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Daniel Sanches
Federal Reserve Bank of Philadelphia

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Banking Panics and Protracted Recessions

Daniel Sanches*

Federal Reserve Bank of Philadelphia

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Abstract

This paper develops a dynamic model of bank liquidity provision to characterize the *ex post* effi cient policy response to a banking panic and study its implications for the behavior of output in the aftermath of a panic. It is shown that the trajectory of real output following a panic episode crucially depends on the cost of converting long-term assets into liquid funds. For small values of this liquidation cost, the recession associated with a banking panic is protracted. For intermediate values, the recession is more severe but short lived. For relatively large values, the contemporaneous decline in real output in the event of a panic is substantial but followed by a vigorous rebound in real activity above the long-run level. I argue that these theoretical predictions are consistent with the observed disparity in crisis-related output losses.

Keywords: banking panic, deposit contract, suspension of convertibility, time-consistent policy

JEL Classification: E32, E42, G21

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*Federal Reserve Bank of Philadelphia, Research Department, Ten Independence Mall, Philadelphia, PA 19106-1574. E-mail address: daniel.sanches@phil.frb.org. The views expressed in this paper are those of the author and do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System. I would like to thank Todd Keister, Shouyong Shi, and seminar participants at the Wharton School of the University of Pennsylvania, Texas A&M, and Tufts University. This paper is available free of charge at www.philadelphiafed.org/research-and-data/publications/working-papers.

1. INTRODUCTION

Economists usually refer to a sudden and apparently unexpected withdrawal of funds from banks as a banking panic. For example, Calomiris and Gorton (1991) define a banking panic as an event in which numerous depositors suddenly choose to exercise the option of converting their checkable deposits into currency from a significant number of banks in the banking system to such an extent that these banks suspend convertibility. Panic episodes are usually associated with significant decline in real activity across several sectors of the economy. For instance, Boyd, Kwak, and Smith (2005) have concluded that recessions associated with banking crises tend to be more severe and persistent, even though they have found considerable disparity in the behavior of real output across different episodes.

In some cases, the recovery from a crisis episode occurs in the following period with a vigorous rebound in real activity. In many episodes, the recession associated with a banking crisis is protracted. A common characteristic in all episodes is that government agencies have intervened to mitigate the adverse effects associated with a systemic run on the banking system. Thus, the observed trajectory of real output following a banking crisis necessarily reflects some form of government intervention in the banking system.

The goal of this paper is to characterize the *ex post* efficient policy response to a banking panic in a dynamic general equilibrium model to investigate its implications for the behavior of output in the aftermath of a panic episode. Ennis and Keister (2009) have shown that a fragile banking system subject to a self-fulfilling panic can be the outcome of an optimal deposit contract when agents form their expectations based on the knowledge of the *ex post* optimal policy response to a panic. In this paper, I characterize the optimal deposit contract given the expectation of an *ex post* optimal policy intervention to study the trajectory of real output following a banking panic and show that interesting intertemporal tradeoffs arise in a dynamic framework.

An important characteristic of the model that follows is that the liabilities issued by private banks circulate as a medium of exchange. The occurrence of a banking panic will result in a contraction of the supply of liquid assets in the economy, affecting transactions in the retail sector. In the event of a panic, a banking authority will intervene to jointly decide the optimal rule for suspending the convertibility of deposits and the fraction of long-term assets that can be prematurely liquidated to respond to a banking panic. This *ex post* efficient policy response to a banking panic will result in different patterns for the evolution of real output.

In the analysis that follows, I show that the trajectory of real output following a panic episode crucially depends on the cost of converting long-term assets into liquid funds. For small values of this liquidation cost, the recession associated with a banking panic is protracted. For intermediate values, the recession is more severe but short-lived. For relatively large values, the contemporaneous decline in real output in the event of a panic is substantial but followed by a vigorous rebound in real activity above the long-run level. Thus, the *ex post* efficient policy intervention implies distinct patterns for the evolution of real output in the event of a systemic run depending on the liquidation cost the banking authority faces when confronted with a bank run. Given these different patterns, it is possible to argue that the model's predictions are consistent with the previously described disparity in crisis-related output losses.

My theoretical framework builds on two apparently distinct strands of the literature on money and banking. The first focuses on the study of panics as an equilibrium outcome under rational expectations. The seminal contributions of Bryant (1980) and Diamond and Dybvig (1983) have initiated a vast literature on the real effects of panics. However, the vast majority of papers in this literature does not account for the fact that bank liabilities are widely used as a medium of exchange. The second strand focuses precisely on the role of money and other assets as a medium of exchange, following the seminal contribution of Kiyotaki and Wright (1989). Following this tradition, Cavalcanti, Erosa, and Temzelides (1999) have modified the original Kiyotaki-Wright framework to study inside money creation (in the form of bank notes). However, the connection between the ability of banks to supply liquid assets and the possibility of panics has not been established.

More recently, some researchers have taken a monetary approach to banking, explicitly accounting for the fact that bank liabilities serve as a medium of exchange. A prominent paper taking this approach is that of Gu, Mattesini, Monnet, and Wright (2013), who study inside money creation in the form of bank deposits that serve as a means of payment. However, there is nothing in their analysis that resembles a banking panic. In this paper, I build on their basic framework and introduce some other elements based on Champ, Smith, and Williamson (1996) to create a socially useful role for a demand deposit contract, as in the Diamond-Dybvig framework. As should be expected, because these elements generate a socially beneficial role for the provision of liquidity insurance by the banking system, in addition to the provision of transaction services, they also open the door to the possibility of self-fulfilling panics.

Very few papers in the literature have attempted to characterize the dynamic effects of a banking panic. A prominent paper that studies the effects of banking panics on capital accumulation and output is that of Ennis and Keister (2003). In a recently published paper, Gertler and Kiyotaki (2015) characterize the real effects of a banking panic in a dynamic framework with an endogenous liquidation price for banking assets. In both studies, the authors do not consider the effects of government intervention on the ensuing trajectory of real output.

2. MODEL

Time $t = 0, 1, 2, ...$ is discrete, and the horizon is infinite. Each period is divided into three subperiods or stages. There exist two symmetric regions that are identical with respect to all fundamentals. There is no communication between these regions. In each region, there are three types of agents, referred to as buyers, sellers, and bankers, who are infinitely lived. There is a $[0, 1]$ continuum of each type in each region.

Agents in each region interact as follows. In the first stage, the group of buyers and the group of bankers get together in a centralized meeting. In the second stage, each buyer is randomly and bilaterally matched with a seller with probability $\lambda \in \left(\frac{1}{2}, 1\right)$. In the third stage, the group of sellers and the group of bankers get together in a centralized meeting. Thus, each type is able to interact with the other two types at each date.

At date 0, a fraction $\varepsilon \in [0,1]$ of buyers in one region is randomly relocated to the other region and vice versa. I refer to a buyer who is relocated as a mover and to a buyer who is not relocated as a nonmover. A buyer finds out whether he is going to be permanently relocated at the end of the first stage, and the actual relocation occurs shortly after the idiosyncratic shock is realized. This shock is independently and identically distributed across agents. Unless otherwise explicitly stated, the relocation status of a buyer is privately observed until the moment he moves to the other location (when it becomes publicly observable). Note that no relocation occurs in periods $t \geq 1$.

There are two perfectly divisible commodities, referred to as good x and good y. A buyer is able to produce good x in the first subperiod. The available technology allows the buyer to produce either zero units or one unit. If good x is not properly stored in the subperiod it is produced, it will depreciate completely. All buyers have access to an *indivisible* storage technology for good x, which can be costlessly liquidated at any moment. In particular, a buyer can store either one unit or nothing. A seller is able to produce good y in the second subperiod. Good y is perishable and cannot be stored, so it must be consumed in the subperiod it is produced.

A banker is unable to produce either good but has access to a *divisible* technology that uses x as input and pays off at the beginning of the following date. Let $F(k)$ denote the payoff in terms of x when $k \in \mathbb{R}_+$ is the amount invested. It follows that

$$F(k) = \begin{cases} (1+\rho)\,k \text{ if } 0 \leq k \leq \bar{\imath}, \\ (1+\rho)\,\bar{\imath} \text{ if } \bar{\imath} < k \leq 1, \end{cases}$$

with $\rho > 0$ and $\frac{1-\lambda}{1+\rho} \leq \bar{\imath} \leq 1-\lambda$. If prematurely liquidated, the technology returns $\delta < 1$. Assume $\delta + \rho > 1$ and $0 < \varepsilon < 1 - \bar{\imath} < \lambda + \rho\bar{\imath}$. In addition, a banker has access to a perfectly *divisible* storage technology for x, which can be costlessly liquidated at any moment. A banker can also access a technology to costlessly create (and destroy) an indivisible, durable, and portable object, referred to as a bank claim, that perfectly identifies the banker as the issuer. An important characteristic of the environment is that a banker can access the productive technology only at the beginning of the period.

Let me now describe preferences. A buyer is a consumer of y, whereas a banker and a seller are consumers of x. Let $x_t \in \{0,1\}$ denote a buyer's production of x at date t, and let $y_t \in \mathbb{R}_+$ denote consumption of y at date t. A buyer's preferences are represented by

$$-\gamma x_t + u\left(y_t\right),$$

where $\gamma \in \mathbb{R}_+$ and $u : \mathbb{R}_+ \to \mathbb{R}_+$ is continuously differentiable, increasing, and strictly concave, with $u\left(0\right) = 0$ and $u'\left(0\right) = \infty$. As previously mentioned, the production technology of x allows a buyer to produce either zero units or one unit at each date. But keep in mind that good x is perfectly divisible.

