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Private Money and Banking Regulation

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Abstract

We show that a competitive banking system is inconsistent with an optimum quantity of private money. Because bankers cannot commit to their promises and the composition of their assets is not publicly observable, a positive franchise value is required to induce the full convertibility of bank liabilities. Under perfect competition, a positive franchise value can be obtained only if the return on bank liabilities is sufficiently low, which imposes a cost on those who hold these liabilities for transaction purposes. If the banking system is monopolistic, then an efficient allocation is incentive-feasible. In this case, the members of the banking system obtain a higher return on assets, making it feasible to pay a sufficiently high return on bank liabilities. Finally, we argue that the regulation of the banking system is required to obtain efficiency.

1. INTRODUCTION

The institutions comprising the banking system do many things, but one of their main functions is to create liquidity. Among many forms of liquidity creation, banks issue liabilities that can be used to facilitate payments and settlement. This is private money. For example, Rockoff (1974), Rolnick and Weber (1983), and Gorton (1999) highlight the free banking era as a period in American monetary history in which privately issued monies circulated as competing media of exchange. After the Civil War, demand deposits, which are another form of private money, expanded vigorously to become the dominant means of payment in the early twentieth century (see Friedman and Schwartz, 1963). More contemporarily, it has been argued by many observers of the recent financial crisis that repurchase agreements are the private monies of our time (e.g., see Gorton and Metrick, 2010, and the explanations therein). Therefore, a primary concern of monetary economists should be to know whether a private banking system is capable of creating enough of this kind of liquidity to allow society to achieve an efficient allocation. In other words, can a private banking system provide a socially efficient quantity of money? And if so, what are the characteristics of such a system? Is it stable? Finally, should we leave the job to the invisible hand or should we regulate the banking system?

To investigate these questions, we construct a model in which a subset of private agents, referred to as bankers, has the ability to issue debt claims, referred to as notes, that circulate as a medium of exchange. In our environment, bankers cannot commit to their promises and their assets are not publicly observable, giving the nonbank public a reason to distrust them.[1] As a result, the use of privately issued notes as a means of payment is endogenously determined so that each agent's decision to refuse to accept notes as a means of payment, if the agent believes the issuers will not fulfill their promises, is sufficient to discipline note creation by private agents.

Our contribution to the literature is to show how the degree of concentration in the

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[1]This is very much in the spirit of the hypotheses made in Cavalcanti, Erosa, and Temzelides (1999); Cavalcanti and Wallace (1999a, 1999b); Boissay (2011); and Gu, Mattesini, Monnet, and Wright (2013b).

banking system influences its members' willingness to supply an optimum quantity of money. We initially show that a competitive banking system is unwilling to supply an optimum quantity of money. In this case, the members of the banking system are willing to maintain the convertibility of their notes (and the nonbank public is willing to hold privately issued notes) only if the franchise value associated with the note-issuing business is sufficiently large. Because the return on the banking sector's assets is relatively low due to competition in the market for bank loans, the only way to implement a positive franchise value is by offering a sufficiently low return on bank liabilities, creating a cost to those who hold them for transaction purposes. For this reason, any equilibrium allocation under perfect competition is necessarily inefficient.

In addition, we show that a competitive banking system is inherently unstable. In our framework, the determination of equilibrium quantities and prices completely depends on agents' beliefs regarding the banker's franchise value, which essentially determines the banker's willingness to maintain the convertibility of the banker's notes. In particular, multiple beliefs regarding the future path of the franchise value are consistent with an equilibrium outcome. These equilibria have undesirable properties: The quantity of money persistently declines over time, agents continuously reduce their demand for money, and trading activity collapses. For this reason, we refer to these equilibria as self-fulfilling crises.

Subsequently, we study the properties of a monopolistic banking system. In particular, we show that an optimum quantity of money requires bankers to earn a sufficiently high return on their assets to ensure a properly large franchise value consistent with the voluntary convertibility of bank liabilities. We show that if the members of the banking system have market power, then they can extract a larger surplus from borrowers, while holding the total gains from trade constant. As a result, an allocation in which the banking system supplies an optimum quantity of money is incentive-feasible because a monopolistic banking system allows its members to sufficiently raise the return on assets. However, the regulation of the banking system is necessary for the implementation of an efficient allocation because a monopolistic banking sector would not choose to voluntarily supply an optimum quantity

of money in the absence of intervention.

Regarding the stability of the banking system, it is possible to show that the presence of concentration does not necessarily result in a stable banking system. This means that self-fulfilling crises are not exclusively associated with perfect competition in the banking system. It is important to emphasize that, in our framework, a banking crisis is associated with a self-fulfilling collapse of the value of private money and of trading activity (obviously, a suboptimal outcome). This type of banking crisis is different from that characterized in Diamond and Dybvig (1983). Their notion of liquidity is one of immediacy: Bank deposits are useful because they can be redeemed on demand when depositors have an urge to consume. So, the banking system is fragile whenever banks cannot fulfill the demand for immediate redemption under a sequential-service constraint. Jacklin (1987), however, considers a solution to banks'inherent fragility; namely, that banks issue tradeable securities. If depositors have an urge to consume, they can sell these securities instead of running to the bank. Interestingly, this notion of liquidity (namely, the ease with which bank liabilities can be traded) is closely related to ours.

2. RELATED LITERATURE

The role of regulation in guaranteeing a high franchise value for banks has been recognized by many experts, and, in this respect, our paper is related to Hellmann, Murdock, and Stiglitz (2000). They consider a model of banks with moral hazard and argue that the best way to ensure a high franchise value is to put a cap on the interest rate paid on deposits. As they write it, by limiting the degree of competition in the deposit market, a deposit rate control will increase per period profits, raising the bank's franchise value. Their analysis does not consider the role of bank liabilities as a means of payment. Although our analysis agrees with the general finding that a high franchise value is necessary for efficiency, we show that this value should not originate from the liability side of a bank's balance sheet.

Our paper is clearly related to the vast literature on the optimal provision of inside money. In this literature, however, the welfare implications of different banking structures are usu-

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ally excluded from the analysis. There are two strands in this literature. The first strand focuses on the role of liquidity as a means of payment. Cavalcanti, Erosa, and Temzelides (1999) and Cavalcanti and Wallace (1999a, 1999b) study private money creation in the context of a random-matching model. Azariadis, Bullard, and Smith (2001) characterize the welfare properties of a private monetary system using an overlapping generations model; Kiyotaki and Moore (2001) propose a theory of inside money based on the possibility of collateralization of part of a debtor's assets; and Monnet (2006) studies the characteristics of the agent that is most able to issue money.[2] The second strand focuses on the role of liquidity as a means of funding investment opportunities. For example, Holmstrom and Tirole (2011) show that a moral hazard problem may limit the ability of firms to refinance their ongoing projects when there is aggregate uncertainty.

Other authors have focused exclusively on the study of competition in bank lending without explicitly accounting for the role of banks as liquidity providers. These include Yanelle (1997) and Winton (1995, 1997). Our results show that the degree of competition in bank lending crucially influences the bankers' willingness to create money. Thus, it is important to consider the interplay between these two activities.

An important paper that accounts for the role of banks as liquidity providers under alternative banking structures is that of Boyd, De Nicolo, and Smith (2004). These authors study the properties of a competitive and monopolistic banking system. In an environment characterized by spatial separation and limited communication, banks offer deposit contracts to agents to provide insurance against the idiosyncratic relocation risk. They find that the probability of a costly banking crisis is always higher under competition than under monopoly. But this advantage of a monopolistic banking system is obtained at the cost of less valuable intertemporal insurance. In contrast to their results, we find that a monopolistic banking system is consistent with an efficient allocation but it does not necessarily imply a stable value of money. But it is important to emphasize that, in our framework, we cannot characterize the probability of a banking crisis as this event depends exclusively

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[2]Other papers in this literature include Williamson (1999); Li (2001, 2006); Kashyap, Rajan, and Stein (2002); Sun (2007); and Andolfatto and Nosal (2009).

on beliefs.

Another paper related to ours is Hart and Zingales (2014), who show that an unregulated private banking system creates too much money. They present an environment similar to Gu, Mattesini, Monnet, and Wright (2013b), to which our paper also bears a resemblance, where a lack of double coincidence of wants, a lack of commitment, and a limited pledge-ability of collateral give rise to an essential role for a medium of exchange. A bank acts as a safekeeping institution for the collateral and issues receipts that can circulate as a means of payment because the bank is able to commit to pay the bearer of a receipt on demand. Hart and Zingales uncover an interesting externality: A bank that issues more money to its customers increases the price level for other agents as well. As a result, too much collateral is stored, and banks create too much money. We depart from their analysis in a fundamental way: While they assume that banks can commit to pay back the bearer of the receipts they have issued, we assume they cannot. This suffices to overturn their result: We show that an unregulated banking system creates too little money.

Empirical work on bank liquidity creation is scant, and the paper by Berger and Bouwman (2009) is, to the best of our knowledge, the only one that measures the amount of liquidity created by the banking system. The authors construct a measure of liquidity creation by comparing how liquid the entries on both sides of a bank's balance sheet are. According to this measure, a bank creates more liquidity the more its liabilities are liquid relative to its assets. Among other interesting things, they find that banks that create more liquidity are valued more highly by investors, as measured by the market-to-book and the price-earnings ratios.

The rest of the paper is structured as follows. In Section 3, we present the model. In Section 4, we characterize efficient allocations. In Section 5, we describe the exchange mechanism in the decentralized economy. In Section 6, we characterize equilibrium allocations in the case of perfect competition. In Section 7, we study the properties of a concentrated banking system. Section 8 provides a discussion of the main results. Section 9 concludes.

3. MODEL

Time $t = 0, 1, 2, \ldots$ is discrete, and the horizon is infinite. Each period is divided into two subperiods. There are three physical commodities: good 1, good 2, and a capital good. Good 1 can be produced only in the first subperiod, and good 2 can be produced only in the second subperiod. If not immediately consumed, good 1 will perish completely. Good 1 can also be used as input in a production process. Specifically, there exists a productive technology that returns $\beta^{-1} > 1$ units of good 1 at date $t + 1$ for each unit of good 1 invested at date t. Good 2 cannot be stored and completely depreciates if not immediately consumed. The capital good can be perfectly stored from the first to the second subperiod. It depreciates completely if stored until the following date or if used in the production process.

There are four types of agents, referred to as buyers, sellers, entrepreneurs, and bankers, with a $[0, 1]$ continuum of each type. Buyers, sellers, and bankers are infinitely lived. Entrepreneurs live for two periods only. At each date t, a new generation of entrepreneurs is born. At date zero, there is a $[0, 1]$ continuum of old entrepreneurs.

