Greed as a Source of Polarization

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Abstract

The political process in the United States appears to be highly polarized: evidence from voting patterns finds that the political positions of legislators have diverged substantially, while the largest campaign contributions come from the most extreme lobby groups and are directed to the most extreme candidates. Is the rise in campaign contributions the cause of the growing polarity of political views? In this paper, we show that, in standard models of lobbying and electoral competition, a free-rider problem amongst potential contributors leads naturally to a divergence in campaign contributors without any divergence in candidates' policy positions. However, we go on to show that a modest departure from standard assumptions — allowing candidates to directly value campaign contributions (because of "ego rents" or because lax auditing allows them to misappropriate some of these funds) — delivers the ability of campaign contributions to cause policy divergence.

Keywords: Polarization; Campaign Contributions; Agendas

JEL Codes: D72, H41

1   Introduction

The political process in the United States appears to be highly polarized. This observation has two dimensions. First, evidence from voting patterns finds that the political positions of legislators have diverged substantially. Second, the largest campaign contributions tend to come from the most extreme lobby groups and are typically directed to the most extreme candidates. Moreover, these trends appear related, with increases in campaign spending coinciding with the increase in the polarization of US politicians. This begs the questions of whether the dominance of extreme lobby groups is the cause of the rising polarization in US politics and, if not, what are the forces that lead to the dominance of extreme lobby groups and of the rising political polarization.

Towards an answer to these questions, we present a theoretical model of policy formation, lobbying and electoral success. In the model, the polarization of campaign contributions arises out of a natural incentive for moderate interest groups to "free ride" on the campaign contributions of more extreme interest groups. Unsurprisingly, candidates with more extreme positions receive larger contributions. However, we show that, under standard assumptions on the objectives of politicians, the polarization of campaign contributions is associated with the *convergence* of politicians' policies. We then show that by departing from standard assumptions and allowing politicians to care about the absolute level of campaign contributions, in addition to their probability of winning an election which is influenced by the relative level of such contributions, we can generate policy divergence in equilibrium.

These results all derive from the same force that leads to the extremity of campaign contributions in the first place. The intuition for this is quite straightforward. As more extreme lobby groups care more intensely about policies than do moderate interest groups, they have an greater incentive to make larger contributions. Moreover, their incentive to contribute (and the amount contributed) is increasing in the degree of polarization of policy platforms. Thus, moving towards an extreme position increases the size of a candidate's campaign chest. However, such a move affects the opponent's contributions as well. And, under standard assumptions, the increase in the opponent's contributions is in fact greater than that in the candidate's own campaign chest. Thus, if politicians care only about their probability of being elected, and if this probability depends on *relative* campaign expenditures, the result is policy *convergence*. However, once politicians begin to care about the absolute size of their contributions — whether because of "ego rents" or intrinsic greed and corruption — the incentive to make policy diverge so as to maximize the absolute level of contributions is restored.

A further surprising result is that when we alter the preferences of the lobbies so as to eliminate the polarization of the contributors (in a generic subgame), we obtain full polarization of the agendas. When the lobby's loss function is concave (as opposed to the convex loss in the standard formulation), the contributing lobbies in

equilibrium are those "targeted" by the candidates, and not (generally) the extreme ones. But eliminating the polarization of contributors for a given set of agendas does not eliminate polarization of the agendas. To the contrary, since the *relative* contributions to the two candidates are unaffected by their agenda choices, while the absolute contributions are increasing with agenda polarization, we obtain complete polarization of the candidates' positions in equilibrium whenever they put any utility weight on the contributions.

This paper is motivated by a substantial empirical literature examining polarization in politics and the role of lobby groups and campaign contributions. Evidence for the increased polarization of legislator positions over the past three decades has been assembled from legislator voting patterns by Poole and Rosenthal (1997) and McCarty, Poole, and Rosenthal (1997, 2006). Our focus on lobbyists' support of candidates with a commitment to a fixed collective agenda, and on the polarization of campaign contributions, is likewise motivated by several empirical findings. Most notably consistent with this framework, Langbein (1993) and Poole and Romer (1985) have found that contributors rarely donate to candidates on both sides of an issue.[1] Similarly, many authors have found that contributors support candidates with similar ideological views[2], with this result being strongest for groups with strong ideological positions (e.g., Langbein (1993)). Similarly, in a series of papers, James M. Snyder (1990, 1992, 1993) has argued that ideological political action committees (PACs) do not fit a quid pro quo model of contributions, while Welch (1979) cites evidence that ideological PACs focus on close races as evidence in favor of models of contributions in support of a candidate with a given position. Finally, Poole and Romer (1985) provide evidence that contributors provide the largest quantity of support for like-minded candidates. Our emphasis on free-riding by interest groups is consistent with a substantial empirical literature that has found free-riding by lobby groups to be important, albeit typically in the context of specific policies. For example, Bloch (1993) finds that the degree of unionization is positively related to support for minimum wage legislation, while Kirchgässner and Pommerehne (1988) find that it is positively related to measures of social expenditure. Other authors have found a positive relationship between producer concentration in an industry and political influence[3].

On the theoretical side, in an important early contribution, Austen-Smith (1987) considered competition between two rival lobbying groups and established a convergence result for candidate agendas. In this paper, we generalize the convergence result to a situation in which the identity and number of contributors is endogenous, and where free-riding leads to divergence of lobby groups. We also establish the limits of this result once candidates are allowed to value the absolute level of

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[1]However, see Schlozman and Tierney (1986) for an alternative view.

[2]For example, Chappell (1982), Gopoian, Smith, and Smith (1984), Saltzman (1987), Welch (1980), Welch (1982) and many others.

[3]For example, Esty and Caves (1983), Gardner (1987), Guttman (1980), Kalt and Zupan (1984) and Trefler (1993). For an opposing view, see Becker (1986) and Pincus (1975).

their contributions. The intuitive idea that lobbyists and campaign contributions may lead to polarization is present in Baron (1994) and Persson and Tabellini (2000) amongst others. Unlike Baron (1994), we focus on the public-good character of campaign contributions and show that polarization may result for collective policies, in addition to particularistic policies. In contrast to the intuition laid out in Persson and Tabellini (2000), we show that polarization of lobbyists may nonetheless lead to policy convergence when candidates care only about electoral success. Herrera, Levine, and Martinelli (2008) explain polarization in a model that differs from ours in that it abstracts from the role of lobbyists in campaign financing. Our divergence result is distinct from other explanations that rely on divergent candidate preferences (Roemer (1991) or Lindbeck and Weibull (1993)), uncertainty about candidate type (Bernhardt and Ingberman (1985)), or the threat of a third-party candidate's entry (Palfrey (1984); see also the survey by Osborne (1995)). Finally, note that Dekel, Jackson, and Wolinsky (2008, 2009) study campaign spending with fixed agendas, while Jackson, Mathevet, and Mattes (2007) establish the possibility of agenda divergence in a model where candidates spend their own resources in the absence of lobbyists.