Let $y_t \in \mathbb{R}_+$ denote a seller's production of y at date t, and let $x_t \in \mathbb{R}_+$ denote consumption of x at date t. A seller's preferences are represented by

$$v\left(x_t\right) - w\left(y_t\right),$$

where $v : \mathbb{R}_+ \to \mathbb{R}_+$ is continuously differentiable, strictly increasing, and concave, with $v\left(0\right) = 0$, and $w : \mathbb{R}_+ \to \mathbb{R}_+$ is continuously differentiable, strictly increasing, and convex, with $w\left(0\right) = 0$. Let $y^* \in \mathbb{R}_+$ denote the quantity satisfying $u'\left(y^*\right) = w'\left(y^*\right)$. Assume $w\left(y^*\right) \geq v\left(1 + \frac{\rho\bar{\iota}}{\lambda}\right)$. Let $\beta \in (0,1)$ denote the common discount factor for buyers and sellers. Assume $\beta\left(1+\rho\right) > 1$.

A banker derives instantaneous utility x_t in period t if his consumption of x is given by $x_t \in \mathbb{R}_+$. Let $\hat{\beta} \in (0,1)$ denote the banker's discount factor. Assume $\hat{\beta}\left(1+\rho\right) \leq 1$.

3. PRELIMINARIES

To see why a banking arrangement is essential in this economy, it is easier to start with the second stage. In this stage, a buyer is randomly matched with a seller with probability λ. A buyer wants y but is unable to produce x for a seller at that time. The pair can trade if the buyer has x in storage. As we have seen, any nonbank agent can convert x into an indivisible unit of storage and vice versa. Is this trading arrangement socially desirable? By adopting this trading strategy, agents hold, at any point in time, an excessive amount of inventories for transaction purposes. These inventories could be either consumed or productively invested.

A superior arrangement can be obtained if a group of bankers is willing to provide a medium of exchange that serves as an alternative to storage. Note that a banker is able to interact with the group of buyers in the first stage and with the group of sellers in the third stage. In the first stage, a buyer can produce one unit and "deposit" it with a banker. In exchange for the buyer's deposit, the banker issues a bank claim certifying the amount originally deposited plus any promised interest payment and entitles the bearer to receive this amount in the third stage. If a seller is willing to accept a privately issued claim in exchange for output, then he is able to redeem this claim in the third stage, so we can think of this stage as the settlement stage.

If a banker is willing to issue a bank claim that promises to pay a higher return than storage, then it is a dominant strategy for a buyer to deposit with a banker. The only problem with this arrangement is that, at date 0, a depositor may need to withdraw funds if he finds out he is a mover. Otherwise, he would have taken into account the inability to withdraw funds on demand when making the deposit decision. Because of a lack of communication across regions, it is impossible to transfer a claim on the banking system in one region to the banking system in the other region. Consequently, a mover needs to hold personal wealth in the form of storage prior to relocation.

Recall that a banker can access the productive technology only at the beginning of the period (before the realization of the idiosyncratic relocation shock). To be able to offer valuable transaction services to depositors, the members of the banking system need to receive deposits at the beginning of the period to make their portfolio decision. At this time, a depositor does not know whether he is going to be permanently relocated to the other region. Thus, the one-shot relocation shock gives rise to a legitimate demand for withdrawals at date 0, with the withdrawal option providing insurance against the relocation risk. At any subsequent date, the withdrawal option is not valuable, so the deposit contract will simply not allow depositors to prematurely withdraw.

A mover who is able to withdraw funds prior to relocation is willing to redeposit these funds in the other region as long as he believes that the banking system there has the ability to pay a higher expected return on deposits than storage. As we shall see, this *expected* flow of resources across regions due to random relocations does not disrupt the investment plans of banks. Although a nonmover does not need to withdraw, I will show that a nonmover will be willing to withdraw if he believes that other nonmovers will also withdraw, given that depositors are sequentially served in random order until the banking system runs out of assets. In this case, the previously described payment mechanism will be severely disrupted.

4. SYMMETRIC INFORMATION

As a useful benchmark, it is helpful to start the analysis by assuming that a depositor's relocation status at date 0 is publicly observable. The members of the banking system offer a demand deposit contract specifying that, in exchange for one unit of x, a depositor receives an *indivisible* bank claim, which is a transferable instrument that entitles the bearer to receive $\phi_t \in \mathbb{R}_+$ units of x in the settlement stage (third stage). Throughout the paper, I assume that there is perfect monitoring of the activities of bankers and that a deposit contract can be perfectly enforced.

If a depositor wishes to withdraw from the banking system after learning his relocation status at date 0, then he is entitled to receive the original deposit amount. As in Diamond and Dybvig (1983), I assume that withdrawal orders are sequentially served in random order until the banking system runs out of assets. In other words, the demand deposit contract satisfies a sequential service constraint.

When there is symmetric information, the members of the banking system are able to perfectly distinguish depositors who have a legitimate motive for exercising the withdrawal option (movers) from depositors who are not going to be relocated and do not need to withdraw (nonmovers). In this case, the banking system can condition the withdrawal option on the depositor's relocation status, so only movers are able to withdraw prior to relocation. As we shall see, there cannot be a banking panic under this type of contract.

4.1. Distributions

To characterize an equilibrium allocation, it is helpful to start by describing the distributions of asset holdings across different types of agents. These distributions can be summarized as follows. Let $m_t^1 \in [0,1]$ denote the measure of buyers holding one unit of asset (either storage or bank deposits) prior to the formation of bilateral matches, let $m_t^2 \in [0,1]$ denote the measure of sellers holding one unit of asset shortly after bilateral matches are dissolved, and let $m_t^3 \in [0,1]$ denote the volume of redemptions in the settlement stage. In what follows, I will demonstrate that all buyers voluntarily choose to deposit with the banking system and that a depositor is willing to hold at most one unit of bank deposit at any given moment.

If each buyer chooses to hold personal wealth in the form of bank deposits, then an equilibrium is consistent with the following invariant distributions:

$$m_t^1 = 1 \tag{1}$$

and

$$m_t^2 = m_t^3 = \lambda \tag{2}$$

for all $t \geq 0$. These distributions imply that each buyer enters the second stage holding a bank claim and that a measure λ of sellers enters the settlement stage holding a bank claim and chooses to redeem these claims. As we shall see, no buyer will choose to use storage for transaction purposes in equilibrium (a mover stores one unit during relocation but chooses to redeposit it in the banking system upon arrival in the new region).

4.2. Buyers

Given these distributions, let me now describe the Bellman equation for a buyer. Let $V_t \in \mathbb{R}$ denote the expected utility of a buyer prior to the formation of bilateral matches at date t. The Bellman equation for a buyer is given by

$$V_t = \lambda \left[u\left(y_t\right) + \beta\left(-\gamma + V_{t+1}\right)\right] + (1-\lambda)\,\beta V_{t+1}. \tag{3}$$

Here, $y_t \in \mathbb{R}_+$ denotes the quantity traded in a bilateral meeting.

With probability λ, a buyer will be matched with a seller and will be able to consume, entering the following period without assets. Then, he will be able to rebalance his portfolio by producing one unit and depositing it in the banking system. With probability $1-\lambda$, a buyer will not find a trading partner, entering the following period with the same asset holdings. If each buyer is willing to trade with a seller and is willing to produce to rebalance his portfolio, then the conjecture $m_t^1 = 1$ for all $t \geq 0$ is consistent with individual behavior.

4.3. Sellers

Let $W_t \in \mathbb{R}$ denote the expected utility of a seller. The Bellman equation for a seller is given by

$$W_t = \lambda \left[-w\left(y_t\right) + v\left(\phi_t\right) + \beta W_{t+1}\right] + (1-\lambda)\,\beta W_{t+1}. \tag{4}$$

Recall that a bank claim entitles the bearer to receive ϕ_t units of x in the settlement stage. In the previous equation, I have conjectured that a seller will redeem a bank claim in the settlement stage instead of holding on to it to claim redemption in a subsequent period. As we shall see, this conjecture is confirmed in equilibrium. If each seller accepts to produce y_t units in exchange for a bank claim, then the conjecture $m_t^2 = \lambda$ for all $t \geq 0$ is consistent with individual behavior.

4.4. Bankers

When a banker issues a bank claim to a buyer, the latter will be able to spend it at the current date with probability λ, so a seller will claim the face value with the same probability. With probability $(1-\lambda)\,\lambda$, a seller will claim the face value at the following date. With probability $(1-\lambda)^2\,\lambda$, a seller will claim the face value two dates after issuance and so on. Because an individual banker faces idiosyncratic risk when issuing a bank claim (i.e., uncertainty regarding the date at which a claim will be redeemed), the members of the banking system have an incentive to engage in a risk-sharing scheme.

An effective arrangement can be constructed as follows. Suppose that all bankers agree that an individual banker who has an opportunity to issue a bank claim is supposed to save a fraction $z_t \in [0, 1]$ of the deposit amount. All bankers then decide how to invest all savings subject to the constraint that all claims presented for redemption in the settlement stage must be retired at the promised value ϕ_t. In other words, a banker is supposed to make a contribution z_t every time he has an opportunity to issue a bank claim in exchange for a disbursement ϕ_t on his behalf every time someone wants to retire a claim issued by him.

