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Term Structures of Inflation Expectations and Real Interest Rates*

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Abstract

In this paper, I use a statistical model to combine various surveys to produce a term structure of inflation expectations — inflation expectations at any horizon — and an associated term structure of real interest rates. Inflation expectations extracted from this model track realized inflation quite well, and in terms of forecast accuracy, they are at par with or superior to some popular alternatives. Looking at the period 2008–2015, I conclude that long-run inflation expectations remained anchored, and the policies of the Federal Reserve provided a large level of monetary stimulus to the economy.

Keywords: surveys, TIPS, inflation swaps, unconventional monetary policy

JEL Codes: C32, E31, E43, E58

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1 Introduction

After almost two decades of being well anchored (and low), inflation expectations in the United States have received increased interest because of the uncertainty created by the Federal Reserve's unprecedented reaction to the financial crisis and the Great Recession between 2008 and 2015.[1] Another important development since the Great Recession is the temporary end of *conventional* monetary policy, in which the Federal Reserve targets short-term interest rates, due to the federal funds rate reaching the zero lower bound (ZLB). The Federal Reserve began lifting its policy rate in December 2015, after seven years at the ZLB, even though realized inflation and near-term inflation expectations were below the Federal Reserve's long-run inflation target throughout this period and remained below target in December 2015. Moreover, much of the aforementioned reaction to the Great Recession was in the form of *unconventional* monetary policy, in which the Federal Reserve purchased various financial assets in unprecedented quantities. All of this makes tracking the term structures of inflation expectations and of real interest rates, which is a key part of the transmission mechanism of monetary policy, very important.

In this paper, I combine forecasts at various horizons from several surveys to obtain a term structure of inflation expectations for consumer price index (CPI) inflation.[2] Further

[1] Many economists, especially in the popular press, have expressed wildly different views about the impact of the expansion of the Federal Reserve's balance sheet on inflation. For example, in an open letter to the Federal Reserve Chairman Ben Bernanke, 23 economists warned about the dangers of this expansion (see "Open Letter to Ben Bernanke," *Real Time Economics* (blog), *Wall Street Journal*, November 15, 2010, blogs.wsj.com/economics/2010/11/15/open-letter-to-ben-bernanke). A number of other economists argued that this expansion is not a problem (see, e.g., Paul Krugman, "The Big Inflation Scare," *New York Times*, May 28, 2009, www.nytimes.com/2009/05/29/opinion/29krugman.html?ref=paulkrugman). This divide is also apparent within the Federal Open Market Committee. For a dovish view, see various 2010 speeches by President and CEO Charles Evans (Federal Reserve Bank of Chicago, www.chicagofed.org/webpages/publications/speeches/2010/index.cfm), which predict that inflation lower than 1.5% in three years' time is a distinct possibility. For a hawkish view, see various 2010 speeches by President and CEO Charles Plosser (Federal Reserve Bank of Philadelphia, www.philadelphiafed.org/publications/speeches/plosser), which call for the winding down of special Fed programs to prevent an increase in inflation in the medium term.

[2] My analysis focuses on CPI inflation as opposed to, for example, personal consumption expenditures (PCE) price index inflation, gross domestic product (GDP) price deflator inflation, or any of the "core" versions that strip out energy and food prices. PCE inflation has been released since the mid-1990s, but it has been scarcely included in commonly followed surveys. See also the discussion in Section 3.1. The same

combining this term structure of inflation expectations with the term structure of nominal interest rates, I obtain a term structure of real interest rates. In particular, I use inflation expectations from the *Survey of Professional Forecasters (SPF)* published by the Federal Reserve Bank of Philadelphia (FRBP) and the *Blue Chip Economic Indicators* and *Blue Chip Financial Forecasts* published by Wolters Kluwer Law & Business. I use the structure of the Nelson-Siegel (NS) model of the yield curve, which summarizes the yield curve with three factors (level, slope, and curvature), and adapt it to the context of inflation expectations. The end result is a monthly inflation expectations curve — inflation expectations at any horizon from three to 120 months — and, once combined with a nominal yield curve, an associated real yield curve from 1998 to the present.

Turning to the results, I show that the model can accurately summarize the information in surveys with reasonably small measurement errors. The inflation expectations curve has a stable long end around 2.4%, and the lower part of the curve fluctuates considerably more. The real interest rate curve produces results that match closely the yields from Treasury Inflation-Protected Securities (TIPS) for the few maturities the latter is available. I find that inflation expectations from the model track actual (ex-post) realizations of inflation quite well. More specifically, with a few minor exceptions, the forecasts from the model outperform alternatives, including those obtained from a simple univariate statistical model or those obtained using financial variables, and in some cases, the difference in forecast accuracy is statistically significant. Finally, I focus on the period 2008–2015 and use these inflation expectations and real yield curves to study how they evolve during various Federal Reserve actions following the financial crisis of 2008, including the initial quantitative easing (QE1), QE2, Operation Twist, QE3, and the announcement of an explicit inflation target. My results show only a minor decline of around 14 basis points in 10-year inflation expectations after 2008. Moreover, much of this decline comes from lower inflation expectations in the

goes for the core versions. Since GDP price deflator is only available quarterly, it is not a very appealing measure. Finally, most financial contracts that use inflation use some variant of CPI inflation.

short to medium run (up to two years) because the year-3-to-year-10 forward forecast does not show a discernible difference before and after 2008. Thus, long-run inflation expectations remain anchored after the crisis and during various Federal Reserve policies.[3] Other parts of the inflation expectations curve and the entire real yield curve show significant declines after 2008. I find that inflation expectations for short to medium horizons recover after QE1 and QE2 but that these policies still lead to substantial declines in real interest rates, with the short-term real interest rate reaching -2% by the end of 2012. Operation Twist reduces long-term real interest rates to a low of -0.8% in the summer of 2012. As a result, the whole real yield curve is below zero in September 2011, roughly -3.5% below precrisis levels, an unprecedented event in the sample I cover. In short, I conclude that these unconventional policies of the Federal Reserve, combined with the zero short-term policy rate, kept long-run inflation expectations anchored and provided a large level of monetary stimulus to the economy.

The results in this paper and the technology that produces them will be useful to policymakers and other observers in describing how inflation expectations and real interest rates evolve both over time and across horizons. The methodology will also be useful to market participants who want to price securities with returns linked to inflation expectations of an arbitrary horizon, including forward inflation expectations, that is, inflation expectations for a period that starts in the future. It is important to emphasize the ease with which expected inflation of an arbitrary horizon can be computed. In 2016, the FRBP started producing an inflation expectations curve and a real interest rate curve using the methodology in this paper. Using the output of this production — only four numbers at any point in time are necessary — anyone can compute all the objects I mention previously using the simple formulas in the paper.

[3]Beechey, Johannsen, and Levin (2011) find that inflation expectations in the Euro area are more firmly anchored than in the U.S. However, they do so using a sample that stops in 2007, and thus their results are not applicable to the central question in this paper.

This paper is related to a number of recent studies. Two of these papers use a variant of the NS model to investigate related but distinct issues. The rest of the papers use structural relationships to link asset prices with inflation expectations. The former two papers are by Christensen, Lopez, and Rudebusch (2010) and Gürkaynak, Sack, and Wright (2010). The first one estimates a variant of the arbitrage-free NS model using both nominal and real (TIPS) yields. As a result of their estimation, the authors can calculate the model-implied inflation expectations and the risk premium. The second paper uses nominal yield data to estimate a nominal term structure and TIPS data to estimate a real term structure, both by using a generalization of the NS structure (the so-called Nelson-Siegel-Svensson form). The authors then define inflation compensation as the difference between these two term structures. By comparing inflation compensation with survey expectations, they show that it is not a good measure of inflation expectations because it is affected by the liquidity premium and an inflation risk premium. I take the opposite route in this paper, in that I construct a term structure of inflation expectations solely from surveys and compare them with measures from financial variables.[4]

Three important papers set out to obtain a term-structure of inflation expectations using structural finance or macro-finance models. Chernov and Mueller (2012) use a no-arbitrage macro-finance model with two observed macro factors (output and inflation) and three latent factors. They estimate their model using nominal yields, inflation, output growth, and inflation forecasts from various surveys as well as TIPS with a sample that ends in 2008. Inflation expectations also have a factor structure, but unlike the model I use, the factors

[4]Two other papers also use a reduced-form approach. Ajello, Benzoni, and Chyruk (2012) use the nominal yields at a given point in time to forecast inflation at various horizons using a dynamic term structure model that has inflation as one of the factors. The important distinction of this paper relative to some others is that the authors separately model the changes in core, energy, and food prices because, as they show, each of these components has different dynamics. Mertens (2011) sets out to extract trend inflation (long-run inflation) from financial variables and surveys. His data consist of long-horizon surveys, realized inflation measures, and long-term nominal yields. He uses a reduced-form factor model with a level and uncertainty factor that captures stochastic volatility in the trend process. My results regarding long-run inflation expectations are similar to his.

in their model are related to the yield curve and macroeconomic fundamentals, except for one factor that the authors loosely label as the "survey factor," the only one that affects the level of inflation expectations.

D'Amico, Kim, and Wei (2010) use a similar multifactor no-arbitrage term structure model estimated with nominal and TIPS yields, inflation, and survey forecasts of interest rates. Their explicit goal is to remove the liquidity premium that existed in the TIPS market for much of its existence in order to obtain a clean break-even inflation rate that may be useful for identifying real yields, inflation expectations, and the inflation risk premium. Their results clearly show the problem of using raw TIPS data due to the often large and time-varying liquidity premium. Haubrich, Pennacchi, and Ritchken (2012) use a model that has one factor for the short-term real interest rate, another factor for expected inflation rate, another factor that models the changing level to which inflation is expected to revert, and four volatility factors. They estimate their model using data that include nominal yields, survey forecasts, and inflation swap rates.

All of these papers extract inflation expectations from financial variables, including a subset of TIPS yields or break-even rates, swap rates, and nominal yields. There are three important reasons why I think the methodology of this paper has significant value added. First, as Ang, Bekaert, and Wei (2007) and Faust and Wright (2013) show, survey-based inflation expectations are known to be superior to those that come from models with financial variables. Because of their nature, available surveys cover the inflation expectations curve very sparsely — at any point in time, these surveys fill only a handful of points on this curve, and often these points are at nonstandard horizons. Combining these surveys to obtain a smooth curve that shows inflation expectations at any arbitrary horizon seems to be a useful exercise, and the NS yield curve model is a parsimonious way of obtaining such a smooth curve. An important part of the reason why surveys have superior forecasts is because the forecasts that come out of the models that use financial variables inherit the inevitable volatility of the underlying financial variables.[5]

Second, all swap and TIPS variables used in these papers have maturities of two years or longer. As such, it is not entirely clear how they can be used to inform the lower part of the inflation expectations curve. They are typically combined with nominal yields with small maturities, but it seems unclear that the relationship between, say, the 10-year TIPS rate and the 10-year nominal yield is informative enough for the one between their one-year counterparts. In my analysis, I have inflation expectations that cover the lower end of the curve as well as the middle and the long end.

Third, the quality of the inflation expectations that come out of the structural models crucially depends on the stability of the relationship among the variables in the model. There are at least three reasons to think that there may have been structural breaks in the data: (1) TIPS and swap markets are relatively young markets, which evolved significantly since their inception; (2) elevated demand for liquidity and safety and increased borrowing by the federal government after the crisis increased the supply of government bonds, which is a point Christensen, Lopez, and Rudebusch (2010) also make; and (3) any model that uses nominal yields needs to take the ZLB seriously in the estimation of the model. As Swanson and Williams (2014) show, in addition to the federal funds rate, most of the yield curve had been constrained at some point in the 2009–2015 period.[6] Of the papers I cite, both Haubrich, Pennacchi, and Ritchken (2012) and D'Amico, Kim, and Wei (2010) ignore the ZLB even though they use nominal yield data from the ZLB period. Chernov and Mueller (2012) use data that end just prior to the ZLB period: As such, it is unclear how their methodology would handle the ZLB. The issue of structural stability will especially be more relevant moving forward when data of various regimes will be mixed in the estimation of

[5] For example, Haubrich, Pennacchi, and Ritchken (2012) use survey data that are similar to mine as well as swap and nominal yield data, and their forecast accuracy is worse than what I obtain, primarily because it is more volatile.

[6] They show that the relationship between macroeconomic surprises and yields that is strong before the crisis weakens or disappears after 2008.

macro-finance models such as these. In my analysis, because I use only inflation forecasts of forecasters, these regime changes do not affect my analysis.

The paper is organized as follows. In Section 2.1, I describe the model used in the estimation, and in Section 2.2, I provide details about the surveys used as inputs in the estimation. Section 2.3 provides a summary of the full state-space model and its estimation, and Section 2.4 explains how I construct the real interest rate curve. Section 3.1 discusses the estimation results, Section 3.2 provides some robustness analysis, and Section 3.3 compares the resulting inflation expectations curve with some alternatives. Section 4 focuses on the period 2008–2015 and discusses how inflation expectations and real yield curves evolve in this period, focusing on the effects of some key Federal Reserve policies. Section 5 provides some concluding remarks. An Appendix contains additional results and details of the analysis.

2 Model

2.1 Term Structure of Inflation Expectations

The Nelson and Siegel (1987) yield curve model (the NS model) is frequently used both in academic studies and by practitioners. As restated by Diebold and Li (2006), the model links the yield of a bond with τ months to maturity, $y_t(\tau)$, to three latent factors, labeled level, slope, and curvature, according to

$$y_t(\tau) = L_t - \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) S_t + \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) C_t + \varepsilon_t, \tag{1}$$

where L_t, S_t, and C_t are the three factors; λ is a parameter; and ε_t is a measurement error. The factors evolve according to a persistent process, inducing persistence on the yields across time. Numerous studies show that the NS model is a very good representation of the yield curve both in the cross section and dynamically.[7] This model is very popular for at least

[7] The original NS model starts with the assumption that the forward rate curve is a variant of a Laguerre polynomial, which results in the function in (1) when converted to yields. As such, it has no economic foundation, unlike some of the papers cited in the introduction that contain asset-pricing models. For an

three reasons. First, the factor loadings for all maturities are characterized by only one parameter, λ. This makes scaling up by adding more maturities relatively costless. Second, the specification is very flexible, capturing many of the possible shapes the yield curve can take: (1) can lead to an upward or downward sloping yield curve, which has at most one peak, whose location depends on the value of λ. Third, it imposes a degree of smoothness on the yield curve that is reasonable; wild swings in the yield curve at a point in time are not common.[8]

Many of the properties of the yield curve, such as smoothness and persistence, are also shared by the term structure of inflation expectations. Thus, at least from a curve-fitting perspective, modeling the latter by the NS model is not too much of a stretch. Defining $\pi_t(\tau)$ as the τ-month inflation expectations from the end of month t to the end of month $t+\tau$, I assume that it follows the process

$$\pi_t(\tau) = L_t - \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) S_t + \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) C_t + \varepsilon_t. \tag{2}$$

According to this specification, L_t captures long-term inflation expectations, S_t captures the difference between long- and short-term inflation expectations, and C_t captures higher or lower medium-term expectations relative to short- and long-term expectations.

I set up a state-space model, following the approach in Diebold, Rudebusch, and Aruoba (2006), where (2) constitutes a generic measurement equation. To complete the state-space representation, I assume that the three latent factors follow the independent AR(3) processes

$$\begin{aligned} L_t &= \mu_L + \rho_{11}(L_{t-1}-\mu_L) + \rho_{12}(L_{t-2}-\mu_L) + \rho_{13}(L_{t-3}-\mu_L) + \eta_t^L \\ S_t &= \mu_S + \rho_{21}(S_{t-1}-\mu_S) + \rho_{22}(S_{t-2}-\mu_S) + \rho_{23}(S_{t-3}-\mu_S) + \eta_t^S \\ C_t &= \mu_C + \rho_{31}(C_{t-1}-\mu_C) + \rho_{32}(C_{t-2}-\mu_C) + \rho_{33}(C_{t-3}-\mu_C) + \eta_t^C, \end{aligned} \tag{3}$$

extensive survey, see Diebold and Rudebusch (2013).

[8] The slope factor in Diebold and Li (2006) is defined as $-S_t$. The three factors are labeled as such because, as Diebold and Li (2006) demonstrate, $L_t = y_t(\infty)$, $S_t = y_t(\infty) - y_t(0)$ (with the definition adopted in this paper), and the loading on C_t starts at zero and decays to zero affecting the middle of the yield curve where the maximum loading is determined by the value of λ.

where $\eta_t^i \sim N(0, \sigma_i^2)$ and $cov\left(\eta_t^i, \eta_t^j\right) = 0$ for $i, j = L, S$, and C and $i \neq j$.[9]

Once the model is cast in state space by combining (2) and (3), estimation and inference are standard. As I explain in the Appendix, due to the timing of surveys, there are many missing observations in the data set I use. As shown in the literature, the Kalman filter and the state-space methods associated with it are well suited to handle them.[10]

Next, I turn to developing some results that will facilitate mapping various observables into measurement equations. First, I define inflation between two arbitrary dates. The Bureau of Labor Statistics, the statistical agency that measures the CPI in the United States, uses the simple growth rate formula to compute inflation. Using this formula in this context, however, leads to a nonlinear state space, which is considerably more difficult to handle. Thus, I define inflation using continuous compounding instead.[11] More specifically, let P_t be the CPI price level at the end of month t. I define

$$\pi_{t \to s} \equiv 100 \times \frac{12}{s-t} \left[\log(P_s) - \log(P_t)\right] \tag{4}$$

as the annualized inflation rate between the end of month t and the end of month s. In terms of the notation in (2), $\pi_t(\tau)$ is represented as $\pi_{t \to t+\tau}$ for $\tau > 0$. This notation is quite flexible. For example, $\pi_{t \to t+12}$ denotes the expected inflation between the end of the month in t to the end of the month in month $t+12$, a conventional one-year-ahead forecast, whereas $\pi_{t+3 \to t+6}$ is the expected quarterly inflation starting from the end of month $t+3$, which is a forward forecast. The former can be immediately written as $\pi_t(12)$, but to convert the latter to this notation, the following result is useful.

Using properties of continuous compounding, if we have $\pi_{t \to t+s}$, $\pi_{t \to t+r}$, and $\pi_{t+r \to t+s}$,

[9] In Section 3.2, I consider a VAR(1) containing all three factors as an alternative. I show that model selection criteria point to the independent AR(3) specification, and I use this as the benchmark.

[10] See, for example, Diebold, Rudebusch, and Aruoba (2006) for the details of estimating the NS model; Aruoba, Diebold, and Scotti (2009) for a specific example with a state-space model with many missing observations; and Durbin and Koopman (2012) for a textbook treatment of both.

[11] In practice, this turns out to be a very minor issue. See footnote 30 in the Appendix.

where $s > r > 0$, then these are related by

$$\pi_{t+r\to t+s} = \frac{s}{s-r}\pi_{t\to t+s} - \frac{r}{s-r}\pi_{t\to t+r}. \tag{5}$$

As an intuitive example of this result, the formula yields $\pi_{t\to t+6} = 0.5\times(\pi_{t\to t+3} + \pi_{t+3\to t+6})$, which shows that the six-month inflation rate is simply the average of the two three-month inflation rates, one from today to three months from now and the other between three and six months from now. In general, inflation between two dates is equal to the average monthly inflation over the period from one to the other.

Finally, to map any inflation measure $\pi_{t+\tau_1\to t+\tau_2}$ with $\tau_2 > \tau_1 \geq 0$ into the factor model in (2), it's easy to show that it can be written as

$$\pi_{t+\tau_1\to t+\tau_2} = L_t + \frac{e^{-\lambda\tau_1} - e^{-\lambda\tau_2}}{\lambda(\tau_2-\tau_1)}(C_t - S_t) + \left(\frac{\tau_1 e^{-\lambda\tau_1} - \tau_2 e^{-\lambda\tau_2}}{\tau_2-\tau_1}\right)C_t. \tag{6}$$

As should be clear from inspecting (6), using continuous compounding, I preserve the linearity of the state-space system, which would not be possible with a simple growth formula. Moreover, computing this for any (τ_1, τ_2) would require knowing only the factors in the current period and the estimated value of λ.

