher than rates on existing debt.

[50] This arises for reasons similar to those in Stiglitz and Weiss (1981). The difference is that we study revolving credit, where the credit limit typically does not bind, while Stiglitz and Weiss examine a standard one-period debt contract.

debt carried over from period 1 and higher rates on new borrowing relative to the unregulated equilibrium, but are as well off in period 2 as in the unregulated equilibrium. The reason, as we discussed previously, is that the outside lenders are unaffected by the regulation and their offers pin down the borrowers' utility. Nevertheless, regulation decreases the rents the inside lenders earn.

First-period rates increase, a result of weaker competition among lenders due to lower second-period profits. The net effect of the regulation is lower borrower utility and a higher deadweight loss.

To summarize, the regulation is indeed successful in forcing lenders to charge lower rates on debt that is carried over into the current period, for the low-risk borrowers. The problem, however, is that lenders respond by raising the rates on new borrowing. Given the informational advantage inside lenders enjoy, they can afford to do so without losing the low-risk borrowers to the outside lenders who are unaffected by the regulation. The high-risk borrowers are unaffected by the regulation because they switch lenders.

The model suggests some promising methods of identification that are unrelated to changes in prices and quantities. In particular, changes in account switching behavior among different segments of consumers are perhaps the clearest indicator that the repricing restrictions of the CARD Act are indeed binding.[51]

A Appendix: Proofs of Lemmas and Propositions

A.1 Proof of Lemma 1

The constrained utility maximization problem can be stated as follows:

$$\max_{\{r^n_{j2}, r^o_{j2}\}} U_2 = u\left(b_2^*\right) - \left(1 + r^n_{j2}\left(i\right)\right) b_2^* p_i - \left(1 + r^o_{j2}\left(i\right)\right)\left(b_1^*\left(1 + r_1\right)\right) p_i$$

subject to:

$$\left((r^n_{j2}(i) - \overline{r})p_i - (1 - p_i)(1 + \overline{r})\right) b_2^* + \left((r^o_{j2}(i) - \overline{r})p_i - (1 - p_i)(1 + \overline{r})\right) b_1^*(1 + r_1) = k.$$

The (absolute) marginal rate of substitution between old and new interest rates, dr^o_{j2}/dr^n_{j2}, using the envelope theorem, holding expected utility constant (indifference curve) is

$$\frac{b_2^*}{b_1(1 + r_1)},$$

[51] Our intuition here was inspired by the results of Calem and Mester (1995) and Calem et al. (2006).

while holding the expected profits constant (iso-profit curve) is

$$\frac{p_i b_2^* + ((r_{j2}^n(i) - \overline{r})p_i - (1-p_i)(1+\overline{r}))\frac{\partial b_2^*}{\partial r_{j2}^n}}{p_i b_1^*(1+r_1)}.$$

It can be easily calculated, using (4.2), that $\partial b_2^*/\partial r_{j2}^n < 0$. The two marginal rates of substitution become equal at $r_{j2}^n(i) = (1+\overline{r}-p_i)/p_i$. This is the one-period break-even interest rate. The unique tangency is optimal because when $r_{j2}^n(i) < (1+\overline{r}-p_i)/p_i$ the iso-profit curve has a steeper slope than the indifference curve and when $r_{j2}^n(i) > (1+\overline{r}-p_i)/p_i$ the slope of the iso-profit is flatter. These imply that any movement away from the tangency, on the iso-profit, will only lower the borrower's expected utility. The optimal rate on old debt, $r_{j2}^o(i)$, is derived from the iso-profit constraint. Therefore, the solution to the previous constrained maximization problem is

$$r_{j2}^n(i) = \frac{1+\overline{r}-p_i}{p_i} \text{ and } r_{j2}^o(i) = \frac{b_1^*(1+r_1)(1+\overline{r}-p_i)+k}{p_i b_1^*(1+r_1)}.$$

A.2 Proof of Proposition 1

We assume that the inside lender moves first and commits to the rate policy. Outsiders observe the insider's policy but not the actual offer. The competitive fringe moves second, as in Sharpe (1990). In what follows, we suppress the dependence of the interest rates on $d = 0$ or $d \neq 0$, but it should be clear from the context which one is the case. Recall that the belief about the fraction of high types in the second period μ depends on whether the borrower carries debt or not. This affects the repayment probabilities $p(s)$ and the interest rates, whenever those depend on the outsiders' signal.

Borrower carries debt. All lenders offer the efficient rate for new debt as given in Lemma 1. The rate on old debt simply transfers surplus. The insider offers $r_{I2}^n(i,s) = \hat{r}_{I2}(i)$ for new debt, $i = L, H$ and $s = \ell, h$. When $i = L$, the insider also offers $r_{I2}^o(L,s) = \hat{r}_{I2}(L)$ for existing debt and for any s, making zero expected profits. The outsiders also offer $r_{O2}^n(s) = r_{O2}^o(s) = \hat{r}_{I2}(L)$ for old and new debt, regardless of the signal they receive. When $i = H$, the insider's offer for existing debt is the one that yields utility to the borrower equal to the utility the borrower would receive had the outsiders offered their break-even rates $\hat{r}_{O2}(s)$, $s = \ell, h$, for new and old debt, that is

$$\begin{aligned} r^o_{I2}(H,s) &= \frac{1+\overline{r}-p(s)}{p(s)} \\ &+ \frac{(u(b^*_2(r^n_{I2}(H,s))) - u(b^*_2(\hat{r}_{O2}(s))))p(s) - (1+\overline{r})(b^*_2(r^n_{I2}(H,s))p(s) - b^*_2(\hat{r}_{O2}(s))p_H)}{p(s)p_H b^*_1(r_1)(1+r_1)}, \end{aligned}$$

where $b^*_2(r^n_{j2})$ is the amount the lender borrows in period 2 (new debt) from lender $j = I, O$.