Let me now describe the investment decisions of the members of the banking system. Let $k_t \in \mathbb{R}_+$ denote per capita investment in the productive technology and let $s_t \in \mathbb{R}_+$ denote per capita investment in storage. At date 0, the resource constraint for the members of the banking system is given by

$$s_0 + k_0 = 1. \tag{5}$$

In addition, we must have $s_0 \geq \varepsilon$ so that the banking system can meet the expected withdrawal demand of movers. In any subsequent period $t \geq 1$, we must have

$$k_t + s_t = F(k_{t-1}) + \lambda z_t + s_{t-1} - \lambda \phi_{t-1} \tag{6}$$

and

$$\lambda \phi_t \leq s_t. \tag{7}$$

At any date $t \geq 1$, a fraction λ of bankers is able to issue a bank claim, so the per capita inflow of funds is given by λz_t. The per capita disbursement due to redemptions is given by $\lambda \phi_t$. Constraint (7) reflects the fact that the productive technology pays off only at the beginning of the following period, so at least part of the amount invested in storage has to be liquidated to meet expected redemptions in the settlement stage. I have implicitly assumed that bankers do not want to prematurely liquidate the productive technology. As we shall see, this is consistent with equilibrium behavior under symmetric information.

Let $J_t \in \mathbb{R}$ denote the expected utility of a banker. At date 0, we have

$$J_0 = 1 - z_0 + \hat{\beta} J_1, \tag{8}$$

given that each banker has an opportunity to issue a bank claim. At any subsequent date $t \geq 1$, we have

$$J_t = \lambda \left(1 - z_t + \hat{\beta} J_{t+1}\right) + (1 - \lambda)\, \hat{\beta} J_{t+1}. \tag{9}$$

A banker is able to consume $1 - z_t$ every time he has an opportunity to issue a bank claim. Because $\hat{\beta}\,(1 + \rho) \leq 1$, a banker is willing to immediately consume any retained earnings. Note that the expected utility of a banker does not depend on the amount of bank claims he has previously issued because of the implementation of a risk-sharing scheme.

4.5. Terms of Trade and Output

Let me now determine the terms of trade in the first and second stages. Start with the second stage. In a bilateral meeting, the terms of trade are determined by Nash bargaining. For simplicity, I assume the buyer makes a take-it-or-leave-it offer to the seller. A buyer is willing to trade provided $u\,(y_t) - \beta\gamma \geq 0$, and a seller is willing to trade provided $-w\,(y_t) + v\,(\phi_t) \geq 0$. Because the seller's participation constraint is binding when the buyer has all the bargaining power, the amount produced is given by

$$y_t = w^{-1}\,(v\,(\phi_t))\,. \tag{10}$$

It is necessary to verify whether a buyer is willing to produce to acquire a bank claim in stage 1. The buyer's participation constraint is given by

$$U\,(\phi_t) \geq \frac{\gamma\,(1 - \beta + \beta\lambda)}{\lambda}, \tag{11}$$

where the function $U : \mathbb{R}_+ \to \mathbb{R}_+$ is defined by

$$U\,(\phi_t) \equiv u\left(w^{-1}\,(v\,(\phi_t))\right).$$

Note that $U\,(\phi_t)$ is increasing and strictly concave in ϕ_t, with $U\,(0) = 0$. Because a buyer has the ability to store goods, it follows that

$$\phi_t \geq 1, \tag{12}$$

which implies that the rate of return on bank deposits must be positive in equilibrium. In other words, bank deposits must command a higher purchasing power than storage to induce a buyer to become a depositor.

The banker's participation constraint is given by $z_t \leq 1$. Throughout the analysis, I assume that the terms of trade in the deposit market are such that a banker earns zero profits in equilibrium, so we must have

$$z_t = 1 \tag{13}$$

for all $t \geq 0$. In addition, the investment plan implemented by the members of the banking system must maximize the expected utility of depositors.

Finally, we need to specify production of x in stage 1. Total output of x is given by

$$x_0 = 1 \tag{14}$$

at date 0 and satisfies the law of motion

$$x_t = \lambda + F\left(k_{t-1}\right) \tag{15}$$

at any subsequent date $t \geq 1$. As previously mentioned, a fraction λ of buyers enters the period without purchasing power and produces one unit to rebalance their portfolio.

4.6. Equilibrium

Given these descriptions of individual behavior and feasibility conditions, it is now possible to provide a formal definition of equilibrium.

Definition 1 *An equilibrium consists of value functions $\{V_t, W_t, J_t\}_{t=0}^{\infty}$, an investment plan $\{k_t, s_t, z_t\}_{t=0}^{\infty}$, a sequence describing the value of bank deposits $\{\phi_t\}_{t=0}^{\infty}$, a sequence specifying sectorial outputs $\{x_t, y_t\}_{t=0}^{\infty}$, and distributions $\left\{m_t^1, m_t^2, m_t^3\right\}_{t=0}^{\infty}$ such that* ***(i)*** *the distributions $\left\{m_t^1, m_t^2, m_t^3\right\}_{t=0}^{\infty}$ satisfy (1)-(2);* ***(ii)*** *the value functions $\{V_t, W_t, J_t\}_{t=0}^{\infty}$ satisfy the Bellman equations (3)-(4) and (8)-(9);* ***(iii)*** *the investment plan $\{k_t, s_t, z_t\}_{t=0}^{\infty}$ satisfies*

(5)-(6) and (13) and is consistent with the maximization of the expected utility of depositors; ***(iv)*** *the sequence of values* $\{\phi_t\}_{t=0}^{\infty}$ *satisfies (7) and (11)-(12); and* ***(v)*** *the quantities* $\{x_t, y_t\}_{t=0}^{\infty}$ *satisfy (10) and (14)-(15).*

The first step toward the characterization of an equilibrium allocation is to derive an investment plan consistent with the maximization of the expected utility of depositors. To derive an optimal investment plan, it is useful to make the following assumption.

Assumption 1 Assume $U'\left(\frac{1-\bar{\iota}}{\lambda}\right) < \beta\left(1+\rho\right)U'\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)$.

This condition is likely to hold when the rate of return on the productive technology is sufficiently large, which is consistent with previously made assumptions. The following lemma describes the optimal investment plan. All proofs are provided in the appendix.

Lemma 2 *Consider the following portfolio choice:* $k_0 = \bar{\iota}$ *and* $s_0 = 1 - \bar{\iota}$ *at date* 0*;* $k_t = \bar{\iota}$ *and* $s_t = \lambda + \rho\bar{\iota}$ *at any subsequent date* $t \geq 1$*. In addition, suppose* $z_t = 1$ *for all* $t \geq 0$*. This investment plan is the unique solution consistent with the maximization of the expected utility of depositors.*

An important property of the optimal investment plan refers to the state of the banking system at the time withdrawal requests can be made. Because the per capita liquidation value of banking assets satisfies

$$s_0 + \delta k_0 = 1 - (1-\delta)\,\bar{\iota} < 1,$$

it is impossible to meet the demand for withdrawals if, for some reason, all depositors choose to exercise the withdrawal option. Thus, we can say that the banking system is illiquid and subject to a self-fulfilling panic. When an agent's relocation status is publicly observable, the fact that the optimal investment plan implies an illiquid banking system is not a problem. Because the members of the banking system can perfectly differentiate movers from nonmovers, it is possible to deny a withdrawal order made by any nonmover to preserve the previously described investment plan, so the fact that the banking system is illiquid has no consequence for the equilibrium allocation.

Note that movers, who temporarily hold storage during relocation, are willing to redeposit their balances upon arrival in the new region, so the previously described investment plan is not disrupted. To formally show existence, I need to make an additional assumption to guarantee that the buyer's participation constraint is satisfied.

Assumption 2 Assume $\lambda\left[U\left(1+\frac{\rho\bar{\iota}+(1-\lambda)}{\lambda}\right)-U\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)\right]\leq\gamma\leq\frac{\lambda U(1)}{1-\beta+\beta\lambda}$.

This assumption also implies that a depositor is willing to hold at most one unit of bank deposit at any moment. I can now formally establish existence.

Proposition 3 *There exists an equilibrium with the property that* $\phi_0=\frac{1-\bar{\iota}}{\lambda}$ *and* $\phi_t=1+\frac{\rho\bar{\iota}}{\lambda}$ *for all* $t\geq 1$. *The ensuing equilibrium allocation is Pareto optimal.*

In this equilibrium, a buyer produces one unit in period 0 and consumes $w^{-1}\left(v\left(\frac{1-\bar{\iota}}{\lambda}\right)\right)$ if he has a trading opportunity. A seller who finds a buyer in period 0 produces $w^{-1}\left(v\left(\frac{1-\bar{\iota}}{\lambda}\right)\right)$ and consumes $\frac{1-\bar{\iota}}{\lambda}$. In subsequent periods, a buyer consumes $w^{-1}\left(v\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)\right)$ when he has a trading opportunity and produces one unit when he needs to rebalance his portfolio, and a seller produces $w^{-1}\left(v\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)\right)$ and consumes $1+\frac{\rho\bar{\iota}}{\lambda}$ when he has an opportunity to trade with a buyer.