Buyers are able to produce good 1 in the first subperiod. Specifically, each buyer has access to a divisible production technology that allows the buyer to produce one unit of good 1 with one unit of effort. Only a buyer wants to consume good 2, and only a seller is able to produce it. Such a technology requires $k \in \mathbb{R}_+$ units of capital and $l \in \mathbb{R}_+$ units of effort to produce $F(k, l)$ units of good 2. Assume that $F : \mathbb{R}_+^2 \to \mathbb{R}_+$ is twice continuously differentiable, increasing in both arguments, and strictly concave, with $F(0, l) = 0$ for all $l \geq 0$ and $F(k, 0) = 0$ for all $k \geq 0$.

Entrepreneurs specialize in the production of the capital good. Each entrepreneur is endowed with a nontradable, indivisible investment project at birth. Each project requires the investment of *exactly* $e \in \mathbb{R}_+$ units of good 1 at date t to produce $\gamma \hat{k}$ units of capital at the beginning of date $t + 1$, where $\hat{k} \in \mathbb{R}_+$ is a constant. Entrepreneurs are heterogeneous with respect to their productivity levels $\gamma \in [0, \bar{\gamma}]$. Specifically, the function $G(\gamma)$ describes the distribution of the productivity levels γ across the population of entrepreneurs. Suppose

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there exists a density function $g(\gamma)$.

We now explicitly describe preferences. Let $y_t \in \mathbb{R}_+$ denote a buyer's production of good 1, and let $q_t \in \mathbb{R}_+$ denote the consumption of good 2. The buyer's preferences are represented by

$$\sum_{t=0}^{\infty} \beta^t \left[-y_t + u(q_t) \right],$$

where $\beta \in (0,1)$. The function $u : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable, increasing, and strictly concave, with $u'(0) = \infty$. Let $x_t^s \in \mathbb{R}_+$ denote a seller's consumption of good 1, and let $l_t \in \mathbb{R}_+$ denote the seller's effort level. The seller's preferences are represented by

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^s - c(l_t) \right].$$

The function $c : \mathbb{R}_+ \to \mathbb{R}_+$ is continuously differentiable, increasing, and convex. Let $x_t \in \mathbb{R}_+$ denote a banker's consumption of good 1. The banker's preferences are represented by

$$\sum_{t=0}^{\infty} \beta^t x_t.$$

Finally, an entrepreneur born at date t wants to consume only at date $t+1$. In particular, each entrepreneur born at date t derives utility x_{t+1}^e if the entrepreneur's consumption of good 1 at date $t+1$ is $x_{t+1}^e \in \mathbb{R}_+$.

Assume that buyers and sellers are anonymous, and their trading histories are privately observable. As in Cavalcanti and Wallace (1999a, 1999b), we assume that the trading histories of bankers are publicly observable. Finally, the amount invested by any individual in the productive technology is privately observable (i.e., other agents do not know how much an individual agent has invested in the productive technology at each date). We provide more details about the implications of these assumptions in the following section.

There exists a centralized location where interactions happen in three stages. At the beginning of each date, all bankers arrive at the centralized location initially and remain there until the end of the first subperiod. Then, all buyers visit the centralized location and have an opportunity to trade with the group of bankers. Before anyone else arrives at the centralized location, all buyers depart from such a location. Subsequently, all sellers

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and old entrepreneurs arrive at the centralized location, have an opportunity to trade, and depart before anyone else arrives. Finally, young entrepreneurs arrive after all sellers and old entrepreneurs have departed from the centralized location. At the end of the first subperiod, all remaining people (i.e., bankers and young entrepreneurs) leave the centralized location.

In the second subperiod, the group of buyers and the group of sellers trade in a competitive Walrasian market, whereas all bankers and all entrepreneurs (young and old) remain idle. Figure 1 depicts the sequence of events within a period.

[Insert Figure 1]

4. EFFICIENT ALLOCATIONS

In this section, we formulate and solve the problem of a social planner with the ability to enforce all transfers at zero cost. This means that any solution to the planner's problem will give us an efficient allocation. Given some minimum utility level $U_t^e \in \mathbb{R}$ assigned to each entrepreneur of generation t, for all generations, and some minimum utility levels $U \in \mathbb{R}$ and $U^s \in \mathbb{R}$ assigned to each banker and each seller at date $t = 0$, respectively, an efficient allocation maximizes the lifetime utility of each buyer subject to resource constraints.

Given the pattern of arrivals and departures previously described, the planner needs to perform the following sequence of transfers in the centralized location. In the first round of interactions, the planner determines the amount $y_t \in \mathbb{R}_+$ of good 1 that each buyer is supposed to produce and transfer to a banker. In the second round of interactions, old entrepreneurs who had received resources to fund their investment projects in the previous period arrive at the centralized location with some amount of capital. The planner instructs each one of them to transfer the capital to a seller. Also, the planner instructs each banker to transfer some amount of good 1 to each seller and to each old entrepreneur. In the third round of interactions, the planner instructs the group of bankers to fund some young entrepreneurs. The planner will fund only the entrepreneurs who are sufficiently productive. This means that each entrepreneur whose productivity level γ is greater than or equal to a specific marginal type $\gamma_t^p \in [0, \bar{\gamma}]$ will receive e units of good 1 to undertake the

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entrepreneur's project at date t, whereas the types $\gamma \in [0, \gamma_t^p)$ will not carry out their projects. We refer to the type γ_t^p as the marginal entrepreneur.

Let $i_t \in \mathbb{R}_+$ denote the amount of resources devoted to the group of entrepreneurs at date t, and let $k_{t+1} \in \mathbb{R}_+$ denote the amount of capital available at the beginning of date $t+1$. Thus, the planner's problem consists of choosing an allocation

$$\{x_t^s, x_t, x_t^e, y_t, q_t, l_t, i_t, k_{t+1}, \gamma_t^p\}_{t=0}^{\infty}$$

to maximize the lifetime utility of the buyer

$$\sum_{t=0}^{\infty} \beta^t \left[-y_t + u\left(q_t\right) \right], \tag{1}$$

subject to the resource constraint for good 1

$$x_t^s + x_t^e + x_t + i_t = y_t, \tag{2}$$

the resource constraint for good 2

$$q_t = F\left(k_t, l_t\right), \tag{3}$$

the law of motion for capital accumulation

$$k_{t+1} = \hat{k} \int_{\gamma_t^p}^{\bar{\gamma}} \gamma g\left(\gamma\right) d\gamma, \tag{4}$$

$$i_t = e\left[1 - G\left(\gamma_t^p\right)\right], \tag{5}$$

the entrepreneurs' required utility levels

$$x_t^e \geq U_{t-1}^e, \tag{6}$$

the banker's required utility level

$$\sum_{t=0}^{\infty} \beta^t x_t \geq U, \tag{7}$$

and the seller's required utility level

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^s - c\left(l_t\right) \right] \geq U^s,$$

taking the initial capital stock $k_0 = k \left(\gamma_{-1}^p \right) > 0$ and the required utility levels $\left\{ U_{t-1}^e \right\}_{t=0}^{\infty}$, U, and U^s as given. Notice that any Pareto optimal allocation solves the problem previously described for a particular choice of required utility levels $\left\{ U_{t-1}^e \right\}_{t=0}^{\infty}$, U, and U^s, and that any solution to the problem above is a Pareto optimal allocation.

Let $k \left(\gamma_t^p \right) \equiv \hat{k} \int_{\gamma_t^p}^{\bar{\gamma}} \gamma g \left(\gamma \right) d\gamma$ denote the aggregate amount of capital available at the beginning of date $t+1$ as a function of the date-t marginal entrepreneur γ_t^p. The first-order conditions are given by

$$\beta u' \left(F \left(k \left(\gamma_t^p \right), l_{t+1} \right) \right) F_k \left(k \left(\gamma_t^p \right), l_{t+1} \right) \hat{k} \gamma_t^p = e, \tag{8}$$

$$u' \left(F \left(k \left(\gamma_{t-1}^p \right), l_t \right) \right) F_l \left(k \left(\gamma_{t-1}^p \right), l_t \right) = c' \left(l_t \right), \tag{9}$$

for all $t \geq 0$. To marginally increase each buyer's consumption at date $t+1$ without changing the effort level that each seller exerts at date $t+1$, the planner needs to give up e units of good 1 at date t at the margin to increase the amount of capital available for production at date $t+1$. The left-hand side in (8) gives the marginal benefit of an extra unit of capital at date $t+1$, whereas the right-hand side gives the marginal resource cost at date t. Similarly, to marginally increase each buyer's consumption at date t given a predetermined amount of capital, the planner needs to instruct each seller to exert more effort in the second subperiod. Condition (9) guarantees that the marginal disutility of effort equals the marginal benefit of consuming an extra unit of good 2.

A stationary solution to the planner's problem involves $\gamma_t^p = \gamma^*$ and $l_t = l^*$ for all $t \geq 0$, with γ^* and l^* satisfying

$$\beta u' \left(F \left(k \left(\gamma^* \right), l^* \right) \right) F_k \left(k \left(\gamma^* \right), l^* \right) \hat{k} \gamma^* = e, \tag{10}$$

$$u' \left(F \left(k \left(\gamma^* \right), l^* \right) \right) F_l \left(k \left(\gamma^* \right), l^* \right) = c' \left(l^* \right). \tag{11}$$

We also need the initial amount of capital to be equal to $k \left(\gamma^* \right)$. In the Appendix, we show the existence and uniqueness of a stationary solution to the planner's problem for at least some specifications of preferences and technologies.

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5. EXCHANGE MECHANISM

In this section, we describe the exchange mechanism in the decentralized economy. Our environment builds on Lagos and Wright (2005) to generate a demand for a medium of exchange.[3] We depart from the standard Lagos-Wright framework, however, in a fundamental way. We assume that the group of buyers and the group of sellers meet *only* in the second subperiod, when the buyer is a consumer and the seller is a producer. As in Freeman (1996a), an important characteristic of the environment is that the group of buyers and the group of sellers do not overlap in the centralized location.[4] This characteristic implies that bankers will play an *essential* intermediation role in the economy. In particular, bankers will provide a payment instrument in the form of notes redeemable on demand.

To understand why trade is difficult in this economy, consider what happens in the second subperiod. A buyer wants to purchase good 2 from a seller but is unable to offer something of value in exchange because the proceeds from investment in the productive technology are unavailable for use in the second subperiod. In addition, because buyers and sellers do not overlap in the centralized location, claims on the proceeds from investment in the productive technology cannot be credibly used as a means of payment. Because a banker has an opportunity to trade sequentially with buyers and sellers, respectively, in the centralized location, he is able to play an essential intermediation role in the economy. Precisely, a banker is able to provide payment services by issuing a transferable debt instrument collateralized by the investment technologies.