Since the rate of the insider on new debt is lower than the rate the outsiders offer, the second-period expected utility of the high-type borrower, from new borrowing only, is higher when he borrows from the insider than the outsider

$$u(b^*_2(r^n_{I2}(H,s))) - (1+\overline{r})b^*_2(r^n_{I2}(H,s)) \geq u(b^*_2(\hat{r}_{O2}(s))) - (1+\overline{r})\frac{p_H b^*_2(\hat{r}_{O2}(s))}{p(s)}.$$

The above inequality implies that the second term in the RHS of (4.14) is positive. Hence, $r^o_{I2}(H,s) \geq \hat{r}_{O2}(s)$, $s = \ell, h$. Given the strategy of the insider, the outsider has no incentive to deviate. To steal the high-type borrower from the insider, it must offer rates lower than $\hat{r}_{O2}(s)$. This strategy will attract both types of borrowers, but the outsider's expected profits are negative. If the outsider's rates are higher than $\hat{r}_{O2}(s)$, it makes a loan to the borrower only in the event the borrower is of low quality. But in this case, expected profit is also negative. Given the response of the competitive fringe, the insider's strategy is optimal. Profits will not increase if the rate offered to the low type changes; if it goes down, profits become negative, and if it goes up, the competitive fringe can make a better and profitable offer. Moreover, if the rates offered to the high type decrease, then profits will decrease (the competitive fringe will not match the offers since expected profits for them conditional on the signal are negative), while if the rates increase the competitive fringe will profitably undercut them.

Borrower carries no debt. The insider offers $r_{I2}(L,s) = \hat{r}_{I2}(L)$, when $i = L$ and regardless of the outsiders' signal. The outsiders also offer $r_{O2}(s) = \hat{r}_{I2}(L)$, no matter what signal they receive. When $i = H$, and assuming that the monopoly interest rate for the inside lender when it lends to the high type is higher than $\hat{r}_{O2}(\ell)$, the insider offers $r_{I2}(H,s) = \hat{r}_{O2}(s)$, $s = \ell, h$. Following a similar logic as in the previous case, no player has an incentive to deviate.

A.3 Proof of Lemma 2

We totally differentiate (4.4) with respect to b_1 and r_1, by noting that equilibrium interest rates in period 2 for the low type are independent of b_1 and r_1, while for the high type the rate for old debt is given by (4.14). We need to compute the effect of b_1 on the second-period rate for the old debt the high type will pay. Using (4.14) we obtain

$$\frac{dr^o_{I2}(H,s)}{db_1} = -\frac{(u(b^*_2(r^n_{I2}(H,s))) - u(b^*_2(\hat{r}_{O2}(s))))p(s) - (1+\bar{r})(b^*_2(r^n_{I2}(H,s))p(s) - b^*_2(\hat{r}_{O2}(s))p_H)}{p(s)p_H(b^*_1(r_1))^2(1+r_1)}. \tag{A.1}$$

From the proof of Proposition 1, the above term is negative. Let

$$A(s) \equiv (u(b^*_2(r^n_{I2}(H,s))) - u(b^*_2(\hat{r}_{O2}(s))))p(s) - (1+\bar{r})(b^*_2(r^n_{I2}(H,s))p(s) - b^*_2(\hat{r}_{O2}(s))p_H).$$

We substitute (A.1), (4.14) and the equilibrium second-period rate for existing debt for the L types into the borrower's first-order condition for b_1, (4.4), also noting that the secondperiod rate on old debt for the low type is not a function of b_1. This yields

$$\begin{aligned}
& u'(b_1) - (1+r_1)\bar{p} \\
- \quad & \delta(1-\bar{p})(1+r_1)\Big(\mu p_H\Big\{\frac{1+\phi}{2}\left(1 + \frac{1+\bar{r}-p(h)}{p(h)} + \frac{A(h)}{p(h)p_H b^*_1(r_1)(1+r_1)}\right) + \\
& \frac{1-\phi}{2}\left(1 + \frac{1+\bar{r}-p(\ell)}{p(\ell)} + \frac{A(\ell)}{p(\ell)p_H b^*_1(r_1)(1+r_1)}\right)\Big\} + (1-\mu)p_L\left(1 + \frac{1+\bar{r}-p_L}{p_L}\right)\Big) \\
+ \quad & \delta(1-\bar{p})b_1(1+r_1)\left(\mu p_H\left\{\frac{1+\phi}{2}\frac{A(h)}{p(h)p_H(b^*_1(r_1))^2(1+r_1)} + \frac{1-\phi}{2}\frac{A(\ell)}{p(\ell)p_H(b^*_1(r_1))^2(1+r_1)}\right\}\right) = 0 \Rightarrow \\
& u'(b_1) - (1+r_1)\bar{p} - \delta(1-\bar{p})(1+r_1)(1+\bar{r})\left[\mu\underbrace{p_H\left\{\frac{1+\phi}{2p(h)} + \frac{1-\phi}{2p(\ell)}\right\}}_{X(\phi)} + (1-\mu)\right] = 0.
\end{aligned} \tag{A.2}$$

As r_1 affects the first-order condition negatively and the utility function $u(\cdot)$ is concave, it follows easily that $db^*_1/dr_1 < 0$.

The $X(\phi)$ term in (A.2) captures the inside lender's informational advantage, and it ranges monotonically from 1 when $\phi = 1$ (no informational advantage) to $p_H/\bar{p} > 1$ when $\phi = 0$ (maximum informational advantage). Finally, it is easy to show that the term in the brackets in (A.2) is inverse

U-shaped in μ, suggesting that the highest uncertainty, and hence market power for the inside lenders, is for intermediate μ's.

A.4 Proof of Proposition 2

First, we account for the outsider signal uncertainty. Using the second-period equilibrium interest rates (see Proposition 1), the expected profits of the insider in the second period, if the borrower is of high type, using (4.5) and (4.6), are

$$E\pi_{I2}(H; d \neq 0) = \pi_{I2}(d \neq 0, h)\frac{1+\phi}{2} + \pi_{I2}(d \neq 0, \ell)\frac{1-\phi}{2},$$

where, using the rates from Proposition 1

$$\begin{aligned} \pi_{I2}(d \neq 0, s) = & \frac{(p_H - p(s))(1+\overline{r})b_1^*(r_1)(1+r_1)}{p(s)} \\ + & \frac{(u(b_2^*(r_{I2}^n(H,s))) - u(b_2^*(\hat{r}_{O2}(s))))p(s) - (1+\overline{r})(b_2^*(r_{I2}^n(H,s))p(s) - b_2^*(\hat{r}_{O2}(s))p_H)}{p(s)} \geq 0 \end{aligned} \tag{A.3}$$