An important property of the previously described equilibrium allocation is that the banking system is able to accumulate the socially efficient amount of capital, which allows it to provide perfect insurance against the relocation risk and to offer a payment instrument with a higher purchasing power. This socially beneficial role of a banking system has been demonstrated by assuming that a depositor's relocation status is publicly observable. As we shall see, this assumption is far from being innocuous.

5. ASYMMETRIC INFORMATION

Suppose now the relocation status of a buyer is privately observable. In this case, the members of the banking system cannot distinguish a mover from a nonmover at the time withdrawal requests can be made. In the absence of an *ex post* suspension of convertibility, it is possible to have a banking panic if all nonmovers decide to prematurely withdraw. This means that, at date 0, the members of the banking system have to make their portfolio decision contemplating the possibility of a banking panic.

In what follows, I allow agents to coordinate their actions based on the realization of a sunspot variable, as in Cooper and Ross (1998), Peck and Shell (2003), Ennis and Keister (2006), and Allen and Gale (2007). There is a publicly observable random variable $S \in \{n, r\}$ with no effects on fundamentals but potentially with an effect on behavior due to expectations. Suppose $\Pr(S = r) = \pi \in (0, 1)$. The realization of S occurs *shortly after* the relocation status of each buyer is privately revealed at date 0.

As we shall see, in equilibrium, all buyers voluntarily choose to hold wealth in the form of deposits. After investment decisions have been made at date 0, a random fraction ε of these depositors is going to be permanently relocated and so chooses to exercise the withdrawal option. Nonmovers choose whether to withdraw depending on the realization of the sunspot variable *and* the state of the banking system. Specifically, nonmovers optimally choose to withdraw when they observe $S = r$ *and* the banking system is illiquid and optimally choose not to withdraw otherwise. Thus, the realization $S = r$ does not trigger a bank run if the banking portfolio is liquid, so the choice of the banking portfolio is crucial for the occurrence of a panic in equilibrium. As previously mentioned, I assume that withdrawal orders are sequentially served in random order until the banking system runs out of assets.

5.1. Distributions

As in the previous section, it is helpful to start by describing the distributions of asset holdings across different types of agents. Let $m_t^1(S) \in [0, 1]$ denote the measure of buyers holding one unit of asset prior to the formation of bilateral matches, let $m_t^2(S) \in [0, 1]$ denote the measure of sellers holding one unit of asset shortly after bilateral matches are dissolved, and let $m_t^3(S) \in [0, 1]$ denote the volume of redemptions in the settlement stage. Note that these distributions may depend on the state S realized at date 0.

If each buyer chooses to hold wealth in the form of bank deposits, then an equilibrium allocation is consistent with the following distributions of asset holdings:

$$m_0^1(S) = \left[1 - \hat{I}(S, k_0, s_0)\right](s_0 + \delta k_0) + \hat{I}(S, k_0, s_0), \tag{16}$$

$$m_0^2(S) = \lambda m_0^1(S), \tag{17}$$

$$m_0^3(S) = m_0^2(S)\,\hat{I}(S, k_0, s_0) \tag{18}$$

for each $S \in \{n, r\}$, with $\hat{I}(S, k_0, s_0)$ representing an indicator function defined by

$$\hat{I}(S, k_0, s_0) = \begin{cases} 0 \text{ if } s_0 + \delta k_0 < 1 \text{ and } S = r, \\ 1 \text{ otherwise.} \end{cases} \tag{19}$$

The per capita liquidation value of the assets of the banking system at the time withdrawal requests can be made is given by $s_0 + \delta k_0$, so the banking portfolio is illiquid if $s_0 + \delta k_0 < 1$.

In the absence of a panic, the nonbank public is able to trade using bank deposits as a means of payment, so the volume of redemptions in the settlement stage is given by λ. In the event of a panic, the banking system is liquidated, so the nonbank public temporarily reverts to storage to settle bilateral transactions. In this case, a seller is able to consume one unit shortly after trading with a buyer, so nothing happens in the settlement stage.

Following the initial date, the distributions are given by

$$m_t^1(n) = m_t^1(r) = 1, \tag{20}$$

$$m_t^2(n) = m_t^2(r) = \lambda, \tag{21}$$

$$m_t^3(n) = m_t^3(r) = \lambda \tag{22}$$

for all $t \geq 1$. Because there is no other shock after date 0, the distributions of asset holdings are invariant given that the banking system does not allow depositors to withdraw.

5.2. Bankers

As previously described, the members of the banking system engage in a risk-sharing scheme when issuing bank claims to the nonbank public. An investment plan consists of a vector (k_0, s_0, z_0) and a sequence

$$\{k_t(S), s_t(S), z_t(S)\}_{t=1}^{\infty}$$

satisfying the following feasibility conditions. At date 0, we must have

$$k_0 + s_0 = z_0, \tag{23}$$

given that no one is a depositor at the beginning of period 0. At date 1, we must have

$$k_1(S) + s_1(S) = F(k_0)\,\hat{I}(S, k_0, s_0) + \left[m_0^3(S) + 1 - \hat{I}(S, k_0, s_0)\right] z_1(S). \tag{24}$$

for each $S \in \{n, r\}$. Note that the feasible set for the members of the banking system at date 1 depends on whether a panic occurred at date 0. I have implicitly assumed that, in the absence of a panic, no one stores goods across periods. In any subsequent period $t \geq 2$, we must have

$$k_t(S) + s_t(S) = F(k_{t-1}(S)) + \lambda z_t(S) \tag{25}$$

for each $S \in \{n, r\}$. In addition, the per capita amount s_0 invested in storage at date 0 must be sufficiently large to meet the expected withdrawal orders of movers: $s_0 \geq \varepsilon$.

The sequence $\{\phi_t(S)\}_{t=0}^{\infty}$ representing the value of a unit of asset must satisfy

$$[\lambda\phi_0(S) - s_0]\,\hat{I}(S, k_0, s_0) = 0$$

at date 0 and $\lambda\phi_t(S) = s_t(S)$ at any subsequent date. When there is no panic, the value of liquid assets is the same as the promised value of bank deposits. When there is a panic, the value of liquid assets is one (i.e., the technological rate of return associated with storage). Thus, the state-dependent value of liquid assets at date 0 is given by

$$\phi_0(S) = \begin{cases} 1 \text{ if } s_0 + \delta k_0 < 1 \text{ and } S = r, \\ \frac{s_0}{\lambda} \text{ otherwise.} \end{cases} \tag{26}$$

At any subsequent date $t \geq 1$, we have

$$\phi_t(S) = \frac{s_t(S)}{\lambda}. \tag{27}$$

In the following section, I will study suspension of convertibility. In this case, a unit of liquid assets can take on two values in the event of a panic depending on whether an agent has been able to withdraw before convertibility has been optimally suspended.

Let $J_0 \in \mathbb{R}$ denote the expected utility of a banker at date 0 and let $J_t(S) \in \mathbb{R}$ denote the expected utility at any subsequent date. At date 0, the value J_0 satisfies

$$J_0 = 1 - z_0 + \hat{\beta}\left[\pi J_1(r) + (1-\pi) J_1(n)\right]. \tag{28}$$

At date 1, the value function is given by

$$J_1(S) = \left[m_0^3(S) + 1 - \hat{I}(S, k_0, s_0)\right]\left[1 - z_1(S)\right] + \hat{\beta} J_2(S). \tag{29}$$

At any subsequent date $t \geq 2$, we have

$$J_t(S) = \lambda\left[1 - z_t(S)\right] + \hat{\beta} J_{t+1}(S). \tag{30}$$

If a panic did not occur at date 0, then a banker is able to issue a bank claim with probability λ at date 1. If a panic occurred at date 0, then each banker is able to issue a bank claim because no one is a depositor at the beginning of period 1. So far, I have conjectured that a depositor is willing to deposit in the banking system knowing that there is the possibility of a panic. Thus, it is necessary to verify whether this conjecture is consistent with individual behavior.

5.3. Buyers

At each date, a buyer has an opportunity to produce x and deposit it in the banking system. A depositor will hold a bank claim until he has an opportunity to spend it. In a bilateral meeting, the buyer's surplus is given by $u(y_t(S)) - \beta\gamma \geq 0$ and the seller's surplus is given by $-w(y_t(S)) + v(\phi_t(S)) \geq 0$. Given that the buyer makes a take-it-or-leave-it offer to the seller, we must have

$$y_t(S) = w^{-1}\left(v\left(\phi_t(S)\right)\right) \tag{31}$$

for each $S \in \{n, r\}$. As in the previous section, it is convenient to work with the utility function $U(\phi) \equiv u\left(w^{-1}\left(v(\phi)\right)\right)$.

Let $V_0 \in \mathbb{R}$ denote the postdeposit expected utility of a buyer at date 0 and let $V_t(S) \in \mathbb{R}$ denote the expected utility at any subsequent date. At date 0, the value V_0 must satisfy

$$\begin{aligned} V_0 &= \pi\left\{-p\beta\gamma + (1-p)\lambda\left[U\left(\phi_0(r)\right) - \beta\gamma\right] + \beta V_1(r)\right\} \\ &\quad + (1-\pi)\left\{\lambda\left[U\left(\phi_0(n)\right) - \beta\gamma\right] + \beta V_1(n)\right\}. \end{aligned} \tag{32}$$

Here, $p \in [0,1]$ represents the probability of loss in the event of a panic, which must satisfy

$$p = \max\left\{0, 1 - s_0 - \delta k_0\right\}.$$

As previously mentioned, a panic does not occur when the banking portfolio is liquid. When $S = r$ and $s_0 + \delta k_0 < 1$, a panic occurs and the banking system in each region is liquidated. Because depositors are sequentially served in random order, an individual depositor is able to withdraw one unit with probability $s_0 + \delta k_0 < 1$.