The remainder of the paper is organized as follows. The basic model is presented in Section 2. Section 3 presents our results on the divergence of contributions with convergent policies. Section 4 introduces greedy politicians into the basic model and derives the polarization results. Section 5 investigates some implications of the model, and Section 6 concludes.

2 The General Model

In this section, we outline a relatively general model of agenda setting, campaign contributions, and electoral outcomes. In succeeding sections, we specialize this model in various ways in order to focus on specific forces that affect the decision-making of both candidates and contributors.

Consider a model with the following elements. There are two political candidates, indexed by $i = 1, 2$, competing for election to one position. The game begins with each candidate selecting a policy platform or "agenda," denoted $a_i \in [0, 1]$. That is, we are allowing candidates to commit to an agenda.

There is a finite (and possibly large) number of lobby groups. Each lobby group is identified with (and indexed by) its preferred agenda $j \in [0, 1]$. A lobby group's preferences over agendas, a, are represented by

$$V_j(a) = -|a - j|^\alpha, \tag{2.1}$$

with a common $\alpha > 0$. It is typical to assume that $\alpha \geq 1$, which ensures that V is concave, so that the marginal distaste of a lobby group for an agenda is increasing as the agenda deviates further from the lobby's preferred agenda. We also allow for the

case of $\alpha \in (0,1)$ in which the marginal distaste for alternative agendas is initially high and decreases as agendas become further removed from the lobby's ideal point.[4]

In the second stage of the game, after observing the agenda choices of each candidate, each lobby group j may elect to contribute a non-negative amount $c_j(i)$ towards candidate i's campaign at a cost to the lobby group of $\phi(c_j(1) + c_j(2))$. For now, we assume only that ϕ is (weakly) convex and strictly increasing in total contributions.

In the third stage of the game, each candidate i chooses how much to spend on the electoral campaign. This choice, S_i, is constrained by both the size of contributions and by the institutions governing the use of campaign funds. For example, in a country with relatively little corruption and accurate auditing of campaign donations, candidates may have to spend all of their contributions on campaigning, while in a country with a great deal of corruption and little auditing, candidates may be able to appropriate some or all of their campaign contributions for their own personal consumption. For now, we represent these constraints abstractly by choice set $B \subset \mathbb{R}_+^2$. I.e., we require $(S_i, C_i) \in B$, where $C_i = \sum_j c_j(i)$ is the total of lobbies' contributions to candidate i.

The preferences of each candidate are likewise expressed somewhat abstractly as the sum of a term that captures the private benefit of campaign contributions net of campaign spending, and a term that reflects the expected benefit from winning the election:

$$U(C_i + \epsilon - S_i) + p_i\left(S_i, S_{-i}, a_i, a_{-i}\right) W, \tag{2.2}$$

where W represents the value the agent places on winning the election, ϵ is the candidate's own wealth,[5] $p_i(.)$ is the probability of i winning the election, and where the notation "$-i$" (for "not i") has been used to denote the rival candidate. For simplicity, we assume that the candidates have no preference over agendas, except insofar as they affect the size of campaign contributions and the probability of electoral victory. We will routinely assume that the candidates' utility function U is continuously differentiable, strictly increasing and strictly concave with

$$\lim_{x \to 0} U'(x) = +\infty. \tag{2.3}$$

The probability of winning the election has been conditioned on both the campaign spending of both candidates and their initial agenda choices, in order to encompass a wide array of voting mechanisms and political economy models. For now, we simply summarize the outcome of the fourth stage of the game — in which agents vote — simply in terms of the probability that a candidate wins the election as a function

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[4]With apologies to Monty Python, the case $\alpha > 1$ corresponds to a world in which the People's Front of Judea and the Judean People's Front are both strongly preferred to the Romans, while the case $\alpha \in (0,1)$ refers to the case where members of the People's Front of Judea despise the Judean People's Front almost as much as they do the Romans.

[5]We introduce candidates' own wealth to guarantee the existence of equilibrium in a subgame where only one of the candidates receives campaign contributions. ϵ should be thought of as an arbitrarily small number.

of both campaign spending and agenda choices, $p_i(S_i, S_{-i}, a_i, a_{-i})$. This reduced form specification allows us to capture a number of alternative, and not necessarily exclusive, possible assumptions about the way campaign expenditures and agendas affect election outcomes. For example, in Appendix B we show that this framework captures both the informed and uninformed voter model of Baron (1994) and the "get out the vote" model of Herrera, Levine, and Martinelli (2008). In each of these examples, the probability of winning the election is continuously differentiable in the campaign spending levels S_i (for strictly positive campaign levels) and in the choice of agenda a_i, and is homogeneous of degree zero in campaign spending:

$$p_i\left(\lambda S_i, \lambda S_{-i}, a_i, a_{-i}\right) = p_i\left(S_i, S_{-i}, a_i, a_{-i}\right) \qquad \forall \lambda > 0. \tag{2.4}$$

Hence, we maintain these assumptions below. We also restrict attention to symmetric p in the sense that if the spending and agendas of the candidates are reversed, the probability of election is reversed as well:

$$p_i\left(S_i, S_{-i}, a_i, a_{-i}\right) = p_{-i}\left(S_{-i}, S_i, a_{-i}, a_i\right).$$

To economize on notation from now on, we will denote the probability of candidate 1 winning the election simply by p. The last condition then becomes

$$p\left(S_i, S_{-i}, a_i, a_{-i}\right) = 1 - p\left(S_{-i}, S_i, a_{-i}, a_i\right).$$

Finally, we assume that campaign spending increases the probability of winning the election, or

$$p\left(1, 0, a_i, a_{-i}\right) > p\left(0, 0, a_i, a_{-i}\right) \qquad \forall a_i, a_{-i}. \tag{2.5}$$

3 Divergent Lobbies and Convergent Agendas

To begin, and to focus attention on the "public good" aspect of campaign contributions, we specialize the above model in a number of ways. First, we assume that the candidates do not value campaign contributions except insofar as these contributions increase the probability of electoral success, and that campaign contributions are the only source of funds for campaign expenditures. This assumption can be implemented either by setting U to 0 everywhere or by simply setting the constraint set $B = \{(S, C) \in \mathbb{R}_+^2 \mid S = C\}$. This is a relatively standard assumption in the literature, although, as we will see below, it has a significant effect on the results.

Second, we assume that the probability of electoral success does not depend on agendas, and is strictly increasing in a candidate's campaign spending. This has the effect of removing an obvious force for the convergence of agendas in equilibrium, and hence strengthens the nature of our convergence result.