This debt instrument, issued in the form of notes redeemable on demand, is extremely useful for a buyer because it allows the buyer to purchase good 2 in the second subperiod. Figure 2 shows how privately issued notes circulate in the economy. Each buyer has an opportunity to acquire notes while visiting the centralized location in the first subperiod. Specifically, a buyer is able to obtain notes by producing and selling good 1 in the market.

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[3]See also Rocheteau and Wright (2005). An alternative tractable framework that also creates a role for a medium of exchange is the large household model in Shi (1997).

[4]See also Freeman (1996b).

In the second subperiod, the buyer has an opportunity to exchange notes for some amount of good 2. In the following period, any seller who holds notes is able to retire them while visiting the centralized location. The retirement of a privately issued note means that the banker who has issued it is supposed to convert it into a certain amount of good 1.

[Insert Figure 2]

Agents are willing to trade a privately issued debt instrument provided that they believe the issuer will be willing to redeem it at the promised face value at a future date. In this case, each seller is willing to accept these privately issued liabilities as a means of payment, so each buyer is willing to use them as a temporary store of value.

What makes it difficult for a buyer or a seller to trust a banker's promise? Another important characteristic of the environment is that agents do not observe the amount of collateral (if any) an individual banker holds in reserve to secure his circulating liabilities. In this respect, the availability of public knowledge of the banker's trading history, together with the possibility of endogenously punishing any banker who reneges on his promises, is crucial for the circulation of privately issued notes. A seller does not trust a buyer's IOU because he knows the latter cannot be (endogenously) punished in case of default. But the same seller is willing to accept a banker's IOU as a means of payment because he knows a banker can be (endogenously) punished if he fails to fulfill the promise of converting his IOUs into goods on demand.

The existence of a centralized location where note holders can claim the face value of privately issued liabilities implies that a banker's notes will be periodically presented for redemption. The banker's willingness to pay note holders today depends on the value of notes in future periods. If future monetary conditions are more favorable for him, then the continuation value of his note-issuing business is higher, so he will be less inclined to renege on his promises. As a result, his ability to raise funds today through the sale of notes increases because his liability holders know that he will have more to lose if he reneges on his promises.

Finally, the spatial separation in the environment implies that only bankers have an

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opportunity to fund young entrepreneurs in exchange for a promised repayment at the following date. An old entrepreneur who has undertaken an investment project is able to sell the capital good to sellers in the centralized location in exchange for good 1. Then, the old entrepreneur can use the proceeds from this sale to repay loans and consume. Throughout the paper, we assume that entrepreneurs can fully commit to repay their loans, so strategic default will not be a problem in this market. Figure 3 provides a depiction of the market for bank loans.

<center>[Insert Figure 3]</center>

6. COMPETITIVE EQUILIBRIUM

In our environment, bankers play an essential role in the functioning of the credit system. To finance investments at date t, a banker is able to raise funds by issuing notes to buyers. Subsequently, the banker uses the proceeds from the sale of notes to supply funds to entrepreneurs or to invest in the productive technology, or both. At date $t + 1$, the banker collects the proceeds from these investments and redeems outstanding notes, consuming or reinvesting the remaining profits. A note issued by a banker at date t provides him with $\phi_t \in \mathbb{R}_+$ units of good 1 and is a promise to pay one unit of good 1 to the note holder at date $t + 1$. Each banker has a technology that allows him to create perfectly divisible notes at zero cost. Notes issued by one banker are perfectly distinguishable from those issued by any other banker so that counterfeiting is not a problem.

Throughout the paper, we restrict attention to symmetric equilibria in which all notes trade at the same price, thus paying the same rate of return. This means that the notes issued by any pair of bankers are perfect substitutes provided that agents believe both bankers will be willing to redeem them at par value. Each agent in the economy takes the sequence of prices $\{\phi_t\}_{t=0}^{\infty}$ as given when making individual decisions.

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6.1. Bank Loans

Given that an entrepreneur's type is publicly observable, let $R_t(\gamma) \in \mathbb{R}_+$ denote the gross interest rate that prevails in the submarket for loans to type-γ entrepreneurs. This means that a type-γ entrepreneur is entitled to receive e units of good 1 at date t in exchange for a repayment of $R_t(\gamma) e$ units of good 1 at date $t+1$. Not all submarkets will be active, however, because an unproductive entrepreneur will not be willing to borrow at the prevailing interest rate for his submarket.

Because of perfect competition among bankers, the return on any bank loan must be equal to the return on the productive technology, so the entrepreneurs will capture all surplus from trade. Thus, the equilibrium interest rate is given by

$$R_t(\gamma) = \beta^{-1} \tag{12}$$

for each submarket $\gamma \in [0, \bar{\gamma}]$. In this case, bankers are willing to supply any amount of good 1 that is demanded in each submarket.

Let us now consider the decision problem of a type-γ entrepreneur. This entrepreneur has a profitable project if and only if $\rho_{t+1} \hat{k} \gamma - e\beta^{-1} \geq 0$, where $\rho_t \in \mathbb{R}_+$ denotes the price of one unit of capital in terms of good 1. Note that $\rho_{t+1} \hat{k} \gamma$ gives the value of the entrepreneur's project at date $t+1$, whereas $e\beta^{-1}$ gives the repayment he needs to make to the creditor at date $t+1$. Thus, a type-γ entrepreneur has a profitable project if and only if the surplus from the project is positive. Given the relative price of capital ρ_{t+1}, any type-γ entrepreneur for whom

$$\rho_{t+1} \hat{k} \gamma \geq e\beta^{-1} \tag{13}$$

finds it optimal to borrow at date t to fund his investment project. Thus, given the relative price ρ_{t+1}, we can define the marginal entrepreneur γ_t^m as the type satisfying

$$\gamma_t^m = \frac{e}{\beta \rho_{t+1} \hat{k}}. \tag{14}$$

This means that any entrepreneur indexed by $\gamma \in [\gamma_t^m, \bar{\gamma}]$ finds it optimal to borrow to fund a project, whereas the types $\gamma \in [0, \gamma_t^m]$ choose not to fund their projects. Thus, the

aggregate loan amount is given by

$$L_t = e\left[1 - G\left(\gamma_t^m\right)\right].$$

In this case, the aggregate amount of capital available for production at date $t+1$ will be given by

$$k_{t+1} = \hat{k} \int_{\gamma_t^m}^{\bar{\gamma}} \gamma g\left(\gamma\right) d\gamma \equiv k\left(\gamma_t^m\right). \tag{15}$$

6.2. Buyer's Problem

Let $W_t^b\left(a\right)$ denote the value function for a buyer who enters the first subperiod holding $a \in \mathbb{R}_+$ privately issued notes, and let $V_t^b\left(a\right)$ denote the value function for a buyer who enters the second subperiod holding $a \in \mathbb{R}_+$ notes. The Bellman equation for a buyer in the first subperiod is given by

$$W_t^b\left(a\right) = \max_{(y,a')\in\mathbb{R}_+^2} \left[-y + V_t^b\left(a'\right)\right],$$

subject to the budget constraint

$$\phi_t a' = y + a.$$

Here, $a' \in \mathbb{R}_+$ denotes the buyer's desired noteholdings in the first subperiod. The buyer has an opportunity to acquire notes in the centralized location at the price $\phi_t \in \mathbb{R}_+$. Recall that a note issued at date t is a promise to pay one unit of good 1 at date $t+1$, so the rate of return on the buyer's noteholdings is given by $1/\phi_t$. If notes did not provide the buyer with transaction services, then the buyer would be willing to hold them only if the expected rate of return on notes (weakly) exceeded the rate of time preference. Because a buyer can use notes to trade with a seller in the second subperiod, he is willing to hold them even if the expected rate of return is less than the rate of time preference.

Assume an interior solution for y. Then, the value $W_t^b\left(a\right)$ is an affine function, $W_t^b\left(a\right) = a + W_t^b\left(0\right)$, with the intercept $W_t^b\left(0\right)$ given by

$$W_t^b\left(0\right) = \max_{a'\in\mathbb{R}_+} \left[-\phi_t a' + V_t^b\left(a'\right)\right]. \tag{16}$$

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Let $p_{t+1} \in \mathbb{R}_+$ denote the price of one unit of good 2 at date t in terms of good 1 at date $t + 1$. Now, we have to consider the buyer's problem in the second subperiod. Each buyer takes the relative price p_{t+1} as given and chooses a demand schedule for good 2. The Bellman equation for a buyer holding $a' \in \mathbb{R}_+$ notes at the beginning of the second subperiod is given by

$$V_t^b \left(a' \right) = \max_{q \in \mathbb{R}_+} \left[u \left(q \right) + \beta W_{t+1}^b \left(a' - p_{t+1}q \right) \right], \tag{17}$$

subject to the liquidity constraint

$$p_{t+1}q \leq a'. \tag{18}$$

This liquidity constraint arises because a buyer needs notes to pay for purchases in the second subperiod, given that sellers do not accept his personal IOUs as a means of payment. Because each privately issued note is a promise to pay one unit of good 1 at the following date, the value of the purchases cannot exceed the face value of the buyer's noteholdings.

Using the fact that $W_t^b \left(a \right)$ is an affine function, we can rewrite the Bellman equation (17) as follows:

$$V_t^b \left(a' \right) = \max_{q \in \mathbb{R}_+} \left[u \left(q \right) - \beta p_{t+1}q \right] + \beta a' + \beta W_{t+1}^b \left(0 \right).$$

Note that the liquidity constraint (18) may either bind or not, depending on the buyer's noteholdings. In particular, it follows that

$$\frac{dV_t^b}{da} \left(a' \right) = \begin{cases} \frac{1}{p_{t+1}} u' \left(\frac{a'}{p_{t+1}} \right) & \text{if } a' < p_{t+1}\hat{q} \left(p_{t+1} \right); \\ \beta & \text{if } a' > p_{t+1}\hat{q} \left(p_{t+1} \right); \end{cases}$$

where $\hat{q} \left(p_{t+1} \right) = \left(u' \right)^{-1} \left(\beta p_{t+1} \right)$. If the liquidity constraint does not bind, then the marginal utility of an additional note equals β, which is simply the discounted value of the payoff of one extra unit of good 1 at date $t+1$. In this case, an extra note does not provide the buyer with additional liquidity services, so the buyer is willing to hold an extra note only if its rate of return is at least the same as the rate of time preference. If the liquidity constraint binds, then the marginal utility of an additional note is greater than β. In this case, an extra note provides the buyer with additional liquidity services, giving rise to a liquidity

premium. Since the buyer can use the productive technology as a store of value and $\phi_t > \beta$, the buyer is willing to hold notes only if they provide him with some liquidity services.