In the event of a panic, only a fraction $s_0 + \delta k_0 < 1$ of buyers enters the second stage holding one unit of x in storage, so the number of trade meetings is given by $\lambda(s_0 + \delta k_0) < \lambda$. Note that a panic affects both the quantity traded in each bilateral meeting (intensive margin) and the total number of trade meetings (extensive margin).

At any subsequent date $t \geq 1$, the values $V_t(S)$ must satisfy

$$V_t(S) = \lambda\left[U\left(\phi_t(S)\right) - \beta\gamma\right] + \beta V_{t+1}(S), \tag{33}$$

given that a panic will not occur in other periods.

So far, I have implicitly assumed that each buyer is willing to deposit in the banking system even though a panic can occur with probability π. A buyer is willing to deposit in the banking system if the following participation constraints are satisfied:

$$\pi(1-p)\lambda U(1) + (1-\pi)\lambda U\left(\phi_0(n)\right) \geq \lambda U(1) + \pi p(1-\lambda)\beta\gamma \tag{34}$$

if $s_0 + \delta k_0 < 1$; and

$$\phi_0(S) \geq 1 \text{ for each } S \in \{n, r\} \tag{35}$$

otherwise. Note that bank claims command a higher purchasing power than storage when a panic does not occur, but a buyer who chooses to store goods is not subject to loss if a panic occurs. Thus, a buyer is willing to hold bank claims provided that the expected rate of return on deposits is sufficiently large to compensate him for the possibility of suffering a loss in the event of a panic.

5.4. Sellers

Let $W_0 \in \mathbb{R}$ denote the expected utility of a seller at date 0 and let $W_t(S) \in \mathbb{R}$ denote the expected utility at any subsequent date. The value W_0 must satisfy

$$W_0 = \lambda m_0^1(S)\left[-w(y_0(S)) + v(\phi_0(S))\right] + \beta\left[\pi W_1(r) + (1-\pi) W_1(n)\right]. \tag{36}$$

At any subsequent date $t \geq 1$, the sequence of value functions satisfies

$$W_t(S) = \lambda\left[-w(y_t(S)) + v(\phi_t(S))\right] + \beta W_{t+1}(S). \tag{37}$$

A seller is willing to produce for a buyer in exchange for a unit of asset provided that the value of the asset is sufficiently large to compensate him for the disutility of production. At date 0, the occurrence of a panic affects the probability with which a seller finds a buyer with purchasing power in the decentralized market.

5.5. Participation Constraints

In addition to the previously described conditions, it must be the case that a buyer is willing to produce to rebalance his portfolio, given the terms of trade in the decentralized market. At date 0, the following participation constraints must hold:

$$\pi(1-p)\lambda U(1) + (1-\pi)\lambda U(\phi_0(n)) \geq (1-\beta+\lambda\beta)\gamma + \pi p(1-\lambda)\beta\gamma \tag{38}$$

if $s_0 + \delta k_0 < 1 < 1$; and

$$\lambda U(\phi_0(S)) \geq (1-\beta+\lambda\beta)\gamma \text{ for each } S \in \{n, r\} \tag{39}$$

otherwise. Note that conditions (34) and (35) imply that both (38) and (39) are necessarily satisfied under Assumption 2.

The banker's participation constraint is given by $z_0 \leq 1$ at date 0 and $z_t(S) \leq 1$ at any subsequent date for each $S \in \{n, r\}$. Because the terms of trade in the deposit market are such that each banker earns zero profits, we must have

$$z_0 = 1 \text{ and } z_t(S) = 1 \text{ at any } t \geq 1. \tag{40}$$

In addition, the investment plan implemented by the members of the banking system must maximize the expected utility of depositors.

5.6. Postdeposit Coordination Game

Consider now the postdeposit coordination game at date 0. All depositors play this game *after* each one of them privately learns his relocation status. It is clear that a mover always chooses to withdraw from the banking system prior to relocation. A nonmover decides whether to withdraw based on his beliefs regarding the actions of other depositors.

It is a best response for a nonmover to withdraw if the banking system is illiquid and he believes all other nonmovers are withdrawing. It is a best response for a nonmover not to withdraw otherwise. Thus, widespread withdrawals are a pure-strategy Nash equilibrium of the coordination game when the banking system is illiquid. In addition, there exists a second pure-strategy Nash equilibrium with the property that movers withdraw and nonmovers do not withdraw.

5.7. Equilibrium

To provide a complete description of equilibrium, I need to specify total output of x. Let $x_0 \in \mathbb{R}_+$ denote sectorial output at date 0 and let $x_t(S) \in \mathbb{R}_+$ represent sectorial output in any subsequent period $t \geq 1$. It follows that

$$x_0 = 1, \tag{41}$$

$$x_1(S) = [\lambda + F(k_0)]\, \hat{I}(S, k_0, s_0) + [\lambda(1-p) + p]\left[1 - \hat{I}(S, k_0, s_0)\right], \tag{42}$$

$$x_t(S) = \lambda + F(k_{t-1}(S)) \text{ for any } t \geq 2. \tag{43}$$

If a panic occurred at date 0, there is no productive investment coming to fruition at date 1, so total output at date 1 is determined by the measure of depositors who suffered losses in the process of liquidation of the banking system and by the measure of depositors who were served and had a consumption opportunity.

Given these descriptions of individual behavior and feasibility conditions, it is now possible to provide a formal definition of equilibrium.

Definition 4 *An equilibrium is a set of values* (V_0, W_0, J_0) *and*

$$\{V_t(S), W_t(S), J_t(S)\}_{t=1}^{\infty},$$

an investment plan (k_0, s_0, z_0) *and*

$$\{k_t(S), s_t(S), z_t(S)\}_{t=1}^{\infty},$$

a sequence $\{\phi_t(S)\}_{t=0}^{\infty}$ *describing the value of liquid assets, a set of quantities* $(x_0, y_0(S))$ *and* $\{x_t(S), y_t(S)\}_{t=1}^{\infty}$ *specifying sectorial outputs, and distributions*

$$\{m_t^1(S), m_t^2(S), m_t^3(S)\}_{t=0}^{\infty}$$

such that ***(i)*** *the distributions satisfy (16)-(22);* ***(ii)*** *the value functions satisfy (28)-(30), (32)-(33), and (36)-(37);* ***(iii)*** *the investment plan satisfies (23)-(25) and (40) and is consistent with the maximization of the expected utility of depositors;* ***(iv)*** *the values* $\{\phi_t(S)\}_{t=0}^{\infty}$ *satisfy (26)-(27) and (34)-(35); and* ***(v)*** *the sectorial outputs satisfy (31) and (41)-(43).*

In what follows, I want to show the existence of an equilibrium with the property that a banking panic occurs at date 0 when the sunspot signal r is realized. In this equilibrium, the distributions of asset holdings are given by

$$m_0^1(n) = 1 > 1 - (1-\delta)\bar{\iota} = m_0^1(r),$$

$$m_0^2(n) = \lambda > \lambda - \lambda(1-\delta)\bar{\iota} = m_0^2(r),$$

$$m_0^3(n) = \lambda > 0 = m_0^3(r).$$

The optimal portfolio choice at date 0 continues to be $k_0 = \bar{\imath}$ and $s_0 = 1 - \bar{\imath}$. At date 1, we have $k_1(n) = k_1(r) = \bar{\imath}$ and

$$s_1(n) = \rho\bar{\imath} + \lambda > 1 - \bar{\imath} = s_1(r).$$

At any subsequent date $t \geq 2$, the banking portfolio is $k_t(n) = k_t(r) = \bar{\imath}$ and $s_t(n) = s_t(r) = \lambda + \rho\bar{\imath}$. These choices imply the values

$$\phi_0(n) = \frac{1-\bar{\imath}}{\lambda} > 1 = \phi_0(r) \tag{44}$$

at date 0,

$$\phi_1(n) = 1 + \frac{\rho\bar{\imath}}{\lambda} > \frac{1-\bar{\imath}}{\lambda} = \phi_1(r) \tag{45}$$

at date 1, and

$$\phi_t(n) = \phi_t(r) = 1 + \frac{\rho\bar{\imath}}{\lambda} \tag{46}$$

at any subsequent date $t \geq 2$. The following lemma establishes the optimality of the previously described banking portfolio.