Under our assumptions, the third stage of the game described above is degenerate. We solve the game consisting of the first two stages by backward induction.

3.1   Campaign Contributions

We first establish the identity of the contributing lobbies and the size of their contributions in an arbitrary subgame for given policy choices of the candidates, (a_1, a_2). For simplicity, and without loss of generality, we will adopt the convention that candidate 1 is to the left of candidate 2, or $a_1 \leqslant a_2$.

Consider the problem of a lobby j, that is considering contributing to candidate 1. The lobby solves the following problem, taking as given the opponent's campaign fund $C_2 = \sum_k c_k(2)$ and the total contributions $C_1(-j) = \sum_{k \neq j} c_k(1)$ of other lobbies to candidate 1's campaign:

$$\max_{c \geqslant 0} -p(c+C_1(-j), C_2)|a_1 - j|^\alpha - (1 - p(c+C_1(-j), C_2))|a_2 - j|^\alpha - \phi(c+c_j(2)), \quad (3.1)$$

which is equivalent to

$$\max_{c \geqslant 0} p(c + C_1(-j), C_2)\Delta_j(a_1, a_2) - \phi(c + c_j(2)), \quad (3.2)$$

where we have defined the added benefit to lobby j of policy a_1 over policy a_2 by

$$\Delta_j(a_1, a_2) = |a_2 - j|^\alpha - |a_1 - j|^\alpha. \quad (3.3)$$

This is a very well behaved convex problem, with the first order condition for an optimum given by

$$\frac{\partial p(C_1, C_2)}{\partial C_1}\Delta_j(a_1, a_2) \leq \phi'(c_j(1) + c_j(2)), \quad (3.4)$$

and symmetrically for contributions to candidate 2

$$\frac{\partial p(C_2, C_1)}{\partial C_2}\Delta_j(a_2, a_1) \leq \phi'(c_j(1) + c_j(2)), \quad (3.5)$$

with each of these conditions holding with equality if the contribution by lobby j to candidate $i = 1, 2$ is positive.

Some results can be established without placing any further restrictions on the cost of funds or the preferences of the lobbies.

Lemma 1 *No lobby ever makes positive contributions to both candidates.*

Proof. Let $c_k(j) > 0$. Then

$$\frac{\partial p(C_j, C_{-j})}{\partial C_j}\Delta_k(a_j, a_{-j}) = \phi'\left(c_k(j) + c_k(-j)\right).$$

Towards a contradiction, let $c_k(-j) > 0$. Then we must also have

$$\frac{\partial p(C_{-j}, C_j)}{\partial C_{-j}}\Delta_k(a_{-j}, a_j) = \phi'\left(c_k(j) + c_k(-j)\right).$$

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Since both derivatives (of p and of ϕ) are strictly positive, both $\Delta_k(a_{-j}, a_j)$ and $\Delta_k(a_j, a_{-j})$ must be strictly positive. But since $\Delta_k(a_{-j}, a_j) = -\Delta_k(a_j, a_{-j})$, that is a contradiction. ∎

The properties of the solution depend on the curvature of the lobby's preferences (given by the size of α), the curvature of the electoral success probability p, and the curvature of the cost of funds function ϕ. We get some of our starkest results when we assume that the cost of funds function ϕ is linear in contributions.

3.1.1 Lobbies with Deep Pockets

The starkest illustration of the key mechanism arises when we assume that lobbies have "deep pockets," i.e., that their cost of funds is linear (rather than strictly convex). The key intuition derived here carries on to the more general case, as we illustrate in Appendix C.1.

Lemma 2 *If $\alpha > 1$ and $\phi(c) = \phi c$, then only the extreme lobbies contribute in any subgame. That is, in every subgame, $C_1 = c_{\underline{j}}$ and $C_2 = c_{\bar{j}}$, where $\underline{j} = \min j$ is the left-most lobby and $\bar{j} = \max j$ is the right-most lobby.*

Proof. Suppose not. Then there exists a j satisfying $\underline{j} < j < \bar{j}$ such that the first order condition for contributing to one candidate holds with equality, or

$$\frac{\partial p(C_1, C_2)}{\partial C_1} \Delta_j(a_1, a_2) = \phi.$$

But under the assumption of the lemma, for any $a_1 < a_2$, $\Delta_j(a_1, a_2)$ is decreasing in j. But then the first order condition for at least one of the extreme lobbies is violated, a contradiction. Suppose then that $a_1 = a_2$. But then $\Delta_j(a_1, a_2) = 0$ for all j and no lobby contributes. ∎

This result follows from the fact that less extreme lobbies have an incentive to free-ride on the contributions of more extreme lobbies. In particular, the most extreme lobbyists contribute up to the point where the marginal benefit from an extra contribution equals its marginal cost. However, since all non-extreme lobbies have a strictly lower marginal benefit, and yet face the same marginal cost, they find it optimal not to contribute.

Assumption (2.5) (combined with assumption (2.4)) guarantees that the extreme lobbies make positive contributions whenever $a_1 \neq a_2$. Moreover, as we establish in in Appendix C.1, these contributions satisfy

$$\frac{C_1}{C_2} = \frac{\Delta_{\underline{j}}(a_1, a_2)}{\Delta_{\bar{j}}(a_2, a_1)}. \tag{3.6}$$

Altering the assumption regarding the preferences of the lobbies changes the identity of contributing lobbies in equilibrium (of a subgame), but not the basic insight regarding the free-riding:

Lemma 3 *If $\alpha < 1$ and $\phi(c) = \phi c$, then at most the lobbies most closely aligned with the candidates' platforms contribute in any subgame. More formally, in every subgame, $C_1 = \sum_{j_1} c_{j_1}$ and $C_2 = \sum_{j_2} c_{j_2}$, where $j_i \in \arg\min_j \left(|a_i - j|^\alpha - |a_{-i} - j|^\alpha\right)$.*

Proof. The proof is analogous to that of the previous lemma. ∎

Note that the contributing lobby is either the lobby most closely aligned with the candidate ($j_i = \arg\min_j |a_i - j|$) or, if the closest lobby is more centrist than the candidate, possibly the second closest.

3.2 Political Agendas

So far, we have studied the outcome of the second stage of the game in which campaign contributions are determined given the agendas of the candidates. Having established the optimal behavior of the lobbies in the second stage of the game, we are now ready to consider the agenda-setting behavior of the candidates. Under the (standard) simplifying assumption that candidates care only about electoral success (and assumption (2.4)), the results above imply that the candidates care only about relative contributions. Using this, we can establish our key results.