2.2 Measurement Equations

With the results of the previous section in hand, all that remains to be done is to map observed measures of inflation expectations into the framework described so far. Letting x_t^i be a generic observable, converted to annualized percentage rates, this amounts to writing

$$x_t^i = \begin{pmatrix} f_L^i & f_S^i & f_C^i \end{pmatrix}\begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + \varepsilon_t^i,$$

where $\{f_L^i, f_S^i, f_C^i\}$ are the loadings on the three factors and $\varepsilon_t^i \sim N(0, \sigma_i^2)$ is an idiosyncratic error term, which accounts for deviations from the factor model. Survey-specific details are provided in the Appendix.

2.3 State-Space System and Methodology

The preceding section and the details in the Appendix show the measurement equations of the observable variables I use in my analysis – all in all, 59 variables. Combining all the measurement equations and the transition equation for the three factors, I obtain a state-space system

$$\begin{aligned} \mathbf{x}_t &= \mathbf{Z}\boldsymbol{\alpha}_t + \boldsymbol{\varepsilon}_t & (7) \\ (\boldsymbol{\alpha}_t - \boldsymbol{\mu}) &= \mathbf{T}(\boldsymbol{\alpha}_{t-1} - \boldsymbol{\mu}) + \boldsymbol{\eta}_t & (8) \end{aligned}$$

with

$$\boldsymbol{\varepsilon}_t \sim N(0, \mathbf{H}) \text{ and } \boldsymbol{\eta}_t \sim N(0, \mathbf{Q}), \tag{9}$$

where the notation follows the standard notation in Durbin and Koopman (2012). The vector $\mathbf{x}_t$ is a 59×1 vector containing all observed variables in period t, and $\boldsymbol{\alpha}_t$ is a 9×1 vector that collects the three inflation expectation factors and their two lags in period t:

$$\boldsymbol{\alpha}_t = \begin{bmatrix} L_t & S_t & C_t & L_{t-1} & S_{t-1} & C_{t-1} & L_{t-2} & S_{t-2} & C_{t-2} \end{bmatrix}'. \tag{10}$$

The vector $\boldsymbol{\varepsilon}_t$ contains the measurement errors, and thus $\mathbf{H}$ is a diagonal matrix with

$$\mathbf{H} = diag\left(\sigma_1^2, \sigma_2^2, ..., \sigma_{35}^2\right). \tag{11}$$

The measurement matrix $\mathbf{Z}$ collects the factor loadings described in the Appendix and is given by

$$\mathbf{Z} = \begin{bmatrix} f_L^1 & f_S^1 & f_C^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ f_L^2 & f_S^2 & f_C^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ f_L^{59} & f_S^{59} & f_C^{59} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{12}$$

The transition matrix $\mathbf{T}$ takes the form

$$\mathbf{T}=\begin{bmatrix} \rho_{11} & 0 & 0 & \rho_{12} & 0 & 0 & \rho_{13} & 0 & 0 \\ 0 & \rho_{21} & 0 & 0 & \rho_{22} & 0 & 0 & \rho_{23} & 0 \\ 0 & 0 & \rho_{31} & 0 & 0 & \rho_{32} & 0 & 0 & \rho_{33} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \tag{13}$$

and the constant $\boldsymbol{\mu}$ is given by

$$\boldsymbol{\mu}=\left[\begin{array}{ccccccccc} \mu_L & \mu_S & \mu_C & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]'. \tag{14}$$

Finally, $\mathbf{Q}$ is a diagonal matrix with

$$\mathbf{Q}=diag\left(\eta_t^L,\eta_t^S,\eta_t^C,0,0,0,0,0,0\right). \tag{15}$$

The model is estimated using maximum likelihood via the prediction-error decomposition and the Kalman filter. A total of 75 parameters are estimated, where all but 16 of these parameters are measurement error variances. Estimates of the level, slope, and curvature factors are obtained using the Kalman smoother because the paper's main application is about a historical analysis. Using the Kalman filter to produce the factors makes them have slightly more high-frequency movements, but the main conclusions still go through.

2.4 Term Structure of Real Interest Rates

The Fisher equation links the nominal interest rate to the real interest rate and expected inflation. Generalizing to a generic maturity, the linearized version can be written as

$$y_t\left(\tau\right)=r_t\left(\tau\right)+\pi_t\left(\tau\right), \tag{16}$$

where $y_t\left(\tau\right)$ and $r_t\left(\tau\right)$ are the nominal and real continuously compounded interest rates or yields for a bond that is purchased in period t and matures in period $t+\tau$.

Since I already obtained $\pi_t(\tau)$, all I need to do to obtain the term structure of *real* interest rates is to obtain the term structure of *nominal* interest rates. To that end, I use the estimated yield curve as computed by the Board of Governors of the Federal Reserve System, following Gürkaynak, Sack, and Wright (2007). This yield curve is estimated using a generalized specification of the NS model, the so-called Nelson-Siegel-Svensson specification, which adds one more factor to the original NS model. Unlike the dynamic approach I take in this paper or the approaches of Diebold and Li (2006) or Diebold, Rudebusch, and Aruoba (2006), Gürkaynak, Sack, and Wright (2007) treat the factors as parameters and estimate their model every period. The relevant parameters have been reported for every day since June 1961, which enables me to compute $y_t(\tau)$ for any arbitrary maturity τ. Since the frequency in this paper is monthly, I take averages over a month to calculate the monthly yields and compute the continuously compounded real interest curve, $r_t(\tau)$, as the difference between $y_t(\tau)$ and $\pi_t(\tau)$ following (16).[12]

3 Inflation Expectations and Real Interest Rate Curves

I estimate the state-space model presented in the previous section on a sample that covers the period from January 1998 through July 2016. I will return to the choice of the start date of the sample and also consider a slightly longer sample starting in 1992 in Section 3.2.

3.1 Estimation Results

Table 1 presents the estimated parameter values. Panel (a) shows the estimated transition equation parameters. All three factors are persistent, but roots of characteristic polynomials (not reported) show that all are comfortably covariance stationary. The long-run average of the level factor is 2.46%. It is well known (see, e.g., Hakkio, 2008) that CPI inflation and

[12] It is important to note that by decomposing the nominal rate as above, I implicitly include the inflation risk premium in $r_t(\tau)$. In Section 4.2, I discuss how this approach may affect my findings.

personal consumption expenditures (PCE) inflation have about a 0.4% difference on average. This means that the average long-run inflation expectation in the sample stays very close to 2% when expressed in terms of the Federal Reserve's preferred inflation measure and its official target. The average slope of the inflation expectations curve, which is defined as the difference between long- and short-term expectations, is mildly positive at 43 basis points. The curvature factor has a mean of -18 basis points, showing that medium-term forecasts are typically lower than short- and long-term forecasts, giving the inflation expectations curve a mild U shape on average, though this estimate is not statistically significant. The variances of the transition equation innovations are small.

Panel (b) of Table 1 shows that λ is estimated as 0.12, which means that the loading on the curvature factor is maximized at just under 15 months. The estimated measurement error variances show that as the forecast horizon of the variable increases, the measurement error variances become smaller, indicating a better fit of the model. All variances are very small, with only a couple of exceptions, indicating a very good fit of the NS model to this expectations data.

Given these estimated parameters, I obtain estimates of the three factors using the Kalman smoother, which are presented in Figure 1.[13] The level factor has a slight downward trend, which is possibly the continuation of the downward trend that starts after the 1980s, as I demonstrate in Section 3.2. This trend, however, is quite negligible (about 1.5 basis points per year in the whole sample) and dies out around 2005. The slope factor is positive for much of the sample, falling below zero briefly just before the 2001 recession, in 2006, and during the Great Recession, prior to the financial crisis of 2008. As I show in detail in Section 4.1, during the financial crisis, the inflation expectations curve sharply steepens,

[13] In all figures, the two National Bureau of Economic Research (NBER) recessions in the sample are shown with gray shading, and September 2008 is shown with a vertical line. The latter is arguably the height of the financial crisis, and significant changes occur in both the inflation forecasts and the financial variables introduced later. Also, where relevant, I use red dashed lines to denote pointwise 95% confidence bands.

with much of the movement coming from the sharp fall of the short end. This is also visible in this figure as the sharp increase in the slope near September 2008. The curvature factor has smaller and nonsystematic fluctuations.

The main output from the estimation that is of interest is the inflation expectations curve itself. In Figure 2, I show the time series of some selected inflation expectations in the full sample: those at a 6-month and at 1-, 5-, and 10-year horizons. It is apparent that as the forecast horizon increases, the forecasts become smoother; note the range of the y-axis in the figures. The financial crisis and the Great Recession clearly change the behavior of these forecasts drastically, and I focus on the post-2008 period in Section 4.1.

Figure 3 shows some of the real interest rates obtained as described in Section 2.4 along with some alternatives where available. The short-term rates (e.g., the six-month rate) show significant cylicality, rising in booms and declining rapidly in recessions. For the one-year and the 10-year rates, I also show the rate reported by the Federal Reserve Bank of Cleveland, following the methodology of Haubrich, Pennacchi, and Ritchken (2012). Focusing on the one-year rate, before the financial crisis the two measures move fairly closely, though the Cleveland measure is substantially more volatile. After the crisis, there is significant disagreement both in the level and in volatility. The nominal yield is close to zero and near constant after 2008. The inflation expectation I extract from surveys at this horizon, shown in panel (a) of Figure 5, is similar to precrisis values and is fairly constant. As a result, the real interest rate I extract is negative and near constant. The Cleveland measure of inflation, shown in Figure 5, is considerably more volatile and given the same near-constant nominal yields induces volatility in the real rate in Figure 3. While it is obviously difficult to pick one as a "better" measure than the other, the differences are striking. It is important to mention that the Cleveland measure uses nominal yields of various maturities, but its swap data have a two-year or higher maturity, in addition to survey forecasts similar to what I use. As such, it is not entirely clear how the inflation expectations and real rates at horizons less than two years are identified using financial variables.

The remaining panels of Figure 3 show the 5- and the 10-year real rates, along with the corresponding TIPS yield, which is supposed to measure the same underlying concept.[14] They disagree significantly during the financial crisis when TIPS yields are pushed to much higher levels due to the developments in the financial markets. Even with this disagreement, the correlation between the two series is 0.94 for the 5-year horizon and 0.96 for the 10-year horizon. This close match is reassuring, and it means the real interest rates I compute at any arbitrary horizon are quite useful, especially because TIPS yields are available only for some limited number of maturities. The Cleveland real rate measure for the 10-year horizon is also highly correlated with that from my model at 0.95, though differences in level persist throughout the sample.

3.2 Robustness of Results

In selecting my benchmark model, I made a number of choices. In this section, I briefly summarize the results under some alternative choices. First, I consider a longer sample, starting in 1992. Given that the 10-year forecast for the *SPF* starts in the last quarter of 1991, starting the estimation earlier than 1992 is not sensible. Figure 4 compares the results of this estimation with the benchmark results. The left panels show the three factors and the right panels show three selected inflation forecasts. When I start the estimation in 1992, the level factor becomes near unit-root (but still stationary) due to the significant downward trend that settles down around 1998. This in turn closely follows the trend in the long-term forecasts in the surveys I use. Evidently, the forecasters did not adjust their long-term forecasts downward quickly even though inflation settled down to much lower levels in the early 1990s. This downward trend is the reason I start my estimation in 1998. Despite

[14] In fact, since TIPS break-even rates are defined as the difference between nominal yields and the TIPS rate, and I define my real rate as the difference between the nominal yields and my inflation expectations, the difference between TIPS yields and my real interest rate is by construction equal to the difference between the break-even rate and my inflation expectations.

this trend, however, the factors and the forecasts are virtually identical to the benchmark estimates in the sample that starts in 1998.

I use independent AR(3)s in the transition equation, which do not allow for any correlation between factors. The extracted factors show some mild correlation: 0.22 between level and slope and -0.21 between slope and curvature. To investigate if the assumed transition equations are too restrictive, I estimate the model with a VAR(1) in factors as the transition equation. The extracted factors remain very close to the benchmark ones with correlations of 0.98 or higher. Comparing the Schwartz or Akaike information criteria the benchmark model is preferred.[15]

In some yield-curve applications, the third factor is only marginally significant, and some authors prefer a two-factor model for parsimony. To investigate this in my model, I reestimate the model without the curvature factor, which eliminates five parameters. Interestingly, the log likelihood falls by 77 log-points. This difference is large enough to overweigh the parsimony it achieves, and the information criteria prefer the benchmark specification.

3.3 Comparison of Forecasts

In this section, I compare the forecasts from the model with some alternatives. First, I present results that compare the model's forecasts with one that comes from an unobserved-components stochastic-volatility model as proposed by Stock and Watson (2007) that is also estimated in Aruoba and Schorfheide (2016). Second, I compare the forecasts from the model with those from the Michigan Survey of Consumers, which was not used in the estimation of the model. Third, I compare the model forecasts with measures obtained from financial variables. These variables are followed widely and considered by many as gauges of the market's inflation expectations.

[15] The two models have the same number of parameters, and thus the difference in the log-likelihood, which is about 22 log points, means that the benchmark model fits the data better, indicating that capturing higher order autoregressive dynamics is much more important than cross-factor correlations.

I present two sets of results. Figures 5 and 6 show plots of the forecasts from the model, the actual inflation that was realized, and a number of alternative measures.[16] Table 2 presents formal forecast comparison test results using realized inflation. The first column reports the root-mean-square error (RMSE) of the model forecast, the second column reports the RMSE of the alternative measure considered, and the third column shows the number of observations available for each comparison.[17] Boldface in a given column indicates the rejection of the null of equal forecast accuracy in favor of the forecast in that column using the Diebold and Mariano (1995) test with the squared-error loss function. Comparing the forecast accuracy of the model with an alternative is useful because the ultimate goal of the technology developed in this paper is to construct good forecasts of inflation at various horizons.[18]

3.3.1 Comparison with the Unobserved-Components Stochastic-Volatility Model

As formulated by Stock and Watson (2007), the unobserved-components stochastic-volatility (UCSV) model decomposes current inflation, π_t, into a slow-moving trend, τ_t, and serially uncorrelated short-run fluctuations where the latter and the innovations to the trend exhibit

[16] The actual inflation measure is the appropriate difference of the natural logarithm of CPI, as extracted from Federal Reserve Economic Data (FRED) in July 2016, with the FRED code CPIAUCSL.

[17] The RMSEs for the model forecast differ accross panels only due to differences in the number of observations and/or sample.

[18] One may take the point of view that if the underlying inflation forecasts are not good in a forecast accuracy sense, then the resulting combination would not be good either, and this is not necessarily a problem. I want to demonstrate, however, that the resulting inflation forecasts indeed have good forecasting properties. It is important to recognize that my model forecast is as good as the inputs that are used to construct it. If the survey forecasts were biased, for example, it would not be surprising to find that the output of my model does not line up well with realized inflation. Thus, any positive results show the joint success of the model that I use, as well as the survey inputs that go into the model.

stochastic volatility. More specifically,

$$\begin{aligned}
\pi_t &= \tau_t + \sigma \exp(h_{\epsilon,t})\epsilon_t, \\
\tau_t &= \tau_{t-1} + (\varphi\sigma)\exp(h_{\eta,t})\eta_t \\
h_{j,t} &= \nu_j h_{j,t-1} + \sqrt{1-\nu_j^2}\sigma_j\omega_{j,t} \\
\epsilon_t, \eta_t, \omega_{j,t} &\sim N(0,1) \text{ with } j \in \{\epsilon, \eta\}.
\end{aligned} \tag{17}$$

I estimate this model and extract a measure of trend denoted by $\hat{\tau}_t$ using Bayesian methods designed for state-space models with stochastic volatility developed by Kim, Shephard, and Chib (1998) that are also used by Schorfheide, Song, and Yaron (2014) and Aruoba and Schorfheide (2016). Parameter estimates are provided in Table 3 in the Appendix. Given the model, the forecast of inflation of any horizon in period t is simply $\hat{\tau}_t$. As Stock and Watson (2007) put it, this model takes current inflation, filters out what it considers to be transitory noise, and uses the remainder as the forecast of inflation at any horizon. Faust and Wright (2013) demonstrate that this simple univariate model has superior or similar forecast accuracy relative to many others, including those that use information from other variables.

The first panel of Table 2 shows the results from comparing the forecast of the UCSV model with the model forecast. In short forecast horizons, the UCSV model provides a better forecast than the model forecast, but the RMSEs are close, and the difference in accuracy is not statistically significant. For forecasts for horizons of two years and longer, the model forecast has lower RMSEs than the UCSV model, and the difference becomes significant for horizons longer than four years. Given the success of the UCSV model established by Faust and Wright (2013), the fact that my model forecast is at least as good as the forecast from the UCSV model is noteworthy.[19]

[19] I also compared the model forecast with a simpler no-change forecast, one that assumes that the forecast of any horizon is equal to the annual inflation at the point of the forecast. The model forecast is superior to this forecast, and this is statistically significant for all horizons.

3.3.2 Comparison with the Michigan Survey

It is well known (see, e.g., Ang, Bekaert, and Wei, 2007) that professional forecasters produce superior forecasts relative to households. The second panel of Table 2 shows the results comparing the two available household-based forecasts from the Michigan Survey with those from my model: a one-year and a five-year forecast. In both cases, the model forecast produces much better forecasts with an RMSE improvement of over 30% and 70%, respectively, with the difference statistically significant.

3.3.3 Comparison with Measures Derived from Financial Variables

It is well understood that many financial variables contain information about the market participants' inflation expectations. Perhaps the two financial instruments that have the most information are inflation swaps and TIPS. An inflation swap is an agreement in which one party makes periodic payments to another party, which are linked to inflation realized in the future, in exchange for a fixed payment up front. In a perfect world – one without risk premia and one in which all assets are arbitrarily liquid – this fixed payment would be a good estimate of the two participants' inflation forecasts. TIPS, on the other hand, are bonds issued by the U.S. Treasury, with yields that are linked to future realized inflation rates. Again, in a perfect world, the difference between the yield on a TIPS at a certain maturity and the U.S. Treasury nominal yield at the same maturity, the so-called break-even rate (or inflation compensation), would be a good estimate for the inflation expectations of the market.[20]

As it turns out, we do not live in such a perfect world (one without risk premia and arbitrarily liquid asset markets). The liquidity of the TIPS market has changed significantly since its inception, which makes it very difficult to use the break-even rate as a direct

[20]The TIPS rate is linked explicitly to seasonally adjusted CPI. Swap rates are linked to seasonally unadjusted CPI. Although there does not seem to be any discernible seasonality in swap rates, this certainly complicates models in which swap rates are used.

estimate of inflation expectations. Similar problems also plague the inflation swaps market.[21] D'Amico, Kim, and Wei (2010) use a no-arbitrage asset pricing model to produce market inflation expectations using the TIPS break-even rate, as well as nominal U.S. Treasury yields. Haubrich, Pennacchi, and Ritchken (2012) follow an approach that is broadly similar in that they consider an asset pricing model that has implications for nominal yields and swap rates, and the model is estimated using these observables, as well as some forecasts from the *Blue Chip Economic Indicators* and the *SPF*. I consider these two forecasts as cleaned versions of the TIPS data (for D'Amico, Kim, and Wei, 2010) and swap rates (for Haubrich, Pennacchi, and Ritchken, 2012).[22]

Figure 5 shows the one-year swap rate and the results from Haubrich, Pennacchi, and Ritchken (2012), labeled "Cleveland Fed." Two things are very clear. First, the Cleveland Fed forecast and the swap rate, when it is available, are significantly more volatile than the model forecast. Second, the swap rate takes a significant dive near the financial crisis, falling to nearly -4%, while the *SPF* and model forecasts remain slightly above 1%. It is evident that the raw swap rate suffers from the problems I list previously. Although not as extreme as the swap rate, the Cleveland Fed forecast also displays similar behavior, falling below zero in early 2009.