Lemma 5 *Consider the following portfolio choice: $k_0 = \bar{\imath}$ and $s_0 = 1-\bar{\imath}$ at date 0; $k_1(n) = k_1(r) = \bar{\imath}$, $s_1(n) = \rho\bar{\imath} + \lambda$, and $s_1(r) = 1 - \bar{\imath}$ at date 1; $k_t(n) = k_t(r) = \bar{\imath}$ and $s_t(n) = s_t(r) = \lambda + \rho\bar{\imath}$ at any subsequent date $t \geq 2$. In addition, suppose $z_0 = 1$ and $z_t(n) = z_t(r) = 1$ at any $t \geq 1$. This investment plan maximizes the expected utility of depositors when agents expect a banking panic to occur at date 0 with probability π provided*

$$\pi \leq \hat{\pi} \equiv \frac{\beta(1+\rho)U'\left(1+\frac{\rho\bar{\imath}}{\lambda}\right) - U'\left(\frac{1-\bar{\imath}}{\lambda}\right)}{(1-\delta)\{\beta\gamma + \lambda[U(1) - \beta\gamma]\} + \beta(1+\rho)U'\left(1+\frac{\rho\bar{\imath}}{\lambda}\right) - U'\left(\frac{1-\bar{\imath}}{\lambda}\right)}. \tag{47}$$

Provided that the probability of a banking panic is sufficiently small, the optimal portfolio choice involves undertaking all productive projects in the economy at any date regardless of the realization of the state S. Because this portfolio choice results in an illiquid banking system, a banking panic occurs when the sunspot signal r is realized, given that agents believe that nonmovers will prematurely withdraw funds from the banking system.

Given the previously described investment plan, sectorial outputs are given by

$$x_0 = 1$$

and

$$y_0(n) = w^{-1}\left(v\left(\frac{1-\bar{\iota}}{\lambda}\right)\right) > w^{-1}(v(1)) = y_0(r)$$

at date 0. In the following period, we have

$$x_1(n) = \lambda + (1+\rho)\bar{\iota} > (1-\lambda)(1-\delta)\bar{\iota} + \lambda = x_1(r)$$

and

$$y_1(n) = w^{-1}\left(v\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)\right) > w^{-1}\left(v\left(\frac{1-\bar{\iota}}{\lambda}\right)\right) = y_1(r).$$

At any subsequent date $t \geq 2$, sectorial outputs are given by

$$x_t(n) = x_t(r) = \lambda + (1+\rho)\bar{\iota}$$

and

$$y_t(n) = y_t(r) = w^{-1}\left(v\left(1+\frac{\rho\bar{\iota}}{\lambda}\right)\right).$$

A formal statement of the existence of an equilibrium allocation with a fragile banking system is provided in the following proposition.

Proposition 6 *There exists an equilibrium with the property that a banking panic occurs at date* 0 *if the sunspot signal r is realized provided $\pi \leq \pi^*$ for some $\pi^* > 0$.*

In the previously described equilibrium, the effects of a banking panic are persistent. Note that production and consumption decline substantially in the event of a panic and remain below their efficient levels in the aftermath of the panic. The efficient level of trading activity is reached only two dates after the onset of the banking panic. Thus, in the absence of a suspension-of-convertibility policy, the occurrence of a banking panic results in a protracted recession.

6. SUSPENSION OF CONVERTIBILITY

It is well known in the banking literature that a simple variation of the demand deposit contract that allows the banking system to suspend convertibility in the event of a panic eliminates the bank-run equilibrium in the Diamond-Dybvig framework. For instance, in the analysis developed in the previous section, it is possible to have a banking authority announcing that it will suspend the convertibility of deposits after ε depositors have been served at date 0. If nonmovers believe that the announcement is credible, this suspension-of-convertibility policy completely removes the incentives to join a run on the banking system.

In an important contribution to the literature, Ennis and Keister (2009) have demonstrated that the effectiveness of a suspension-of-convertibility policy in removing any incentive to join a run relies heavily on the assumption that the banking authority responsible for implementing such a suspension can fully commit to its *ex ante* policies. In particular, they have shown that an optimal *ex post* suspension of convertibility may not eliminate the bank-run equilibrium. In this section, I want to restrict attention to time-consistent suspension-of-convertibility policies.

Following Ennis and Keister, suppose that there exists a benevolent banking authority endowed with the power to freeze deposits at date 0 (after types have been privately revealed). Let $\hat{\varepsilon} \in [0, 1]$ denote the freeze point, that is, the fraction of depositors that the banking authority would choose to serve before suspending convertibility at date 0. In addition, let $\hat{k} \in \mathbb{R}_+$ denote the amount of productive investments the banking authority chooses to prematurely liquidate at date 0. As in Ennis and Keister, I want to characterize the optimal *ex post* freeze point. The banking authority chooses $\left(\hat{\varepsilon}, \hat{k}\right)$ to maximize the expected utility of depositors

$$\hat{\varepsilon}\lambda \left[U(1) - \beta\gamma\right] + (1 - \hat{\varepsilon}) \left\{-\varepsilon\beta\gamma + (1 - \varepsilon)\lambda \left[U\left(\phi\left(\hat{\varepsilon}, \hat{k}\right)\right) - \beta\gamma\right]\right\} + \beta\lambda \left[U\left(\phi_+\left(\hat{\varepsilon}, \hat{k}\right)\right) - \beta\gamma\right]$$

subject to

$$\varepsilon \leq \hat{\varepsilon} \leq 1 - \bar{\iota} + \delta\hat{k},$$

$$0 \leq \hat{k} \leq \bar{\iota},$$

$$U\left(\phi\left(\hat{\varepsilon}, \hat{k}\right)\right) - \beta\gamma \geq \beta\lambda \left[U\left(\phi_+\left(\hat{\varepsilon}, \hat{k}\right)\right) - \beta\gamma\right], \tag{48}$$

$$\phi\left(\hat{\varepsilon}, \hat{k}\right) = \frac{1 - \bar{\iota} + \delta\hat{k} - \hat{\varepsilon}}{\lambda(1 - \hat{\varepsilon})(1 - \varepsilon)},$$

and

$$\phi_{+}\left(\hat{\varepsilon},\hat{k}\right)=\frac{(1+\rho)\left(\bar{\iota}-\hat{k}\right)+1-(1-\lambda)(1-\varepsilon)(1-\hat{\varepsilon})-\bar{\iota}}{\lambda}.$$

The first and second constraints are feasibility constraints. The freeze point cannot exceed the available liquid funds. In addition, the fraction of productive investments the banking authority chooses to prematurely liquidate to respond to a panic episode cannot exceed the amount previously invested in the productive technology. The third constraint is the participation constraint for a depositor who had his deposit account frozen in stage 1 and currently has a trading opportunity in stage 2. This constraint arises because such a depositor can wait until date 1 to try to spend a bank claim.

The tradeoffs in the previously described optimization problem are as follows. When the banking authority chooses a larger value for the freeze point, it increases the proportion of movers in the economy with purchasing power. In addition, it increases the inflow of funds into the banking system at date 1, allowing it to raise the purchasing power of deposits at date 1. Because any nonmover with a frozen bank account who does not have a trading opportunity at date 0 will remain a deposit holder at date 1, a larger value for the freeze point implies that fewer agents will enter date 1 as deposit holders, which results in a larger inflow of *new* deposits into banking system. The cost associated with the decision to select a larger value for the freeze point is to reduce the purchasing power of any nonmover with a frozen bank account who has a trading opportunity at date 0.

The decision to prematurely liquidate productive investments provides the banking authority with more resources to deal with the banking panic but lowers total output at the following date, given that a smaller amount of capital will come to fruition at date 1 when some premature liquidation occurs at date 0. As a result, the decision to prematurely liquidate productive investments reduces the purchasing power of deposits at date 1.

If the solution to the optimization problem implies $\frac{1-\bar{\iota}+\delta\hat{k}-\hat{\varepsilon}}{\lambda(1-\hat{\varepsilon})(1-\varepsilon)}\geq 1$, then nonmovers are better off if they do not attempt to withdraw. In this case, a banking panic does not materialize and the ensuing allocation is the Pareto optimal allocation described in Proposition 3. In other words, a time-consistent suspension-of-convertibility policy successfully eliminates panics.

If the optimal freeze point implies $\frac{1-\bar{\iota}+\delta\hat{k}-\hat{\varepsilon}}{\lambda(1-\hat{\varepsilon})(1-\varepsilon)} < 1$, then a nonmover will rationally choose to withdraw when he or she believes that other nonmovers will do the same. In this case, a banking panic occurs if the banking system is illiquid and the signal r is realized. In the event of a panic, a fraction $\hat{\varepsilon}$ of depositors is served so that each one of them holds one unit in storage before entering the decentralized retail market in stage 2. A fraction $(1-\hat{\varepsilon})\,\varepsilon$ of depositors consists of movers who are not served. These agents arrive at the new region without purchasing power and need to wait until the following period to rebalance their portfolio. Finally, a fraction $(1-\hat{\varepsilon})\,(1-\varepsilon)$ of depositors consists of nonmovers who submitted a withdrawal order but were not served. These agents enter the second stage holding a claim worth $\frac{1-\bar{\iota}+\delta\hat{k}-\hat{\varepsilon}}{\lambda(1-\hat{\varepsilon})(1-\varepsilon)} < 1$, so a deposit holder who has an opportunity to trade with a seller is able to purchase a smaller amount at date 0.

A nonmover who has been served in the event of a panic can potentially choose to redeposit funds in the banking system after relocated agents from the other region arrive. If $\frac{1-\bar{\iota}+\delta\hat{k}-\hat{\varepsilon}}{\lambda(1-\hat{\varepsilon})(1-\varepsilon)} < 1$, these nonmovers will optimally choose not to redeposit in the banking system, given the agents' knowledge of the optimal freeze point chosen by the banking authority. Similarly, the fraction $\hat{\varepsilon}$ of movers with purchasing power chooses not to redeposit in the banking system upon arrival in the new region provided $\frac{1-\bar{\iota}+\delta\hat{k}-\hat{\varepsilon}}{\lambda(1-\hat{\varepsilon})(1-\varepsilon)} < 1$.