We begin with some helpful observations:

Lemma 4 *If $\phi(c) = \phi c$ and $\alpha > 1$, no candidate ever chooses a platform that is located further from the center (the other candidate) than the preferred point j of the lobby that contributes to the candidate's campaign in equilibrium.*

Proof. Candidate 1 aims to maximize $C_1/C_2 = \Delta_1/\Delta_2$. Under our assumptions, at most two lobbies, which we denote $j_1 \leq j_2$ without loss of generality, contribute. But $\Delta_{j_1}(a_1, a_2)$ is increasing in a_1 for $a_1 < j_1$, while $\Delta_{j_2}(a_2, a_1)$ is decreasing in a_1 in this range. That is, by locating further from the other candidate than the supporting lobby's preferred point, a candidate would both lower her own campaign contributions and increase those of the opponent. This contradicts optimization by candidate 1. The same logic applies to candidate 2. ∎

Lemma 5 *If $\alpha < 1$, no candidate ever chooses a platform that is not located at a preferred point j of some lobby (which contributes to the candidate's campaign in equilibrium). That is, candidates do not locate (choose platforms) at points where there are no lobbies.*

Proof. Due to the concavity of the value function, moving away from the preferred point of a supporting coalition lowers the candidate's contribution more than the opponent's. ∎

The last lemma implies that when $\alpha < 1$, only one lobby contributes to each candidate.

We now establish the first key result — the convergence of agendas when the candidates' sole objective is winning the election. In particular, we show that, for all specifications, there is convergence in agendas to some "central" agenda. We also show that, in general, the convergence will *not* be to a median agenda, and provide conditions under which agendas converge to the one preferred by the average lobby group.

Theorem 1 *If $\alpha > 1$ and the candidates' sole objective is winning the election, the unique equilibrium has both candidates locating (choosing platforms) in the mid-point between the two extreme lobbies, at $j_m = \frac{\bar{j} + \underline{j}}{2}$. Contributions are zero in equilibrium.*

Proof. This is an equilibrium because moving away from the midpoint increases one's opponent's contributions more than one's own. A candidate i, whose only objective is to win the election, will (strategically) maximize C_i / C_{-i}. That is, the candidate will take into account the effect of her choice of platform a_i on the contributions to her opponent. So, the problem of the (left) candidate 1 is:

$$\max_{a_1 \leqslant a_2} \frac{(a_2 - \underline{j})^\alpha - (a_1 - \underline{j})^\alpha}{(\bar{j} - a_1)^\alpha - (\bar{j} - a_2)^\alpha} \tag{3.7}$$

If $a_{-i} < \frac{\underline{j} + \bar{j}}{2}$, then candidate i will choose to locate to the right of a_{-i} ($a_i > a_{-i}$). To see this, simply observe that $\Delta_{\underline{j}}(a, a_{-i}) < \Delta_{\bar{j}}(a_{-i}, a)$ for $a < a_{-i}$ and $\Delta_{\bar{j}}(a, a_{-i}) > \Delta_{\underline{j}}(a_{-i}, a)$ for $a_{-i} < a \leqslant j_m$. Similarly, if $a_{-i} > \frac{\underline{j} + \bar{j}}{2}$, then candidate i will choose to locate to the left of a_{-i} ($a_i < a_{-i}$). Either way, candidate i can guarantee herself more than a 50% chance of winning the election. Thus, choosing any platform other than j_m cannot be part of a pure strategy equilibrium. In fact, since choosing j_m guarantees at least a 50% chance of winning the election (regardless of what the opponent's platform is), the only equilibrium has both candidates choosing j_m. ∎

It is important to note that the above result is not a median voter result. The midpoint to which the platforms converge need not be the preferred point of a median voter (or a median lobby for that matter).

Theorem 2 *If $\alpha < 1$ and the candidates' sole objective is winning the election and the number of lobbies is N, then there are N^2 distinct equilibria. The two candidates choose some lobbies' (not necessarily distinct) preferred points as their platforms. The contributions are $C_1 = C_2 = \frac{\Delta}{4\phi}$, where Δ is given by equation (3.3).*

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Proof. The proof follows from Lemma 5. ∎

Lastly, if $\alpha = 1$, we have a continuum of equilibria with platforms locating anywhere on $[\underline{j}, \overline{j}]$ and the identity (and number) of contributing lobbies being indeterminant in general.

The multiplicity of equilibria when $\alpha \leq 1$ is not robust to allowing the probability of an election victory to also depend directly on agendas. In particular, if there are any informed voters (who vote sincerely and are not affected by campaign spending), and there is a lobby that has the same preferred point as the median voter, then platforms converge to the median voter's preferred point.

4   Greedy Candidates and Divergent Agendas

In the previous section we established the key result that, at first glance, seems surprising: Even though the private provision of publicly valuable contributions leads to extreme lobbies being the largest (and in some cases, the only) contributors, and even though the contributions to a candidate are increasing with polarization, the agendas of competing candidates converge in equilibrium. Upon reflection, the result is quite intuitive: although polarization increases the absolute level of a candidate's contributions, it increases the level of the candidate's opponent's contributions even more, so that relative contributions decline. Under our (standard) assumption that it is relative contributions that matter for electoral success, and that politicians care only about electoral success, we obtain policy convergence.

This logic also suggests that in order for polarization to arise in equilibrium, the candidates in the model must value the absolute level of their contributions in addition to (or possibly instead of) their relative contributions. There are a large number of more or less compelling reasons why this might be the case. For example, candidates may derive some pure utility ("ego rents") from receiving a large quantity of contributions. Alternatively, if contributions to a campaign need not be spent on the campaign, and may instead be used to finance the candidate's consumption, then candidates will also value a large absolute level of contributions. Finally, to the extent that candidates may use their own funds to support their campaign, the larger the absolute level of contributions, the less of a candidate's own money will be spent on the campaign, leading candidates to value the absolute level of contributions.

In this section, we establish our agenda polarization result, first for the general case, and then in detail for a simple example.

4.1   General Case

In the previous section, we assumed that the probability of election depended only on campaign contributions in order to remove the most obvious force for convergence of

policies, and hence to focus attention on our mechanism for convergence. Now that we seek to establish the divergence of policies, we allow the probability of winning the election to depend both on campaign spending *and* on the candidates' agendas, so that the incentive to choose divergent policies to maximize campaign contributions must offset the incentive to set agendas at the point preferred by the median voter. We maintain the assumptions on the probability of election that were introduced in Section 2.

Theorem 3 *Under the assumptions* (2.3)*,* (2.4)*,* (2.5)*, and* $B = \{(S, C) \in \mathbb{R}_+^2 | S \leq C\}$*, the candidates' agendas diverge in equilibrium:* $a_1 \neq a_2$*.*

Proof. See Appendix A.1. ∎

The intuition for this result is straightforward. If there is no divergence in agendas, then campaign contributions are zero, and the candidate cannot divert anything for personal consumption. A small divergence in policies generated by one candidate produces positive campaign contributions that can be consumed and that, because of our assumption on candidate preferences, offset the reduction in the probability of winning the election.