and[52]

$$\begin{aligned} E\pi_{I2}(H; d = 0) &= \pi_{I2}(d = 0, h)\frac{1+\phi}{2} + \pi_{I2}(d = 0, \ell)\frac{1-\phi}{2} \\ &= \frac{1+\phi}{2}\left(\left(\hat{r}_{O2}(h) - \overline{r}\right)p_H - (1-p_H)\left(1+\overline{r}\right)\right)b_2^*(\hat{r}_{O2}(h)) \\ &+ \frac{1-\phi}{2}\left(\left(\hat{r}_{O2}(l) - \overline{r}\right)p_H - (1-p_H)\left(1+\overline{r}\right)\right)b_2^*(\hat{r}_{O2}(l)) \\ &= \frac{(1-\mu)(p_H - p_L)(1+\overline{r})(1-\phi^2)b_2^*(\hat{r}_{O2}(h))}{2\left(p_H\mu(1+\phi) + p_L(1-\mu)(1-\phi)\right)} \\ &+ \frac{(1-\mu)(p_H - p_L)(1+\overline{r})(1-\phi^2)b_2^*(\hat{r}_{O2}(\ell))}{2\left(p_H\mu(1-\phi) + p_L(1-\mu)(1+\phi)\right)} \geq 0. \end{aligned} \tag{A.4}$$

When the borrower is of the low type (high risk), profits are $E\pi_{I2}(L) = 0$, regardless of the level of debt.

[52]Note that $\pi_{I2}(d \neq 0, s)$ decreases as r_1 decreases, because the $b_1^*(r_1)(1+r_1)$ term decreases. A standard argument from basic monopoly theory would suggest that the rate that maximizes total expected revenue, $p(1+r)b^*(r)$, is higher than the one where the monopolist breaks even. Now observe that, in our model, when $\phi = 1$ the first-period interest rate, $r_1^* = (1+\overline{r}-\overline{p})/\overline{p}$, is the one where the lender breaks even in period 1, with a repayment probability $p = \overline{p}$. So, r_1^* is below the rate that maximizes total revenue (which does not depend on p_i), and the relevant range for r_1 must be on the upward part of the total revenue curve (assuming it is inverse U-shaped), for any p_i. This is true for all ϕ since as ϕ decreases r_1^* decreases as well.

Second, we account for the uncertainty over the true probability p_i and the level of debt d_1. Therefore, second-period expected profits are given by

$$E\pi_{I2} = \mu \left[p_H E\pi_{I2}(H; d = 0) + (1 - p_H) E\pi_{I2}(H; d \neq 0) \right] \geq 0. \tag{A.5}$$

The representative lender's expected profits at the beginning of period 1 are

$$E\pi = \left((r_1 - \overline{r})\overline{p} - (1 - \overline{p})(1 + \overline{r}) \right) b_1^* + \delta E\pi_{I2}.$$

Because borrowers are ex-ante identical and lenders are homogeneous, competition in the first period will drive expected profits down to zero. The first period equilibrium interest rate r_1^* must satisfy $E\pi = 0$, which yields

$$r_1^* = \frac{1 + \overline{r} - \overline{p}}{\overline{p}} - \frac{\delta E\pi_{I2}}{\overline{p} b_1^*(r_1^*)}. \tag{A.6}$$

Note that: i) $E\pi_{I2}(H; d = 0)$ does not depend on r_1, ii) $E\pi_{I2}(H; d \neq 0)$ is increasing in r_1 and iii) $db_1^*(r_1)/dr_1 < 0$ (see Lemma 2). Therefore, the term $\delta E\pi_{I2}/(\overline{p} b_1^*(r_1))$ is increasing in r_1. So, the right-hand side of (A.6) is decreasing in r_1. This implies that there exists a unique r_1^* (not necessarily positive) that makes $E\pi$ zero.

Next, we examine the effect of ϕ on r_1^*. It is clear that as ϕ increases, the informational advantage of the insider over the outsiders diminishes and second-period expected profits decrease. Also, from (A.2), it follows that b_1 increases as ϕ increases, holding r_1 fixed. This implies that as ϕ increases, the RHS of (A.6) increases as well and hence r_1^* increases.

A.5 Proof of Proposition 4

Second-period expected profits for the inside lender will be positive. Therefore, first-period rate r_1^{reg} must be less that $(1 + \overline{r} - \overline{p})/\overline{p}$; otherwise first-period profits are also positive, which contradicts the fact that overall expected profits must be zero in equilibrium. Hence, the following ranking holds

$$r_1^{reg} < \frac{1 + \overline{r} - \overline{p}}{\overline{p}} < \frac{1 + \overline{r} - p(\ell)}{p(\ell)} < \frac{1 + \overline{r} - p_L}{p_L}. \tag{A.7}$$

We use $\tilde{r}$ to denote the second-period rates under regulation. In what follows, we assume that the borrower carries debt into the second period. Otherwise, the regulation does not affect the equilibrium described in Propositions 1 and 2.

Low type. From (A.7), the break-even rate for the low type, $\hat{r}_{I2}(L) \equiv (1+\overline{r}-p_L)/p_L$, is higher than the first-period rate, r_1^{reg}. The inside lender is constrained by the regulation not to raise the second-period rate for existing debt from r_1^{reg}. Thus, in equilibrium, the insider offers $\tilde{r}_{I2}^o(L,s) = r_1^{reg}$ for existing debt and

$$\tilde{r}_{I2}^n(L,s) = \frac{1+\overline{r}-p_L}{p_L} + \frac{b_1(1+r_1^{reg})\,(1+\overline{r}-p_L(1+r_1^{reg}))}{p_L b_2}, \tag{A.8}$$

$s = \ell, h$, for new debt so that expected profits are zero.[53] Denote the utility to the borrower by $U_2(L)$. The outside lenders have an advantage over the insider and can offer to the borrower strictly higher utility than $U_2(L)$, while their expected profits are zero. This follows from Lemma 1, which demonstrates that the rate for new debt that maximizes the surplus in the lender-borrower relationship is $\hat{r}_{I2}(L) \equiv (1+\overline{r}-p_L)/p_L$. The rate on existing debt simply transfers surplus between the two parties. Due to regulation, the insider offers a rate on new debt that is higher than the efficient rate, $\tilde{r}_{I2}^n(L,s) > \hat{r}_{I2}(L)$. In equilibrium, the outsiders offer $\tilde{r}_{O2}^n(s) = \tilde{r}_{O2}^o(s) = \hat{r}_{I2}(L)$, $s = \ell, h$, which provides to the borrower strictly higher utility than the insider[54] (but the same as in the unregulated case) while, due to Bertrand-type competition, make zero expected profits.