The solution to the previously described optimization problem crucially depends on the liquidation cost $1-\delta$. If the liquidation cost is relatively small, then it is likely that the solution involves the premature liquidation of a substantial fraction of productive investments in the event of a panic. As previously described, the benefits of premature liquidation are twofold. First, premature liquidation allows the banking authority to serve more depositors in the event of a panic in an attempt to maximize the number of movers who are able to withdraw prior to relocation. Second, premature liquidation allows the banking authority to increase the value of deposits for those with a frozen bank account who currently have a trading opportunity in the decentralized market.

If the liquidation cost is sufficiently large, then it is likely that the banking authority will optimally choose to have no premature liquidation of productive investments. An interesting property of the optimal government intervention is that, when the solution involves no premature liquidation, the panic-induced contraction in real activity is necessarily followed by a vigorous rebound in real activity above the long-run level.

Proposition 7 *If* $\hat{k} = 0$ *holds at the optimum, then* $\phi_+ (\hat{\varepsilon}, 0) > 1 + \frac{\rho \bar{\iota}}{\lambda}$.

Because the banking authority optimally chooses to preserve productive capital coming to fruition at date 1 when the liquidation cost is large, it follows that, for a sufficiently large liquidation cost, the occurrence of a banking panic leads to a sharp contemporaneous decline in output that is followed by a vigorous expansion in real activity above the long-run level.

It is helpful to numerically solve the previously described optimization problem to illustrate some important properties of the optimal policy response to a banking panic. In what follows, suppose $u(y) = (1-\sigma)^{-1} y^{1-\sigma}$ and $v(x) = (1-\eta)^{-1} x^{1-\eta}$, with $0 < \sigma < 1$ and $0 < \eta < 1$. For simplicity, assume $w(y) = y$. In addition, suppose $\beta = .96$, $\hat{\beta} = .8$, $\bar{\iota} = .3$, $\varepsilon = .25$, $\sigma = .5$, $\gamma = .5$, $\eta = .5$, and $\rho = .5$. Table 1 provides the solution to the optimal freezing problem for different values of the liquidation cost $1 - \delta$. As should be expected, the optimal freeze point and the amount of capital prematurely liquidated in the event of a panic are both decreasing in the liquidation cost.

Table 1: *Ex Post* Optimal Intervention

$1-\delta$	welfare	$\hat{k}$	$\hat{\varepsilon}$	$\phi\left(\hat{\varepsilon}, \hat{k}\right)$	$\phi_+\left(\hat{\varepsilon}, \hat{k}\right)$
.02	203.78	.29	.97	.70	1.04
.04	203.15	.24	.88	.71	1.11
.06	202.66	.18	.80	.72	1.17
.08	202.30	.13	.70	.73	1.25

Note that the value of frozen deposits at a panic date is increasing in the liquidation cost. Because it is costly to prematurely liquidate productive investments when the liquidation cost is relatively large, the banking authority will allow a large intertemporal variation in the value of deposits following a banking panic. Interestingly, as the liquidation cost rises, the banking authority will not permit the value of frozen deposits to rise by a large amount. The large intertemporal disparity in the value of deposits comes from the sharp increase in the value of deposits in the aftermath of the crisis. Note that there is a maximum intertemporal dispersion consistent with a solution to the optimization problem given that the participation constraint (48) must be satisfied.

Let me now investigate the behavior of aggregate output when the banking authority optimally intervenes to mitigate the effects of a banking panic. In particular, I will focus on retail-sector output (i.e., the sum of outputs across different matches in stage 2). Figure 1 plots the deviation of retail-sector output from the socially efficient level. In Panel (a), I show the evolution of real activity in the retail sector when the liquidation cost is relatively small ($1 - \delta = 0.02$). In this case, the decline in output associated with a banking panic is followed by a recovery period characterized by a suboptimal level of real activity, so we can say that the recession associated with a systemic run on the banking system is protracted. The government's optimal response involves considerable premature liquidation of productive investments to mitigate the adverse effects of a banking panic.

In Panel (b), the liquidation cost is set at 4 percent. It is clear that the contemporaneous decline in output is larger than that depicted in Panel (a). However, it is possible to say that the recovery from a panic episode occurs in the subsequent period. Although the level of output remains slightly below the socially efficient level at date 1, from a practical perspective, we can confidently say that real activity quickly recovers from a panic-induced recession in this case. As we have seen, a change in the liquidation cost from 2 to 4 percent leads the banking authority to optimally liquidate a smaller fraction of productive investments in the event of a panic. Given this optimal policy response, we can conclude that, for an intermediate range of values for the liquidation cost, the recession associated with a banking panic is short-lived.

Panels (c) and (d) plot the deviation in output for larger values of the liquidation cost. As we can see, the occurrence of a panic causes a severe contemporaneous decline in retail-sector output. In both cases, the decline in real activity is followed by a surge in retail-sector output, given the banking authority's decision to preserve a larger fraction of productive projects. This decision also results in an unusually large inflow of funds in the banking system as agents optimally rebalance their portfolio following a panic episode. These two effects contribute to yield a vigorous rebound in real activity in the post-panic period.

7. CONCLUSIONS

This paper has developed a dynamic model of bank liquidity provision to characterize the *ex post* efficient policy response to a banking panic and study its implications for the behavior of output in the aftermath of a banking panic. As we have seen, the equilibrium trajectory of real output in the event of a panic can follow very different patterns depending on the liquidation cost that a banking authority faces when jointly deciding the optimal rule for suspending the convertibility of deposits and the fraction of long-term assets that can be prematurely liquidated to respond to a banking panic. Specifically, a protracted recession is the ensuing outcome of an optimal *ex post* intervention when the liquidation cost is sufficiently low. For intermediate values of the liquidation cost, the contemporaneous contraction in output is more severe but the recession associated with a banking panic is short-lived, given that the economy fully recovers in the post-panic period. When the liquidation cost is sufficiently large, the contemporaneous decline in real output in the event of a panic is substantial but followed by a vigorous rebound in real activity above the long-run efficient level. Finally, I have argued that these theoretical predictions are consistent with the observed disparity in crisis-related output losses.

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APPENDIX

A.1. Proof of Lemma 2

To establish the optimality of the proposed portfolio when agents do not expect the occurrence of a panic, consider the following variational argument. At date 0, the marginal change in the expected utility of a depositor is given by

$$-U'\left(\frac{1-k_0}{\lambda}\right)+\beta\left(1+\rho\right)U'\left(1+\frac{\left(1+\rho\right)k_0-\bar{\iota}}{\lambda}\right).$$

Because

$$U'\left(\frac{1-\bar{\iota}}{\lambda}\right)<\beta\left(1+\rho\right)U'\left(1+\frac{\rho\bar{\iota}}{\lambda}\right),$$

it follows that

$$-U'\left(\frac{1-k_0}{\lambda}\right)+\beta\left(1+\rho\right)U'\left(1+\frac{\left(1+\rho\right)k_0-\bar{\iota}}{\lambda}\right)>0$$

for any $k_0<\bar{\iota}$. Because the productive technology pays off nothing for anything invested above $\bar{\iota}$, we must have $k_0=\bar{\iota}$ at the optimum.

In any subsequent date $t\geq 1$, the marginal change in the expected utility of a depositor is given by

$$-U'\left(1+\frac{\left(1+\rho\right)\bar{\iota}-k_0}{\lambda}\right)+\beta\left(1+\rho\right)U'\left(1+\frac{\left(1+\rho\right)k_0-\bar{\iota}}{\lambda}\right)$$

for any $k_0<\bar{\iota}$. Note that

$$U'\left(1+\frac{\left(1+\rho\right)k_0-\bar{\iota}}{\lambda}\right)>U'\left(1+\frac{\left(1+\rho\right)\bar{\iota}-k_0}{\lambda}\right)$$

for any $k_0<\bar{\iota}$. Because $\beta\left(1+\rho\right)>1$, it follows that

$$-U'\left(1+\frac{\left(1+\rho\right)\bar{\iota}-k_0}{\lambda}\right)+\beta\left(1+\rho\right)U'\left(1+\frac{\left(1+\rho\right)k_0-\bar{\iota}}{\lambda}\right)>0$$

for any $k_0<\bar{\iota}$. Given that the productive technology pays off nothing for anything invested above $\bar{\iota}$, we must have $k_t=\bar{\iota}$ at the optimum for any $t\geq 1$. **Q.E.D.**

A.2. Proof of Proposition 3

Because the members of the banking system maximize the expected utility of depositors, it must be the case that condition (7) holds with equality in equilibrium. Given the investment plan described in Lemma 2, it follows that $\phi_0 = \frac{1-\bar{\iota}}{\lambda}$ and $\phi_t = 1 + \frac{\rho\bar{\iota}}{\lambda}$ for all $t \geq 1$.