4.2 Illustrative Extension

Consider first an extremely simplified extension: Allow the candidates to consume fraction $1 - \gamma$ of the contributions they collect, and make their preferences increasing in consumption and independent of winning the election.

Now, the candidates will choose their platforms with the sole goal of maximizing their own contributions. The candidates are no longer concerned with their opponents' contributions, since they do not care about winning the elections. We immediately obtain the desired result:

Theorem 4 *The unique equilibrium (for any* $\alpha > 0$*) has two candidates tailoring to the extreme lobbies. That is,* $a_1 = \underline{j}$ *and* $a_2 = \overline{j}$*.*

It is worth noting that allowing the candidates to consume a fixed portion of endowments does not affect the (subgame) equilibrium contributions as a function of the platforms. That is, equation (3.6) still holds and does not include γ. The basic intuition for this (somewhat surprising) result comes from the fact that the marginal productivity of the contributions (in affecting the probability of election victory) remains unchanged. While the left candidate's consumption lowers the productivity of the left lobby's contributions, the right candidate's consumption of her (right lobby's) contributions raises the productivity right back up. This observation will be important in allowing us to characterize the equilibria in a richer model of the next subsection (see the derivation (4.6)).

4.3 Richer Model

Again, the only aspect of the model we will alter is the preferences of the candidates. They will now care about *both* personal consumption (out of the campaign contributions) *and* winning the elections. The candidates will get to decide how much to consume out of their campaign fund:

$$\max_{S_i \in [0,C]} v(C - S_i) + p(S_i, S_{-i})W, \tag{4.1}$$

where S_i is the amount the candidate i actually spends on the campaign, C is the amount contributed to the candidate by the lobbies, S_{-i} is campaign spending (net of consumption) of the opponent, and W is the value of winning the election. Of course, this is the problem of a candidate in the third stage of the game. In the first stage, the candidates still get to choose their platforms. We will not allow the candidates in stage 1 to commit to restricting their consumption at the later stage.[6]

The first order condition of the candidate's (ex-post) problem (4.1) is

$$v'(C - S_i) = W\frac{\partial p(S_i, S_{-i})}{\partial S_i} = \frac{WS_{-i}}{(S_i + S_{-i})^2}. \tag{4.2}$$

In order to get a closed form solution, we will consider a particular functional form of the candidates' utility function:

$$v(h) = \ln h.$$

This assumption dramatically simplifies the analysis, as it implies that, from the perspective of the contributors, the candidates' behavior resembles that in the illustrative example above, and the equilibrium in the second stage is still characterized by equation (3.6). The first order conditions (4.2) for the campaign spending become:

$$\frac{1}{C_1 - S_1} = \frac{WS_2}{(S_1 + S_2)^2},$$
$$\frac{1}{C_2 - S_2} = \frac{WS_1}{(S_1 + S_2)^2}.$$

It follows that

$$\frac{S_1}{C_1 - S_1} = \frac{S_2}{C_2 - S_2} = \frac{WS_1 S_2}{(S_1 + S_2)^2}. \tag{4.3}$$

That is, the candidates spend the same fraction γ of their contributions on their campaigns (and consume the rest)! The fraction γ spent on the campaigns solves

$$\frac{1}{\gamma} = \frac{(C_1 + C_2)^2}{WC_1 C_2} + 1. \tag{4.4}$$

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[6]However, the model may be well-suited to study some campaign finance regulation that does restrict such consumption.

Now, the problem in stage 2 of the (extreme) contributor to candidate i is the familiar

$$\max_{c_i} p(S_i(c_i, C_{-i}), S_{-i}(c_i, C_{-i}))\Delta_i - \phi c_i \qquad (4.5)$$

and the intuition developed in Section 4.2 applies. It is worth noting that the level of contributions does affect the fraction γ spent on the campaign. While the contributors do recognize this fact, their behavior is still captured by the familiar equation (3.6), since they are not concerned with the campaign spending per se, but only with the *relative* campaign spending of their preferred candidate (relative to the opponent's).[7] More formally, the marginal effect of the contributions on the probability of winning the election (taking the effect on candidates' behavior into account) is

$$\begin{aligned}
\frac{\partial p(S_i(c_i, C_{-i}), S_{-i}(c_i, C_{-i}))}{\partial c_i} &= \frac{\partial p(S_i, S_{-i})}{\partial S_i}\frac{\partial S_i}{\partial c_i} + \frac{\partial p(S_i, S_{-i})}{\partial S_{-i}}\frac{\partial S_{-i}}{\partial c_i} \\
&= \frac{S_{-i}}{(S_i + S_{-i})^2}\left(\gamma + \frac{\partial\gamma}{\partial c_i}c_i\right) - \frac{S_i}{(S_i + S_{-i})^2}\frac{\partial\gamma}{\partial c_i}C_{-i} \\
&= \gamma\frac{S_{-i}}{(S_i + S_{-i})^2} + \frac{\partial\gamma}{\partial c_i}\left(\frac{\gamma C_{-i}c_i}{(S_i + S_{-i})^2} - \frac{\gamma c_i C_{-i}}{(S_i + S_{-i})^2}\right) \\
&= \frac{\gamma^2 C_{-i}}{\gamma^2(c_i + C_{-i})^2} \quad = \quad \frac{C_{-i}}{(c_i + C_{-i})^2}, \quad (4.6)
\end{aligned}$$

which is exactly what we had in section 3.2. This allows us to arrive at the following:

Theorem 5 *If $\alpha > 1$, the degree of polarization $|a_1 - a_2|$ is decreasing in the value W that candidates attribute to winning the election.*

Proof. See Appendix A.2. ■

Theorem 6 *If $\alpha \leqslant 1$, the equilibrium features complete polarization, regardless of the value of W.*

Proof. This follows directly from Theorem 2. ■

4.4 Generalization

While the nice closed-form solutions of the last subsection were derived under specific functional form assumptions, the key results do carry over to more general environments.

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[7]The key is the fact that the candidates spend the *same* fraction γ of their contributions on campaigning, which was established by equation (4.3). We can thus use the expression $S_j = \gamma(c_i, C_{-i})C_j$.

Consider an environment with the general specification of the candidates' preferences given by equation (2.2). To keep the analysis tractable, we will not let the candidates choose how much to consume — the fraction of contributions spent on campaigning, γ, is given exogenously, as in section 4.2. This is still a meaningful model, since the sophisticated preferences of the candidates affect their platform choices.