Figure 6 shows the 10-year swap rate, the TIPS break-even rate, and the results from D'Amico, Kim, and Wei (2010), labeled "DKW Inflation Expectation." Also shown is the Cleveland Fed forecast, with TIPS-related variables in the top panel and the swap rate-related variables in the bottom panel. The TIPS break-even rate clearly displays very differ-

[21] Lucca and Schaumburg (2011) provide a good summary of these problems and some others that make TIPS and swap rates noisy indicators of inflation expectations.

[22] Both of these papers start their estimations prior to the introduction of the respective financial asset, using only nominal yields. As such, their reported inflation expectations can be considered as being related to TIPS and swaps only after 1999 for TIPS and 2004 for swaps. The forecasts of D'Amico, Kim, and Wei (2010) are graciously provided by the Federal Reserve Board. The forecasts of Haubrich, Pennacchi, and Ritchken (2012) are available from the website of the Federal Reserve Bank of Cleveland (www.clevelandfed.org). The forecasts of other studies cited in the Introduction are not publicly available; therefore, I am not able to use them in this comparison.

ent behavior before 2003 and again after the financial crisis in 2008 relative to both actual inflation and the model forecast. A similar conclusion also applies for the swap rate, which is available for a shorter sample. Both rates fall below zero during the financial crisis. Both DKW and the Cleveland Fed forecasts behave much better relative to the raw financial variables, although they are still more volatile relative to the model forecast. The same conclusions apply for the five-year forecasts (not shown).

The rest of Table 2 shows forecast comparison results for the variables discussed in this section. The raw financial variables, shown in the third and fifth panels, produce substantially worse forecasts relative to the model forecast, with improvements in the RMSEs of the latter as large as 53% for the five-year TIPS break-even rate. Looking deeper into the source of the large RMSE for this particular variable, it is twice as volatile as the model forecast and has a larger bias. The DKW and Cleveland Fed forecasts produce results that are much better, with RMSEs that are roughly half to two-thirds of the raw financial variables and near the values attained by the model forecast. The model forecast is significantly more accurate than the 5-year and 10-year DKW forecasts. The two-year DKW forecast has a lower RMSE than that of the model forecast, and the difference is marginally statistically significant. The model forecast comes out significantly more accurate than the 10-year Cleveland Fed forecast, with the model forecast producing better RMSEs in all other cases. This is especially interesting because the Cleveland Fed model uses survey forecasts as I do and adds nominal yields and swap rates as additional observables to a macro-finance model; this evidently reduces the forecast accuracy of the model relative to a simple model such as mine that uses only surveys.

I view the results of this section as making a strong case for the usefulness of the model forecast relative to a number of alternatives related to the financial markets. This strong case is also why I chose not to use any financial variables in the model developed in this paper. The results in this section are also a confirmation and generalization of the results of Gürkaynak, Sack, and Wright (2010), who show that inflation compensation from TIPS has been far more volatile than survey expectations from the *Blue Chip* surveys and that the two have no consistent relationship.

4 Unconventional Monetary Policy: 2008–2015

The results so far show that the inflation expectations curve obtained from the model and the real yield curve obtained following the procedure discussed in Section 2.4, to the extent that they can be compared with alternatives or actuals, seem to be good measures. In this section, I put them to use and analyze how they evolved between 2008 and 2015. This is a very important period on which to focus. The Federal Reserve by and large had succeeded in anchoring long-run inflation expectations starting in the late 1990s. Between 1999 and 2007, the 10-year forecast from the model fluctuates between 2.3% and 2.7%, with an average of 2.47%. Since *conventional* monetary policy, targeting the federal funds rate, which was the main tool that the Fed used to keep inflation (and its expectations) under control in this period, ceased to be effective with the nominal interest rates hitting the ZLB in early 2009, it is important to see whether long-run inflation expectations become unhinged. Moreover, since the Federal Reserve started conducting various *unconventional* policies in response to and following the financial crisis, it is also interesting to analyze the impact of these policies on the inflation expectations curve.[23] Finally, some, if not most, of these unconventional policies were aimed at providing additional stimulus for the real side of the economy, and by looking at the real yield curve, we can investigate whether these policies succeeded.[24]

[23] At first pass, one may think that some (perhaps all) of this analysis can be conducted by looking at the survey forecasts I use in my model and at some financial variables. This, however, is not very reasonable for three reasons. First, the survey variables for the long-horizon forecasts are only available quarterly, which would make it difficult to pin down to the month when they change. Second, given the sparse structure of forecasts in surveys, it will be difficult to detect patterns at certain horizons. Third, as shown in the previous section, even the "clean" version of variables based on financial variables does not perform well in forecast comparisons, and in particular, they are too volatile to detect patterns.

[24] Here I have in mind a very general model in which the real interest rate is one of the key determinants of current economic activity. A decline in the real interest rate stimulates private consumption demand by

I focus on six specific events in this period: the financial crisis, QE1, QE2, Operation Twist, the Federal Reserve's explicit adoption of an inflation target, and QE3.[25] Although determining the start of the financial crisis is difficult, I use September 2008, which is when many of the major events happened. QE1, which consists of a number of large-scale asset purchase programs, started in December 2008 and concluded by August 2010, after increasing the balance sheet of the Fed by some $1.75 trillion. QE2, which consisted of a plan to purchase $600 billion of long-term Treasury securities, was put in place between November 2010 and June 2011. Operation Twist (formally known as the Maturity Extension Program) aimed at increasing the maturity of the Fed's assets and was in effect between September 2011 and December 2012. The Federal Reserve announced a formal inflation target of 2% (based on annual changes in the personal consumption expenditures price index) on January 25, 2012. Finally, a final round of QE was announced on September 2012, and it ended in October 2014. June 2013 is also an important month in that the chairman of the Federal Reserve announced that the Fed would start "tapering" its purchases later in the year. This caused a major reaction by financial markets, one that seemed unreasonably large, and is referred to as the "taper tantrum." In the next section, I look at how the inflation expectations and real interest rate curves change around these dates.

Before I do so, however, I want to be clear that my goal is not to do a formal event study or a statistical analysis that singles out the *effects* of a policy change but more a descriptive analysis based on the outputs of my model. Doing the former would require a strong model (perhaps a dynamic stochastic general equilibrium model or an otherwise structural model) to compute the counterfactual of what would have happened in the absence of these events or policies. While interesting, this is very challenging and beyond the scope of this paper.

making consumption cheaper today as opposed to the future, and it also boosts investment by reducing the opportunity cost of funds used for investment.

[25] Inflation expectations and real interest rates at various key horizons are shown in Tables 4 and 5 in the Appendix for every month starting in January 2008.

4.1 Inflation Expectations Curve

Figure 7 plots inflation expectations obtained from the model for some key horizons, and Figure 9 in the Appendix shows the same information but with the whole inflation expectations curve plotted for various key dates. In Figures 7 and 8, discussed next, the financial crisis is shown with a vertical line in September 2008, the announcement of the formal inflation target is denoted with a dashed-dotted vertical line, and the taper tantrum is shown with a dashed vertical line. The four Fed programs are shown with yellow shading, and green shading is used to show the overlap between Operation Twist and QE3.

The financial crisis causes drastic changes in inflation expectations in the short and medium horizons but not as much in the long horizon. In March 2008, the inflation expectations curve is almost flat with the short end at just under 2.4%. In the summer, the short end (e.g., the six-month horizon) reaches 2.8%, reflecting the concern about rising energy prices at the time. After the onset of the crisis and the decline of energy prices in September, the short end of the curve takes a big plunge, falling to just above 1.2% in February of 2009. Throughout the crisis, the long end of the curve remains stable, especially compared with the short end: The 10-year forecast decreases by only 7 basis points between July 2008 and February 2009.

When viewed in a low frequency, the 10-year forecast does not seem to be affected significantly by the financial crisis: In the period from September 2008 to December 2015, it fluctuates between 2.19% and 2.47%, with an average of 2.33%, a decline of only 14 basis points relative to the period prior to the crisis. Considering the earlier discussion about the downward trend in the level factor that dies out in 2005, the decline during this period is negligible. Using the model, I am also able to compute the year-3-to-year-10 (starting in year three, and ending in year 10) and year-6-to-year-10 forward forecasts, which are shown as dashed lines in this figure. These forecasts remain at the precrisis levels of 2.4%. Thus, much of the (already small) decline in the 10-year expectations arises from the expected decline during the first two to five years. My results are in line with Mertens' (2011) results, which show that trend inflation does not change by much during the crisis.

During the period that the Fed ran QE1, the short end of the inflation expectations curve recovers from the decline due to the financial crisis — the six-month expectation went up from 1.23% in February 2009 to 1.7% in June 2010 — and the long end falls somewhat. The latter, however, should be seen as a consequence of having lower-than-average expectations for the next few years. During QE2, the short end continues to recover, increasing above 2.1% in the summer of 2011. Since the short end is recovering, the long end shows some small increases as well. None of these changes, however, bring the inflation expectations curve close to its precrisis shape despite the strong level of mean reversion that is built into the model. One way to see this is by computing a counterfactual in which the model evolves without shocks starting in December 2008 and then comparing it with the actual evolution. This approach reveals that inflation expectations of all horizons are consistently below the counterfactual even though they increase relative to the onset of the crisis.

The inflation expectations curve does not seem to change much during Operation Twist. The announcement of the formal inflation target in January 2012 seems to shift the whole inflation expectations curve by a few basis points in the long end and by about 10 basis points in the short end. As for the period during QE3, there is a decline of about 10 basis points in the long end of the inflation expectations curve with smaller changes in the short end.

4.2 Real Interest Rate Curve

Paralleling the analysis in the previous section, Figure 8 plots some key real interest rates over the period 2008–2015, and Figure 10 in the Appendix shows the full real interest rate curve at various key points in time.

Before turning to the results, I want to explain how I handle the issue pointed out in Section 2.4. The real interest rate that I compute via (16) inherits the inflation risk premium from nominal yields. Identifying and removing the risk premium require a financial model. Fortunately, as I explained earlier, the goal of D'Amico, Kim, and Wei (2010) is precisely to identify the risk premium and liquidity premium components of the TIPS inflation compensation to arrive at inflation expectations. These components are available for the 2-year, 5-year, and 10-year measures. The risk premia, at least for these somewhat longer maturities, are small enough not to matter for the conclusions I reach below.[26] Duffee (2014) also concludes that inflation risk is responsible for at most 10% of the fluctuations of nominal yields since the mid-1980s.

In January 2008, as the U.S. economy is already slowing down, but before the peak of the financial crisis, all real interest rates are positive with a U-shaped real interest curve, dipping to 0.2% at the two-year maturity. As the Fed reduces the short end of the nominal yield curve throughout 2008, the whole real yield curve also shifts downward significantly, almost in a parallel fashion. As of December 2008, the real interest rate for horizons up to seven years is negative, with the two-year rate around −1.4%. Thus, the combination of the financial crisis and the Fed's conventional response leads to a downward shift of the real yield curve, with the short end of the curve remaining higher than the middle.

During QE1, the short end of the curve falls, bringing the six-month rate to around −1.3% by the end of the program, a decline of almost a full percentage point relative to August 2008. Despite fluctuations throughout the program, real interest rates of longer maturities remain largely the same after the program ends, but at lower levels relative to those before the financial crisis. During QE2, the short end of the curve falls by another 0.6%, with the longer maturities behaving the same way as they did during QE1. From the end of QE2 until the end of QE3, the six-month real interest rates hover just above −2%.

[26] Over the period 1998–2013, the period I can compute these premia, the average (maximum absolute) risk premium for the 2-year horizon is 0.03% (0.24%), for the 5-year horizon is 0.08% (0.33%), and for the 10-year horizon is 0.17% (0.43%).

Considering that the average six-month real interest rate in the period 1998–2007 is 1.5% with a minimum of −0.8% obtained in December 2003, this very low level seems to be highly stimulative.

The long end of the real interest rate curve remains positive through 2011, and just as Operation Twist is starting in September 2011, it falls into negative territory, falling to −0.8% in July 2012. Considering the average 10-year real interest rate of 2.6% prior to the crisis, these are exceptionally low levels.

Interestingly, during all QE episodes, the long end of the real interest rate curve displays a hump shape, initially increasing and subsequently falling, with negligible differences between the start and the end points. The taper tantrum seems to create an acceleration of the increase in real interest rates during QE3, especially the medium-term rates such as the five-year rate.

4.3 Summary

Here is a summary of the findings in this section:

- The financial crisis and the Fed's conventional response reduced both inflation expectations and real interest rates significantly, with the distinct exception of inflation expectations of long horizons in which the decline was very mild at best.

- During QE1, to some extent due to mean reversion, inflation expectations of short maturities rise by about 0.5% and by another 0.5% following QE2. They do not change during QE3. Short-term real rates continue to fall, reaching a low of −2% by the end of 2012 and staying there through the end of QE3. This rate is about 3.5% below the precrisis average.

- During QE1, longer horizon inflation expectations fall about 0.1%; during QE2, they rise about 0.2%; and during QE3, they fall about 0.1%. The long-term real rates do not change when one compares the beginning and end of these programs, but they display a hump-shaped behavior, increasing by as much as 1% before returning to previous levels.

- Long-term real interest rates fall by about 0.6% during Operation Twist, reaching −0.8% in the summer of 2012. This rate is about 3.4% below the precrisis average.
- The announcement of the formal inflation target generates a small increase in inflation expectations of all horizons but does not change the real interest rates in any significant way.
- At the end of 2015, the short end of the inflation expectations curve is still about 15 basis points below its precrisis levels, while the long end is about 18 basis points below.

5 Conclusions

Starting in 2008, the Federal Reserve enacted unprecedented policies in response to the biggest decline in economic activity since the Great Depression. The impact of these policies on medium- to long-term inflation is yet to be seen. In this paper, I provide a statistically efficient and accurate way of aggregating survey-based inflation expectations into an inflation expectations curve. I also compute a term structure of real interest rates by combining the inflation expectations curve with a nominal yield curve.

The resulting term structure of inflation expectations proves capable of providing superior forecasts relative to some of the popular alternatives. Thus, moving forward, this approach seems to be a useful tool to gauge inflation expectations at any arbitrary horizon.

I find that long-run inflation expectations remained anchored during both the financial crisis of 2008 and the various Federal Reserve programs that were enacted in the aftermath of the crisis, despite large changes in shorter horizons. The Federal Reserve's unconventional policies, especially QE1 and QE2, are associated with large changes on the slope of the inflation expectations curve, where short-term expectations increased. As a result, these same policies are associated with a decline of short-term real rates to unprecedented levels. All in all, real interest rates of all horizons are about 3.5% lower than their precrisis averages, indicating a massive stimulus to the economy, with no significant change in long-run inflation expectations.

From here, a number of further directions are possible. First, a reasonable approach may be to consider non-Gaussian errors or stochastic volatility (or both) in the model. Second, although the model in this paper explicitly excludes financial variables, there may be ways of introducing them without worsening performance. For example, similar to but distinctly different from what Christensen, Lopez, and Rudebusch (2012) do, one could model inflation expectations as I do here and add nominal yields that follow an NS structure with different factors, however, by explicitly introducing ZLB into the model. Finally, one could introduce information from various online prediction markets. I leave these directions for future work.

References

[1] Ajello, A., L. Benzoni, and O. Chyruk (2012), "Core and 'Crust': Consumer Prices and the Term Structure of Interest Rates," mimeo, Federal Reserve Bank of Chicago.

[2] Ang, A., G. Bekaert, and M. Wei (2007), "Do Macro Variables, Asset Markets, or Surveys Forecast Inflation Better?" *Journal of Monetary Economics*, 54(4), 1163-1212.

[3] Aruoba, S.B., F.X. Diebold, and C. Scotti (2009), "Real-Time Measurement of Business Conditions," *Journal of Business and Economic Statistics*, 27(4), 417-427.

[4] Aruoba, S.B., and F. Schorfheide (2016), "Inflation During and After the Zero Lower Bound," *2015 Jackson Hole Symposium Volume*, Federal Reserve Bank of Kansas City.

[5] Beechey, M.J., B.K. Johannsen, and A.T. Levin (2011), "Are Long-Run Inflation Expectations Anchored More Firmly in the Euro Area Than in the United States?," *American Economic Journal: Macroeconomics*, 104-129.

[6] Chernov, M., and P. Mueller (2012), "The Term Structure of Inflation Expectations," *Journal of Financial Economics*, 106(2), 367-394.

[7] Christensen, J.H.E., J.A. Lopez, and G.D. Rudebusch (2010), "Inflation Expectations and Risk Premiums in an Arbitrage-Free Model of Nominal and Real Bond Yields," *Journal of Money, Credit and Banking*, 42(S1), 143-178.

[8] Christensen, J.H.E., J.A. Lopez, and G.D. Rudebusch (2012), "Extracting Deflation Probability Forecasts from Treasury Yields," *International Journal of Central Banking*, 8(4), 21-60.

[9] D'Amico, S., D.H. Kim, and M. Wei (2010), "Tips from TIPS: The Informational Content of Treasury Inflation-Protected Security Prices," Board of Governors of the Federal Reserve System Finance and Economics Discussion Series*,* 2010-19.

[10] Diebold, F.X., and C. Li (2006), "Forecasting the Term Structure of Government Bond Yields," *Journal of Econometrics*, 130(2), 337-364.

[11] Diebold, F.X., and R.S. Mariano (1995), "Comparing Predictive Accuracy," *Journal of Business and Economic Statistics*, 13(3), 253-263.

[12] Diebold, F.X., and G.D. Rudebusch (2013), *Yield Curve Modeling and Forecasting: The Dynamic Nelson-Siegel Approach*, Princeton University Press.

[13] Diebold, F.X., G.D. Rudebusch, and S.B. Aruoba (2006), "The Macroeconomy and the Yield Curve: A Dynamic Latent Factor Approach," *Journal of Econometrics*, 131(1-2), 309-338.

[14] Duffee, G.F. (2014), "Expected Inflation and Other Determinants of Treasury Yields," mimeo.

[15] Durbin, J., and S.J. Koopman (2012), *Time Series Analysis by State Space Methods*, 2nd edition, Oxford University Press.

[16] Faust, J., and J.H. Wright (2013), "Forecasting Inflation," in *Handbook of Economic Forecasting*, vol. 2A, eds. G. Elliott and A. Timmermann, 2-56, St. Louis, Elsevier.

[17] Gürkaynak, R.S., B. Sack, and J.H. Wright (2007), "The U.S. Treasury Yield Curve: 1961 to the Present," *Journal of Monetary Economics*, 54(8), 2291-2304.

[18] Gürkaynak, R.S., B. Sack, and J.H. Wright (2010), "The TIPS Yield Curve and Inflation Compensation," *American Economic Journal: Macroeconomics*, 2(1), 70-92.

[19] Hakkio, C.S. (2008), "PCE and CPI Inflation Differentials: Converting Inflation Forecasts," *Federal Reserve Bank of Kansas City Economic Review,* First Quarter, 51-68.

[20] Haubrich, J.G., G. Pennacchi, and P. Ritchken (2012), "Inflation Expectations, Real Rates, and Risk Premia: Evidence from Inflation Swaps," *Review of Financial Studies*, 25(5), 1588-1629.

[21] Kim, S. N. Shephard, and S. Chib (1998), "Stochastic Volatility: Likelihood Inference and Comparison with ARCH Models," *Review of Economic Studies*, 65(3), 361-393.

[22] Lucca, David, and Ernst Schaumburg (2011), "What to Make of Market Measures of Inflation Expectations?" *Liberty Street Economics* (blog), Federal Reserve Bank of New York, August 15, 2011, http://libertystreeteconomics.newyorkfed.org/2011/08/what-to-make-of-market-measures-of-inflation-expectations.html.

[23] Mertens, E. (2011), "Measuring the Level and Uncertainty of Trend Inflation," Board of Governors of the Federal Reserve System Finance and Economics Discussion Series, 2011-42.

[24] Nelson, C.R., and A.F. Siegel (1987), "Parsimonious Modeling of Yield Curves," *Journal of Business*, 60(4), 473-489.