In sum, regulation has no effect on the second-period equilibrium rates offered to the low type (same as in Proposition 1). Also, all lenders make zero expected profits, as in the unregulated case. The only effect regulation has is that it induces the low type to switch lenders. The low type, however, is as well off as he was prior to regulation (assuming switching is costless). This is true as long as no high type switches lenders, so the outsiders are certain that they are lending to the low types.

High type. The rate for existing debt prior to regulation is given by (4.14). It follows from (A.7) that regulation is certainly binding for the insider when $i = H$ and $s = \ell$, that is, r_1^{reg} is below $r_{I2}^o(H,\ell)$, the rate the insider offers under no regulation to the H type when the marker's signal is ℓ. It may or may not be below $r_{I2}^o(H,h)$, depending on the informational asymmetry ϕ.[55]

We assume that $r_1^{reg} > \hat{r}_{I2}(H) \equiv (1+\overline{r}-p_H)/p_H$, which guarantees strictly positive expected profits of the inside lenders when they lend to the high types. Hence, no high-type borrower switches lenders. This assumption is confirmed by the numerical example of Section B and is certainly true for

[53] It can be verified that (A.8) is greater than $(1+\overline{r}-p_L)/p_L$ if and only if $r_1^{reg} < (1+\overline{r}-p_L)/p_L$.

[54] Due to the rate inefficiency for new debt, while the insider's expected profit is still zero.

[55] In the numerical example in Section B, regulation is not binding for the inside lender when it lends to the H type when ϕ exceeds a threshold.

high ϕ. In what follows, we assume that $r_1^{reg} < r_{I2}^o(H,h) < r_{I2}^o(H,\ell)$, but the arguments do not change if $r_1^{reg} > r_{I2}^o(H,h)$.

In equilibrium, the insider makes interest rate offers to the borrower for existing and new debt so that, if the outsiders were to match the borrower's utility, their profits would be zero (the same as when they lend to the low type exclusively). Since the outsiders face no constraints, a deviator's offer for new debt would be equal to $\hat{r}_{O2}(s)$, $s=\ell,h$ and for old debt greater than or equal to $\hat{r}_{O2}(s)$, $s=\ell,h$. To ensure that there does not exist a profitable deviation on part of the outsiders, when regulation is binding, the rate the insider offers for new debt should match the expected utility the borrower obtains if he borrows from an outsider instead at the zero-profit-for-an-outsider deviation rates $\hat{r}_{O2}(s)$, $s=\ell,h$, and it is given (implicitly) by[56]

$$\begin{aligned}\tilde{r}_{I2}^n(H,s) &= -1+\left(\frac{(1+\hat{r}_{O2}(s))(1+r_1^{reg})}{b_2^*(\tilde{r}_{I2}^n(H,s))}-\frac{(1+r_1^{reg})^2}{b_2^*(\tilde{r}_{I2}^n(H,s))}\right)b_1^* \\ &+ \frac{u(b_2^*(\tilde{r}_{I2}^n(H,s)))-u(b_2^*(\hat{r}_{O2}(s)))}{b_2^*(\tilde{r}_{I2}^n(H,s))p_H}+\frac{b_2^*(\hat{r}_{O2}(s))(1+\hat{r}_{O2})}{b_2^*(\tilde{r}_{I2}^n(H,s))p(H)}. \qquad \text{(A.9)}\end{aligned}$$

Due to the insider's inability to raise the rate on old debt from r_1^{reg}, the rate on new debt can be higher than the efficient rate.[57] Given that the inside lender can no longer offer the efficient rate for new debt and the borrower's utility stays the same between the two regimes (determined by the outsiders' potential offers), it follows that the inside lender is worse off under the regulation in the second period, that is, $E\pi_{I2}^{reg}(H;d\neq 0) \leq E\pi_{I2}(H;d\neq 0)$, where $E\pi_{I2}(H;d\neq 0)$ was derived when we analyzed the unregulated equilibrium.

Combining the case of positive second-period debt we just examined with that of no debt, the expected profits in the second period for the inside lender are

$$E\pi_{I2}^{reg} = \mu\left[p_H E\pi_{I2}(H;d=0)+(1-p_H)E\pi_{I2}^{reg}(H;d\neq 0)\right] > 0.$$

[56]Note that such deviation will attract both types of borrowers. That is why in computing the expected profit of an outsider, we use $p(s)$ as the probability of repayment.

[57]The utility borrowers receive is the same as in the unregulated equilibrium, since it is determined by what the competitive fringe offers. These offers have not been affected by the regulation for two reasons: First, the regulation applies to the insider and second, competition among the outsiders disciplines their behavior. Given a downward sloping indifference curve in the space of old and new interest rates, if regulation is binding for the insider – which it definitely is if the signal of the outsiders is low – then the rate on new debt must increase relative to the unregulated equilibrium.