To demonstrate that the equilibrium allocation is Pareto optimal, note that it is impossible to make a banker better off without making a depositor worse off. It remains to verify whether it is possible to achieve a higher level of expected utility for a depositor without making other agents worse off. There is one relevant feasible deviation that I need to check to conclude that the allocation is indeed Pareto optimal. Suppose that a buyer who enters the period as a deposit holder decides to produce one unit of x and transfer it to a banker with the expectation that the banker can raise the purchasing power of *existing* deposits (i.e., no additional deposit is issued). Note that it is infeasible to increase the level of investment in the productive technology given that the economywide productive capacity is fully utilized. Thus, these additional resources are necessarily invested in storage. In this case, it is feasible to implement the value

$$1 + \frac{\rho\bar{\iota}}{\lambda} + \frac{1-\lambda}{\lambda} = 1 + \frac{\rho\bar{\iota} + (1-\lambda)}{\lambda}.$$

Note that each banker remains indifferent and that the original investment plan is not altered in other periods. Now I need to verify whether a buyer who enters the period as a deposit holder is willing to produce in order to increase the purchasing power of deposits in this way. A depositor is willing to produce provided that

$$-\gamma + \lambda U\left(1 + \frac{\rho\bar{\iota} + (1-\lambda)}{\lambda}\right) > \lambda U\left(1 + \frac{\rho\bar{\iota}}{\lambda}\right).$$

Rearranging this expression, we obtain the following condition:

$$\gamma < \lambda\left[U\left(1 + \frac{\rho\bar{\iota} + (1-\lambda)}{\lambda}\right) - U\left(1 + \frac{\rho\bar{\iota}}{\lambda}\right)\right].$$

If $\gamma \geq \lambda\left[U\left(1 + \frac{\rho\bar{\iota}+(1-\lambda)}{\lambda}\right) - U\left(1 + \frac{\rho\bar{\iota}}{\lambda}\right)\right]$, then a deposit holder is better off if he does not produce one unit of the good to raise the purchasing power of deposits. As a result, there is no feasible deviation that can increase the expected utility of a depositor without making other agents worse off, which means that the aforementioned equilibrium allocation is Pareto optimal. **Q.E.D.**

A.3. Proof of Lemma 5

To show the optimality of the proposed portfolio choice when agents expect the occurrence of a banking panic with probability π, consider the following variational argument at date 0. Note that the marginal change in the expected utility of a depositor is given by

$$\begin{aligned} &-\pi(1-\delta)\beta\gamma - \pi(1-\delta)\lambda[U(1)-\beta\gamma] + \\ &+(1-\pi)\left[-U'\left(\frac{1-k_0}{\lambda}\right)+\beta(1+\rho)U'\left(1+\frac{(1+\rho)k_0-\bar{\iota}}{\lambda}\right)\right] \end{aligned}$$

for any $k_0 \in (0, \bar{\iota})$. If the probability π associated with the realization r satisfies (47), then the previously described marginal change is strictly positive, indicating a corner solution (i.e., $k_0 = \bar{\iota}$) to the decision problem when agents contemplate the possibility of a banking panic. **Q.E.D.**

A.4. Proof of Proposition 6

Given the investment plan described in Lemma 5, the equilibrium value of liquid assets is described by (44)-(46). It remains to verify whether a buyer is willing to deposit in the banking system knowing that a banking panic occurs with probability π at date 0. The buyer is willing to deposit provided

$$(1-\pi)\lambda\left[U\left(\frac{1-\bar{\iota}}{\lambda}\right)-U(1)\right] \geq \pi(1-\delta)\bar{\iota}\{\beta\gamma+\lambda[U(1)-\beta\gamma]\}.$$

This conditions holds if and only if

$$\pi \leq \bar{\pi} \equiv \frac{\lambda\left[U\left(\frac{1-\bar{\iota}}{\lambda}\right)-U(1)\right]}{(1-\delta)\bar{\iota}\{\beta\gamma+\lambda[U(1)-\beta\gamma]\}+\lambda\left[U\left(\frac{1-\bar{\iota}}{\lambda}\right)-U(1)\right]}.$$

Given that the proposed investment plan maximizes the expected utility of depositors only if the probability π associated with the realization r satisfies (47), existence requires

$$\pi \leq \pi^* = \min\{\bar{\pi}, \hat{\pi}\}.$$

In this case, we obtain an equilibrium with the property that a banking panic occurs if the sunspot signal r is realized. **Q.E.D.**

A.5. Proof of Proposition 7

Note that

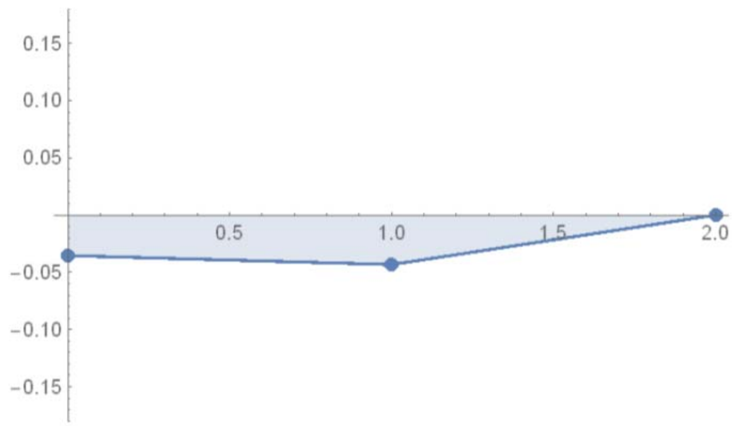
$$\phi_+(\hat{\varepsilon}, 0) = \frac{1 - (1-\lambda)(1-\varepsilon)(1-\hat{\varepsilon})}{\lambda} + \frac{\rho\bar{\iota}}{\lambda}.$$

Because $1 - (1-\lambda)(1-\varepsilon)(1-\hat{\varepsilon}) > \lambda$, it follows that

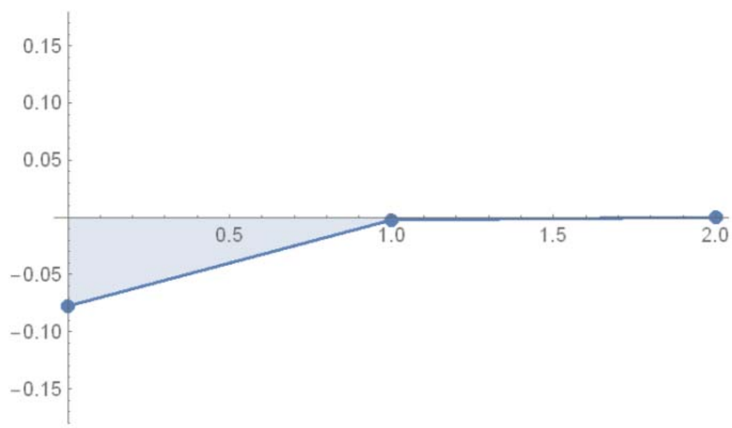
$$\phi_+(\hat{\varepsilon}, 0) > 1 + \frac{\rho\bar{\iota}}{\lambda}$$

as claimed. **Q.E.D.**

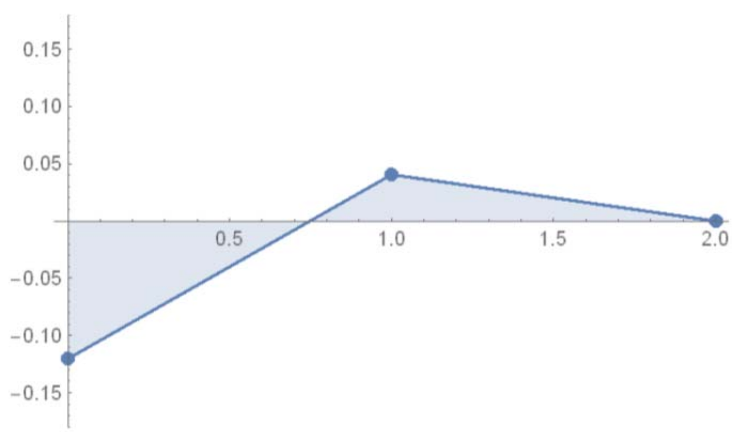
Figure 1: Output Deviation in the Event of a Banking Panic



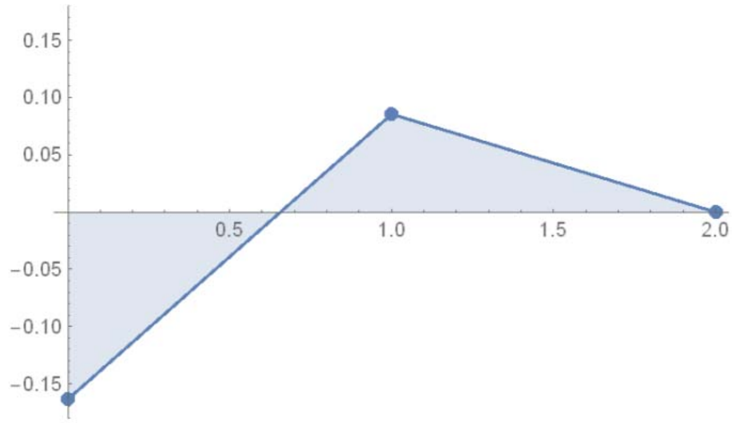
(a) $1 - \delta = .02$



(b) $1 - \delta = .04$



(c) $1 - \delta = .06$



(d) $1 - \delta = .08$