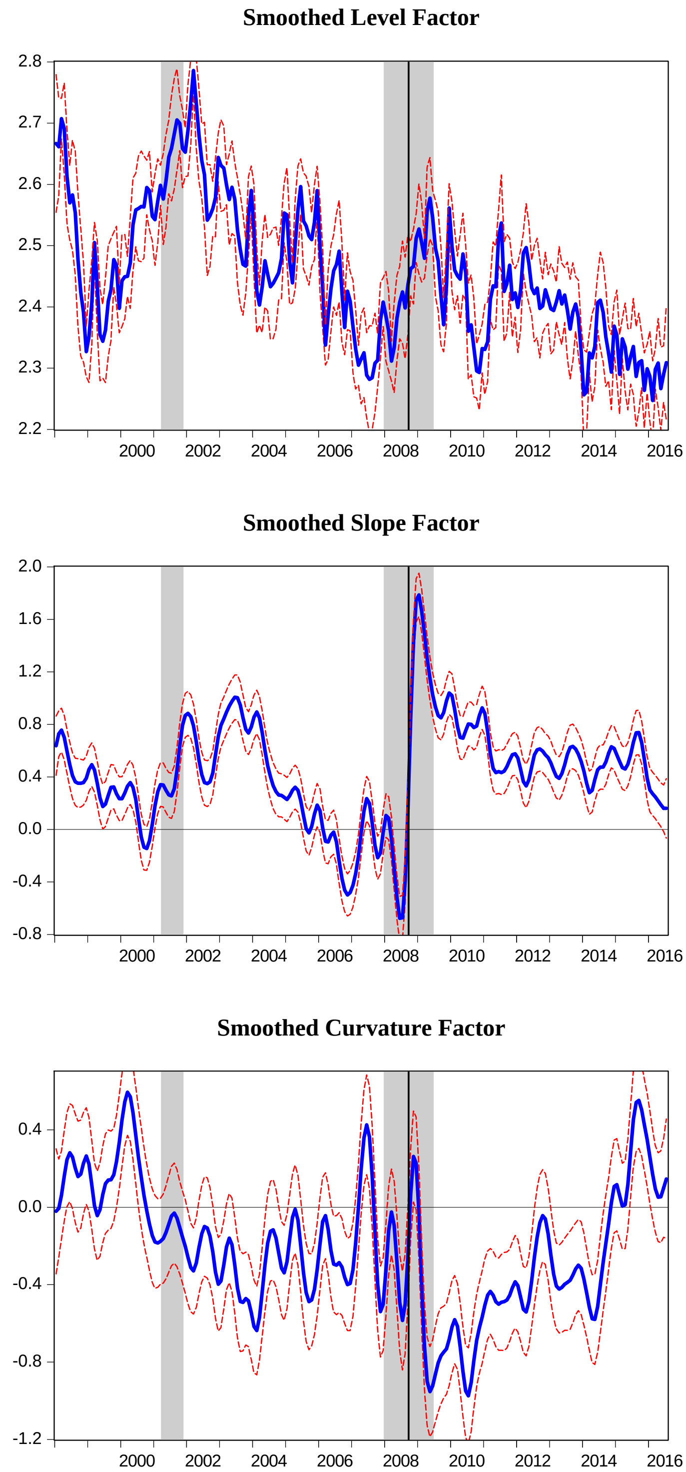
[25] Schorfheide, F., D. Song, and A. Yaron (2014), "Identifying Long-Run Risks: A Bayesian Mixed-Frequency Approach," *NBER Working Paper 20303*.

[26] Stark, T. (2010), "Realistic Evaluation of Real-Time Forecasts in the Survey of Professional Forecasters," Federal Reserve Bank of Philadelphia *Research Rap* Special Report.

[27] Stock, J.H., and M.W. Watson (2007), "Why Has US Inflation Become Harder to Forecast?" *Journal of Money, Credit and Banking,* 39(1), 3-33.

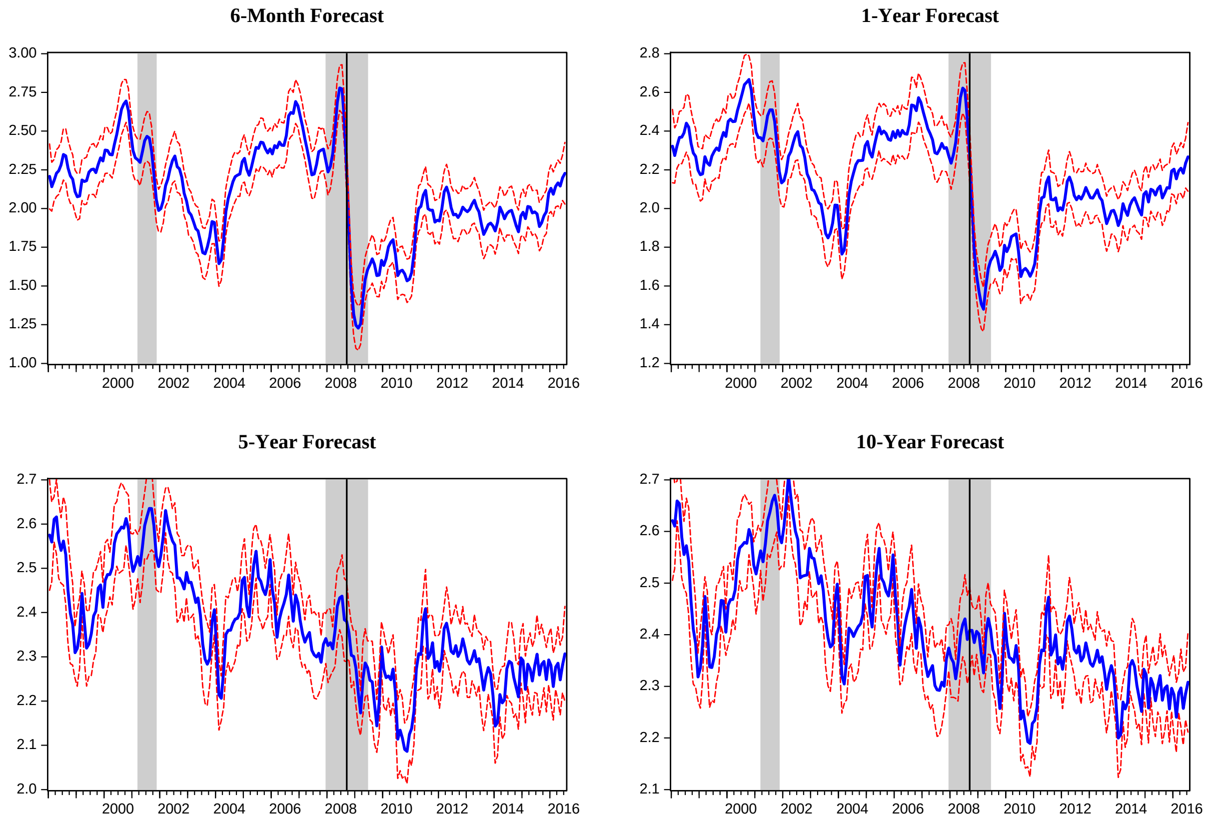
[28] Swanson, E.T., and J.C. Williams (2014), "Measuring the Effect of the Zero Lower Bound on Medium- and Longer-Term Interest Rates," *American Economic Review*, 104(10), 3154-3185.

Figure 1: Extracted Factors



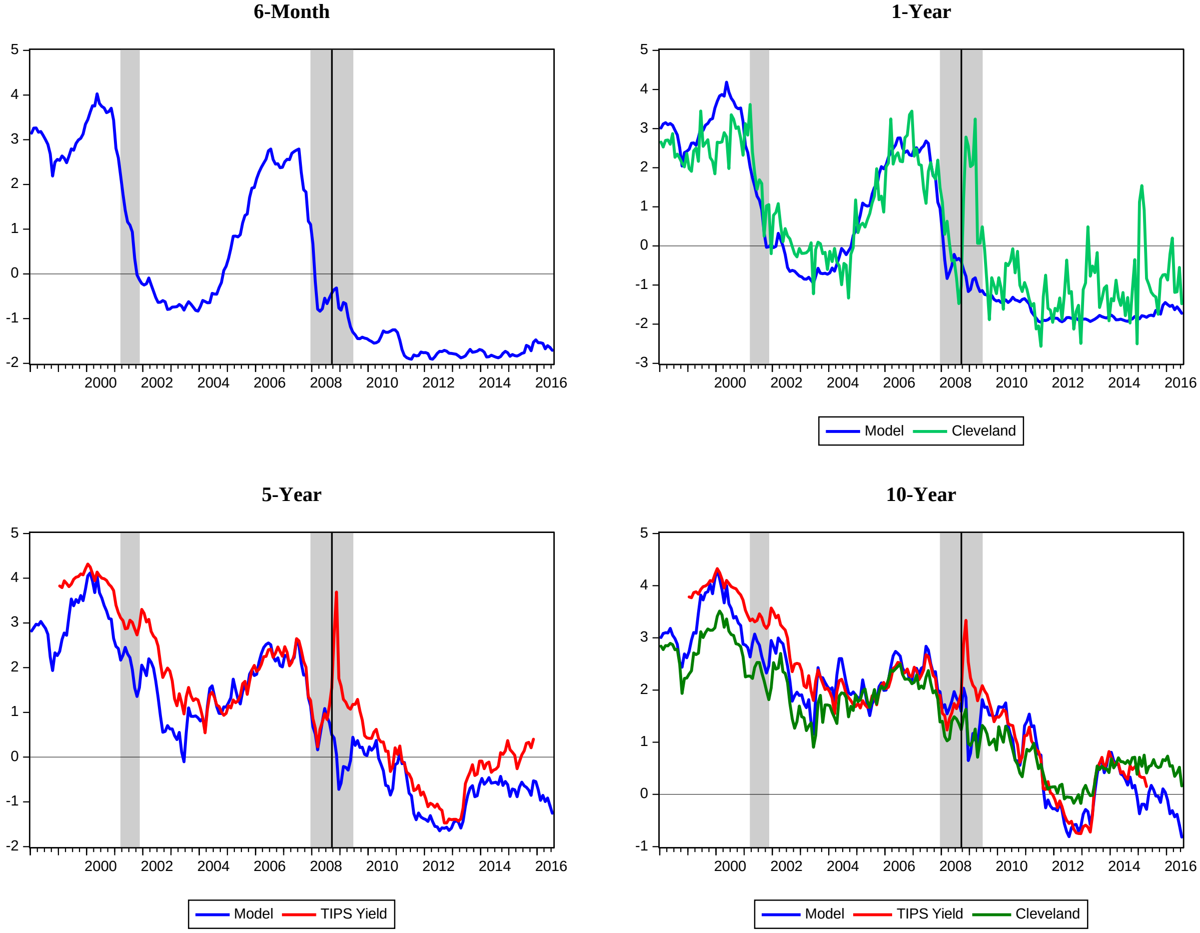
Notes: The gray bars denote NBER recessions. The vertical line denotes September 2008. The blue lines denote the smoothed factors, and the red dashed lines show their pointwise 95% confidence bands.

Figure 2: Selected Inflation Expectations



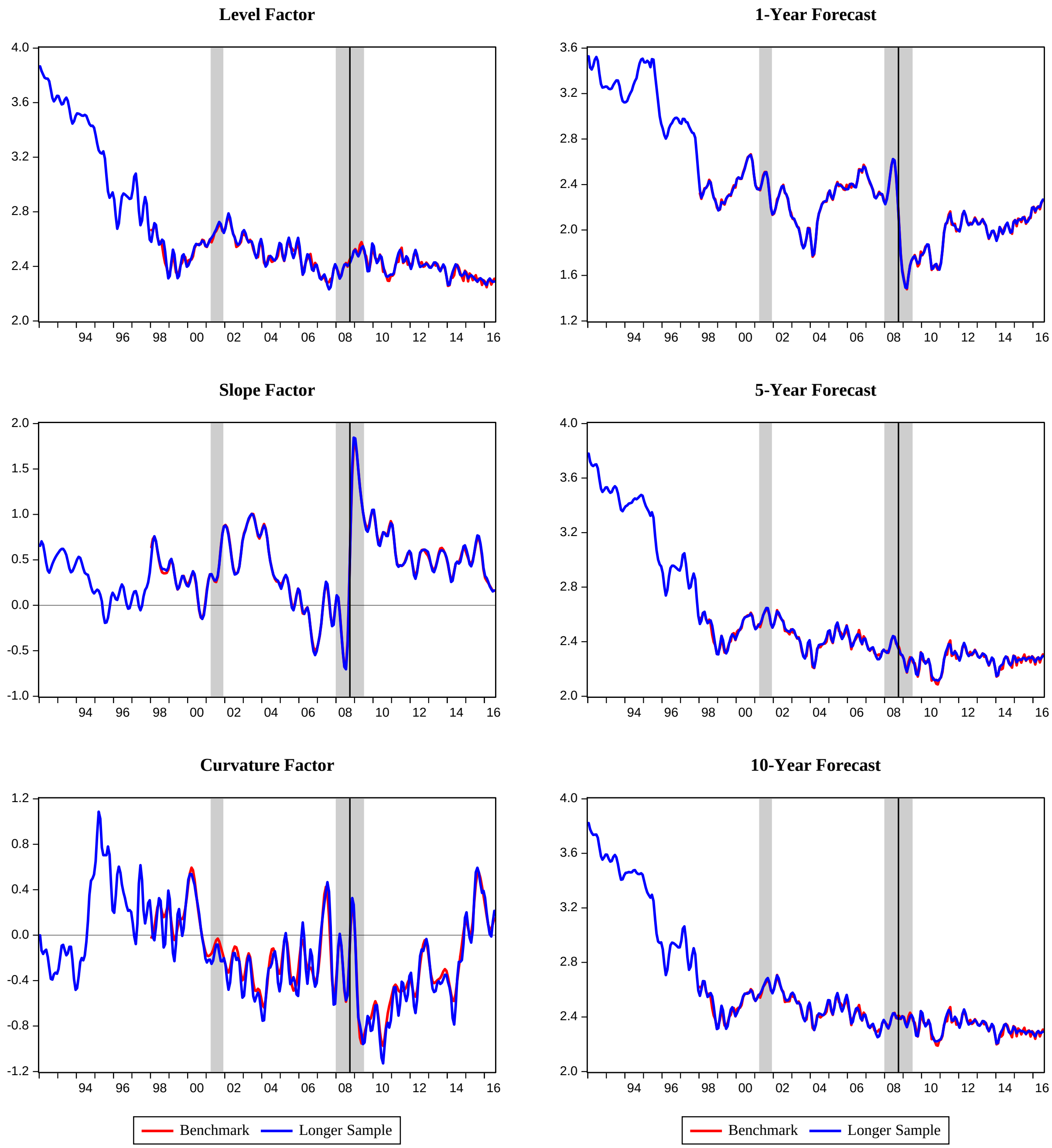
Notes: The gray bars denote NBER recessions. The vertical line denotes September 2008. The blue lines denote the smoothed factors, and the red dashed lines show their pointwise 95% confidence bands.

Figure 3: Selected Real Interest Rates



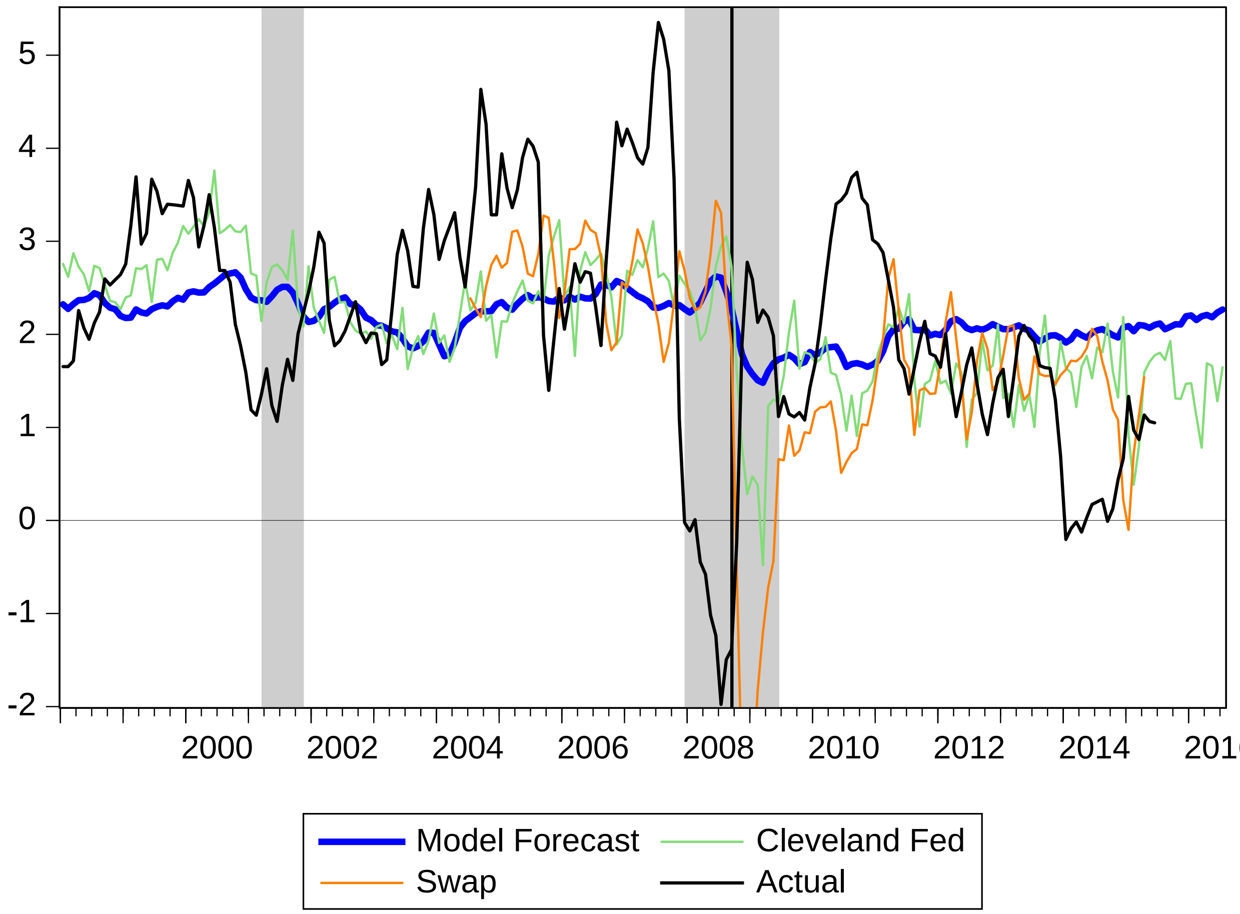
Notes: The gray bars denote NBER recessions. The vertical line denotes September 2008. The green line in the 1-Year panel is the real interest rate computed by the Federal Reserve Bank of Cleveland, and the red lines in the 5-Year and 10-Year panels are the corresponding TIPS yields.

Figure 4: Comparison of Factors and Forecasts with Those from a Longer Sample



Notes: The gray bars denote NBER recessions. The vertical line denotes September 2008.