Theorem 7 *If $\alpha > 1$, we obtain partial polarization in equilibrium. The extent of polarization is decreasing in the value of winning the elections, W.*

In contrast,

Theorem 8 *If $\alpha \leqslant 1$ and candidates put any weight on their private consumption, we obtain complete polarization in equilibrium.*

Proof. Recall from the analysis in section 3 that a candidate's choice of platform affects the willingness to contribute of her own and her opponent's lobbies symmetrically. Thus, there is no cost to polarization, while there is still the benefit of raising the amount contributed (both to oneself and to the opponent). ∎

This result is especially striking, since the basic mechanism determining the contributions is exactly that of Baron (1994).

5   Implications: Corruption and Polarization

The effect of corruption on polarization in our model depends critically on what is meant by "corruption." On the one hand, corruption can be thought of as the ability of candidates to divert campaign contributions to private consumption. Mechanically, it can then be modeled as a low value of the parameter γ in Section 4.2. From this perspective, corruption is unequivocally associated with a greater degree of polarization.

But on the other hand, corruption can be thought of as the ability of a *successful* candidate to extract large office rents following the election. This then corresponds to a large value of the parameter W in our model. And this form of "corruption" unequivocally implies a *lower* degree of polarization (see Theorem 5).

Any empirical investigation of the relation between corruption and polarization has to take great care in defining the concept of corruption.

6 Conclusion

Most basic models of electoral competition predict that candidate policies should converge. Yet in practice, we observe a great deal of polarization in both candidate policies and in the identity of the lobby groups that support them. In this paper, we have shown that polarization in the form of campaign contributions from extreme lobby groups arises naturally in models of policy formation, lobbying and electoral success, as a result of the public-good characteristic of campaign contributions. In contrast to a widely held intuition, we also show that under standard assumptions lobbyist divergence is consistent with complete policy convergence, albeit to a midpoint or average lobby group rather than a median voter, as candidates seek to maximize the *relative*, and not *absolute*, level of their campaign contributions. However, we go on to show that a small modification of our standard model that allows candidates to value the absolute level of their contributions in addition to their probability of election, either because of "ego rents" or because they are able to divert some contributions for private consumption, restores the incentive for policy divergence.

The amount of policy polarization observed in practice will depend on the relative strength of the motive for maximizing *relative* contributions (to maximize electoral success) versus the strength of the motive for maximizing *absolute* contributions. Differences in these incentives produced by, for example, differences in political institutions across countries or by differences in the rules and technologies that govern campaign spending over time, may explain different outcomes in practice. However, the predictions of the model can be quite subtle. For example, if countries differ in their level of corruption, the result is likely to depend upon the form that corruption takes: when corruption increases the value of an election victory relative to the value of diverting campaign funds, candidate policies might be expected to converge and the absolute level of contributions might fall; if alternatively corruption increases the value candidates place on the absolute level of their campaign contributions, we might expect both the level of contributions and the level of polarization to increase.

References

AUSTEN-SMITH, D. (1987): "Interest Groups, Campaign Contributions, and Probabilistic Voting," *Public Choice*, 54, 123–139.

BARON, D. P. (1994): "Electoral Competition with Informed and Uninformed Voters," *American Political Science Review*, 88(1), 33–47.

BECKER, G. S. (1986): "The Public Interest Hypothesis Revisited: A New Test of Peltzman's Theory of Regulation," *Public Choice*, 49, 223–234.

BERNHARDT, M. D., AND D. E. INGBERMAN (1985): "Candidate Reputations and the 'Incumbency Effect'," *Journal of Public Economics*, 27(1), 47–67.

BLOCH, F. (1993): "Political Support for Minimum Wage Legislation," *Journal of Labor Research*, 14, 187–190.

CHAPPELL, H. (1982): "Campaign Contributions and Congressional Voting: A Simultaneous Probit-Tobit Model," *Review of Economics and Statistics*, 61, 77–83.

DEKEL, E., M. O. JACKSON, AND A. WOLINSKY (2008): "Vote Buying: General Elections," *Journal of Political Economy*, 116(2), 351–380.

——— (2009): "Vote Buying: Legislatures and Lobbying," *Quarterly Journal of Political Science*, 4(2), 103–128.

ESTY, D. C., AND R. E. CAVES (1983): "Market Structure and Political Influence: New Data on Political Expenditures, Activity and Success," *Economic Inquiry*, 21, 24–38.

GARDNER, B. L. (1987): "Causes of U.S. Farm Commodity Programs," *Journal of Political Economy*, 95, 290–310.

GOPOIAN, J. D., H. SMITH, AND W. SMITH (1984): "What Makes PACs Tick? An Analysis of the Allocation Patterns of Economic Interest Groups," *American Journal of Political Science*, 28(2), 259–281.

GUTTMAN, J. M. (1980): "Villages as Interest Groups: The Demand for Agricultural Extension Services in India," *Kyklos*, pp. 122–141.

HERRERA, H., D. K. LEVINE, AND C. MARTINELLI (2008): "Policy Platforms, Campaign Spending and Voter Participation," *Journal of Public Economics*, 92(3-4), 501–513.

JACKSON, M. O., L. MATHEVET, AND K. MATTES (2007): "Nomination Processes and Policy Outcomes," *Quarterly Journal of Political Science*, 2(1), 67–94.

JAMES M. SNYDER, J. (1990): "Campaign Contributions as Investments: The U.S. House of Representatives, 1980-1986," *Journal of Political Economy*, 98, 1195–1227.

——— (1992): "Long-Term Investing in Politicians: Or, Give Early, Give Often," *Journal of Law and Economics*, 35, 15–43.

——— (1993): "The Market for Campaign Contributions: Evidence for the U.S. Senate 1980-1986," *Economics and Politics*, 5, 219–240.

KALT, J. P., AND M. A. ZUPAN (1984): "Capture and Ideology in the Economic Theory of Politics," *American Economic Review*, 74, 279–300.

KIRCHGÄSSNER, G., AND W. W. POMMEREHNE (1988): "Government Spending in Federal Systems: A Comparison Between Switzerland and Germany," in *Explaining the Growth of Government*, ed. by J. Lybeck, and M. Henrekson, vol. 171 of *Contributions to Economic Analysis*, pp. 327–356. Elsevier.

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LANGBEIN, L. I. (1993): "PACs, Lobbies and Political Conflict: The Case of Gun Control," *Public Choice*, 77, 551–572.

LINDBECK, A., AND J. W. WEIBULL (1993): "A Model of Political Equilibrium in a Representative Democracy," *Journal of Public Economics*, 51(2), 195–209.

MCCARTY, N. M., K. T. POOLE, AND H. ROSENTHAL (1997): *Income Redistribution and the Realignment of American Politics*. AEI Press.

——— (2006): *Polarized America: The Dance of Ideology and Unequal Riches*. MIT Press.

OSBORNE, M. J. (1995): "Spatial Models of Political Competition Under Plurality Rule: A Survey of Some Explanations of the Number of Candidates and the Positions They Take," *Canadian Journal of Economics*, 28, 261–301.