Analogous to the unregulated case, see (A.6), the first-period equilibrium rate must satisfy

$$r_1^{reg} = \frac{1+\bar{r}-\bar{p}}{\bar{p}} - \frac{\delta E\pi_{I2}^{reg}}{\bar{p}b_1^*(r_1^{reg})}. \tag{A.10}$$

The $\delta E\pi_{I2}^{reg}/(\bar{p}b_1^*(r_1^{reg}))$ term decreases as r_1^{reg} decreases. As we argued previously, the numerator decreases, while the denominator increases, $db_1^*(r_1^{reg})/dr_1^{reg} < 0$, as we show next. First, we differentiate (4.3) with respect to b_1 taking into account that b_1 affects the rates for new borrowing in period 2, as it is demonstrated by (A.9). This yields

$$\begin{aligned} & u'(b_1) - (1+r_1^{reg})\bar{p} - \delta(1-\bar{p})(1+r_1^{reg})\left[\left(\mu p_H\left(1+R_{j2}^o(H)\right) + (1-\mu)p_L\left(1+R_{j2}^o(L)\right)\right)\right] \\ - \ & \delta(1-\bar{p})\left(\mu p_H \frac{\partial R_{j2}^n(H)}{\partial b_1} b_2^*(p_H)\right) = 0. \end{aligned}$$

Using the equilibrium rates and how (A.9) responds to changes in b_1, we obtain

$$\begin{aligned} & u'(b_1) - (1+r_1^{reg})\bar{p} - \delta(1-\bar{p})\left[\left(\mu p_H(1+r_1^{reg})^2\right) + (1-\mu)(1+\bar{r})(1+r_1^{reg})\right] \\ - \ & \delta(1-\bar{p})\mu p_H\left(\frac{1+\phi}{2}\left(\frac{(1+\bar{r})(1+r_1^{reg})}{b_2^*(r_1^{reg})p(h)} - \frac{(1+r_1^{reg})^2}{b_2^*(r_1^{reg})}\right)b_2^*(r_1^{reg})\right) \\ - \ & \delta(1-\bar{p})\mu p_H\left(\frac{1-\phi}{2}\left(\frac{(1+\bar{r})(1+r_1^{reg})}{b_2^*(r_1^{reg})p(\ell)} - \frac{(1+r_1^{reg})^2}{b_2^*(r_1^{reg})}\right)b_2^*(r_1^{reg})\right) = 0 \Rightarrow \\ & u'(b_1) - (1+r_1^{reg})\bar{p} \\ - \ & \delta(1-\bar{p})(1+\bar{r})(1+r_1^{reg})\left(\frac{1+\phi}{2}\frac{\mu p_H}{p(h)} + \frac{1-\phi}{2}\frac{\mu p_H}{p(\ell)} + (1-\mu)\right) = 0. \end{aligned} \tag{A.11}$$

Using (A.11), we can derive db_1^*/dr_1^{reg}

$$\frac{db_1^*}{dr_1^{reg}} = \frac{\bar{p} + \delta(1-\bar{p})(1+\bar{r})\left(\frac{1+\phi}{2}\frac{\mu p_H}{p(h)} + \frac{1-\phi}{2}\frac{\mu p_H}{p(\ell)}\right)}{u''(b_1)} < 0.$$

Thus, the RHS of (A.10) is increasing as r_1^{reg} decreases. Therefore, there exists a unique r_1^{reg} that satisfies the equilibrium condition (A.10).

How does the regulated equilibrium first-period rate compare with that in the unregulated equilibrium? Because $E\pi_{I2} \geq E\pi_{I2}^{reg}$, the equilibrium interest rate under regulation is higher than the first-period rate under no regulation.

A.6 Proof of Proposition 5

The first-order condition of (6.1) with respect to b_2 is

$$
\begin{aligned}
& u'(b_2) - p_i C^{nd} f(C^{nd}) \frac{dC^{nd}}{db_2} + p_i C^{nd} f(C^{nd}) \frac{dC^{nd}}{db_2} - p_i \frac{dC^{nd}}{db_2}(1 - F(C^{nd})) = 0 \\
& \Rightarrow u'(b_2) - p_i(1 + r^{n/nd})(1 - F(C^{nd})) = 0.
\end{aligned} \tag{A.12}
$$

Let $b_2^{nd}(r^{n/nd})$ denote the solution to the borrower's optimization problem. The effect of the interest rate on the amount borrowed is (where the superscript of r is suppressed)

$$
\frac{db_2^{nd}}{dr} = \frac{p_i \left((1 - F(C^{nd})) - (1 + r) f(C^{nd}) b_2\right)}{u''(b_2) + p_i (1 + r)^2 f(C^{nd})}.
$$

The denominator must be negative (from the second order condition). The numerator is positive if and only if

$$
b_2 < \frac{1 - F(C^{nd})}{(1 + r) f(C^{nd})}. \tag{A.13}
$$

If we assume that $F(\cdot)$ satisfies the monotone hazard rate (MHR) property, then the RHS of (A.13) is decreasing in b_2, because C^{nd} is increasing in b_2. When $b_2 = 0$, the above inequality is satisfied, but for some b_2, it may not. At that point, demand for credit will become upward sloping. So there may exist a b_2, denoted by $\bar{b}$ that satisfies the above expression with equality, such that beyond this level $db_2/dr > 0$. We want to rule out this possibility. This is what we do next.

From the first-order condition of the borrower's problem, see (A.12), the following expression must be satisfied

$$
p_i = \frac{u'(b_2)}{(1 + r)(1 - F(C^{nd}))}. \tag{A.14}
$$

We substitute the above into the borrower's second-order condition

$$
\begin{aligned}
& u^{''}(b_2) + \frac{u'(b_2)}{(1 + r)(1 - F(C^{nd}))}(1 + r)^2 f(C^{nd}) < 0 \Rightarrow u^{''}(b_2) + \frac{u'(b_2)(1 + r) f(C^{nd})}{(1 - F(C^{nd}))} < 0 \\
& \Rightarrow -\frac{u^{''}(b_2)}{u^{'}(b_2)} > \frac{(1 + r) f(C^{nd})}{1 - F(C^{nd})}.
\end{aligned} \tag{A.15}
$$

If the above condition is satisfied, then the borrower's utility function is concave at the point where the first-order condition is satisfied. This suggests that for every interest rate there is a unique (interior) level of borrowing that maximizes utility. The LHS of (A.15) is the Arrow-Pratt coefficient of absolute risk aversion, which – given our assumption – is decreasing in b_2. On the other hand, the RHS is increasing in b_2 given that $F(\cdot)$ satisfies the MHR property. Furthermore, we assume that the inequality (A.15) is strictly satisfied at $b_2 = 0$.[58] It follows that there must exist a unique $b_2 > 0$ that satisfies (A.15) with equality. For b_2's higher than this threshold, whenever the first-order condition is satisfied, the borrower's utility function is convex. Moreover, from that point on, the expected utility of the borrower is monotonically increasing in b_2. The probability of default is very high and the borrower chooses to borrow a lot knowing that default is almost certain. In sum, the borrower's expected utility may exhibit a sideways “S"-shape with a unique interior maximum.[59]

Let $\overline{B}$ be the level of borrowing that attains the same expected utility as the interior maximum, see Figure 2. The lender, to prevent the borrower from borrowing too much and then defaulting, will impose a credit limit somewhere to the right of the point that satisfies the first-order condition for the interior maximum and to the left of $\overline{B}$. If $\Gamma \leq (1+r)\overline{B}$, then the probability of default is 1 when borrowing exceeds $\overline{B}$. Hence, a lender will never find it optimal to set a credit limit that exceeds $\overline{B}$, as this will induce a certain default. Therefore, the first-order condition as given by (A.12) characterizes the borrower's decision. When the probability of default is exogenously given, as we have assumed up to this section, a credit limit is not needed, since the borrower's utility assumes a unique interior maximum.