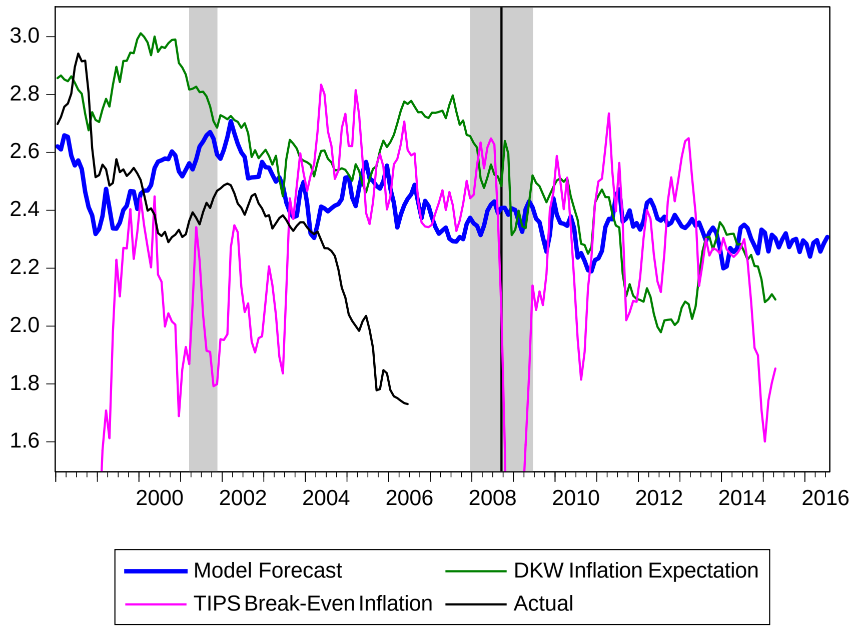
Figure 5: Comparison of One-Year Inflation Expectations with Financial Variables, and Actual



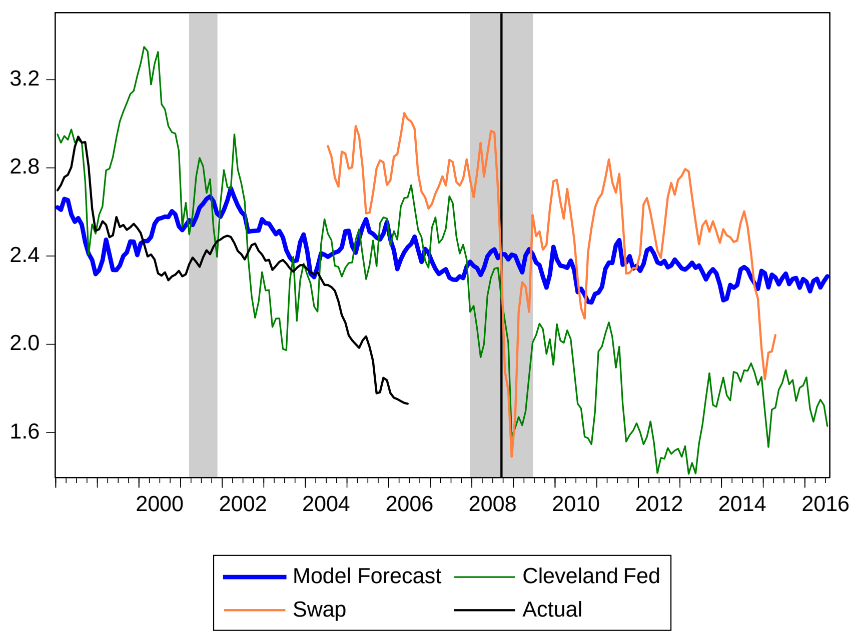
Notes: The gray bars denote NBER recessions. The vertical line denotes September 2008. The swap rate (orange line) falls to −3.83% in December 2008, but the graph is truncated at −2%.

Figure 6: Comparison of 10-Year Inflation Expectations with Inputs, Financial Variables, and Actual

(a) Model Forecast, TIPS-Based Financial Variables and Actual

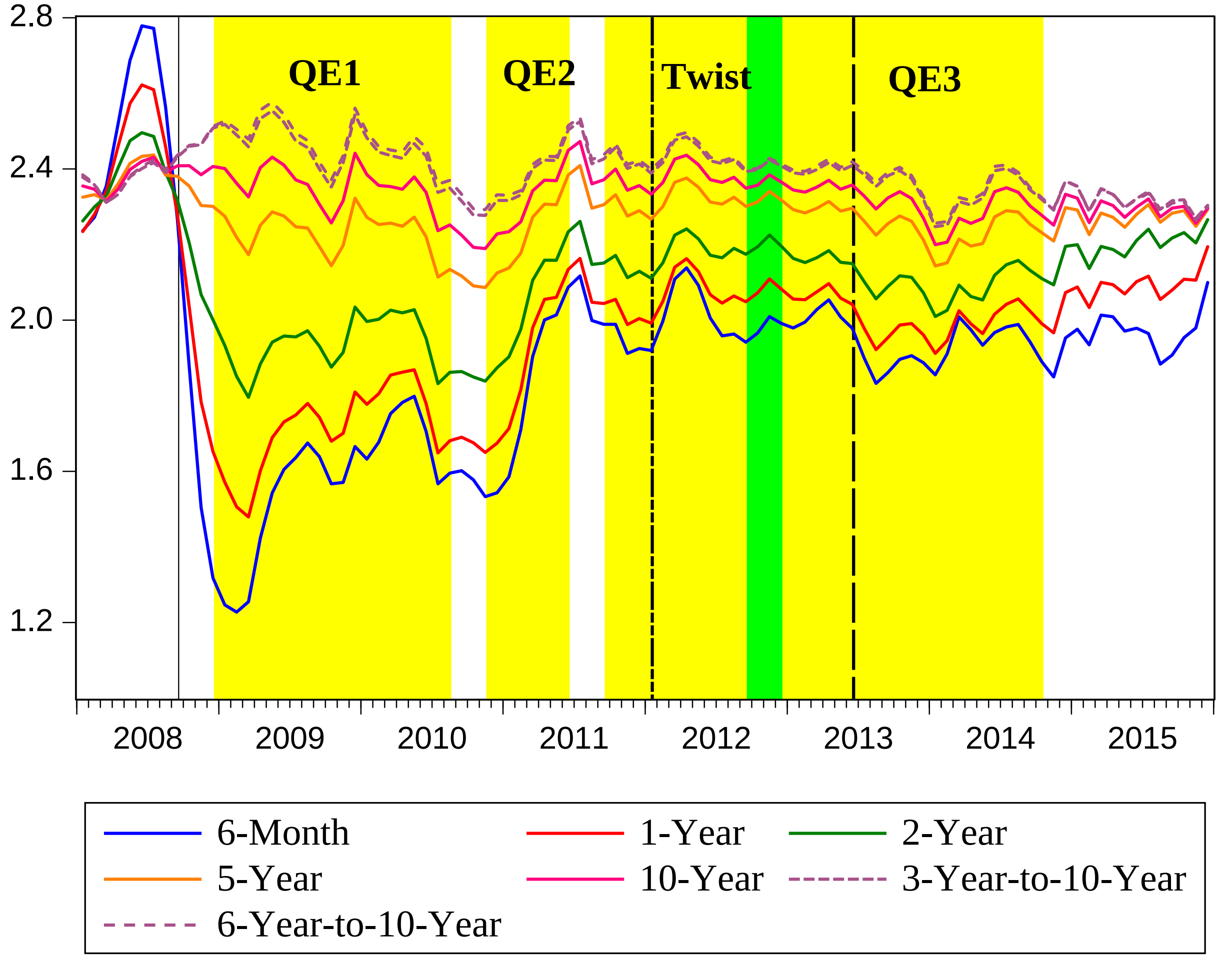


(b) Model Forecast, Swap-Based Financial Variables and Actual



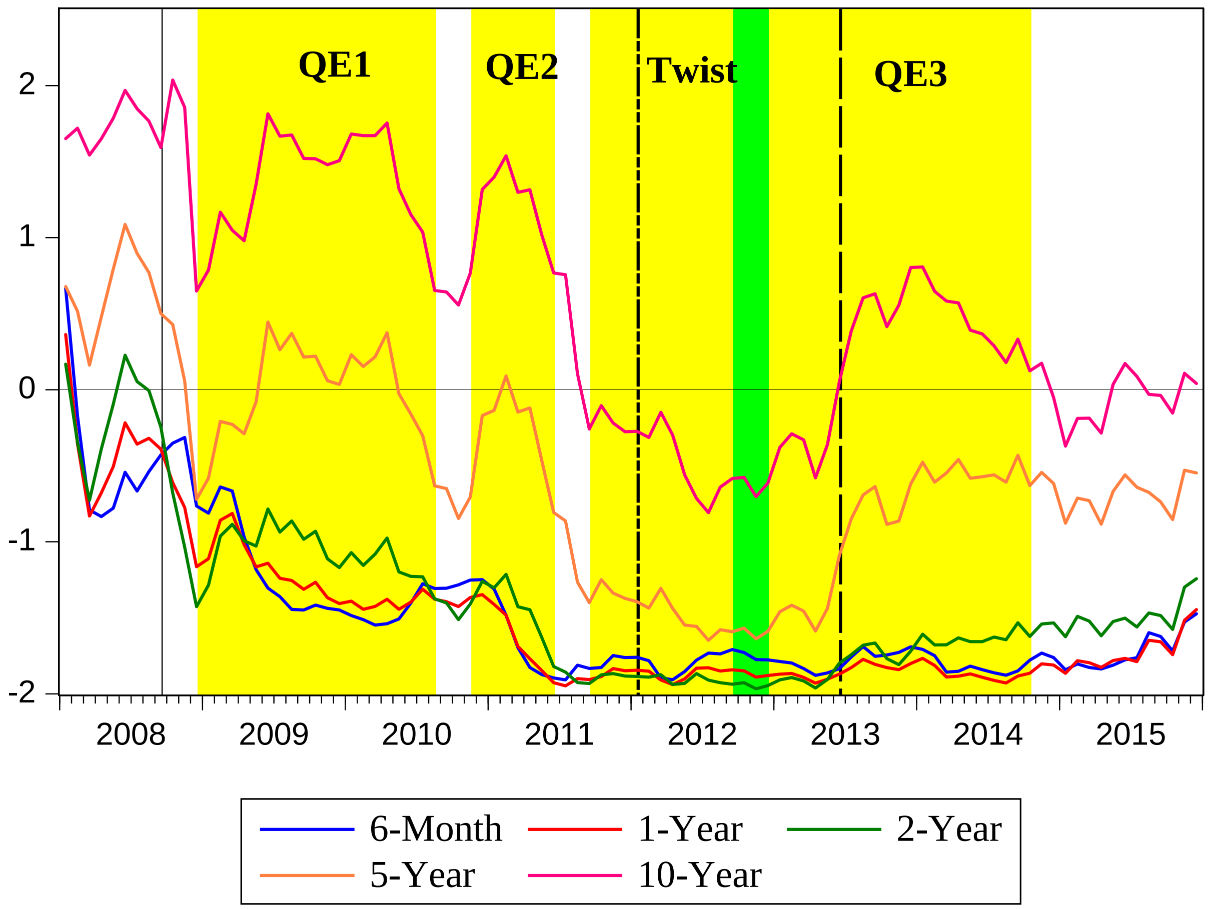
Notes: The gray bars denote NBER recessions. The vertical line denotes September 2008. In panel (a), the TIPS break-even rate (purple line) falls to 0.52%, but the graph is truncated at 1.5%.

Figure 7: Key Inflation Expectations and Fed Programs



Notes: The solid vertical line denotes September 2008. The yellow shading indicates the months when the particular Fed program was active. During the green shaded area, both Operation Twist and QE3 were active. The dashed-dotted vertical line shows January 2012, when the formal inflation target was announced. The dashed vertical line shows the "taper tantrum" episode in June 2013. The two dashed purple lines show the year-3-to-year-10 and year-6-to-year-10 forecasts.

Figure 8: Key Real Interest Rates and Fed Programs



Notes: The solid vertical line denotes September 2008. The yellow shading indicates the months when the particular Fed program was active. During the green shaded area, both Operation Twist and QE3 were active. The dashed-dotted vertical line shows January 2012, when the formal inflation target was announced. The dashed vertical line shows the "taper tantrum" episode in June 2013.

Table 1: Estimation Results

(a) Transition Equation

Level		Slope		Curvature	
ρ_{11}	**0.99**	ρ_{21}	**2.11**	ρ_{31}	**2.19**
ρ_{12}	-0.28	ρ_{22}	**-1.65**	ρ_{32}	**-1.87**
ρ_{13}	0.20	ρ_{23}	0.51	ρ_{33}	**0.65**
μ_1	**2.46**	μ_2	**0.43**	μ_3	-0.18
σ_L^2	**0.003**	σ_S^2	0.004	σ_C^2	0.003

(b) Measurement Equation

$\lambda = \mathbf{0.12}$

SPF Quarterly		Blue Chip Short-Run		Blue Chip Long-Run	
σ_1^2	**0.034**	σ_{21}^2	**0.008**	σ_{36}^2	0.002
σ_2^2	**0.011**	σ_{22}^2	**0.016**	σ_{37}^2	0.003
σ_3^2	**0.007**	σ_{23}^2	**0.007**	σ_{38}^2	0.003
σ_4^2	**0.006**	σ_{24}^2	**0.004**	σ_{39}^2	0.001
SPF Annual		σ_{25}^2	**0.007**	σ_{40}^2	0.001
σ_5^2	**0.009**	σ_{26}^2	**0.020**	σ_{41}^2	0.011
σ_6^2	0.005	σ_{27}^2	**0.017**	σ_{42}^2	0.005
σ_7^2	**0.008**	σ_{28}^2	**0.009**	σ_{43}^2	0.003
σ_8^2	0.006	σ_{29}^2	**0.005**	σ_{44}^2	**0.003**
σ_9^2	**0.008**	σ_{30}^2	**0.007**	σ_{45}^2	0.004
σ_{10}^2	0.006	σ_{31}^2	**0.089**	σ_{46}^2	0.006
σ_{11}^2	0.005	σ_{32}^2	**0.011**	σ_{47}^2	0.004
σ_{12}^2	0.005	σ_{33}^2	**0.009**	σ_{48}^2	0.003
SPF 5-Year		σ_{34}^2	**0.005**	σ_{49}^2	0.003
σ_{13}^2	0.012	σ_{35}^2	**0.008**	σ_{50}^2	0.002
σ_{14}^2	0.009			σ_{51}^2	0.011
σ_{15}^2	0.012			σ_{52}^2	0.010
σ_{16}^2	0.011			σ_{53}^2	**0.001**
SPF 10-Year				σ_{54}^2	0.001
σ_{17}^2	0.009			σ_{55}^2	**0.004**
σ_{18}^2	0.009			σ_{56}^2	**0.002**
σ_{19}^2	0.015			σ_{57}^2	**0.002**
σ_{20}^2	**0.009**			σ_{58}^2	0.002
				σ_{59}^2	0.001

Notes: Boldface indicates significance at the 5% level.

Table 2: **Forecast Comparison Results**

Forecast	**RMSE - Model**	**RMSE - Alternative**	**T**
UC-SV 1-year	1.29	1.21	210
UC-SV 2-year	0.89	0.93	198
UC-SV 5-year	**0.43**	0.79	162
UC-SV 10-year	**0.27**	0.69	102
Michigan 1-year	**1.29**	1.80	210
Michigan 5-year	**0.45**	1.39	162
SWAP 1-year	1.51	1.89	130
SWAP 2-year	1.01	1.30	119
SWAP 5-year	**0.51**	0.77	83
Cleveland 1-year	1.29	1.33	210
Cleveland 2-year	0.89	0.90	198
Cleveland 5-year	0.45	0.54	162
Cleveland 10-year	**0.31**	0.45	102
TIPS BE 2-year	1.02	1.43	126
TIPS BE 5-year	**0.46**	0.97	150
TIPS BE 10-year	**0.31**	0.50	90
DKW 2-year	0.89	**0.82**	198
DKW 5-year	**0.45**	0.52	162
DKW 10-year	**0.31**	0.45	102

Notes: Boldface for RMSEs indicates rejection of the null of equal accuracy at the 5% level using the Diebold-Mariano (1995) test with squared errors in favor of the model that has the boldface RMSE.

A Appendix

A.1 Measurement Equations

All the data I use in the estimation come from surveys. In virtually all cases, the question asked of the forecasters does not correspond exactly to a simple τ-month-ahead forecast, in the form of $\pi_t(\tau)$ for some $\tau > 0$, so I do some transformations as I explain in detail below. I convert all raw data to annualized percentage points to conform with the previous notation. Unless otherwise noted, all data start in 1998. In all cases, the forecasters are asked to forecast the seasonally adjusted CPI inflation rate. Both the *SPF* and *Blue Chip* forecasts are released around the middle of the month, with the forecasts due a fews days prior to the release.[27] I will thus consider both of the forecasts released in month t as forecasts made in month t.

As will be clear below, in some cases what is asked of the forecasters is a mixture of realized (past) inflation and a forecast of future inflation. Most of the realized inflation will be in the form of $\pi_{s\to r}$, where $s < r \leq t-2$ so that in period t the forecasts are able to observe the official data release before making their forecasts.[28] I use Archival Federal Reserve Economic Data (ALFRED) at the Federal Reserve Bank of St. Louis to obtain the exact inflation rate the forecasters would have observed in real time.[29] Furthermore, there will be instances in which I need $\pi_{t-2\to t-1}$, $\pi_{t-1\to t}$, or $\pi_{t-2\to t}$, none of which are observed by the time forecasters make their forecasts in period t. Since it is difficult to know explicitly what the forecasters knew when they sent their forecasts in month t about these inflation rates that are realized (but not yet released by statistical agencies), I assume that these expectations are equal to the longer horizon being forecast. Once I show the equations

[27] See Figure 1 in Stark (2010) that shows the timing of *SPF* forecasts. Similar information is confirmed for the *Blue Chip* forecasts.

[28] For example, $\pi_{t-3\to t-2}$ would involve P_{t-3} and P_{t-2}, and the latter is released (and perhaps the former is revised) in the second half of the month $t-1$. Remember that both the *SPF* and *Blue Chip* forecasts are made in the first half of month t, before P_{t-1} is released.

[29] The data are available at http://alfred.stlouisfed.org/series?seid=CPIAUCSL.

below, what I mean by this will be clear.

A.1.1 *Survey of Professional Forecasters*

The *SPF* is a quarterly survey that has been conducted by the FRBP since 1990. The forecasters are asked to make forecasts for a number of key macroeconomic indicators several quarters into the future, and in the case of CPI inflation, they are also asked to make 5-year and 10-year forecasts. I use the median of these forecasts.

***SPF* Quarterly Forecasts** The *SPF* reports six quarterly forecasts ranging from "minus 1 quarter" to "plus 4 quarters" from the current quarter. The forecasts labeled "3," "4," "5," and "6" are forecasts for one, two, three, and four quarters after the current quarter, respectively.[30] More specifically, the forecasters are asked to forecast the annualized percentage change in the quarterly average of the CPI price level. Using my notation, the "4" forecast made in period t is

$$\text{SPF-4}_t = 100 \left[\left(\frac{\frac{P_{t+5}+P_{t+6}+P_{t+7}}{3}}{\frac{P_{t+2}+P_{t+3}+P_{t+4}}{3}} \right)^4 - 1 \right],$$

where the numerator is the average CPI price level in the second quarter following the current one and the denominator is the average CPI price level for the next quarter. Using continuous compounding and geometric averaging, this forecast can be written as[31]

$$\begin{aligned}
\text{SPF-4}_t &\approx 400 \left\{ \log \left[(P_{t+5}P_{t+6}P_{t+7})^{1/3} \right] - \log \left[(P_{t+2}P_{t+3}P_{t+4})^{1/3} \right] \right\} \\
&= \frac{400}{3} \left\{ \log \left[(P_{t+5}P_{t+6}P_{t+7}) \right] - \log \left[(P_{t+2}P_{t+3}P_{t+4}) \right] \right\} \\
&= \frac{400}{3} \left[\log (P_{t+5}) - \log (P_{t+2}) + \log (P_{t+6}) - \log (P_{t+3}) + \log (P_{t+7}) - \log (P_{t+4}) \right] \\
&= \frac{\pi_{t+2 \to t+5} + \pi_{t+3 \to t+6} + \pi_{t+4 \to t+7}}{3},
\end{aligned}$$

[30] The "1" and "2" forecasts contain at least some realized inflation rates, and I do not use them since I want to focus as much as possible on pure forecasts.

[31] The correlation of actual inflation computed using the exact formula and the approximation I use is 0.9993.

which is the arithmetic average of three quarterly inflation rates.