PALFREY, T. R. (1984): "Spatial Equilibrium with Entry," *Review of Economic Studies*, 51, 139–156.

PERSSON, T., AND G. TABELLINI (2000): *Political Economics*. MIT Press.

PINCUS, J. (1975): "Pressure Groups and the Pattern of Tariffs," *Journal of Political Economy*, 83, 757–777.

POOLE, K. T., AND T. ROMER (1985): "Patterns of Political Action Committee Contributions to the 1980 Campaigns for the United States House of Representatives," *Public Choice*, 47(1), 63–111.

POOLE, K. T., AND H. ROSENTHAL (1997): *Congress: A Political-Economic History of Roll Call Voting*. Oxford University Press.

ROEMER, J. E. (1991): "A Theory of Class-Differentiated Politics in an Electoral Democracy," UC Davis - Institute of Governmental Affairs, Paper 384.

SALTZMAN, G. M. (1987): "Congressional Voting on Labor Issues: The Role of PACs," *Industrial and Labor Relations Review*, 40, 163–179.

SCHLOZMAN, K. L., AND J. T. TIERNEY (1986): *Organized Interests and American Democracy*. Harper and Row, New York, NY.

TREFLER, D. (1993): "Trade Liberalization and the Theory of Endogenous Protection: An Econometric Study of U.S. Import Policy," *Journal of Political Economy*, 101, 138–160.

WELCH, W. (1979): "Patterns of Contributions: Economic Interest and Ideological Groups," in *Political Finance (SAGE Electoral Studies Yearbook)*, ed. by H. E. Alexander. SAGE, Beverly Hills, CA.

——— (1980): "Allocation of Political Monies: Economic Interest Groups," *Public Choice*, 35, 97–120.

——— (1982): "Campaign Contributions and Legislative Voting: Milk Money and Dairy Price Supports," *Westem Political Quarterly*, 35, 478–495.

A Proofs

A.1 Proof of Theorem 3

The proof is structured along the lines of backwards induction: First, we establish that both candidates will invest strictly positive amounts into their campaigns in all equilibria of every subgame where they have received positive contributions. Second, we establish that the (extreme) lobbies contribute strictly positive amounts to the candidates whenever the candidates' agendas are not identical. Lastly, we obtain the result that the candidates will choose different agendas in any pure strategy equilibrium.

One technical detail needs to be highlighted up front: When we refer to the "equilibrium" of the model, we have in mind the subgame perfect equilibrium of the game. However, strictly speaking, the subgame perfect equilibrium does not exist when the candidates' wealth $\epsilon = 0$. This non-existence is purely technical and has nothing to do with the core mechanism. It arises solely because a class of subgames, in which only one of the candidates has received a strictly positive contribution, have no equilibrium.[8] But these subgames would not arise on an equilibrium path under any reasonable assumption regarding their outcome. To deal with this technical issue we define "equilibrium" as the subgame perfect equilibrium of the game whenever $\epsilon > 0$, and when $\epsilon = 0$, we define equilibrium as the limit of (outcomes of) subgame perfect equilibria of the game as ϵ approaches 0.

Lemma 6 *Campaign spending by both candidates is strictly positive in every equilibrium of every subgame where the candidates had received positive contributions.*

Suppose not. Suppose there is a Nash equilibrium of some such subgame where one of the candidates consumes all contributions and does not campaign. But then contributions of the other candidate cannot be strictly positive — cutting campaign spending in half would increase consumption without changing the probability of winning the election (by assumption (2.4)).

However, zero campaign spending by both candidates is not an equilibrium of the subgame either, as investing an infinitesimal amount would generate a discontinuously larger probability of winning the election (by assumption (2.5)). □

Lemma 7 *Contributions to both candidates are strictly positive in every equilibrium of every subgame where the candidates' agendas are distinct.*

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[8]This non-existence of equilibrium in the subgame is simply a matter of non-existence of the smallest real number greater than 0. However, there is only one sensible outcome of such a subgame — the candidate with the positive contribution wins the election with probability $p(1, 0, \cdot, \cdot)$ and consumes (almost) all of the contribution. Thus, the equilibrium we characterize could be called an "almost subgame perfect" equilibrium.

The logic of the proof of the previous Lemma applies here directly. If only one of the candidates receives a contribution, then the contributors can cut their contributions without suffering a reduced probability of winning. Zero contributions to both candidates is not an equilibrium either, as long as (some) lobbies are not indifferent between the candidates, as an infinitesimal contribution changes payoffs discontinuously (based on the assumptions (2.5) and (2.4) and the previous Lemma). $\square$

Lastly, we establish that the candidates' agendas are not identical in equilibrium. Suppose not. Suppose that there is an equilibrium where $a_1 = a_2 = a^*$. In that case, contributions are 0, as all lobbies are completely indifferent between the two candidates. Consider an infinitesimal deviation by agent 1: $a_1' = a^* - \delta$. As we have already established, such a deviation will generate strictly positive contributions. As we have established in Section 4.3, the candidates will spend the same fraction of their contributions on campaigning, and that implies that the relative contributions are given by equation (3.6).

We have two possible cases to consider. First, suppose that the probability of winning the election changes continuously. This would be the case if $a^* = j_m = \frac{\bar{j}+\underline{j}}{2}$, since this would result in $\frac{\Delta_{\bar{j}}}{\Delta_{\underline{j}}} \approx 1$. In this case, there exists a δ small enough that the gain in utility from consumption will outweigh the possible loss in the probability of winning the election (by assumptions (2.3)). Thus, a^* was not an equilibrium.

In principal, we could have a situation where the probability of winning moves discontinuously in favor of one of the candidates. That would happen if $a^* \neq j_m$. But such a^* could not be an equilibrium, as one of the candidates would have a strict incentive to deviate from a^*. This would be the candidate who would receive greater contributions — if $a^* > j_m$, the deviator would be the left candidate, while in the opposite case, the right candidate would benefit by deviating (toward the midpoint j_m). $\blacksquare$

It is worth noting that the above argument does not hold when there are informed voters (who are unaffected by the campaign spending) voting *deterministically*.[9] In that environment, moving away from the median-voter position lowers the probability of winning the election *discontinuously*.

A.2   Proof of Theorem 5

To establish that polarization increases with greed (decreases with W) when $\alpha > 1$, start with an (interior) equilibrium (a_1^*, a_2^*) which occurs when $W = W^*$. Consider the candidate i's ex-ante maximization problem of choosing the agenda:

$$\max_{a_i} U((1-\gamma)C_i(a_i, a_{-i})) + p_i(\gamma C_i(a_i, a_{-i}), \gamma C_{-i}(a_i, a_{-i}))W, \qquad \text{(A.1)}$$

---

[9]This is *not* the case in Baron (1994), where the voting has to be thought of as probabilistic.

where the fraction γ of contribution spent on campaigning (by both candidates) is an outcome of the third-stage game (see problem (4.1)). It is straightforward to show (and rather intuitive) that γ is increasing in W.