Further, we assume that the Arrow-Pratt degree of relative risk aversion for $u(\cdot)$ does not exceed one

$$-\frac{u^{''}(b_2)b_2}{u^{'}(b_2)} \leq 1.$$

This, together with (A.15), imply

$$\frac{1}{b_2} \geq -\frac{u^{''}(b_2)}{u^{'}(b_2)} > \frac{(1+r)f(C^{nd})}{1-F(C^{nd})}.$$

[58] This is true, for example, when the utility is as in the examples in footnote 46 and $f(0)$ is not too high.

[59] Alternatively, the RHS of (A.14) is decreasing in b_2 as long as (A.15) is satisfied. (Marginal utility is decreasing faster than the probability of no default, as borrowing increases.) Also, given our assumption that $u'(0)$ is high, the RHS of (A.14) exceeds p_i when $b_2 = 0$. At a certain b_2, the LHS (which is independent of b_2) and RHS of (A.14) become equal, and this is where the first-order condition for an interior maximum is satisfied, while the RHS of (A.14) is still decreasing in b_2. After a threshold, as we argued previously, the inequality given by (A.15) is reversed and the RHS of (A.14) becomes increasing in b_2. The next b_2 at which it meets p_i corresponds to a local minimum of the expected utility. Beyond this point, expected utility keeps rising, see Figure 2.

Therefore, (A.13) is satisfied. Demand for credit is downward sloping, when the solution to a borrower's utility maximization problem is interior (i.e., credit limit is not binding). Moreover, given our assumption $\Gamma \leq (1+r)\overline{B}$, the credit limit is not binding.

Now suppose the borrower carries debt from the first period. The first-order condition of (6.2) with respect to b_2 is

$$
\begin{aligned}
& u'(b_2) - p_i C^d f(C^d)\frac{dC^d}{db_2} + p_i C^d f(C^{nd})\frac{dC^d}{db_2} - p_i \frac{dC^d}{db_2}(1-F(C^d)) = 0 \\
& \Rightarrow u'(b_2) - p_i(1+r^{n/d})(1-F(C^d)) = 0. \qquad \text{(A.16)}
\end{aligned}
$$

Let $b_2^d(r^{n/d}, r^o)$ denote the solution to the borrower's optimization problem. It follows easily, by comparing (A.12) with (A.16), that $b_2^d \geq b_2^{nd}$ if the interest rates on new debt are the same. This is because default is more likely when the consumer carries debt, which makes borrowing in the second period cheaper. The effect of interest rates on the amount borrowed is

$$
\begin{aligned}
\frac{db_2^d}{dr^{n/d}} &= \frac{p_i\left((1-F(C^d)) - (1+r^{n/d})f(C^d)b_2\right)}{u''(b_2) + p_i(1+r^{n/d})^2 f(C^d)} \\
\frac{db_2^d}{dr^o} &= \frac{p_i\left((1-F(C^d)) - (1+r^{n/d})f(C^d)b_1(1+r_1)\right)}{u''(b_2) + p_i(1+r^n)^2 f(C^d)}. \qquad \text{(A.17)}
\end{aligned}
$$

Both $r^{n/d}$ and r^o affect second-period borrowing. The rate on old debt affects new borrowing because it affects the default probability. As in the case of no debt, $db_2/dr^{n/d}$ is negative provided that the borrower's utility has an Arrow-Pratt degree of relative risk aversion is not greater than one. For db_2/dr^o to be negative, it must be that $b_1(1+r_1)$ is low enough.

A.7 Proof of Lemma 3

The break-even rates must (implicitly) satisfy[60]

$$
\hat{r}^{n/d} = \frac{1+\overline{r} - p_i(1-F(C^d))}{p_i(1-F(C^d))} \text{ and } \hat{r}^o = \frac{1+\overline{r} - p_i(1-F(C^d))}{p_i(1-F(C^d))}. \qquad \text{(A.18)}
$$

These rates resemble the break-even interest rates from the case of exogenous default (see (4.7)) but are modified to account for the probability of no default $(1-F(\cdot))$.

[60]These are not the only rates that yield zero expected profit. They yield zero profit from new and old borrowing separately, but there are other rates that yield overall zero expected profit, as we will see next.

The first-order condition with respect to $r^{n/d}$ is

$$\begin{aligned}
&\frac{d\pi_2^d}{dr^{n/d}} = \\
&p_i(1-F(C^d))b_2 - p_i f(C^d)\left((1+r^{n/d})b_2^d + (1+r^o)(1+r_1)b_1\right)\left(b_2 + (1+r^{n/d})\frac{db_2}{dr^{n/d}}\right) \\
+ \quad & p_i(1-F(C^d))(1+r^{n/d})\frac{db_2}{dr^{n/d}} - (1+\overline{r})\frac{db_2}{dr^{n/d}} = 0.
\end{aligned}$$

The first-order condition with respect to r^o is

$$\begin{aligned}
&\frac{d\pi_2^d}{dr^o} = \\
&p_i(1-F(C^d))(1+r_1)b_1 - p_i f(C^d)\left((1+r^{n/d})b_2^d + (1+r^o)(1+r_1)b_1\right)\left(b_1(1+r_1) + (1+r^{n/d})\frac{db_2}{dr^o}\right) \\
+ \quad & p_i(1-F(C^d))(1+r^{n/d})\frac{db_2}{dr^o} - (1+\overline{r})\frac{db_2}{dr^o} = 0.
\end{aligned}$$