The SPF-3 forecast is special (as will be the other *SPF* forecasts I turn to next) in that a part of the object being forecast refers to the past and not to the future. Using similar derivations as above, the SPF-3 forecast in period t can be written as

$$\begin{aligned}
\text{SPF-3}_t &= \frac{\pi_{t-1\to t+2} + \pi_{t\to t+3} + \pi_{t+1\to t+4}}{3} \\
&= \frac{\left(\frac{\pi_{t-1\to t}+2\pi_{t\to t+2}}{3}\right) + \pi_{t\to t+3} + \pi_{t+1\to t+4}}{3} \\
&= \frac{1}{9}\left(\pi_{t-1\to t} + 2\pi_{t\to t+2} + 3\pi_{t\to t+3} + 3\pi_{t+1\to t+4}\right) \\
&= \frac{1}{8}\left(2\pi_{t\to t+2} + 3\pi_{t\to t+3} + 3\pi_{t+1\to t+4}\right),
\end{aligned}$$

where in the last line I replace $\pi_{t-1\to t}$ with SPF-3$_t$. This is the assumption I will maintain whenever formulas call for $\pi_{t-2\to t-1}$, $\pi_{t-1\to t}$, or $\pi_{t-2\to t}$ – I will assume each of them are equal to the main object being forecast.[32]

Using similar derivations for the "5" and "6" forecasts, and using definitions $x_t^1 \equiv$SPF-3$_t$, $x_t^2 \equiv$SPF-4$_t$, $x_t^3 \equiv$SPF-5$_t$, and $x_t^4 \equiv$SPF-6$_t$, the measurement equations for the quarterly *SPF* forecasts are

$$\begin{aligned}
x_t^1 &= \frac{1}{8}\left(2\pi_{t\to t+2} + 3\pi_{t\to t+3} + 3\pi_{t+1\to t+4}\right) + \varepsilon_t^1 \\
x_t^2 &= \frac{1}{3}\left(\pi_{t+2\to t+5} + \pi_{t+3\to t+6} + \pi_{t+4\to t+7}\right) + \varepsilon_t^2 \\
x_t^3 &= \frac{1}{3}\left(\pi_{t+5\to t+8} + \pi_{t+6\to t+9} + \pi_{t+7\to t+10}\right) + \varepsilon_t^3 \\
x_t^4 &= \frac{1}{3}\left(\pi_{t+8\to t+11} + \pi_{t+9\to t+12} + \pi_{t+10\to t+13}\right) + \varepsilon_t^4.
\end{aligned}$$

Once stated as combinations of $\pi_{t+\tau_1\to t+\tau_2}$, it is straightforward, though somewhat tedious,

[32] This creates a small inconsistency across different forecasts when the same object, say $\pi_{t-2\to t-1}$, is set equal to different forecasts with different values. Given that these terms receive small weights and the absence of a clearly better alternative, I choose this route.

to write the full measurement equations for these forecasts using (6).[33]

***SPF* Annual Forecasts** The *SPF* provides three annual forecasts, one for the survey calendar year and one each for the next two calendar years. I use the latter two, the "B" forecast and the "C" forecast, since they are (mostly) pure forecasts into the future. The "C" forecast is available starting in 2005Q3. More specifically, in every quarter of the survey year, for the "B" forecast the forecasters are asked to forecast the change in average price level of the last quarter of the year after the survey year relative to the last quarter of the survey year. Similarly for the "C" forecast, they need to forecast the change in the average price level of the last quarter of the year that is two years after the survey year, relative to the last quarter of the year that is one year after the survey year. As such, as we progress further into the current year, the distance between the period being forecast and the point of forecast gets shorter. This requires me to define forecasts made in particular quarters as separate variables.[34]

The "B" forecast released in February (denoted by t) is thus

$$\text{SPF-B-Q1}_t = 100\left[\left(\frac{P_{t+20}+P_{t+21}+P_{t+22}}{P_{t+8}+P_{t+9}+P_{t+10}}\right)-1\right].$$

Using the same derivations as in $SPF4$, this simplifies to

$$\text{SPF-B-Q1}_t \approx \frac{\pi_{t+8\to t+20}+\pi_{t+9\to t+21}+\pi_{t+10\to t+22}}{3}.$$

Doing the same derivations for the other quarters 1 through 3, and using definitions $x_t^5 \equiv$SPF-

[33]For example, the second measurement equation will be

$$\begin{aligned} x_t^2 \quad = \quad & L_t + \left[\frac{e^{-2\lambda}-e^{-5\lambda}+e^{-3\lambda}-e^{-6\lambda}+e^{-4\lambda}-e^{-7\lambda}}{9\lambda}\right](C_t - S_t) \\ & + \left(\frac{2e^{-2\lambda}-5e^{-5\lambda}+3e^{-3\lambda}-6e^{-6\lambda}+4e^{-4\lambda}-7e^{-7\lambda}}{9}\right)C_t + \varepsilon_t^2. \end{aligned}$$

[34]To be clear, I split each variable into four variables, each of which is observed only once a year.

B-Q1$_t$, x_t^6 $\equiv$SPF-B-Q2$_t$, and x_t^7 $\equiv$SPF-B-Q3$_t$, I get the measurement equations

$$\begin{aligned} x_t^5 &= \frac{1}{3}\left(\pi_{t+8\to t+20} + \pi_{t+9\to t+21} + \pi_{t+10\to t+22}\right) + \varepsilon_t^5 \\ x_t^6 &= \frac{1}{3}\left(\pi_{t+5\to t+17} + \pi_{t+6\to t+18} + \pi_{t+7\to t+19}\right) + \varepsilon_t^6 \\ x_t^7 &= \frac{1}{3}\left(\pi_{t+2\to t+14} + \pi_{t+3\to t+15} + \pi_{t+4\to t+16}\right) + \varepsilon_t^7. \end{aligned}$$

For the last quarter, I need to take into account that a small part of the object being forecast is realized by the time the forecast is made. In particular, the Q4 forecast is

$$\text{SPF-B-Q4}_t = \frac{1}{35}\left(11\pi_{t\to t+11} + 12\pi_{t\to t+12} + 12\pi_{t+1\to t+13}\right),$$

where, again, I replaced $\pi_{t-1\to t}$ with SPF-B-Q4$_t$. Defining x_t^8 $\equiv$SPF-B-Q4$_t$, I get the measurement equation

$$x_t^8 = \frac{1}{35}\left(11\pi_{t\to t+11} + 12\pi_{t\to t+12} + 12\pi_{t+1\to t+13}\right) + \varepsilon_t^8.$$

For the "C" forecast, in the first quarter of a year, the expression being forecast is

$$\text{SPF-C-Q1}_t = 100\left[\left(\frac{P_{t+32} + P_{t+33} + P_{t+34}}{P_{t+20} + P_{t+21} + P_{t+22}}\right) - 1\right],$$

and defining x_t^9 $\equiv$SPF-C-Q1$_t$, x_t^{10} $\equiv$SPF-C-Q2$_t$, x_t^{11} $\equiv$SPF-C-Q3$_t$, x_t^{12} $\equiv$SPF-C-Q4$_t$, the measurement equations are

$$\begin{aligned} x_t^9 &= \frac{1}{3}\left(\pi_{t+20\to t+32} + \pi_{t+21\to t+33} + \pi_{t+22\to t+34}\right) + \varepsilon_t^9 \\ x_t^{10} &= \frac{1}{3}\left(\pi_{t+17\to t+29} + \pi_{t+18\to t+30} + \pi_{t+19\to t+31}\right) + \varepsilon_t^{10} \\ x_t^{11} &= \frac{1}{3}\left(\pi_{t+14\to t+26} + \pi_{t+15\to t+27} + \pi_{t+16\to t+28}\right) + \varepsilon_t^{11} \\ x_t^{12} &= \frac{1}{3}\left(\pi_{t+11\to t+23} + \pi_{t+12\to t+24} + \pi_{t+13\to t+25}\right) + \varepsilon_t^{12}. \end{aligned}$$

Given these expressions, I directly apply (6) to get the measurement equations.

***SPF* 5-Year and 10-Year Forecasts** Although much of the *SPF* contains short- to medium-term forecasts, the forecasters are asked to provide five-year and 10-year forecasts for inflation as well. In particular, they are asked to forecast five and 10 years into the future, starting from the last quarter of the previous year. In other words, as with the other forecasts, these forecasts compare the average price level over the last quarter of the previous year with the average price level over the last quarter of four or nine years following the current year. Similar to the annual forecasts, I divide the 10-year and the five-year forecasts into four separate variables, taking into account the different forecast horizons at each quarter. The 10-year forecast has been a part of the *SPF* since 1991Q4, and the five-year forecast was added in 2005Q3.

In all cases below, what is reported by the forecasters includes some inflation that is already realized. Denoting February with period t, the five-year forecast made in the first quarter is

$$
\begin{aligned}
\text{SPF-5YR-Q1}_t &= 100\left[\left(\frac{P_{t+56}+P_{t+57}+P_{t+58}}{P_{t-4}+P_{t-3}+P_{t-2}}\right)^{\frac{1}{5}}-1\right] \\
&\approx \frac{\pi_{t-4\to t+56}+\pi_{t-3\to t+57}+\pi_{t-2\to t+58}}{3} \\
&= \frac{1}{3}\left\{\left[\frac{4}{60}\pi_{t-4\to t}+\frac{56}{60}\pi_{t\to t+56}\right]+\left[\frac{3}{60}\pi_{t-3\to t}+\frac{57}{60}\pi_{t\to t+57}\right]\right. \\
&\quad \left.+\left[\frac{2}{60}\pi_{t-2\to t}+\frac{58}{60}\pi_{t\to t+58}\right]\right\} \\
&= \frac{1}{180}\left(\pi_{t-4\to t-3}+\pi_{t-3\to t-2}+2\pi_{t-2\to t}\right)+\frac{1}{180}\left(\pi_{t-3\to t-2}+2\pi_{t-2\to t}\right) \\
&\quad +\frac{1}{180}2\pi_{t-2\to t}+\frac{1}{180}\left(56\pi_{t\to t+56}+57\pi_{t\to t+57}+58\pi_{t\to t+58}\right) \\
&= \frac{1}{180}\left(\pi_{t-4\to t-3}+2\pi_{t-3\to t-2}+6\pi_{t-2\to t}\right)+\frac{1}{180}\left(56\pi_{t\to t+56}+57\pi_{t\to t+57}+58\pi_{t\to t+58}\right) \\
&= \frac{1}{174}\left(\pi_{t-4\to t-3}+2\pi_{t-3\to t-2}\right)+\frac{1}{174}\left(56\pi_{t\to t+56}+57\pi_{t\to t+57}+58\pi_{t\to t+58}\right),
\end{aligned}
$$

where I use the properties of continuous compounding to simplify the expressions, and the last line uses $\pi_{t-2\to t}$ =SPF-5YR-Q1$_t$. Note that this forecast contains two realized inflation terms and three terms that are forecasts from period t onward.

Turning to the second-quarter forecast, where period t now denotes May, I have

$$\begin{aligned}
\text{SPF-5YR-Q2}_t &= 100\left[\left(\frac{P_{t+53}+P_{t+54}+P_{t+55}}{P_{t-7}+P_{t-6}+P_{t-5}}\right)^{\frac{1}{5}}-1\right] \\
&\approx \frac{1}{174}\left(\pi_{t-7\to t-6}+2\pi_{t-6\to t-5}+9\pi_{t-5\to t-2}\right) \\
&\quad+\frac{1}{174}\left(53\pi_{t\to t+53}+54\pi_{t\to t+54}+55\pi_{t\to t+55}\right).
\end{aligned}$$

The third-quarter forecast, where period t now denotes August, is given by

$$\begin{aligned}
\text{SPF-5YR-Q3}_t &= 100\left[\left(\frac{P_{t+50}+P_{t+51}+P_{t+52}}{P_{t-10}+P_{t-9}+P_{t-8}}\right)^{\frac{1}{5}}-1\right] \\
&\approx \frac{1}{174}\left(\pi_{t-10\to t-9}+2\pi_{t-9\to t-8}+18\pi_{t-8\to t-2}\right) \\
&\quad+\frac{1}{174}\left(50\pi_{t\to t+50}+51\pi_{t\to t+51}+52\pi_{t\to t+52}\right).
\end{aligned}$$

Finally, the fourth-quarter forecast, with t denoting November, is given by

$$\begin{aligned}
\text{SPF-5YR-Q4}_t &= 100\left[\left(\frac{P_{t+47}+P_{t+48}+P_{t+49}}{P_{t-13}+P_{t-12}+P_{t-11}}\right)^{\frac{1}{5}}-1\right] \\
&\approx \frac{1}{174}\left(\pi_{t-13\to t-12}+2\pi_{t-12\to t-11}+27\pi_{t-11\to t-2}\right) \\
&\quad+\frac{1}{174}\left(47\pi_{t\to t+47}+48\pi_{t\to t+48}+49\pi_{t\to t+49}\right).
\end{aligned}$$

Using the definitions

$$\begin{aligned}
x_t^{13} &\equiv \text{SPF-5YR-Q1}_t-\frac{1}{174}\left(\pi_{t-4\to t-3}+2\pi_{t-3\to t-2}\right) \\
x_t^{14} &\equiv \text{SPF-5YR-Q2}_t-\frac{1}{174}\left(\pi_{t-7\to t-6}+2\pi_{t-6\to t-5}+9\pi_{t-5\to t-2}\right) \\
x_t^{15} &\equiv \text{SPF-5YR-Q3}_t-\frac{1}{174}\left(\pi_{t-10\to t-9}+2\pi_{t-9\to t-8}+18\pi_{t-8\to t-2}\right) \\
x_t^{16} &\equiv \text{SPF-5YR-Q4}_t-\frac{1}{174}\left(\pi_{t-13\to t-12}+2\pi_{t-12\to t-11}+27\pi_{t-11\to t-2}\right).
\end{aligned}$$

The measurement equations for the 5-year forecasts are

$$
\begin{aligned}
x_t^{13} &= \frac{1}{174}\left(56\pi_{t\to t+56} + 57\pi_{t\to t+57} + 58\pi_{t\to t+58}\right) + \varepsilon_t^{13} \\
x_t^{14} &= \frac{1}{174}\left(53\pi_{t\to t+53} + 54\pi_{t\to t+54} + 55\pi_{t\to t+55}\right) + \varepsilon_t^{14} \\
x_t^{15} &= \frac{1}{174}\left(50\pi_{t\to t+50} + 51\pi_{t\to t+51} + 52\pi_{t\to t+52}\right) + \varepsilon_t^{15} \\
x_t^{16} &= \frac{1}{174}\left(47\pi_{t\to t+47} + 48\pi_{t\to t+48} + 49\pi_{t\to t+49}\right) + \varepsilon_t^{16}.
\end{aligned}
$$

Applying the same idea to 10-year forecasts, I define

$$
\begin{aligned}
x_t^{17} &\equiv \text{SPF-10YR-Q1}_t - \frac{1}{354}\left(\pi_{t-4\to t-3} + 2\pi_{t-3\to t-2}\right) \\
x_t^{18} &\equiv \text{SPF-10YR-Q2}_t - \frac{1}{354}\left(\pi_{t-7\to t-6} + 2\pi_{t-6\to t-5} + 9\pi_{t-5\to t-2}\right) \\
x_t^{19} &\equiv \text{SPF-10YR-Q3}_t - \frac{1}{354}\left(\pi_{t-10\to t-9} + 2\pi_{t-9\to t-8} + 18\pi_{t-8\to t-2}\right) \\
x_t^{20} &\equiv \text{SPF-10YR-Q4}_t - \frac{1}{354}\left(\pi_{t-13\to t-12} + 2\pi_{t-12\to t-11} + 27\pi_{t-11\to t-2}\right),
\end{aligned}
$$

and the remaining measurement equations are

$$
\begin{aligned}
x_t^{17} &= \frac{1}{354}\left(116\pi_{t\to t+116} + 117\pi_{t\to t+117} + 118\pi_{t\to t+118}\right) + \varepsilon_t^{17} \\
x_t^{18} &= \frac{1}{354}\left(113\pi_{t\to t+113} + 114\pi_{t\to t+114} + 115\pi_{t\to t+115}\right) + \varepsilon_t^{18} \\
x_t^{19} &= \frac{1}{354}\left(110\pi_{t\to t+110} + 111\pi_{t\to t+111} + 112\pi_{t\to t+112}\right) + \varepsilon_t^{19} \\
x_t^{20} &= \frac{1}{354}\left(107\pi_{t\to t+107} + 108\pi_{t\to t+108} + 109\pi_{t\to t+109}\right) + \varepsilon_t^{20}.
\end{aligned}
$$

The full measurement equations follow from applying (6) to the right-hand sides of these equations.

A.1.2 *Blue Chip* Quarterly Forecasts

I use the quarterly forecasts published in the *Blue Chip Economic Indicators*. Every month, the forecasters are asked to make between five and nine short-term forecasts. The table below summarizes the availability of forecasts for the year 2016 as an example in which "X" denotes a forecast, "-" reflects the absence of a forecast, and "*" next to an "X" shows a forecast that I use in this paper.

	2015Q4	2016Q1	2016Q2	2016Q3	2016Q4	2017Q1	2017Q2	2017Q3	2017Q4
January	X	X	X*	X*	X*	X*	X*	X	X
February	-	X	X*	X*	X*	X*	X*	X	X
March	-	X	X*	X*	X*	X*	X*	X	X
April	-	X	X	X*	X*	X*	X*	X*	X
May	-	-	X	X*	X*	X*	X*	X*	X
June	-	-	X	X*	X*	X*	X*	X*	X
July	-	-	X	X	X*	X*	X*	X*	X*
August	-	-	-	X	X*	X*	X*	X*	X*
September	-	-	-	X	X*	X*	X*	X*	X*
October	-	-	-	X	X	X*	X*	X*	X*
November	-	-	-	-	X	X*	X*	X*	X*
December	-	-	-	-	X	X*	X*	X*	X*

The table shows that in most months I use five forecasts, each of which reflects the change in the average level of CPI over a quarter, relative to the previous quarter. The first quarter forecast I use is the one that follows the month the forecast is made – for example, in months that are in the first quarter, I use forecasts that are about the second, third, and fourth quarters of the current year and the first and second quarters of the following year. In the fourth quarter, I use only four forecasts. These choices follow from the timing and the unbalanced structure of the survey.

The first four of the forecasts I use follow the "3," "4," "5," and "6" forecasts of the *SPF*. Since the *SPF* forecasts were done in the second month of a quarter, I treated them identically. However, *Blue Chip* forecasts are made monthly and thus in different months of a quarter. This means I need to create three different versions of each *Blue Chip* forecast, depending on which month of the quarter it is made.

Starting with those made in the second month of a quarter, the expressions exactly mimic those from the *SPF* – for example the next-quarter forecast in February, May, August or November will be

$$\text{BC-1Q-M2}_t = \frac{1}{8}\left(2\pi_{t\to t+2} + 3\pi_{t\to t+3} + 3\pi_{t+1\to t+4}\right),$$

where the notation is BC-XQ-MY means the forecast made in the Y^{th} month of a quarter covering the quarter that is X quarters after the current one. And the BC-5Q-M2 forecast, which was not available in the *SPF*, is given by

$$\text{BC-5Q-M2}_t = \frac{1}{3}\left(\pi_{t+11\to t+14} + \pi_{t+12\to t+15} + \pi_{t+13\to t+16}\right).$$

A forecast made in the first month of a quarter for the next quarter is

$$\begin{aligned}\text{BC-1Q-M1}_t &= 100\left[\left(\frac{P_{t+3}+P_{t+4}+P_{t+5}}{P_t+P_{t+1}+P_{t+2}}\right)^4 - 1\right] \\ &\approx \frac{\pi_{t\to t+3} + \pi_{t+1\to t+4} + \pi_{t+2\to t+5}}{3},\end{aligned}$$

and others are obtained as

$$\begin{aligned}\text{BC-2Q-M1}_t &= \frac{\pi_{t+3\to t+6} + \pi_{t+4\to t+7} + \pi_{t+5\to t+8}}{3} \\ \text{BC-3Q-M1}_t &= \frac{\pi_{t+6\to t+9} + \pi_{t+7\to t+10} + \pi_{t+8\to t+11}}{3} \\ \text{BC-4Q-M1}_t &= \frac{\pi_{t+9\to t+12} + \pi_{t+10\to t+13} + \pi_{t+11\to t+14}}{3} \\ \text{BC-5Q-M1}_t &= \frac{\pi_{t+12\to t+15} + \pi_{t+13\to t+16} + \pi_{t+14\to t+17}}{3}.\end{aligned}$$

Finally, a forecast made in the last month of a quarter for the next quarter is

$$\begin{aligned}\text{BC-1Q-M3}_t &= 100\left[\left(\frac{P_{t+1}+P_{t+2}+P_{t+3}}{P_{t-2}+P_{t-1}+P_t}\right)^4 - 1\right] \\ &\approx \frac{1}{6}\left(\pi_{t\to t+1} + 2\pi_{t\to t+2} + 3\pi_{t\to t+3}\right),\end{aligned}$$

where in the last line I used $2\pi_{t-2\to t} + \pi_{t-1\to t} = 3\text{BC-1Q-M3}_t$.