The first-order condition for the optimal choice of noteholdings is given by

$$-\phi_t + \frac{dV_t^b}{da}\left(a'\right) \leq 0,$$

with equality if $a' > 0$. If $\phi_t > \beta$, then the optimal choice of noteholdings is given by

$$u'\left(\frac{a'}{p_{t+1}}\right) = \phi_t p_{t+1}, \tag{19}$$

which means that notes offer a liquidity premium. This condition gives the individual demand for notes as a function of the relative price of good 2 and the price of notes.

Because the demand for notes depends only on the aggregate prices p_{t+1} and ϕ_t, the desired noteholdings are the same for each buyer. Thus, condition (19) also gives the aggregate demand for notes as a function of the prices p_{t+1} and ϕ_t. Holding p_{t+1} constant, it follows from (19) that a higher price of notes reduces the demand for notes. In this case, the rate of return on notes is lower, which in turn reduces its exchange value. The effect of the relative price p_{t+1} on the demand for notes depends on the curvature of the utility function $u(q)$. If $-\left[u''(q)q\right]/u'(q) < 1$, then an increase in p_{t+1} reduces the demand for notes, holding ϕ_t constant. If $-\left[u''(q)q\right]/u'(q) > 1$, then an increase in p_{t+1} results in a higher demand for notes.

6.3. Seller's Problem

Let $W_t^s(a)$ denote the value function for a seller who enters the first subperiod holding $a \in \mathbb{R}_+$ notes, and let $V_t^s(k, a)$ denote the value function for a seller who enters the second subperiod holding $k \in \mathbb{R}_+$ units of capital and $a \in \mathbb{R}_+$ notes. The Bellman equation for a seller in the first subperiod is given by

$$W_t^s(a) = \max_{(x,k',a')\in\mathbb{R}_+^3} \left[x + V_t^s\left(k', a'\right)\right],$$

subject to the budget constraint

$$x + \rho_t k' + \phi_t a' = a.$$

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Here $k' \in \mathbb{R}_+$ denotes the amount of capital the seller accumulates in the first subperiod, and $a' \in \mathbb{R}_+$ denotes his noteholdings. While visiting the centralized location, a seller has an opportunity to rebalance these noteholdings and buy capital from old entrepreneurs in a competitive market. The seller also has an opportunity to redeem previously accumulated notes (i.e., the proceeds from previous transactions).

Assume an interior solution for x. Then, the value $W_t^s(a)$ is an affine function, $W_t^s(a) = a + W_t^s(0)$, with the intercept $W_t^s(0)$ given by

$$W_t^s(0) = \max_{(k',a') \in \mathbb{R}_+^2} \left[-\rho_t k' - \phi_t a' + V_t^s(k', a') \right]. \tag{20}$$

Now, we have to consider the seller's problem in the second subperiod. Each seller takes the relative price p_{t+1} as given and chooses the amount of good 2 he is willing to supply. The Bellman equation for a seller holding $k' \in \mathbb{R}_+$ units of capital and $a' \in \mathbb{R}_+$ notes at the beginning of the second subperiod is given by

$$V_t^s(k', a') = \max_{l \in \mathbb{R}_+} \left[-c(n) + \beta W_{t+1}^s \left(p_{t+1} F(k', l) + a' \right) \right]. \tag{21}$$

Using the fact that $W_t^s(a)$ is an affine function, we can rewrite the right-hand side of (21) as follows:

$$\max_{l \in \mathbb{R}_+} \left[-c(l) + \beta p_{t+1} F(k', l) \right] + \beta a' + \beta W_{t+1}^s(0).$$

The first-order condition for the optimal choice of effort in the second subperiod is given by

$$c'(l) = \beta p_{t+1} F_l(k', l). \tag{22}$$

Because $(\partial V_t^s / \partial k)(k', a') = \beta p_{t+1} F_k(k', l)$, the first-order condition for the optimal choice of capital on the right-hand side of (20) is given by

$$\rho_t = \beta p_{t+1} F_k(k', l). \tag{23}$$

Thus, conditions (22) and (23) determine the demand for capital and the effort decision as a function of the relative price of good 2 and the relative price of capital. Combining conditions (22) and (23), we obtain the following condition:

$$\frac{\rho_t}{c'(l)} = \frac{F_k(k', l)}{F_l(k', l)}. \tag{24}$$

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Finally, the first-order condition for the optimal choice of notes is given by

$$-\phi_t + \beta \leq 0,$$

with equality if $a' > 0$. This means that a seller does not hold notes if $\phi_t > \beta$. If $\phi_t = \beta$, then he is indifferent. Without loss of generality, we assume that a seller does not hold notes.

So far, we have implicitly assumed that sellers voluntarily accept privately issued notes in exchange for their output in the second subperiod. A seller's decision to accept a banker's notes as a means of payment depends on his beliefs about the issuer's willingness to redeem them at the promised face value. Specifically, a seller is willing to accept privately issued notes as a means of payment provided that the amount of notes issued by each banker does not exceed an upper bound $\bar{B}_t \in \mathbb{R}_+$ at each date $t \geq 0$. If this upper bound is exceeded at some date t, then each seller refuses to accept privately issued notes as a means of payment. This means that a seller's acceptance rule depends on the current bound on note issue and on all future bounds. As we will show later, it is possible to construct a sequence $\{\bar{B}_t\}_{t=0}^{\infty}$ such that the seller's acceptance rule is individually rational.

6.4. Banker's Problem

Let $J_t(n, s)$ denote the value function for a banker who issued $n \in \mathbb{R}_+$ notes at the previous date and who enters date t holding $s \in \mathbb{R}_+$ assets. The banker's assets at the beginning of date t consist of loans made at date $t - 1$ and claims on the proceeds from the productive technology. As we have seen, the marginal return on assets is given by β^{-1}, whether he makes loans to entrepreneurs or invests in the productive technology. The banker's decision problem can be formulated as follows:

$$J_t(n, s) = \max_{(x, \hat{n}, \hat{s}) \in \mathbb{R}_+^3} [x + \beta J_{t+1}(\hat{n}, \hat{s})],$$

subject to the budget constraint

$$\hat{s} + x + n = \beta^{-1} s + \phi_t \hat{n}$$

and the upper bound on the number of notes that can be issued at each date

$$\hat{n} \le \bar{B}_t.$$

Here, $\hat{s} \in \mathbb{R}_+$ represents the amount of good 1 the banker devotes to the purchase of assets at the current date, $x \in \mathbb{R}_+$ represents his current consumption, and $\hat{n} \in \mathbb{R}_+$ represents the number of notes he decides to issue at the current date. The constraint $\hat{n} \le \bar{B}_t$ incorporates the seller's acceptance rule into the banker's decision problem. Thus, when making decisions at each date, a banker takes as given the sequence of prices $\{\phi_t\}_{t=0}^{\infty}$ as well as the sequence of individual limits on note issue $\{\bar{B}_t\}_{t=0}^{\infty}$.

If $\phi_t > \beta$, then each banker finds it optimal to issue as many notes as possible (i.e., he chooses $\hat{n} = \bar{B}_t$). Because the rate of return paid on notes (the cost of funds) is lower than the rate of return on assets, the banker makes a positive profit by issuing notes to finance the purchase of assets. Also, note that because the return on assets equals the rate of time preference, the banker is indifferent between immediately consuming and reinvesting the proceeds from these earnings. Therefore, an optimal investment decision is given by $\hat{s} = \bar{B}_t$, which can be interpreted as the decision to voluntarily hold in reserve all proceeds from the sale of notes in the current period. In this case, the banker's consumption at date t is given by

$$x_t = \bar{B}_{t-1} \left(\beta^{-1} \phi_{t-1} - 1 \right).$$

We define the franchise value as the lifetime utility associated with a particular choice of the return on the banker's assets, the sequence of limits on note issue, and the sequence of prices. At each date t, the franchise value is given by

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} \bar{B}_{\tau-1} \left(\beta^{-1} \phi_{\tau-1} - 1 \right).$$

Due to competition in the market for bank loans, the return on assets is the smallest possible, given by β^{-1} at each date, reducing the franchise value.

6.5. Aggregate Noteholdings

Let $a_t \in \mathbb{R}_+$ denote the aggregate noteholdings at date t. For any price $\phi_t > \beta$, the liquidity constraint (18) is binding, so the value of all notes in circulation must equal the value of aggregate production in the second subperiod:

$$a_t = p_{t+1} F\left(k\left(\gamma_{t-1}^m\right), l_t\right). \tag{25}$$

Note that the aggregate production depends on the total amount of capital and the effort level that each seller is willing to exert at the current date. Combining (19) with (25), we obtain

$$u'\left(F\left(k\left(\gamma_{t-1}^m\right), l_t\right)\right) = \phi_t p_{t+1}.$$

Using (22) to substitute for p_{t+1}, we obtain the following equilibrium condition:

$$u'\left(F\left(k\left(\gamma_{t-1}^m\right), l_t\right)\right) = \frac{\phi_t}{\beta} \frac{c'\left(l_t\right)}{F_l\left(k\left(\gamma_{t-1}^m\right), l_t\right)}. \tag{26}$$

This condition determines the equilibrium effort decision, given the predetermined capital stock. The price of notes ϕ_t influences this decision in the following way: A lower price of notes increases the return on notes and, consequently, the buyer's expenditure decision, raising the relative price p_{t+1} and inducing each seller to exert more effort.

As we have seen, the choice of the marginal entrepreneur is given by (14). Using (24) to substitute for ρ_{t+1}, we obtain the following equilibrium condition:

$$\beta u'\left(F\left(k\left(\gamma_t^m\right), l_{t+1}\right)\right) F_k\left(k\left(\gamma_t^m\right), l_{t+1}\right) \hat{k}\gamma_t^m = e\frac{\phi_{t+1}}{\beta}. \tag{27}$$

This condition determines the equilibrium amount of capital at date t, given the effort decision at date $t + 1$. Notice that a lower anticipated value for ϕ_{t+1} results in a larger amount of capital available for production at date $t + 1$, holding l_{t+1} constant.