Let's now turn to the borrower's utility. Using (6.2), we can derive the (absolute) marginal rate of substitution between old and new interest rates, dr^o/dr^n, using the envelope theorem, and holding expected utility constant (indifference curve). This yields

$$\frac{dr^o}{dr^{n/d}} = \frac{b_2^*}{b_1(1+r_1)}.$$

The iso-profit of the lender has slope (using $C^d \equiv (1+r^{n/d})b_2 + (1+r^o)b_1(1+r_1)$)

$$\begin{aligned}
&\frac{dr^o}{dr^n} = \frac{d\pi/dr^{n/d}}{d\pi/dr^o} = \\
&\frac{(p_i(1-F(C^d)) - p_i f(C^d)C^d)b_2 + \left(p_i(1-F(C^d))(1+r^{n/d}) - (1+\overline{r}) - p_i f(C^d)C^d\right)\frac{db_2}{dr^{n/d}}}{(p_i(1-F(C^d)) - p_i f(C^d)C^d)b_1(1+r_1) + \left(p_i(1-F(C^d))(1+r^/dn) - (1+\overline{r}) - p_i f(C^d)C^d\right)\frac{db_2}{dr^o}}.
\end{aligned}$$

If $r^{n/d}$ satisfies

$$r^{n/d} = \frac{1+\overline{r} - p_i(1-F(C^d))}{p_i(1-F(C^d))} + \frac{f(C^d)C^d}{1-F(C^d)}, \tag{A.19}$$

then the slope of the iso-profit equals the slope of the indifference curve. The interest rates associated with this tangency maximize borrower utility subject to a constant lender profit constraint. The idea is

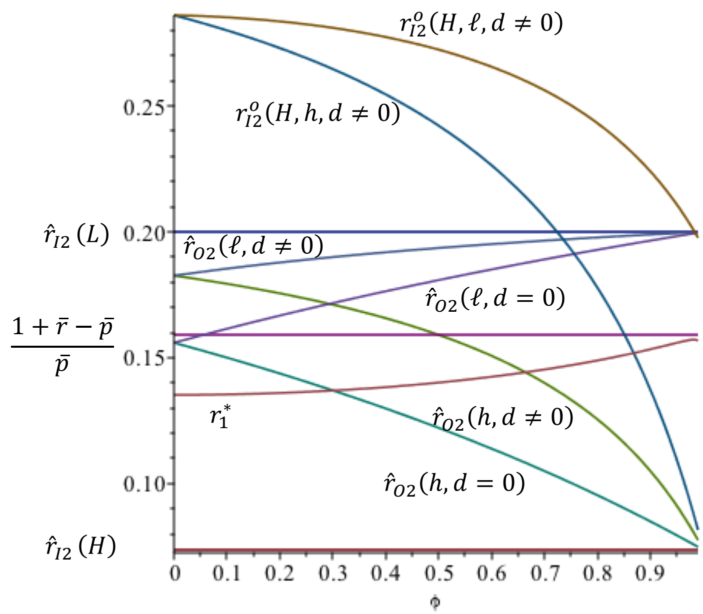

Figure 3: Equilibrium second- and first-period interest rates as a function of the outsiders' signal accuracy ϕ, the borrower's type $i = H, L$, the outsiders' signal $s = h, \ell$, and the level of debt, $d = 0$ or $d \neq 0$

similar to the one used in the proof of Lemma 1 and uses the fact we state in Proposition 5 that demand for credit is downward sloping.

Observe that the rate implied by (A.19) is higher than the break-even rate for new borrowing implied by (A.18). Therefore, for the lender to break even, the rate on existing debt must decrease, suggesting that the efficient break-even rates must satisfy

$$\tilde{r}^{n/d} = \frac{1+\bar{r}-p_i(1-F(C^d))}{p_i(1-F(C^d))} + \frac{f(C^d)C^d}{1-F(C^d)} \text{ and } \tilde{r}^o < \frac{1+\bar{r}-p_i(1-F(C^d))}{p_i(1-F(C^d))}.$$

B Appendix: A Numerical Example

We assume a CRRA utility function $u(b_t) = \frac{b_t^{1-\gamma}}{1-\gamma}$, $t = 1, 2$. We choose the parameter values so that the equilibrium interest rates are reasonably close to what is observed in real data. We set $\gamma = 1/2$, $\mu = 0.3$, $p_H = 0.95$, $p_L = 0.85$, $\bar{r} = 0.02$, and $\delta = 0.8$.

Figure 3 depicts the unregulated equilibrium second- and first-period rates, see Propositions 1 and 2.

Figure 4 depicts the first- and second-period equilibrium and first-best borrowing. First-period equilibrium borrowing can be above or below what a social planner prefers, depending on the extent of the informational asymmetry. In the second period, while the social planner prefers smoothing of borrowing

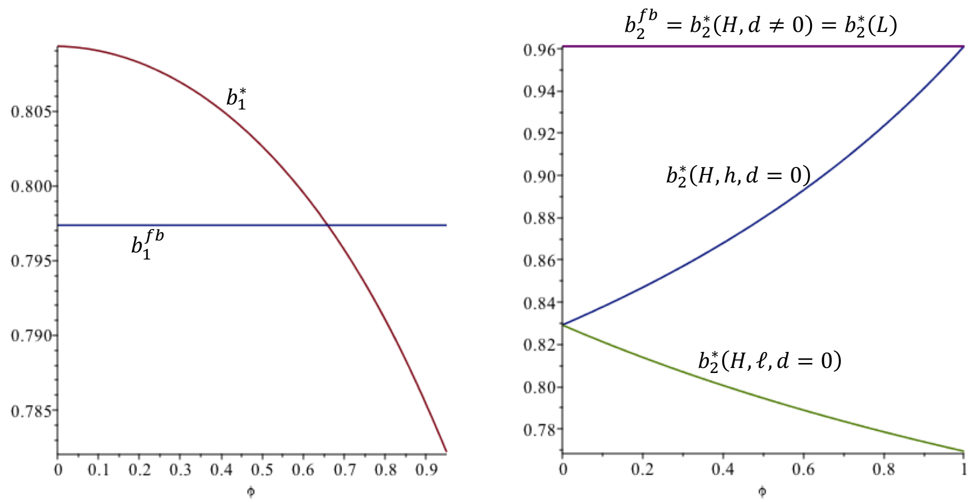

Figure 4: Equilibrium (*) versus the first-best (fb) borrowing (b) in periods 1 and 2 as a function of the outsiders' signal accuracy ϕ, the borrower's type $i = H, L$, the outsiders' signal $s = h, \ell$ and the level of debt, $d = 0$ or $d \neq 0$

across types the market equilibrium does not deliver it. In addition, there is underborrowing relative to the first best. When $\phi = 1$, and hence the inside lenders have no market power, the equilibrium borrowing in the second period is at the first-best levels,[61] but there is still inefficiency in period 1 borrowing. As we discuss in Section 4.3, this is because the social planner solution may imply negative profit for the lenders.