Other forecasts follow from

$$
\begin{aligned}
\text{BC-2Q-M3}_t &= \frac{\pi_{t+1\to t+4} + \pi_{t+2\to t+5} + \pi_{t+3\to t+6}}{3} \\
\text{BC-3Q-M3}_t &= \frac{\pi_{t+4\to t+7} + \pi_{t+5\to t+8} + \pi_{t+6\to t+9}}{3} \\
\text{BC-4Q-M3}_t &= \frac{\pi_{t+7\to t+10} + \pi_{t+8\to t+11} + \pi_{t+9\to t+12}}{3} \\
\text{BC-5Q-M3}_t &= \frac{\pi_{t+10\to t+13} + \pi_{t+11\to t+14} + \pi_{t+12\to t+15}}{3}.
\end{aligned}
$$

Collecting all these, I define

$$
\begin{aligned}
x_t^{21} &\equiv \text{BC-1Q-M1}_t,\ x_t^{22} \equiv \text{BC-2Q-M1}_t,\ x_t^{23} \equiv \text{BC-3Q-M1}_t,\ x_t^{24} \equiv \text{BC-4Q-M1}_t, \\
x_t^{25} &\equiv \text{BC-5Q-M1}_t,\ x_t^{26} \equiv \text{BC-1Q-M2}_t,\ x_t^{27} \equiv \text{BC-2Q-M2}_t,\ x_t^{28} \equiv \text{BC-3Q-M2}_t, \\
x_t^{29} &\equiv \text{BC-4Q-M2}_t,\ x_t^{30} \equiv \text{BC-5Q-M2}_t,\ x_t^{31} \equiv \text{BC-1Q-M3}_t,\ x_t^{32} \equiv \text{BC-2Q-M3}_t, \\
x_t^{33} &\equiv \text{BC-3Q-M3}_t,\ x_t^{34} \equiv \text{BC-4Q-M3}_t,\ x_t^{35} \equiv \text{BC-5Q-M3}_t.
\end{aligned}
$$

The measurement equations are

$$
\begin{aligned}
x_t^{21} &= \frac{1}{3}\left(\pi_{t\to t+3} + \pi_{t+1\to t+4} + \pi_{t+2\to t+5}\right) + \varepsilon_t^{21} \\
x_t^{22} &= \frac{1}{3}\left(\pi_{t+3\to t+6} + \pi_{t+4\to t+7} + \pi_{t+5\to t+8}\right) + \varepsilon_t^{22} \\
x_t^{23} &= \frac{1}{3}\left(\pi_{t+6\to t+9} + \pi_{t+7\to t+10} + \pi_{t+8\to t+11}\right) + \varepsilon_t^{23} \\
x_t^{24} &= \frac{1}{3}\left(\pi_{t+9\to t+12} + \pi_{t+10\to t+13} + \pi_{t+11\to t+14}\right) + \varepsilon_t^{24} \\
x_t^{25} &= \frac{1}{3}\left(\pi_{t+12\to t+15} + \pi_{t+13\to t+16} + \pi_{t+14\to t+17}\right) + \varepsilon_t^{25}
\end{aligned}
$$

$$
\begin{aligned}
x_t^{26} &= \frac{1}{8}\left(2\pi_{t\to t+2} + 3\pi_{t\to t+3} + 3\pi_{t+1\to t+4}\right) + \varepsilon_t^{26} \\
x_t^{27} &= \frac{1}{3}\left(\pi_{t+2\to t+5} + \pi_{t+3\to t+6} + \pi_{t+4\to t+7}\right) + \varepsilon_t^{27} \\
x_t^{28} &= \frac{1}{3}\left(\pi_{t+5\to t+8} + \pi_{t+6\to t+9} + \pi_{t+7\to t+10}\right) + \varepsilon_t^{28} \\
x_t^{29} &= \frac{1}{3}\left(\pi_{t+8\to t+11} + \pi_{t+9\to t+12} + \pi_{t+10\to t+13}\right) + \varepsilon_t^{29} \\
x_t^{30} &= \frac{1}{3}\left(\pi_{t+11\to t+14} + \pi_{t+12\to t+15} + \pi_{t+13\to t+16}\right) + \varepsilon_t^{30}
\end{aligned}
$$

$$
\begin{aligned}
x_t^{31} &= \frac{1}{6}\left(\pi_{t\to t+1} + 2\pi_{t\to t+2} + 3\pi_{t\to t+3}\right) + \varepsilon_t^{31} \\
x_t^{32} &= \frac{1}{3}\left(\pi_{t+1\to t+4} + \pi_{t+2\to t+5} + \pi_{t+3\to t+6}\right) + \varepsilon_t^{32} \\
x_t^{33} &= \frac{1}{3}\left(\pi_{t+4\to t+7} + \pi_{t+5\to t+8} + \pi_{t+6\to t+9}\right) + \varepsilon_t^{33} \\
x_t^{34} &= \frac{1}{3}\left(\pi_{t+7\to t+10} + \pi_{t+8\to t+11} + \pi_{t+9\to t+12}\right) + \varepsilon_t^{34} \\
x_t^{35} &= \frac{1}{3}\left(\pi_{t+10\to t+13} + \pi_{t+11\to t+14} + \pi_{t+12\to t+15}\right) + \varepsilon_t^{35}.
\end{aligned}
$$

A.1.3 *Blue Chip* Long-Range Forecasts

In the March and October issues of the *Blue Chip Economic Indicators* and the June and December issues of the *Blue Chip Financial Forecasts*, the forecasters are asked about their long-term forecasts. They are asked to make six forecasts of long-range inflation: five annual forecasts, each covering one calendar year, and one five-year forecast covering the five years following the five years in the last forecast. The annual forecasts are labeled as "year-over-year" forecasts, which means they are the percentage change in the average price level across years. More specifically, since October 2008, both of these publications ask the forecasters to forecast five years following the next year – for 2008 this would be years 2010, 2011, 2012, 2013, and 2014 – as well as the the five-year forward forecast of 2015-2019. Prior to October 2008, in most years the format remained the same, but in some years the horizon shifted earlier by one year. To keep variables consistent throughout the sample, I use the format since 2008, and in years in which there is a shift, I use missing observations where appropriate.

In March of a year, the first object being forecast is defined as

$$\begin{aligned}
\text{BCLR-2Y-M}_t &= 100\left[\left(\frac{\sum_{s=22}^{33} P_{t+s}}{\sum_{s=10}^{21} P_{t+s}}\right)-1\right] \\
&\approx \frac{1}{12}\sum_{s=10}^{21}\pi_{t+s\rightarrow t+s+12},
\end{aligned}$$

which I label as a two-year forecast simply because the forecast window is in the calendar year following the next. I will continue using the same notation for the rest of the three months in which these publications are released to keep things simple even though the forecasting window moves and it approaches the period in which the forecast is made. The "M" at the end of the variable name reflects the "March" forecast, and I use "J," "O," and "D" to represent June, October, and December, respectively, below. The first annual forecast made in June, October, and December refer to the year that is 19, 15, and 13 months following the month the forecast is made, respectively.

Defining

$$\begin{aligned}
x_t^{36} &\equiv \text{BCLR-2Y-M}_t,\ x_t^{37} \equiv \text{BCLR-3Y-M}_t,\ x_t^{38} \equiv \text{BCLR-4Y-M}_t,\ x_t^{39} \equiv \text{BCLR-5Y-M}_t \\
x_t^{40} &\equiv \text{BCLR-6Y-M}_t,\ x_t^{41} \equiv \text{BCLR-2Y-J}_t,\ x_t^{42} \equiv \text{BCLR-3Y-J}_t,\ x_t^{43} \equiv \text{BCLR-4Y-J}_t \\
x_t^{44} &\equiv \text{BCLR-5Y-J}_t,\ x_t^{45} \equiv \text{BCLR-6Y-J}_t,\ x_t^{46} \equiv \text{BCLR-2Y-O}_t,\ x_t^{47} \equiv \text{BCLR-3Y-O}_t \\
x_t^{48} &\equiv \text{BCLR-4Y-O}_t,\ x_t^{49} \equiv \text{BCLR-5Y-O}_t,\ x_t^{50} \equiv \text{BCLR-6Y-O}_t,\ x_t^{51} \equiv \text{BCLR-2Y-D}_t \\
x_t^{52} &\equiv \text{BCLR-3Y-D}_t,\ x_t^{53} \equiv \text{BCLR-4Y-D}_t,\ x_t^{54} \equiv \text{BCLR-5Y-D}_t,\ x_t^{55} \equiv \text{BCLR-6Y-D}_t,
\end{aligned}$$

the measurement equations for March are

$$
\begin{aligned}
x_t^{36} &= \frac{1}{12}\sum_{s=10}^{21}\pi_{t+s\to t+s+12}+\varepsilon_t^{36}\\
x_t^{37} &= \frac{1}{12}\sum_{s=22}^{33}\pi_{t+s\to t+s+12}+\varepsilon_t^{37}\\
x_t^{38} &= \frac{1}{12}\sum_{s=34}^{45}\pi_{t+s\to t+s+12}+\varepsilon_t^{38}\\
x_t^{39} &= \frac{1}{12}\sum_{s=46}^{57}\pi_{t+s\to t+s+12}+\varepsilon_t^{39}\\
x_t^{40} &= \frac{1}{12}\sum_{s=58}^{69}\pi_{t+s\to t+s+12}+\varepsilon_t^{40}.
\end{aligned}
$$

Then the measurement equations for June are

$$
\begin{aligned}
x_t^{41} &= \frac{1}{12}\sum_{s=7}^{18}\pi_{t+s\to t+s+12}+\varepsilon_t^{41}\\
x_t^{42} &= \frac{1}{12}\sum_{s=19}^{30}\pi_{t+s\to t+s+12}+\varepsilon_t^{42}\\
x_t^{43} &= \frac{1}{12}\sum_{s=31}^{42}\pi_{t+s\to t+s+12}+\varepsilon_t^{43}\\
x_t^{44} &= \frac{1}{12}\sum_{s=43}^{54}\pi_{t+s\to t+s+12}+\varepsilon_t^{44}\\
x_t^{45} &= \frac{1}{12}\sum_{s=55}^{66}\pi_{t+s\to t+s+12}+\varepsilon_t^{45}.
\end{aligned}
$$

And for October the measurement equations are

$$
\begin{aligned}
x_t^{46} &= \frac{1}{12}\sum_{s=3}^{14}\pi_{t+s\to t+s+12} + \varepsilon_t^{46} \\
x_t^{47} &= \frac{1}{12}\sum_{s=15}^{26}\pi_{t+s\to t+s+12} + \varepsilon_t^{47} \\
x_t^{48} &= \frac{1}{12}\sum_{s=27}^{38}\pi_{t+s\to t+s+12} + \varepsilon_t^{48} \\
x_t^{49} &= \frac{1}{12}\sum_{s=39}^{50}\pi_{t+s\to t+s+12} + \varepsilon_t^{49} \\
x_t^{50} &= \frac{1}{12}\sum_{s=51}^{62}\pi_{t+s\to t+s+12} + \varepsilon_t^{50}.
\end{aligned}
$$

And finally December forecasts use

$$
\begin{aligned}
x_t^{51} &= \frac{1}{12}\sum_{s=1}^{12}\pi_{t+s\to t+s+12} + \varepsilon_t^{51} \\
x_t^{52} &= \frac{1}{12}\sum_{s=13}^{24}\pi_{t+s\to t+s+12} + \varepsilon_t^{52} \\
x_t^{53} &= \frac{1}{12}\sum_{s=25}^{36}\pi_{t+s\to t+s+12} + \varepsilon_t^{53} \\
x_t^{54} &= \frac{1}{12}\sum_{s=37}^{48}\pi_{t+s\to t+s+12} + \varepsilon_t^{54} \\
x_t^{55} &= \frac{1}{12}\sum_{s=49}^{60}\pi_{t+s\to t+s+12} + \varepsilon_t^{55}.
\end{aligned}
$$

These publications contain two more forecasts. Once again using the October 2008 issue as an example, there are forecasts for "2010-2014" and "2015-2019." The former is an arithmetic average of the five annual forecasts I use and thus is not independently useful. In order to use the latter, I take its simple average with the former and label this the forecast for the 10-year period of 2010-2019. This forecast is defined as the average of the 10 annual price changes, each of which is in the format I use above – annual change in the average price level between two years. The March forecast can be written as

$$
\begin{aligned}
\text{BCLR-10Y-M}_t &= \frac{1}{10}\left\{\frac{1}{12}\left[\sum_{s=10}^{21}\pi_{t+s\to t+s+12} + \sum_{s=22}^{33}\pi_{t+s\to t+s+12} + \sum_{s=34}^{45}\pi_{t+s\to t+s+12}\right.\right. \\
&\quad + \sum_{s=46}^{57}\pi_{t+s\to t+s+12} + \sum_{s=58}^{69}\pi_{t+s\to t+s+12} + \sum_{s=70}^{81}\pi_{t+s\to t+s+12}\sum_{s=82}^{93}\pi_{t+s\to t+s+12} \\
&\quad \left.\left. + \sum_{s=94}^{105}\pi_{t+s\to t+s+12} + \sum_{s=106}^{117}\pi_{t+s\to t+s+12} + \sum_{s=118}^{129}\pi_{t+s\to t+s+12}\right]\right\} \\
&= \frac{1}{12}\sum_{s=10}^{21}\pi_{t+s\to t+s+120},
\end{aligned}
$$

where the last equality follows from the properties of continuous compounding.[35] Denoting $x_t^{56} \equiv$BCLR-10Y-M$_t$, $x_t^{57} \equiv$BCLR-10Y-J$_t$, $x_t^{58} \equiv$BCLR-10Y-O$_t$, and $x_t^{59} \equiv$BCLR-10Y-D$_t$, the measurement equations are

$$
\begin{aligned}
x_t^{56} &= \frac{1}{12}\sum_{s=10}^{21}\pi_{t+s\to t+s+120} + \varepsilon_t^{56} \\
x_t^{57} &= \frac{1}{12}\sum_{s=7}^{18}\pi_{t+s\to t+s+120} + \varepsilon_t^{57} \\
x_t^{58} &= \frac{1}{12}\sum_{s=3}^{14}\pi_{t+s\to t+s+120} + \varepsilon_t^{58} \\
x_t^{59} &= \frac{1}{12}\sum_{s=1}^{12}\pi_{t+s\to t+s+120} + \varepsilon_t^{59}.
\end{aligned}
$$

The measurement equations are directly obtained from (6) with measurement errors ε_t^i.

[35] To see this, consider the simplified example

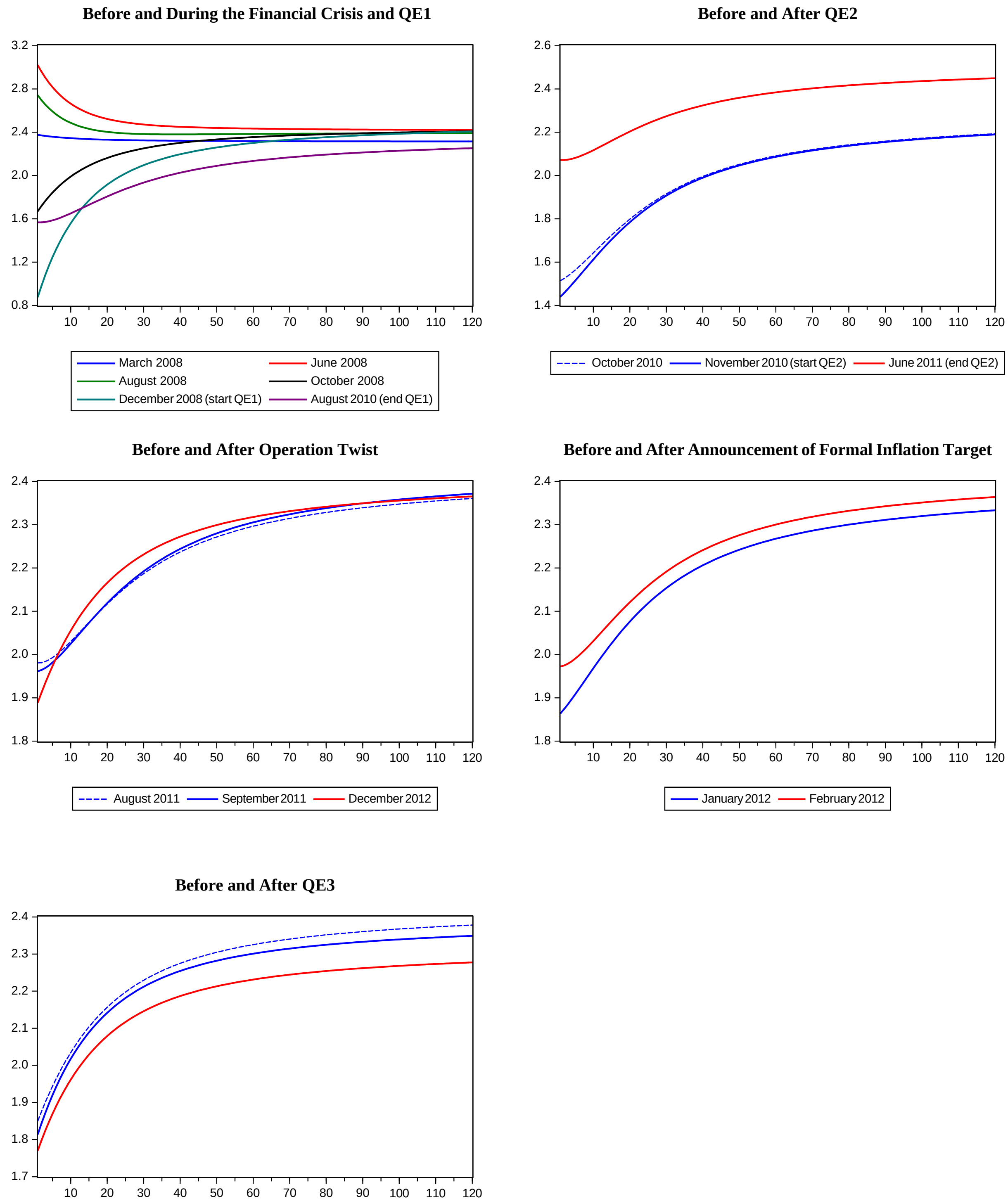
$$
\begin{aligned}
\frac{1}{2}\left(\frac{1}{2}\sum_{s=13}^{14}\pi_{t+s\to t+s+2} + \frac{1}{2}\sum_{s=15}^{16}\pi_{t+s\to t+s+2}\right) &= \frac{1}{4}\left(\pi_{t+13\to t+15} + \pi_{t+14\to t+16} + \pi_{t+15\to t+17} + \pi_{t+16\to t+18}\right) \\
&= \frac{1}{4}\left(\pi_{t+13\to t+15} + \pi_{t+15\to t+17} + \pi_{t+14\to t+16} + \pi_{t+16\to t+18}\right) \\
&= \frac{1}{2}\left(\pi_{t+13\to t+17} + \pi_{t+14\to t+18}\right) \\
&= \frac{1}{2}\sum_{s=13}^{14}\pi_{t+s\to t+s+4}.
\end{aligned}
$$

Table 3: Parameter Estimates for the UCSV Model

	Prior	5%	**Median**	95%
φ	U[0,1]	0.08	0.14	0.20
σ	IG(3,5)	1.25	1.39	1.65
ρ_η	N(0.9,5)	-0.40	0.33	0.80
ρ_ϵ	N(0.9 5)	0.85	0.93	0.97
σ_{ν_η}	IG(3,0.1)	0.14	0.24	0.44
σ_{ν_ϵ}	IG(3,0.1)	0.51	0.65	0.90