We can use (26) and (27) to implicitly define the functions $\gamma_{t-1}^m = \gamma^m\left(\phi_t\right)$ and $l_t = l\left(\phi_t\right)$. Using these functions, we can define the aggregate production of good 2 by $q\left(\phi_t\right) = F\left(k\left(\gamma^m\left(\phi_t\right)\right), l\left(\phi_t\right)\right)$. Then, the aggregate noteholdings as a function of the price ϕ_t are given by

$$a\left(\phi_t\right) = \frac{u'\left(q\left(\phi_t\right)\right) q\left(\phi_t\right)}{\phi_t}. \tag{28}$$

6.6. Equilibrium

To define an equilibrium, we need to specify the sequence of limits on note issue $\left\{\bar{B}_t\right\}_{t=0}^{\infty}$ in such a way that each banker is willing to supply the amount of notes other agents demand and is willing to voluntarily redeem notes at the promised face value. We take two steps to define a sequence of limits on note issue satisfying these two conditions. First, for any given sequence of prices $\{\phi_t\}_{t=0}^{\infty}$, we set

$$\bar{B}_t = a(\phi_t) \tag{29}$$

at each date t. This condition guarantees that each banker is willing to supply the amount of notes in (28) at the price ϕ_t. Then, given this choice for the individual limits on note issue, we need to verify whether a particular choice for the price sequence $\{\phi_t\}_{t=0}^{\infty}$ implies that each banker does not want to divert resources at any date. Thus, a particular price sequence $\{\phi_t\}_{t=0}^{\infty}$ is consistent with voluntary convertibility if and only if

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} a\left(\phi_{\tau-1}\right)\left(\beta^{-1}\phi_{\tau-1} - 1\right) \geq a\left(\phi_{t-1}\right)\left(\beta^{-1}\phi_{t-1} - 1\right) + \phi_t a(\phi_t)$$

holds at each date $t \geq 1$.

As in Kehoe and Levine (1993) and Alvarez and Jermann (2000), these constraints allow each banker to issue as many notes as possible without inducing the banker to opportunistically renege on any promises. The left-hand side gives the banker's beginning-of-period lifetime utility. The right-hand side gives the short-term payoff received if the banker decides *not* to hold in reserve the proceeds from the sale of notes at date t. In this case, the banker can increase current consumption by the amount $\phi_t a(\phi_t)$, but he will inevitably suspend convertibility at date $t+1$, resulting in the autarkic payoff from date $t+1$ onward. This happens because sellers will refuse to accept the banker's notes as a means of payment in future transactions.

We can rewrite the previous convertibility constraints as follows:

$$-\phi_t a(\phi_t) + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} a\left(\phi_{\tau-1}\right)\left(\beta^{-1}\phi_{\tau-1} - 1\right) \geq 0. \tag{30}$$

As in Alvarez and Jermann, we want to set limits on note issue that are not too tight so that condition (30) holds with equality.

As previously mentioned, a seller's decision rule specifies that the seller is willing to accept privately issued notes only if the amount of notes issued by each banker does not exceed the upper bound $\bar{B}_t = a(\phi_t)$ at each date, given a sequence of prices $\{\phi_t\}_{t=0}^{\infty}$ satisfying (30). Thus, we can interpret the banker's decision to suspend convertibility as the dissolution of the banker's note-issuing business, given that nobody will be willing to produce to acquire these notes in future periods if the banker defaults on this obligation today.

Let $J_t = \sum_{\tau=t}^{\infty} \beta^{\tau-t} a(\phi_{\tau-1})(\beta^{-1}\phi_{\tau-1}-1)$ denote the banker's discounted lifetime utility at the beginning of date t. Then, the equations describing the equilibrium dynamic behavior of J_t and ϕ_t are given by

$$J_t = a(\phi_{t-1})(\beta^{-1}\phi_{t-1}-1) + \beta J_{t+1} \tag{31}$$

and

$$\phi_t a(\phi_t) = \beta J_{t+1}. \tag{32}$$

Note that (32) is simply the convertibility constraint holding with equality. Combining these two conditions, we can define an equilibrium as a sequence of prices $\{\phi_t\}_{t=0}^{\infty}$ satisfying

$$\phi_{t+1} a(\phi_{t+1}) = a(\phi_t) \tag{33}$$

at each date $t \geq 0$. Note that, at date zero, the relative price of capital ρ_0 adjusts to ensure that the demand for capital is consistent with the initial stock of capital. The seller's effort decision at date zero also adjusts to changes in the price ϕ_0, given the initial stock of capital. A formal definition is now provided.

Definition 1 *A competitive equilibrium can be defined as a sequence of prices $\{\phi_t\}_{t=0}^{\infty}$ satisfying $\phi_t \geq \beta$ and (33).*

An important property of the dynamic system in (33) is that there is no condition to pin down the initial choice of the price sequence. As we shall see, multiple beliefs regarding the value of notes in future periods will be consistent with an equilibrium outcome.

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6.7. Welfare

Now we want to show an important property of any equilibrium allocation (even though we have not shown existence yet). If we compare equations (26) and (27) with the solution to the planner's problem, given by equations (8) and (9), we realize that setting $\phi_t = \beta$ at each date $t \geq 0$ makes the choices of the marginal entrepreneur and the effort level exactly the same as those in the planner's solution. Thus, $\phi_t = \beta$ for all $t \geq 0$ is a necessary condition for efficiency so that the optimum return is given by β^{-1}. But condition (30) implies that the convertibility constraint is necessarily violated in this case, so we cannot have an equilibrium with $\phi_t = \beta$ for all $t \geq 0$. This means that any allocation that can be implemented in a competitive equilibrium is necessarily inefficient. We summarize these findings in the following proposition.

Proposition 2 *Any equilibrium allocation in the case of perfect competition is necessarily inefficient.*

Why are the bankers unwilling to supply a socially efficient quantity of money? As we have seen, the return on the banker's assets is the same as the return on the productive technology and the rate of time preference. Because of competition among bankers, there is no markup over the return on the productive technology. Given this rate of return on assets, there exists an upper bound on the return bankers are willing to offer on their liabilities without inducing them to voluntarily exit the note-issuing business. Any return above this bound gives an individual banker an incentive to strategically suspend the convertibility of notes, which will lead nonbank agents to refuse to use the banker's notes in future transactions. The problem is that an optimum quantity of money requires a rate of return on noteholdings that is greater than the upper bound consistent with a competitive equilibrium. In other words, the optimum return on notes can only be implemented if the franchise value is zero, which is clearly inconsistent with the convertibility constraints in the case of perfect competition.

The previous result also says that any kind of regulation that seeks to restrict competition on the liability side of banks' balance sheets, such as the interest rate cap proposed by Hellmann, Murdock, and Stiglitz (2000), will result in an inefficient amount of private money, regardless of the kind of intervention that is carried out on the asset side. Regulation Q in the U.S. is an example of a regulatory measure aimed at restricting the return that banks are allowed to pay to their depositors. Our analysis thus predicts that these measures necessarily lead to an inefficient amount of private money in the economy.

6.8. Existence and Stability

Let us initially characterize stationary equilibria with the property that the aggregate amount of notes is constant over time. In this case, we have $\phi_t = \phi$ for all $t \geq 0$. We can use (26) and (27) to define the choices of the marginal entrepreneur γ^m and the effort level l as a function of the price ϕ. Then, we can use (28) to define the aggregate noteholdings a as a function of the price ϕ. Finally, any stationary equilibrium must also satisfy the convertibility constraints (30). In particular, a stationary solution ϕ satisfies the convertibility constraints if and only if

$$-\phi a\left(\phi\right) + \frac{\beta}{1-\beta} a\left(\phi\right) \left(\beta^{-1}\phi - 1\right) \geq 0. \tag{34}$$

Because $a\left(\phi\right) > 0$ for any $\phi > \beta$, condition (34) holds if and only if

$$\phi \geq 1.$$

This means that each banker is willing to supply any quantity of notes for which the return on notes is nonpositive. In a competitive equilibrium, each banker is required to charge for the liquidity services he provides to the nonbank public to guarantee that the convertibility constraints are satisfied at each date. Thus, the voluntary convertibility of notes imposes a minimum franchise value consistent with the existence of equilibrium.

The following proposition establishes existence and uniqueness for some specifications of preferences and technologies.

Proposition 3 *Suppose that $u(q) = (1 - \sigma)^{-1} \left(q^{1-\sigma} - 1 \right)$, with $0 < \sigma < 1$, $c(l) = l$, and $F(k,l) = k^\alpha l^{1-\alpha}$, with $0 < \alpha < 1$. Suppose also that $g(\gamma) = 1$ for all $0 \leq \gamma \leq 1$ and $g(\gamma) = 0$ otherwise. Then, there exists a unique interior stationary equilibrium for which $\phi_t = 1$ for all $t \geq 0$.*

Under these specifications of preferences and technologies, it is straightforward to show that the aggregate amount of notes in circulation $a(\phi)$ is strictly decreasing in ϕ, so a competitive equilibrium results in an inefficiently small amount of money creation. In other words, the rate of return on money will be too low to allow society to achieve a Pareto optimal allocation.

We now turn to nonstationary equilibria. The following proposition establishes the existence of nonstationary equilibria.

Proposition 4 *Suppose that $u(q) = (1 - \sigma)^{-1} \left(q^{1-\sigma} - 1 \right)$, with $0 < \sigma < 1$, $c(l) = l$, and $F(k,l) = k^\alpha l^{1-\alpha}$, with $0 < \alpha < 1$. Suppose also that $g(\gamma) = 1$ for any $0 \leq \gamma \leq 1$ and $g(\gamma) = 0$ otherwise. Then, there exist nonstationary equilibria with the property that the rate of return on notes converges monotonically to zero and the economy converges to autarky.*

We interpret this kind of equilibrium as a self-fulfilling collapse of the banking system. As previously mentioned, the determination of equilibrium quantities and prices completely depends on agents' beliefs regarding future monetary conditions. Because agents believe the exchange value of notes will persistently depreciate over time, the amount of funds devoted to each banker is lower at the current date, so the number of notes in circulation today is lower. In fact, the number of notes in circulation monotonically decreases over time, resulting in decreasing trading activity. From a buyer's standpoint, his demand for notes decreases over time because he expects the purchasing power of notes to depreciate over time, allowing him to purchase ever smaller amounts of goods.[5]

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[5]In these nonstationary equilibria, individual limits on note issue monotonically decrease over time, similar to what happens in Gu, Mattesini, Monnet, and Wright (2013a).

The existence of nonstationary equilibria with the property that the value of private money collapses necessarily implies that a competitive banking system is unstable. As previously mentioned, there is no condition to pin down the initial value of notes, so multiple beliefs regarding the value of notes in future periods are consistent with an equilibrium outcome.

7. MARKET POWER

In the previous section, we characterized the properties of a competitive banking system. In this section, we want to study the properties of a monopolistic banking system. In principle, we could have simply assumed that there is only a large bank that behaves monopolistically in the economy. To be consistent with the model used in the previous section, however, let us assume that bankers are able to coordinate their actions to form a coalition.[6] As a practical matter, their coalition can be seen as a large monopolist bank. Because there is no risk of confusion, we will refer to the bank coalition in the sequel simply as "the bank."