Figure 5 plots the regulated first-period rate, r_1^{reg}, together with the equilibrium, r_1^*, and the first-best, r_1^{fb}, first-period rates, for all values of ϕ.

Borrowers pay a higher first-period rate but a lower rate on existing borrowing carried over in period 2 under regulation than in the unregulated equilibrium. As Figure 6 illustrates, the overall effect of regulation on first-period borrowing is negative. Less first-period borrowing can be social welfare enhancing or damaging depending on the value of ϕ. Second-period borrowing for the H types who carry debt under regulation falls below the first best, since the rate on new debt under regulation is higher than the efficient rate. Moreover, these rates, as it can be seen from (A.9), depend on s and therefore regulation also introduces variability in the borrowing of the H types in period 2. Finally, as we know, regulation increases the deadweight loss, see Figure 7.

[61]Note that in this case the probability of $s = \ell$, given $i = H$ is zero.

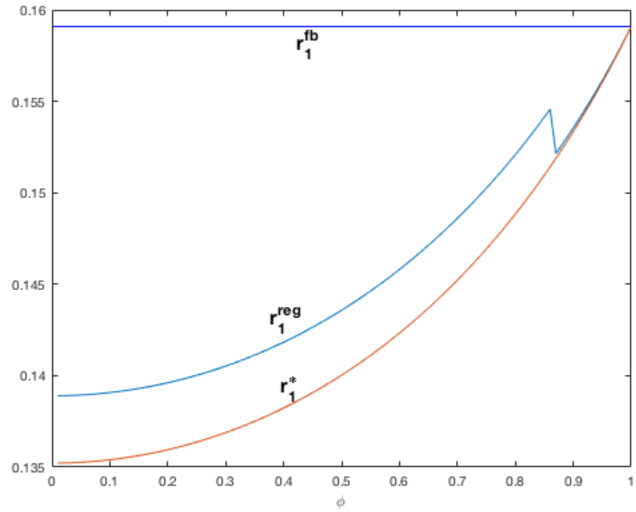

Figure 5: The first-period rates: r_1^* under no regulation, r_1^{reg} under regulation and $r_1^{fb} = \frac{1+\bar{r}-\bar{p}}{\bar{p}}$, the first-best first-period rate. For ϕ less than (approximately) 0.81, the regulation is binding for the H type regardless of the outsiders' signal, i.e., $r_1^{reg} < r_{I2}^o(H, h, d \neq 0) < r_{I2}^o(H, \ell, d \neq 0)$. For $\phi > 0.81$, r_1^{reg} is higher than $r_{I2}^o(H, h, d \neq 0)$ and therefore the regulation does not bind for the H type when $s = h$. In this case, the inside lender switches to the unregulated equilibrium rates.

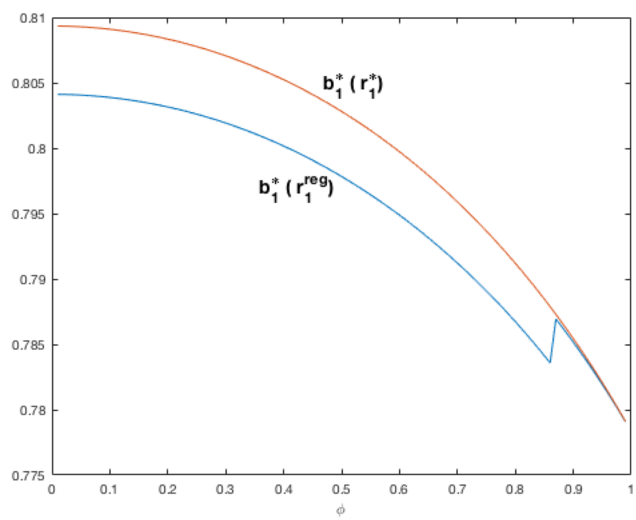

Figure 6: Borrowers borrow less in the first period under regulation than no regulation.

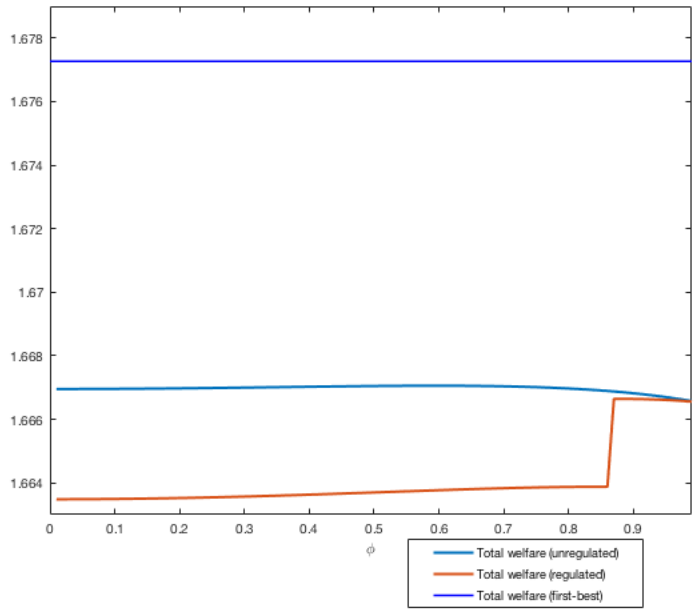

Figure 7: Total welfare comparisons: Regulation exacerbates the inefficiency relative to the unregulated equilibrium. When there is no asymmetric information between the inside and the outside lenders, $\phi = 1$, then total welfare is the same between the regulated and the unregulated equilibrium.

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