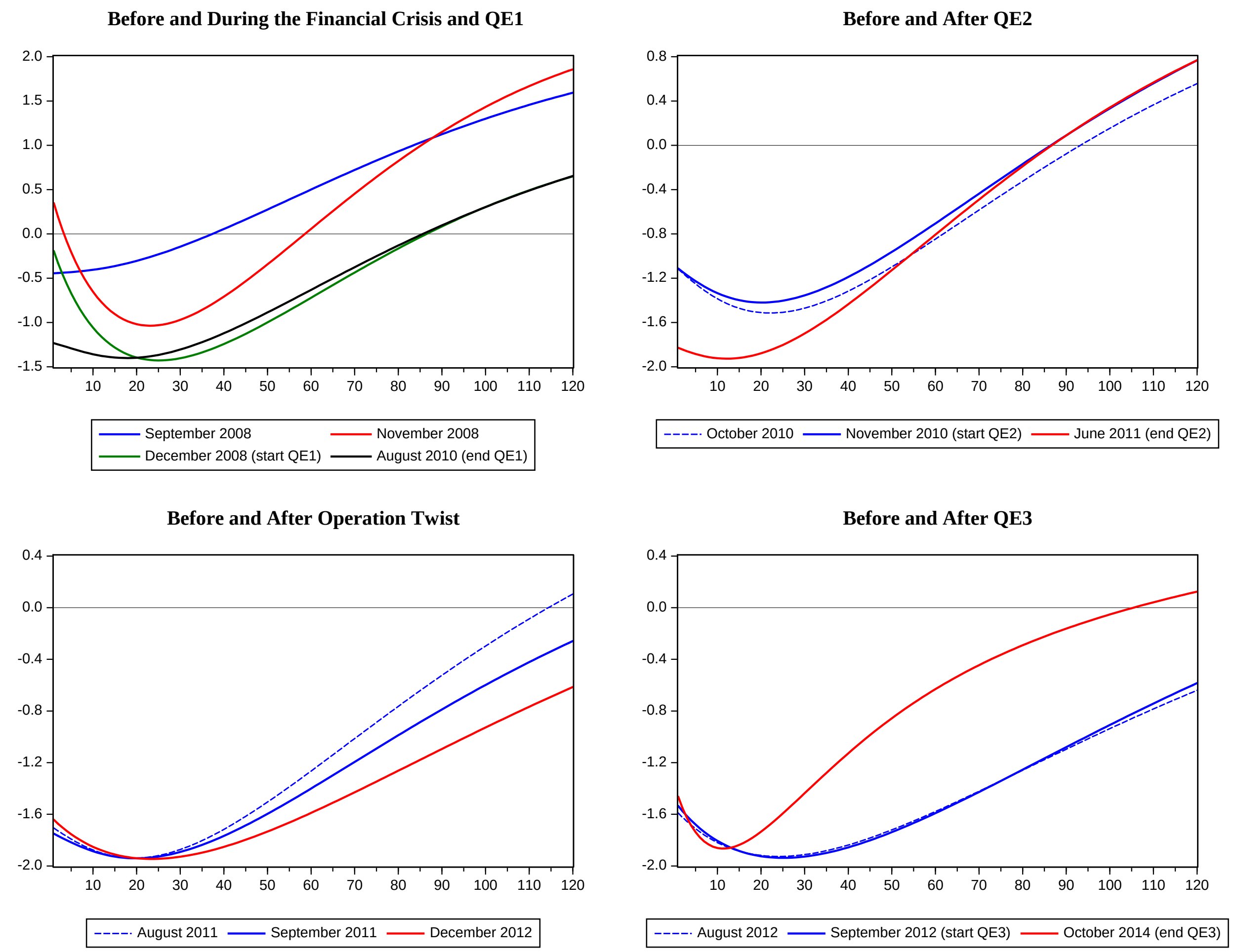
Notes: The first column shows the marginal prior distribution for each parameter where $U[a,b]$ means the uniform distribution between a and b, $N(a,b)$ means the normal distribution with mean a and variance b, and IG means the inverse gamma distribution $IG(a,b)$, which is parameterized as $p_{IG}(\sigma \mid a, b) \propto \sigma^{-a-1}\exp(b/\sigma)$. The priors for ρ_η and ρ_ϵ are truncated to ensure stationarity. The remaining columns show the given percentiles of the posterior distribution. Estimation uses data from January 1984 to December 2015.

Figure 9: Inflation Expectations Curves



Notes: The x-axis is in months, and each figure starts with the three-month forecast and goes up to the 10-year forecast.

Figure 10: Real Interest Rate Curves



Notes: The x-axis is in months, and each figure starts with the three-month forecast and goes up to the 10-year interest rate.

Table 4: **Inflation Expectations, 2008–2016**

	1-Year	2-Year	5-Year	10-Year	Year-3-to-Year-10	Year-6-to-Year-10
2008M01	2.23	2.26	2.33	2.35	2.38	2.38
2008M02	2.28	2.30	2.33	2.35	2.36	2.36
2008M03	2.34	2.33	2.32	2.31	2.31	2.31
2008M04	2.46	2.40	2.36	2.35	2.33	2.33
2008M05	2.57	2.47	2.41	2.40	2.38	2.38
2008M06	2.62	2.50	2.43	2.42	2.40	2.41
2008M07	2.61	2.49	2.44	2.43	2.42	2.42
2008M08	2.46	2.39	2.38	2.39	2.39	2.40
2008M09	2.28	2.32	2.38	2.41	2.43	2.44
2008M10	2.03	2.20	2.35	2.41	2.46	2.46
2008M11	1.78	2.07	2.30	2.38	2.46	2.47
2008M12	1.65	2.00	2.30	2.41	2.51	2.51
2009M01	1.57	1.93	2.27	2.40	2.52	2.53
2009M02	1.51	1.85	2.22	2.36	2.49	2.50
2009M03	1.48	1.80	2.17	2.33	2.46	2.48
2009M04	1.60	1.88	2.25	2.40	2.53	2.56
2009M05	1.69	1.94	2.29	2.43	2.55	2.58
2009M06	1.73	1.96	2.28	2.41	2.52	2.54
2009M07	1.75	1.96	2.25	2.37	2.47	2.49
2009M08	1.78	1.97	2.24	2.36	2.46	2.47
2009M09	1.74	1.93	2.19	2.31	2.40	2.42
2009M10	1.68	1.88	2.14	2.26	2.35	2.37
2009M11	1.70	1.91	2.20	2.32	2.42	2.43
2009M12	1.81	2.03	2.32	2.44	2.54	2.56
2010M01	1.78	2.00	2.27	2.39	2.48	2.50
2010M02	1.81	2.00	2.25	2.36	2.44	2.46
2010M03	1.85	2.03	2.26	2.35	2.44	2.45
2010M04	1.86	2.02	2.25	2.35	2.43	2.44
2010M05	1.87	2.03	2.27	2.38	2.47	2.49
2010M06	1.78	1.95	2.22	2.34	2.43	2.46
2010M07	1.65	1.83	2.11	2.24	2.34	2.36
2010M08	1.68	1.86	2.13	2.25	2.35	2.37
2010M09	1.69	1.86	2.12	2.22	2.31	2.33
2010M10	1.68	1.85	2.09	2.19	2.28	2.29
2010M11	1.65	1.84	2.09	2.19	2.28	2.29
2010M12	1.67	1.87	2.12	2.23	2.32	2.33
2011M01	1.71	1.90	2.14	2.23	2.32	2.33
2011M02	1.82	1.98	2.18	2.26	2.33	2.34
2011M03	1.98	2.11	2.27	2.34	2.40	2.41
2011M04	2.06	2.16	2.31	2.37	2.42	2.43

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Table 4: **Inflation Expectations, 2008–2016** ***(continued)***

	1-Year	2-Year	5-Year	10-Year	Year-3-to-Year-10	Year-6-to-Year-10
2011M05	2.06	2.16	2.31	2.37	2.42	2.43
2011M06	2.13	2.23	2.38	2.45	2.50	2.51
2011M07	2.16	2.26	2.41	2.47	2.53	2.54
2011M08	2.05	2.15	2.30	2.36	2.41	2.42
2011M09	2.04	2.15	2.31	2.37	2.43	2.44
2011M10	2.05	2.17	2.33	2.40	2.46	2.47
2011M11	1.99	2.11	2.28	2.34	2.40	2.41
2011M12	2.00	2.13	2.29	2.36	2.41	2.42
2012M01	1.99	2.11	2.27	2.33	2.39	2.40
2012M02	2.05	2.15	2.30	2.36	2.42	2.43
2012M03	2.14	2.23	2.36	2.43	2.48	2.49
2012M04	2.16	2.24	2.38	2.44	2.48	2.50
2012M05	2.13	2.21	2.35	2.41	2.46	2.47
2012M06	2.07	2.17	2.31	2.37	2.42	2.43
2012M07	2.04	2.17	2.31	2.36	2.41	2.42
2012M08	2.06	2.19	2.33	2.38	2.43	2.43
2012M09	2.05	2.17	2.30	2.35	2.39	2.40
2012M10	2.07	2.19	2.31	2.36	2.40	2.40
2012M11	2.11	2.23	2.34	2.38	2.42	2.43
2012M12	2.08	2.20	2.32	2.37	2.41	2.41
2013M01	2.06	2.16	2.29	2.34	2.39	2.40
2013M02	2.05	2.15	2.28	2.34	2.39	2.39
2013M03	2.07	2.17	2.30	2.35	2.40	2.41
2013M04	2.10	2.18	2.31	2.37	2.42	2.43
2013M05	2.06	2.15	2.29	2.35	2.39	2.40
2013M06	2.04	2.15	2.30	2.36	2.41	2.42
2013M07	1.98	2.10	2.26	2.33	2.38	2.39
2013M08	1.92	2.06	2.22	2.29	2.35	2.36
2013M09	1.95	2.09	2.26	2.32	2.38	2.39
2013M10	1.99	2.12	2.28	2.34	2.40	2.40
2013M11	1.99	2.11	2.26	2.32	2.37	2.38
2013M12	1.96	2.07	2.21	2.27	2.32	2.33
2014M01	1.91	2.01	2.14	2.20	2.25	2.26
2014M02	1.95	2.03	2.15	2.21	2.25	2.26
2014M03	2.03	2.09	2.21	2.27	2.31	2.32
2014M04	1.99	2.06	2.20	2.26	2.30	2.32
2014M05	1.96	2.05	2.20	2.27	2.32	2.34
2014M06	2.02	2.12	2.27	2.34	2.40	2.41
2014M07	2.04	2.15	2.29	2.35	2.40	2.41
2014M08	2.06	2.16	2.29	2.34	2.38	2.39
2014M09	2.02	2.13	2.25	2.30	2.34	2.35

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Table 4: **Inflation Expectations, 2008–2016** *(continued)*

	1-Year	**2-Year**	**5-Year**	**10-Year**	**Year-3-to-Year-10**	**Year-6-to-Year-10**
2014M10	1.99	2.11	2.23	2.28	2.32	2.32
2014M11	1.97	2.09	2.21	2.25	2.29	2.29
2014M12	2.07	2.20	2.30	2.33	2.37	2.37
2015M01	2.09	2.20	2.29	2.32	2.35	2.35
2015M02	2.03	2.14	2.23	2.26	2.29	2.29
2015M03	2.10	2.19	2.28	2.32	2.35	2.35
2015M04	2.09	2.19	2.27	2.30	2.33	2.33
2015M05	2.07	2.17	2.25	2.27	2.30	2.30
2015M06	2.10	2.21	2.28	2.30	2.32	2.32
2015M07	2.12	2.24	2.31	2.32	2.34	2.34
2015M08	2.05	2.19	2.26	2.27	2.29	2.29
2015M09	2.08	2.22	2.28	2.30	2.32	2.31
2015M10	2.11	2.23	2.29	2.30	2.32	2.31
2015M11	2.11	2.20	2.25	2.26	2.27	2.26
2015M12	2.19	2.27	2.29	2.30	2.30	2.30
2016M01	2.20	2.26	2.28	2.28	2.29	2.29
2016M02	2.15	2.20	2.23	2.24	2.25	2.25
2016M03	2.19	2.24	2.28	2.29	2.30	2.30
2016M04	2.21	2.25	2.29	2.30	2.31	2.31
2016M05	2.18	2.22	2.25	2.26	2.27	2.27
2016M06	2.23	2.26	2.28	2.29	2.29	2.29
2016M07	2.27	2.30	2.31	2.31	2.31	2.31

Table 5: Real Interest Rates, 2008–2016

	6-Month	1-Year	2-Year	5-Year	10-Year
2008M01	0.67	0.36	0.17	0.68	1.65
2008M02	-0.17	-0.36	-0.35	0.52	1.72
2008M03	-0.79	-0.83	-0.73	0.16	1.54
2008M04	-0.83	-0.68	-0.39	0.48	1.65
2008M05	-0.78	-0.51	-0.10	0.79	1.79
2008M06	-0.54	-0.22	0.23	1.09	1.97
2008M07	-0.67	-0.36	0.05	0.90	1.85
2008M08	-0.54	-0.32	-0.01	0.77	1.77
2008M09	-0.43	-0.39	-0.25	0.50	1.59
2008M10	-0.35	-0.61	-0.68	0.43	2.04
2008M11	-0.31	-0.77	-1.04	0.06	1.86
2008M12	-0.76	-1.16	-1.43	-0.72	0.65
2009M01	-0.81	-1.11	-1.29	-0.58	0.79
2009M02	-0.64	-0.86	-0.96	-0.21	1.17
2009M03	-0.67	-0.81	-0.89	-0.23	1.05
2009M04	-0.97	-1.02	-0.99	-0.29	0.98
2009M05	-1.18	-1.17	-1.03	-0.08	1.35
2009M06	-1.31	-1.14	-0.78	0.44	1.82
2009M07	-1.36	-1.24	-0.94	0.26	1.67
2009M08	-1.45	-1.25	-0.86	0.37	1.68
2009M09	-1.45	-1.31	-0.98	0.21	1.52
2009M10	-1.42	-1.27	-0.93	0.22	1.52
2009M11	-1.44	-1.37	-1.11	0.06	1.48
2009M12	-1.45	-1.41	-1.17	0.03	1.51
2010M01	-1.49	-1.39	-1.07	0.23	1.68
2010M02	-1.51	-1.44	-1.16	0.15	1.67
2010M03	-1.55	-1.43	-1.08	0.22	1.67
2010M04	-1.54	-1.38	-0.98	0.37	1.75
2010M05	-1.51	-1.44	-1.20	-0.03	1.32
2010M06	-1.40	-1.40	-1.23	-0.16	1.15
2010M07	-1.28	-1.31	-1.23	-0.30	1.04
2010M08	-1.31	-1.38	-1.38	-0.63	0.65
2010M09	-1.31	-1.39	-1.40	-0.65	0.64
2010M10	-1.28	-1.43	-1.51	-0.85	0.56
2010M11	-1.25	-1.37	-1.41	-0.71	0.77
2010M12	-1.25	-1.35	-1.26	-0.17	1.32
2011M01	-1.31	-1.41	-1.30	-0.14	1.40
2011M02	-1.48	-1.48	-1.21	0.09	1.54
2011M03	-1.70	-1.69	-1.43	-0.15	1.30
2011M04	-1.83	-1.77	-1.45	-0.12	1.31

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Table 5: Real Interest Rates, 2008–2016 *(continued)*

	6-Month	1-Year	2-Year	5-Year	10-Year
2011M05	-1.87	-1.85	-1.63	-0.46	1.02
2011M06	-1.90	-1.93	-1.82	-0.81	0.77
2011M07	-1.91	-1.95	-1.86	-0.86	0.76
2011M08	-1.81	-1.90	-1.93	-1.27	0.11
2011M09	-1.83	-1.91	-1.93	-1.40	-0.26
2011M10	-1.83	-1.89	-1.88	-1.25	-0.10
2011M11	-1.75	-1.83	-1.87	-1.34	-0.22
2011M12	-1.76	-1.85	-1.88	-1.37	-0.28
2012M01	-1.76	-1.84	-1.89	-1.39	-0.27
2012M02	-1.78	-1.85	-1.89	-1.44	-0.31
2012M03	-1.90	-1.91	-1.87	-1.31	-0.15
2012M04	-1.91	-1.94	-1.94	-1.44	-0.30
2012M05	-1.85	-1.90	-1.93	-1.55	-0.56
2012M06	-1.78	-1.83	-1.87	-1.56	-0.72
2012M07	-1.73	-1.83	-1.91	-1.65	-0.81
2012M08	-1.74	-1.85	-1.93	-1.58	-0.64
2012M09	-1.71	-1.84	-1.94	-1.59	-0.58
2012M10	-1.73	-1.85	-1.93	-1.57	-0.58
2012M11	-1.78	-1.89	-1.97	-1.64	-0.70
2012M12	-1.78	-1.88	-1.95	-1.59	-0.61
2013M01	-1.79	-1.87	-1.91	-1.46	-0.38
2013M02	-1.80	-1.87	-1.89	-1.42	-0.29
2013M03	-1.83	-1.89	-1.92	-1.46	-0.33
2013M04	-1.88	-1.93	-1.96	-1.59	-0.58
2013M05	-1.86	-1.90	-1.91	-1.44	-0.36
2013M06	-1.84	-1.87	-1.80	-1.09	0.05
2013M07	-1.76	-1.83	-1.74	-0.85	0.39
2013M08	-1.69	-1.77	-1.68	-0.69	0.60
2013M09	-1.75	-1.81	-1.67	-0.64	0.63
2013M10	-1.75	-1.83	-1.77	-0.89	0.41
2013M11	-1.73	-1.84	-1.81	-0.86	0.56
2013M12	-1.69	-1.80	-1.72	-0.62	0.80
2014M01	-1.71	-1.77	-1.61	-0.48	0.81
2014M02	-1.75	-1.81	-1.68	-0.61	0.65
2014M03	-1.86	-1.89	-1.68	-0.55	0.58
2014M04	-1.85	-1.88	-1.63	-0.46	0.57
2014M05	-1.82	-1.87	-1.66	-0.58	0.39
2014M06	-1.84	-1.89	-1.66	-0.57	0.37
2014M07	-1.86	-1.91	-1.63	-0.56	0.29
2014M08	-1.88	-1.93	-1.64	-0.61	0.18
2014M09	-1.85	-1.88	-1.53	-0.43	0.33

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Table 5: Real Interest Rates, 2008–2016 *(continued)*

	6-Month	1-Year	2-Year	5-Year	10-Year
2014M10	-1.78	-1.86	-1.62	-0.63	0.12
2014M11	-1.73	-1.80	-1.54	-0.54	0.17
2014M12	-1.76	-1.81	-1.53	-0.62	-0.05
2015M01	-1.84	-1.87	-1.63	-0.88	-0.37
2015M02	-1.80	-1.78	-1.49	-0.71	-0.19
2015M03	-1.83	-1.80	-1.52	-0.73	-0.19
2015M04	-1.84	-1.83	-1.62	-0.89	-0.29
2015M05	-1.81	-1.78	-1.52	-0.67	0.03
2015M06	-1.78	-1.77	-1.50	-0.56	0.17
2015M07	-1.76	-1.79	-1.56	-0.64	0.09
2015M08	-1.60	-1.65	-1.47	-0.68	-0.03
2015M09	-1.62	-1.66	-1.49	-0.74	-0.04
2015M10	-1.72	-1.74	-1.58	-0.85	-0.15
2015M11	-1.53	-1.52	-1.30	-0.53	0.11
2015M12	-1.47	-1.45	-1.24	-0.55	0.04
2016M01	-1.54	-1.50	-1.32	-0.72	-0.11
2016M02	-1.54	-1.55	-1.45	-0.97	-0.37
2016M03	-1.56	-1.52	-1.37	-0.85	-0.31
2016M04	-1.68	-1.63	-1.49	-0.99	-0.43
2016M05	-1.61	-1.56	-1.41	-0.91	-0.38
2016M06	-1.65	-1.63	-1.53	-1.09	-0.59
2016M07	-1.71	-1.72	-1.65	-1.26	-0.82