Because the bank is the only source of funding for entrepreneurs, it is able to extract all surplus from each borrower by charging the following gross interest rate:

$$R_t\left(\gamma\right) = \begin{cases} \beta^{-1} \text{ if } \gamma < \gamma^m\left(\phi_{t+1}\right), \\ \beta^{-1}\frac{\gamma}{\gamma^m\left(\phi_{t+1}\right)} \text{ if } \gamma \geq \gamma^m\left(\phi_{t+1}\right). \end{cases}$$

Given these interest rates, any entrepreneur who is more productive than the marginal type is willing to borrow from the bank, but the interest rate on the loan is such that the entrepreneur is indifferent between borrowing funds and remaining idle.

Let $s_{t+1} \in \mathbb{R}_+$ denote the per capita amount of good 1 that the bank decides to invest at date t. Anticipating that $s_{t+1} \geq e\left[1 - G\left(\gamma^m\left(\phi_{t+1}\right)\right)\right]$, the bank devotes the amount

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[6]For simplicity, we do not describe the strategic behavior of bankers, i.e., their incentives to form and maintain a bank coalition. Instead, we simply assume that they are willing to form and maintain a bank coalition. This assumption is certainly consistent with the outcome of an explicit game-theoretic approach provided that agents are sufficiently patient and there is a finite number of bankers.

$e\left[1-G\left(\gamma^{m}\left(\phi_{t+1}\right)\right)\right]$ to fund all projects up to the marginal entrepreneur and uses the remaining resources to invest in the productive technology. Therefore, the amount of resources available to the bank at the following date is given by

$$\beta^{-1}e\frac{\int_{\gamma^{m}(\phi_{t+1})}^{\bar{\gamma}}\gamma g\left(\gamma\right)d\gamma}{\gamma^{m}\left(\phi_{t+1}\right)}+\beta^{-1}\left\{s_{t+1}-e\left[1-G\left(\gamma^{m}\left(\phi_{t+1}\right)\right)\right]\right\}.$$

The first term is the sum of all proceeds $\beta^{-1}\frac{\gamma}{\gamma^{m}(\phi_{t+1})}$ from individual loans, up to the marginal entrepreneur. The second term gives the proceeds from investing any excess over $e\left[1-G\left(\gamma^{m}\left(\phi_{t+1}\right)\right)\right]$ in the productive technology.[7]

As in the previous section, each member of the coalition is able to borrow up to the member's endogenous limit on note issue, given by $\bar{B}_{t}$, and is willing to invest the per capita amount $s_{t+1}=\phi_{t}\bar{B}_{t}$. Then, setting $\bar{B}_{t}=a\left(\phi_{t}\right)$ at each date $t\geq0$ implies that each member is willing to maintain convertibility. In this case, the coalition's per capita profit is given by

$$\Pi\left(\phi_{t+1},\phi_{t}\right)-a\left(\phi_{t}\right),$$

where the revenue function $\Pi\left(\phi_{t+1},\phi_{t}\right)$ is given by

$$\Pi\left(\phi_{t},\phi_{t+1}\right)=\beta^{-1}\phi_{t}a\left(\phi_{t}\right)+\beta^{-1}e\frac{\int_{\gamma^{m}(\phi_{t+1})}^{\bar{\gamma}}\left[\gamma-\gamma^{m}\left(\phi_{t+1}\right)\right]g\left(\gamma\right)d\gamma}{\gamma^{m}\left(\phi_{t+1}\right)}. \tag{35}$$

In other words, the bank's revenue consists of the proceeds from investment in the productive technology and the proceeds from loans to entrepreneurs.

Given this revenue function, the convertibility constraints are given by

$$-\phi_{t}a\left(\phi_{t}\right)+\sum_{\tau=t+1}^{\infty}\beta^{\tau-t}\left[\Pi\left(\phi_{\tau-1},\phi_{\tau}\right)-a\left(\phi_{\tau-1}\right)\right]\geq0 \tag{36}$$

at each date $t\geq0$. One immediate consequence of having a monopolistic banking system is that the average return on assets is higher than the average return obtained in a competitive equilibrium. Specifically, for any given sequence of prices $\{\phi_{t}\}_{t=0}^{\infty}$, the coalition's revenue

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[7]Note that the bank is willing to supply at least the amount of resources required to fund all entrepreneurs for whom $\gamma\geq\gamma^{m}\left(\phi_{t+1}\right)$ because, for any $s_{t+1}<e\left[1-G\left(\gamma^{m}\left(\phi_{t+1}\right)\right)\right]$, the rate of return to each incremental amount invested at date t is greater than the rate of time preference.

exceeds the aggregate revenue obtained in the case of perfect competition by the second term on the right-hand side of (35), which gives the additional amount of resources owing to the fact that the monopolist is able to extract all surplus from entrepreneurs. As a consequence, the set of prices satisfying the convertibility constraints must be larger than the one we obtained in the case of perfect competition because a higher return on assets essentially relaxes the convertibility constraints.

Finally, it is possible to formulate the monopolist's problem as the choice of a sequence of prices $\{\phi_t\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=1}^{\infty} \beta^t \left[\Pi \left(\phi_{t-1}, \phi_t \right) - a \left(\phi_{t-1} \right) \right] \tag{37}$$

subject to $\phi_t \geq \beta$ and the convertibility constraints (36). A solution to this problem gives an equilibrium outcome under a monopolistic banking system.

7.1. Incentive-Feasible Allocations

In this subsection, we want to verify whether the set of incentive-feasible allocations has been expanded in a nontrivial way. We initially restrict attention to stationary allocations. Before we proceed, it is necessary to show that, at any given stationary price ϕ, the members of the banking system will be able to raise enough resources from the sale of notes to finance all entrepreneurs whose projects have a positive surplus. The following Lemma delivers this result.

Lemma 5 *For any given $\phi > \beta$, we have $\phi a \left(\phi \right) > e \left[1 - G \left(\gamma^m \left(\phi \right) \right) \right]$.*

Given the previous result, note that any stationary price $\phi \geq \beta$ satisfying (36) implies that the redemption of notes at par value is individually rational for each banker. In this case, we can say that the allocation associated with the stationary price ϕ is incentive-feasible. The following proposition establishes that it is possible to have a strictly positive rate of return on notes under a monopolistic banking system.

Proposition 6 *There exists a stationary value $\bar{\phi} < 1$ for the price of notes that satisfies (36).*

30

With a positive markup, it is possible to have an allocation with a strictly positive return on notes. Because a positive markup raises the return on banking assets, it essentially mitigates the commitment problem associated with the note-issuing business. Thus, it is possible to implement an allocation in which the rate of return on notes is higher and the aggregate money supply is larger than those obtained under perfect competition. It may be necessary, however, to regulate the banking system in order to implement an allocation with a strictly positive return on notes.

7.2. Welfare and Stability

We now turn to the welfare implications of having a monopolistic banking sector. In particular, we want to know whether the existence of market power is consistent with the implementation of the optimum return on notes (i.e., the return that eliminates the opportunity cost of holding notes for transaction purposes). The following proposition establishes that it is possible to implement the optimum return on notes provided that agents are sufficiently patient.

Proposition 7 *Suppose that* $u(q) = (1 - \sigma)^{-1} (q^{1-\sigma} - 1)$, *with* $0 < \sigma < 1$, $c(l) = l$, *and* $F(k, l) = k^{\alpha} l^{1-\alpha}$, *with* $0 < \alpha < 1$. *Suppose also that* $g(\gamma) = 1$ *for all* $0 \leq \gamma \leq 1$ *and* $g(\gamma) = 0$ *otherwise. If* β *is sufficiently close to one, then the allocation associated with the stationary price* $\bar{\phi} = \beta$ *is incentive-feasible.*

Given the monopolist markup, it is possible to have an allocation in which the return on notes equals the rate of time preference. In this case, the opportunity cost of holding notes for transaction purposes is eliminated, maximizing the surplus from trade in private transactions. Because any other allocation that makes at least one entrepreneur better off necessarily makes a banker worse off, we conclude that the stationary allocation with $\phi_t = \beta$ for all $t \geq 0$ is Pareto optimal.

Regarding the stability of the banking system, it is possible to show that the presence of concentration does not *necessarily* result in a stable banking system. Suppose that $u(q) = (1 - \sigma)^{-1} (q^{1-\sigma} - 1)$, with $0 < \sigma < 1$, $c(l) = l$, and $F(k, l) = k^{\alpha} l^{1-\alpha}$, with

$0 < \alpha < 1$. Suppose also that $g\left(\gamma\right) = 1$ for any $0 \leq \gamma \leq 1$ and $g\left(\gamma\right) = 0$ otherwise. Now restrict attention to incentive-feasible allocations for which the convertibility constraints are binding at each date. In this case, the equations defining the dynamic behavior of J_t and ϕ_t are given by (32) and

$$J_t = a\left(\phi_{t-1}\right)\left(\beta^{-1}\phi_{t-1} - 1\right) + e\beta^{-1}\frac{\left[1 - \gamma^m\left(\phi_t\right)\right]^2}{2\gamma^m\left(\phi_t\right)} + \beta J_{t+1}. \tag{38}$$

Combining (32) with (38), we can reduce the dynamic system to a single equation:

$$a\left(\phi_{t-1}\right) = e\beta^{-1}\frac{\left[1 - \gamma^m\left(\phi_t\right)\right]^2}{2\gamma^m\left(\phi_t\right)} + \phi_t a\left(\phi_t\right). \tag{39}$$

Note that, for β sufficiently close to one, $\phi_t = \beta$ for all $t \geq 0$ is a stationary solution. If this is the unique interior stationary solution, for any initial choice $\phi_0 > \beta$, the individual limits on note issue, given by $\bar{B}_t = a\left(\phi_t\right)$, shrink over time and the return on notes converges monotonically to zero as the economy approaches autarky. Although an efficient allocation is incentive-feasible under a monopolistic banking system, there exist other equilibria with the property that the value of private money collapses as a result of self-fulfilling beliefs. Because it is possible to construct other incentive-feasible allocations with undesirable properties, the presence of concentration does not necessarily result in a stable banking system.

8. DISCUSSION

Our analysis has shown that the existence of market power in the banking system has a surprising welfare implication. In the absence of market power, bankers compete on the asset side of their balance sheets and can only obtain a positive franchise value if they offer a sufficiently low return on their liabilities. We have shown how the existence of market power can increase the return on the banking sector's assets and, consequently, allow bankers to increase the return paid on their liabilities, favoring the provision of liquidity. As we have seen, bankers are willing to supply an optimum quantity of money only if the average return on assets is sufficiently close to the return that a monopolist banker would obtain.

It is important to emphasize that a monopolistic banking sector would not necessarily implement an efficient allocation because it would certainly not choose the price of its

liabilities to be $\phi_t = \beta$ at each date. This would imply a corner solution to the maximization problem previously defined, which is unlikely to be the case. Unfortunately, it is not possible to fully characterize the solution to the monopolist's maximization problem. But this is not crucial for our analysis. What is relevant for our analysis is to show that an allocation with the property that the rate of return on notes equals the rate of time preference is incentive-feasible under a monopolistic banking system when agents are sufficiently patient. This allocation may not be an equilibrium outcome in the absence of regulation but is certainly consistent with the voluntary convertibility of bank liabilities. This means that it is possible to implement such an allocation in the decentralized economy provided that the government is willing to intervene in the banking sector to ensure that its members pay the socially efficient return on money.