The first-order condition of the problem (A.1) can be expressed as

$$U'((1-\gamma)C_i)\frac{1-\gamma}{\gamma}\frac{\partial C_i}{\partial a_i} = \left[-\frac{\partial p_i}{\partial S_{-i}}\frac{\partial C_{-i}}{\partial a_i} - \frac{\partial p_i}{\partial S_i}\frac{\partial C_i}{\partial a_i}\right]W, \qquad (A.2)$$

where the left-hand side is increasing in polarization, and the right-hand side is decreasing in polarization. Increasing greed (i.e., decreasing W) raises the left-hand side of (A.2) and lowers the right-hand side. To restore the optimality, polarization must then increase.∎

B   Mapping Existing Frameworks into Our Model

Example 1. Informed and Uninformed Voters.   The example closest to our model is that of Baron (1994). However, we are interested only in collective policies (those that affect everyone), and choose to drop the particularistic policy considerations. The mapping from Baron (1994) into our framework is then quite simple: Normalize the total number of voters to one and assume that they are divided into separate groups of informed and uninformed voters with the measure of uninformed voters given by θ. The probability that candidate 1 wins the election, given spending levels and agendas, is then given by

$$p^{UV}(S_1, S_2, a_1, a_2) = \theta\frac{S_1}{S_1 + S_2} + (1-\theta)\frac{a_1 + a_2}{2}. \qquad (B.1)$$

Example 2. Spending to "Get Out the Vote."   Another model that fits neatly into our general framework is a slight modification of Herrera, Levine, and Martinelli (2008), in which campaign spending increases the proportion of potential voters who turn up to vote.[10] There are two office-motivated candidates, who first simultaneously choose their agendas and then their spending. The voters have both idiosyncratic and aggregate (unknown) candidate bias. The voters also care about the policy choices — they have Euclidean preferences with their ideal points distributed uniformly on $[0,1]$. Campaign spending by the candidates is necessary to motivate the electorate to vote. However, the spending is not perfectly targeted and brings some of the opponent's supporters to the polling stations. The fraction of candidate i's supporters who turn out to vote is then $(tS_i + (1-t)S_{-i})$, where $t \in (\frac{1}{2}, 1]$ is the

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[10]We omit the policy preferences of the candidates (parties) which are present in Herrera, Levine, and Martinelli (2008).

accuracy of campaign targeting. As Herrera, Levine, and Martinelli (2008) show, the probability of the (left) candidate 1 winning the election is

$$p^{GV}(S_1, S_2, a_1, a_2) = F\left(a_1 - a_1^2 - a_2 + a_2^2 + 2\beta\left(t - \frac{1}{2}\right)\frac{S_1 - S_2}{S_1 + S_2}\right), \qquad \text{(B.2)}$$

where F is the c.d.f. of the distribution of the aggregate bias for candidate 1, and β is measure of the dispersion of the idiosyncratic bias (which is distributed uniformly on $[-\beta, \beta]$).

C   Extensions and Generalizations

This appendix highlights that the key findings of the paper do not rely on the stark assumptions we made for illustrative purposes.

C.1   Lobbies with Increasing Marginal Cost of Funds

The key to the extreme free-riding results obtained in section 3.1.1 is that the marginal cost of contributing the first dollar is strictly positive, and that the extreme lobbies never tire of contributing (that their marginal cost of doing so does not increase). If the marginal cost of contributing the first dollar is small and/or the marginal cost of contributing rises with the level of contributions, we obtain less extreme results in which more than one lobby may contribute to each candidate. For the purposes of generalization, let the preferences of a lobby j be now represented by

$$u_j(a) = -|a - j|^\alpha - \phi c^\sigma, \qquad \text{(C.1)}$$

where $\sigma > 1$, and note that

$$\lim_{c \to 0} \frac{d\phi(c)}{dc} = \lim_{c \to 0} \sigma \phi c^{\sigma-1} = 0.$$

In this case, the only lobby that may *not* contribute to any candidate is the one that is indifferent between the candidates. For all other lobbies, the first order condition with respect to contribution holds with equality for at least one candidate, or

$$\begin{array}{ll} \frac{\partial p(C_1, C_2)}{\partial C_1} \Delta_j(a_1, a_2) = \phi \sigma c_j(1)^{\sigma-1} & \text{whenever } \Delta_j(a_1, a_2) > 0, \\ \frac{\partial p(C_2, C_1)}{\partial C_2} \Delta_j(a_2, a_1) = \phi \sigma c_j(2)^{\sigma-1} & \text{whenever } \Delta_j(a_2, a_1) > 0. \end{array} \qquad \text{(C.2)}$$

Denoting the marginal productivity of contributions to candidate 1 by $\theta_1 = \partial p(C_1, C_2)/\partial C_1$, and defining θ_2 analogously, we obtain

$$c_j(i) = \left(\frac{\theta_i}{\phi \sigma} \Delta_j(a_i, a_{-i})\right)^{\frac{1}{\sigma-1}} \quad \text{whenever } \Delta_j(a_i, a_{-i}) > 0, \qquad \text{(C.3)}$$

so that the most extreme candidates are still the largest contributors. To complete the characterization, we simply need to note that

$$C_i = \sum_{\Delta_j(a_i, a_{-i}) > 0} c_j(i). \tag{C.4}$$

Given our assumptions on p, it is relative contributions that matter for the probability of election. Noting that by homogeneity of degree zero, $p(C_1, C_2) = p(C_1/C_2, 1) \equiv \tilde{p}(C_1/C_2)$, and hence using symmetry we obtain

$$\frac{\partial p(C_1, C_2)}{\partial C_1} = \tilde{p}'\left(\frac{C_1}{C_2}\right)\frac{1}{C_2},$$
$$\frac{\partial p(C_2, C_1)}{\partial C_2} = \tilde{p}'\left(\frac{C_1}{C_2}\right)\frac{C_1}{C_2^2},$$

so that we can solve for relative contributions

$$\frac{C_1}{C_2} = \frac{\sum_j \left(\max\{\Delta_j(a_1, a_2), 0\}\right)^{1/(\sigma-1)}}{\sum_j \left(\max\{\Delta_j(a_2, a_1), 0\}\right)^{1/(\sigma-1)}}, \tag{C.5}$$

which reduces to

$$\frac{C_1}{C_2} = \frac{\Delta_{\underline{j}}(a_1, a_2)}{\Delta_{\overline{j}}(a_2, a_1)}, \tag{C.6}$$

when $\phi(c) = \phi c$ and $\alpha > 1$.