9. CONCLUSION

We showed that a competitive banking system is unwilling to supply an optimum quantity of money. Because bankers cannot commit to their promises and their assets are not publicly observable, the voluntary convertibility of bank liabilities is consistent with an equilibrium outcome only if the members of the banking system receive a strictly positive franchise value. Because the return on the banking sector's assets is relatively low due to competition in the market for bank loans, the only way to implement a positive franchise value is by offering a relatively low return on bank liabilities, imposing a cost on those who hold these liabilities for transaction purposes. For this reason, any equilibrium allocation under perfect competition is necessarily inefficient.

Then, we characterized the properties of a monopolistic banking system. In particular, we showed that an optimum quantity of money requires bankers to earn a **sufficiently** high return on assets to ensure a properly large franchise value consistent with the voluntary convertibility of bank liabilities. If the members of the banking system have market power, then they can extract a larger surplus from borrowers while holding the total gains from trade constant. As a result, an allocation with the property that the banking system supplies

an optimum quantity of money is incentive-feasible because a monopolistic banking system allows its members to sufficiently raise the return on assets. Finally, we showed that the regulation of the banking sector is necessary for the implementation of an efficient allocation because a monopolistic banking sector would not choose to voluntarily pay the optimum return on money in the absence of intervention.

So far, we have left aside the role of banks as risk transformers, whereby banks undertake risky investments but issue relatively safe debt, or alternatively whereby banks' assets are information sensitive while they issue information-insensitive liabilities (an idea that dates back to Gorton and Pennacchi, 1990, but has regained some traction recently; see Gorton, 2010). This is clearly an important issue that will impact the optimal provision of liquidity, and we leave it for future work.

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APPENDIX

A.1. Existence of a Unique Stationary Solution to the Planner's Problem

Here, we show the existence of a unique stationary solution to the planner's problem for some specifications of preferences and technologies. In particular, we assume that $u(q) = (1-\sigma)^{-1}(q^{1-\sigma}-1)$, with $0 < \sigma < 1$, $c(l) = l$, and $F(k,l) = k^{\alpha}l^{1-\alpha}$, with $0 < \alpha < 1$. We also assume that $g(\gamma) = 1$ for any $0 \le \gamma \le 1$ and $g(\gamma) = 0$ otherwise. In this case, conditions (10) and (11) become

$$l = \chi \left[\gamma^{-1}(1-\gamma^2)^{1-\alpha+\alpha\sigma} \right]^{\frac{1}{(1-\alpha)(1-\sigma)}} \equiv Z(\gamma), \tag{40}$$

$$l = \lambda \left(1-\gamma^2\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \equiv H(\gamma), \tag{41}$$

respectively, where the constants χ and λ are given by

$$\chi \equiv \left(\frac{1}{2}\right)^{\frac{1-\alpha+\alpha\sigma}{(1-\alpha)(1-\sigma)}} \left[\frac{e}{\alpha\beta\hat{k}^{\alpha(1-\sigma)}}\right]^{\frac{1}{(1-\alpha)(1-\sigma)}}, \tag{42}$$

$$\lambda \equiv \left[(1-\alpha)\left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)}\right]^{\frac{1}{\alpha+\sigma(1-\alpha)}}. \tag{43}$$

Note that $Z'(\gamma) < 0$ for all $\gamma \in (0,1)$. Also, we have $\lim_{\gamma\to 0} Z(\gamma) = \infty$ and $\lim_{\gamma\to 1} Z(\gamma) = 0$. This means that $Z(\gamma)$ is strictly decreasing in the open interval $(0,1)$. With respect to the function $H(\gamma)$, we have $H'(\gamma) < 0$ and $H''(\gamma) < 0$ for all $\gamma \in (0,1)$. Also, it follows that $\lim_{\gamma\to 0} H(\gamma) = \lambda$ and $\lim_{\gamma\to 1} H(\gamma) = 0$. This means that $H(\gamma)$ is strictly decreasing and concave in the open interval $(0,1)$. Thus, a unique interior solution exists.

A.2. Proof of Proposition 3

Suppose that $u(q) = (1-\sigma)^{-1}(q^{1-\sigma}-1)$, with $0 < \sigma < 1$, $c(l) = l$, and $F(k,n) = k^{\alpha}l^{1-\alpha}$, with $0 < \alpha < 1$. Suppose also that $g(\gamma) = 1$ for any $0 \le \gamma \le 1$ and $g(\gamma) = 0$

otherwise. In this case, conditions (26) and (27) become

$$l = \chi^e (\phi) \left[\gamma^{-1} \left(1 - \gamma^2 \right)^{1-\alpha+\alpha\sigma} \right]^{\frac{1}{(1-\alpha)(1-\sigma)}} \equiv Z^e (\gamma, \phi) , \tag{44}$$

$$l = \lambda^e (\phi) \left(1 - \gamma^2 \right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \equiv H^e (\gamma, \phi) , \tag{45}$$

respectively, where the functions $\chi^e (\phi)$ and $\lambda^e (\phi)$ are given by

$$\chi^e (\phi) = \left(\frac{1}{2} \right)^{\frac{1-\alpha+\alpha\sigma}{(1-\alpha)(1-\sigma)}} \left[\frac{e\phi}{\alpha\beta^2 \hat{k}^{\alpha(1-\sigma)}} \right]^{\frac{1}{(1-\alpha)(1-\sigma)}} ,$$

$$\lambda^e (\phi) = \left[(1 - \alpha) \frac{\beta}{\phi} \left(\frac{\hat{k}}{2} \right)^{\alpha(1-\sigma)} \right]^{\frac{1}{\alpha+\sigma(1-\alpha)}} .$$

Note that $d\chi^e/d\phi > 0$, whereas $d\lambda^e/d\phi < 0$. Also, we have $\chi^e (\beta) = \chi$ and $\lambda^e (\beta) = \lambda$, where χ and λ are given by (42) and (43), respectively. For any fixed $\phi > \beta$, we have $\partial Z^e/\partial\gamma < 0$ for all $\gamma \in (0, 1)$, $\lim_{\gamma \to 0} Z^e (\gamma, \phi) = \infty$, and $\lim_{\gamma \to 1} Z^e (\gamma, \phi) = 0$. For any fixed $\phi > \beta$, we also have $\partial H^e/\partial\gamma < 0$ and $\partial^2 H^e/\partial\gamma^2 < 0$ for all $\gamma \in (0, 1)$, $\lim_{\gamma \to 0} H^e (\gamma, \phi) = \lambda^e (\phi)$, and $\lim_{\gamma \to 1} H^e (\gamma, \phi) = 0$. Thus, for any fixed $\phi > \beta$, a unique interior solution exists. Moreover, the Implicit Function Theorem implies $d\gamma^m/d\phi > 0$ and $dl/d\phi < 0$.

As we have seen, condition (34) holds if and only if $\phi \geq 1$. In particular, it holds with equality if and only if $\phi = 1$. Thus, there exists a unique stationary equilibrium for which $\gamma^m = \gamma^m (1)$, $l = l (1)$, $a = a (1)$, where $a (1)$ is given by

$$a (1) = \left(\frac{\hat{k}}{2} \right)^{\alpha(1-\sigma)} \left[1 - \gamma^m (1)^2 \right]^{\alpha(1-\sigma)} l (1)^{(1-\alpha)(1-\sigma)} . \tag{46}$$

Q.E.D.

A.3. Proof of Proposition 4

Suppose that $u (q) = (1 - \sigma)^{-1} \left(q^{1-\sigma} - 1 \right)$, with $0 < \sigma < 1$, $c (l) = l$, and $F (k, l) = k^\alpha l^{1-\alpha}$, with $0 < \alpha < 1$. Suppose also that $g (\gamma) = 1$ for any $0 \leq \gamma \leq 1$ and $g (\gamma) = 0$

otherwise. Using the Implicit Function Theorem, we find that

$$\frac{d\phi_t}{d\phi_{t-1}} = \frac{a'\left(\phi_{t-1}\right)}{\phi_t a'\left(\phi_t\right) + a\left(\phi_t\right)} > 0.$$

In particular, we have

$$\left.\frac{d\phi_t}{d\phi_{t-1}}\right|_{\phi_{t-1}=\phi_t=1} = \frac{a'\left(1\right)}{a'\left(1\right) + a\left(1\right)} > 1.$$

If $\phi_{t-1} = \phi_t = 1$ is the unique interior stationary solution, then we have that, for any initial value $\phi_0 > 1$, the equilibrium return on notes, given by $1/\phi_t$, converges monotonically to zero, so the equilibrium allocation approaches autarky as $t \to \infty$. Along this equilibrium path, the limits on note issue, given by $\bar{B}_t = a\left(\phi_t\right)$, also converge monotonically to zero. This happens because autarky is always an equilibrium outcome. The convergence is monotone provided that we choose an initial value ϕ_0 sufficiently away from the lower bound β. **Q.E.D.**

A.4. Proof of Lemma 5

Note that we can rewrite the expression for the aggregate noteholdings as follows:

$$\phi a\left(\phi\right) = \frac{e\phi F\left(k\left(\gamma^m\left(\phi\right)\right), l\left(\phi\right)\right)}{\beta^2 \hat{k}\gamma^m\left(\phi\right) F_k\left(k\left(\gamma^m\left(\phi\right)\right), l\left(\phi\right)\right)}.$$

For any price $\phi > \beta$, we have

$$
\begin{aligned}
\frac{e\phi F\left(k\left(\gamma^m\left(\phi\right)\right), l\left(\phi\right)\right)}{\beta^2 \hat{k}\gamma^m\left(\phi\right) F_k\left(k\left(\gamma^m\left(\phi\right)\right), l\left(\phi\right)\right)} \quad &> \quad \frac{e\phi k\left(\gamma^m\left(\phi\right)\right)}{\beta^2 \hat{k}\gamma^m\left(\phi\right)} \\
&> \quad \frac{ek\left(\gamma^m\left(\phi\right)\right)}{\beta \hat{k}\gamma^m\left(\phi\right)} \\
&> \quad \frac{ek\left(\gamma^m\left(\phi\right)\right)}{\hat{k}\gamma^m\left(\phi\right)} \\
&= \quad \frac{e\int_{\gamma^m\left(\phi\right)}^{\bar{\gamma}} \gamma g\left(\gamma\right) d\gamma}{\gamma^m\left(\phi\right)} \\
&> \quad e\left[1 - G\left(\gamma^m\left(\phi\right)\right)\right].
\end{aligned}
$$

Q.E.D.

A.5. Proof of Proposition 6

Note that

$$\frac{\int_{\gamma^m(\phi)}^{\bar{\gamma}} \gamma g(\gamma) \, d\gamma}{\gamma^m(\phi)} - [1 - G(\gamma^m(\phi))] > \int_{\gamma^m(\phi)}^{\bar{\gamma}} g(\gamma) \, d\gamma - [1 - G(\gamma^m(\phi))] = 0$$

for any $\phi \geq 0$. We have already shown that

$$-\phi a(\phi) + \frac{\beta}{1-\beta} a(\phi) \left(\phi \beta^{-1} - 1\right) \geq 0$$

if and only if $\phi \geq 1$. This means that there exists a value $\bar{\phi} < 1$ such that the convertibility constraint (36) is satisfied. **Q.E.D.**

A.6. Proof of Proposition 7

Suppose that $u(q) = (1-\sigma)^{-1} \left(q^{1-\sigma} - 1\right)$, with $0 < \sigma < 1$, $c(l) = l$, and $F(k,l) = k^\alpha l^{1-\alpha}$, with $0 < \alpha < 1$. Suppose also that $g(\gamma) = 1$ for any $0 \leq \gamma \leq 1$ and $g(\gamma) = 0$ otherwise. In this case, (36) can be written as
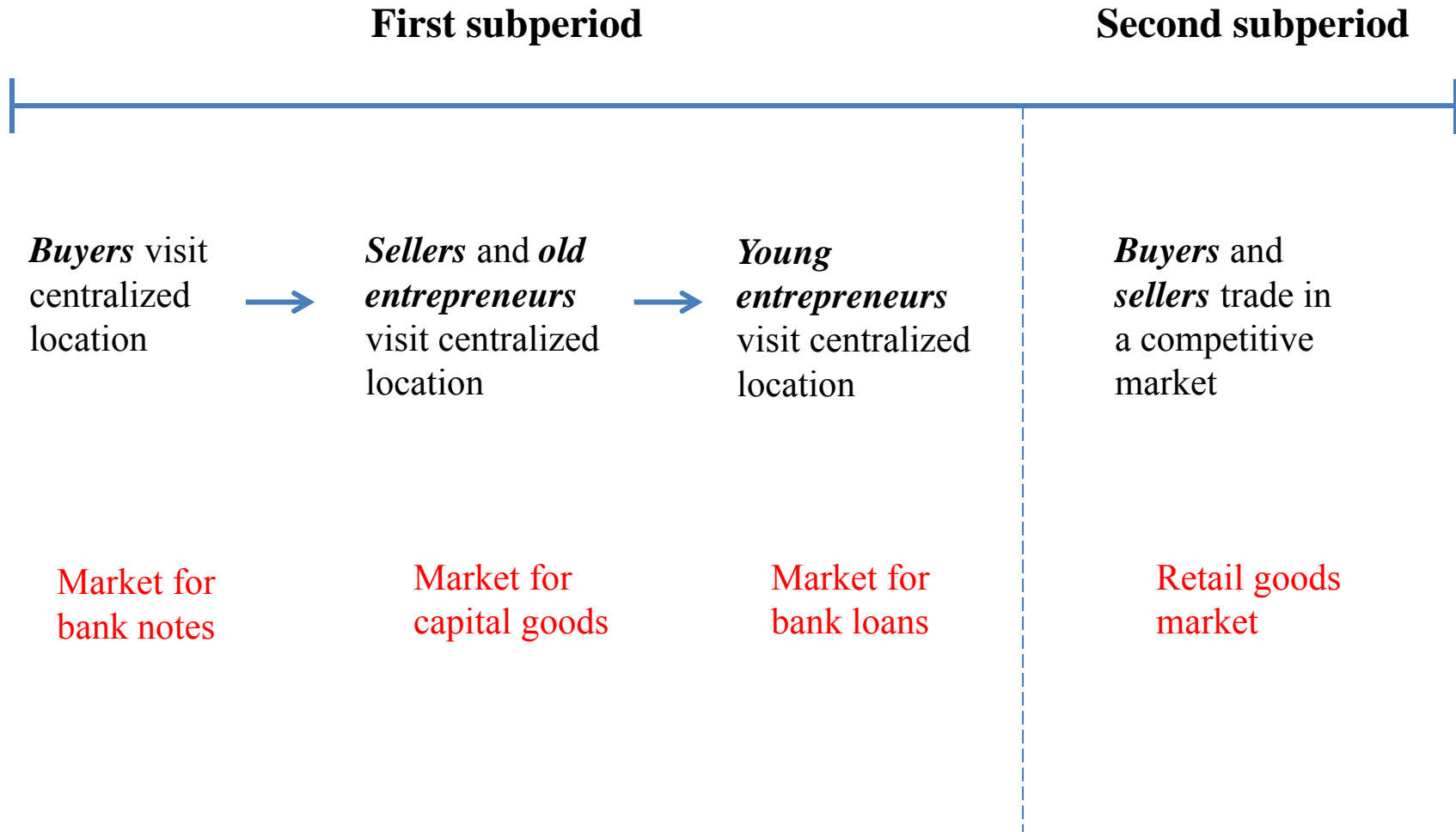
$$\frac{e}{\beta} \left[\frac{1}{2\gamma^m(\phi)} + \frac{\gamma^m(\phi)}{2} - 1 \right] - \left(\frac{1-\phi}{\phi}\right) \left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)} \left[1 - \gamma^m(\phi)^2\right]^{\alpha(1-\sigma)} l(\phi)^{(1-\alpha)(1-\sigma)} \geq 0.$$

Taking the limit as $\phi \to \beta$ from above, the left-hand side of this expression converges to

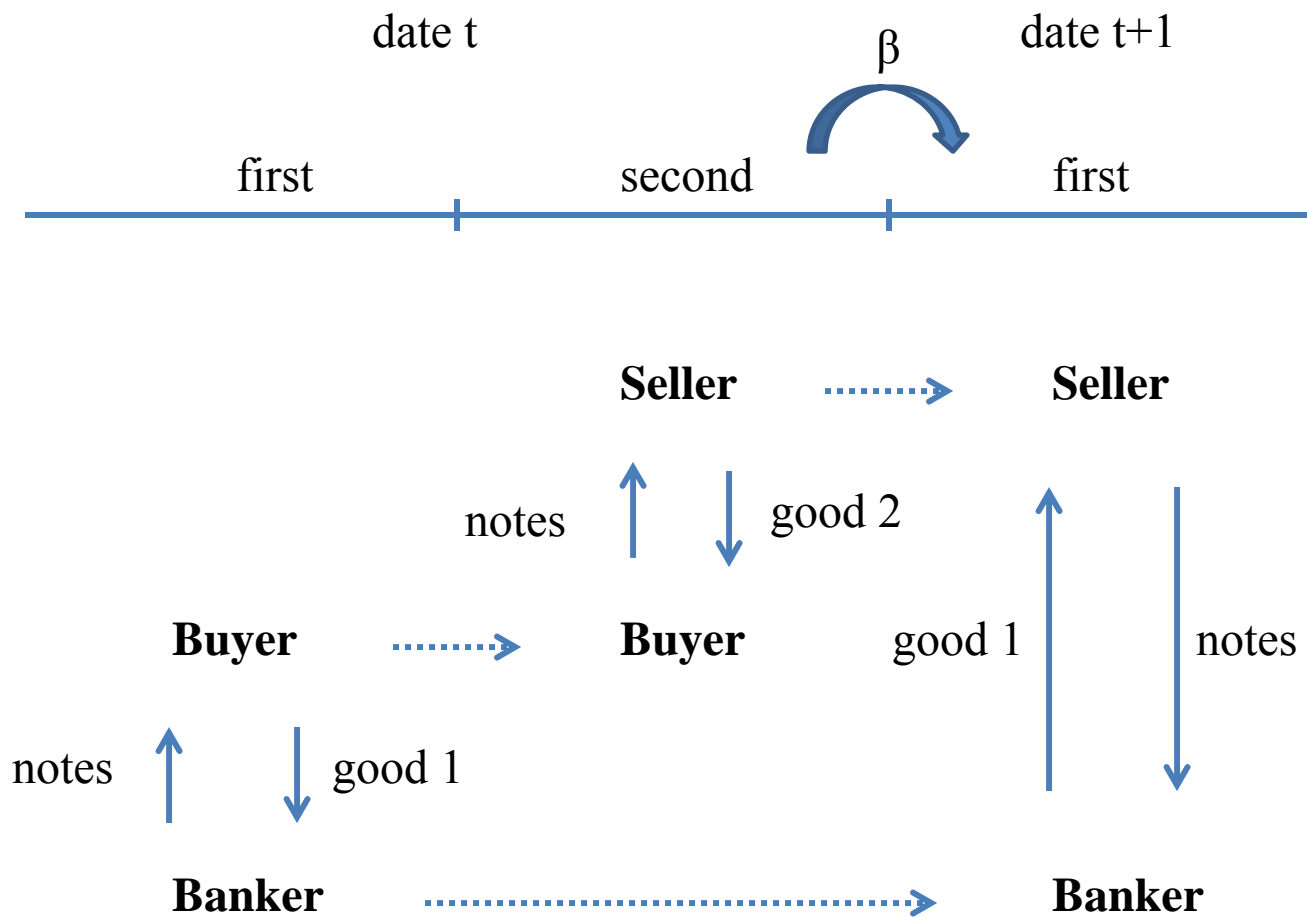$$e\left(\frac{1}{2\gamma_\beta^*} + \frac{\gamma_\beta^*}{2} - 1\right) - (1-\beta) \left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)} \left[1 - (\gamma_\beta^*)^2\right]^{\alpha(1-\sigma)} (l_\beta^*)^{(1-\alpha)(1-\sigma)},$$

where $\left(\gamma_\beta^*, l_\beta^*\right)$ denotes the solution to the planner's problem [i.e., the unique interior solution to the system (40)-(41)] for any given discount factor $\beta < 1$. As $\beta \to 1$ from below, we have $0 < \lim_{\beta \to 1} \gamma_\beta^* < 1$. This means that there exists $\beta < 1$ sufficiently close to one such that the expression above is strictly positive. **Q.E.D.**

Figure 1: Sequence of Events Within a Period

First subperiod   **Second subperiod**

Buyers visit centralized location → *Sellers* and *old entrepreneurs* visit centralized location → *Young entrepreneurs* visit centralized location | *Buyers* and *sellers* trade in a competitive market

Market for bank notes   Market for capital goods   Market for bank loans   Retail goods market

Figure 2: Circulation of Private Notes

Figure 3: Bank Loans